Stefan-Boltzmann's law of radiation with Cobra3

Related Topics
Black body radiation, thermoelectric e.m.f. (electro magnetic force), temperature dependence of resistances.

Principle
The energy emitted by a black body per unit area and unit time is proportional to the fourth power of the body's absolute temperature (Stefan-Boltzmann law). This is also valid for a grey body. A grey body has a surface with a wavelength-independent absorption coefficient of less than one. In this experiment the filament of an incandescent lamp is taken as a model for a grey body and its emission is investigated as a function of its temperature.

Equipment

Cobra3 Basic Unit, USB	12150.50	1
Power supply, 12 V-	12151.99	1
Cobra3 Universal Writer Software	14504.61	1
Thermopile, molltype	08479.00	1
Shielding tube, for 08479.00	08479.01	1
Power supply var.15VAC/12VDC/5A	13530.93	1
Lamp holder E 14, on stem	06175.00	1
Filament lamp 6 V/5 A, E14	06158.00	3
Connection box	06030.23	1
Resistor in plug-in box 100 Ohm	06057.10	1
Digital multimeter	07122.00	1
Connecting cord, 500 mm, blue	07361.04	3
Connecting cord, 500 mm, red	07361.01	2
Barrel base -PASS-	02006.55	2
Meter scale, l = 1000 mm	03001.00	1
PC, Windows® 95 or higher		

Tasks
1. Measure the resistance of the filament of the incandescent lamp at room temperature and calculate the filament's resistance R_0 at 0°C.
2. Measure the energy flux density of the incandescent lamp at different values of lamp current. Determine the corresponding filament temperature by the resistance calculated from the measured values of lamp current and lamp voltage assuming a second-order temperature dependence of the filament resistance.

Set-up and Procedure
1. Connect the Cobra3 Basic Unit to the computer USB port and start the program "measure". Select "Gauge" > "Universal Writer". Select the "Fast measurement" chart and set the parameters according to Fig. 2.

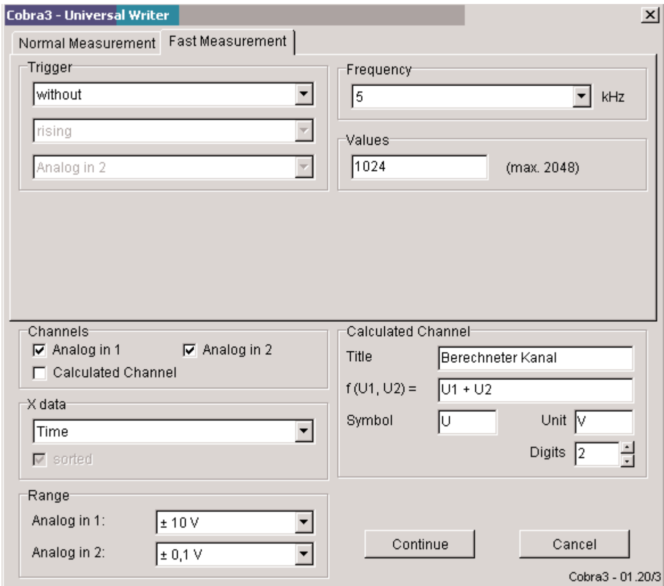

Fig. 2: Universal Writer settings

Fig. 1: Experimental set-up for part two

First measure the lamp filament's resistance at room temperature using the circuit shown in Fig.3. For the set-up of the circuit use the connection box and the 100 Ohm resistor.

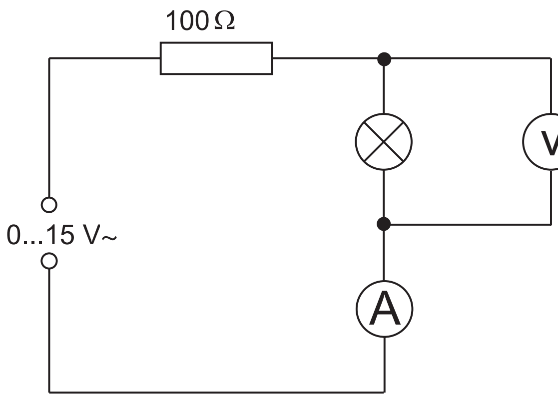

Fig. 3: Circuit for measuring the resistance of the lamp at room temperature.

The voltage U on the lamp is to be measured with the "Analog In 2 / S2" and the current I through the lamp with the digital multimeter. Use the power supply as AC source. The 100 Ohm resistor is needed for fine adjustment of the current. Be sure you use the "AC" setting of the digital multimeter. Adjust the current to 100 mA AC and start the measurement with the "Continue" button.
After ending the measurement use the "Survey" function to evaluate the peak-to-peak voltage. The function "Analysis" > "Smooth" may help with the evaluation to improve the visibility of the peaks.
Divide the peak-to-peak voltage by two and by the square root of two to determine the effective voltage on the lamp.
Do the same for a lamp current of 50 mA AC. These currents should be low enough not to considerably heat the filament.
Calculate the resistance $R(t_R)$ at room temperature t_R (in °C) (by Ohm's law $R = U/I$) and from this value the resistance at 0°C R_0 by

$$R_0 = R(t_R) / (1 + \alpha\, t_R + \beta\, t_R^2)$$

with

$\alpha = 4.82 \cdot 10^{-3}$ K^{-1} and
$\beta = 6.67 \cdot 10^{-7}$ K^{-2}.

a and b are material constants of tungsten.

2. Now set up the equipment according to Fig. 1 and align the thermopile in a way that it receives the lamp's radiation with the distance between lamp and thermopile less than 20 cm. The helix of the filament should be at right angle to the thermopile.
The digital multimeter is to measure the current through the lamp and "Analog In 1 / S1" is to measure the voltage on the lamp.
Set the AC current so that the digital multimeter shows 1 A. Wait a minute until the thermopile has tempered and start a measurement with the "Continue" button.
Increase the current through the lamp in steps of 0.5 A taking a measurement for each current strength up to 5.5 A. U_2 is proportional to the energy flux from the lamp if there are no other sources detected by the thermopile as disturbing background. Wait always at least one minute for tempering of the thermopile.
For evaluation use the function "Survey" to measure the amplitude of "Analog In 1" voltage U1 in the just recorded measurement (see Fig. 4). Take down the effective value, that is the amplitude (half the peak-to-peak value) divided by the square root of two. Use the "Show average value" function to evaluate the "Analog In 2" voltage U2. Note down both results in the "measure" program using "Measurement" > "Enter data manually…" with the current I as x-data set and two channels (for U1 and U2) – measure the new values and continue with the next current step.

Theory and evaluation
If the energy flux density L of a black body, e.g. energy emitted per unit area and unit time at temperature T and wavelength λ within the interval $d\lambda$, is designated by $dL(T, \lambda)/d\lambda$,

Planck's formula states:

$$\frac{dL(\lambda, T)}{d\lambda} = \frac{2c^2\, h\lambda^{-5}}{e^{\frac{hc}{\lambda kt}} - 1} \qquad (1)$$

with: c = velocity of light ($3.00 \cdot 10^8$ [m/s])

h = Planck's constant ($6.62 \cdot 10^{-34}$ [J · s])

k = Boltzmann's constant ($1.381 \cdot 10^{-23}$ [J · K^{-1}])

Integration of equation (1) over the total wavelength-range from $\lambda = 0$ to $\lambda = \infty$ gives the flux density $L(T)$ (Stefan-Boltzmann's law).

$$L(T) = \frac{2\pi^5}{15} \cdot \frac{k^4}{c^2\, h^3} \cdot T^4 \qquad (2)$$

respectively $L(T) = \sigma \cdot T^4$

with $\sigma = 5.67 \cdot 10^{-8}$ [W · m^{-2} · K^{-4}]

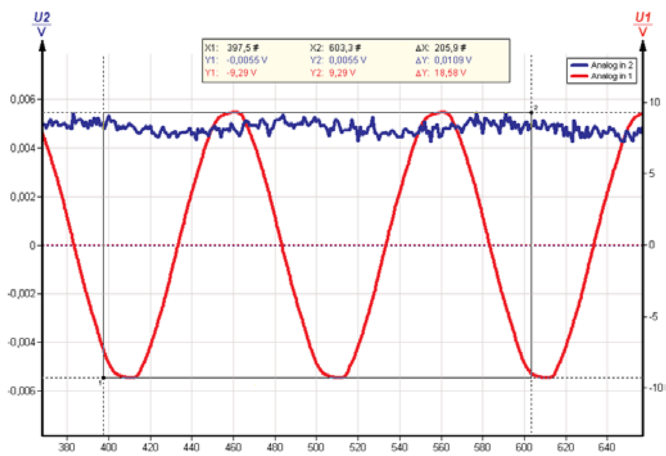

Fig. 4: Example for the use of the "Survey function" for evaluation – the amplitude is half the peak-to-peak value

The proportionality $L \sim T^4$ is also valid for a so-called "grey" body whose surface shows a wavelength-independent absorption-coefficient of less than one.

To prove the validity of Stefan-Boltzmann's law, we measure the radiation emitted by the filament of an incandescent lamp which represents a "grey" body fairly well. For a fixed distance between filament and thermopile, the energy flux ϕ which hits the thermopile is proportional to $L(T)$.

$$\phi \sim L(T)$$

Because of the proportionality between ϕ and the thermo-electric e.m.f., U_{therm} of the thermopile, we can also write:

$$U_{therm} \sim T^4$$

if the thermopile is at a temperature of zero degrees Kelvin. Since the thermopile is at room temperature T_R it also radiates due to the T^4 law so that we have to write:

$$U_{therm} \sim (T^4 - T_R^4)$$

Under the present circumstances, we can neglect T_R^4 against T^4 so that we should get a straight line with slope "4" when representing the function $U_{therm} = f(T)$ double logarithmically.

$$\lg U_{therm} = 4 \lg T + \text{const.} \tag{3}$$

The absolute temperature $T = t + 273$ of the filament is calculated from the measured resistances $R(t)$ of the tungsten filament (t = temperature in centigrade). For the tungsten filament resistance, we have the following temperature dependence:

$$R(t) = R_0 \, (1 + \alpha t + \beta t^2) \tag{4}$$

with R_0 = resistance at 0 °C

α = $4.82 \cdot 10^{-3}$ K^{-1}

β = $6.76 \cdot 10^{-7}$ K^{-2}

The resistance R_0 at 0 °C can be found by using the relation:

$$R_0 = \frac{R(t_R)}{1 + \alpha \cdot t_R + \beta \cdot t_R^2} \tag{5}$$

Solving $R(t)$ with respect to t and using the relation $T = t + 273$ gives:

$$T = 273 + \frac{1}{2\beta}\left[\sqrt{\alpha^2 + 4\beta\left(\frac{R(t)}{R_0} - 1\right)} - \alpha\right] \tag{6}$$

$R(t_R)$ and $R(t)$ are found by applying Ohm's law, e.g. by voltage and current measurements across the filament.

For evaluation of the data add the calculated resistance $R(t)$ as a new channel to the manually created measurement by "Analysis" > "Channel modification…" dividing the "Analog In 1 / S1" values, i.e. the voltage on the lamp U_1, by the current I values. Add the temperature T, which was calculated from the resistance values, to that measurement in the same manner: Equation (6) has to be written in a form suitable for the channel modification with the actual symbol for R and the numerical value for R_0 inserted (which was measured in 1.) like

f := 273 + 7396,45 * ((0,2323 + 0,02704 * (R/R0 – 1))^0,5 – 0,482).

See Fig. 5 and Fig. 6 (in this case is R_0 = 0.16 Ohm).

Now use "Measurement" > "Channel manager…" and set the temperature data to the x-axis so as to create a plot of the thermopile voltage vs. temperature T. A result may look like Fig. 7.

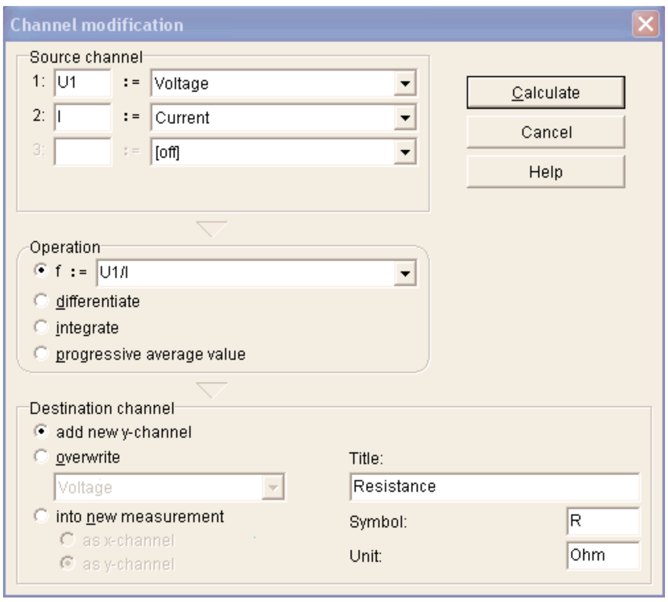

Fig. 5

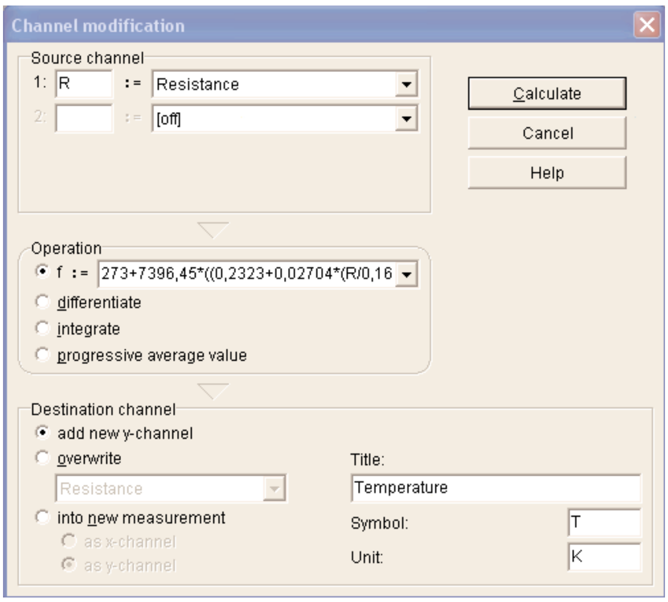

Fig. 6

The following table shows the corresponding measurement values, the resistance R_0 was calculated as 0.16 Ohm, and the slope of the regression line is with 3.97 close to the theoretical value of four for the exponent of Stefan-Boltzmann's law.

Current I/A	Voltage U_{eff}/V	Energy flux U_{therm}/mV	Temperature T/K
1.00	0.25	0.02	384.74
1.50	0.50	0.05	491.09
2.00	1.03	0.12	704.29
2.50	1.70	0.40	893.31
3.00	2.55	0.75	1075.62
3.50	3.39	1.35	1202.10
4.00	4.42	2.30	1339.01
4.50	5.45	3.40	1442.76
5.00	6.65	4.80	1557.97
5.50	7.92	6.60	1662.25

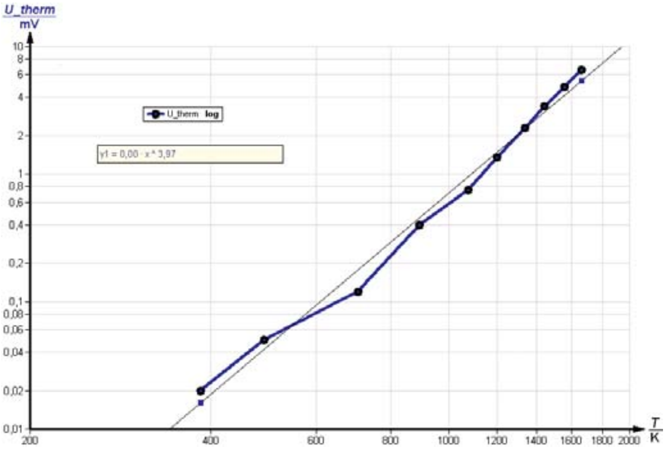

Fig. 7: Example of measurement results