promotes metamorphic reactions and has a major impact on the thermodynamics of re-equilibration.

Problems

1. Estimate Table 7.2 for 4000 and 4500 Ma. Discuss the implications of the relative heat-generation values in this table, particularly with respect to the Archaean Earth.
2. Calculate the phase difference between the daily and annual surface temperature variations that would be measured at depths of 2 and 5 m in a sandstone.
3. Taking T_0 as 40 °C for the annual variation in surface temperature, calculate the depth at which the variation is 5 °C. What is the phase difference (in weeks) between the surface and this depth?
4. Calculate an equilibrium geotherm for $0 \le z \le d$ from the one-dimensional heat-flow equation, given the following boundary conditions:
   (i) $T = 0$ at $z = 0$ and
   (ii) $T = T_d$ at $z = d$.
   Assume that there is no internal heat generation.
5. Calculate an equilibrium geotherm from the one-dimensional heat-flow equation given the following boundary conditions:
   (i) $\partial T/\partial z = 30\,°\mathrm{C\,km^{-1}}$ at $z = 0$ km and
   (ii) $T = 700$ °C at $z = 35$ km.
   Assume that the internal heat generation is 1 $\mu\mathrm{W\,m^{-3}}$ and the thermal conductivity is 3 $\mathrm{W\,m^{-1}\,°C^{-1}}$.
6. On missions to Venus the surface temperature was measured to be 740 K, and at three sites heat-producing elements were measured (in percentage of total volume) as follows (ppm, parts per million).

	Venera 8	Venera 9	Venera 10
K (%)	0.47 ± 0.08	0.30 ± 0.16	4 ± 1.2
U (ppm)	0.60 ± 0.16	0.46 ± 0.26	2.2 ± 0.2
Th (ppm)	3.65 ± 0.42	0.70 ± 0.34	6.5 ± 2

   The density of the Venusian crust can be taken, from a measurement by Venera 9, to be $2.8 \times 10^3\,\mathrm{kg\,m^{-3}}$. Calculate the heat generation in $\mu\mathrm{W\,m^{-3}}$ at each site. (From Nisbet and Fowler (1982).)
7. Using the one-dimensional equilibrium heat-conduction equation, calculate and plot the Venus geotherms (Aphroditotherms) of Problem 6 down to 50 km depth at each site. Assume that the conductivity is 2.5 $\mathrm{W\,m^{-1}\,°C^{-1}}$ (a typical value for silicates) and that, at a depth of 50 km, the heat flow from the mantle and deep lithosphere of Venus is $21 \times 10^{-3}\,\mathrm{W\,m^{-2}}$. What have you assumed in making this calculation? What do these Aphroditotherms suggest about the internal structure of the planet?

8. Calculate the geotherms for the models shown in Fig. 7.14. Discuss the reason for the difference at depth between these geotherms and the geotherm shown as a solid line in Fig. 7.14.
9. Calculate an equilibrium geotherm for the model Archaean crust shown in Fig. 7.4. Discuss your estimates.
10. To what depth are temperatures in the Earth affected by ice ages? (Use thermal conductivity $2.5\,\mathrm{W\ m^{-1}\,{}^\circ C^{-1}}$ and specific heat $10^3\,\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$.)
11. Calculate the equilibrium geotherm for a two-layered crust. The upper layer, 10 km thick, has an internal heat generation of $2.5\,\mu\mathrm{W\,m^{-3}}$, and the lower layer, 25 km thick, has no internal heat generation. Assume that the heat flow at the base of the crust is $20 \times 10^{-3}\,\mathrm{W\,m^{-2}}$ and that the thermal conductivity is $2.5\,\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$.
12. Repeat the calculation of Problem 11 when the upper layer has no internal heat generation and the lower layer has internal heat generation of $1\,\mu\mathrm{W\,m^{-3}}$. Comment on the effect that the distribution of heat-generating elements has on geotherms.
13. Calculate geotherms for a layered continental crust and comment on the significance of your results for the following cases.
    (a) A 10-km-thick upper layer with heat generation of $2.5\,\mu\mathrm{W\,m^{-3}}$ overlying a 30-km-thick layer with heat generation of $0.4\,\mu\mathrm{W\,m^{-3}}$.
    (b) A 30-km-thick upper layer with heat generation of $0.4\,\mu\mathrm{W\,m^{-3}}$ overlying a 10-km-thick layer with heat generation of $2.5\,\mu\mathrm{W\,m^{-3}}$. For both cases, assume a surface temperature of zero, heat flow from the mantle of $20 \times 10^{-3}\,\mathrm{W\,m^{-2}}$ and thermal conductivity of $2.5\,\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$.
14. A 1-m-wide dyke with a temperature of 1050 °C is intruded into country rock at a temperature of 50 °C.
    (a) Calculate how long the dyke will take to solidify.
    (b) After two weeks, what will the temperature of the dyke be?
    (Assume a diffusivity of $10^{-5}\,\mathrm{m^{-2}\,s^{-1}}$ and a solidus temperature of 800 °C.)
15. Volcanic flood basalts can be several kilometres thick and extend over very large areas (the Karoo basalt in southern Africa is one example). A 2-km-thick basalt is erupted at 1200 °C. If the solidus temperature is 900 °C, estimate the time required for the basalt to solidify. If this basalt is later eroded and the underlying rocks exposed, indicate how far you would expect the metamorphism to extend from the basalt. State all your assumptions in answering this question.
16. (a) Calculate the difference in depth of the seabed at the intersection of a mid-ocean ridge and a transform fault. Assume that the ridge is spreading at $5\,\mathrm{cm\,yr^{-1}}$ and that the ridge axis is offset 200 km by the transform fault.
    (b) Calculate the difference in depth on either side of the same fault 1000 km from the ridge axis and 3000 km from the ridge axis. (See Section 9.5 for information on transform faults.)
17. Calculate the 60-Ma geotherm in the oceanic lithosphere for the simple model of Section 7.5.2. What is the thickness of the 60-Ma-old lithosphere? Use an asthenosphere temperature of 1300 °C and assume a temperature of 1150 °C for the base of the lithosphere

18. Assume that the Earth is solid and that all heat transfer is by conduction. What value of internal heat generation distributed uniformly throughout the Earth is necessary to account for the Earth's mean surface heat flow of $87 \times 10^{-3}\,\mathrm{W\,m^{-2}}$? How does this value compare with the actual estimated values for the crust and mantle?
19. Calculate the rate at which heat is produced in (a) the crust and (b) the mantle. Assume that the crust is 10 km thick and that the volumetric heat-generation rates are $1.5 \times 10^{-6}\,\mathrm{W\,m^{-3}}$ in the crust and $1.5 \times 10^{-8}\,\mathrm{W\,m^{-3}}$ in the mantle.
20. Calculate the steady-state surface heat flow for a model solid Earth with the following constant thermal properties: k, $4\,\mathrm{W\,m^{-1}\,{}^\circ C^{-1}}$; and A, $2 \times 10^{-8}\,\mathrm{W\,m^{-3}}$.
21. It takes about 4 min to boil a hen's egg of mass 60 g to make it edible for most people. For how long would it be advisable to boil an ostrich egg weighing about 1.4 kg? (From Thompson (1987).)
22. (a) Calculate the conductive characteristic time for the whole Earth.
    (b) Calculate the thickness of the layer that has a characteristic time of 4500 Ma.
    (c) Comment on your answers to (a) and (b).
23. (a) A sphere has radius r and uniform density ρ. What is the gravitational energy released by bringing material from infinitely far away and adding a spherical shell, of density ρ and thickness δr, to the original shell?
    (b) By integrating the expression for the gravitational energy over r from 0 to R, calculate the gravitational energy released in assembling a sphere of density ρ and radius R.
    (c) Use the result of (b) to estimate the gravitational energy released as a result of the accretion of the Earth.
    (d) Assume that all the energy calculated in (c) became heat and estimate the rise in temperature of the primaeval Earth. Comment on your answer.

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