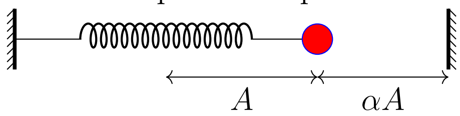
1. Consider a spring-mass oscillator of time period T. A wall is placed αA distance to the right from the equilibrium position of the oscillator, as shown below. [5]



The oscillator is given an initial displacement A towards the left and released from the rest. Consider all collisions to be elastic.

(a) Find the time it takes for the oscillator to move from the equilibrium to the wall. [2]

(b) Now find the time period of the oscillator in terms of α and T? [2]

(c) What is the period if $\alpha = \sqrt{3}/2$? [1]

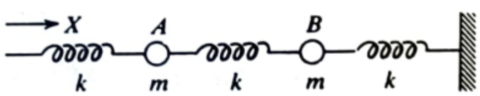
2. Consider two pendulums, a and b, with the same string length L, but with different bob masses, M and $3M$. They are coupled by a spring of spring constant K which is attached to the bobs. Assuming small angle oscillations, [5]

(a) Find the equations of motion using angles of the pendulums (w.r.t. the vertical) as dynamical variables. [2]

(b) Find the normal mode frequencies. [2]

(c) Find the ratios between normal mode amplitudes. [1]

3. Two equal masses m are connected to three identical springs (spring constant k) on a frictionless horizontal surface (see figure). One end of the system is fixed; the other is driven back and forth via a displacement $X = X_\circ \cos \omega t$. [5]



(a) Find the equations of motion of both the masses. [2]

(b) What are the normal frequencies? Hint: assume the left boundary to be fixed with $X_\circ = 0$ for this part of the problem. [2]

(c) What are the ratio of amplitudes of the masses when ω is equal to the lowest normal mode angular frequency? [1]

4. While moving, a mass m experiences a resistive force $-m\gamma v$ where v is the velocity, and γ is a constant, but no spring-like restoring force. [5]

(a) Show that its displacement as a function of time is of the form [2]

$$x = C - \frac{v_0}{\gamma} e^{-\gamma t},$$

where, C and v_0 are constants.

(b) At $t = 0$ the mass is at rest at $x = 0$. At this instant a driving force $F = F_0 \cos \omega t$ is switch on. Find the values of A and δ in the steady state solution $x = A \cos(\omega t - \delta)$. [1]

(c) Write down the general solution [the sum of parts (a) and (b)] and find the values of C and v_0 from the conditions that $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$, and $\omega = \gamma$. [2]