Experiment No. 2
Velocity of light

AIM: Determination of velocity of light and refractive index of the transparent slab provided.

Experimental Procedure:

1. **READ THE MANUAL STARTING NEXT PAGE.**
2. Switch on the apparatus. Put the slider with two mirrors at the end of the track and check that the red light emitted by the LED falls back on the detector [photodiode (PD)] after reflection from the two mirrors on the slider.
3. Slowly bring the slider with mirror close to the LED and PD and check that the reflected light always falls on the PD for all position of the mirror slider on the track.
4. Bring the slider to zero position (nearest to the LED and PD).
5. Adjust the phase dial such that the Lissajous figure on the oscilloscope (CRO) becomes a straight line.
6. Move the slider on the track slowly away from the LED and PD. Lissajous figure on CRO screen will continuously change in shape.
7. Continue to move until you get back another straight line Lissajous figure with opposite slope.
8. Note the distance Δx by which you moved the slider.
9. Repeat steps 3 to 8 ten times. Take average of 10 measured distances and calculate error.
10. Calculate speed of light in air using $c = 4 f \Delta x$, where f = modulation frequency = 50.1 MHz.
11. Insert the transparent material slab near the LED in one of the light path. Keep the slider next to the slab. Note the position of the slider.
12. Adjust the phase dial such that the Lissajous figure on the oscilloscope (CRO) becomes a straight line.
13. Remove the transparent material slab. Move the slider slowly backwards (away from the LED) until you get back again a straight line Lissajous figure with same slope.
14. Note the distance Δx by which you moved the slider.
15. Each student repeat steps 11 to 14 ten times. Take average of 10 measured distances and also do error calculation in measurement.
16. Calculate refractive index (and error in it) of the material using $n = 1 + 2 \Delta x / L$, where L = light path length (geometrical) within the material.

Measuring the velocity of light

Related topics

Refractive index, wavelength, frequency, phase, modulation, electric field constant, magnetic field constant.

Principle

The intensity of the light is modulated and the phase relationship of the transmitter and receiver signal compared. The velocity of light is calculated from the relationship between the changes in the phase and the light path.

Equipment

Light velocity measuring app.	11224.93	1
Screened cable, BNC, l = 1500 mm	07542.12	2
Oscilloscope, 30 MHz, 2 channels	11459.95	1
Block, synthetic resin	06870.00	1

Tasks

1. To determine the velocity of light in air.
2. To determine the velocity of light in water and synthetic resin and to calculate the refractive indices.

Set-up and procedure

The deviating mirror and the lenses are set up in such a way that the incident and emergent light rays are parallel to the base plate (Fig. 1) and a maximum signal reaches the receiving diode (detailed directions can be found in the operating instructions).

The modulation frequency of 50.1 MHz (quartz stabilised) is reduced, to the approximately 50 kHz so that the transmitter and receiver signals can be displayed on the oscilloscope.

1. First of all, the mirror is placed as close to the operating unit as possible (zero point on the scale). A Lissajous figure appears on the oscilloscope (XY-operation) and is transformed into a straight line using the ‘phase’ knob on the operating unit. The mirror is then slid along the graduated scale until the phase has changed by π, i.e. until a straight line sloping in the opposite direction is obtained.

The mirror displacement Δx is measured; the measurement should be repeated several times.

2. The water-filled tube or the synthetic resin block is placed in the path of the ray so that its end faces are perpendicular to the optic axis; the mirror is placed directly behind them (top of Fig. 3). A supporting block can be used with the resin block so that the light passes through it in both directions.

A straight line is obtained on the oscilloscope again with the ‘phase’ knob. The medium is then taken out of the path of the rays and the mirror moved until the Lissajous figure agains shows the same phase difference.

The mirror displacement Δx is measured several times.

Theory and evaluation

The velocity of light is obtained as follows from Maxwell’s equations:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (1)$$

where $\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{F}{\text{m}}$ is the electric field constant, $\mu_0 = 1.257 \cdot 10^{-6} \frac{H}{\text{m}}$ is the magnetic field constant, ε the relative permittivity of the medium and μ its permeability.

The refractive index of a medium is the quotient of the light velocity in a vacuum and in the medium.

$$n = \sqrt{\varepsilon \cdot \mu} \qquad (2)$$

μ = 1 for most transparent substances.

Relative permittivity and refractive index are dependent of frequency (dispersion) because of the natural vibration of atoms and molecules. Red light (LED) is used in the experiment. The phase relationship between transmitter and receiver signal is represented by a Lissajous figure on the oscilloscope. If it is a straight line, the phase difference is 0 in the case of a positive slope and π in the case of a negative one.

Fig. 1: Experimental set-up for measuring the velocity of light in synthetic resin.

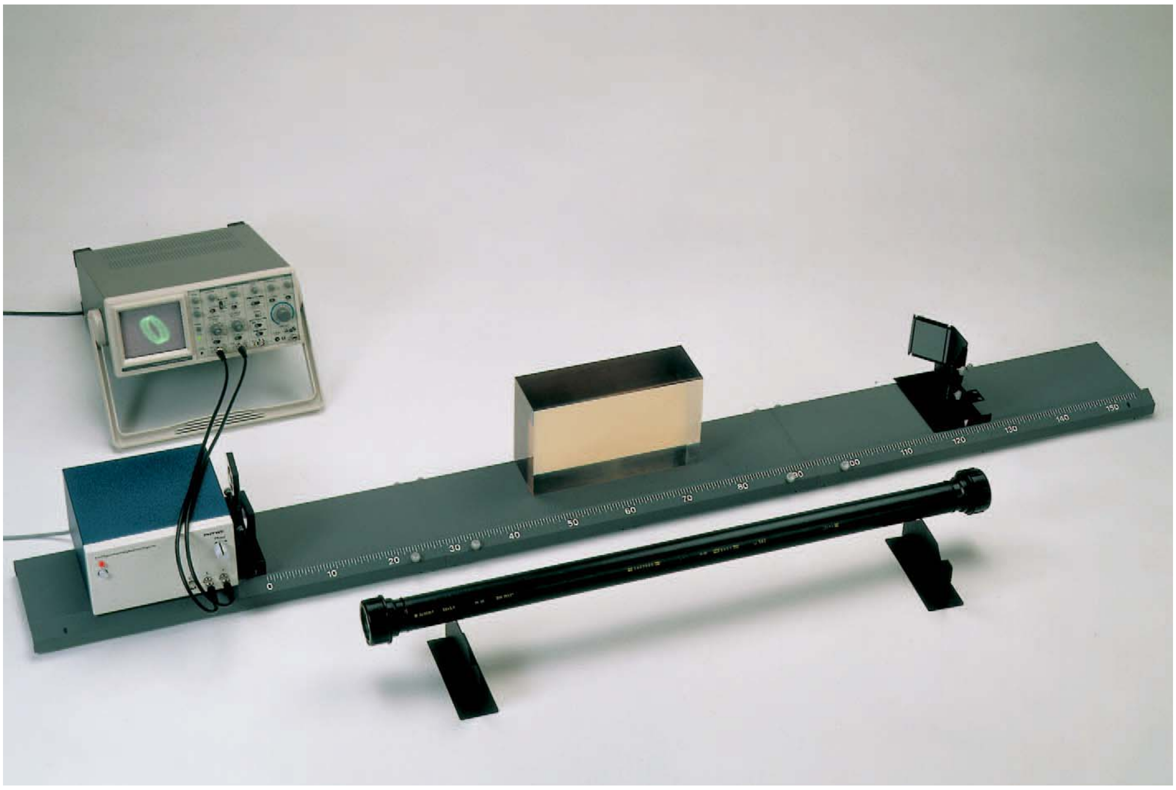
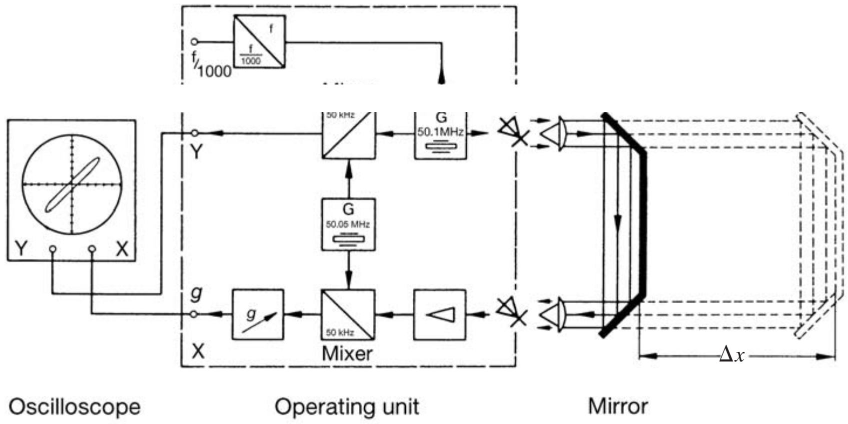


Fig. 2: Diagram of the experimental set-up for measuring the velocity of light in air.



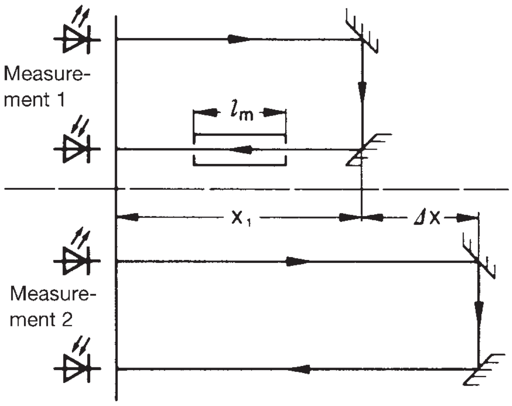


Fig. 3: Measuring the velocity of light in other media.

1. In order to measure the velocity of light in air, the light path is extended by

$$\Delta l = 2 \cdot \Delta x$$

(Fig. 2), to produce a phase change of π: i.e. to travel this distance the light requires a time

$$\Delta t = \frac{1}{2f}$$

where f = 50.1 MHz, the modulation frequency.
The velocity of light in air is thus expressed by

$$c_L = \frac{\Delta l}{\Delta t} = 4f \cdot \Delta x \qquad (3)$$

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Value taken from literature:

$$c_L = 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

2. The velocity of light in water or synthetic resin, c_M, is measured by comparing it with the velocity of light in air c_L (Fig. 3).

In the first measurement (with the medium), the light travels a distance l_1 in time t_1.

$$l_1 = 2x_1$$

$$t_1 = \frac{1}{c_L}(l_1 - l_m) + \frac{1}{c_M} l_m$$

In the second measurement (no medium), the light travels a distance

$$l_2 = l_1 + 2\Delta x$$

in time

$$t_1 = \frac{1}{c_L}(l_1 + 2\Delta x)$$

The phase relationship between transmitter and receiver signal is the same in both cases, so that

$$t_1 = t_2 + \frac{k}{f} \; ; k = 0, 1, 2 \ldots$$

We thus obtain the refractive index

$$n = \frac{c_L}{c_M} = \frac{2 \cdot \Delta x}{l_m} + 1 + \frac{k \cdot c_L}{f \cdot l_m} \qquad (4)$$

In water, the distance measured l_m = 1 m, so that the term

$$k \cdot \frac{c_L}{f \cdot l_m} \simeq 6 \cdot k$$

In synthetic resin, for a distance m of 30 cm, the term

$$k \cdot \frac{c_L}{f \cdot l_m} \simeq 20 \cdot k$$

From the expected magnitude for the refractive index we can deduce that

$$k = 0, \text{ therefore } t_1 = t_2 \qquad (5)$$

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Values from literature:

$$n_{H_2O} = 1.333$$

$$c_{H_2O} = 2.248 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

For the synthetic resin block

$$n_{\text{synthetic resin}} = 1.597$$

$$c_{\text{synthetic resin}} = 1.87 \cdot 10^8 \quad \frac{\text{m}}{\text{s}}$$

Detailed Instruction

Fig. 1: Emitting and receiving unit of the light velocity measuring apparatus 11224.93.

1. PURPOSE AND DESCRIPTION

The light velocity measuring apparatus serves to determine the propagation speed of visible (red) light in air, or in transparent liquids or solids.

A light-emitting diode is used as the light source. The light intensity of this diode is modulated by a high frequency alternating voltage of approx. 50 MHz, so that the light beam exhibits time markers. After the light beam has travelled a certain distance, it is directed towards a photodiode by an angular mirror and generates an alternating voltage of the same frequency there. This alternating voltage is phase shifted in comparison with the original signal, however, in dependance on the length of the light path. The phase shift can be easily visualized and determined from the Lissajous figure using an oscilloscope. The velocity of light can be calculated from the phase shift.

The back-reflection of the light beam by an angular mirror, which can be moved along the direction of the beam, offers the advantage that the light path required for the light beam is halved. In addition, a phase relationship between the two signals which is clear and definable from a measurement point of view (e.g. phase coincidence) can be realized with the angular mirror positioned either near to the emitter-receiver unit or far away from it with the help of a phase shifter. This enables the total length of the apparatus to be kept at less than 2 m.

2. PARTS AND OPERATING ELEMENTS

The complete light velocity measuring apparatus consists of the following components (Fig. 2; digital counter, oscilloscope and BNC cable are not supplied with the apparatus):

1 Base plate of painted steel sheet, made up of three parts which can be taken apart. The base plate is fitted out with a scale and with a guiding edge for the exact movement of the angular mirror along it.

2 Emitting and receiving unit (see Fig. 1)
2.1 Mains switch
2.2 Mains control light
2.3 Light emitting diode (emitter)
2.4 Photodiode (receiver)
2.5 Adjusting knob „Phase“
2.6 BNC output „Y“ for the receiver alternating voltage
2.7 BNC output „X“ for the emitter alternating voltage
2.8 BNC output „f/10^3“ for the emitter alternating voltage, frequency-reduced by 1000:1, so that a control of the modulation frequency can be made with a normal digital counter.

3 Two plano-convex lenses. Each of these has a holder with magnetic base for fixing it on the base plate. One lens is positioned in front of the light-emitting diode to make the emitted divergent bundle of light parallel. The second lens is positioned to focus the light beam which has been turned twice by the angular mirror onto the light sensitive area of the photodiode.

4 Angular mirror with which the parallel light bundle from the light-emitting diode is moved approx. 10 cm to one side and reflected back to the photodiode. The three adjusting screws on the back are used to adjust the mirrors in the factory, so that they enclose a 90° angle and their faces are in the same plane.

5 Tube cell with clear plastic windows which can be screwed onto the ends. For the measurement of the velocity of light in transparent liquids, and for the determination of their refractive indices.

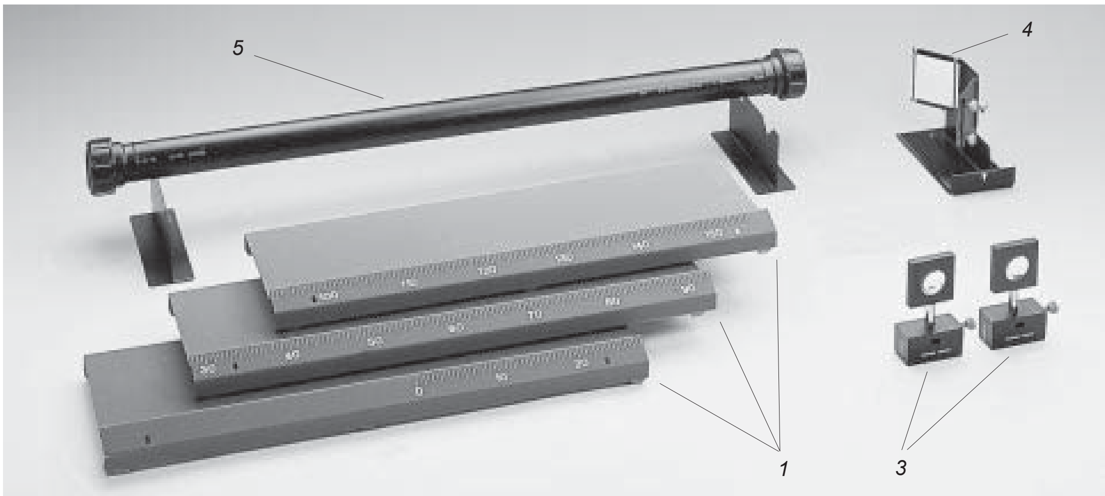

Fig. 2: Accessories for the light velocity measuring apparatus (supplied with 11224.93)

3 MEASUREMENT PRINCIPLE

The light-emitting diode is fed with such a high frequency alternating voltage, that the emitted light is periodically intensity modulated. With a modulation frequency of 50 MHz, this corresponds to a period time of 2 x 10^{-8} s. After travelling a certain distance, the light hits a photodiode and generates there an alternating voltage which has the same frequency as the initial signal but a differing phase position. If the light path is known for which the phase relationship between the emitted and received signal is 0, and a second light path, with which the phase is shifted by 180° (= π) can be determined, then, when the modulation frequency is known, the velocity of light can be determined from the path difference Δx, because the light path 2 x Δx is travelled in the time $T = 1/(2f)$. The velocity of light c is then given by:

$$c = \frac{2 \cdot \Delta x}{1/(2f)} = 4f \cdot \Delta x$$

(As the light beam is reflected back by the angular mirror, the light path x must be doubled). An oscilloscope used in the X,Y mode allows a very sensitive phase comparison. Two voltages of the same frequency generate an ellipse as Lissajous figure on the screen of the oscilloscope, as generally the voltage amplitudes are not exactly equal. The shape of the ellipse depends upon the phase relationship of the two voltages. When this is 0 or π, then the ellipse degenerates to an inclined straight line.

Note

Because of the construction, electrical overvoltages between emitting and receiving diodes can cause the wanted line to degenerate to a diagonally lying, very flat figure „8“. This effect is not significant for the actual measurement, however. The mirror should always be so moved for the measurement so that the junction of the flat „8“ is positioned exactly at the middle of the „8“.

In order to be able to use a favourably priced oscilloscope for the determination of the phase position, a further high frequency voltage (f = 50.05 MHz) is superimposed on the voltages of the emitter and receiver (f = 50.00 MHz). The differences are filtered out, so that two voltages of the same frequency of approximately 50 kHz are available for the phase comparison.

It is now possible, using the phase shifter of the emitting and receiving unit, to adjust a phase condition of zero or even of with the angular mirror at the start of the scale or at the end of it. The change in the light path required for a phase shift of 180° can be realized by moving the angular mirror out by approximately 1.5 m on the base plate.

4 HANDLING

4.1 Assembly

A plane and horizontal surface of a length of at least 2 m is first necessary. Connect the three parts of the base plate together, using the four connecting elements, whereby the angled pieces must be fixed at the front of the plate and the flat pieces at the back, all on the inside. When tightening the screws, take care that no difference in level occurs where the parts are held against each other.

Place the emitting and receiving unit with front plate on the end of the base plate which is not scaled.

Mount the lenses in the magnetic holders and position one of them in front of the emitter and the other in front of the receiver, each with the plane lens side facing towards the respective diode. We recommend, that the lenses be put on the base plate with the magnetic feet longwise on the plate, not across it, and that a narrow paper strip be placed under them to simplify the fine adjustment of the position of the lenses.

Place the angular mirror on the base plate so that the mirrors face towards the diodes.

4.2 Electrical connections

Connect the X and Y outputs of the emitting and receiving unit with the corresponding inputs of the oscilloscope. Set the oscilloscope sensitivity for the X channel to 200 mV/cm. Select the highest sensitivity at first for the Y input, but this must be at least 20 mV/cm.

If, for didactic purposes, the modulation frequency is to be measured, connect the „$f/10^3$“ output of the emitting and receiving unit to the corresponding input of a digital counter. The reduction factor must be taken into account when the frequency measurement is evaluated.

The connecting cords used must not be longer than 3 m.

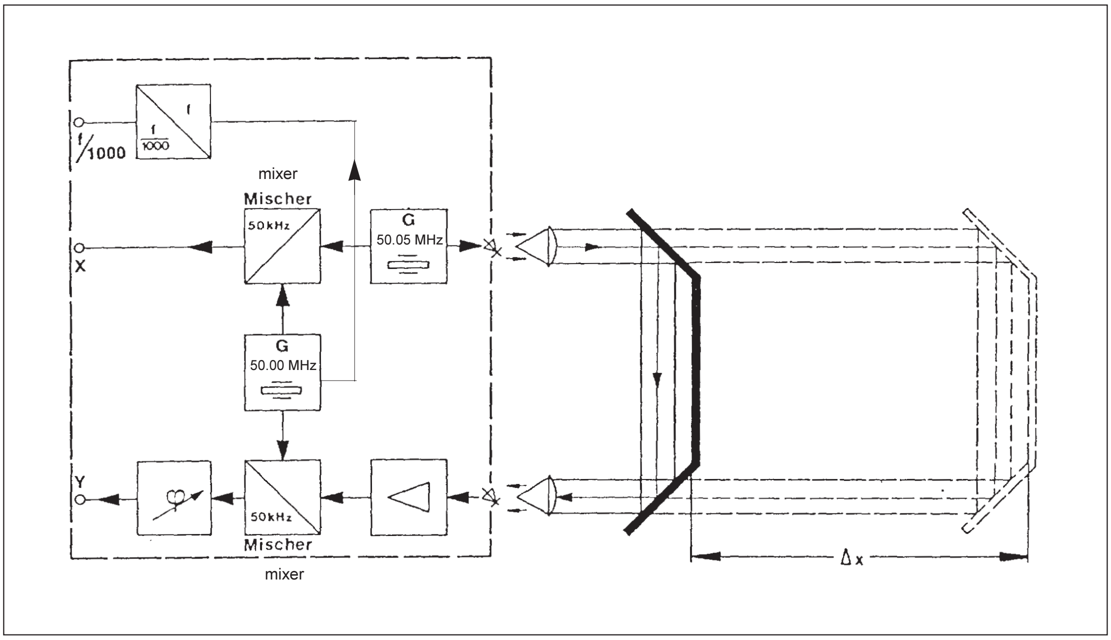

Fig. 3: Operating principle of the measurement of the velocity of light

5. ADJUSTMENT

5.1 Preliminary work

Set the lenses, which are positioned with their plane surfaces parallel to the side of the emitting and receiving unit, so that each is at a distance of 3.5 to 4 cm from the diode facing it. Adjust the height of the lenses, and move them sideways, to centre them by eye on the facing diode.

The angular mirror has been correctly adjusted in the factory. Should one or the other of the mirrors go out of adjustment after some time, it can be re-adjusted so that it is again parallel to its carrier plate by means of the adjusting screws on the back of it. First adjust the vertical position with the two outer screws, then use the middle screw to adjust the horizontal position.

When each mirror is parallel to its carrier plate, the angle contained by the mirrors is 90°. When this is the case and you look along the symmetrical axis of the double mirror, you will see the accustomed face width and distance between the eyes.

5.2 Adjustment under normal room lighting

A partial darkening of the room is only recommended when the room is very bright due to direct sunlight. Keep a sheet of white paper available, this is the only item required for the adjusment.

Step 1: Alignment of the condenser lens and the angular mirror

Position the angular mirror first at the outer end of the base plate. Follow the course of the light bundle which must centrally illuminate the mirror on the emitter side. To attain this, adjust the converging lens in front of the light-emitting diode vertically and horizontally; position the lens in the direction of the light beam so that the light-emitting diode is sharply pictured on the sheet of paper held in front of the mirror. Follow the light beam to the photodiode, where a patch of light is to be seen. Should, in exceptional cases, the photodiode be far away from the center of the patch of light, this is normally simply due to the inclination of the mirror on the receiver side.

Step 2: Alignment of the image-forming lens

This lens is positioned with its plane surface facing the emitting and receiving unit, at a distance of approximately 5 cm from the window of the photodiode. First move the lens to one side of the axis, so that the picture of the light-emitting diode can be observed on the white ring surrounding the photodiode. Now move the lens in the direction of the light beam until the picture of the light-emitting diode is in the form of a disc whose diameter is slightly less than that of the window of the photodiode. Now move the lens back sideways, and adjust the height if necessary, so that the picture is optimally positioned on the photodiode. For exact adjustment, observe the connected oscilloscope, watching for the maximum amplitude on the screen.

6. MEASUREMENT OF THE VELOCITY OF LIGHT

6.1 Measurement in air

We recommend that you start the measurement with the longest possible light path, as the system has now been optimally adjusted for this. The angular mirror is so positioned on the base plate that it is directly touching the end edge of it.

Select the sensitivities on the oscilloscope, set for X,Y operation, so that the Lissajous figure which is shown almost fills the screen. Use the adjusting knob „Phase“ to change the figure to an inclined straight line. The phase shift between the emitted and received signals is now 0° or 180°. This phase adjustment must remain unchanged during the further course of the experiment.

Now slide the angular mirror away from the end of the base plate towards the emitting and receiving unit. The Lissajous straight line opens up to an ellipse, whose main axis changes its slope until a straight line is again formed, but this time inclined in the opposite direction to the initial straight line. The phase shift is now 180° or 0°.

The signal amplitutde of the receiver diode has now increased considerably. Compensate for this by reducing the Y sensitivity of the oscilloscope. Should over-drive occur and cause distortion of the signal, reduce the light intensity which is incident on the receiver diode by defocussing. To do this, move the lens which is in front of the photodiode in

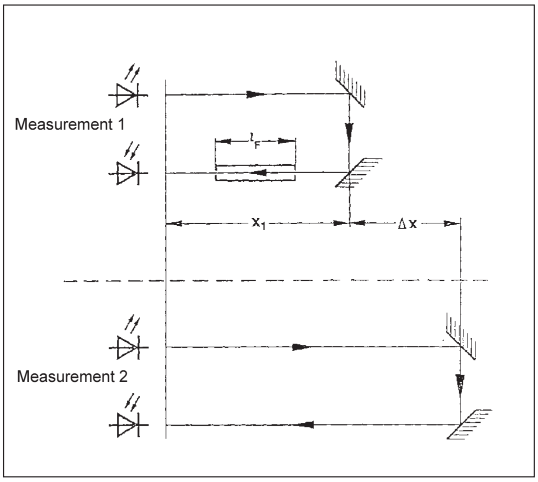

Fig. 4: Measurement proinciple in liquids and in solid substances

the direction of the light beam until the signal is no longer distorted.
When the path difference from the starting point to the angular mirror is Δx, then the total difference in the length of the light paths is (because the light travels to one end and back again) $\Delta l = 2\Delta x$. This path difference has changed the phase position by π, which corresponds to half a period of the quartz-stabilized modulation frequency.
c is given by:

$$c = \Delta l / \Delta t.$$

when $\Delta t = 1/(2f)$ and $\Delta l = 2\Delta x$, then the velocity of light is given by:

$$c = \frac{2 \cdot \Delta x}{1/(2f)} = 4f \cdot \Delta x$$

Calculation example:
$\Delta x = (1490 \pm 10)$ mm; f = (50,1 ± 0,01) MHz;
c = (298600 ± 2000) km/s; $\Delta c/c = \pm 0{,}7$ %

6.2 Measurements in transparent liquids or solids

The tube cell which is supplied with the light velocity measuring apparatus has parallel windows at its ends. Fill it with the liquid to be examined and mount it on the supports, stood against the ends of the base plate, so that the light beam passes through the cell, either on the emitter or the receiver side.
It is purposeful to position the angular mirror close behind the cell and to maximize the Y amplitude by a fine re-adjustment. Use the adjusting knob „Phase“ to generate a Lissajous figure on the screen of the oscilloscope. Read off the position of the angular mirror x_1 from the scale. Remove the cell and move the angular mirror away from the emitting and receiving unit until a straight line with the same slope is obtained. Read off the position x_2. As the phase shift between the emitted and the received signal was the same for both angular mirror positions, the light has taken the same time t_1 to travel each of the light paths.
The light path travelled once in the liquid is l_F.
The light has travelled the distance l_1 in the first measurement and the distance $l_1 + 2\Delta x$ in the second measurement, where $\Delta x = x_2 - x_1$.
When c_L is the velocity of light in air and c_F the velocity of light in the liquid, then we have, for the travelling time t_1 of the light in each measurement

Measurement 1:

$$t_1 = \frac{l_1}{c_L} - \frac{l_F}{c_L} + \frac{l_F}{c_F}$$

Measurement 2:

$$t_1 = \frac{l_1}{c_L} + \frac{2\Delta x}{c_L}$$

By equating and converting we have:

$$\frac{c_L}{c_F} = \frac{2\Delta x}{l_F} + 1$$

When the velocity of light in air c_L is known, then the velocity of light in the liquid c_F can be determined.
The quotient $c_L/c_F = n$ gives the refractive index of the liquid.

Calculation example (for water): $\Delta x = (165 \pm 10)$ mm; l_F = (1000 ± 5) mm; c_F = (225000 ± 5000) km/s; $\Delta c_F/c_F = \pm 2\%$; $n = 1{,}33 \pm 0{,}02$.

The velocity of light in a transparent solid can be determined in an analogous way as described above.
The block of synthetic resin which is included in the „List of equipment“ below is particularly suitable as test substance.

7. SOURCES OF ERROR

7.1 Measurement of the path length

Check that the continuity of the imprinted scale is not disturbed at the junctions of the three parts of the base plate. If necessary, correct for this by comparison with another ruler.
To help you decide exactly when the Lissajous figure has become a straight line when moving the angular mirror, we recommend that you first bring the figure to the middle of the oscilloscope screen and then increase the sensitivity of the oscilloscope until only the magnified middle of the screen is to be seen. This limits the inaccuracy in the determination of the position of the angular mirror to about ± 1 mm.

7.2 Interference voltage at the receiver

A small interference signal U_s can be recognized at the receiver when the light beam is interrupted. This results from an „over-communication“ between emitter and receiver within the electronics which is difficult to avoid. This interference voltage adds itself to the useful signal U_n. In an unfavourable phase position, the maximum error

$$F_{max} = U_s / 2U_n.$$

The Y amplitude corresponds to the useful signal U_n; the larger this is, the smaller the error.

7.3 Sources of error in experiments with the tube cell

The tube cell must be so positioned in the light beam, that a maximum Y signal is obtained. Rotation of the cell around its longitudinal axis can assist in this.
For the determination of the velocity of light in liquids, it is only the length of the liquid column which is decisive. When the total length of the tube cell is measured, the sum of the thicknesses of the two plasic windows must be subtracted from this length.
To avoid dirtying of the tube cell, we recommend that you use deionized or distilled water instead of normal tap water. Take care that no air bubbles are enclosed in the tube cell.