Distributions

class – 8 (29.8.24)

LS2103 (Autumn 2024)

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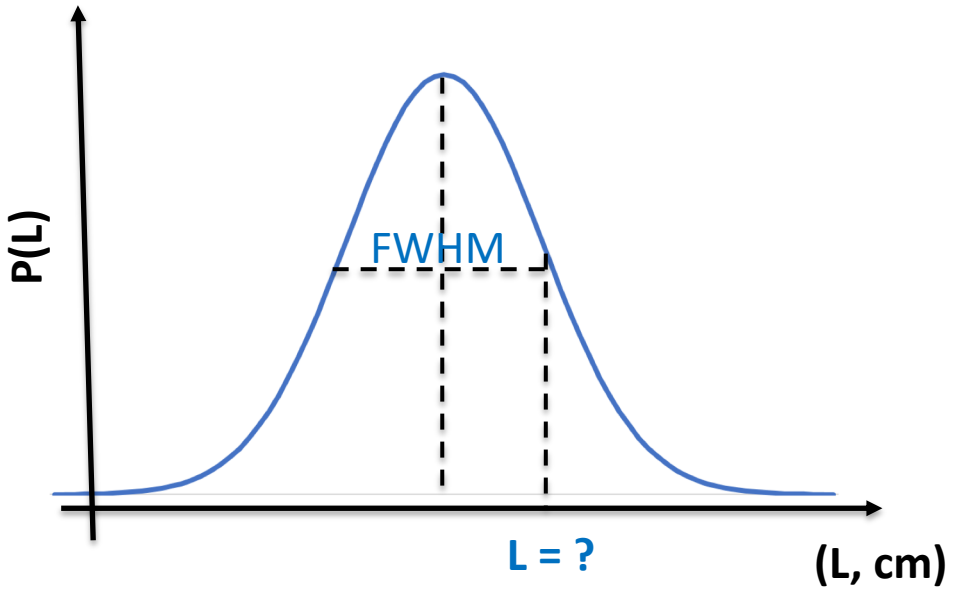
Prob. The length of a certain type of eel, L cm, is assumed to be a *Normal variable* with a mean of 77 and a standard deviation of 8.



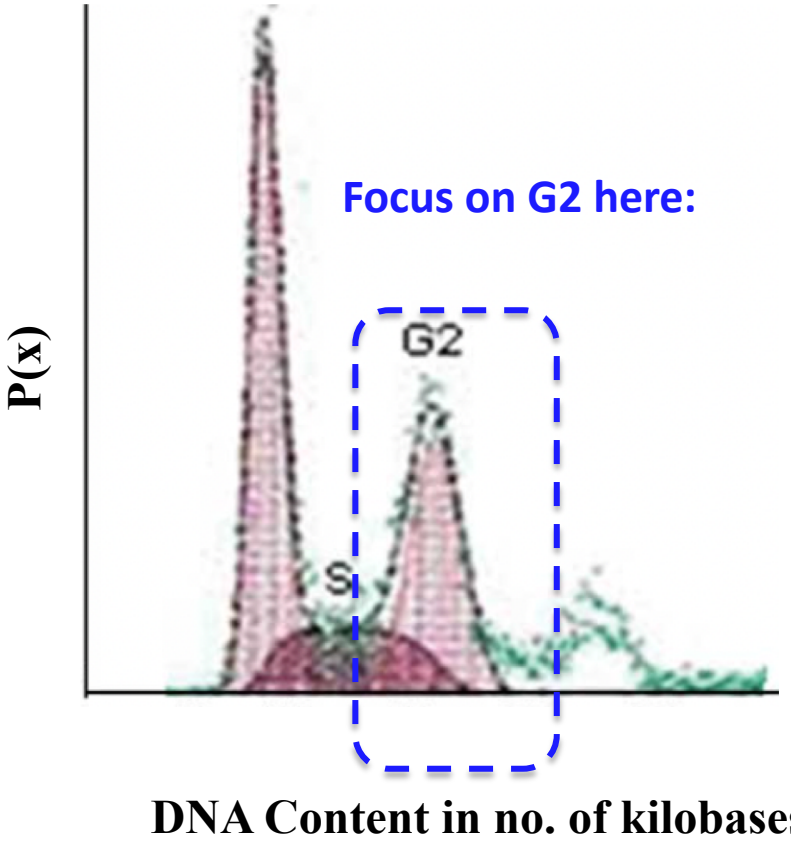
1) Find the *probability density* at L = 65 cm.

2) In a randomly selected catch of 1500 eels, write down expressions for how many you expect to be:

- at least 65 cm in length?
- below 65 cm in length?
- Shade these regions in the plot above

Prob. In treatment of colorectal cancer, the cellular response to irradiation varies depending on the expression of tumor suppressor p53, which can be estimated by the corresponding "DNA content".

The raw data was converted to a zero-centered normal distribution with $\sigma = 1$



- Let's hypothesize that patient survival (in years) scales with square of the DNA content (x) in the G2-phase, with a suitable prefactor, A = 25
- **Find the mean patient survival time.**
  - **Ans: A**
- What are the units of A?
  - Ans: years

Binomial Distribution

Only **2 possible outcomes** of an event with ***N*** attempts:

N → trials .

→ **Success**, probability ***s***

→ **Failure**, probability ***f = (1 – s)***

Mean success, $\langle n \rangle = s \cdot N$

n = No. of success

Variance in success, $\sigma^2 = s(1-s)N$

Standard deviation, $\sigma = \sqrt{s(1-s)N}$

The ratio of std. dev. to mean $= \sqrt{\frac{1-s}{sN}} \propto \frac{1}{\sqrt{N}}$

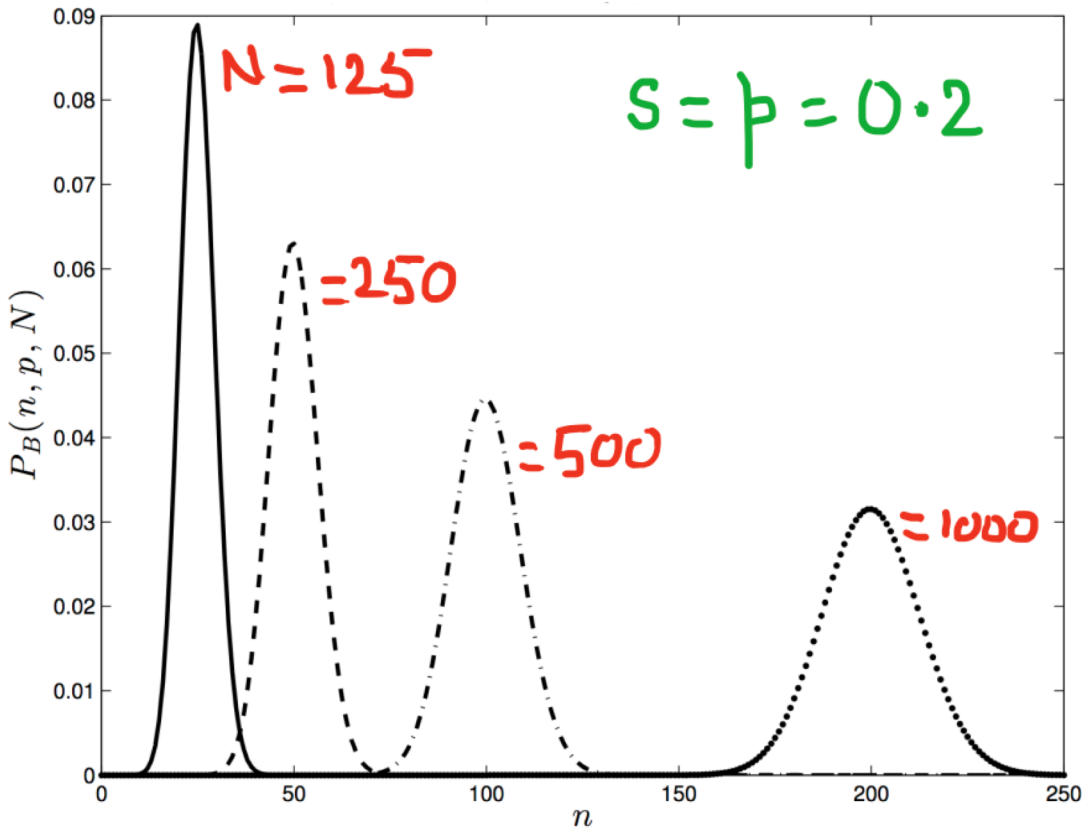
$r = \frac{\sigma}{\mu}$

High no. of trials (N):
Normal approximation to Binomial Disbn.

N trials:
2 possible outcomes
at each trial

Success, probability ***s***

Failure, probability ***f = (1 – s)***



n = No. of success

High no. of trials (N):
Normal approximation to Binomial Disbn.

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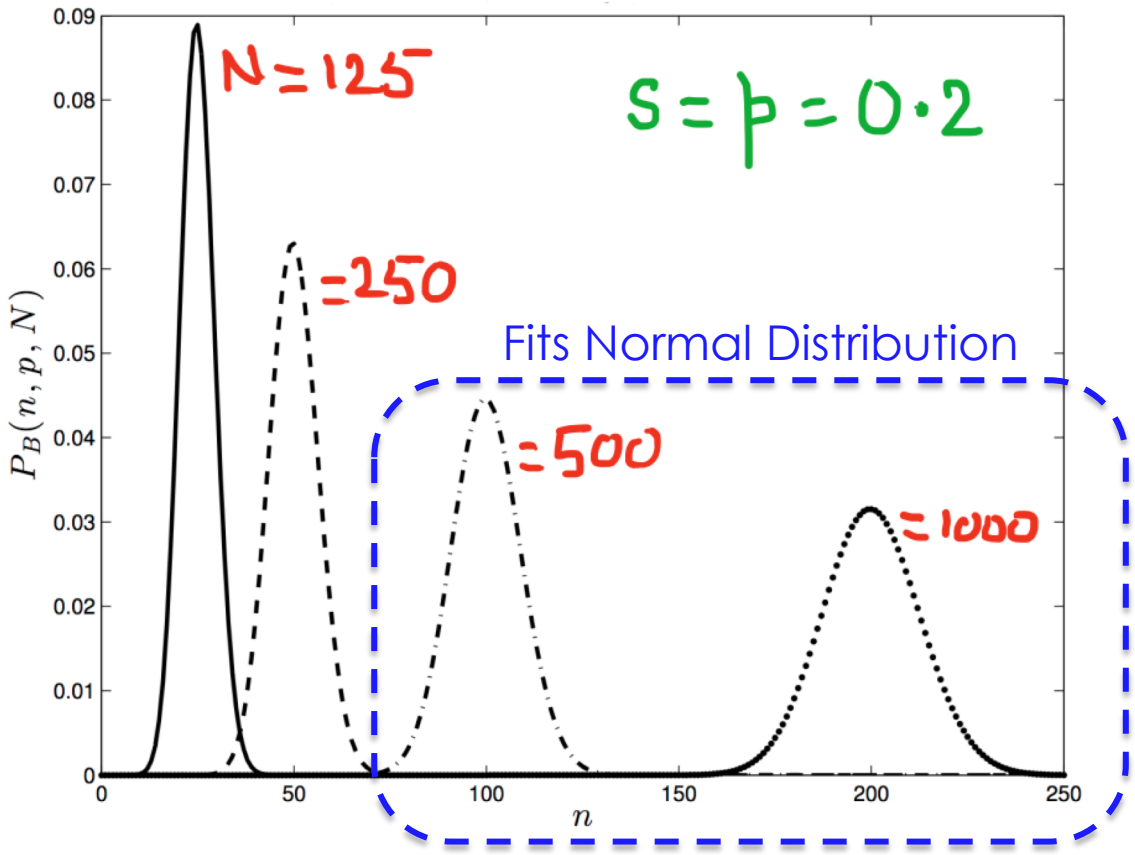


n = No. of success

High no. of trials (N):
Normal approximation to Binomial Disbn.

Example.
From years of observation,
a biologist knows that the *probability is 0.65*
that an Arctic tern will survive migration from
its summer nesting area to its
winter feeding grounds.

Random sample of 500 birds were banded at a summer nesting area.

- What is the mean and std. dev (σ) of survival?
- What is probability density of survival when 310 birds are considered?

- What is the probability density of death for 310 birds?

High no. of trials (N):
Normal approximation to Binomial Disbn.

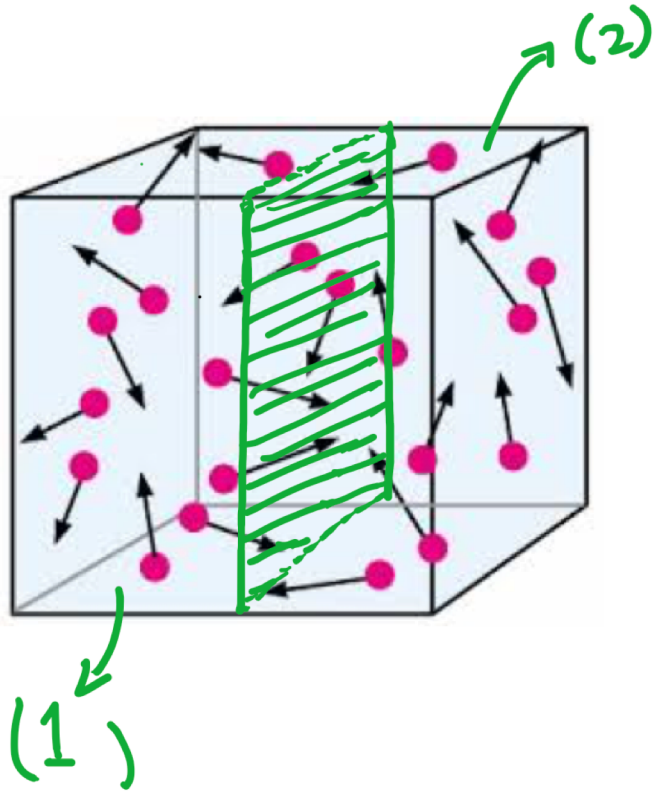
N no. of gas molecules under normal conditions is left in a cubical box marked in equal compartments.

- Find the *ratio* of the *uncertainty* (σ) *to the mean probability* (μ) of finding any molecule in one of the compartments.

- How does the ratio vary when:

a) $N = 1000$

b) $N = 6 \times 10^{23}$

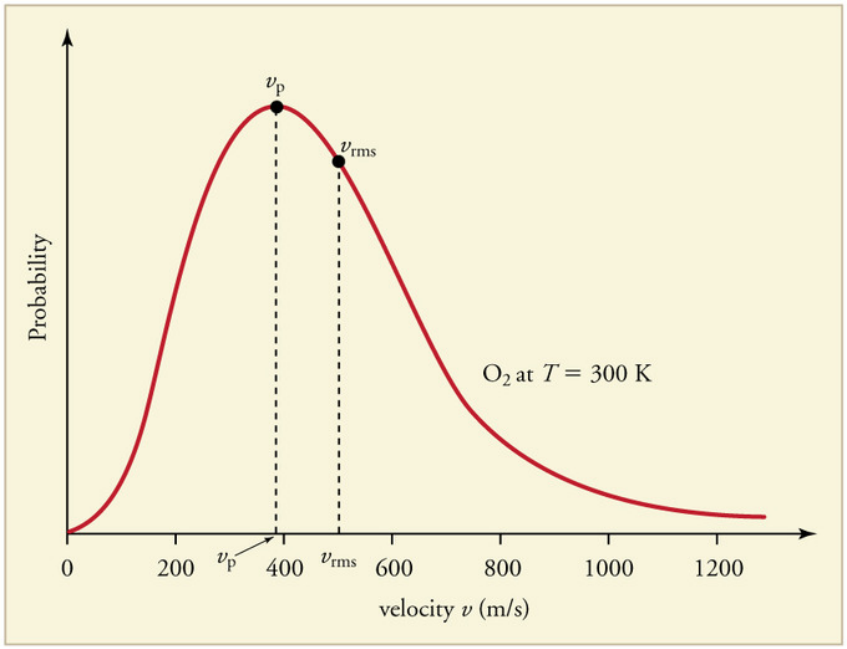


Maxwell-Boltzmann velocity distributions

$$v_{m.p} = \sqrt{\frac{2k_BT}{m}}$$

$$\bar{v} = v_{av.} = \sqrt{\frac{8k_BT}{\pi m}}$$

$$v_{r.m.s} = \sqrt{\frac{3k_BT}{m}}$$



$$\frac{k_BT}{m} \equiv \frac{RT}{M}$$

m ↓ mass of molecule

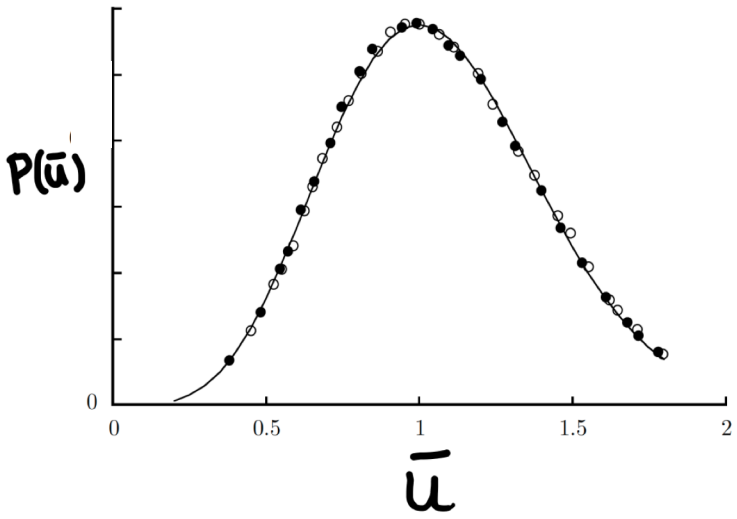
M ↓ molar mass

Prob. Write an expression comparing $\boldsymbol{v_{rms}}$ of dilute CO_2 gas to a dilute protein solution.

State assumptions, if any.

- Molecular masses for calculation:
  - CO_2: 44 g/mol;
  - Protein Ubiquitin: 8.6 kiloDalton (kDa)
  - 1kDa ~ 10^3 g/mol

What is Equilibrium?



- **Physics: No net force acting on system**

Characteristics in many-particle system (eg. gas or dilute solution):

- The measured state variables (pressure, volume, temperature) are constant

- No flux (entry / exit) of mass or energy into the system

- Distribution functions unchanged; are independent of the instantaneous particle positions and velocities

Equilibrium. vs. Steady State.

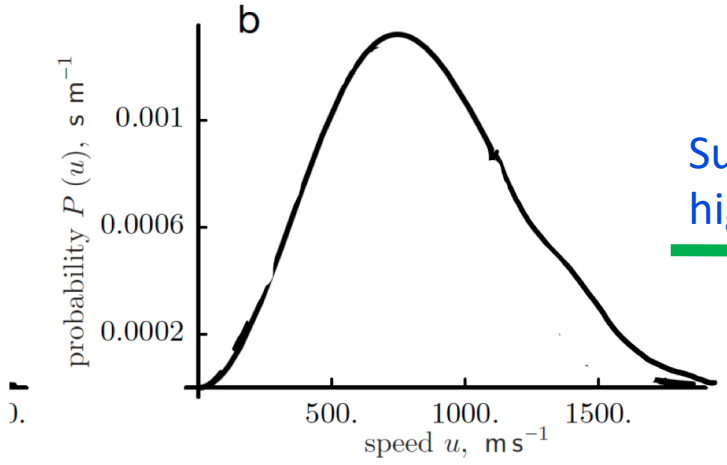
- Distribution remains constant,
- The state variables (pressure, volume, temperature) are constant
- No flux (entry/exit) of mass or energy into the system
- No net force acting on the system
- Generally closed system

- Distributions remains constant
- The state variables (pressure, volume, temperature) are constant
- Entry / exit of mass or energy into the system is allowed
- There could be net forces
- Open system

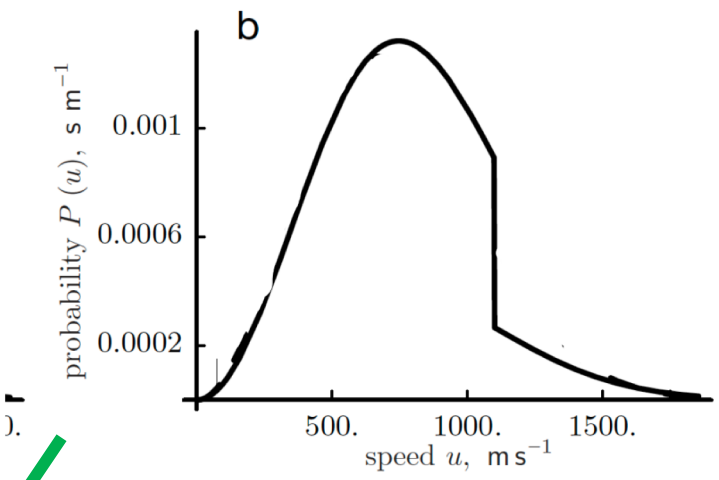
Example: Consider a suspension of bacterial cells in a solution of nutrients, with constant dilution and growth rate.

Is the system in equilibrium?

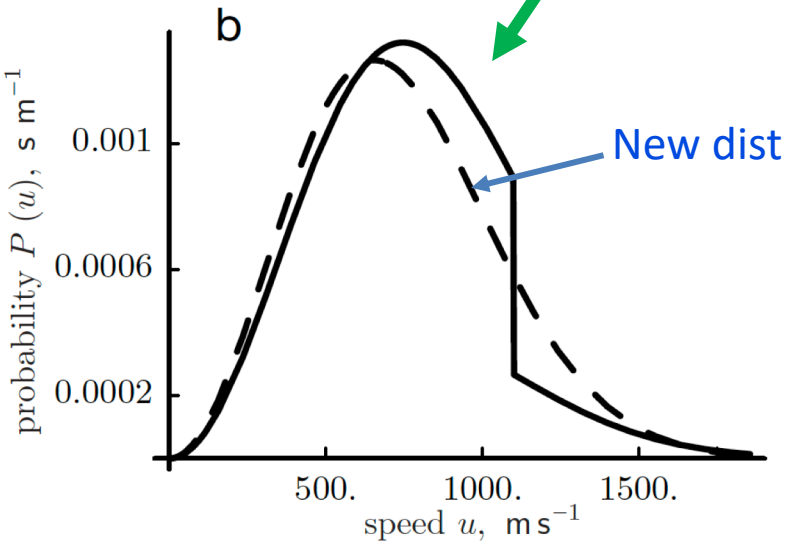
M-B Distribution signifies equilibrium



Sudden removal of high temp. molecules

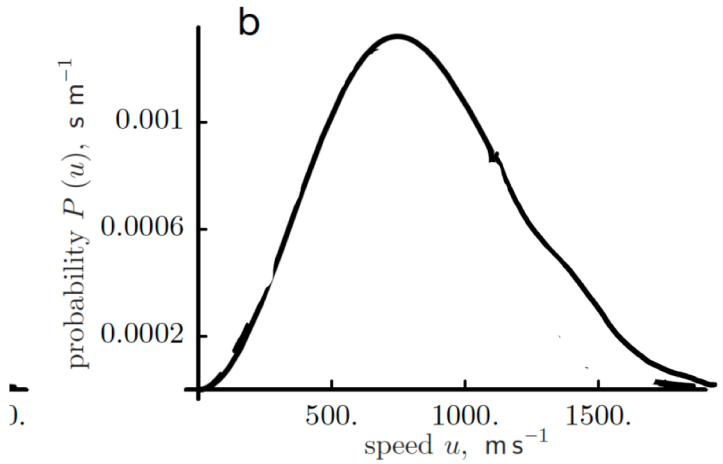


Relaxation to new state



New distribution

M-B Distribution signifies equilibrium



Change in ENTROPY

Sudden addition of very fast molecules

New state