CARDIFF UNIVERSITY

EXAMINATION PAPER

Academic Year: **2011/2012**

Semester: **AUTUMN**

Module Code: **BST169**

Module Title: **ECONOMETRICS**

Duration: **3 Hours**

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Structure of Examination Paper:

There are 4 pages.
There are 8 questions in total.

The maximum mark for the examination paper is 100%.

Students to be provided with:

Answer book

Instructions to Students:

Answer **FOUR** questions.

The use of a translation dictionary between English or Welsh and another language, provided that it bears an appropriate departmental stamp, is permitted in this examination.

Please turn over

1. Consider the model $y=X\beta+u$ where y is a $T\times 1$ vector, X is a $T\times K$ matrix, β is a $K\times 1$ vector and u is a vector of mean zero, constant variance, uncorrelated random errors. An investigator wishes to test the hypothesis that the model is subject to q restrictions summarised as $R\beta=r$.

   (a) Determine the Restricted Least Squares estimator of β and provide a Wald test for the validity of the restrictions based on the value of the associated Lagrange Multiplier.

   (b) Derive a test for the null hypothesis based on a comparison of the residual sums of squares of the restricted and the unrestricted regressions.

   (c) Reformulate the model so that the validity of the restrictions can be tested via q exclusion restrictions.

   (d) Comment on the large sample justification of these tests.

2. Consider the regression model

$$y = X\beta + u$$

where the usual assumptions hold except that

$$E\left(uu'\right) = \Omega$$

where Ω is a positive definite matrix.

   (a) Examine the properties of the Ordinary Least Squares estimator and compare them to the Generalised Least Squares Estimator.

   (b) Suppose that Ω is a diagonal matrix and let the t^{th} term on the diagonal be ω_t^2. What is the Maximum Likelihood estimator of β?

   (c) Describe how one might carry out a Lagrange Multiplier test for the hypothesis that the ω_t are constant.

3. Consider the model $y_t = x_t'\beta + u_t$ where $u_t \sim IN(0, \sigma^2)$ and suppose we want to estimate the value of a linear combination, $\mu = c'\beta$, of the regression parameters using a linear function, $\hat{\mu} = a'y$, of the observed y.

   (a) Find the mean and variance of $a'y$, What condition must be fulfilled for this to be an unbiased estimator of $c'\beta$?

   (b) Show that the minimum variance unbiased estimator of μ is $\hat{\mu} = c'\hat{\beta}$, where $\hat{\beta}$ is the least square estimator of β.

   (c) Let $y_{T+j}^f = x_{T+j}'\hat{\beta}$ be a forecast value of based on a known vector of regressors, x_{T+j}'. Find the mean and variance of the forecast error, $y_{I+j} - y_{T+j}^f$.

   (d) Comment on the factors which influence the accuracy of forecasts based on such a regression.

4. When is a regression model 'well-specified'? What range of test would you carry out to test for correct specification?

5. Consider the model

$$y = X\beta_1 + Z\beta_2 + u \quad u \sim N\left(0, \sigma^2 I\right)$$

Let b_1^U and b_2^U be the (unrestricted) *OLS* estimators of β_1 and β_2. Let b_1^R be the restricted estimator of β_1 obtained by regressing y on the columns of X.

   (a) Show that the relationship between the restricted and unrestricted estimator of β_1 can be written $b_1^R = b_1^U + Qb_2^U$ for some matrix, Q. Comment on this result.

   (b) Show that the variance, $V\left(b_1^R\right)$, of the restricted estimator is no larger than the variance, $V\left(b_1^U\right)$ of the unrestricted estimator in the sense that $V\left(b_1^U\right) - V\left(b_1^R\right)$ is a positive semidefinite matrix. (Hint: It is simpler to compare the precision of the estimators).

   (c) Compare the properties of the OLS estimator for the cases where (i) the restricted model is correct and the additional variables have been unnecessarily included, and (ii) the unrestricted model is correct and the regressors in *Z* has been incorrectly excluded.

6. What issues for estimation arise when the error term in a regression is serially correlated? Examine how these may be addressed with particular reference to estimation and testing in a Maximum Likelihood framework.

7. Consider the model

$$y = X\beta + u$$

where X is a $T \times K$ matrix. Let P_X be the matrix which projects orthogonally on to the columns of X and let z denote the unit vector with the t^{th} entry equal to one and all other entries zero.

(a) The 'leverage', h_t, of observation t is defined to be the t^{th} term on the principal diagonal of P_X. Show that

$$h_t = x_t \left(X'X\right)^{-1} x_t' = z' P_X z$$

(b) Show that h_t lies between zero and one.

(c) Suppose we estimate the model

$$y = X\beta + z\delta + \tilde{u}$$

with fitted equation

$$\tilde{y} = X\tilde{\beta} + z\tilde{\delta} + \tilde{u}$$

Show that the residual for observation t is zero
Show that $\tilde{\beta}$ can be obtained by regressing y on X with the t^{th} observation excluded from the data set and that

$$\tilde{\delta} = \frac{u_t}{1 - h_t}$$

8. Using appropriate examples, examine the meaning and use of instrumental variables estimation in econometrics.