Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 4–A
Double Integrals and change of order of integration

In this experiment, we consider a continuous function f such that $f(x,y) \geq 0$ for all (x,y) in a region R in the $xy-$plane, then the volume of the solid region that lies above R and below the graph of f is defined as the double integral $V = \iint_R f(x,y)dA$, where R is the region bounded by the curves $y=\phi_1(x)$ and $y=\phi_2(x)$ between $x=a$ and $x=b$.

In this case the inner integration is with respect to y and outer integration is with respect to x. Hence

$$V = \iint_R f(x,y)dA = \int_{x=a}^{b} [\int_{y=\phi_1(x)}^{\phi_2(x)} f(x,y)dy]dx$$

MATLAB Syntax

`int(int(f(x,y),y, phi1, phi2),x,a,b)` where y is the inner variable, x is the outer variable.

When R is a region bounded by the curves $x=\psi_1(y)$ and $x=\psi_2(y)$ between $y=c$ and $y=d$, i.e., the inner integration is with respect to x and outer integration is with respect to y. Then

$$V = \iint_R f(x,y)dA = \int_{y=c}^{d} [\int_{x=\psi_1(y)}^{\psi_2(y)} f(x,y)dx]dy$$

MATLAB Syntax

`int(int(f,x, psi1, psi2),y,c,d)` where x is the inner variable, y is the outer variable.

Supporting files required:

To visualize the surfaces two additional m-files viz., `viewSolid, viewSolidone` are required. These files are to be included in the current working directory before execution.

Syntax for double integral:

```
viewSolid(z,0+0*x+0*y,f,y,phi1,phi2,x,a,b)

viewSolidone (z,0+0*x+0*y,f,x,psi1,psi2,y,c,d)
```

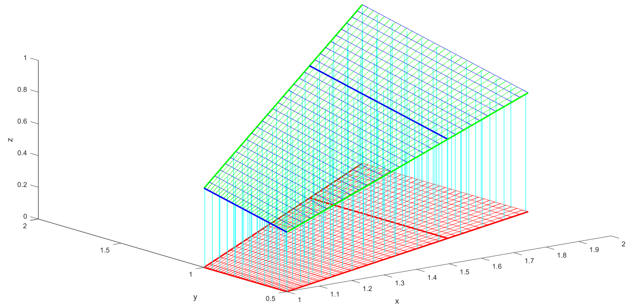
Example. 1

To find $\int_{1}^{2}\int_{x/2}^{x} \frac{x+y}{4} dydx$.

```
syms x y z
int(int((x+y)/4,y,x/2,x),x,1,2)
viewSolid(z,0+0*x+0*y,(x+y)/4,y,x/2,x,x,1,2)
```

Output

```
ans =
49/96
```



In this figure the required volume is above the plane z=0 (shown in red) and above the surface $z=\frac{x+y}{4}$ (shown in green) .

Example. 2

To find the volume of the prism whose base is the triangle in the $xy-$plane bounded by the $x-$axis and the lines $y=x$ and $x=1$ and whose top lies in the plane $z=f(x,y)=3-x-y$. The limits of integration here are $y=0$ to 1 while $x=y$ to 1. Hence

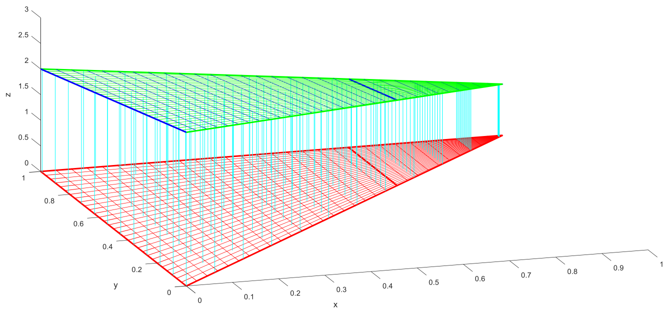
$$\iint_R (3-x-y)dA=\int_0^1\int_y^1 (3-x-y)dxdy$$

MATLAB code:

```
syms x y z
int(int(3-x-y,x,y,1),y,0,1)
viewSolidone(z,0+0*x+0*y,3-x-y,x,y,1,y,0,1)
```

Output:

```
ans =
1
```



In this figure the triangular region on the *xy* plane is shown in red, while the plane surface *z*=3–*x*–*y* above the *xy* plane is shown in green .

Example 3

Evaluate the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)dydx$ by changing the order of integration.

As per the given limits of integration $x = 0$ to 2 while $y = x^2$ to $2x$.

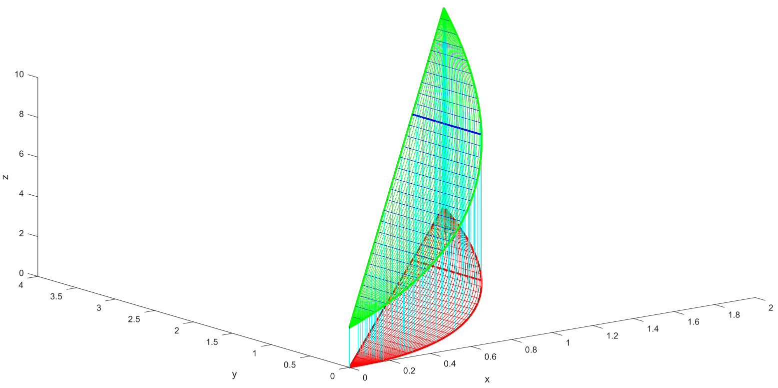
MATLAB Code:

```
syms x y z
int(int((4*x+2),y,x^2,2*x),x,0,2)
viewSolid(z,0+0*x+0*y, 4*x+2,y,x^2,2*x,x,0,2)
```

Output

```
ans =

8
```

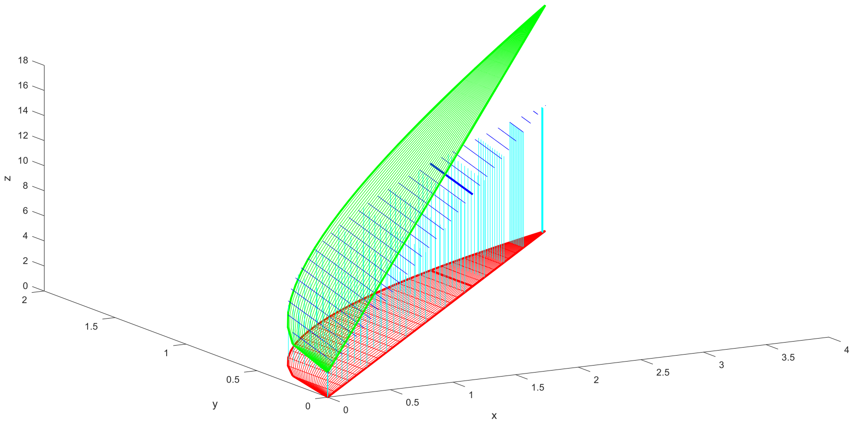


By changing the order of integration, the limits are $y = 0$ to 4 while $x = \frac{y}{2}$ to $\sqrt{y}$.

```
int(int(4*x+2,x,y/2,sqrt(y)),y,0,4)

ans =

8
```



Example 4:

Consider the following mathematical problem

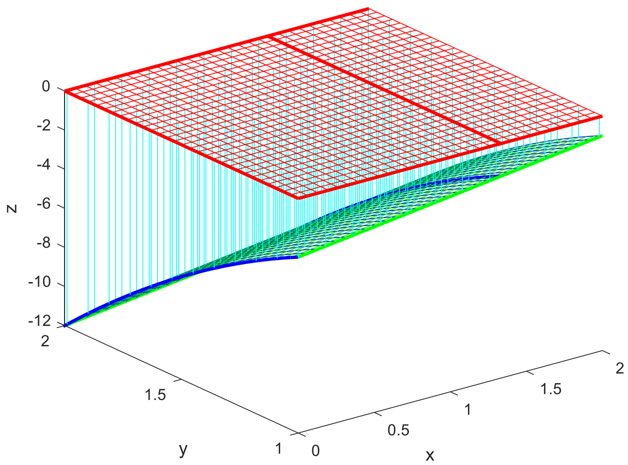
$$\text{Evaluate } \iint_R (x-3y^2)\,dA \text{ where } R=\{(x,y)\,|\,0\le x\le 2,\,1\le y\le 2\}$$

MATLAB Code:

```
clc
clear all
syms x y z
viewSolid(z, 0+0*x+0*y, x-3*y^2,y,1+0*x, 2+0*x,x,0,2)
int(int(x-3*y^2,y,1,2),x,0,2)
```

Output:

```
>> ans
     -12
```



In this figure the required volume is below the plane $z=0$ (shown in red) and above the surface $z = x - 3y^2$ (shown in green). The reason why the answer is negative is that the surface $z = x - 3y^2$ is below $z=0$ for the given domain of integration.

Example 5:

Consider the next mathematical problem

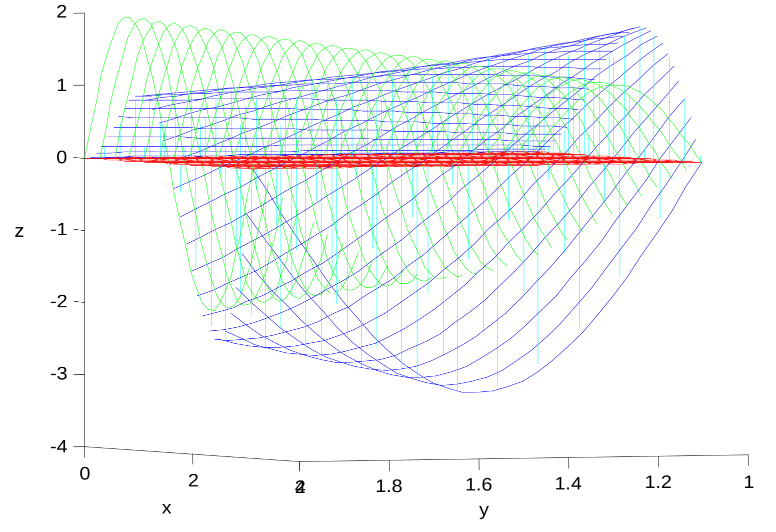
Evaluate $\iint\limits_R y\sin(xy)\, dA$ where $R=[1,2]\times[0,\pi]$

MATLAB Code:

```
clc
clear all
syms x y z
viewSolidone(z, 0+0*x+0*y, y*sin(x*y),x,1+0*y, 2+0*y,y,0,pi)
int(int(y*sin(x*y),x,1,2),y,0,pi)
```

Output:

```
>>  ans
    0
```



For a function $f(x,y)$ that takes on both positive and negative values $\iint_R f(x,y)dA$ is a difference of volumes V_1-V_2, V_1 is the volume above R and below the graph of f and V_2 is the volume below R and above the graph. The integral in this example is 0 means V_1=V_2

Example 6:

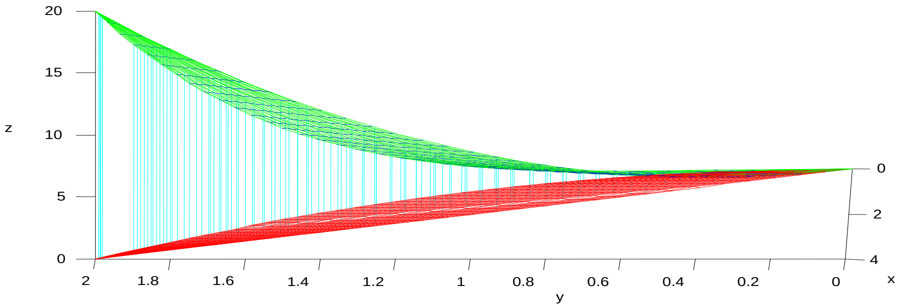
Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the lines $y = 2x$ and the parabola $y = x^2$

MATLAB Code:

```
clc
clear all
syms x y z
viewSolidone(z, 0+0*x+0*y, x^2+y^2,x,y/2,sqrt(y),y,0,4)
int(int(x^2+y^2,x,y/2,sqrt(y)),y,0,4)
```

Output:

```
>>  ans
   216/35
```



Exercise:

1. Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ *and* $y = 2$, and the three coordinate planes.
2. Evaluate $\iint_R \sin x \cos y \, dA$ where $R = [0, \pi/2] \times [0, \pi/2]$