Quantification across scales

class – 3 (8.8.24)

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Dr. Neelanjana Sengupta
Associate Professor, DBS
https://www.iiserkol.ac.in/~n.sengupta/

Orders of Magnitude

How would you compare the “rate of movement” of E. coli with that of fastest humans?

E. coli

- Speed: ~20 μm/s
- ~10 body lengths/s
- Water:
  ‘high resistance’ medium

Average walker

- Speed: ~1.3 m/s
- ~1 body length/s
- Air:
  ‘low resistance’

’Fastest’ humans (outliers)

- Speed: ~12.4 m/s
- ~10 body lengths/s
- Air:
  ‘low resistance’

What does this information suggest?

'Rule of Thumb': Length Scales

Table 1.1: Rules of thumb for biological estimates.

	Quantity of interest	Symbol	Rule of thumb
E. coli			
	Cell volume	$V_{E.\,coli}$	$\approx 1\ \mu m^3$
	Cell mass	$m_{E.\,coli}$	≈ 1 pg
	Cell cycle time	$t_{E.\,coli}$	≈ 3000 s
	Cell surface area	$A_{E.\,coli}$	$\approx 6\ \mu m^2$
	Macromolecule concentration in cytoplasm	$c_{E.\,coli}^{macromol}$	≈ 300 mg/mL
	Genome length	$N_{bp}^{E.\,coli}$	$\approx 5 \times 10^6$ bp
	Swimming speed	$v_{E.\,coli}$	$\approx 20\ \mu m/s$
Yeast			
	Volume of cell	V_{yeast}	$\approx 60\ \mu m^3$
	Mass of cell	m_{yeast}	≈ 60 pg
	Diameter of cell	d_{yeast}	$\approx 5\ \mu m$
	Cell cycle time	t_{yeast}	≈ 200 min
	Genome length	N_{bp}^{yeast}	$\approx 10^7$ bp
Organelles			
	Diameter of nucleus	$d_{nucleus}$	$\approx 5\ \mu m$
	Length of mitochondrion	l_{mito}	$\approx 2\ \mu m$
	Diameter of transport vesicles	$d_{vesicle}$	≈ 50 nm

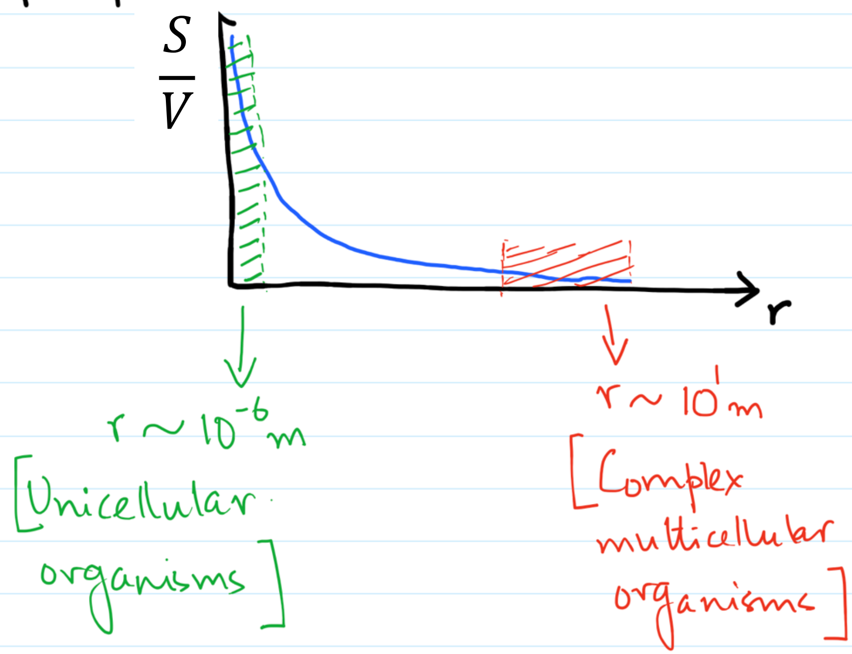
Phys. Bio. of Cell., Garland Science, Ch. 1

'Rule of Thumb': Length Scales

Surface-to-Volume ratios of organisms (in general)

In general, for any entity of spatial scale 'r',

$$\frac{S}{V} \propto \frac{1}{r}$$



- The (S/A) ratio is high for prokaryotes
- It reduces in complex multicellular organisms
  - *It is not very sensitive to size in multi-cellular organisms*

'Rule of Thumb': Length Scales

Table 1.1: Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
Water		
Volume of molecule	V_{H_2O}	$\approx 10^{-2}$ nm^3
Density of water	ρ	1 g/cm^3
Viscosity of water	η	$\approx$1 centipoise (10^{-2} g/(cm s))
Hydrophobic embedding energy	$\approx E_{hydr}$	2500 cal/(mol nm^2)

- What is the approx. radius of water molecule?
- What is the "E_{hydr}" given above?

'Rule of Thumb': Length Scales

Table 1.1: Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
Amino acids and proteins		
Radius of "average" protein	r_{protein}	≈ 2 nm
Volume of "average" protein	V_{protein}	$\approx 25\ \text{nm}^3$
Mass of "average" amino acid	M_{aa}	≈ 100 Da
Mass of "average" protein	M_{protein}	$\approx 30{,}000$ Da
Protein concentration in cytoplasm	c_{protein}	≈ 150 mg/mL
Characteristic force of protein motor	F_{motor}	≈ 5 pN
Characteristic speed of protein motor	v_{motor}	≈ 200 nm/s
Diffusion constant of "average" protein in cytoplasm	D_{protein}	$\approx 10\ \mu\text{m}^2/\text{s}$

- How does mass of a typical amino acid, and a typical protein, compare with mass of a water molecule?

'Rule of Thumb': Length Scales

Table 1.1: Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
Lipid bilayers		
Thickness of lipid bilayer	d	≈ 5 nm
Area per molecule	A_{lipid}	$\approx \frac{1}{2}$ nm^2
Mass of lipid molecule	m_{lipid}	≈ 800 Da

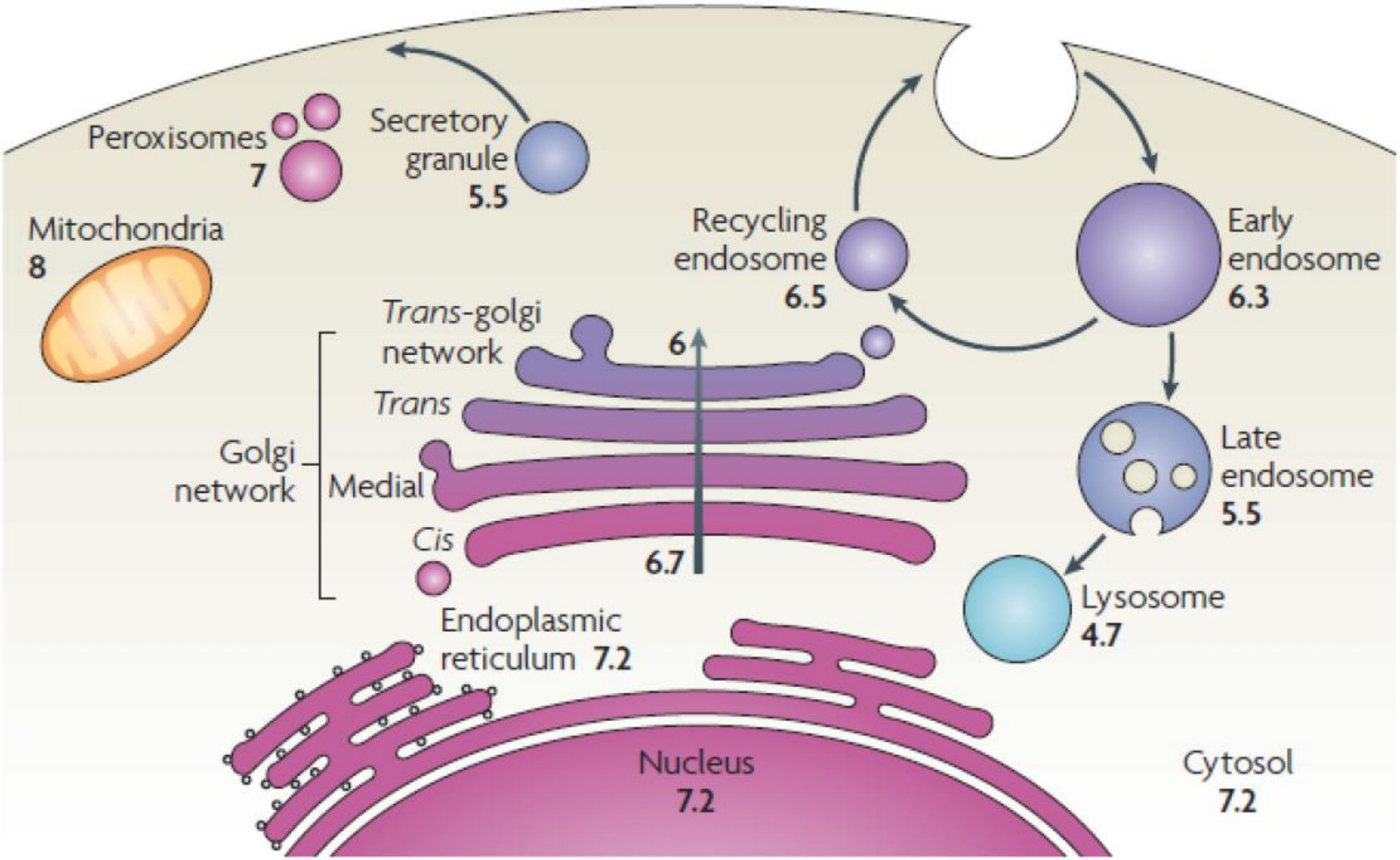
- Consider that a protein of radius 1 nm displaces water to embed into a lipid bilayer.
- Estimate the energetic cost in kcal/mol.

Quantity of interest	Symbol	Rule of thumb
DNA		
Length per base pair	l_{bp}	$\approx 1/3$ nm
Volume per base pair	V_{bp}	≈ 1 nm^3
Charge density	λ_{DNA}	$2\,e/0.34$ nm
Persistence length	ξ_{p}	50 nm

Numerical estimates: pH

$pH = -\log_{10}[H^+]$

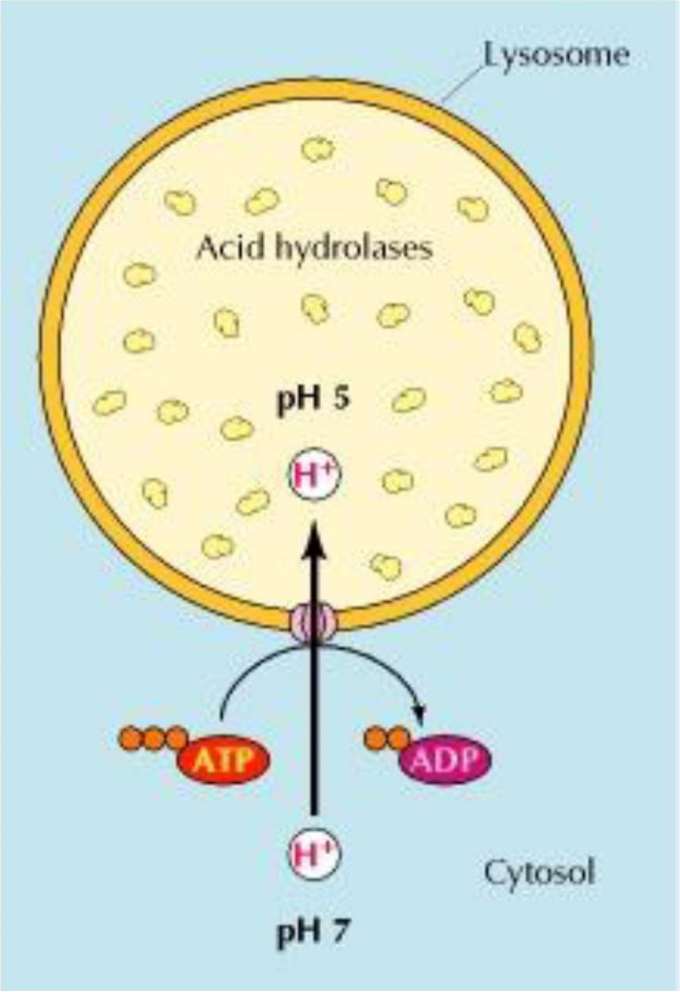
Eukaryotic cell: typical pH values of subcellular compartments



Numerical estimates: pH

$pH = -\log_{10}[H^+]$

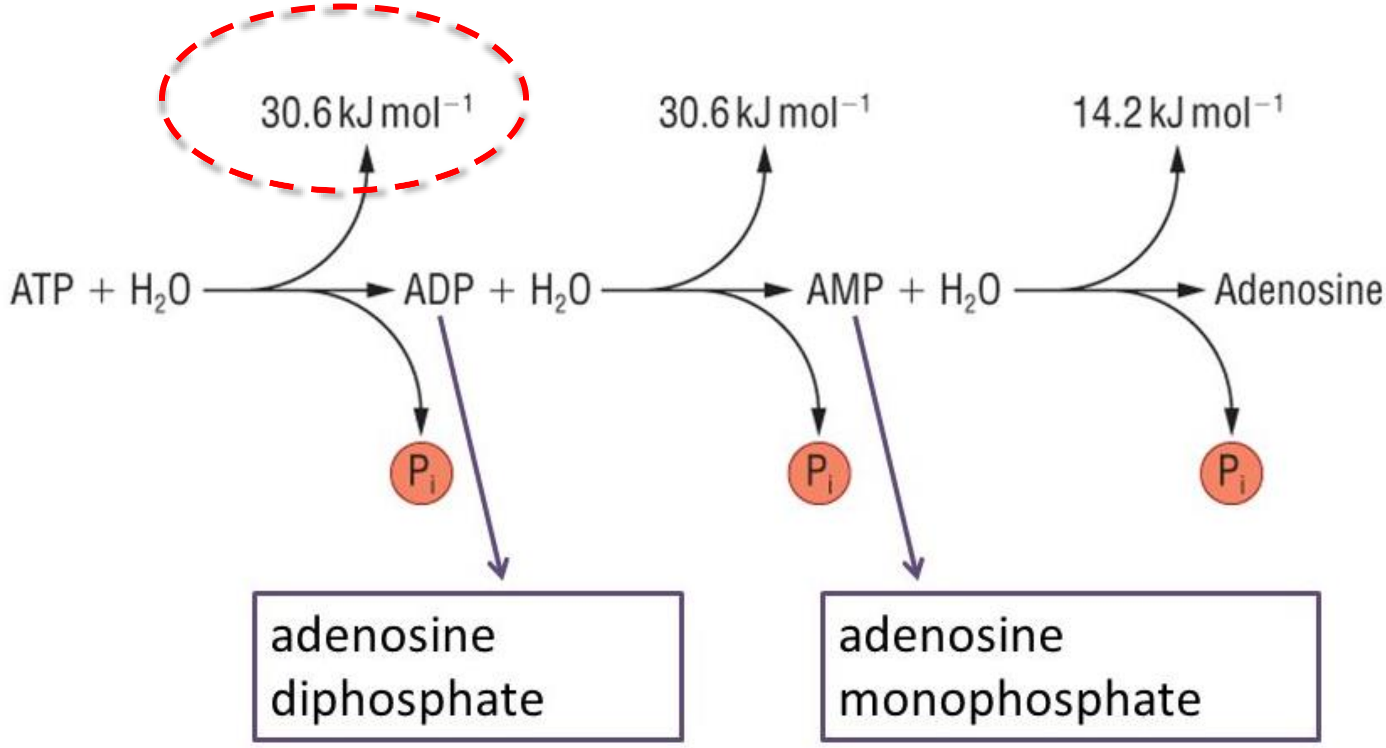
HW. Calculate the number of H^+ ions for a lysosome of radius 500 nm at steady state



(RT) is the energy scale in *molecular* biology

B. ATP hydrolysis: "Energy currency" in Biology

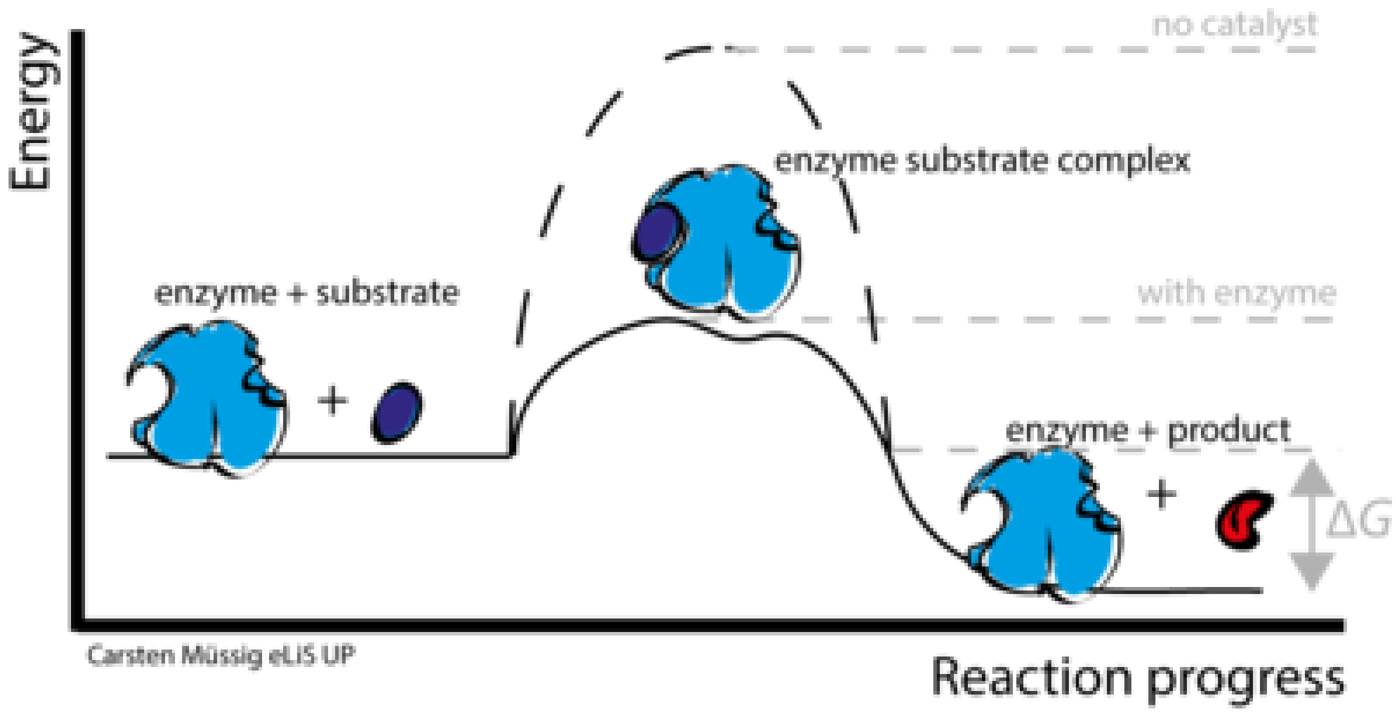
How many *physiological* k_BT?



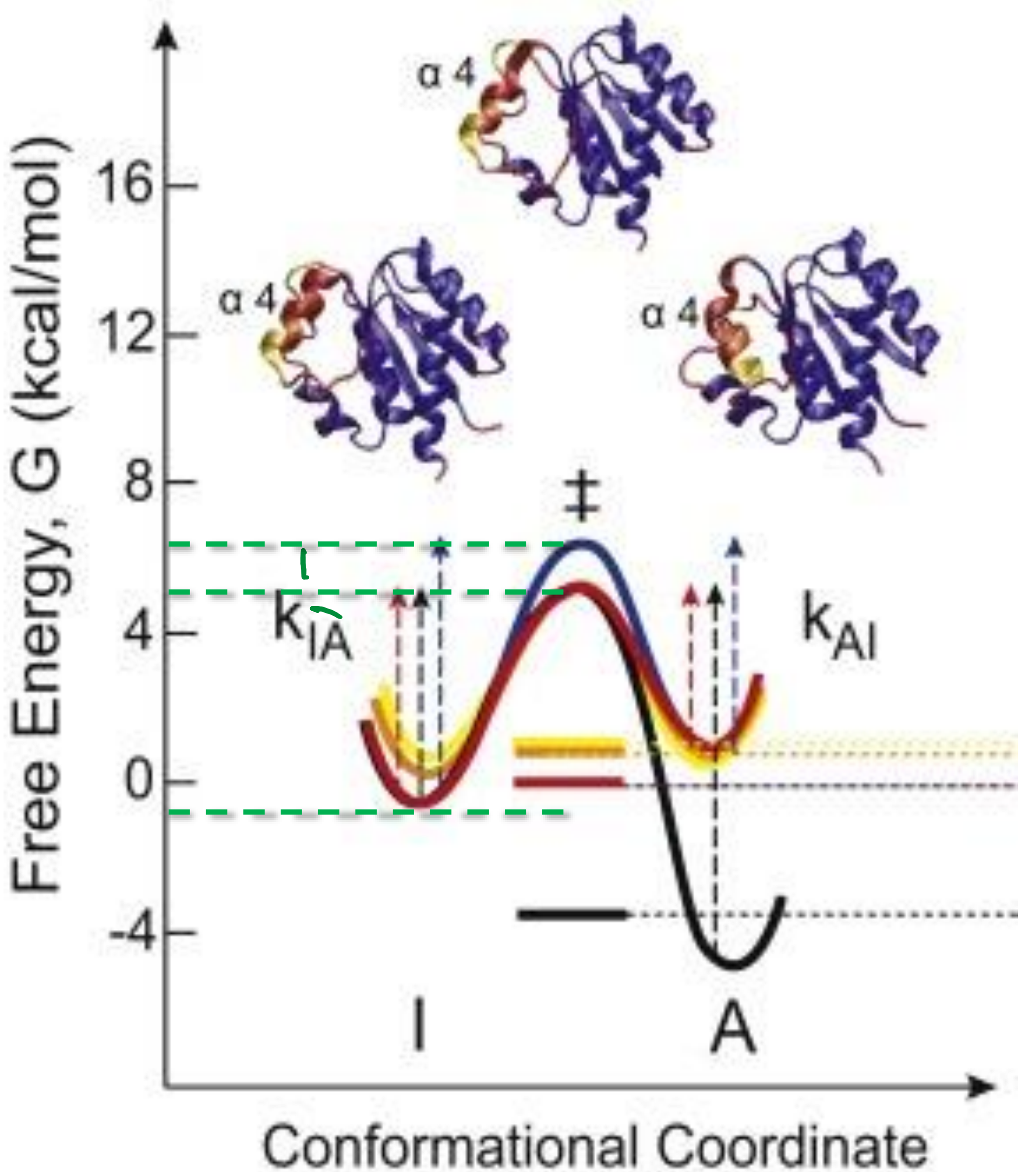
Note: Values may fluctuate (within the order of magnitude)

(RT) is the energy scale in *molecular* biology

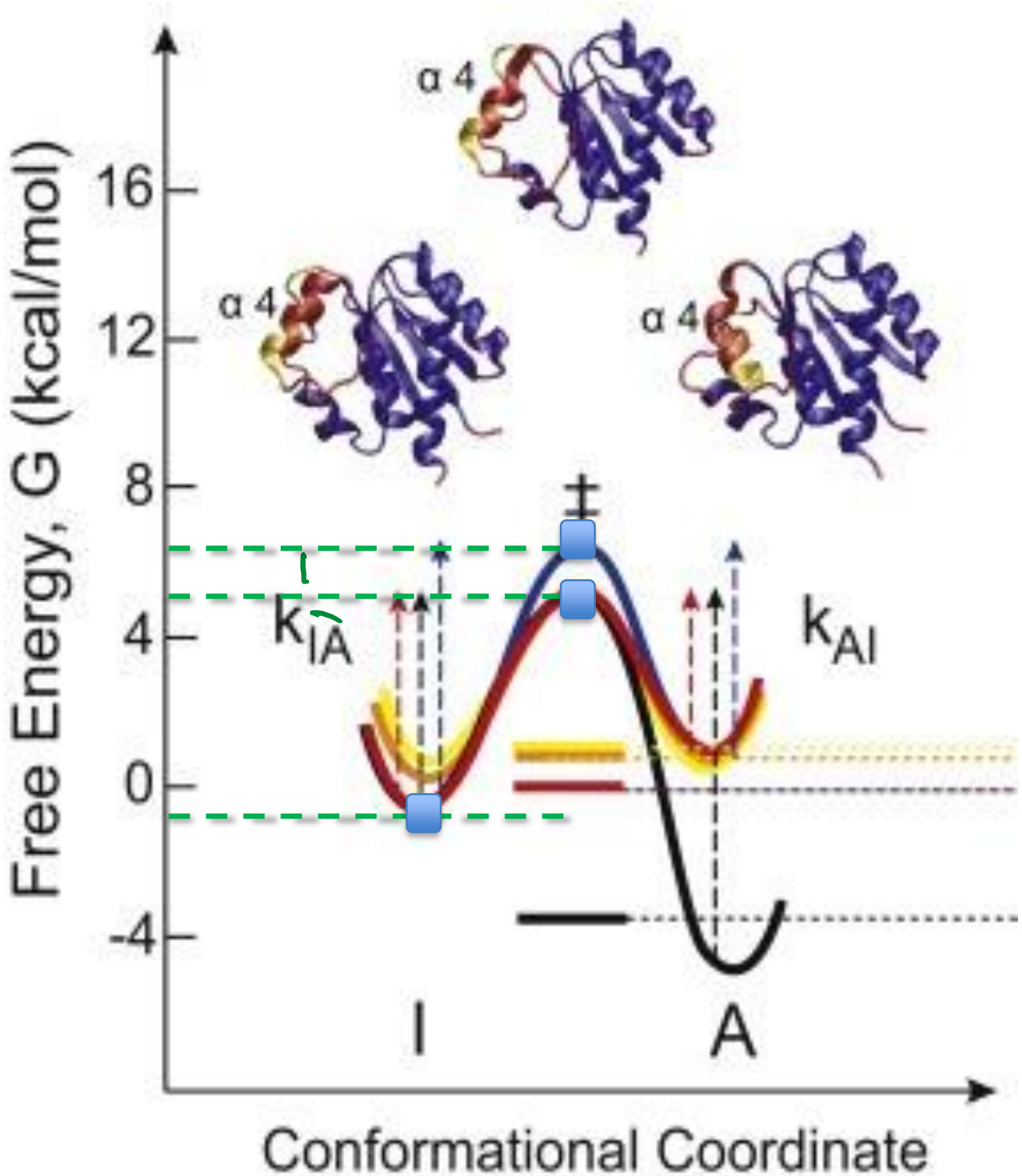
Eg. Enzymatic reactions



(RT) is the energy scale in *molecular* biology



(RT) is the energy scale in *molecular* biology



By what factor does the rate change?

(RT) is the energy scale in *molecular* biology

Bjerrum length (λ_B): Distance between charges where the electrostatic interactions are comparable to thermal energy ($k_B T$)

$$\boxed{\lambda_B = \frac{e^2}{4\pi\epsilon_0\epsilon_r k_B T}}$$

e: Elementary charge

i. Show that the equation is dimensionally correct.

ii. Find λ_B at 300 K in water (ϵ_r ~ 80)

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ N\ m^2 C^2$$

Dimensional Analysis

Extracting mathematical relationships between physical quantities

Fundamental Quantity	Dimension	SI Units
Mass	[M]	Kilograms (kg)
Length	[L]	Metre (m)
Time	[T]	Second (s)
Charge	[Q]	Coulomb (C)
Temperature	[K]	Kelvin (K)

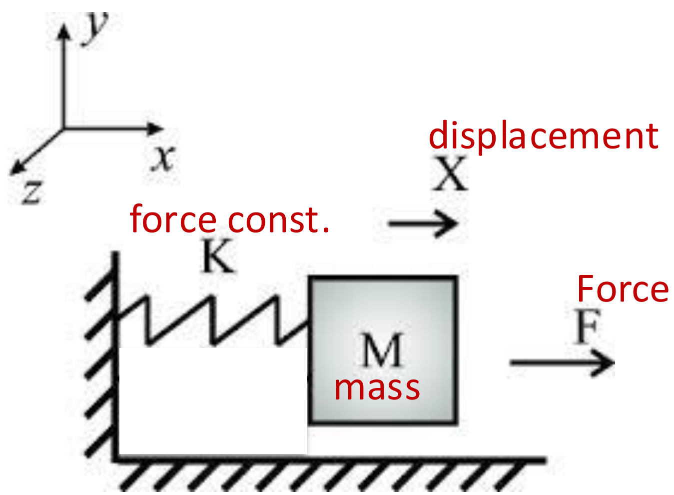
$$A\,[M]^{\alpha}\,[L]^{\beta}\,[T]^{\gamma}\,[Q]^{\delta}\,[K]^{\kappa} = B\,[M]^{\pi}\,[L]^{\theta}\,[T]^{\rho}\,[Q]^{\sigma}\,[K]^{\upsilon}$$

Dimensional Analysis: examples

$$A\,[M]^{\alpha}\,[L]^{\beta}\,[T]^{\gamma}\,[Q]^{\delta}\,[K]^{\kappa} = B\,[M]^{\pi}\,[L]^{\theta}\,[T]^{\rho}\,[Q]^{\sigma}\,[K]^{\upsilon}$$

1. Natural frequency of a simple spring

Find the dependence of Time Period (τ) and Frequency (ν) on the spring's characteristic parameters using dim. analys.



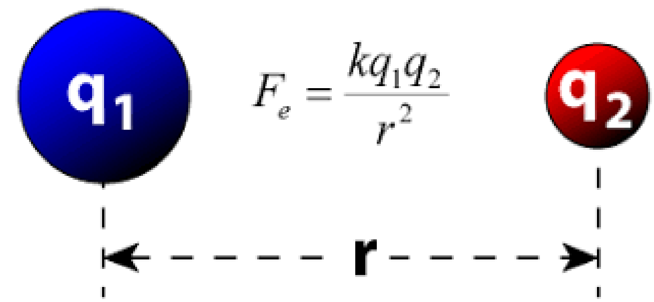
Start with *known physics (Hooke's Law):*

$$\mathbf{F = K\ X}$$

Dimensional Analysis: examples

2. Based on **Coulomb's Law**, find:

- *Dimensions of the medium characteristic ‘k’*
- *What should be the S.I units of ‘k’?*

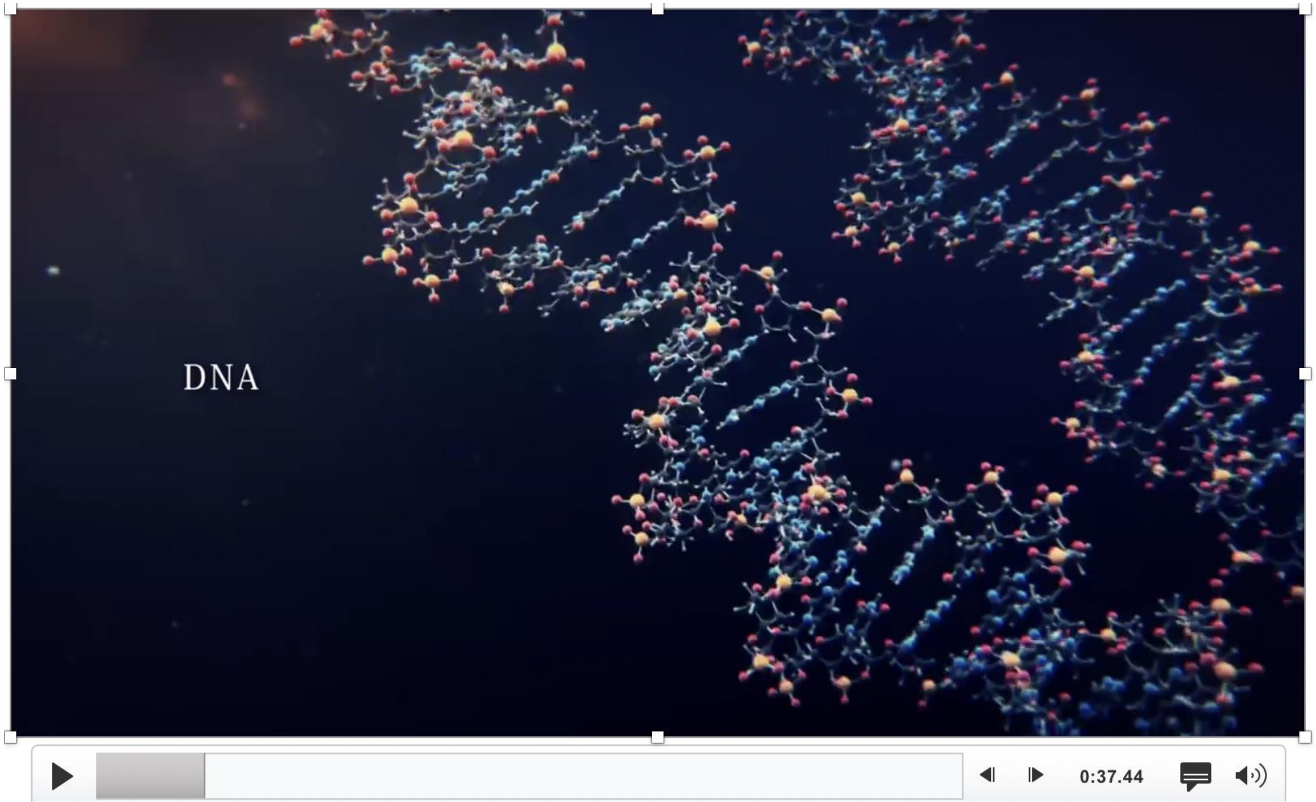


(RT) is the energy scale in *molecular* biology

Bjerrum length (λ_B): Distance between charges where the electrostatic interaction is comparable to thermal energy (*RT*)

Biomolecules often exist and function in highly charged environments

https://www.youtube.com/watch?v=4Z4KwuUfh0A



Supercomput. Life Sci., Riken. http://www.scls.riken.jp/en/