PH2102 Mid Semester Examination

Full marks : 20 Time : **90 minutes**

Answer any two

Q 1) Bob is standing at the back of a train compartment which he observes to be 16 m long. The train is moving at a speed of $0.6c$ with respect to Alice, who is standing beside the track. He throws a ball at a speed of $0.8c$ towards the front wall of the carriage. The ball hits the front wall and rebounds perfectly elastically back to him. Consider the three events -

- $\mathcal{E}_1$: Bob throws the ball
- $\mathcal{E}_2$: The ball hits the front wall
- $\mathcal{E}_3$: The ball returns to Bob.

a) What is the temporal difference between $\mathcal{E}_2$ and $\mathcal{E}_1$ according to Alice?

b) What is the temporal difference between $\mathcal{E}_3$ and $\mathcal{E}_2$ according to Alice?

c) For which pair of events is one of the two measuring proper time?

d) Show that your results above is consistent with the time dilation formula (where it is appropriate).

For this problem use a lt-m= $\frac{1\text{ m}}{c}$ as the unit of time. *Hint : it may be a better idea to work with fractions (rather than use a calculator)* $(3+3+1+3)$

Q 2) a) Show how the length contraction formula can be derived using the K-calculus.

b) Derive an expression for the acceleration four-vector $A^\mu \equiv \frac{dU^\mu}{d\tau}$ in terms of velocity and acceleration 3 vectors. Is this four-vector time-like, light-like or space-like? *Hint : the last part can be answered quickly if you take into account the fact that this classification is* observer independent.$(4 + (4 + 2))$

Q 3) A Lorentz transformation can be written in the compact form $\mathbf{x}' = L\mathbf{x}$ (where $\mathbf{x}$ is a 4×1 column vector with elements $x^0 = ct, x^1 = x, x^2 = y$ and $x^3 = z$). Let $\Lambda^{\mu}{}_{\nu}$ represent the μ-th row, ν-th column element of the 4×4 matrix L.

a) Use the fact that $\mathbf{x}^T \eta \mathbf{x}$ is an invariant, prove that $L^T \eta L = \eta$.

b) Write the equation in part (a) in terms of the components $\Lambda^{\mu}{}_{\nu}$ of the matrix L.

c) Show that $\det L$ must be either +1 or -1.

d) Show that $|\Lambda^0{}_0| \geq 1$.

e) It is given that the matrix L given below represents a particular Lorentz transformation:

$$\frac{1}{64}\begin{pmatrix} 125 & -48 & -60 & 75 \\ -75 & 80 & 36 & -45 \\ -60 & 0 & 80 & -36 \\ 48 & 0 & 0 & 80 \end{pmatrix}$$

Calculate L^{-1}. *Hint : it should take you no more than a minute or two, at most!* $(3 + 1 + 2 + 2 + 2)$