

Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 3–A

Plotting 3D curves and surfaces, Taylor series of function of two variables

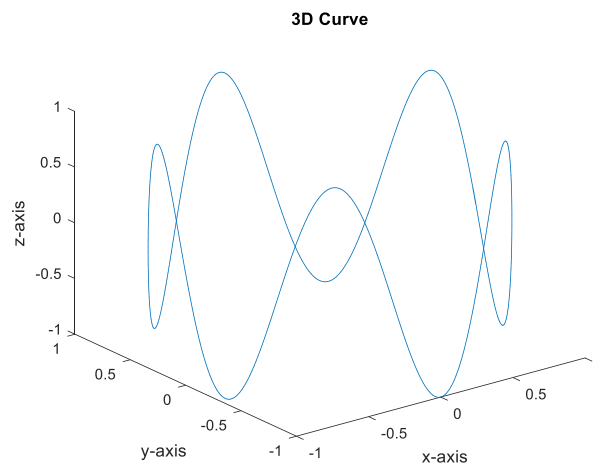
Commands	Description
<code>plot3(x,y,z)</code>	displays a three-dimensional plot of a set of data points
<code>comet3(x,y,z)</code>	displays a comet graph of the curve through the points $[x(i),y(i),z(i)]$
<code>ezplot3(funx,funy,funz)</code>	plots the spatial curve $\text{funx}(t)$, $\text{funy}(t)$, and $\text{funz}(t)$ over the default domain $0 < t < 2\pi$.
<code>ezplot3(fx,fy,fz,[tmin,tmax])</code>	plots the curve $f_x(t)$, $f_y(t)$, and $f_z(t)$ over the domain $t_{\min} < t < t_{\max}$
<code>[X,Y] = meshgrid(xgv,ygv)</code>	$[X,Y] = \text{meshgrid}(xgv,ygv)$ replicates the grid vectors xgv and ygv to produce a full grid. This grid is represented by the output coordinate arrays X and Y . The output coordinate arrays X and Y contain copies of the grid vectors xgv and ygv respectively. The sizes of the output arrays are determined by the length of the grid vectors. For grid vectors xgv and ygv of length M and N respectively, X and Y will have N rows and M columns.
<code>surf(X,Y,Z)</code>	Uses Z for the colour data and surface height. X and Y are vectors or matrices defining the x and y components of a surface. If X and Y are vectors, $\text{length}(X) = n$ and $\text{length}(Y) = m$, where $[m,n] = \text{size}(Z)$. In this case, the vertices of the surface faces are $(X(j), Y(i), Z(i,j))$ triples. To create X and Y matrices for arbitrary domains, use the <code>meshgrid</code> function.
<code>ezsurf(fun)</code>	creates a graph of $\text{fun}(x,y)$ using the <code>surf</code> function. fun is plotted over the default domain: $-2\pi < x < 2\pi$, $-2\pi < y < 2\pi$.
<code>[X,Y,Z] = sphere(n)</code>	returns the coordinates of a sphere in three matrices that are $(n+1)$ -by- $(n+1)$ in size.
<code>sphere(n)</code>	draws a surf plot of an n -by- n sphere in the current figure.

Example.1

Using MATLAB plot the curves $x=\cos(t)$, $y=\sin(t)$, $z=\sin(5*t)$.

```
t=linspace(0,2*pi,500);
x=cos(t);
y=sin(t);
z=sin(5*t);
comet3(x,y,z);
plot3(x,y,z);
xlabel('x-axis');
ylabel('y-axis');
zlabel('z-axis');
title('3D Curve');
```

Output

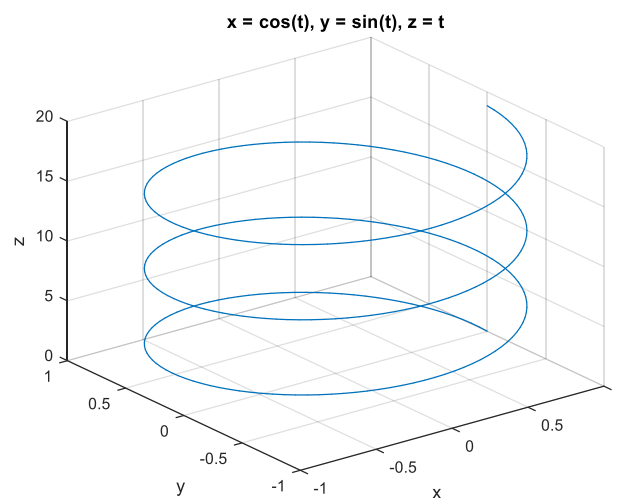


Example.2

Plot the helix define by the parametric equations: $x = \cos(t)$, $y = \sin(t)$, $z = t$

```
>> syms t  
>> ezplot3(cos(t), sin(t), t, [0, 6*pi])
```

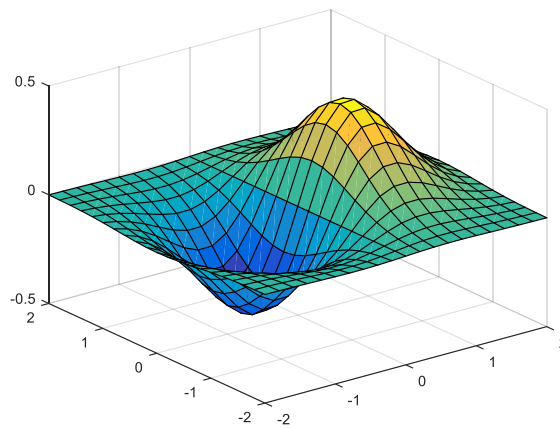
Output



Example.3

```
[x,y] = meshgrid(-2:.2:2);  
g = x .* exp(-x.^2 - y.^2);  
surf(x, y, g)
```

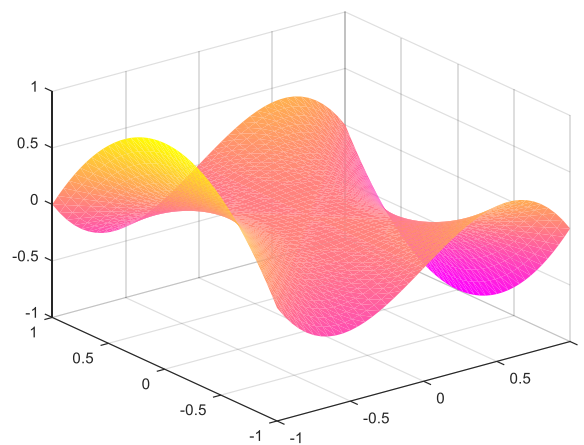
Output



Example.4

```
x=-1:.05:1;  
y=-1:.05:1;  
[x,y]=meshgrid(x,y);  
z=x.*y.^2-x.^3  
surf(x,y,z);  
colormap spring  
shading interp
```

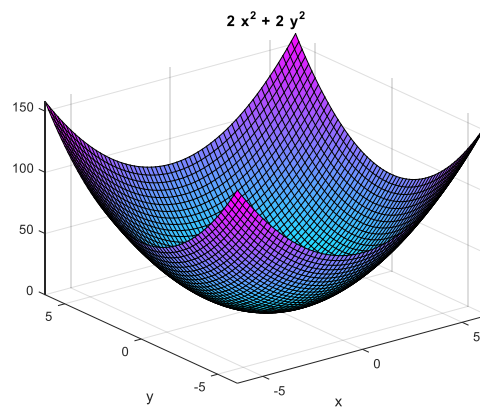
Output



Example.5

```
syms x y  
f = 2*(x^2+y^2)  
ezsurf(f)  
colormap cool
```

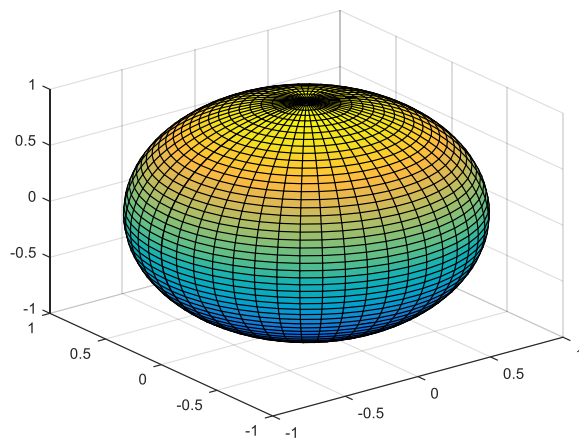
Output



Example.6

```
syms x y z
n=50;
[x y z]=sphere (n);
sphere(n)
```

Output



Taylor Series for a two variable functions:

Let $f(x,y)$ be a function of two variables x and y then the Taylor series expansion of f about the point (a,b) is given by

$$f(x,y) = f(a,b) + [(x-a)f_x(a,b) + (y-b)f_y(a,b)] \\ + \frac{1}{2!} [(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)] + \dots$$

Mat Lab code

```
clc
clearvars
close all
syms x y
f = input('Enter the function f(x,y): ');
I = input('Enter the point [a,b] at which Taylor series is sought: ');
a = I(1); b = I(2);
n = input('enter the order of series:');
tayser = taylor(f, [x,y], [a,b], 'order', n);
subplot(1,2,1);
ezsurf(f);
subplot(1,2,2);
ezsurf(tayser);
```

Example

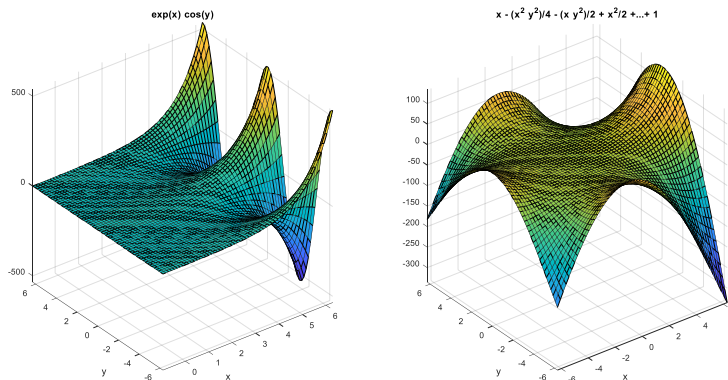
Find Taylor series of $f(x,y) = e^x \cos y$ about the origin.

Input

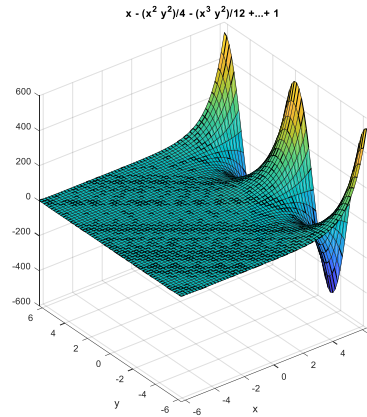
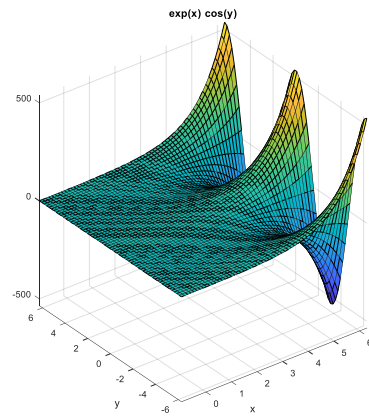
```
Enter the function f(x,y): exp(x)*cos(y)
Enter the point [a,b] around which Taylor series is sought: [0 0]
enter the order of series:5
```

Output

```
tayser =
x^4/24 + x^3/6 - (x^2*y^2)/4 + x^2/2 - (x*y^2)/2 + x + y^4/24 - y^2/2 + 1
```



Note: The above program when executed with higher order ($n=20$), produces a better approximation than with order 5, the same is shown in the figure below.



Exercise:

1. Draw the surface of the function $f(x,y)=e^x+e^y$ using ezsurf.
2. Draw the 3-D plot for the function $f(t)=(t, t^2, t^3)$, where $0 \leq t \leq 100$.
3. Using 'surf' plot the surface $f(x, y) = x(x^2 + y^2)$.
4. Expand $f(x, y) = e^x \ln(1 + y)$ in terms of x and y upto the terms of 3rd degree using Taylor series.
5. Expand $e^{x,y}$ in Taylor series the neighbourhood of $(1,1)$.