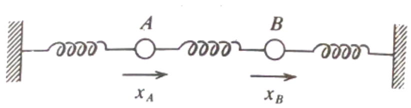
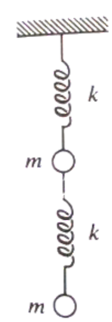
1. A mass m is subject to a resistive force $-bv$, but no spring-like restoring force.
   (a) Show that its displacement as a function of time is of the form
   $$x = C - \frac{v_0}{\gamma} e^{-\gamma t},$$
   where, γ is equal to b/m.
   (b) At $t = 0$ the mass is at rest at $x = 0$. At this instant a driving force $F = F_0 \cos \omega t$ is switch on. Find the values of a and δ in the steady state solution $x = A \cos(\omega t - \delta)$.
   (c) Write down the general solution [the sum of parts (a) and (b)] and find the values of C and v_0 from the conditions that $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$. Sketch x as a function of t.

2. Two equal masses on an effectively frictionless horizontal air track are held between rigid supports by three identical springs, as shown. The displacements from equilibrium along the line of the springs are describe by coordinates x_A and x_B, as shown. If either of the masses is clamped, the period $T = 2\pi/omega$ for one complete vibration of the mass is 3 s.

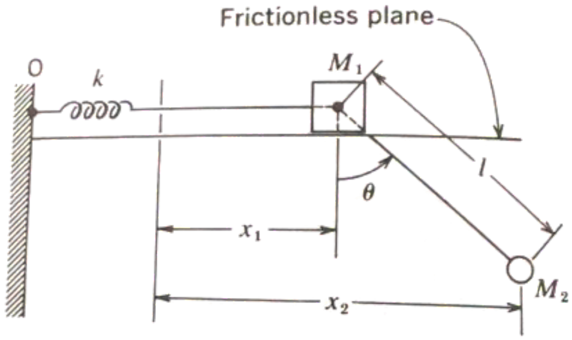


   (a) If both masses are free, what are the periods of the two normal modes of the system? Sketch graphs of x_A and x_B versus t in each mode. At $t = 0$ mass A is at its normal resting position and mass B is pulled aside a distance of 5 cm. The masses are released from rest at this instant.
   (b) Write an equation for the subsequent displacement of each mass as a function of time.
   (c) What length of time (in seconds) characterizes the periodic transfer of the motion from B to A and back again? After one cycle is the situation at $t = 0$ exactly reproduced? Explain.

3. Two equal masses are connected as shown, with two identical massless strings of spring constant k. Considering only motion in the vertical direction, show that the angular frequency of the two normal modes are given by $\omega^2 = (3 \pm \sqrt{5})\frac{k}{2m}$ and hence that the ratio of the normal mode frequency is $(\sqrt{5}+1)/(\sqrt{5}-1)$. Find the ratio of the amplitude of the two masses in each separate mode. (Note: you need not consider the gravitational force acting on the masses because they are independent of the displacements and hence do not contribute to the restoring forces that cause the oscillations. The gravitational force is merely cause a shift in the equilibrium position of the masses and you do not have to find what the shifts are.)



4. The sketch shows a mass m on a frictionless plane connected to support O by a spring of stiffness k. Mass M_2 is supported by a string of length l from M_1.



(a) Using the approximation of small acceleration

$$\sin\theta \approx \tan\theta \approx (x_2 - x_1)/l$$

and starting from $F = ma$ derive the equation for motion of M_1 and M_2.

(b) For $M_1 = M_2 = M$ use the equations to obtain the normal frequencies of the system.

(c) What are the normal mode motions for $M_1 = M_2 = M$ $g/l \gg k/m$?