Chapter 7

Tie Strength & Resilience

Summary

- Tie Strength
- Resilience/Robustness
- Failures and Attacks

Reading

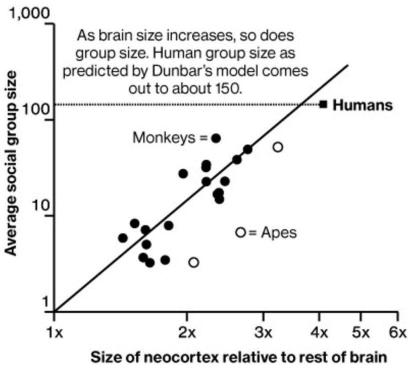
- Chapter 8 of Barabasi’s book
- Chapter 3 of Kleinberg’s book

How many friends does one person needs?

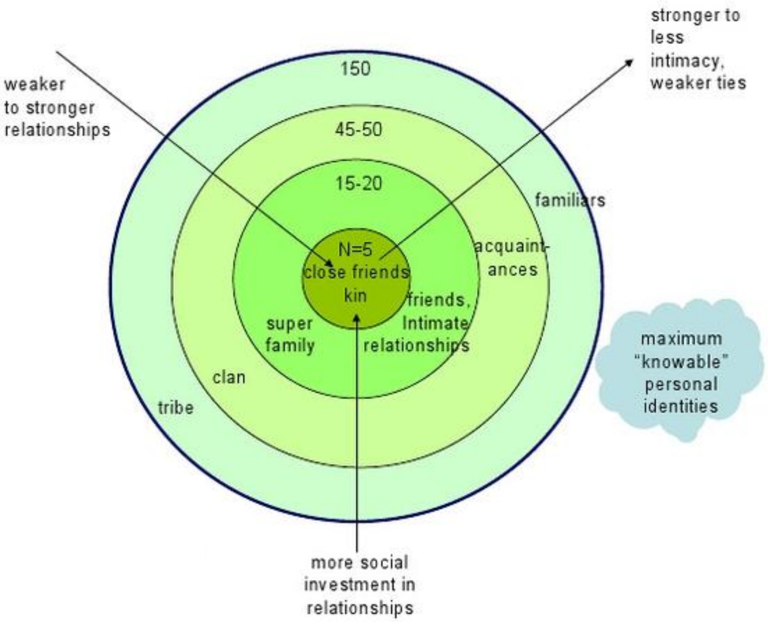
Not all ties in a social graph are the same

Dunbar's Number (Sociological Theory)

a suggested cognitive limit to the number of people with whom one can maintain stable social relationships



Considering the average human brain size and extrapolating from the results of primates, humans can comfortably maintain 150 stable relationships



In Dunbar's own words:

"the number of people you would not feel embarrassed about joining uninvited for a drink if you happened to bump into them in a bar"

Dunbar, Robin IM. Neocortex size as a constraint on group size in primates. Journal of human evolution 22.6 (1992): 469-493.

Dunbar, Robin. How many friends does one person need?: Dunbar's number and other evolutionary quirks. Faber & Faber, 2010.

The Strength of Weak Ties

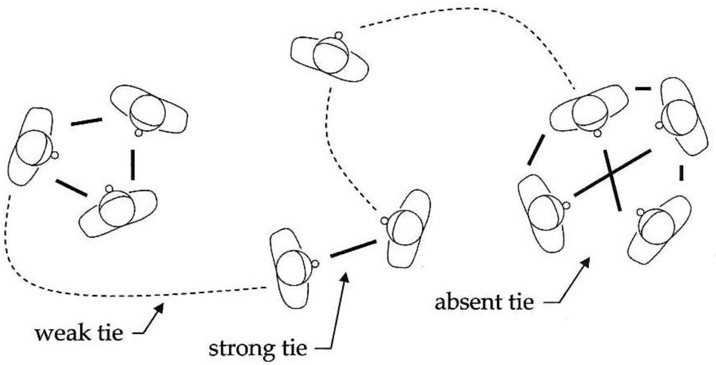
The strength of weak ties

Mark S. **Granovetter**, 1973

- (PhD Thesis)
  "How people get to know about new jobs?"
- Answer: Through personal contacts

Unexpected result:

Often acquaintances, **not** close friends… but *why*?



How to measure tie strength?

Granovetter's **dimensions of tie strength**:

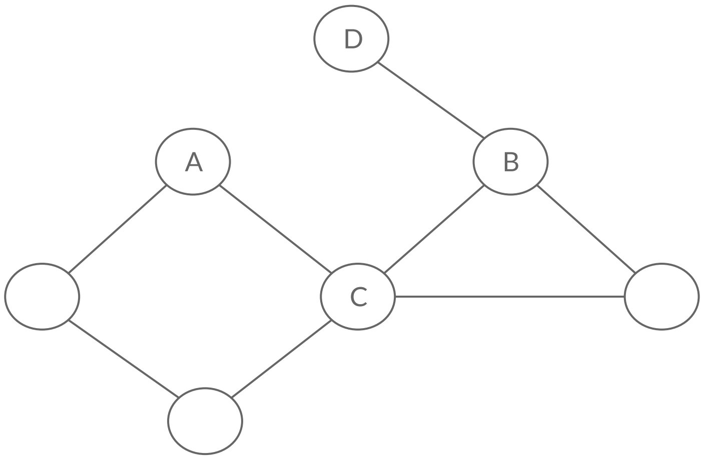
- the amount of time spent interacting with someone,
- the level of intimacy,
- the level of emotional intensity,
- and the level of reciprocity.

Granovetter, Mark S. "The strength of weak ties." Social networks. Academic Press, 1977. 347-367.

Triadic Closure

Social Intuition:

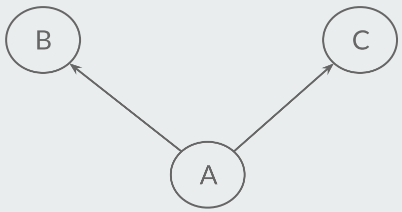
if two people in a network have a friend in common there is an increased likelihood that they will become friends themselves



Which is more likely to appear (A,B) or (A,D)?

Triadic Closure

Triadic Closure
implies
High Clustering Coefficient



(Social) Reasons for triadic closures

If B and C have a friend A in common then:

- B is more likely to meet C
  (since they spend time with A)
- B and C trust each other
  (since they have a friend in common)
- A has incentive to bring B and C together
  (as it is hard for A to maintain two disjoint relationships)

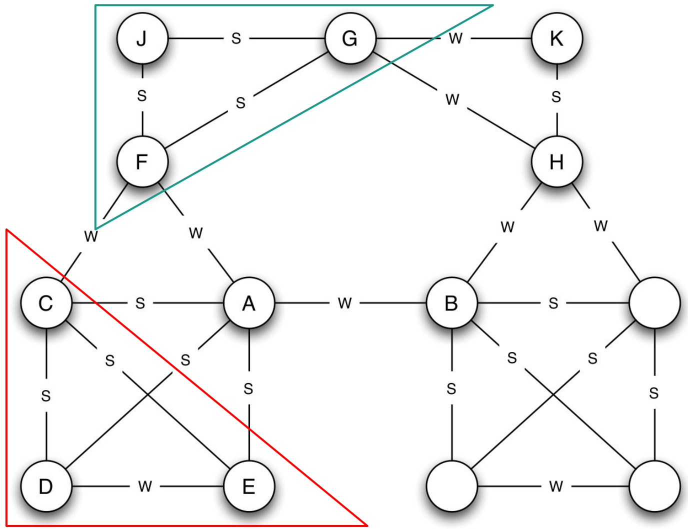
Strong Triadic Closure

Links in networks have strength;

- Friendship, Communication

We characterize links as either:

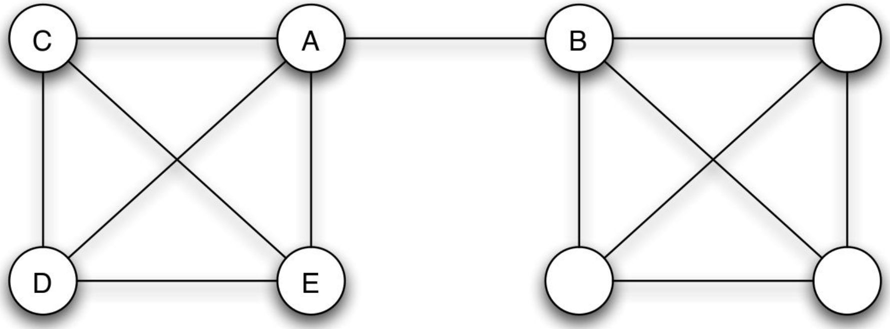
- Strong (friends), or
- Weak (acquaintances)



Strong Triadic Closure Property:

if A has strong links to B and C then there must be a link (B,C) (that can be strong or weak)

Bridges and Local Bridges

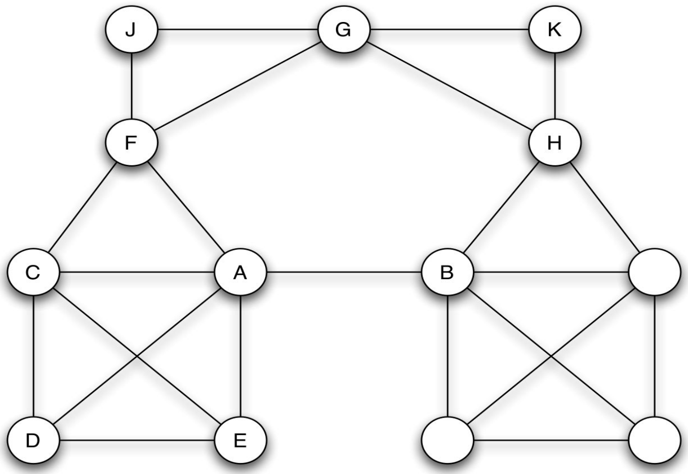


Strong Definition

Edge (A,B) is a bridge if deleting it would make A and B in two separate connected components

Bridges and Local Bridges

Edge (A,B) is a local bridge since A and B have no friends in common



The span of a local bridge is the distance of the edge endpoints if the edge is deleted

Local bridges with long span are like real bridges

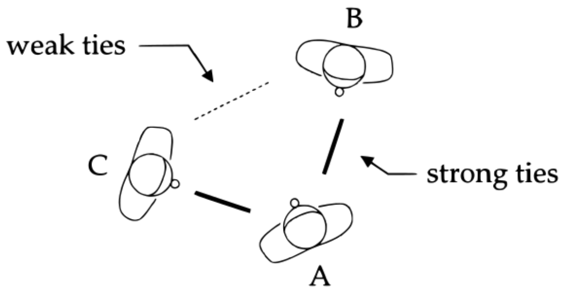
Local Bridges and Weak ties

Claim:

If a node A satisfies Strong Triadic Closure and it is involved in at least two strong ties, then any local bridge adjacent to A must be a weak tie

Proof (by contradiction):

- A satisfies Strong Triadic Closure
- Let (A,B) be a local bridge and a strong tie
- Then (B,C) must exist because of Strong Triadic Closure
- But then (A,B) is not a bridge



Measuring Tie Strength in Real Data

Social proximity and tie strength

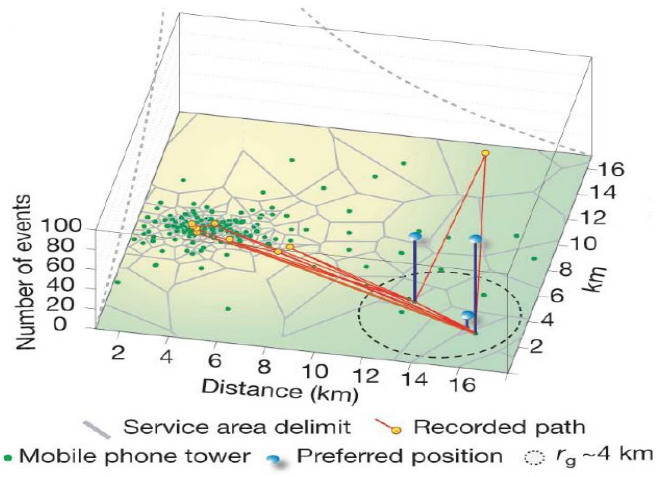
How connected are u and v in the social network.

- Various well-established measures of network proximity, based on the common neighbors (Jaccard, Adamic-Adar) or the structure of the paths (Katz) connecting u and v in the who-calls-whom network.

How intense is the interaction between u and v.

- Number of calls as **strength of tie**

Cell-phone network of 20% of country's population



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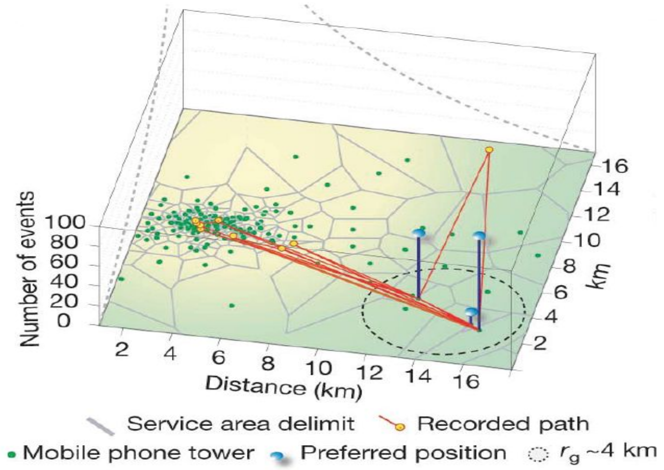
Strength of weak ties

First large scale empirical validation of Granovetter's theory

- Social proximity increases with tie strength
- Weak ties span across different communities

Cell-phone network of 20% of country's population



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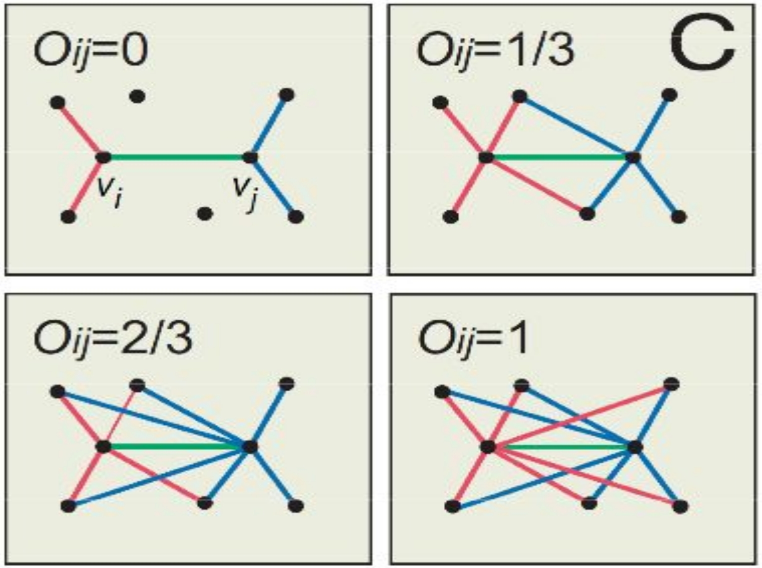
Measuring Tie Strength

Neighborhood Overlap

$$\mathbf{O}_{\mathrm{ij}} = \frac{n(\mathrm{i}) \cap \mathbf{n}(\mathrm{j})}{\mathbf{n}(\mathrm{i}) \cup \mathbf{n}(\mathrm{j})}$$

where n(i) is the neighbor set of i

If Overlap = 0 the edge is a local bridge



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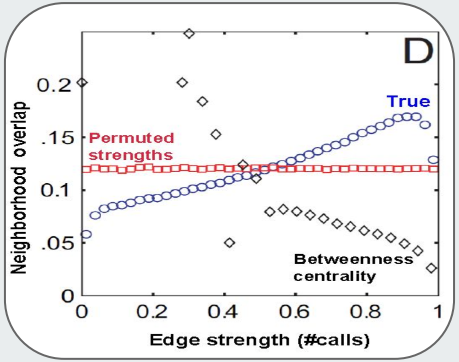
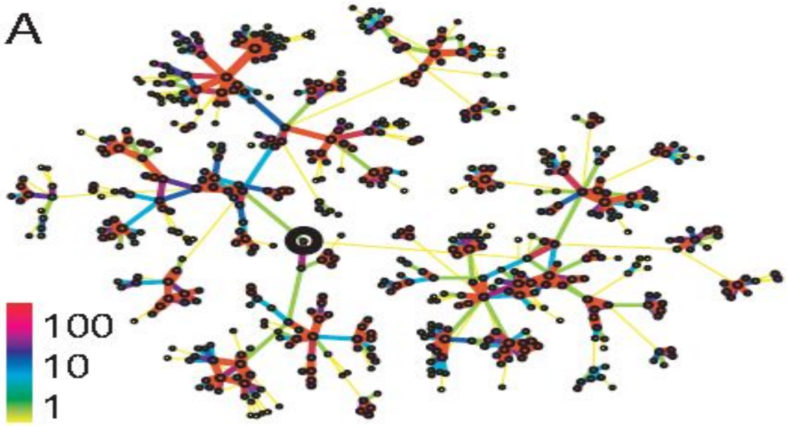
Strength, overlap and betweenness

Overlap and Betweenness are inversely correlated:

- Weak ties: the higher the number of shortest paths crossing an edge, the lower the overlap among the endpoints of the edge





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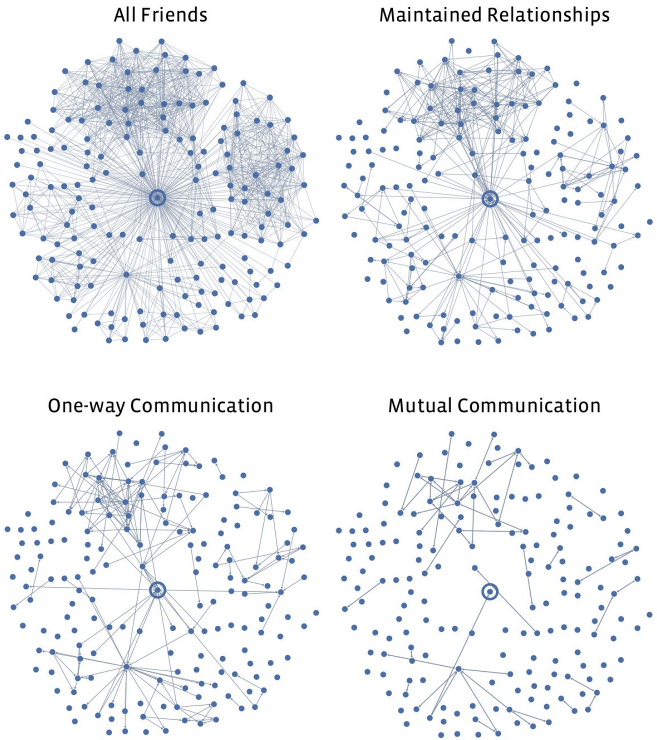
Ties Strength on Facebook

Different types of connections

- **Mutual communication**:
  both user sent messages eachother
- **One-way communication**:
  user messages where not reciprocated
- **Maintained relationship**:
  user clicked on content produced by his friend (no communication)

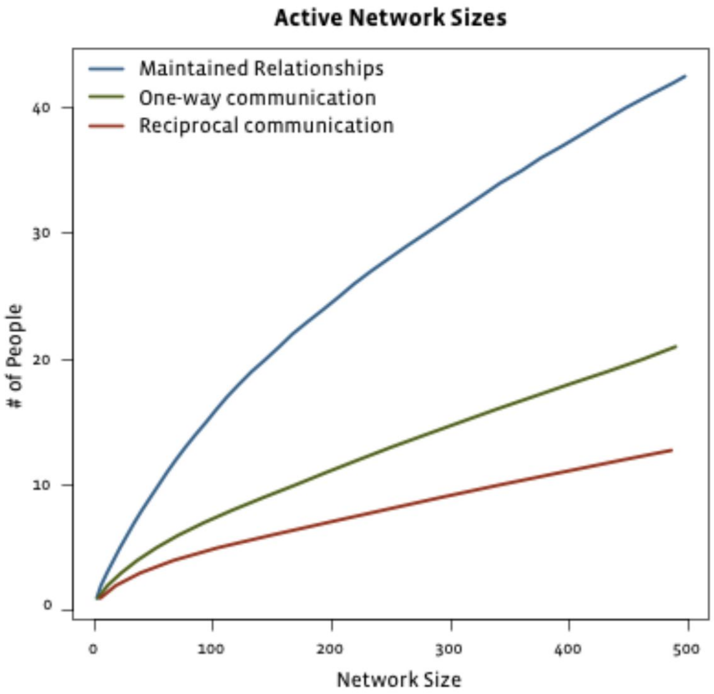


Cameron Marlow, Lee Byron, Tom Lento, and Itamar Rosenn. Maintained relationships on Facebook, 2009. http://overstated. net/2009/03/09/maintainedrelationships-on-facebook

Does tie strength affect network size?

Tie strength allows to:

- discriminate different type of contacts,
- categorize them by the involvement required to nurture them



Network Resilience

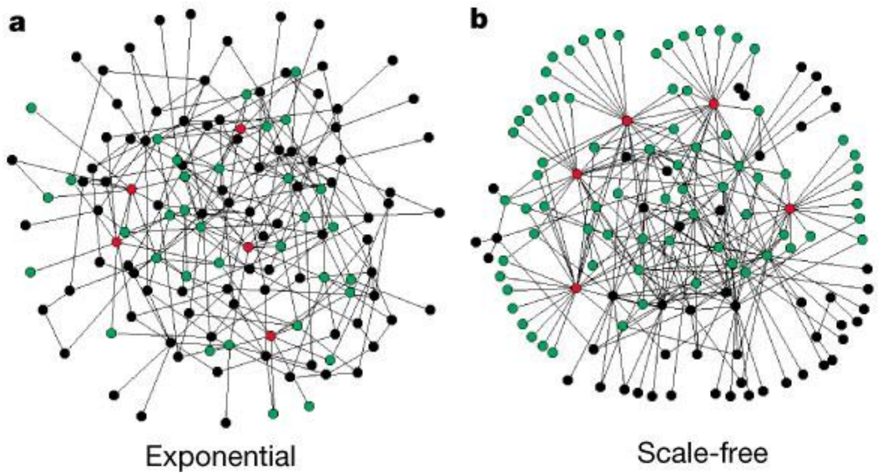
How robust is a complex network to node failures/attacks?

Network robustness and attack tolerance

How network topology is resistant against failure and targeted attacks

Numerical experiment:

1. Take a connected network
2. Remove nodes one at time
3. Observe the size of LCC
   (largest connected component)

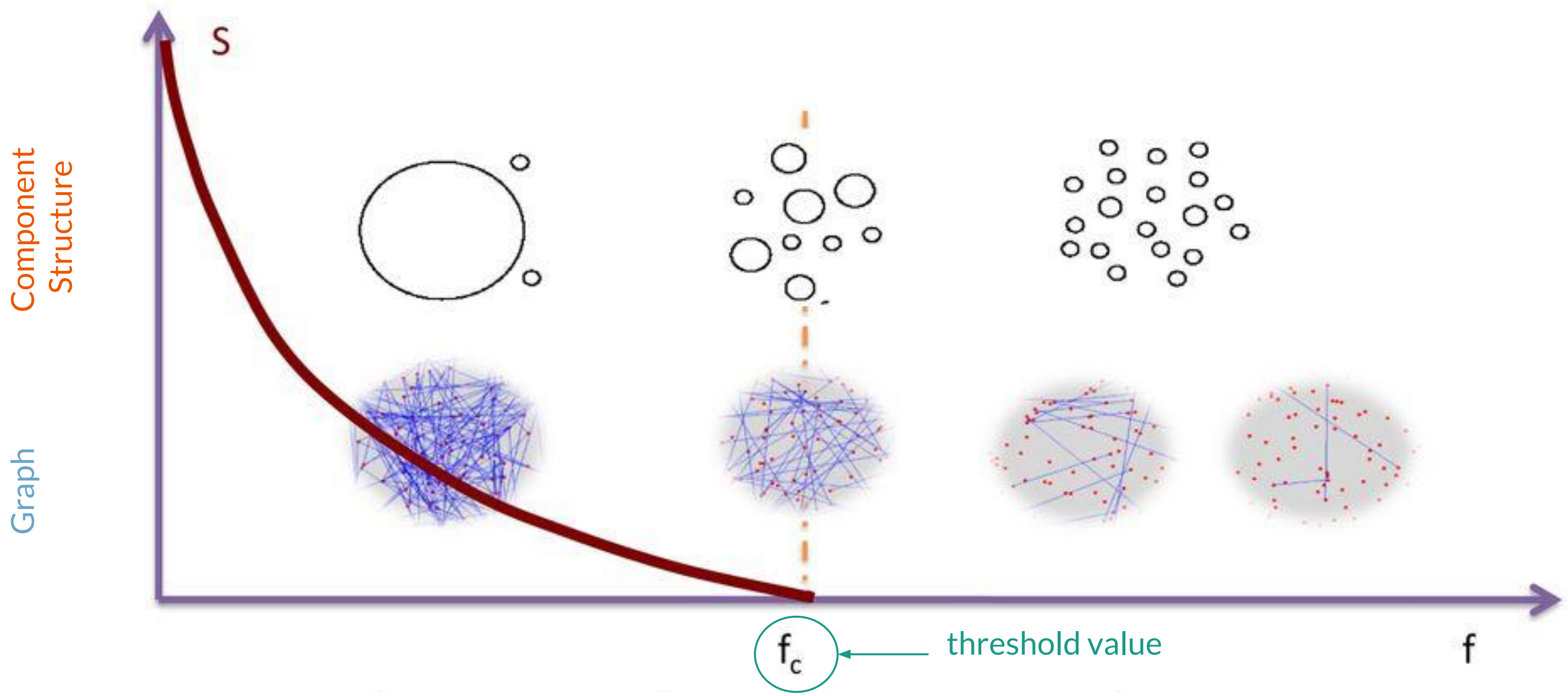


Different topologies, same parameters (N=130, <k>=3.3)

Node removal strategies:

1. Random removal ("failures")
   e.g., random failure of internet routers

f = fraction of removed nodes



Inverse Percolation problem

Molloy-Reed criterion for giant components

A giant cluster exists if each node is connected to at least two other nodes.

Or, equivalently:
The average degree of a node *i* linked to the GC, must be at least 2.

Can be shown to correspond to the following relation:

$$\kappa \equiv \frac{\langle k^2 \rangle}{\langle k \rangle} = 2$$

second moment of the degree distribution (i.e., variance)

- κ>2: Giant component exist
- κ<2: Many disconnected cluster

Malloy, Reed,
Random Structures and Algorithms (1995);
Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).

Breakdown threshold

ER graphs

Random node removal changes

- The degree of individual nodes decrease by losing links via node removal (e.g., [k -> k’ ≤ k])

- A node with degree k becomes a node with degree k’ with probability

$$\binom{k}{k} f^{k-k'} (1-f)^{k'}$$

where k’ ≤ k

Remove k-k’ links, each with probability f

Leave k’ links untouched, each with probability 1-f

the degree distribution [P(k) -> P’(k’)] after random removal of f fraction of nodes becomes

$$P'(k') = \sum_{k=k'}^{\infty} P(k) \binom{k}{k'} f^{k-k} (1-f)^{k}$$

thus,

$$\langle k' \rangle_f = (1-f)\langle k \rangle$$

$$\langle k^2 \rangle_f = (1-f)^2 \langle k^2 \rangle + f(1-f)\langle k \rangle$$

Breakdown threshold

ER graphs

We know that,

$$\langle k' \rangle_f = (1-f)\langle k \rangle$$

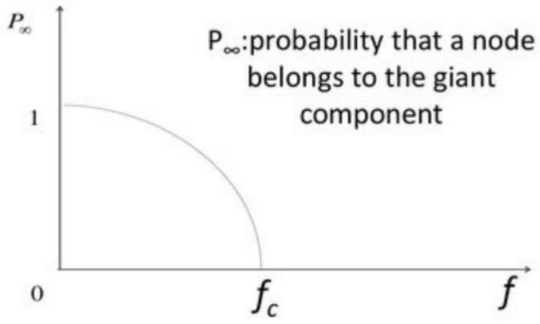
$$\langle k^2 \rangle_f = (1-f)^2\langle k^2 \rangle + f(1-f)\langle k \rangle$$

$$\kappa \equiv \frac{\langle k^2 \rangle}{\langle k \rangle} = 2$$

- κ>2: Giant component exist
- κ<2: Many disconnected cluster

Thus, the breakdown threshold becomes:

$$f_c = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$



$$\kappa \equiv 1 - CN^{-\frac{3-\gamma}{\gamma-1}}$$

Breakdown threshold

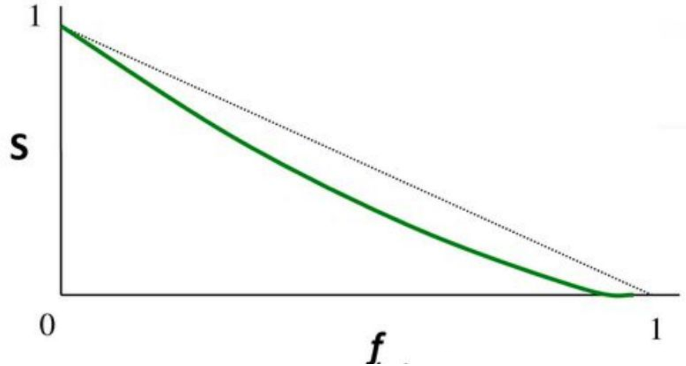
Resilience of Scale-free networks

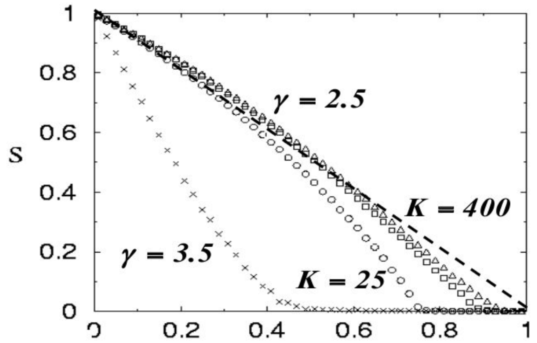
Scale-free graph with

$P(k) = Ak^{-\gamma}$ with $k = m, \ldots, K$

Scale-free networks do not appear to break apart under random failures.

Reason:
The likelihood of removing a hub is small.





Example Internet:

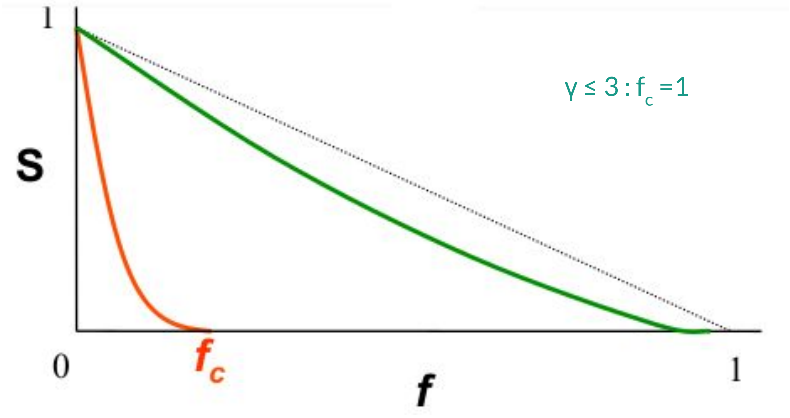
- Router level map, N=228,263, $\gamma=2.1\pm0.1$, k=28 -> f_c**=0.962**
- AS level map, N=11, $\gamma=2.1\pm0.1$, k=264 -> f_c**=0.996**

Achille’s Heel of Scale-free networks

The robustness of scale free networks is due to the hubs, which are difficult to hit by chance

Node removal strategies:

1. Random removal (“failures”)
   e.g., random failure of internet routers
2. Remove nodes in descending order of their degrees (“attacks”)
   i.e., hubs first



Examples:

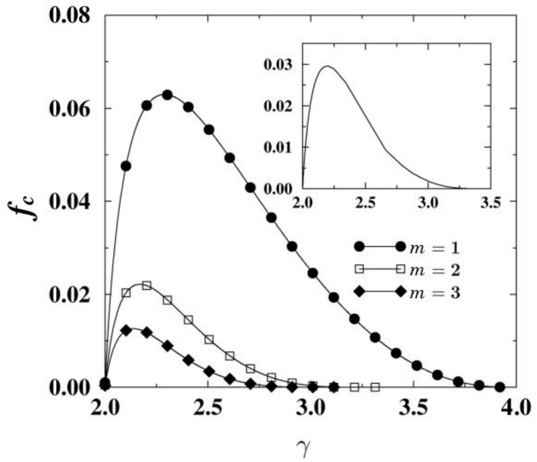
- Terrorist attacks
- Efficient vaccination in epidemics

Attack threshold for Scale-free networks

Attack problem:

- What if we remove a fraction *f* of the hubs?
- At what threshold f_c will the network fall apart (no giant component)?

$$f_c^{\frac{2-\gamma}{1-\gamma}} = 2 + \frac{2-\gamma}{3-\gamma} K_{\min} \left(f_c^{\frac{3-\gamma}{1-\gamma}} - 1 \right)$$



- f_c depends on γ;
  it reaches its max for γ<3

- f_c depends on K_{min}
  (m in the figure)

- f_c is tiny.
  Its maximum reaches only 6%, i.e. the removal of 6% of nodes can destroy the network in an attack mode.

Example:

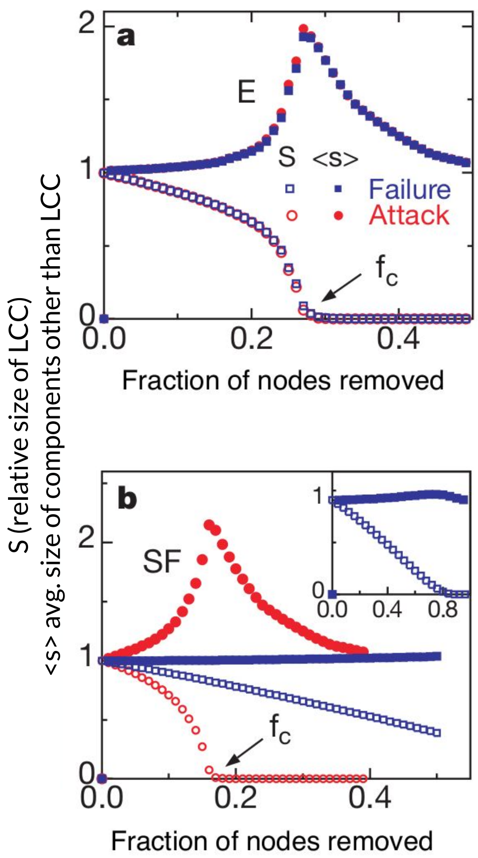
Internet γ=2.1, so 4.7% is the threshold

Network robustness & attack tolerance

Role of Hubs

Observations:

- Internet still works even when several servers are out of service
- Random vaccination is not effective in case of epidemic spreading



Poisson random graph

Both removal methods give the same result

The network falls apart after a finite fraction of nodes are removed

Scale-free network

Robust against random removal (blue)

Vulnerable against targeted attacks

Connecting the dots...

Resilience and Tie Strength

How to target a network?

Not only nodes can fail/being targeted

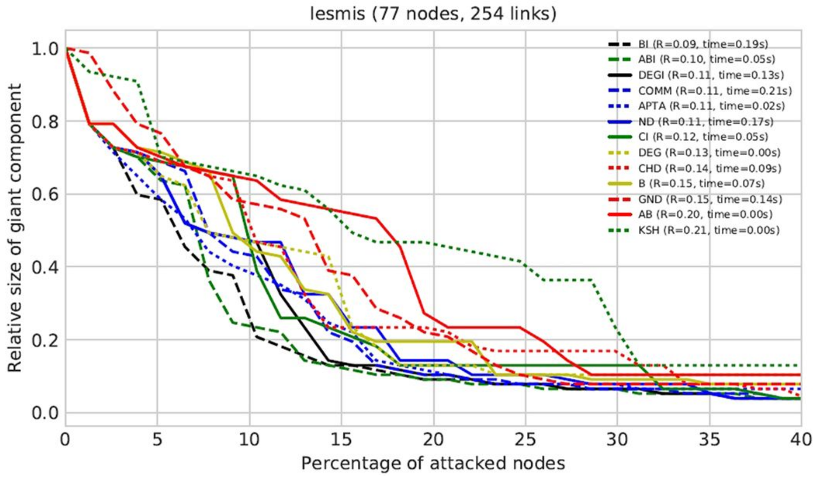
Identifying local/real bridges can lead to extremely efficient attacks

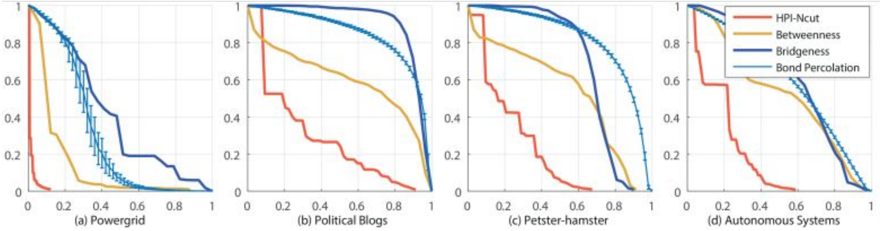
Node attacks:

- Centrality based
- Community based
- ...

Edge attacks:

- Edge Betweenness centrality removal
- Neighbour overlap removal
- ...





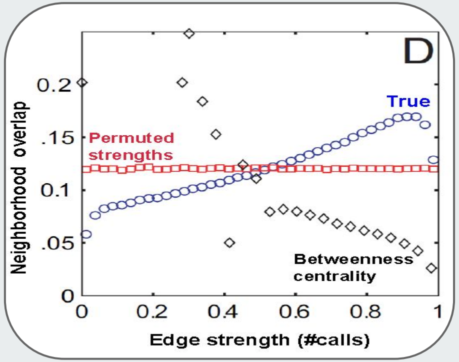
Ren, Xiao-Long, et al. "Underestimated cost of targeted attacks on complex networks." Complexity 2018 (2018).

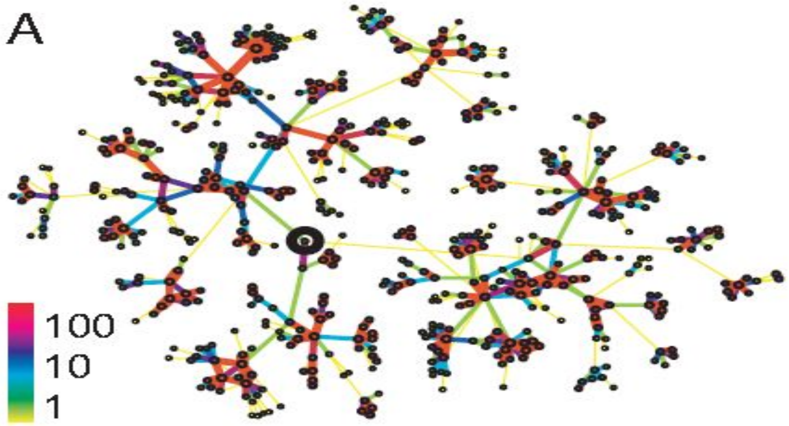
Wandelt, Sebastian, et al. "A comparative analysis of approaches to network-dismantling." Scientific reports 8.1 (2018): 13513.

Strength, overlap and betweenness

Overlap and Betweenness are inversely correlated:

- Weak ties: the higher the number of shortest paths crossing an edge, the lower the overlap among the endpoints of the edge





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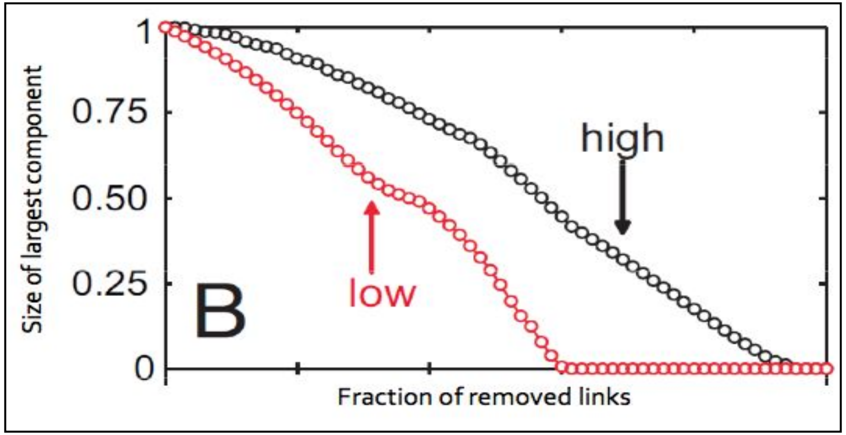
Removing links based on overlap

Two strategy of edge removal w.r.t. neighborhood overlap

- Low to high
- High to low

Removing bridges first (low overlap) destroy the network structure faster



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Chapter 7

Conclusion

Take Away Messages

1. Weak ties are not so weak…
2. Different topologies suffers from different vulnerabilities (node failures & attacks)
3. Even ties can be used to dismantle a complex networks

Suggested Readings

- Chapter 8 of Barabasi's book
- Chapter 3 of Kleinberg's book

What's Next

Chapter 8:
Networks Beyond Pairwise interactions