CARDIFF UNIVERSITY

EXAMINATION PAPER

Academic Year:	**2013/2014**
Examination Period	**AUTUMN**
Examination Paper Number:	**BST169**
Examination Paper Title:	**ECONOMETRICS**
Duration:	**3 HOURS**

Do not turn this page over until instructed to do so by the Senior Invigilator

Structure of Examination Paper:

There are 3 pages.
There are 5 questions in total.

There are no appendices

The maximum mark for the examination paper is 100%. The percentage for each question is given in parentheses.

Students to be provided with:

Answer book

Instructions to Students:

All questions must be answered.

The use of non-electronic translation dictionaries between English or Welsh and a foreign language bearing an appropriate departmental stamp is permitted in this examination.

Please turn over

1) For the linear regression model:
$y= X_1\beta_1 + x_2\beta_2+\varepsilon$
where $y = (y_1, y_2, \ldots, y_N)'$, X_1 is an $N \times K$ matrix of explanatory variables with ones in the first column, x_2 is an $N \times 1$ vector and $Var(\varepsilon)=\sigma^2 I_N$.

a) Find separately the OLS estimators of $\hat{\beta}_1$ and $\hat{\beta}_2$. (6%)

b) Define $X = [X_1, x_2]$, $M_{X1} = I - X_1(X_1'X_1)^{-1}X_1'$, $M_X = I - X(X'X)^{-1}X'$ and $M_0 = I - \iota(\iota'\iota)^{-1}\iota'$, where ι is an $N \times 1$ vector of ones. Show that $y'M_{X1}y = y'M_Xy + \frac{(y'M_{X1}x_2)^2}{x_2'M_{X1}x_2}$. (6%)

c) Show that the adjusted R-squared will rise when x_2 is excluded from the regression, i.e. $1-\frac{y'M_Xy/(N-K-1)}{y'M_0y/(N-1)} < 1-\frac{y'M_{X1}y/(N-K)}{y'M_0y/(N-1)}$, if the t statistic for H_0: $\beta_2=0$ has absolute value smaller than 1. (8%)

2) Explain the principle of Generalized Method of Moments (GMM) by an example. (8%) Are the estimators obtained from GMM unbiased in general? Illustrate your answer. (4%) Show how the hypothesis tests on linear restrictions of the model parameters can be implemented? (8%)

3) For the linear regression model,

$$y_i=\beta_0+\beta_1 x_i+\varepsilon_i, \qquad [3.1]$$
$$x_i=\alpha_0+ \alpha_1 z_i+ \alpha_2\varepsilon_i+u_i, \qquad [3.2]$$
$$i=1,2,..,N,$$

where ε_i and u_i are the unobserved errors, $E(\varepsilon_i|z_i)=0$, $E(u_i| z_i)=0$, $Var(z_i)=\gamma^2$, $Cov(\varepsilon_i, u_i)=0$, $Var(\varepsilon_i)=\sigma^2$ and $Var(u_i)=\omega^2$. y_i, x_i and z_i are the observed data. All parameters in both equations are scalars.

a) If we run an OLS regression in [3.1], show what the probability limits of $\widehat{\beta_1}$ and $\widehat{\sigma^2}$ are. (6%)

b) Consider the following regression
$$y_i=\beta_0+\beta_1 x_i+\beta_2\widehat{s_i}+v_i, \qquad [3.3]$$
where $\widehat{s_i}= x_i-\widehat{\alpha_0}-\widehat{\alpha_1}z_i$. $\widehat{\alpha_0}$ and $\widehat{\alpha_1}$ are the OLS estimates from [3.2] obtained by regressing x_i on a constant and z_i. Show that the OLS estimator from [3.3], $\widetilde{\beta_1}$ the same as the two stage least square estimator. (6%)

c) Calculate the probability limit of the estimator for $Var(v_i)$ in [3.3], i.e. $\frac{SSR}{N}$, where SSR is the sum of squared residuals from the OLS regression in [3.3]. (8%)

4) Consider the following linear regression model,

$$y_i=\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+\ldots+\beta_K x_{Ki}+\varepsilon_i, \quad i=1,2,..,N, \qquad [4.1]$$

a) Define $\varepsilon=(\varepsilon_1, \varepsilon_2,\ldots \varepsilon_N)'$. If $\varepsilon \sim N(0, \sigma^2 I_N)$, what distribution does $\frac{c'\varepsilon/\sqrt{c'c}}{\sqrt{\varepsilon' A\varepsilon/q}}$ A is an N $\times$N orthogonal projection matrix whose rank is q, c is an N $\times$1 vector and Ac=0. (4%)

b) If $\varepsilon \sim N(0, \sigma^2 I_N)$, what distribution does $\frac{\widehat{\beta_1}}{\sqrt{\frac{SSR}{SSR_1(N-K-1)}}}$ follow? Prove it. SSR is sum of squared residuals obtained in [4.1] and SSR_1 is obtained by regressing on a constant and other explanatory variables. (8%)

c) If ε_i does not follow normal distribution, would your answer for the previous question be the same given a large sample size? Discuss the assumptions required? (8%)

5) Suppose {Zi} is a sequence of independently and identically distributed (i.i.d.) random variables,

a) What are the conditions for $\frac{1}{N}\sum_{i=1}^{N} Z_i$ to converge to μ almost surely given that μ=E(Zi)? (4%)

b) $\{y_i\}$ is a sequence of i.i.d. random variables with density function $p(y_i|\theta)$. What are the regularity conditions if we want to estimate θ by maximum likelihood? (8%)

c) What is the asymptotic distribution for the maximum likelihood estimator $\widehat{\theta_{MLE}}$? Derive your results. (8%)