Derivative

The derivative of the function $f(x)$ with respect to the variable x is the function $f'(x)$ whose value is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ whenever the limit exits.

The derivative of symbolic function $f(x)$ can be evaluated in MATLAB with the command `diff`.

Below code illustrates the derivative of $y = \sqrt{x}$.

```
>> syms x
>> y=sqrt(x);
>> diff(y)
ans =
1/(2*x^(1/2))
```

Similarly the derivative of $(3x^2 + 1)^2$ is

```
>> syms x
>> y=(3*x^2+1)^2
>> diff(y)
ans =
12*x*(3*x^2 + 1)
```

Tangent Line

The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with the slope $m = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

The following code illustrate the plotting of the tangent to the curve $y = x^2$ at the point $(2,4)$.

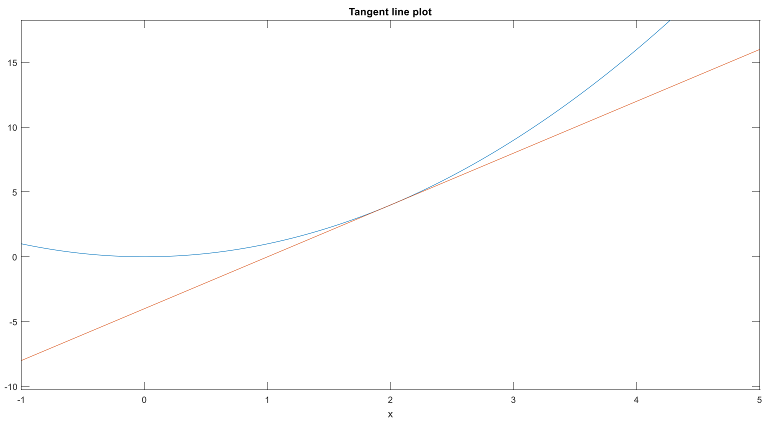
```
clear all
clc
syms x y
f = input('Enter the given function in variable x: ');
x0 = input('Enter the x-coordinate of the point: ');
y0 = subs(f,x,x0);
fx = diff(f,x);
m = subs(fx,x,x0);
tangent = y0 + m*(x-x0);
t_line=y-tangent;
plotrange = [x0-3,x0+3];
ezplot(f,plotrange);
hold on;
ezplot(tangent,plotrange)
title('Tangent line plot')
t=sprintf('The tangent to the curve y= %s at (%d,%d) is y=%s',
f,x0,y0,tangent);
disp(t)
```

Input:

```
Enter the given function in variable x: x^2
Enter the x-coordinate of the point:2
```

Output:

```
The tangent to the curve y= x^2 at (2,4) is y= 4*x - 4
```



Mean value theorem:

Rolle's theorem:

Suppose that the function $y = f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point in (a,b) and if $f(a) = f(b)$, then there is at least one number c in (a,b) at which $f'(c) = 0$.

The below code illustrates the verification of Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ on the interval $[-3,3]$.

```
clear all;
clc;
syms x c;
f=input('Enter the function: ');
I=input('Enter the interval [a,b]: ');
a=I(1); b=I(2);
fa=subs(f,x,a);fb=subs(f,x,b);
df=diff(f,x); dfc=subs(df,x,c);
if fa==fb
    c=solve(dfc);
count=0;
for i=1:length(c)
    if c(i)>a && c(i)<b
        count=count+1;
        r(count)=c(i);
    end
end
values=sprintf('The values of c between %d and %d which satisfy Rolles theorem are x=',a,b);
disp(values)
disp(r)
else
    disp('f(a) and f(b) are not equal, function doesnot satisfy conditions for Rolles theorem');
end
tval=subs(f,x,r);
xval=linspace(a,b,100);
yval=subs(f,x,xval);
plot(xval,yval);
[p,q]=size(xval);
for i=1:length(tval)
```

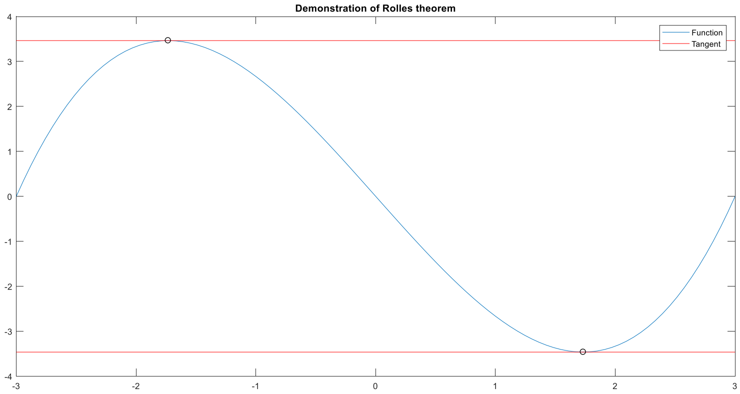
```
    hold on;
plot(xval,tval(i)*ones(p,q),'r');
 hold on;
plot(r(i),tval(i),'ok');
end
hold off;
legend('Function','Tangent');
 title('Demonstration of Rolles theorem');
```

Input:

```
Enter the function: x^3/3-3*x
Enter the interval [a,b]: [-3,3]
```

Output:

```
The values of c between -3 and 3 which satisfy Rolles theorem are x=
[3^(1/2), -3^(1/2)]
```



Lagrange's mean value theorem:

Suppose that the function $y = f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point in (a,b), then there is at least one number c in (a,b) so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The below code illustrates the verification of Lagrange's theorem for the function $f(x) = x^3 - 3x^2 + 2x + 1$ on the interval $[-3,3]$.

```
clear all;
clc;
syms x c;
f=input('Enter the function: ');
I=input('Enter the interval [a,b]: ');
a=I(1); b=I(2);
fa=subs(f,x,a);fb=subs(f,x,b);
df=diff(f,x);
dfc=subs(df,x,c);
 LM=dfc-(fb-fa)/(b-a);
    c=solve(LM);
count=0;
for i=1:length(c)
    if c(i)>a && c(i)<b
        count=count+1;
        tx(count)=c(i);
    end
end
fprintf('The values of c between %d and %d which satisfy LMVT are
x=',a,b);
disp(double(tx))
```

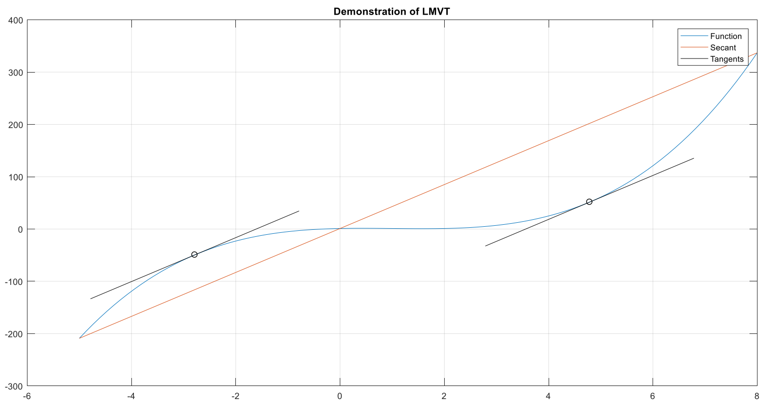
```
xval=linspace(a,b,100);
yval=subs(f,x,xval);
m=subs(df,tx) ; % Slopes of tangents at the points between a and b.
ty=subs(f,x,tx) ;
plot(xval,yval);
hold on;
secant_slope=(fb-fa)/(b-a);
secant_line=fa+secant_slope*(x-a);
sx_val=xval;
sy_val=subs(secant_line,x,sx_val);
plot(sx_val,sy_val);
hold on;
for i=1:length(tx)
txval=linspace(tx(i)-1,tx(i)+1,20);
t_line=ty(i)+m(i)*(x-tx(i));
tyval=subs(t_line,x,txval);
plot(txval,tyval,'k');
hold on
plot(tx(i),ty(i),'ok');
end
hold off;
grid on
legend('Function','Secant','Tangents');
 title('Demonstration of LMVT');
```

Input:

```
Enter the function: x^3-3*x^2+2*x+1
Enter the interval [a,b]: [-5,8]
```

Output:

```
The values of c between -5 and 8 which satisfy LMVT are x=-2.7859  4.7859
```



Exercise:

1. Using MATLAB find the tangent to the curves $y=\sqrt{x}$ at $x=4$ and show graphically.
2. Using MATLAB find the tangent to the curves $y=-\sin(x/2)$ at the origin and and show graphically.
3. Verify Rolle's theorem for the function $(x+2)^3(x-3)^4$ in the interval $[-2,3]$. Plot the curve along with the secant joining the end points and the tangents at points which satisfy Rolle's theorem.
4. Verify Lagrange's mean value theorem for the function $f(x)=x+e^{3x}$ in the interval $[0,1]$. Plot the curve along with the secant joining the end points and the tangents at points which satisfy Lagrange's mean value theorem.