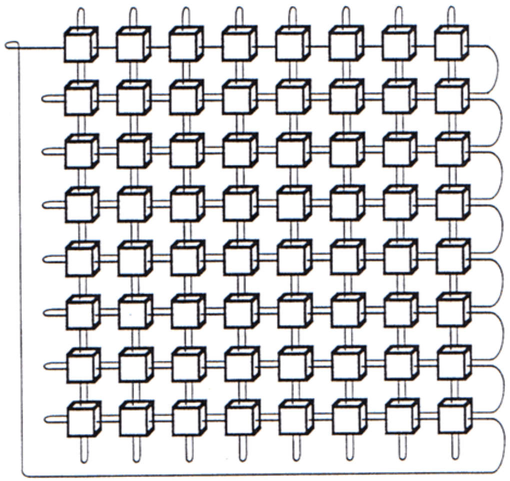
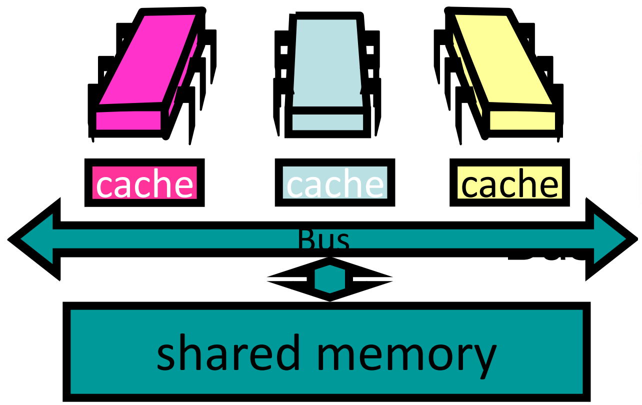
Shared Memory

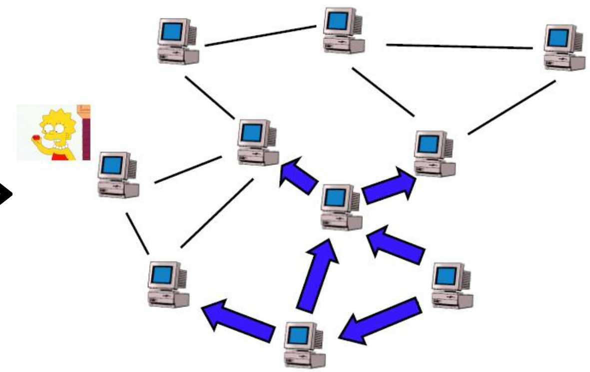
Spectrum of Distributed (Computer) Systems



E.g., graphical processing units (GPUs) and specialized devices, in which large arrays of *simple processors* work in lock-step ("Gleichschritt"), PRAM, ...



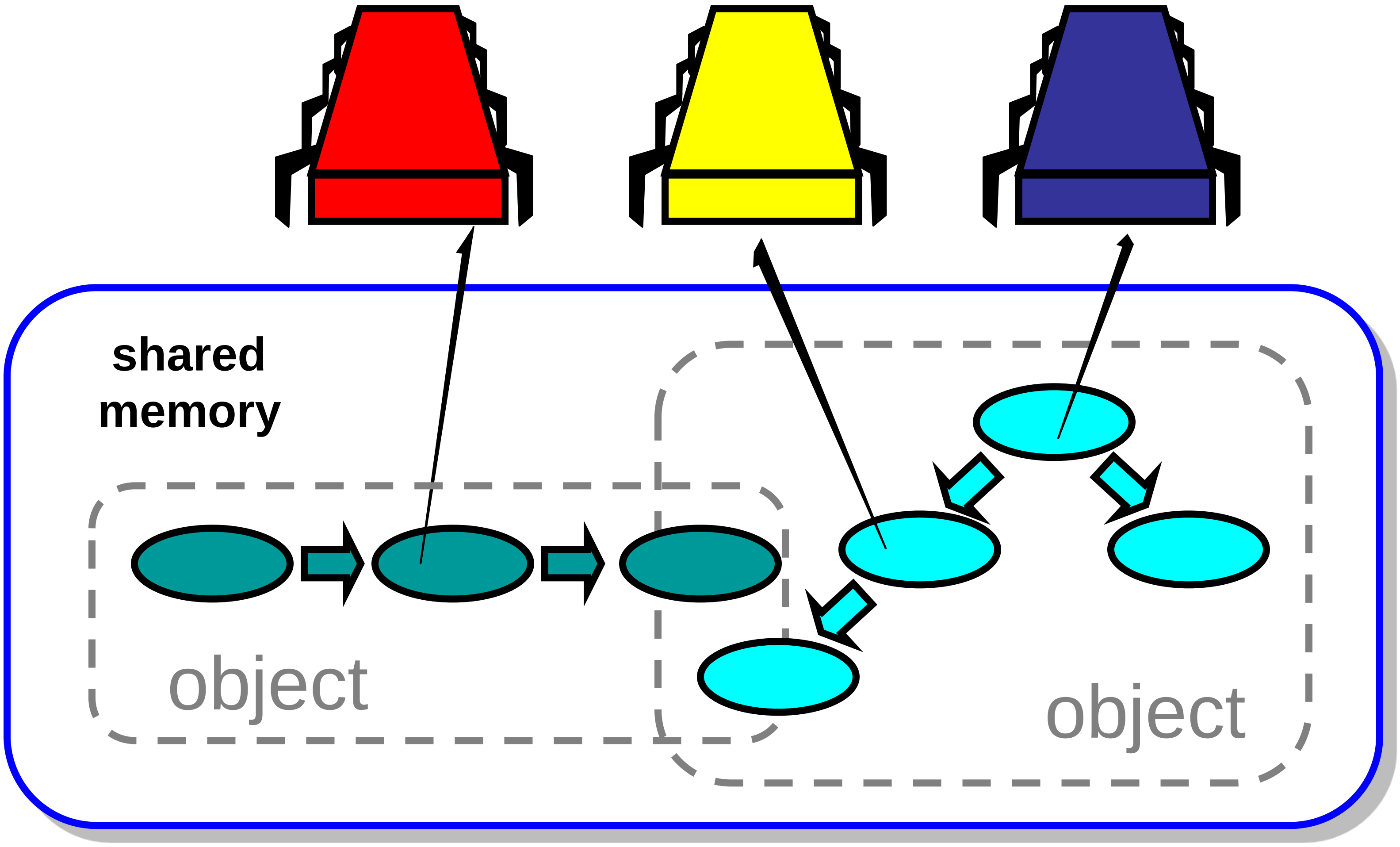
Multi-threaded + multi-core servers/desktops with *shared memory for communication*.



Loosely-coupled *peer-to-peer* systems with message passing communication

small/synchronous/... wide-area/decoupled/...

The Shared Memory Model



Shared memory consists of registers.

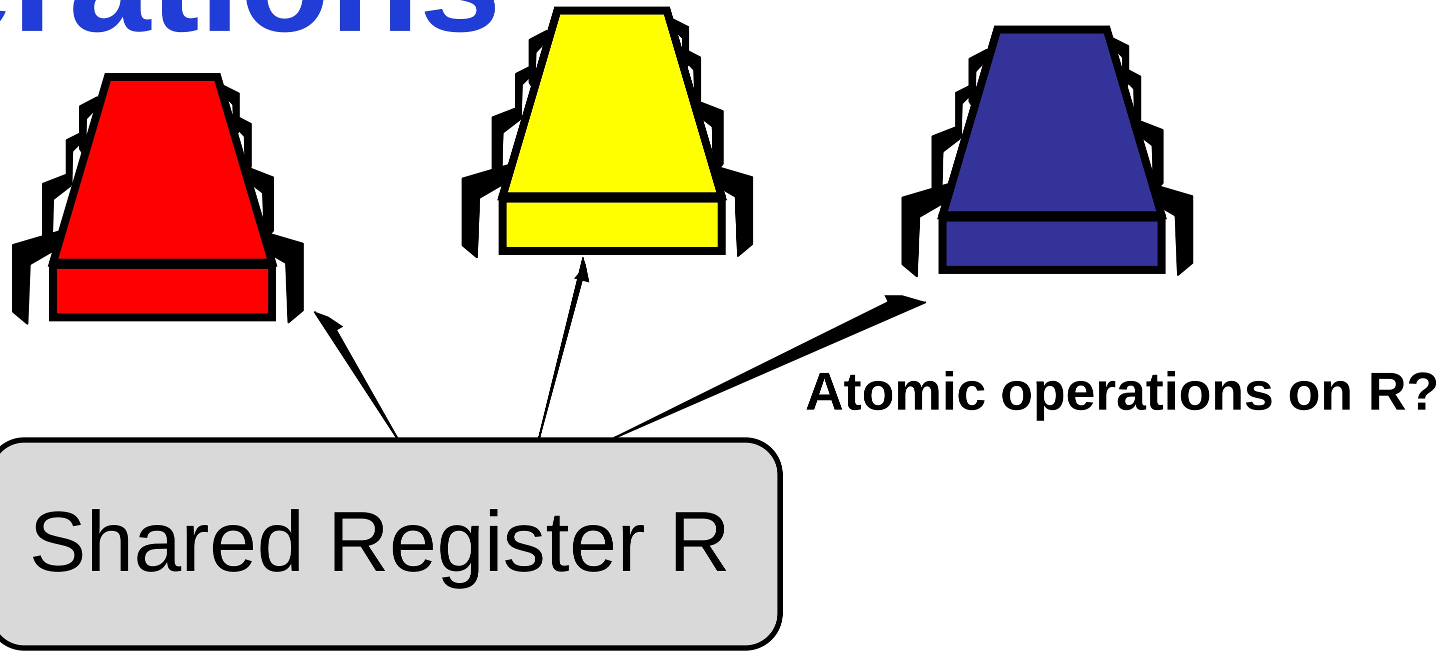
Formal Definition

Shared Memory

A shared memory system is a system that consists of asynchronous processes that access a *common (shared) memory*. A process can atomically access a register in the shared memory through a set of *predefined operations*. An atomic modification appears to the rest of the system instantaneously. Apart from this shared memory, processes can also have some *local (private) memory*.

Often a useful, simpler alternative model to reason about distributed systems!

Operations



<u>Examples:</u> (a.k.a. Data Types, Mealy Machines)

(1) Test-and-Set(R): t := R; R := 1; return t

(2) Fetch-and-Add(R; x): t := R; R := R + x; return t

(3) Compare-and-Swap(R; x; y):

if R = x then R := y; return true;

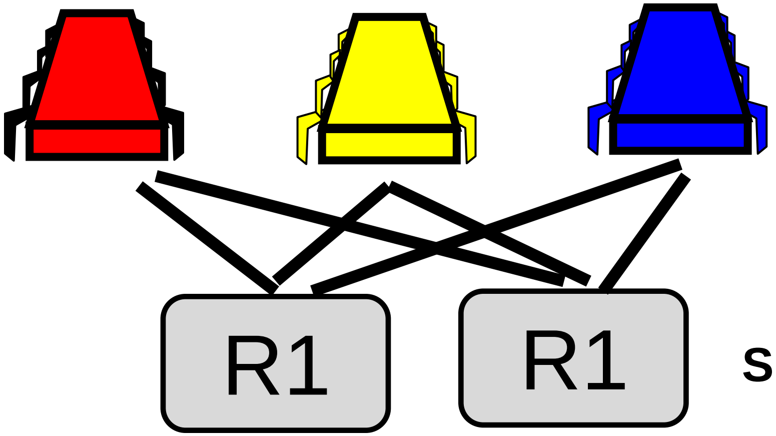
else return false; endif

Why Shared Memory?

- Programming a shared memory system is *easier*: programmers access global variables directly!

- Because of this, even message passing systems often programmed through a *shared memory middleware*!

- From a message passing perspective, shared memory model is like a *bipartite graph*:



Nodes (asynchronous, may fail)

Shared registers (no failures, no delay)

The Power of RMW

The power of a shared memory system is determined by the *Consensus Number* (“universality of consensus".)

Consensus Number

The power of the RMW variant is measured by the consensus number. Consensus number k defines whether one can solve *consensus for k processes* (but not k+1).

Examples:

- *Test-and-Set* has consensus number 2
  (one can solve consensus with 2 processes, but not 3)
- *Compare-and-Swap* has an infinite Consensus Numbers!

Desirable Properties of Distributed Systems

Safety, Liveness

Safety: “Something bad will never happen”,
Examples: some invariant holds
(function never returns *-1* values),
serializability for DB transactions,
linearizability

Liveness: “Eventually something good happens”,
“system makes progress”

Precedes / Follows

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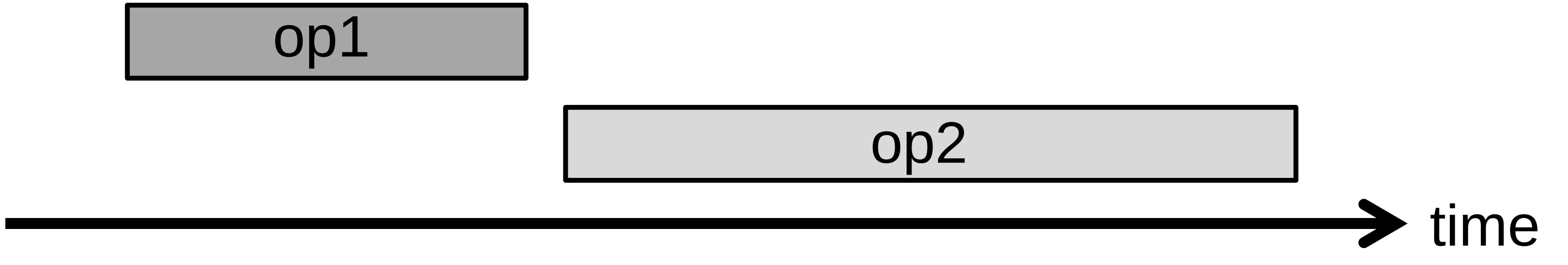
An operation op1 *precedes* an operation op2 iff op1 terminates before op2 starts.

An operation op2 *follows* operation op1 iff op1 precedes.



A Classic Shared Memory Problem

Fundamental *synchronization problem*: access to a resource

Mutual Exclusion

Each process executes the following code sections:

<Entry> → <Critical Section> → <Exit> → <Remaining Code>

A mutual exclusion algorithm consists of code for entry and exit sections, such that the following holds:

(1) **Mutual Exclusion (Property?):** At most one process is in the critical section.

(2) **No deadlock (Property?):** If some process manages to get to the entry section, later some (possibly different) process will get to the critical section.

Sometimes we in addition ask for

(3) **No lockout (Property?):** If some process manages to get to the entry section, later *the same process* will get to the critical section. (“Fairness”)

(4) **Unobstructed exit (Property?):** No process can get stuck in the exit section.

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Mutex

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Repeat:

r := test-and-set(R)

Until r=0

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…

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Set register to 1, then check whether it was so already.

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Correct Mutex?

No deadlock?

No lockout?

Unobstructed exit?

Mutex

(1) Mutex: ?

(2) Deadlock free: ?

(3) Lockout: ?

(4) Unobstructed exit: ?

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(1) Mutex: **ok!**

(2) Deadlock free: ?

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(1) Mutex: R initially 0. Let p_i be the i-th process to successfully execute the test-and-set (i.e., result 0) at time t_i, and say at time t'_i, p_i resets R:=0. Between these times, nobody else can execute CS.

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(2) Deadlock: One of the processes waiting in the entry section will successfully test-and-set as soon as the process in the critical section exited.

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(1) Mutex: **ok!**

(2) Deadlock free: **ok!**

(3) Lockout: ?

(4) Unobstructed exit: **ok!**

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(2) Deadlock: One of the processes waiting in the entry section will successfully test-and-set as soon as the process in the critical section exited.

(4) Exit: Since the exit section only consists of a single instruction (no potential infinite loops) we have unobstructed exit.

Lockout

Test-and-Set(R): t := R; R := 1; return t

May be *unfair*!

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<Remaining Code>

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Always same process may win!

Solution?

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Solution: make FIFO queue…

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What about weaker objects?

Can I do without atomic RMW?

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What about weaker objects?

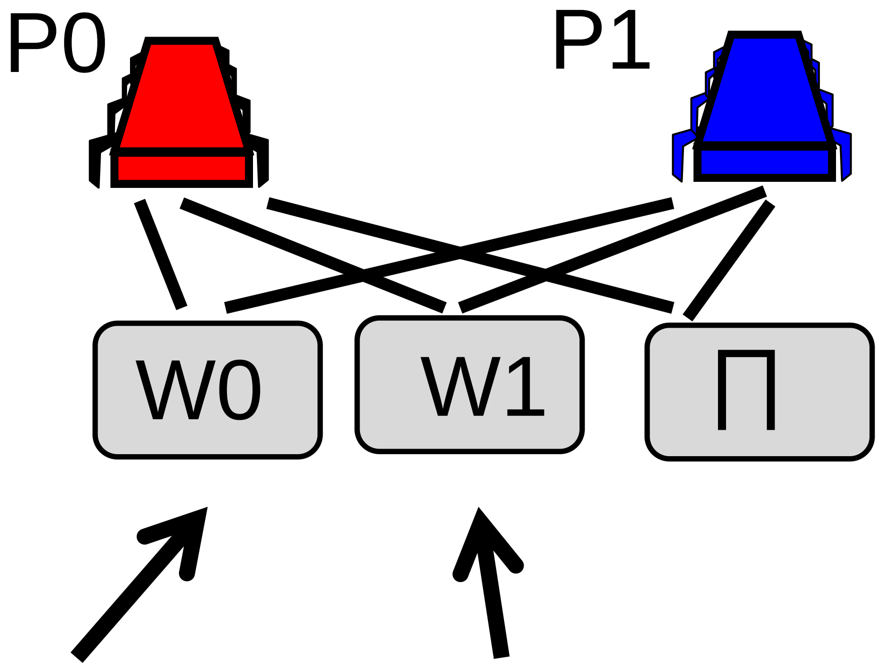
Can I do without atomic RMW?

Yes: Peterson's algorithm!

Peterson's Algo

Assume: *two processes* only!

Need *three* registers (init: 0).



"P0 wants CS"
Written only by P0.
Read by both.

"P1 wants CS"
Written only by P1.
Read by both.

Peterson's Algo

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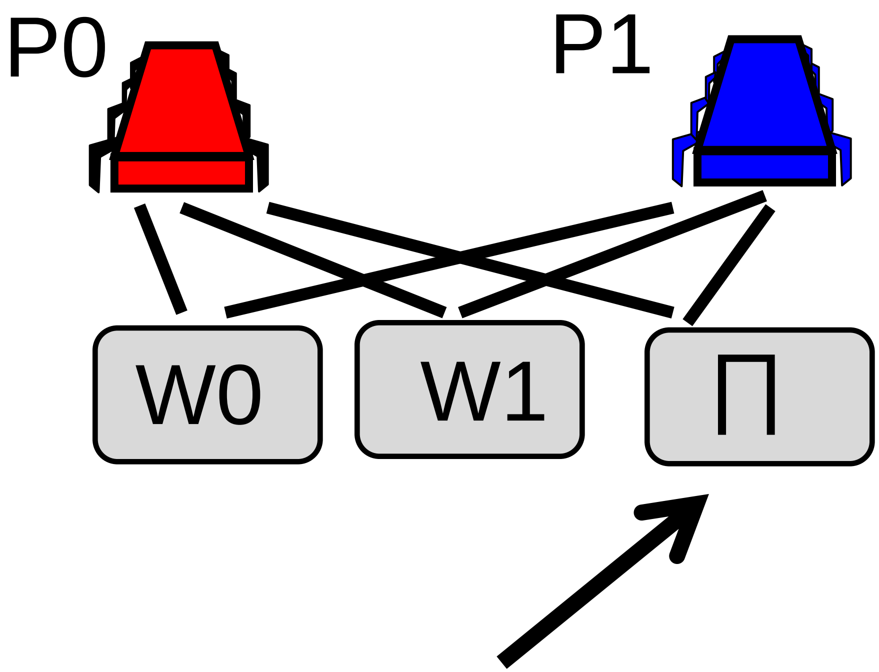

"Who has priority at the moment?"

Written by both.

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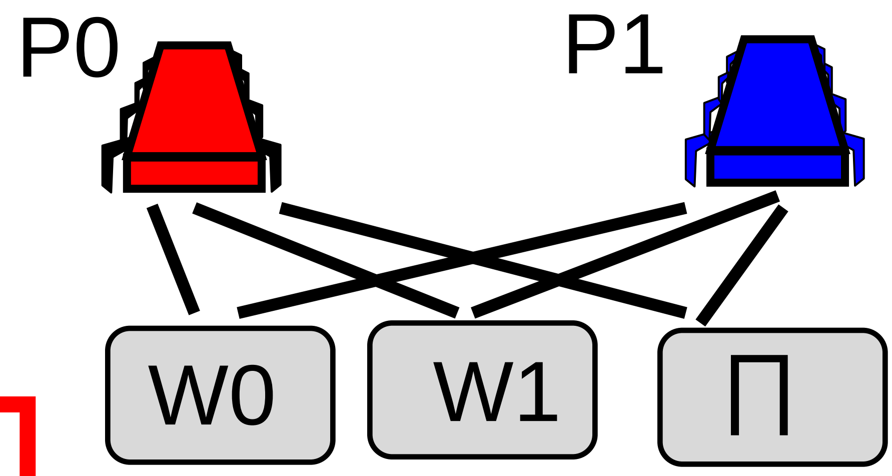

Peterson's Mutex

Code for process P_i

<Entry>

$W_i := 1$

$\prod := 1-i$

Loop until $\prod = i$ or $W_{1-i} = 0$

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$W_i := 0$

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Process indicates that it wants to enter CS in "*Want-Register*". Can only do if other process does not want, or I have *priority (shared variable!)*.

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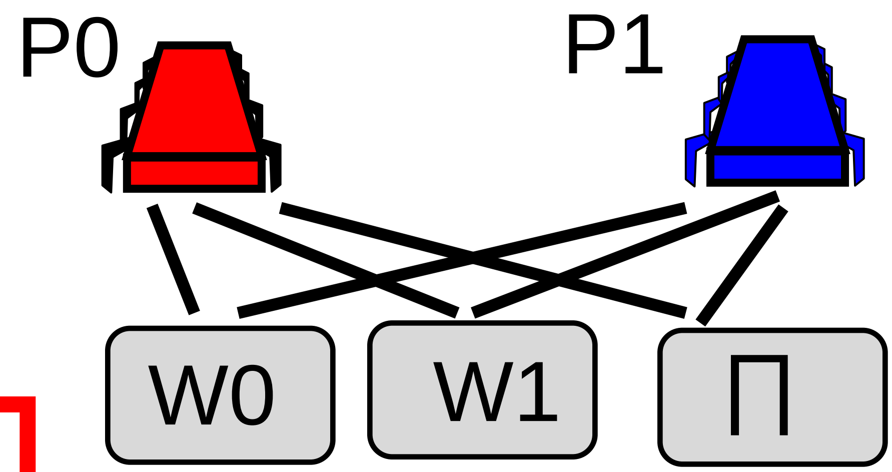

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Process indicates that it wants to enter CS in "*Want-Register*". Can only do if other process does not want, or I have *priority (shared variable!)*.

Spin-Lock! (Busy-wait)

Priority register used to avoid deadlock!

Peterson

Peterson gives (1) mutex, (2) no deadlock, (3) no lockout, (4) unobstructed.

Peterson's Mutex

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$\prod := 1\text{-}i$
Loop until $\prod = i$ or $W_{1-i} = 0$
<Critical Section>
…
<Exit>
$W_i := 0$
<Remaining Code>
…

Proof:

(1) Mutex: If both compete ("want"), only one can get priority and access CS.

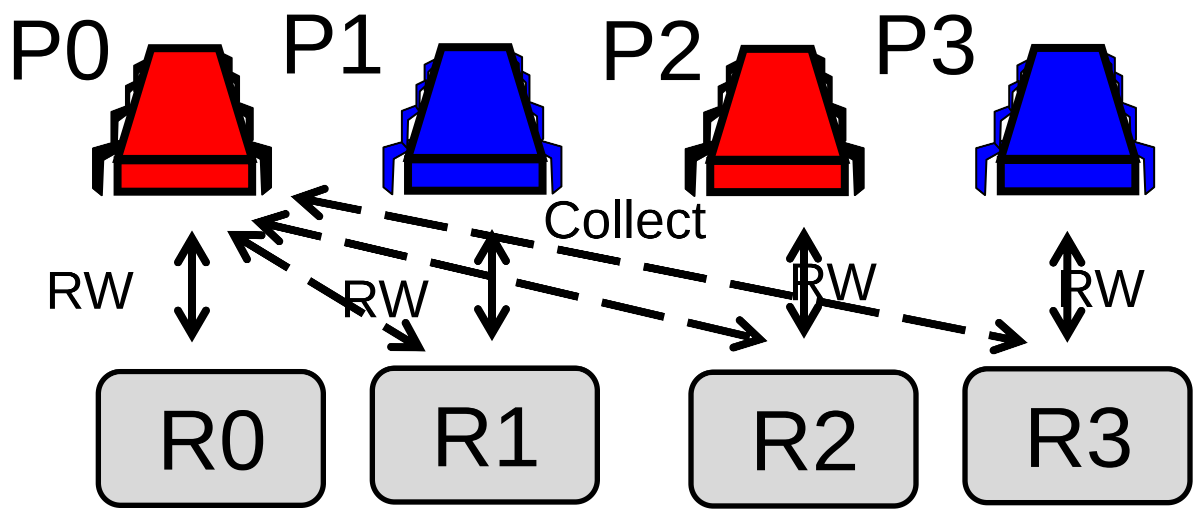
(2) No Deadlock: If both in loop and want, one process must have priority and it gets direct access to the critical section.

(3) Fairness: Non-priority process waiting in loop gets priority when other process starts again! Shared variable.

(4) Unobstructed Exit: Exit only a single instruction.

QED

Another fundamental task: Store&Collect



Goal: Collect up-to-date infos about other processes!
With *atomic RW* only.

Two operations:

- **sop(val)**: Process p_i *stores* val to be the latest value of its own register Ri. ("1:1 write", so *single-writer model*)
- **cop()**: *Collects* a "view", a function V where V(pi) is the latest value stored by p_i, for each process p_i.

Store&Collect

Assume: registers initialized to «?».

Note: Collect has no sequential specification and cannot be linearized.

Our goal here: A collect operation **cop** should never read from the future or miss a preceding store operation **sop.**

Store&Collect

For a collect operation cop, the following validity properties must hold for every process p_i :

1. If $V(p_i)$ = “?”, then no store operation by pi *precedes* cop.
2. If $V(p_i)$ = v, then v is the value of a sop operation of p_i that does not *follow* cop, and there is no store operation by p_i that *follows* sop and *precedes* cop.

Complexity Meaure

We measure the following complexity.

Step Complexity

Step complexity of an operation is the number of accesses to the registers in the shared memory.

How to implement a valid Collect() operation?

Simple Algorithm

Step sop() & cop()

Operation STORE(val), by proc. p_i

Ri := val;

Operation COLLECT:

for i:= 1 to n do

V(p_i):=Ri (* read register Ri *)

end

Works (atomic read/write).
Complexity?

Simple Algorithm

Step sop() & cop()

Operation STORE(val), by proc. p_i

Ri := val;

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for i:= 1 to n do

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Works (atomic read/write).

Complexity?

STORE is 1 step

COLLECT is n steps

Adaptive Algorithm

If only two processes wrote some value, COLLECT is too costly!
How to make an operation adaptive to the number of processes that were active in the execution?

Adaptive Operation

If up to time t, $k \leq n$ processes have started or finished at least one operation, an operation is called *adaptive* if step complexity depends on k but not on n.

How to make our algorithms adaptive?

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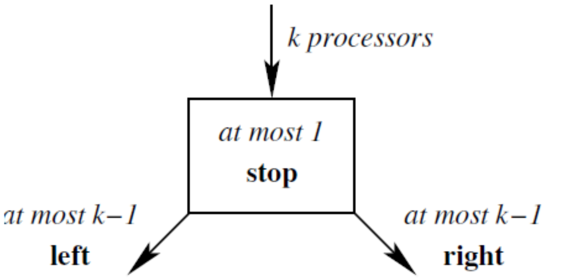
How to make our algorithms adaptive?
We need *Splitters*…

Splitter



Splitter

Synchronization primitive:

- Process entering it exits with *stop, left* or *right*
- If k processes enter, at most one exists with stop, and *at most k-1* processes exit with left and at most k-1 processes exit with right.
- If single process enters it, *stop for sure*.

Not perfect balance, but there are two processes that obtain *different values* (stop, left, right).

How to implement splitter?

Splitter Algo

Splitter

Two shared registers X: {?, 1, ..., n}, Y: bool

Initialization: X=?, Y=false

Splitter access by pi:

X:=i

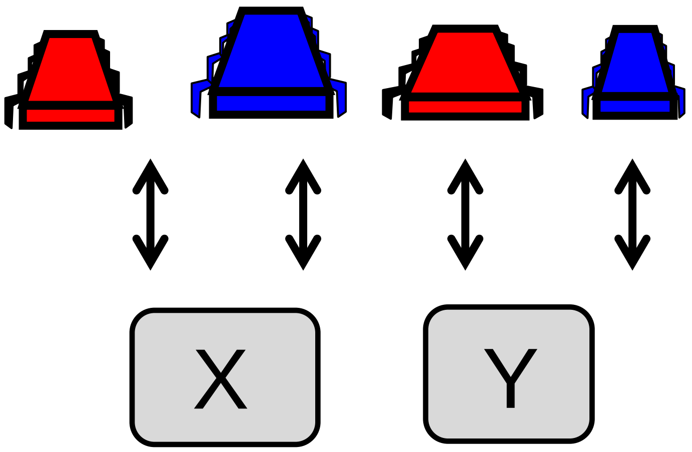
If Y then *return right*

Else

  Y:=true

  if X=i then *return stop*

  else *return left*



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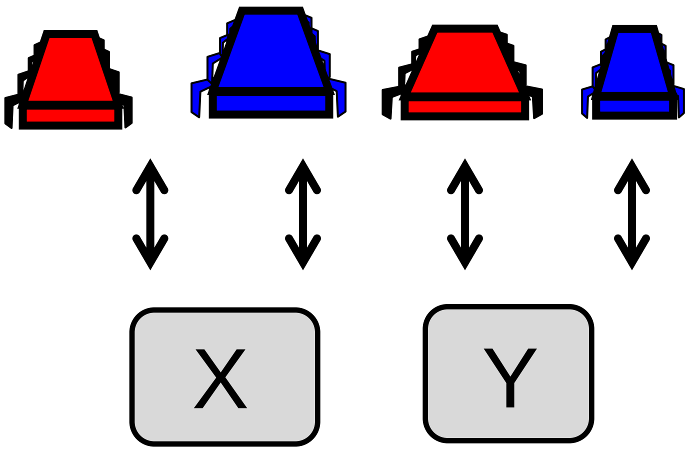

Why correct?

Correctness

A single process *always stops*:

- Clear: check solo-run

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At most k-1 return right:
- **First process** checking Y will *not return right*

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At most k-1 return left:
- Assume process p is **last** to set X:=i
- If p does not return right, it will find its own value later and *stop*: it does *not return left*!

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At most one process stops:
- Assume contrary: both processes pi and pj return stop, and w.l.o.g. assume pi sets X:=i before pj sets X:=j.
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Correctness

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-

At

-

n

How to realize adaptive collect now?

Splitter trees!

Splitter

Two shared registers X: {?, 1, ..., n}, Y: bool

Initialization: X=?, Y=false

else *return left*

At most k-1 return left:

- Assume process p is **last** to set X:=i
- If p does not return right, it will find its own value later and *stop*: it does *not return left*!

At most one process stops:

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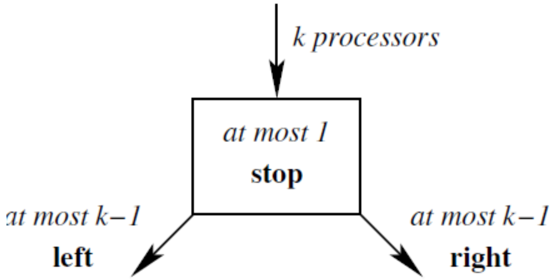
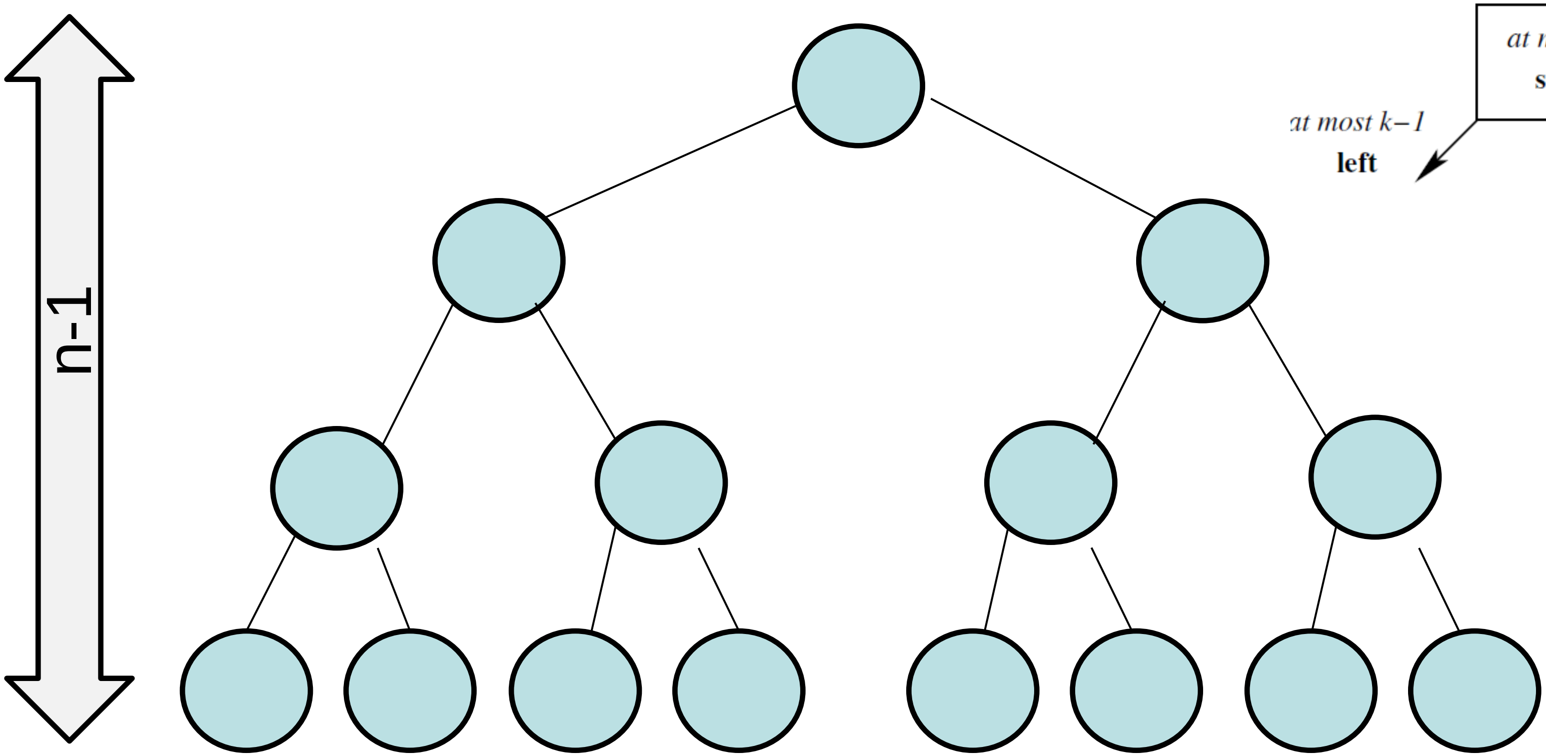
Splitter Tree

Assume we have 2^n-1 splitters, arranged in complete binary tree:





Let S(v) be splitter of node v in tree.
Additionally: for every splitter, shared variables Z_S: {?,1,…,n} and boolean M_S.
A splitter is marked if M_S=true.

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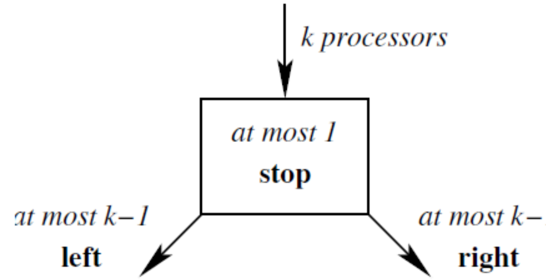

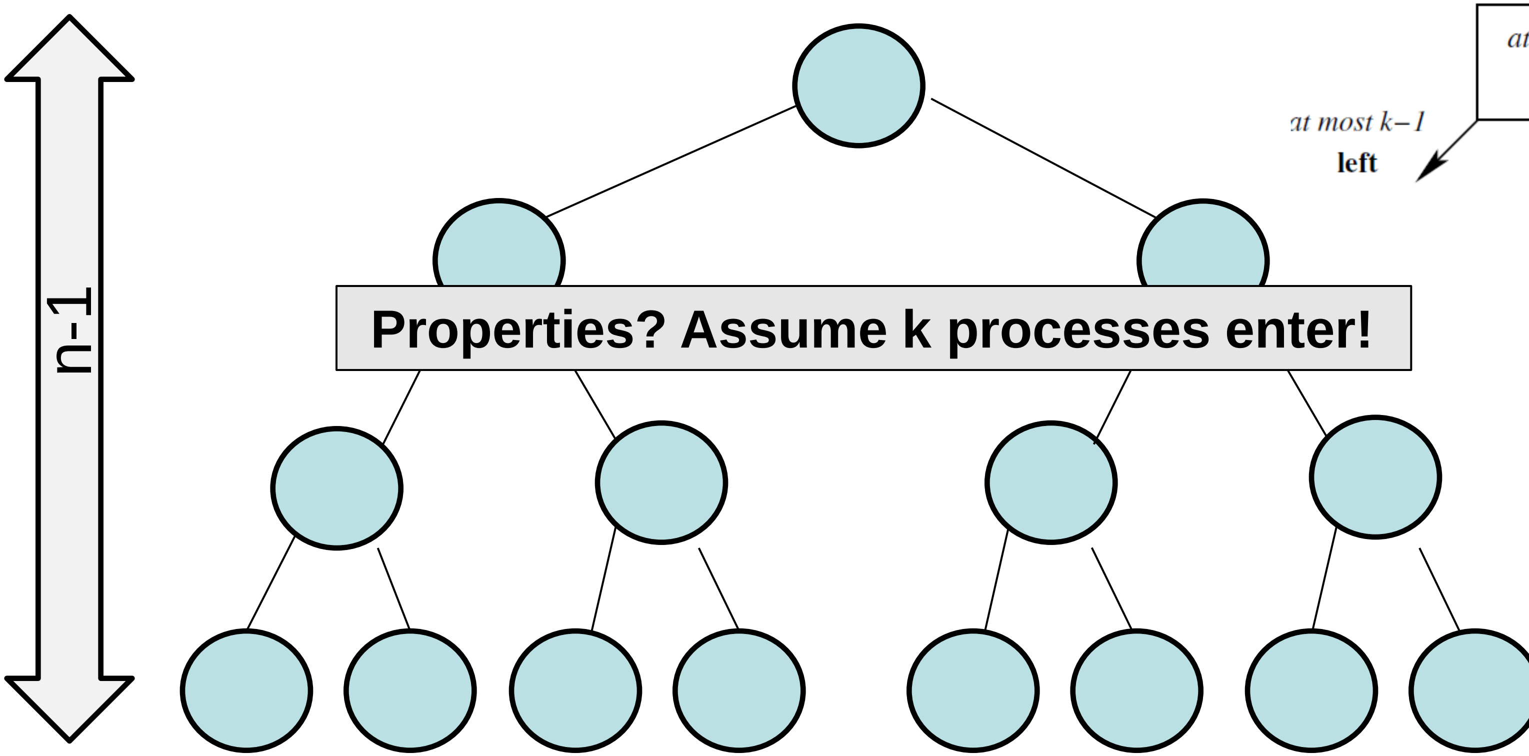

Properties? Assume k processes enter!

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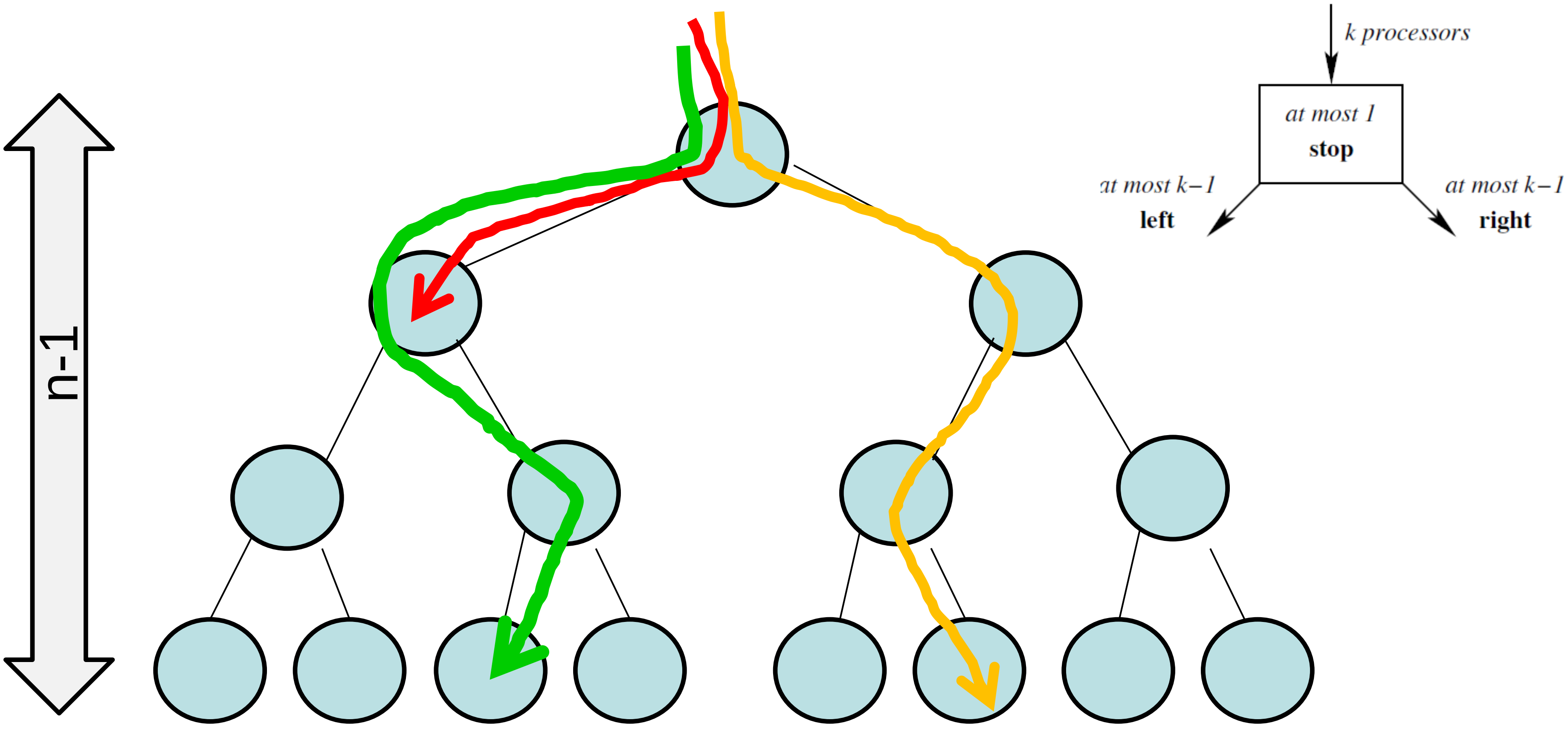
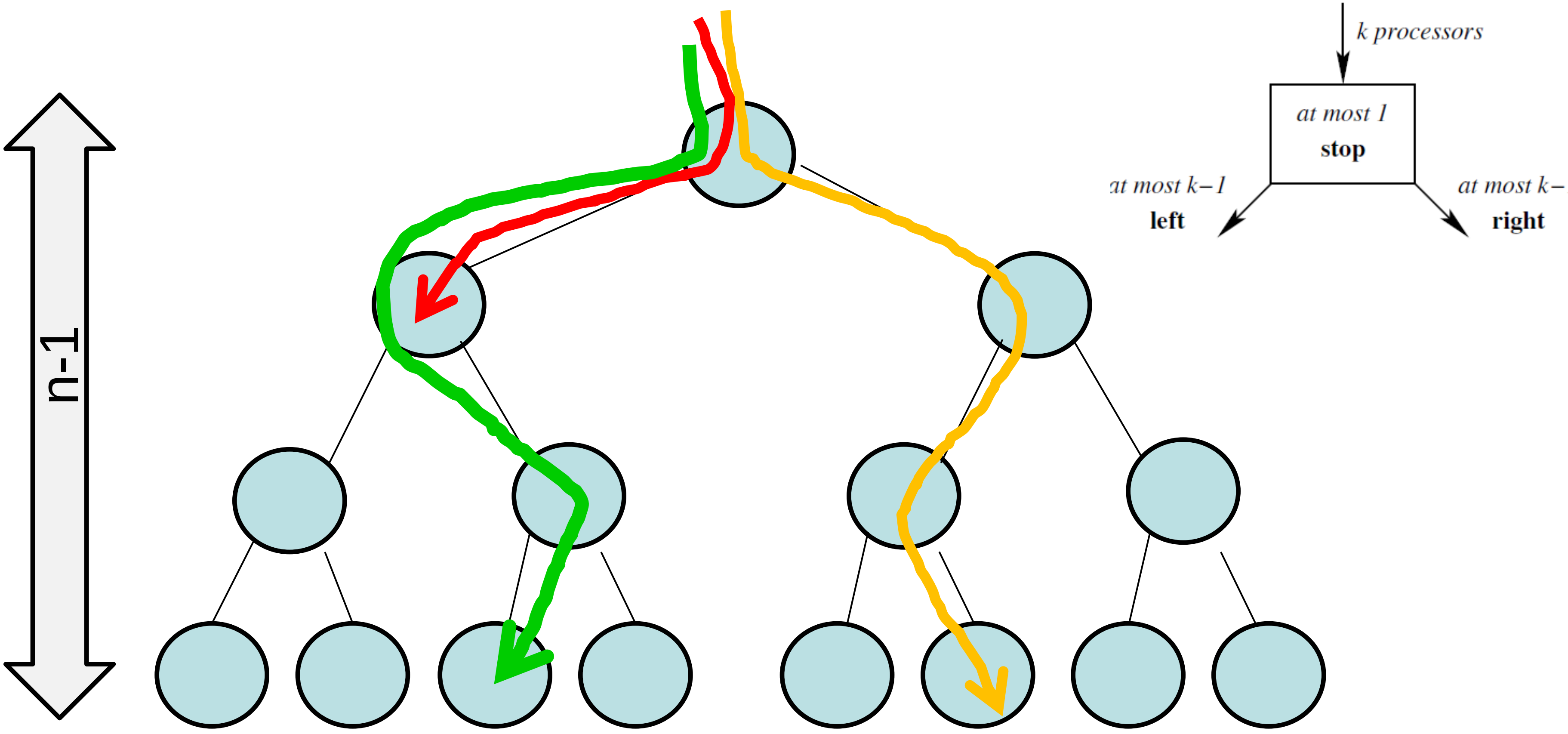


k processes *traversing splitter tree*:

- At most one process can stop at some given splitter
- Every process stops at some splitter at depth at most k-1. Why?



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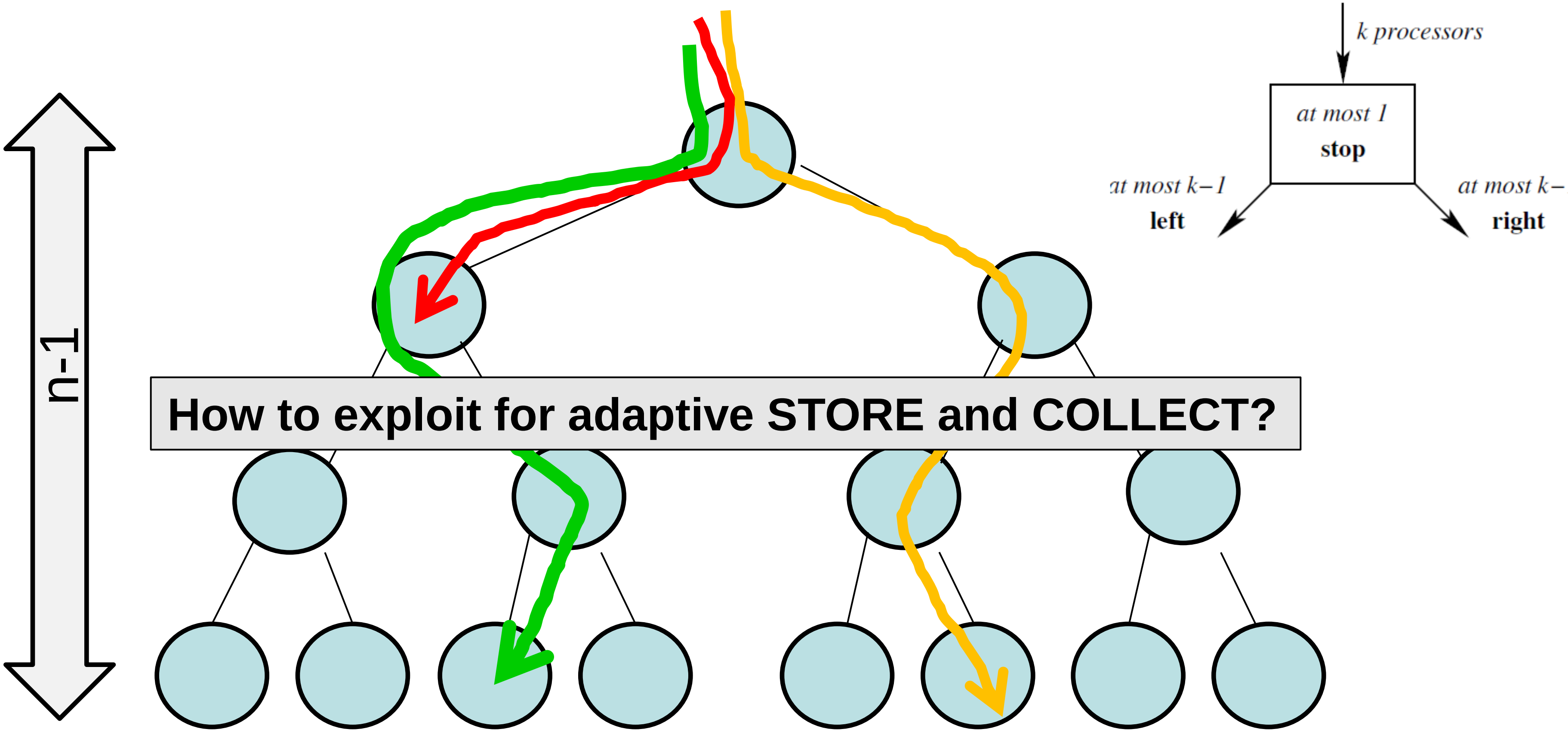

How to exploit for adaptive STORE and COLLECT?

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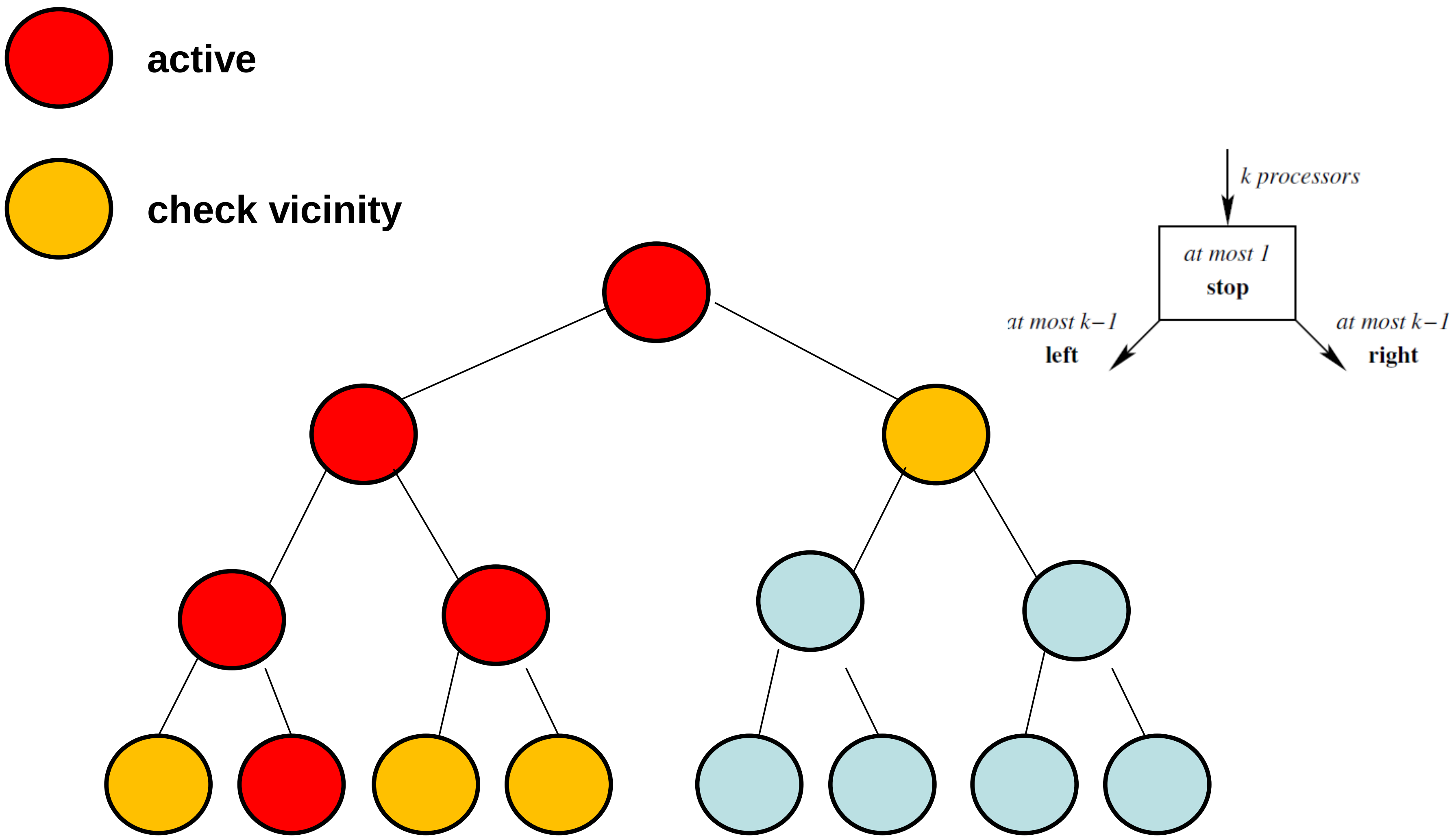
- By definition, k processes enter the root splitter at depth 0
- If k-i processes enter splitter at root of subtree at depth i (induction hypothesis), at most k-i-1 obtain left and at most k-i-1 right.

Upon *first store operation* of a process, *traverse* splitter tree:
data structure to mark which nodes were already active!
A connected component: *collect only needs to check this part*!



Tree Ops

Operation STORE(val) (by process p_i) **:**

$R_i := val$
if first STORE operation by p_i **then**
  $v :=$ root node of binary tree
  $\alpha :=$ result of entering splitter $S(v)$;
  $M_{S(v)} :=$ **true**
  while $\alpha \neq$ stop **do**
    if $\alpha =$ left **then**
      $v :=$ left child of v
    else
      $v :=$ right child of v
    end if
    $\alpha :=$ result of entering splitter $S(v)$;
    $M_{S(v)} :=$ **true**
  end while
  $Z_{S(v)} := i$
end if

Traverse splitter tree from top, mark traversed nodes (M) and store splitter where pi stopped.

Operation COLLECT**:**
Traverse marked part of binary tree:

for all marked splitters S **do**
  if $Z_S \neq \perp$ **then**
    $i := Z_S$; $V(p_i) := R_i$
  end if
end for

Only collect marked parts!

Tree Ops

Operation STORE(val) (by process p_i) **:**

```
R_i := val
if first STORE operation by p_i then
    v := root node of binary tree
    α := result of entering splitter S(v);
    M_S(v) := true
    while α ≠ stop do
        if α = left then
            v := left child of v
        else
        end if
        α := result of entering splitter S(v);
        M_S(v) := true
    end while
    Z_S(v) := i
end if
```

Traverse splitter tree from top, mark traversed nodes (M) and store splitter

Complexity of solution?

Operation COLLECT**:**

Traverse marked part of binary tree:

```
for all marked splitters S do
    if Z_S ≠ ⊥ then
        i := Z_S; V(p_i) := R_i
    end if
end for
```

Only collect marked parts!

Adaptive Collect

Step complexity of first STORE is O(k), subsequent ones are O(1). COLLECT has step complexity O(k).

Proof.

- First *STORE*: splitter tree traversal to find "my" location, at most at depth k
- From then on, will always *STORE* there: O(1)
- *COLLECT*:
  - Only need to check marked part and their neighbors
  - Marked part of the tree is connected
  - At most 2k-1 nodes are marked:
    - By induction: a single process entering a splitter will always stop
    - The right and left child of the root are subtrees too, first node will stop at first splitter.

Adaptive Collect

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Disadvantage?

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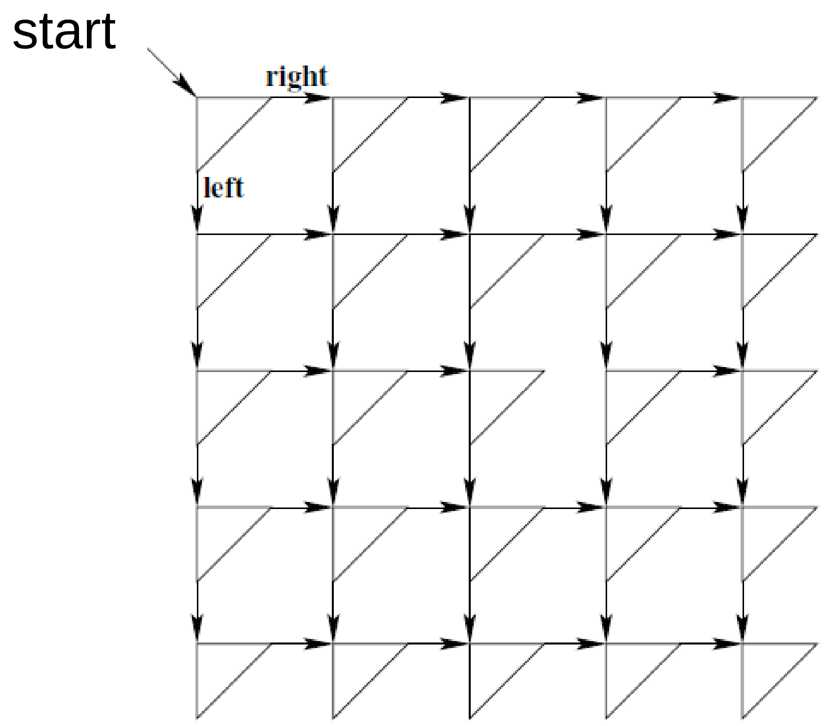
    - The right and left child of the root are subtrees too, first node will stop at first splitter.

Disadvantage? Space complexity! Store O(2^n) tree in memory...

Splitter Matrix

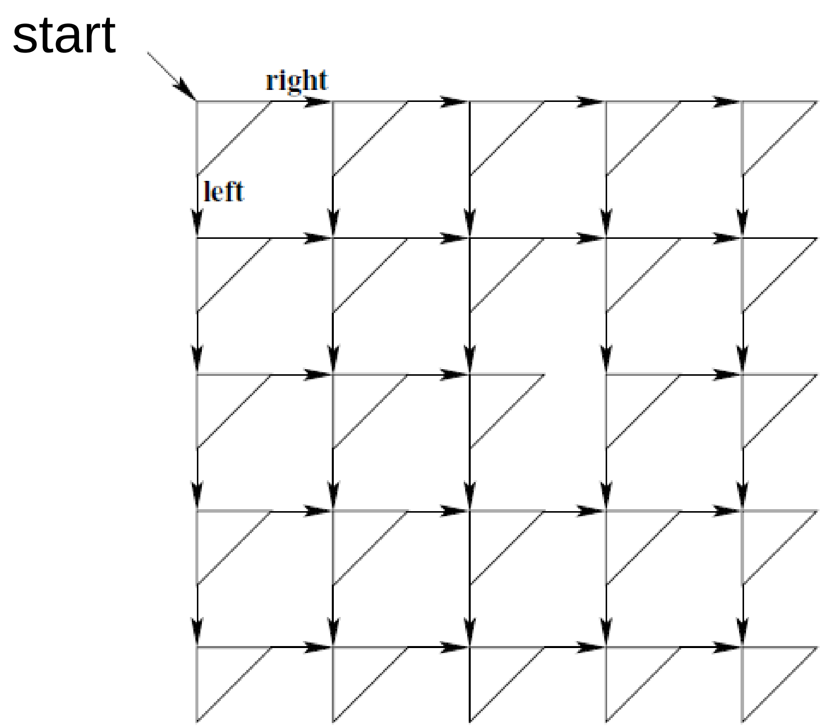
Idea: Instead of arranging splitters in **2^n** binary tree, arrange them in *n x n matrix*:



5x5 splitter matrix

Splitter Matrix

Idea: Instead of arranging splitters in **2^n** binary tree,
arrange them in *n x n matrix*:



5x5 splitter matrix

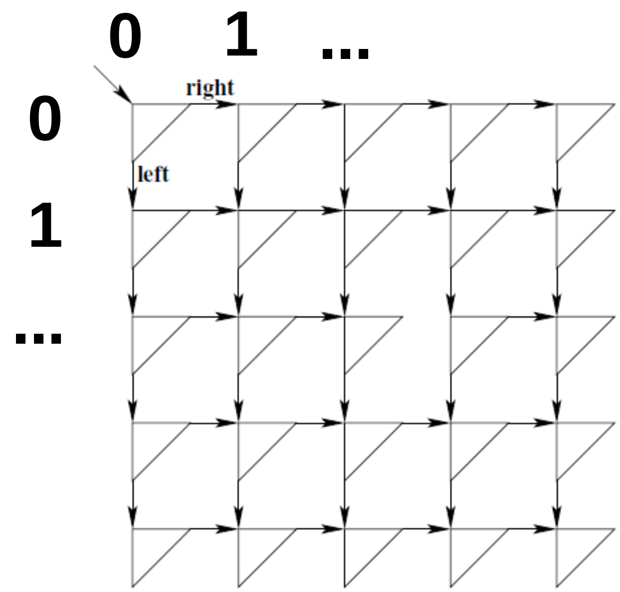
Space complexity n^2. Step complexity?

Matrix Collect

Step complexity of first *STORE* is O(k), subsequent ones are O(1). *COLLECT* has step complexity $O(k^2)$.

Proof.

- Let xi be number of procs entering row i. By induction on i, xi **≤** k-i.
- Of course: x0 **≤** k
- Let j be largest column s.t. at least one process visits the splitter at (i-1,j).
- Not all processes go left, so xi **≤** k-i.
- Same for column.
- So *every process stops the latest in row k-1 and column k-1*.
- The number of marked splitters is at most *k^2*: complexity of COLLECT.
- The longest path in matrix is 2k, so STORE complexity at most *O(k)*.



QED

Remarks

- Randomized algorithms can achieve binary trees of depth O(log n) only.

- O(k) step complexity and $O(n^2)$ space complexity is possible for COLLECT, even deterministically

End of Lecture