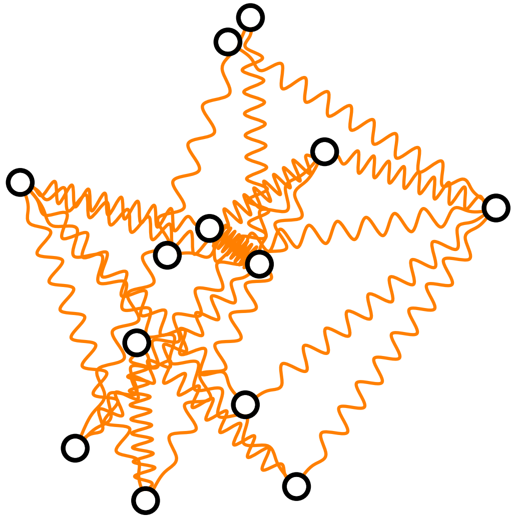
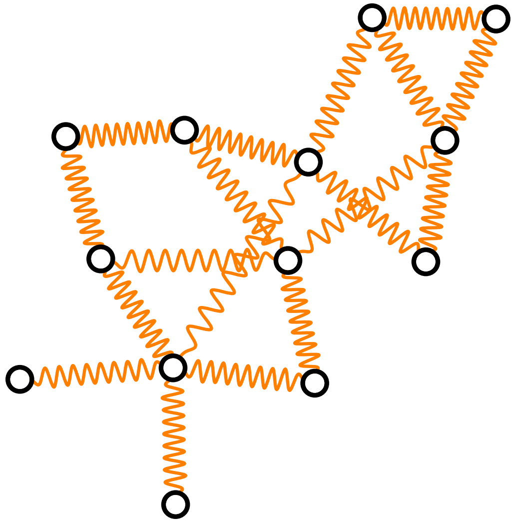
CS F402: Computational Geometry

Lecture 9:
GD - Force-Directed Drawing Algorithms-I



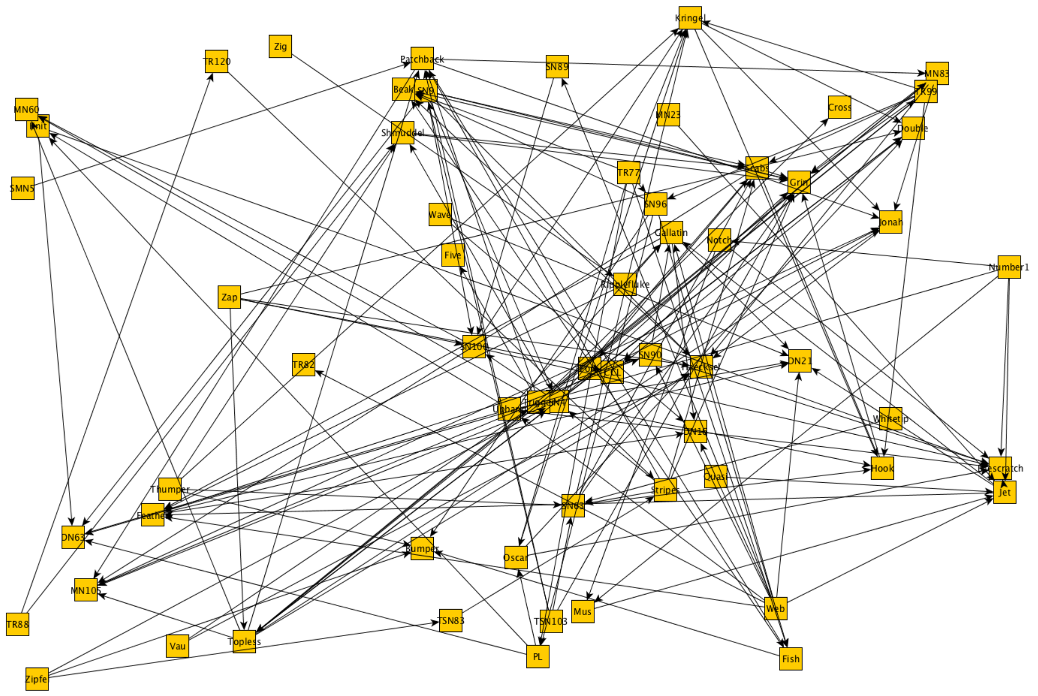
Siddharth Gupta

February 12+14, 2025



General Layout Problem

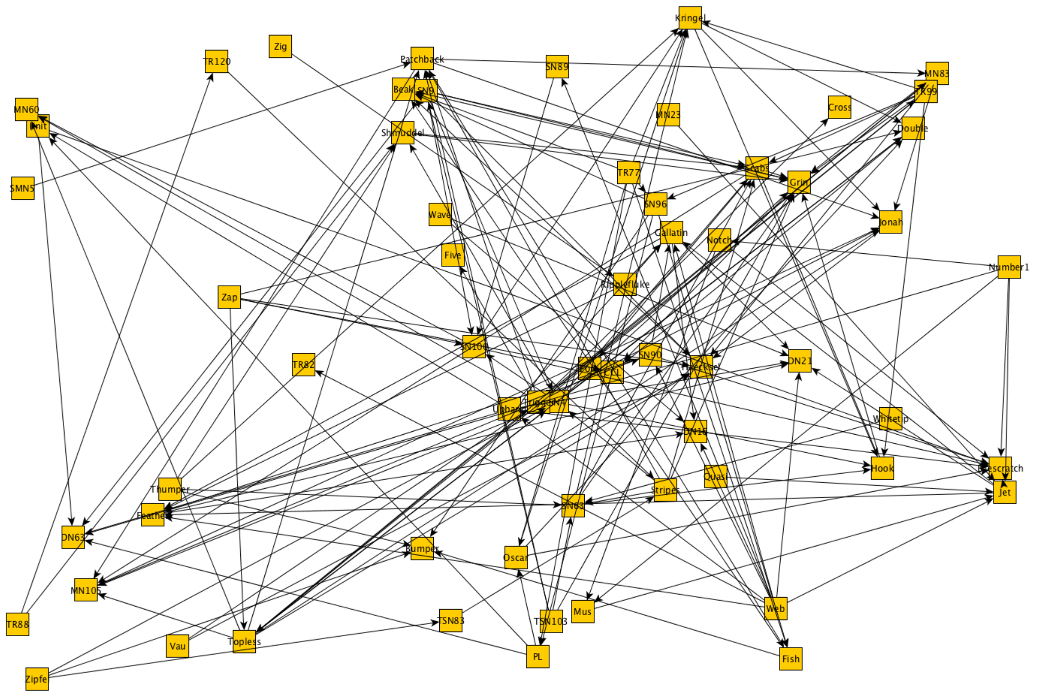
Input: Graph $G = (V, E)$

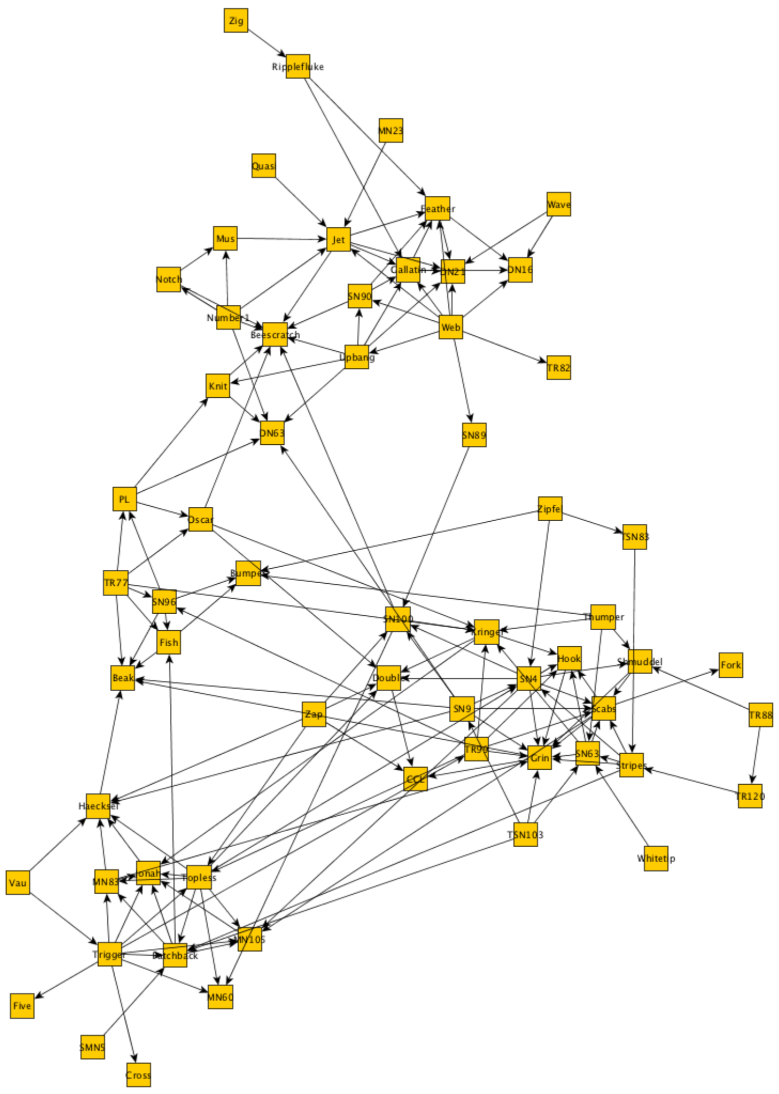


General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G



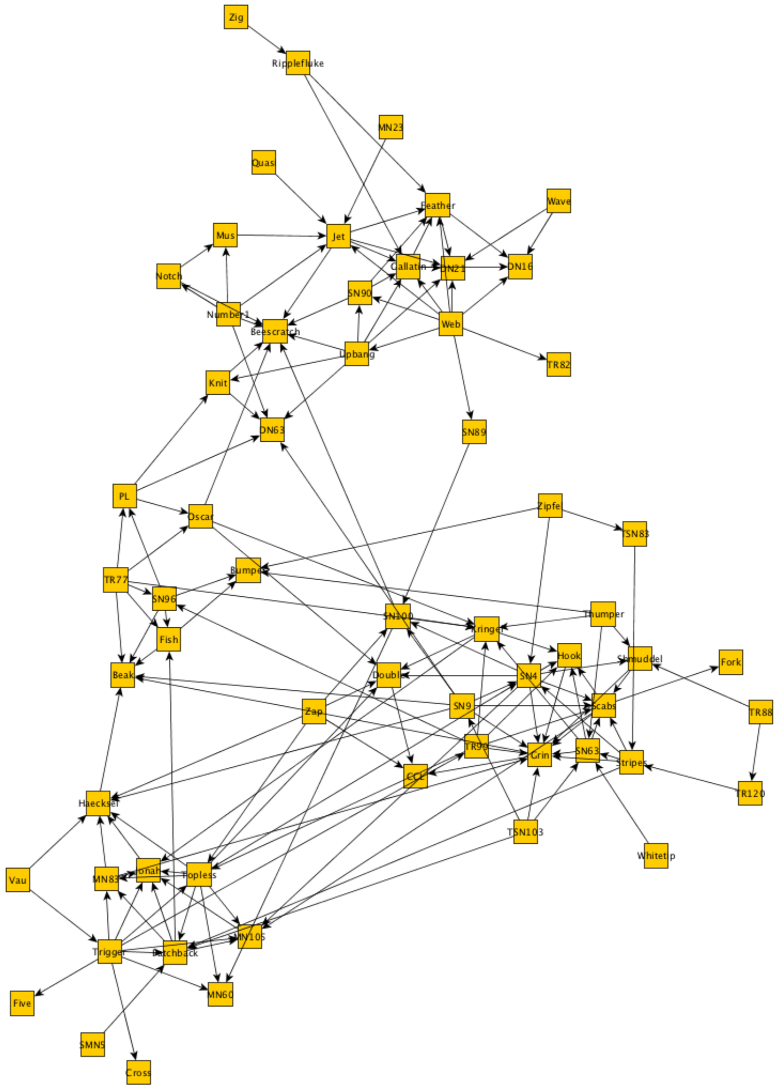


General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:



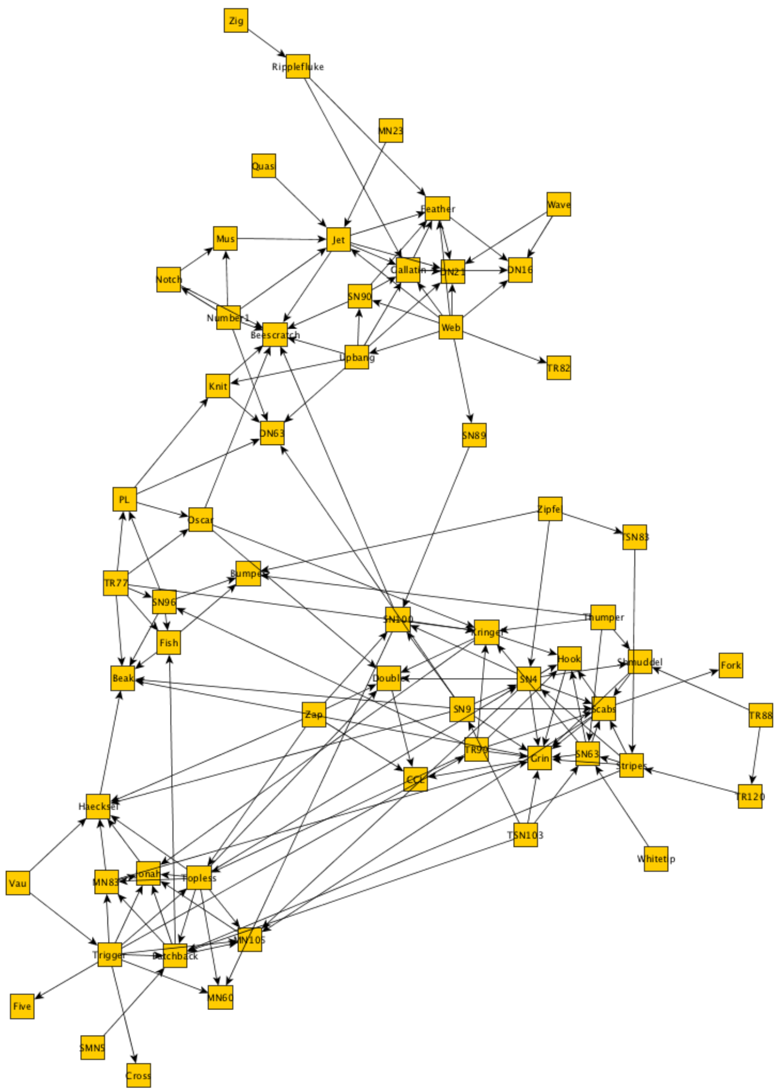
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close



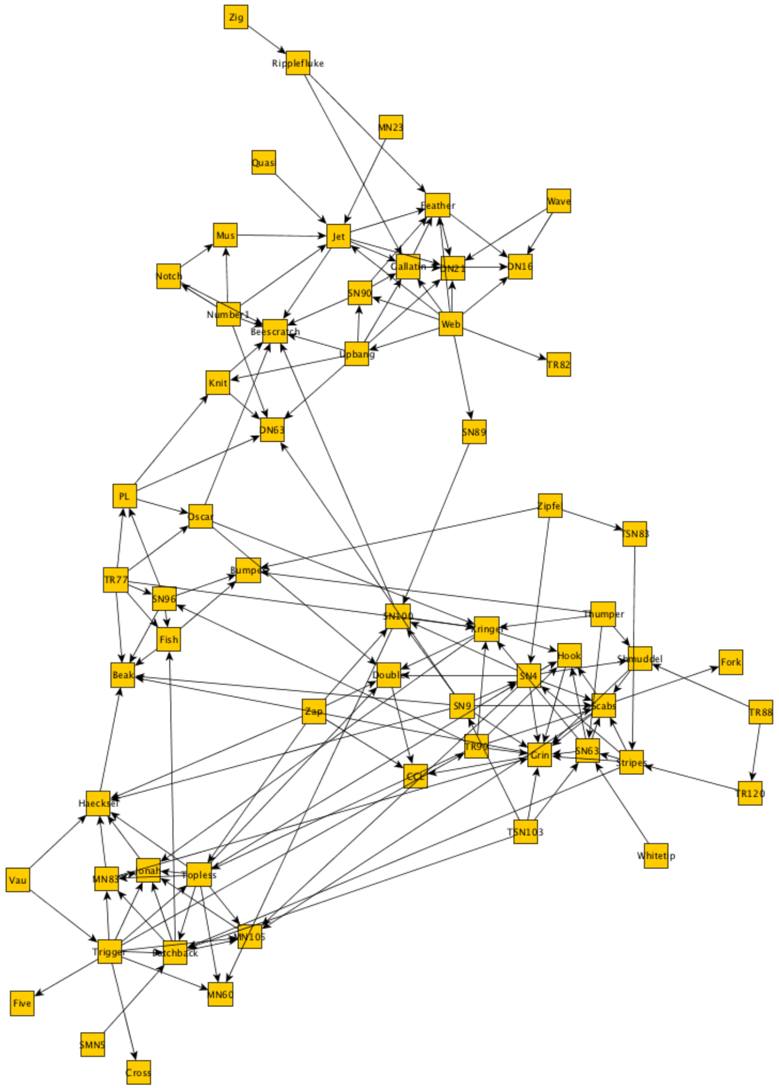
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart



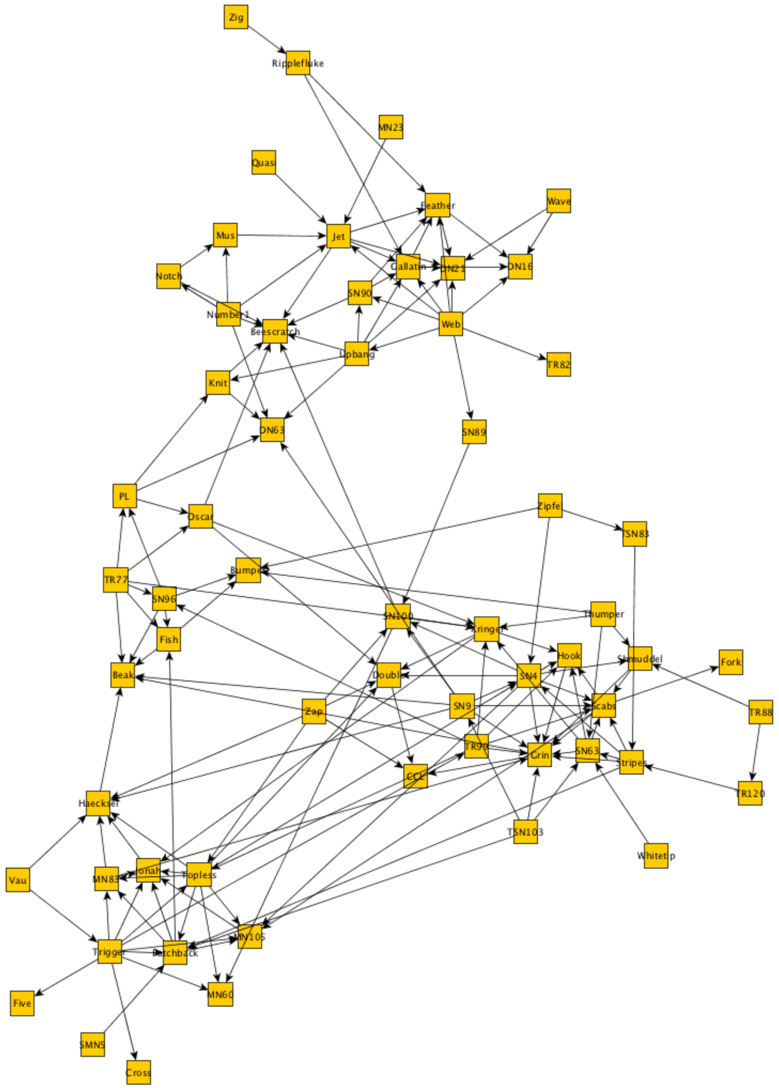
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart
- edges short, straight-line, **similar length**



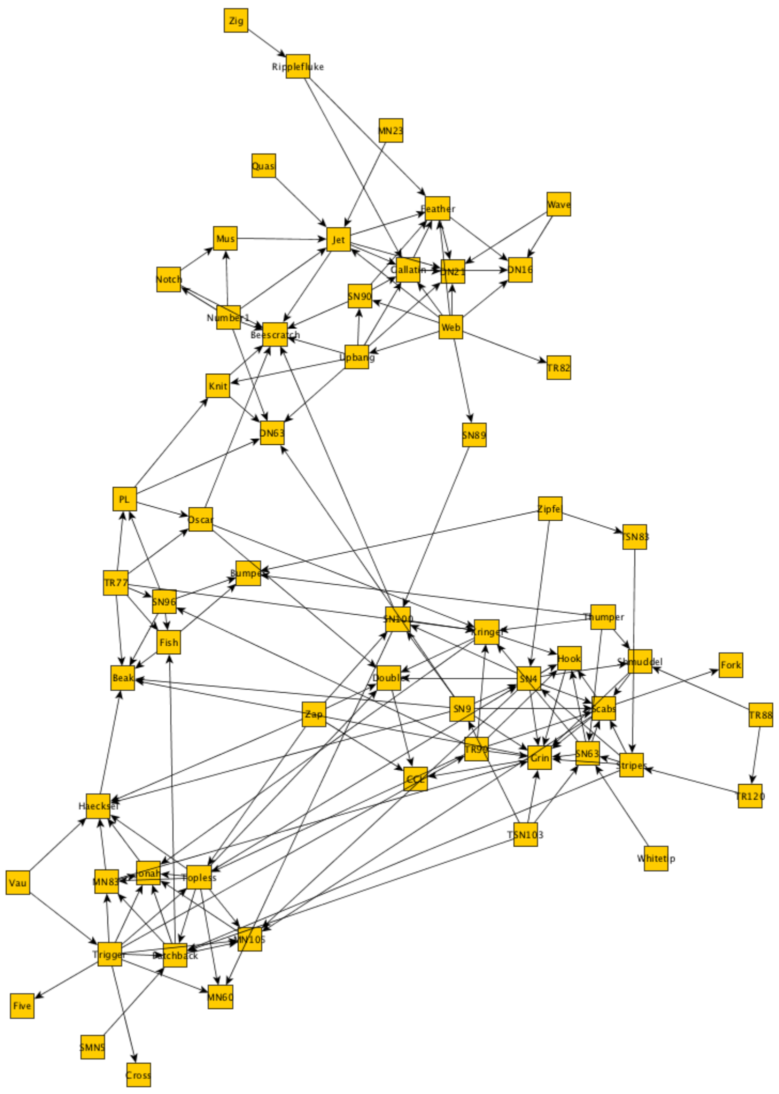
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart
- edges short, straight-line, **similar length**
- densely connected parts (clusters) form communities



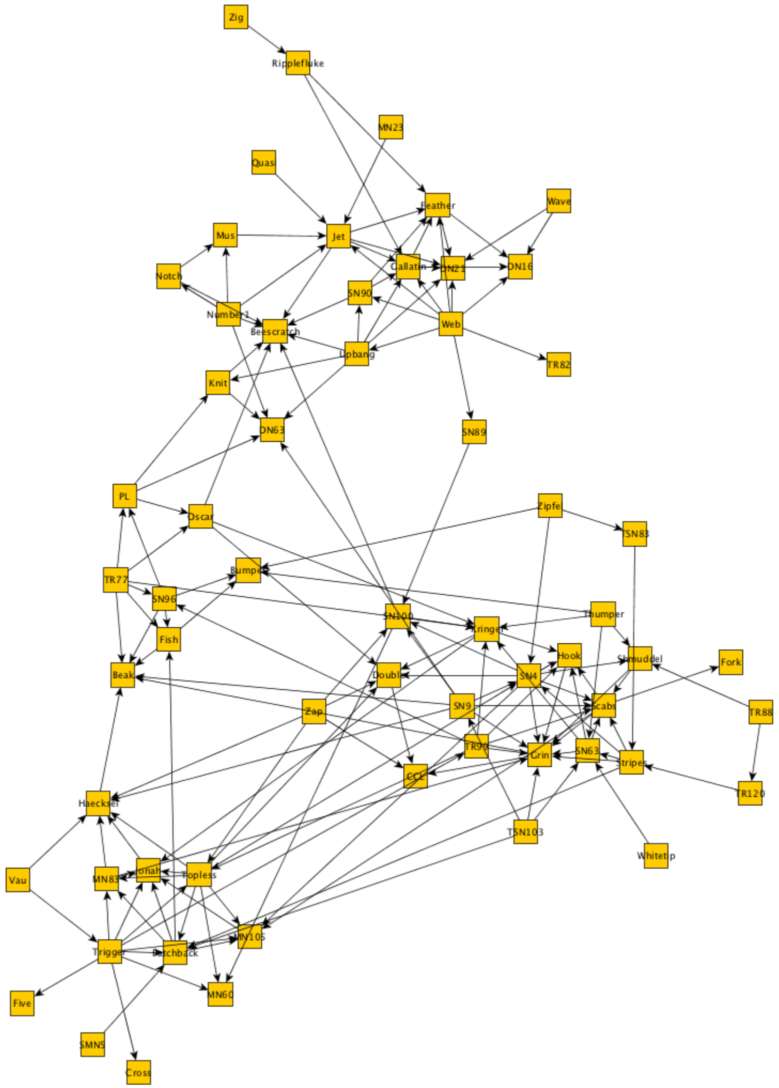
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart
- edges short, straight-line, **similar length**
- densely connected parts (clusters) form communities
- as few crossings as possible



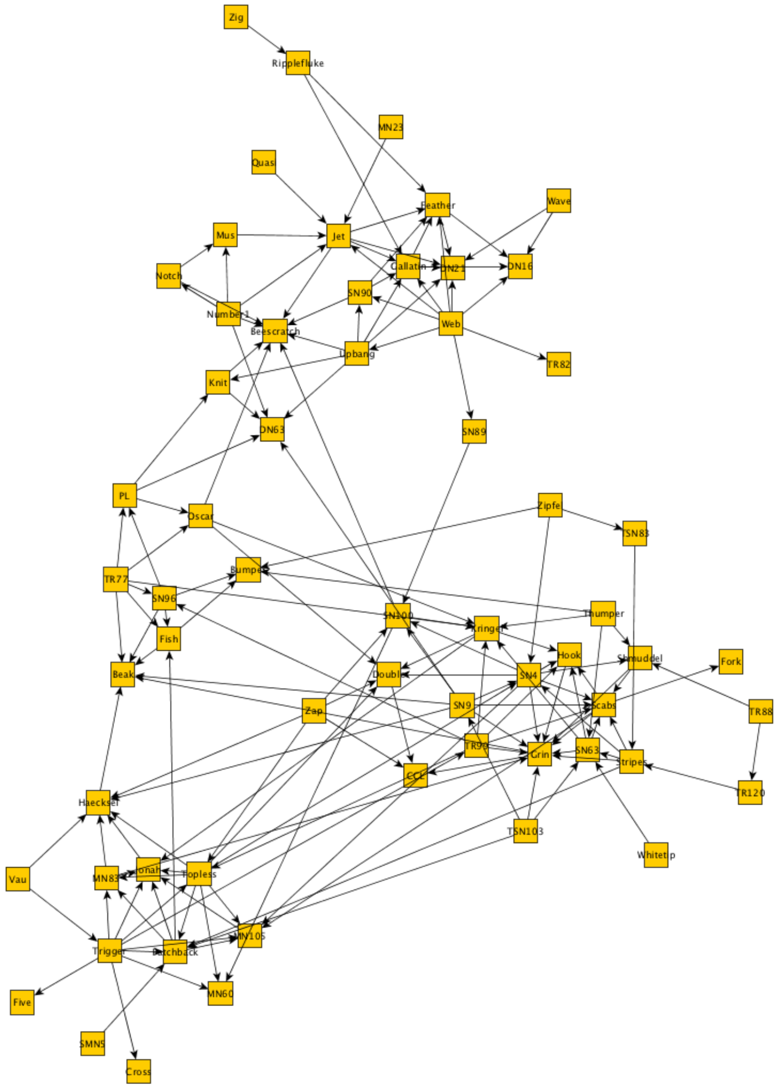
General Layout Problem

Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart
- edges short, straight-line, **similar length**
- densely connected parts (clusters) form communities
- as few crossings as possible
- nodes distributed evenly



General Layout Problem

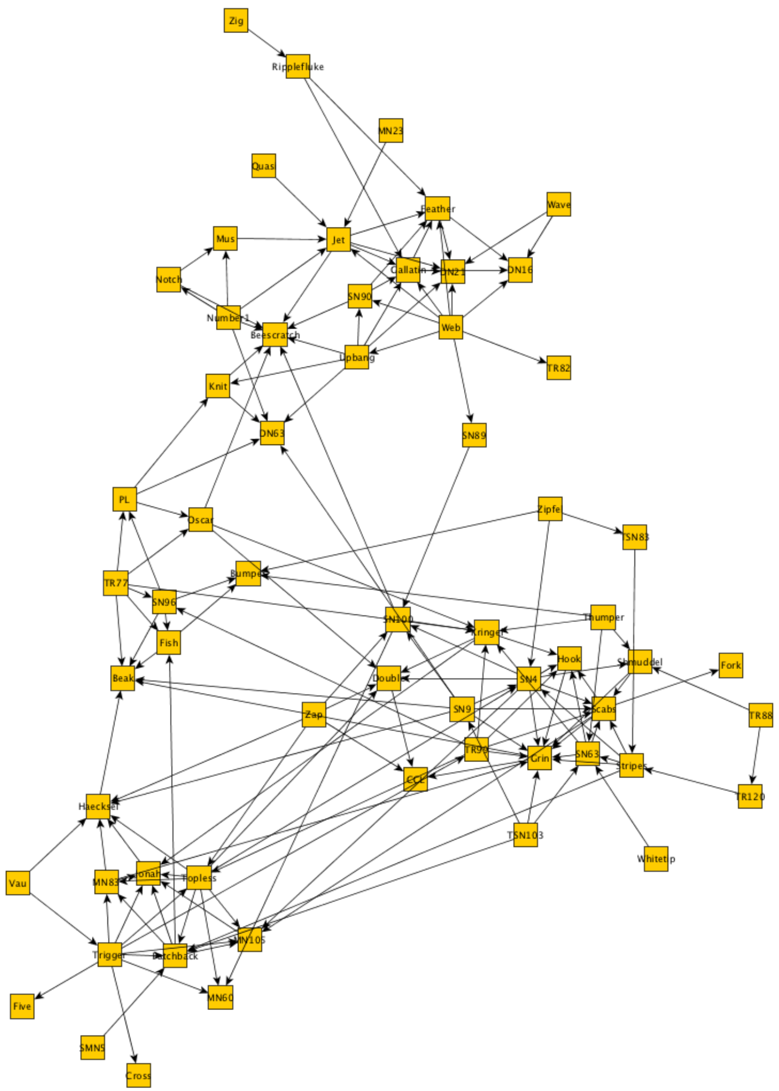
Input: Graph $G = (V, E)$

Output: Clear and readable straight-line drawing of G

Drawing aesthetics:

- adjacent vertices are close
- non-adjacent vertices are far apart
- edges short, straight-line, **similar length**
- densely connected parts (clusters) form communities
- as few crossings as possible
- nodes distributed evenly

Optimization criteria partially contradict each other



Fixed Edge Lengths?

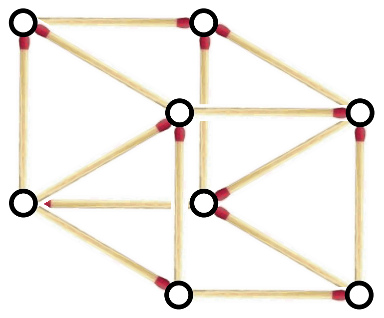
Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

Output: Drawing of G which realizes all the edge lengths

Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

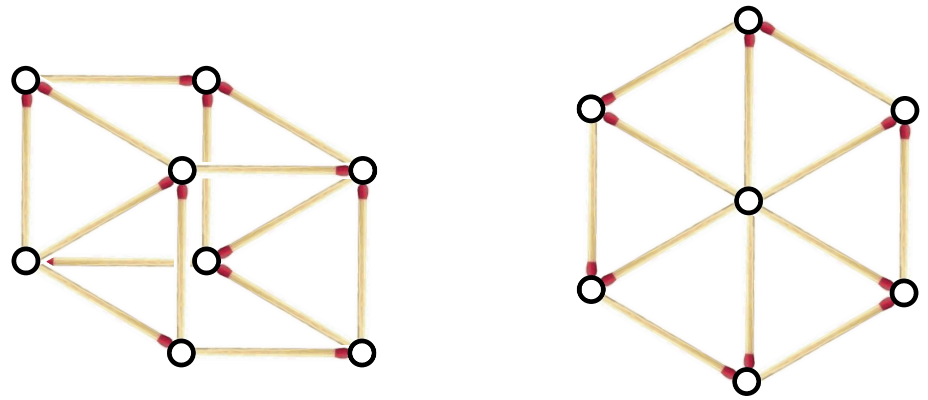
Output: Drawing of G which realizes all the edge lengths



Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

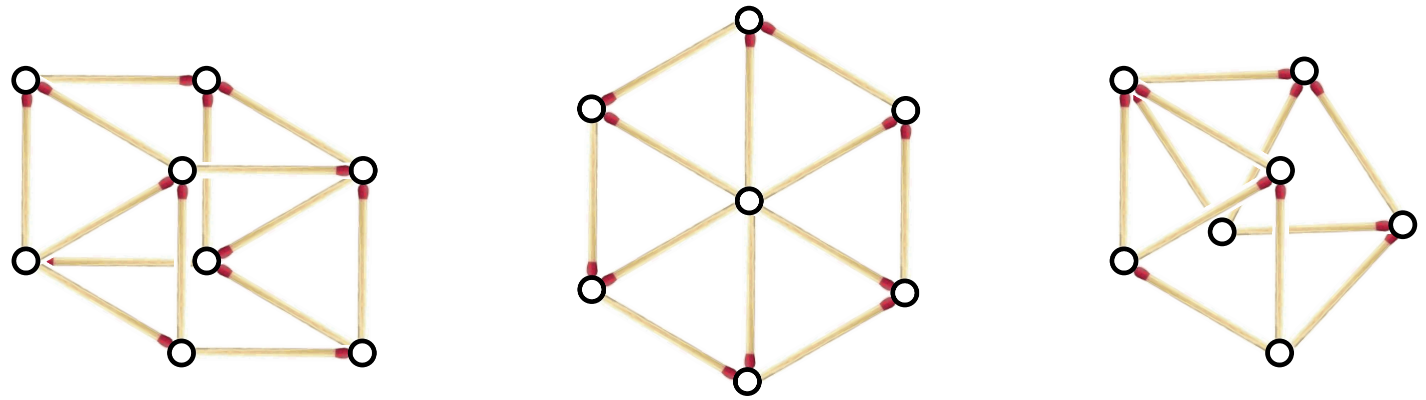
Output: Drawing of G which realizes all the edge lengths



Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

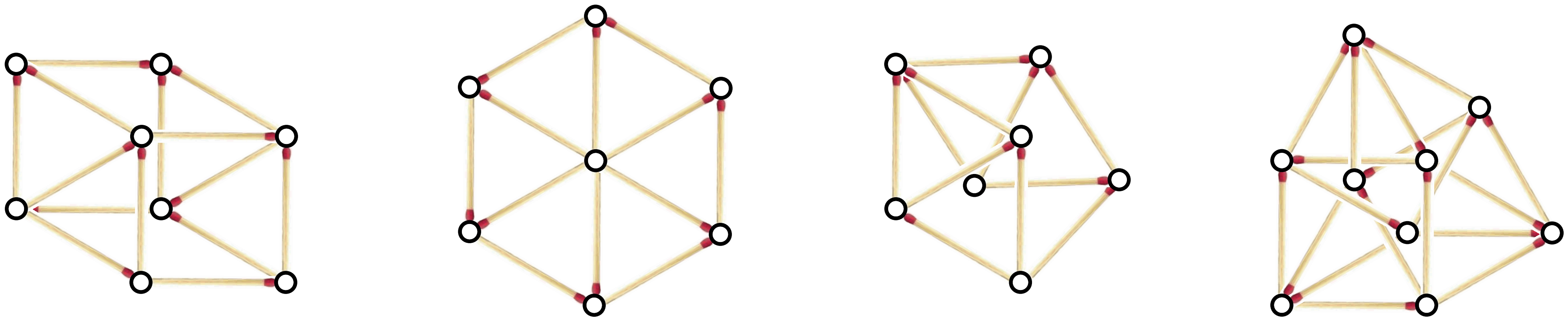
Output: Drawing of G which realizes all the edge lengths



Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

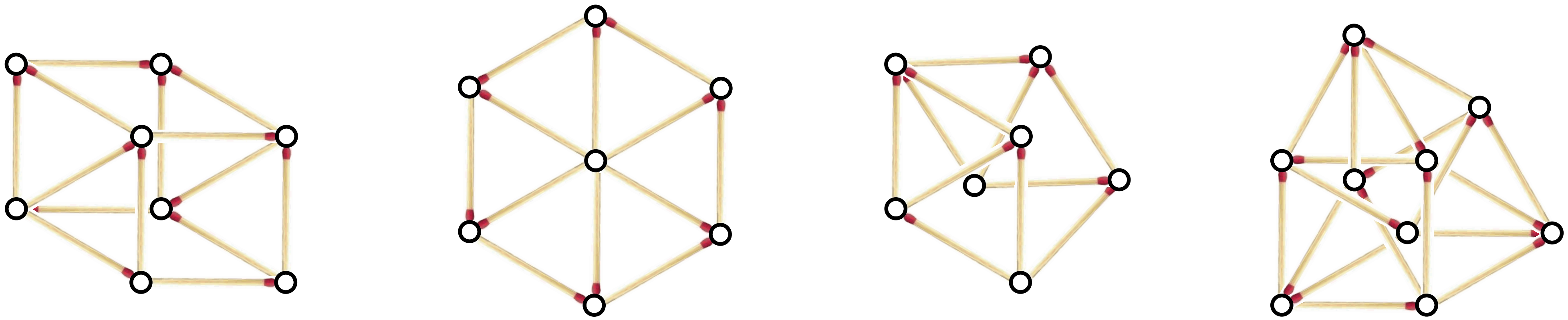
Output: Drawing of G which realizes all the edge lengths



Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

Output: Drawing of G which realizes all the edge lengths

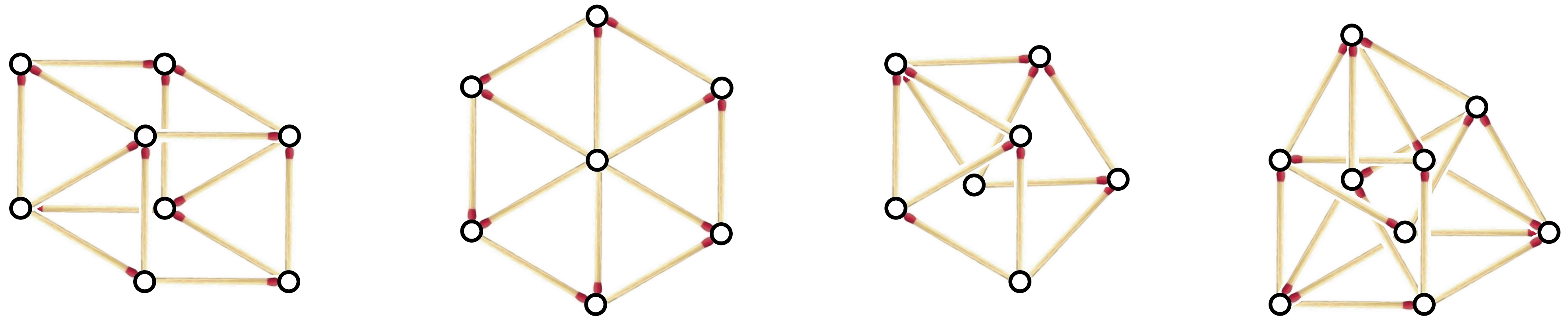


NP-hard for

Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

Output: Drawing of G which realizes all the edge lengths



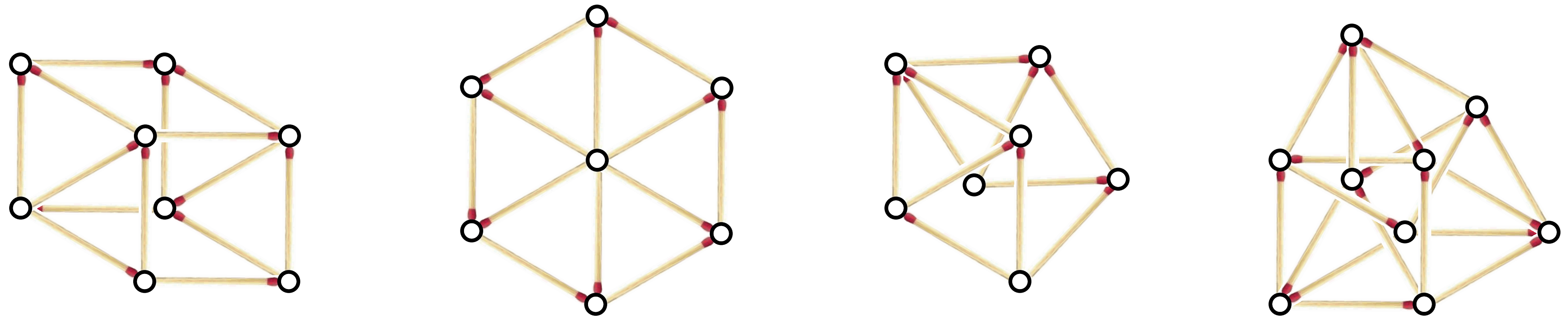
NP-hard for

- uniform edge lengths in any dimension [Johnson '82]

Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

Output: Drawing of G which realizes all the edge lengths



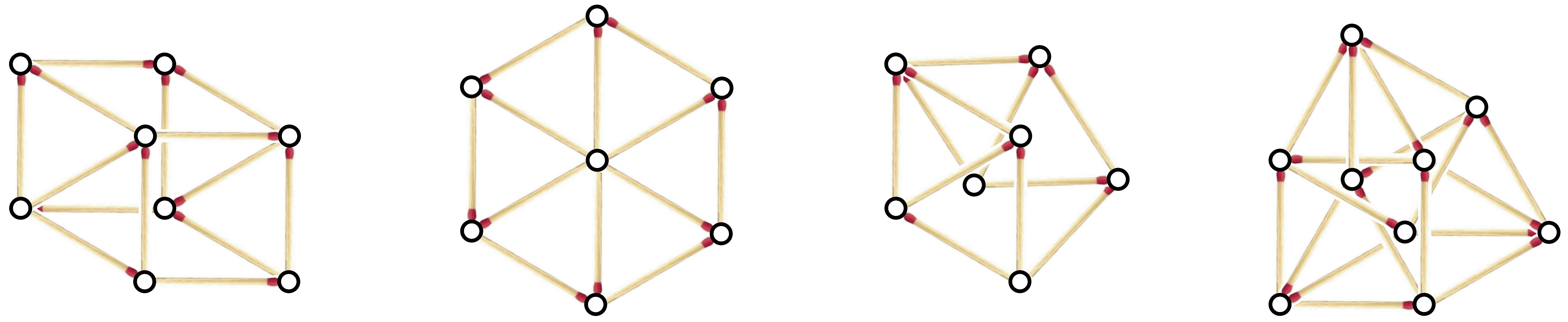
NP-hard for

- uniform edge lengths in any dimension [Johnson '82]
- uniform edge lengths in planar drawings [Eades, Wormald '90]

Fixed Edge Lengths?

Input: Graph $G = (V, E)$, required edge length $\ell(e)$, $\forall e \in E$

Output: Drawing of G which realizes all the edge lengths



NP-hard for

- uniform edge lengths in any dimension [Johnson '82]
- uniform edge lengths in planar drawings [Eades, Wormald '90]
- edge lengths $\{1, 2\}$ [Saxe '80]

Physical Analogy

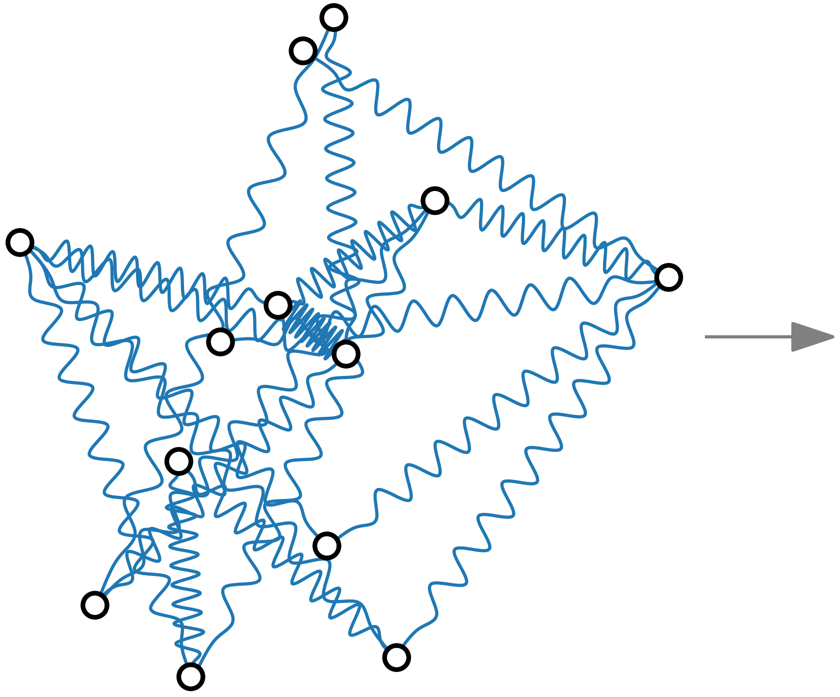
Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system .

Physical Analogy

Idea. [Eades ’84]

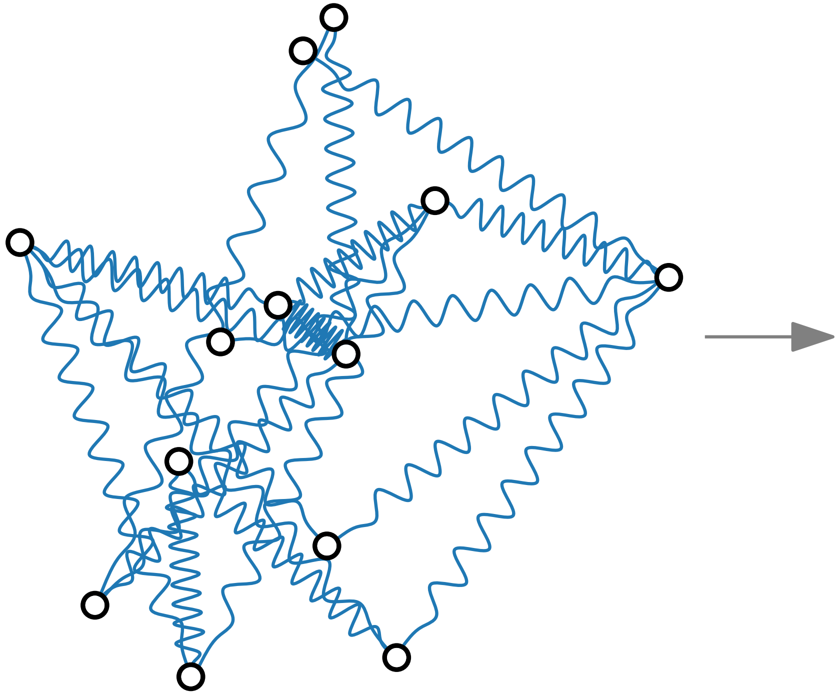
“To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system .



Physical Analogy

Idea. [Eades ’84]

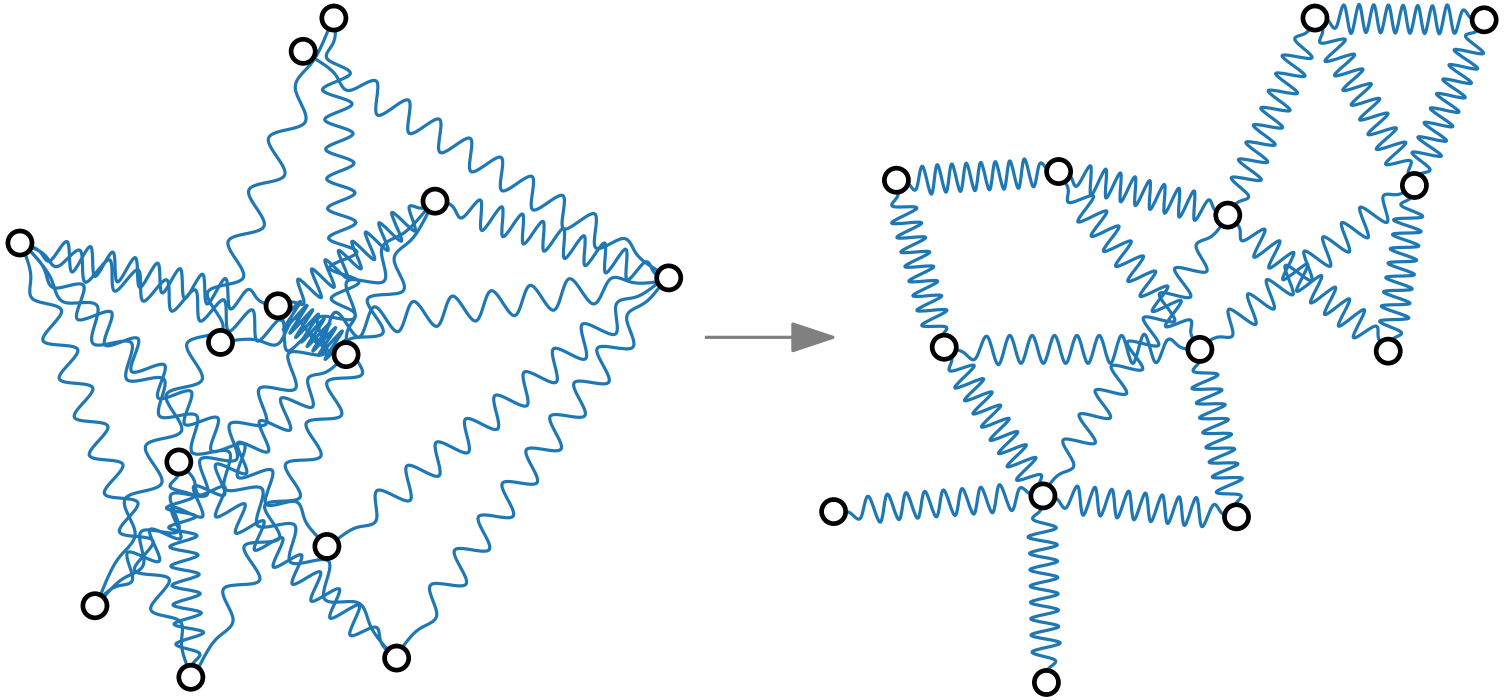
“To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state.”



Physical Analogy

Idea. [Eades '84]

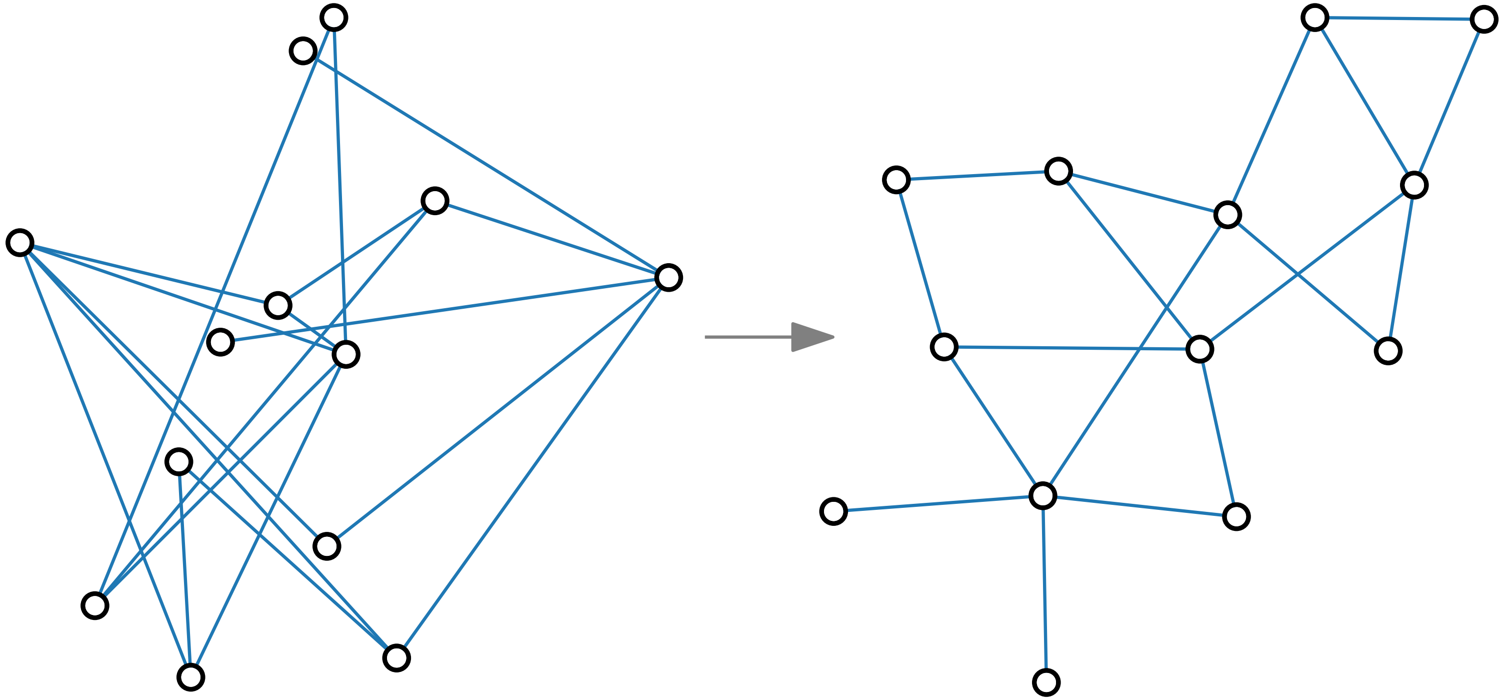
"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



Physical Analogy

Idea. [Eades ’84]

“To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state.”

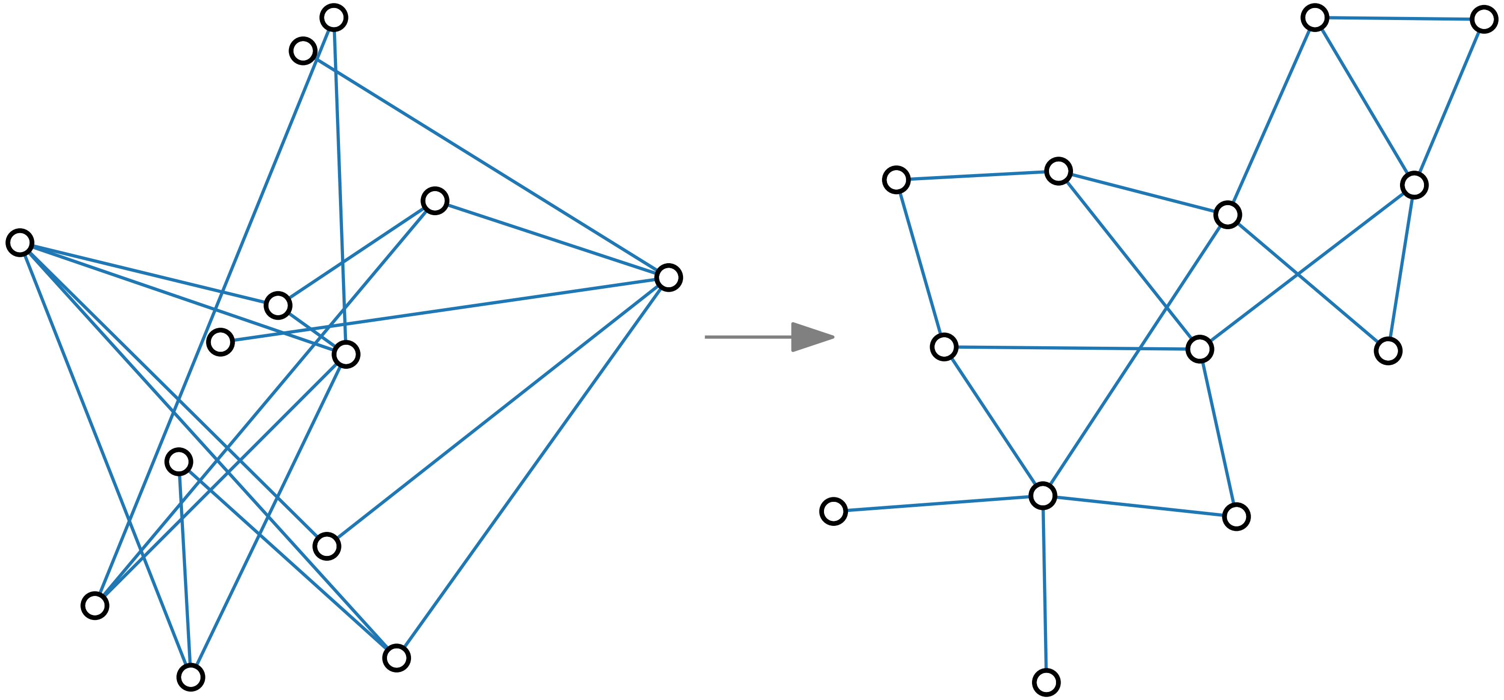


Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."

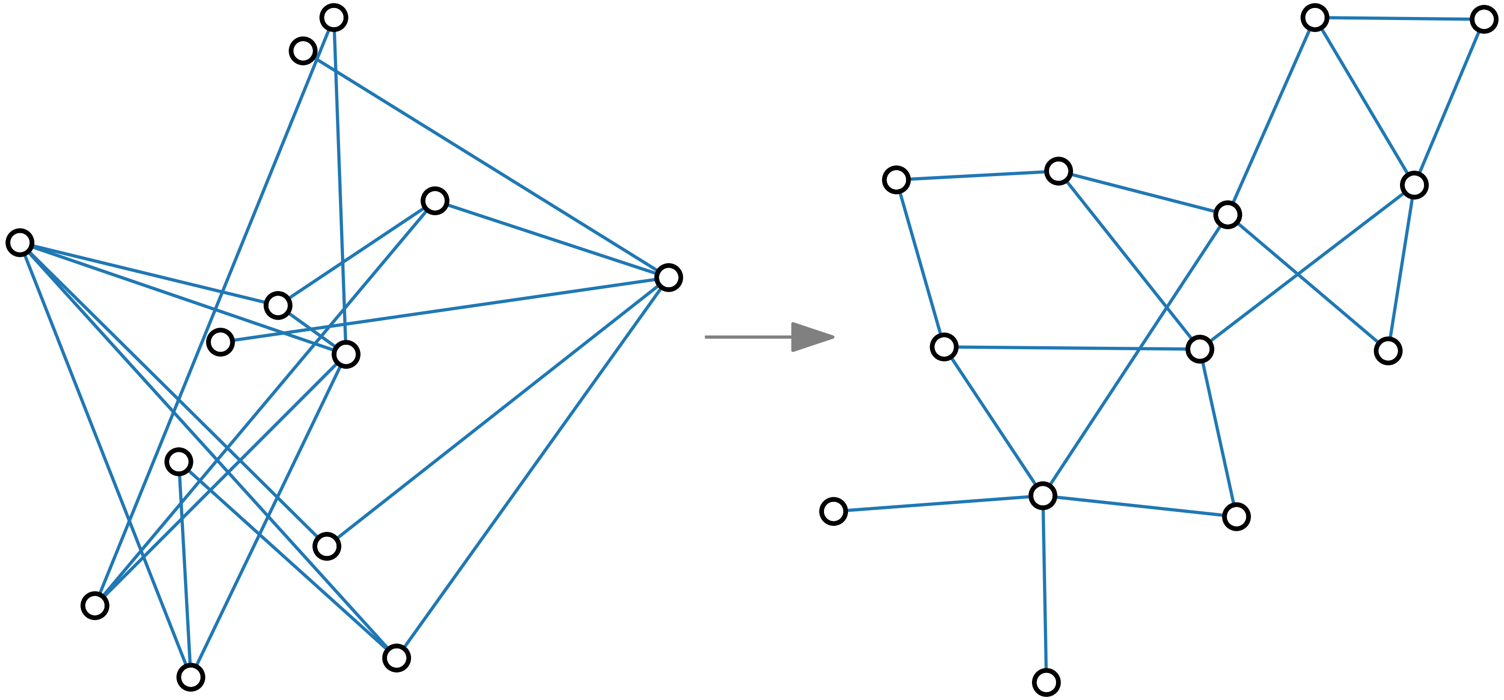
Attractive forces.



Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system ... The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



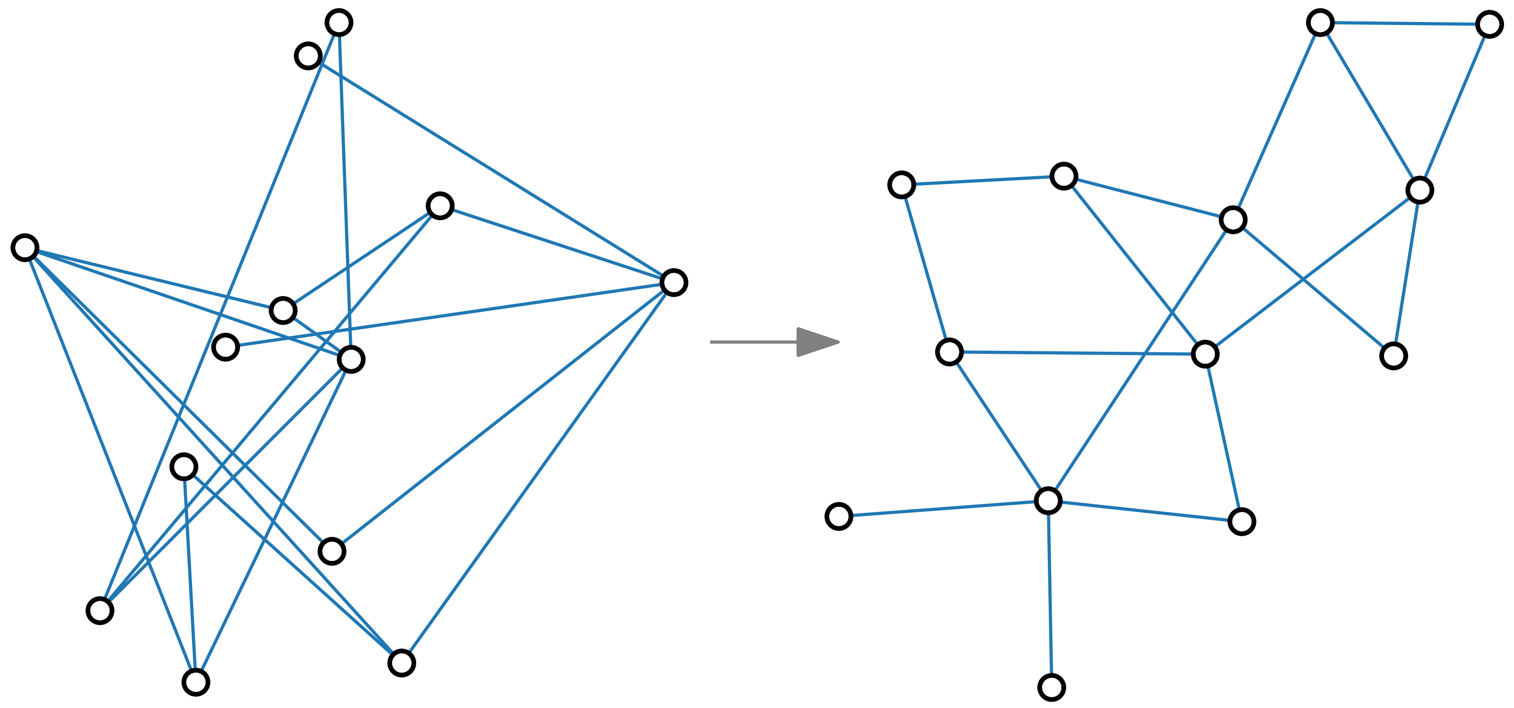
Attractive forces.

adjacent vertices u and v:

Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system ... The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



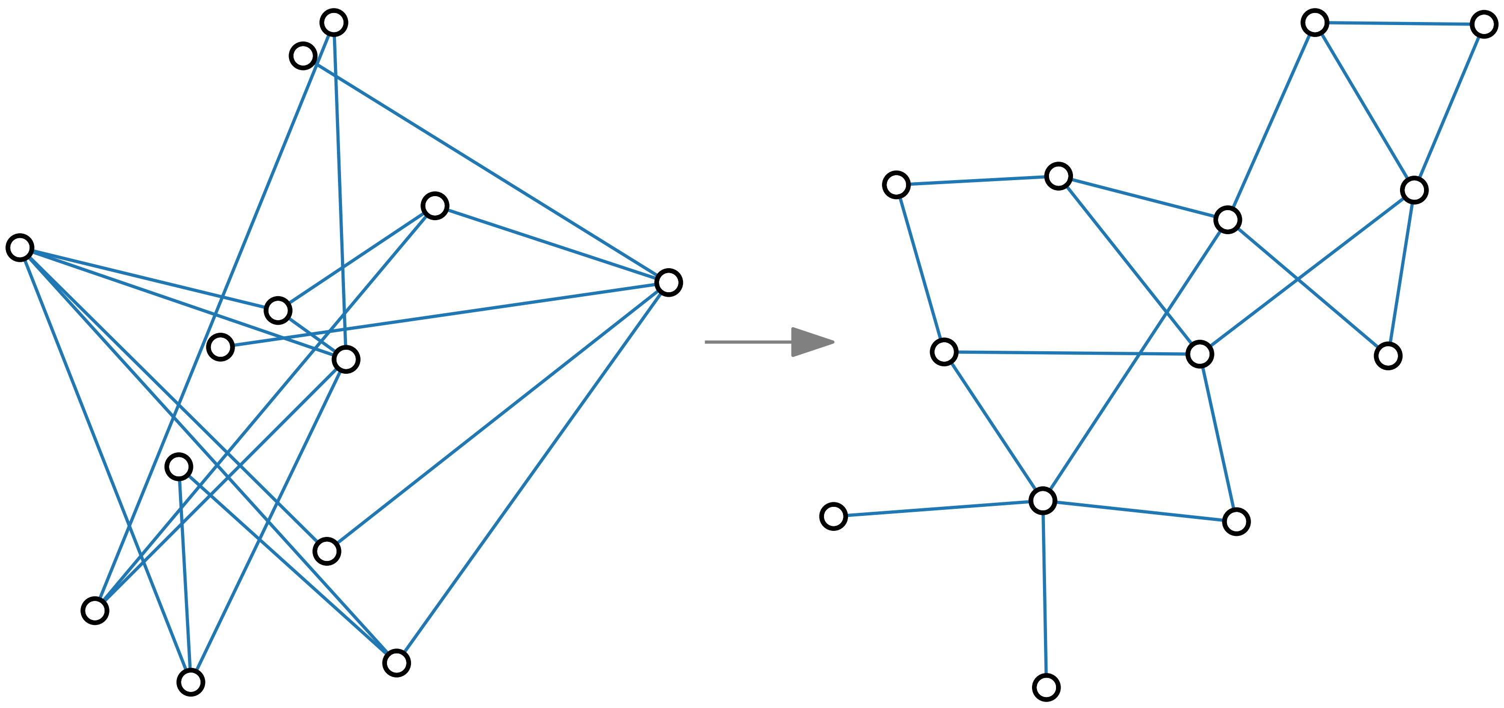
Attractive forces.

adjacent vertices u and v:

Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



Attractive forces.

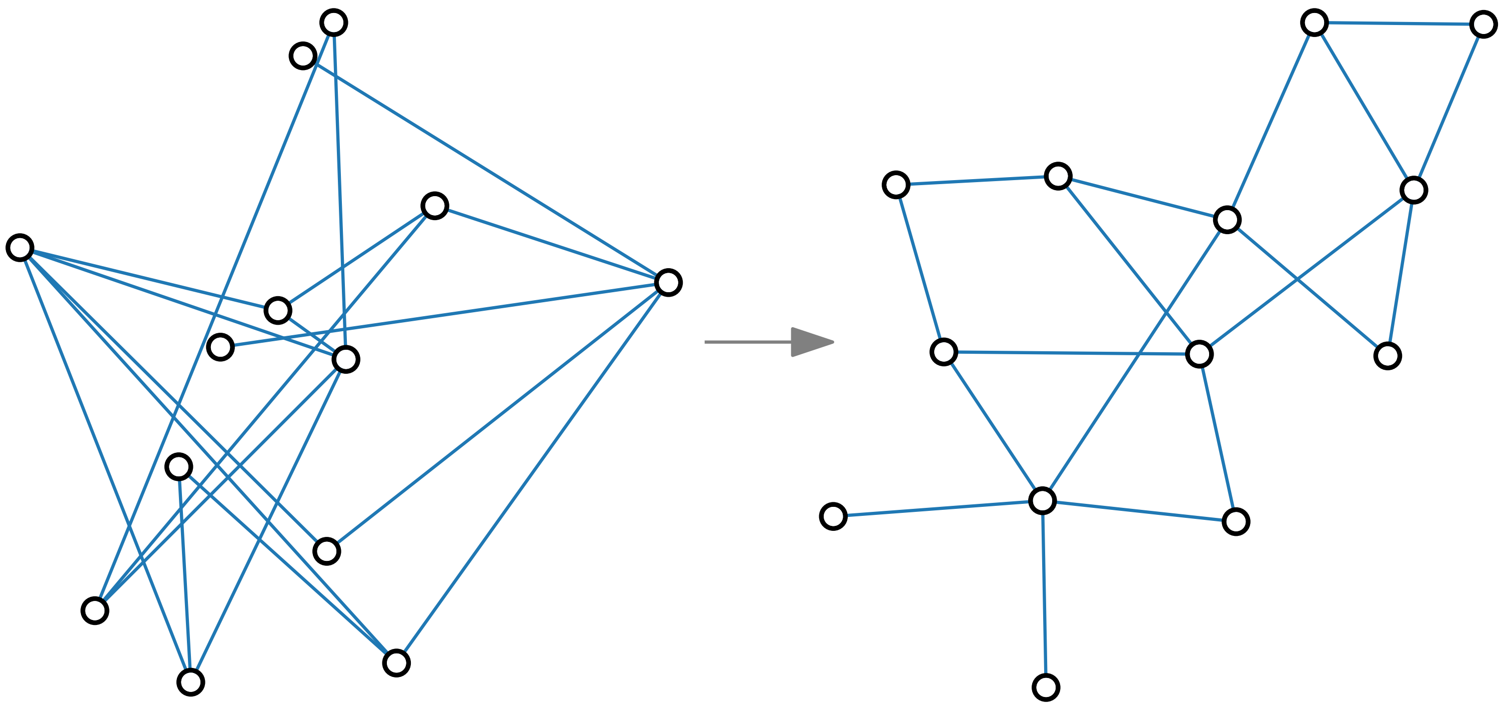
adjacent vertices u and v:

Repulsive forces.

Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system ... The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



Attractive forces.

adjacent vertices u and v:

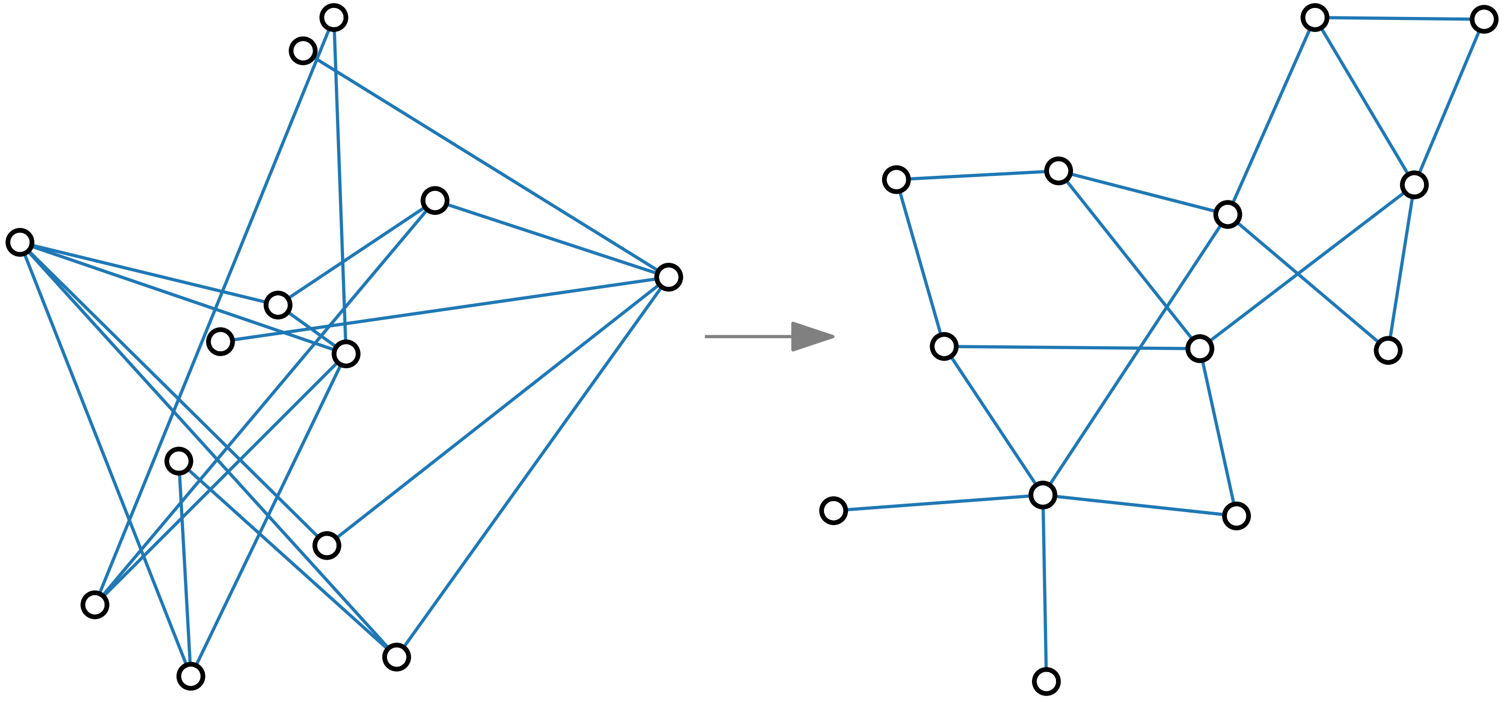
Repulsive forces.

all vertices x and y:

Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



Attractive forces.

adjacent vertices u and v:

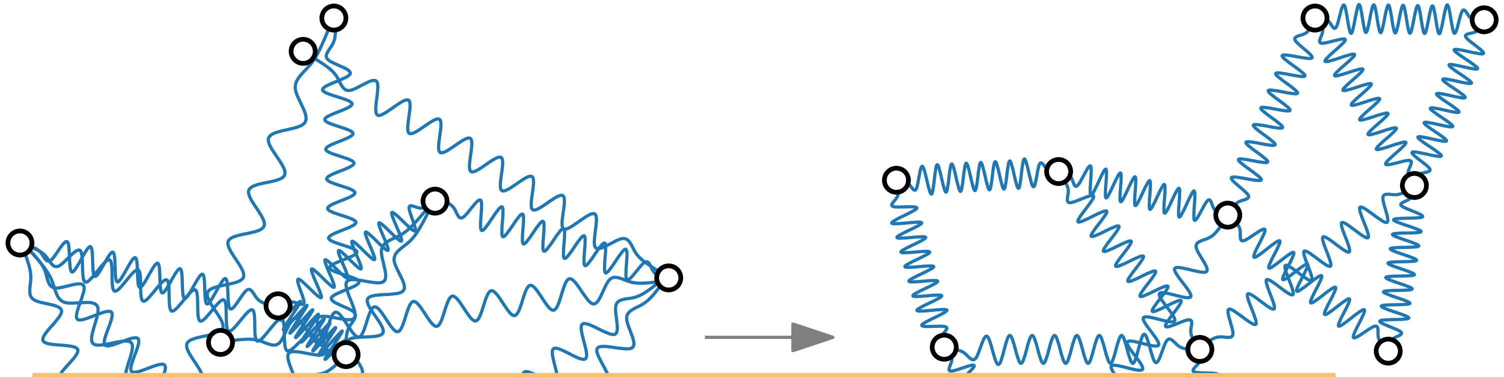
Repulsive forces.

all vertices x and y:

Physical Analogy

Idea. [Eades '84]

"To embed a graph we replace the vertices by steel rings and replace each edge with a **spring** to form a mechanical system . . . The vertices are placed in some initial layout and let go so that the spring forces on the rings move the system to a minimal energy state."



Attractive forces.

adjacent vertices u and v:

Repulsive forces.

all vertices x and y:

So-called **spring embedders** or **force-directed** algorithms that work according to this or similar principles are among the most frequently used graph-drawing methods in practice.

Force-Directed Algorithms

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

return p

Force-Directed Algorithms

initial layout

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

return p

Force-Directed Algorithms

initial layout

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

return p

end layout

Force-Directed Algorithms

initial layout

threshold

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

return p

end layout

Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

return p

end layout

Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

$t \leftarrow t + 1$

return p

end layout

Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

  $t \leftarrow t + 1$

return p

end layout

u ∘

Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected$(G = (V, E), p = (p_v)_{v \in V}, \varepsilon > 0, K \in \mathbb{N})$

  $t \leftarrow 1$

  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

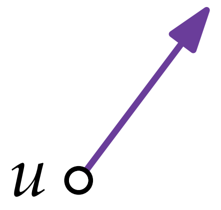
    foreach $u \in V$ **do**

      $F_u(t) \leftarrow$

    $t \leftarrow t + 1$

  return p

end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected$(G = (V, E), p = (p_v)_{v \in V}, \varepsilon > 0, K \in \mathbb{N})$

  $t \leftarrow 1$

  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

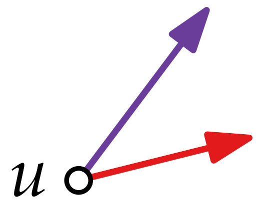
    foreach $u \in V$ **do**

      $F_u(t) \leftarrow \sum_{v \in V} f_{\mathrm{rep}}(u, v) +$

    $t \leftarrow t + 1$

  return p

end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

  $t \leftarrow 1$

  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

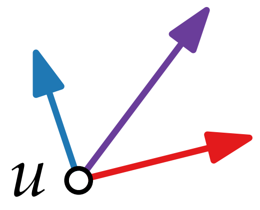
    foreach $u \in V$ **do**

      $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$

    $t \leftarrow t + 1$

  return p

end layout



Force-Directed Algorithms

initial layout

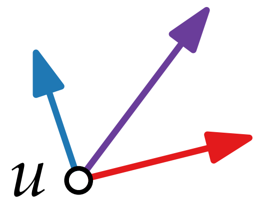
threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

- $t \leftarrow 1$
- **while** $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
  - **foreach** $u \in V$ **do**
    - $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
  - **foreach** $u \in V$ **do**
    - $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
  - $t \leftarrow t + 1$
- **return** p

end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

    $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$

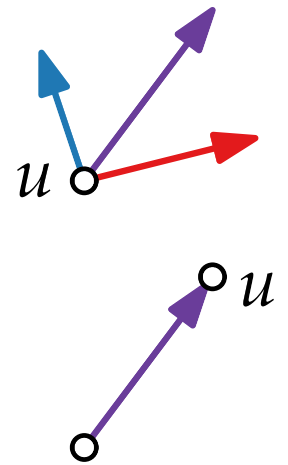
  foreach $u \in V$ **do**

    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$

  $t \leftarrow t + 1$

return p

end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected$(G = (V, E), p = (p_v)_{v \in V}, \varepsilon > 0, K \in \mathbb{N})$

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

    $F_u(t) \leftarrow \sum_{v \in V} f_{\mathrm{rep}}(u, v) + \sum_{uv \in E} f_{\mathrm{attr}}(u, v)$

  foreach $u \in V$ **do**

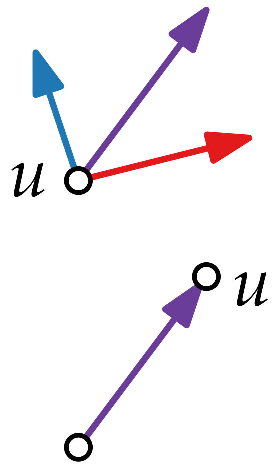
    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$

  $t \leftarrow t + 1$

return p

cooling factor

end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

    $F_u(t) \leftarrow \sum_{v \in V} f_{\mathrm{rep}}(u, v) + \sum_{uv \in E} f_{\mathrm{attr}}(u, v)$

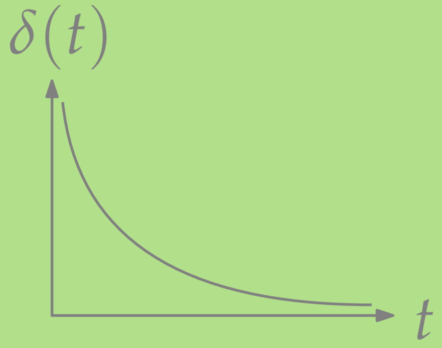
  foreach $u \in V$ **do**

    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$

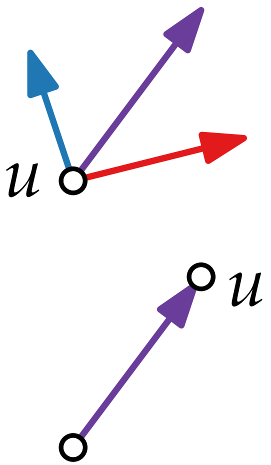
  $t \leftarrow t + 1$

return p

cooling factor



end layout



Force-Directed Algorithms

initial layout

threshold

max # iterations

ForceDirected$(G = (V, E), p = (p_v)_{v \in V}, \varepsilon > 0, K \in \mathbb{N})$

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

    $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$

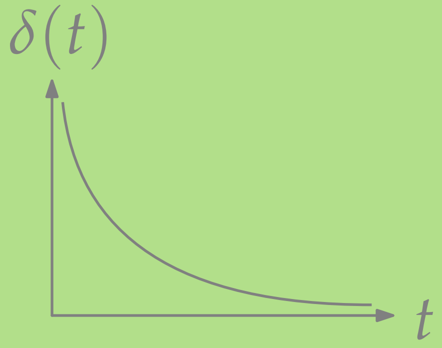
  foreach $u \in V$ **do**

    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$

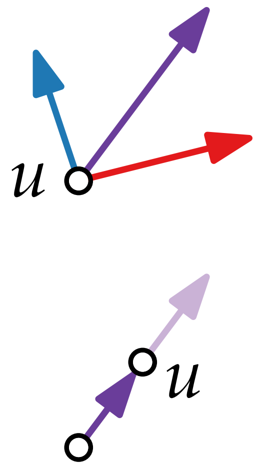
  $t \leftarrow t + 1$

return p

cooling factor



end layout



Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$

while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**

  foreach $u \in V$ **do**

    $F_u(t) \leftarrow \sum_{v \in V} f_{\mathrm{rep}}(u, v) + \sum_{uv \in E} f_{\mathrm{attr}}(u, v)$

  foreach $u \in V$ **do**

    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$

  $t \leftarrow t + 1$

return p

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

$t \leftarrow 1$
while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
  foreach $u \in V$ **do**
    $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
  foreach $u \in V$ **do**
    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
  $t \leftarrow t + 1$
return p

- **Repulsive forces**

- **Attractive forces**

- **Resulting displacement vector**

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

```
t ← 1
while t < K and max_{v∈V} ||F_v(t)|| > ε do
    foreach u ∈ V do
        F_u(t) ← Σ_{v∈V} f_rep(u,v) + Σ_{uv∈E} f_attr(u,v)
    foreach u ∈ V do
        p_u ← p_u + δ(t) · F_u(t)
    t ← t + 1
return p
```

- **Repulsive forces**
- **Attractive forces**
- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)
  $t \leftarrow 1$
  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
    foreach $u \in V$ **do**
      $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
    foreach $u \in V$ **do**
      $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
    $t \leftarrow t + 1$
  return p

- **Repulsive forces**

$$f_{\text{rep}}(u, v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)

```
t ← 1
while t < K and max_{v∈V} ||F_v(t)|| > ε do
    foreach u ∈ V do
        F_u(t) ← Σ_{v∈V} f_rep(u, v) + Σ_{uv∈E} f_attr(u, v)
    foreach u ∈ V do
        p_u ← p_u + δ(t) · F_u(t)
    t ← t + 1
return p
```

- **Repulsive forces**

$$f_{\text{rep}}(u, v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)
  $t \leftarrow 1$
  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
    foreach $u \in V$ **do**
      $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
    foreach $u \in V$ **do**
      $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
    $t \leftarrow t + 1$
  return p

- **Repulsive forces**

$$f_{\text{rep}}(u, v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v

Spring Embedder by Eades – Model

```
ForceDirected(G = (V, E), p = (p_v)_{v∈V}, ε > 0, K ∈ ℕ)
  t ← 1
  while t < K and max_{v∈V} ||F_v(t)|| > ε do
      foreach u ∈ V do
          F_u(t) ← Σ_{v∈V} f_rep(u, v) + Σ_{uv∈E} f_attr(u, v)
      foreach u ∈ V do
          p_u ← p_u + δ(t) · F_u(t)
      t ← t + 1
  return p
```

- **Repulsive forces**

repulsion constant (e.g. 2.0)

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u,v) + \sum_{uv \in E} f_{\text{attr}}(u,v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v

Spring Embedder by Eades – Model

```
ForceDirected(G = (V, E), p = (p_v)_{v∈V}, ε > 0, K ∈ ℕ)
  t ← 1
  while t < K and max_{v∈V} ||F_v(t)|| > ε do
    foreach u ∈ V do
      F_u(t) ← Σ_{v∈V} f_rep(u, v) + Σ_{uv∈E} f_attr(u, v)
    foreach u ∈ V do
      p_u ← p_u + δ(t) · F_u(t)
    t ← t + 1
  return p
```

- **Repulsive forces**

repulsion constant (e.g. 2.0)

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

$$f_{\text{spring}}(u,v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u,v) + \sum_{uv \in E} f_{\text{attr}}(u,v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v

Spring Embedder by Eades – Model

$\text{ForceDirected}(G = (V, E), p = (p_v)_{v \in V}, \varepsilon > 0, K \in \mathbb{N})$

$t \leftarrow 1$
while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
  foreach $u \in V$ **do**
    $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
  foreach $u \in V$ **do**
    $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
  $t \leftarrow t + 1$
return p

- **Repulsive forces**

repulsion constant (e.g. 2.0)

$$f_{\text{rep}}(u, v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

$$f_{\text{spring}}(u, v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v
- ℓ = ideal spring length for edges

Spring Embedder by Eades – Model

```
ForceDirected(G = (V, E), p = (p_v)_{v∈V}, ε > 0, K ∈ ℕ)
  t ← 1
  while t < K and max_{v∈V} ||F_v(t)|| > ε do
    foreach u ∈ V do
      F_u(t) ← Σ_{v∈V} f_rep(u, v) + Σ_{uv∈E} f_attr(u, v)
    foreach u ∈ V do
      p_u ← p_u + δ(t) · F_u(t)
    t ← t + 1
  return p
```

- **Repulsive forces**

repulsion constant (e.g. 2.0)

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

spring constant (e.g. 1.0)

$$f_{\text{spring}}(u,v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u,v) + \sum_{uv \in E} f_{\text{attr}}(u,v)$$

Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v
- ℓ = ideal spring length for edges

Spring Embedder by Eades – Model

ForceDirected($G = (V, E)$, $p = (p_v)_{v \in V}$, $\varepsilon > 0$, $K \in \mathbb{N}$)
  $t \leftarrow 1$
  while $t < K$ **and** $\max_{v \in V} \|F_v(t)\| > \varepsilon$ **do**
    foreach $u \in V$ **do**
      $F_u(t) \leftarrow \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$
    foreach $u \in V$ **do**
      $p_u \leftarrow p_u + \delta(t) \cdot F_u(t)$
    $t \leftarrow t + 1$
  return p

- **Repulsive forces**

repulsion constant (e.g. 2.0)

$$f_{\text{rep}}(u, v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

- **Attractive forces**

spring constant (e.g. 1.0)

$$f_{\text{spring}}(u, v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

$$f_{\text{attr}}(u, v) = f_{\text{spring}}(u, v) - f_{\text{rep}}(u, v)$$

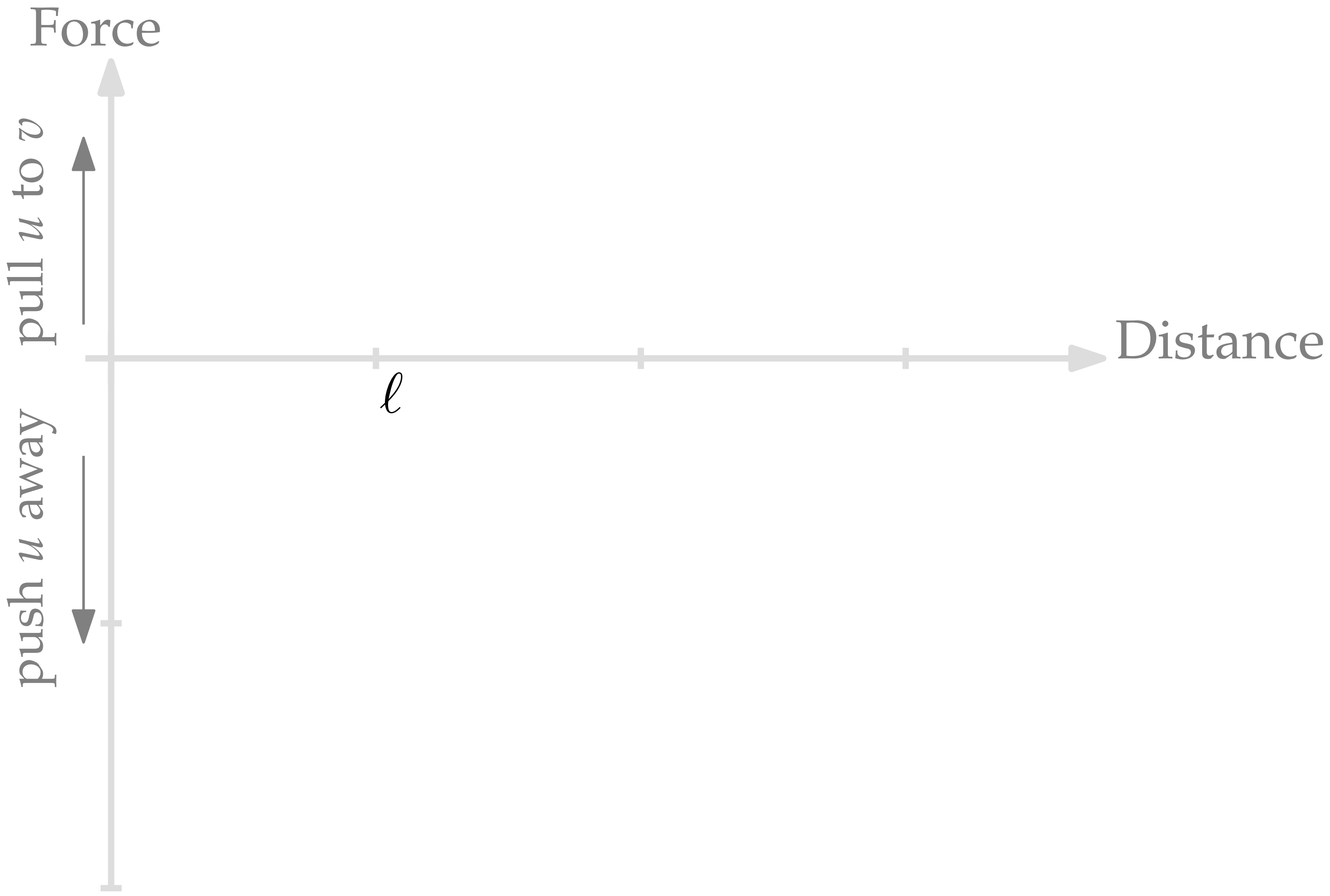
- **Resulting displacement vector**

$$F_u = \sum_{v \in V} f_{\text{rep}}(u, v) + \sum_{uv \in E} f_{\text{attr}}(u, v)$$

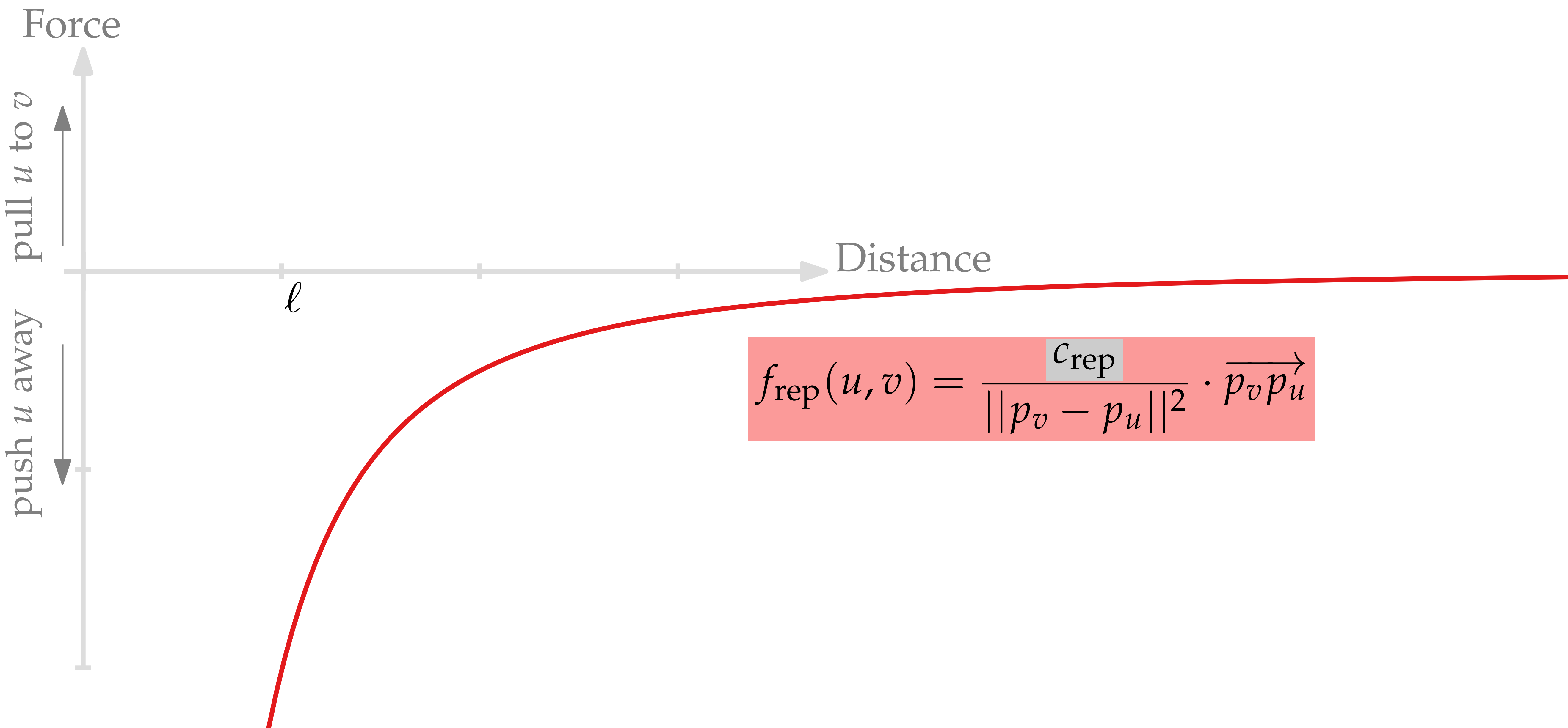
Notation.

- $||p_u - p_v||$ = Euclidean distance between u and v
- $\overrightarrow{p_u p_v}$ = unit vector pointing from u to v
- ℓ = ideal spring length for edges

Spring Embedder by Eades – Force Diagram

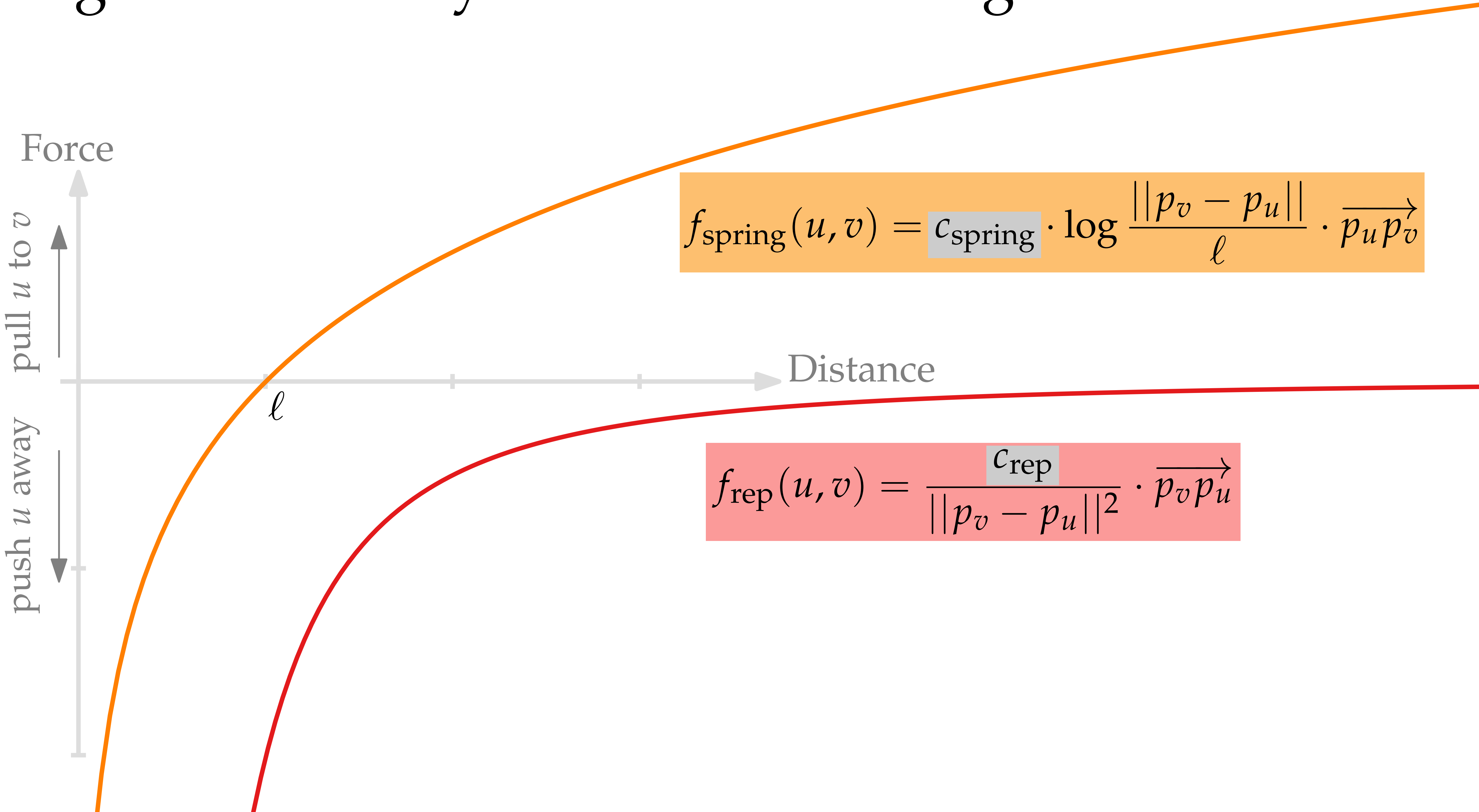


Spring Embedder by Eades – Force Diagram



$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

Spring Embedder by Eades – Force Diagram

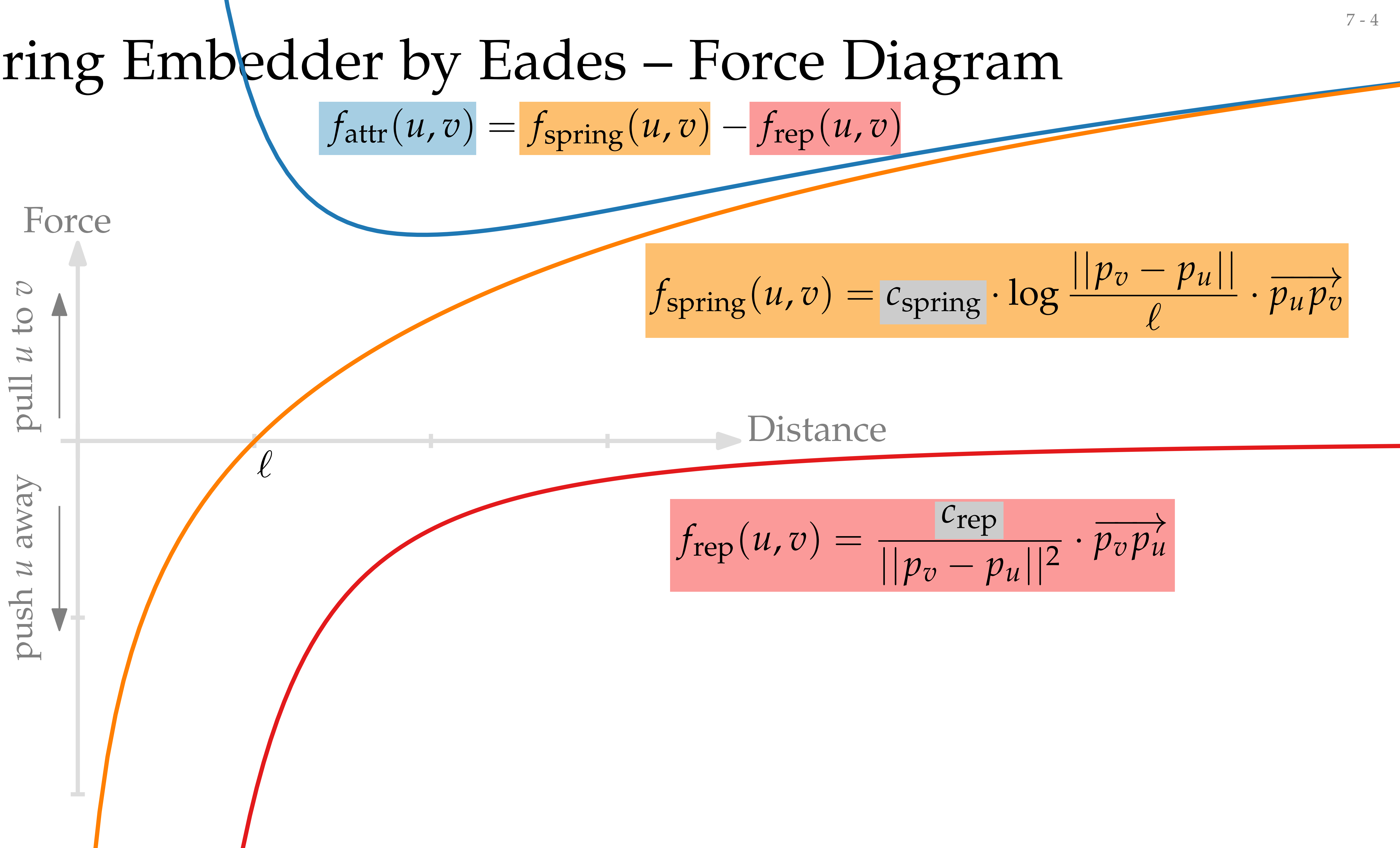


$$f_{\text{spring}}(u,v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

Spring Embedder by Eades – Force Diagram

$$f_{\text{attr}}(u,v) = f_{\text{spring}}(u,v) - f_{\text{rep}}(u,v)$$

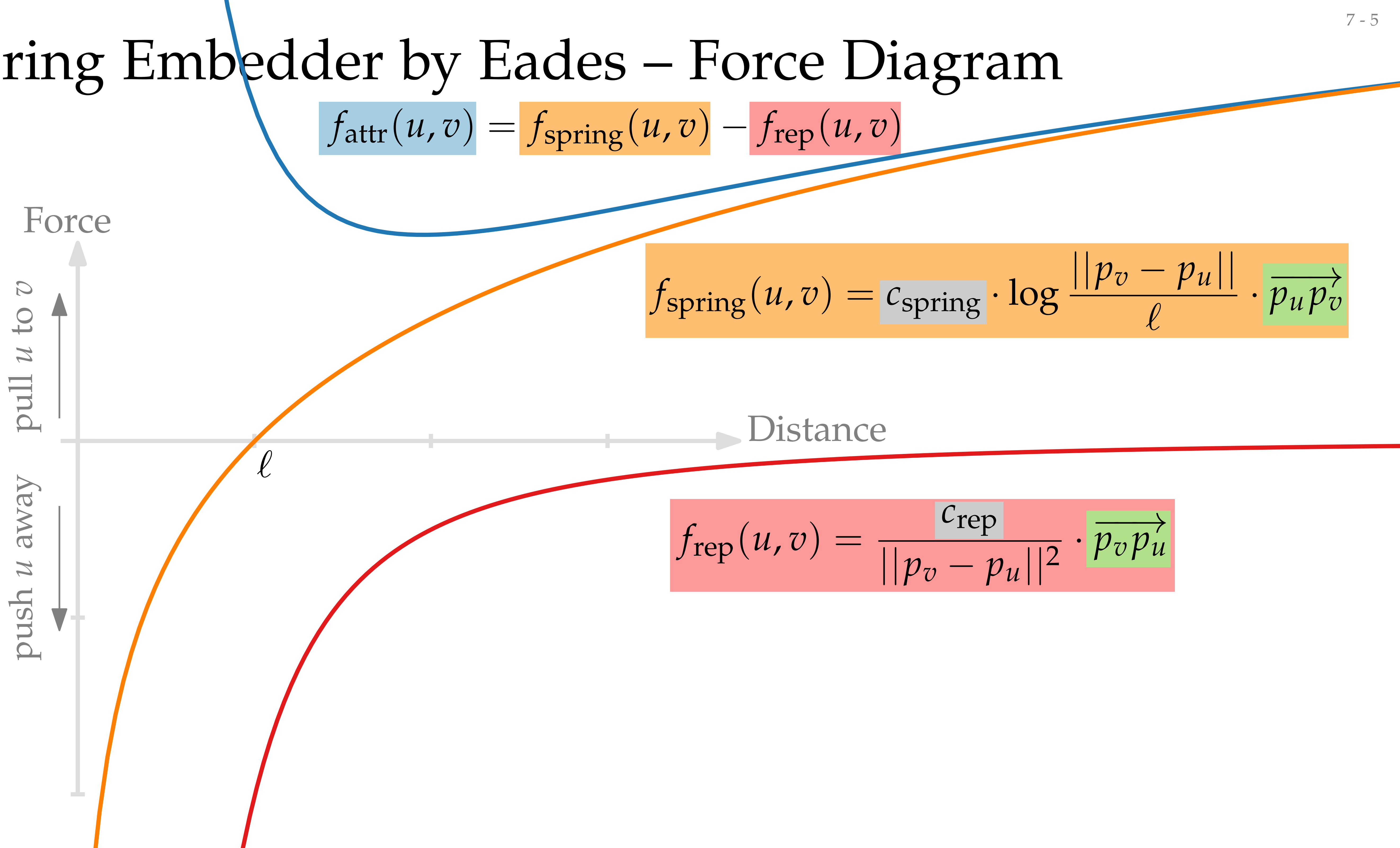


$$f_{\text{spring}}(u,v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

Spring Embedder by Eades – Force Diagram

$$f_{\text{attr}}(u,v) = f_{\text{spring}}(u,v) - f_{\text{rep}}(u,v)$$



$$f_{\text{spring}}(u,v) = c_{\text{spring}} \cdot \log \frac{||p_v - p_u||}{\ell} \cdot \overrightarrow{p_u p_v}$$

$$f_{\text{rep}}(u,v) = \frac{c_{\text{rep}}}{||p_v - p_u||^2} \cdot \overrightarrow{p_v p_u}$$

Spring Embedder by Eades – Discussion

Advantages.

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end
- converging to local minima

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end
- converging to local minima
- timewise f_{spring} in $\mathcal{O}(|E|)$ and f_{rep} in $\mathcal{O}(|V|^2)$

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end
- converging to local minima
- timewise f_{spring} in $\mathcal{O}(|E|)$ and f_{rep} in $\mathcal{O}(|V|^2)$

Influence.

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end
- converging to local minima
- timewise f_{spring} in $\mathcal{O}(|E|)$ and f_{rep} in $\mathcal{O}(|V|^2)$

Influence.

- original paper by Peter Eades [Eades '84] got $\sim$ 2000 citations

Spring Embedder by Eades – Discussion

Advantages.

- very simple algorithm
- good results for small and medium-sized graphs
- empirically good representation of symmetry and structure

Disadvantages.

- system is not stable at the end
- converging to local minima
- timewise f_{spring} in $\mathcal{O}(|E|)$ and f_{rep} in $\mathcal{O}(|V|^2)$

Influence.

- original paper by Peter Eades [Eades '84] got $\sim$ 2000 citations
- basis for many further ideas