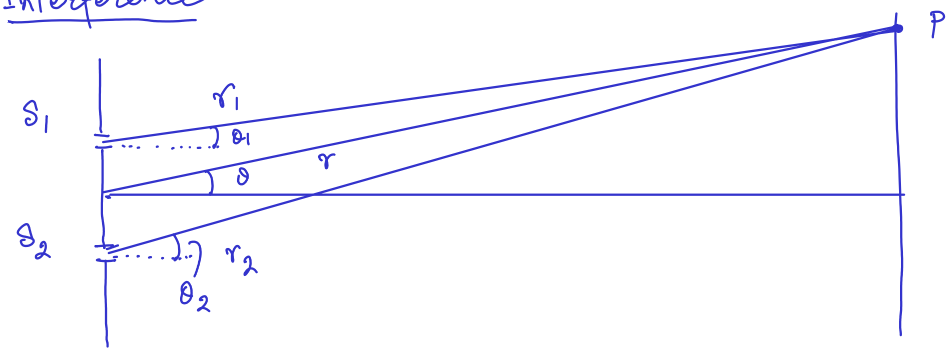
Interference and diffractions

Interference → Multiple discrete sources

Diffractions → Continuous source

Both rely on the superposition principle.

Interference



S_1, S_2 are sources. Both emit EM waves with the same amplitude A.

EM waves from S_1 and S_2 arrive at P.

E field at P is given by,

$$E = A e^{i(kr_1 - \omega t + \phi_1)} + A e^{i(kr_2 - \omega t + \phi_2)}$$

$$= A e^{-i\omega t}\left[e^{i(kr_1 + \phi_1)} + e^{i(kr_2 + \phi_2)} \right]$$

$$= A e^{-i\omega t} e^{i\left(k\frac{r_1+r_2}{2} + \frac{\phi_1+\phi_2}{2}\right)} \left[e^{i\left(k \cdot \frac{r_1-r_2}{2} + \frac{\phi_1-\phi_2}{2}\right)} + e^{-i\left(k\frac{r_1-r_2}{2} + \frac{\phi_1-\phi_2}{2}\right)} \right]$$

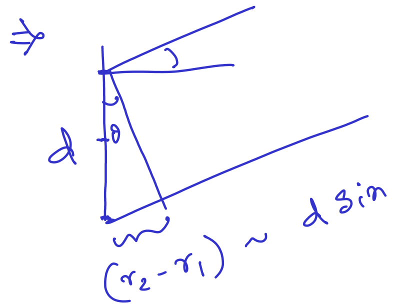
$$= A e^{-i\omega t} e^{i(kr + \phi_{av})}\, 2\cos\left(k\frac{\Delta r}{2} + \frac{1}{2}\Delta\phi\right)$$

$\Delta\phi = \phi_1 - \phi_2$

$\Delta r = r_1 - r_2$

Let, $\theta_1 \sim \theta_2 \sim \theta \implies$ Far-field approx

$\Rightarrow$



$$\text{Intensity} \propto |E|^2$$

$$\propto 4A^2 \cos^2\left(\frac{1}{2}kd\sin\theta + \frac{1}{2}\Delta\phi\right)$$

For coherent sources, $\Delta\phi = 0$

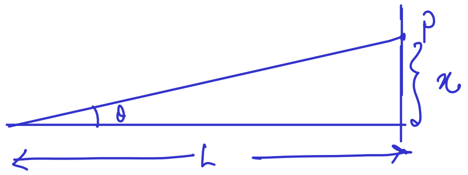
$$\Rightarrow \text{Intensity} \propto 4A^2\cos^2\left(\pi\frac{d}{\lambda}\sin\theta\right)$$

In general, minima occur when,

$$\frac{1}{2}kd\sin\theta = \boxed{\pi\frac{d}{\lambda}\sin\theta = (2n+1)\frac{\pi}{2}} \qquad n = 0, 1, 2, \cdots$$

maxima occur when,

$$\frac{1}{2}kd\sin\theta = \boxed{\pi\frac{d}{\lambda}\sin\theta = n\pi} \qquad n = 0, 1, 2, \cdots$$



For small θ,

$$\sin\theta \sim \tan\theta \sim \theta \sim \frac{x}{L}$$

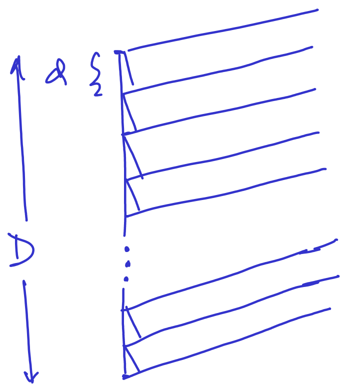
$$\Rightarrow \text{Intensity} \propto 4A^2 \cos^2\left(\frac{\pi d}{\lambda}\cdot\frac{x}{L}\right)$$

At $x=0$, central maxima.

$x = \frac{\lambda L}{2d}(2n+1)$ for $n = 0, 1, 2, \cdots$, minima

* Adding $\Delta\phi$ introduces shift of the pattern.

Diffraction:



For an extended source of width D, we divide it into N segments with $d = D/N$.
$\Rightarrow$ There are $(N+1)$ beams (see figure); we are counting them from the edges. N is large.

Amplitude at a far away point (small angle holds and all angles are approximated by the average angle) is given by,

$$E = \frac{A}{(N+1)} e^{i(kr-\omega t)}\left[1 + e^{ikd\sin\theta} + e^{ik2d\sin\theta} + \cdots + e^{ikNd\sin\theta}\right]$$

$$= \frac{A}{(N+1)} e^{i(kr-\omega t)}\left(\frac{e^{ikNd\sin\theta}-1}{e^{ikd\sin\theta}-1}\right)$$

$$= \frac{A}{(N+1)} e^{i(kr-\omega t)} \frac{e^{i\frac{kNd}{2}\sin\theta}}{e^{i\frac{kd}{2}\sin\theta}} \cdot \frac{e^{i\frac{kNd}{2}\sin\theta} - e^{-i\frac{kNd}{2}\sin\theta}}{e^{i\frac{kd}{2}\sin\theta} - e^{-i\frac{kd}{2}\sin\theta}}$$

$$= \frac{A}{(N+1)} \cdot e^{i(kr-\omega t)}\, e^{ik(N-1)\frac{d}{2}\sin\theta} \cdot \frac{\sin\frac{Nkd\sin\theta}{2}}{\sin\frac{kd\sin\theta}{2}}$$

$$\Rightarrow I \propto |E|^2 = \frac{A^2}{(N+1)^2}\cdot\frac{\sin^2\frac{Nkd\sin\theta}{2}}{\sin^2\frac{kd\sin\theta}{2}} = \frac{A^2}{(N+1)^2}\cdot\frac{\sin^2 N\phi}{\sin^2\phi}$$

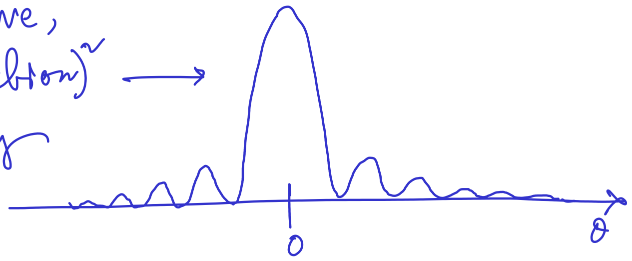
where, $\phi = \frac{kd\sin\theta}{2} = \pi\frac{d}{\lambda}\sin\theta$.

For very large N, we can have a small ϕ and $\sin\phi \sim \phi$ and $(N+1)^2 \sim N^2$

$$\Rightarrow I \propto \frac{A^2}{N^2} \cdot \frac{\sin^2\left(\pi \frac{Nd}{2\lambda} \sin\theta\right)}{\left(\pi \frac{d}{2\lambda} \sin\theta\right)^2} = A^2 \frac{\sin^2\left(\pi \frac{D}{2\lambda} \sin\theta\right)}{\left(\frac{\pi D}{2\lambda} \sin\theta\right)^2} \approx A^2 \frac{\sin^2\left(\frac{\pi D}{2\lambda}\theta\right)}{\left(\frac{\pi D}{2\lambda}\theta\right)^2}$$

For small angle

So, we have, a (sinc function)^2 like intensity distribution.



* Sinc function has a θ-dependent width
  $\Rightarrow$ the beam "spreds" out as a result of the diffraction.