The Laplace Transform of a function $f(t)$ is defined as

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)dt \ ,$$

provided the integral exists.

Command	Purpose
`laplace(f)`	To find the Laplace transform of a scalar symbol f with default independent variable t. The default return is a function of s.
`laplace(f,w)`	Returns the Laplace transform of f in symbol w instead of the default s.
`laplace(f,x,w)`	Assumes f as a function of the symbolic variable x and returns the Laplace transform as a function of w.
`ilaplace(F)`	To find the inverse Laplace transform of the scalar symbolic object F with default independent variable s. The default return is a function of t.
`ilaplace(F,x)`	Returns the inverse Laplace transform of the function F as a function of x instead of the default t.
`ilaplace(F,w,x)`	Assumes F as a function of the symbolic variable w and returns the inverse Laplace transform of F as a function of x.
`heaviside(t-a)`	To input the heaviside's unit step function $H(t-a)$.
`dirac(t-a)`	To input the dirac delta function δ(t−a).

Example 1. The following MATLAB code finds the Laplace transform of a function $f(t)$

```
clear all
clc
syms t
f=input('Enter the function of t: ');
F=laplace(f);
disp(['L{',char(f),'}=',char(F)]);
```

Input:
```
Enter the function of t: t^2
```
Output:
```
L{t^2}=2/s^3
```

Example 2. The following MATLAB code finds the Laplace transform of $f(t)$ in terms of w.

```
clear all
clc
syms t w
f=input('Enter the function of t: ');
F=laplace(f,w);
```

```
disp(['L{',char(f),'}=',char(F)]);
```

Input:
```
Enter the function of t: sin(t)
```
Output:
```
L{sin(t)}=1/(w^2 + 1)
```

Example 3. The following MATLAB code finds the Laplace transform of $x^3 e^{-3x}$ in terms of w.
```
clear all
clc
syms x w
f=input('Enter the function of x: ');
F=laplace(f,x,w);
disp(['L{',char(f),'}=',char(F)]);
```
Input:
```
Enter the function of x: x^3*exp(-3*x)
```
Output:
```
L{x^3*exp(-3*x)}=6/(w + 3)^4
```

Example 4: The following MATLAB code computes the Laplace Transform of

$$f(t) = \begin{cases} t^2, 0 < t < 2 \\ t-1, 2 < t < 3 \\ 7, t > 3 \end{cases}$$

```
clear all
clc
syms t
f=input('Enter the function of t: ');
F=laplace(f);
F=simplify(F);
disp(['L{f(t)}=',char(F)]);
```

Input:
```
Enter the function of t: t^2*(heaviside(t)-heaviside(t-2))+
(t-1)*(heaviside(t-2)-heaviside(t-3))+7*heaviside(t-3)
```
Output:
```
L{f(t)}=-(exp(-3*s)*(s-2*exp(3*s)+2*exp(s)+3*s^2*exp(s)+3*s*exp(s)-
5*s^2))/s^3
```

Example 5. The following MATLAB code computes the inverse Laplace transform of $F(s)$.
```
clear all
clc
syms s
F=input('Enter the function of s: ');
f=ilaplace(F);
disp(['f(t)=',char(f)]);
```

Input:
```
Enter the function of s: 6/(s^3+2*s^2-s-2)
```
Ouput:
```
f(t)=2*exp(-2*t)-3*exp(-t)+exp(t)
```

Example 6. Write MATLAB commands to find the following:
    (i) $L[\delta(t)]$ (ii) $L[\delta(t-a)]$ (iii) $L[\delta(t-a)\sin(t)]$

Solution:

(i)
```
syms t
F=laplace(dirac(t))
```

Output:
```
F =
1
```

(ii)
```
syms t a
F=laplace(dirac(t-a))
```
Output:
```
F =
piecewise(a < 0, 0, 0 <= a, exp(-a*s))
```

(iii)
```
syms t a
F=laplace(dirac(t-a)*sin(t))
```
Output:
```
F =
piecewise(a<0,0,0<=a,exp(-a*s)*sin(a))
```

Example 7. Write MATLAB commands to find (i) $L^{-1}\left[\dfrac{s}{s-a}\right]$ (ii) $L^{-1}\left[\dfrac{se^{-s}+ae^{-2s}}{s^2+a^2}\right]$

(i)
```
syms s a
f=ilaplace(s/(s-a))
```

Output
```
f=
dirac(t) + a*exp(a*t)
```

(ii)
```
syms s a
f= ilaplace((s*exp(-s)+a*exp(-2*s))/(s^2+a^2))
```
Output
```
f=
heaviside(t-1)*cos((a^2)^(1/2)*(t-1))+(a*heaviside(t-2)*
sin((a^2)^(1/2)*(t-2)))/(a^2)^(1/2)
```

Exercise.
1. Find the Laplace transforms of the following functions:

(i) $f(t) = 1 + 2\sqrt{t} + \dfrac{3}{\sqrt{t}}$

(ii) $f(t) = \begin{cases} \sin t & ; \quad 0 \le t \le \pi \\ 0 & ; \quad \pi \le t \le 2\pi \end{cases}$

(iii) $f(t) = \sin^3 t$

(iv) $f(t) = \sin 2t \sin 3t$

(v) $f(t) = e^{-t} \sin^2 t$

(vi) $f(t) = \dfrac{\cos 2t - \cos 3t}{t}$

2. Find the inverse Laplace transforms of the following functions:

(i) $F(s) = \dfrac{6}{s^2 + 2s - 8}$

(ii) $F(s) = \dfrac{4s + 5}{(s-1)^2 (s+2)}$

(iii) $F(s) = \dfrac{s^2 + 2s - 4}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$

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