Department of Mathematics
School of Advanced Sciences
MAT 1011 – Calculus for Engineers (MATLAB)
Experiment 2–A
Applications of Integration: finding area, volume of solid of revolution

Area between the curves

If f and g are continuous with $f(x) \geq g(x)$ for $x \in [a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral

$$A = \int_a^b [f(x) - g(x)]dx.$$

Also, if a region's bounding curves f and g are described by functions of y, where f denotes the right hand curve and g denotes the left hand curve, $f(y) - g(y)$ being non negative, then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from $y = c$ to d is the integral

$$A = \int_c^d [f(y) - g(y)]dy.$$

Example 1.

The area bounded by the curves $y = 2 - x^2$ and the line $y = -x$, from $x = -1$ to 2 is given by the following code:

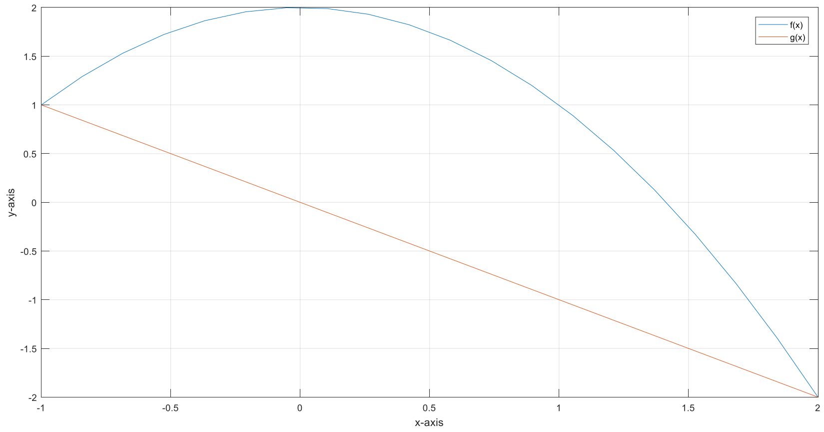
```
clear all
clc
syms x
f=input('Enter the upper curve f(x): ');
g=input('Enter the lower curve g(x): ');
L=input('Enter the limits of integration for x [a,b]:');
a=L(1); b=L(2);
Area=int(f-g,x,a,b);
disp(['Area bounded by the curves f(x) and g(x) is: ',char(Area)]);
x1=linspace(a,b,20);y1=subs(f,x,x1);
x2=x1;y2=subs(g,x,x1);
plot(x1,y1);hold on; plot(x2,y2);hold off;
xlabel('x-axis');ylabel('y-axis');
legend('f(x)','g(x)');grid on;
```

Input

```
Enter the upper curve f(x): 2-x^2
Enter the lower curve g(x): -x
Enter the limits of integration for x [a,b]:[-1,2]
```

Output

```
Area bounded by the curves f(x) and g(x) is: 9/2
```



Example 2.

To find the area of the region bounded by the curves $y^2 = x$, $y = x - 2$ in the first quadrant.

Here the right curve is the straight line $x = 2 + y$, the left curve is $x = y^2$. The limits of integration being $y = 0$ to 2

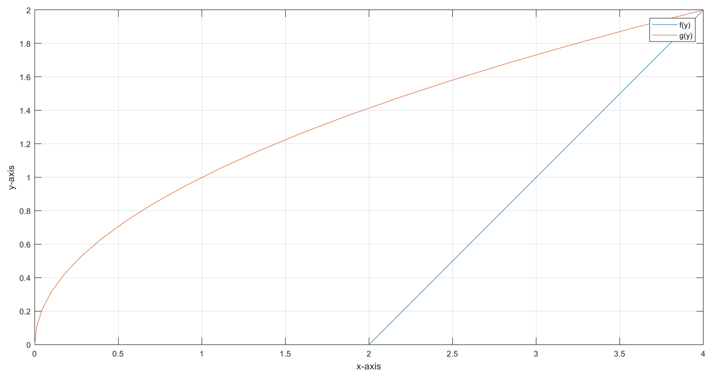
```
clear all
clc
syms y
f=input('Enter the right curve f(y): ');
g=input('Enter the left curve g(y): ');
L=input('Enter the limits of integration for y [c,d]:');
c=L(1); d=L(2);
Area=int(f-g,y,c,d);
disp(['Area bounded by the curves f(y) and g(y) is: ',char(Area)]);
y1=linspace(c,d,20);x1=subs(f,y,y1);
y2=y1;x2=subs(g,y,y1);
plot(x1,y1);hold on;
plot(x2,y2);hold off;
xlabel('x-axis');ylabel('y-axis');
legend('f(y)','g(y)');grid on;
```

Input

```
Enter the right curve f(y): 2+y
Enter the left curve g(y): y^2
Enter the limits of integration for y [c,d]:[0,2]
```

Output

```
Area bounded by the curves f(y) and g(y) is: 10/3
```



Volume of solid of revolution – Disc method

When a plane curve is revolved about an axis, a solid is formed which is called a solid of revolution. The volume of the solid generated by revolving a curve $y = f(x)$ about $x-$axis from $x = a$ to $x = b$ is given by

$$V = \int_a^b \pi y^2 dx$$

If the solid is formed by revolving the curve $y = f(x)$ about a line $y = c$ (parallel to the $x-$axis, then the volume of the solid is given by

$$V = \int_a^b \pi [y-c]^2 dx$$

Example 3.

The volume of the solid generated by the revolving the curve $y = \sqrt{x}$ about the line $y = 1$ from $x = 1$ to $x = 4$ is given by the following code:

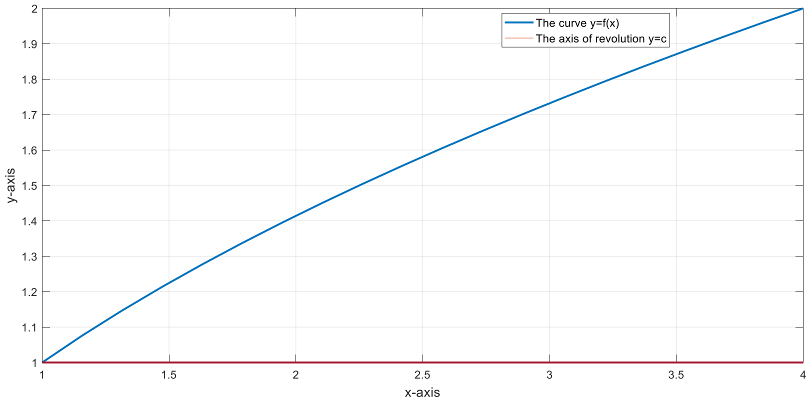
```
clc
syms x
f=input('Enter the function f(x): ');
c=input('Enter the axis of rotation y = c (enter only c value): ');
iL=input('Enter the integration limits: ');
a=iL(1);b=iL(2);
vol=pi*int((f-c)^2,a,b);
disp(['Volume of solid of revolution is: ',char(vol)]);
x1=linspace(a,b,20); y1=subs(f,x,x1);
x2=x1; y2=c*ones(length(x1));
plot(x1,y1);hold on;
plot(x2,y2);hold off;
xlabel('x-axis');ylabel('y-axis')
legend('The curve y=f(x)','The axis of revolution y=c');
grid on;
```

Input

```
Enter the function f(x)sqrt(x)
Enter the axis of rotation y = c (enter only c value): 1
Enter the integration limits: [1,4]
```

Output

```
Volume of solid of revolution is: (7*pi)/6
```



Exercise:

1. Find the area of the region bounded by the curve $y = x^2 - 2x$ and the line $y = x$.
2. Find the area of the region bounded by the curves $x = y^3$ and $x = y^2$.
3. Find the volume of a sphere formed by rotating a semicircle of radius 2 units about $x-$axis.
4. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the curve $y = \dfrac{4}{x^2+4}$, the $x-$axis, and the lines $x = 0$ and $x = 2$.

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