PH2102 Problem Set 5

Q 1) A blob of hot material is ejected from a star with a velocity v which makes an angle θ with the transverse direction to the line of sight from an earth bound observer. A time t later the blob has been displaced to a new position. Note that the distance between the earth and the star is much, much larger than the magnitude of this displacement.

a) What will the time interval be between the light from the two events reaching the observer on the earth, assuming that the longitudinal component is directed towards the earth?

b) Given that the observer on the earth can only observe the motion in the transverse direction, shoa that the apparent speed seen by the observer is given by

$$v_{\text{app}} = c\frac{\beta\cos\theta}{1-\beta\sin\theta}$$

where $\beta = \frac{v}{c}$.

c) From this, show that for the motion may appear superluminal ($v_{\text{app}} > c$) only if $\beta > \frac{1}{\sqrt{2}}$.

d) In the class we talked about a case where $v_{\text{app}} = 6.25c$. Find the lower bound on the actual value of β.

Q 2) Consider a rotation through an angle θ (for the purpose of this problem, as well as usually, the angle is measured in a direction seen to be counterclockwise when looking towards the origin along the axis) about an axis denoted by the unit vector $\hat{n}$. In this problem, we will try to figure out the relation between a vector $\vec{r}$ and its rotated version $\vec{r}'$. For this purpose, it will be convenient to break the vector $\vec{r}$ into components - $\vec{r}_\parallel$, parallel to $\hat{n}$, and $\vec{r}_\perp$, perpendicular to $\hat{n}$.

a) Show that $\vec{r}_\parallel = (\hat{n}\cdot\vec{r})\,\hat{n}$ and $\vec{r}_\perp = \hat{n}\times(\vec{r}\times\vec{n})$.

b) Upon the rotation it is obvious that $\vec{r}_{\parallel}$ remains unchaged, $\vec{r}'_{\parallel} = \vec{r}_{\parallel}$. On the other hand $\vec{r}_{\perp}$ turns through an angle θ. Show that this implies that

$$\vec{r}'_{\perp} = \vec{r}_{\perp} \cos\theta + (\hat{n} \times \vec{r}_{\perp}) \sin\theta$$

c) Show that this means that the rotated vector is given by

$$\vec{r}' = \vec{r} \cos\theta + (\hat{n} \cdot \vec{r})\, \hat{n}\, (1 - \cos\theta) + \hat{n} \times \vec{r} \sin\theta$$

d) Check that this expression reduces to the known result when $\hat{n} = \hat{k}$.

e) By writing this expression in terms of components, write doen the corresponding rotation matrix $R(\hat{n}, \theta)$ explicitly in terms of the components n_1, n_2 and $n_3$of $\hat{n}$, and the angle θ.