

Solve all questions

Q 1) Draw the t and x axes of the spacetime coordinates of an observer \mathcal{O} and then draw (for this problem $c = 1$, both x and t are measured in meters):

1. The world line of \mathcal{O} 's clock at $x = 1$ m.
2. The world line of a particle moving with the velocity $dx/dt = 0.4$, and which is at $x = 0.5$ m when $t = 0$.
3. The locus of events whose interval Δs^2 from the origin is -1 m².
4. The world line of a photon which is emitted at the event $t = -1$ m, $x = 0$, travels in the negative x direction, is reflected at a mirror located at $x' = -1$ m, and is absorbed when it encounters a detector located at $x = 0.75$ m [4]

Q 2) The world line of a particle is described by the parametric equation

$$ct(\lambda) = \alpha \sinh\left(\frac{\lambda}{\alpha}\right), \quad x(\lambda) = \alpha \cosh\left(\frac{\lambda}{\alpha}\right)$$

where λ is a parameter and α is a constant. Calculate its 4-velocity and 4-acceleration. Interpret λ and α physically. [6]

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}, \quad \frac{d}{dx} \cosh x = \sinh x, \quad \frac{d}{dx} \sinh x = \cosh x$$