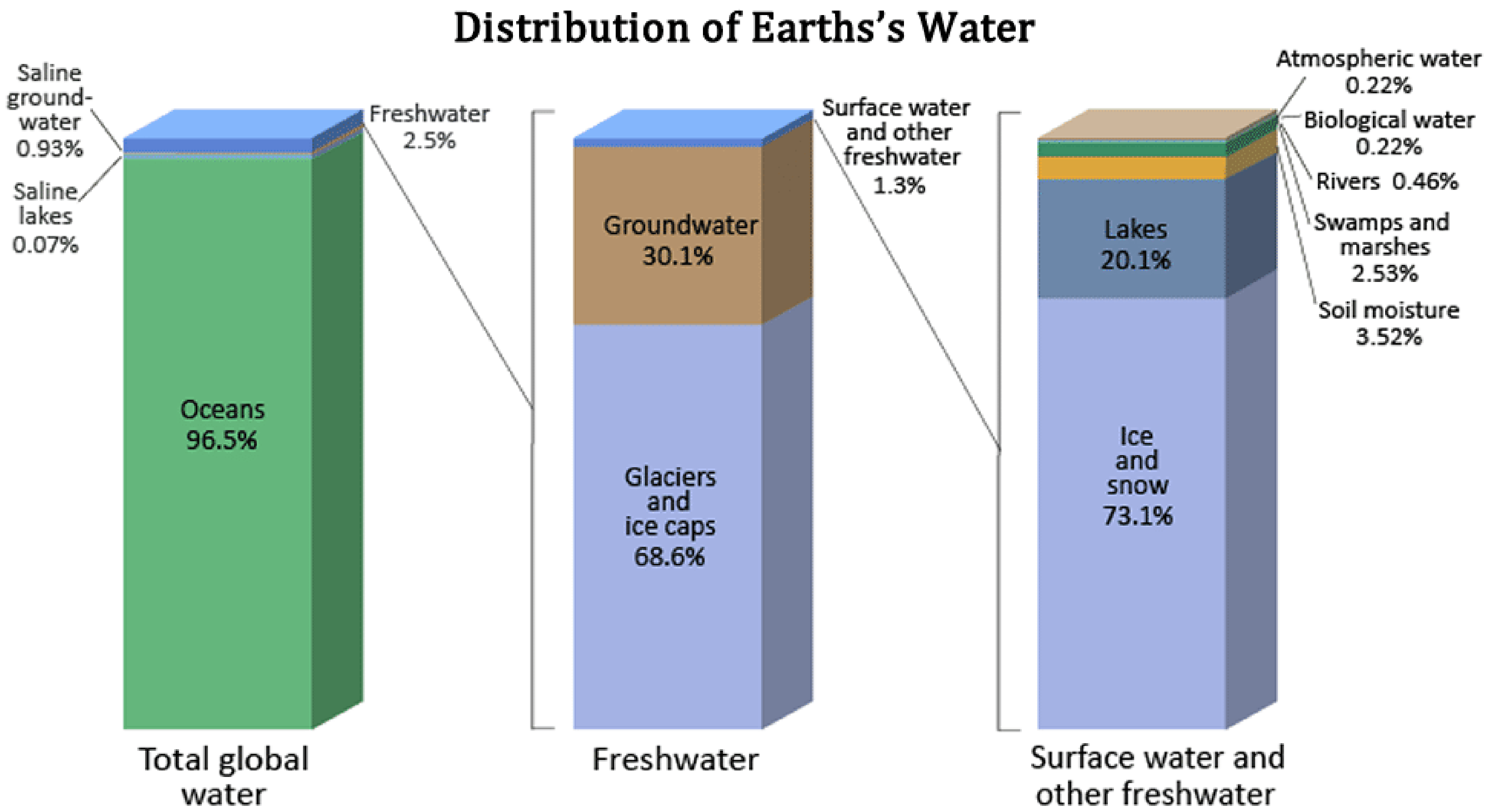
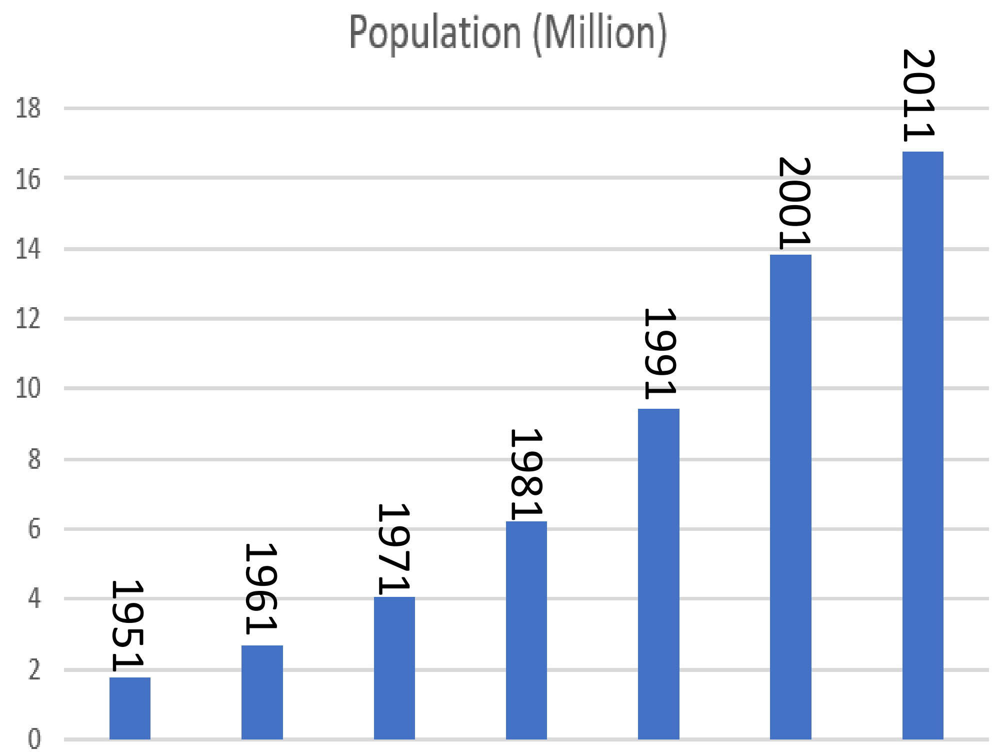
Distribution of Water



Increase in Population in Delhi

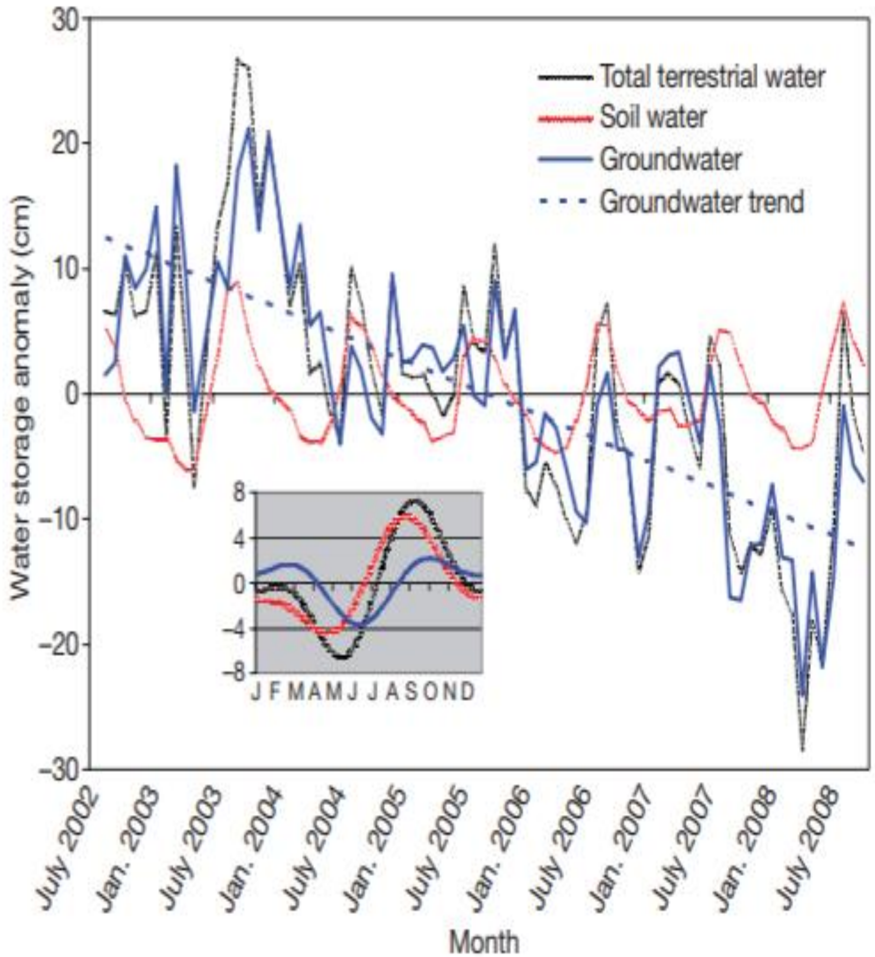


Source : India Census 2011

The population density of Delhi had grown from 13.85 to 16.78 million from 2001 to 2011 with population density 11,320 person per square kilometre.

Between 2006-2016 the water demand has increased by 39% [Source: DJB].

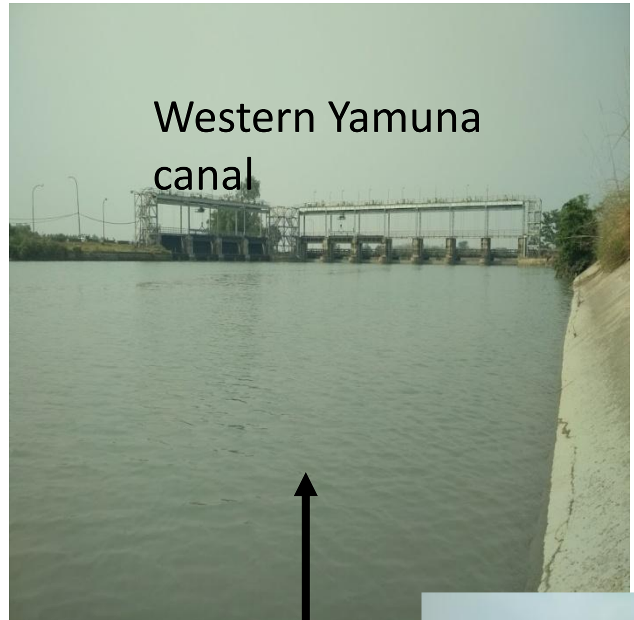
Overexploitation of ground water in NW India

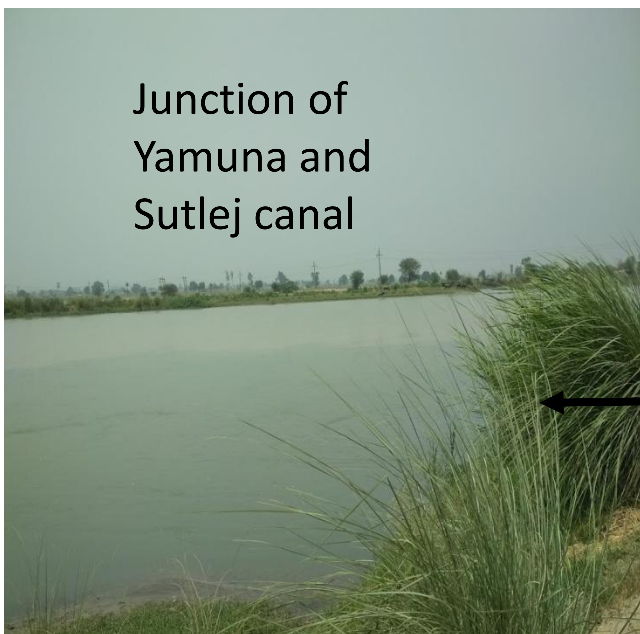


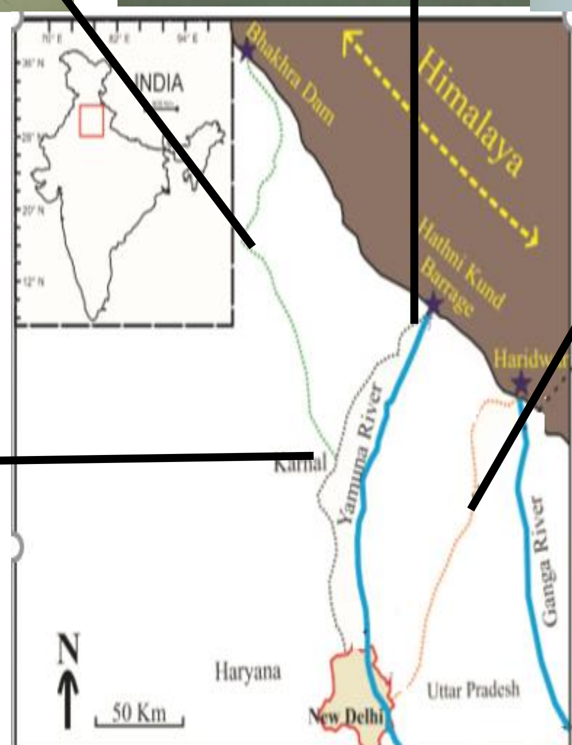
- groundwater withdrawals exceed recharge

Source: Rodell et al 2009 (Nature)

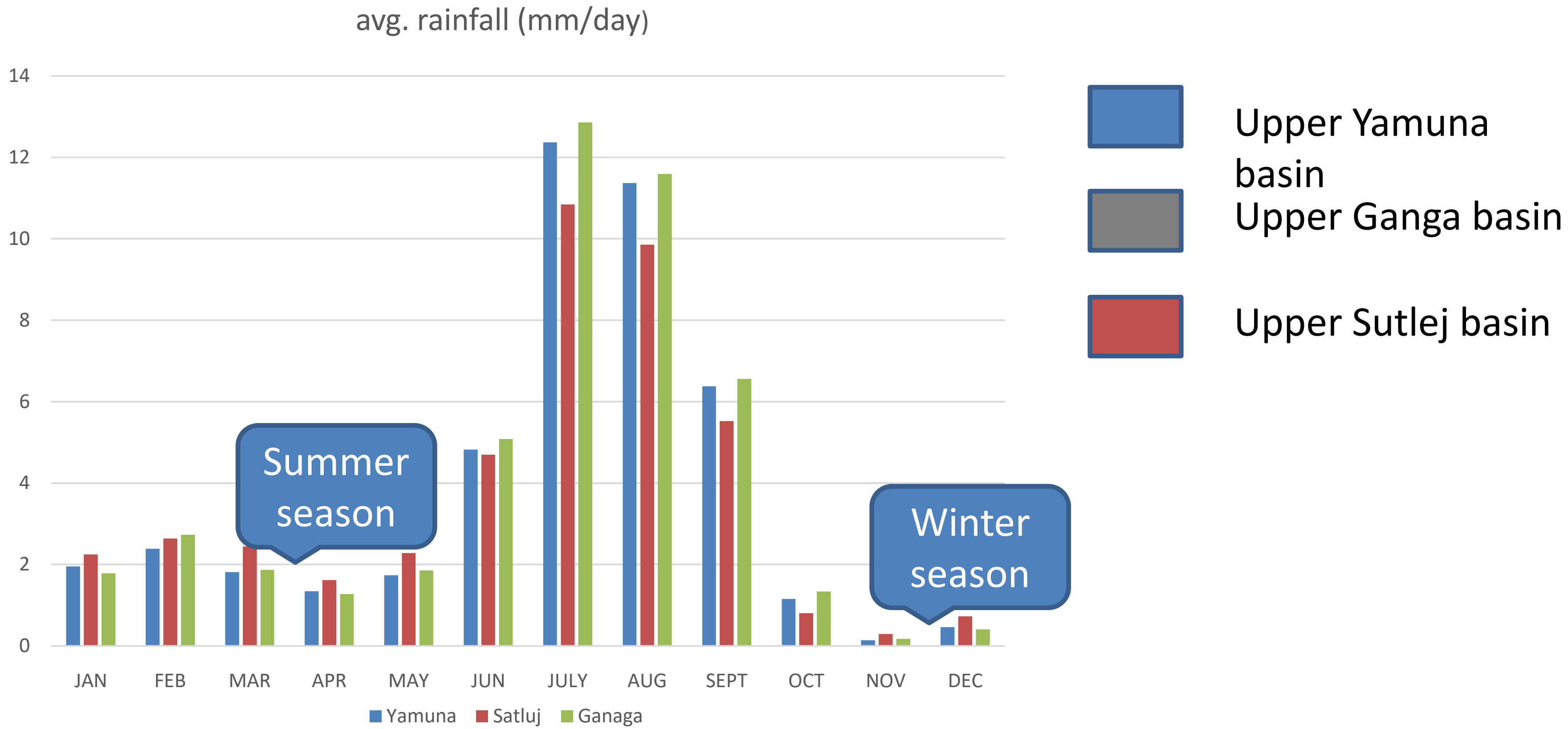
Sources of water in Delhi







Rainfall distribution in Delhi

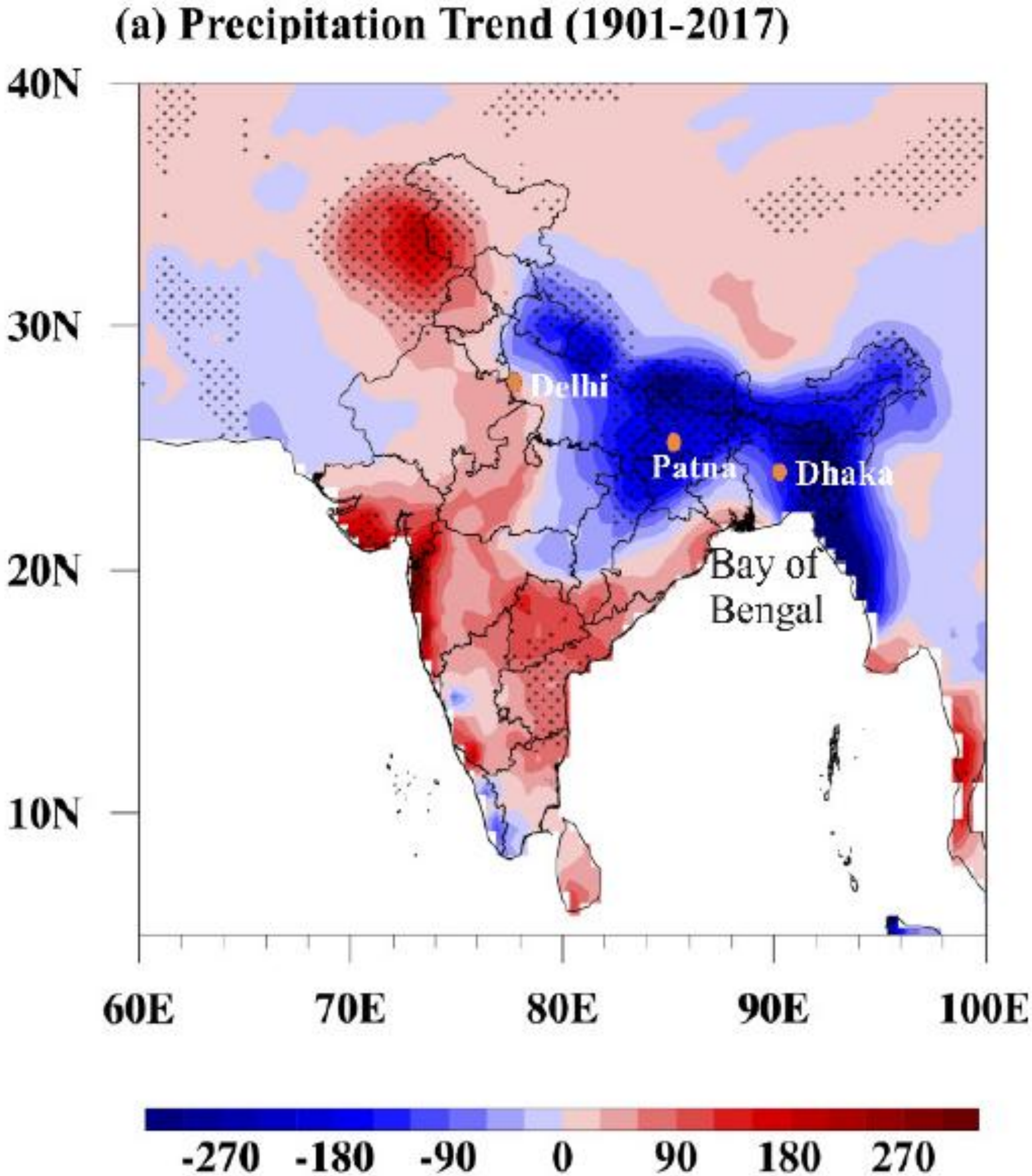


Winter season

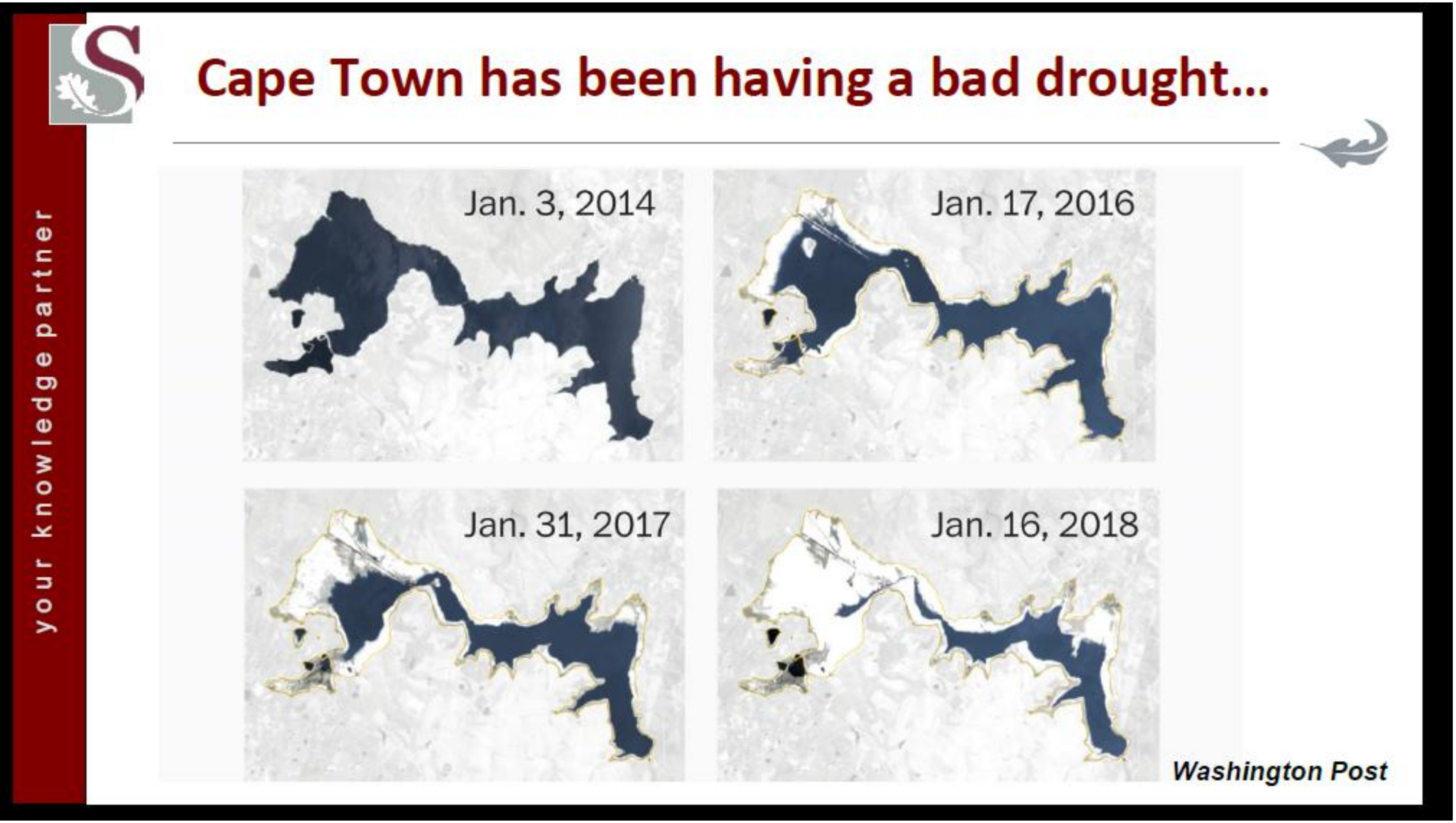
- During Nov-Dec rainwater contribution in basin can be neglected.
- Only two end member ,i.e. groundwater and glacier water
- Moreover in this season (Dec-Jan) not much glacier water melts
- Assumption : In Dec. only ground water contributes to river

Summer season

- During Apr-May , rainwater contribution can be neglected.
- Basin is recharge through mainly glacierwater and ground water
- Glacierwater isotopic value can be estimated (the SWI value of ground water will be same as of winter season.


(a) Precipitation Trend (1901-2017)
40N
30N
20N
10N
60E
70E
80E
90E
100E
Delhi
Patna
Dhaka
Bay of
Bengal
-270
-180
-90
0
90
180
270

South Africa



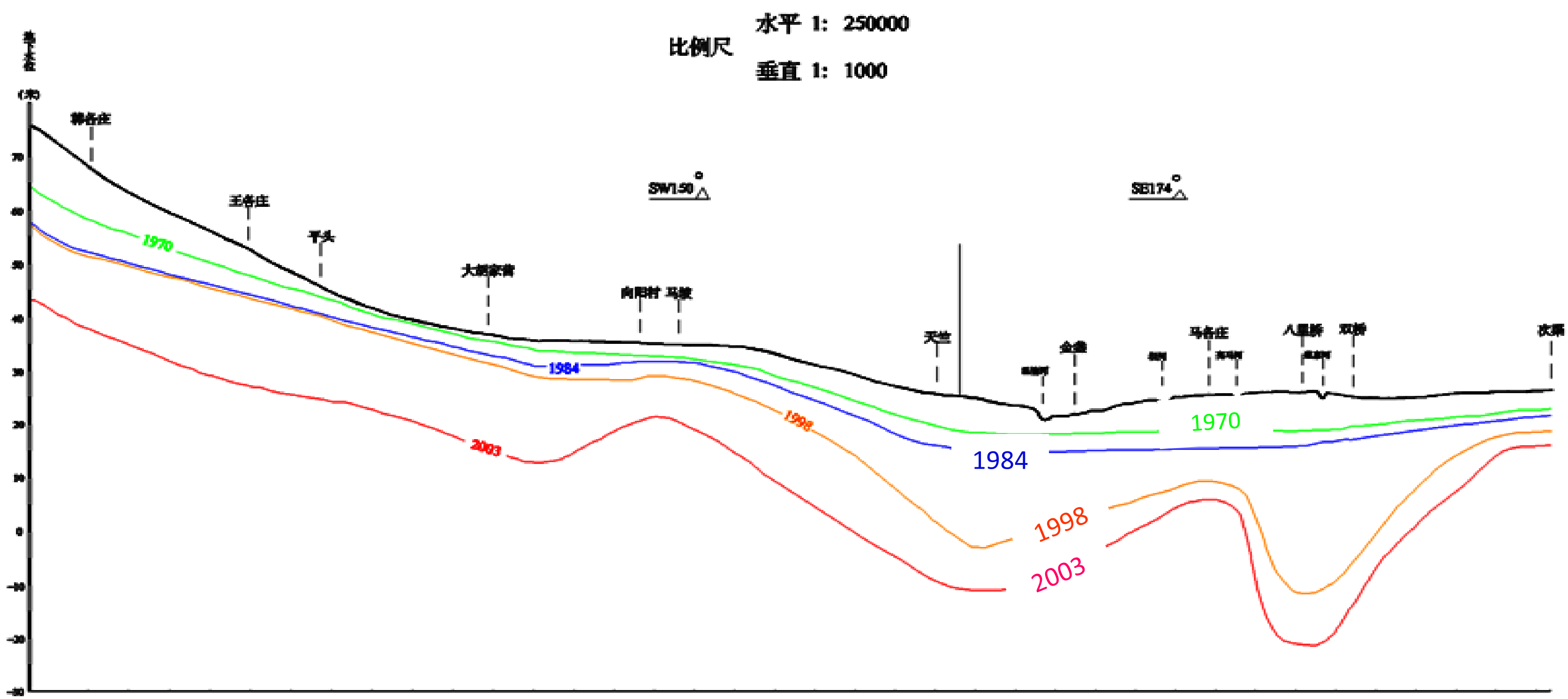
Theewatersloof Dam 54% CoCT water supply

Photography credit Greg Gordon

Drought Crisis

Beijing

Groundwater level



Groundwater level decline: 40m (1970—2003)

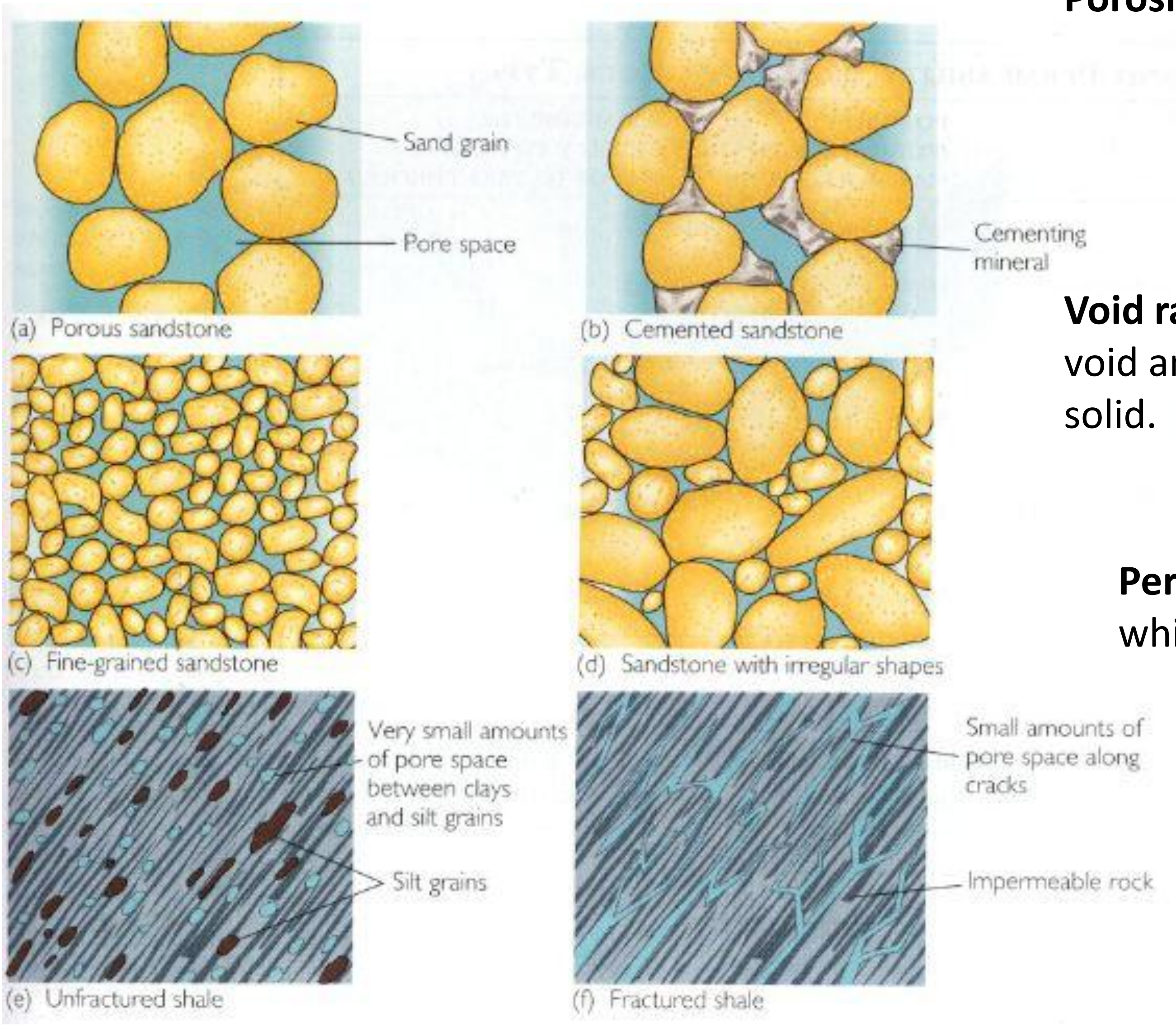
How will groundwater flow regime and groundwater evolution change under such conditions/stress?

Textural Classification of grains

$$\emptyset = -log_2^D$$

Millimeters	Phi (Φ)	Wentworth Size Class	
		Boulder	GRAVEL
256	-8		
		Cobble	
64	-4		
		Pebble	
2.0	-1		
		Very coarse sand	SAND
1.0	0		
		Coarse sand	
1/2 0.5	1		
		Medium sand	
1/4 0.25	2		
		Fine sand	
1/8 0.125	3		
		Very fine sand	
1/16 0.0625	4		
		Coarse Silt	MUD
1/32 0.0310	5		
		Medium Silt	
1/64 0.0156	6		
		Fine Silt	
1/128 0.0078	7		
		Very fine silt	
1/256 0.0039	8		
		Clay	

Pore Spaces and their connectivity



Porosity = (Volume of Voids / Total Volume) x100%.

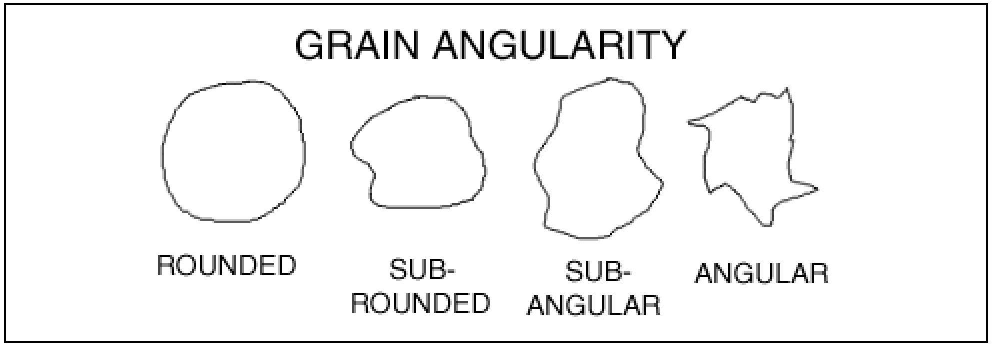
Void ratio: the total volume of void and the volume of the solid.

Permeability = Ease with which water can flow

Permeability: Interconnectivity of pore space

Clay: …Angular

Sand and other coarse grains: Angularity lost by impact during transportation



Clay: Small Size

Surface area/volume ratio high

More number of grains can be in contact with one grain which blocks the connectivity of pores

Mineral composition also plays role: shall discuss later

Types of openings in rocks

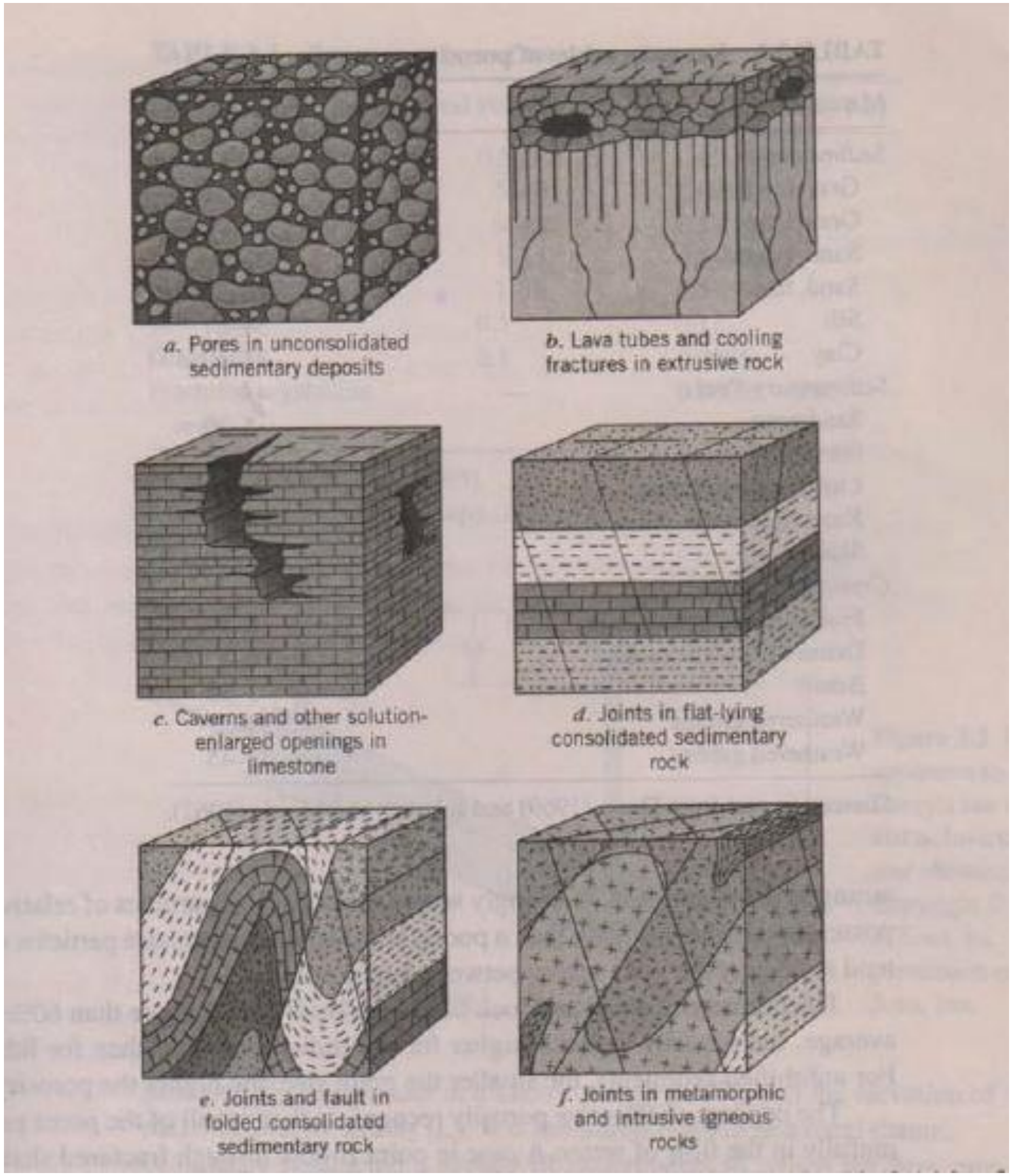
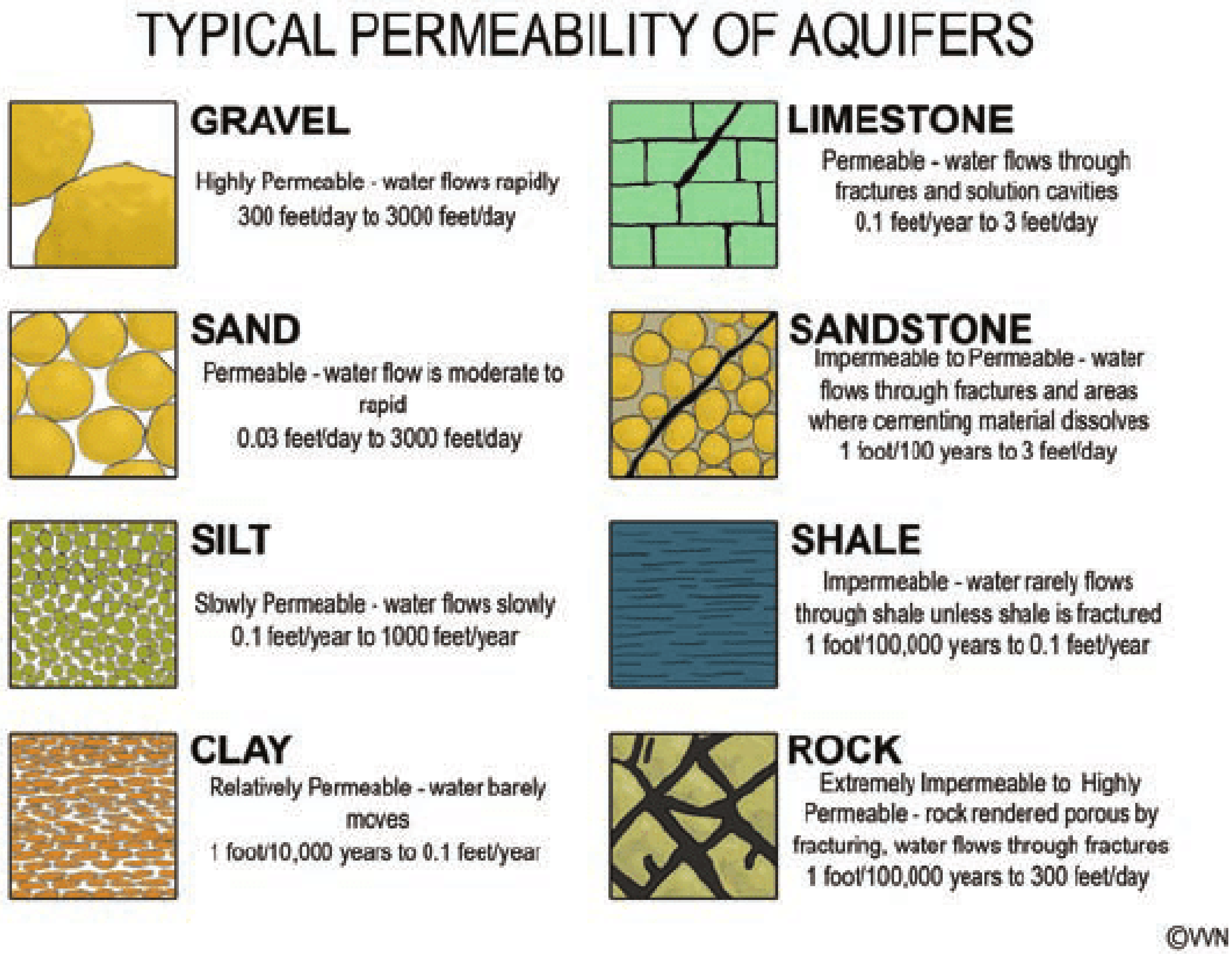


TABLE 3.1 Range in values of porosity

Material	Porosity (%)
Sedimentary	
Gravel, coarse	24–36
Gravel, fine	25–38
Sand, coarse	31–46
Sand, fine	26–53
Silt	34–61
Clay	34–60
Sedimentary Rocks	
Sandstone	5–30
Siltstone	21–41
Limestone, dolomite	0–40
Karst limestone	0–40
Shale	0–10
Crystalline Rocks	
Fractured crystalline rocks	0–10
Dense crystalline rocks	0–5
Basalt	3–35
Weathered granite	34–57
Weathered gabbro	42–45

Source: In part from Davis (1969) and Johnson and Morris (1962).


TYPICAL PERMEABILITY OF AQUIFERS
GRAVEL
Highly Permeable - water flows rapidly
300 feet/day to 3000 feet/day
LIMESTONE
Permeable - water flows through fractures and solution cavities
0.1 feet/year to 3 feet/day
SAND
Permeable - water flow is moderate to rapid
0.03 feet/day to 3000 feet/day
SANDSTONE
Impermeable to Permeable - water flows through fractures and areas where cementing material dissolves
1 foot/100 years to 3 feet/day
SILT
Slowly Permeable - water flows slowly
0.1 feet/year to 1000 feet/year
SHALE
Impermeable - water rarely flows through shale unless shale is fractured
1 foot/100,000 years to 0.1 feet/year
CLAY
Relatively Permeable - water barely moves
1 foot/10,000 years to 0.1 feet/year
ROCK
Extremely Impermeable to Highly Permeable - rock rendered porous by fracturing, water flows through fractures
1 foot/100,000 years to 300 feet/day
©VVN

Topics to be covered

1. Aquifer types: Confined, Unconfined

2. Parameters to define the flow of water: K,T,S,Ss, Sy, Sr, h……

3. Flow of water: Darcy's Law, Limitations

4. Derivation of groundwater flow equation: Laplace equation etc.

5. Solution of groundwater flow equation: Boundary Conditions: Uni-directional, Radial, Confined, Unconfined

6. Groundwater flow in fractured medium: Flow of water in Hard rock

7. Modeling of groundwater flow: Different types especially Finite Difference method

8. Technique of groundwater exploration: Mostly Electrical conductivity

9. Age determination of groundwater: Radio carbon (^{14}C), Tritium (^{3}H)

10. Recharge method of groundwater

11. Pollution in groundwater in the context of Arsenic

Aquifer

Aquifer

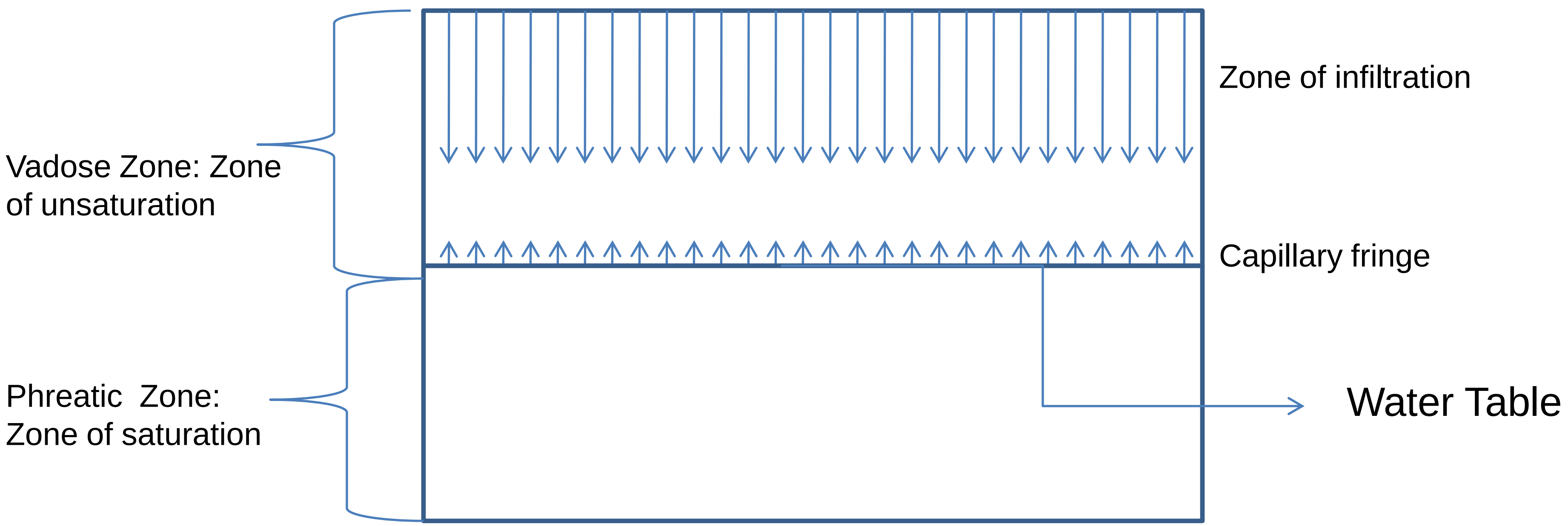
Aquitard

Decreasing Sand content

Aquiclude

Aquifuge

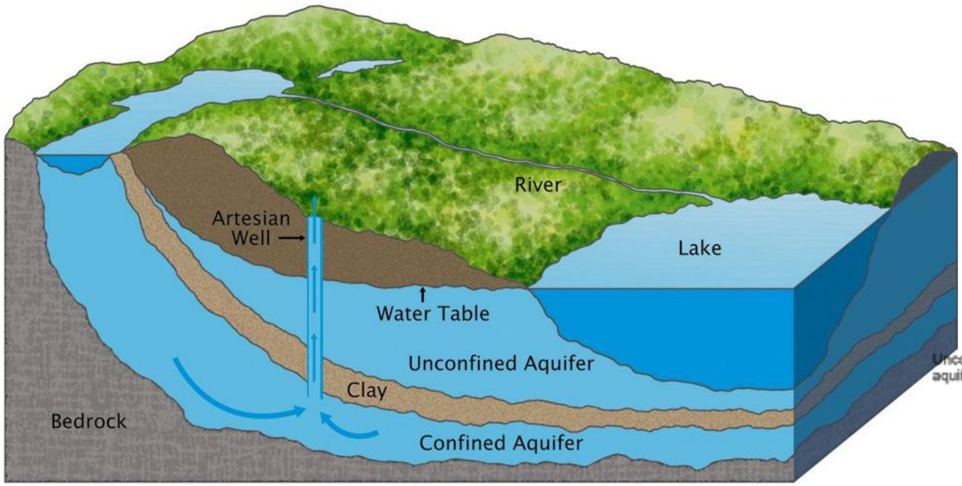
Internal structure of Aquifer: Saturation based

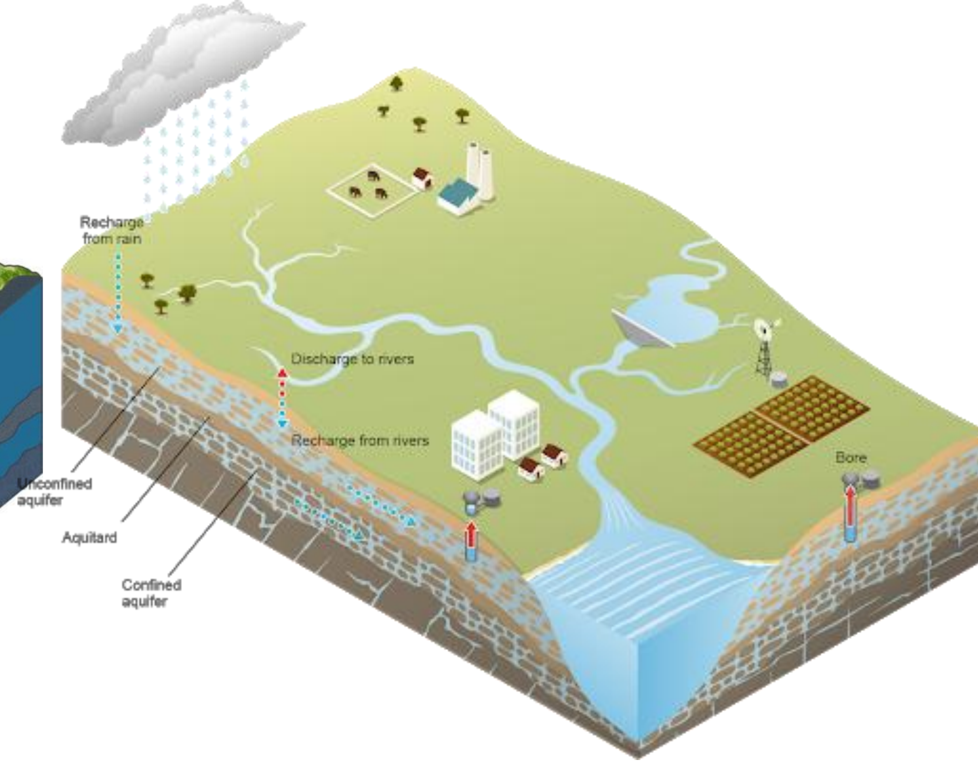


https://www.youtube.com/watch?v=YoF7RuBZIpM

Water Table: Atmospheric Pressure=hydrostatic pressure

Aquifer Type: Confined and Unconfined



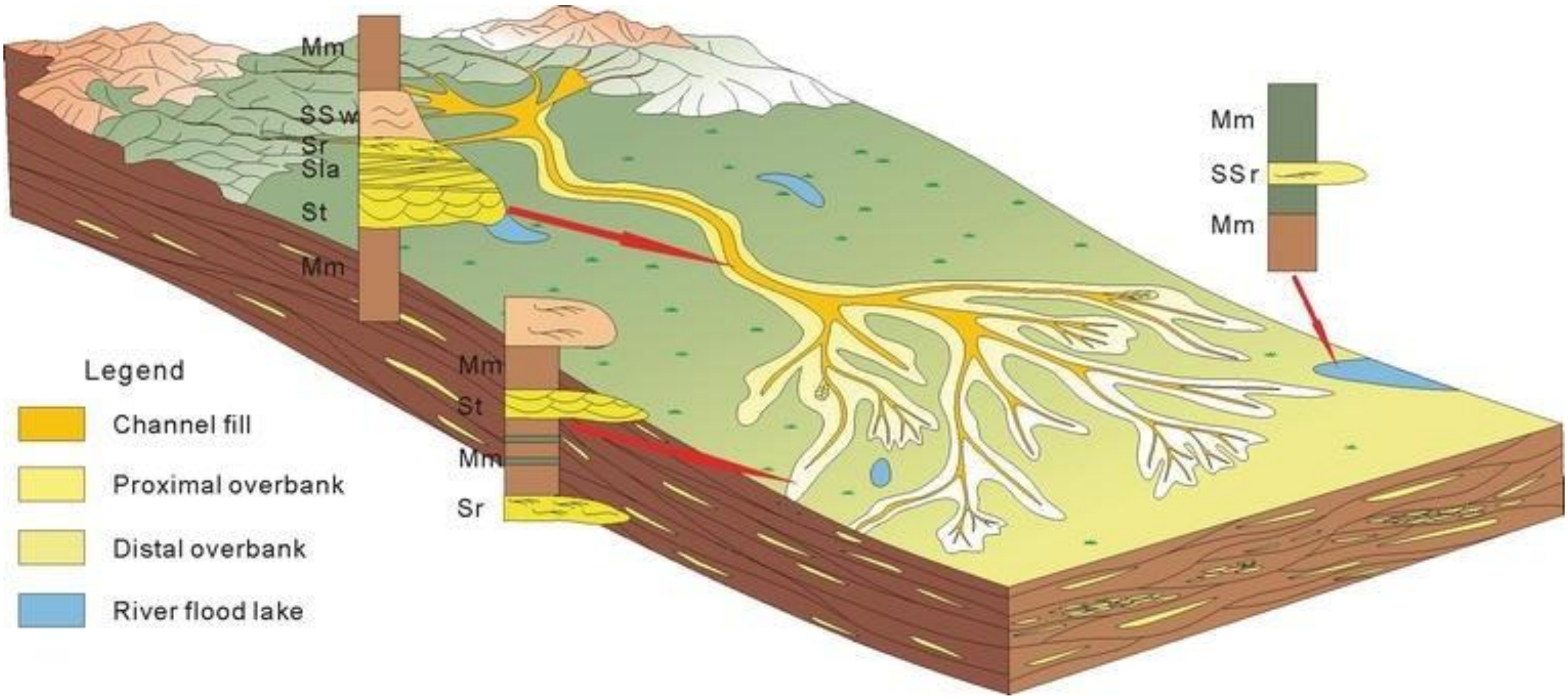


Confined Aquifer: Layers of impermeable material are both above and below the aquifer

Unconfined Aquifer: Groundwater is in direct contact with the atmosphere through the open pore spaces of the overlying soil or rock

Water Table: Unconfined aquifer

Piezometric surface: Imaginary surface, **Confined aquifer**
Atmospheric Pressure=hydrostatic pressure


Mm
SS w
Sr
Sla
St
Mm
Mm
SS r
Mm
Mm
St
Mm
Sr
Legend
Channel fill
Proximal overbank
Distal overbank
River flood lake

Hydraulic head

$$h = z + \frac{P}{\rho_w g} + \frac{v^2}{2g}$$

Hydraulic Head : is a concept that relates the energy in an incompressible fluid

The total energy:

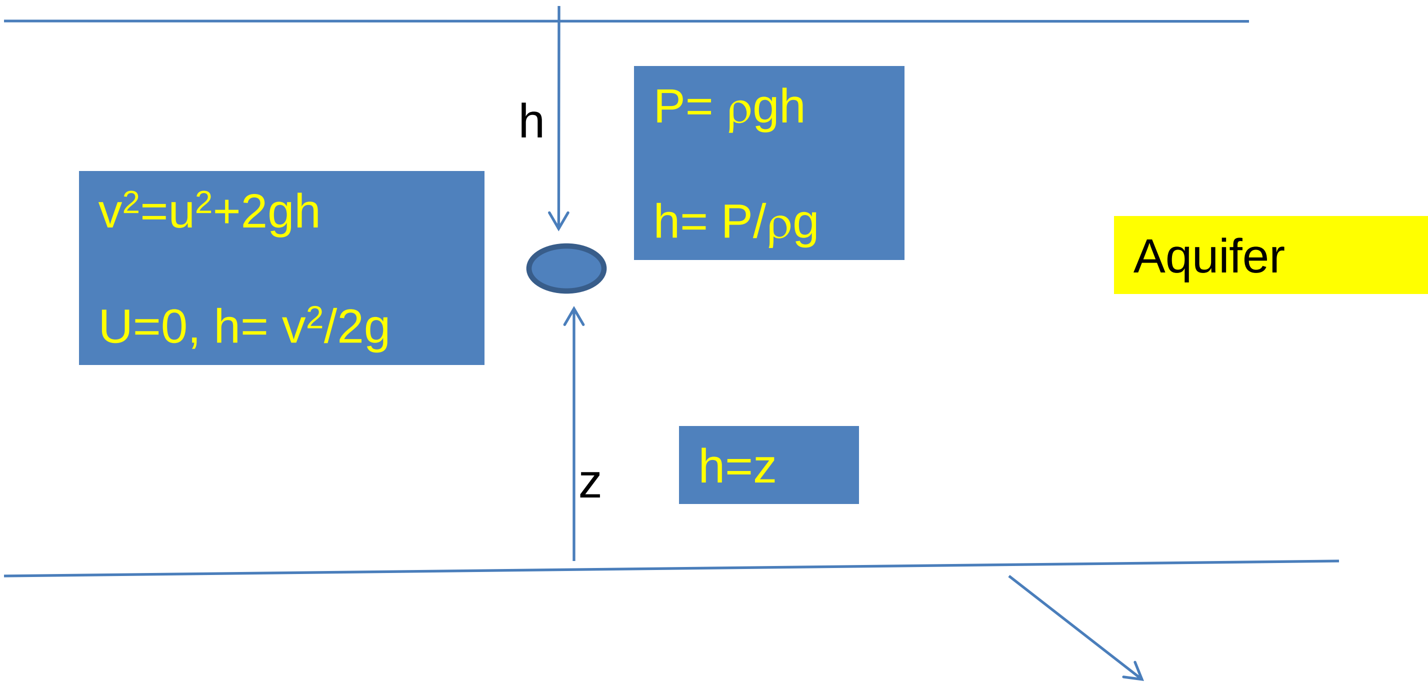
1) energy associated with the movement of the fluid,
2) energy from static pressure in the fluid,
3) energy from the height of the fluid relative to an arbitrary datum.

Head is expressed in units of height such as meters or feet.

Velocity head: is due to the bulk motion of a fluid (kinetic energy).
Pressure head: Its pressure head correspondent is the dynamic pressure.
Elevation head: It is due to the fluid's weight, the gravitational force acting on a column of fluid.

Hydraulic head

$$h = z + \frac{P}{\rho_w g} + \frac{v^2}{2g}$$



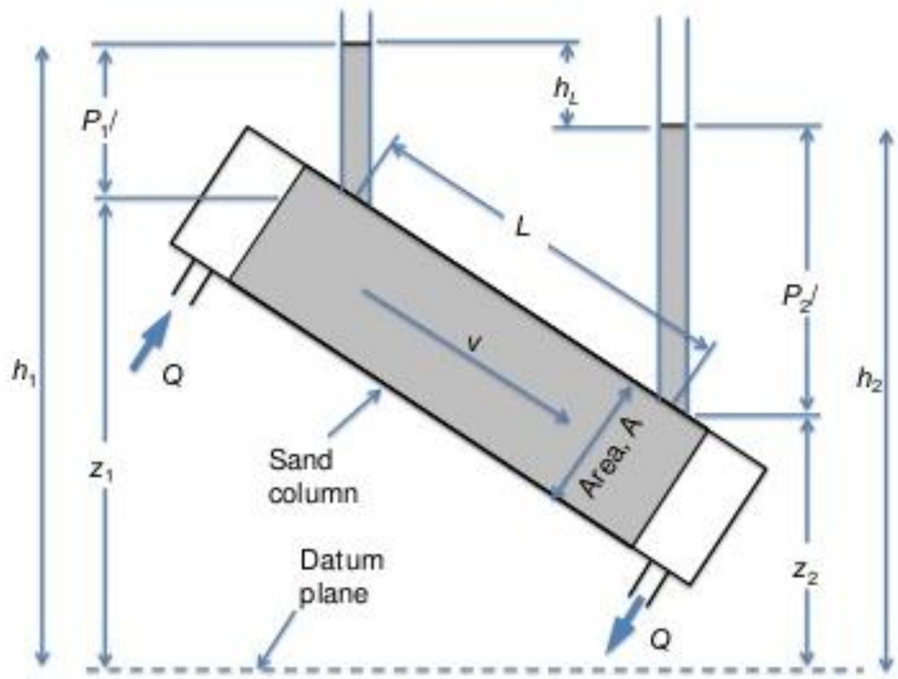
Datum plane or Reference plane (Can be impervious layer)

At water table, Hydraulic head= Elevation head

Darcy's Law

- Discharge is
  Proportional to
  - Area
  - Head difference

  Inversely proportional to
  - Length
- Coefficient of proportionality is
  K = hydraulic conductivity



$$Q \propto A\frac{h_1 - h_2}{L} \qquad Q = KA\frac{h_2 \quad h_1}{L} \qquad Q = KA\frac{h}{L}$$

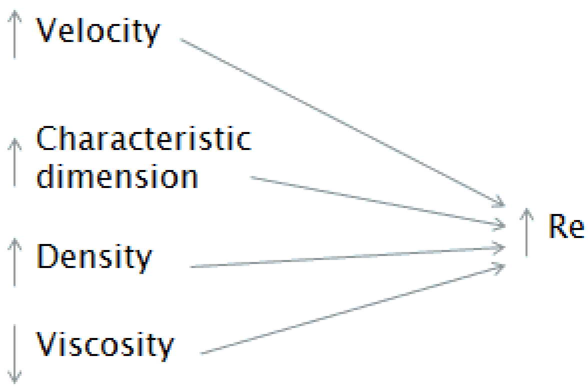
Q= v/t

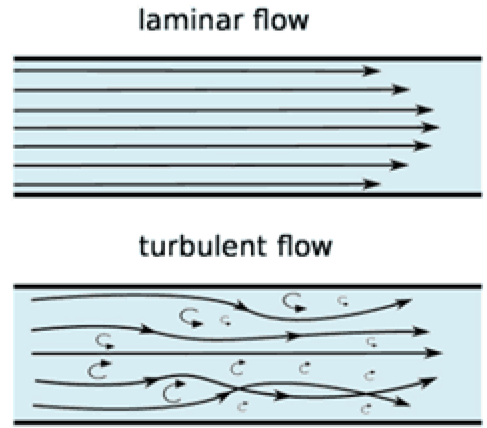
Darcy Velocity: Flow per unit area= q= K.h/L

Laminar vs Turbulent flow

$$q = \frac{Q}{A} = -K\frac{dh}{dl},$$

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho \,.\, V \,.\, D}{\mu}$$





$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

Re > 1; Turbulent flow

Re < 1; Laminar flow

https://www.youtube.com/watch?v=rfRAI49iAJ8&t=38s

Intrinsic Permeability (k)= $K\mu/\rho g$

$$[k] = \frac{(\mathrm{m/sec})(\mathrm{kg/(m.sec)})}{(\mathrm{kg/m^3})(\mathrm{m/sec^2})(\mathrm{m/m})} = m^2$$

Transmissivity (T)= K.b Unit= l^2/t

It is another very important transmission property of an aquifer, which is different from hydraulic conductivity in that it includes the whole saturation thickness, b, of the aquifer while K is defined for unit saturation thickness only.

Storativity or Coefficient of storage (S)

$$S = \frac{\text{volume of water}}{(\text{unit area})(\text{unit head change})}$$

Specific Storage (Ss) =

$$\frac{\text{volume for water}}{(\text{unit area})(\text{unit aquifer thickness})(\text{unit head change})}$$

Specific Yield (Sy)=

$$S_y = \frac{V_d}{V_T}$$

where S_y is the specific yield and V_d is the volume of water that drains from a total volume of V_T.

Specific Retention (Sr)=

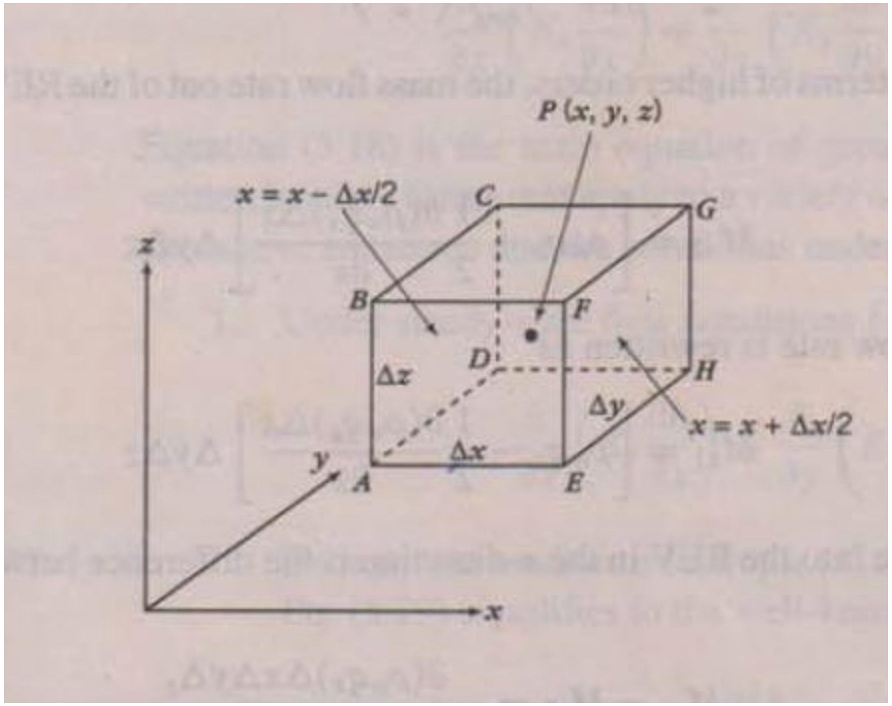
$$S_r = \frac{V_r}{V_T}$$

where S_r is the specific retention and V_r is the volume water retained against gravity. The defined in Section 3.1 is related to specific yield and specific retention by

$$n = S_y + S_r$$

Ground water flow equation

Representative Elementary Volume (REV) with dimension Δx, Δy and ΔZ



Darcy velocity is known at the centre of REV

Mass flow: It is the movement of mass per unit time

Principle of Mass conservation

mass inflow rate − mass outflow rate = change of mass storage with time

$$M_{x1} - M_{x2} + M_{y1} - M_{y_2} + M_{z1} - M_{z2} = \frac{\partial}{\partial t}(n\rho_w \Delta x \Delta y \Delta z)$$

Mass inflow rate at direction *i*

$$M_i = \rho_w q_i \Delta S_i$$

Mass Flow Rate = (density)*(velocity)*(area of the cross section)
m = ρ v A   (Unit: mass/time)

Where we have:
ρ: Density of the fluid

v: Velocity of the fluid= Darcy Velocity $(\frac{Q}{A} = q(DarcyVelocity) = K\frac{dh}{dl})$

A: Area or cross section   (Δx*Δy; Δz*Δy; Δx*Δz)

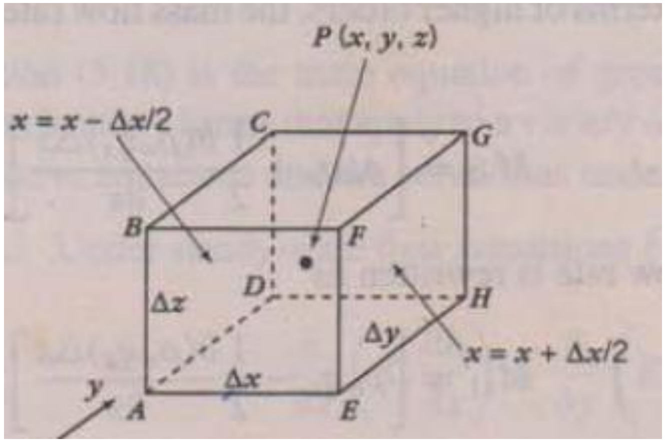
Taylor Series for $f(x)$ about $x = a$ is,

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$q_{x1} = q_x - \frac{\partial q_x}{\partial x}\left(\Delta x - \frac{\Delta x}{2}\right)$$

$$q_{x2} = q_x + \frac{\partial q_x}{\partial x}\left(\Delta x - \frac{\Delta x}{2}\right)$$



Mass inflow at face x_2

$$M_{x2} = \left[\rho_w q_x + \frac{1}{2}\frac{\partial(\rho_w q_x)\Delta x}{\partial x}\right]\Delta y \Delta z$$

$$M_i = \rho_w q_i \Delta S_i$$

Mass inflow at face x_1

$$M_{x1} = \left[\rho_w q_x - \frac{1}{2}\frac{\partial(\rho_w q_x)\Delta x}{\partial x}\right]\Delta y \Delta z$$

Net inflow rate in the REV at x-direction

$$M_{x1} - M_{x2} = -\frac{\partial(\rho_w q_x)\Delta x \Delta y \Delta_z}{\partial x}$$

Net inflow rate in the REV at y and z-direction

$$M_{y1} - M_{y2} = -\frac{\partial(\rho_w q_y)\Delta x \Delta y \Delta_z}{\partial y}$$

$$M_{z1} - M_{z2} = -\frac{\partial(\rho_w q_z)\Delta x \Delta y \Delta_z}{\partial z}$$

The sum of water inflow rate minus the sum of water outflow rate for the REV is

$$M_{x1} - M_{x2} + M_{y1} - M_{y2} + M_{z1} - M_{z2} = -\left[\frac{\partial(\rho_w q_x)}{\partial x} + \frac{\partial(\rho_w q_y)}{\partial y} + \frac{\partial(\rho_w q_z)}{\partial z}\right]\Delta x \Delta y \Delta$$

The change in ground-water storage within the REV is

$$\text{change of water storage per unit time} = \frac{\partial(\rho_w n)}{\partial t}\Delta x \Delta y \Delta z$$

$$-\left[\frac{\partial(\rho_w q_x)}{\partial x} + \frac{\partial(\rho_w q_y)}{\partial y} + \frac{\partial(\rho_w q_z)}{\partial z}\right] = \frac{\partial(\rho_w n)}{\partial t}$$

As Δx.Δy.ΔZ≠0

If density does not vary spatially:

$$-\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] = \frac{1}{\rho_w}\frac{\partial(\rho_w n)}{\partial t}$$

$$\frac{1}{\rho_w}\frac{\partial(\rho_w n)}{\partial t} = S_s\frac{\partial h}{\partial t}$$

$$-\left[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}\right]=\frac{1}{\rho_w}\frac{\partial(\rho_w n)}{\partial t}=S_s\frac{\partial h}{\partial t}$$

$$q=-K.\frac{dh}{dx}$$

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)=S_s\frac{\partial h}{\partial t}$$

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)=S_s\frac{\partial h}{\partial t}$$

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S_s}{K}\frac{\partial h}{dt}$$

Isotropic and Homogeneous: $K_x=K_y=K_z=K$

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S_s\times b}{K\times b}\frac{\partial h}{dt}$$

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S}{T}\frac{\partial h}{dt}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{dt}$$

Non steady state, Confined aquifer, Homogeneous , Isotropic

Under steady state condition:

$$\frac{dh}{dt} = 0$$

If the porous medium is isotropic (K_x, K_y, K_z) and homogeneous ($K_{x,y,z}$ = constant), Eq. (5.19) simplifies to the well-known Laplace equation

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (5.20)$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

$$\frac{\partial^2 h}{\partial x^2} = 0$$

Confined Aquifer: Steady state, Unidirectional flow

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) = 0$$

$$\partial\left(\frac{\partial h}{\partial x}\right) = 0.\partial x$$

$$\int \partial\left(\frac{\partial h}{\partial x}\right) = \int 0.\partial x$$

$$\frac{\partial h}{\partial x} = C_1$$

$$\partial h = C_1.\partial x$$

$$h = C_1 x + C_2$$

At, x=0, h=h_0 and x=L, h=h_L

$$h = h_0 + (h_L - h_0)\frac{x}{L}$$

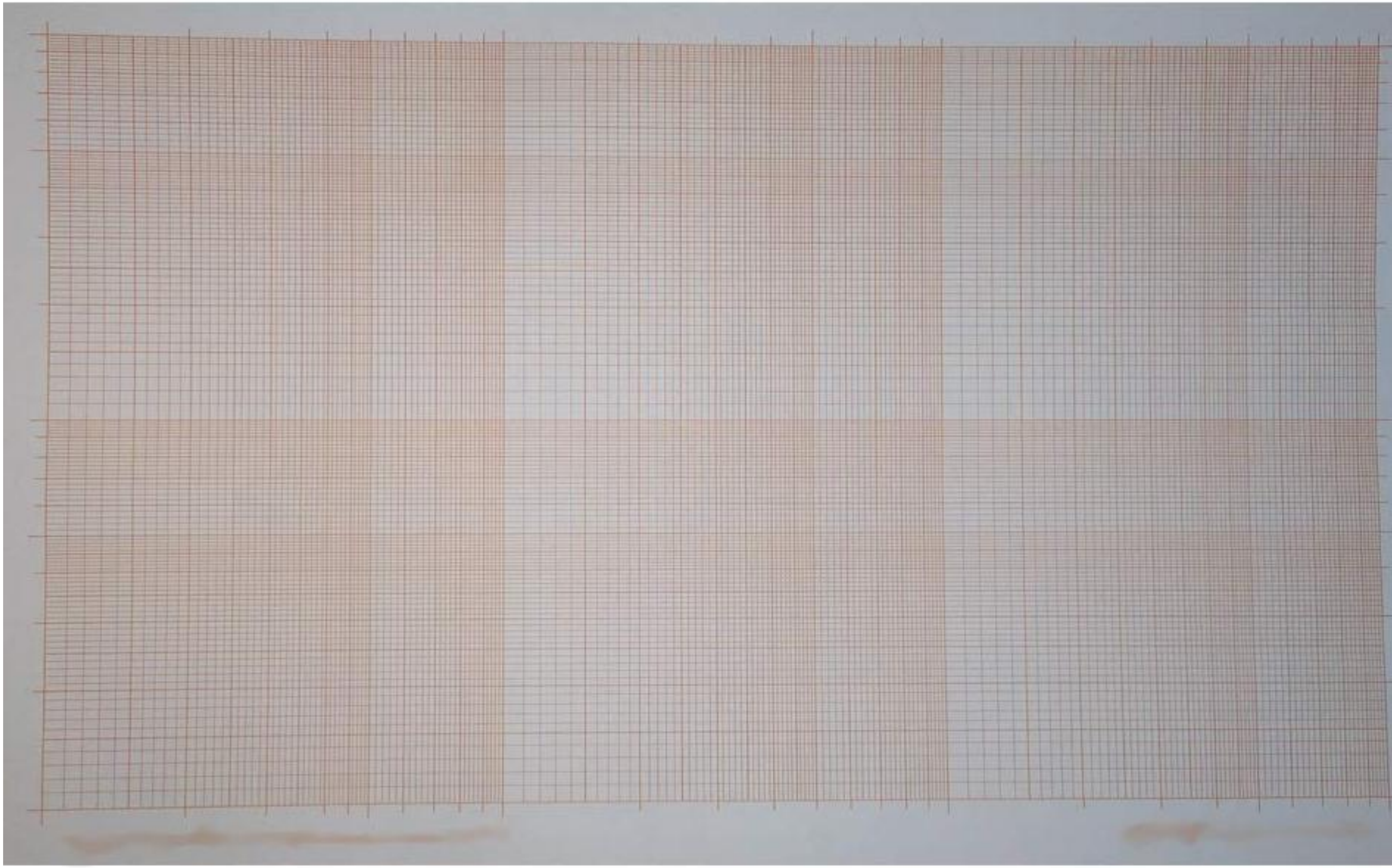
Darcy Velocity= $q_x = K\frac{h_0 - h_L}{L}$

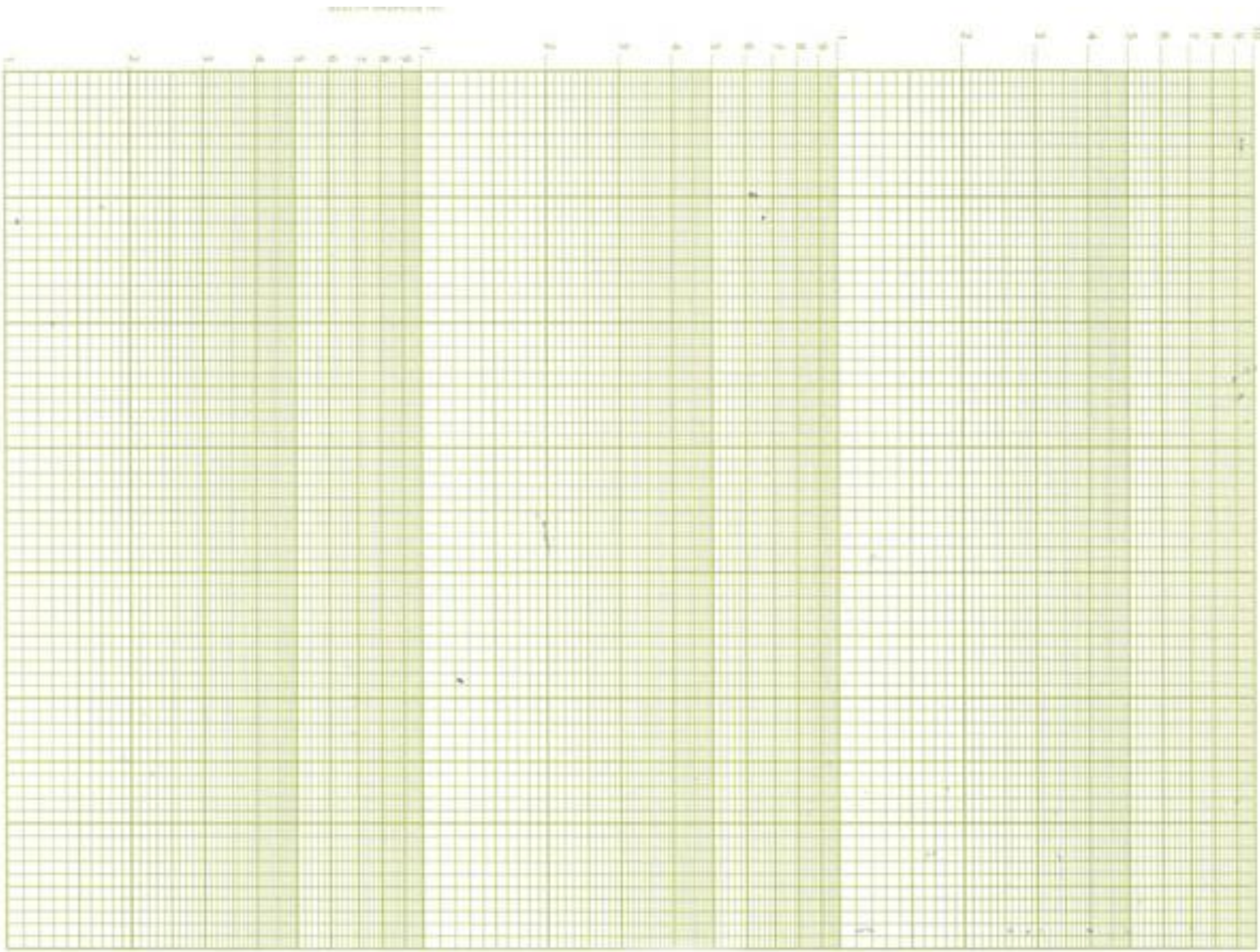
Unidirectional flow in Unconfined Aquifer:

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = 0$$

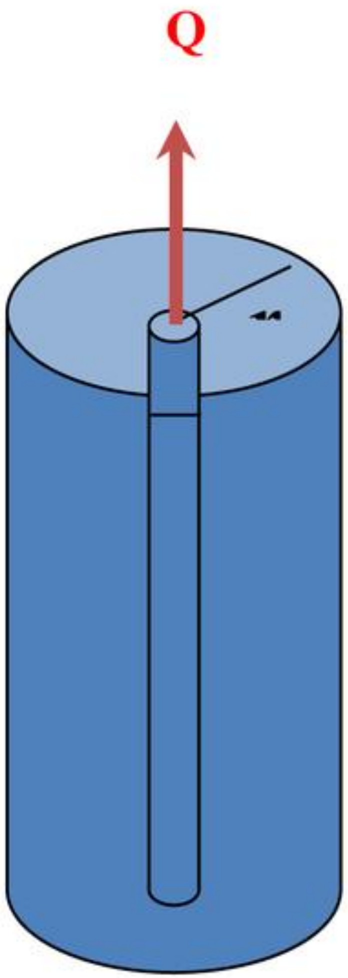
$$h = \sqrt{h_0^2 + (h_L^2 - h_0^2)\frac{x}{L}}$$

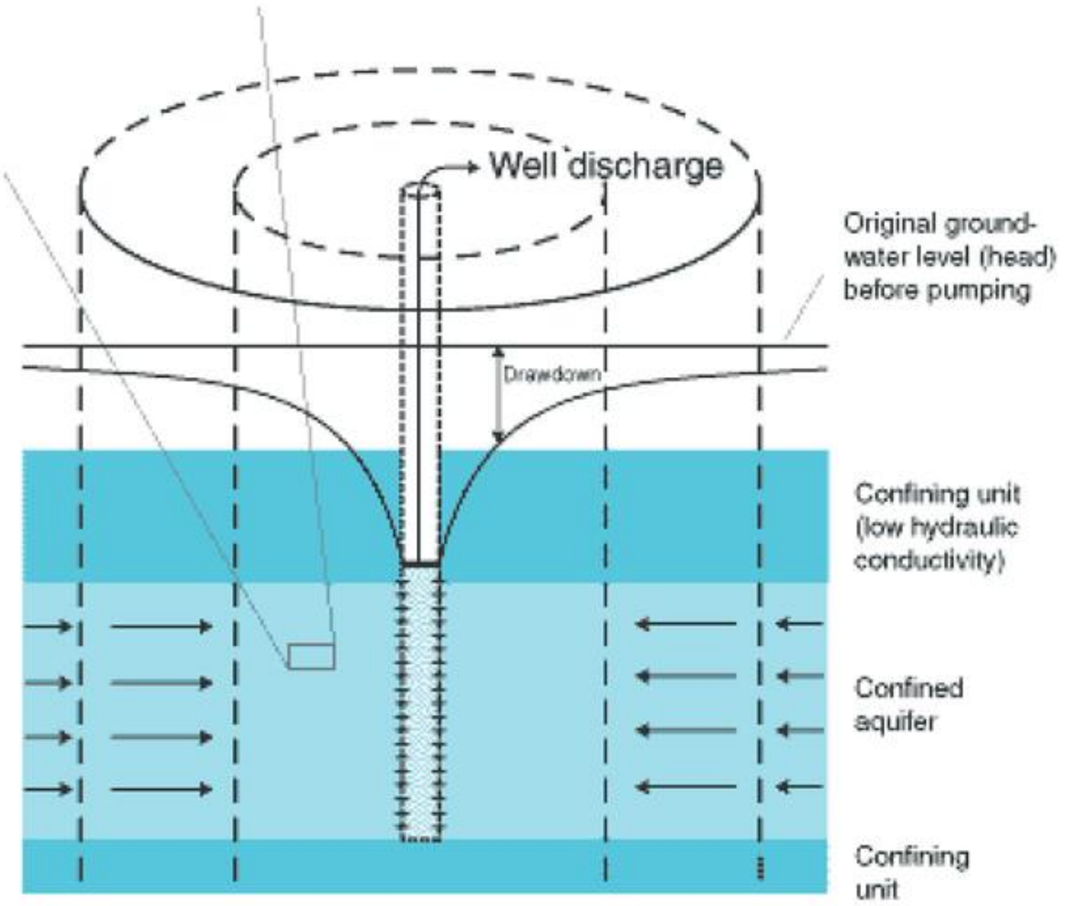
$$q = K\frac{h_0^2 - h_L^2}{2Lh}$$





Radial Flow


Q

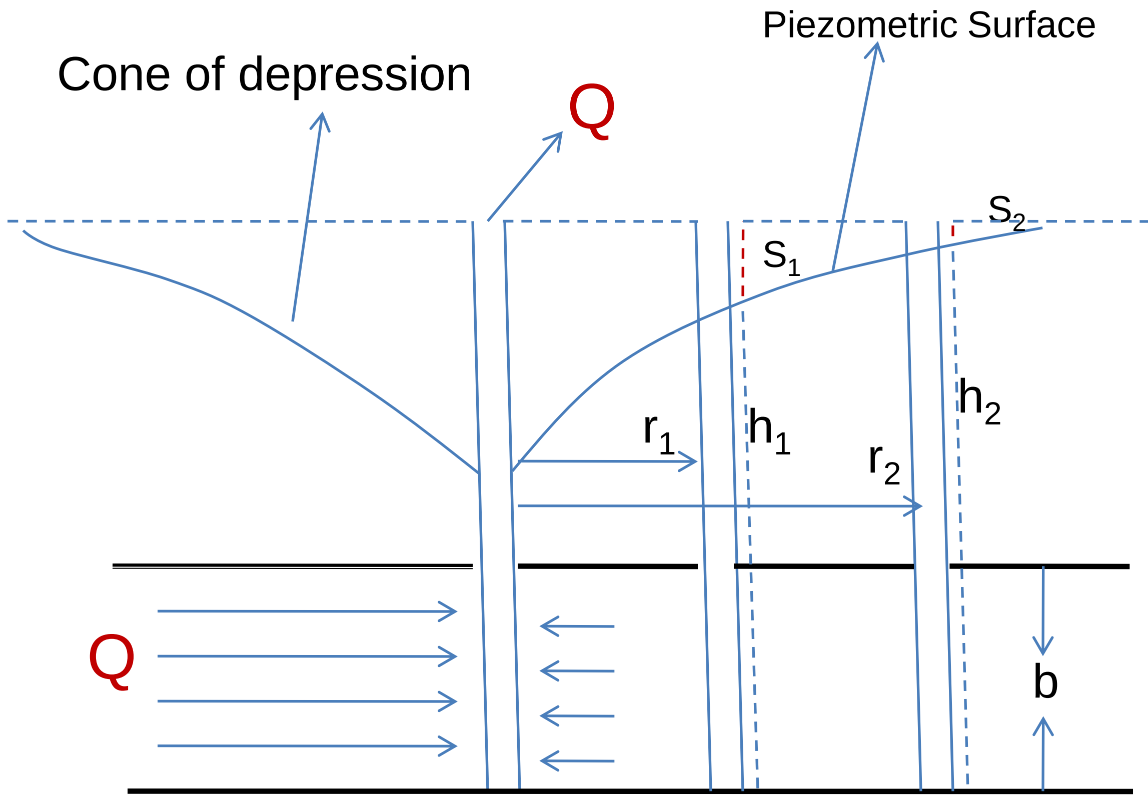

Well discharge
Original ground-water level (head) before pumping
Drawdown
Confining unit (low hydraulic conductivity)
Confined aquifer
Confining unit

Steady State Flow to Well in Confined Aquifer

$$Q = K.A.\frac{\partial h}{\partial r}$$

$$A = 2\pi rb$$

$$Q = K.2\pi rb.\frac{\partial h}{\partial r}$$



$+s_1$

$+s_2$

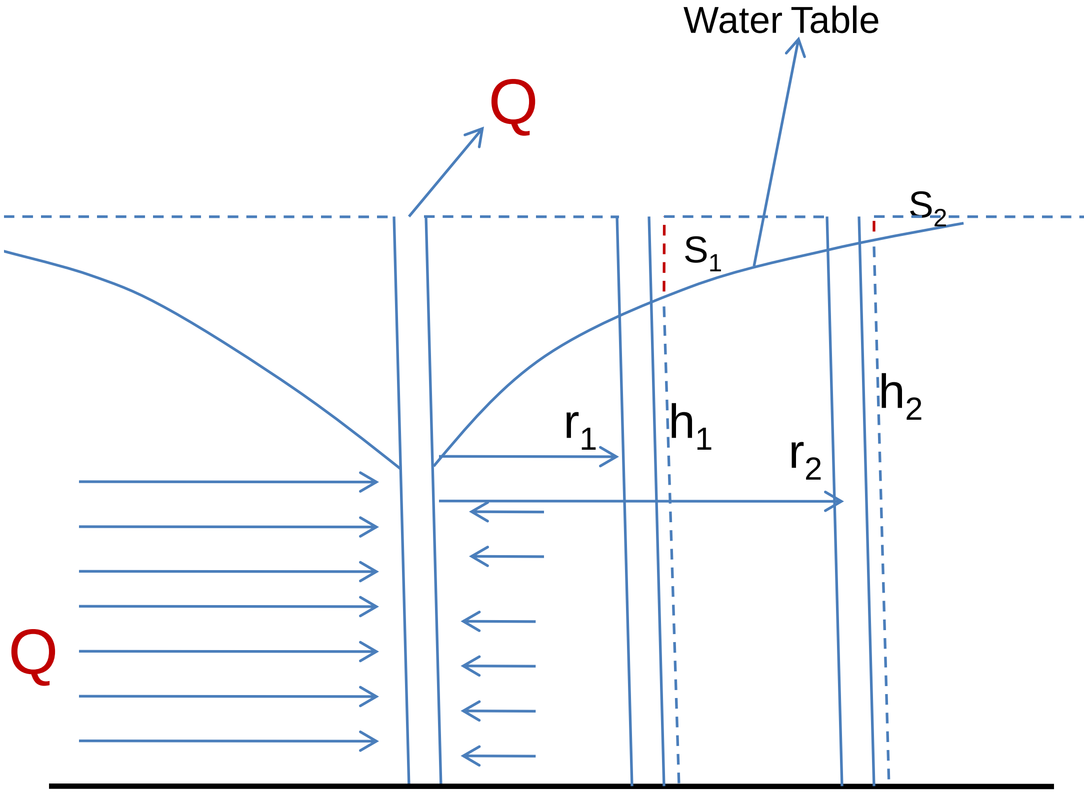
$$K = \frac{Q.\ln\dfrac{r_2}{r_1}}{2\pi b.[s_1 - s_2]}$$

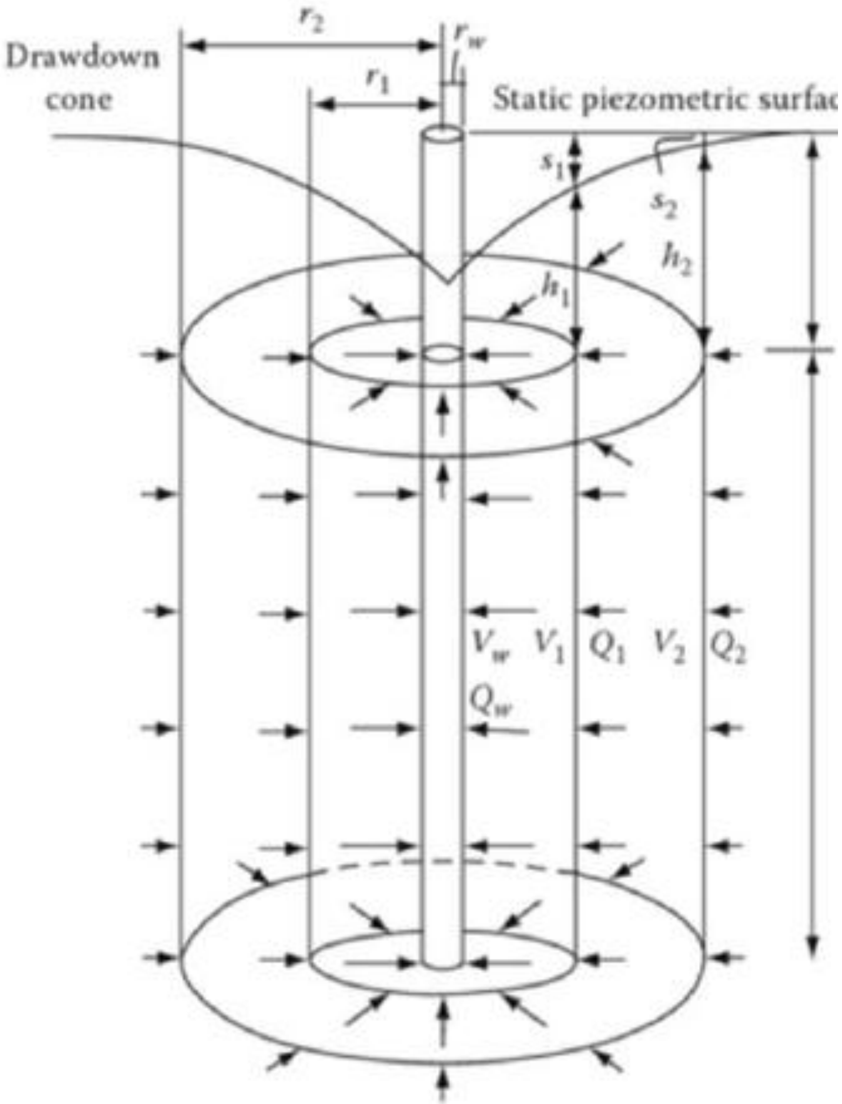
Steady State Flow to Well in Un-confined Aquifer

$$Q = K.A.\frac{\partial h}{\partial r}$$

$$A = 2.\pi rh$$

$$Q = K.2.\pi rh.\frac{\partial h}{\partial r}$$





r_2
r_w
Drawdown cone
r_1
Static piezometric surfac
s_1
s_2
h_2
h_1
V_w V_1 Q_1 V_2 Q_2
Q_w

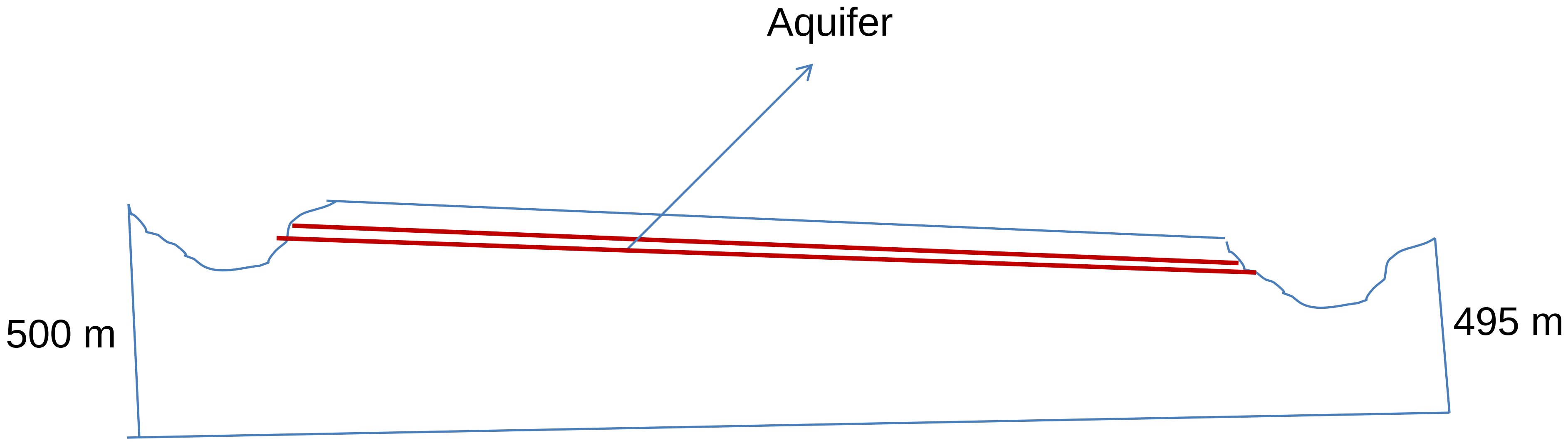
1. A water saturated medium has a hydraulic conductivity of 20 m/day, what will be the intrinsic permeability of that medium? The density and dynamic viscosity of water are 998.2 kg/m^3 and $1.002*10^{-3}$ kg/m.sec.

2. After a soil sample is drained by gravity, the weight of the soil sample is 85 g. After the sample is oven-dried, the sample weighs 80 g. The bulk density of the wet soil is 1.65 g/cm3, and the density of water is 1 g/cm3. Calculate the specific yield, specific retention and porosity of the sample. Assume water that was drained by gravity is 20g.

3. Two rivers 1000 m apart penetrate a confined aquifer 20 m thick. The hydraulic conductivity is 20 m/day. The stages of the two rivers are 500 m and 495 m above sea level, respectively. What is the Darcy flow velocity for groundwater in the aquifer (0.1 m/day). Assume that the aquifer is unconfined, calculate the hydraulic head in the aquifer at the midpoint, Darcy flow velocity at x=500 m

4. A fully penetrating well of 30 cm diameter is installed in confined aquifer of 15 m thickness, is discharged at the rate of 56 m^3/hour. Calculate the hydraulic conductivity of aquifer, if draw-down in the piezometers located at the radial distance of 10 and 20 m from the well is 2.4 and 0.75 m, respectively under steady-state condition.

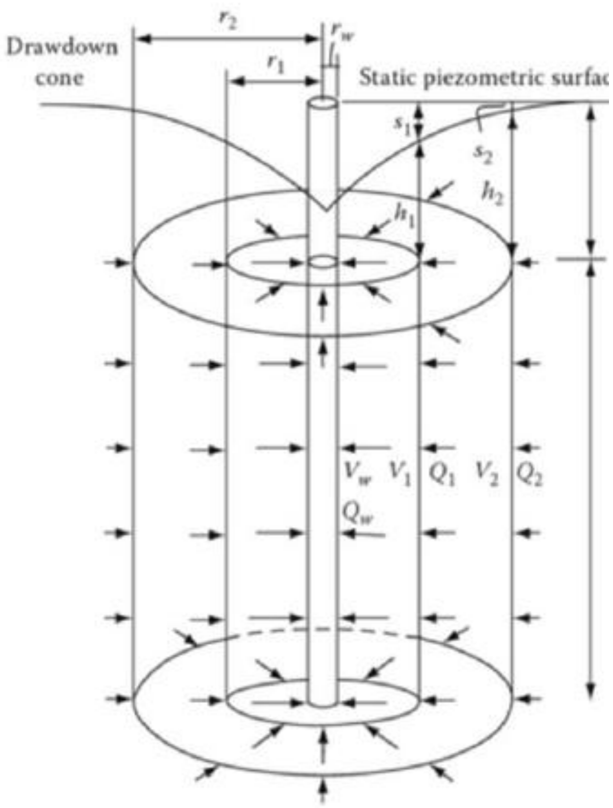
Two rivers 1000 m apart penetrate a confined aquifer 20 m thick. The hydraulic conductivity is 20 m/day. The stages of the two rivers are 500 m and 495 m above sea level, respectively. What is the Darcy flow velocity for groundwater in the aquifer (0.1 m/day). Assume that the aquifer is unconfined, calculate the hydraulic head in the aquifer at the midpoint, Darcy flow velocity at x=500 m



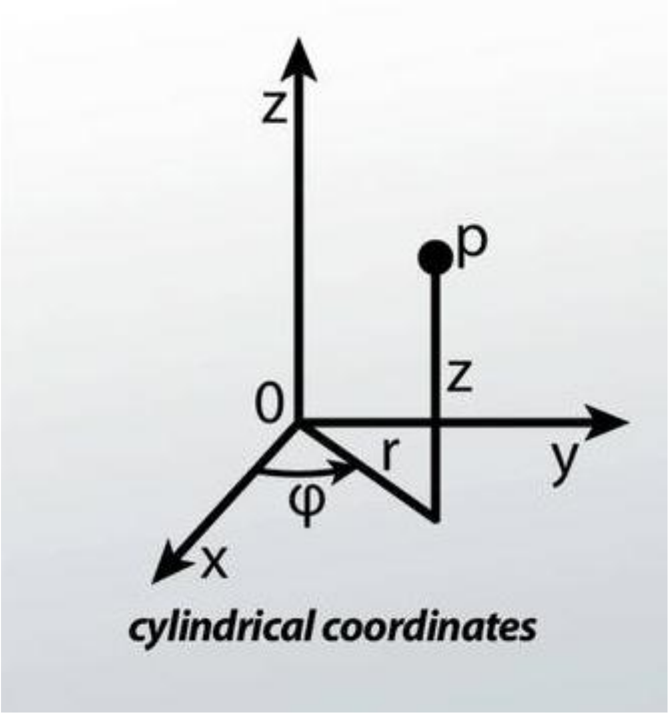
Non steady state flow/ Transient Flow

Theis Method

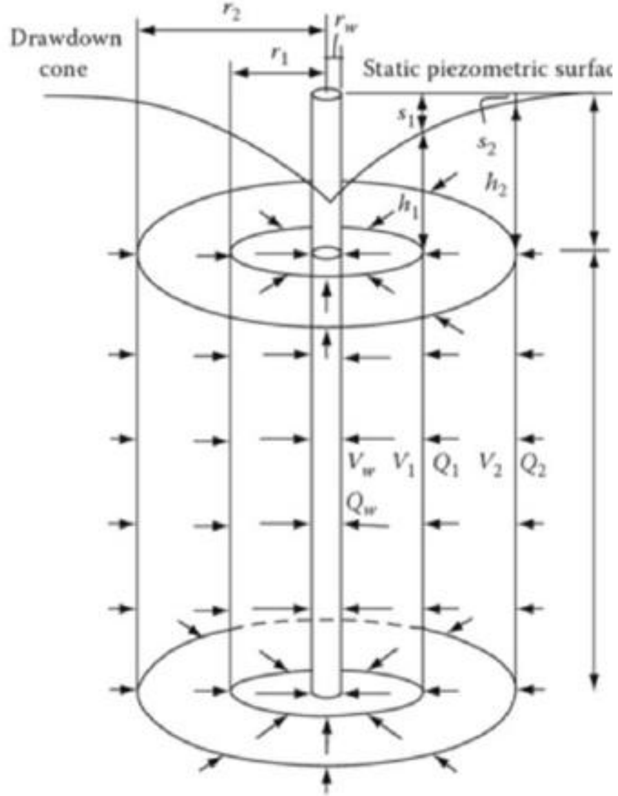
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{dt}$$

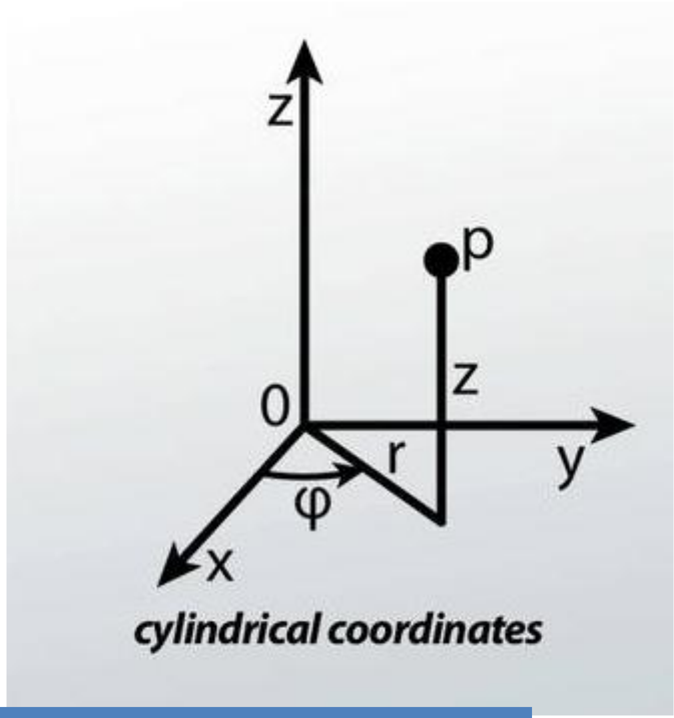


Cylindrical Polar Coordinate: r,φ,z



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r.\frac{\partial h}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 h}{\partial \varphi}+\frac{\partial^2 h}{\partial z^2}=\frac{S}{T}\frac{\partial h}{\partial t}$$





φ and Z not required

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r.\frac{\partial h}{\partial r}\right)=\frac{S}{T}\frac{\partial h}{\partial t}$$

Transient Flow in confined Aquifer

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r.\frac{\partial h}{\partial r}\right)=\frac{S}{T}\frac{\partial h}{\partial t}$$

$$\frac{1}{r}\times r\times\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\times\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}$$

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}$$

r = radial distance of observational well from a pumped well

t= time since pumping started

h= hydraulic head T= Transmissivity S=Storativity

Boundary Condition

$$h(r,0) = h_0$$

$$h(\infty,t) = h_0$$

$$\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right) = \frac{Q}{2\pi T}$$

$$h_0 - h = s = \frac{Q}{4\pi T}W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y}dy = -0.577216 - \ln(u) + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \cdots$$

$$u = \frac{r^2 S}{4Tt}$$

Well Function

$u = 4.6 x 10^{-4}$ $W(u) = 7.12$

Table 4.4.1 Values of $W(u)$ for Values of u

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

Type Curve

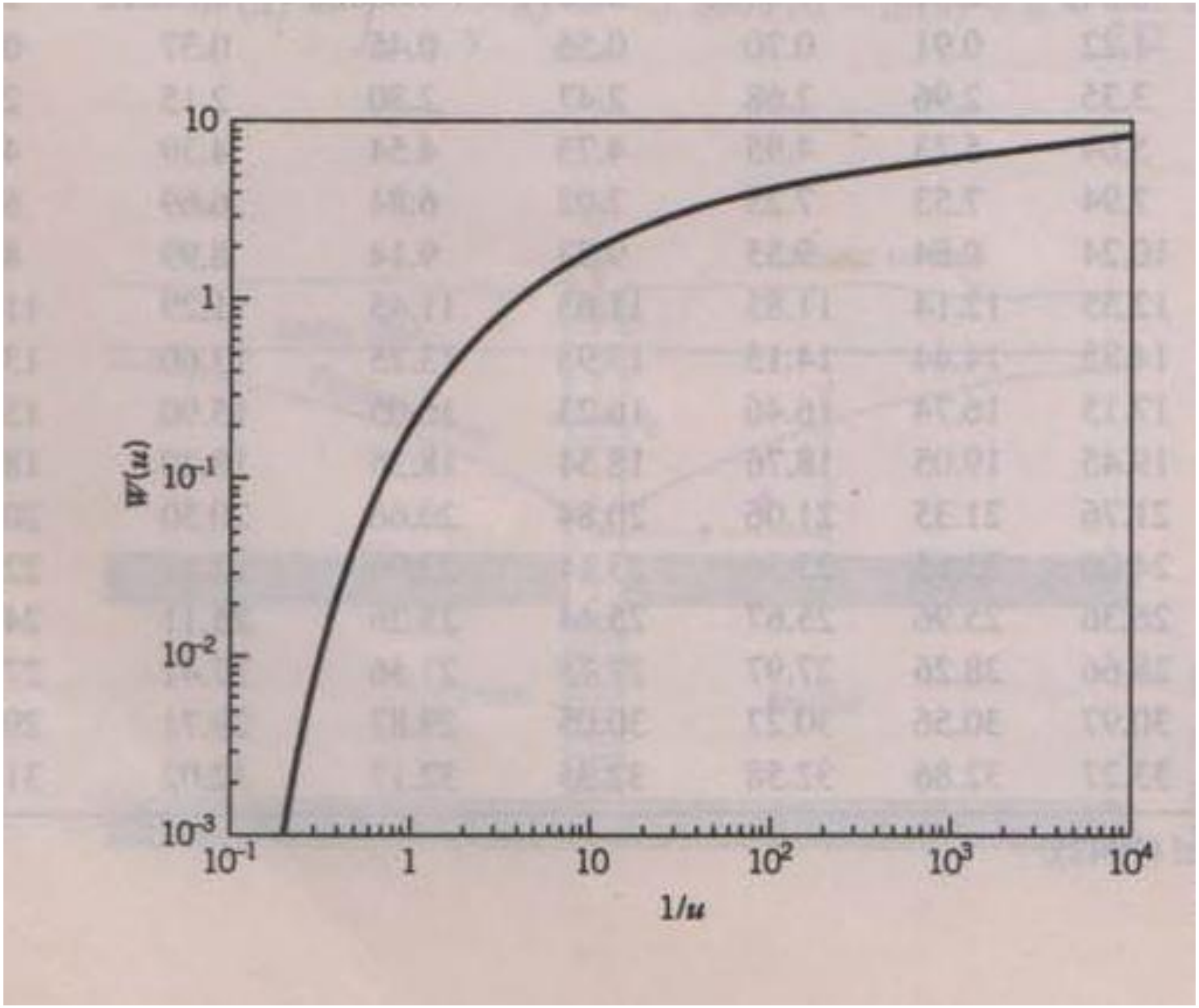
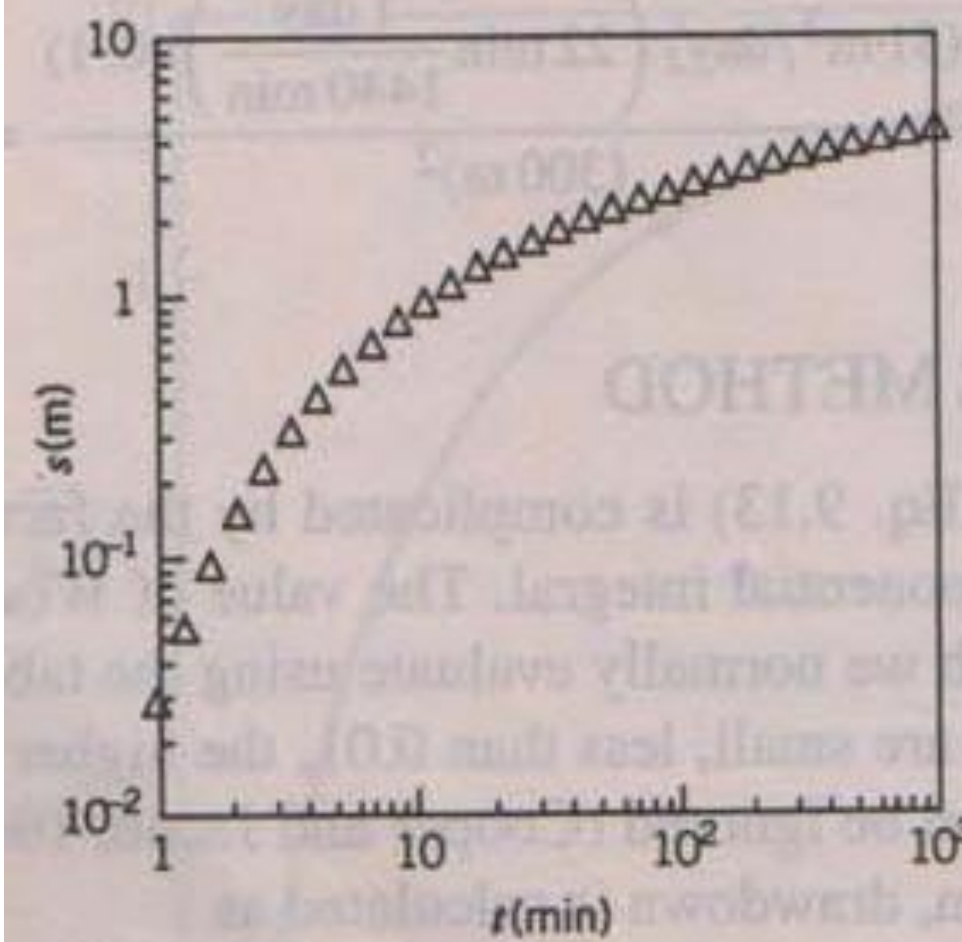
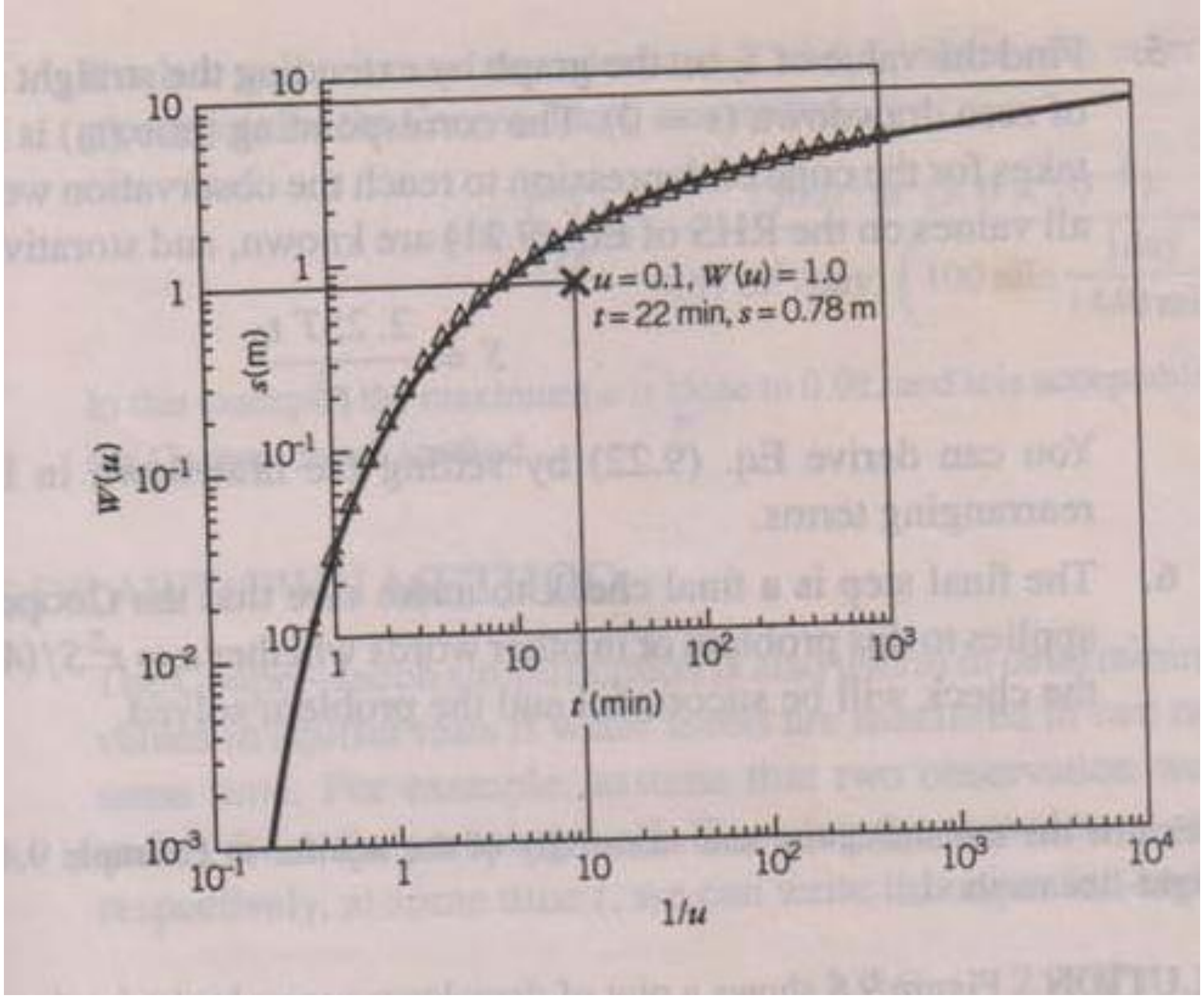


TABLE 9.3 Drawdowns measured at an observation well 300 m away

Time (min)	S(m)
1.00	0.03
1.27	0.05
1.61	0.09
2.04	0.15
2.59	0.22
3.29	0.31
4.18	0.41
5.30	0.53
6.72	0.66
8.53	0.80
10.83	0.95
13.74	1.11
17.43	1.27
22.12	1.44
28.07	1.61
35.62	1.79
45.20	1.97
57.36	2.15
72.79	2.33
92.37	2.52
117.21	2.70
148.74	2.89
188.74	3.07
239.50	3.26
303.92	3.45
385.66	3.64
489.39	3.83
621.02	4.02
788.05	4.21
1000.0	4.39





W(u), 1/u, s, t

$$s = \frac{Q}{4\pi T} W(u)$$

Q is known

$$u = \frac{r^2 S}{4 T t}$$

T and S

$$s = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

$$\log W(u) = \log(s. \frac{4\pi T}{Q})$$

$$\log W(u) = \log\ s + \log\ \frac{4\pi T}{Q}$$

$$\log \frac{1}{u} = \log\left(\frac{4Tt}{r^2 S}\right)$$

$$\log \frac{1}{u} = \log(t) + \log\left(\frac{4T}{r^2 S}\right)$$

Copper -Jacob Method of straight line

$$h_0 - h = s = \frac{Q}{4\pi T} W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy = -0.577216 - \ln(u) + u - \frac{u^2}{2!2} + \frac{u^3}{3!3} - \frac{u^4}{4!4} + \cdots$$

$$u = \frac{r^2 S}{4Tt}$$

$$W(u) = \int_u^\infty \frac{e^{-y}}{y} dy = -0.577216 - \ln(u)$$

$$s = \frac{Q}{4\pi T}\left(-0.577216 - \ln(u)\right)$$

$$s = \frac{Q}{4\pi T}\left(\ln 0.56 - \ln(u)\right)$$

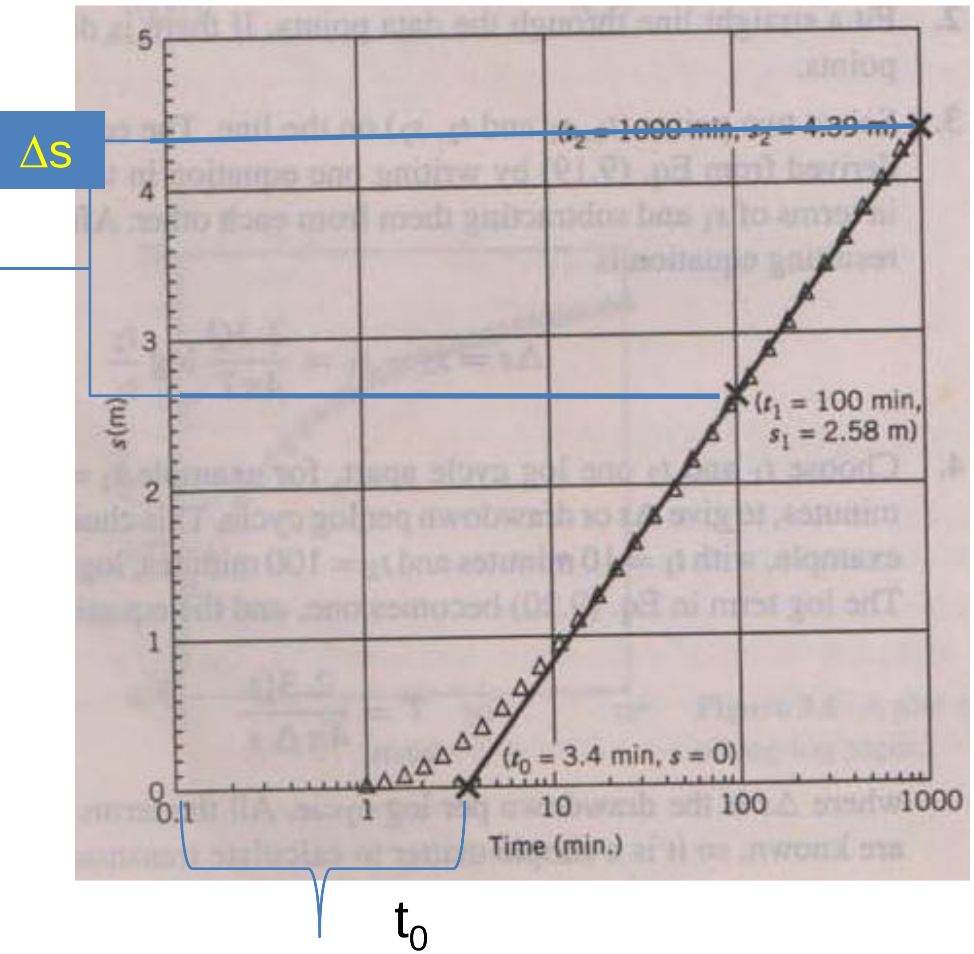
$$s = \frac{Q}{4\pi T}\left(\ln \frac{0.56}{u}\right)$$

$$s = \frac{Q}{4\pi T}\left(\ln \frac{0.56 \times 4Tt}{r^2 S}\right)$$

$$s = \frac{Q}{4\pi T}\left(\ln \frac{2.25Tt}{r^2 S}\right)$$

$$s = \frac{Q}{4\pi T}\left(\ln \frac{2.25Tt}{r^2 S}\right)$$

$$s = \frac{Q}{4\pi T}\left(\ln(t) + \ln(\frac{2.25T}{r^2 S})\right)$$



$$s_1 = \frac{Q}{4\pi T}\left(\ln(t_1) + \ln(\frac{2.25T}{r^2 S})\right)$$

$$s_2 = \frac{Q}{4\pi T}\left(\ln(t_2) + \ln(\frac{2.25T}{r^2 S})\right)$$

$$\Delta s = s_2 - s_1 = \frac{2.3Q}{4\pi T} \log \frac{t_2}{t_1}$$

$$T = \frac{2.3Q}{4\pi \Delta s}$$

$$S = \frac{2.25T t_0}{r^2}$$

Distance-Drawdown method

$$s = \frac{Q}{4\pi T}\left(\ln\frac{2.25Tt}{r^2 S}\right)$$

$$s_1 = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_1^2 S}$$

$$s_2 = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_2^2 S}$$

$$s_1 - s_2 = \frac{2.3Q}{2\pi T}\log\frac{r_2}{r_1}$$

$$T = \frac{2.3Q}{2\pi(\Delta s)}$$

$$S = \frac{2.25Tt}{r_0^2}$$

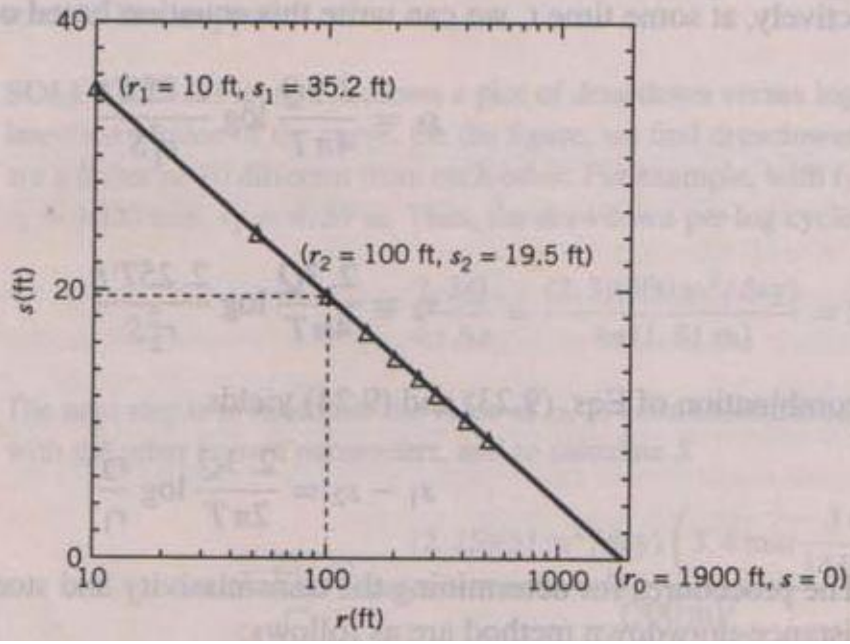
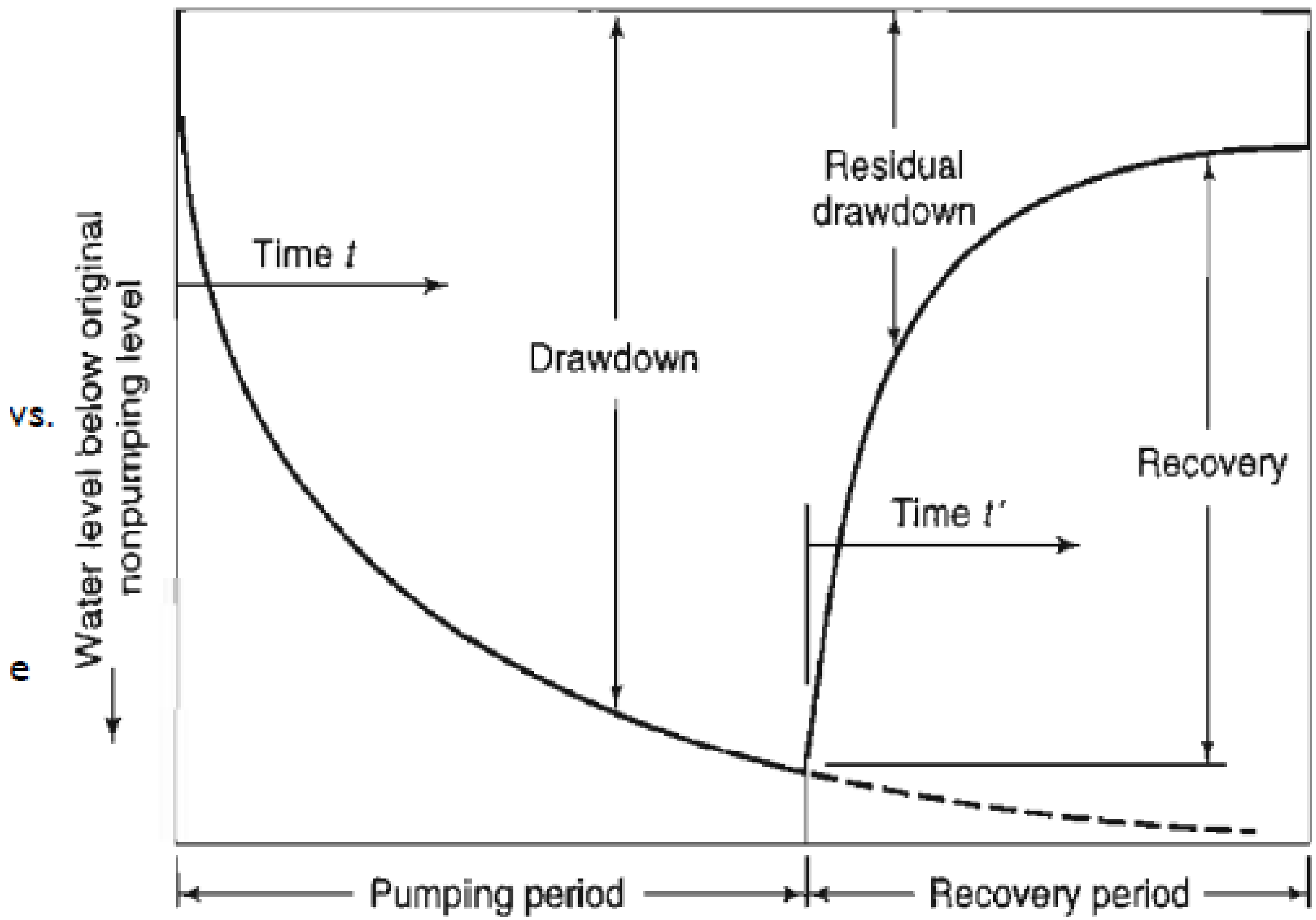


TABLE 9.4 Values of drawdown versus distance measured at 220 minutes

r(ft)	s(ft)
10	35.20
50	24.35
100	19.68
150	16.96
200	15.03
250	13.54
300	12.32
400	10.42
500	8.97

The residual drawdown measurement at any time during the recovery period is the difference between the observed water level and the prepumping static water level.



Why should we identify old groundwaters?

- To determine the time and place of recharge (recharge may already be stopped)
- Mean residence time
- Exploitation induced recharge
- To understand the geochemical and hydrological processes

Cosmic Rays

- Cosmic rays are high-energy (several GeV up to 10^{19} eV) atomic nuclei,
- *Spallation Reaction:* in which a nucleus struck by a high energy particle shatters into two or more pieces, including stable and unstable nuclei, as well as protons and neutrons. The interaction of a cosmic ray with a nucleus sets of a chain reaction of sorts as the secondary particles and nuclear fragments, which themselves have very high energies, then strike other nuclei producing additional reactions of lower energy.

ISOTOPES OF CARBON

$^{12}C = 98.89\%$ $^{13}C = 1.11\ \%$ $^{14}C = 10^{-12}$

$$^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}p$$

- $A_t = A_0 e^{-\lambda t}$

Decay of Carbon ^{14}C

Radioactive carbon (^{14}C) decays back to nitrogen (^{14}N) emitting an electron (e^{-}) and an antineutrino ($\bar{\nu}$) with no mass or charge.

$$^{14}_{6}C \Rightarrow {}^{14}_{7}N + {}^{0}_{-1}e^{-} + {}^{0}_{0}\bar{\nu}$$

Basis of ^{14}C age determination

- Radioactive decay (discovered by Libby in 1946, Nobel Prize).
- Half-life of ^{14}C is 5730 ±40 (years).
- Decay equation:

$$A_t = A_0 \times e^{-\lambda t}$$

- A_0 and A_t are ^{14}C initial activity, and activity after time ‘t’, λ is decay constant.

Factors affecting the production of ^{14}C

Solar Cycle: 11 years

The highest rate of carbon-14 production takes place at altitudes of 9 to 15 km (30,000 to 49,000 ft) and at high geomagnetic latitudes.

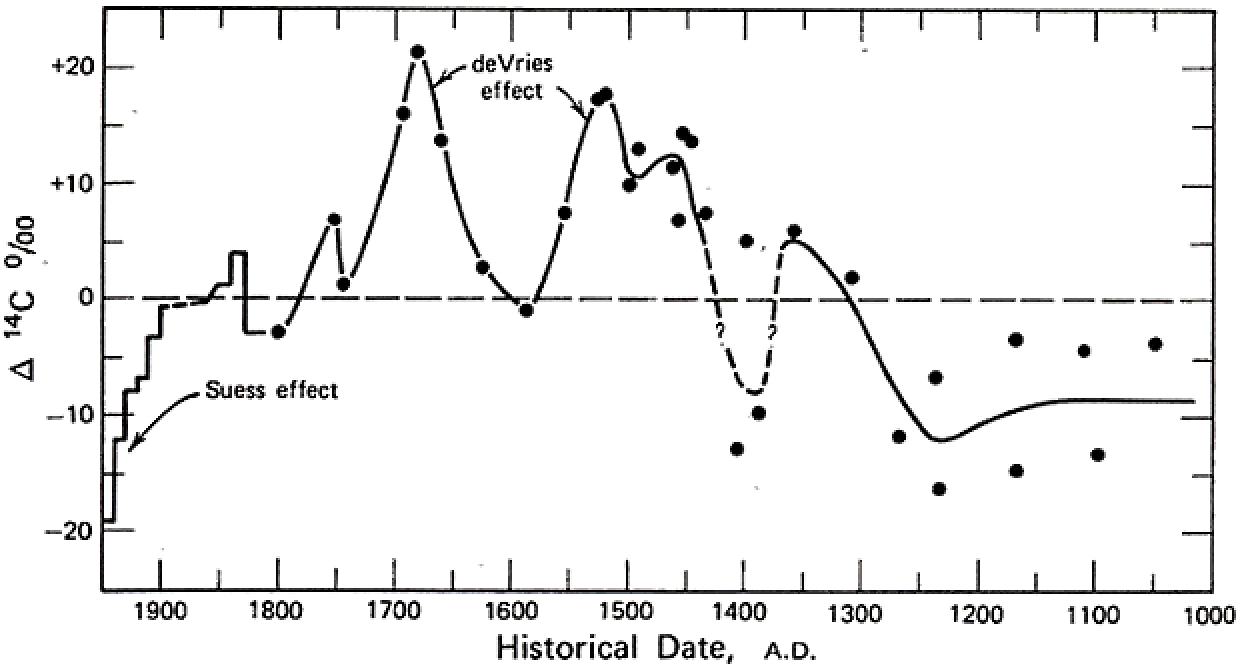
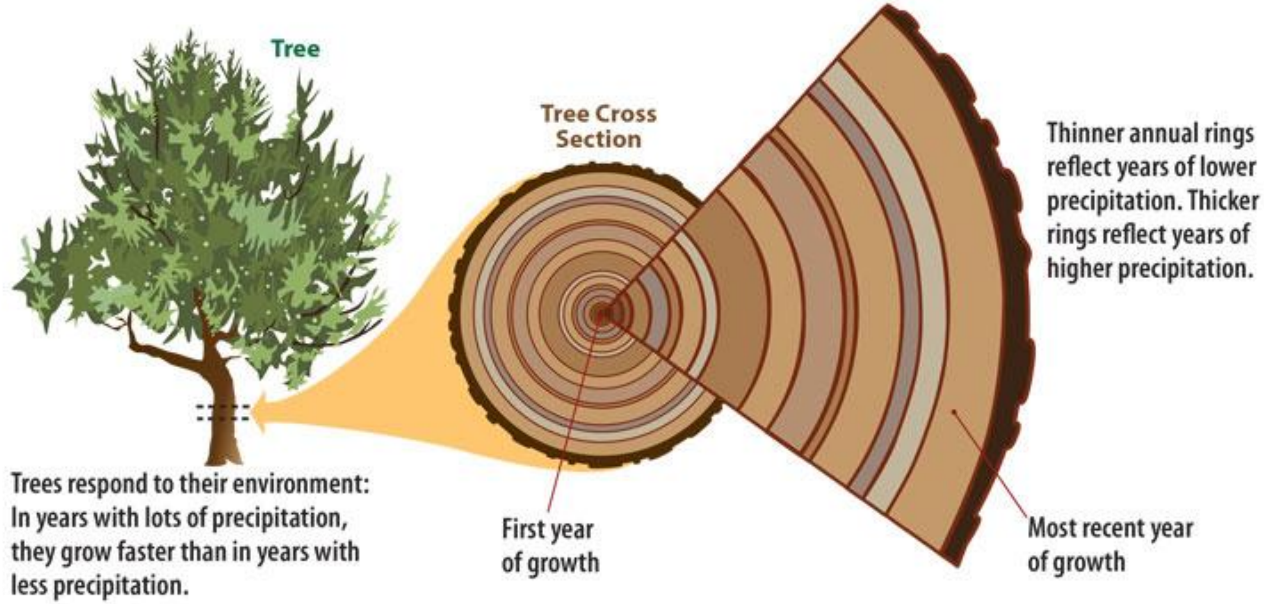
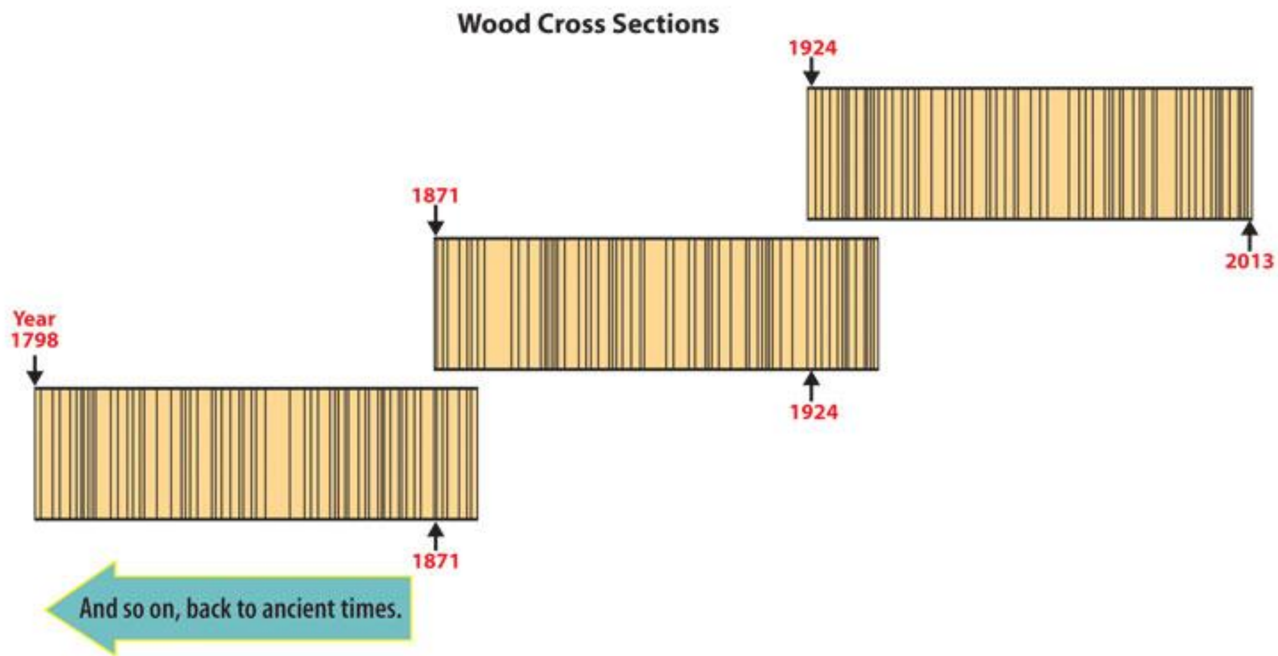
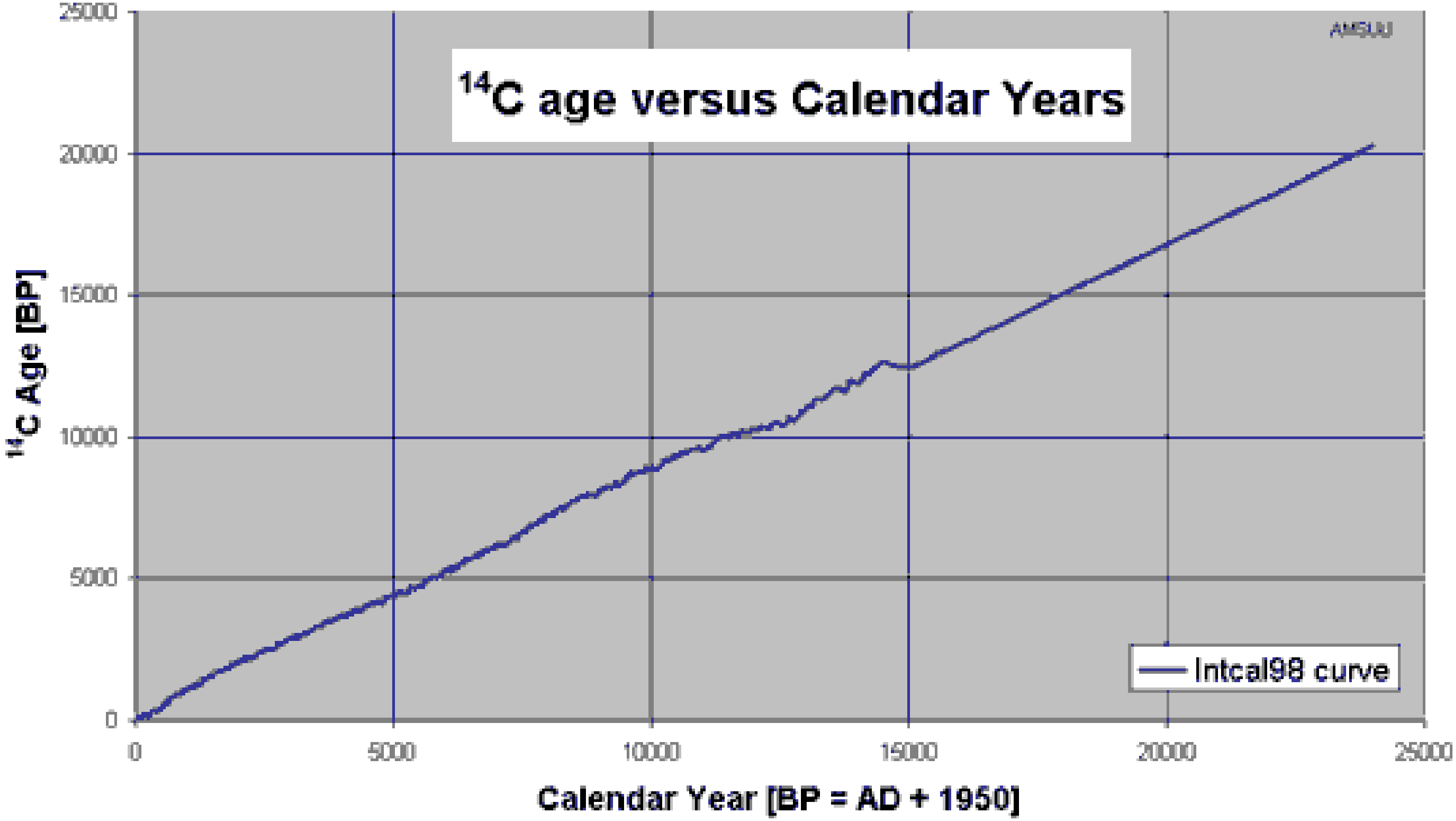


Figure 22.4 Deviation of the initial radiocarbon activity in per mil of wood samples of known age relative to 95 percent of the activity of the oxalic acid standard of the National Bureau of Standards. The observed activities were corrected for carbon isotope fractionation and recalculated using a value of 5730 years for the half-life of ^{14}C. The decline in the radiocarbon content starting at about 1900 results from the introduction of fossil CO_2 into the atmosphere by the combustion of fossil fuels (Suess effect). The anomalously high radiocarbon activity around 1710 and 1500 A.D. is known as the "de Vries effect." Its causes are not understood. (Data from Table 6a and b of Lerman et al., 1970.)



Scientists build tree-ring chronologies by starting with living trees and then finding progressively older specimens—including archaeological wood—whose outer rings overlap with the inner rings of more-recent specimens.





$$A_t = A_0 \times e^{-\lambda t}$$

$CaCO_3$-CO_2-H_2O Equilibrium

$$H_2O + CO_2 \leftrightarrow H_2CO_3$$

$$H_2CO_3 \leftrightarrow H^+ + HCO_3^-$$

$$CO_3^{2-} + H^+ \leftrightarrow HCO_3^-$$

$$CaCO_3 \leftrightarrow Ca^{2+} + CO_3^{2-}$$

$$CaCO_3 + H_2O + CO_2 \leftrightarrow Ca^{2+} + 2HCO_3^-$$

Residence time: Tr = V/I [T], a measure of the average **time** a molecule of water spends in a reservoir. The **residence time** defined for steady-state systems is equal to the reservoir volume divided by the inflow or outflow rate.

Hydrogen Isotopes

Name	Symbol	Atomic number	Mass number	Relative abundance	Nature radioactive or non-radioactive
Protium or Hydrogen	$^{1}_{1}H$ or H	1	1	99.99%	Non-radioactive
Deuterium	$^{2}_{1}H$ or D	1	2	0.015%	Non-radioactive
Tritium	$^{3}_{1}H$ or T	1	3	10-18%	Radioactive

^{3}H= in TU= **1 tritium** atom in a mixture of 10^{18} atoms of all hydrogen isotopes

$H^{3}HO$, $D^{3}HO$

Production and Decay of Tritium

Tritium Production Methods

$$^{14}N + {^1n} \longrightarrow {^{12}C} + {^3\mathbf{H}}$$

$$^{2}H + {^2H} \longrightarrow {^3\mathbf{H}} + {^1H}$$

$$^{235}U + {^1n} \longrightarrow {^3\mathbf{H}} + \text{other fragments}$$

$$^{6}Li + {^1n} \longrightarrow {^4He} + {^3\mathbf{H}}$$

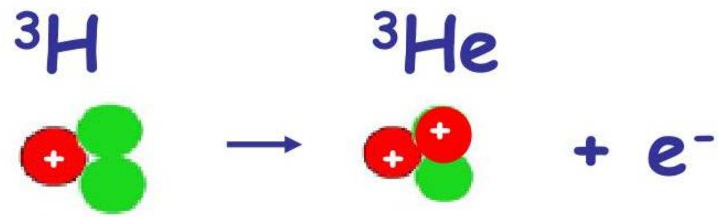
$$^{7}Li + {^1n} \longrightarrow {^4He} + {^3\mathbf{H}} + {^1n}$$

$$D_2O + 2\,{^1n} \longrightarrow \mathbf{T_2}O$$

$$D_2O + {^1n} \longrightarrow \mathbf{T}DO$$

$$^{10}B + {^1n} \longrightarrow 2\,{^4He} + {^3\mathbf{H}}$$

Tritium Beta decay



$$^3_1H \rightarrow {^3_2He} + e^-$$

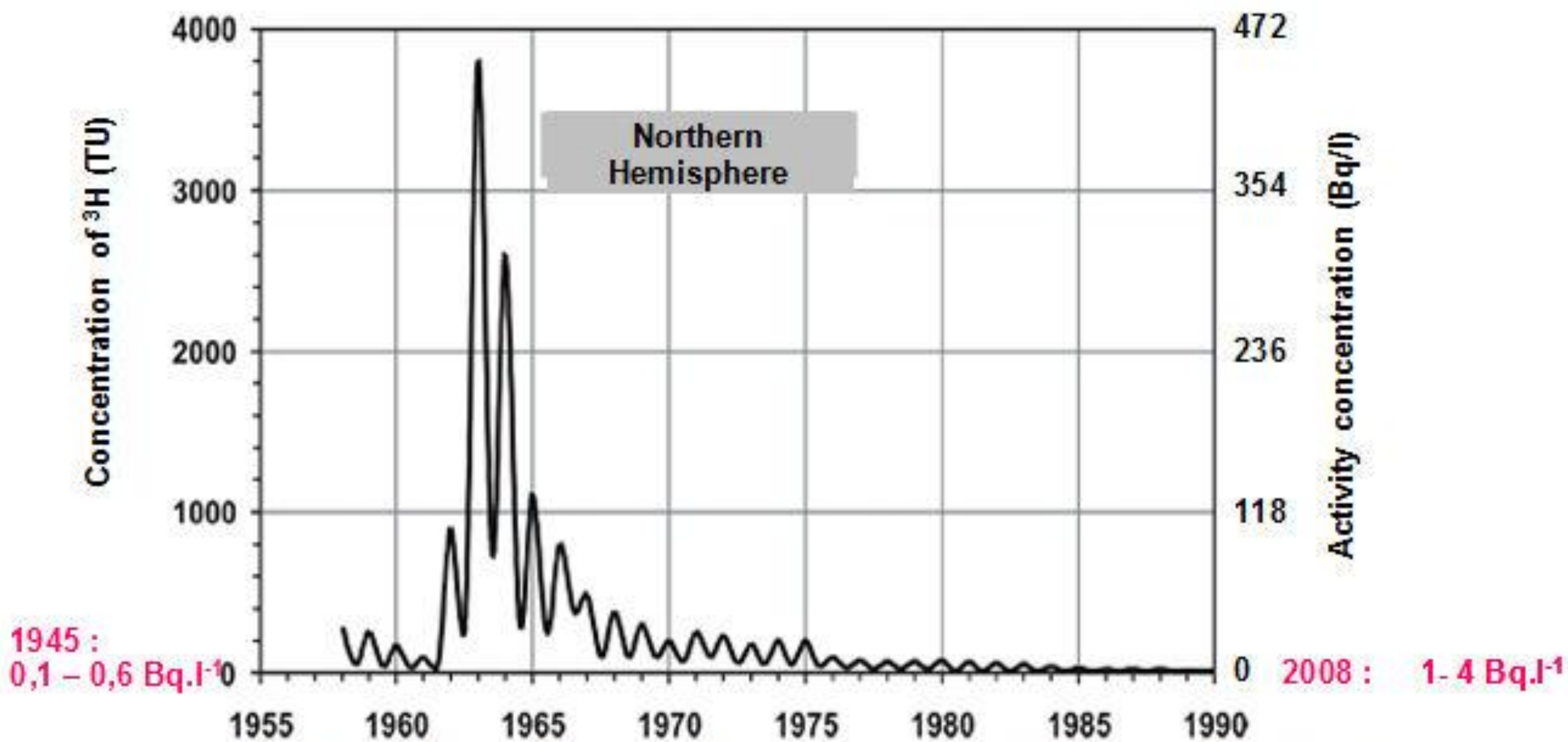
Age Determination of water by Tritium

$$A_t = A_0 \times e^{-\lambda t}$$

A_0 and A_t are ^3H initial activity, and activity after time ‘t’, λ is decay constant

Half Life 12.32 years

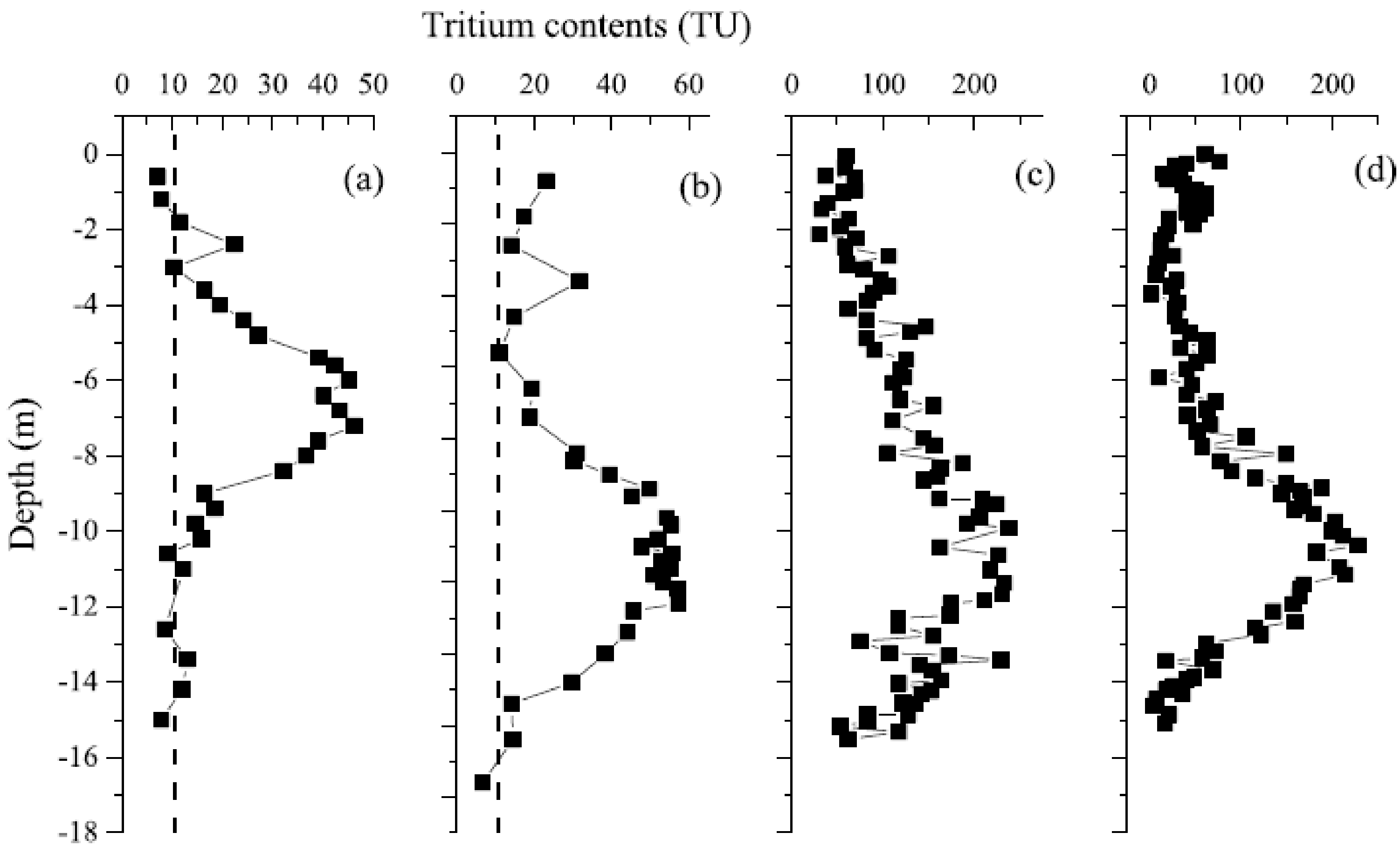
Temporal variation in Tritium production (Variation in A_0 value)



Smooth curve showing the average ^3H concentrations in precipitation over the continental surface in the Northern hemisphere. ***Source = IAEA Isotope hydrology, 2006***

$$A_t = A_0 \times e^{-\lambda t}$$

Infiltration Rate determination by Tritium



Infiltration rate= Depth/time

Helium-Tritium Method

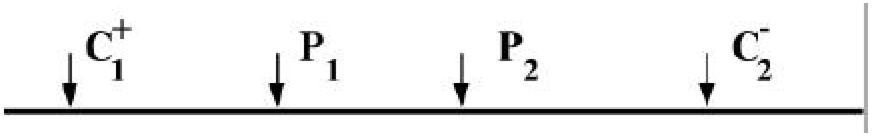
$$D = Ne^{\lambda t} - N$$

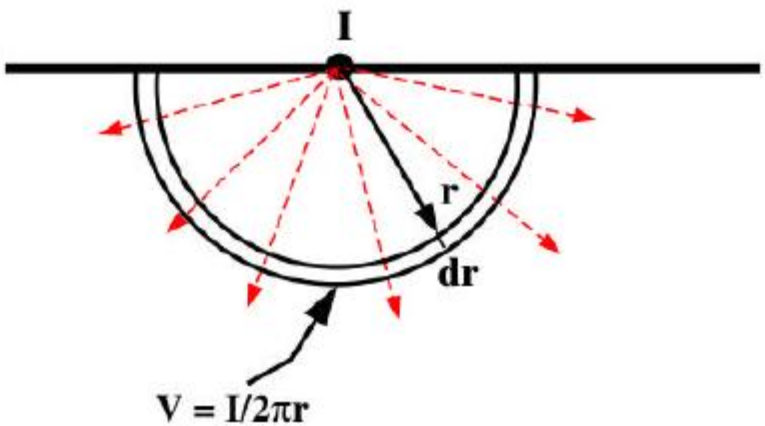
$$^3He = {}^3He^{\lambda t} - {}^3H$$

$$^3He = {}^3H\left(e^{\lambda t} - 1\right)$$

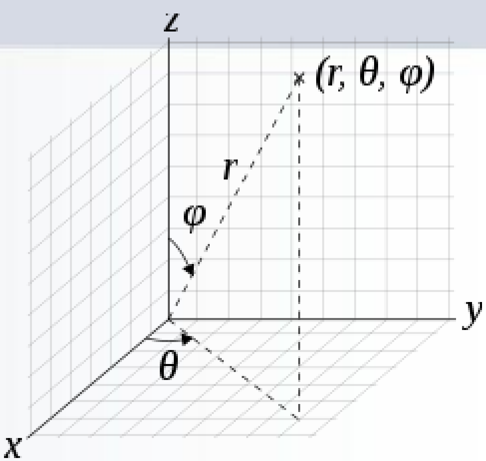
- Atmospheric ^4He concentration is 5.24 ppmv (Glueckauf, 1946)
- Atmospheric $^3He/^4He$ ratio is $1.3 \cdot 10^{-6}$ (Coon, 1949, cited in Andrews, 1987)
- The solubility of atmospheric helium is temperature dependent, and for 10°C is $4.75 \cdot 10^{-8}$ cm^3 STP/cm^3 H_2O (Fig. 7-11).
- ^4He is slightly more soluble in water, with a fractionation factor, $\alpha_{w\text{-}air} \sim 0.983$ (Benson and Krause, 1976)

Material	Resistivity (Ohm-meter)
Air	∞
Pyrite	3×10^{-1}
Galena	2×10^{-3}
Quartz	$4 \times 10^{10} - 2 \times 10^{14}$
Calcite	$1 \times 10^{12} - 1 \times 10^{13}$
Rock Salt	$30 - 1 \times 10^{13}$
Mica	$9 \times 10^{12} - 1 \times 10^{14}$
Granite	$100 - 1 \times 10^{6}$
Gabbro	$1 \times 10^{3} - 1 \times 10^{6}$
Basalt	$10 - 1 \times 10^{7}$
Limestones	$50 - 1 \times 10^{7}$
Sandstones	$1 - 1 \times 10^{8}$
Shales	$20 - 2 \times 10^{3}$
Dolomite	100 - 10,000
Sand	1 - 1,000
Clay	1 - 100
Ground Water	0.5 - 300
Sea Water	0.2





The Laplace Equation



- Cartesian coordinates

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

  - V is potential
  - Harmonic!
- Spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

  - r is the radius
  - θ is the angle between the z-axis and the vector we're considering
  - ϕ is the angle between the x-axis and our vector

The electrical potential depends on distance, not direction, so it is appropriate to write LaPlace's equation in spherical coordinates with an r -dependence only:

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)\right]=0, \qquad (4.15)$$

which has a solution:

$$V=\frac{A}{r}+B. \qquad (4.16)$$

Voltage is function of : Geological materials

Electrode spacing

Laplace equation in spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}=0$$

$$\boxed{\nabla^2 V=0}$$

Electric potential depends on distance not on direction,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)=0$$

$$\partial\left(r^2\frac{\partial V}{\partial r}\right)=0$$

$$r^2\frac{\partial V}{\partial r}=C$$

$$\Rightarrow \frac{\partial V}{\partial r}=Cr^2$$

$$\Rightarrow \partial V=Cr^2$$

$$\Rightarrow V=\frac{C}{r}+B$$

Potential must go to zero a great distance from the current source, it is obvious that $B=0$

$$\Rightarrow V=\frac{C}{r}$$

$$E = -\frac{\partial V}{\partial r}$$

$$j = \sigma \frac{\partial V}{\partial r}$$

$$\frac{\partial V}{\partial r} = \frac{c}{r^2}$$

$$\Rightarrow \frac{j}{\sigma} = \frac{c}{r^2}$$

$$\Rightarrow j\rho = \frac{c}{r^2}$$

$$c = j\rho r^2$$

$$V = \frac{c}{r}$$

$$V = \frac{j\rho r^2}{r}$$

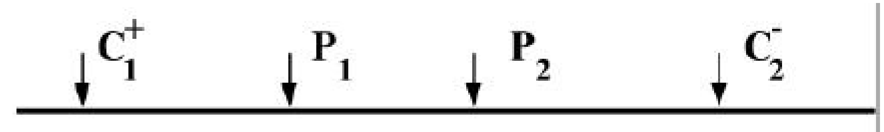
$$V = j\rho r$$

$$\boxed{j = \frac{I}{4\pi r^2}}$$

$$V = \frac{I}{4\pi r^2} \times r$$

$$V = \frac{I}{4\pi r}$$

$$\boxed{V = \frac{I}{2\pi r}}$$



Using the basic

$$V = \frac{\rho I}{2\pi r} \tag{4.27}$$

we have

$$V_1 = \frac{\rho I}{2\pi}\left(\frac{1}{r_{11}} - \frac{1}{r_{21}}\right) \tag{4.28}$$

Note the minus sigh because of the current convention. We also have

$$V_2 = \frac{\rho I}{2\pi}\left(\frac{1}{r_{12}} - \frac{1}{r_{22}}\right) \tag{4.29}$$

All we can really measure is the <u>voltage drop</u> between two electrodes, ΔV:

$$\Delta V = V_1 - V_2 = \frac{\rho I}{2\pi}\left(\frac{1}{r_{11}} + \frac{1}{r_{22}} - \frac{1}{r_{21}} - \frac{1}{r_{12}}\right) \tag{4.30}$$

Now we can solve for the resistivity of the subsurface:

$$\rho = 2\pi \frac{\Delta V}{I} k \tag{4.31}$$

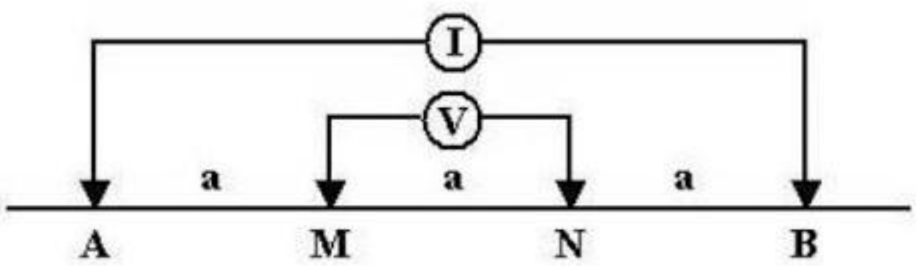
where k is the *array constant* or *geometrical factor*:

$$\frac{1}{k}=\frac{1}{r_{11}}+\frac{1}{r_{22}}-\frac{1}{r_{21}}-\frac{1}{r_{12}} \tag{4.32}$$

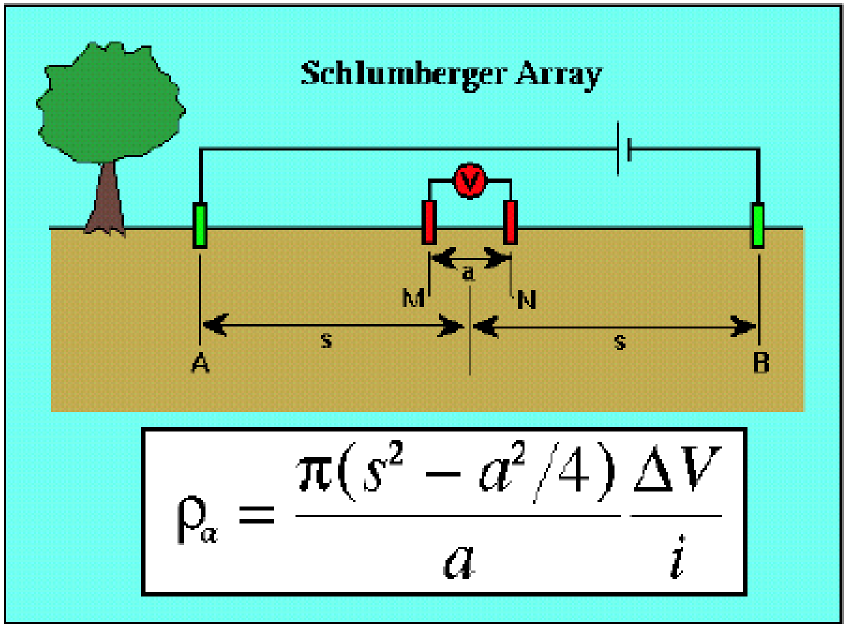
But what if the subsurface is not homogeneous of a single resistivity, ρ. Then we define an apparent resistivity, ρ_a. ρ_a is the value obtained from equation (4.31) and will only equal the true resistivity if the subsurface is inhomogeneous:

$$\rho_a=2\pi\frac{\Delta V}{I}k \tag{4.33}$$

Wenner Configuration

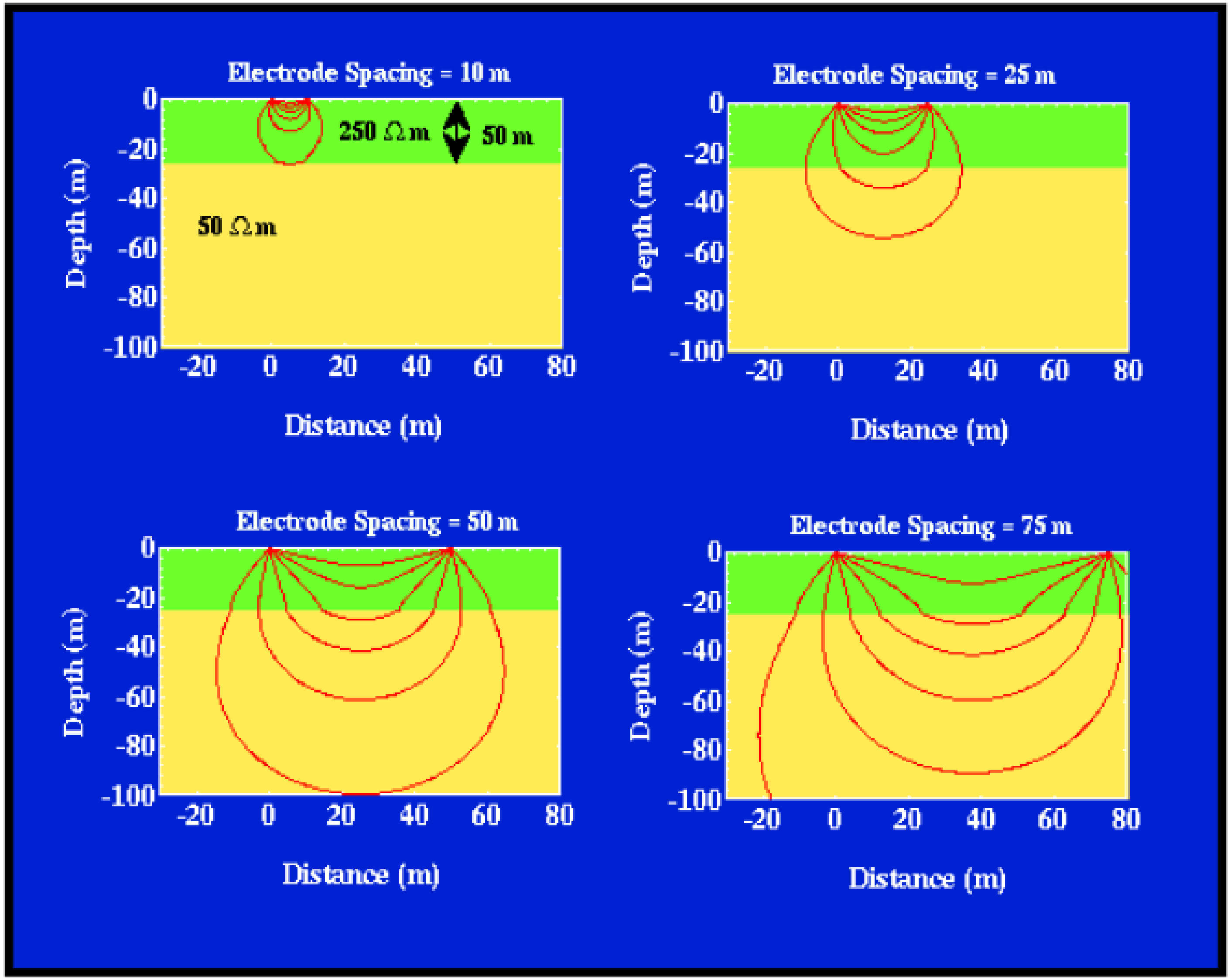


$$\rho_A=2\pi\ a\frac{V}{I}$$

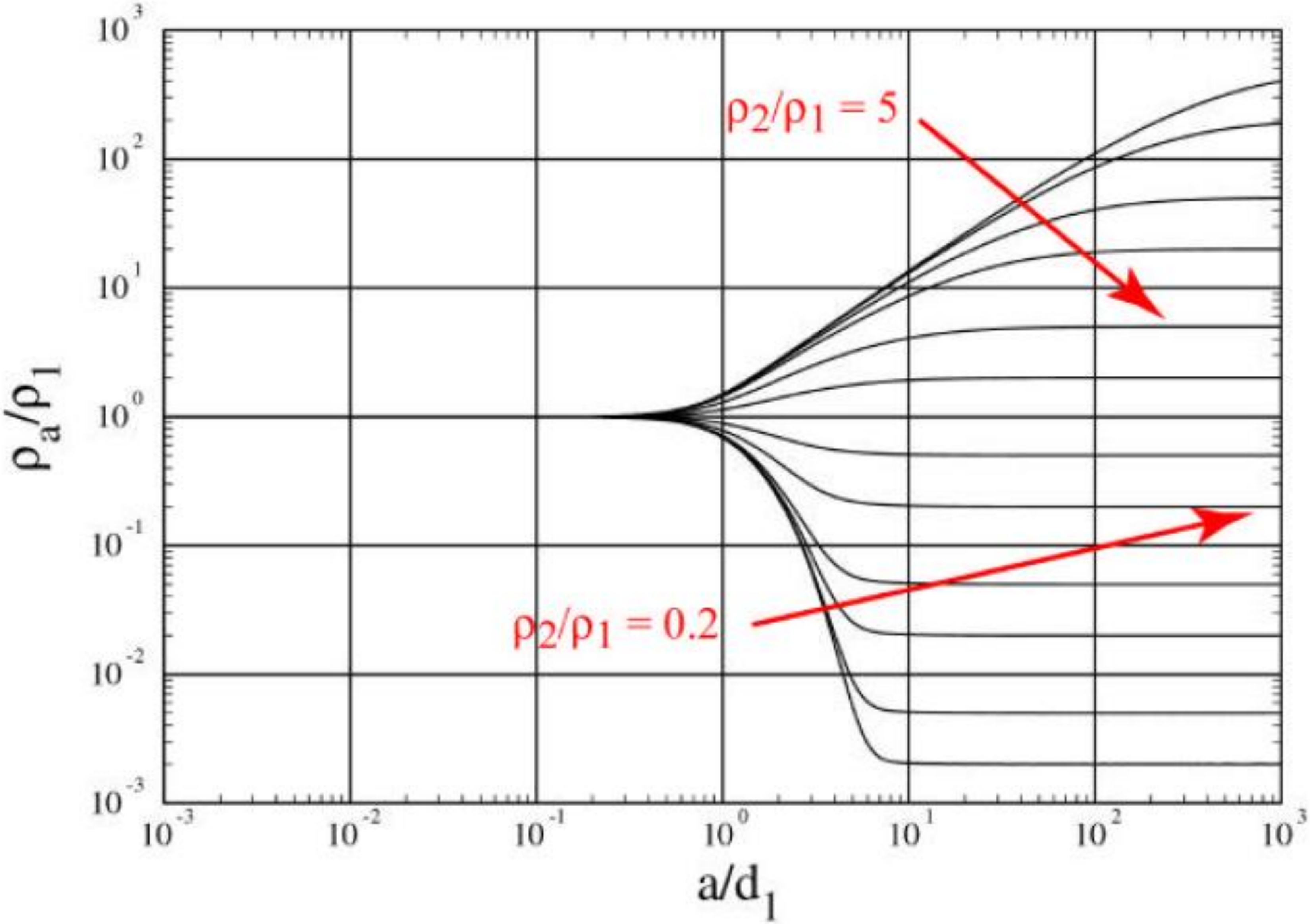


Sounding

Profiling



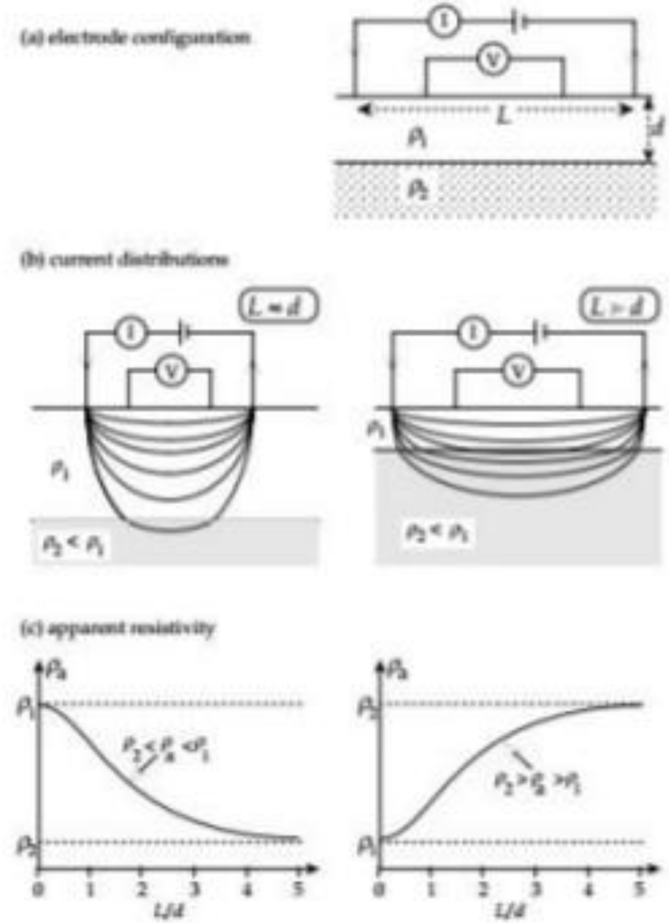
Electrode Spacing = 10 m
250 Ω m
50 m
50 Ω m
Depth (m)
Distance (m)
Electrode Spacing = 25 m
Depth (m)
Distance (m)
Electrode Spacing = 50 m
Depth (m)
Distance (m)
Electrode Spacing = 75 m
Depth (m)
Distance (m)

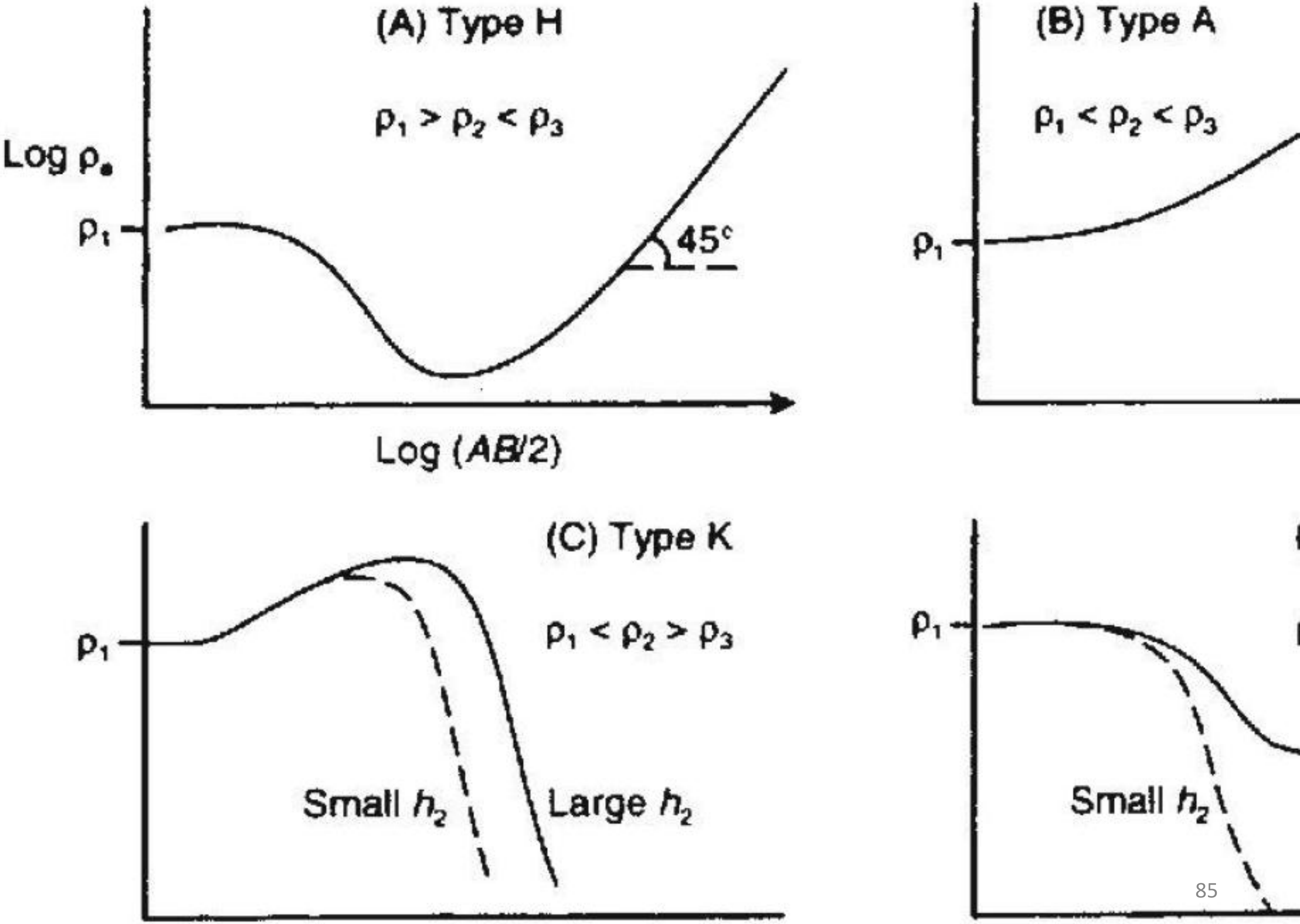


10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
ρ_a/ρ_1
$\rho_2/\rho_1 = 5$
$\rho_2/\rho_1 = 0.2$
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
a/d_1

Apparent Resistivity

- In the idealized case of a perfectly uniform conducting half-space the current flow lines resemble a dipole pattern and the resistivity determined with a four-electrode configuration is the true resistivity of the half-space.
- But in real situations the resistivity is determined by different lithologies and geological structures and so maybe very inhomogeneous. This complexity is not taken into account when measuring resistivity with a four-electrode method, which assumes that the ground is uniform. The result of such a measurement is the *apparent resistivity* of an equivalent uniform half-space and generally does not represent the true resistivity of any part of the ground.

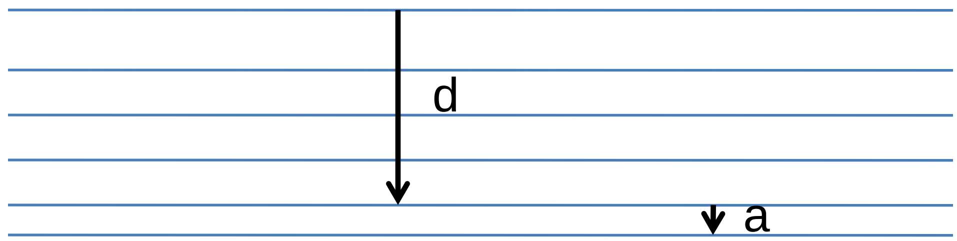




(A) Type H
ρ1 > ρ2 < ρ3
Log ρa
ρ1
45°
Log (AB/2)
(B) Type A
ρ1 < ρ2 < ρ3
ρ1
(C) Type K
ρ1 < ρ2 > ρ3
ρ1
Small h2
Large h2
ρ1
Small h2

Limitations: Resistivity survey

Resolution: Relative Thickness (ratio of the thickness of the layer to the depth of top layer) ,



a/d is less than 0.1

Suppression: If resistivity contrast is inadequate

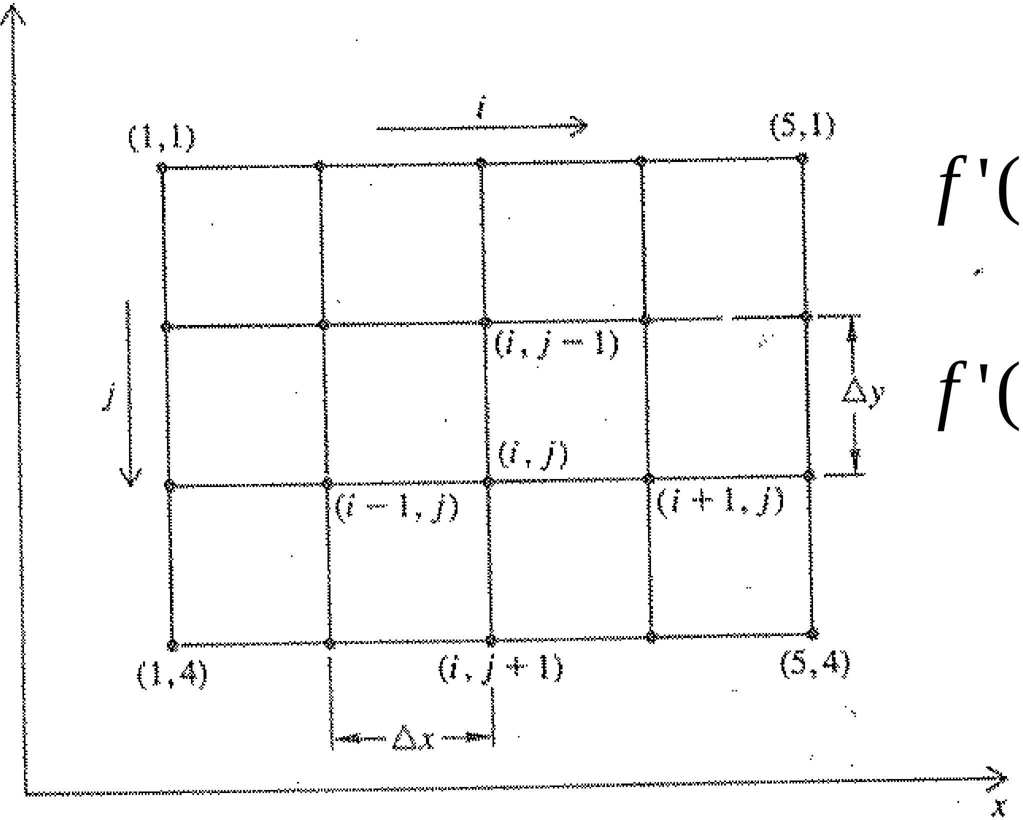
$$(0.1 < RT < 1)$$

Equivalence:

Modelling of groundwater flow: Simplified version of reality

Finite Difference method

Finite Difference Method



$$f'(x) = \frac{dy}{dx} =_{\Delta x \to 0} \frac{f(x+h) - f(x)}{\Delta x}$$

$$f'(x) = \frac{dy}{dx} = \frac{f(x+h) - f(x)}{\Delta x}$$

$$f'(x) = \frac{h_{i+1,j} - h_{i,j}}{\Delta x}$$

$$f'(y) = \frac{h_{i,j+1} - h_{i,j}}{\Delta y}$$

$$\frac{\partial^2 h}{\partial x^2} \simeq \frac{\dfrac{h_{i+1,j} - h_{i,j}}{\Delta x} - \dfrac{h_{i,j} - h_{i-1,j}}{\Delta x}}{\Delta x} \tag{2.2}$$

which simplifies to

$$\frac{\partial^2 h}{\partial x^2} \simeq \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2} \tag{2.3}$$

Similarly,

$$\frac{\partial^2 h}{\partial y^2} \simeq \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{(\Delta y)^2} \tag{2.4}$$

According to Laplace's equation, we must add the preceding two equations and set the result equal to zero. If we consider a square grid of points such that $\Delta x = \Delta y$, then the finite difference approximation for Laplace's equation at the point (i, j) simplifies to

$$h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j} = 0 \tag{2.5}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

vation well and are given in Figure 2.3. We have now specified the

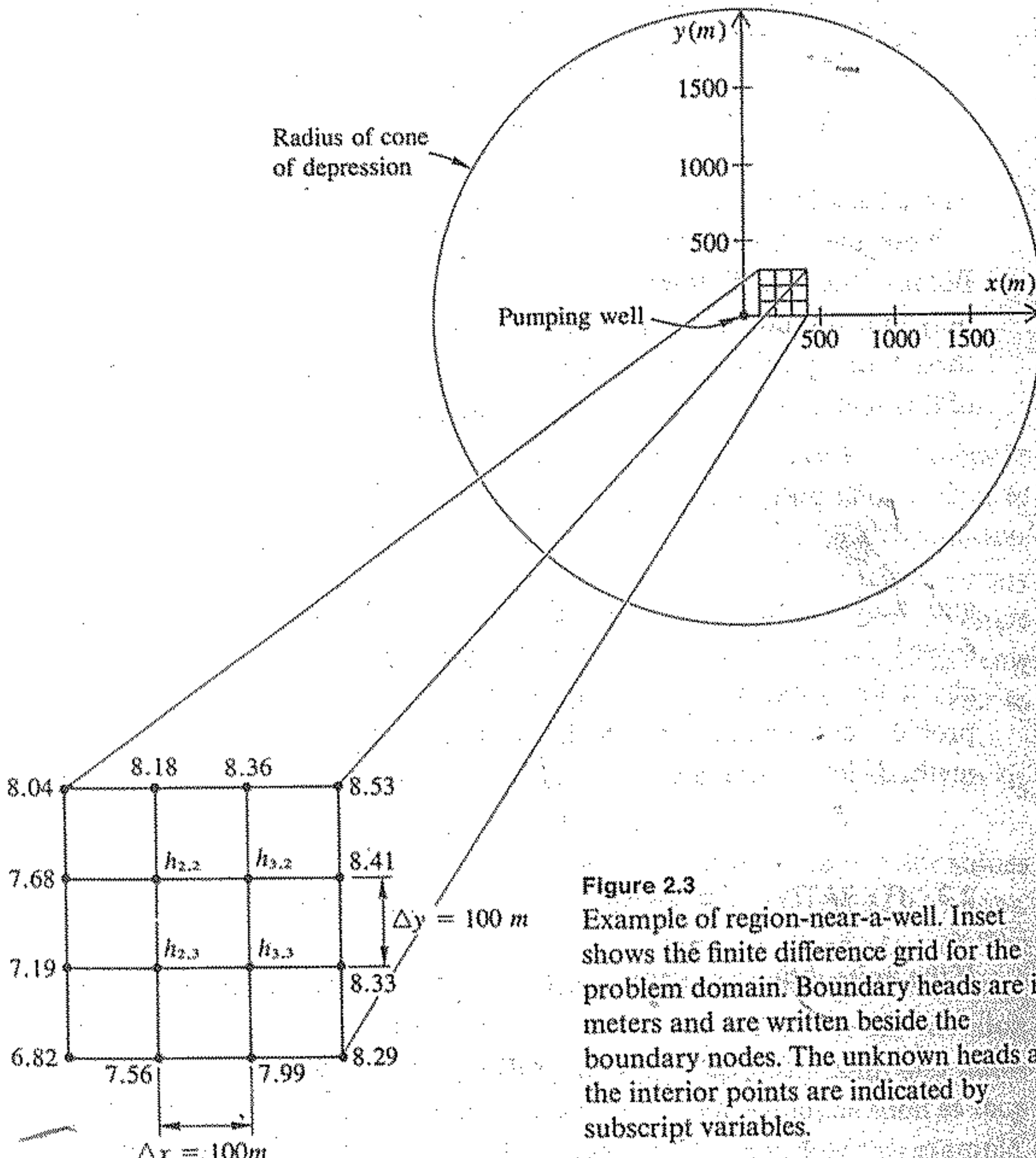
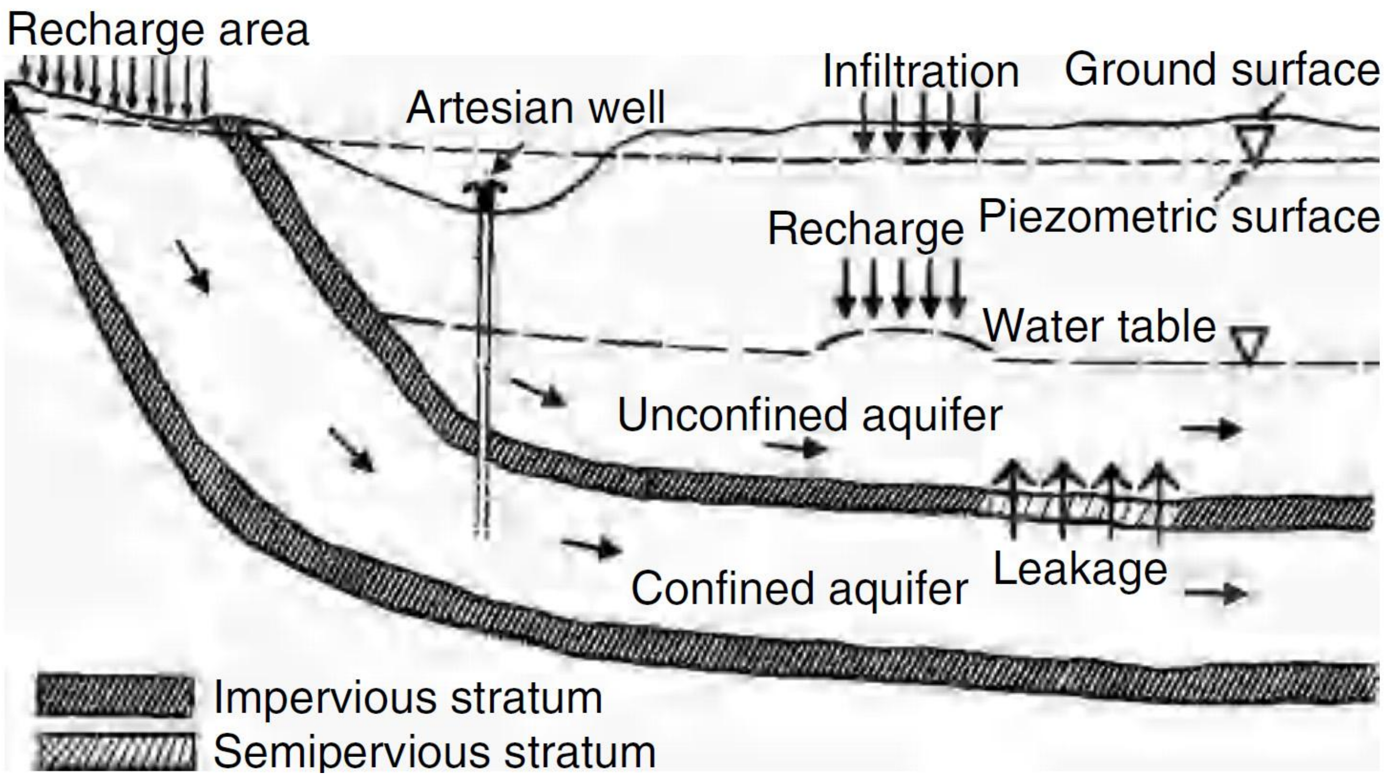


Figure 2.3
Example of region-near-a-well. Inset shows the finite difference grid for the problem domain. Boundary heads are in meters and are written beside the boundary nodes. The unknown heads at the interior points are indicated by subscript variables.


Recharge area
Artesian well
Infiltration
Ground surface
Recharge
Piezometric surface
Water table
Unconfined aquifer
Leakage
Confined aquifer
Impervious stratum
Semipervious stratum

15968

$Q \propto b^3$

$Q = K.A.\frac{dh}{dl}$

$= \left(\frac{\rho g b^2}{12\mu}\right).(bw).\frac{dh}{dl}$

$\boxed{K = \left(\frac{\rho g b^2}{12\mu}\right)}$

Fracture porosity $\boxed{\phi_f = \frac{b}{s}}$

$\boxed{K = \frac{b^3}{12}N}$

$\boxed{N = \frac{1}{s}}$

W = fracture width in the flow

b = fracture opening

N = Fracture frequency

= Number of fracture in unit length

s = Fracture spacing

s unit — 1

$1 \rightarrow \frac{1}{s}$

$\boxed{N = \frac{1}{s}}$

$K = k.\frac{\rho g}{\mu}$

$= \frac{b^3}{12}N.\frac{\rho g}{\mu}$

$= \frac{\rho g b^2}{12\mu}.\frac{b}{s}$

$K = K_{sing}.\phi_f$