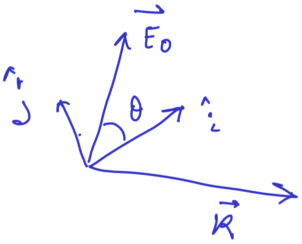
Polarization

Consider the plane formed by the displacement vector and the wavevector $\vec{k}$. If the displacement vector remains in the same direction as one moves along $\vec{k}$, we have a linearly polarized light.

$$\vec{E} = \vec{E}_0 \cos(kz - \omega t)$$

$\vec{E}_0$ is not a function of z or t.



We choose the axes as shown in the figure.

$$\Rightarrow \vec{E}_0 = E_0 \cos\theta\, \hat{i} + E_0 \sin\theta\, \hat{j}$$

and

$$\vec{E} = \underbrace{\left(E_0 \cos\theta\, \hat{i} + E_0 \sin\theta\, \hat{j}\right)}_{\text{amplitude.}} \cos(kz - \omega t)$$

Where we choose x axis such that xz is the plane of incidence.

$$= \left(E_0^p\, \hat{i} + E_0^s\, \hat{j}\right) \cos(kz - \omega t)$$

amplitude of E_0^p: p-type, p-polarized, p-component

amplitude of E_0^s: s-type, s-polarized, s-component

Continued in the next page.

Now, Fresnel's equations

$$\frac{E_R^p}{E_I^p} = \frac{\alpha - \beta}{\alpha + \beta} \qquad \boxed{\frac{E_R^s}{E_I^s} = \frac{1 - \alpha\beta}{1 + \alpha\beta}}$$

$$\frac{E_T^p}{E_I^p} = \frac{2}{\alpha + \beta} \qquad \boxed{\frac{E_T^s}{E_I^s} = \frac{2}{1 + \alpha\beta}}$$

Fresnel's equations along with the decomposition to "s" and "p" types provide complete description.

* Case $\alpha = \beta \Rightarrow E_R^p = 0$ and $E_R^s \neq 0$ → Brewster's angle

⇒ Reflected light is linearly polarized.

How polarizers work:

Explanation based on free electrons in "wire".

Now, consider

$$\vec{E} = E_0 \hat{i} \cos(kz - \omega t) = \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\sin(kz-\omega t)\right\} + \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) - \hat{j}\sin(kz-\omega t)\right\}$$

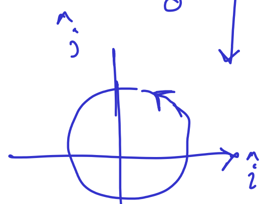
$$= \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t-\pi/2)\right\} + \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t+\pi/2)\right\}$$

using complex notation

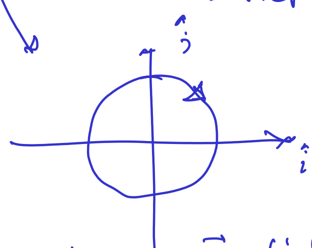
$$= \frac{E_0}{2}\left\{\hat{i}\, e^{i(kz-\omega t)} + \hat{j}\, e^{-i\pi/2} e^{i(kz-\omega t)}\right\} + \frac{E_0}{2}\left\{\hat{i}\, e^{i(kz-\omega t)} + \hat{j}\, e^{+i\pi/2} e^{i(kz-\omega t)}\right\}$$

$$= \frac{E_0}{2}\left\{(\hat{i} + i\hat{j})\, e^{i(kz-\omega t)}\right\} + \frac{E_0}{2}\left\{(\hat{i} - i\hat{j})\, e^{i(kz-\omega t)}\right\}$$

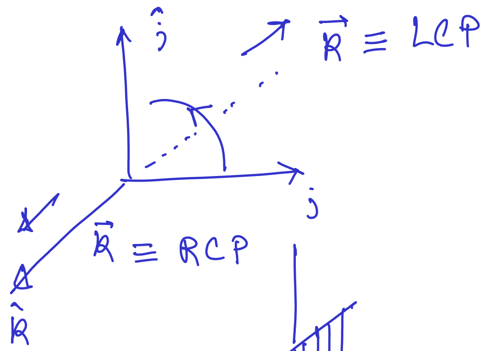
$$= \vec{E}_+ + \vec{E}_-$$



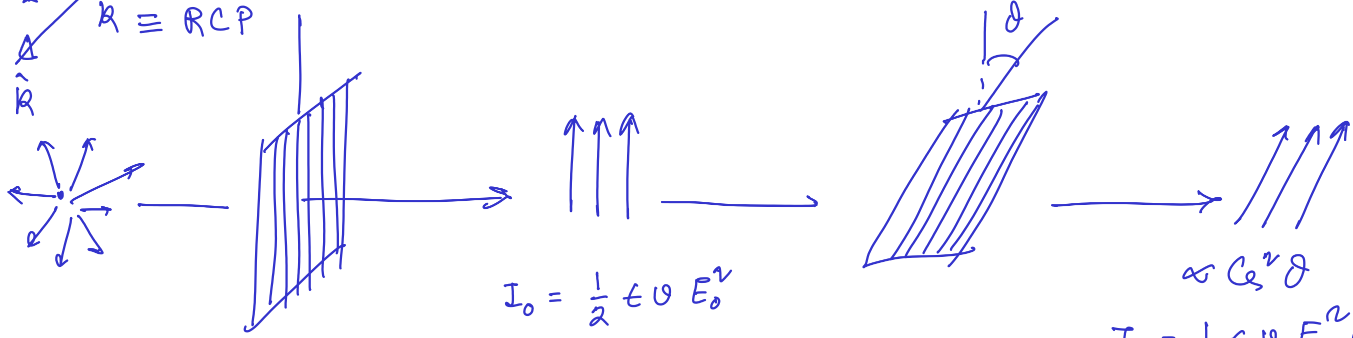
tip of $\vec{E}$ field at a given t as one moves along z → RCP



tip of $\vec{E}$ field at a given t as one moves along z → LCP



⊛



$I_0 = \frac{1}{2}\epsilon v E_0^2$

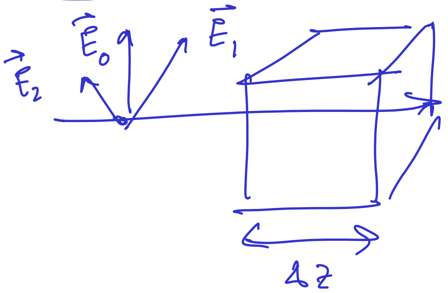
$\propto \cos^2\theta$

$I = \frac{1}{2}\epsilon v E_0^2 \cos^2\theta$

Malu's law.

⊛ Reflection of RCP gives LCP

⊛ Retardation plates. (has two optical axes with diff. n values).



$$\frac{\omega}{K} = v = \frac{c}{n} \Rightarrow K = n\frac{\omega}{c}$$

$$\Rightarrow \text{Phase lag} = K\Delta z = n\frac{\omega}{c}\Delta z$$

Lag between two axes

$$= n_1\frac{\omega}{c}\Delta z - n_2\frac{\omega}{c}\Delta z$$

$$= \Delta n \frac{\omega}{c}\cdot\Delta z = \Delta n \frac{2\pi}{\lambda_0}\cdot\Delta z$$

$$\boxed{\phi = 2\pi\cdot\Delta n\cdot\frac{\Delta z}{\lambda_0}}$$

$\phi \to \pi/2 \to$ quarterwave plate $\to$ linear to circular

$\phi \to \pi \to$ half-wave plate $\to$ RCP $\rightleftarrows$ LCP