

PH2102 Problem Set 4

Q 1) Use the velocity transformation law

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

to show that if both u and v are smaller than c , then u' is smaller than c .

For the next two problems assume that $c = 1$. This is not essential - but simplifies the algebra somewhat

Q 2) a) Write down the special Lorentz transformation equation in 1+1 dimensions in terms of the rapidity $\eta = \frac{1}{2} \ln \left(\frac{1+\beta}{1-\beta} \right)$ in place of v . *Your result should involve hyperbolic functions of η .*

b) Write down the form that these equations take for small η .

Q 3) Consider a situation where we have three observers - Alice, Bob and Charlie moving in in spatial dimension. The spacetime coordinates for an event according to them are (x, t) , (x', t') and (x'', t'') . If the rapidity of Bob with respect to Alice is η_1 and that of Charlie with respect to Alice is η_2 , write down the transformation equations for (x', t') in terms of (x, t) and (x'', t'') in terms of (x', t') . Combine these to obtain the relation between (x'', t'') and (x, t) . Use this final transformation to determine the rapidity of Charlie with respect to Alice.