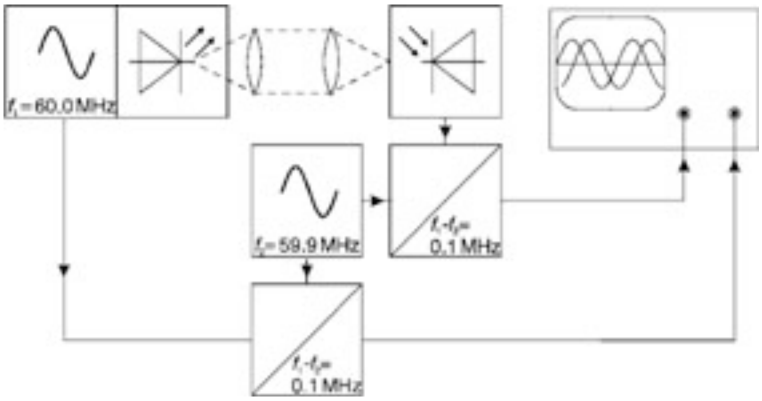
P5.6.3 Measuring with an electronically modulated signal

P5.6.3.1 Determining the velocity of light using a periodical light signal at a short measuring distance

In the experiment P5.6.3.1, the apparent transit time Δt' is measured as a function of the measuring distance Δs, and the velocity of light in the air is calculated according to the formula



Determining the velocity of light in various materials (P5.6.3.2_c)

476301 Light transmitter and receiver

For measuring the velocity of light and also the refractive indices of optically clear liquids as well as optically clear solids using an electronic modulation method. The light only needs to travel short distances (a distance of just 2.5 m between the transmitter and the receiver provides measured values with an accuracy of ± 1 %).
Scope of delivery:
1 Light transmitter with condenser
1 Receiver with power supply unit
1 BNC-cable, 6 m
2 BNC-cables, 2 m
Technical data:
Light transmitter:
Transmitter: light-emitting diode (red, 670 nm)
with condenser
Modulation frequency: 60.0 MHz ± 5 kHz
Power supply: via cable from power supply of the receiver
Dimensions: approx. 12 cm x 7 cm Ø
Rod: 10 mm Ø, 115 mm length
Weight: 0.8 kg
Receiver and power supply unit:
Sensor: silicon PIN photodiode
Output voltage:
Reference channel approx. 2 V_{pp}
Working channel approx. 2 V_{pp}
Signal-to-noise ratio: 46 dB
Load resistor: higher than 2 kΩ
Mains supply: 230 V, 50/60 Hz
Power consumption: 15 VA
Fuse: T 0.125 B
Dimensions: 21 cm x 20 cm x 23 cm
Weight: 3.5 kg

575212 Two-channel oscilloscope 400

Bandwidth: 0...40 MHz (-3 dB)
Input impedance: 1 MOhm, 15 pF, max. 400 V
Screen: 8 x 10 cm with internal graticule
Vertical deflection: 1 mV/cm...20 V/cm (14 steps)
Time base: 0.1 µs/cm...0.2 s/cm (20 steps),
with X-magnification x10 to 10 ns/cs
Trigger sources: Ch1, Ch2, line, ext.
Operating modes:
Ch1, Ch2, Ch1+Ch2 (alternate or chopped), Ch1/Ch2 sum or difference, Ch2 inv., XY-Mode
Built-in component tester
Dimensions: 28.5 x 12.5 x 38.0 cm (WxHxD)
Mains supply: 105...253 V, 50/60 Hz ±10%, Cat II
Without probes

46008 Lens in frame f = +150 mm

The focal lengths of the lens is engraved on the frame; on rod.
Focal length: 150 mm
Diameter of lens: 75 mm
Diameter of frame: 13 cm
Stand rod diameter: 10 mm

30011 Saddle Base

cylindrical stand base with clamping screw for holding of rods and plates.
Jaw width for rods: up to 14 mm
Jaw width for plates: up to 9.5 mm
Dimensions: 5.5 cm x 6 cm Ø
Weight: 0.75 kg

31102 Metal rule, l = 1 m

Width: 25 mm, Divisions: dm, cm and mm

46025 Prism table

For attaching prisms, plate glass cells and similar objects to an optical bench. With adjustable spring clips and stand rod

46043A Optical bench large, l = 2 m

Steel rail (four-sided profile with triangular groove, for attachment to a stand rod via a swivel joint at the side and a tommy screw. The rail can be adjusted to any desired angle of inclination. Leybold clamps are used as riders and can be moved freely over the entire length. Coupling of two benches can be accomplished via a swivel joint with protractor scale (460 40).

47634 Transparent plastic block

For measurement of the refractive index using an electronic modulation method; polished faces.

Determining the velocity of light with a periodic light signal at small distances

Objects of the experiments

- Measuring the phase shift $\Delta\varphi$ of a periodic light signal on a path Δs.
- Determining the velocity of light c.

Principles

A periodic light signal with the time-dependent intensity

$$I = I_0 + \Delta I_0 \cdot \cos(2\pi \cdot \nu \cdot t) \quad \text{(I)}$$

is extremely well suited for determining the velocity of light c. The light signal is measured with a receiver, which converts the signal into an alternating voltage with the time behaviour

$$U = a \cdot \cos(2\pi \cdot \nu \cdot t) \quad \text{(II)}.$$

If the receiver is at the distance Δs from the light transmitter, the time delay

$$\Delta t = \frac{\Delta s}{c} \quad \text{(III)},$$

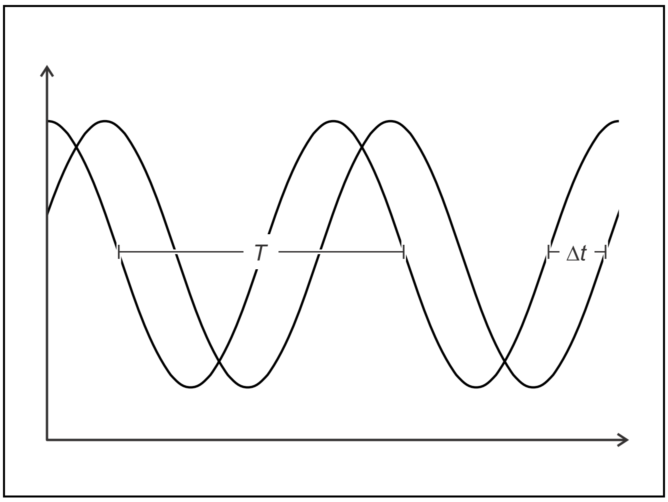
of the signal along the path Δs leads to the phase shift

$$\Delta\varphi = 2\pi \cdot \nu \cdot \Delta t = 2\pi \cdot \frac{\Delta t}{T} \quad \text{(IV)}.$$

ν: modulation frequency
T: period

Excepting possible losses in intensity, the receiver measures the phase-shifted signal

$$U = a \cdot \cos(2\pi \cdot \nu \cdot t - \Delta\varphi) \quad \text{(V)}.$$



From (III) and (IV) the following equation, which determines the velocity of light, is obtained

$$c = \frac{\Delta s}{\Delta\varphi} \cdot 2\pi \cdot \nu \quad \text{(VI)}.$$

If the modulation frequency is very high, considerable phase shifts $\Delta\varphi$ are achieved on short paths Δs. In the experiment, ν is equal to 60 MHz. The path Δs = 5 m therefore corresponds to a phase shift by a full period.

However, the high frequency makes it more difficult to display the receiver signal with an oscilloscope. As a simple oscilloscope will be used for determining the phase shift, the receiver signal will be mixed (multiplied) electronically with a signal of the frequency ν' = 59.9 MHz. From the addition formula

$$\cos\alpha \cdot \cos\alpha' = \frac{1}{2} \cdot (\cos(\alpha + \alpha') + \cos(\alpha - \alpha'))$$

it follows that the mixed signal

$$U = a \cdot \cos(2\pi \cdot \nu \cdot t - \Delta\varphi) \cdot \cos(2\pi \cdot \nu' \cdot t) \quad \text{(VII)}$$

can be represented as the sum of two signals, one with the frequency $\nu + \nu'$ and one with the difference frequency $\nu_1 = \nu - \nu'$. The high-frequency part is suppressed with a low-pass filter. Therefore only

$$U_1 = \frac{1}{2} a \cdot \cos\left(2\pi \cdot \nu_1 \cdot t - \Delta\varphi\right) \quad \text{(VIII)}$$

is left as receiver signal. It can be displayed with a simple oscilloscope since the frequency ν_1 is only 100 kHz. The phase shift $\Delta\varphi$ has not been changed by the mixing, but it now corresponds to an *apparent propagation time* Δt_1. The period T_1 of the mixed signal is also read from the oscilloscope, and the phase shift can be calculated:

$$\Delta\varphi = 2\pi \cdot \frac{\Delta t_1}{T_1} \quad \text{(IX)}.$$

Fig. 1 Phase shift of a periodic light signal

0910-Sel/Wei

Apparatus

1 light transmitter and receiver	476 30
1 lens, f = + 150 mm	460 08
2 saddle bases	300 11
1 two-channel oscilloscope 1004	575 221
1 metal scale, 1 m long	311 02

For the *actual propagation time* Δt of the light signal along the path Δs

$$\Delta t = \Delta t_1 \cdot \frac{T}{T_1} = \frac{\Delta t_1}{T_1 \cdot \nu} \qquad \text{(X)}$$

is obtained from (IV) and (IX), and this leads to an equation which determines the velocity of light:

$$c = \frac{\Delta s}{\Delta t_1} \cdot \frac{T_1}{T} = \frac{\Delta s}{\Delta t_1} \cdot T_1 \cdot \nu \qquad \text{(XI)}.$$

For an exact determination of the phase shift $\Delta\varphi$ a reference signal is available in the experiment. This signal is synchronized with the intensity of the light transmitter. It is mixed with the same 59.9-MHz signal and filtered in the same way as the receiver signal (see Fig. 2). As the propagation times of the signals in connection leads and in the apparatus cannot be neglected, the light transmitter is first set up at a distance s from the receiver. In this arrangement, the reference signal is synchronized with the receiver signal by means of an electronic phase shift. Then the light transmitter is displaced by the path Δs = 1 m away from the receiver. The phase shift $\Delta\varphi$ which is now observed is due to the propagation time Δt.

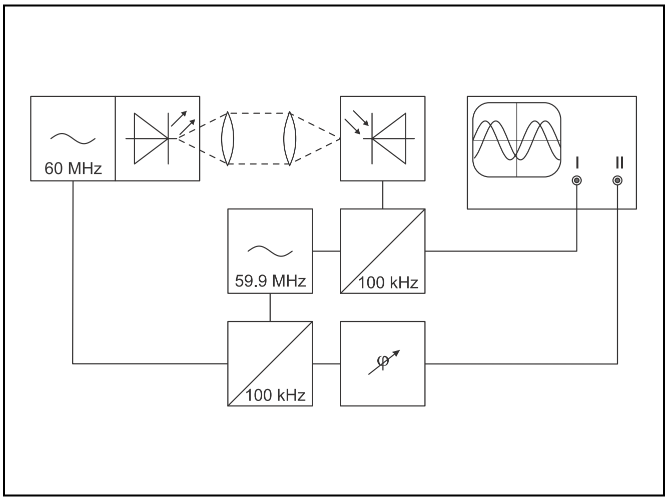


Fig. 2 Block diagram illustrating the measurement of the velocity of light

Setup

The experimental setup is illustrated in Figs. 3 and 4.

– Place the light transmitter at a distance of approx. 1 m from the receiver, connect it to the output **(a)** of the receiver via a 6 m long coaxial cable, and switch the receiver on.
– Image the red light patch of the light transmitter on the front plate of the receiver and displace the insert **(e)** relative to the condensor **(d)** so that the red light patch is illuminated as evenly as possible.
– Reduce the distance between the light transmitter and the receiver to 50 cm, a place the lens in the ray path.
– Align the light transmitter and the lens so that the red light patch impinges on the entrance aperture of the receiver. If necessary, optimize the alignment of the light transmitter with the knurled screws **(f)**.
– Connect the output **(c)** of the receiver to channel II of the oscilloscope.

Oscilloscope settings:

Coupling of channel II:	AC
Trigger:	channel II
Time base:	2 μs/DIV

– Observe the receiver signal on the oscilloscope and optimize the alignment of the light transmitter and the lens once more.

If the receiver signal is distorted due to overload:

– Slightly defocus the light bundle by displacing the lens.

– Mark the position of the light transmitter on the table as position 1.
– Displace the light transmitter along the optical axis by Δs = 100 cm, check the alignment of the light transmitter, and mark the second position, too.

Carrying out the experiment

Remark:

A satisfactory accuracy of the result can only be achieved if the light transmitter and the receiver are thermally stable. Start the experiment only half an hour after switching on the light transmitter and the receiver.

Synchronizing the phases of the reference and the receiver signal:

– Connect the output **(b)** of the receiver to channel I of the oscilloscope and look at channel I (reference signal) and channel II (receiver signal) simultaneously.

Oscilloscope settings:

Coupling channel I and II:	AC
Trigger:	channel I
Time base:	2 μs/DIV

– Adjust the vertical positions of channels I and II so that they are as symmetric as possible with respect to the horizontal centre line of the screen.

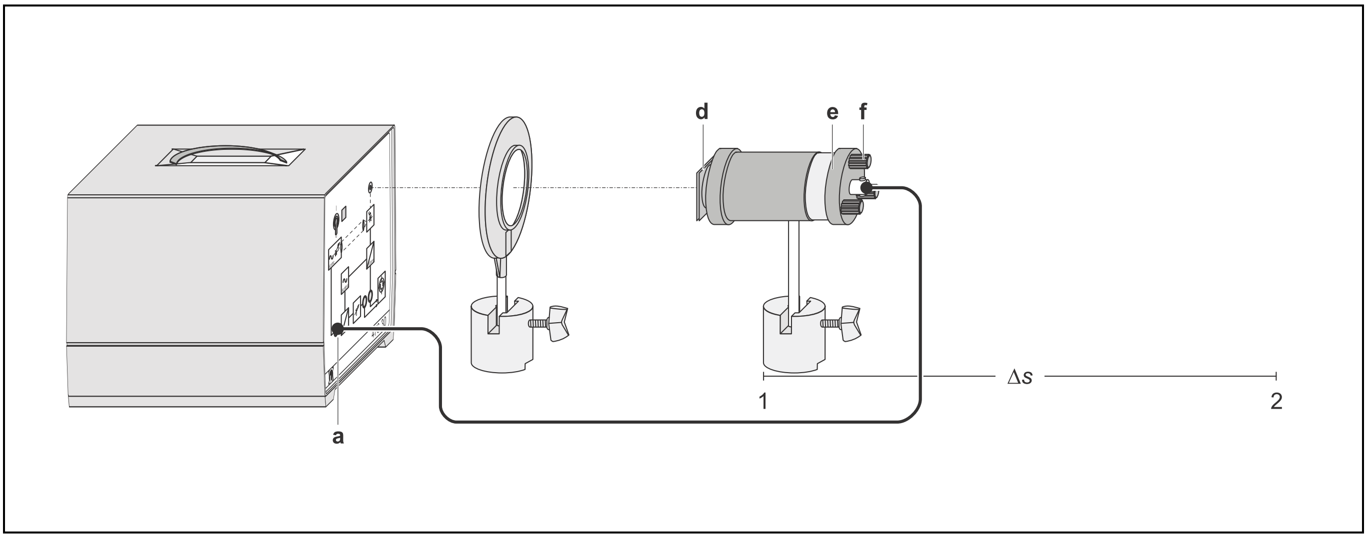


Fig. 3 Optical setup for determining the velocity of light with a periodic light signal.

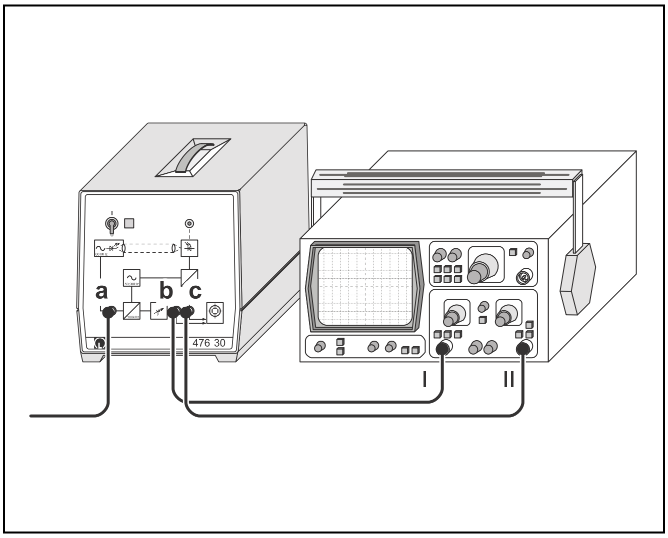


Fig. 4 Connection of the oscilloscope for measuring the phase shift of the periodic light signal.

– For the sake of control adjust the vertical deflections with the fine adjustment control so that the maxima of both signals touch the same horizontal line.
– Adjust the two signals with the phase shifter φ so that they are in phase as exactly as possible.
– Choose a suitable horizontal position of the signals, and determine the period T_1.

Determining the phase shift $\Delta\varphi$:

– Displace the light transmitter back to position 1, and observe the two signals.
– Set the time base 0.5 µs/DIV, shift the zero of the reference signal exactly to the vertical centre line, and determine the *apparent propagation time* Δt_1.
– Displace the light transmitter back and forth several times, and determine the mean value of the measured values Δt_1.

Measuring example

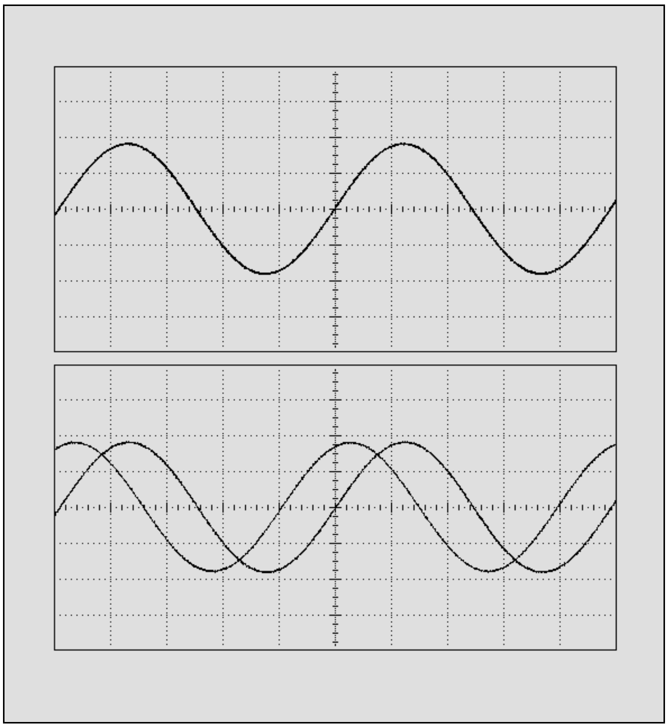


Fig. 5 Reference signal and receiver signal for the positions 1 (above) and 2 (below) of the light transmitter, Δs = 100 cm, time base 2 µs/DIV.

Δs = (100 ± 1) cm

Δt_1 = (3.9 ± 0.1) DIV · 0.5 µs/DIV = (1.95 ± 0.05) µs

T_1 = (4.90 ± 0.05) DIV · 2 µs/DIV = (9.8 ± 0.1) µs

Since the modulation frequency ν = 60 MHz is timed by a quartz, it need not be measured.

Evaluation

With (X) the propagation time of the light signal along the path Δs = 1 m is calculated:

$\Delta t = (3.32 \pm 0.09)$ ns

The velocity of light is obtained from (XI):

$$c = (3.02 \pm 0.09) \cdot 10^8 \frac{\text{m}}{\text{s}}$$

Values quoted in the literature:

Velocity of light in vacuum:

$$c_0 = 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

Velocity of light in air:

$$c = \frac{c_0}{n} = 2.997 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

(refractive index n = 1.003 under standard conditions)

Results

The finite propagation time of light along a certain path manifests itself in the phase shift of a periodic light signal. From the phase shift and the path length the velocity of light can be determined if the period or the frequency, respectively, of the light signal is known.

Determining the velocity of light in various materials

Objects of the experiments

- Determining the velocity of light and the refractive index for water.
- Determining the velocity of light and the refractive index for organic liquids.
- Determining the velocity of light and the refractive index for glass.

Principles

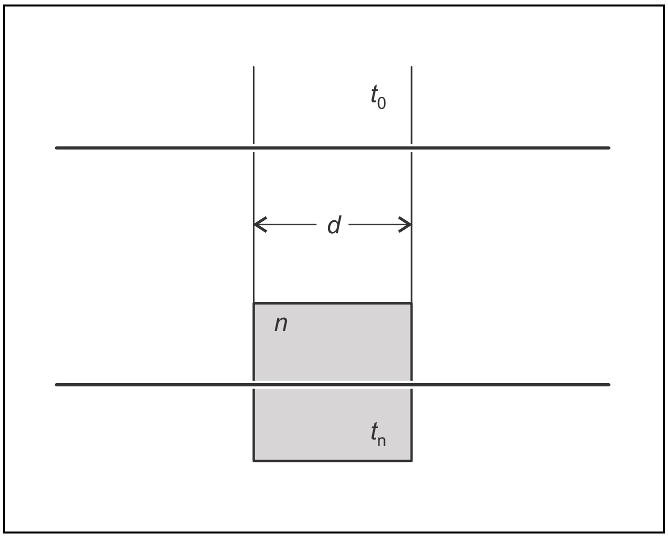


Fig. 1 In a medium with the refractive index n light propagates at a lower velocity than in vacuum. This leads to a change in the propagation time t of light along a path of length d.

The velocity of light c_n in a medium depends on the refractive index n of that medium. This material-dependent quantity is a measure for the optical density of the medium, and it relates the velocity of light in the medium to that in vacuum via

$$c_n = \frac{c_0}{n} \qquad \text{(I)}$$

$c_0 = 2.998 \cdot 10^8 \frac{m}{s}$: velocity of light in vacuum

Different velocities of light correspond to different propagation times along a certain path length d. The propagation time of light in a medium is

$$t_n = \frac{d}{c_n} \qquad \text{(II)},$$

whereas in vacuum the propagation time is

$$t_0 = \frac{d}{c_0} \qquad \text{(III)}$$

(see Fig. 1). With the definition

$\Delta t = t_n - t_0$

and (II) and (III), the velocity of light in the medium is determined by the equation

$$c_n = \frac{c_0}{1 + \frac{\Delta t}{d} \cdot c_0} \qquad \text{(IV).}$$

By comparison with (I)

$$n = 1 + \frac{\Delta t}{d} \cdot c_0 \qquad \text{(V)}$$

is obtained. The change of the velocity of light in the medium is measured in the experiment with a periodic light signal with the modulation frequency ν = 60 MHz. The propagation time difference Δt is observed as a phase shift

$$\Delta\varphi = 2\pi \cdot \nu \cdot \Delta t = 2\pi \cdot \frac{\Delta t}{T} \qquad \text{(VI)}$$

T: period

of the signal. The phase shift is measured with a receiver, which converts the light signal into an alternating voltage with the time behaviour

$$U = a \cdot \cos\left(2\pi \cdot \nu \cdot t - \Delta\varphi\right) \qquad \text{(VII).}$$

A reference signal which oscillates synchronously with the intensity of the light transmitter is synchronized with the receiver signal by means of an electronic phase shift while the light transmitter and the receiver are at a distance s from each other in air (refractive index n = 1.003 under standard conditions). If then a medium of sufficient optical density is put into the ray path covering the partial path length d, this causes a change Δt in the propagation time of the light signal. This change can be measured as a phase shift $\Delta\varphi$ between the reference and the receiver signal.

0910-Wei

Apparatus

1 light transmitter and receiver	476 30
1 lens, f = + 150 mm	460 08
2 saddle bases	300 11
1 two-channel oscilloscope 1004	575 221
1 metal scale, 1 m	311 02
velocity of light in water:	
1 tube with 2 end windows	47635
2 saddle bases	300 11
distilled or demineralized water	
velocity of light in organic liquids:	
1 ethanol, denaturated, 1 l	671 972
1 glycerine, 99 %, 250 ml	672 121
1 plate glass cell, 50×50×50 mm	477 03
1 prism table	460 25
1 saddle base	300 11
velocity of light in glass:	
1 acrylic glass block	476 34
1 prism table	460 25
1 saddle base	300 11

As a simple oscilloscope will be used for determining the phase shift, the two signals will be mixed (multiplied) electronically with a signal of the frequency ν' = 59.9 MHz, and the high-frequency part of the mixed signal will be suppressed. The receiver signal then has the form

$$U_1 = \frac{1}{2} a \cdot \cos(2\pi \cdot \nu_1 \cdot t - \Delta\varphi) \qquad \text{(VIII)}$$

with $\nu_1 = \nu - \nu'$.

This signal can be displayed with a simple oscilloscope since the frequency ν_1 is only 100 kHz. The phase shift $\Delta\varphi$ is not changed by the mixing, but it corresponds to an *apparent change* Δt_1 in the propagation time. The period T_1 of the mixed signal is read from the oscilloscope. Then

$$\Delta\varphi = 2\pi \cdot \frac{\Delta t_1}{T_1} \qquad \text{(IX)}$$

or

$$\Delta t = \Delta t_1 \cdot \frac{T}{T_1} = \frac{\Delta t_1}{T_1 \cdot \nu} \qquad \text{(X)}$$

is obtained. Inserting in (IV) and (V) renders he velocity of light in the medium

$$c_n = \frac{c_0}{1 + \frac{c_0}{d \cdot \nu} \cdot \frac{\Delta t_1}{T_1}} \qquad \text{(XI)}$$

and the refractive index

$$n = 1 + \frac{c_0}{d \cdot \nu} \cdot \frac{\Delta t_1}{T_1} \qquad \text{(XII).}$$

Setup

The experimental setup is illustrated in Figs. 2 and 3.

– Set the light transmitter up at a distance of approx. 1.5 m from the receiver, connect it to the output **(a)** of the receiver, and switch the receiver on.
– Image the red light patch of the light transmitter on the front plate of the receiver and displace the insert **(e)** relative to the condensor **(d)** so that the red light patch is illuminated as evenly as possible.
– Place the lens in the ray path.
– Align the light transmitter and the lens so that the red light patch impinges on the entrance aperture of the receiver. If necessary, optimize the alignment of the light transmitter with the knurled screws **(f)**.
– Connect the output **(c)** of the receiver to channel II of the oscilloscope.

Oscilloscope settings:

Coupling channel II:	AC
Trigger:	channel II
Time base:	2 μs/DIV

– Observe the receiver signal on the oscilloscope and optimize the alignment of the light transmitter and the lens once more.

a) Velocity of light in water:

– Clamp the tube holder in the a saddle base, put the tube with 2 end windows on the holder, and fix the tube with two elastic bands as shown in Fig. 4.
– Put a piece of tubing with a funnel on the hose connector on the bottom of the tube, and open the two stopcocks.
– Hold the funnel over the tube, and fill the tube with distilled water as bubble-free as possible.

Remark:

The above stopcock should remain open to avoid changes in pressure inside the tube in the case of fluctuations of the temperature.

b) Velocity of light in organic liquids:

– Mount the prism table in a saddle base, and clamp the plate glass cell on the prism table with the metal holder.
– Place the empty plate glass cell in the ray path immediately in front of the light transmitter.

c) Velocity of light in glass:

– Mount the prism table in a saddle base, and clamp the acrylic glass block on the prism table with the metal holder.

Fig. 2 Optical setup for determining the velocity of light.

Carrying out the experiment

Remark:

A satisfactory accuracy of the results can only be achieved if the light transmitter and the receiver are thermally stable. Start the experiment only half an hour after switching on the light transmitter and the receiver.

Since the modulation frequency ν = 60 MHz is timed by a quartz, it need not be measured.

Synchronizing the phases of the reference and the receiver signal:

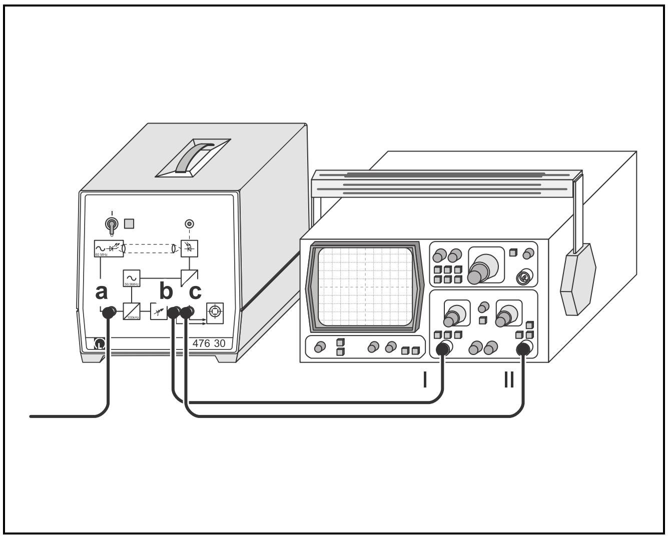
– Connect the output **(b)** of the receiver to channel I of the oscilloscope and look at channel I (reference signal) and channel II (receiver signal) simultaneously.

Oscilloscope settings:

Coupling channel I and II:	AC
Trigger:	channel I
Time base:	2 μs/DIV

– Adjust the vertical positions of channels I and II so that they are as symmetric as possible with respect to the horizontal centre line of the screen.
– For the sake of control adjust the vertical deflections with the fine adjustment control so that the maxima of both signals touch the same horizontal line.
– Adjust the two signals with the phase shifter φ so that they are in phase as exactly as possible.
– Choose a suitable horizontal position of the signals, and determine the period T_1.

Fig. 3 Connection of the oscilloscope for measuring the phase shift of the periodic light signal.



a) Velocity of light in water:

– Put the tube filled with water into the ray path immediately in front of the light transmitter.
– Set the time base 1 μs/DIV, read the distance between the zero passages, and determine the *apparent change in propagation time* Δt_1.
– Repeat the measurement several times, and determine the mean value of the measured values Δt_1.

Fig. 4 Determining the velocity of light in water.

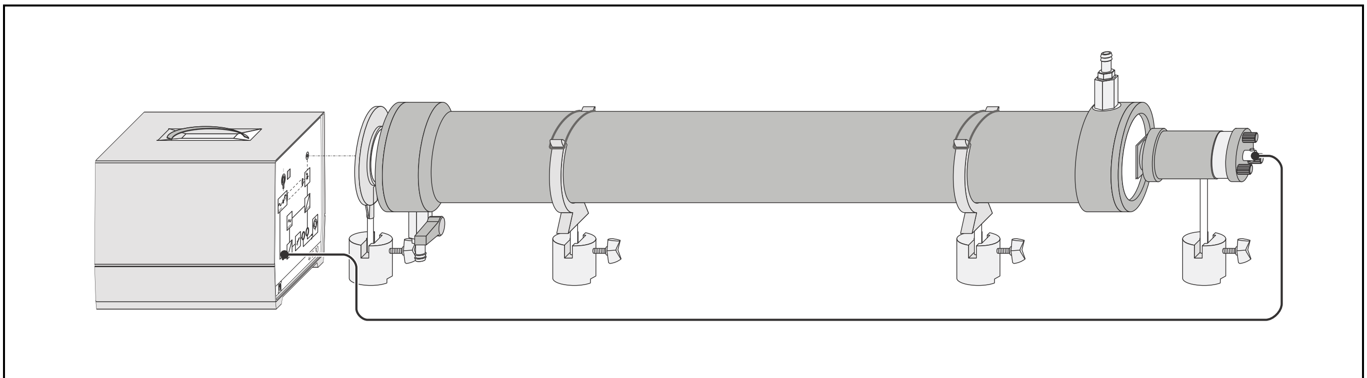


Fig. 5 Determining the velocity of light in organic liquids.

b) Velocity of light in organic liquids:

– In order to achieve an improved display of the phase shift, enlarge the section for both signals.

Oscilloscope settings:	
Coupling channel I and II:	AC
Trigger:	channel I
Automatic adjustment of the trigger threshold:	off
Time base:	0.1 µs/DIV
Amplitude:	0.1 mV/DIV

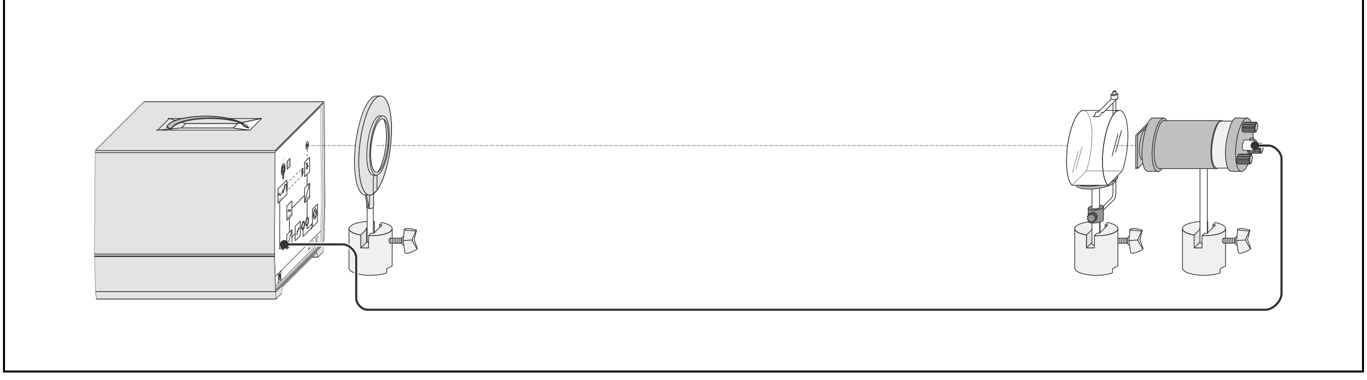
– Choose a suitable horizontal position of the signals (e.g. shift the zero passage to the centre of the screen).
– Fill the plate glass cell with ethanol, and observe the displacement of the signals.
– Read the distance between the zero passages, and determine the *apparent propagation time* Δt_1.

– Empty the plate glass cell, and adjust the signals with the phase shifter φ so that they are in phase again.
– Fill the plate glass cell with glycerine, and repeat the measurement.

c) Velocity of light in acrylic glass:

– Perform the measurement as described in section b) using the acrylic glass block instead of the plate glass cell.

Fig. 6 Determining the velocity of light in glass.



Measuring example

a) Velocity of light in water:

$d = (100 \pm 1)$ cm

$\Delta t_1 = (0.70 \pm 0.05)$ DIV · 1 µs/DIV = (0.70 ± 0.05) µs

$T_1 = (4.90 \pm 0.05)$ DIV · 2 µs/DIV = (9.8 ± 0.1) µs

b) Velocity of light in organic liquids:

The reference and the receiver signals for ethanol and glycerine look similar to those shown in Fig. 8.

Ethanol:

$d = (5.0 \pm 0.1)$ cm

$\Delta t_1 = (0.3 \pm 0.1)$ DIV · 0.1 µs/DIV = (0.03 ± 0.01) µs

$T_1 = (4.90 \pm 0.05)$ DIV · 2 µs/DIV = (9.8 ± 0.1) µs

Glycerine:

d = (5.0 ± 0.1) cm

$\Delta t_1 = (0.4 \pm 0.1)$ DIV · 0.1 µs/DIV = (0.04 ± 0.01) µs

$T_1 = (4.90 \pm 0.05)$ DIV · 2 µs/DIV = (9.8 ± 0.1) µs

c) Velocity of light in acrylic glass:

$d = (5.0 \pm 0.1)$ cm

$\Delta t_1 = (0.6 \pm 0.1)$ DIV · 0.1 µs/DIV = (0.06 ± 0.01) µs

$T_1 = (4.90 \pm 0.05)$ DIV · 2 µs/DIV = (9.8 ± 0.1) µs

Evaluation

a) Velocity of light in water:

The refractive index n of water can be determined with equation (XII):

$n = 1.36 \pm 0.03$

The velocity of light in water is calculated from the refractive index

$c = (2.20 \pm 0.05) \cdot 10^8 \frac{\text{m}}{\text{s}}$

Value quoted in the literature:

Refractive index of water: $n = 1.333$

b) Velocity of light in organic liquids:

The refractive indices n of ethanol and glycerine can be determined with equation (XII):

Ethanol: $n = 1.3 \pm 0.1$

Glycerine: $n = 1.4 \pm 0.1$

The velocities of light in ethanol and glycerine are calculated from the refractive indices

Ethanol: $c = (2.29 \pm 0.16) \cdot 10^8 \frac{\text{m}}{\text{s}}$

Glycerine: $c = (2.13 \pm 0.14) \cdot 10^8 \frac{\text{m}}{\text{s}}$

Values quoted in the literature:

Refractive index of ethanol $n = 1.36$

Refractive index of glycerine $n = 1.47$

c) Velocity of light in acrylic glass:

The refractive index n of acrylic glass can be determined with equation (XII):

$n = 1.6 \pm 0.1$

The velocity of light in acrylic glass is calculated from the refractive index

$c = (1.86 \pm 0.14) \cdot 10^8 \frac{\text{m}}{\text{s}}$

Value quoted in the literature:

Refractive index of acrylic glass $n = 1.5$

Results

If a light ray passes an optically dense medium, the propagation time of the light is changed. The change in propagation time manifests itself as a phase shift of a periodic light signal. From the phase shift the refractive index of the medium can be determined.

Fig. 7 Reference and receiver signal for water, d = 100 cm, horizontal deflection 1 µs/DIV.

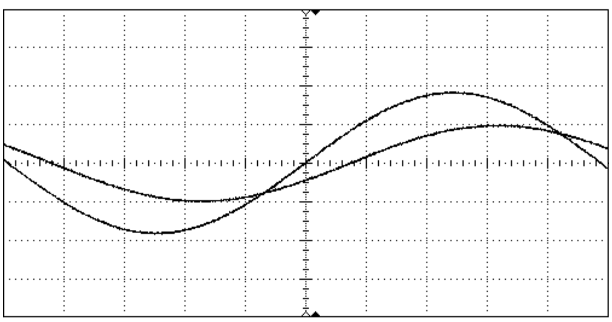


Fig. 8 Reference and receiver signal for acrylic glass, d = 5 cm, horizontal deflection 0.1 µs/DIV.