# PH2102 Problem Set 4

**Q 1)** Use the velocity transformation law

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

to show that if both $u$ and $v$ are smaller than $c$, then $u'$ is smaller than $c$.

For the next two problems assume that $c = 1$. This is not essential - but simplifies the algebra somewhat

**Q 2) a)** Write down the special Lorentz transformation equation in 1+1 dimensions in terms of the rapidity $\eta = \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$ in place of $v$. *Your result should involve hyperbolic functions of* $\eta$.

**b)** Write down the form that these equations take for small $\eta$.

**Q 3)** Consider a situation where we have three observers - Alice, Bob and Charlie moving in in spatial dimension. The spacetime coordinates for an event according to them are $(x, t)$ , $(x', t')$ and $(x'', t'')$. If the rapidity of Bob with respect to Alice is $\eta_1$ and that of Charlie with respect to Bob is $\eta_2$, write down the transformation equations for $(x', t')$ in terms of $(x, t)$ and $(x'', t'')$in terms of $(x', t')$. Combine these to obtain the relation between $(x'', t'')$ and $(x, t)$. Use this final transformation to determine the rapidity of Charlie with respect to Bob.