Diffusion in Biological Systems

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LS2103 (Autumn 2024)

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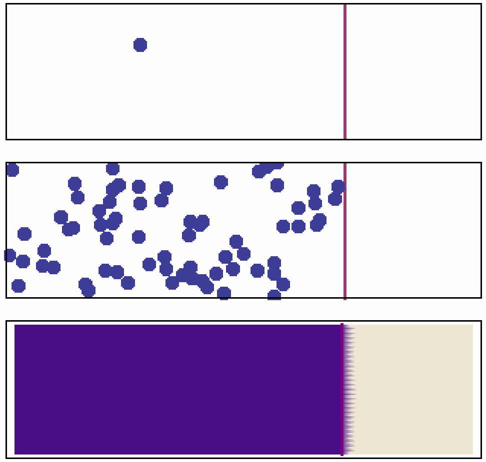
Diffusion: movement of particles in unbiased random walks in any dimension



$$\langle r_N^2 \rangle = (d)Na^2$$

τ : time taken for step $(+a)$ or $(-a)$

T: total elapsed time

No. of steps, $N = \dfrac{T}{\tau}$

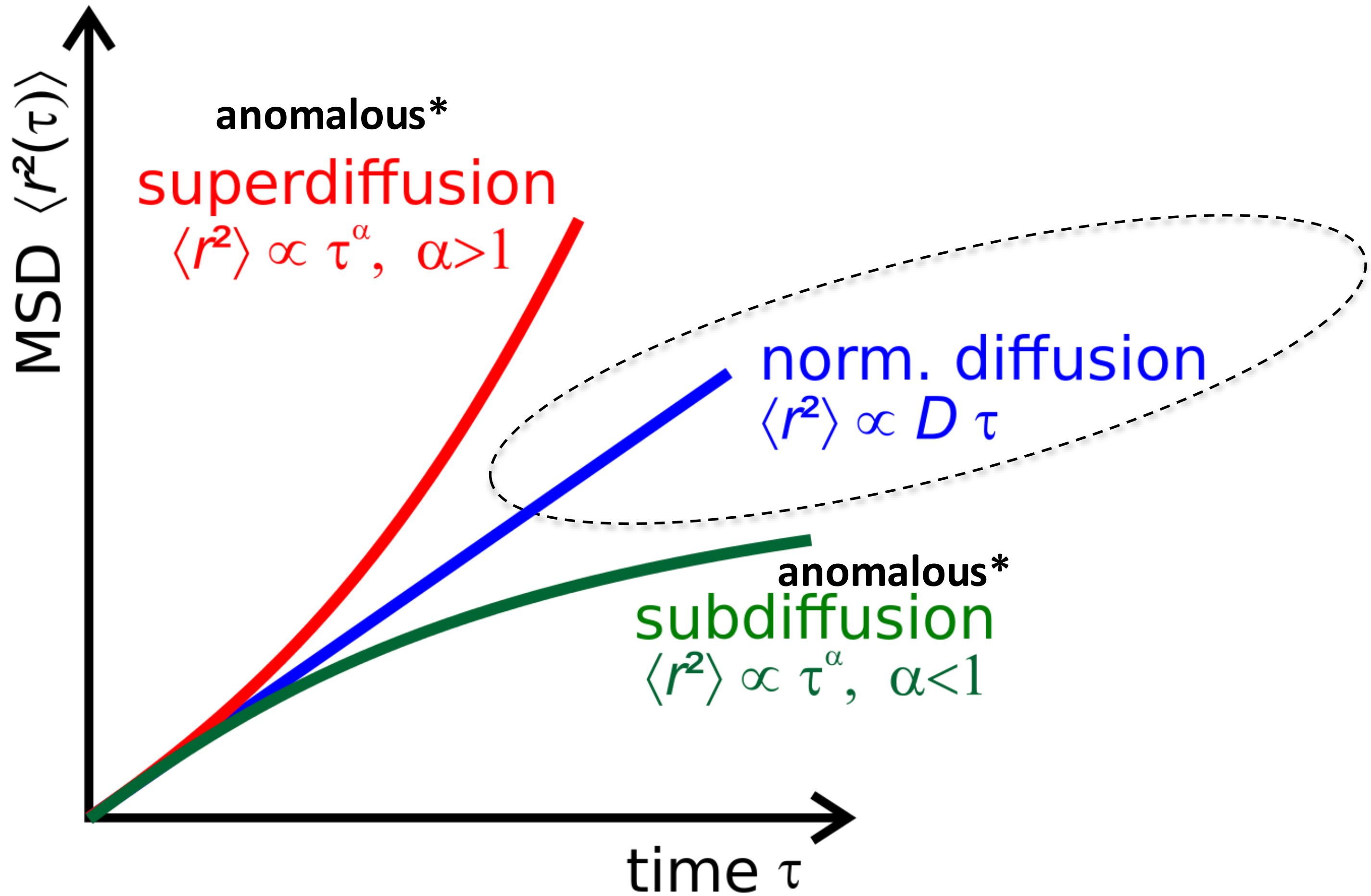
define **Diffusion Coefficient:** $D \equiv \dfrac{a^2}{2\tau}$

Diffusion Relationship:

$$\boxed{\langle r_N^2 \rangle = (2d).D.(\text{elapsed time})}$$

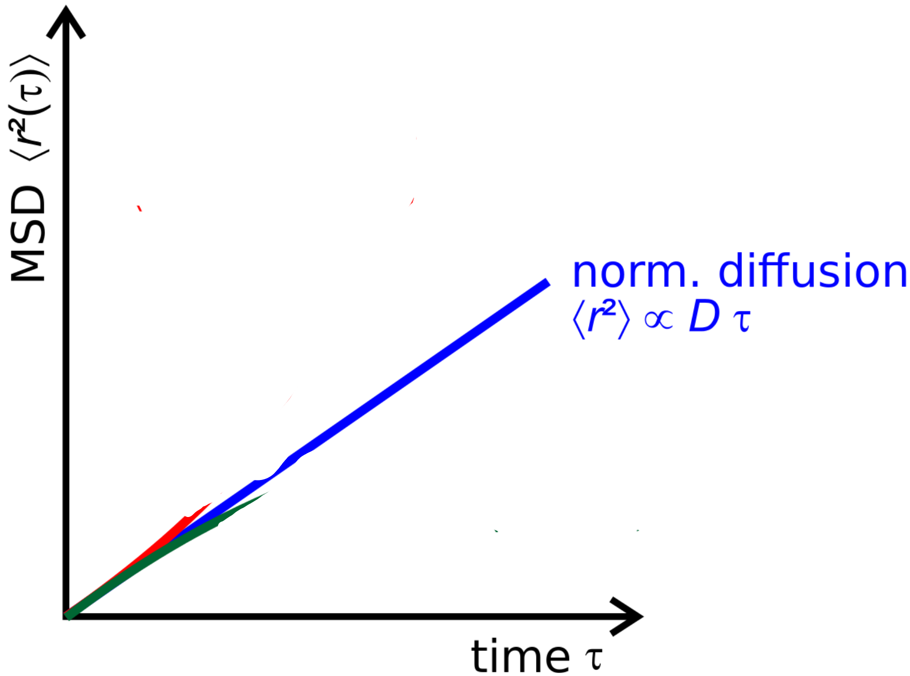
There is large number of independent random walks!

Diffusion: movement of particles in unbiased random walks in any dimension



*** Not a true random walk; external influences at play**

Diffusion: movement of particles in unbiased random walks in any dimension



Diffusion is related to friction

EINSTEIN RELATIONSHIP

$$\zeta D = k_B T$$

Viscous Friction Coefficient

STOKES-EINSTEIN RELATIONSHIP

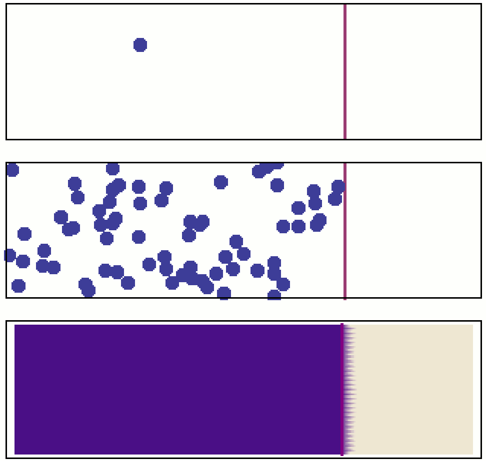
viscosity

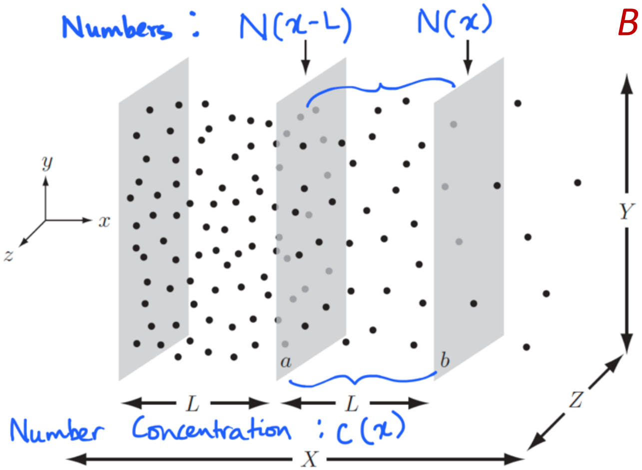
$$6\pi\eta a D = k_B T$$

radius

At a given temperature, viscosity is inversely proportional to the diffusion coefficient.

Flux *(j)*: Number of particles through unit area in unit time





Both space and time variables involved!

$$j \longrightarrow j(x,t)$$

$$c \longrightarrow c(x,t)$$

Under steady state conditions:

1. Fick's Law

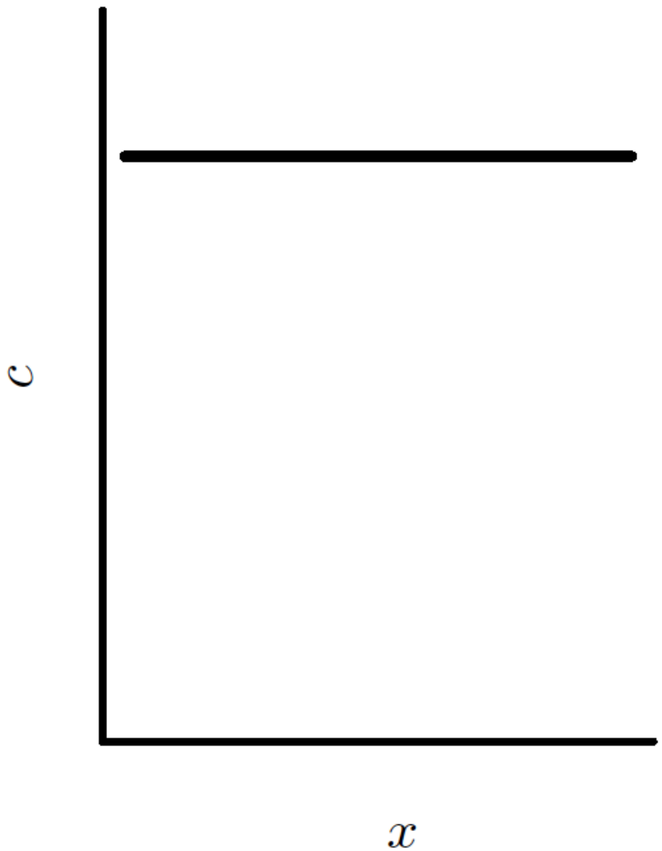
$$\boxed{j = -D\,\frac{dc(x)}{dx}}$$

When concentration is time-dependent:

2. Fick's Diffusion Equation

$$\boxed{\frac{\partial c(x,t)}{\partial t} = +D\,\frac{\partial^2 c(x,t)}{\partial x^2}}$$

Steady state:



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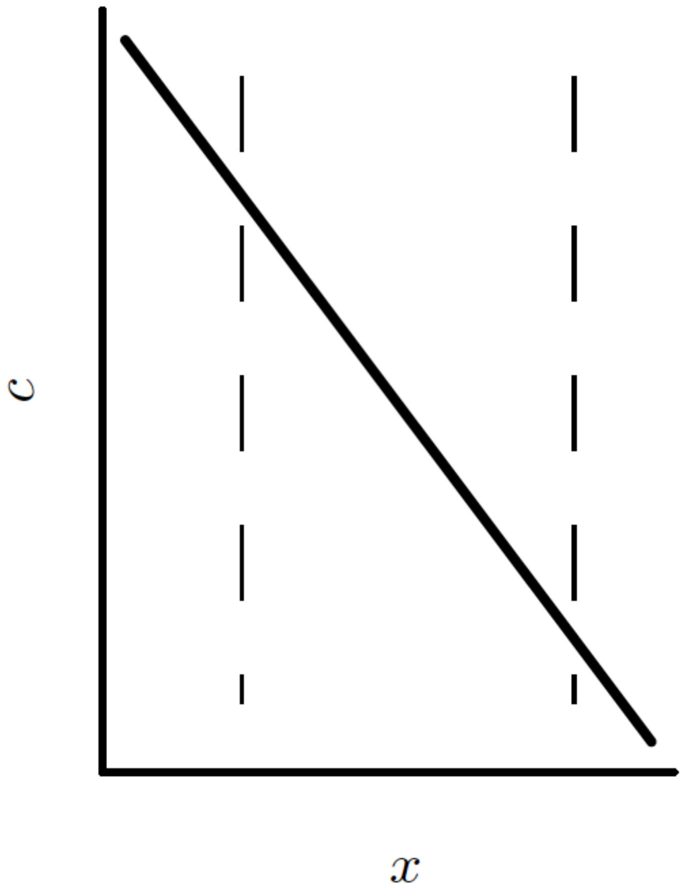

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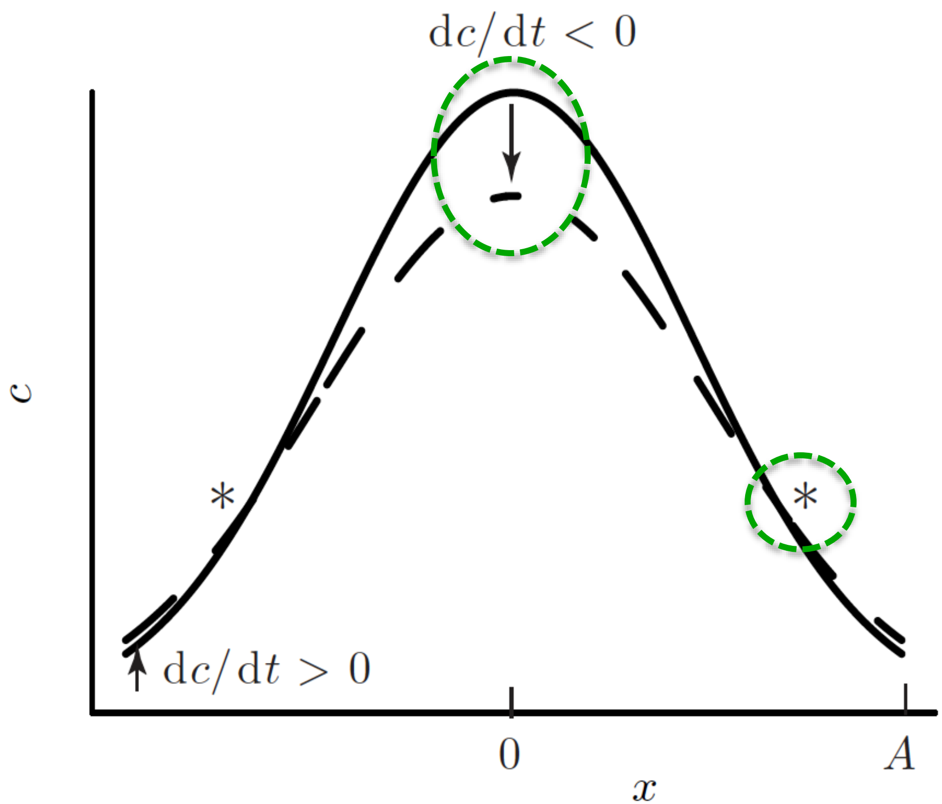
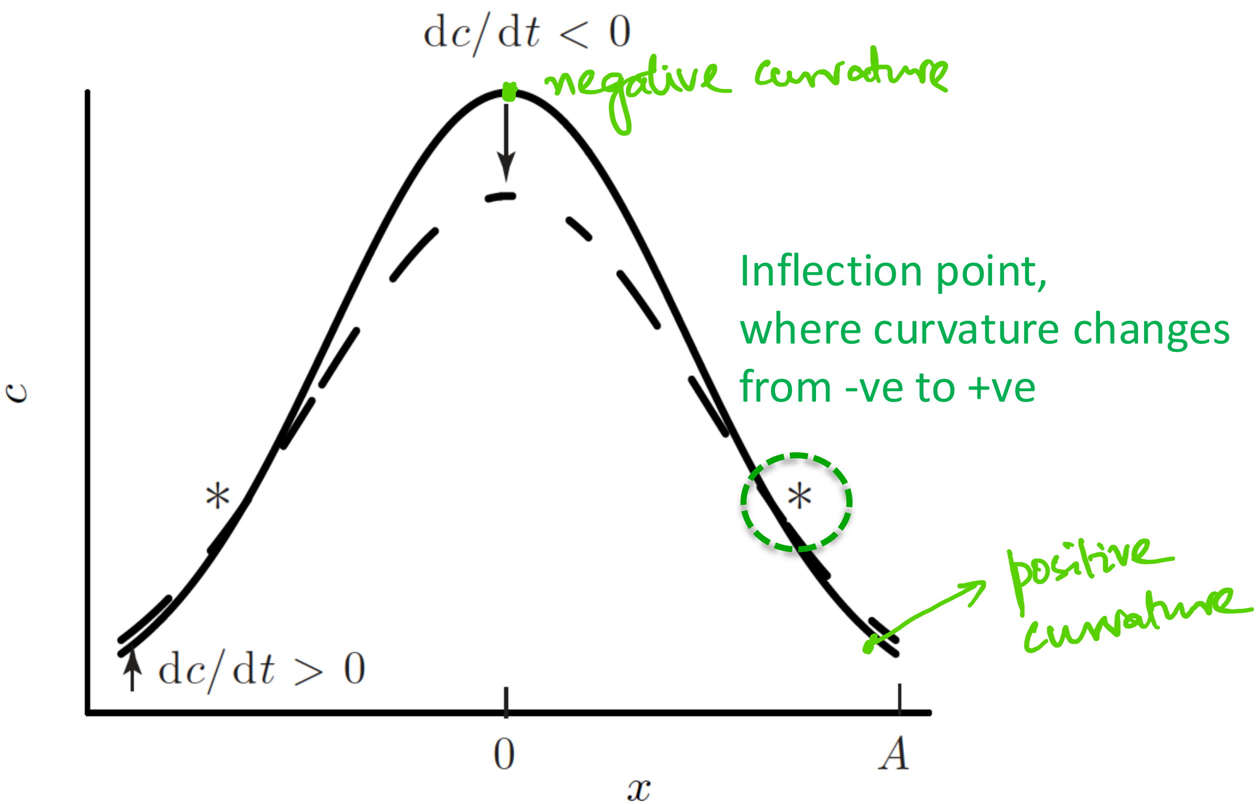
Non steady state, eg. a pulse of nutrients created at x = 0:



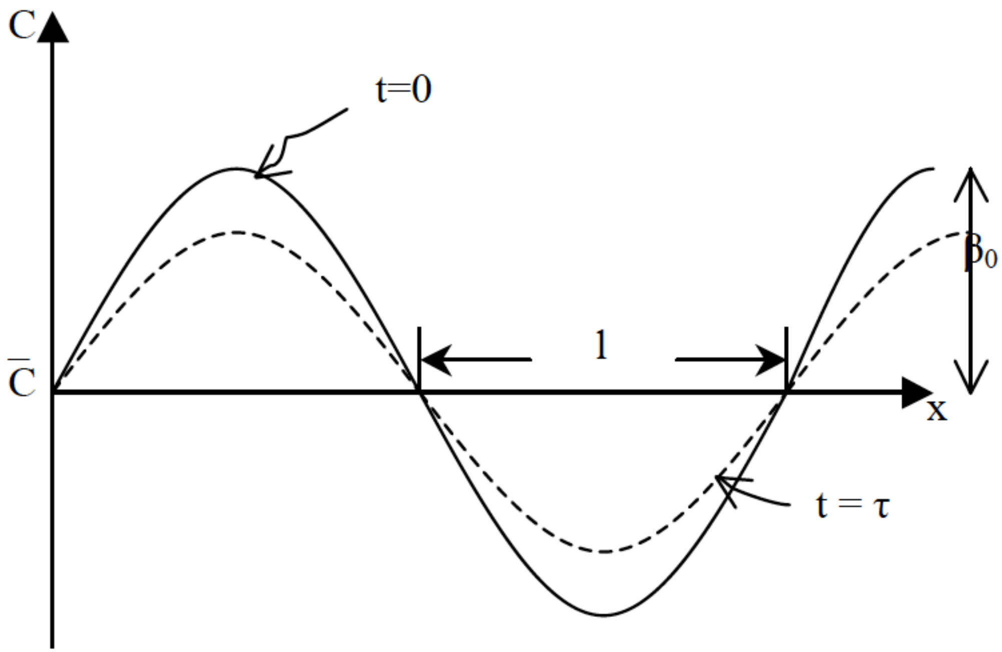
$$\frac{\partial c(x,t)}{\partial t} = +D\frac{\partial^2 c(x,t)}{\partial x^2}$$

Non steady state, eg. a pulse of nutrients created at x = 0:



$$\boxed{\frac{\partial c(x,t)}{\partial t} = +D\frac{\partial^2 c(x,t)}{\partial x^2}}$$

Non steady state cases (advanced applications):



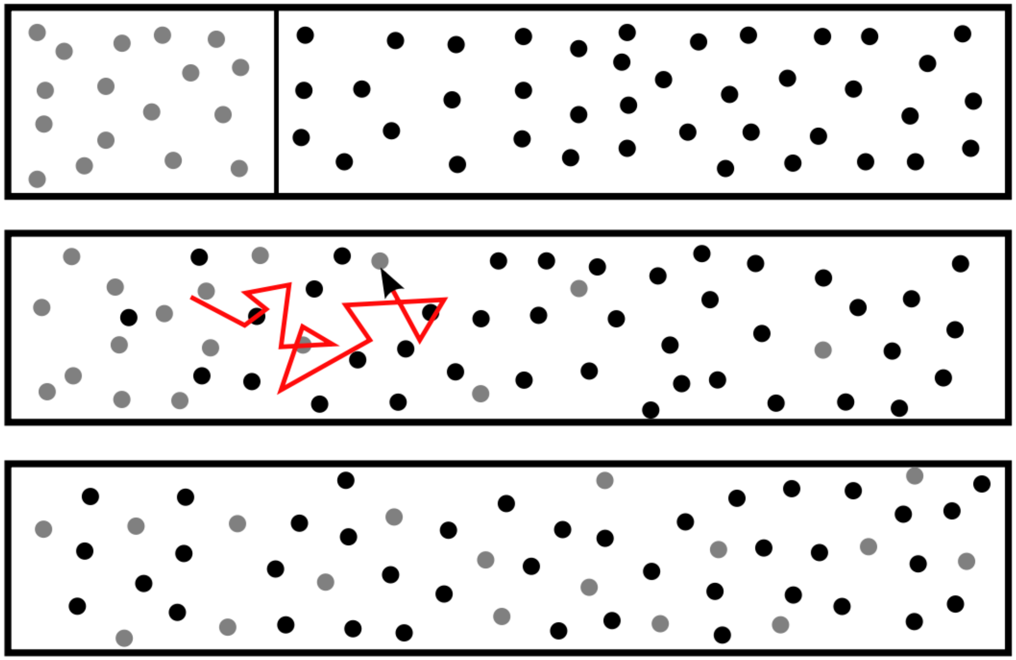
1) Find c(x, 0)

2) Show that the Fick's Diffusion equation is satisfied

At time t, $$c(x,t) = \bar{c} + \beta_0 \sin \frac{\pi x}{l} \times e^{-t/\tau}$$

Where $\tau = l^2/\pi^2 D$, τ is defined as the *relaxation time*.

Offsetting Diffusive Effects with:



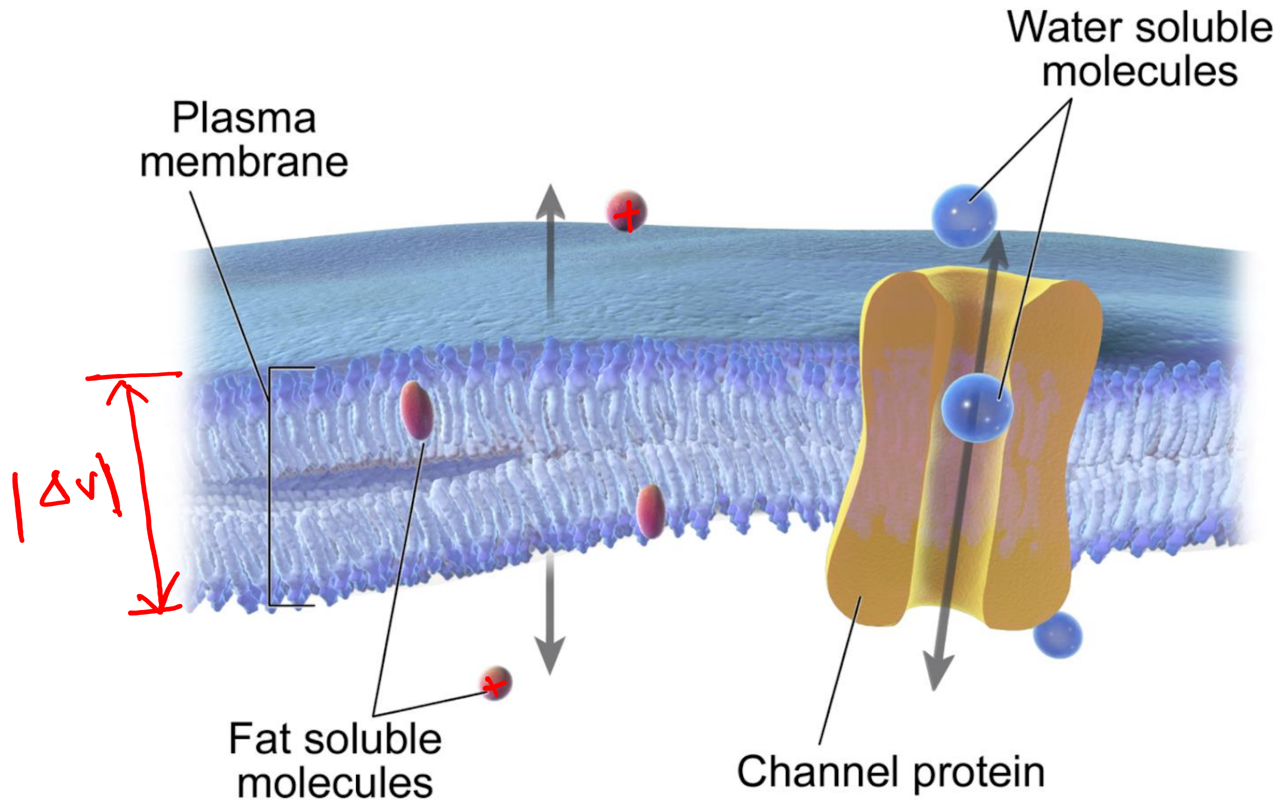
1. Electrostatic potential

2. Gravity

3. Centrifugal force

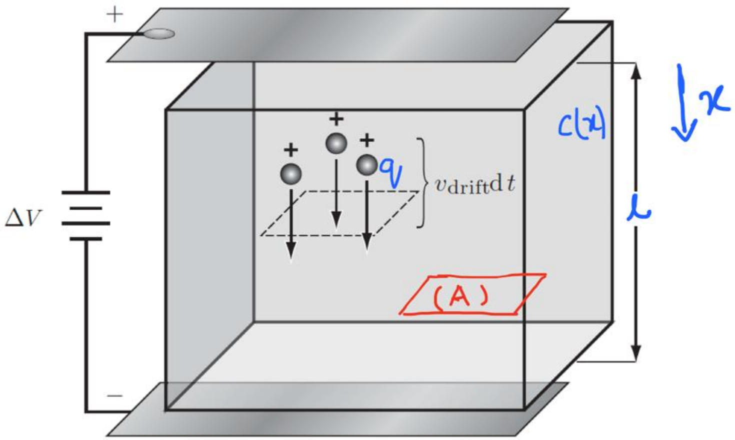
Offsetting Diffusive Effects:

1. Electrostatic potential:



Channels have "selective permittivity" towards molecules

Nernst Relationship: Flux of charges



The net flux,

No external effect

$$j_{net} = \left(-D \frac{\partial c}{\partial x} + j' \right)$$

Electric field, $\xi = \frac{\Delta V}{l}$

Force on each charge, (q)

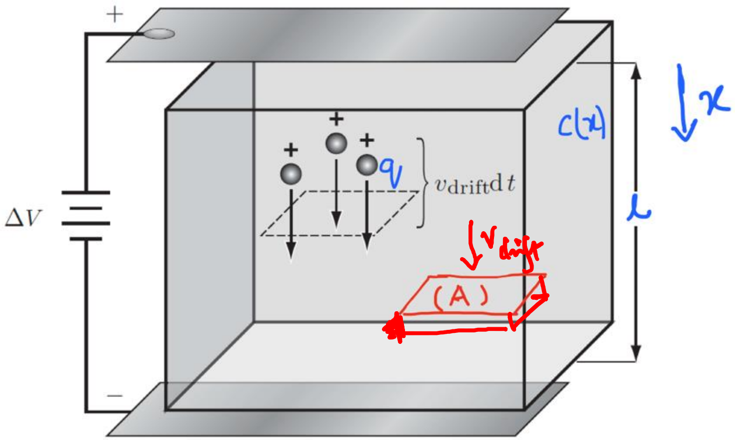
$$f_\xi = q\xi$$

Drift velocity induced by the force,

$$V_{drift} = \frac{f_\xi}{\zeta} = \frac{q \Delta V}{l \zeta}$$

friction coeff.

Nernst Relationship: Flux of charges



The net flux,

No external effect

$$j_{net} = \left(-D\frac{\partial c}{\partial x} + j'\right)$$

No. of ions passing an area 'A' in time Δt,

$$n = (V_{drift} \cdot \Delta t)(A)(c_{ion})$$

Hence,

$$j' = \frac{n}{(\Delta t)\cdot A} = V_{drift}\, c_{ion}$$

$$= q\left(\frac{\Delta V}{l}\right)\frac{1}{\xi}\cdot c_{ion}$$

$$= \frac{q \cdot c_{ion} \cdot \xi}{\zeta}$$

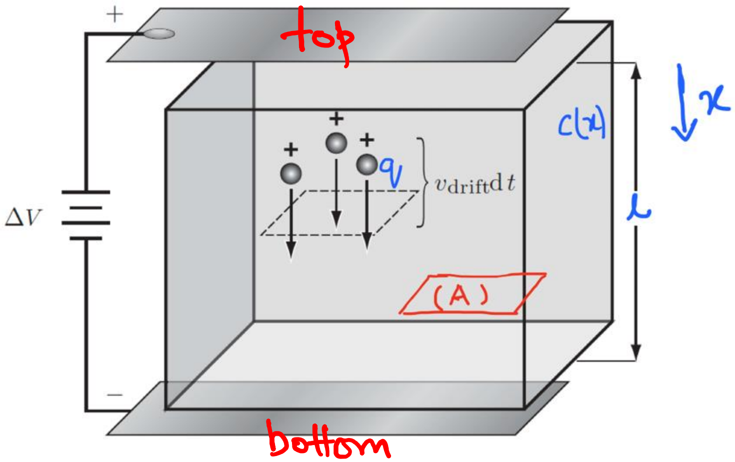
Electric field, $\xi = \frac{\Delta V}{l}$

Force on each charge, $f_\xi = q\xi$

Drift velocity induced by the force,

$$V_{drift} = \frac{f_\xi}{\zeta} = \frac{q\,\Delta V}{l\,\zeta}$$

Nernst Relationship: Flux of charges



Modified flux under the electric field,

NERNST-PLANCK FORMULA

$$\boxed{j = D\left(-\frac{\partial c_{ion}}{\partial x} + \frac{q\xi c_{ion}}{k_B T}\right)}$$

When the net movement if *exactly offset* by the electric field,

$$\frac{\partial c_{ion}}{\partial x} = \frac{q \cdot \xi(x) \cdot c_{ion}}{k_B T}$$

$$\int_{top}^{bottom} \frac{dc}{c} = \frac{q}{k_B T}\int_{top}^{bottom} \xi(x)\,dx$$

Nernst relationship at equilibrium

$$\boxed{\Delta(\ln c) = -\frac{q\Delta V}{k_B T}}$$

Nernst Relationship: Flux of charges

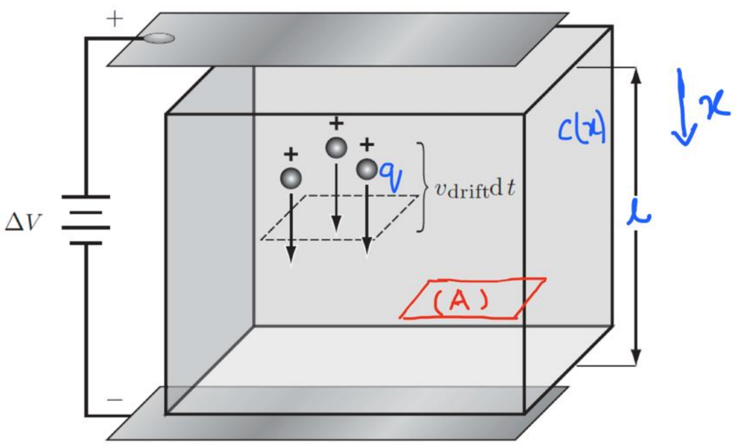
$$C_2 = C_1 \left[e^{-q\Delta V / k_B T} \right]$$

Nernst relationship at equilibrium

$$\boxed{\Delta(\ln C) = -\frac{q\Delta V}{k_B T}}$$

$$\ln\left(\frac{C_2}{C_1}\right) = \ln C_2 - \ln C_1$$

Nernst Relationship: Flux of charges



Modified flux under the electric field,

NERNST-PLANCK FORMULA

$$\boxed{j = D\left(-\frac{\partial c_{ion}}{\partial x} + \frac{q\xi\, c_{ion}}{k_B T}\right)}$$

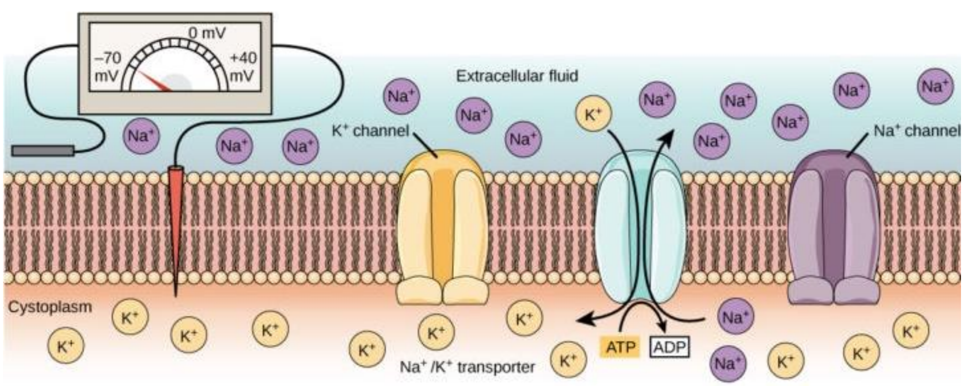
$$c_2 = c_1\left[e^{-q\Delta V/k_B T}\right]$$

Sets the scale on the *electrostatic potential difference across a membrane* under physiological conditions

Nernst relationship at equilibrium

$$\boxed{\Delta(\ln c) = -\frac{q\Delta V}{k_B T}}$$

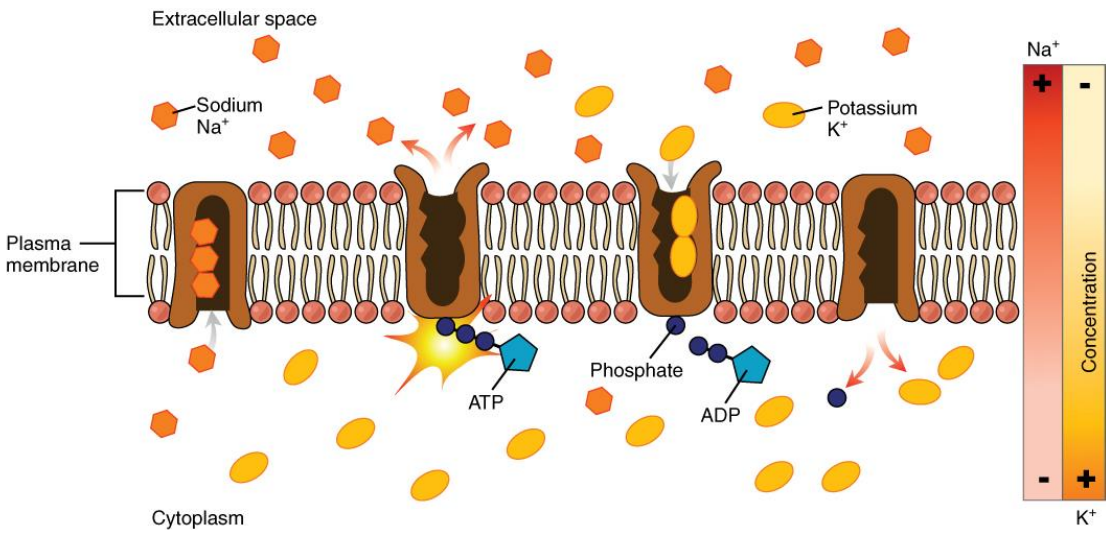
$$\ln\left(\frac{c_2}{c_1}\right) = \ln c_2 - \ln c_1$$



Courtesy: LumenLearning *via* Google Images

HW: The phosphate ion (PO_4^{3-}) is one of the most abundant minerals in the body, and maintenance of extracellular and intracellular phosphate levels is critically important.

- Estimate the potential difference required to maintain an intracellular PO_4^{3-} concentration 5 times higher than the extracellular one at physiological temperature (310 K).



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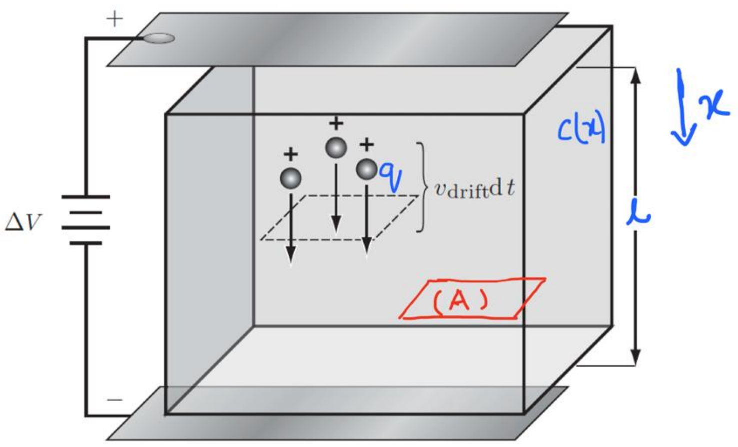
Nernst relationship

$$\Delta(\ln c) = -\frac{q\,\Delta V}{k_B T}$$

Offsetting Diffusive Effects:



Modified flux under the electric field,

NERNST-PLANCK FORMULA

$$\boxed{j = D\left(-\frac{\partial c_{ion}}{\partial x} + \frac{q\xi c_{ion}}{k_B T}\right)}$$

Generalizing:

$$\frac{\partial c}{\partial x} = \frac{q \cdot \xi(x) \cdot c}{k_B T}$$

Force

$$\frac{dc}{c} = \frac{q\xi}{k_B T}\, dx$$

$$\int_{c_1}^{c_2} \frac{dc}{c} = \int \frac{(\text{force})(dx)}{(k_B T)}$$

$$= \ln\left(\frac{c_2}{c_1}\right)$$

Offsetting Diffusive Effects:

With constant force, we may generalize to, $\ln\left(\frac{c_2}{c_1}\right) \equiv \frac{[\text{force}][\text{length}]}{(k_B T)}$

2. Gravity:

$$\ln\left(\frac{c_2}{c_1}\right) \equiv \frac{[mg][h]}{(k_B T)}$$

Ink molecule ~800 g/mol

Vs.

Large protein: 100 kDa

Offsetting Diffusive Effects:

With constant force, we may generalize to, $$\ln\left(\frac{c_2}{c_1}\right) \equiv \frac{[\text{force}][\text{length}]}{(k_B T)}$$

3. Centrifugal force:

At a distance *r*,

$$\ln\left(\frac{c_r}{c_0}\right) \equiv \frac{[m\omega^2 r][r]}{(k_B T)}$$

$$\equiv \left(\frac{1}{k_B T}\right) \times \left[\frac{4\pi^2 (r.p.m)^2}{3600} \times r^2\right] (m)$$

Angular velocity in rotations per minute

Biocompare.com

Prob. Consider a solution of proteins that are of mass 50 kiloDa (Note: 1 Da ~ 1 g/mol). The solution is spun in a low-powered centrifuge that achieves the highest rotation per minute (rpm) of 100.
Find the concentration ratio in the centrifuge tube at (r = 0 cm) with that at (r = 5 cm).