Introduction to Reinforcement Learning

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#wearetothepoint, # AMLD2020

Outline

What is Reinforcement Learning (RL)

- RL is one of **three main learning problems** studied in AI:
- Learning by trial and error and improving over time;
- Natural form of learning, ubiquitous in biology and psychology:

What is Reinforcement Learning?

- **Supervised learning** needs teacher (labeled data!)
- **RL learns on the job:** interact, explore, exploit experience!

What makes RL attractive and interesting?

Natural way to learn complex skills, can we copy this?

RL Example 1: Playing a song by ear

- **Actions and transitions:** Moving from key to key...
- **Immediate reward:** correct note or not;
- **Cumulative reward:** Being able to play whole song
- Relatively easy, one gets immediate valuable feedback...

RL Example 2: Cooking

- **States:**
- **Actions:** e.g. cooking, seasoning, stirring, etc.
- **Transitions:** e.g. from raw to cooked
- **Immediate reward** e.g. smell, look
- **Final reward:** enjoyable dinner
- **Problem:** what's the best sequence of actions (i.e. optimal policy)?

RL Examples: Playing Chess

- **States:** board configurations
- **Actions:** legal moves
- **Value of state/action:** very valuable to determine next move (action),
- **Final reward:** Win(2), lose(0), draw(1)
- **Problem:** what's the optimal action to take in each state? (policy)

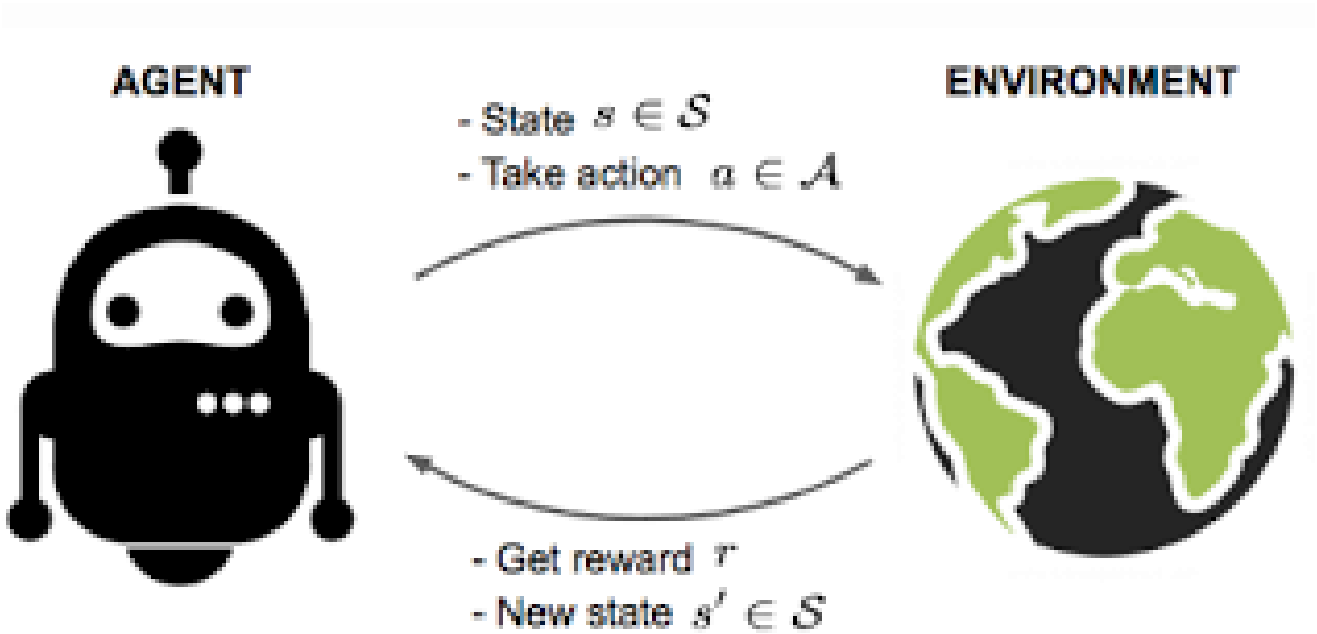
Reinforcement Learning: from **examples** to **abstraction**

- **RL:** Learning from (sequential) interaction:
  take actions to move from state to (better!) state;
- No labeled data, but **feedback** (reward, possibly delayed)
- **Goal:** optimise **end-result** (i.e. long-term/cumulative reward)

RL: Even more abstract version

Environment:

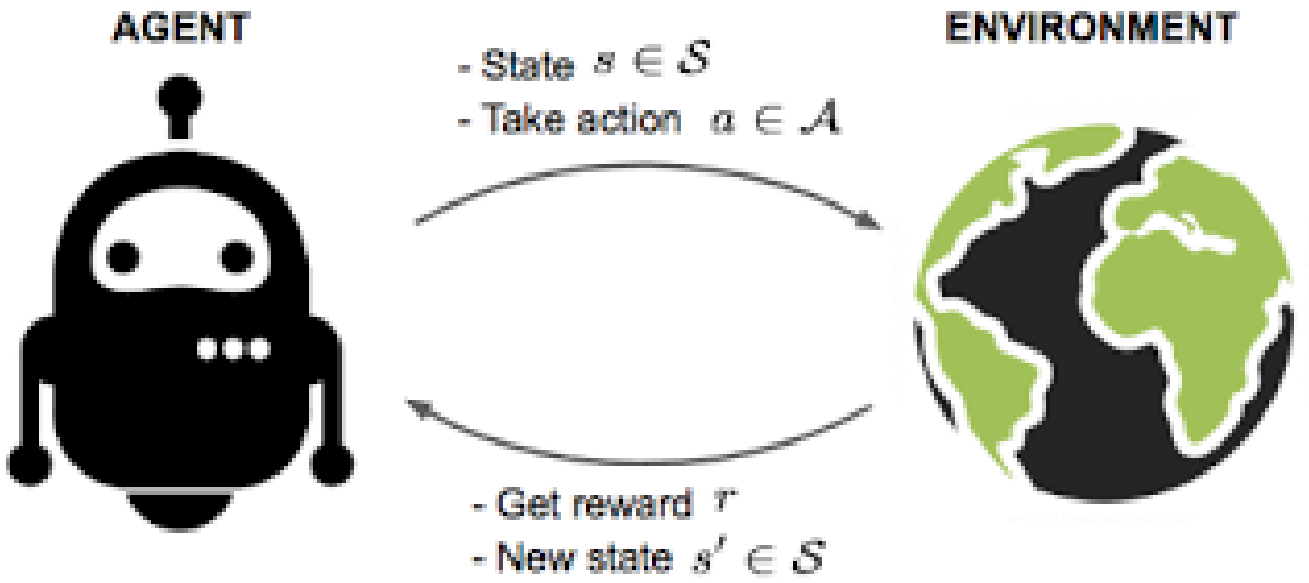
- states(s), actions (a) and transitions: $s \xrightarrow{a} s'$
- (immediate) reward $r(s, a, s')$



RL GOAL: Find optimal policy!

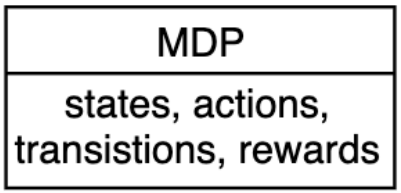
Policy π, tells for each state (s), which action (a) to take:

$$s \xrightarrow{\pi} a$$



RL Goal: Given environment, determine **optimal policy** π^*:
what **action** *to choose in* **each state** *to achieve* **best FINAL reward***?*

Overview



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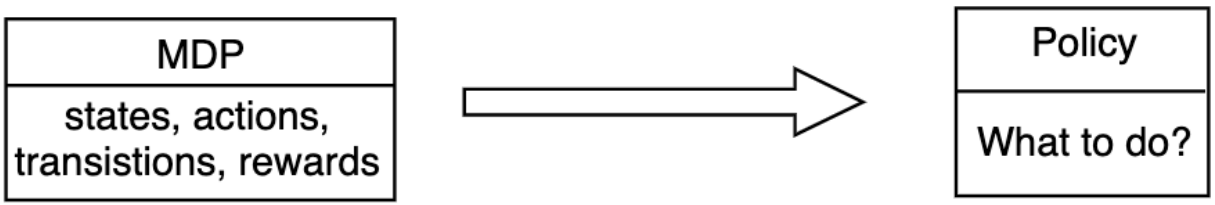

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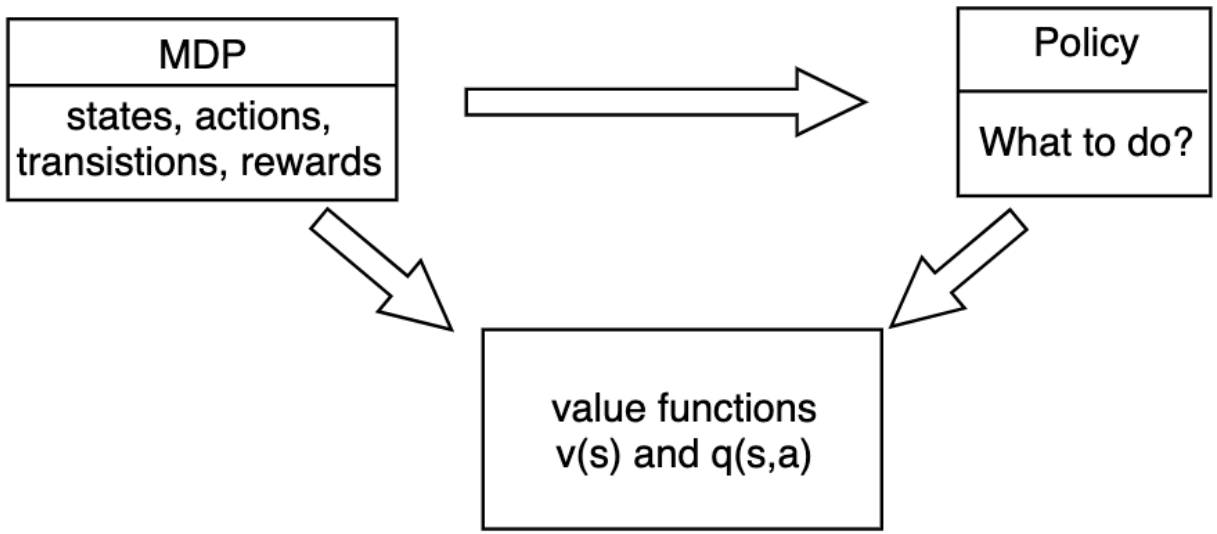
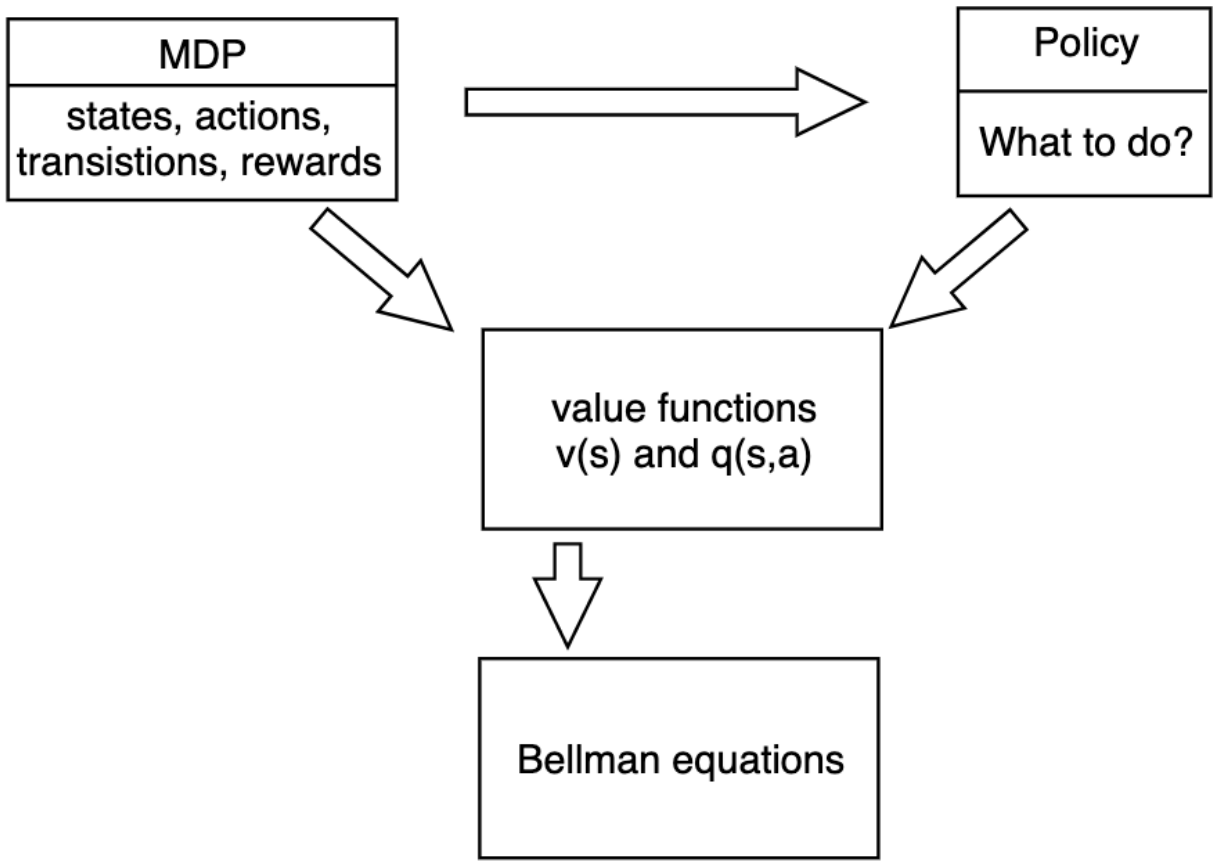

MDP
states, actions,
transistions, rewards
Policy
What to do?
value functions
v(s) and q(s,a)

Overview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Bellman equations

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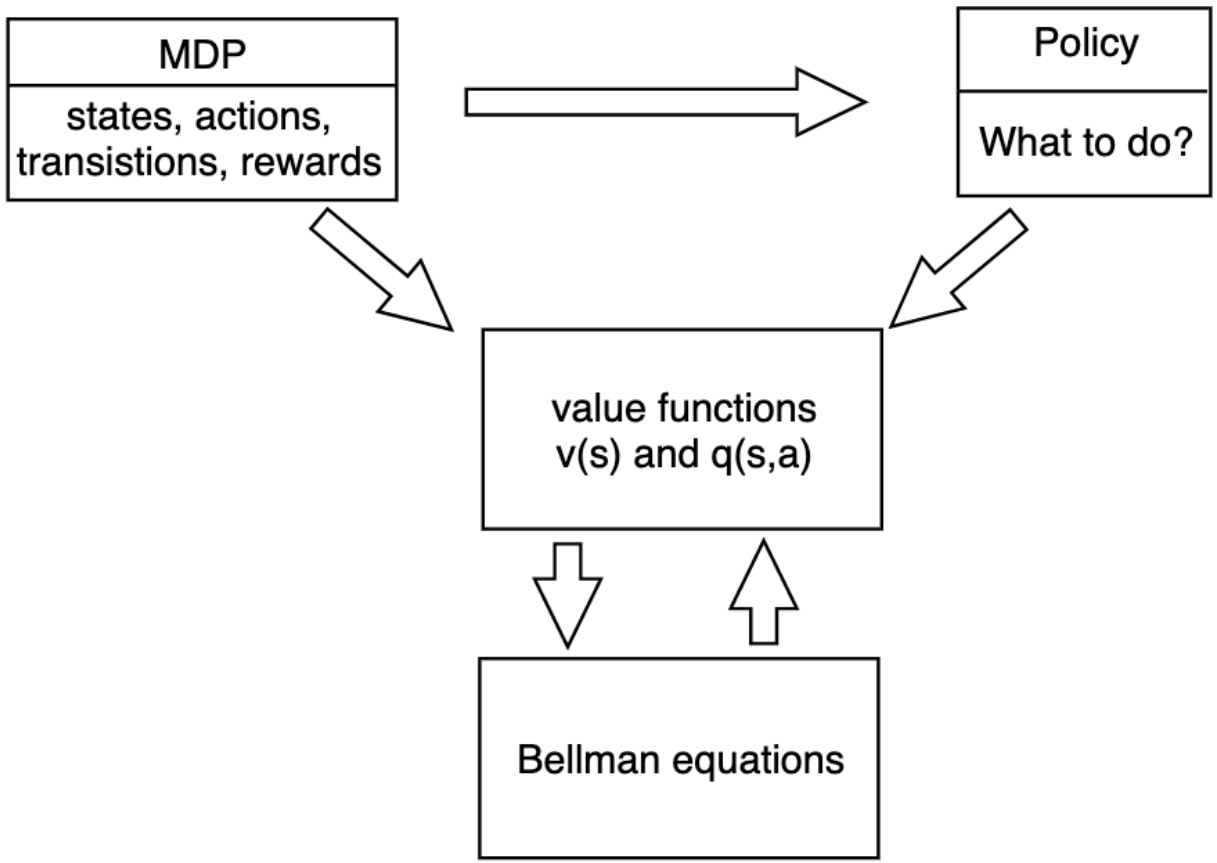
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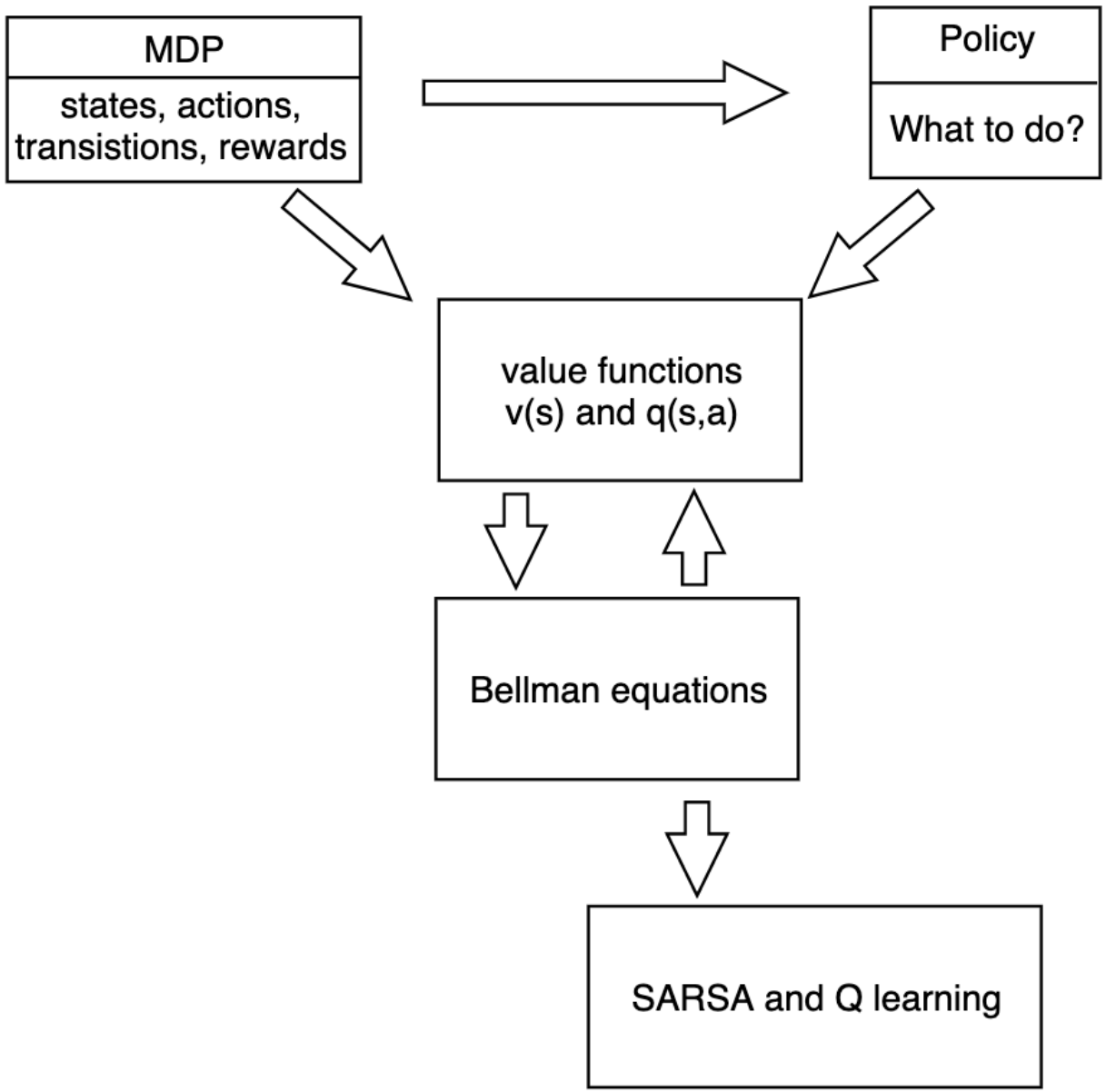
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SARSA and Q learning

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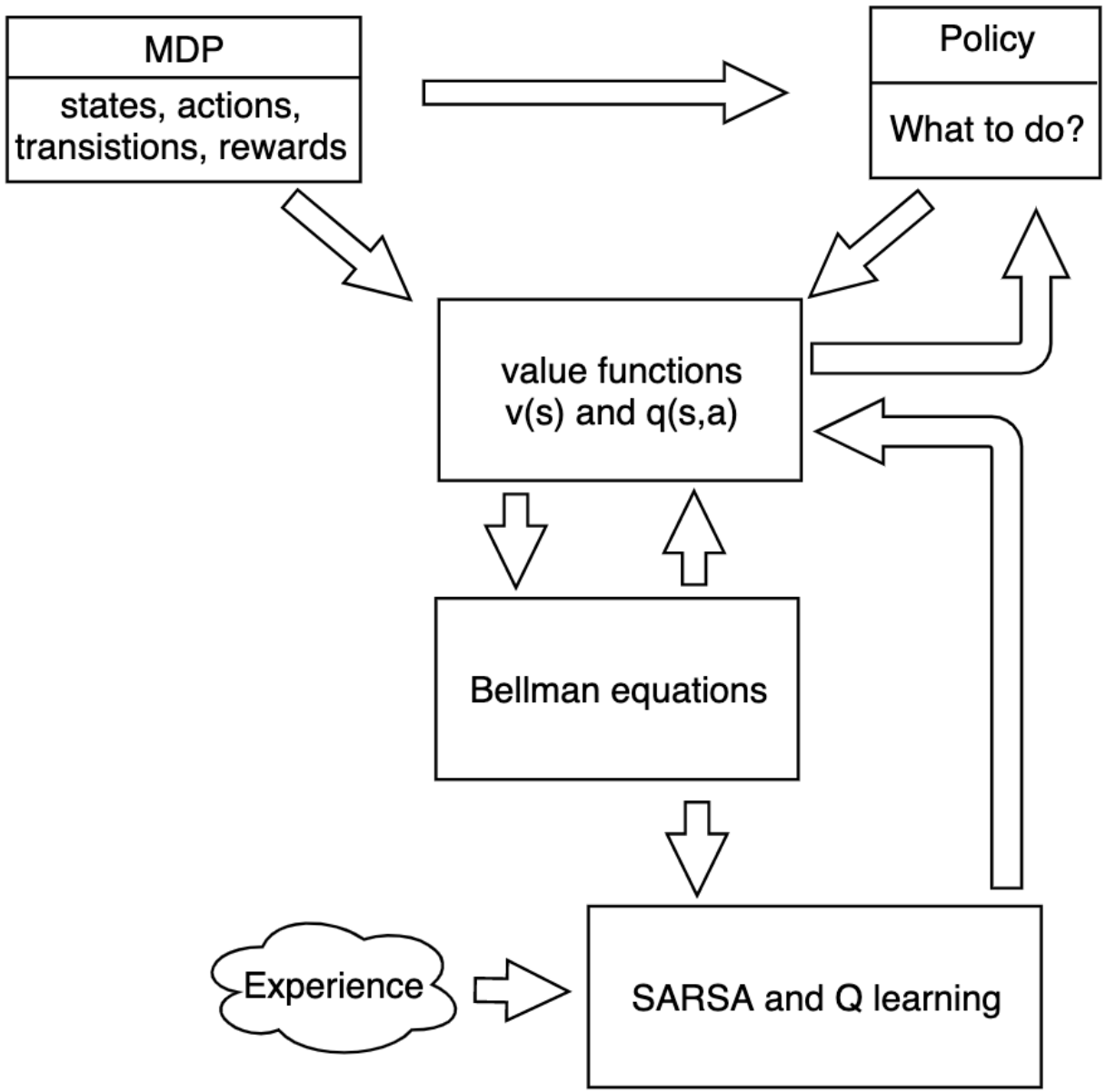
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Experience
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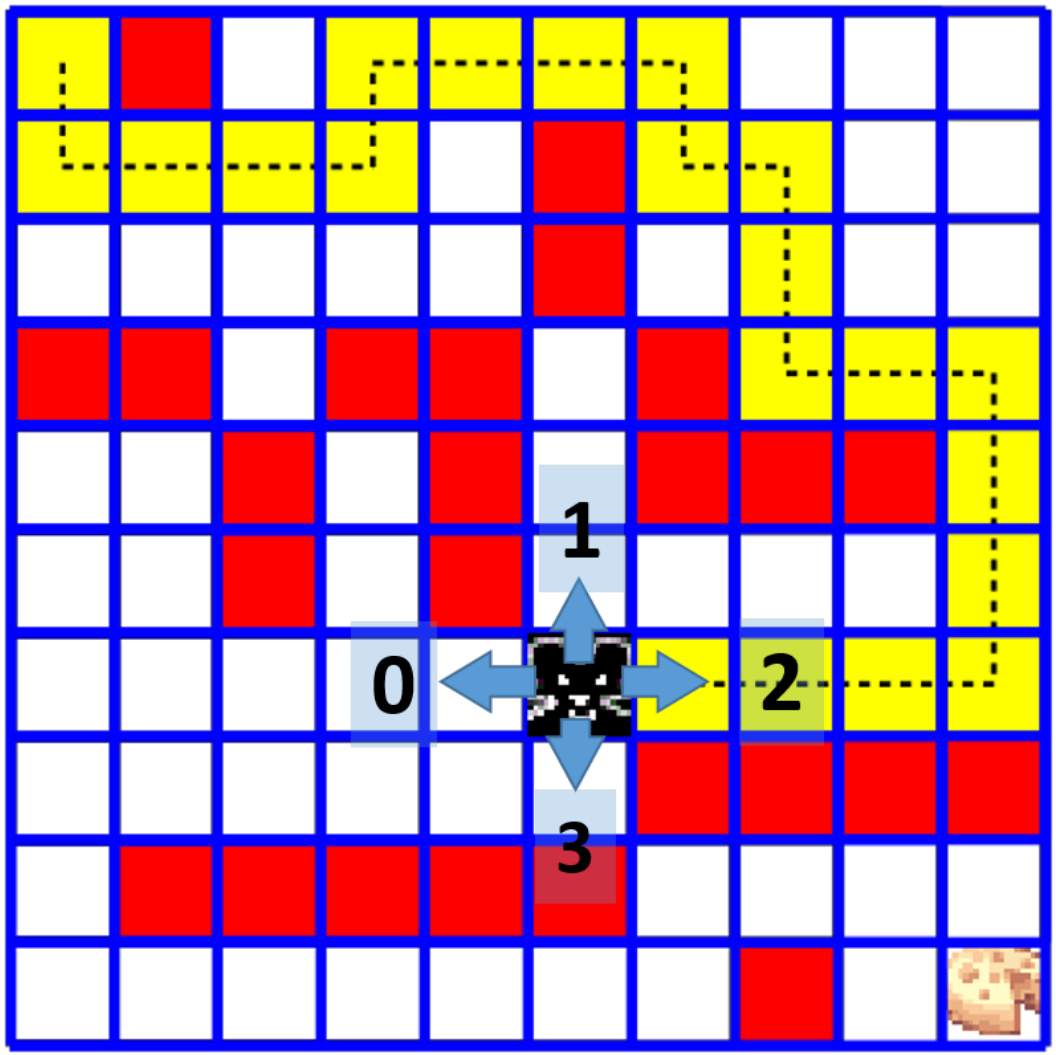
Signature Challenges for RL

- **Evaluative feedback** (reward), not prescriptive;
- **Sequential** (often **delayed**) consequences of actions;
- **Online learning:** improve policies while interacting with environment
  - In *off-line learning:* find optimal strategy **prior** to interaction!
- Need for **trial and error**, combination of exploration and exploitation;

Gridworld Maze as Prototypical RL Example



Outline

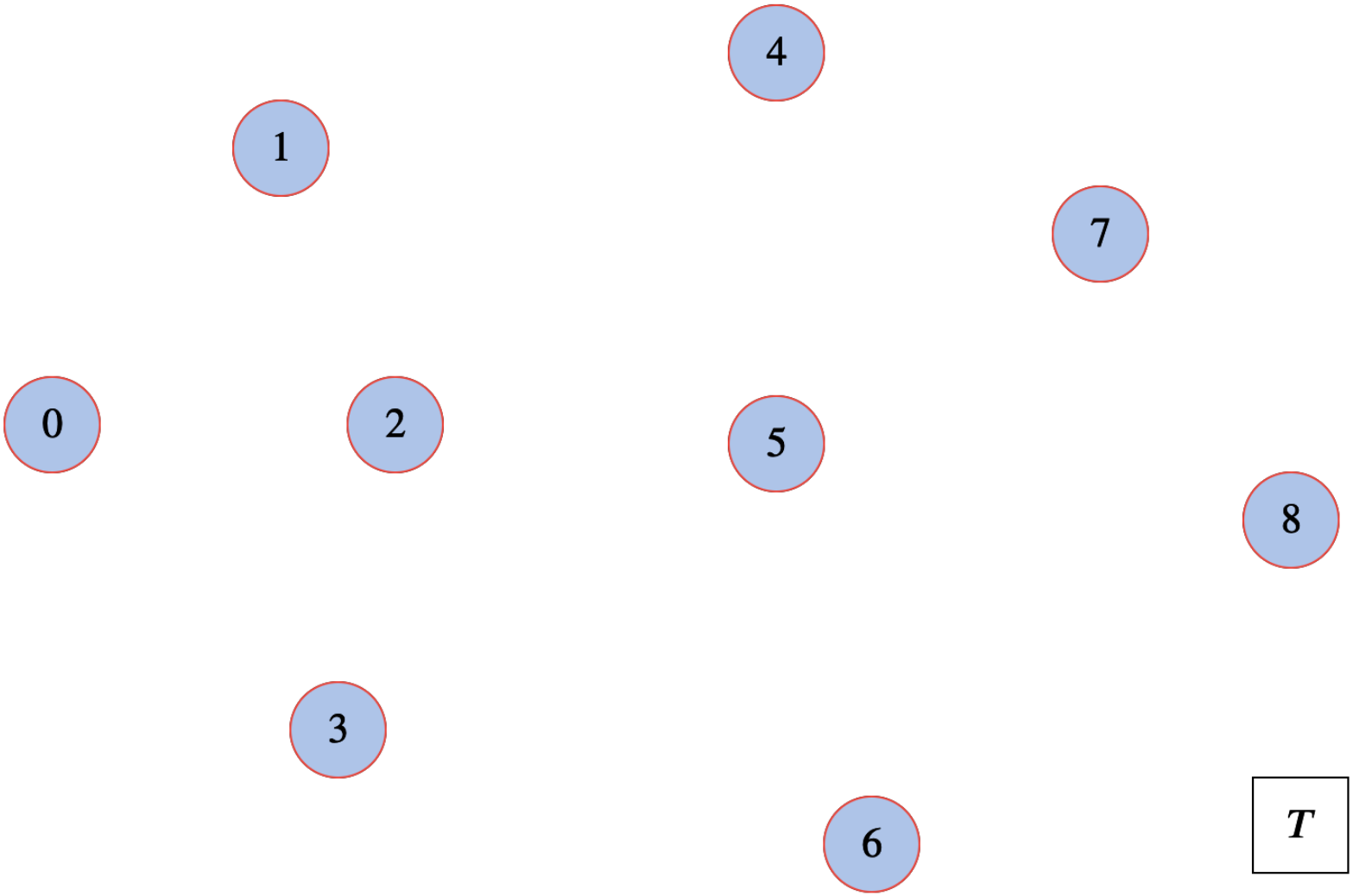
Markov Decision Process (MDP)

Mathematical formalism to capture the **essential characteristics** of the RL problem:

- MDP = states, actions, transitions, rewards, (discount factor)

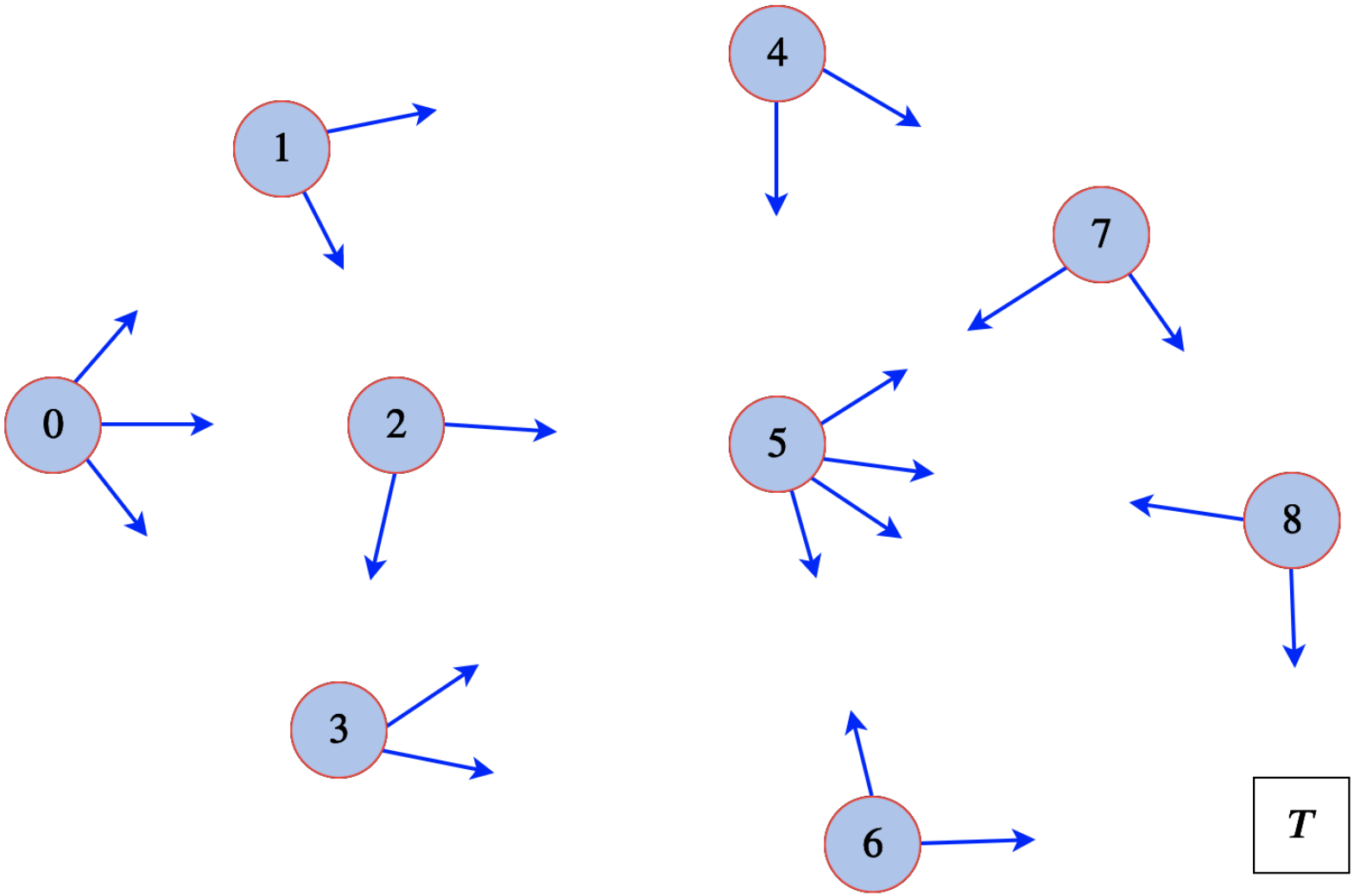
Markov Decision Process (MDP)

States

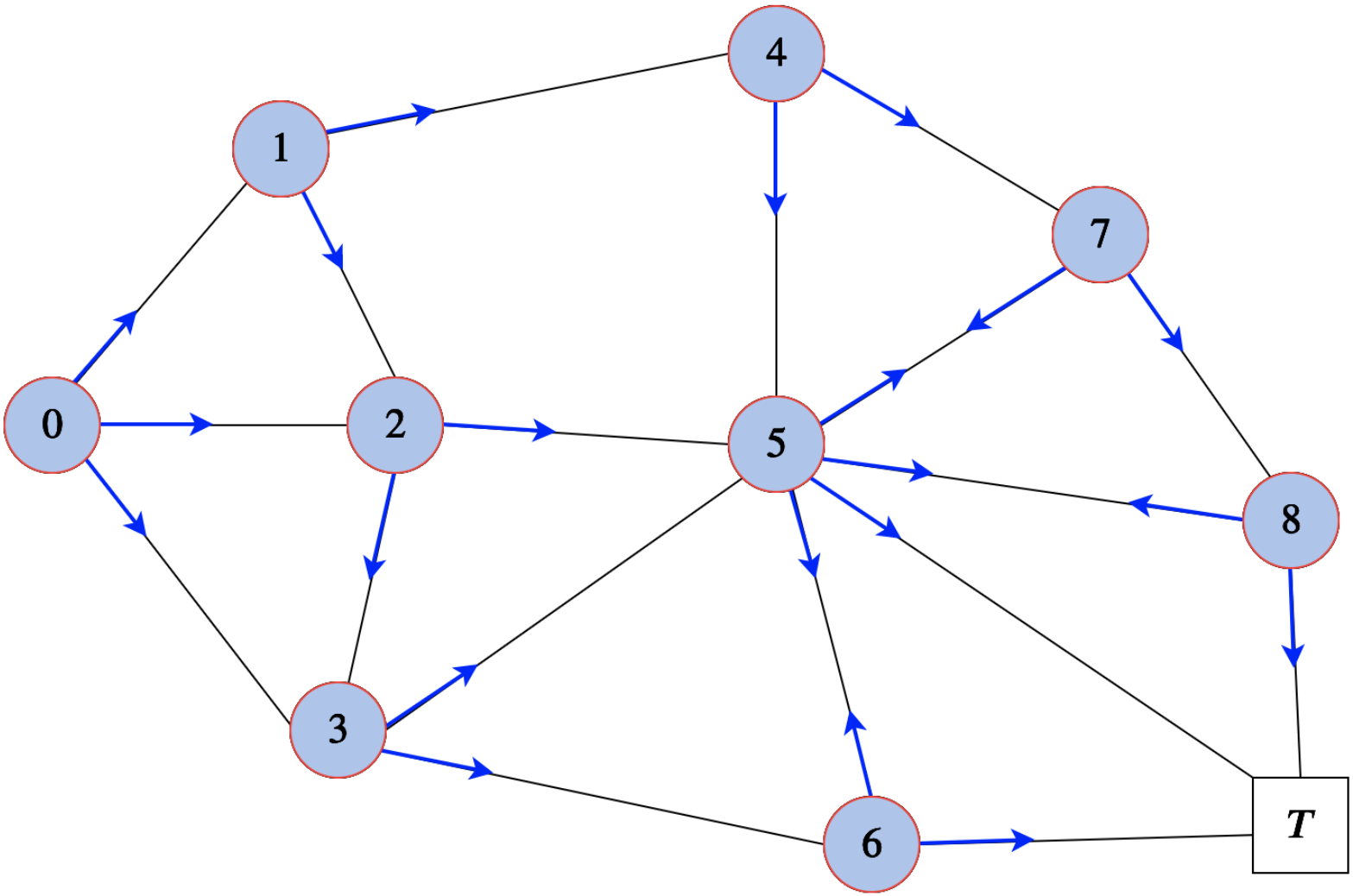


Markov Decision Process (MDP)

States + Actions

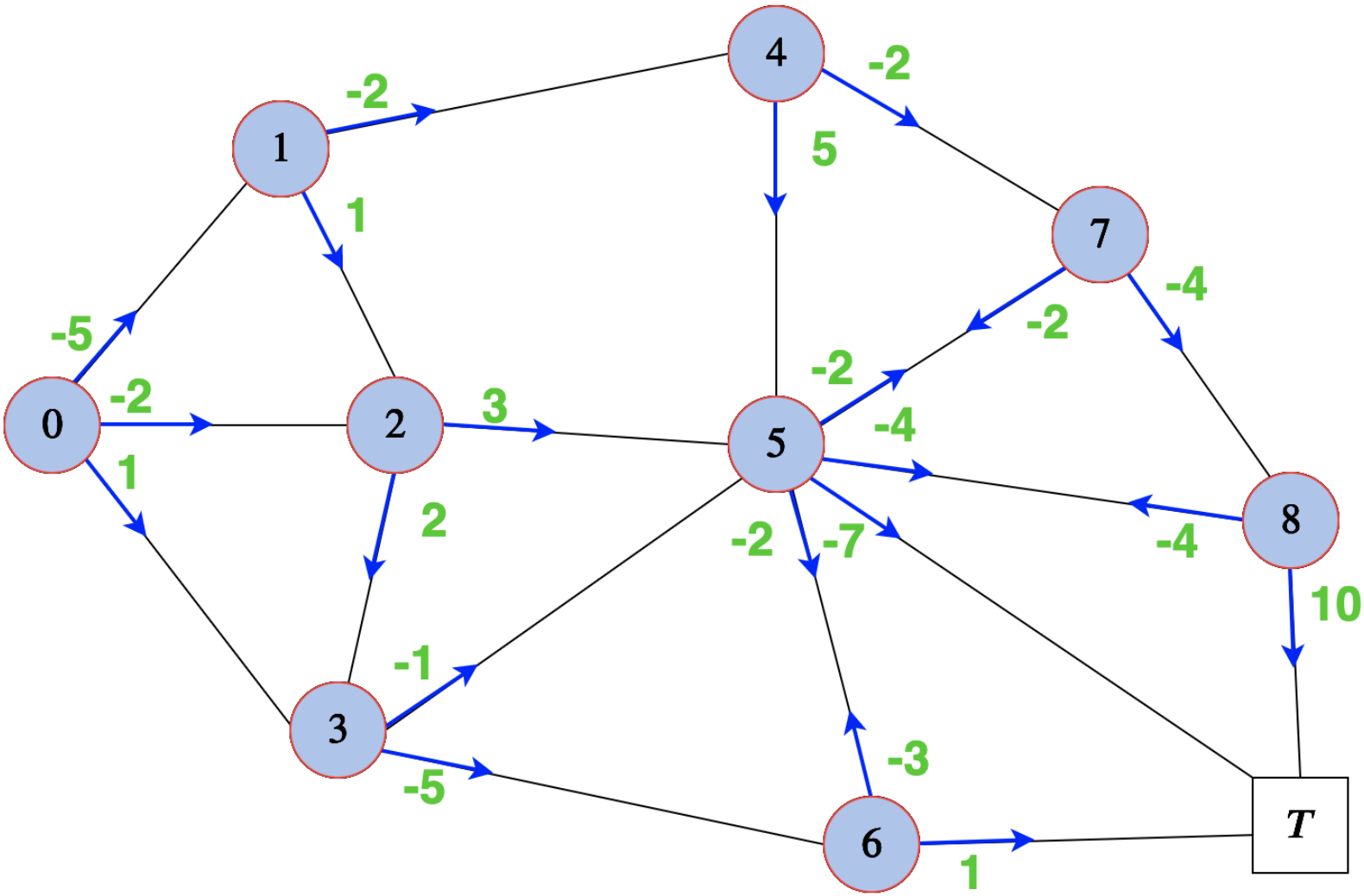


Markov Decision Process (MDP)
States + Actions + Transitions

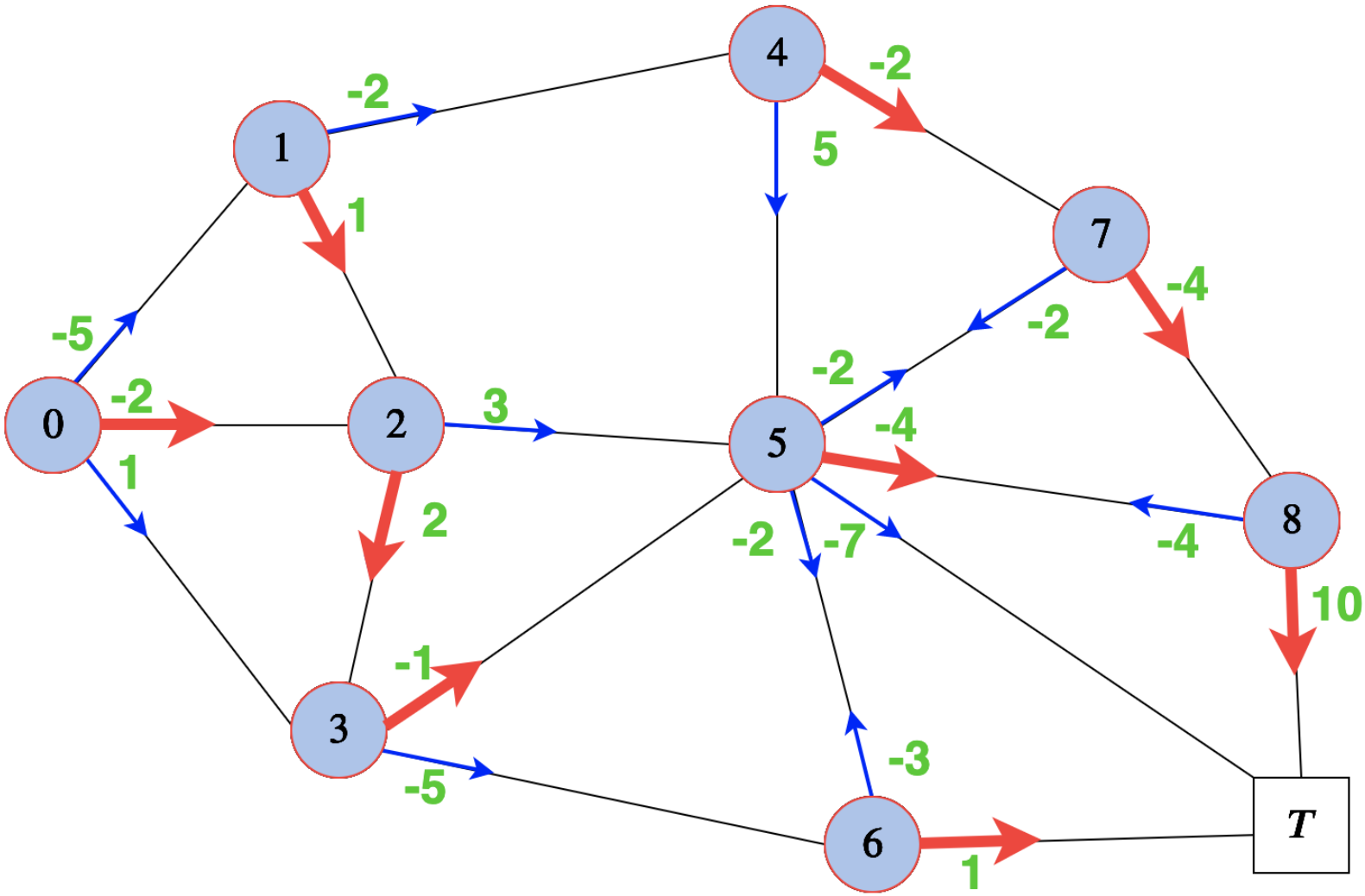


Markov Decision Process (MDP)

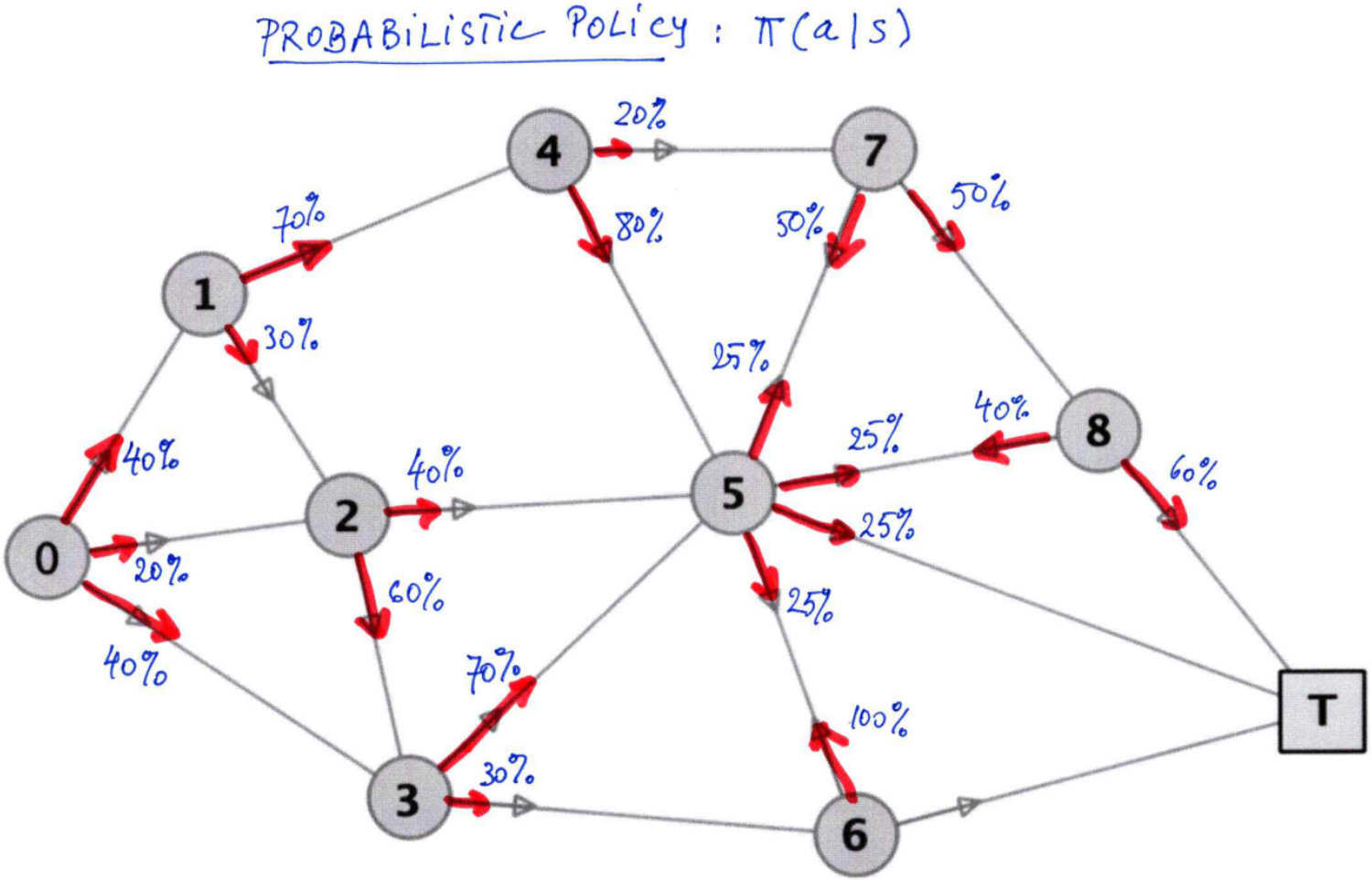
States + Actions + Transitions + Rewards



MDP + Policy (deterministic)

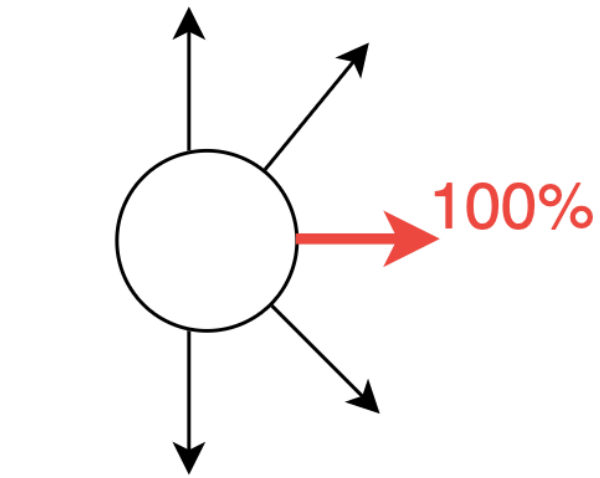


MDP + Policy (Probabilistic/Stochastic)

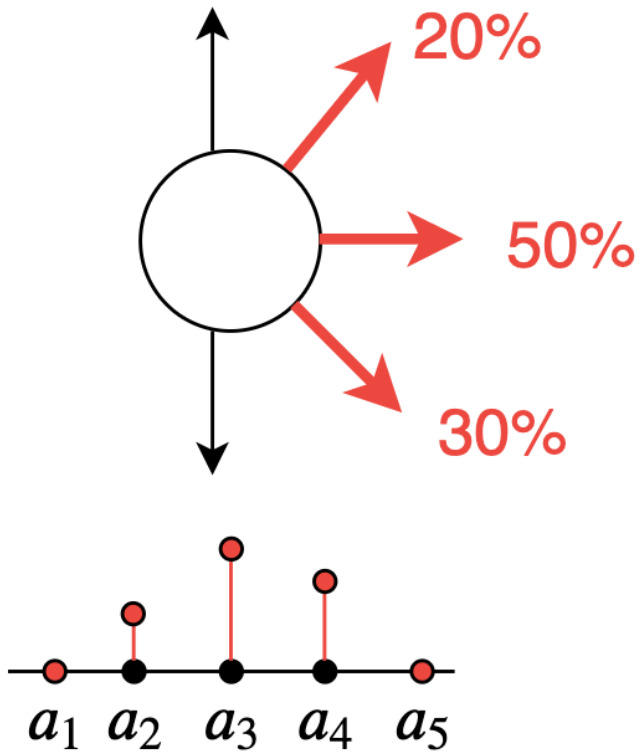


Deterministic vs. Probabilistic Policy

deterministic



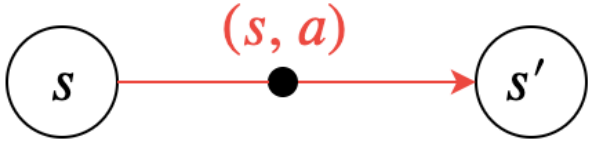
probabilistic



$$s \rightarrow \pi(a \mid s)$$

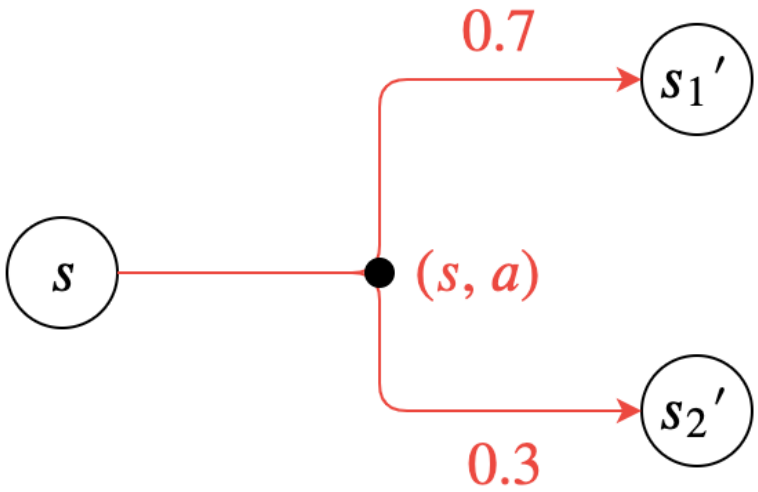
Deterministic vs. Probabilistic Transitions

Deterministic



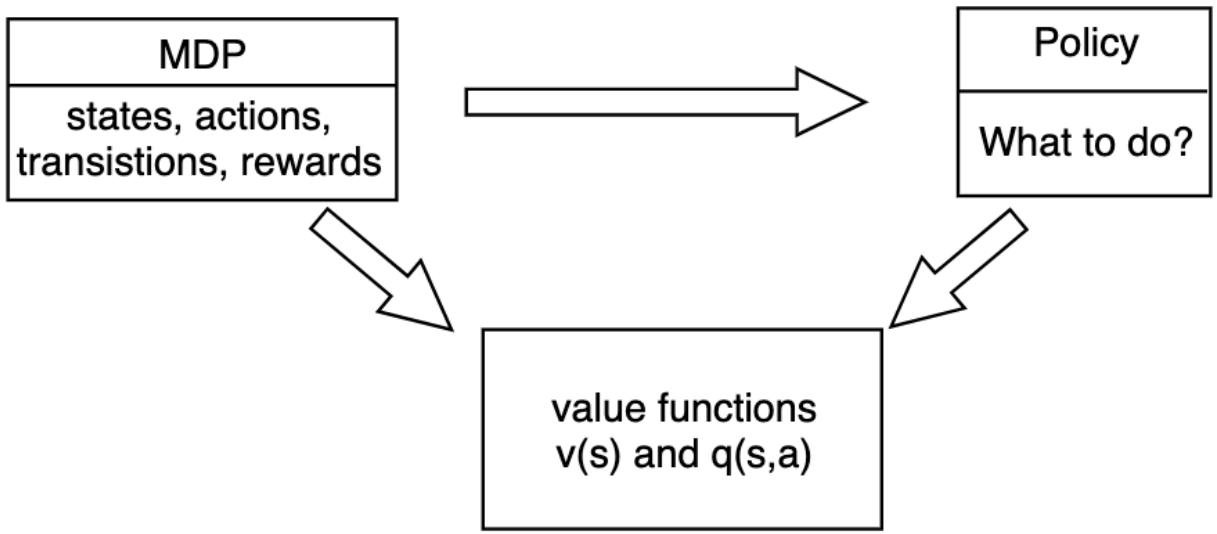
$$p(s' \mid s, a) = 1$$

Probabilistic



$$p(s_1' \mid s, a) = 0.7$$
$$p(s_2' \mid s, a) = 0.3$$

Overview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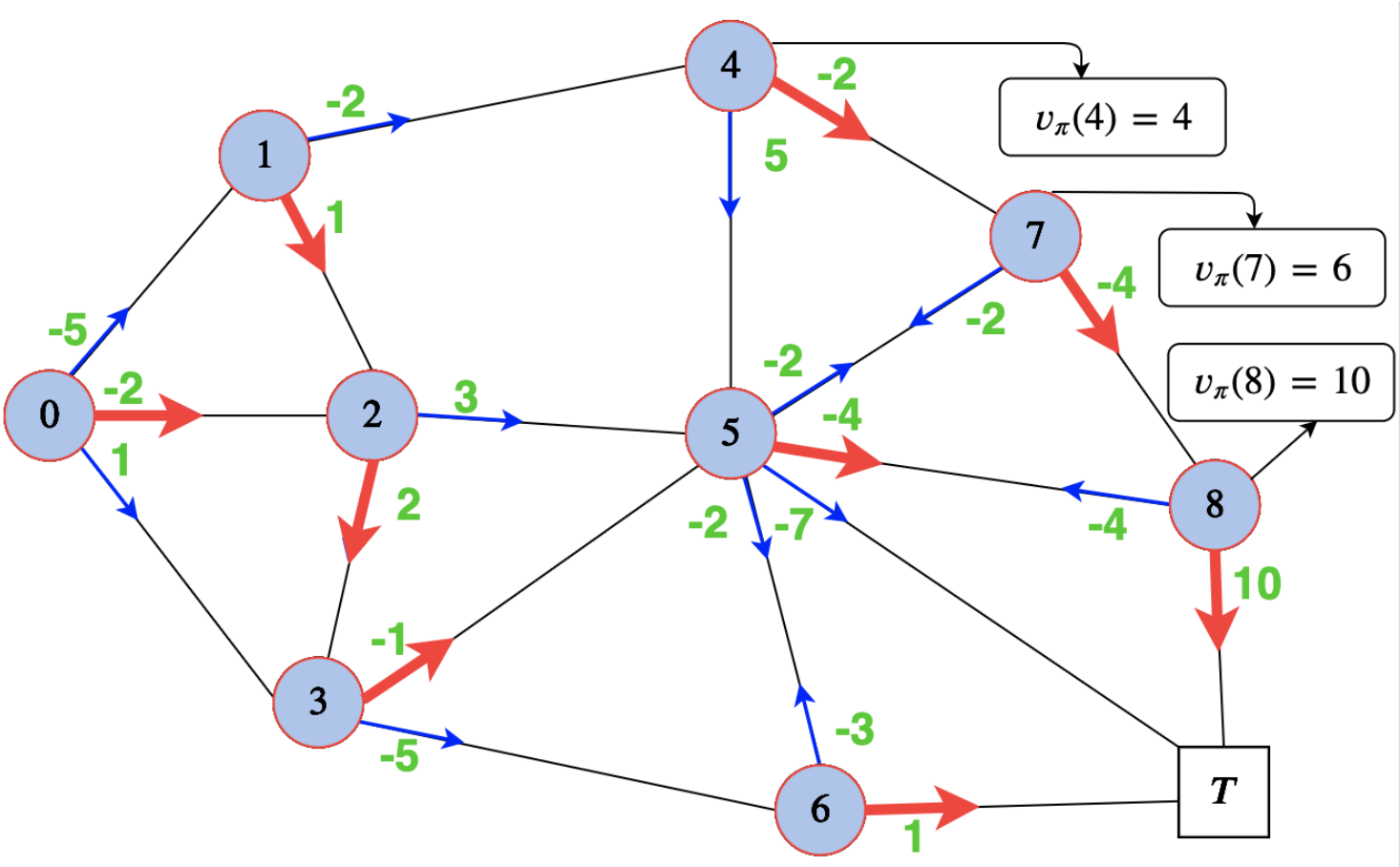
MDP + Policy $\longrightarrow$ state value function $v_\pi(s)$

$v_\pi(s)$: value of state s when actions are specified by policy π:

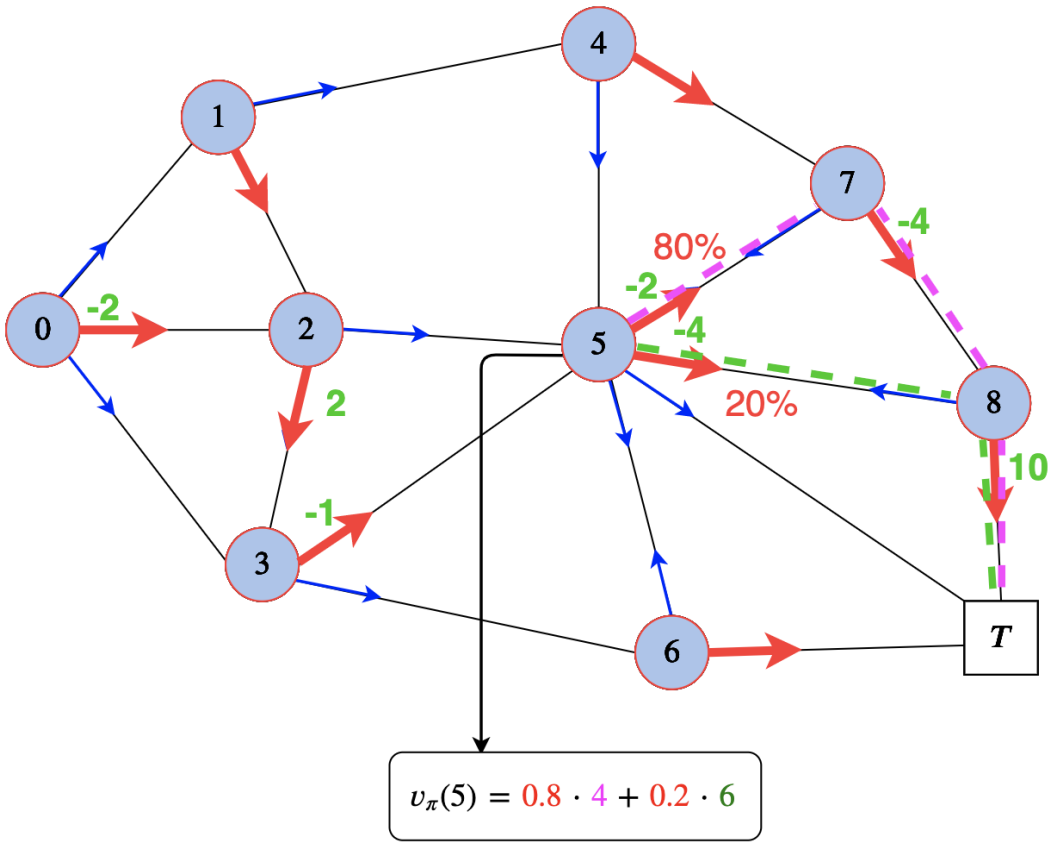
$$v_\pi(s) = \sum_{t=1}^{T} R_t \quad \text{given } s_0 = s$$



MDP + Policy $\longrightarrow$ state value function $v_\pi(s)$

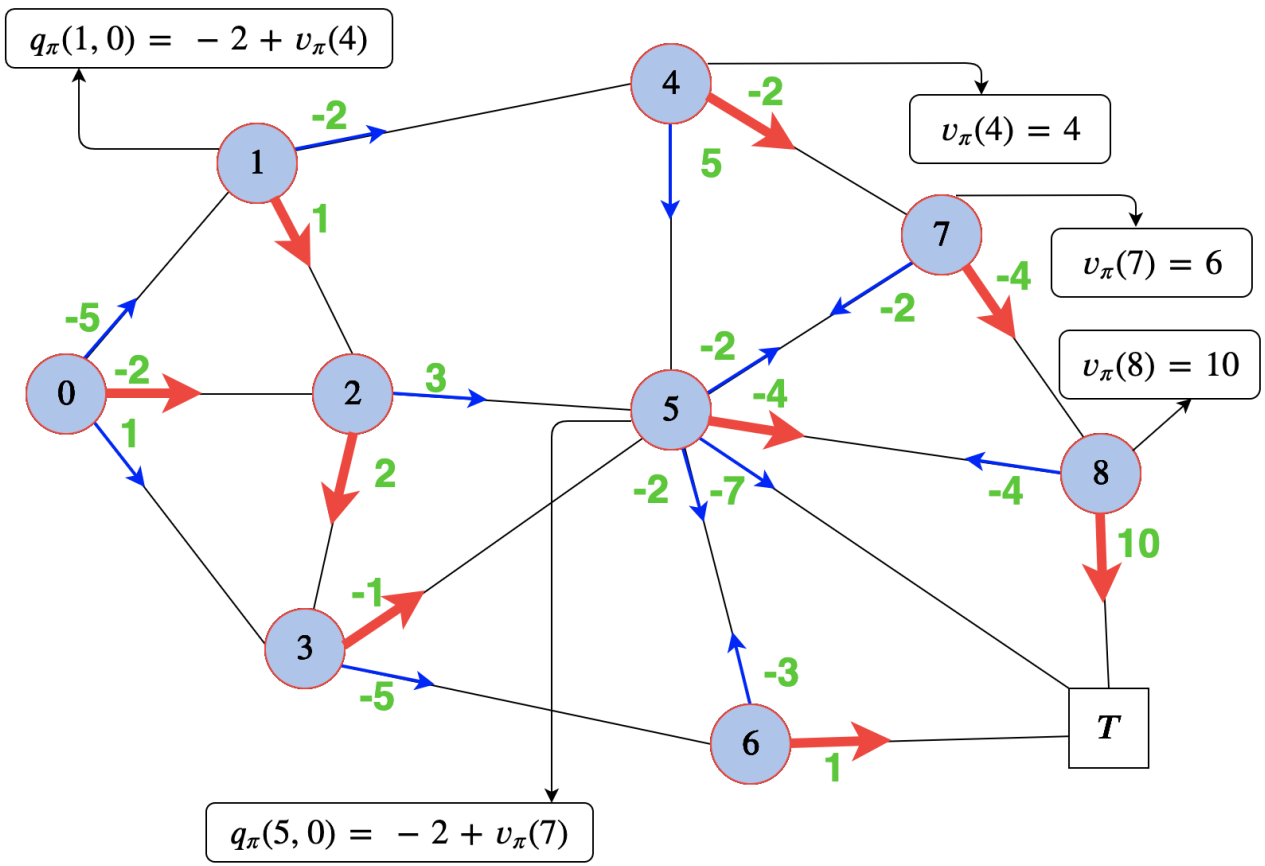
$v_\pi(s)$: expected value of state s when actions are specified by π:

$$v_\pi(s) = E_\pi\left(\sum_{t=1}^{T} R_t \,|\, s_0 = s\right)$$

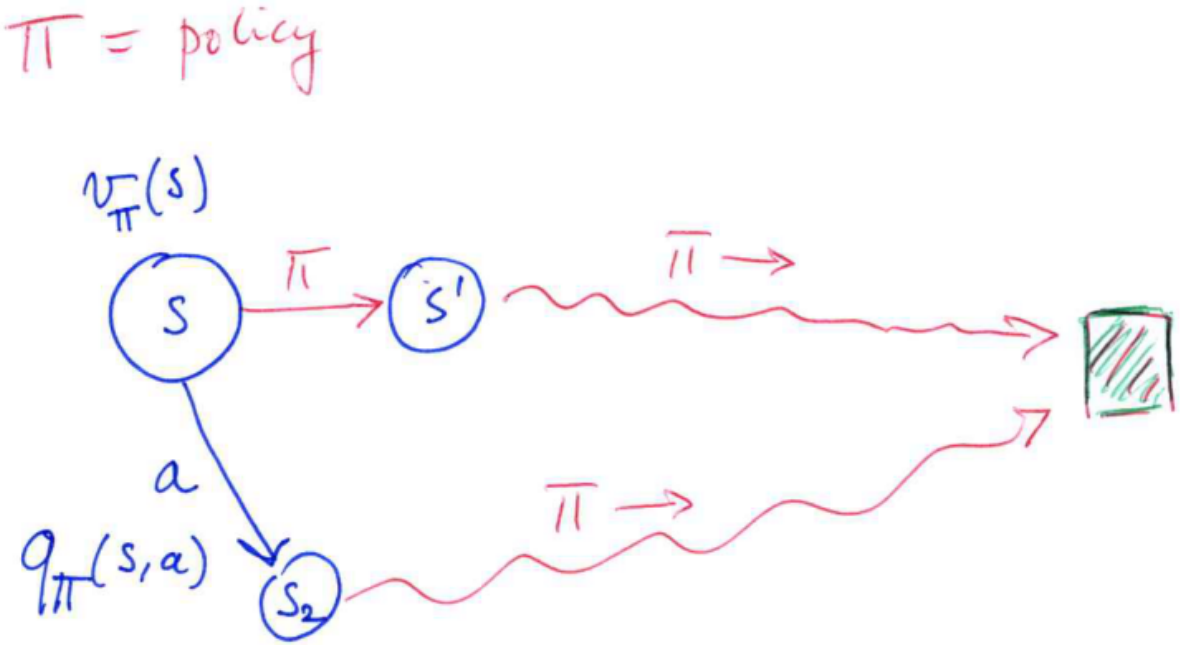


MDP + Policy $\longrightarrow$ state-action value function $q_\pi(s, a)$

$$q_\pi(s, a) = E_\pi \left(\sum_{t=1}^{T} R_t \,|\, s_0 = s,\ a_0 = a \right)$$



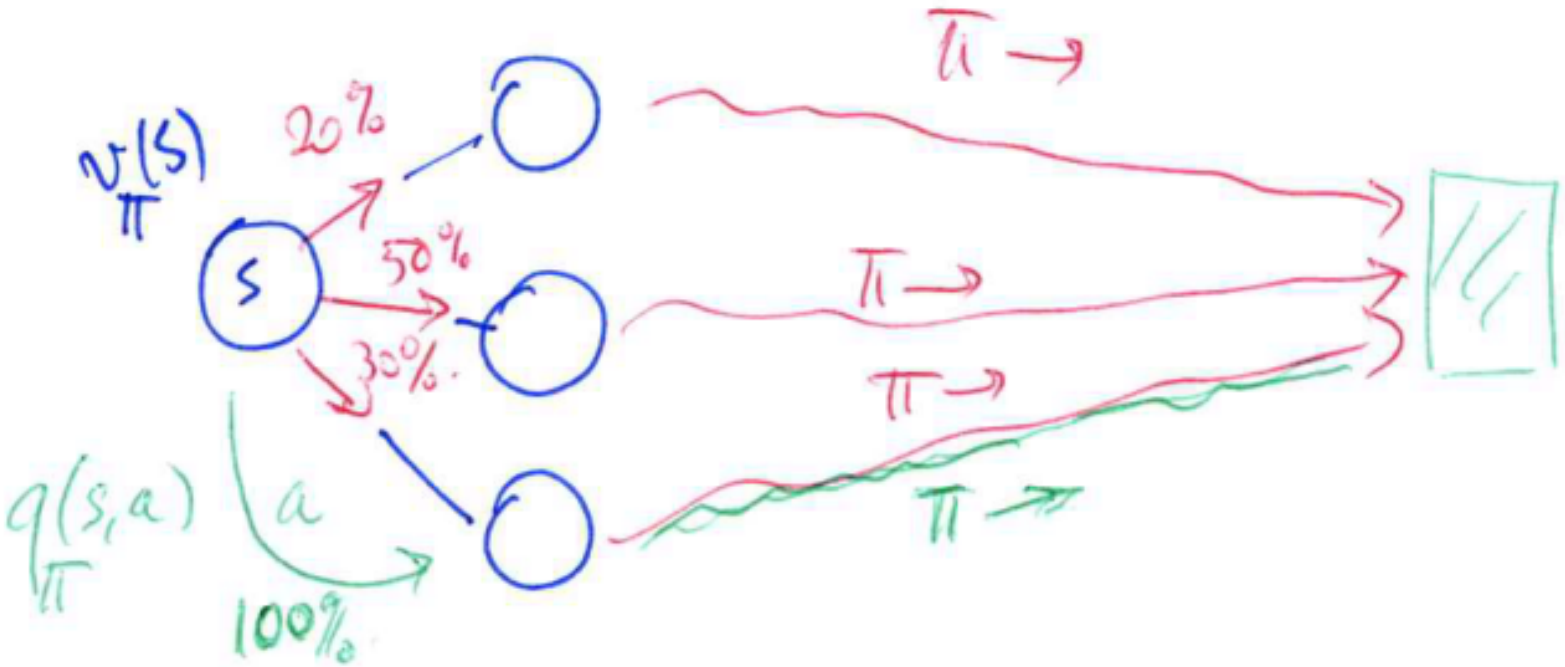
Difference between value functions $v_\pi(s)$ and $q_\pi(s, a)$



- $v_\pi(s)$: allow the policy π to dictate every action along the path;
- $q_\pi(s, a)$: first action a is taken independently of the policy π, from then onward, π dictates the remaining actions taken along the rest of the path.

Difference between value functions $v_\pi(s)$ and $q_\pi(s, a)$

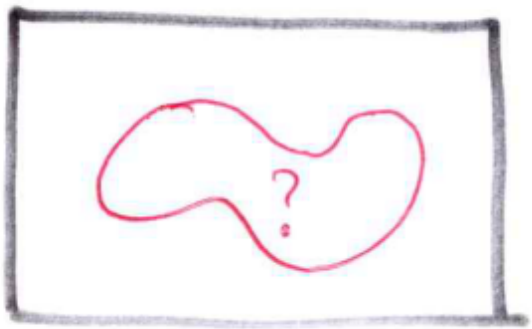
Similar interpretation for stochastic policy.

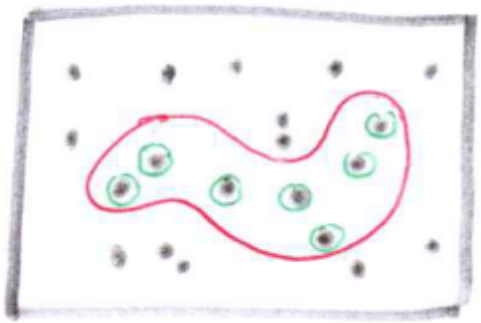


Outline

Coding excursion 1: Monte Carlo (MC) for value estimation

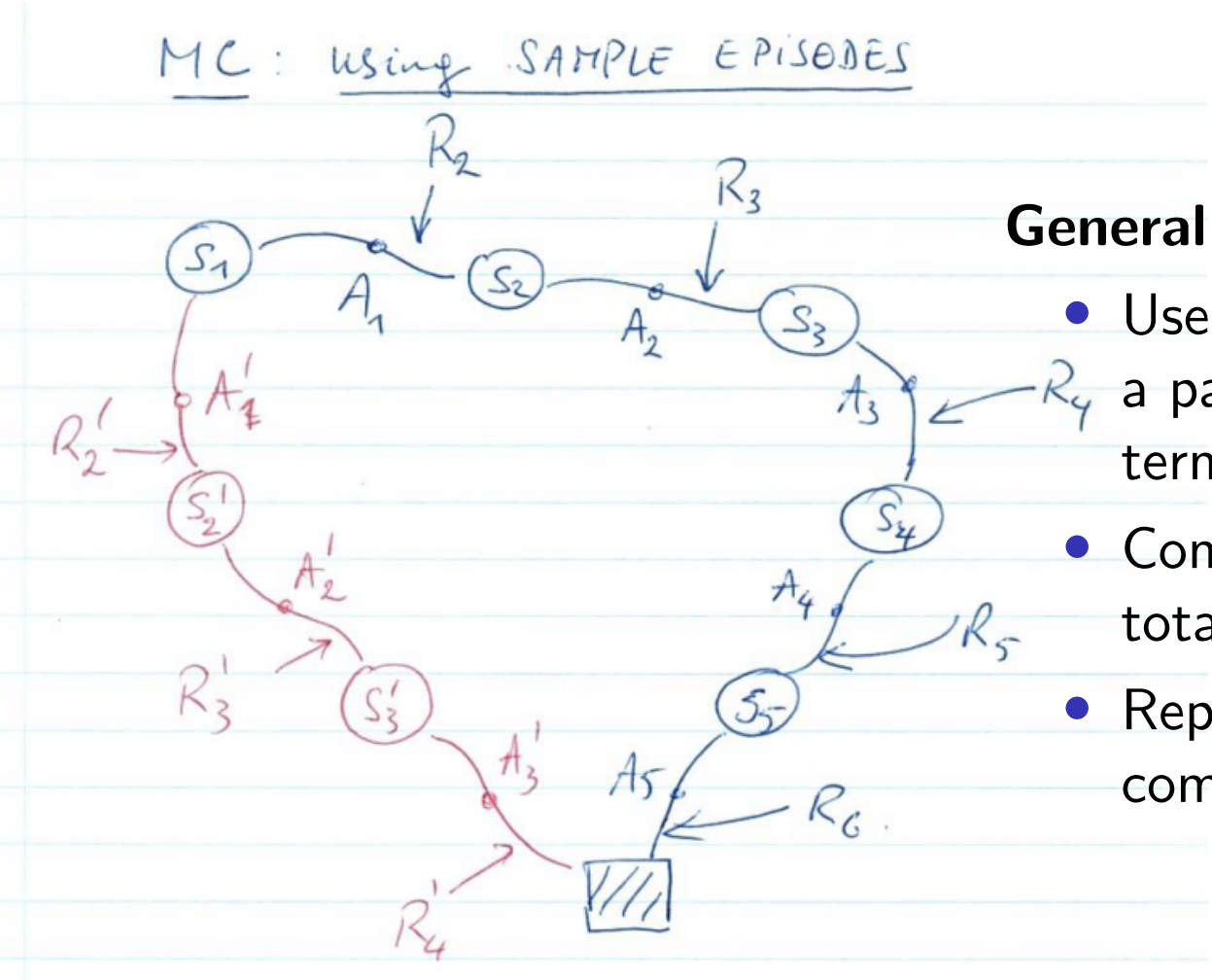
- Monte Carlo (MC): just do it!
- **Computational technique** for finding (approximate) solution to (difficult) problems using **random sampling**
- **Illustration:** Estimating area inside irregular contour:





- MC is costly, sometimes we can do better!

Coding excursion 1: Monte Carlo (MC) for value estimation

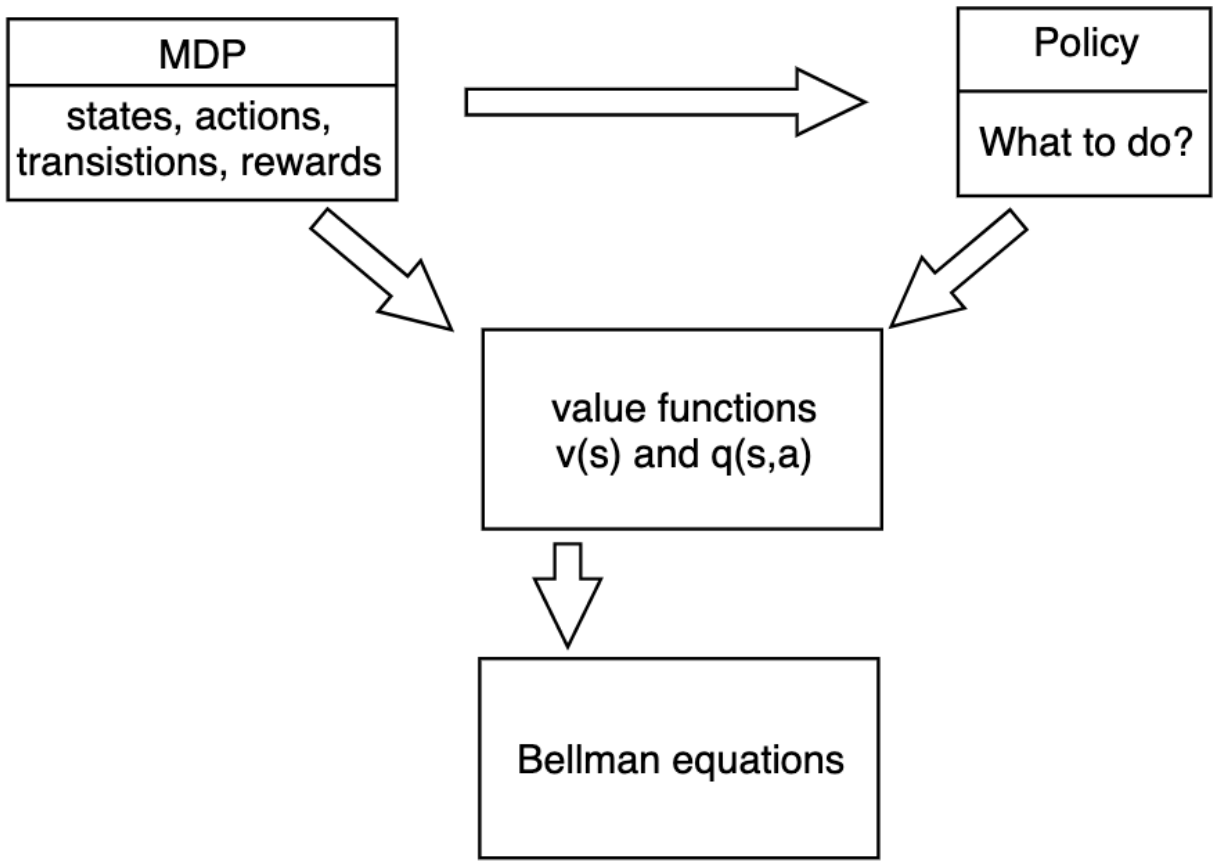


General principle:

- Use the policy to roll-out a path from initial to terminal state;
- Compute the total/cumulative reward;
- Repeat many times and compute mean

Outline

Overview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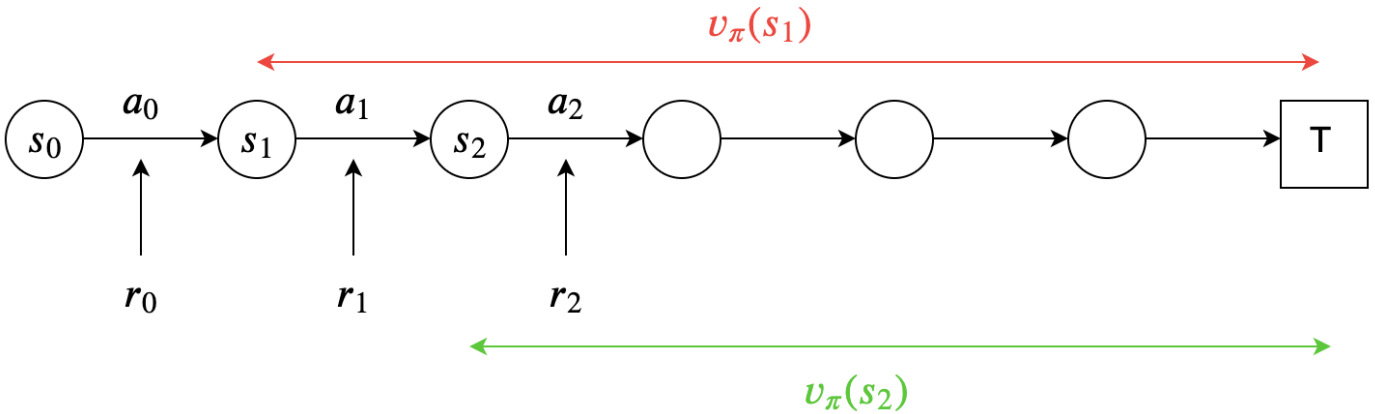
Bellman equations

Bellman equation for value function $v_\pi(s)$



For **deterministic policy:** if $s \xrightarrow{a} s'$ then $v_\pi(s) = r(s, a, s') + v_\pi(s')$

Bellman equation for value function $v_\pi(s)$

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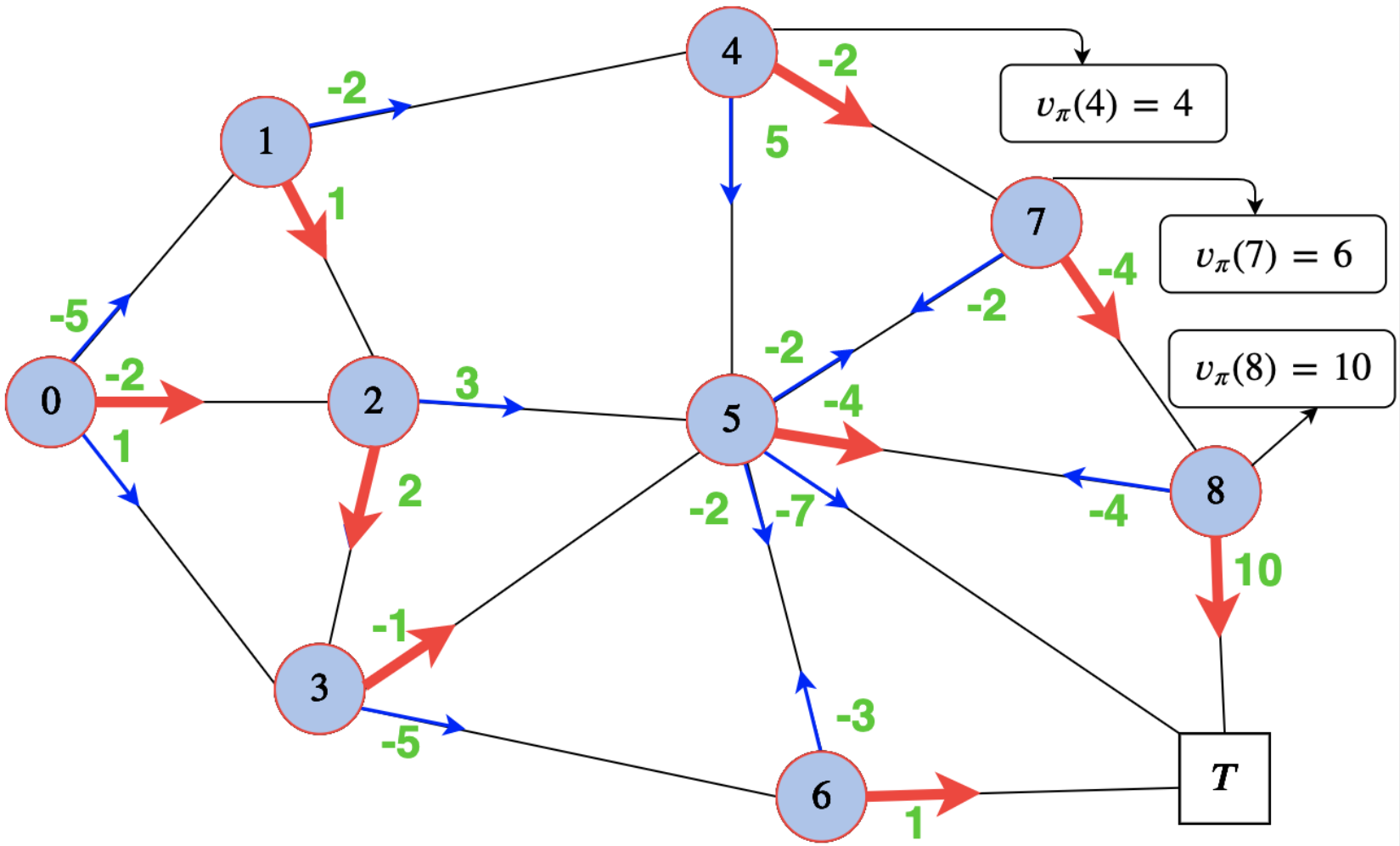
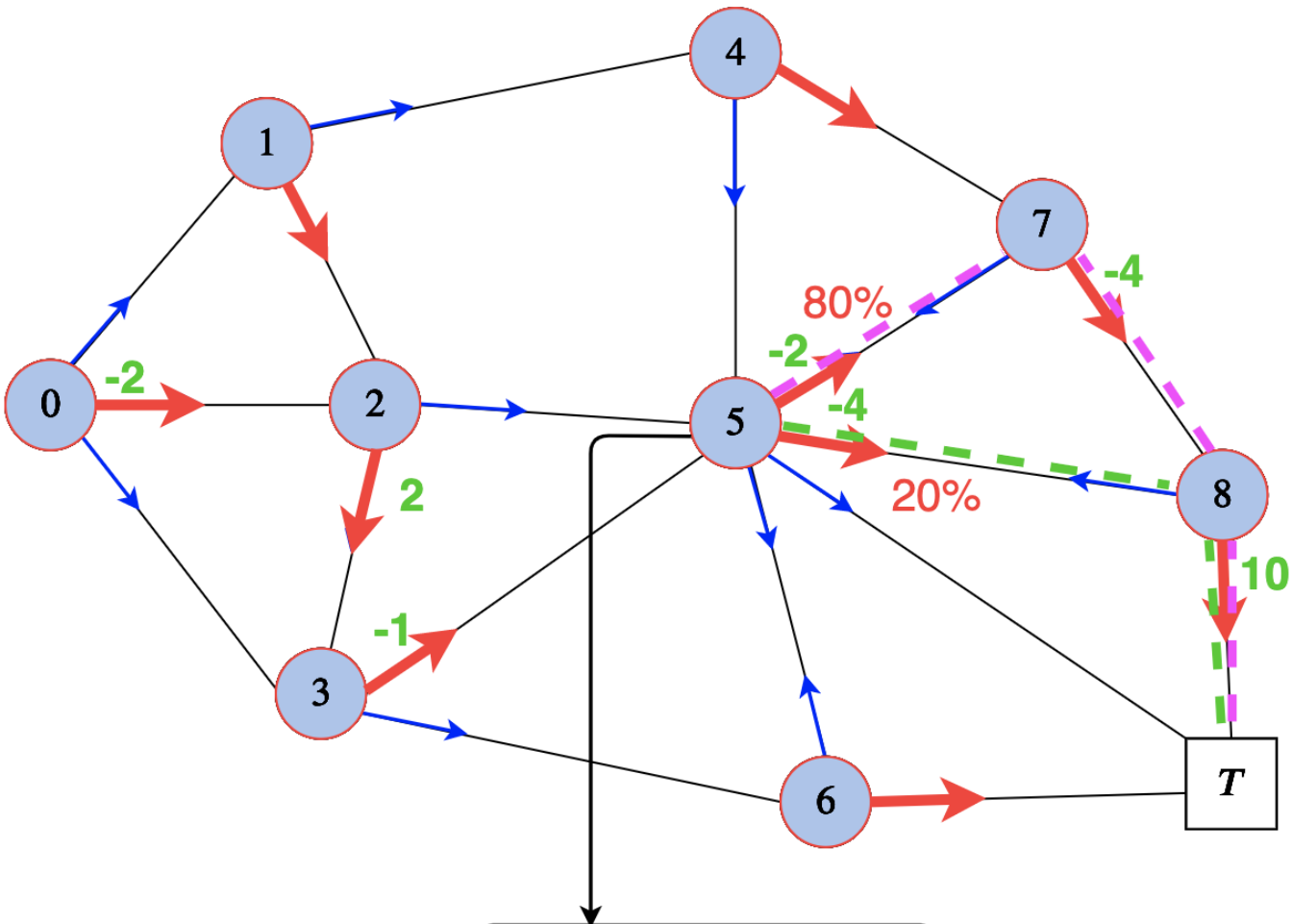


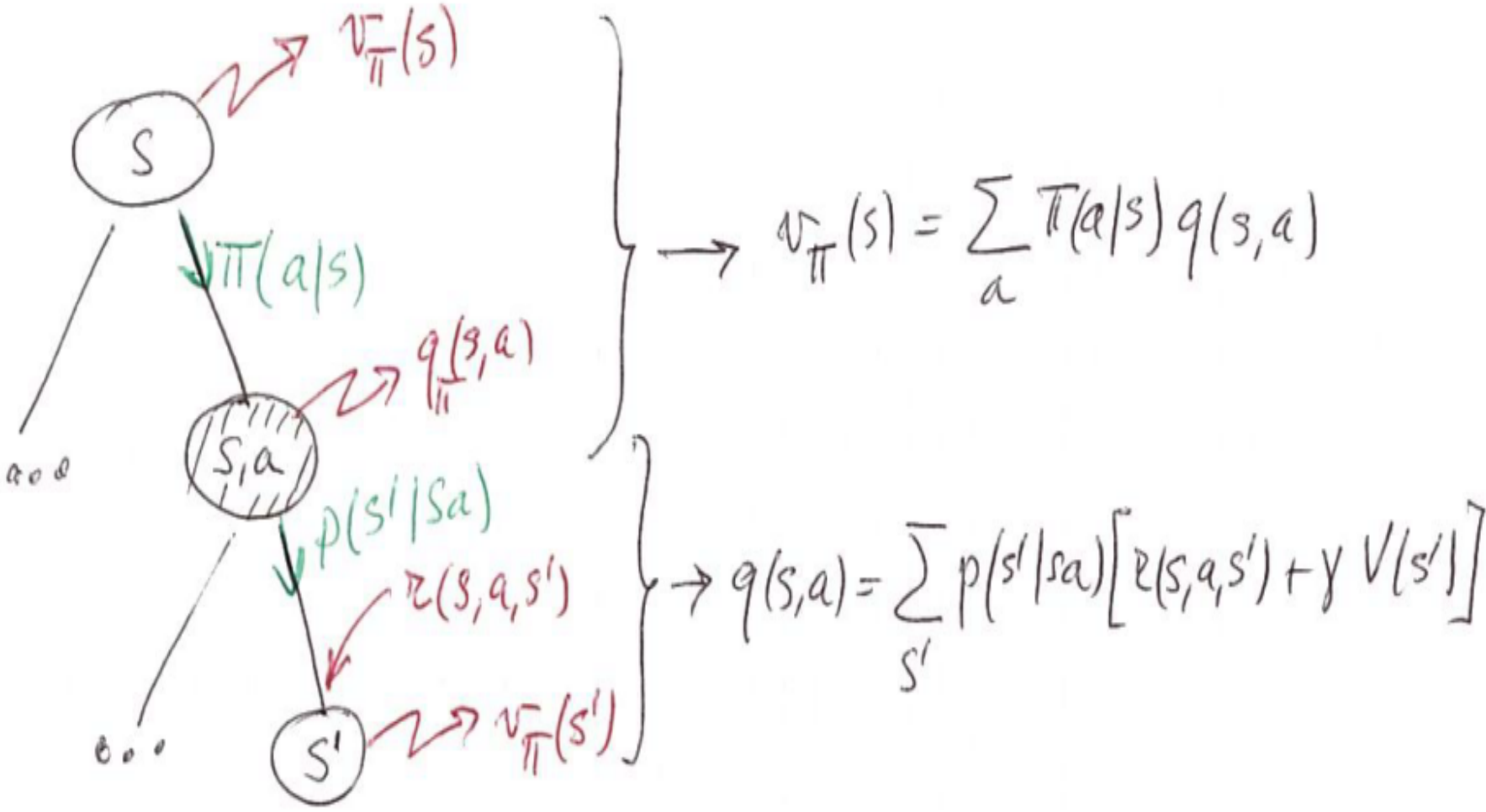
Bellman equation for value function $v_\pi(s)$

For **stochastic policy:**



$$v_\pi(5) = 0.8 \cdot 4 + 0.2 \cdot 6$$

Bellman equations (schematically): Backup Diagram



Bellman equation: Summary

- **Back-up**

$$v_\pi(s) = \sum_a \pi(a \mid s) q(s, a)$$

$$q(s, a) = \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma v(s') \right]$$

- **Combined**

$$v_\pi(s) = \sum_a \pi(a \mid s) \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma v_\pi(s') \Big]$$

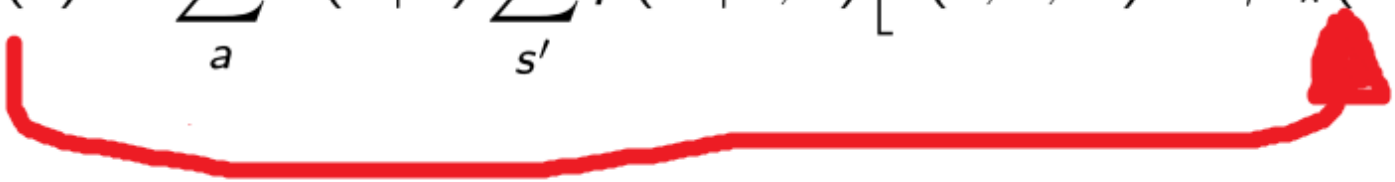
$$q_\pi(s, a) = \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma \sum_{a'} \pi(a' \mid s') q_\pi(s', a') \Big]$$

Outline

Coding excursion 2: Bellman equations and Dynamic Programming

- If model for environment (MDP) is fully known then Bellman equations can be solved for given policy π;
  - **MDP:** states, actions,
  - **MDP:** transitions $p(s' \mid s, a)$, rewards $r(s, a, s')$
  - **Policy:** $\pi(a \mid s)$
- Iterative solution (fixpoint solution)

$$v_\pi(s) = \sum_a \pi(a \mid s) \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma v_\pi(s') \Big]$$



Coding excursion 2: Bellman equations and Dynamic Programming

Iterative solution (fixpoint solution)

$$v_\pi(s) = \sum_a \pi(a \mid s) \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma v_\pi(s') \Big]$$

Pseudo code

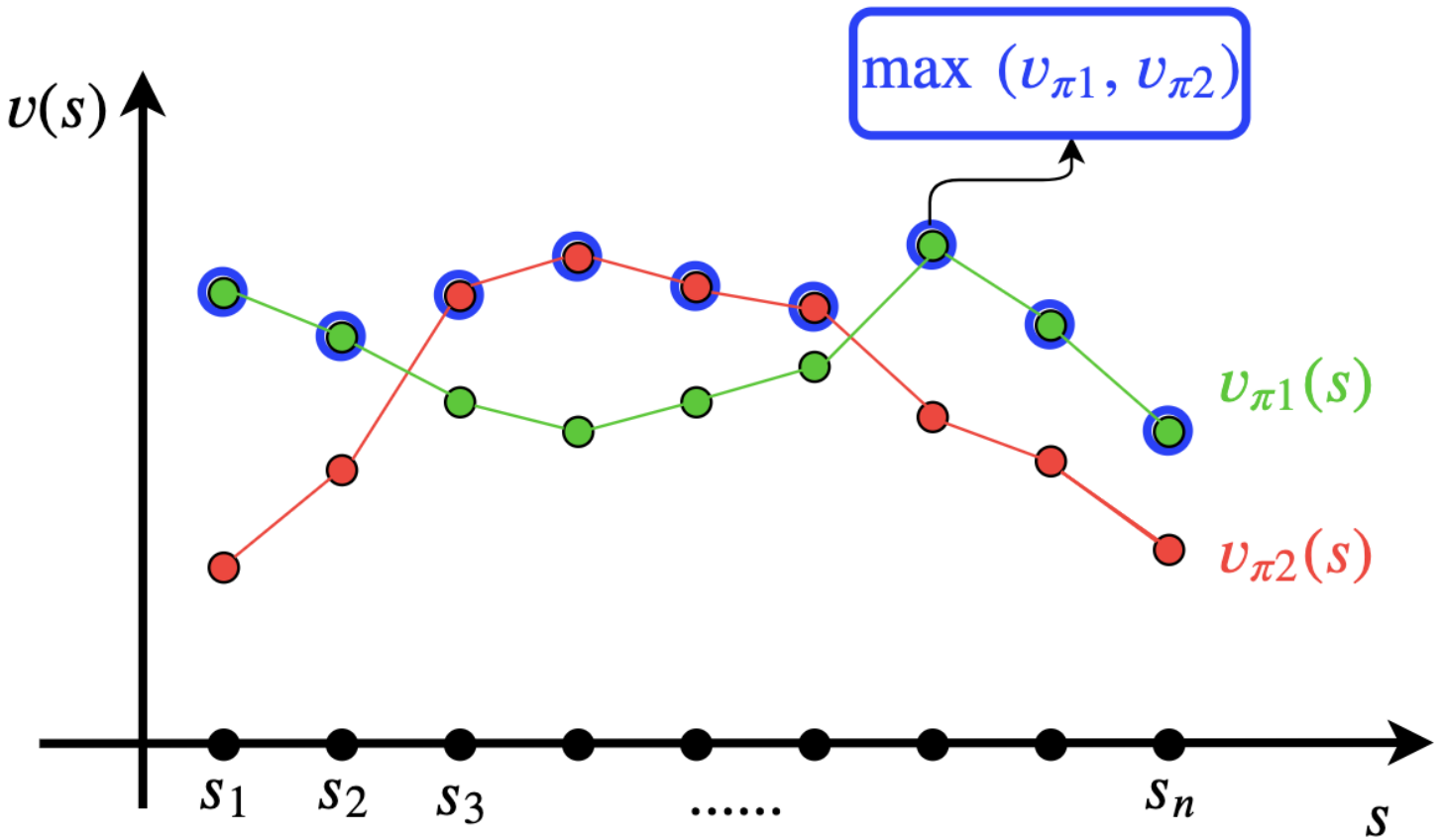
- Initialise $v_\pi(s)$ (zero or randomly) for all states s;
- Repeat until convergence:
  - Pick a random state s
  - Compute right-hand side of equation using policy π and MDP-parameters;
  - Update $v_\pi(s)$

Outline

Optimal value functions

The optimal value functions are defined by the **pointwise maximum over policies**:

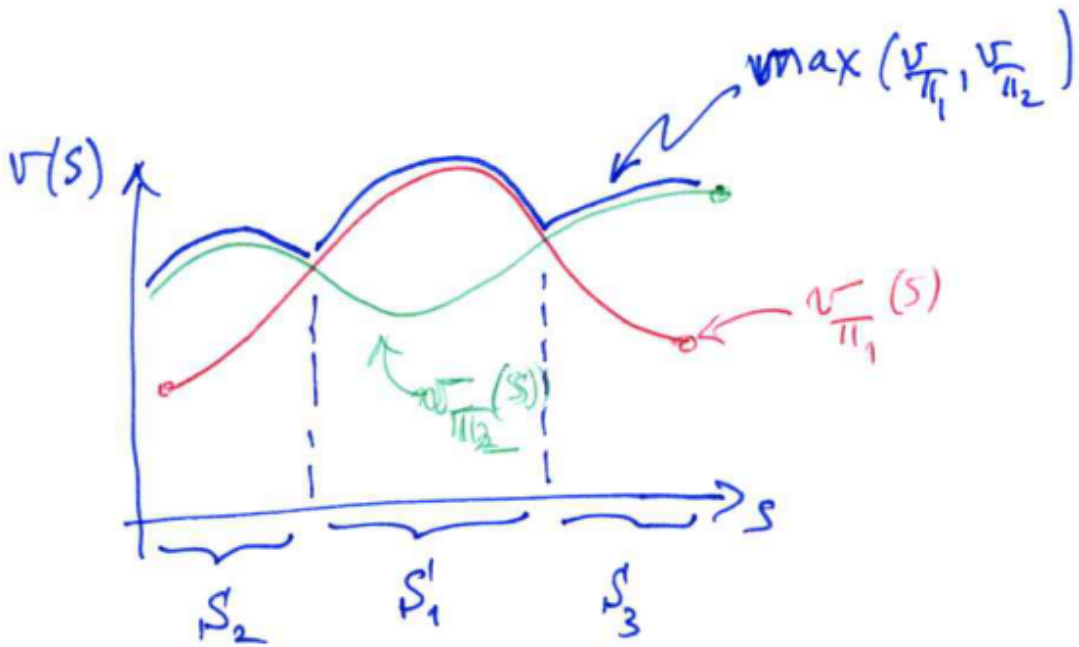
$$\forall s, a: \quad v^*(s) := \max_\pi v_\pi(s) \qquad \text{and} \quad q^*(s,a) := \max_\pi q(s,a)$$



Optimal policy π^*

There exists an **optimal policy** π^* such that its value functions corresponds to the optimal value functions:

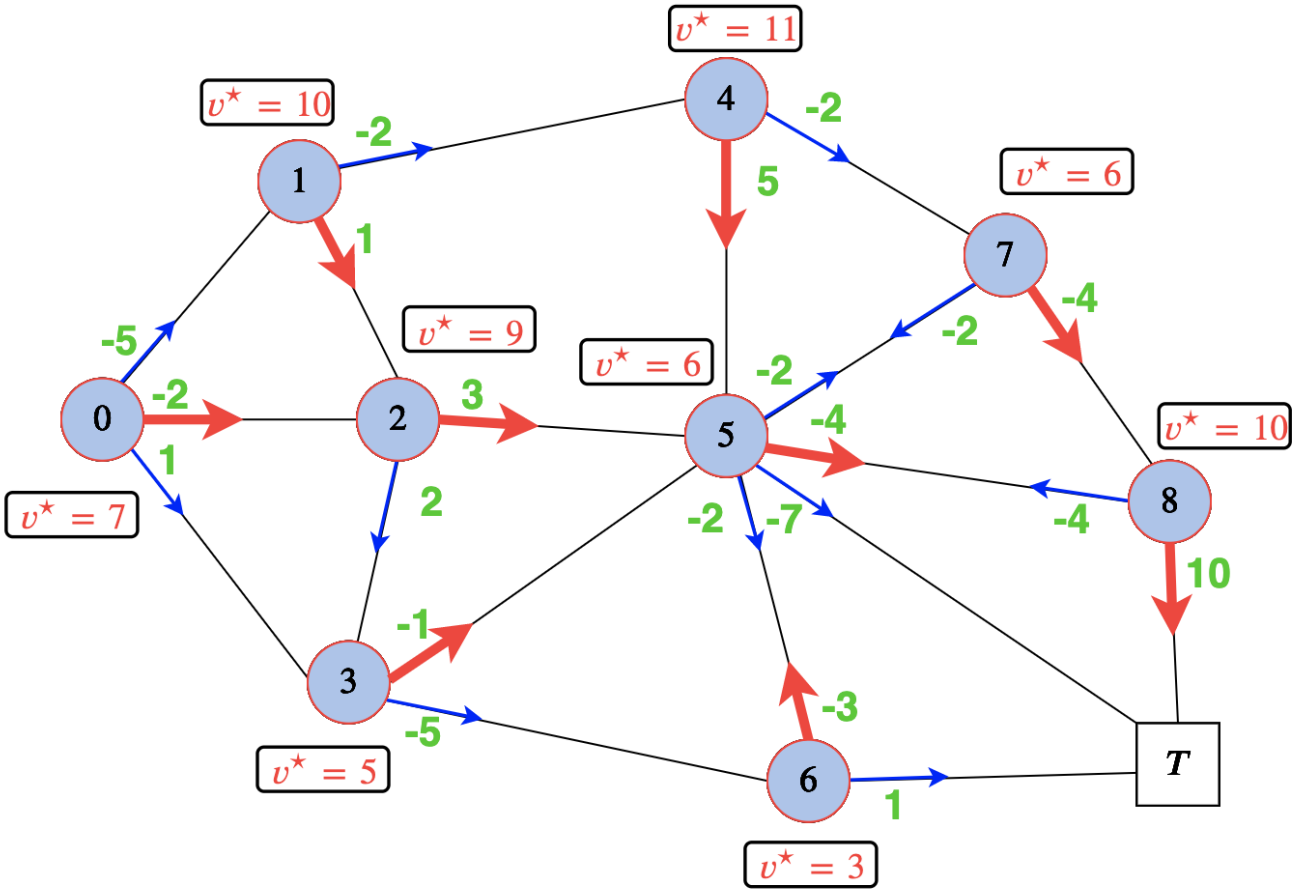
$$v_{\pi^*}(s) = v^*(s) \qquad \text{and} \qquad q_{\pi^*}(s,a) = q^*(s,a)$$



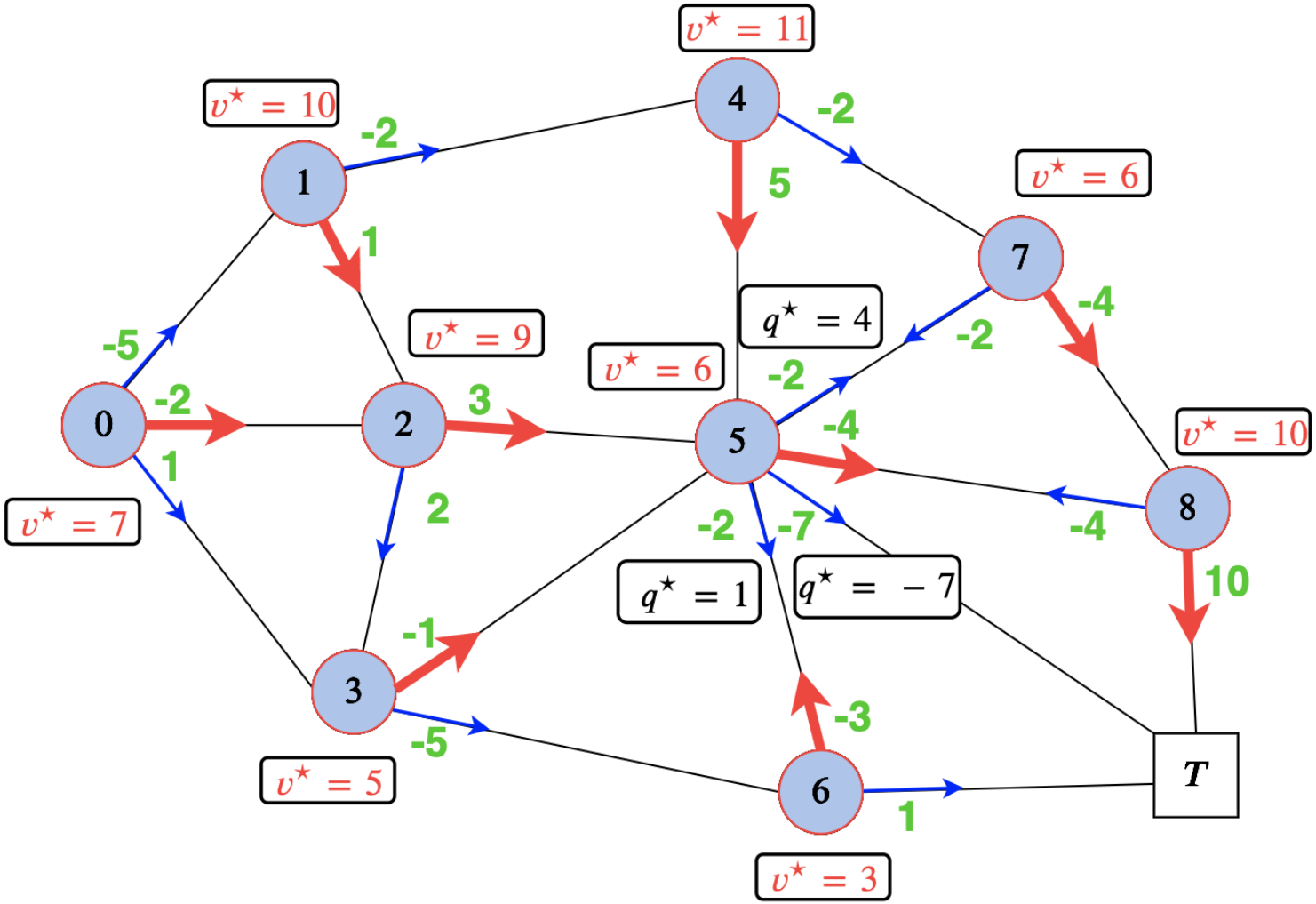
$$\pi^*(s) = \begin{cases} \pi_1(s) & \text{if } s \in S_2 \\ \pi_2(s) & \text{if } s \in S_1 \cup S_3 \end{cases}$$

Optimal policy

An optimal policy π^* assign optimal values v^* to all states

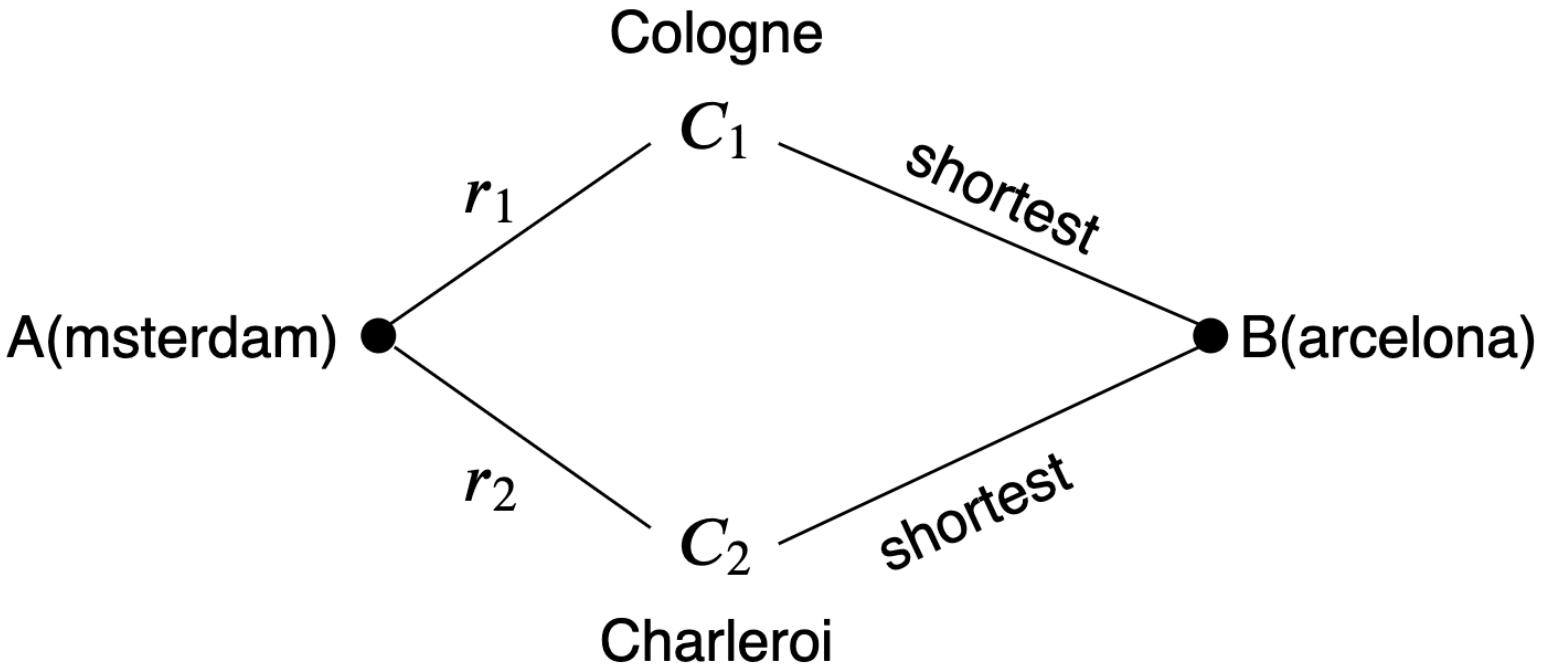


Bellman Optimality conditions



$$v^*(s) = \max_a q^*(s, a)$$

Bellman Optimality equation: Travel Distance Analogy



$$d^*(A, B) = \min_{C_i} \{r_i + d^*(C_i, B)\}$$

Bellman optimality conditions (for deterministic transitions)

- Travel Analogy: Shortest distance paths:

$$d^*(A, B) = \min_{C_i} \{r_i + d^*(C_i, B)\}$$

- Deterministic transitions: $s \xrightarrow{a} s'$

$$v^*(s) = \max_a \left\{ r(s, a, s') + v^*(s') \right\}$$

$$q^*(s, a) = r(s, a, s') + v^*(s') = r(s, a, s') + \max_{a'} q^*(s', a')$$

Bellman optimality equations (General form)

- **Optimal state value function**

$$v^*(s) = \max_a q^*(s, a)$$

- **Optimal state-action value function**

$$q^*(s, a) = \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma v^*(s') \right]$$

- **Combined**

$$v^*(s) = \max_{a \in A} \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma v^*(s') \Big]$$

$$q^*(s, a) = \sum_{s'} p(s' \mid s, a) \Big[r(s, a, s') + \gamma \max_{a' \in A} q^*(s', a') \Big]$$

Backup Diagram for Bellman Optimality Equations

Optimize over actions!

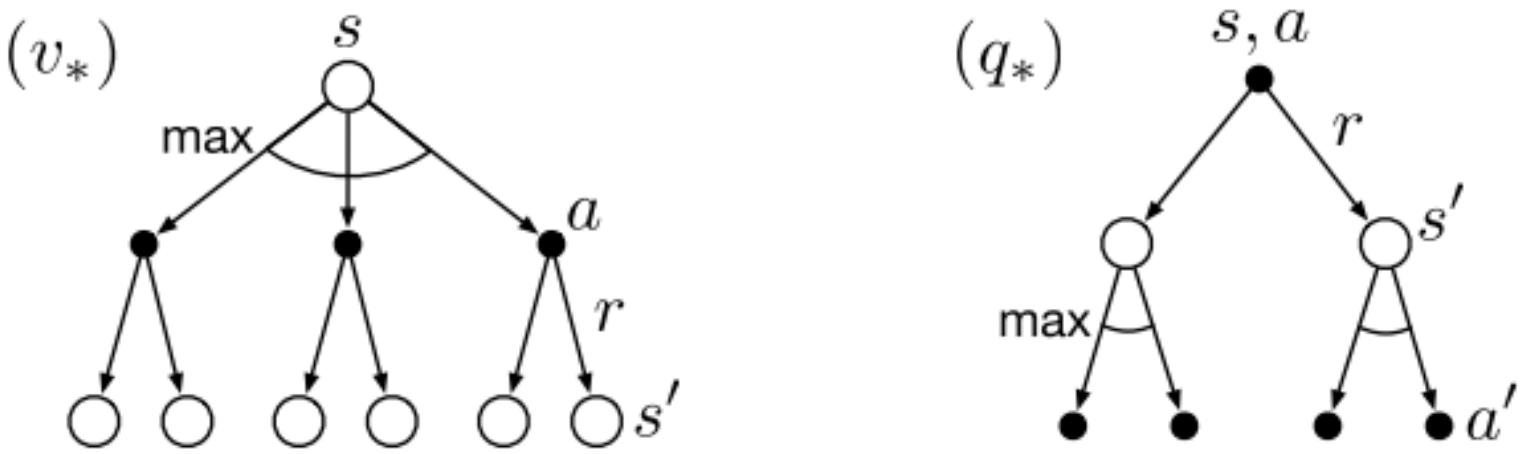


Figure 3.5: Backup diagrams for v_* and q_*

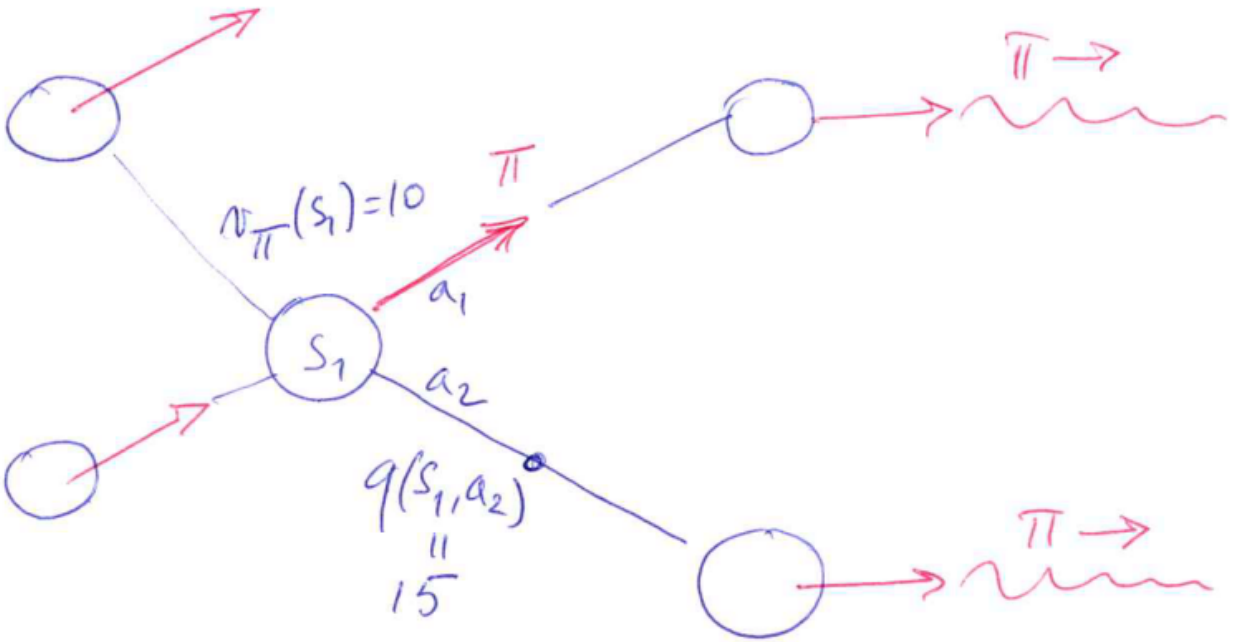
Bellman Optimality Conditions

- v^* and q^* satisfy a set of (non-linear) equations analoguous to Bellman equations for v_π and q_π;
- these non-linear equations do **not refer** explicitly to the optimal policy;
- As a consequence we can find the optimal policy by first solving these equations to find $v^*(s)$ (and $q^*(s,a)$) and then use **greedification** to determine the corresponding optimal policy π^*:

$$\forall s: \quad a_{opt} = \arg\max_a q^*(s,a)$$

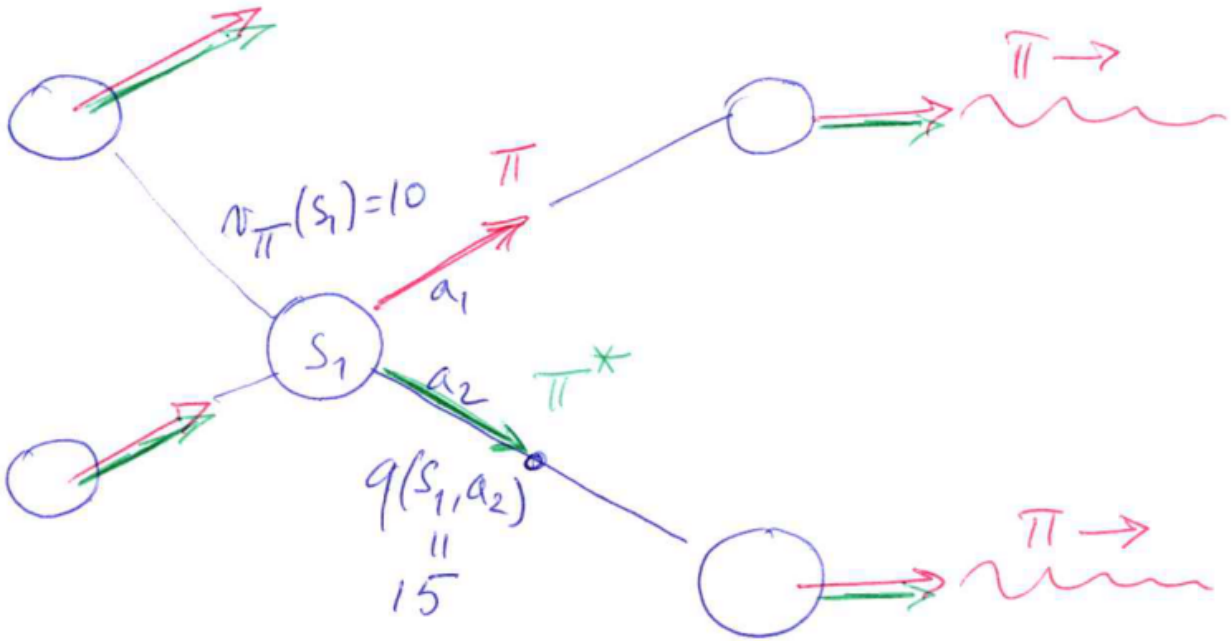
Outline

Incremental policy improvement



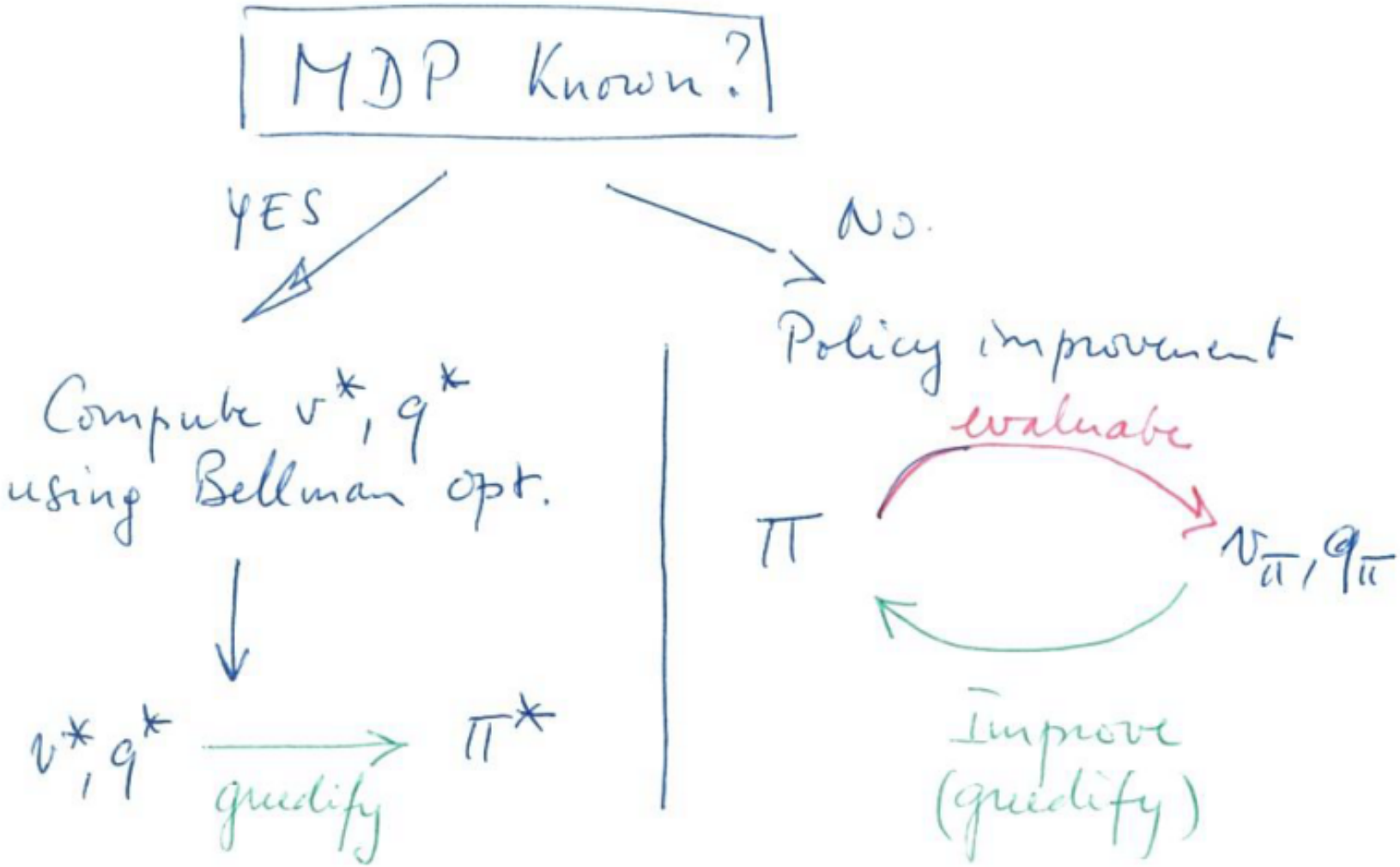
Room for improvement: In state s_1 we have $v_\pi(s_1) < q_\pi(s_1, a_2)$, hence switch from action a_1 to a_2.

Policy improvement for deterministic policy



$$\pi \longrightarrow \pi^* \qquad \text{where} \qquad \pi^*(s) = \begin{cases} a_2 & \text{if} \quad s = s_1 \\ \pi(s) & \text{if} \quad s \neq s_1 \end{cases}$$

Finding the optimal policy π^*



Outline

Coding Excursion 3: Finding an optimal policy

Bellman optimality equations: (General form)

- **Optimal state value function**

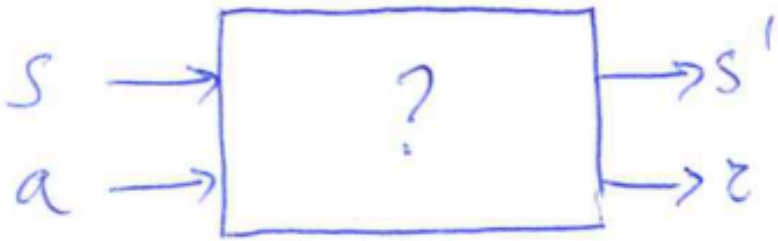
$$v^*(s) = \max_a q^*(s, a)$$

- **Optimal state-action value function**

$$q^*(s, a) = \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma v^*(s') \right]$$

Outline

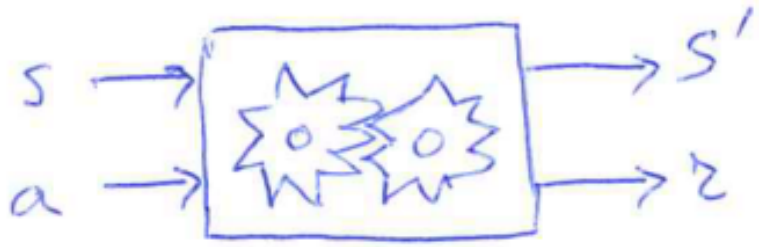
Model-free vs. model-based



Model-free

Black box

LEARNING



Model-based.

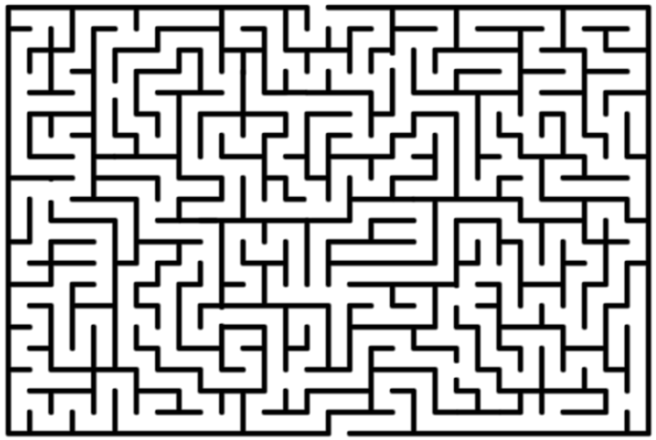
White box.

PLANNING

(Dyn. progr)

Planning (model-based) versus Learning (model-free)

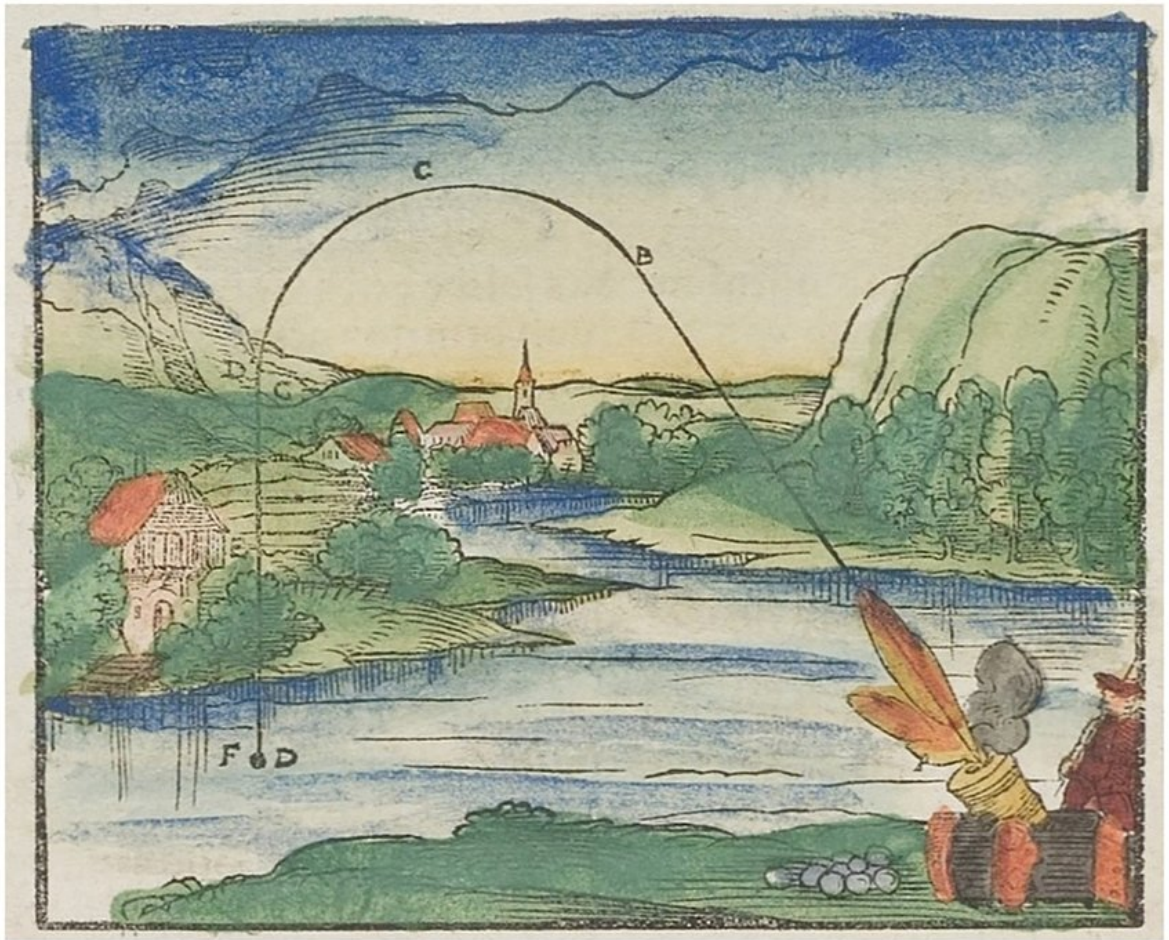
Model-based



Model-free

Model-free versus model-based

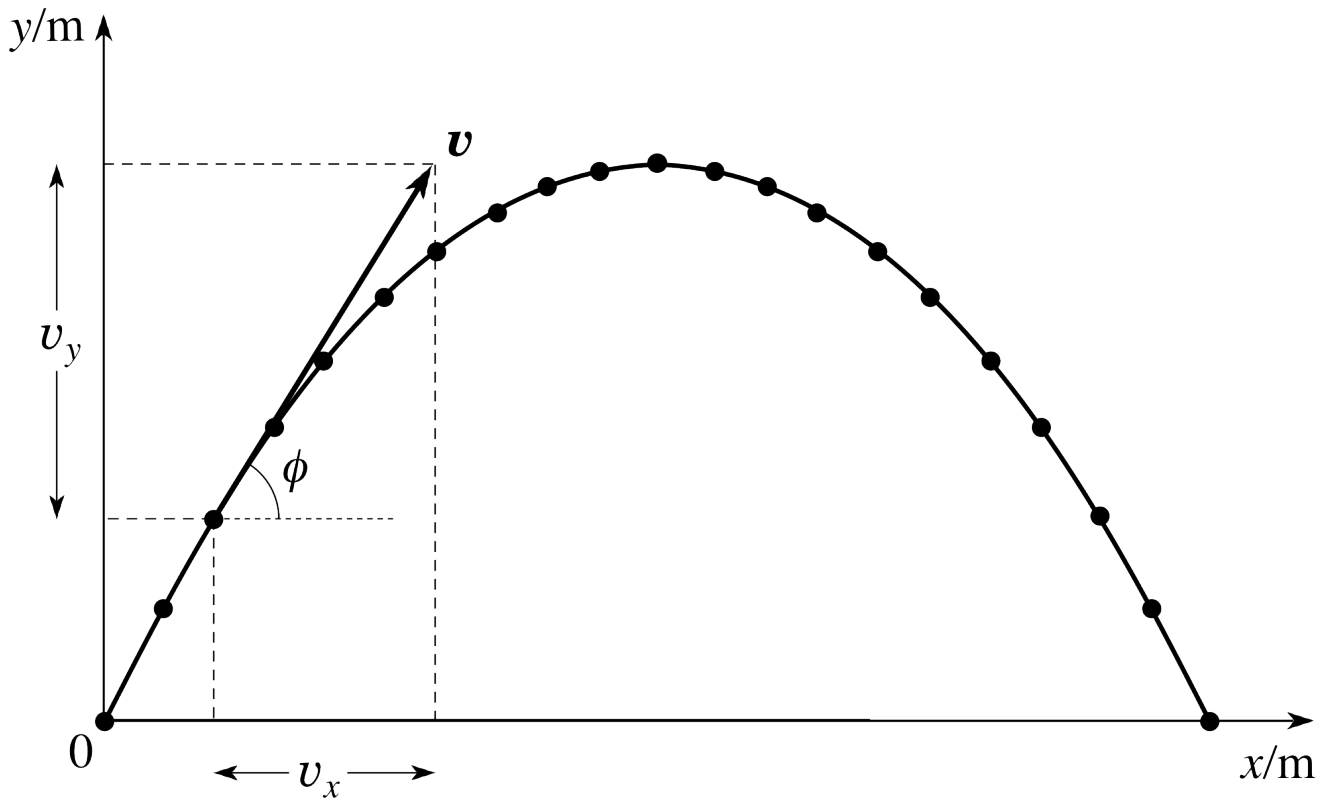
Model-free: Ballistics in the Middle Ages: Trial and Error



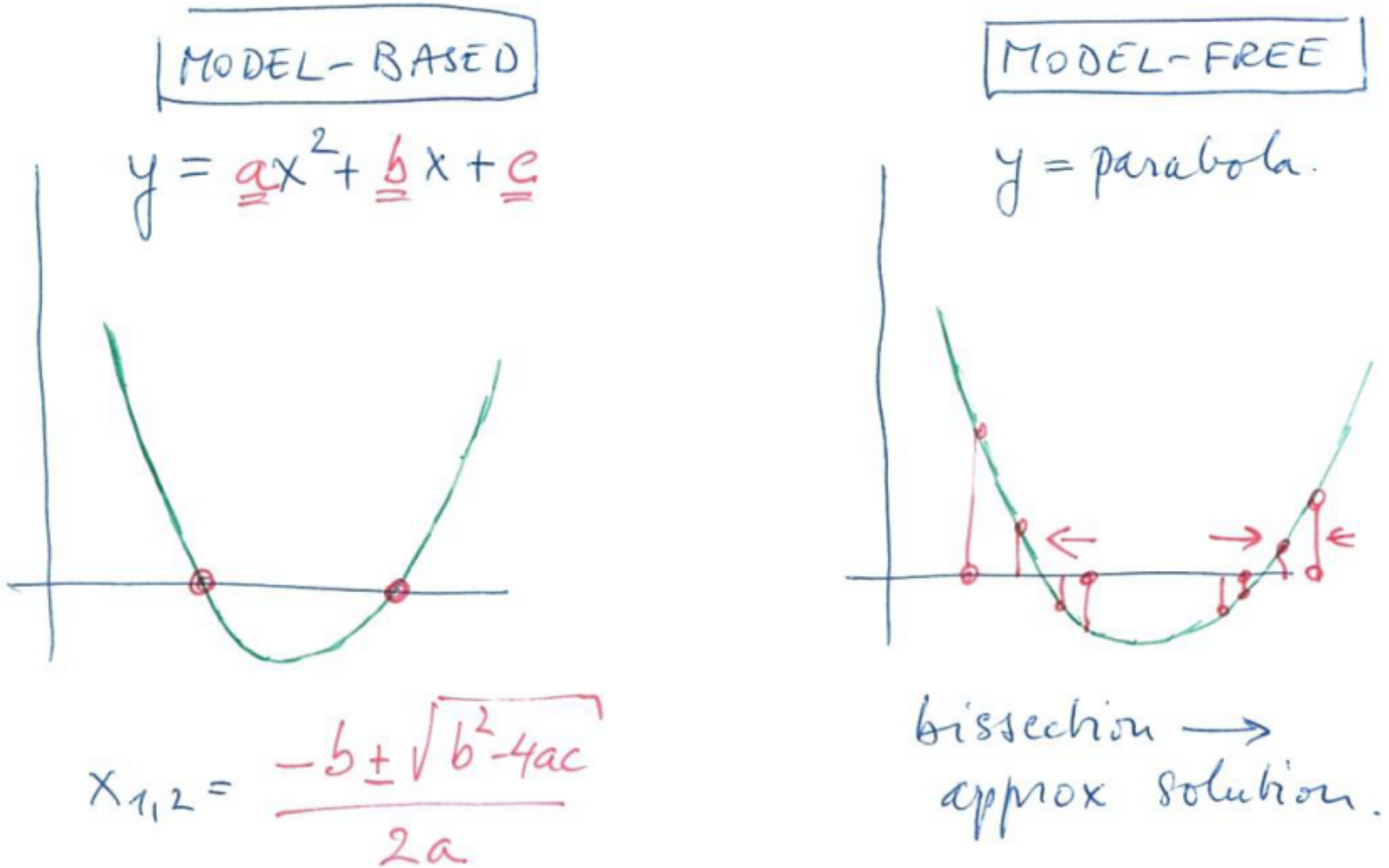
Model-free versus model-based

Model-based:

- Newton's laws for ballistics;
- Friction, Coriolis acceleration, etc.



Model-based vs. Model-free
Analogy: Computing zeros for quadratic equation



Model-based vs model-free

- **Model-based:** the MDP $= (\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)$ is completely specified;
  - Solve the Bellman equations!

- **Model-free:** only **direct experience**, i.e. sample paths (states, actions and rewards) are given. Put differently, only experience-based information is given!
  - Random search but Bellman equations allow to propagate values!

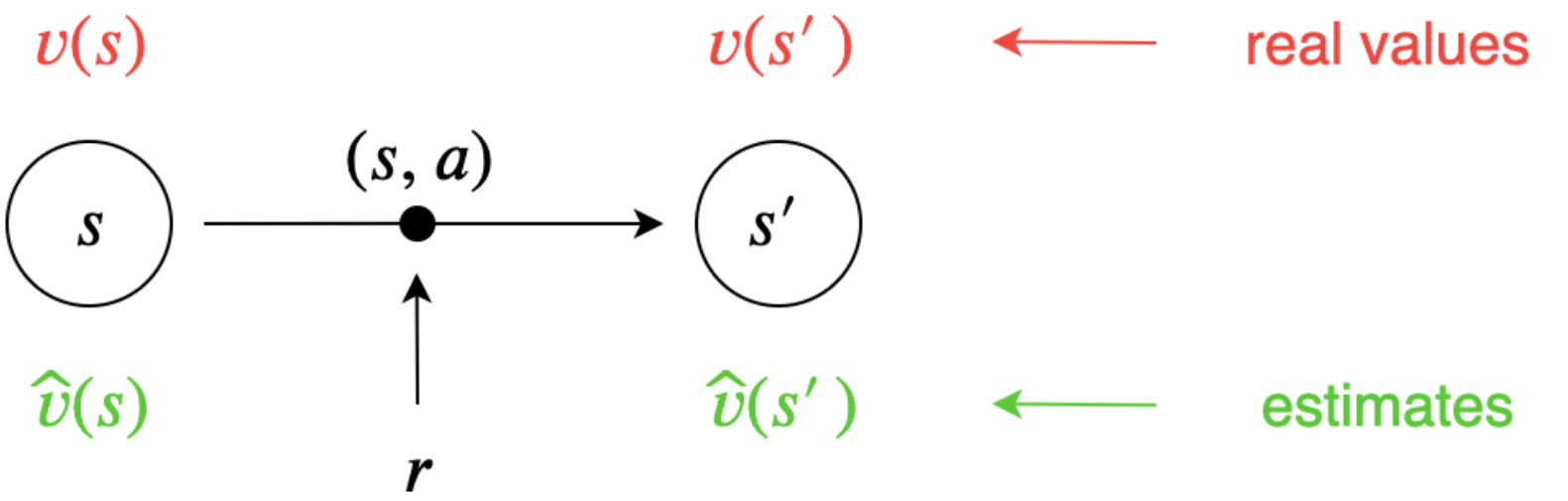
Outline

Solving model-free RL problems

- The agent has **no prior knowledge** about states, actions, rewards, transitions!
- By **acting in the world**, the agent gains experience.
  An experience can be expressed as a 4-tuple: (s, a, r, s')
- Over time the agent collects a list of experiences that he uses to find value functions (and corresponding policy) . . . HOW?
- **Key observation:** Bellman equations link values in neighbouring states along paths!

Temporal differencing (TD): Use Bellman eqs to propagate values!

Temporal-differencing: Exploiting Bellman eqs.



time t

$\hat{v}(s)$

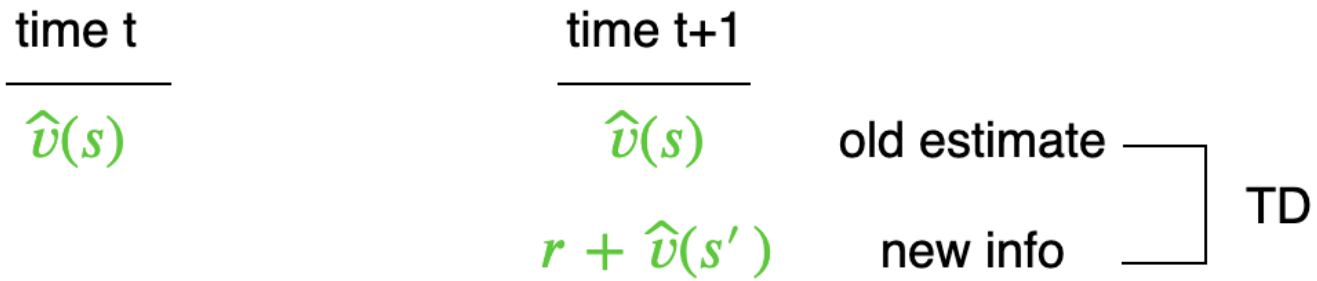
time t+1

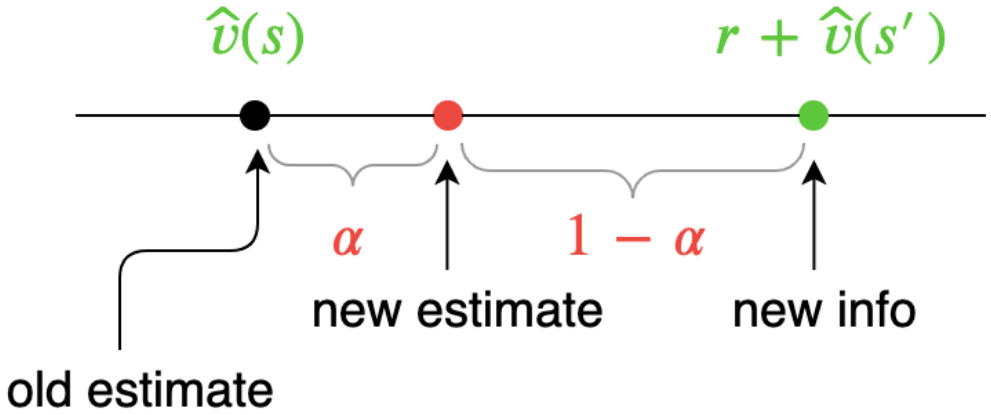
$\hat{v}(s)$ old estimate

$r + \hat{v}(s')$ new info

TD

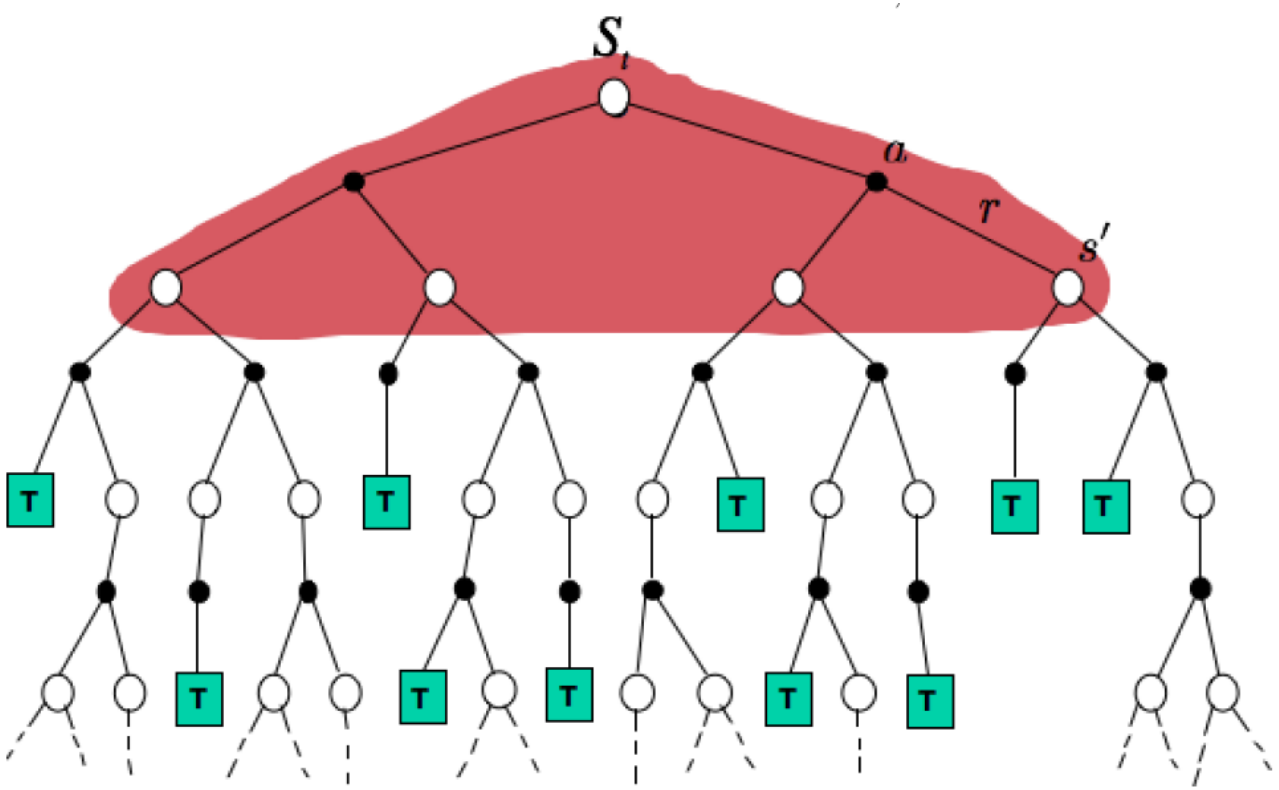
Temporal-differencing: Exploiting Bellman eqs.





$$\hat{v}_{\text{new}}(s) = (1 - \alpha)\hat{v}_{old}(s) + \alpha\left(r + \hat{v}(s')\right)$$

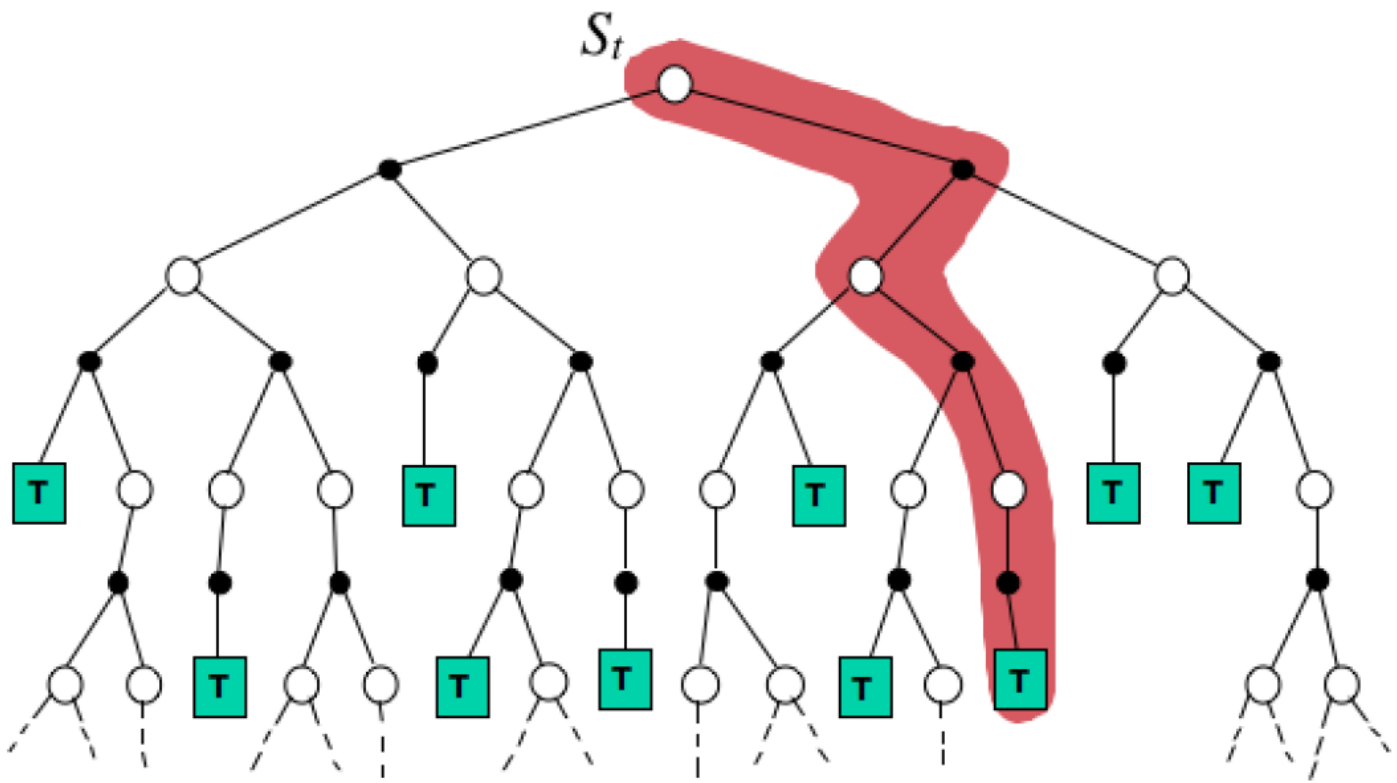
Dynamic Programming (DP): Backup diagram



Monte-Carlo (MC): Backup diagram

Update rule: $V(S_t) \leftarrow (1-\alpha)V(S_t) + \alpha G_t$

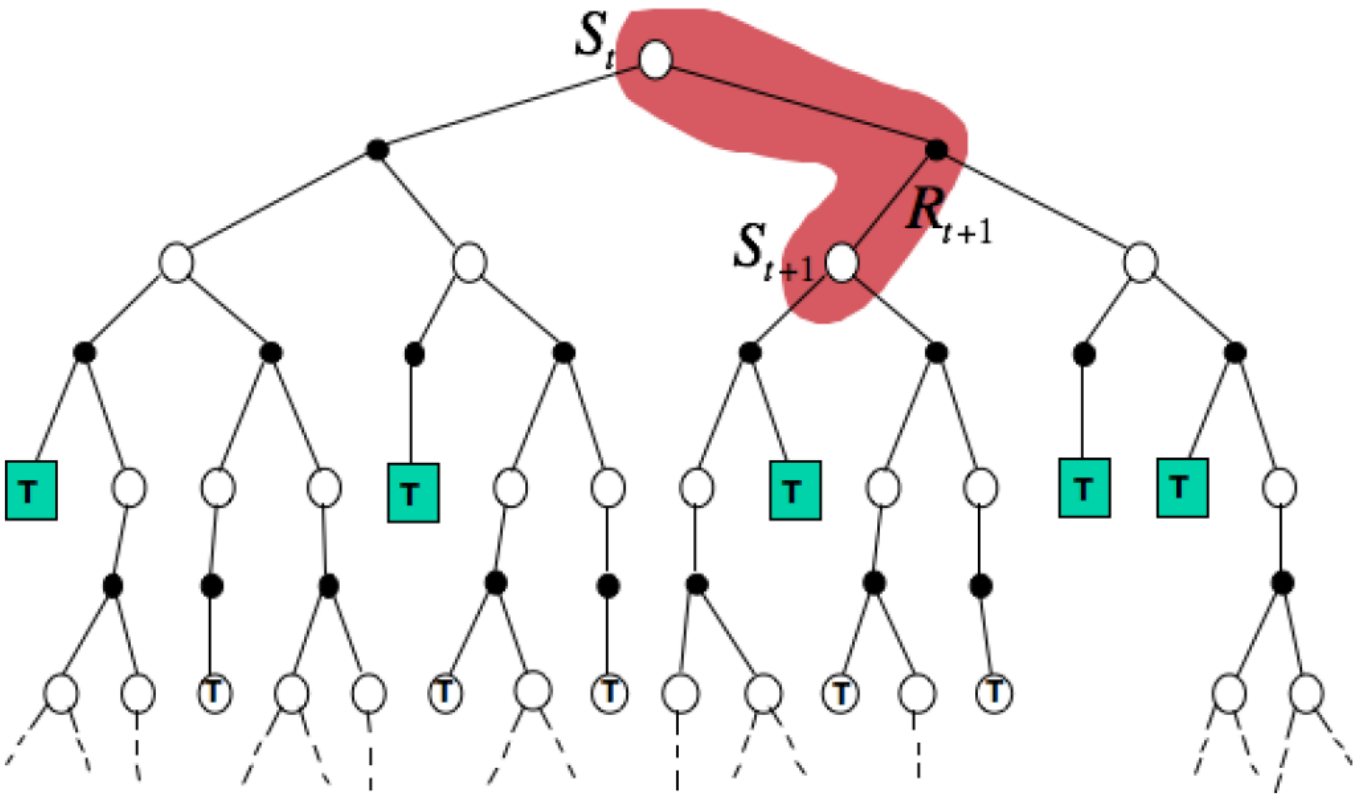
$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t - V(S_t)\right)$$



Temporal Difference (TD): Backup diagram

Update rule: $V(S_t) \leftarrow (1-\alpha)V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}))$

$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$



Caveats when addressing model-free RL problems

1. **We need to compute $q(s, a)$ rather than $v(s)$**
   - To **improve a policy**, we need to know for each state s which action a results in the highest $q(s, a)$:

     construct $\pi : s \mapsto a_{max}$ where $q(s, a_{max}) = \max_{a'} q(s, a')$

     - **Model-based:** $q(s, a)$ can be computed from $v(s)$:

       $$q(s, a) = r(s, a, s') + v(s')$$

     - **Model-free:** $q(s, a)$ needs to be estimated explicitly!
2. **Keep exploring!**
   - Balance exploration versus exploitation
   - E.g. ϵ-greedy, soft-max, etc.

Applying TD to model-free RL problems

Recall: Model-free, hence focus on value function $q(s, a)$

1. **SARSA**: Evaluating a given policy π
   - Bellman: $q_\pi(s, a) = r(s, a, s') + \sum_{a'} \pi(a' \mid s') q_\pi(s', a')$
   - SARSA update rule (sample-based):

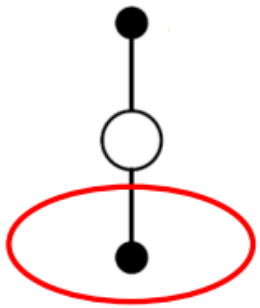
$$q(s, a) \leftarrow (1 - \alpha) q_\pi(s, a) + \alpha(r(s, a, s') + q(s', a'))$$

2. **Q-learning:** Estimating the **optimal** value function $q^*(s, a)$
   - Bellman: $q^*(s, a) = r(s, a, s') + \max_{a'} q^*(s', a')$
   - Q-learning update rule (sample-based):

$$q^*(s_t, a_t) \leftarrow (1 - \alpha) q^*(s_t, a_t) + \alpha(r(s, a, s') + \max_{a'} q^*(s', a'))$$

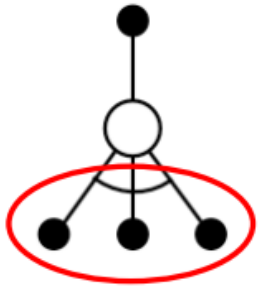
SARSA vs. Q-learning: On-Policy vs. Off-Policy

Sarsa:



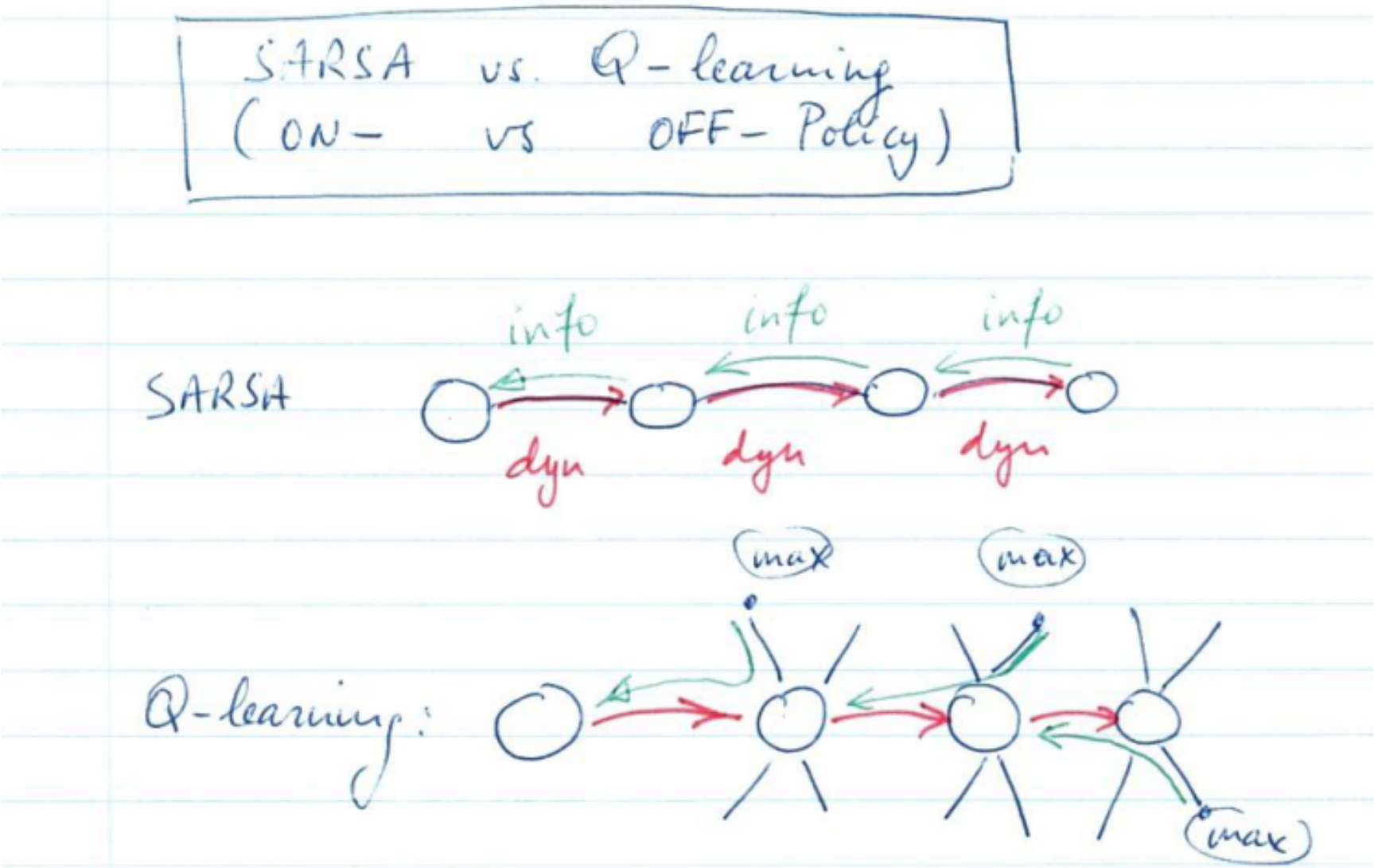
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

Q-learning:



$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha\left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)\right]$$

SARSA (on-policy) vs. Q-Learning (off-policy)



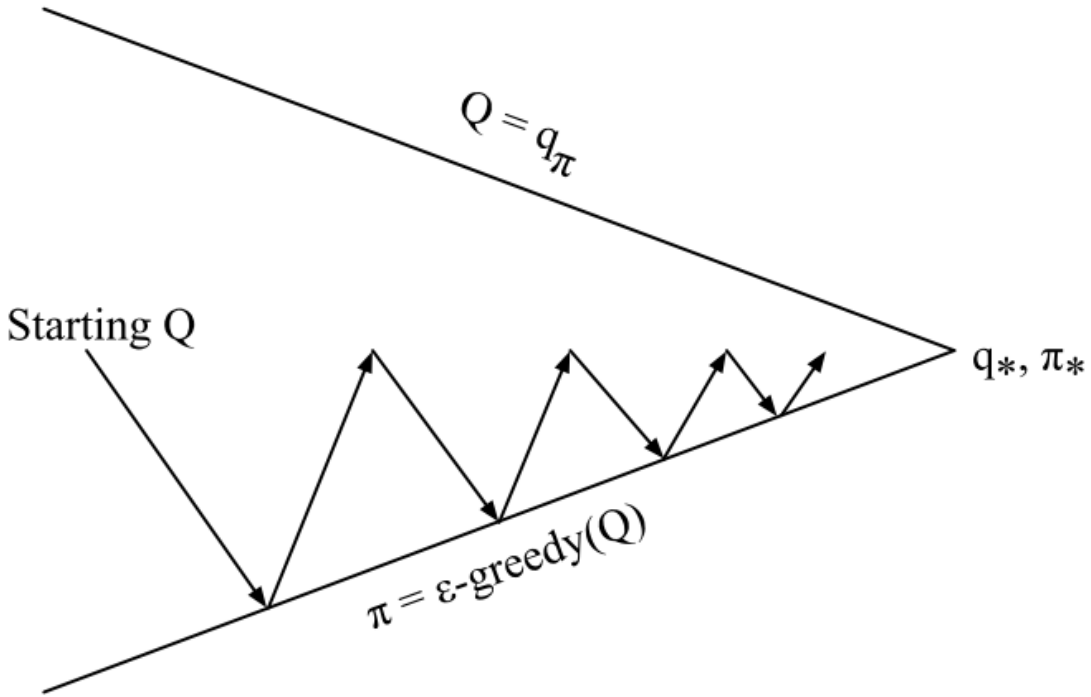
SARSA: Pseudo-code for model-free control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Initialize $Q(s,a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ϵ-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ϵ-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma Q(S',A') - Q(S,A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

SARSA model-free control: Schematically

SARSA only evaluates the value function q_π for a given policy π, then use policy improvement to find a better policy. Repeat!



Every time-step:

Policy evaluation Sarsa, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

Q-learning: Pseudo-code for off-policy TD control

Make TD off-policy: bootstrap with best action, not actual action:

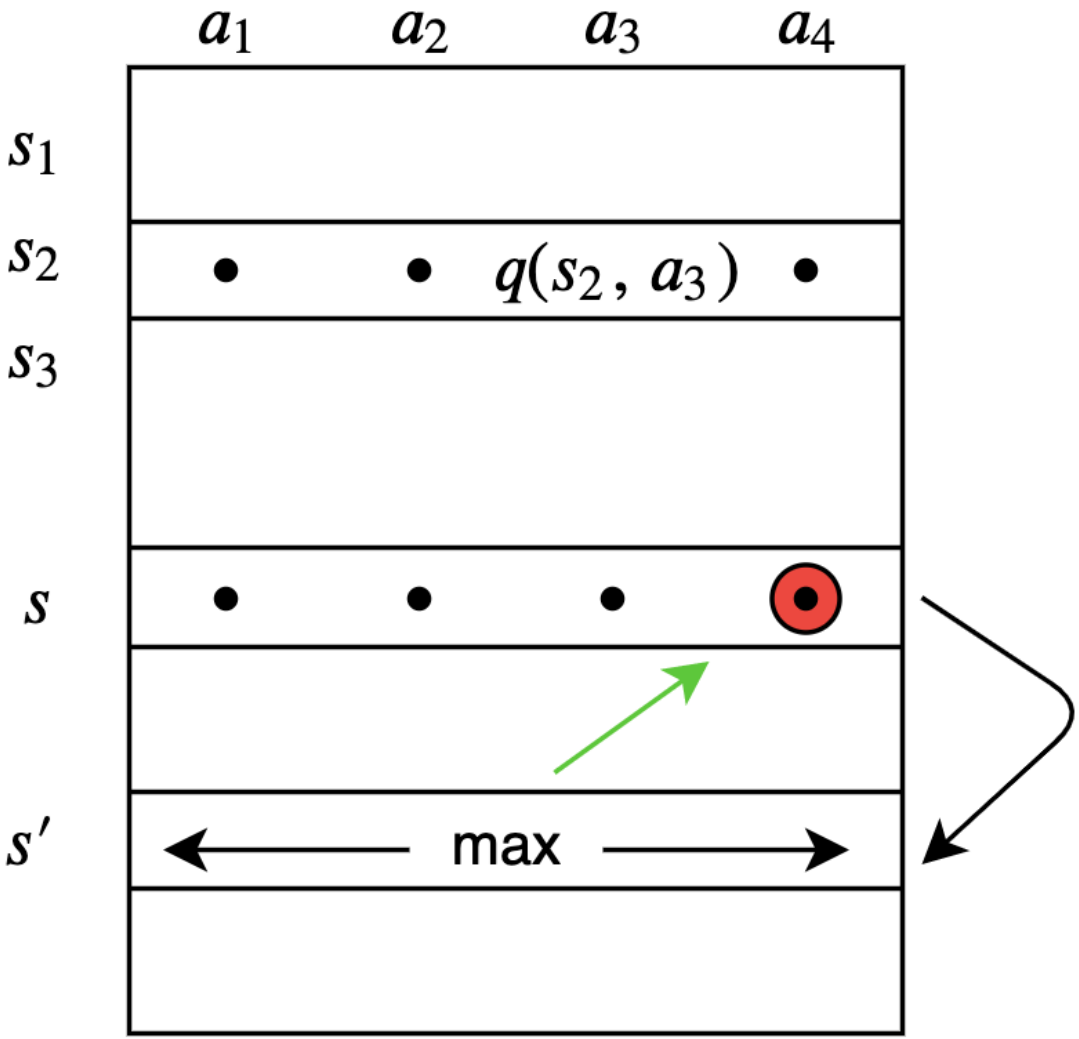
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state},$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ϵ-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal

Outline

Coding excursion 4: Implement SARSA and Q-Learning



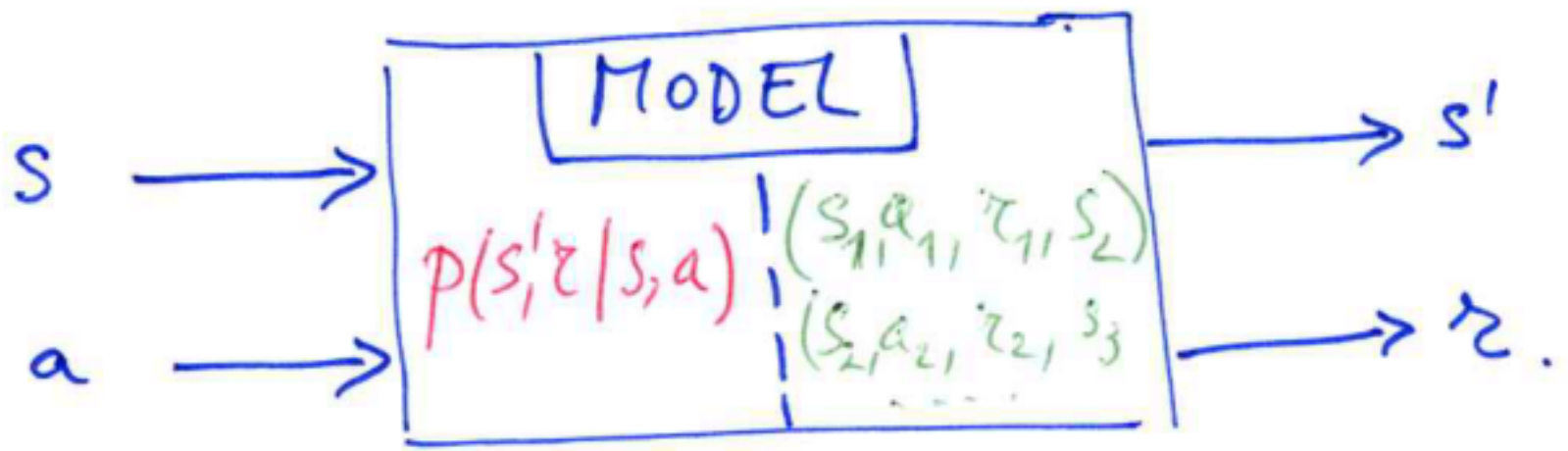
Outline

Dyna-Q; Integrating planning and learning

Model: tells agent what will happen next ...

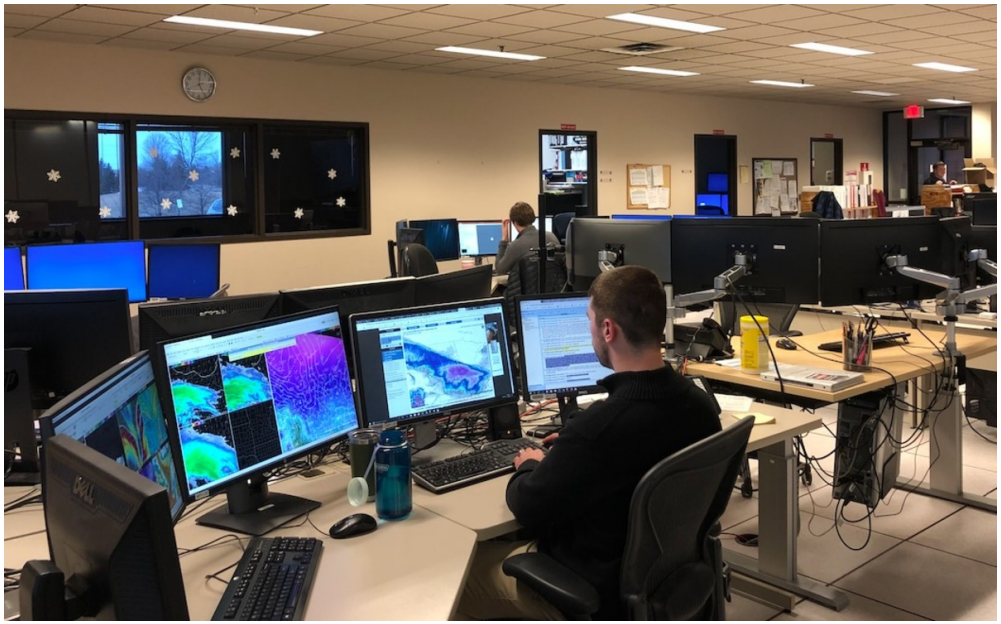
- **Model-based**: planning
- **Model-free**: learning

Distributional vs. Sample-based Model



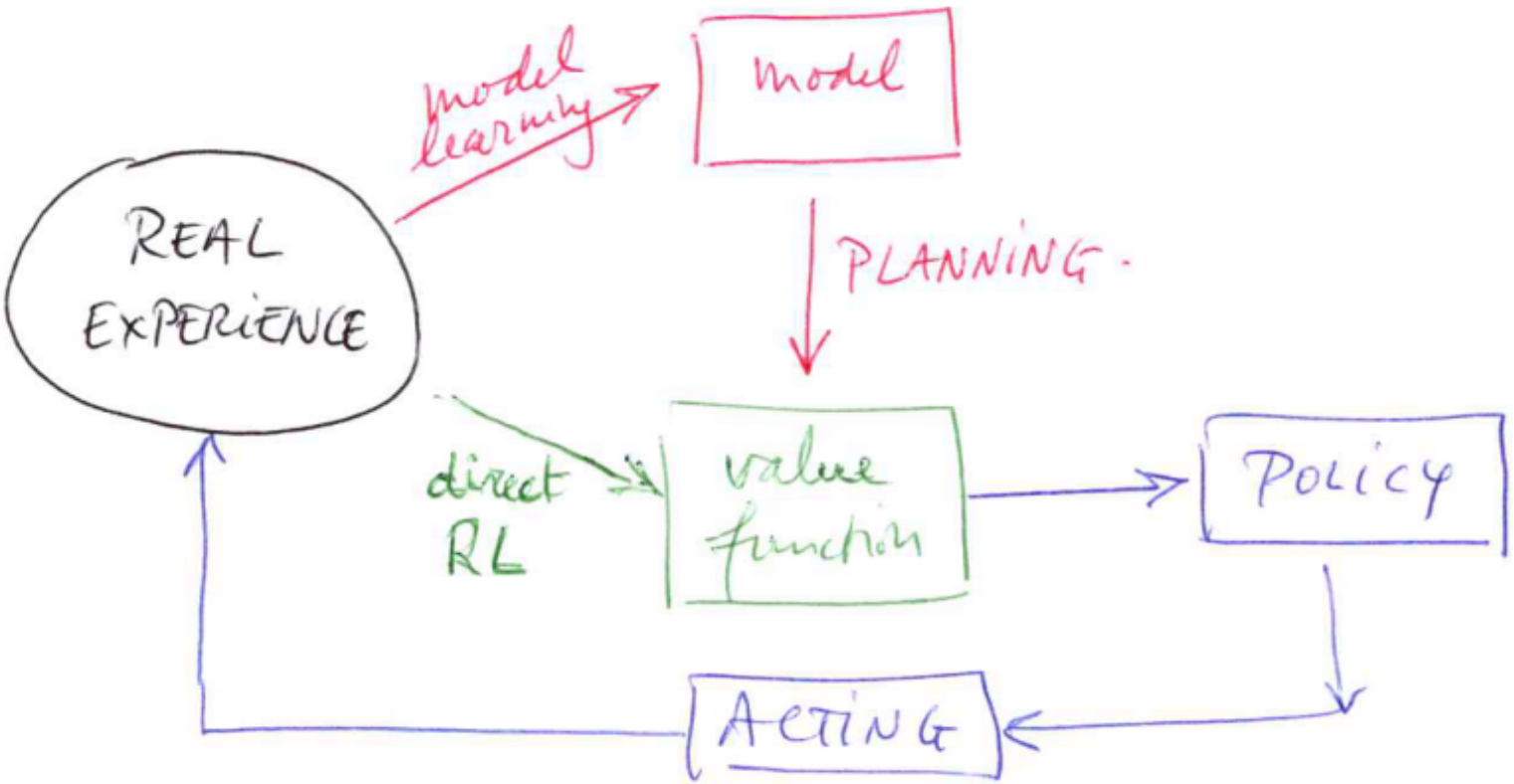
Model-based learning

- Until now:Model = fully specified MDP!
- More generally: anything that helps the agent to plan:
- More accurate models are more effective (myths vs. science):
  - Folklore and weather saying:
    wet and windy May fills the barn with corn and hay.
  - Meteorological models running on supercomputer



Dyna-Q; Integrating planning and learning

Real experience can be used in two ways:



DYNA-Q: Algorithm

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

Do forever:

(a) $S \leftarrow$ current (nonterminal) state

(b) $A \leftarrow \varepsilon$-greedy(S,Q)

(c) Execute action A; observe resultant reward, R, and state, S'

(d) $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

(e) $Model(S,A) \leftarrow R, S'$ (assuming deterministic environment)

(f) Repeat n times:

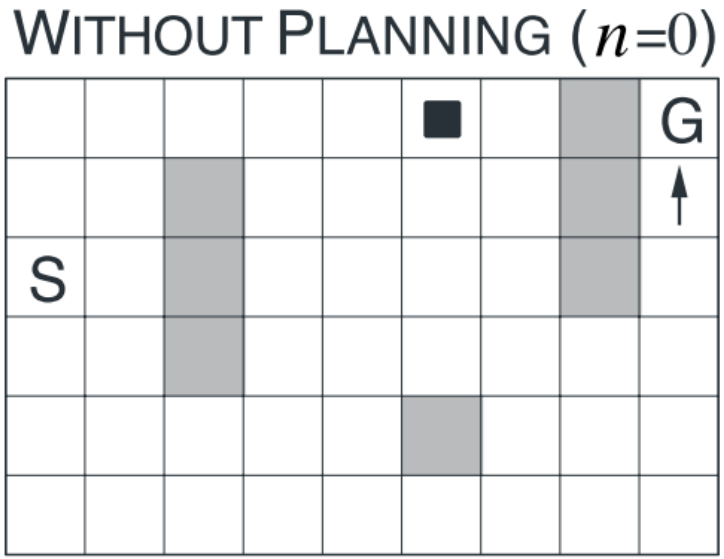
$S \leftarrow$ random previously observed state

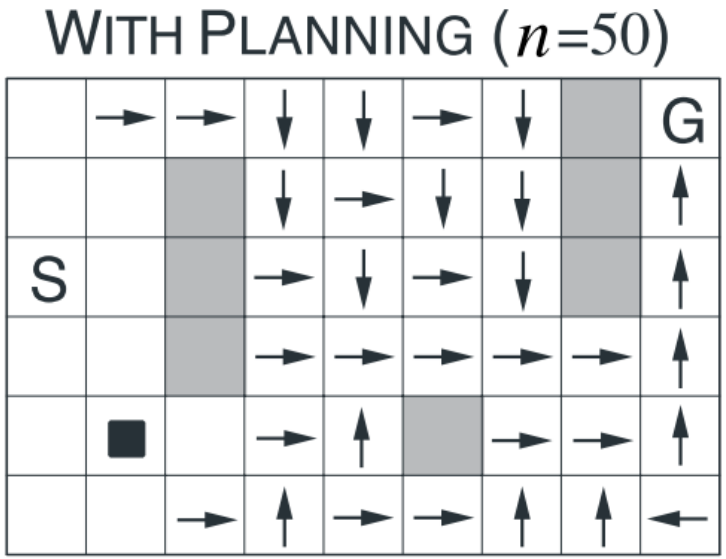
$A \leftarrow$ random action previously taken in S

$R, S' \leftarrow Model(S,A)$

$Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

DYNA-Q: Maze example

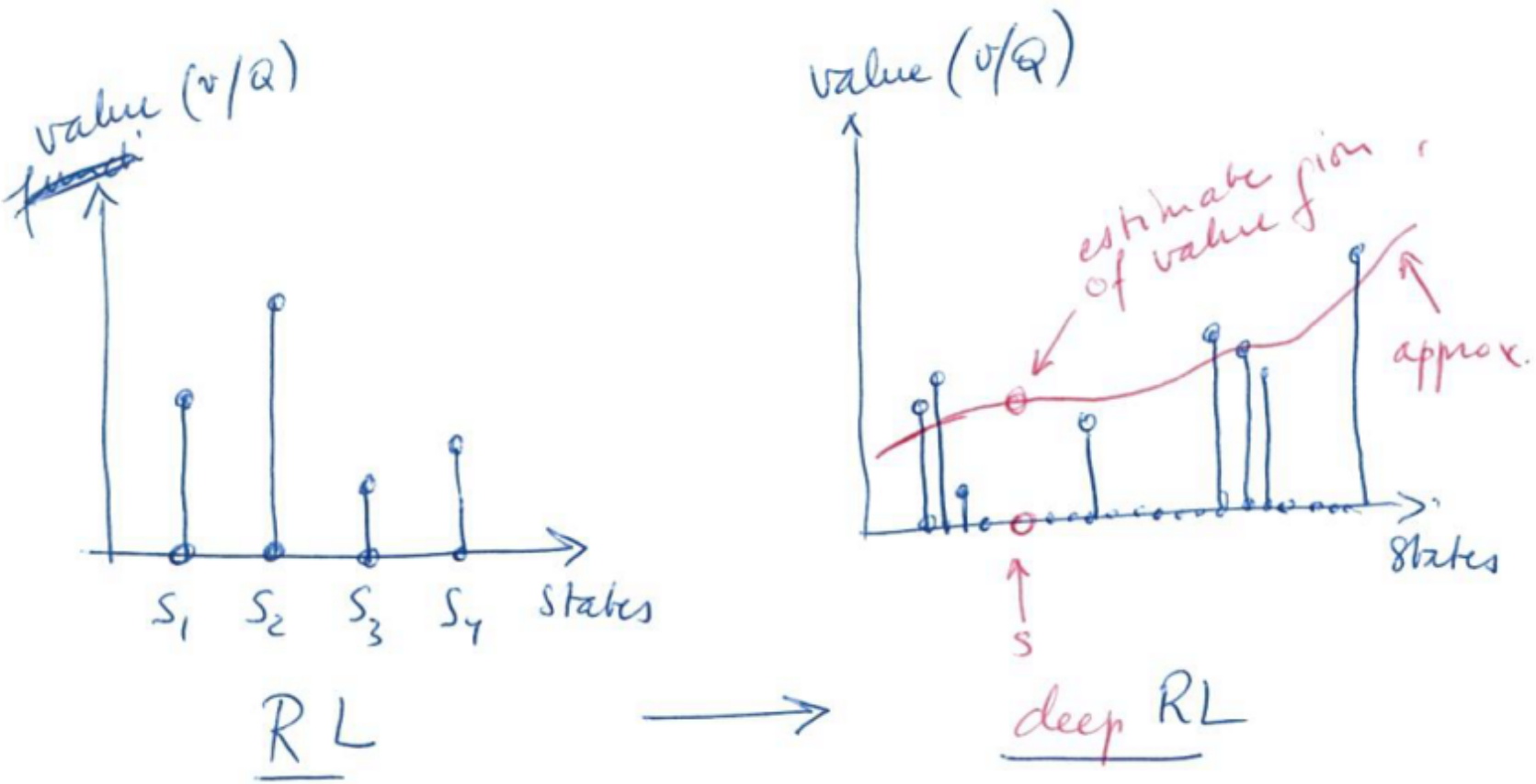




Outline

Deep RL

- What if number of states or actions is **huge**!
- Use function approximation (i.e. generalisation) to estimate value functions in unseen states;



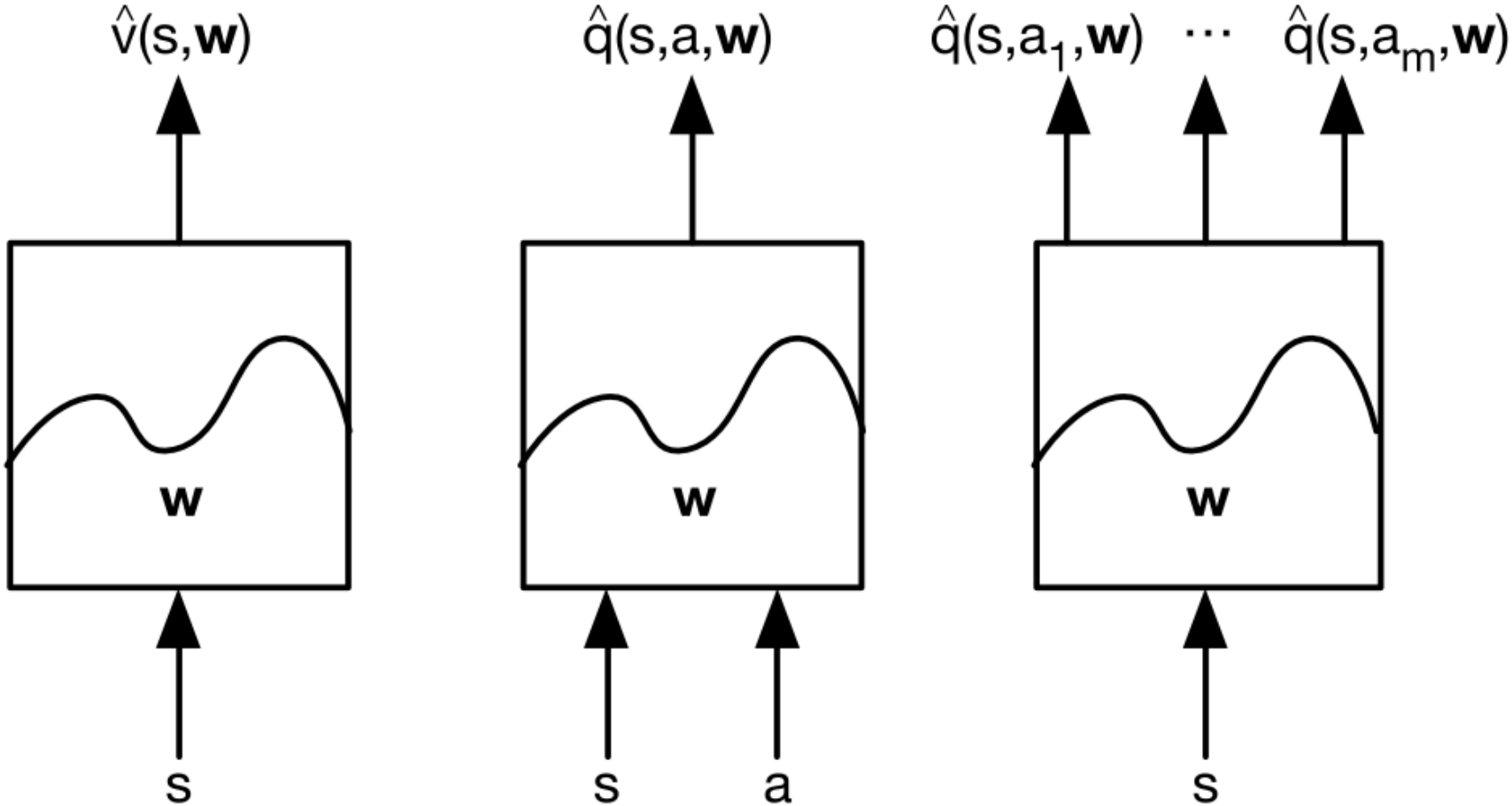
Value Function Approximation for Large RL Problems

- So far we have represented value function by a *lookup table*
  - Every state s has an entry $V(s)$
  - Or every state-action pair s, a has an entry $Q(s, a)$
- Problem with large MDPs:
  - There are too many states and/or actions to store in memory
  - It is too slow to learn the value of each state individually
- Solution for large MDPs:
  - Estimate value function with *function approximation*

$$\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$$
$$\text{or } \hat{q}(s, a, \mathbf{w}) \approx q_\pi(s, a)$$

  - *Generalise* from seen states to unseen states
  - *Update* parameter $\mathbf{w}$ using MC or TD learning

Types of Value Function Approximation



Which function approximation?

Plenty of choice:

- Linear combinations of features (basis functions)
- Neural networks (ANNs): universal function approximator;
- Miscellaneous techniques: e.g. decision or regression trees,
- etc. ...

Gradient-based solution methods:

use differentiable approximators

Stochastic Gradient Descent

- Goal: find parameter vector $\mathbf{w}$ minimising mean-squared error between approximate value fn $\hat{v}(s, \mathbf{w})$ and true value fn $v_\pi(s)$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$$

- Gradient descent finds a local minimum

$$\begin{aligned} \Delta \mathbf{w} &= -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \\ &= \alpha \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \right] \end{aligned}$$

- Stochastic gradient descent *samples* the gradient

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- Expected update is equal to full gradient update

Incremental Prediction Algorithms

- Have assumed true value function $v_\pi(s)$ given by supervisor
- But in RL there is no supervisor, only rewards
- In practice, we substitute a *target* for $v_\pi(s)$
  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

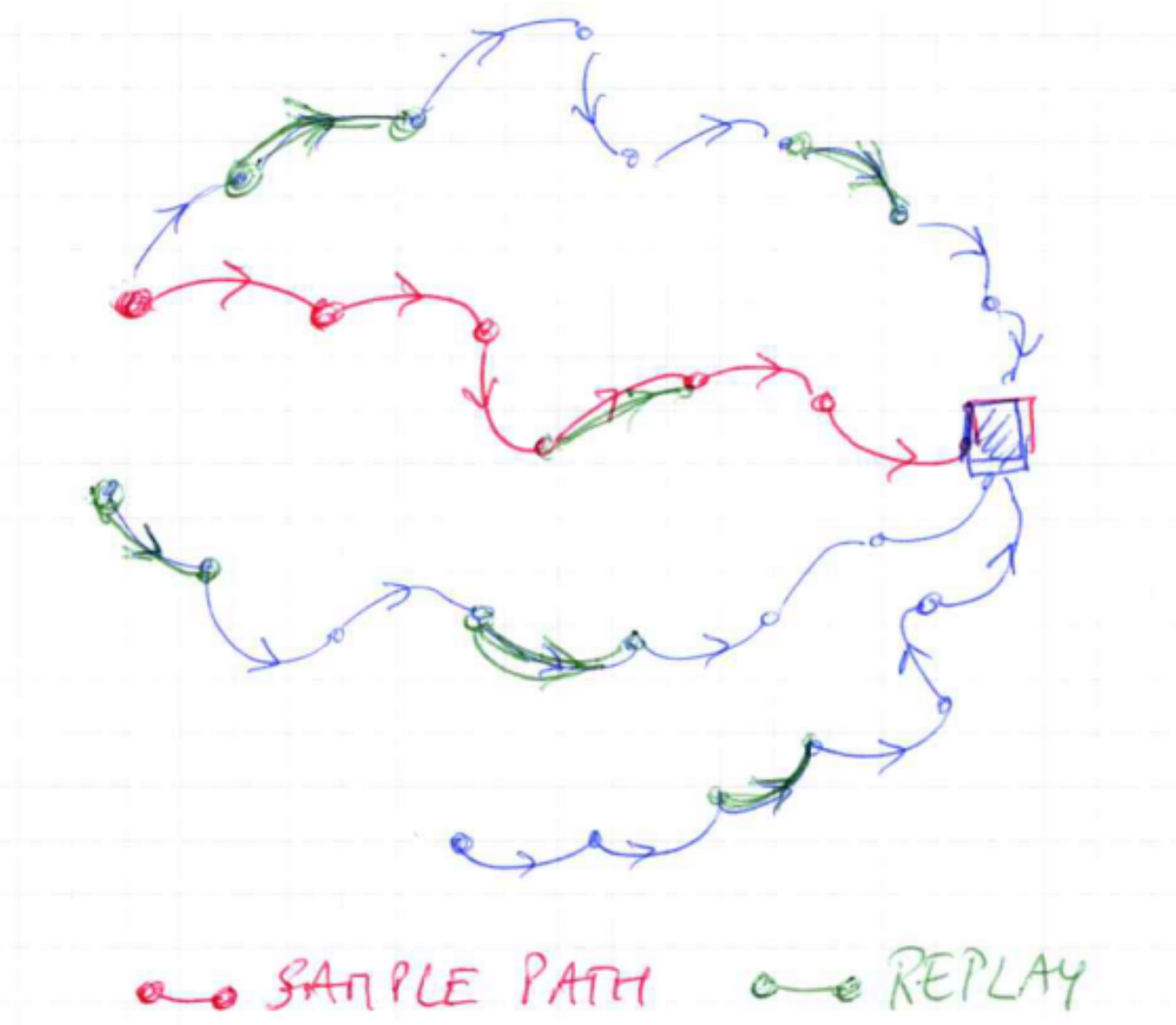
  - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

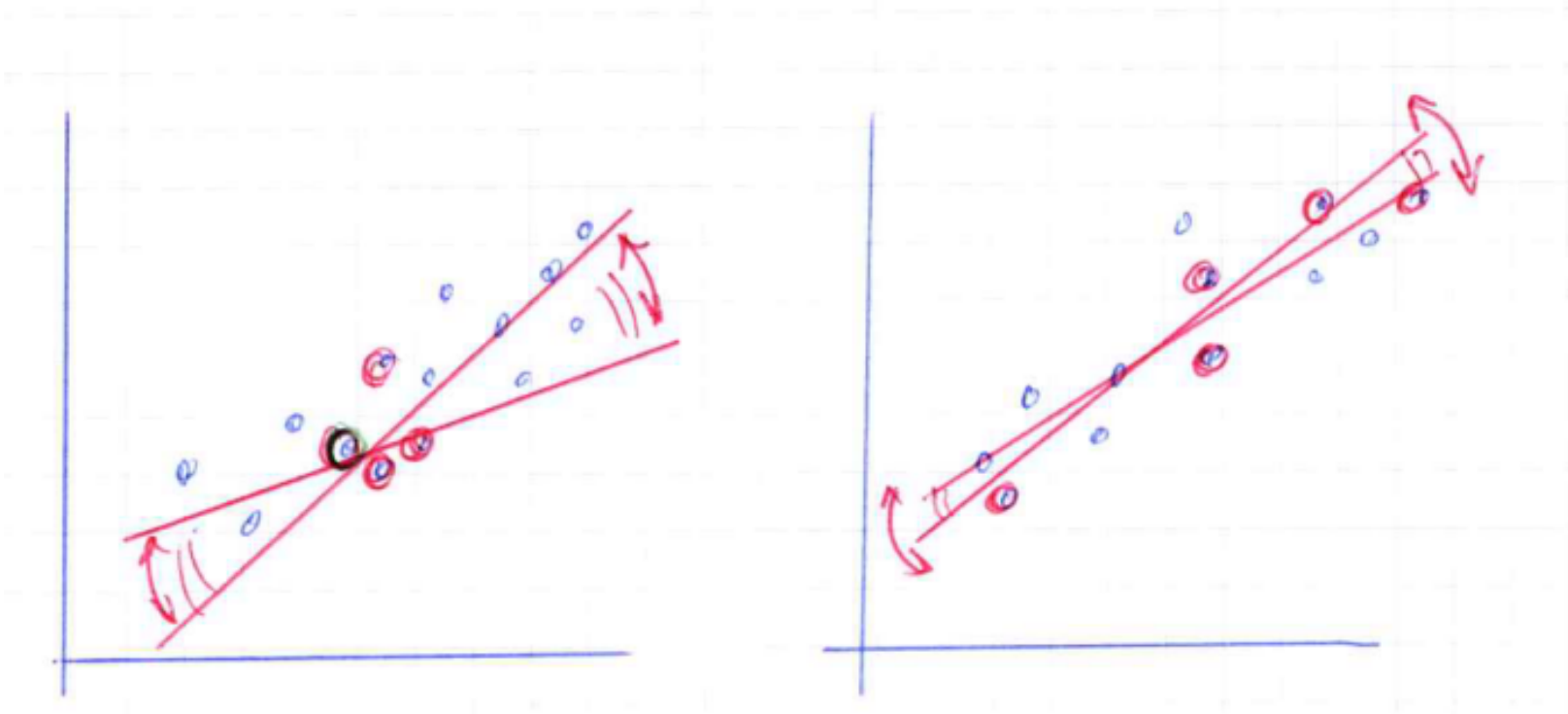
Batch methods and Experience Replay

- Standard Q-learning: Data are NOT used optimally:
  - sample paths are not re-used;
  - values along sample paths are strongly correlated (unstable!)!
- Batch methods seek to find best-fitting solution
- given all of the agent's experience (experience replay)

Sample path versus experience replay



Correlated vs. uncorrelated



DQN: Experience replay in Deep Q-Networks

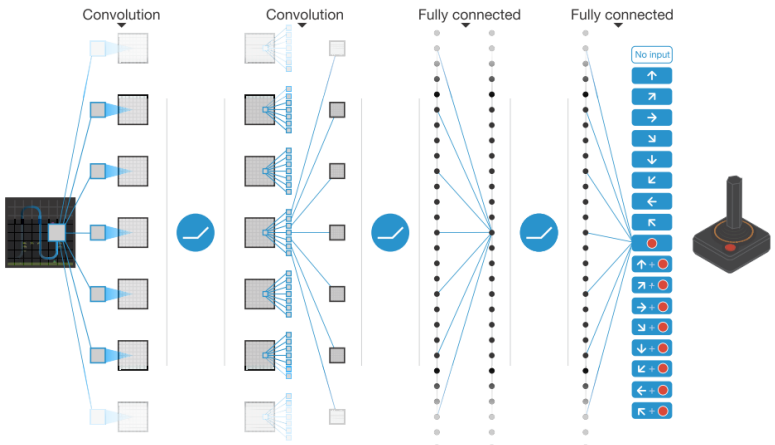
- DQN uses experience replay and fixed Q-targets
- Store transition (s_t,a_t,r_{t+1},s_{t+1}) in replay memory D
- Sample random mini-batch of transitions (s,a,r,s′) from D
- Compute Q-learning targets w.r.t. old, fixed parameters w−

- Optimize MSE between Q-network and Q-learning targets

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i}\left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a'; w_i^-)}_{\text{Q-learning target}} - \underbrace{Q(s, a; w_i)}_{\text{Q-network}}\right)^2\right]$$

DQN: Deep Q-Network, *(Mnih et al, Nature 2015)*

- Single RL architecture achieved (super)human performance on suite of classic ATARI games;



- **Key features and improvements:**
  - Function approximation based on deep NN;
  - Experience replay (data re-use and de-correlation)
  - Use of target network for stabilisation

DQN: Deep Q-Network, *(Mnih et al, Nature 2015)*

- **Loss function**

$$L_i(\theta_i) = \mathbb{E}_{(s,a,r,s') \sim \mathrm{U}(D)} \left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i) \right)^2 \right]$$

- **Gradient**

$$\nabla_{\theta_i} L(\theta_i) = \mathbb{E}_{s,a,r,s'} \left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i) \right) \nabla_{\theta_i} Q(s, a; \theta_i) \right].$$

DQN Pseudo-code

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$

For episode $= 1, M$ **do**

Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1$,T **do**

With probability ε select a random action a_t

otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

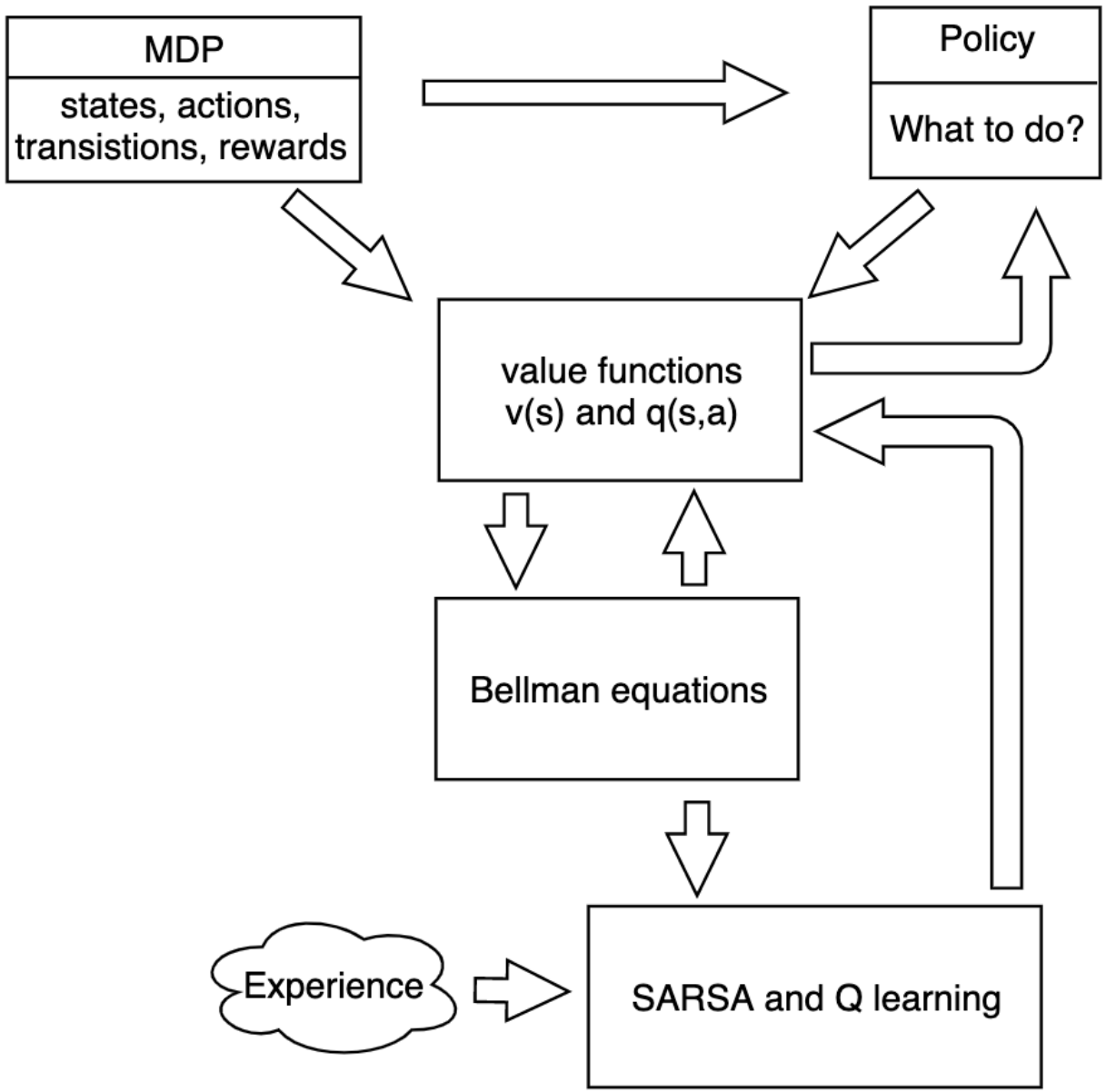
Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^{-}) & \text{otherwise} \end{cases}$

Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ

Every C steps reset $\hat{Q} = Q$

Outline

Summary


MDP
states, actions, transistions, rewards
Policy
What to do?
value functions v(s) and q(s,a)
Bellman equations
Experience
SARSA and Q learning

Summary

- RL: learning from sequential interaction
- RL formalism: Markov Decision Process (MDP):
  MDP = states + actions + rewards + transitions ($+\delta$)
- Bellman equations for value functions (internal consistency)
- Model-based (planning) versus model-free (learning)
- Bellman optimality condition
- Gists of Recent Deep RL Algorithms
- etc