Distributions

class – 6 (22.8.24)

LS2103 (Autumn 2024)

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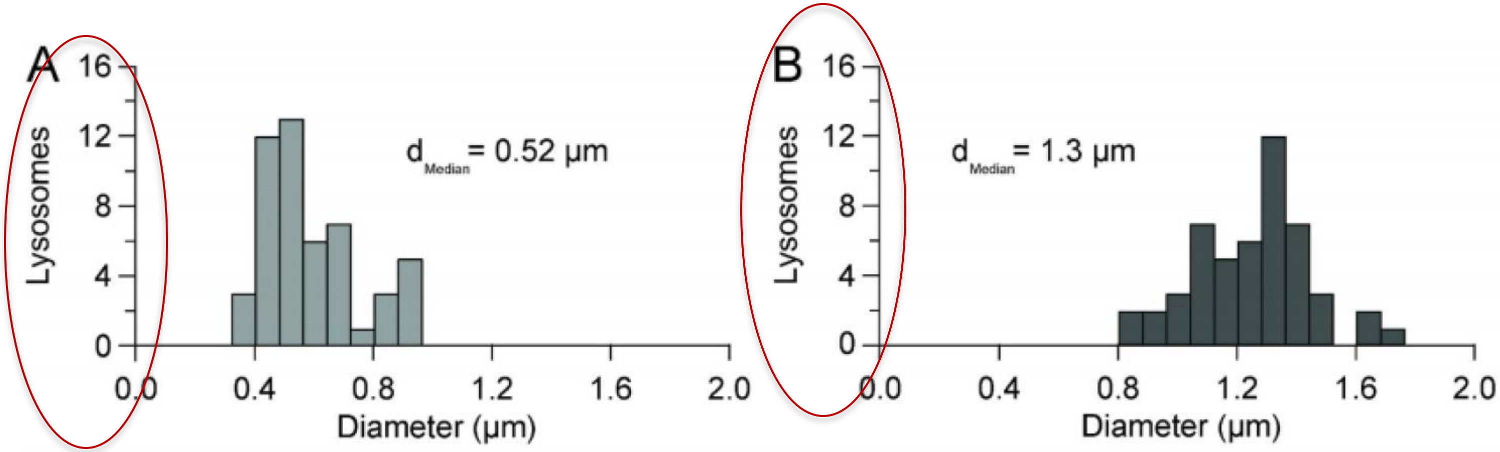


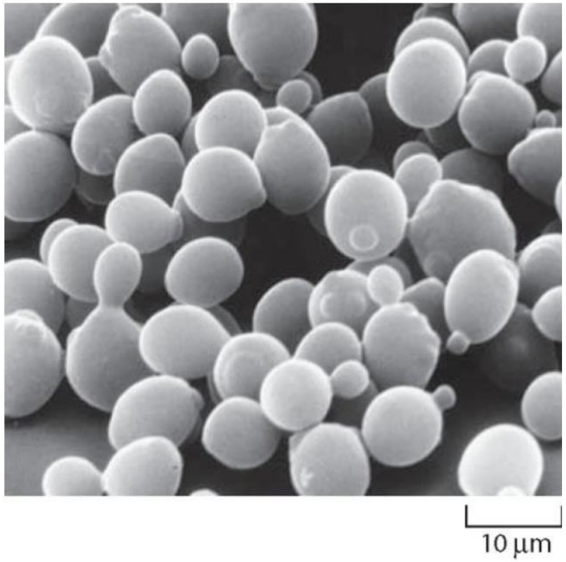
Figure 2. Distribution of lysosome diameters. (*A*) Distribution of lysosome diameters measured in control, untreated cells. (*B*) Incubation with sucrose shifts the distribution of lysosome diameters to greater values. For both plots, n = 50 lysosomes from 3 cells.
doi:10.1371/journal.pone.0086847.g002

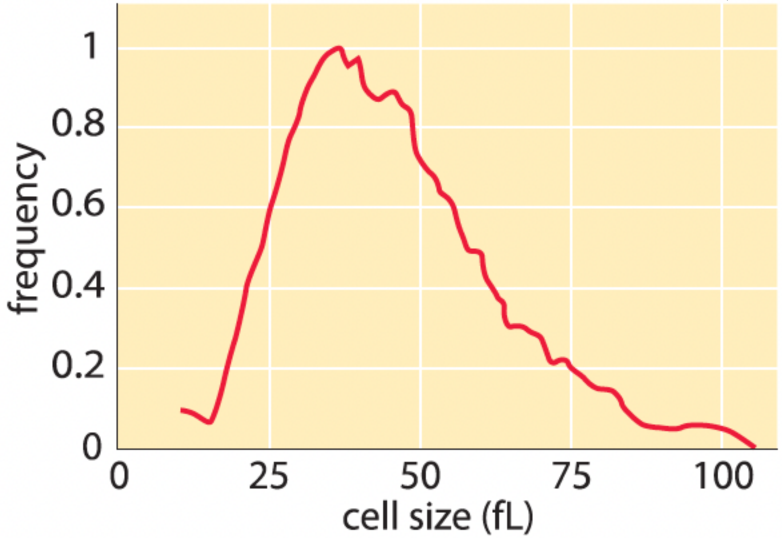
What should you get when you *add* the y-axis values?

Diameter Distributions

Yeast cells:

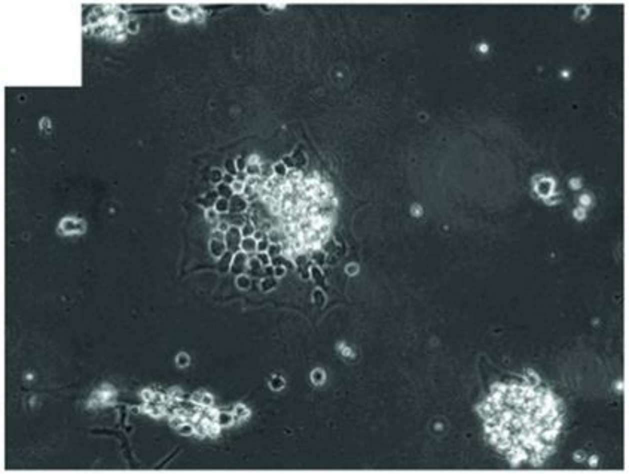
$1\ \mu m^3 = 1$ femtoLitre (fL)

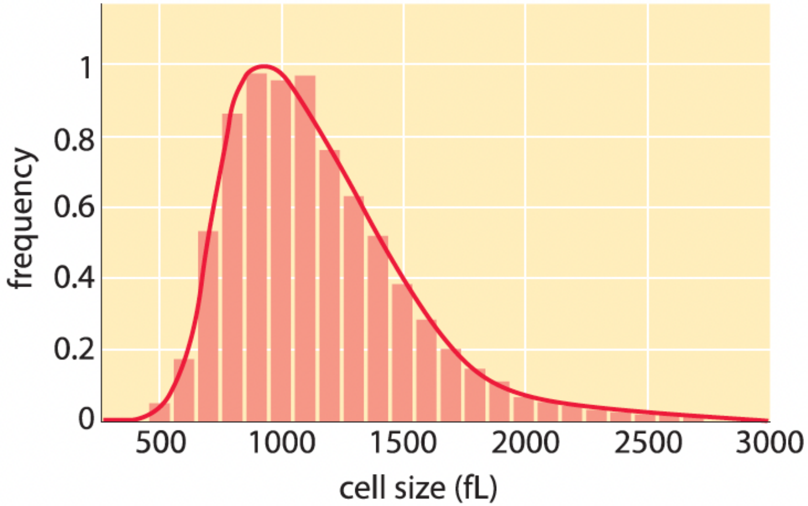




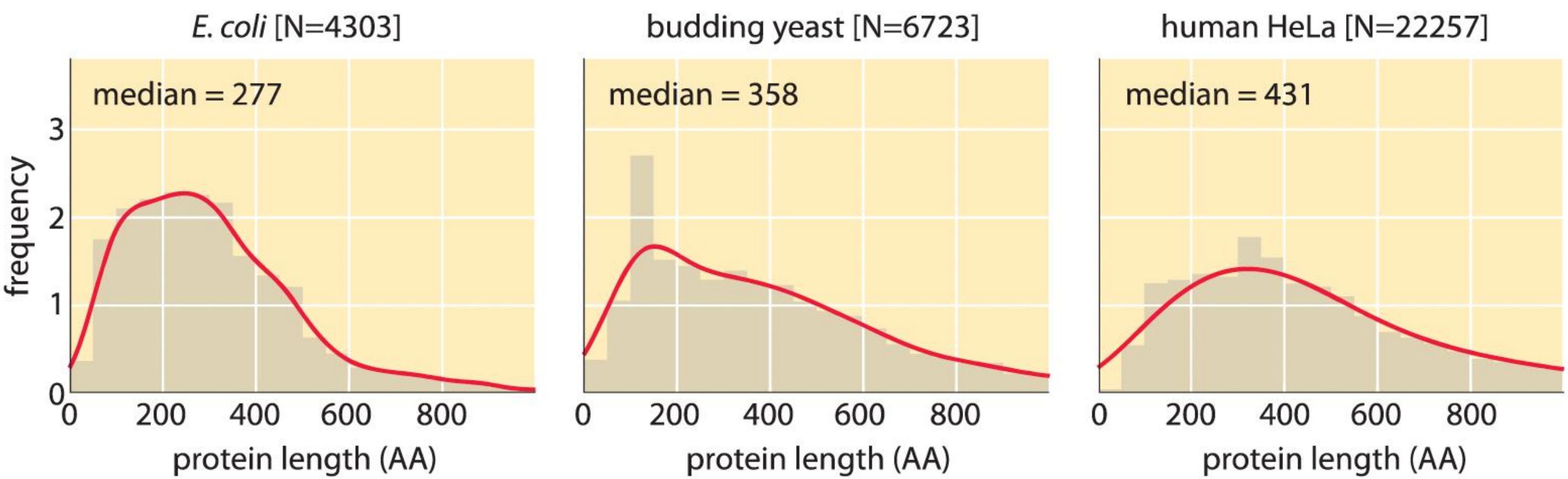
Cell. Biol. by the Numbers.

Lymphoblast cells:





Diameter Distributions

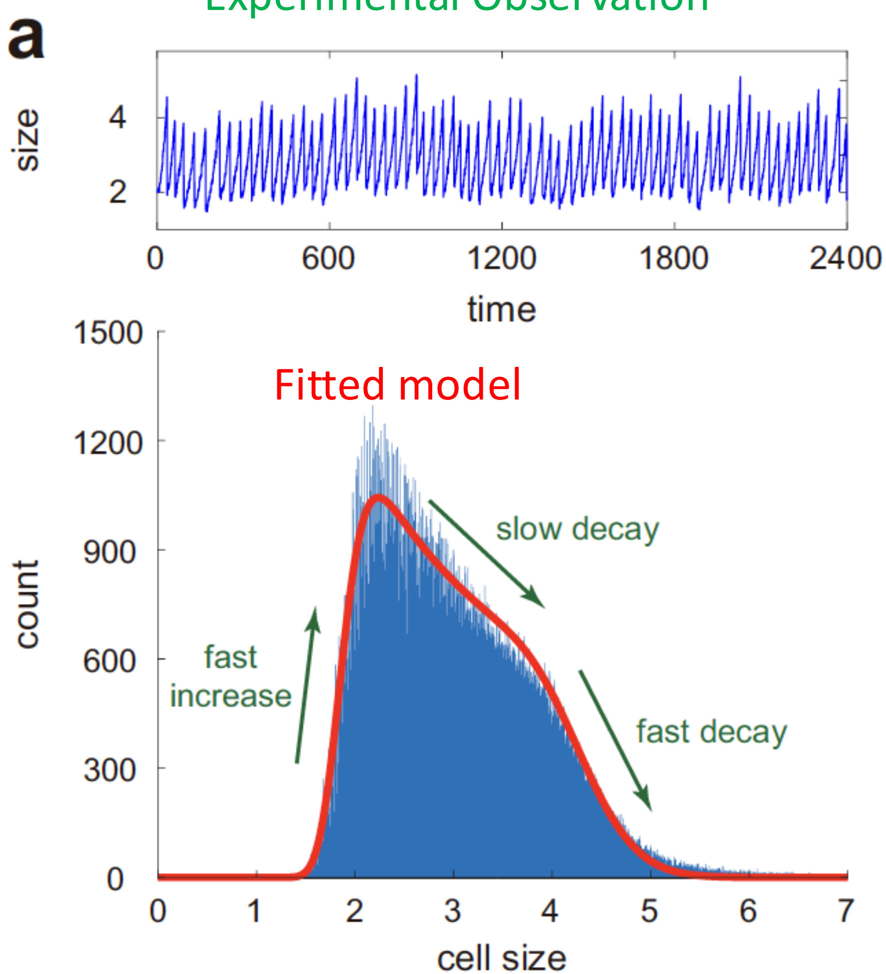


Cell. Biol. by the Numbers.

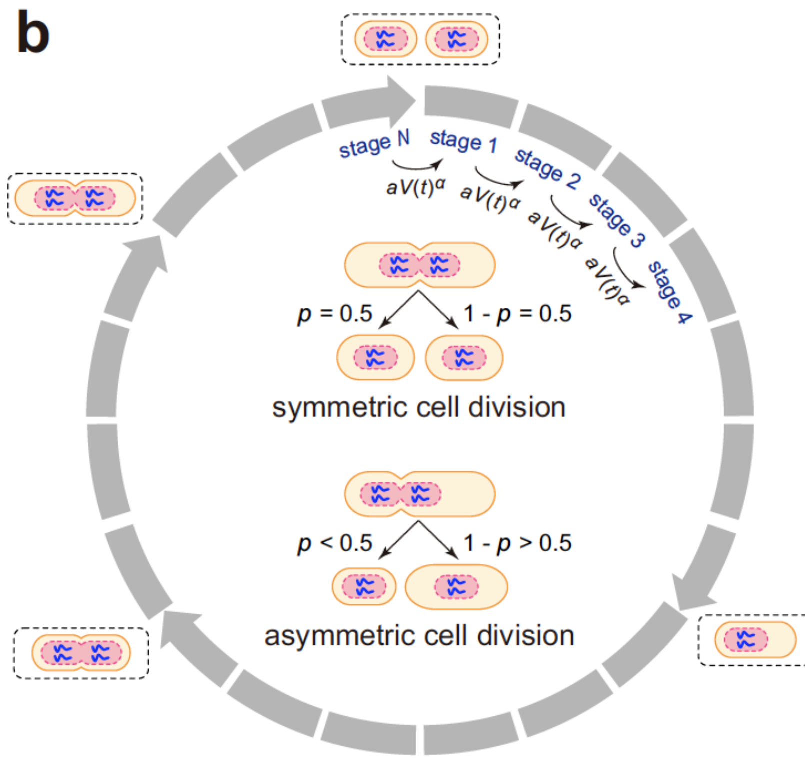
Distributions aid understanding (model building)

Eg. How do cells maintain their size across cell cycles?

Experimental Observation



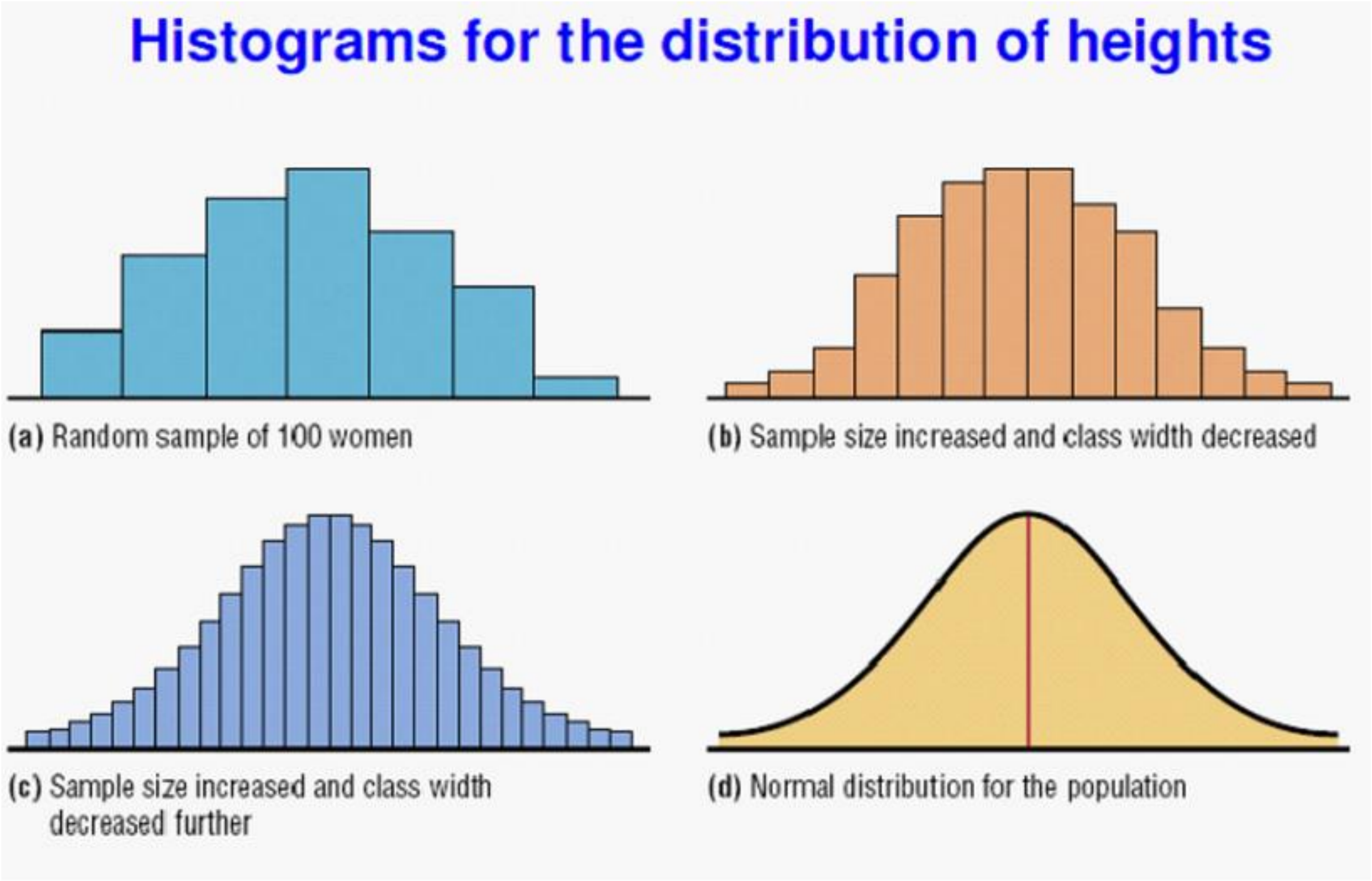
Explanation



Jia et al, iScience 2020, https://doi.org/10.1016/j.isci.2021.102220

Discrete vs. Continuous distributions

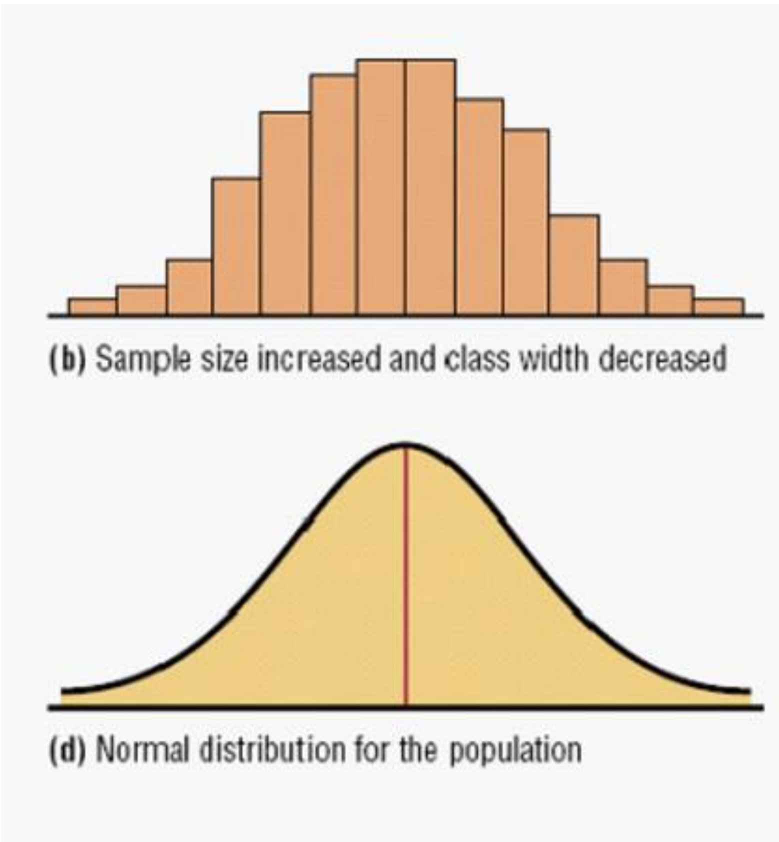
Eg.



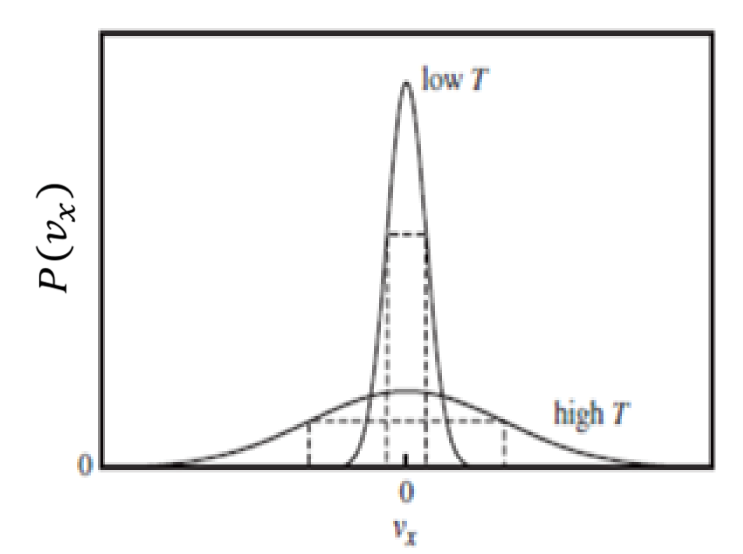
http://rchsbowman.wordpress.com/2009/11/29/statistics-notes-%E2%80%93-properties-of-normal-distribution-2/

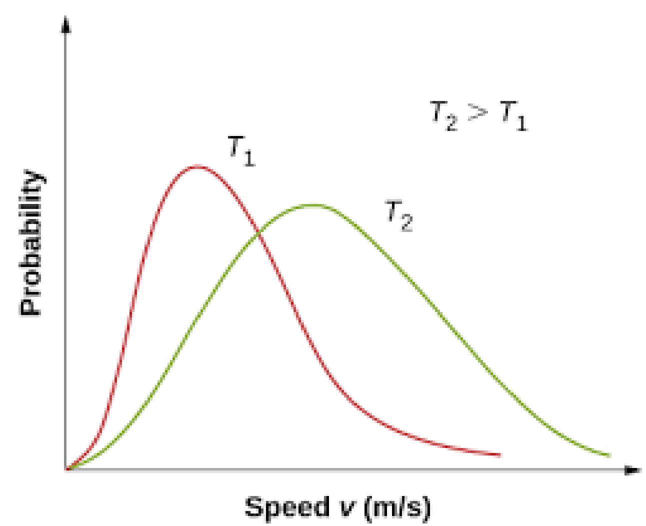
Discrete vs. Continuous distributions

Height distributions of population



Molecular velocity distributions





Discrete vs. Continuous distributions

Monday, 31 August 2020 3:49 PM

- Consider an event with 'N' number of outcomes:

$$\{x_1, x_2, \ldots\ldots, x_j, x_{j+1}, \ldots\ldots x_N\} \quad \text{— (1a)}$$

- The PROBABILITIES of these outcomes are :

$$\{p_1, p_2, \ldots\ldots, p_j, p_{j+1}, \ldots\ldots p_N\} \quad \text{— (1b)}$$

Such that :-

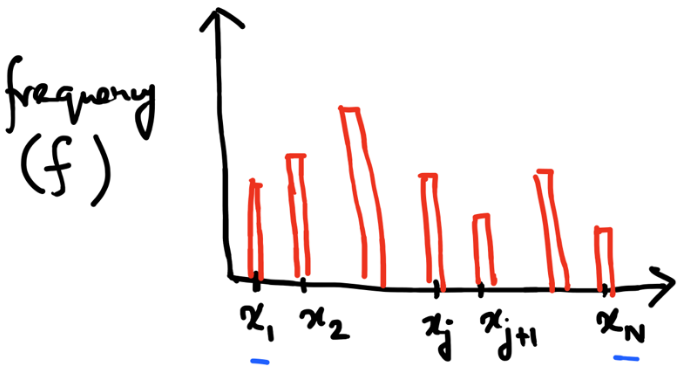
$$\boxed{\sum_{j=1}^{N} p_j = 1} \quad \text{— (1c)}$$

Example : Roll of a dice with 6 sides. Here, $p_1 = p_2 = \ldots\ldots = p_6 = \frac{1}{6}$

Discrete vs. Continuous distributions

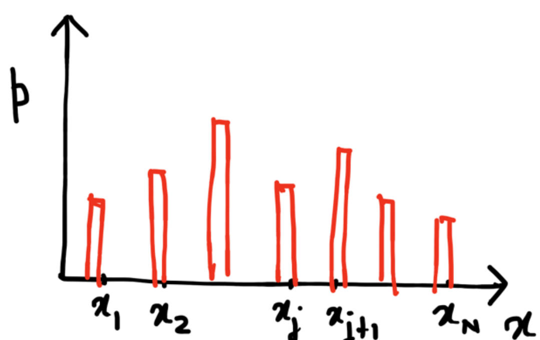
Thursday, 3 September 2020 2:04 PM

If this "event" takes place enough ($\underline{\underline{M}}$) number of times, we can obtain a DISCRETE distribution :-



$$\sum_{j=1}^{N} f_j = M$$

4 PM



$$\boxed{\sum_{j=1}^{N} p_j = \sum_{j=1}^{N} \frac{f_j}{M}}$$

Discrete vs. Continuous distributions

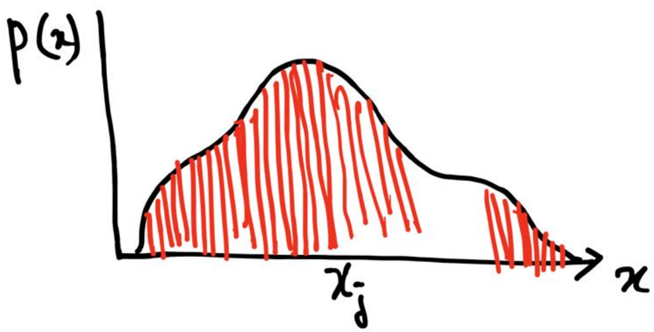
Now, if the possible "events" were:

i) Placed very close together, ie. $x_{j+1} - x_j \sim 0$

ii) 'N' was very large

iii) 'M' was very large

$$\sum_j p(x_j) = 1$$



Discrete vs. Continuous distributions

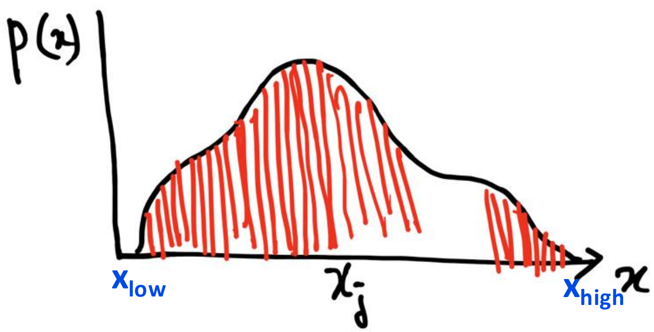
Now, if the possible "events" were:

i) Placed very close together, ie. $x_{j+1} - x_j \sim 0$

For vanishingly small class intervals

ii) 'N' was very large

iii) 'M' was very large



$$\sum_j p(x_j) = 1$$

$$\downarrow$$

$$\int_{x_{low}}^{x_{high}} p(x)\,dx = 1$$

Continuous distributions

What are the UNITS of $p(x)$ for a continuous probability distribution?

$$\int_{x_{low}}^{x_{high}} p(x)\,dx = 1$$

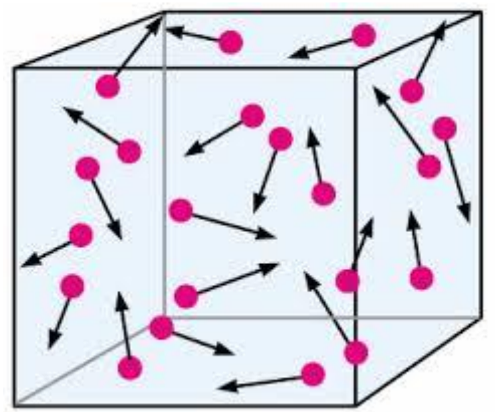
$$[p(x)]\,[dx] \longrightarrow \text{dimensionless}$$

Units & dimensions of $p(x) \longrightarrow [dx]^{-1} \equiv [x]^{-1}$

Q. What are the units of $f(x_i)$ or $P(x_i)$ in a discrete distribution?

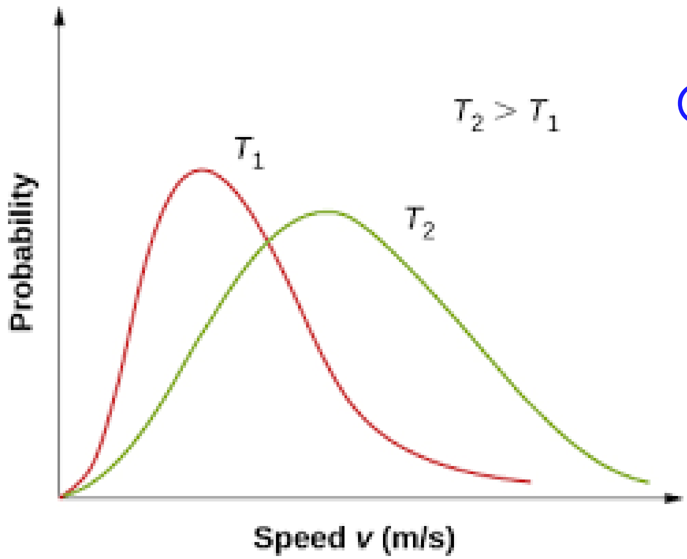
M-B Velocity Distribution is a *Continuous Disbn*.

$$P(v_x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v_x-0)^2}{2\sigma^2}}$$



Considering the velocity magnitude,

$$P(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$



Compare the dimensions of $P(v_x)$ and $P(v)$

Continuous distributions

The probability of obtaining a value *within an interval*:

$$\int_{x_j}^{x_k} p(x)\,dx = P_{jk} < 1$$

Therefore,

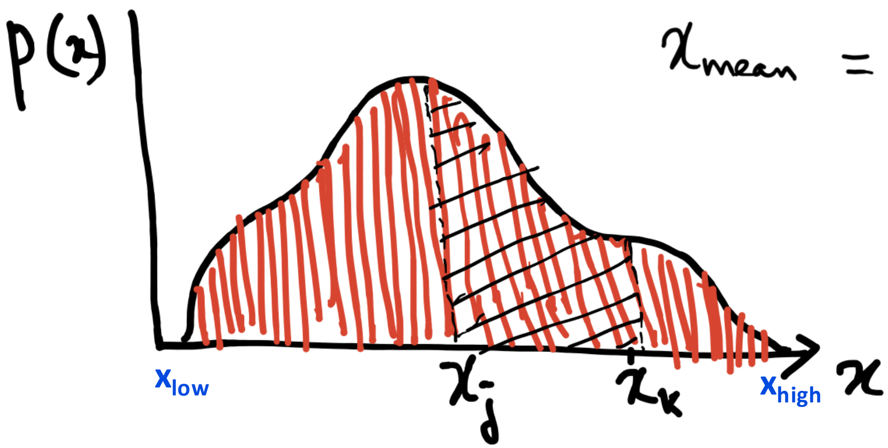
$$\int_{x_{low}}^{x_j} p(x)\,dx + \int_{x_j}^{x_k} p(x)\,dx + \int_{x_k}^{x_{high}} p(x)\,dx = 1$$

Q. What are x_{low} and x_{high} for M-B distribution?

Continuous distributions

The *mean* value of a measurement,

$$x_{mean} = \bar{x} = \langle x \rangle = \int_{x_{low}}^{x_{high}} x\, p(x)\, dx$$



The *variance*, $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

$$= \int_{x_{low}}^{x_{high}} x^2\, p(x)\, dx - \left[\int_{x_{low}}^{x_{high}} x\, p(x)\, dx \right]^2$$

Q. Write down the expression for *standard deviation* of a continuous distribution

Continuous distributions

Consider a quantity f that depends on the variable x, *ie.*

$$f \longrightarrow f(x)$$

The mean value of the f,

$$\bar{f} = \int_{x_{low}}^{x_{high}} f(x)\, p(x)\, dx$$

The mean value of the f^2,

$$\overline{f^2} = \int_{x_{low}}^{x_{high}} [f(x)]^2\, p(x)\, dx$$

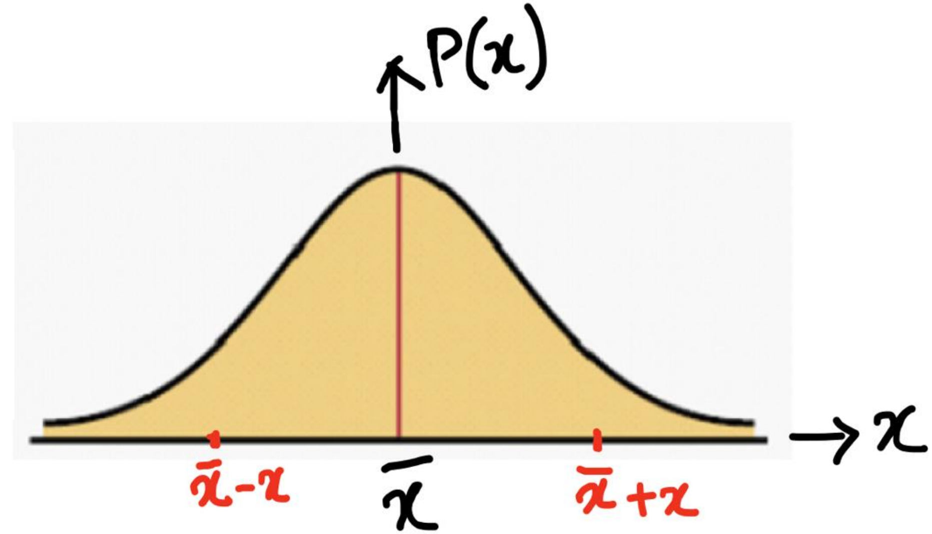
The variance,

$$\sigma_f^2 = \overline{f^2} - \left(\bar{f}\right)^2$$

Continuous distributions

The Gaussian (Normal) Distribution:

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\bar{x})^2/2\sigma^2}$$



Symmetric function: $P(\bar{x}+x) = P(\bar{x}-x)$

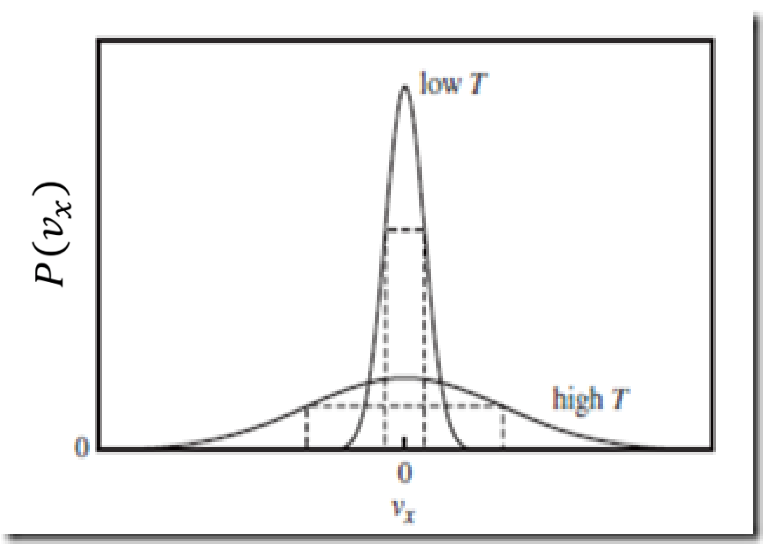
If centered at the origin, it becomes an *even function*

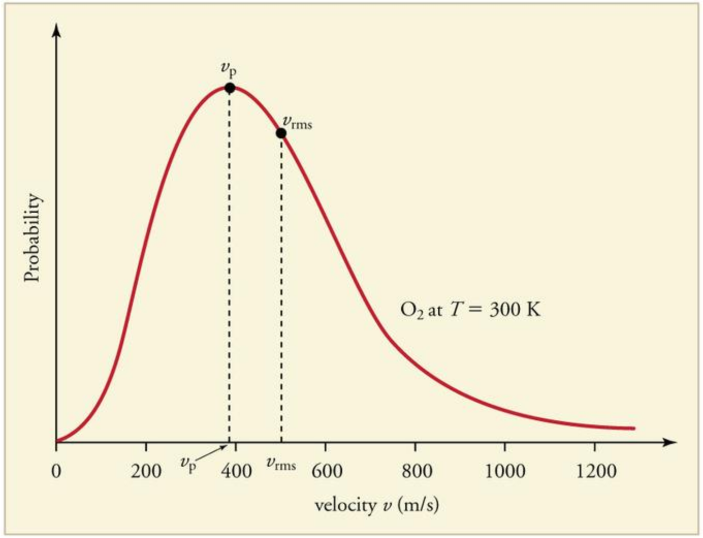
HW. Show that the *variance* of the function is given by σ^2

Why is the full M-B velocity distribution NOT a Normal Distribution?

$$P(v_x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v_x - 0)^2}{2\sigma^2}}$$

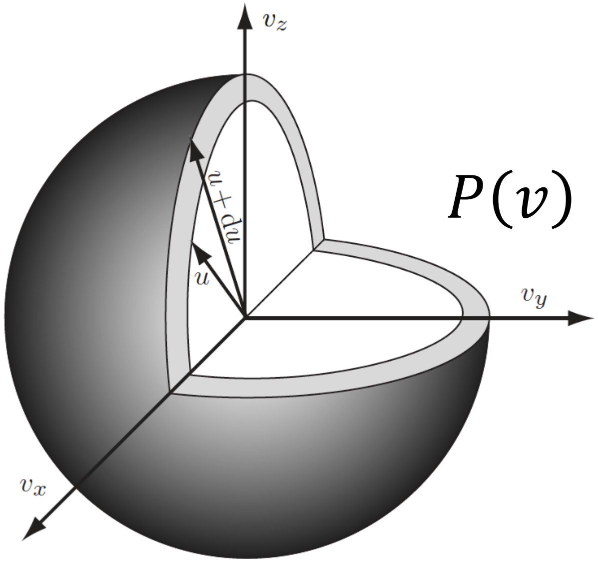
$$P(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$





Maxwell-Boltzmann Velocity Distribution

Why is the full velocity distribution NOT a Gaussian (Normal) Distribution?



$$P(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$

Figure 3.4: (Sketch.) The set of all vectors $\mathbf{v}$ of length u is a sphere. The set of all vectors with length between u and $u + du$ is a spherical shell.

$$P(v)\,dv \equiv P(v_x) \cdot P(v_y) \cdot P(v_z) \times \left[\text{Vol. of shell "}dv\text{" in velocity space}\right]$$

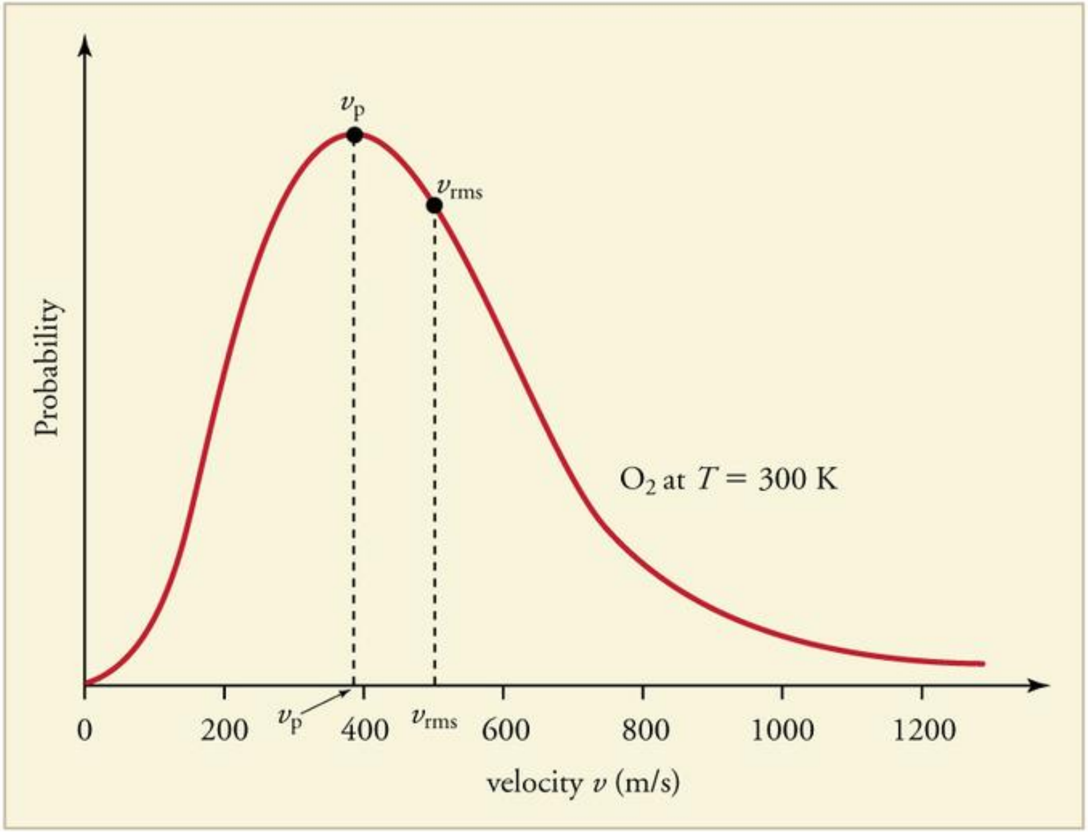
$$= P(v_x) \cdot P(v_y) \cdot P(v_z) \times \left[4\pi v^2 dv\right]$$

HW. Derive these relationships

$$v_{m.p} = \sqrt{\frac{2k_BT}{m}}$$

$$\bar{v} = v_{av.} = \sqrt{\frac{8k_BT}{\pi m}}$$

$$v_{r.m.s} = \sqrt{\frac{3k_BT}{m}}$$



$$\frac{k_BT}{m} \equiv \frac{RT}{M}$$

m → mass of molecule

M → molar mass

The **GAMMA FUNCTION**
is a friendly aid!
(…if you practice a bit)

$$\boxed{\Gamma(n) = \int_0^\infty e^{-x}\, x^{n-1}\, dx}$$

1. For positive *integer n*,

$$\boxed{\Gamma(n) = (n-1)!}$$

2. For any positive *n*,

$$\boxed{\Gamma(n+1) = n\,\Gamma(n)}$$

3. For *n* = *1/2*,

$$\boxed{\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}}$$