Reynold's Number
Laminar vs. Turbulent Flow

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IISER Kolkata

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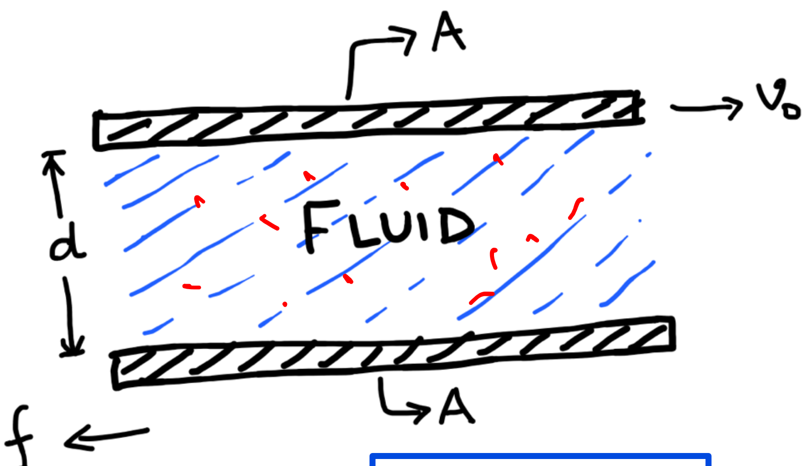
Laminar or Turbulent?

SHEARING MOTION



$$\boxed{f = -(\eta)\left(\frac{A V_o}{d}\right)}$$

variables

viscosity of the medium [property]

Critical force, $\boxed{f_{critical} = \frac{\eta^2}{\rho_m}}$

Laminar: $f < f_{critical}$

Turbulent: $f > f_{critical}$

Laminar or Turbulent?

Critical force, $\boxed{f_{critical} = \frac{\eta^2}{\rho_m}}$

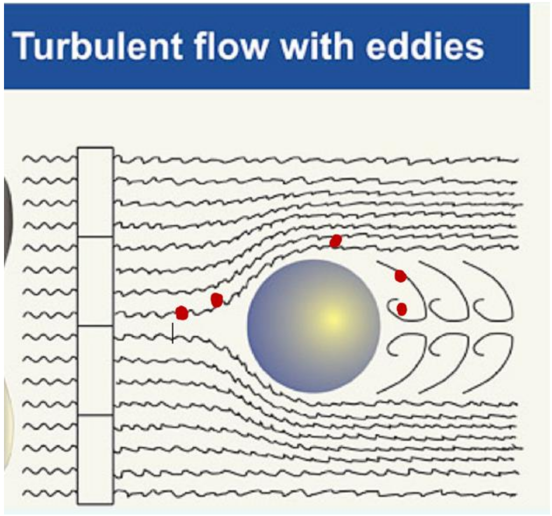
$$\boxed{f = -(\eta)\left(\frac{A V_0}{d}\right)}$$

~Room temperature data orders of magnitude difference.

Fluid	Density ($kg\ m^{-3}$)	η (Pa-s)	$f_{critical}$ (N)
Water	1000	0.0018	3.24×10^{-9}
Olive oil	900	0.08	$\sim 7 \times 10^{-6}$
Corn syrup	~ 1500 1000 → check (?)	5	$\sim 2.5 \times 10^{-2}$

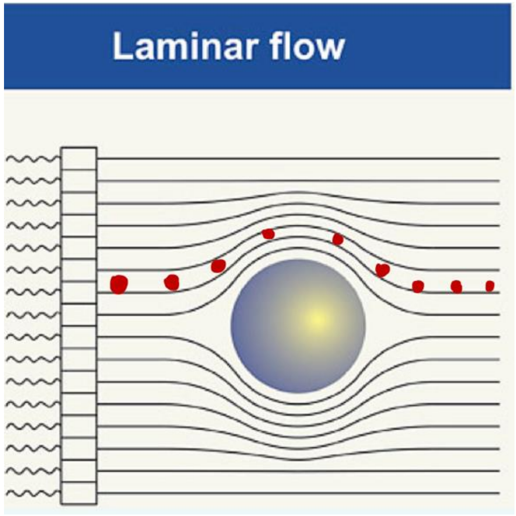
Reynolds Number (Re): The ratio of inertial to viscous forces

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$



Vs.

$$f_{critical} = \frac{\eta^2}{\rho_m}$$



The transition from laminar to turbulent flow occurs at Re ~1000

Reynolds Number (Re): The ratio of inertial to viscous forces

$$f_{critical} = \frac{\eta^2}{\rho_m}$$

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$

speed size

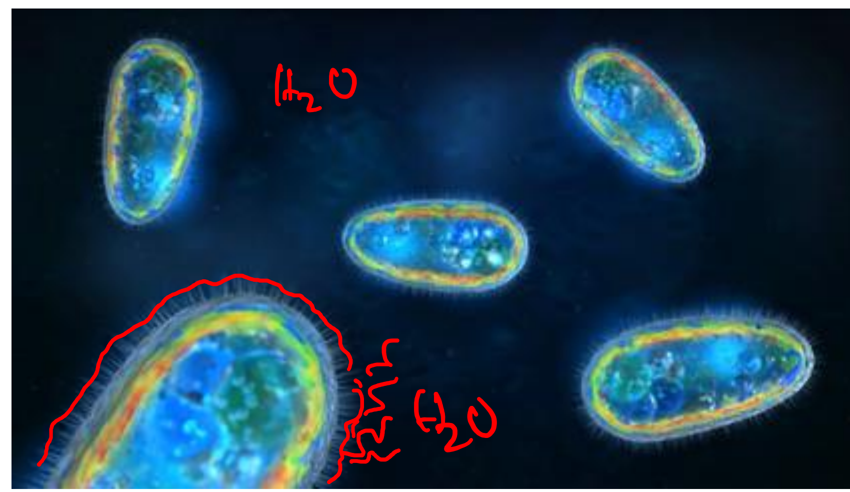
$$Re = \frac{\rho_m\, v\, a}{\eta}$$

dynamic viscosity

$$= \frac{v\, a}{\nu}$$

kinematic viscosity

Vs.



The transition from laminar to turbulent flow occurs at Re ~1000

Prob 1.

a) Consider an insect measuring 1.5 cm, and swimming at a speed of 5 times its own length per second in pure ethanol at a temperature of 25 °C.

Ethanol: density is 0.789 g/cc; viscosity is 1.1 centiPoise

$\eta_{EtOH} = 1.1\,cP = 1.1 \times 10^{-2}$ Poise

$= 1.1 \times 10^{-3}$ Pa-s

b) Consider a unicellular organism of size 10 μm moving at 100 μm s^{-1} in water at 25 °C.

In relative terms, which situation is more laminar (or turbulent)?

$\eta_{H_2O} = 1\,cP$

$= 10^{-3}$ Pa-s

a) $R_e \sim 0.8 \times 10^{3}$

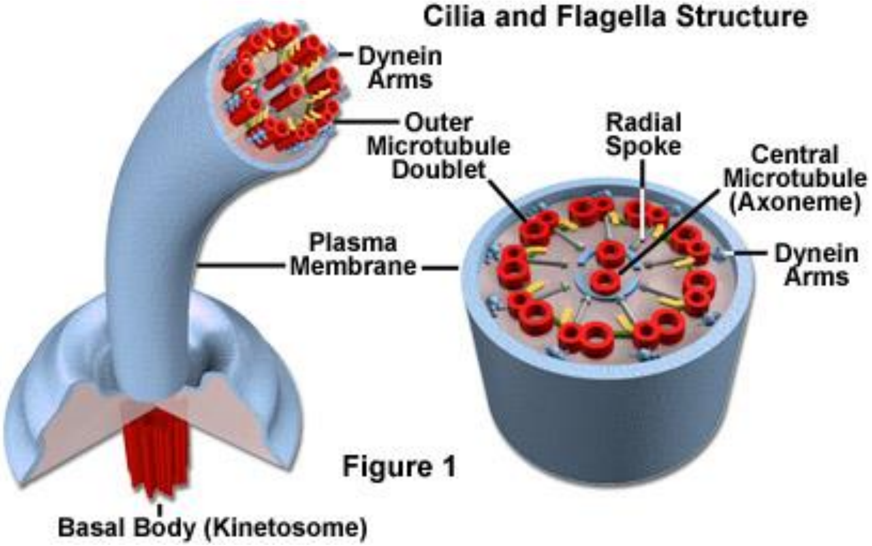
b) $R_e \sim 10^{-3}$

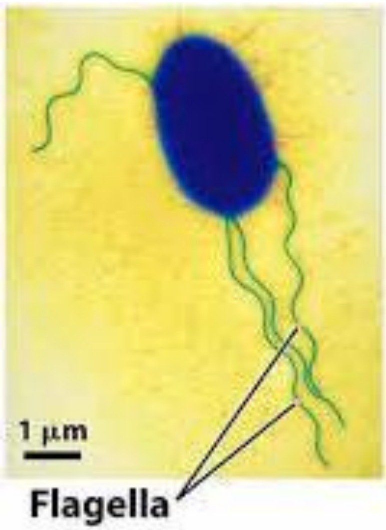
Flagella

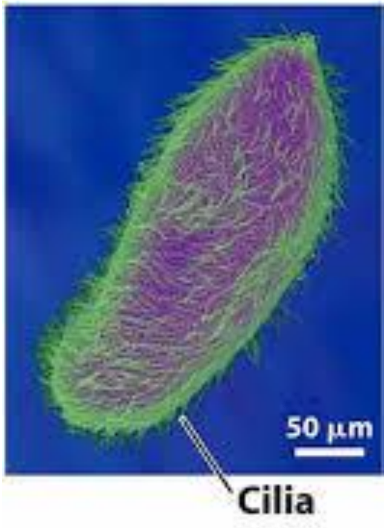
- Microtubule ring wrapped in membrane
- Shorter (~1 μm)
- Typically isolated
- ~10s per cell
- **Rigid "crank" motions**

Cilia

- Microtubule ring wrapped in membrane
- Longer (~tens of μm)
- In clumps
- ~100s per cell
- **Back and forth motion**

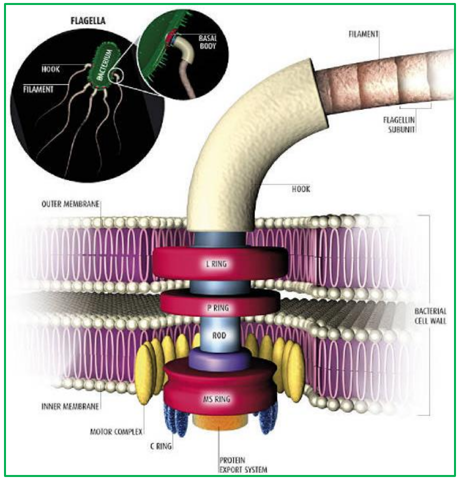


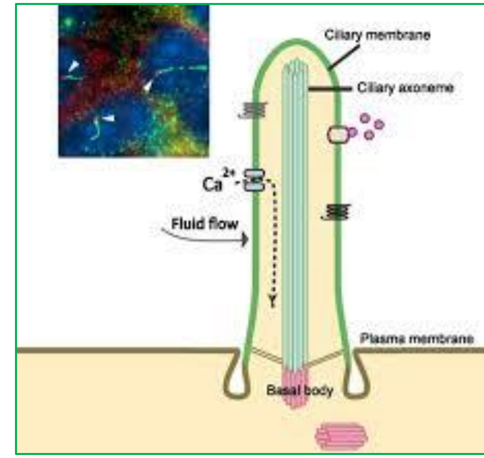




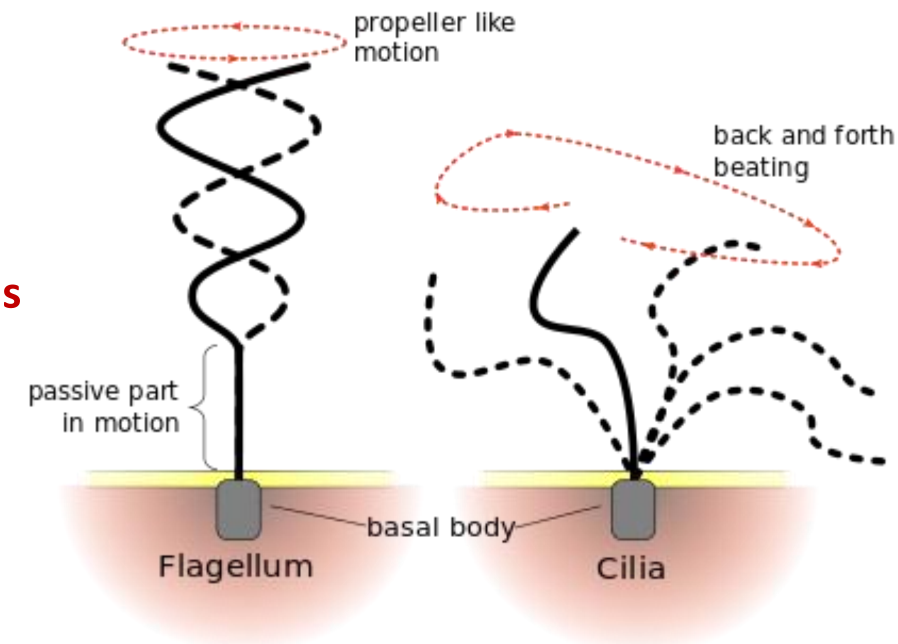
Flagella

Cilia

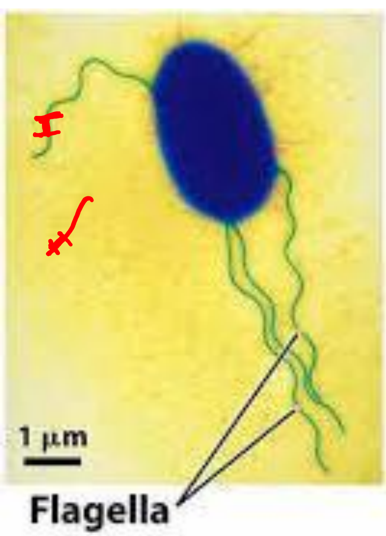




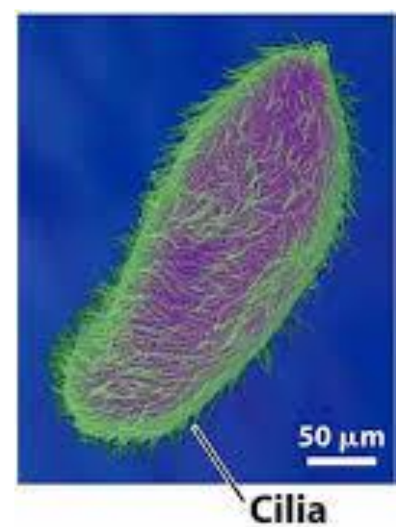
Both create local turbulence



- **Rigid "crank" motions**



- **Back and forth motion**



E. Coli flagella: Rigid objects that "crank" like a rotary engine

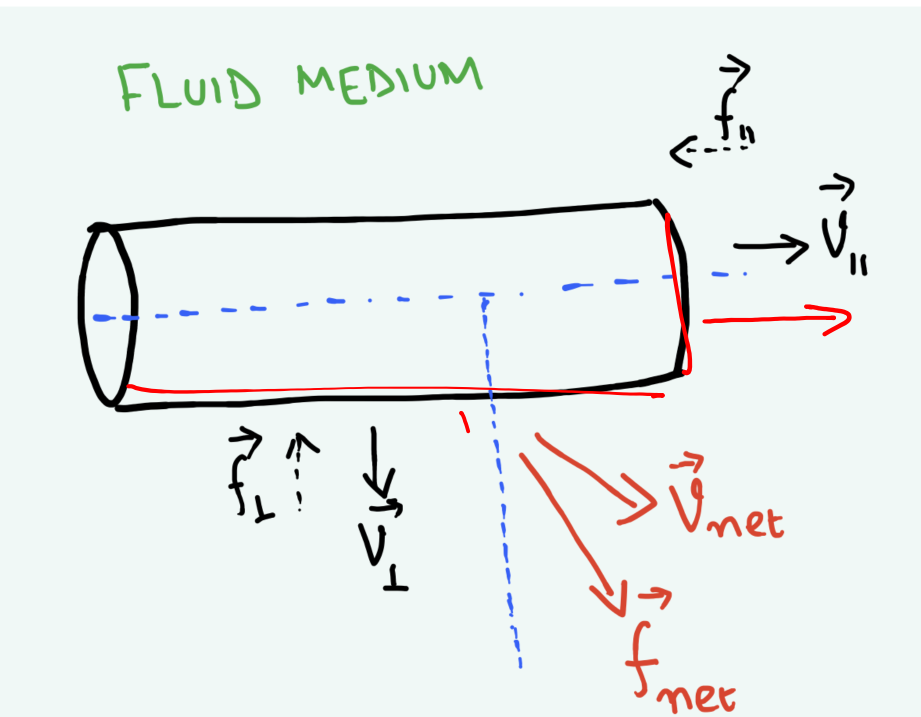
$f \propto \frac{A v}{d}$

$|f| \equiv (K)\, \xi \frac{v}{d}$

$A_\perp > A_\parallel$

from Fluid Dynamics, for the same fluid (viscosity η),

$$\boxed{\xi_\perp > \xi_\parallel}$$



Key consequences:

- Axial resistive force is smaller in magnitude:

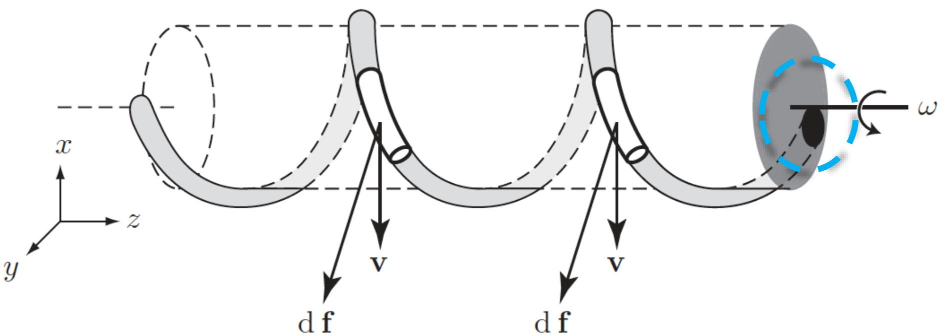
$$\boxed{|f_\perp| > |f_\parallel|}$$

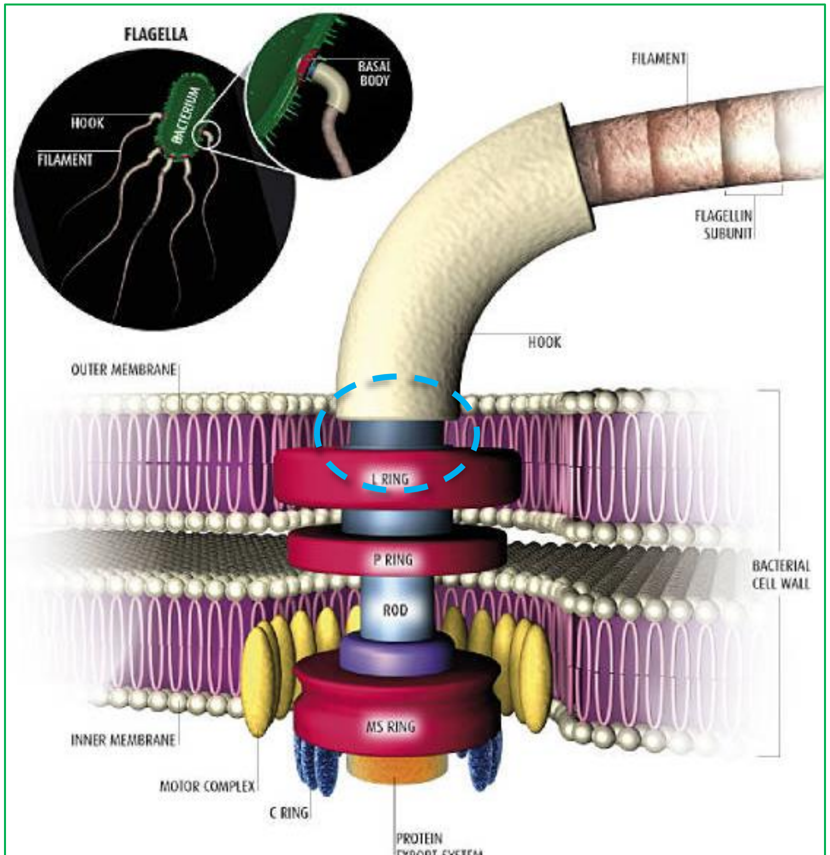
- **Direction of net force and net velocity not identical:**

$$\boxed{\hat{v}_{net} \neq \hat{f}_{net}}$$

- **Force direction closer to the normal**

Flagellary propulsion



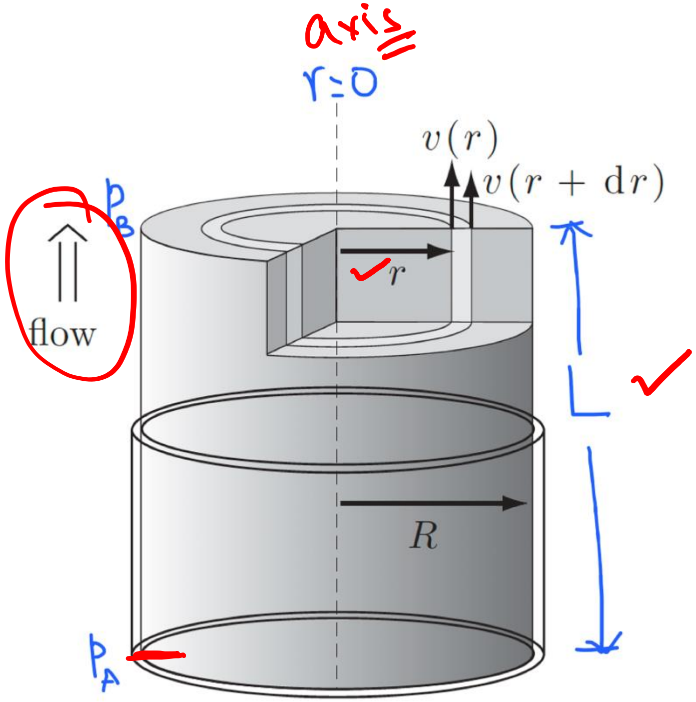


Key consequences:

- **Velocity components, *except* those in axial (-z) direction, are cancelled out**
- **Net "propulsion" in the axial direction**

- Axial resistive force is smaller in magnitude: $\boxed{|f_\perp| > |f_\parallel|}$
- Direction of net force and net velocity not identical: $\boxed{\hat{v}_{net} \neq \hat{f}_{net}}$
- Force direction closer to the normal

Vascular Networks:

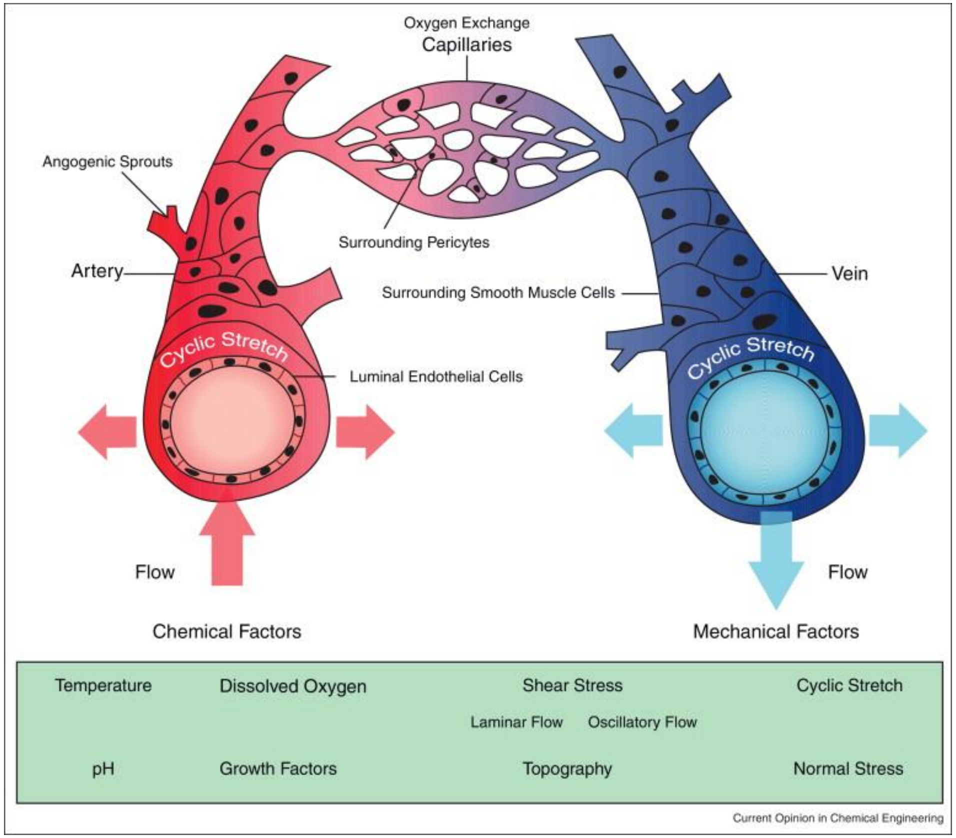


$\Delta p = p_B - p_A$ ✓

Speed,

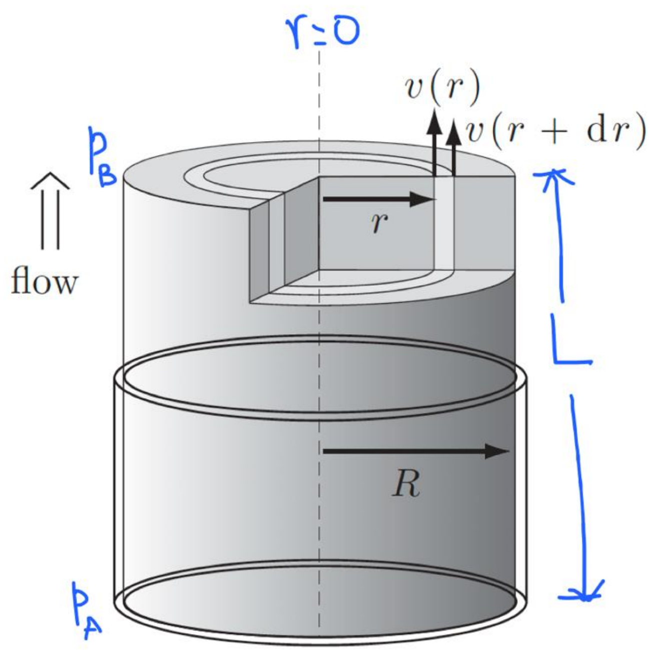
$$v(r) = \frac{(R^2 - r^2)\Delta p}{4L\eta}$$

Blood flow must be laminar



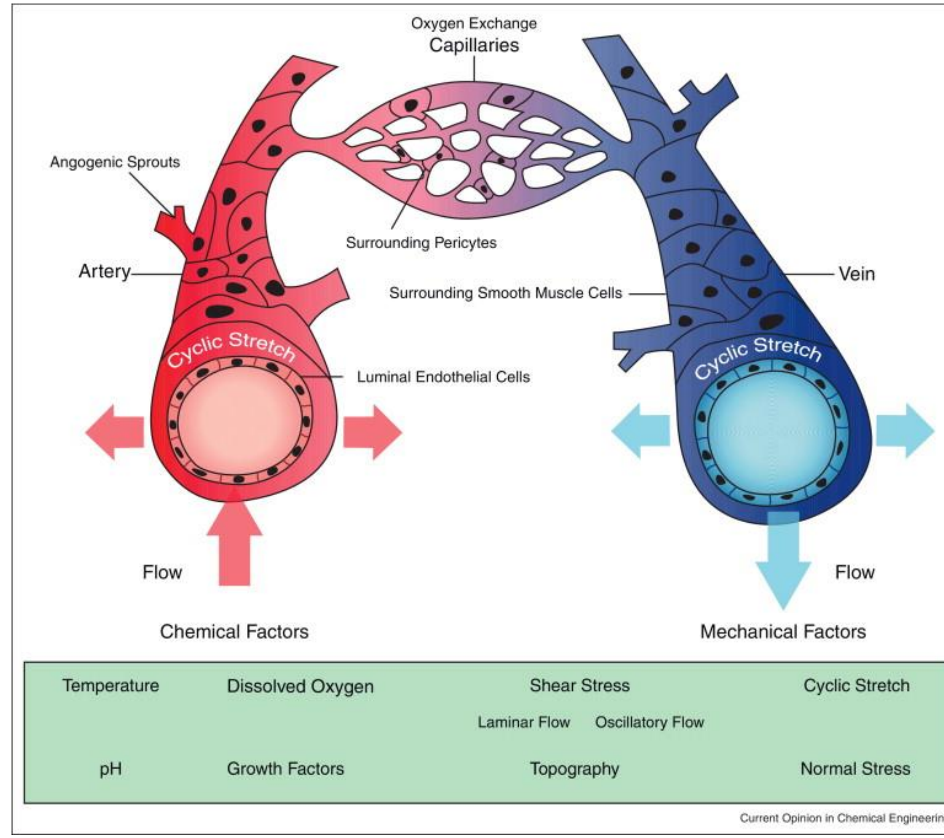
Vascular Networks:

Blood flow must be laminar



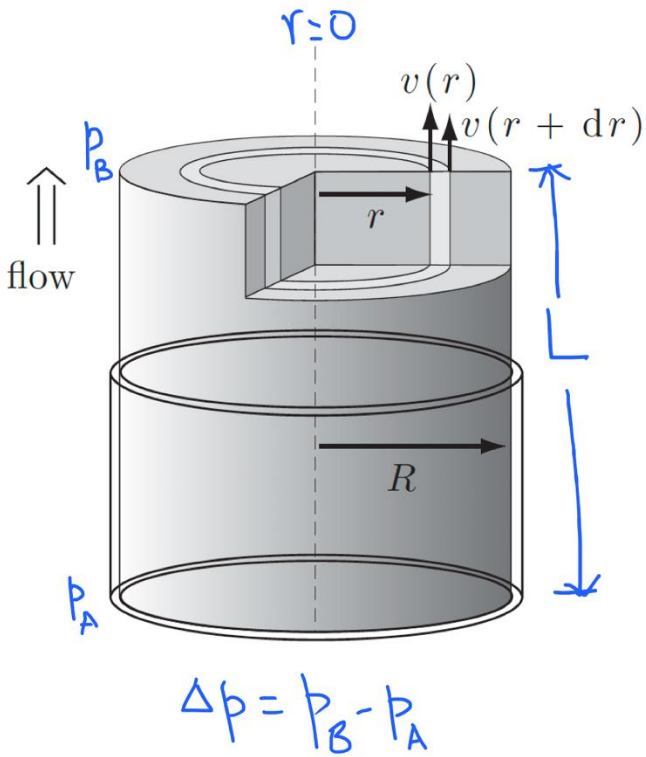
$$\Delta p = p_B - p_A$$

$$\boxed{v(r) = \frac{(R^2 - r^2)\,\Delta p}{4L\eta}}$$



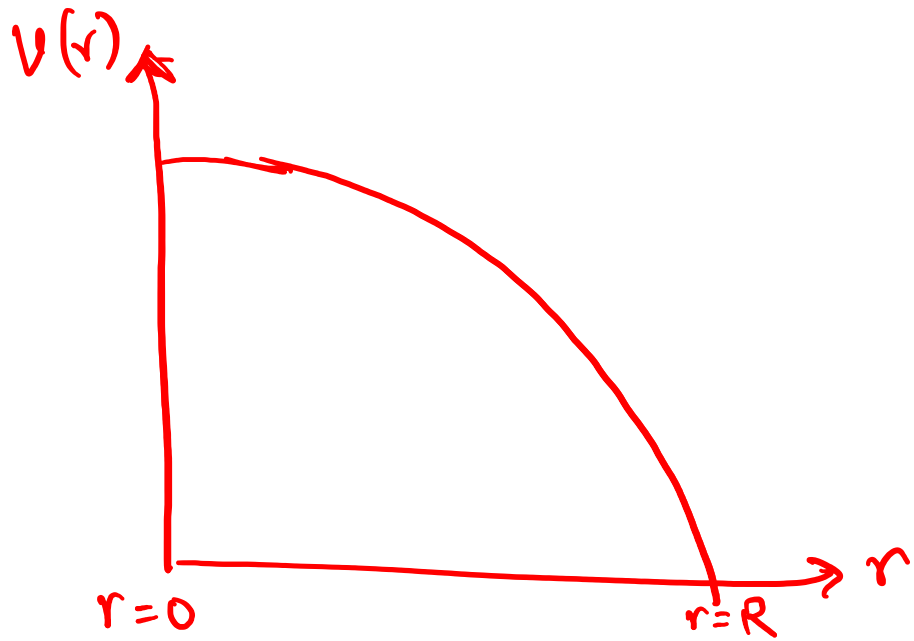
- Where is the flow fastest?
- How is the flow at the periphery?

Vascular Networks:



$$v(r) = \frac{(R^2 - r^2)\,\Delta p}{4L\eta}$$

- What is the shape of the 'velocity profile'?



Vascular Networks:

$$v(r) = \frac{(R^2 - r^2)\Delta p}{4L\eta}$$

Prob 2. Human blood has viscosity ~3.2 cP. Consider a blood vessel of length 2 cm and radius 1 mm. At a systolic blood pressure of 120 mm of Hg, calculate the v_{max} in the vessel.

$$\Delta p = \frac{120\,mm}{760\,mm} \times (1\,atm)$$

$$= \frac{120}{760} \times (1.01 \times 10^5)\,Pa$$

$$\approx 1.6 \times 10^4\,Pa.$$

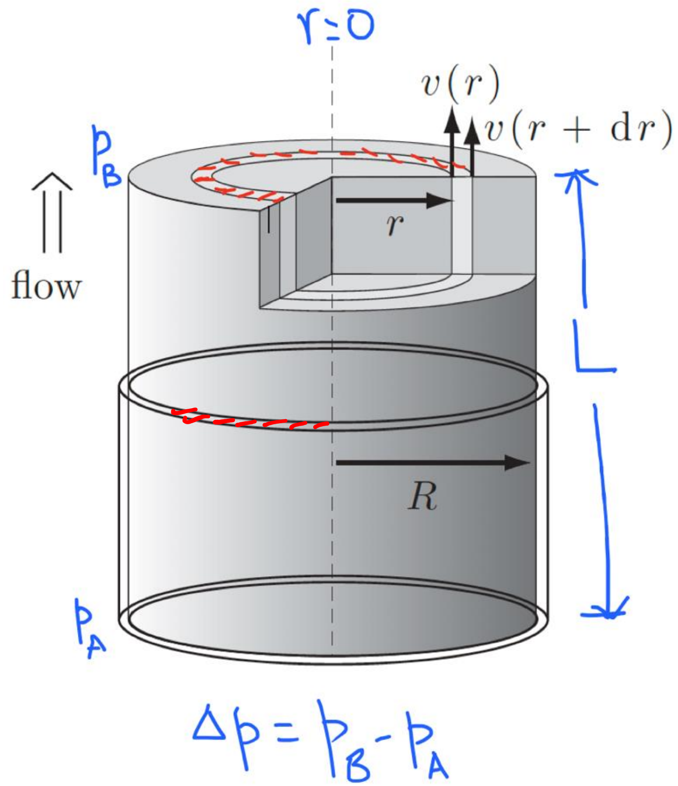
$$\eta = 3.2 \times 10^{-3}\,Pa\text{-}s$$

$$R = 10^{-3}\,m$$

$$v_{max} = v(r = 0)$$

$$= \underline{\hspace{3cm}}\ ms^{-1}$$

Vascular Networks:



$\Delta p = p_B - p_A$

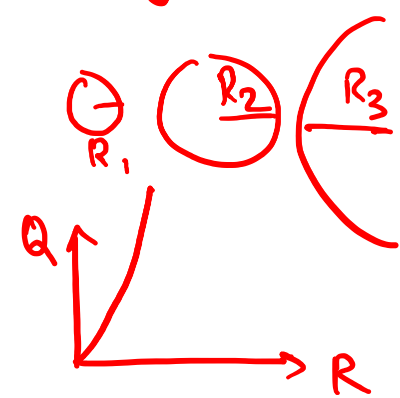
Volume flowing per unit time, or ***flow rate*** is,

$$Q = \int_{r=0}^{r=R} (2\pi r \cdot dr)\, v(r)$$

$$= \frac{2\pi}{4L\eta}\int_0^R \Delta p\,(R^2 - r^2)\, r\, dr$$

Hagen-Poiseuille's Equation

$$\boxed{Q = \frac{\pi(\Delta p)R^4}{8L\eta}}$$



What are the consequences of a reduction of blood vessel radius, say by 5%?

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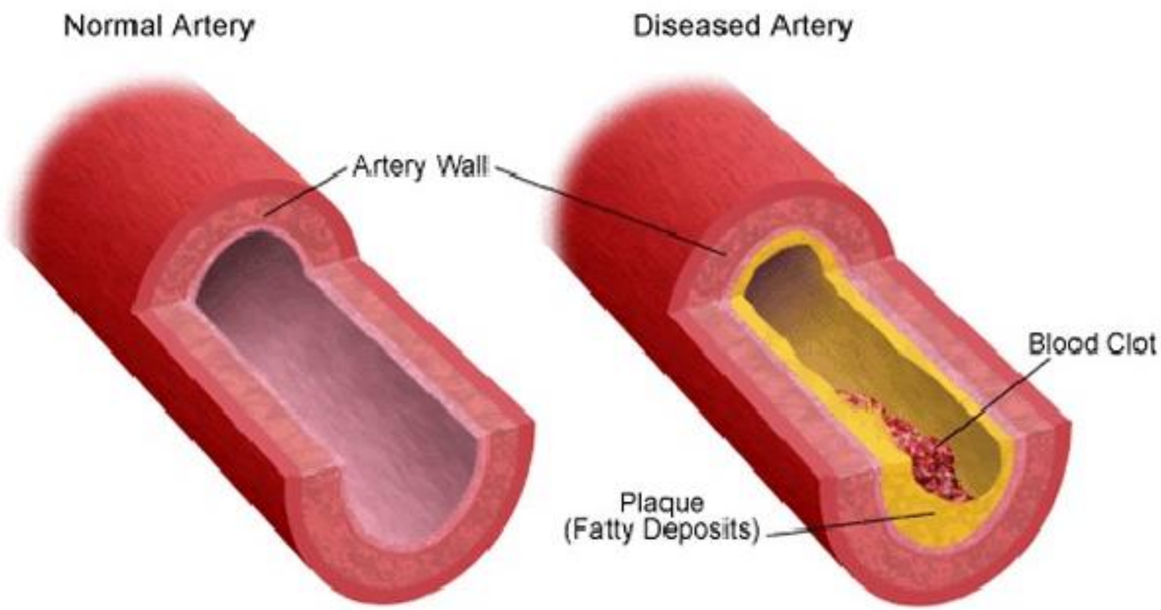

$$\boxed{Q = \frac{\pi (\Delta p) R^4}{8 L \eta}}$$

Say, $R' = 0.95R$.

$Q' =$ approx. $0.81\,Q$