Session1-5 for Econometrics

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[1] "a string of characters"

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1 Session1-5	
1.1 Basic Objects	
1.1.1 list enviornment	
ls()# to list the current environment	
## character(0)	
1.2 Vector and Matrix	
1.2.1 Vector	
to create Vector based on $\mathbf{c}()$ function $\mathbf{c}()$ function is the function to create tuple or called Vector	
# $c()$: create a tuple, also can be seen a Vector if it is numberic tuple. because the tuple is $nv=c(6,5,4,3,2,1)$ $cv=c("a string of characters", "also a string of characters") nv[3]$	the lis
## [1] 4	
cv[1]	

```
vec=vector(mode = "list", length = 2) # in the mode, there are three options: "list", "experession" and "a
length(nv) #show the number of elements
## [1] 6
min(nv) #minimum value of nv
## [1] 1
which.min(nv) #the index of the minimum value of nv
## [1] 6
max(nv) #maximum value of nv
## [1] 6
which.max(nv) #the index of the maximum value of nv sort(nv, decreasing=TRUE) #list nv's elements of nv
## [1] 1
nv[2:5] #print 2-5 elements of nv
## [1] 5 4 3 2
the code for matrix is matrix():it look like a dataframe
m1=matrix(nv,nrow=3,ncol=2)
m1
        [,1] [,2]
##
## [1,]
## [2,]
           5
                2
## [3,]
          4
class(nv)#class() function is to show the class of target element, like the type function in Python
## [1] "numeric"
class(m1)
## [1] "matrix"
#show the dimension
attributes(nv)
## NULL
```

```
attributes(m1)# show the dimension of element:(row,colmn)
## $dim
## [1] 3 2
dim(m1)#another function can show the dimension as well:(row,colmn)
## [1] 3 2
m1[3,2] #print the (3, 2) element of m1
## [1] 1
m1[,2] #print the 2nd column of m1
## [1] 3 2 1
2*m1 #scalar multiplication
##
        [,1] [,2]
## [1,]
         12
## [2,]
         10
                4
## [3,]
diag(2) #identity matrix of dimension 2
##
        [,1] [,2]
## [1,]
         1
## [2,]
rep(1,2) #replicate 1 twice, this function can create the identity matrix
## [1] 1 1
cbind(rep(1,6),nv) #put two vectors vertically together to make a matrix
## [1,] 1 6
## [2,] 1 5
## [3,] 1 4
## [4,] 1 3
## [5,] 1 2
## [6,] 1 1
m3=m1\%*\%t(m1) #t() is to transpose a matrix and \%*\% is for matrix multiplication
qr(m3)#A=QR,use QR Decomposition: an orthogonal matrix Q and an upper triangular matrix R. QR decomposi
```

```
## $qr
##
                          [,2]
                                      [,3]
              [,1]
## [1,] -63.6396103 -51.1945310 -38.7494516
## [2,]
         0.5656854 -0.3464102 -0.6928203
## [3,]
         0.4242641
                     0.9517782
                                 0.0000000
##
## $rank
## [1] 2
##
## $qraux
## [1] 1.707107 1.306787 0.000000
## $pivot
## [1] 1 2 3
##
## attr(,"class")
## [1] "qr"
qr(m3)$rank #rank of m3
## [1] 2
diag(m3)#extract the diagonal of a matrix(m3)
## [1] 45 29 17
sum(diag(m3)) #trace of m3: the trace of an n-by-n square matrix A is defined to be the sum of the elem
## [1] 91
diag(m3)<-1#replace the diagonal of matrix with 1
solve(m3) #inverse of m3
              [,1]
                          [,2]
                                      [,3]
## [2,] 0.01385991 -0.01808246 0.02359662
## [3,] 0.01900149 0.02359662 -0.03216592
     Infinite Values and Missing Data
the computer software exist several limit on the precision format and 32-bit/64-bit limit
```

```
typeof(1.9)# another kind of class funtion with more detail in the numeric part, it categories the type
## [1] "double"
as.integer(1.9)# this function transfer the 1.9(double numeric) into the integer number
## [1] 1
```

```
is.integer(1.9) # this function judege the type of this number(a), return the pool value: (FASLE or TRUE)

## [1] FALSE

as.integer(2^31 - 1) #transfer (2^31 - 1) into integer number , larger than this number can not be cal

## [1] 2147483647

2^1024 #this is also too larger than the upper limits which is based on the 32-bit limit of software

## [1] Inf
```

1.4 Probability Related Functions

```
runif(n=10,min = 1,max = 10)# generate 10 random numbers uniformly from 1 to 10

## [1] 4.037550 5.823609 6.375857 6.889476 8.248140 2.389865 1.203745

## [8] 6.520246 3.269259 5.277858

rnorm(n=10,mean=0,sd=1)# generate 10 normally distributed random numbers with mean=0 and standard devia

## [1] 0.4269484 -0.7474745 1.5034915 0.5583608 2.2922638 -1.0136342

## [7] 0.2130808 1.3395750 -1.5583255 1.0500899

rt(n=10,df=2) # generate 10 random numbers from student t-distribution with df=2 degress of freedom

## [1] -1.0503787 10.5623084 0.3140259 0.2112197 -1.8549355 0.4716793

## [7] 1.0865071 0.5964313 -3.0527752 -1.7637586

dnorm(0.005) # return the density function value of standard normal distribution evaluated at 0.005

## [1] 0.3989373

pnorm(0.5) # return the cumulative distribution function of standard normal distribution eveluated at 0.005

## [1] 0.6914625

qnorm(p=0.25) # return the 0.25th quantile of standard normal distribution(0<=p<=1)

## [1] -0.6744898
```

1.5 Other Useful Objetcs

1.5.1 Factor object

organize the data point into different categories.- tag as character variable retag the data point with factor tag,

```
gender<-factor(c("male","male","male","female","female"))# create two level categories: male and female</pre>
levels(gender)# show the categories.
## [1] "female" "male"
1.5.2 Time Series Object
use ts() function to create a special vector with a time index.
ts(data, start=c(start-year, start-mounth), frequency) will create a series of random normally dir-
tributed numbers (n=100) and order them by mounths, and start mouths is april 1987 (12 mouths as a
circle; mounthly times series). if change the freq as 4, so it will create quarterly times series. other freq will
not be defined as times series, but based on year.
myts1=ts(data=rnorm(100), start=c(1987,4), freq = 12) #create a series of random normally dirtributed numb
tsp(myts1) # return the start, end time(in numeric format) and frequency
## [1] 1987.25 1995.50
                          12.00
1.5.3 List Object
use list objects(list()) to contain the data with different types/classes
lobj<-list(name="Jack", stid=123, score=100) # create a list with different attributes, it likes the dictio
lobj$name# will print the value of name in list of lobj
## [1] "Jack"
lobj[2]# will print the 2rd element, include the name and value
## $stid
## [1] 123
lobj[[2]] # will print the value of 2rd element, if there not exist the name, will use the order instea
## [1] 123
lobj2 =vector(mode="list",length = 2)# in the mode, there are three options: "list", "experession" and "
lobj2[[1]]<-lobj # put the lobj into the 2rd element of lobj2
lobj2[[1]]<-list(name="Jack",stid=123,score=100)# equivalently lobj2[[1]]<-lobj
lobj2[[2]] <-list(name="Mike",stid=245,score=60)</pre>
lobj2[[1]]$stid # the find student's ID number
## [1] 123
lobj2[[2]][[3]] # the second student's score
```

[1] 60

1.5.4 Dataframe Object

the dataframe is the most important function for data mining and data analysis, and it can help us to contain extern data file into R. and almost all the data will be handle in the dataframe use setwd() function to set up the working directory, that can help us more easy to import the data and #### set up working directory and import data

```
setwd("/Users/snOwfree/Dropbox/PhD(1st)/snOwfree.github.io/BST169_Econometrics/computing/session1-5")#s
london<-read.csv('london.csv')# import data from london.csv
class(london)

## [1] "data.frame"

typeof(london)</pre>
```

1.5.4.1 structure, head and colname

[1] "list"

now about look the structure of data, head of data and the label or colmn's name

str(london)# show the structure of data, include the types, observation and variables

```
## 'data.frame':
                   1519 obs. of 11 variables:
   $ X... : int 1 2 3 4 5 6 7 8 9 10 ...
   $ wfood : num 0.427 0.374 0.194 0.444 0.333 ...
   $ wfuel : num  0.1342 0.1686 0.4056 0.1258 0.0824 ...
   $ wcloth: num  0 0.0091 0.0012 0.0539 0.0399 ...
##
   $ walc : num 0.0106 0.0825 0.0513 0.0397 0.1571 ...
##
   $ wtrans: num  0.1458 0.1215 0.2063 0.0652 0.2403 ...
   $ wother: num  0.282 0.244 0.141 0.272 0.147 ...
##
   $ totexp: int 50 90 180 80 90 70 140 50 100 90 ...
##
   $ income: int
                  130 150 230 100 100 70 190 100 260 110 ...
                  25 39 47 33 31 24 46 25 30 41 ...
##
            : int
                  2 2 2 2 1 1 1 1 1 1 ...
   $ nk
            : int
```

head(london)# show first 6 row

```
X... wfood wfuel wcloth
                                 walc wtrans wother totexp income age nk
## 1
        1 0.4272 0.1342 0.0000 0.0106 0.1458 0.2822
                                                          50
                                                                130
                                                                     25
## 2
        2 0.3739 0.1686 0.0091 0.0825 0.1215 0.2444
                                                          90
                                                                150
                                                                     39
                                                                         2
        3 0.1941 0.4056 0.0012 0.0513 0.2063 0.1415
                                                         180
                                                                230
                                                                     47
        4 0.4438 0.1258 0.0539 0.0397 0.0652 0.2716
## 4
                                                          80
                                                                100
                                                                     33
                                                                         2
        5 0.3331 0.0824 0.0399 0.1571 0.2403 0.1473
                                                          90
                                                                100
                                                                     31
                                                                         1
## 6
        6 0.3752 0.0481 0.1170 0.0210 0.0955 0.3431
                                                          70
                                                                 70
                                                                     24
                                                                         1
```

colnames(london) # show the label or colmn's name, also called the variables' name

```
## [1] "X..." "wfood" "wfuel" "wcloth" "walc" "wtrans" "wother" ## [8] "totexp" "income" "age" "nk"
```

```
##
      [1] 0.0000 0.0091 0.0012 0.0539 0.0399 0.1170 0.0453 0.1131 0.1671
##
              NA 0.2547 0.1617 0.0018 0.0728 0.2010 0.0501 0.0722 0.0727
##
     [19] 0.1511 0.0454 0.0931 0.0246 0.0506 0.1451 0.0054 0.0180 0.2388
##
     [28] 0.0392 0.0788 0.3875 0.0077 0.0085 0.1310 0.1428 0.0458 0.2943
     [37] 0.0662 0.1530 0.2702 0.1808 0.1046 0.0033 0.0058 0.0419 0.0662
##
##
     [46] 0.0542 0.1605 0.1109 0.0000 0.0796 0.1156 0.1846 0.2889 0.1610
##
     [55] 0.0928 0.2845 0.1902 0.1102 0.0517 0.1495 0.1060 0.1012 0.0254
     [64] 0.2637 0.0155 0.0049 0.1491 0.0834 0.0098 0.1980 0.0147 0.1582
     [73] 0.0596 0.0419 0.1914 0.0200 0.0927 0.0217 0.2294 0.0017 0.2278
##
     [82] 0.1393 0.0144 0.0451 0.0000 0.0168 0.0306 0.0038 0.2158 0.1370
##
     [91] 0.0717 0.0834 0.0446 0.0000 0.1680 0.0000 0.0856 0.0674 0.3036
##
    [100] 0.0985 0.1172 0.0030 0.0184 0.0456 0.0693 0.1423 0.0213 0.0363
    [109] 0.1034 0.0755 0.0000 0.1856 0.0049 0.2395 0.2728 0.0613 0.0863
##
##
    [118] 0.1169 0.3933 0.0453 0.2002 0.1613 0.0616 0.0830 0.0623 0.0054
    [127] 0.1744 0.1345 0.0493 0.0579 0.0545 0.2106 0.1114 0.1848 0.1940
##
   [136] 0.0842 0.1344 0.1353 0.1261 0.0825 0.1731 0.1140 0.0985 0.0447
    [145] 0.0167 0.1440 0.2382 0.0126 0.1297 0.3082 0.0000 0.0452 0.0004
##
##
   [154] 0.0134 0.1747 0.0319 0.1465 0.0317 0.0110 0.0000 0.0981 0.0872
   [163] 0.1310 0.1583 0.0371 0.1875 0.0645 0.1037 0.0639 0.1647 0.0952
   [172] 0.1739 0.0714 0.0717 0.2046 0.0499 0.0260 0.0970 0.0599 0.0806
    [181] 0.0861 0.1077 0.1823 0.1086 0.0883 0.0510 0.1519 0.0398 0.1463
##
    [190] 0.0000 0.0155 0.0780 0.1368 0.2587 0.0030 0.1253 0.3148 0.0033
##
    [199] 0.0252 0.0509 0.0621 0.0109 0.0205 0.0932 0.0738 0.1237 0.0000
##
    [208] 0.0245 0.0136 0.0517 0.0173 0.2086 0.0991 0.0264 0.1863 0.0129
    [217] 0.0247 0.0751 0.1545 0.0447 0.0570 0.0233 0.2344 0.2543 0.0054
   [226] 0.2590 0.0888 0.0076 0.0000 0.1498 0.1533 0.0369 0.1174 0.0935
   [235] 0.0128 0.0000 0.1243 0.0673 0.2385 0.0000 0.0090 0.0612 0.0395
    [244] 0.1426 0.1055 0.1012 0.2756 0.0550 0.0542 0.1584 0.1196 0.1759
##
    [253] 0.0000 0.0483 0.1092 0.0317 0.0253 0.1200 0.0497 0.0030 0.2278
##
    [262] 0.1715 0.0787 0.0000 0.0805 0.1438 0.0992 0.1458 0.0451 0.0791
   [271] 0.0769 0.0000 0.0861 0.0032 0.1735 0.0907 0.0400 0.1325 0.0332
    [280] 0.0839 0.1406 0.2199 0.1409 0.0365 0.0499 0.2540 0.0054 0.0459
##
##
   [289] 0.2309 0.0039 0.0558 0.0306 0.0241 0.0074 0.0247 0.0255 0.0739
##
   [298] 0.0278 0.1066 0.0036 0.2119 0.1293 0.1089 0.0316 0.0332 0.0735
##
    [307] 0.1710 0.0957 0.2256 0.1929 0.0310 0.0058 0.0990 0.0215 0.0670
##
    [316] 0.0185 0.0080 0.1803 0.0890 0.0624 0.0039 0.0931 0.2183 0.0143
##
    [325] 0.0345 0.0671 0.3472 0.1578 0.2345 0.1626 0.0958 0.2317 0.0051
    [334] 0.0097 0.1050 0.0076 0.0000 0.0503 0.1922 0.1527 0.0946 0.3358
   [343] 0.0410 0.0501 0.3022 0.2068 0.2739 0.1492 0.1270 0.1206 0.0407
##
    [352] 0.0987 0.0980 0.2278 0.4057 0.0283 0.2279 0.0022 0.0021 0.1774
##
    [361] 0.0897 0.1733 0.0281 0.0785 0.0264 0.0816 0.1732 0.2212 0.1285
   [370] 0.0000 0.1788 0.3340 0.0401 0.0196 0.1217 0.1516 0.0105 0.0556
    [379] 0.1451 0.0482 0.0395 0.0693 0.0069 0.0856 0.1761 0.1004 0.2709
##
    [388] 0.1397 0.0804 0.1061 0.2733 0.2905 0.2035 0.1251 0.1241 0.0981
##
##
    [397] 0.0085 0.1777 0.1640 0.0000 0.1037 0.0275 0.0308 0.0495 0.0879
   [406] 0.0436 0.2345 0.2118 0.1671 0.0430 0.0000 0.0094 0.0899 0.1994
    [415] 0.0878 0.0100 0.0035 0.0320 0.1326 0.0131 0.0744 0.0705 0.1424
##
   [424] 0.2661 0.1269 0.0039 0.1201 0.0108 0.0389 0.0892 0.0330 0.0295
   [433] 0.0517 0.0498 0.0542 0.0690 0.3070 0.1726 0.1367 0.1939 0.1939
##
   [442] 0.0145 0.3252 0.0241 0.0550 0.0287 0.0651 0.0393 0.1118 0.0328
   [451] 0.1370 0.2926 0.1177 0.0234 0.0660 0.2948 0.1543 0.3283 0.0000
```

```
[460] 0.0544 0.0075 0.0000 0.3010 0.0116 0.0114 0.4293 0.3883 0.0553
    [469] 0.0217 0.0527 0.1686 0.0272 0.2200 0.1388 0.1021 0.2824 0.0189
##
    [478] 0.1566 0.1158 0.0946 0.1186 0.0605 0.0102 0.0345 0.0731 0.2002
    [487] 0.0853 0.2052 0.1642 0.0418 0.0051 0.0617 0.0052 0.0117 0.1409
##
    [496] 0.0355 0.2354 0.0120 0.0202 0.0727 0.0219 0.0069 0.1284 0.0040
    [505] 0.0017 0.3018 0.0645 0.0681 0.1285 0.0886 0.2214 0.0338 0.1649
##
    [514] 0.1013 0.0494 0.2073 0.0993 0.4169 0.4340 0.0000 0.1194 0.1113
    [523] 0.1014 0.0583 0.1235 0.3414 0.0052 0.1332 0.0092 0.0144 0.1371
##
##
    [532] 0.0157 0.0631 0.1298 0.0482 0.1672 0.2472 0.0000 0.0857 0.1746
    [541] 0.0881 0.0314 0.0472 0.0593 0.1313 0.0042 0.0000 0.2277 0.0021
##
    [550] 0.0023 0.0696 0.3152 0.1185 0.1557 0.0531 0.2338 0.1215 0.0536
    [559] 0.2163 0.1253 0.1033 0.0277 0.1105 0.0321 0.3703 0.0693 0.1924
##
    [568] 0.0048 0.2136 0.0000 0.0134 0.3270 0.0079 0.0906 0.0137 0.0000
    [577] 0.1324 0.2193 0.1358 0.0574 0.1613 0.0128 0.1385 0.0171 0.3181
    [586] 0.0593 0.0000 0.1879 0.2261 0.0339 0.0659 0.0881 0.0297 0.0463
##
    [595] 0.2694 0.1568 0.0395 0.0937 0.1088 0.0413 0.0539 0.0221 0.1333
    [604] 0.1506 0.1911 0.0028 0.0798 0.0461 0.0135 0.0102 0.0457 0.0054
##
    [613] 0.0990 0.0133 0.0881 0.1171 0.0183 0.1291 0.0412 0.0200 0.0028
    [622] 0.0314 0.2163 0.1608 0.0722 0.0584 0.0582 0.0956 0.0233 0.1609
##
    [631] 0.0942 0.2269 0.1650 0.1910 0.0494 0.0880 0.0000 0.1344 0.1025
##
    [640] 0.0000 0.0362 0.0148 0.2505 0.0114 0.0888 0.0974 0.0769 0.2404
    [649] 0.1119 0.2758 0.2433 0.1310 0.1184 0.0000 0.1461 0.0666 0.1734
    [658] \ 0.0098 \ 0.0755 \ 0.0972 \ 0.0000 \ 0.4282 \ 0.0721 \ 0.0658 \ 0.1143 \ 0.0860
##
    [667] 0.2098 0.1157 0.1302 0.0855 0.0451 0.2350 0.1038 0.2097 0.0000
##
    [676] 0.0842 0.0858 0.2600 0.0867 0.0296 0.0572 0.0112 0.0524 0.0676
##
    [685] 0.1697 0.1498 0.1566 0.0677 0.0090 0.0142 0.0596 0.1096 0.3897
    [694] 0.4039 0.2443 0.0616 0.0602 0.1741 0.2704 0.1163 0.0387 0.1991
##
    [703] 0.0239 0.0213 0.3842 0.1591 0.0322 0.0579 0.1565 0.0561 0.2043
    [712] 0.0713 0.0043 0.1927 0.1074 0.0854 0.0861 0.0000 0.0000 0.0691
    [721] 0.1442 0.1295 0.0000 0.0086 0.1283 0.0231 0.0172 0.0267 0.0623
##
    [730] 0.1982 0.0851 0.1302 0.0353 0.1034 0.1111 0.0678 0.0197 0.0337
##
    [739] 0.0048 0.1665 0.0416 0.0656 0.1603 0.0698 0.0200 0.0907 0.0935
    [748] 0.0947 0.0988 0.1147 0.3009 0.1472 0.3911 0.1041 0.0277 0.1816
    [757] 0.1376 0.0396 0.0288 0.0742 0.1270 0.0000 0.0619 0.0832 0.1752
##
    [766] 0.0072 0.0000 0.2805 0.1672 0.0081 0.1357 0.0348 0.1628 0.3721
##
    [775] 0.0625 0.1381 0.1706 0.0000 0.0403 0.1397 0.0985 0.3093 0.2034
##
    [784] 0.0689 0.1126 0.2445 0.0195 0.0237 0.0440 0.2673 0.0083 0.0570
##
    [793] 0.0000 0.0546 0.0110 0.2702 0.3519 0.1101 0.0079 0.1070 0.0944
    [802] 0.0324 0.1894 0.2239 0.1534 0.0995 0.0888 0.1290 0.1075 0.1035
##
    [811] 0.0878 0.5035 0.0812 0.0444 0.0000 0.0123 0.2948 0.0691 0.0572
##
    [820] 0.0210 0.0625 0.1818 0.1027 0.1743 0.0275 0.1977 0.0677 0.0420
    [829] 0.3106 0.0000 0.1211 0.1125 0.1100 0.0036 0.0623 0.4381 0.0248
##
    [838] 0.0477 0.0668 0.0000 0.0536 0.0210 0.0396 0.0493 0.3620 0.2331
    [847] 0.1566 0.0090 0.1053 0.1557 0.0791 0.1404 0.2086 0.0410 0.1234
##
    [856] 0.1163 0.2170 0.1714 0.1611 0.0781 0.0783 0.0000 0.2178 0.1690
    [865] 0.0141 0.1072 0.0671 0.1516 0.3622 0.2153 0.0038 0.0845 0.0040
##
    [874] 0.0000 0.0573 0.1859 0.1162 0.0000 0.0381 0.0642 0.2645 0.1988
##
    [883] 0.2198 0.3612 0.0240 0.0978 0.0996 0.1653 0.0117 0.0164 0.1303
    [892] 0.0094 0.0520 0.0251 0.2327 0.0824 0.2497 0.0000 0.1905 0.1797
##
    [901] 0.0646 0.1574 0.0267 0.1614 0.0080 0.0991 0.0853 0.1320 0.0019
    [910] 0.0503 0.0386 0.0062 0.1685 0.2202 0.0019 0.0408 0.0306 0.1546
##
    [919] 0.0285 0.1199 0.0000 0.1388 0.0000 0.1261 0.3058 0.2840 0.0649
##
##
    [928] 0.1306 0.0000 0.1833 0.0193 0.1555 0.0243 0.0461 0.1784 0.1752
    [937] 0.1707 0.2214 0.0259 0.1913 0.0759 0.2610 0.2882 0.1820 0.1289
```

```
[946] 0.0749 0.0550 0.0574 0.0502 0.1183 0.1753 0.1099 0.0592 0.1557
    [955] 0.1664 0.1760 0.1892 0.1523 0.0799 0.0212 0.0955 0.0728 0.0822
##
    [964] 0.0204 0.1180 0.0078 0.0552 0.2315 0.2306 0.4280 0.0345 0.0517
   [973] 0.1254 0.0164 0.3321 0.0764 0.0604 0.1145 0.1076 0.2357 0.0000
    [982] 0.0743 0.1091 0.2973 0.0464 0.1207 0.0378 0.0692 0.0000 0.0070
   [991] 0.0884 0.2602 0.1670 0.0789 0.2640 0.0740 0.1178 0.1485 0.1105
## [1000] 0.0495 0.1431 0.0192 0.0281 0.0449 0.0057 0.1692 0.0000 0.0266
## [1009] 0.0538 0.1888 0.0079 0.3188 0.7602 0.0937 0.0261 0.0119 0.0501
## [1018] 0.0032 0.0588 0.0416 0.1294 0.1868 0.1612 0.2364 0.0000 0.0517
## [1027] 0.3128 0.3284 0.1852 0.0982 0.2266 0.1059 0.0804 0.1844 0.2679
## [1036] 0.1386 0.0411 0.0120 0.1118 0.1950 0.0661 0.0354 0.0145 0.2622
## [1045] 0.1661 0.0735 0.0220 0.1125 0.1985 0.0000 0.0563 0.1319 0.0000
## [1054] 0.0335 0.0673 0.2093 0.0285 0.0484 0.0225 0.1932 0.0411 0.1723
## [1063] 0.3913 0.1613 0.0259 0.1851 0.0470 0.1259 0.0453 0.0088 0.3873
## [1072] 0.2734 0.1921 0.0595 0.1781 0.2069 0.1684 0.0764 0.0054 0.0148
## [1081] 0.0788 0.0802 0.1065 0.0508 0.0848 0.1533 0.2415 0.2660 0.0099
## [1090] 0.1241 0.4789 0.0808 0.0945 0.0361 0.0705 0.0501 0.1160 0.0431
## [1099] 0.1342 0.1683 0.0730 0.0174 0.1268 0.0000 0.0112 0.1245 0.1679
## [1108] 0.0743 0.0067 0.3573 0.0000 0.1575 0.1123 0.0620 0.0439 0.2444
## [1117] 0.2151 0.1262 0.2311 0.2090 0.0549 0.0108 0.2635 0.0412 0.0921
## [1126] 0.1630 0.1324 0.1125 0.1047 0.0485 0.0063 0.0073 0.0000 0.0953
## [1135] 0.0488 0.1418 0.0000 0.0244 0.1109 0.1532 0.0600 0.0440 0.0810
## [1144] 0.0000 0.1512 0.0204 0.0691 0.2310 0.0143 0.0000 0.1356 0.0708
## [1153] 0.2209 0.3643 0.0405 0.2610 0.3136 0.0322 0.0000 0.2243 0.1164
## [1162] 0.3520 0.0763 0.1290 0.0044 0.1303 0.1323 0.0000 0.1454 0.3624
## [1171] 0.1720 0.1033 0.0711 0.0000 0.0000 0.3622 0.0663 0.1479 0.0806
## [1180] 0.0000 0.1547 0.0876 0.1518 0.1438 0.0466 0.0342 0.6155 0.1900
## [1189] 0.0106 0.2177 0.1740 0.0484 0.1871 0.1122 0.0677 0.0653 0.0801
## [1198] 0.0779 0.0184 0.1719 0.0000 0.1491 0.3208 0.0460 0.0000 0.3157
## [1207] 0.0000 0.0344 0.0057 0.0000 0.0362 0.0763 0.1267 0.0695 0.0000
## [1216] 0.1473 0.1923 0.2847 0.0745 0.1615 0.1738 0.2082 0.1155 0.1863
## [1225] 0.0170 0.0000 0.1450 0.1360 0.1345 0.0678 0.1197 0.1736 0.2642
## [1234] 0.0531 0.3034 0.1731 0.1145 0.0122 0.1586 0.1803 0.0738 0.1100
## [1243] 0.0613 0.4454 0.0354 0.1184 0.2160 0.0029 0.1445 0.1153 0.0000
## [1252] 0.1706 0.1050 0.3499 0.0112 0.0578 0.0512 0.0721 0.3145 0.0612
## [1261] 0.2210 0.0000 0.0358 0.2305 0.1592 0.0786 0.0755 0.1184 0.2754
## [1270] 0.0748 0.1196 0.0800 0.1648 0.0200 0.0000 0.0766 0.0363 0.2508
## [1279] 0.3349 0.1539 0.0879 0.3341 0.0062 0.0241 0.1050 0.0577 0.0807
## [1288] 0.0000 0.2620 0.1258 0.0916 0.0625 0.0987 0.2052 0.0555 0.0000
## [1297] 0.0051 0.1943 0.1136 0.1246 0.0719 0.1495 0.3718 0.1912 0.0000
## [1306] 0.1827 0.1989 0.3034 0.0738 0.0250 0.0337 0.0250 0.1438 0.0221
## [1315] 0.0012 0.0022 0.0000 0.1171 0.0685 0.0000 0.0100 0.1035 0.0000
## [1324] 0.3053 0.1827 0.1154 0.1969 0.0946 0.1306 0.1006 0.0752 0.0682
## [1333] 0.0062 0.0000 0.0046 0.0526 0.0378 0.1534 0.0438 0.1780 0.2310
## [1342] 0.0169 0.0658 0.1863 0.0856 0.1431 0.0271 0.0498 0.0682 0.1253
## [1351] 0.1556 0.1132 0.1179 0.3404 0.0000 0.0659 0.0572 0.1118 0.1492
## [1360] 0.0164 0.0936 0.0000 0.2092 0.0265 0.1368 0.0247 0.0000 0.1978
## [1369] 0.1244 0.1198 0.0350 0.0000 0.0767 0.1265 0.0527 0.2224 0.0045
## [1378] 0.0768 0.0035 0.1941 0.1878 0.0837 0.2087 0.1523 0.0026 0.2892
## [1387] 0.0038 0.0713 0.0094 0.0656 0.1082 0.1262 0.1319 0.1906 0.2457
## [1396] 0.0000 0.1707 0.1759 0.1350 0.0862 0.0419 0.1765 0.0925 0.4824
## [1405] 0.0034 0.1255 0.0044 0.1952 0.0538 0.0910 0.2430 0.0455 0.1410
## [1414] 0.1804 0.0714 0.1858 0.0289 0.1689 0.1712 0.0000 0.0576 0.2003
## [1423] 0.0396 0.1115 0.0167 0.1045 0.0341 0.0000 0.1231 0.0506 0.1288
```

```
## [1432] 0.3873 0.2335 0.0719 0.2019 0.3601 0.1015 0.0655 0.1856 0.0000 ## [1441] 0.0107 0.2918 0.0990 0.0972 0.1368 0.0315 0.1410 0.3747 0.0250 ## [1450] 0.0671 0.2153 0.0206 0.0196 0.0320 0.0745 0.1465 0.1965 0.1284 ## [1459] 0.1329 0.0000 0.0764 0.0028 0.0105 0.2763 0.0000 0.1138 0.0109 ## [1468] 0.0247 0.0334 0.1162 0.0292 0.0782 0.1205 0.0122 0.1349 0.0258 ## [1477] 0.0201 0.2029 0.0811 0.1291 0.0492 0.0248 0.2538 0.0347 0.0000 ## [1486] 0.1072 0.1481 0.0233 0.1730 0.3352 0.1821 0.0310 0.1103 0.1106 ## [1495] 0.1131 0.1292 0.2737 0.0000 0.0346 0.1544 0.0674 0.0430 0.0119 ## [1504] 0.0726 0.0114 0.3681 0.1715 0.0315 0.0089 0.0402 0.1740 0.0899 ## [1513] 0.2391 0.0000 0.0054 0.0176 0.2566 0.0787 0.1279
```

london[,1]=as.character(london[,1])# reset the 1st column as.character

1.5.5 Some Statistical Functions

Some useful Statistical Functions

```
sum(london$income) #sum of the income of all obervations in london dataset

## [1] 206960

mean(london$income)

## [1] 136.2475

var(london$income)

## [1] 3728.466

sd(london$income)

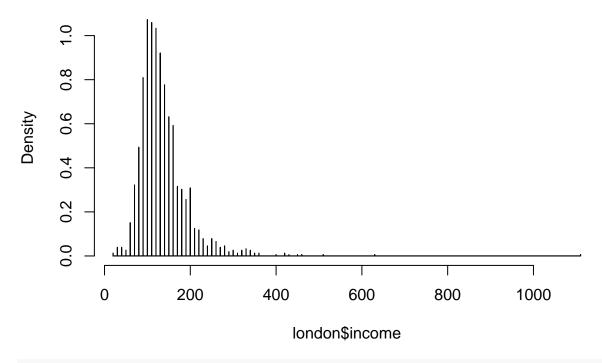
## [1] 61.06116

quantile(london$income)
```

0% 25% 50% 75% 100% ## 20 100 120 160 1110

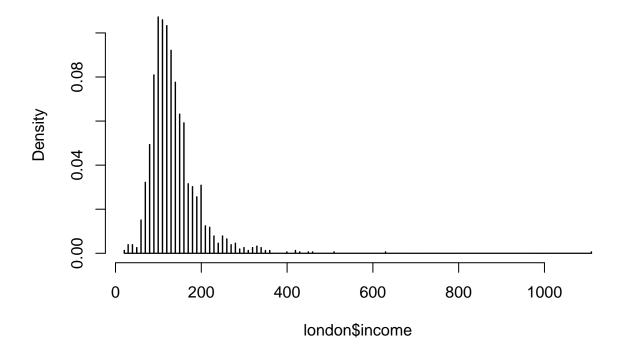
hist(london\$income, breaks=10000, freq = FALSE)# draw a histogram based on london\$income, the breaks is t

Histogram of london\$income

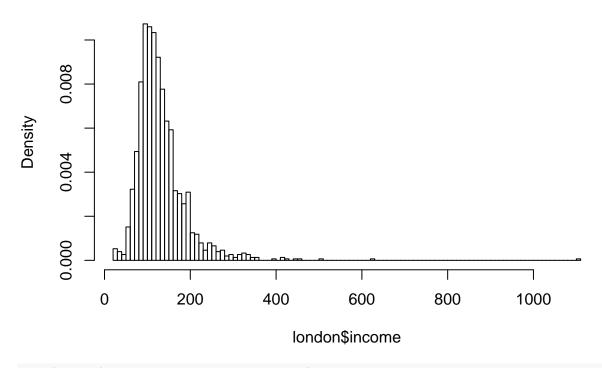


hist(london\$income,breaks=1000,freq = FALSE)

Histogram of london\$income

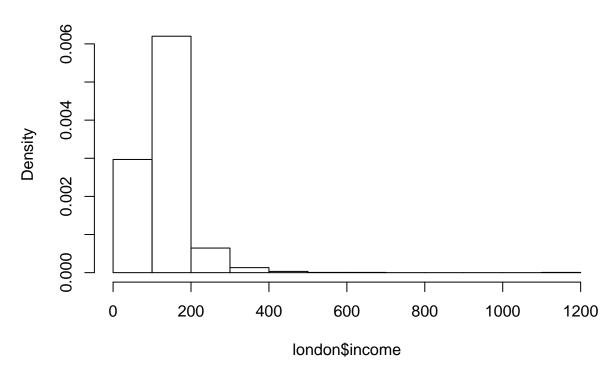


Histogram of london\$income

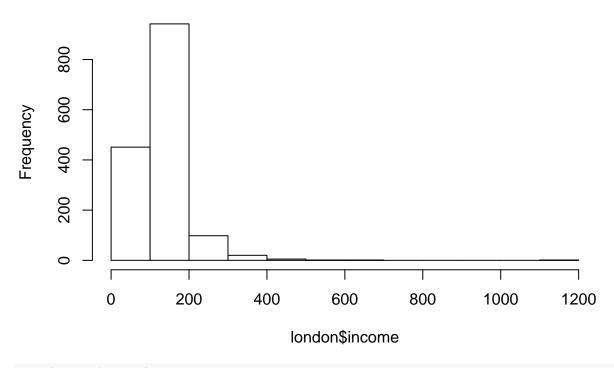


hist(london\$income,breaks=10,freq = FALSE)

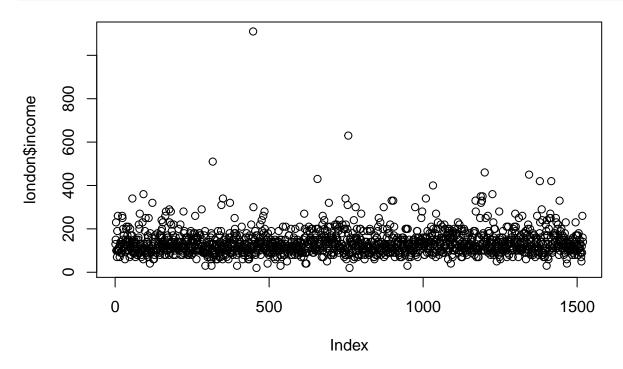
Histogram of london\$income



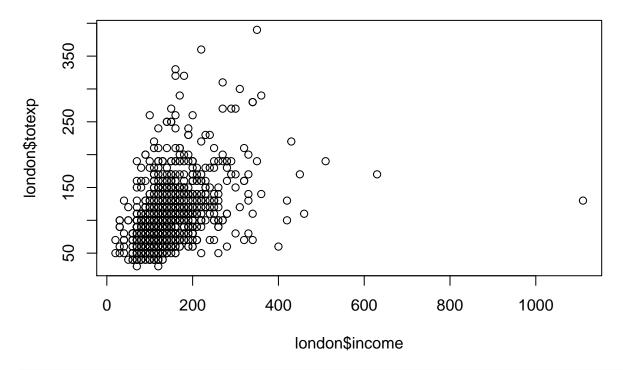
Histogram of Iondon\$income



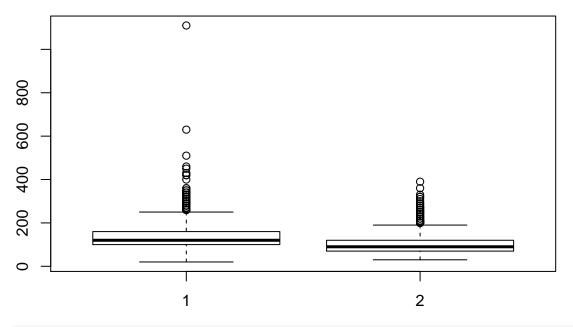
plot(london\$income)



plot(london\$income,london\$totexp)

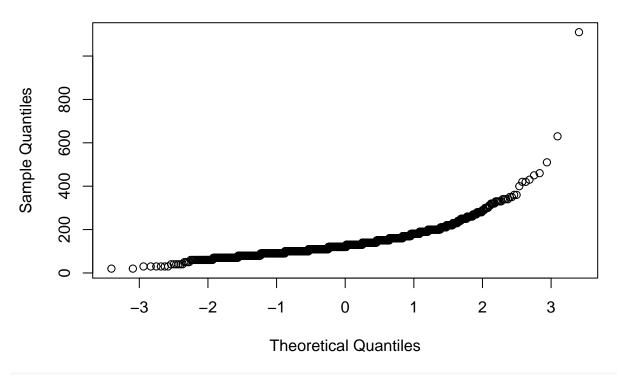


boxplot(london\$income,london\$totexp)#draw a boxplot



qqnorm(london\$income)#plot the qq-plot

Normal Q-Q Plot



which(london\$income>140) #return the indices for observations with income higher than 140.

##	[1]	2	3	7	9	23	24	25	26	28	34	36	37	40	43
##	[15]	44	49	51	55	56	62	69	75	76	77	79	80	89	92
##	[29]	97	99	106	107	109	114	119	121	130	131	138	145	147	148
##	[43]	150	151	152	155	161	163	165	169	170	172	173	176	177	181
##	[57]	182	184	188	190	192	197	206	209	212	222	224	225	228	230
##	[71]	233	234	238	244	246	248	255	258	259	260	262	265	268	273
##	[85]	274	276	277	281	283	289	292	310	312	315	317	325	334	340
##	[99]	344	345	350	353	355	358	369	372	373	374	384	386	387	388
##	[113]	389	391	392	395	402	404	411	419	424	425	427	433	437	442
##	[127]	447	448	449	453	455	458	463	471	472	476	478	479	481	482
##	[141]	485	486	488	492	496	497	506	507	519	531	534	540	545	553
##	[155]	554	564	565	571	580	582	585	592	599	611	614	615	616	619
##	[169]	623	629	631	632	633	634	635	637	640	643	644	649	651	653
##	[183]	657	658	659	666	668	669	670	672	673	674	678	679	680	681
##	[197]	682	683	684	689	691	694	695	697	698	699	700	702	706	707
##	[211]	708	714	716	717	718	719	720	722	723	725	728	729	736	738
##	[225]	742	746	748	749	753	754	755	756	757	762	765	771	775	780
##	[239]	781	783	784	785	786	794	795	796	797	798	799	805	819	827
##	[253]	828	829	843	844	849	850	853	854	856	861	869	872	882	883
##	[267]	885	888	890	893	898	900	901	903	905	906	912	913	917	927
##	[281]	928	932	934	935	940	943	946	950	955	968	971	974	976	980
##	[295]	981	983	986	992	994	995	997	999	1006	1009	1011	1012	1014	1021
##	[309]	1022	1026	1028	1029	1032	1033	1039	1042	1043	1045	1046	1048	1049	1050
##	[323]	1057	1058	1061	1062	1064	1066	1069	1071	1074	1077	1079	1080	1083	1084
##	[337]	1085	1086	1089	1092	1093	1097	1098	1100	1101	1107	1108	1110	1111	1112
##	[351]	1113	1115	1118	1119	1125	1126	1133	1135	1138	1145	1151	1155	1160	1161
##	[365]	1164	1166	1168	1170	1171	1176	1181	1185	1187	1188	1190	1191	1193	1195

```
## [379] 1197 1198 1200 1203 1204 1208 1210 1212 1214 1216 1218 1224 1225 1228 ## [393] 1230 1231 1233 1238 1247 1249 1250 1256 1258 1259 1271 1273 1276 1282 ## [407] 1286 1287 1288 1293 1296 1297 1298 1299 1300 1301 1306 1311 1312 1317 ## [421] 1322 1325 1326 1327 1331 1332 1333 1338 1340 1341 1344 1346 1347 1350 ## [435] 1356 1358 1361 1365 1368 1369 1370 1378 1379 1380 1381 1385 1387 1394 ## [449] 1398 1400 1402 1403 1404 1408 1411 1413 1414 1415 1416 1422 1424 1426 ## [463] 1427 1428 1430 1431 1433 1436 1438 1442 1443 1445 1446 1449 1450 1452 ## [477] 1453 1456 1458 1460 1463 1473 1483 1487 1495 1496 1500 1503 1505 1508 ## [491] 1517 1518
```

cov(london\$income,london\$totexp)# show the sample covariance between income and total expenditure.

[1] 1183.511

cor(london[,2:11])#sample correlation matrix for the 2-11 variables(columns) in london

```
##
                wfood
                           wfuel wcloth walc
                                                   wtrans
                                                                wother
## wfood
          1.00000000 0.10156032
                                     NA
                                          NA -0.333761601 -0.353580774
          0.10156032 1.00000000
## wfuel
                                     NA
                                          NA -0.160910408 -0.132764775
## wcloth
                  NA
                              NA
                                      1
                                          NA
                                                       NA
                                                                    NA
## walc
                  NA
                                                                    NΑ
                                     NA
                                           1
                                                       NA
                                          NA 1.000000000 -0.295970628
## wtrans -0.33376160 -0.16091041
                                     NA
## wother -0.35358077 -0.13276478
                                     NA
                                          NA -0.295970628 1.000000000
## totexp -0.47874726 -0.31937891
                                     NA
                                          NA 0.148271628 0.157614011
## income -0.23466674 -0.02870070
                                          NA 0.008442807 0.153370720
                                     NA
## age
          0.02140373 -0.03951842
                                     NA
                                          NA 0.026928761 0.026064288
## nk
          0.10178633 -0.02723013
                                     NA
                                          NA -0.043608816 -0.005305898
##
              totexp
                           income
                                           age
## wfood -0.47874726 -0.234666736 0.021403728 0.101786327
## wfuel -0.31937891 -0.028700703 -0.039518424 -0.027230131
## wcloth
                  NA
                               NA
                                            NA
## walc
                  NA
                               NA
                                            NA
                                                         NΑ
## wtrans
          0.14827163 0.008442807
                                   0.026928761 -0.043608816
          0.15761401 0.153370720
                                   0.026064288 -0.005305898
## wother
## totexp
          1.00000000 0.448740348
                                   0.189450718 0.071415282
## income
          0.44874035 1.000000000
                                   0.218494045
                                                0.025439105
## age
          0.18945072 0.218494045
                                   1.000000000
                                                0.008092095
          0.07141528 0.025439105 0.008092095
                                               1.000000000
## nk
```

summary(london[,2:11]) #summary some statistics for the 2-11 variables(columns) in london

```
##
        wfood
                         wfuel
                                           wcloth
                                                               walc
  Min.
          :0.0571
                     Min.
                            :0.00000
                                       Min.
                                              :0.00000
                                                         Min.
                                                                 :0.00000
  1st Qu.:0.2817
                     1st Qu.:0.05530
                                       1st Qu.:0.03220
                                                         1st Qu.:0.01252
## Median :0.3540
                     Median :0.08000
                                       Median :0.08575
                                                         Median: 0.04220
##
  Mean
           :0.3565
                            :0.09101
                                       Mean
                                              :0.10729
                     Mean
                                                         Mean
                                                                 :0.06057
   3rd Qu.:0.4258
                     3rd Qu.:0.11365
                                       3rd Qu.:0.15827
                                                          3rd Qu.:0.08983
                                       Max.
##
  \mathtt{Max}.
           :0.7890
                     Max.
                            :0.48030
                                               :0.76020
                                                         Max.
                                                                 :0.42810
##
                                       NA's
                                              :1
                                                          NA's
                                                                 :1
##
        wtrans
                          wother
                                           totexp
                                                            income
           :0.00000
                            :0.0361
                                       Min. : 30.0
  Min.
                      Min.
                                                       Min. : 20.0
  1st Qu.:0.05695
                                       1st Qu.: 70.0
                      1st Qu.:0.1785
                                                       1st Qu.: 100.0
```

```
## Median :0.11530 Median :0.2397
                                      Median: 90.0
                                                      Median : 120.0
         :0.13235 Mean :0.2523
                                      Mean : 98.7
## Mean
                                                       Mean
                                                            : 136.2
   3rd Qu.:0.17940
                     3rd Qu.:0.3100
                                       3rd Qu.:120.0
                                                       3rd Qu.: 160.0
## Max.
          :0.76380
                             :0.8066
                                       Max.
                                              :390.0
                                                       Max.
                                                              :1110.0
                     {\tt Max.}
##
##
        age
                         nk
##
  Min.
         :19.00
                  Min.
                          :1.000
##
   1st Qu.:30.00
                   1st Qu.:1.000
## Median :35.00
                   Median :2.000
## Mean
         :35.78
                   Mean :1.609
  3rd Qu.:40.00
                    3rd Qu.:2.000
          :60.00
                          :2.000
## Max.
                   Max.
##
colSums(london[,2:11]) # return the column sums of the 2-11 variables in
##
         wfood
                     wfuel
                                wcloth
                                              walc
                                                                    wother
                                                        wtrans
##
     541.4616
                  138.2480
                                    NA
                                                NA
                                                      201.0404
                                                                  383.3165
##
                    income
        totexp
                                   age
                                                nk
## 149920.0000 206960.0000 54348.0000
                                         2444.0000
#rowSums(london[2:11,])# return the row sums of the 2-11 rows in london
Question 1: Can you find out which variables have missing obervation(s)? in what positions?
1st: use manual way to seek the missing data
#colSums(london)# 2 columns:wcloth and walc
#rowSums(london)# 2 rows: 10 and 15
london$wcloth[10] #NA
## [1] NA
london$walc[15] #NA
## [1] NA
is.na(london$wcloth[1:20])
## [1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE TRUE FALSE
## [12] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
2rd: use auto way to seek the missing data
setwd("/Users/sn0wfree/Dropbox/PhD(1st)/sn0wfree.github.io/BST169_Econometrics/computing/session1-5")#s
london<-read.csv('london.csv')# import data from london.csv</pre>
compare_missing_data_function<-function(symbol){</pre>
  b<-c()
  for (i in 1:length(symbol)){
```

```
if(is.na(symbol[i])==TRUE){
      b=append(b,i)
    }
 }
  return(b)
seek_missing_data<- function(data){</pre>
  missing_data_location<-c()
  missing_col_number<-compare_missing_data_function(colSums(data))
  missing_row_number<-compare_missing_data_function(rowSums(data))
  for(i in missing_col_number){
    for(j in missing_row_number){
      if (is.na(data[j,i])==TRUE){
        missing_data_location<-append(missing_data_location,c(names(data)[i],j))}</pre>
      }
 }
  return(missing_data_location)
}
seek_missing_data(london)
```

```
## [1] "wcloth" "10" "walc" "15"
```

1.6 Least Squares Regression

1.6.1 OLS

```
lmobj<-lm(wfood~log(totexp)+log(income),data=london)
summary(lmobj)#summary of the fitted model</pre>
```

```
##
## lm(formula = wfood ~ log(totexp) + log(income), data = london)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
## -0.33339 -0.06201 -0.00229 0.06075 0.32267
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.964691 0.033414 28.871
                                            <2e-16 ***
## log(totexp) -0.133104
                          0.006924 -19.224
                                             <2e-16 ***
                                             0.827
## log(income) -0.001564
                         0.007142 -0.219
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.09139 on 1516 degrees of freedom
## Multiple R-squared: 0.2452, Adjusted R-squared: 0.2442
## F-statistic: 246.2 on 2 and 1516 DF, p-value: < 2.2e-16
```

```
typeof(lmobj)# is a list
## [1] "list"
class(lmobj)# is a "lm"
## [1] "lm"
typeof(lmobj) and class(lmobj) have different return value
coef(lmobj)#estimated model parameters (estimated_beta of OLS)
## (Intercept) log(totexp) log(income)
## 0.964691151 -0.133104065 -0.001564379
head(resid(lmobj)) #estimated residuals M_{x,y}
##
                                                                  5
## -0.009170321 0.015990337 -0.070880272 0.069578635 -0.025443964
## -0.017352911
head(fitted(lmobj)) #fitted values of the model (P_{x,y})
## 0.4363703 0.3579097 0.2649803 0.3742214 0.3585440 0.3925529
deviance(lmobj) #the residual sum of squares (SSR or y'M_{x,y})
## [1] 12.66322
confint(lmobj) #confidence interval
                     2.5 %
                                97.5 %
## (Intercept) 0.89914931 1.03023299
## log(totexp) -0.14668549 -0.11952264
## log(income) -0.01557339 0.01244463
logLik(lmobj) #value of the log likelihood function (assume normal er- ror)
## 'log Lik.' 1480.439 (df=4)
AIC(lmobj,k=2) #information criterion, k=2 for AIC and k=log(N) for BIC (assume normal error, to be
## [1] -2952.879
```

vcov(lmobj)# estimated-beta OLS's estimated variance-covariance matrix (\espison^{hat}_{2}(X'X)_{-1})

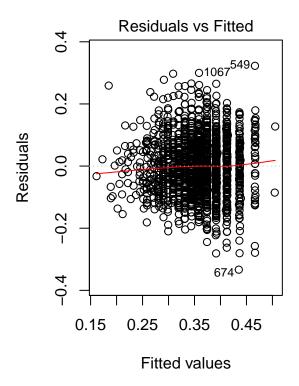
```
## (Intercept) log(totexp) log(income)
## (Intercept) 1.116471e-03 -9.885454e-05 -1.373587e-04
## log(totexp) -9.885454e-05 4.794031e-05 -2.427202e-05
## log(income) -1.373587e-04 -2.427202e-05 5.100646e-05
```

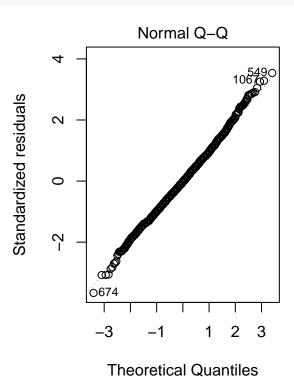
anova(lmobj) #returns an anova table; ANOVA analysis

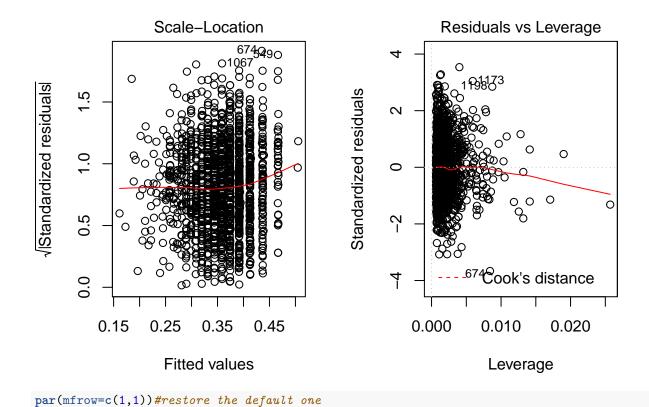
predict(lmobj,data.frame(totexp=1,income=1)) #returns predictions

1 ## 0.9646912

```
par(mfrow=c(1,2))#par graphes by 2*2
plot(lmobj) #create diagnostic plots
```







Question 2: Can you use matrix algebra in R to reproduce the outputs of the following functions:

- 1. coef(lmobj)
- 2. resid(lmobj)
- 3. fitted(lmobj)
- 4. deviance(lmboj)
- 5. vcov(lmboj)

Equation:

1). the normal form:

$$wfood = \beta_0 + \beta_1 * totexp + \beta_2 * income + \epsilon$$

2). the matrix form:

$$wfood = x * \beta + \epsilon$$

here,

$$x = |\beta_0 \quad totexp \quad income|$$

because the matrix form can be easy solved out thus, i show the solution of this OLS methods: the OLS mainly aims to solve the β , based on the minimum of Sum Squared Residuals(SSR): in normal form

$$SSR = \sum_{i=1}^{N} \epsilon^2$$

in matrix form

$$SSR = (wfood - x\beta)' * (wfood - x\beta)$$

and minimize the SSR

$$\min SSR = \min (wfood - x\beta)' * (wfood - x\beta)$$

for minimum, i make the 1st different should as zero. Thus,

$$\frac{\mathrm{d}(wfood-x\beta)'*(wfood-x\beta)}{\mathrm{d}\beta} = 0$$

$$\frac{\mathrm{d}(wfood-x\beta)'*(wfood-x\beta)}{\mathrm{d}\beta} = \frac{\mathrm{d}(wfood'-\beta'x')*(wfood-x\beta)}{\mathrm{d}\beta} = 0$$

$$\frac{\mathrm{d}(wfood-x\beta)'*(wfood-x\beta)}{\mathrm{d}\beta} = \frac{\mathrm{d}(wfood'*wfood-2\beta'x'y+\beta'x'x\beta)}{\mathrm{d}\beta} = 0$$

here, the wfood'*wfood and $\beta'x'y$ are non-beta martix or single-beta matrix, after differentiate on β , there will left 0 and the coefficient of single-beta:x'y. Thus,

$$\frac{\mathrm{d}(wfood - x\beta)' * (wfood - x\beta)}{\mathrm{d}\beta} = 0 - 2x'y + 2x'x\beta = 0$$

and

$$\beta = (x'x)^{-1}x'y$$

that is the solution of beta from OLS methods

```
 x < -cbind(rep(1, length(london\$wfood)), log(london\$totexp), log(london\$income)) \# create the martix x in the coef_of_lmboj < -solve(t(x)%*%x)%*%t(x)%*%london$wfood#coef(lmobj) coef(lmobj)
```

```
## (Intercept) log(totexp) log(income)
## 0.964691151 -0.133104065 -0.001564379
```

```
coef_of_lmboj
```

```
## [,1]
## [1,] 0.964691151
## [2,] -0.133104065
## [3,] -0.001564379
```

```
fitted_of_lmobj<-x%*%coef_of_lmboj#fitted(lmobj)
head(fitted(lmobj))</pre>
```

```
head(fitted_of_lmobj)
##
             [,1]
## [1,] 0.4363703
## [2,] 0.3579097
## [3,] 0.2649803
## [4,] 0.3742214
## [5,] 0.3585440
## [6,] 0.3925529
resid_of_lmobj<-london$wfood-x%*%coef_of_lmboj#resid(lmobj)</pre>
head(resid(lmobj))
                                         3
                                                                   5
## -0.009170321 0.015990337 -0.070880272 0.069578635 -0.025443964
## -0.017352911
head(resid_of_lmobj)
                [,1]
##
## [1,] -0.009170321
## [2,] 0.015990337
## [3,] -0.070880272
## [4,] 0.069578635
## [5,] -0.025443964
## [6,] -0.017352911
deviance_of_lmobj<-t(resid_of_lmobj)%*%resid_of_lmobj#just calulate the SSR,equal to deviance(lmobj)
deviance(lmobj)
## [1] 12.66322
deviance_of_lmobj
##
            [,1]
## [1,] 12.66322
vcov(lmobj)
                 (Intercept)
                               log(totexp)
                                             log(income)
## (Intercept) 1.116471e-03 -9.885454e-05 -1.373587e-04
## log(totexp) -9.885454e-05 4.794031e-05 -2.427202e-05
## log(income) -1.373587e-04 -2.427202e-05 5.100646e-05
```