

# Quantification across scales

class – 3 (8.8.24)

LS2103 (Autumn 2024)

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## Details of LS2103 (Autumn 2024)

Level: 2

Type: Theory

Credits: 3.0

Course Code	Course Name	Instructor(s)
LS2103	Biophysics	Neelanjana Sengupta

### Syllabus

- Heat as a form of energy: Concept of free energy; free energy transduction; order/disorder in biology; forces and energies
- Molecular interactions: Physical basis and implications in biology
- Dimensions and Units: Dimensional analysis; biomolecules dimensions, arrangements, internal energies
- Special properties of water: Importance in biology
- Overview of structures inside cells: Dimensions, crowding, basic functioning principles, timescales of cellular processes; energies/forces inside live cells. Modes of information transfer;
- Distributions in nature: Origin, implications

# Course modules:

I. Introduction: What is Biophysics?

II. Quantification in Biology:

- Time and Length scales; Dimensional analysis
- Avogadro's Number; pH; concentrations

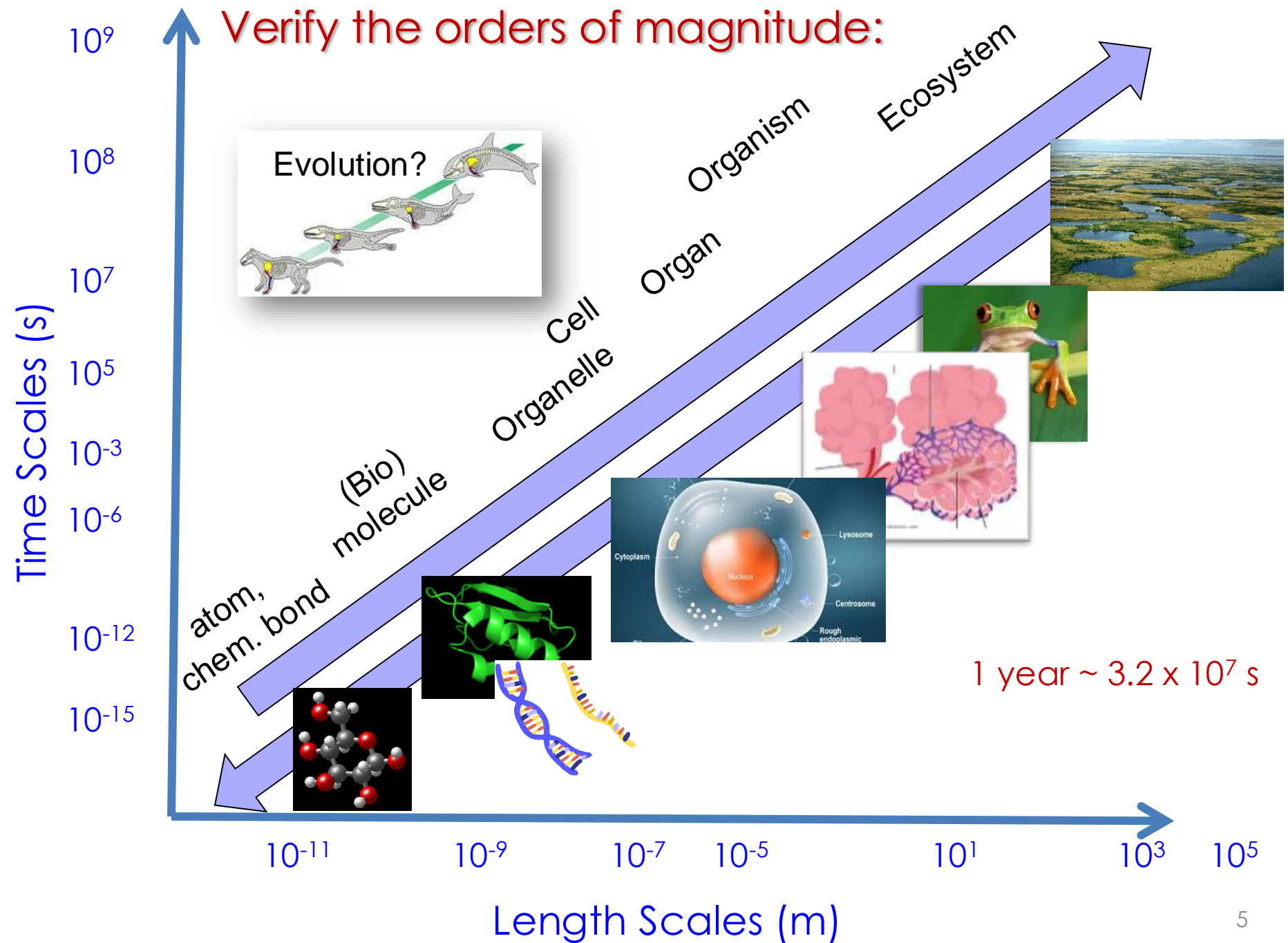
III. Distributions:

- Thermal motion and velocity distribution
- Normal distributions, deviations
- Ideal Gas Laws; Equilibrium; Boltzmann distribution
- Equipartition; role of  $k_B T$  in biology
- Entropy in Biology
- Free Energy, Activation, Scaling Barriers

# Course modules:

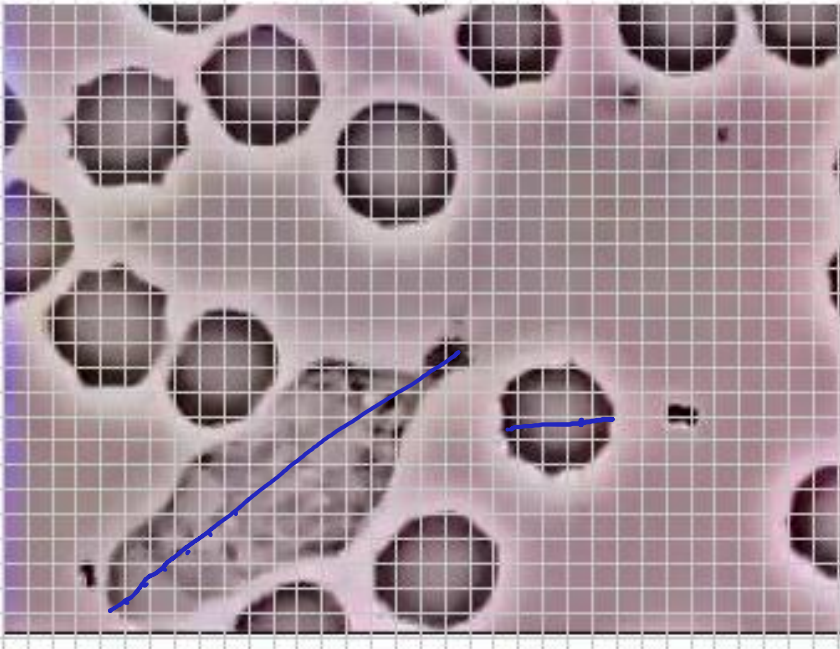
## IV. Connecting the Microscopic and Macroscopic Pictures

- Brownian Motion
- Diffusion(\*) and Dissipation
- Reynold's number
- Osmotic machine
- Hydrophobicity
- Depletion forces



# Neutrophil chasing bacteria

Red blood cell  $\sim 8\ \mu\text{m}$



What are the approx. sizes of:

- a) the neutrophil?
- b) the bacteria it is chasing?
- c) If the movie was in “real time”, what would be the average speed of the bacterium? Does that match with the neutrophil’s speed?

# 'Rule of Thumb': Length Scales

**Table 1.1:** Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
<i>E. coli</i>		
Cell volume	$V_{E. coli}$	$\approx 1 \mu\text{m}^3$
Cell mass	$m_{E. coli}$	$\approx 1 \text{ pg}$
Cell cycle time	$t_{E. coli}$	$\approx 3000 \text{ s}$
Cell surface area	$A_{E. coli}$	$\approx 6 \mu\text{m}^2$
Macromolecule concentration in cytoplasm	$c_{E. coli}^{\text{macromol}}$	$\approx 300 \text{ mg/mL}$
Genome length	$N_{bp}^{E. coli}$	$\approx 5 \times 10^6 \text{ bp}$
Swimming speed	$v_{E. coli}$	$\approx 20 \mu\text{m/s}$
Yeast		
Volume of cell	$V_{\text{yeast}}$	$\approx 60 \mu\text{m}^3$
Mass of cell	$m_{\text{yeast}}$	$\approx 60 \text{ pg}$
Diameter of cell	$d_{\text{yeast}}$	$\approx 5 \mu\text{m}$
Cell cycle time	$t_{\text{yeast}}$	$\approx 200 \text{ min}$
Genome length	$N_{bp}^{\text{yeast}}$	$\approx 10^7 \text{ bp}$
Organelles		
Diameter of nucleus	$d_{\text{nucleus}}$	$\approx 5 \mu\text{m}$
Length of mitochondrion	$l_{\text{mito}}$	$\approx 2 \mu\text{m}$
Diameter of transport vesicles	$d_{\text{vesicle}}$	$\approx 50 \text{ nm}$

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Genome length	$N_{bp}^{E. coli}$	$\approx 5 \times 10^6 \text{ bp}$
Swimming speed	$v_{E. coli}$	$\approx 20 \mu\text{m/s}$

- I. What is the “**surface-to-volume**” ratio of *E. coli*?
  - Unit?
  - How is this expected to scale with size of organism?
- II. What is the average cell cycle duration of *E. coli* in days?
- III. Approx. how many times it's 'body size' does *E. coli* cover per second?
  - How would you compare this movement with that of a fastest humans?  
(Usain Bolt's speed  $\sim 12.4 \text{ m/s}$ , ie.  $\sim 10 \times 1.2 \text{ m/s}$ )



# 'Rule of Thumb': Length Scales

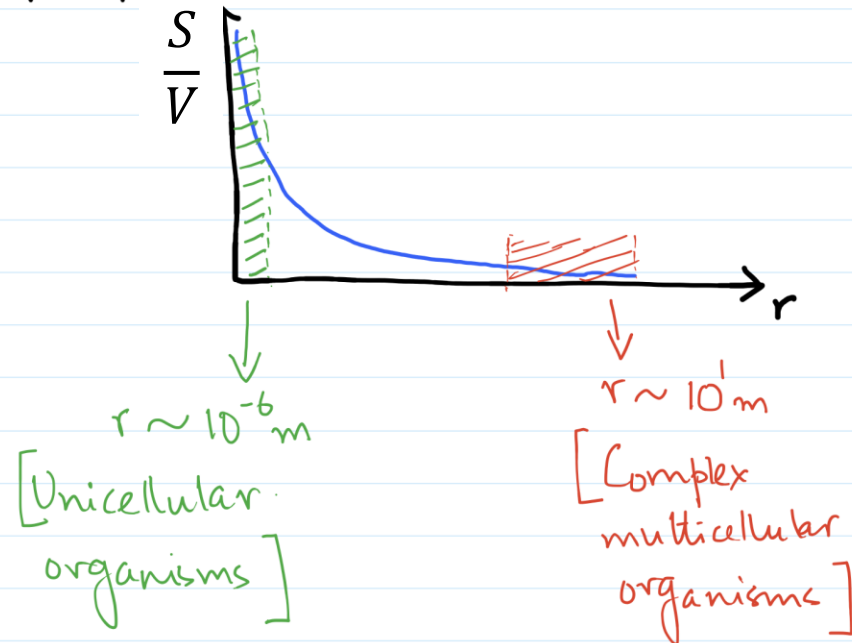
- I. Construct a model for the “**surface-to-volume**” ratio of *E. coli*

# 'Rule of Thumb': Length Scales

Surface-to-Volume ratios of organisms (in general)

In general, for any entity of spatial scale 'r',

$$\frac{S}{V} \propto \frac{1}{r}$$

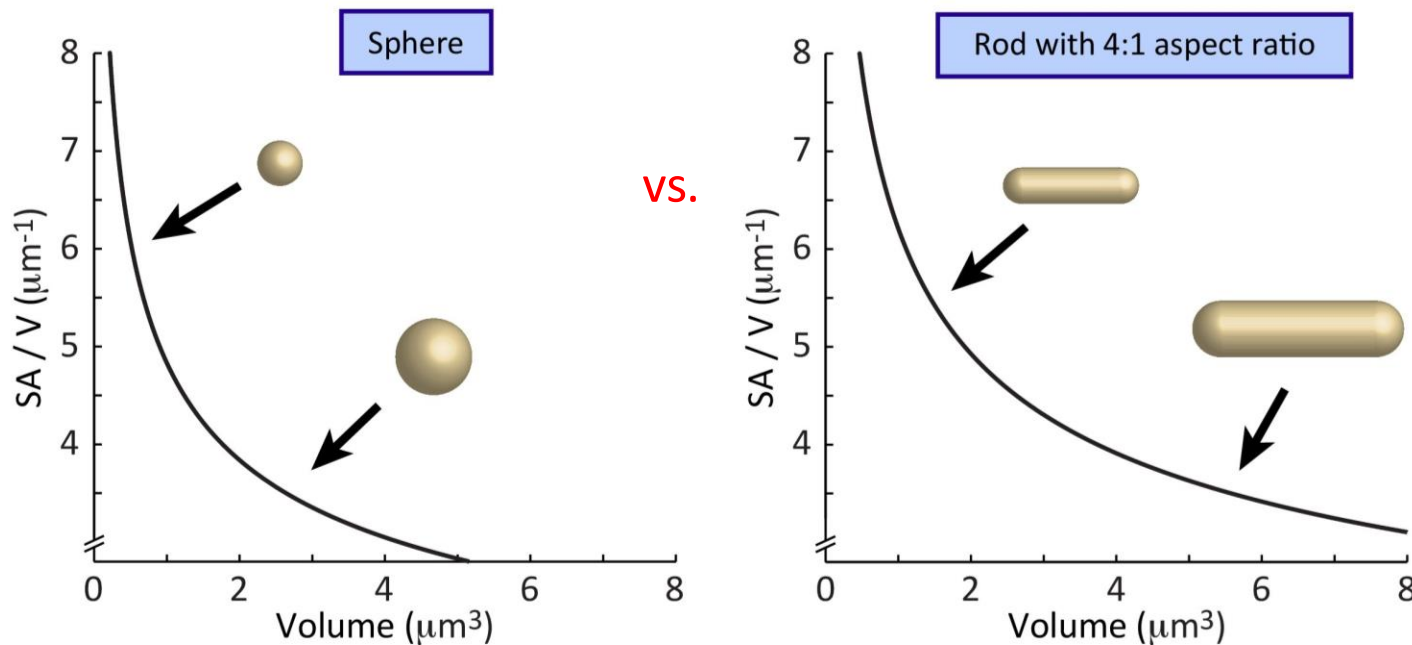


- The (S/A) ratio is high for prokaryotes
- It reduces in complex multicellular organisms
  - *It is not very sensitive to size in multi-cellular organisms*

# Bacteria: Surface to Volume (S/V) ratio

- Within a species, size can be variable
- But (S/V) generally maintained if the conditions do not change
- What triggers bacteria shape and size changes?

(B) Hold shape constant



# 'Rule of Thumb': Length Scales

**Table 1.1:** Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
Water		
Volume of molecule	$V_{\text{H}_2\text{O}}$	$\approx 10^{-2} \text{ nm}^3$
Density of water	$\rho$	$1 \text{ g/cm}^3$
Viscosity of water	$\eta$	$\approx 1 \text{ centipoise}$ $(10^{-2} \text{ g/(cm s)})$
Hydrophobic embedding energy	$\approx E_{\text{hydr}}$	$2500 \text{ cal/(mol nm}^2\text{)}$

- What is the approx. radius of water molecule?
- What is the " $E_{\text{hydr}}$ " given above?

# 'Rule of Thumb': Length Scales

**Table 1.1:** Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
Amino acids and proteins		
Radius of "average" protein	$r_{\text{protein}}$	$\approx 2 \text{ nm}$
Volume of "average" protein	$V_{\text{protein}}$	$\approx 25 \text{ nm}^3$
Mass of "average" amino acid	$M_{\text{aa}}$	$\approx 100 \text{ Da}$
Mass of "average" protein	$M_{\text{protein}}$	$\approx 30,000 \text{ Da}$
Protein concentration in cytoplasm	$c_{\text{protein}}$	$\approx 150 \text{ mg/mL}$
Characteristic force of protein motor	$F_{\text{motor}}$	$\approx 5 \text{ pN}$
Characteristic speed of protein motor	$v_{\text{motor}}$	$\approx 200 \text{ nm/s}$
Diffusion constant of "average" protein in cytoplasm	$D_{\text{protein}}$	$\approx 10 \mu\text{m}^2/\text{s}$

- How does mass of a typical amino acid, and a typical protein, compare with mass of a water molecule?

# 'Rule of Thumb': Length Scales

**Table 1.1:** Rules of thumb for biological estimates.

Quantity of interest	Symbol	Rule of thumb
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## Lipid bilayers

Thickness of lipid bilayer	$d$	$\approx 5 \text{ nm}$
Area per molecule	$A_{\text{lipid}}$	$\approx \frac{1}{2} \text{ nm}^2$
Mass of lipid molecule	$m_{\text{lipid}}$	$\approx 800 \text{ Da}$

- Consider that a protein of radius 1 nm displaces water to embed into a lipid bilayer.
- Estimate the energetic cost in kcal/mol.

## DNA

Length per base pair	$l_{\text{bp}}$	$\approx 1/3 \text{ nm}$
Volume per base pair	$V_{\text{bp}}$	$\approx 1 \text{ nm}^3$
Charge density	$\lambda_{\text{DNA}}$	$2 e/0.34 \text{ nm}$
Persistence length	$\xi_p$	$50 \text{ nm}$