Boundary conditions

We begin with the Maxwell's equations in the integral form.

(i) $$\oint_S \vec{D} \cdot d\vec{a} = Q_{free,\, enclosed}$$

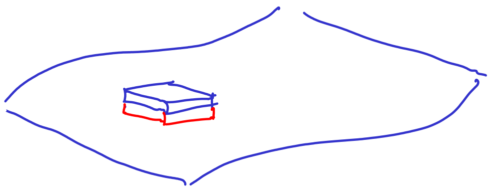
(ii) $$\oint_S \vec{B} \cdot d\vec{a} = 0$$

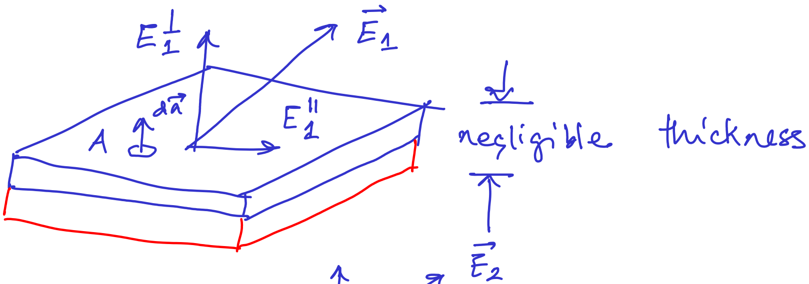
(iii) $$\oint_P \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a}$$

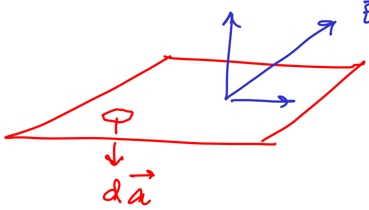
(iv) $$\oint_P \vec{H} \cdot d\vec{l} = I_{free,\, enclosed} + \frac{d}{dt}\int_S \vec{D} \cdot d\vec{a}$$

From, (i)

We use a thin Gaussian box around the boundary.







$$\epsilon_1 E_1^\perp A - \epsilon_2 E_2^\perp A = 0$$

$$\Rightarrow \boxed{\epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp} \quad \text{--- (B1)}$$

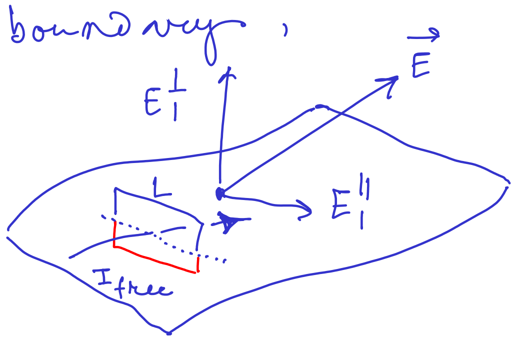
From (ii),

Exactly the same way, from (i) above we get,

$$\boxed{B_1^\perp = B_2^\perp} \quad \text{---- (B2)}$$

From (iii)

We use a narrow rectangular loop around the boundary,



$$E_1^\parallel L - E_2^\parallel L = 0$$

For narrow loop the flux vanishes.

$$\Rightarrow \boxed{E_1^\parallel = E_2^\parallel} \quad \text{--- (B3)}$$

From (iv),

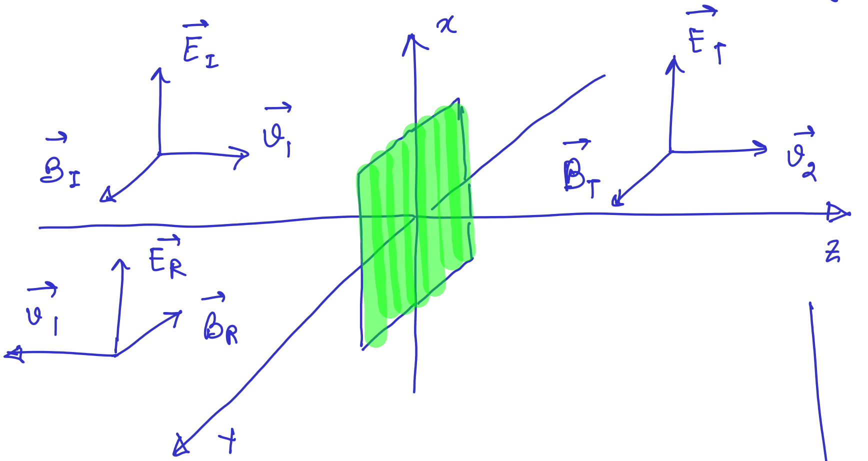
we use the narrow loop above to get,

$$\frac{1}{\mu_1} B_1^{\parallel} L - \frac{1}{\mu_2} B_2^{\parallel} L = 0$$ ← no free current and using (i) (no free charge).

$$\Rightarrow \boxed{\frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_1^{\parallel}} \quad \text{.... (B4)}$$

We shall use (B1) to (B4) to get the laws of geometric optics.

Consider the following boundary



$$\vec{E}_I = E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_I = B_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y} = \frac{1}{v_1} E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

$$\vec{E}_R = E_{0R}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_R = -\frac{1}{v_1} E_{0I}\, e^{i(-k_1 z - \omega t)}\, \hat{y}$$

and similarly $\vec{E}_T$ & $\vec{B}_T$

(B1) and (B2) is trivially zero.

From (B3) we get,

$$E_{0I} + E_{0R} = E_{0T}$$

and from (B4) we get,

$$\frac{1}{\mu_1}\left(\frac{1}{v_1} E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{0T} = \frac{1}{\mu_2 v_2}\left(E_{0I} + E_{0R}\right)$$

$$\Rightarrow \left(\frac{1}{\mu_1 v_1} + \frac{1}{\mu_2 v_2}\right) E_{0R} = \left(\frac{1}{\mu_1 v_1} - \frac{1}{\mu_2 v_2}\right) E_{0I}$$

$$\Rightarrow E_{0R} = \frac{1 - \frac{\mu_1 v_1}{\mu_2 v_2}}{1 + \frac{\mu_1 v_1}{\mu_2 v_2}} \cdot E_{0I} = \frac{1-\beta}{1+\beta} E_{0I}$$

where, $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$ | using, $\frac{n_1}{n_2} = \frac{v_2}{v_1}$

$$\Rightarrow \quad E_{0T} = (1+R)E_{0I} = \left(1 + \frac{1-\beta}{1+\beta}\right)E_{0I} = \left(\frac{2}{1+\beta}\right)E_{0I}$$

Intensity of a light beam

$$= \boxed{\frac{1}{2}\epsilon v E_0^2}$$

$$\Rightarrow \quad R = \frac{I_R}{I_I} = \frac{E_{0R}^2}{E_{0I}^2} = \left(\frac{1-\beta}{1+\beta}\right)^2 \sim \left(\frac{1 - n_2/n_1}{1 + n_2/n_1}\right)^2$$

as $\mu \approx \mu_0$ for most materials

$$= \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2 E_{0T}^2}{\epsilon_1 v_1 E_{0I}^2} = \frac{n_2^2\, n_1}{n_1^2\, n_2}\,\frac{(2n_1)^2}{(n_1+n_2)^2}$$

$$= \frac{4 n_1 n_2}{(n_1+n_2)^2}$$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\frac{\epsilon_1}{\epsilon_2} = \frac{\epsilon_1 \mu}{\epsilon_2 \mu} = \frac{v_2^2}{v_1^2} = \frac{n_1^2}{n_2^2}$$

So, $R + T = 1$.

For, $n_1 = 1$, $n_2 = 1.5$, $R = 0.04$ and $T = 0.96$.

Wavevector

We have $\vec{E} = \vec{E}_0\, e^{i\overbrace{(kz - \omega t)}^{\text{phase}}}$

$kz \equiv k\hat{k}\cdot z\hat{k} \longrightarrow$ assumption the EM wave is propagating along z.

In general, we have

instead of $k\hat{k} \rightarrow k_x\hat{i} + k_y\hat{j} + k_z\hat{k} = \vec{k} = k\hat{k}$

and $z\hat{k} \rightarrow x\hat{i} + y\hat{j} + z\hat{k} = \vec{r} = r\hat{r}$

$$\Rightarrow \quad \vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= \vec{E}_0\, e^{i(k\hat{k}\cdot\vec{r} - \omega t)}$$

and $\boxed{v = \frac{\omega}{k}}$