Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 5–A

Divergence, Curl and Gradient and visualization of vector field

Aim:

- To write Matlab codes to visualize the vector field of 2-Dimensions as well as 3-Dimensions.
- To find the gradient, divergence and curl of a vector field and visualize it with contour curves.

Mathematical form:

- Draw the two dimensional vector field for the vector $\vec{\boldsymbol{F}}=\vec{\boldsymbol{f_1}}(x, y) + \vec{\boldsymbol{f_2}}(x,y)$
- Draw the three dimensional vector field for the vector $\vec{\boldsymbol{F}}=\vec{\boldsymbol{f_1}}(x,y,z)+ \vec{\boldsymbol{f_2}}(x,y,z) +\vec{\boldsymbol{f_3}}(x,y,z)$
- **Gradient vector of a scalar function** $f(x, y)$

  The vector function ∇f is defined as the gradient of the scalar function f and is written as *grad f.*

  grad f = $\nabla f = (\partial f/\partial x)\vec{i} +(\partial f/\partial y)\vec{j}+(\partial f/\partial z)\vec{k}$.
- **Divergence of a vector $\vec{\boldsymbol{F}}$**

  Divergence of a continuously differentiable vector point function $\vec{\boldsymbol{F}}$ is denoted by *div* $\vec{\boldsymbol{F}}$ and is defined as

  div $\vec{\boldsymbol{F}} = \nabla.\vec{\boldsymbol{F}} = \vec{i}.(\partial\vec{\boldsymbol{F}}/ \partial x) +\vec{j}.(\partial\vec{\boldsymbol{F}}/ \partial y)+\vec{k}.(\partial\vec{\boldsymbol{F}}/ \partial z)$
- **Curl of a vector $\vec{\boldsymbol{F}}$**

  Curl of a continuously differentiable vector point function $\vec{\boldsymbol{F}}$ is denoted by *curl* $\vec{\boldsymbol{F}}$ and is defined as

  curl $\vec{\boldsymbol{F}} = \nabla \times \vec{\boldsymbol{F}} = \vec{i} \times (\partial\vec{\boldsymbol{F}}/ \partial x) + \vec{j} \times (\partial\vec{\boldsymbol{F}}/ \partial y)+ \vec{k} \times (\partial\vec{\boldsymbol{F}}/ \partial z)$

MATLAB Syntax used:

inline(expr)	Constructs an inline function object from the MATLAB expression contained in the string expr.
vectorize(fun)	Inserts a . before any ^, * or / in s. The result is a character string
quiver(x,y,u,v)	Displays velocity vectors as arrows with components (u,v) at the points (x,y)
quiver3(x,y,z,u,v,w)	Plots vectors with components (u,v,w) at the points (x,y,z))
`gradient(f,v)`	Finds the gradient vector of the scalar function f with respect to vector v in Cartesian coordinates.
`div = divergence(X,Y,Z,U,V,W)`	Computes the divergence of a 3-D vector field having vector components U, V, W. The arrays X, Y, and Z, which define the coordinates for the vector components U, V, and W, must be monotonic, but do not need to be uniformly spaced. X, Y, and Z must have the same number of elements.
`curl(V,X)`	Returns the curl of the vector field V with respect to the vector X. The vector field V and the vector X are both three-dimensional.
`pcolor(x,y,C)`	When x,y and C are matrices of the same size, pcolor(x,y,C) plots the colored patches of vertices (x(i,j), y(i,j)) and color C(i,j).

Example 1: Draw the two dimensional vector field for the vector $\boldsymbol{x}\vec{\boldsymbol{i}} + \boldsymbol{y}\vec{\boldsymbol{j}}$

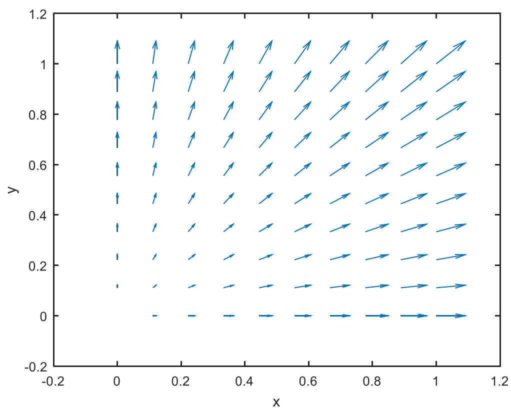
MATLAB Code:

```
syms x  y
F=input( 'enter the vector as i,j order in vector form:');
P = inline(vectorize(F(1)), 'x', 'y');
Q = inline(vectorize(F(2)), 'x', 'y');
x = linspace(0, 1, 10); y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V)
axis on
xlabel('x'); ylabel('y');
```

Input:

```
Enter the vector as i, j order in vector form:[x y]
```

Output:



Example 2: Draw the three dimensional vector field for the vector $\boldsymbol{x}\vec{\boldsymbol{i}} + \boldsymbol{y}\vec{\boldsymbol{j}} + \boldsymbol{z}\vec{\boldsymbol{k}}$

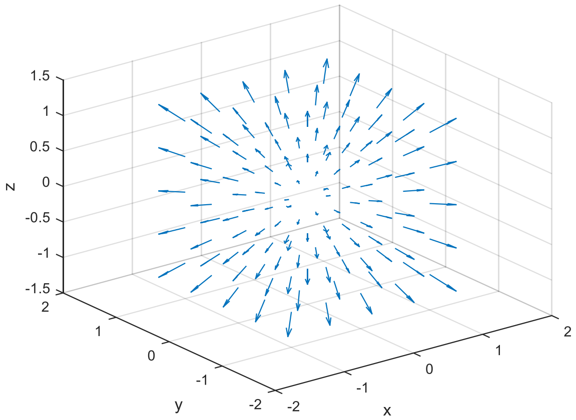
MATLAB Code:

```
syms x  y  z
F=input( 'Enter the vector as i,j and k order in vector form:')
P = inline(vectorize(F(1)), 'x', 'y','z');
Q = inline(vectorize(F(2)), 'x', 'y','z');
R = inline(vectorize(F(3)), 'x', 'y','z');
x = linspace(-1, 1, 5); y = x;
z=x;
[X,Y,Z] = meshgrid(x,y,z);
U = P(X,Y,Z);
V = Q(X,Y,Z);
W = R(X,Y,Z);
quiver3(X,Y,Z,U,V,W)
axis on
xlabel('x');
ylabel('y');
zlabel('z');
```

Input:

```
Enter the vector as i, j and k order in vector form: [x  y   z]
F =[ x, y, z]
```

Output:



Example 3: Find the Gradient of the function $f = 2xy$

MATLAB Code:

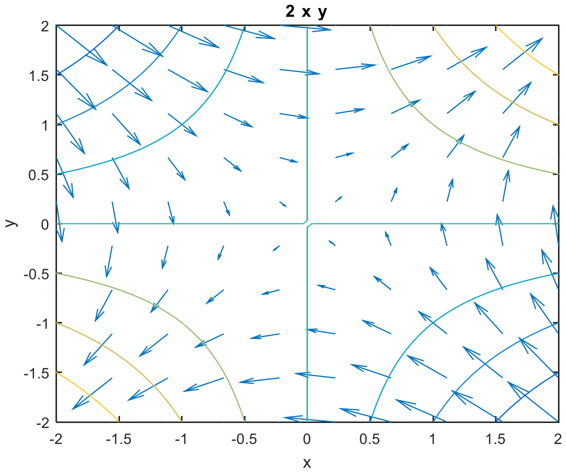
```
syms x y
f=input( 'Enter the function f(x,y):');
grad=gradient(f,[x,y])
f1=diff(f,x);
f2=diff(f,y);
P = inline(vectorize(f1), 'x', 'y');
Q = inline(vectorize(f2), 'x','y');
x = linspace(-2, 2, 10);
y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V,1)
axis on
xlabel('x')
ylabel('y')
hold on
ezcontour(f,[-2 2])
```

Input:

```
Enter the function f(x,y):2*x*y
```

Output:

```
grad =
 2*y
 2*x
```



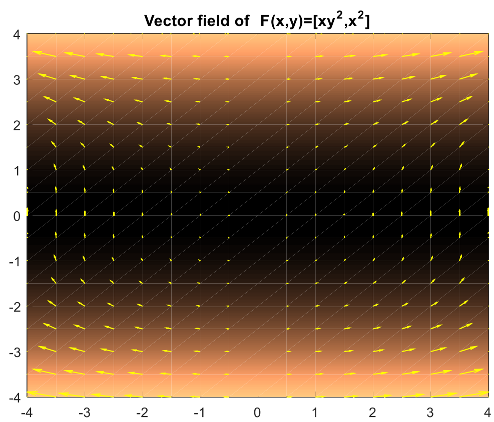
Inference:
The gradient vectors are orthogonal to the contours. It will be lengthier if the level curves are close to each other.

Example 4: Find the divergence of the vector field $\vec{F} = xy^2\vec{i} + x^2\vec{j}$ and visualize it.

MATLAB Code:

```
x=-4:0.5:4;
y=x;
[X Y]=meshgrid (x,y);
Div=divergence(X,Y,X.*Y.^2, X.^2);
figure
pcolor(X,Y,Div);
shading interp
hold on;
quiver(X,Y, X.*Y.^2, X.^2,'Y');
hold off;
colormap copper
title('Vector field of F(x,y)=[xy^2,x^2]');
```

Output:



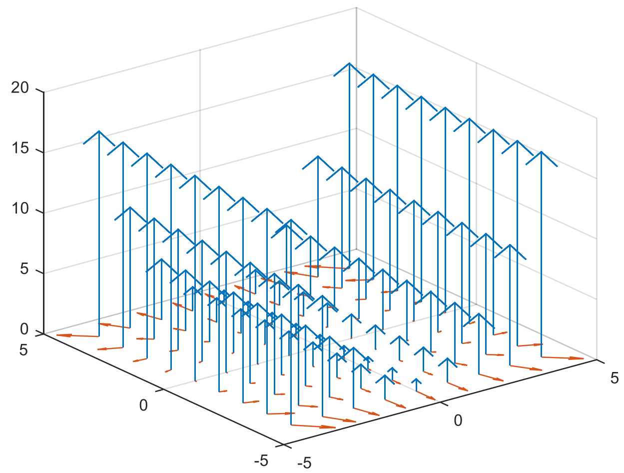
Example 5:

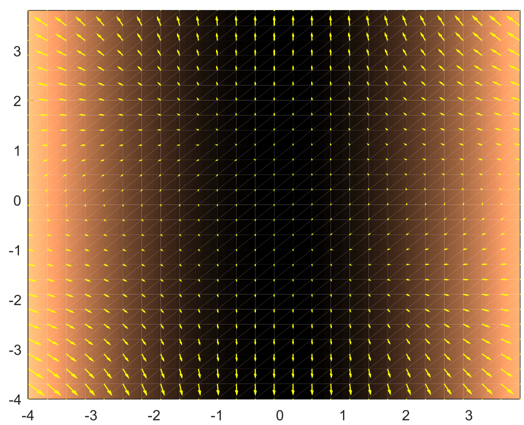
Find the curl of the vector field $\vec{F} = -yx^2\vec{i} + (x + y^3)\vec{j}$ and visualize it.

MATLAB Code:

```
x=-4:4;
y=x;z=x;
[X, Y, Z]=meshgrid (x,y,z);
[Cx, Cy, Cz]=curl(X,Y,Z,-Y.*X.^2,X+Y.^3,zeros(size(X)));
% Note that as there are no z-values in our field, we supplied
zeros for z
figure
quiver3(X,Y,zeros(size(X)),Cx,Cy,Cz,0);
hold on;
[X Y]=meshgrid (x,y);
quiver(X,Y, -Y.*X.^2,X+Y.^3);
figure
[X Y]=meshgrid (-4:.3:4);
crl=curl(X,Y,-Y.*X.^2,X+Y.^3);
pcolor(X,Y,crl);
shading interp
hold on;
quiver(X,Y, -Y.*X.^2,X+Y.^3,'y');
hold off;
colormap copper
```

Output:





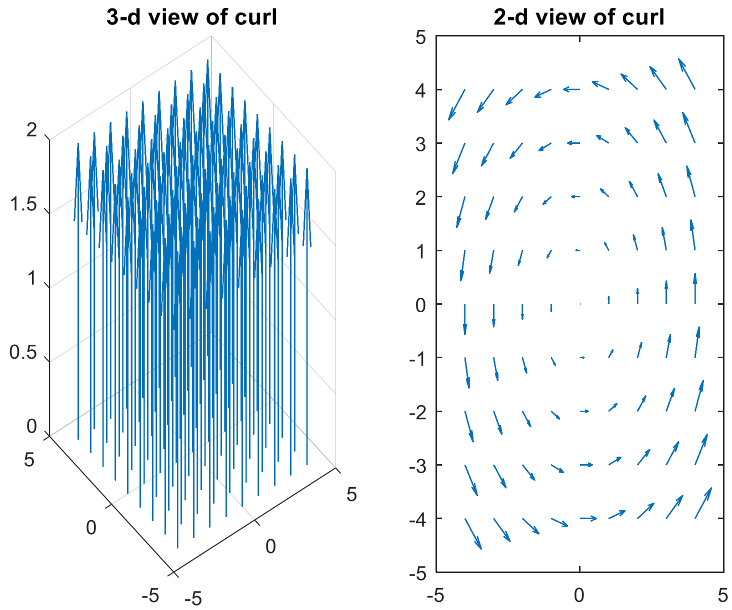
Example-8. To visualize the curl of a vector function F=[-y, x]

MATLAB Code

```
clear all
close all
clc
X=-4:4; Y=X;Z=X;
[x,y,z]=meshgrid(X,Y,Z);
[cx,cy,cz]=curl(x,y,z,-y,x,zeros(size(x)));
figure;
subplot(1,2,1);
quiver3(x,y,zeros(size(x)),cx,cy,cz,0);
```

```
title('3-d view of curl');
subplot(1,2,2);
quiver(x,y,-y,x);
title('2-d view of curl');
```

Output in the Figure Window



Exercise:

1. Draw the two dimensional vector field for the vector $2xi + 3yj$.
2. Find the Gradient of the function $f = x^2 y^3 - 4y$.
3. Find the divergence of a vector field $f = [xy, x^2]$.
4. Visualize the curl of a vector function $f = [yz, 3zx, z]$.