Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 1–B
Maxima and Minima of a function of one variable

Absolute/Global Extrema

- Let $f(x)$ be a function defined in $[a,b]$. Then f has an absolute maximum value on $[a,b]$ at a point $c \in [a,b]$ if $f(x) \le f(c)$ for all $x \in [a,b]$.
- Let $f(x)$ be a function defined in $[a,b]$. Then f has an absolute minimum value on $[a,b]$ at a point $c \in [a,b]$ if $f(x) \ge f(c)$ for all $x \in [a,b]$.

Extreme value theorem

If f is continuous on a closed interval $[a,b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a,b]$.

Relative/Local Extreme values

- A function f has a local maximum value at an interior point c of its domain if $f(x) \le f(c)$ for all x in some open interval containing c.
- A function f has a local minimum value at an interior point c of its domain if $f(x) \ge f(c)$ for all x in some open interval containing c.

Categorization of Extema

For a continuous function f on a bounded interval $[a,b]$, suppose c is a global extemum (a global minimum or a global maximum). Then c must satisfy one of the following:

- $f'(c) = 0$.
- $f'(c)$ does not exist.
- c is an endpoint.

MATLAB syntax used in the code:

`vectorize(s)`	Inserts a `.` before any `^`, `*` or / in string expression `'s'`
`inline(expr)`	Constructs an inline function object from the MATLAB expression contained in the string `expr`. The result is a character string.
`x=fzero(fun,x0)`	Tries to find a point x where `fun(x)=0`. This solution is where `fun(x)` changes sign.
`n=numel(A)`	Returns the number of elements, `n`, in array `A`.
`c=unique(A)`	Returns the same data as in `A`, but with no repetitions. The values of `'c'` are in sorted order.

The following MATLAB code illustrates the evaluation and visualization of global and local extrema of a function $f(x)$ in an interval (a,b).

MATLAB code

```
clear all
clc
syms x
f = input('Enter the function f(x):');
I = input('Enter the interval: ');
a=I(1);b=I(2);
df = diff(f,x);
ddf = diff(df,x);
f = inline(vectorize(f));
df = inline(vectorize(df));
ddf = inline(vectorize(ddf));
range = linspace(a,b,100);
plot(range,f(range),'-b','LineWidth',2);
legstr = {'Function Plot'}; % Legend String
hold on;
%%%%%%%
% Due to limitations in symbolic toolbox we find the zeros of
% f'(x) numerically.
%%%%%%%%%
guesses = linspace(a,b,5);
root = zeros(size(guesses));
for i=1:numel(guesses)
root(i) = fzero(df,guesses(i));
end
root = root(a <= root & root <=b);
root = unique(round(root,4));
plot(root,f(root),'ro','MarkerSize',10);
legstr = [legstr, {'Critical Points'}];
disp(['Critical Points of f(x) are: ',num2str(root)])
%%%%%%%
%We categorize the critical points by the second derivative test
%%%%%%%%%%
maxp = root(ddf(root) < 0);
if(numel(maxp) ~= 0)
disp(['Local maximum of f(x) occurs at: ',num2str(maxp)])
end
minp = root(ddf(root) > 0);
if(numel(minp) ~= 0)
disp(['Local minimum of f(x) occurs at: ',num2str(minp)])
end
fval = f(root);
if(numel(maxp) ~= 0)
gmax = root(fval == max(fval));
disp(['Global maximum of f(x) occurs at: ',num2str(gmax),' and its value is:', num2str(max(fval))])
plot(gmax,f(gmax),'m+','MarkerSize',10);
legstr = [legstr, {'Global Maximum'}];
end
if(numel(minp) ~= 0)
gmin = root(fval == min(fval));
disp(['Global minimum of f(x) occurs at: ',num2str(gmin),' and its value is: ', num2str(min(fval))])
plot(gmin,f(gmin),'m*','MarkerSize',10);
legstr = [legstr, {'Global Minimum'}];
end
legend(legstr,'Location','Best')
```

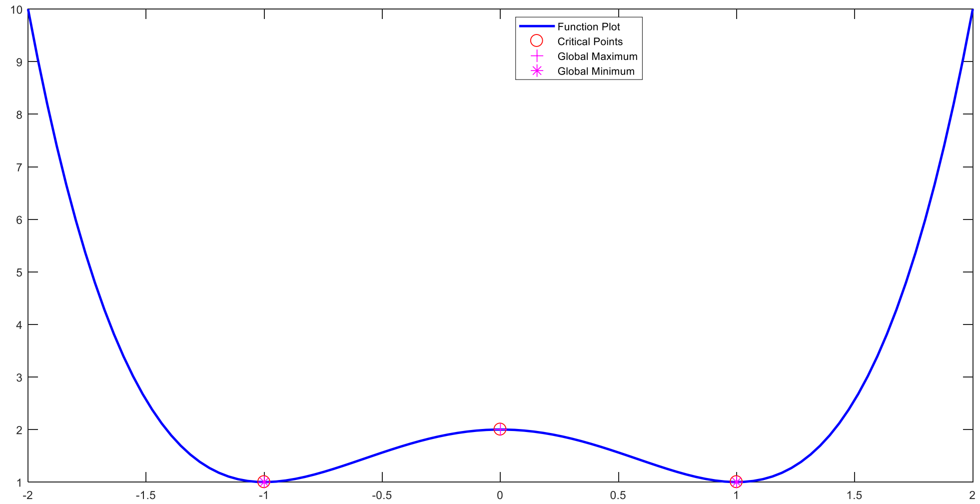
Example: Find the local and global maxima and minima of the function $f(x) = x^4 - 2x^2 + 2$ in the interval $(-2,2)$.

Input

```
Enter the function f(x):x^4-2*x^2+2
Enter the interval: [-2,2]
```

Output

```
Critical Points of f(x) are: -1   0   1
Local maximum of f(x) occurs at: 0
Local minimum of f(x) occurs at: -1   1
Global maximum of f(x) occurs at: 0 and its value is:2
Global minimum of f(x) occurs at: -1   1 and its value is: 1
```



Exercise

1. Find the local and global maxima and minima for the function $x^3 - 12x - 5$, $x \in (-4,4)$.
2. Find the global extrema of the function $f(x) = x^2 e^{\sin x} - \dfrac{x}{x^3+1}$ on the interval $[0,5]$.

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