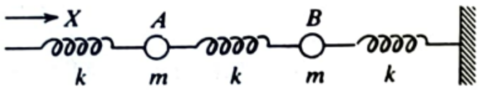
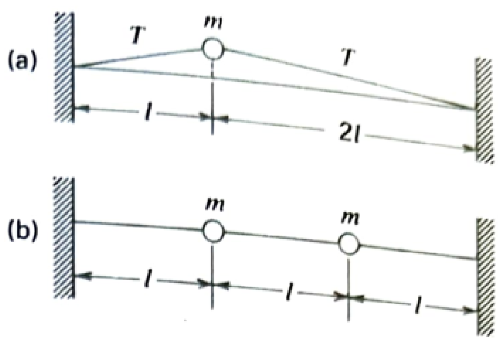
1. Two equal masses m are connected to three identical springs (spring constant k) on a frictionless horizontal surface (see figure). One end of the system is fixed; the other is driven back and forth via a displacement $X = X_\circ \cos \omega t$. Find and sketch graphs of the resulting displacements of the two masses.



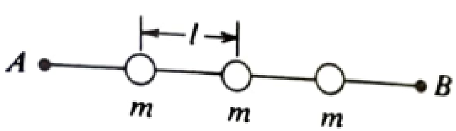
2. A string of length 3ℓ and negligible mass is attached to two fixed supports at its ends. The tension in the string is T.
   (a) A particle of mass m is attached at a distance ℓ from one end of the spring, as shown. Set up the equation for small transverse oscillation of m, and find the period.
   (b) An additional particle of mass m is connected to the string as shown, dividing it into three equal segments each with tension T. Sketch the appearance of the string and masses in the two separate normal modes of transverse oscillations.
   (c) Calculate ω for the normal mode which has the higher frequency.



3. To get a feeling for the use of the equation,

$$A_n^{(m)} = C \sin\left(\frac{nm\pi}{N+1}\right)$$

which describes the amplitudes of connected particles in various normal modes, take the case $N = 3$ and tabulate, in a 3×3 array, the relative numerical values of the amplitudes of the particles ($n = 1, 2, 3$) in each of the normal modes ($m = 1, 2, 3$).



4. An elastic string of negligible mass, stretched so as to have a tension T, is attached to fixed points A and B, a distance 4ℓ apart, and carries three equally spaced particle of mass m, as shown.
   (a) Suppose that the particles have small transverse displacements y_1, y_2, and y_3; respectively, at some instant. Write down the differential equation for each mass.

(b) The appearance of the normal modes can be found by drawing the sine curve that pass through A and B. Sketch such curves so as to find the relative values and signs of A_1, A_2 and A_3 in each of the possible modes of the system.

(c) Putting $y_1 = A_1 \cos \omega t$, $y_2 = A_2 \cos \omega t$, and $y_3 = A_3 \cos \omega t$, in the above equations, use the ratios from part b to find the angular frequencies of the separate modes.