1. Consider how to solve the steady-state motion of a forced oscillator if the driving force is of the form $F = F_0 \sin \omega t$ instead of $F_0 \cos \omega t$.

2. An object of mass 0.2 kg is hung from a spring who spring constant is 80 Nm^{-1}. The body is subjected to a resistive force given by $-bv$, where v is its velocity in ms^{-1} and $b = 4\ \text{Nm}^{-1}\text{s}$.
   (a) Set up the differential equations of motion for free oscillations of the system and find the period of such oscillations.
   (b) The object is subjected to a sinusoidal driving force given by $F(t) = F_0 \sin \omega t$, where $F_0 = 2$ N and $\omega = 30\ \text{rad s}^{-1}$. In the steady state, what is the amplitude of the forced oscillation?

3. A block of mass m is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system:
   1. If the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.
   2. The viscous resistive force is equal to mg if the block moves with a certain known speed u.

   (a) For this complete system, write the differential equation governing horizontal oscillations of the mass in terms of m, g, h, and u.
   Answer the following for the case that $u = 3\sqrt{gh}$:
   (b) What is the angular frequency of the damped oscillation?
   (c) After what time, expressed as a multiple of $\sqrt{h/g}$, is the energy down by a factor $1/e$?
   (d) What is the Q factor of the oscillator?
   (e) This oscillator, initially in its rest position, is suddenly set into motion at $t = 0$ by a bullet of negligible mass but non-negligible momentum travelling in the positive x direction. Find the value of the phase angle δ in the equation $x = Ae^{-\gamma t/2} \cos(\omega t - \delta)$ that describes the subsequent motion and sketch x versus t for the first few cycles.
   (f) If the oscillator is driven with the force $mg \cos \omega t$ where $\omega = \sqrt{2g/h}$ what is the amplitude of the steady state response?

4. A simple pendulum has a length of 1 m. In free vibration the amplitude of its swings falls off by a factor e in 50 swings. The pendulum is set into forced vibration by moving its point of suspension horizontally in S.H.M. with an amplitude of 1 mm.
   (a) Show that if the horizontal displacement of the pendulum bob is x and the horizontal displacement of the support is ξ, the equation of motion of the bob for small oscillation is

   $$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{g}{l}x = \frac{g}{l}\xi$$

   Solve this equation for steady-state motion if $\xi = \xi_0 \cos \omega t$. (Put $\omega_0 = g/l$.)
   (b) At exact resonance, what is the aptitude of the motion of the pendulum bob? (First, use the given information to find Q.)
   (c) At what angular frequencies is the amplitude half its resonant value?

5. Imagine a simple seismograph consisting of a mass m hung from a spring on a rigid framework attached to the earth, as shown. The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but the dynamically significant acceleration is the acceleration of m relative to the fixed stars (inertial frame).

   (a) Using y to denote the displacement of m relative to the earth and η to denote the displacement of the earth's surface itself, show the equation of motion is

   $$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 x = -\frac{d^2\eta}{dt^2}$$

   (b) Solve for y (steady-state vibration) if $\eta = C\cos\omega t$.

   (c) Sketch a graph of the amplitude A of the displacement y as a function of omega (supposing C the same for all ω).

   (d) A typical long-period seismometer has a period of about 30 second and a Q of about 2. As the result of a violent earthquake, the earth's surface may oscillate with a period of about 20 minute and with an amplitude such that the maximum acceleration is about 10^{-9} ms^{-2}. How small a value of A must be observable if this is to be detected?