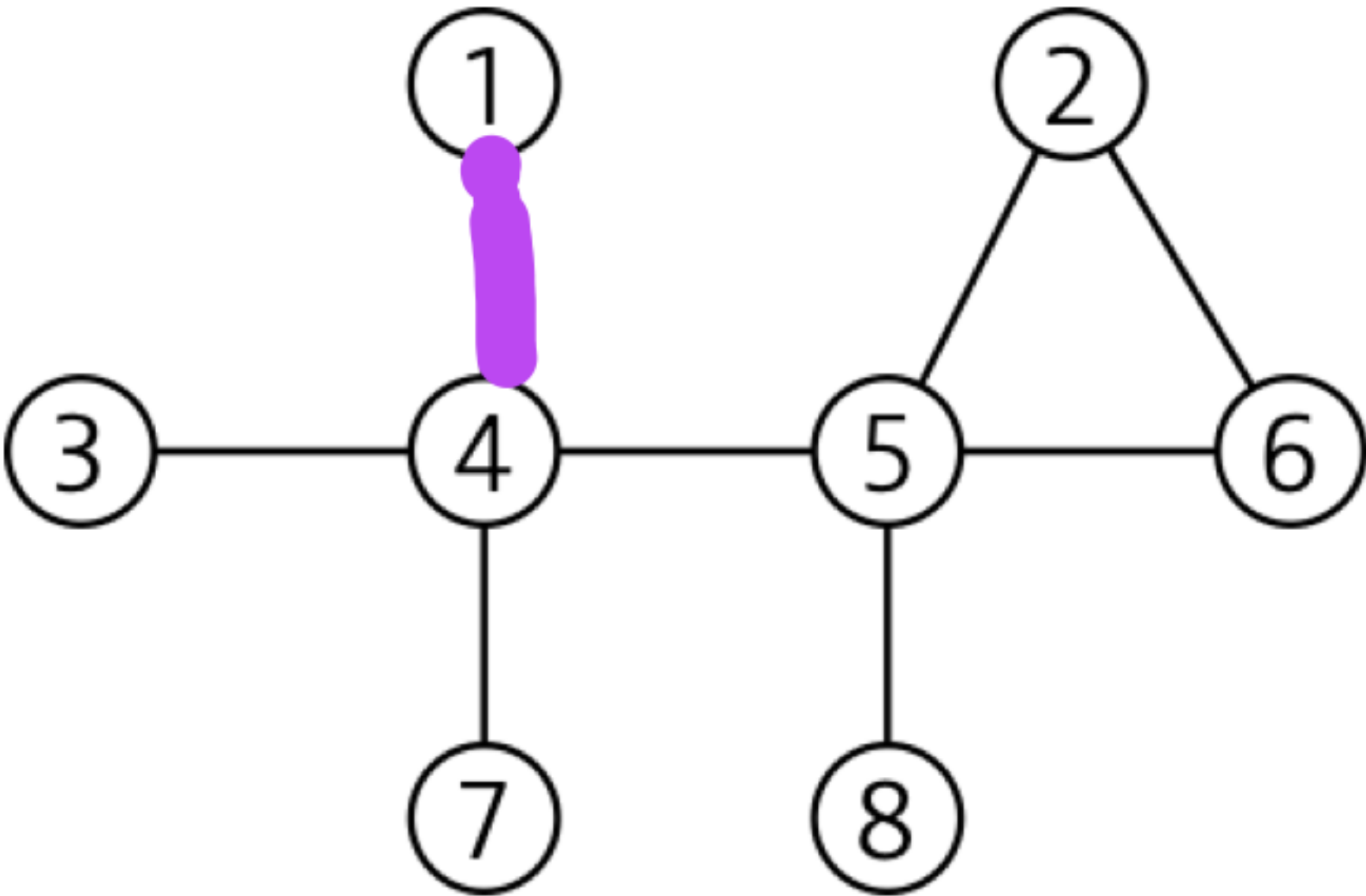
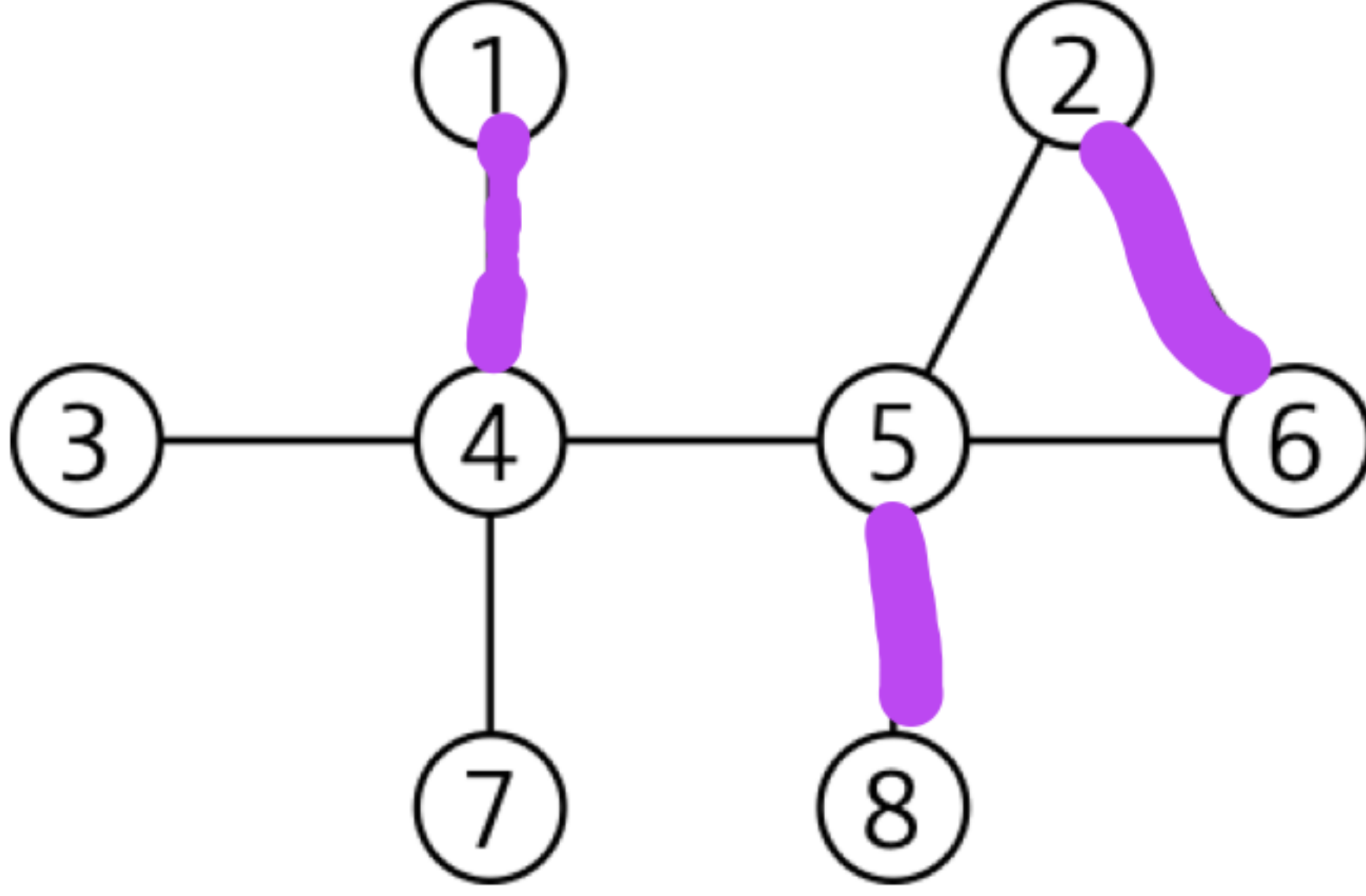


Matching



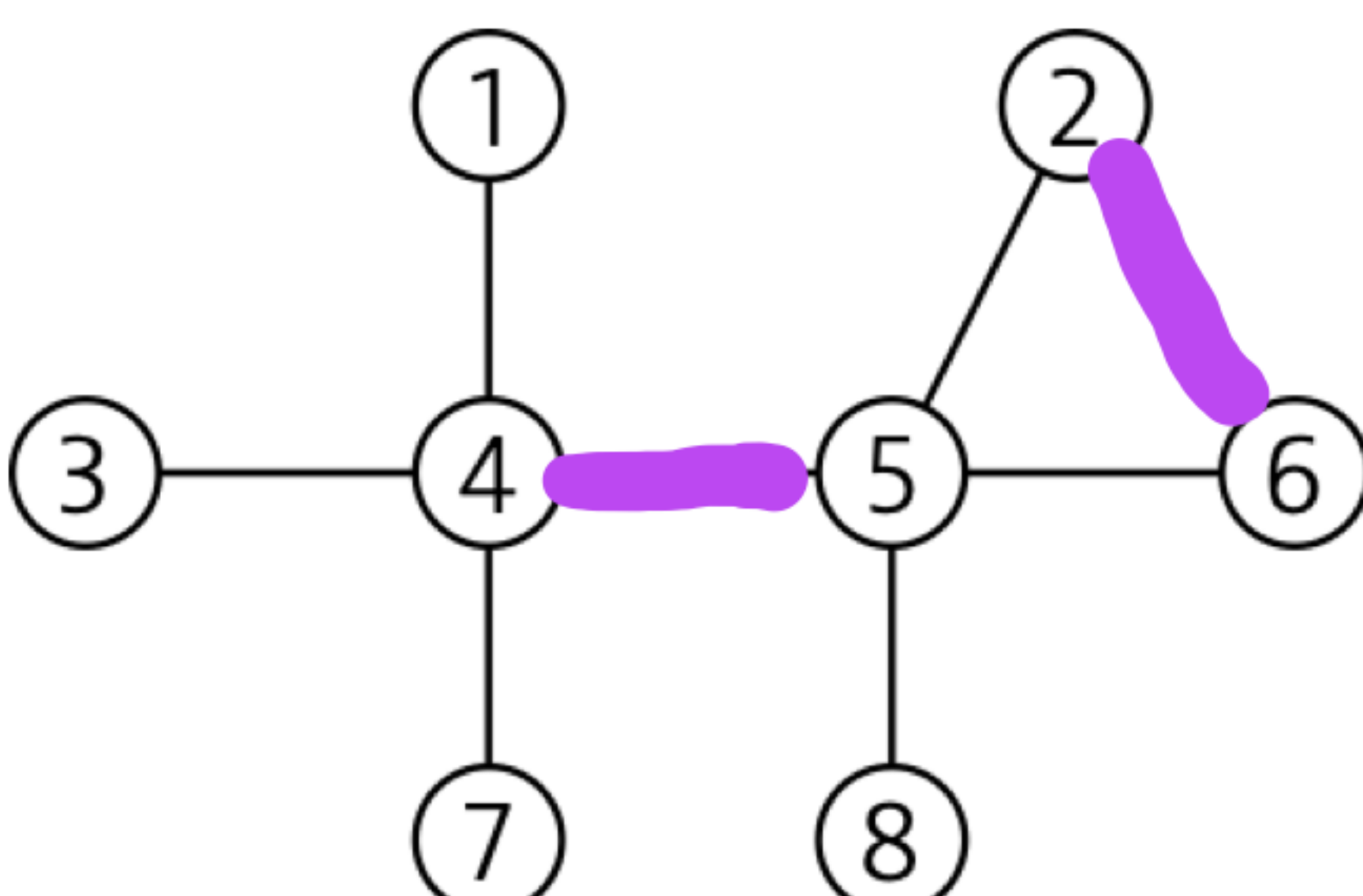
set  $M \subseteq E$  is a *matching* if no two edges in  $M$  share a common endpoint

Maximum matching



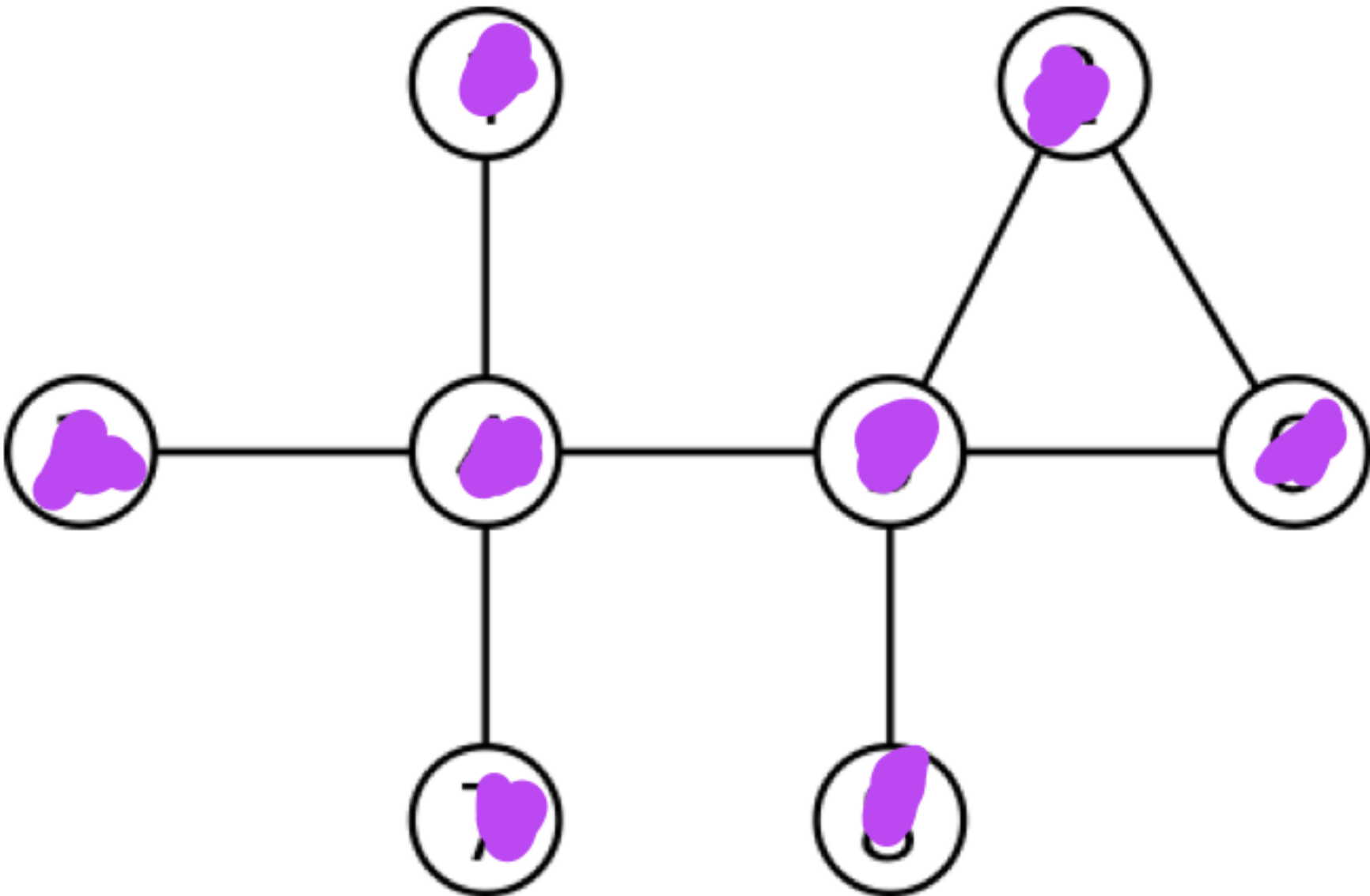
*maximum matching* = matching with the largest possible number of edges

Maximal matching



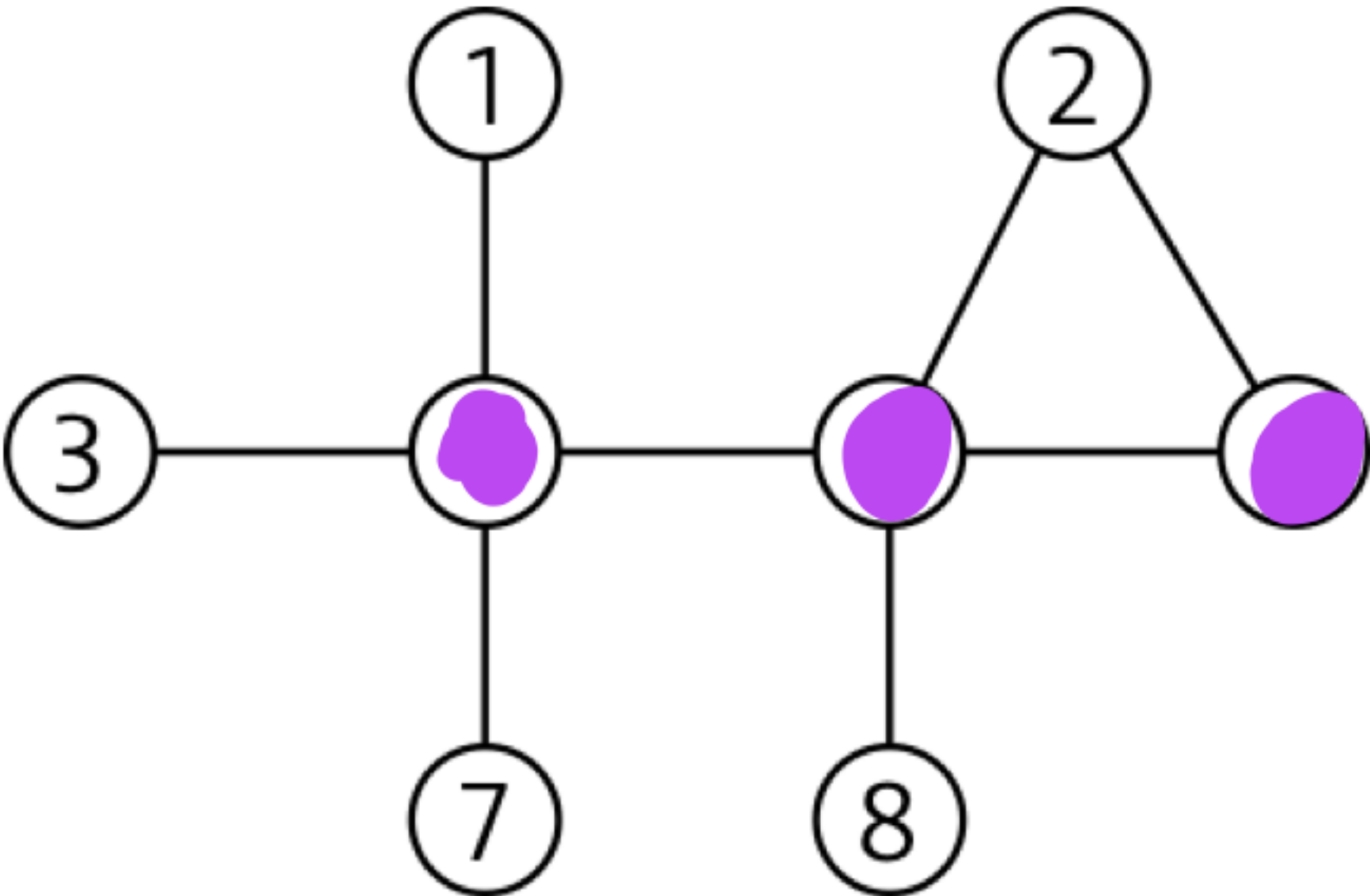
*maximal matching* = largest matching with the given set of edges

Vertex Cover



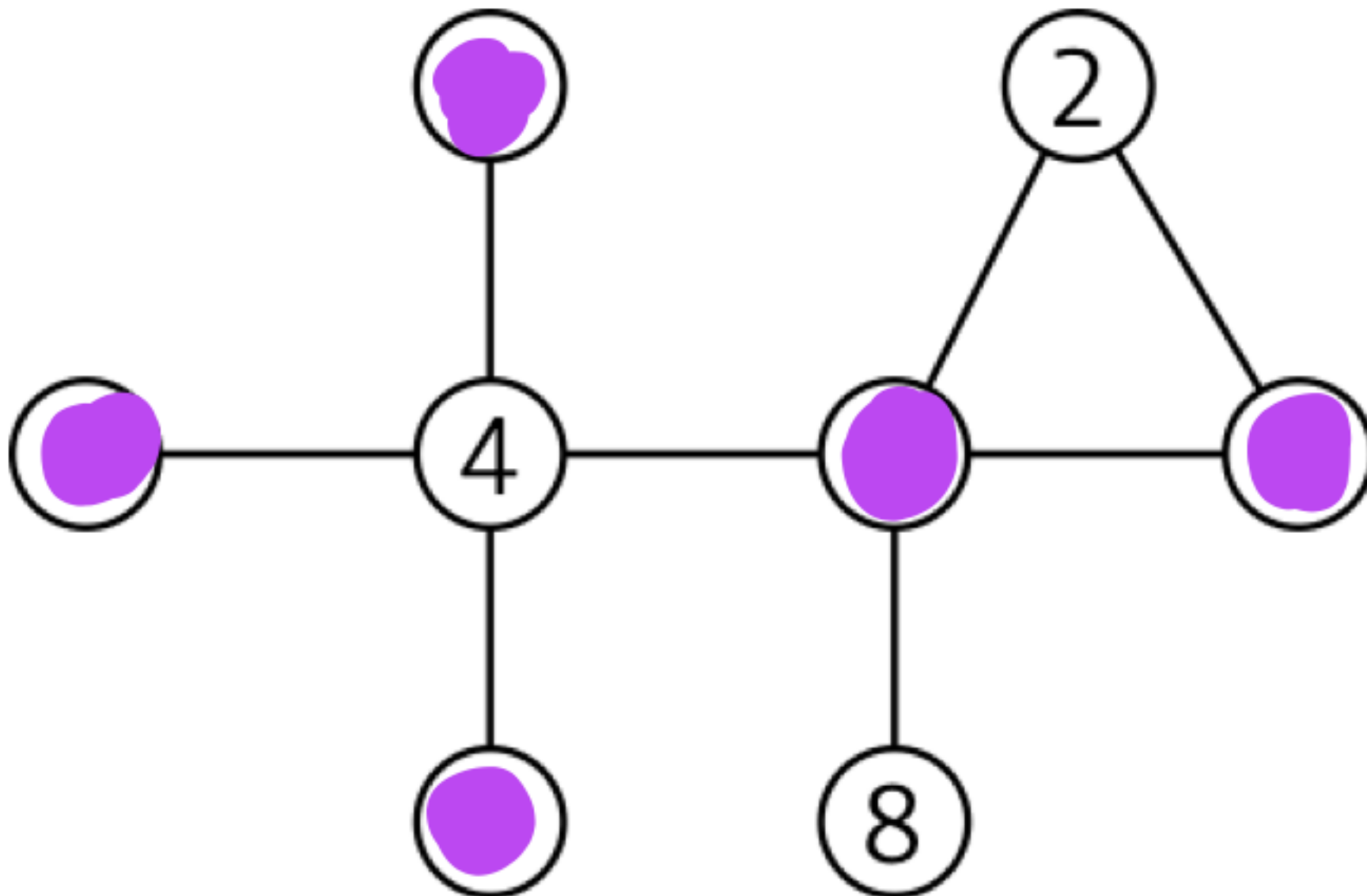
set  $C \subseteq V$  is a *vertex cover* if every edge has at least one endpoint in  $C$

Minimum vertex cover



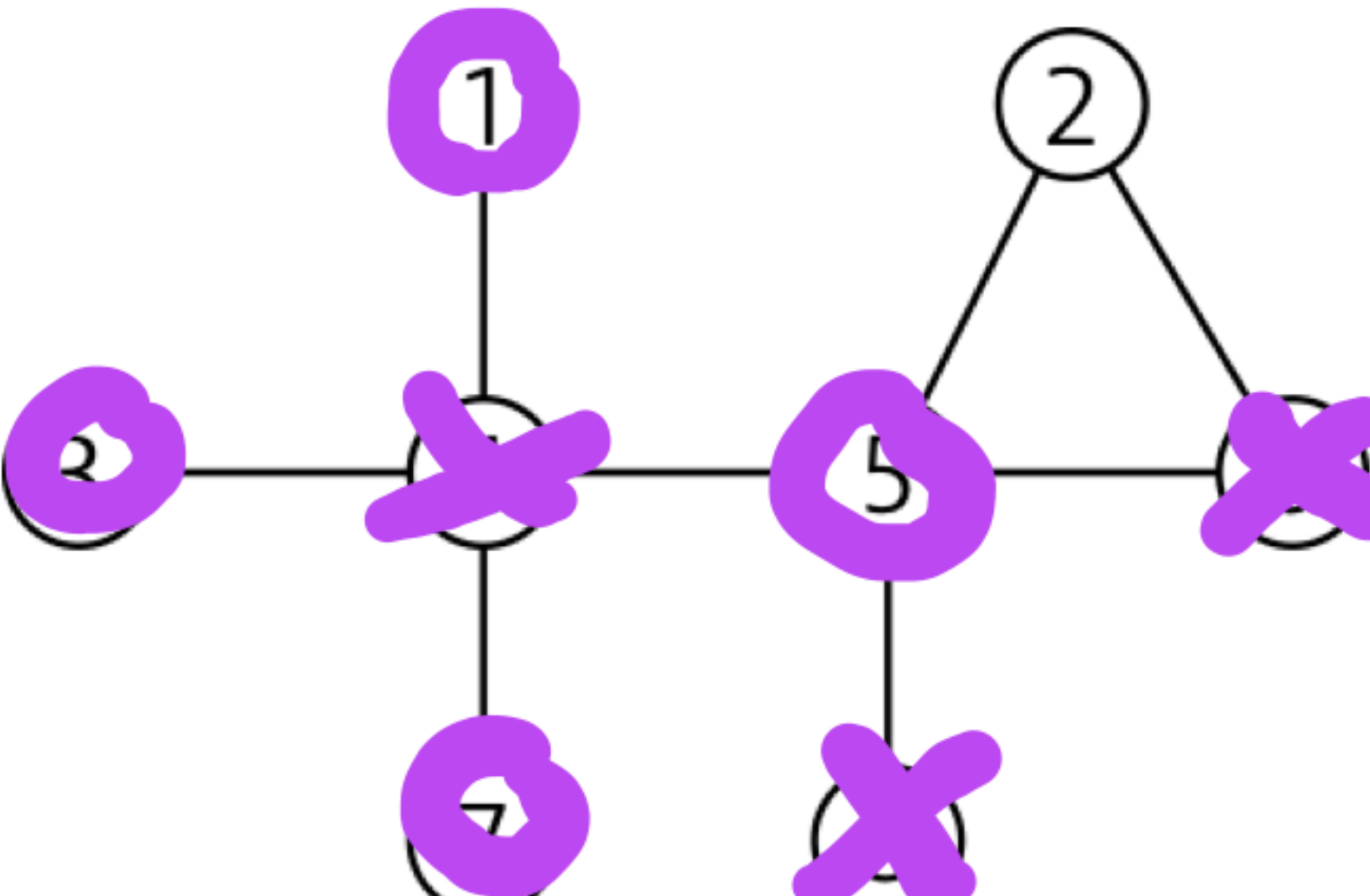
*minimum vertex cover* = vertex cover with smallest possible number of nodes

Minimal vertex cover



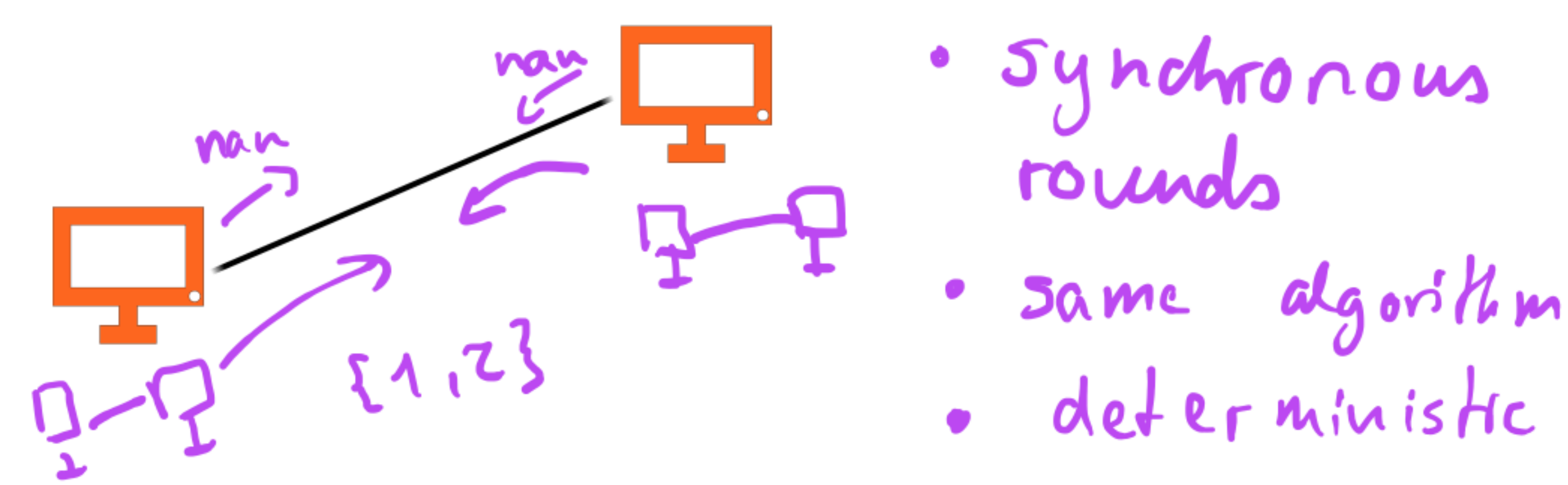
*minimal vertex cover* = smallest vertex cover with the given set of nodes

Vertex coloring



*vertex coloring* = assign a color to each node of the graph such neighbors have different colors

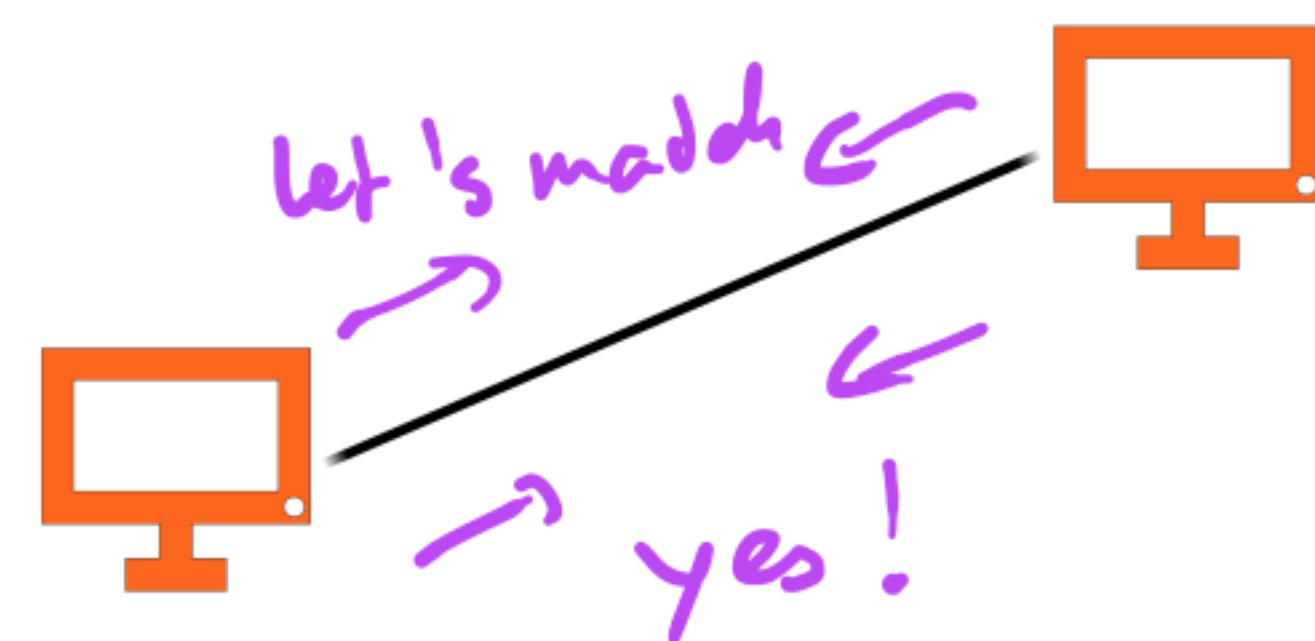
Coloring



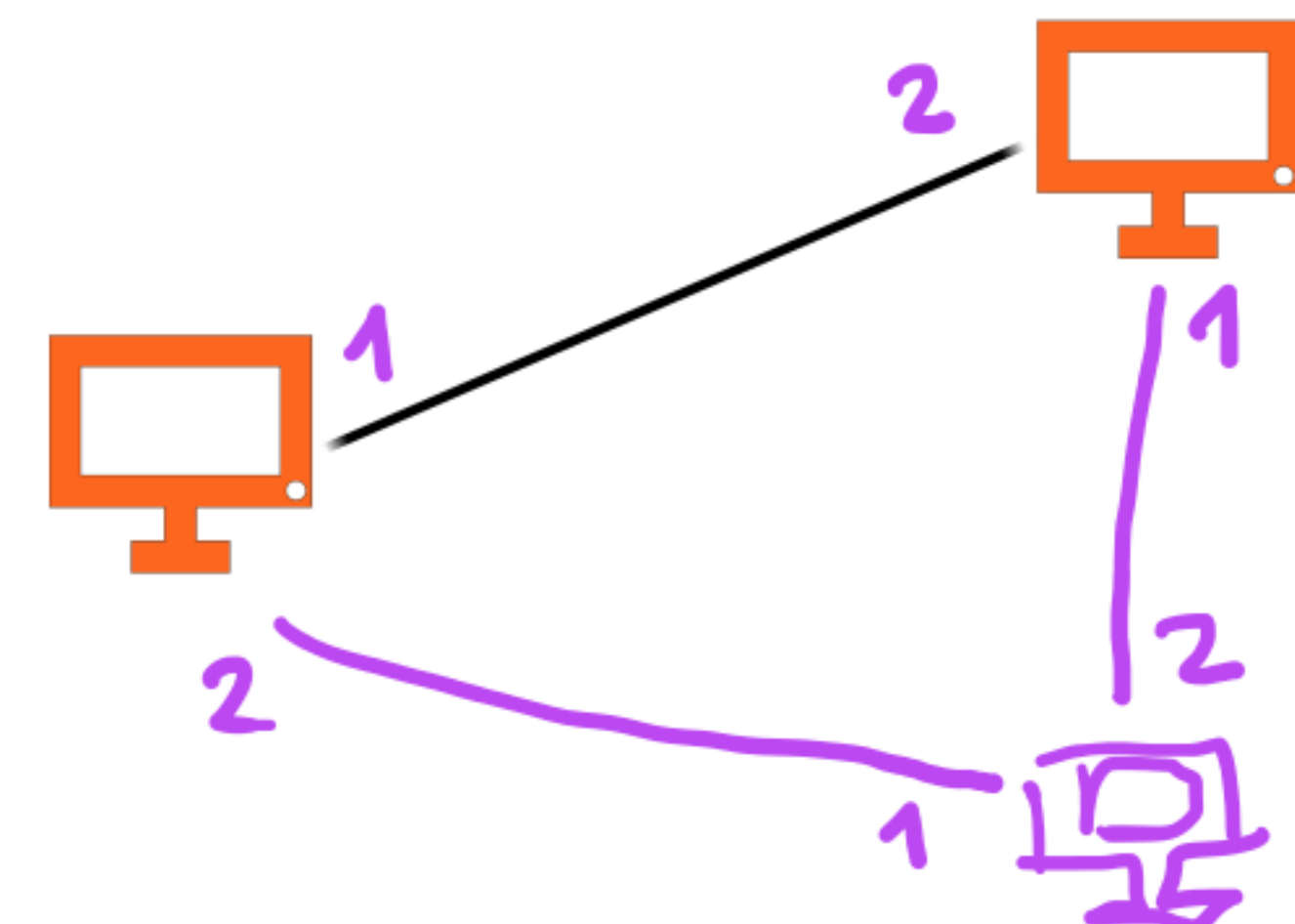
Minimum vertex cover



Non-empty matching

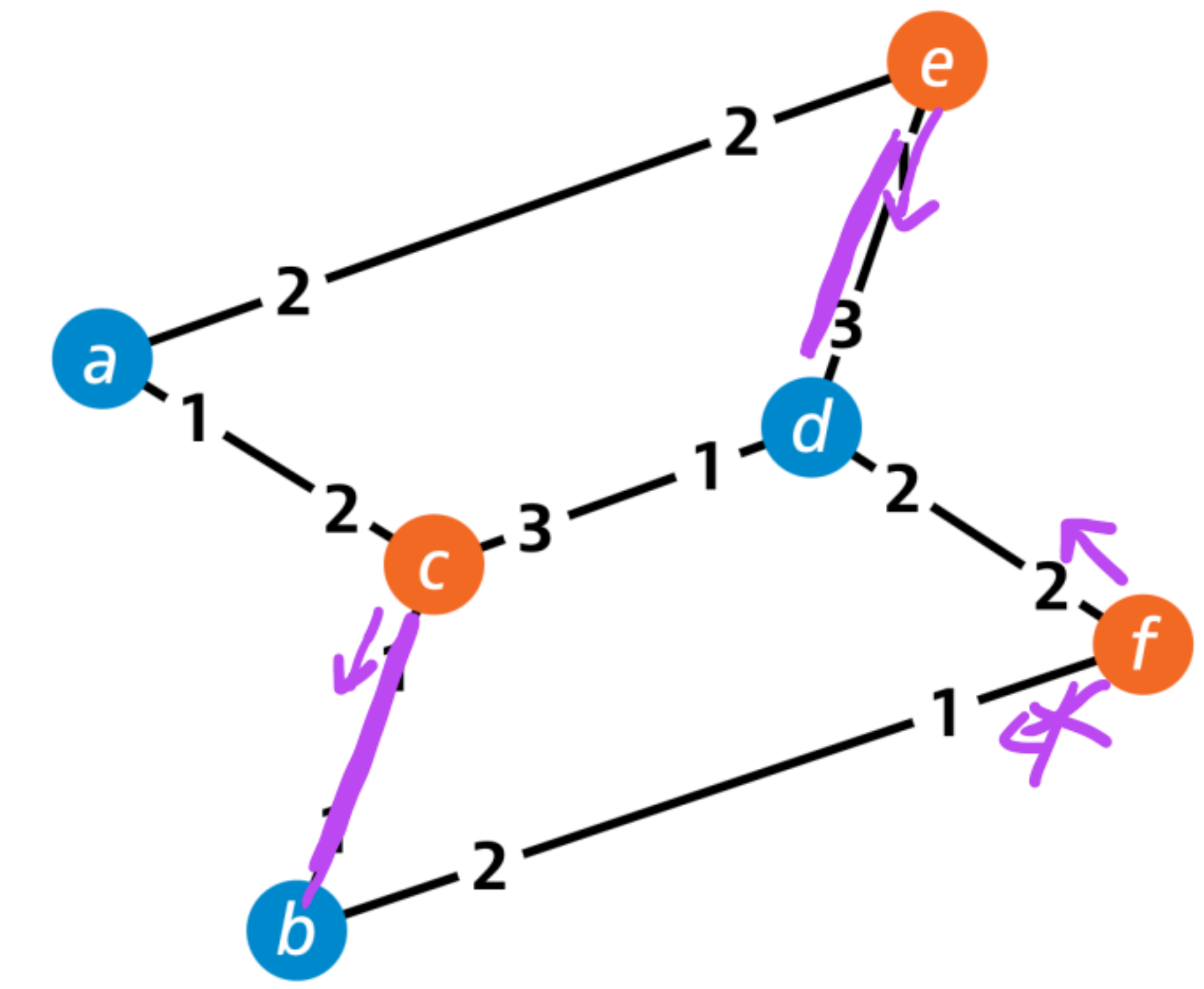
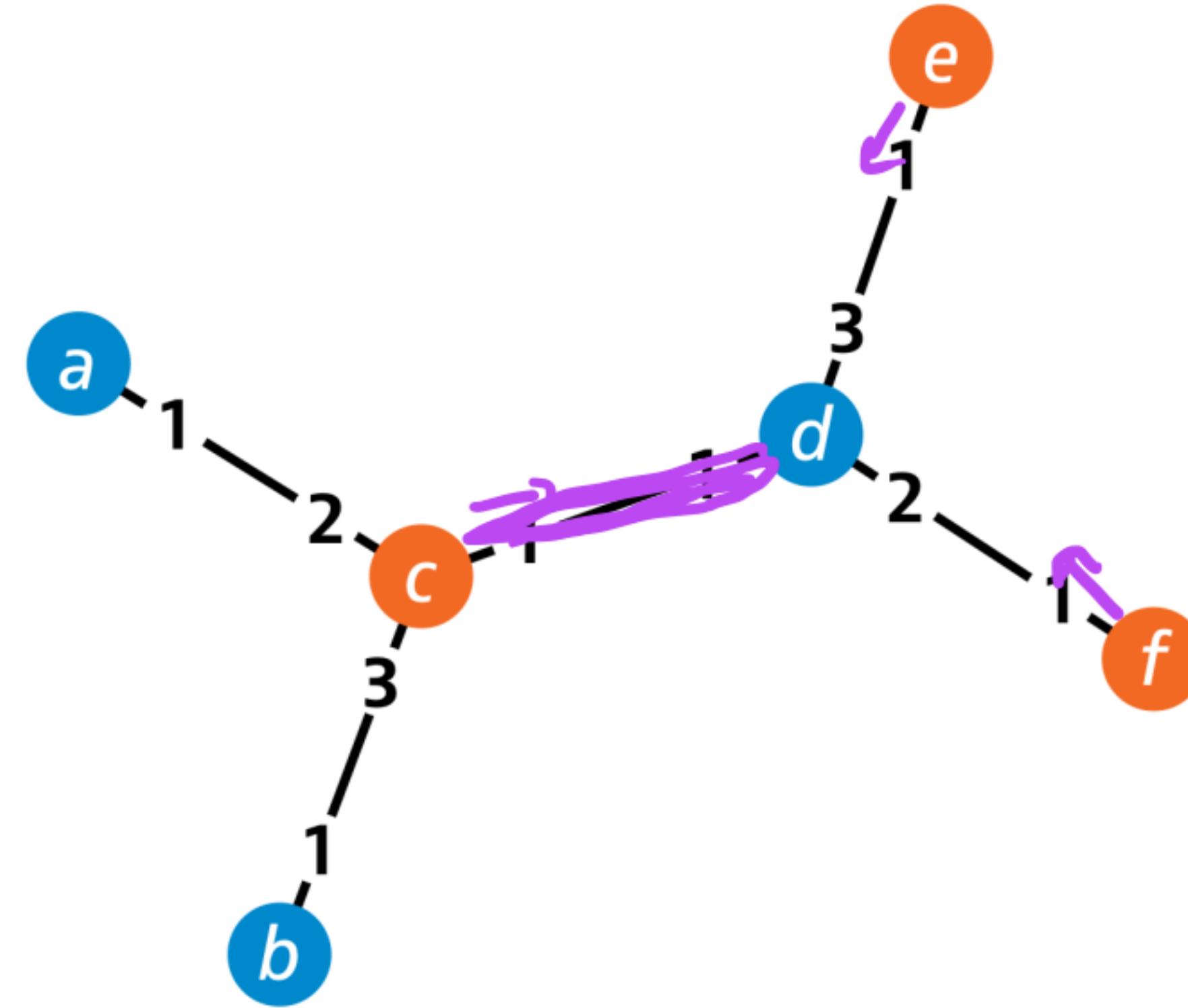
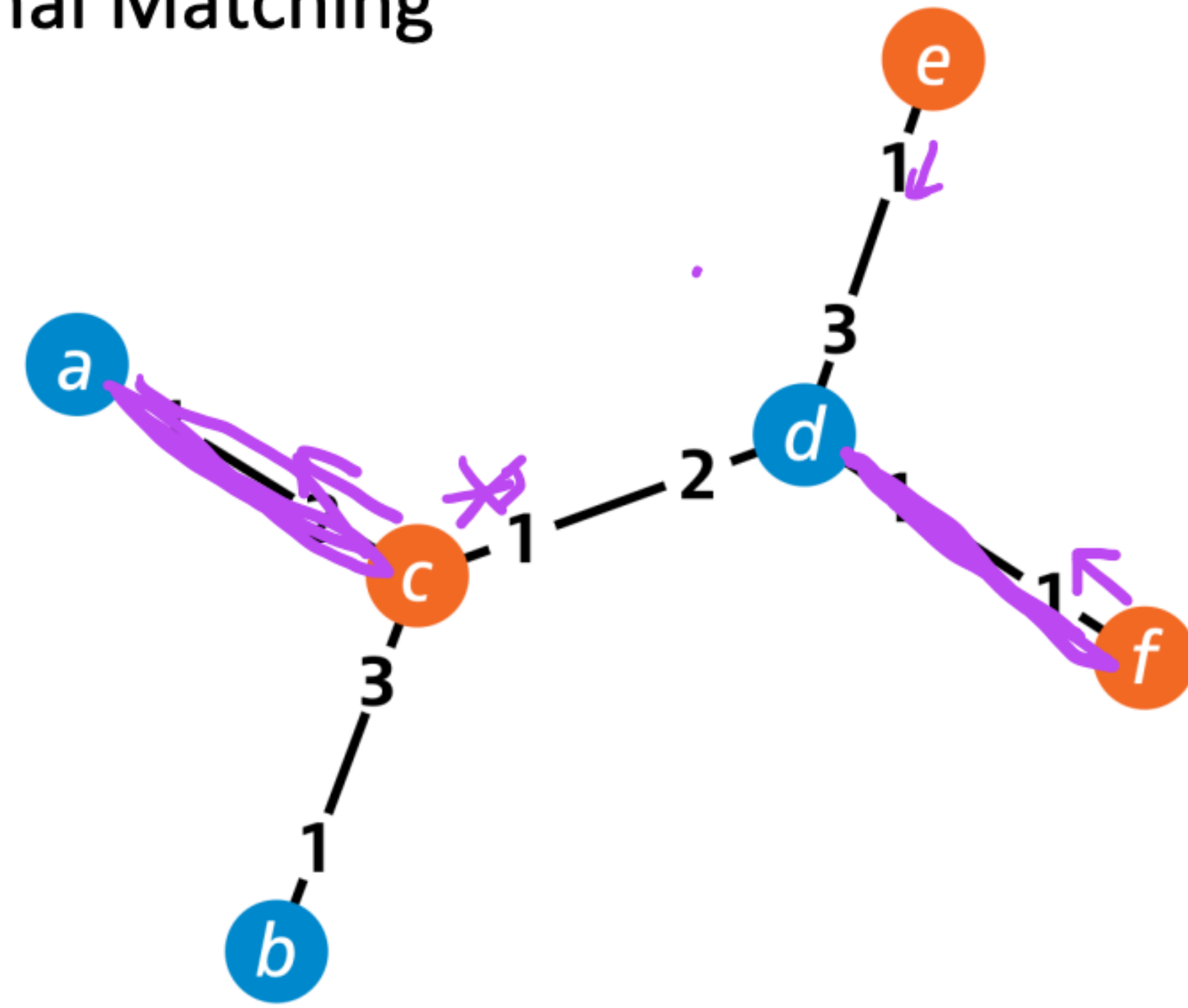


Non-empty matching





## Bipartite Maximal Matching



Correctness of the bipartite maximal matching algorithm

- Algorithm terminates

orange nodes propose to  $\Delta$  nodes

$\Delta$ : maximal degree of a graph

# rounds:  $\Delta + \Delta + 1 = O(\Delta)$   
propose answers

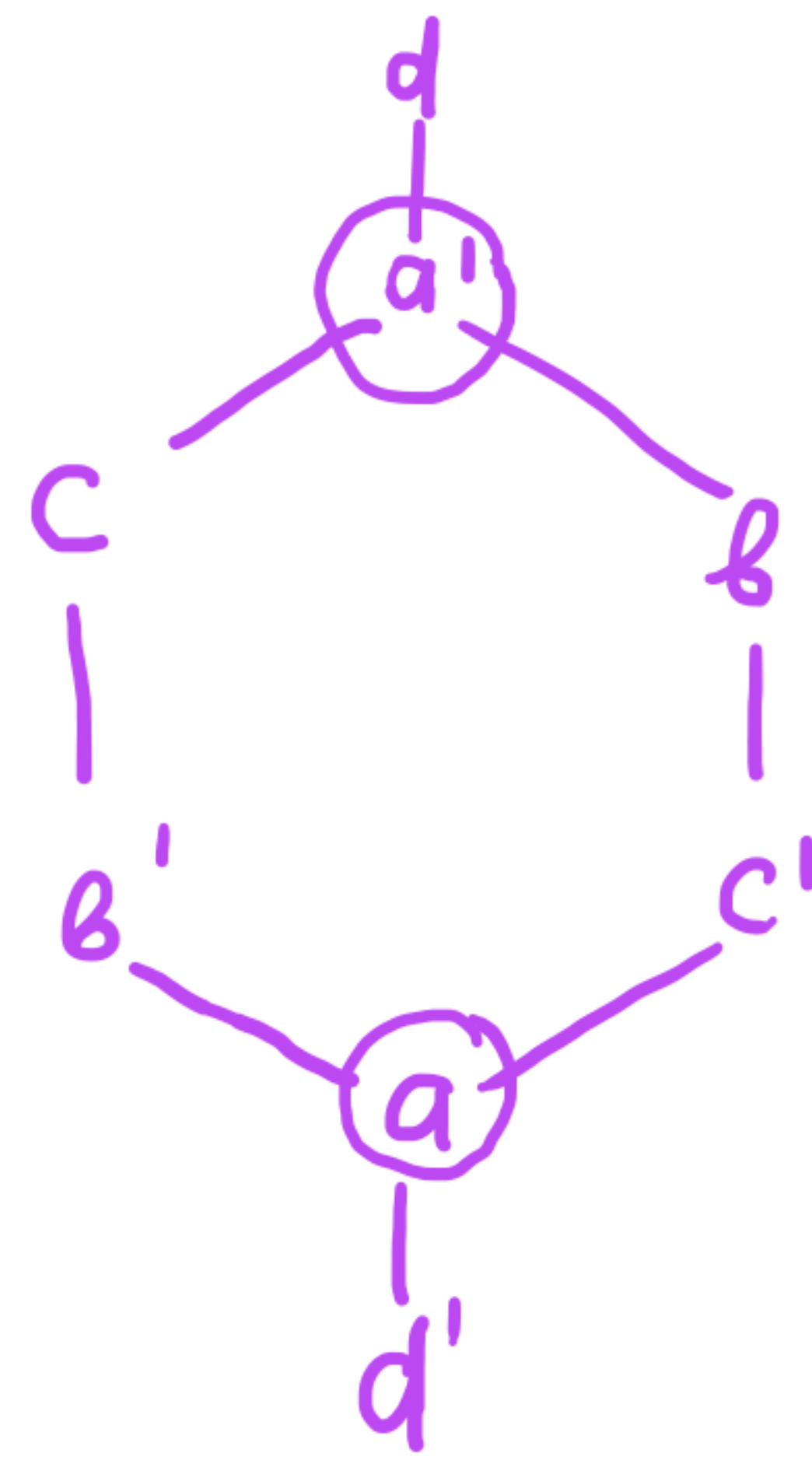
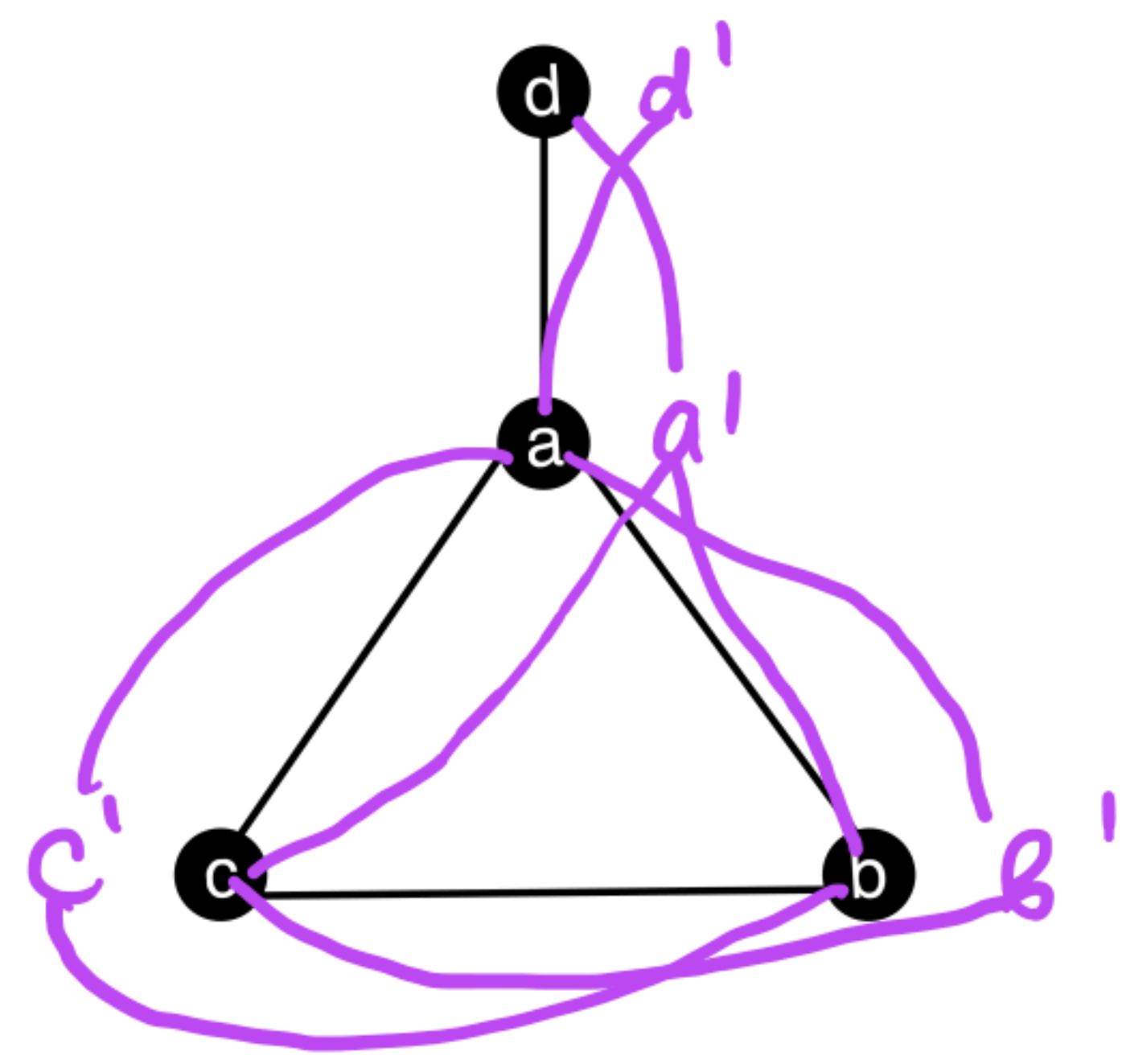
- Algorithm finds a maximal matching

orange nodes: if not matched, then all blue neighbors were matched

blue nodes: if not matched, then did not receive proposals

no matching edges could be added

Example: bipartite double cover



Correctness of the vertex cover algorithm

• Algorithm terminates

clearly, maximal matching terminates

• Algorithm computes a vertex cover

Idea: endpoints of any maximal matching give us a vertex cover

Proof by contradiction:

$G'$  Assume that there is an uncovered edge

$G \hookrightarrow$  we can add it to the matching  $\nexists$

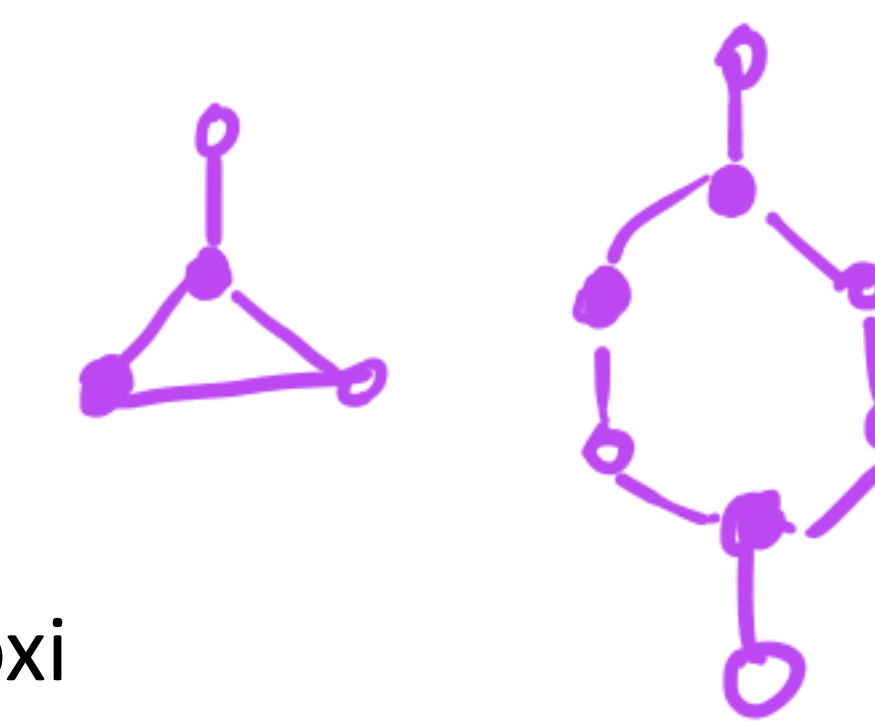


• ... it computes a 4-approximation of the minimum vertex cover



Idea: any maximal matching is a 2-approximation of the minimum vertex cover

a vertex cover must cover matching edges



$\leadsto$  4-approx

• ... it computes a 3-approx of the minimum vertex cover

