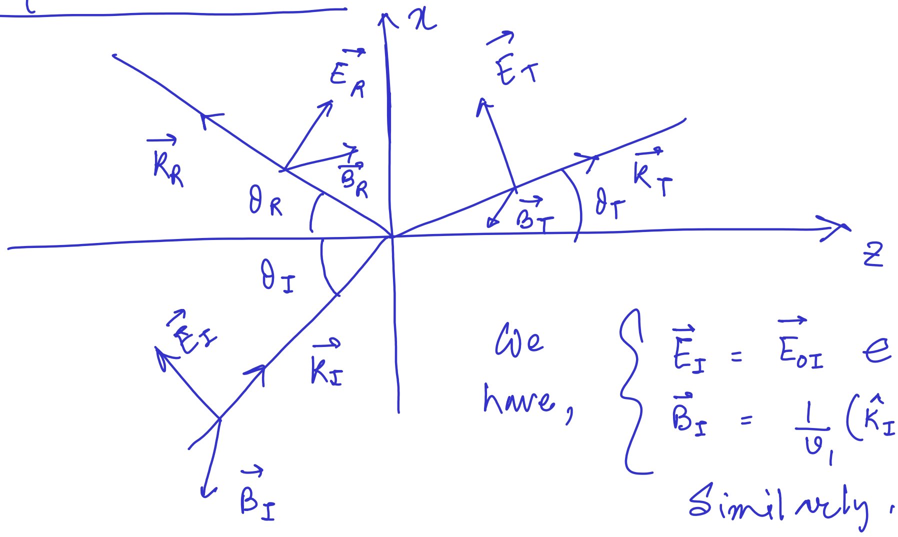
Oblique incidence



We have,

$$\begin{cases} \vec{E}_I = \vec{E}_{0I}\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} \\ \vec{B}_I = \frac{1}{v_1}(\hat{K}_I \times \vec{E}_{0I})\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} \end{cases}$$

Similarly, for R and T

We note,

$$k_I = \frac{\omega}{v_1} = k_R \quad \text{and} \quad K_T = \frac{\omega}{v_2} = \frac{v_1}{v_2} k_I = \frac{n_2}{n_1} k_I$$

Matching boundary conditions will lead to

$$(\ )_I\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} + (\ )_R\, e^{i(\vec{K}_R \cdot \vec{r} - \omega t)} = (\ )_T\, e^{i(\vec{K}_T \cdot \vec{r} - \omega t)} \quad \text{at } z = 0$$

The oscillatory part must match on both sides.

$$\Rightarrow \vec{K}_I \cdot \vec{r} = \vec{K}_R \cdot \vec{r} = \vec{K}_T \cdot \vec{r} \quad \text{when } z = 0$$

$$\Rightarrow x K_{Ix} + y K_{Iy} = x K_{Rx} + y K_{Ry} = x K_{Tx} + y K_{Ty}$$

$\Rightarrow$ For $y = 0$, $\quad K_{Ix} = K_{Rx} = K_{Tx}$

and for $x = 0$, $\quad K_{Iy} = K_{Ry} = K_{Ty}$

Now, if we set $K_{Iy} = 0$ (choosing an axes for a given incidence),

we have $K_{Ry} = K_{Ty} = 0$

$\Rightarrow$ $\vec{K}$ vectors are all on a single (xz in this case) plane $\longrightarrow$ plane of incidence. First law.

Now, $K_{Ix} = K_{Rx} = K_{Tx}$

$$\Rightarrow k_I \sin\theta_I = k_R \sin\theta_R = k_T \sin\theta_T$$

$\Rightarrow \theta_I = \theta_R$ as $k_I = k_R$ $\longrightarrow$ Second law / law of reflection

and $\dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{K_I}{K_T} = \dfrac{n_1}{n_2}$ $\longrightarrow$ Third law / Snell's law.

Boundary conditions

$$
\left.\begin{array}{lll}
(B1) & \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} & \rightarrow \text{for } z \\
(B2) & B_1^{\perp} = B_2^{\perp} & \rightarrow \text{for } z \\
(B3) & E_1^{\parallel} = E_2^{\parallel} & \rightarrow \text{for both } x, y \\
(B4) & \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} & \rightarrow \text{for both } x, y
\end{array}\right\} \text{for the chosen boundary}
$$

$(B1) \Rightarrow \quad \epsilon_1 \left(-E_{0I} \sin\theta_I + E_{0R} \sin\theta_R\right) = \epsilon_2 \left(-E_{0T} \sin\theta_T\right) \quad \ldots \quad (1)$

$(B2) \Rightarrow \quad 0 = 0$

$(B3) \Rightarrow \quad \left(E_{0I} \cos\theta_I + E_{0R} \cos\theta_R\right) = E_{0T} \cos\theta_T \quad \ldots \quad (2)$

$(B4) \Rightarrow \quad \frac{1}{\mu_1}\frac{1}{v_1}\left(E_{0I} - E_{0R}\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{0T} \quad \ldots \quad (3)$

$(1) \Rightarrow \quad E_{0I} - E_{0R} = \frac{\epsilon_2}{\epsilon_1}\frac{\sin\theta_T}{\sin\theta_I} E_{0T} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} E_{0T} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} \qquad \epsilon_1 = \frac{1}{\mu_1 v_1^2}$

$= \beta E_{0T}$

$(3) \Rightarrow \quad E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} = \beta E_{0T}$

$(2) \Rightarrow \quad E_{0I} + E_{0R} = \frac{\cos\theta_T}{\cos\theta_I} E_{0T} = \alpha E_{0T}$

$$\boxed{E_{0R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) E_{0I}, \qquad E_{0T} = \left(\frac{2}{\alpha + \beta}\right) E_{0I}}$$

Fresnel's equations

* Transmitted beam $\rightarrow$ always in-phase with the incident beam
* Reflected beam $\rightarrow$ either in-phase or out-of-phase with the incident beam

$\alpha = \beta \Rightarrow$ No reflection condition!

$$\boxed{\sin^2\theta_B = \frac{1 - \beta^2}{\left(\frac{n_1}{n_2}\right)^2 - \beta^2}}$$

$$\boxed{\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I}}{\cos\theta_I}}$$

$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$

$n_1 \sin\theta_I = n_2 \sin\theta_T$

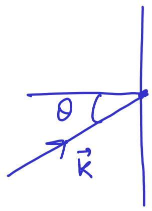
$\sin\theta_T = \frac{n_1}{n_2}\sin\theta_I$

$\Rightarrow \cos\theta_T = \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_I\right)^2}$

Intensity

$$I \propto \frac{1}{2}\epsilon v E^2 \underline{\cos\theta}$$

$\uparrow$ Component of the beam perpendicular to the boundary



Intensity of the reflected wave,

$$\frac{I_R}{I_I} = \frac{\epsilon_1 v_1 E_{0R}^2}{\epsilon_1 v_1 E_{0I}^2} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

Intensity of the transmitted wave,

$$\frac{I_T}{I_I} = \frac{\epsilon_1 v_1 E_{0T}^2 \cos\theta_T}{\epsilon_2 v_2 E_{0I}^2 \cos\theta_I} = \frac{\mu_1 v_1}{\mu_2 v_2} \cdot \frac{\cos\theta_T}{\cos\theta_I} \cdot \left(\frac{2}{1+\beta}\right)^2 = \beta \cdot \alpha \cdot \frac{4}{(1+\beta)^2}$$

* **Brewster's angle**

$\alpha = \beta \Rightarrow$ no reflection

$$\Rightarrow \frac{\cos\theta_T^\circ}{\cos\theta_I^\circ} = \frac{\mu_1 v_1}{\mu_2 v_2} \approx \frac{n_2}{n_1}$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2} = \frac{1}{\beta}$$

$$\Rightarrow \frac{\sqrt{1-\sin^2\theta_T^\circ}}{\cos\theta_I^\circ} = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I^\circ}}{\cos\theta_I^\circ} = \frac{n_2}{n_1} = \beta$$

$$\Rightarrow \sin^2\theta_I^\circ = \frac{\beta^2}{1+\beta^2} \Rightarrow \tan\theta_I^\circ = \beta = \frac{n_2}{n_1} \Rightarrow \theta_I^\circ = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

Total internal reflection

For $n_2 > n_1$, $\theta_T < \theta_I$

For $n_2 < n_1$, $\theta_T > \theta_I$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$

For a particular θ_I° for $n_2 < n_1$,

if $\sin\theta_T^\circ = 1$ i.e. $\theta_T^\circ = \pi/2$, we have,

$$\sin\theta_I^\circ = \frac{n_2}{n_1} \cdot \sin\theta_T^\circ = \frac{n_2}{n_1} \Rightarrow \theta_I^\circ = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

We get this again from

$$\frac{I_T}{I_I} = \frac{4\alpha\beta}{(\alpha+\beta)^2} \quad \text{with} \quad \alpha = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)\sin^2\theta_I}}{\cos\theta_I} = 0$$

For α to be real,

$$\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I \leq 1$$

or $\theta_I \leq \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Equality $\longrightarrow$ threshold for <u>total internal reflection</u>

For all $\theta_I > \theta_I^0$, we have "exponential wave" instead of a regular transmission.

Also,

$$\frac{E_{0R}}{E_{0I}} = \frac{\alpha - \beta}{\alpha + \beta} = \frac{\frac{\cos\theta_T}{\cos\theta_I} - \frac{n_2}{n_1}}{\frac{\cos\theta_T}{\cos\theta_I} + \frac{n_2}{n_1}} = \left(\frac{n_1\cos\theta_T - n_2\cos\theta_I}{n_1\cos\theta_T + n_2\cos\theta_I}\right)$$

<u>For "p-type" polarization</u>