Quantification across scales

class – 5 (21.8.24)

LS2103 (Autumn 2024)

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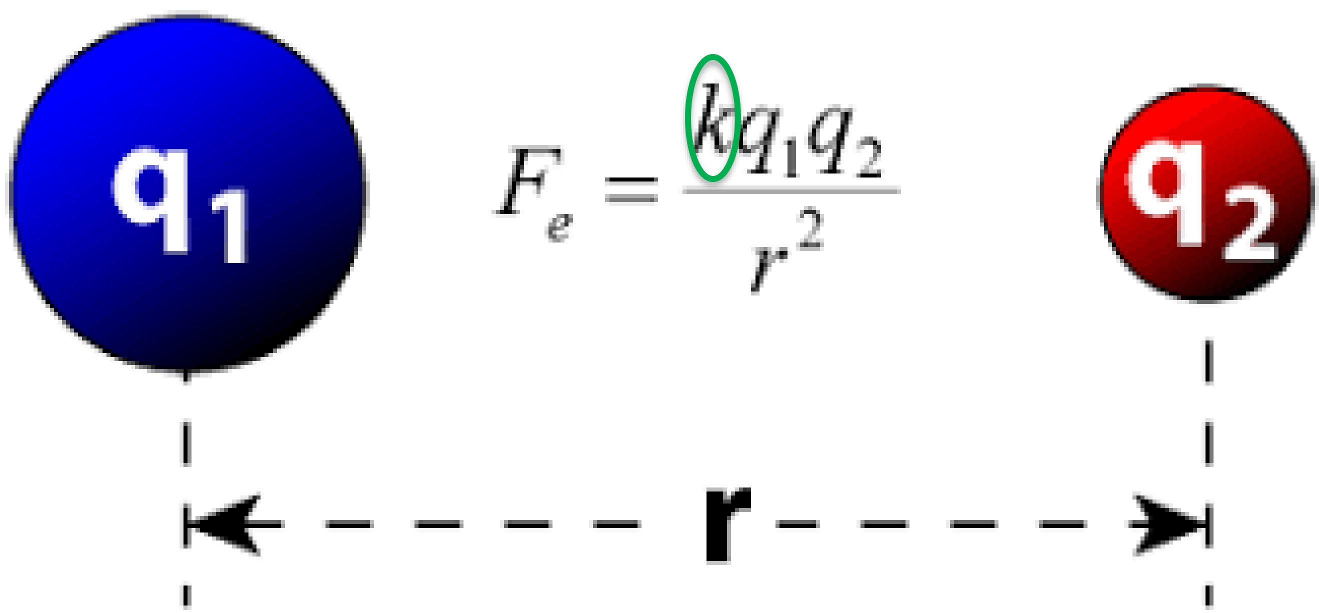
Dimensional Analysis: examples

2. Based on **Coulomb's Law**, find:

- *Dimensions of the medium characteristic ‘k’*
- *What should be the S.I units of ‘k□?*



(RT) is the energy scale in *molecular* biology

Bjerrum length (λ_B):
Use the 'k' from the previous analysis. Verify the equation via dim. analysis.

$$\boxed{\lambda_B = \left(\frac{k}{\epsilon_r}\right)\frac{e^2}{k_B T}}$$

e: Elementary charge

i. Show that the equation is dimensionally correct.
ii. Find λ_B at 300 K in water (ϵ_r ~ 80)

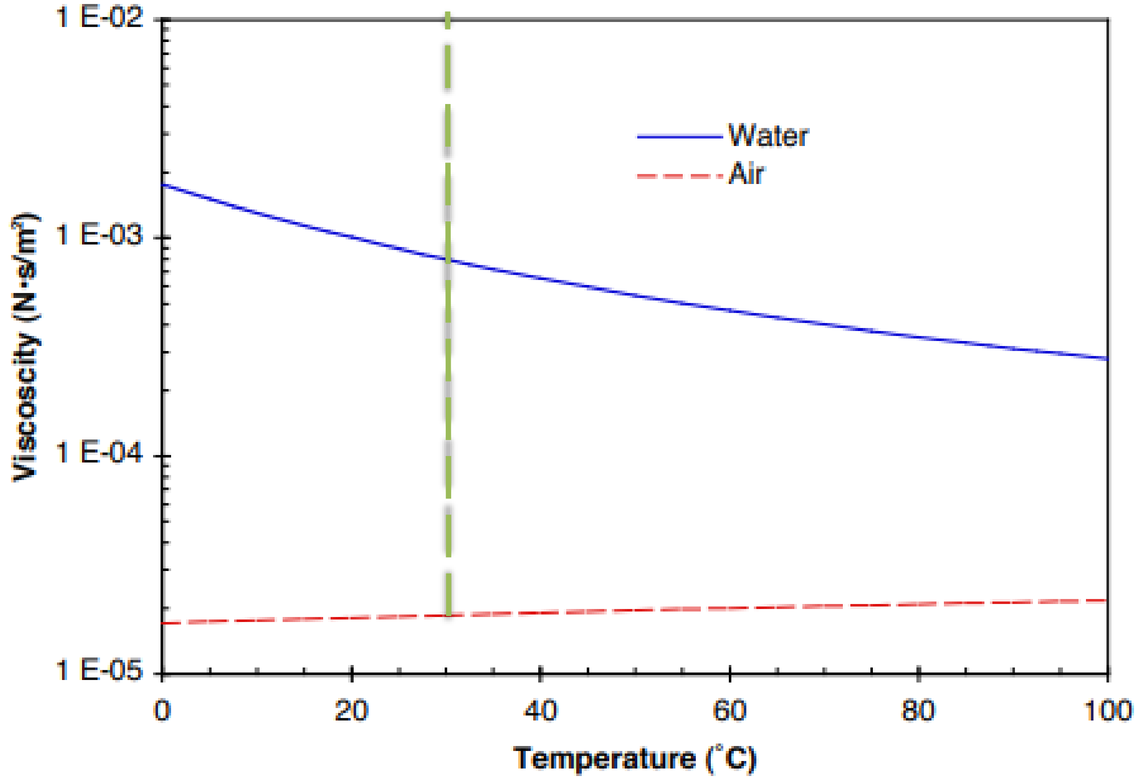
$k = 9 \times 10^9$ ***SI units.***

Movement of E. coli and humans

E. coli (in aq. environment**)**

Average jogger (in air**)**

Room temperature



$$\frac{\text{(Viscosity of water)}}{\text{(Viscosity of air)}} \sim 10^2$$

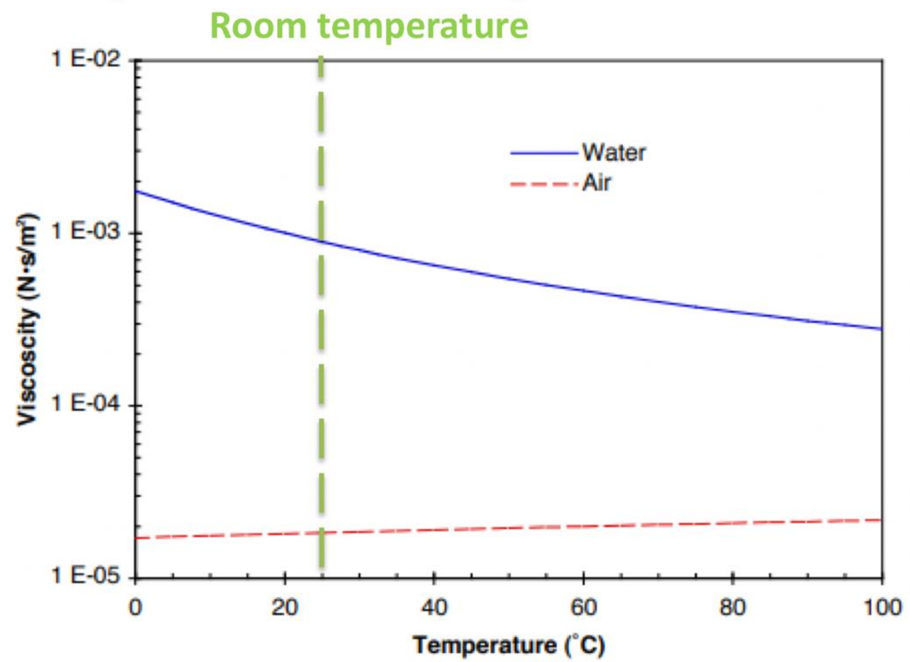
Dimensional Analysis: examples

Reynold's number

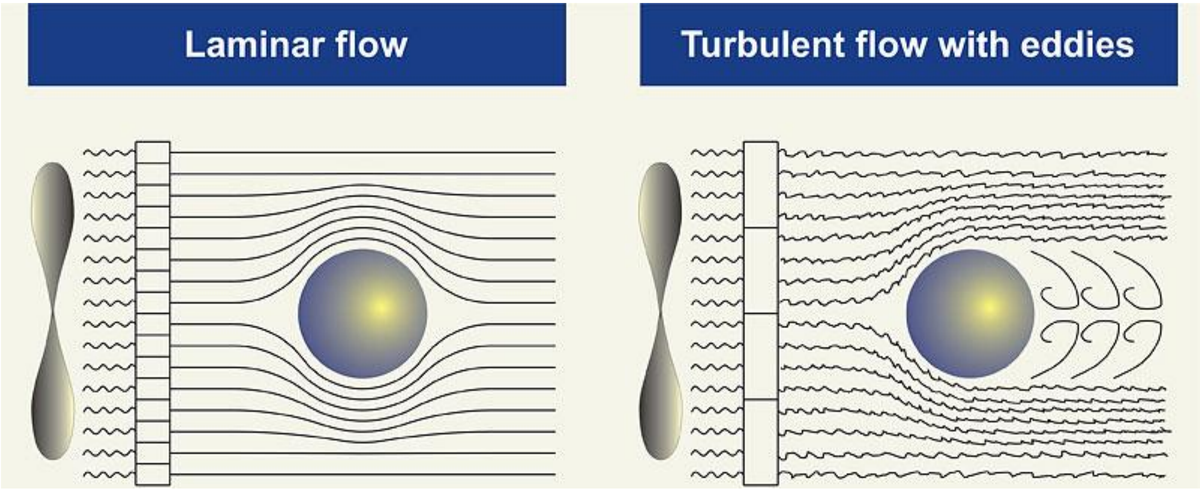
$$\mathrm{Re} = \frac{\textit{inertial}}{\textit{viscous}}$$

mass x acceleration

depends on viscosity of fluid; velocity; and size of the body.



Re determines the shift from **Laminar (streamlined)** to **Turbulent (chaotic)** flow of fluid around a body.



Low Re

High Re > 1000

Dimensional Analysis: examples

Viscosity and Reynold's number

$$Re = \frac{inertial}{viscous}$$

mass x acceleration

depends on viscosity of fluid; velocity; and size of the body.

What are the dimensions of Re?

$$\boxed{Re = \frac{\rho\, v\, L}{\mu}}$$

ρ: fluid density

v: speed of body

L: characteristic linear dimensions

μ: dynamic viscosity (SI units: N-s/m^2)

Find an approx. ratio of Re of E.coli in water to that of an average human jogging. State your reasonings.

Air density ~1.2 kg m^{-3}

Numerical estimates: pH

On average, after sucrose incubation, how many protons needed to be pumped in to maintain the pH?

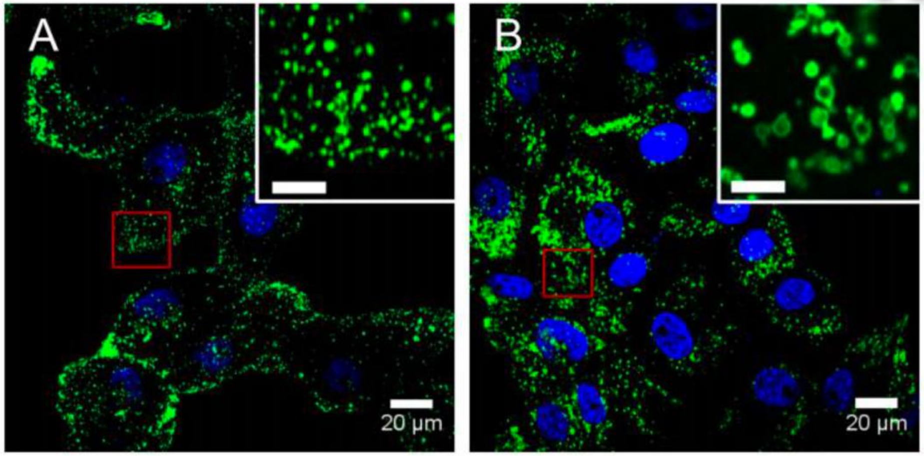


Figure 1. Incubation of cells with sucrose results in enlargement of lysosomes. *(A)* Confocal fluorescence microscopy image of untreated BS-C-1 cells shows the normal cellular distribution and punctate appearance of lysosomes (green) labeled with EYFP. The nuclei are stained with DAPI (blue). *(B)* Incubation with sucrose (50 mM, 12 h) leads to enlargement of lysosomes. The increased diameter gives the lysosomes a circular

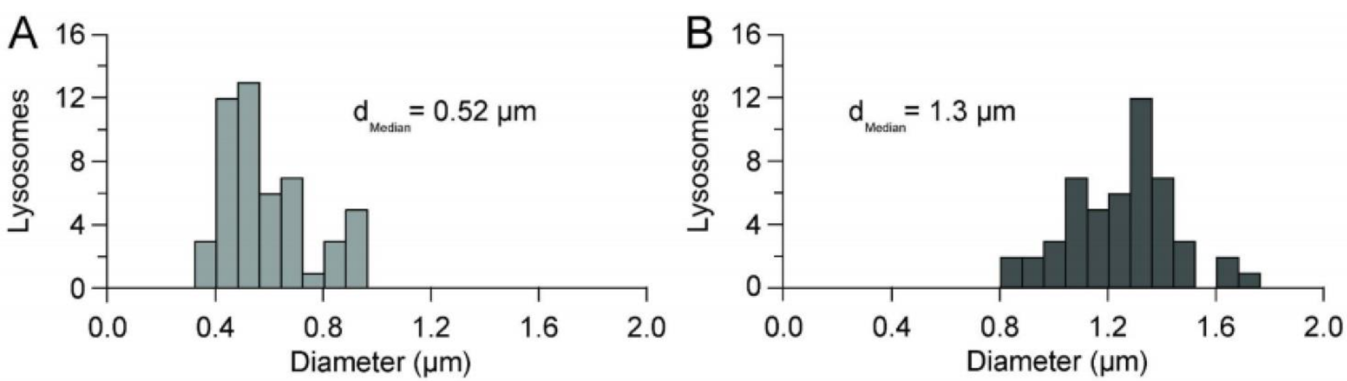


Figure 2. Distribution of lysosome diameters. *(A)* Distribution of lysosome diameters measured in control, untreated cells. *(B)* Incubation with sucrose shifts the distribution of lysosome diameters to greater values. For both plots, n = 50 lysosomes from 3 cells.
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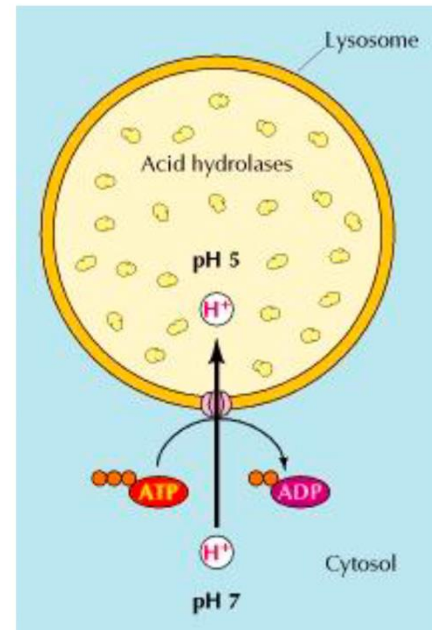

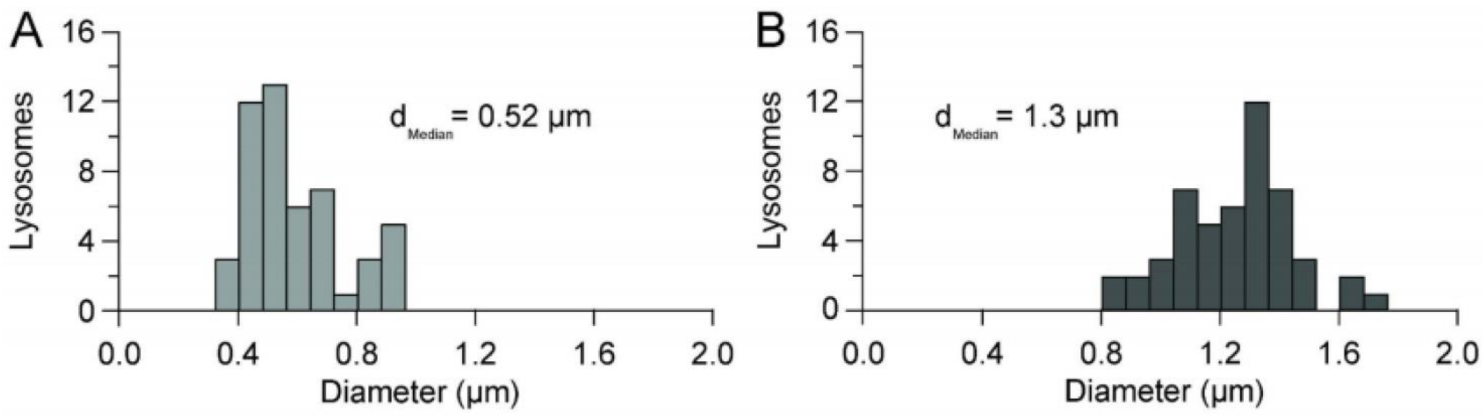

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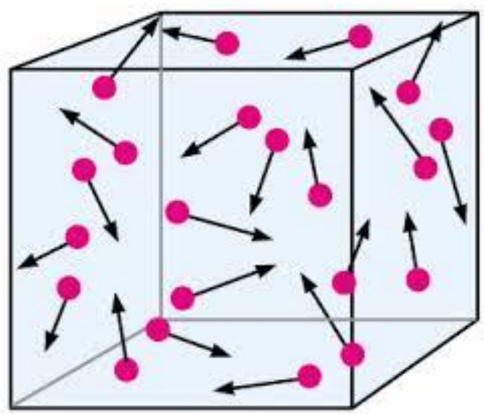
Distributions

Kinetic energy and Temperature

Equipartition theorem: Energy is shared equally amongst all energetically accessible degrees of freedom of a system.

***k_B*:**
Boltzmann's Constant



- Consider a system of ***N*** ideal, monoatomic gas molecules at temperature ***T***
- No interaction, and hence no potential energy

For a given molecule, the Kinetic energy along the 3 degrees of freedom are equal, ie.

$$\frac{1}{2}m\langle v_x^2\rangle = \frac{1}{2}m\langle v_y^2\rangle = \frac{1}{2}m\langle v_z^2\rangle$$

It can be shown that each of these quantities equals $\frac{1}{2}k_BT$

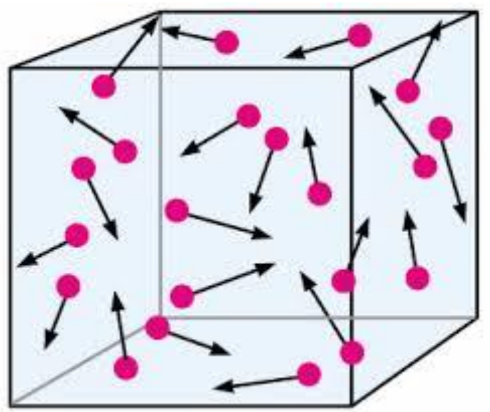
Note the averages!

$\boxed{\text{So, the total (kinetic) energy of the molecule is } \frac{3}{2}k_BT}$

Total (kinetic) energy of the system $= \frac{3}{2}Nk_BT$

Kinetic energy and Temperature

- All gas molecules at temperature *T* do not move at identical speeds
- **Maxwell-Boltzmann Velocity Distribution**

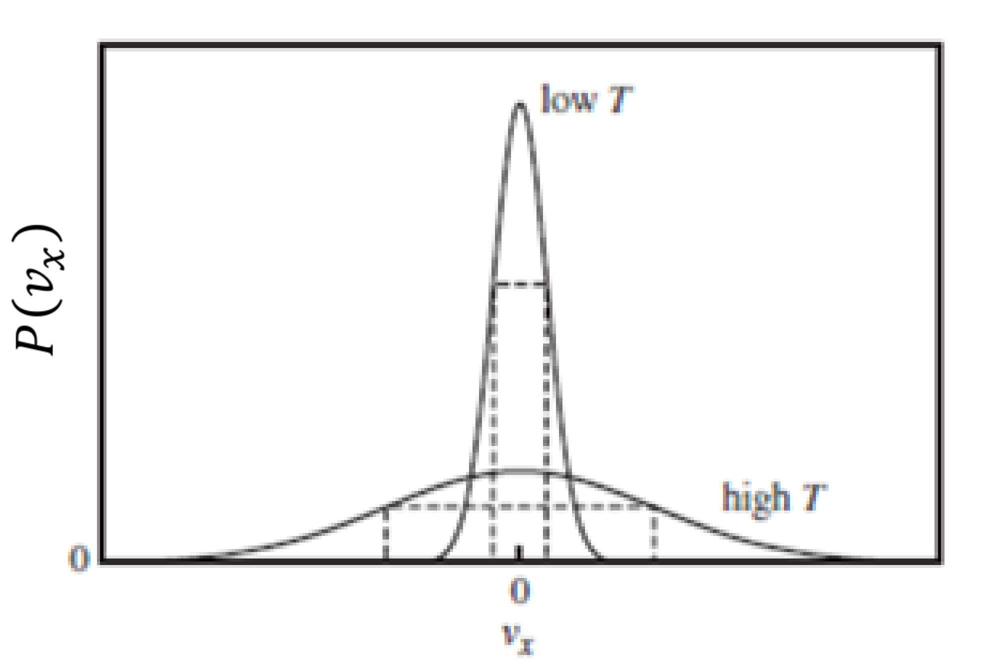


- Mean value of velocity component, $v_x = 0$
- Standard deviation, $\sigma = \sqrt{\frac{k_B T}{m}}$
- What are the dimensions?

Gaussian (Normal) Distribution

$$P(v_x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v_x - 0)^2}{2\sigma^2}}$$

- How do *m* and *T* affect the component distributions?



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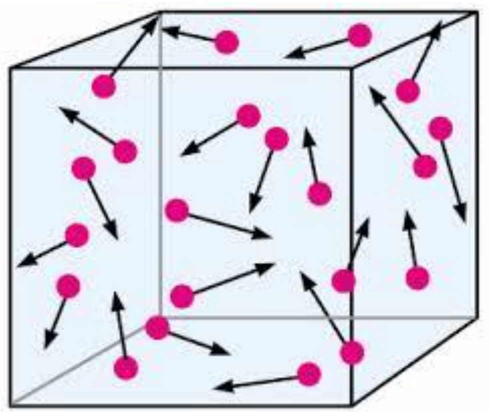

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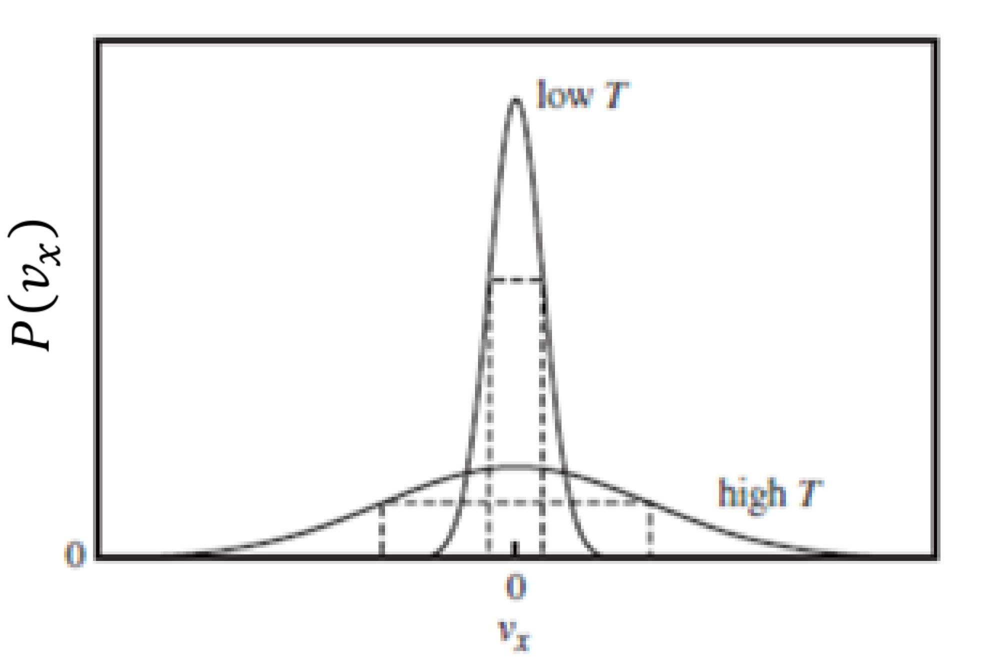
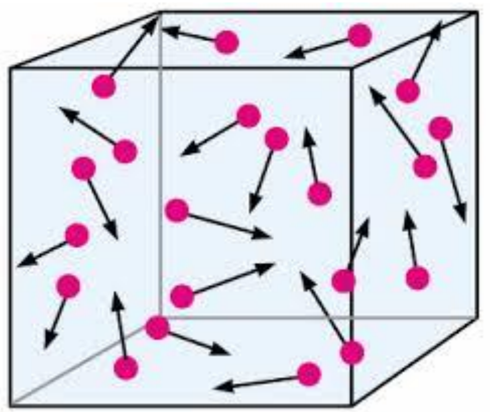


HW. Find σ for an oxygen molecule at room temperature (~300 K)

Kinetic energy and Temperature

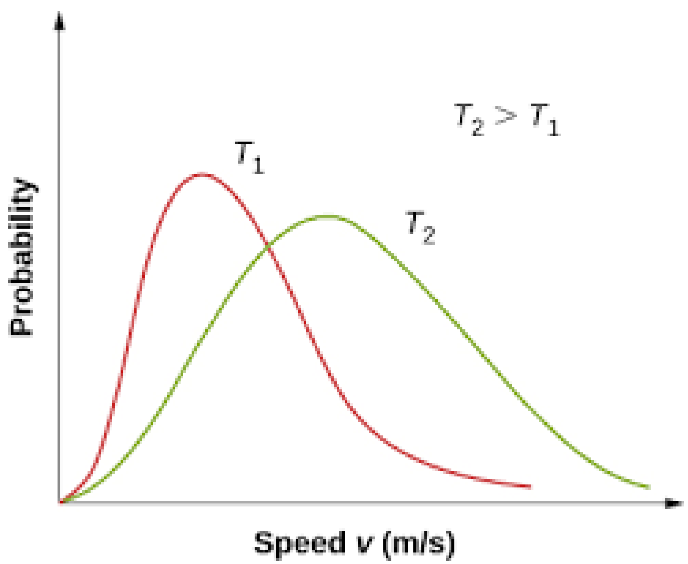
- All gas molecules at temperature *T* do not move at identical speeds
- **Maxwell-Boltzmann Velocity Distribution**



Considering the velocity magnitude,

$$P(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$

- P(v) = 0, as v → 0
- P(v) = 0, as v → inf.



- **Compare the dimensions of *P*(v_x) and *P*(*v*)**

HW. How do *m* and *T* affect the distribution?