Viscosity: Laminar vs. Turbulent Flow

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Diffusion Equations:

$$\langle \boldsymbol{r}_N^{\,2} \rangle = (2d)\,(D)\,(T)$$

$$\zeta D = k_B T$$

$$6 \pi \eta a D = k_B T$$

Dimensions and Units of dynamic viscosity

$$[\eta] = [\text{pressure}]\,[\text{time}]$$

$$\equiv [M L^{-1} T^{-1}]$$

SI units: Pa-s $\equiv 1\ \text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$

Commonly used Units: Poise

$$1 \text{ poise} = 1\ \text{g}\,\text{cm}^{-1}\,\text{s}^{-1}$$
$$= 0.1\ \text{Pa}\cdot\text{s}$$

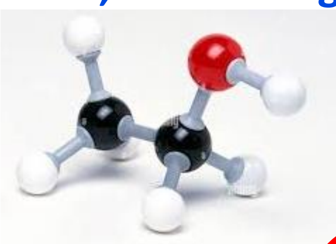
$$1 \text{ centipoise } (1 \text{ cP}) = 10^{-2} \times 0.1\ \text{Pa-s}$$
$$= 10^{-3}\ \text{Pa-s}$$

Diffusion Equations:

$$\langle r_N^2 \rangle = (2d)\,(D)\,(T)$$

$$6\pi\eta a D = k_B T$$

radius, a = 4.4 Angs



η at 25°C:
1.074 centipoise (cP)

Eg 1. **self-diffusion of pure Ethanol**

- How will you approximately estimate the time to diffuse across a given length, say 5 cm?

- What does your answer indicate?

$$D = \frac{k_B T}{6\pi\eta a}$$

$$= \frac{(1.38\times10^{-23})\times(298)}{6\times(3.14)\times(1.074\times10^{-3})\times(4.4\times10^{-10})}$$

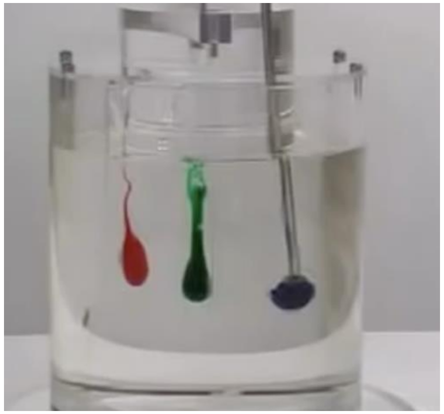
$$= (4.62)\times10^{-10}\ m^2 s^{-1}$$

$$\langle r^2 \rangle = 25\times10^{-4}\ m^2 = 6\times(4.62\times10^{-10})(\text{time})$$

$$(\text{time}) \equiv 0.9\times10^{6}\ \text{sec}$$

Ink droplets in corn syrup can be *unmixed* by reverse stirring

Viscosity, $\eta_{\text{corn-syrup}}$ = 5 Pa-s (at 298 K)



Stir

(apply force)

'Reverse' Stir

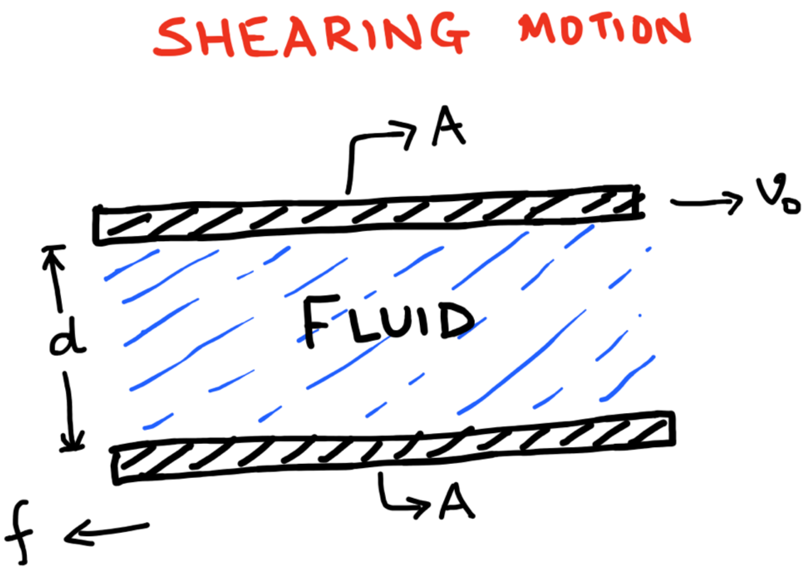
But not milk in coffee

Viscosity, η_{water} = 0.0018 Pa-s == 1.8 centipoise (at Room Temperature)

Stir
(apply force)

'Reverse' Stir

What is viscosity?



$$f \propto A,\quad \propto v_0,\quad \propto \frac{1}{d}$$

Plate dragged with speed v_0 gives rise to resistive viscous force (f) in the opposite direction.

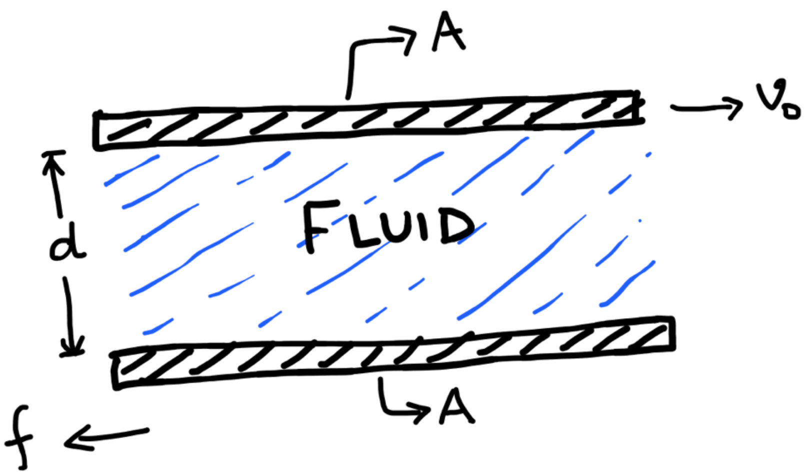
$$\boxed{f = -(\eta)\left(\frac{A v_0}{d}\right)}$$

Bulk property of the fluid

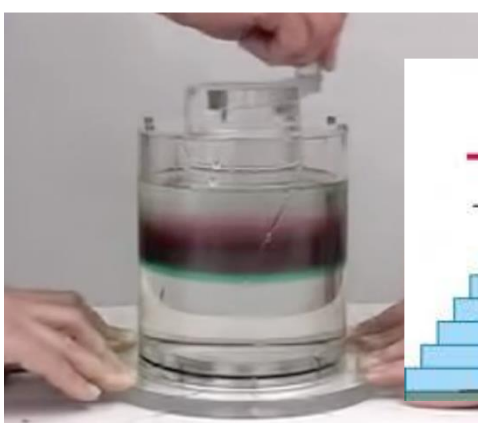
Note:
Drag (resistive) *force* felt in water will be much smaller than in corn syrup and glycerin

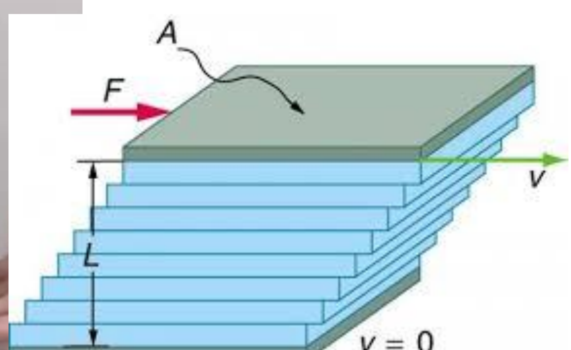
What is viscosity?

SHEARING MOTION



Laminar (organized) Flow





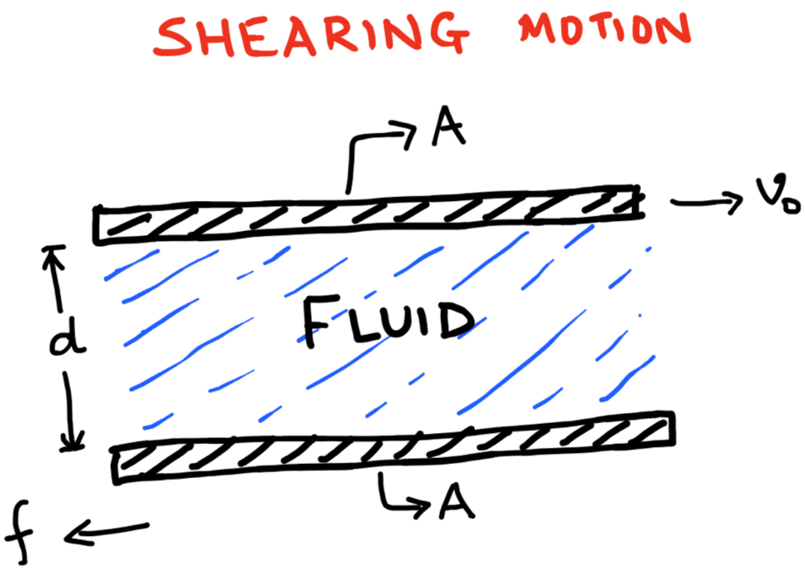
$$\boxed{f = -(\eta)\left(\frac{A v_0}{d}\right)}$$

Large

Small

$$\boxed{6 \pi \eta a D = k_B T}$$

What is viscosity?



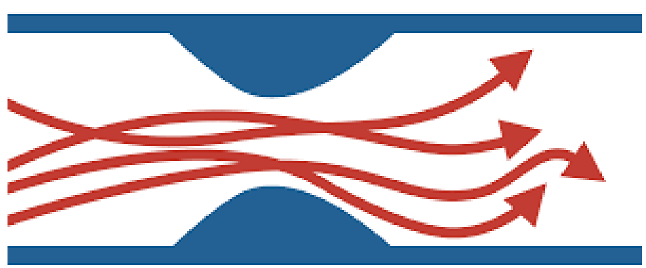
$$\boxed{f = -(\eta)\left(\frac{A v_0}{d}\right)}$$

Small

Turbulent (chaotic) flow

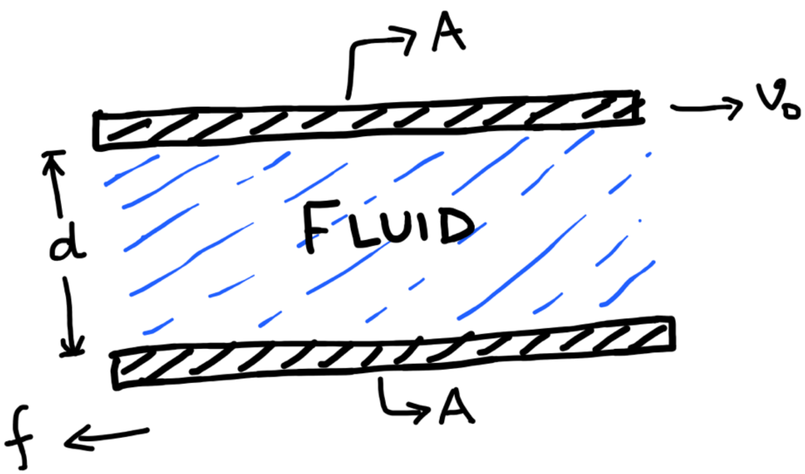
Large

$$\boxed{6 \pi \eta a D = k_B T}$$



Laminar or Turbulent?

SHEARING MOTION



$$f = -(\eta)\left(\frac{A v_0}{d}\right)$$

Laminar → Turbulent

Can there be a transition?

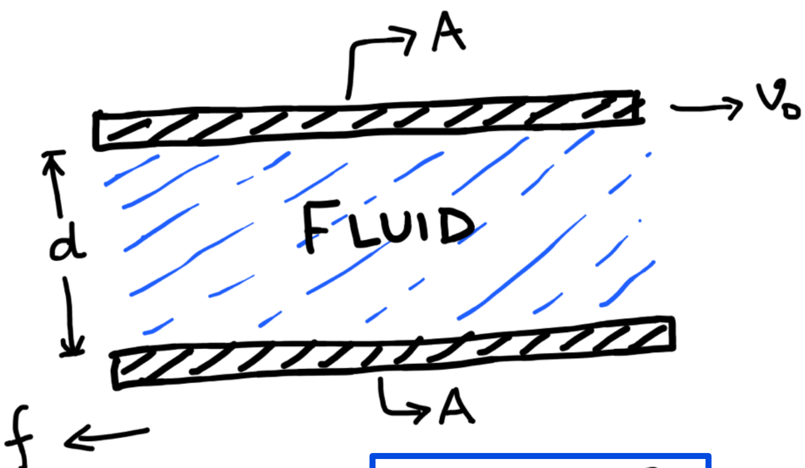
Let $Q \equiv \frac{\eta^2}{\rho_m}$

$$[Q] \equiv ?$$

$$\equiv [MLT^{-2}]$$

Laminar or Turbulent?

SHEARING MOTION



$$\boxed{f = -(\eta)\left(\frac{A V_0}{d}\right)}$$

Situation and system dependent

Critical force, $\boxed{f_{critical} = \frac{\eta^2}{\rho_m}}$

Can there be a transition?

Laminar: $f < f_{critical}$

Turbulent: $f > f_{critical}$

Laminar or Turbulent?

Critical force, $$f_{critical} = \frac{\eta^2}{\rho_m}$$

$$f = -(\eta)\left(\frac{A V_0}{d}\right)$$

Situation and system dependent

- What magnitude of force is needed for turbulence in these liquids?

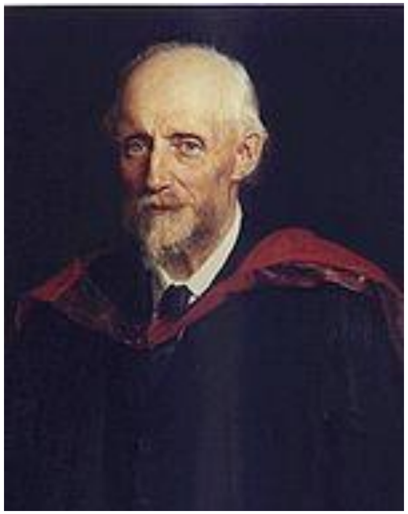
Fluid	Density (kg m^{-3})	η (Pa-s)	$f_{critical}$ (N)
Water	1000	0.0018	3.24×10^{-9} N ?
Olive oil	900	0.08	?
Corn syrup	1000	5	?

Room temperature data

By what order of magnitude do they vary?

Reynolds Number (Re)

The ratio of inertial to viscous forces



Osborne Reynolds
1842-1912

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$

(mass) x (acceleration)

depends on viscosity of fluid; velocity; and size of the body.

High Re: inertial forces dominate, flow is more **turbulent**

Low Re: viscous forces dominate, flow is more **laminar**

The transition from laminar to turbulent occurs at Re ~1000

Reynolds Number (Re)

The ratio of inertial to viscous forces

Work out the dimensions

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$

$$\nu = \frac{\eta}{\rho_m}$$

$$[\nu] = ?$$

$$Re = \frac{\rho_m v a}{\boxed{\eta}}$$

speed

size

dynamic viscosity

$$= \frac{v a}{\nu}$$

kinematic viscosity

The transition from laminar to turbulent occurs at Re ~1000

Reynolds Number (Re)

The ratio of inertial to viscous forces

$$f_{critical} = \frac{\eta^2}{\rho_m}$$

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$

- For **water**,
  $f_{critical}$ is ~**10^{-9} Newtons**!
- **Water is easily 'turbulent' for macroscopic objects**

Swimming is easily turbulent (for us)

>A ~2 kg load (say, your arm) at acceleration of 0.1 g involves a force of 0.2 Newtons

> A ~60 kg swimmer in water generates very large forces

The transition from laminar to turbulent occurs at Re ~1000

Reynolds Number (Re)

The ratio of inertial to viscous forces

$$f_{critical} = \frac{\eta^2}{\rho_m}$$

$$Re = \frac{|\text{Inertial force}|}{|\text{Viscous force}|}$$

- For **water**, **$f_{critical}$** is ~**10^{-9} Newtons**!
- **Water is easily 'turbulent' for macroscopic objects**

$$Re = \frac{\rho_m V a}{\eta} =$$

What about unicellular organisms?

> Typical parasitic protozoa size: ~50 μm

> Floating speed ~100 μm/second ✓

- ➢ Surrounding flow appears **laminar**
- ➢ What are the consequences? $Re \sim 10^{-3}$

The transition from laminar to turbulent occurs at Re ~1000

Prob.

c) Plankton are passive organisms found in *sea water*. The largest plankton measure about 20 mm, the smallest measure about 2 micrometers, and they can cover roughly their own length per second in water.

What are the ranges of their Reynolds number ~~at 20 °C~~?

Which forces dominate the motion of plankton in calm sea water?

η_1 viscosity of pure water at 25 °C = 8.90×10^{-4} Pascal-seconds.

η_2 viscosity of *sea water* at 20 °C = 1.07×10^{-3} Pascal-seconds.

ρ_2 density of *sea water* at 20 °C = ~~$1.2\ \mathrm{kg\ m^{-3}}$~~ $1.2 \times 10^3\ \mathrm{kg/m^3}$

$a = 20\ \mathrm{mm}$
$= 20 \times 10^{-3}\ \mathrm{m}$

	smallest Plankton	largest Plankton
Pure H_2O	4.5×10^{-6}	$4.5 \times 10^{+2}$
Sea water	4.5×10^{-6}	4.5×10^{2}