

Chapter 9

Dynamics Of Networks

Summary

- Representing Dynamic Topologies
- Analyzing Dynamic Networks
- Stream Graphs
- Random Models

Reading

- “Temporal Networks” Holme et al.



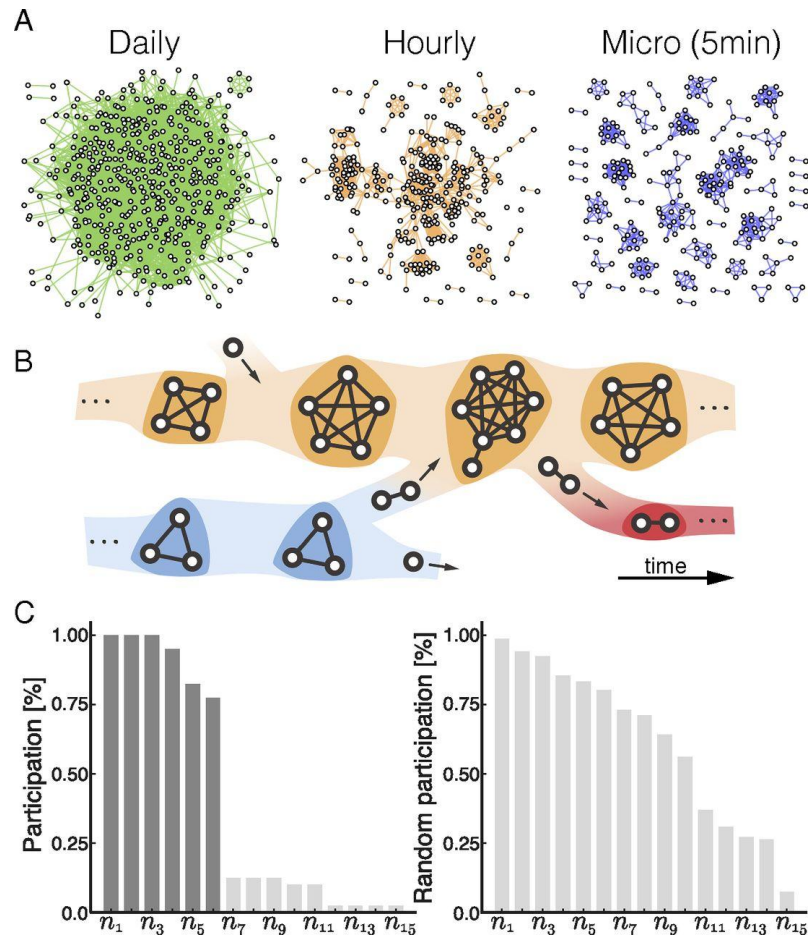
Representing Dynamic Topologies



Why bother of time?

Most real world networks are **dynamic**

- Facebook friendship
 - People joining/leaving
 - Friend/Unfriend
- Twitter mention network
 - Each mention has a timestamp
 - Aggregated every day/month/year => still dynamic
- World Wide Web
- Urban networks
- Protein-protein interactions
- Brain networks
- ...



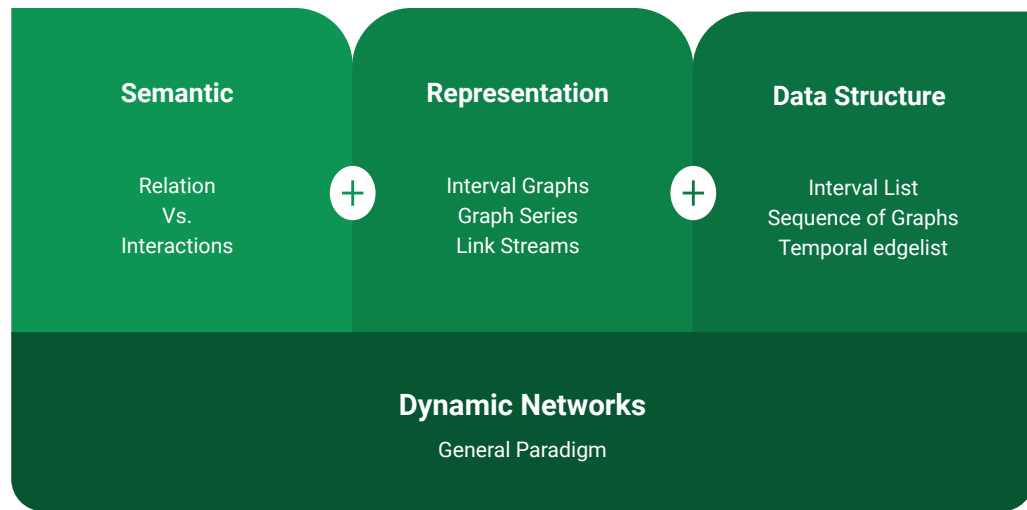
Evolving Topologies

- Nodes can appear/disappear
- Edges can appear/disappear
- Nature of relations can change

How to **represent** those changes?

How to **manipulate** dynamic networks?

Three different levels of abstraction



Semantic

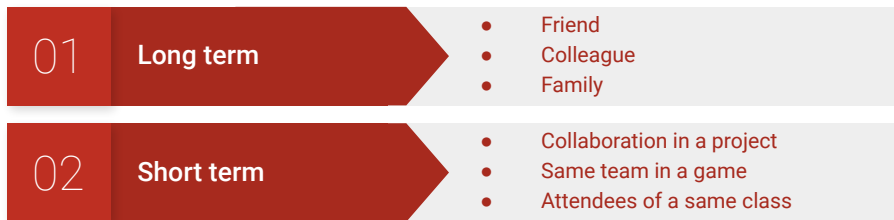
Relations Vs. Interactions

Topological perturbations may have different **temporal scales** depending on their intrinsic **semantic value**.

Two families:

- Relations (stable ties)
- Interactions (unstable ties)

Relations



Interactions



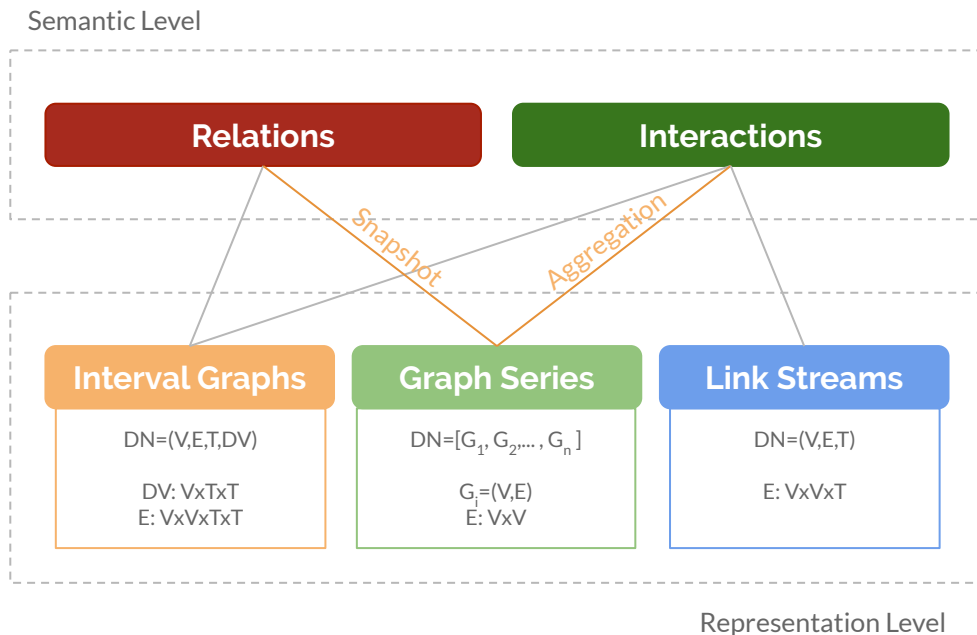
Semantics and how to represent them

Relations

The graph is more and more stable, until most observations are completely similar to previous/later ones (frequency faster than change rate)

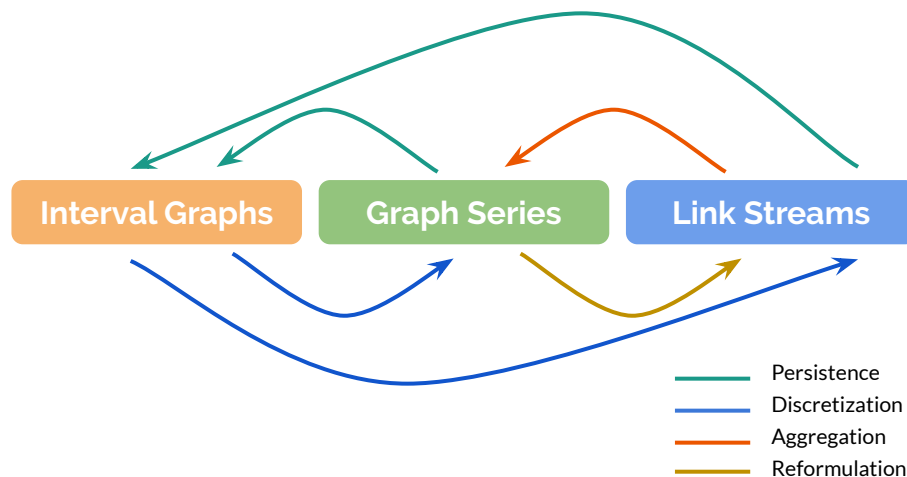
Interactions

The graph is less and less stable, until each observation is a graph in itself, thus completely different from previous/later ones (frequency faster than observed events rate)



Changing Representation

Alternative representations can be, to some extent, **converted** among them by applying appropriate data **transformations**



Analyzing Dynamic Networks

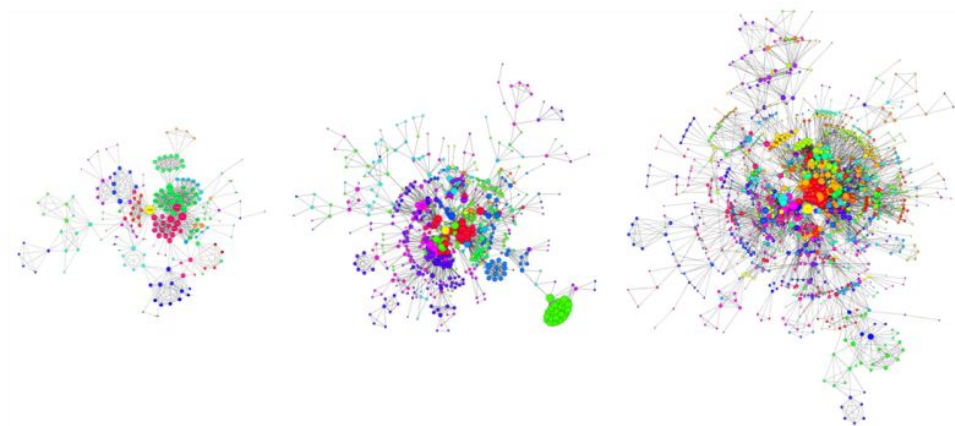
A brief Introduction



Unstable Snapshots

The evolution is represented as a series of a few snapshots

- Many changes between snapshots
(Cannot be visualized as a “movie”)
- Each snapshot can be studied as a static graph
- Evolution of node properties can be studied
“independently”
(e.g., node i had low centrality in snapshot t and high centrality in snapshot $t+n$)



Stable Network

Edges change (relatively) slowly

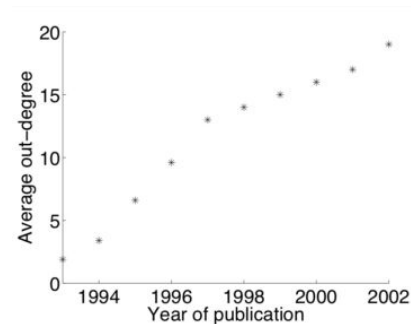
The network is well defined at any t

- Temporal network: nodes/edges described by (long lasting) intervals
- Enough snapshots to track nodes

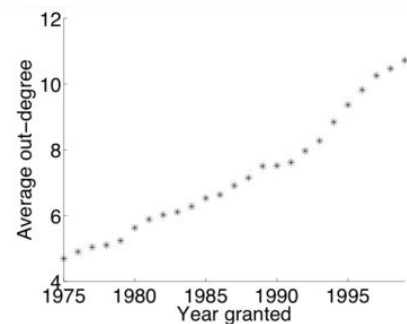
A static analysis at every (relevant) t gives a dynamic vision

No formal distinction with previous case (higher observation frequency)

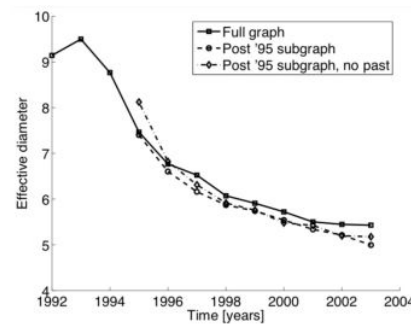
Properties can be analyzed as time series



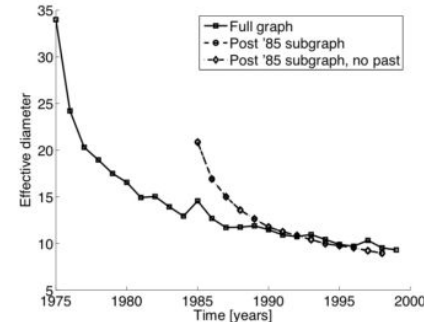
(a) arXiv



(b) Patents



(a) arXiv citation graph



(c) Patents citation graph

Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graph evolution: Densification and shrinking diameters." ACM Transactions on Knowledge Discovery from Data. (2007)

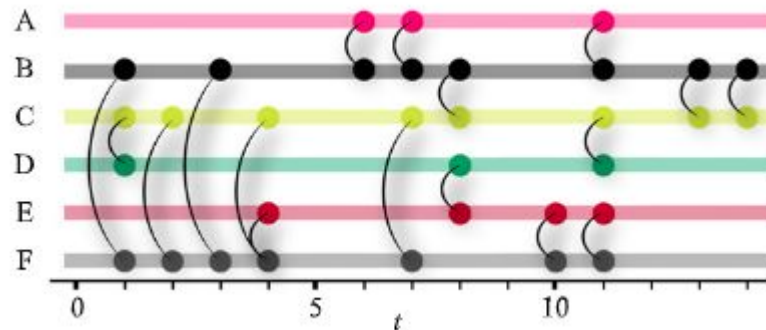
Unstable Temporal Network

The network at a given t is not meaningful

How to analyze such a network?

Until recently, network was transformed using aggregation/sliding windows

- Information loss
- How to choose a proper aggregation window size?



Stream Graphs

Network Properties, Centralities, Paths & Components



Stream Graph

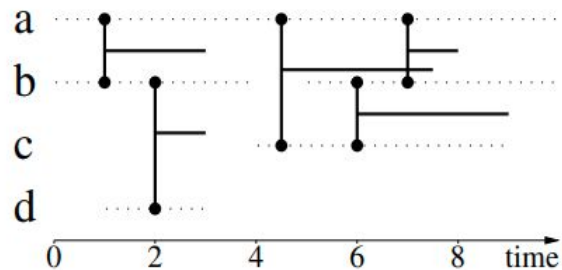
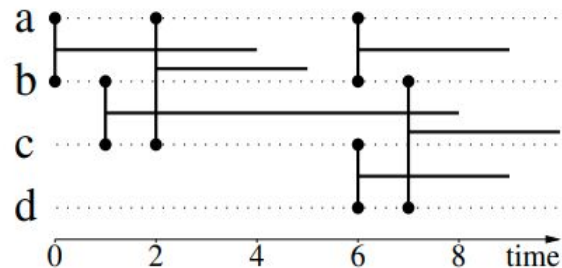
$$S = (T, V, W, E)$$

T: Possible Time

V: vertices

W: Vertices presence time

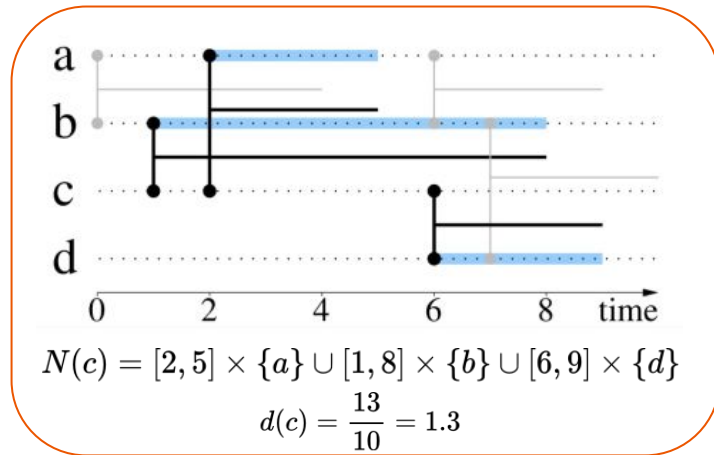
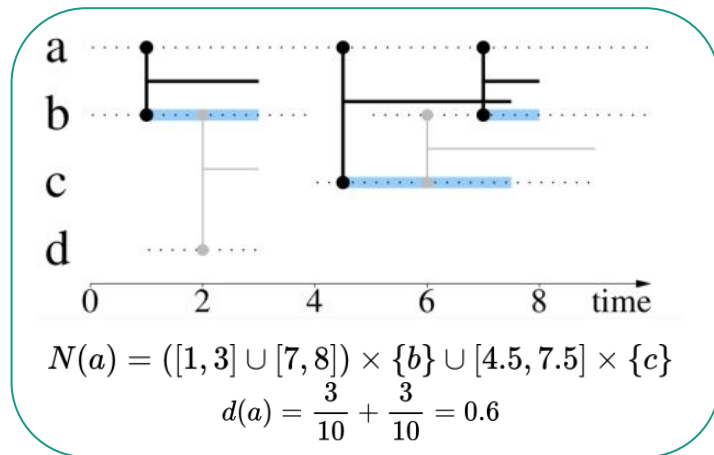
E: Edges presence time



Latapy, M., Viard, T., & Magnien, C. (2018). *Stream graphs and link streams for the modeling of interactions over time*. Social Network Analysis and Mining, 8(1), 61.

Stream Graph Indices

Number of nodes	$n = \sum_{v \in V} n_v = \frac{ W }{ T }$
Number of edges	$m = \sum_{uv \in V \otimes V} m_{uv} = \frac{ E }{ T }$
Neighbors of a node	$N(v) = \{(t, u), (t, uv) \in E\}$
Degree of a node	$d(v) = \frac{ N(v) }{ T } = \sum_{u \in V} \frac{ T_{uv} }{ T }$

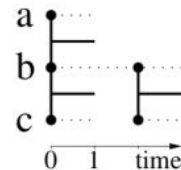


Stream Graph Indices

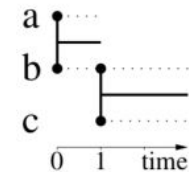
Average Degree	$d(V) = \frac{1}{n} \cdot \sum_{v \in V} n_v \cdot d(v) = \sum_{v \in V} \frac{ T_v }{ W } \cdot d(v)$
Clustering Coefficient	$\frac{\sum_{u,v \in V \otimes V} T_{vu} \cap T_{vw} \cap T_{uw} }{\sum_{u,v \in V \otimes V} T_{vu} \cap T_{vw} }$
Density	$\delta(S) = \frac{\sum_{u,v \in V \otimes V} T_{uv} }{\sum_{u,v \in V \otimes V} T_u \cap T_v }$

Total Edge presence

Total overlapping time between each pair of nodes



$n = 2$
 $m = 1$
 $\delta = 0.75$



$n = 2$
 $m = 1$
 $\delta = 1$

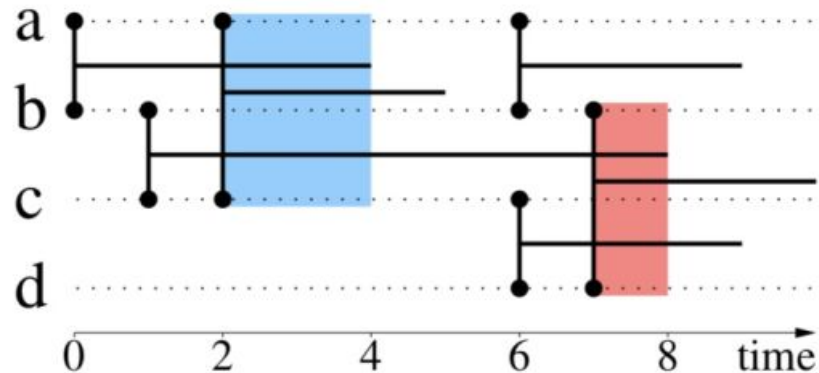
Stream Graph

Cliques

Temporal cluster having density equal to 1

- All pairs of nodes in a clique are linked together in S

A clique is maximal if there is no other clique in S that contains it

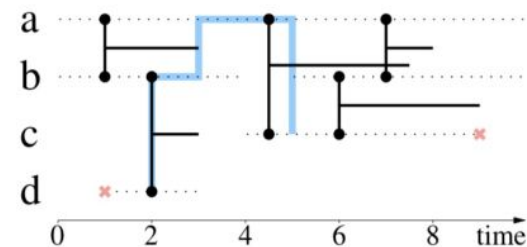


Stream Graph

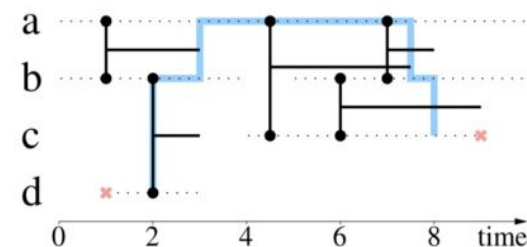
Paths and Distances

A path in a stream graphs

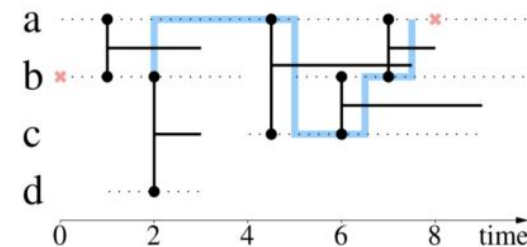
- Starts at a node and a date
- Ends at a node and a date
- Has a length
(number of hops)
- Has a duration
(duration from leaving node to reaching node)



Path: (d,1)(c,9)
Length: 3
Duration: 3



Path: (d,2)(c,8)
Length: 4
Duration: 6



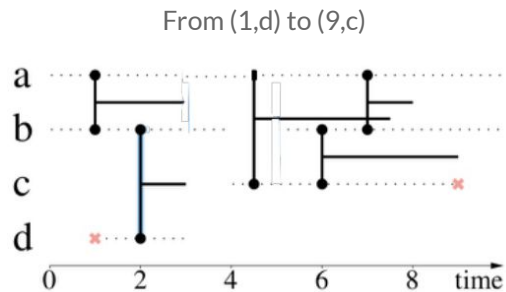
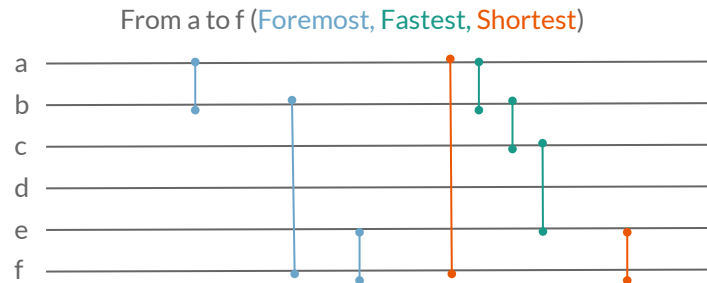
Path: (b,0)(a,8)
Length: 4
Duration: 5

Stream Graph

Paths and Distances

Several types of shortest paths in Stream graphs:

Type	Description
Shortest path	Minimal length
Fastest path	Minimal Duration
Foremost path	First to reach
Fastest shortest paths	Minimum duration among minimum length
Shortest fastest paths	Minimum length among minimum duration



Shortest path (2.5, d, b) (3, b, a) (7, a, c)

Fastest path (3, d, b) (3, b, a) (4.5, a, c)

Foremost path (2, d, b) (2, b, a) (4.5, a, c)

Fastest shortest path (3, d, b) (3, b, a) (4.5, a, c)

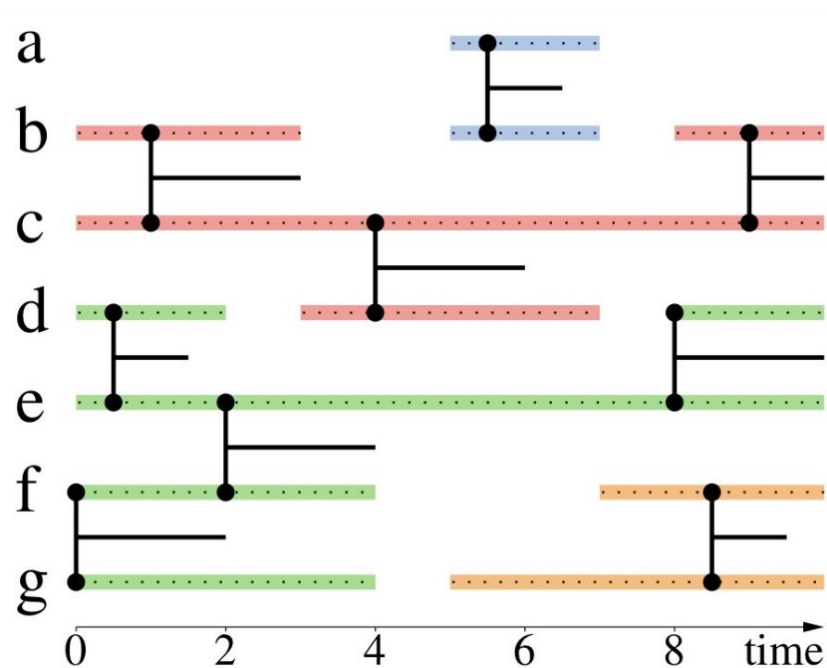
Shortest Fastest path from (3, d, b) (3, b, a) (4.5, a, c)

Stream Graph

Connected Components

Weakly connected component:

- There is at least a non-temporally respecting path among each pair of node (component in the static, undirected, graph)



Random Models for Dynamic Graphs



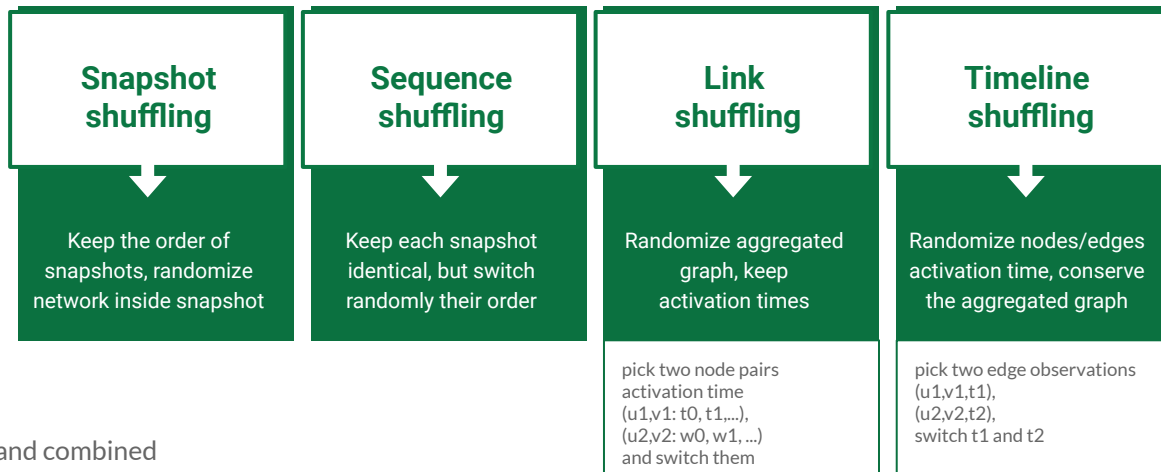
Random Models

Random models for static graphs:

- ER, BA, WS, Configuration,...
- Each one preserving different characteristics

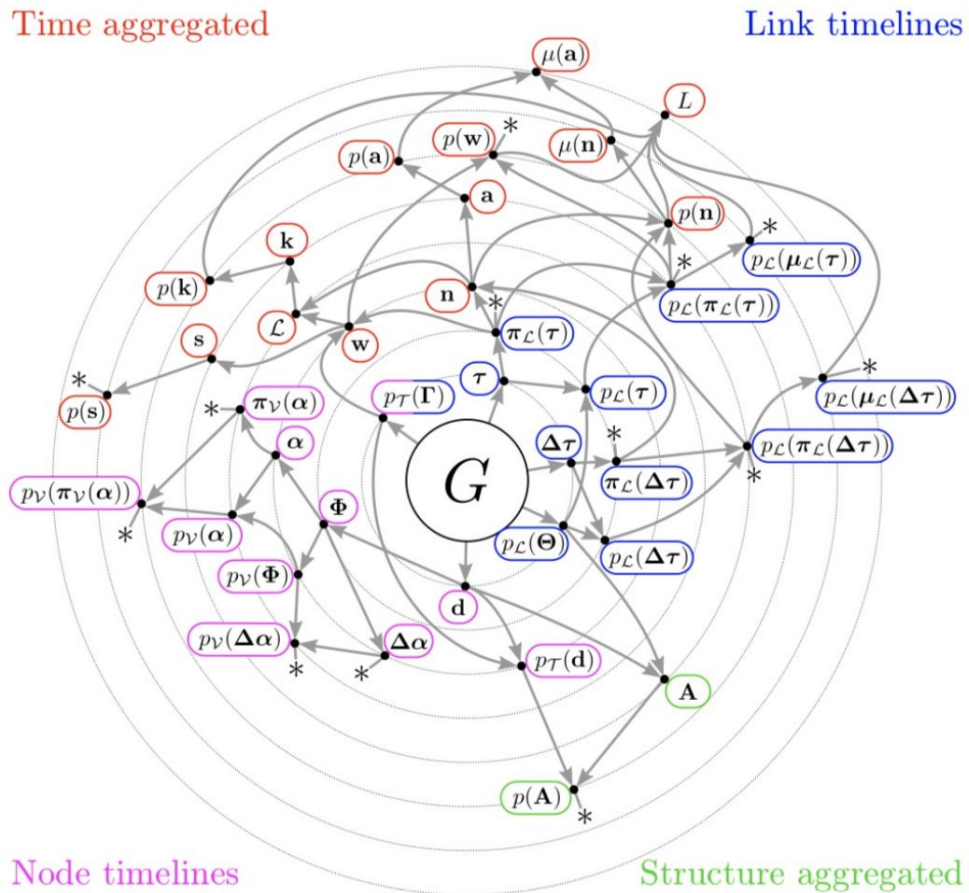
In dynamic networks, everything becomes more complex...

Different Shuffling strategies can be leveraged and combined



Note:

Different Shuffling strategies can and combined



Activity Driven Model

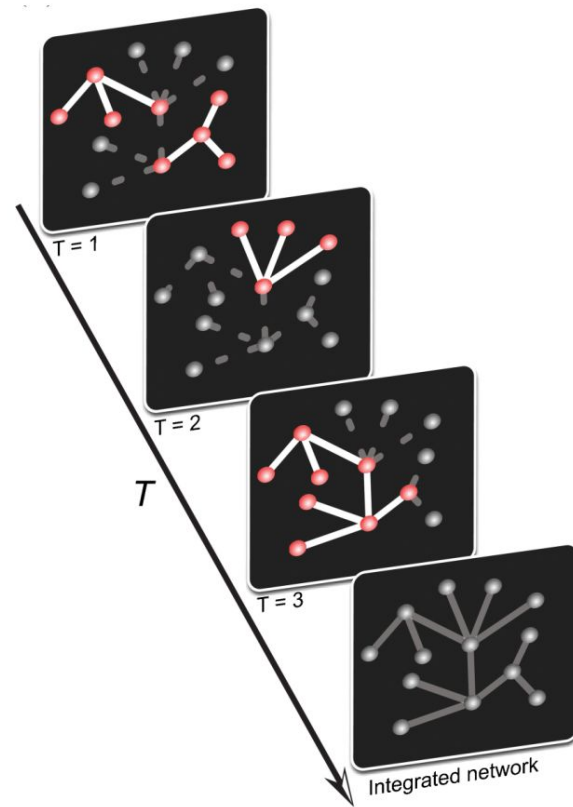
Agent based model of temporal interactions

General framework

- additional mechanisms can be added
- capable of simulating dynamical processes
co-evolving with the contact dynamics

single a-priori assumption:

- agents have different activity potentials



Activity Driven Model

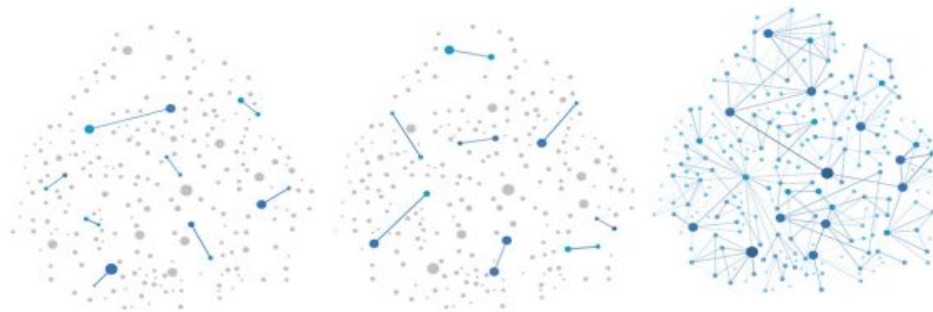
Setup:

N disconnected nodes, with pre-assigned activity rates

$$a_i = \eta x_i$$

where

- x_i is the activity potential of node i
(sampled from an arbitrary distribution $F(x)$ and $x_i \in [\varepsilon, 1]$)
- η is a scaling factor



Each time step t start with N disconnected nodes:

1. With probability a_i node i is activated and connects to m other nodes randomly
2. With probability $1-a_i$ node i remains inactive (still can receive connections from other active nodes)

At the end of each time step we delete each link and start the loop over again

The structure of the actual network at each t will be a **random network**

The emerging degree distribution of the integrated network will follow the same scaling form as the pre-assigned activity distribution

Memory and Social Reinforcement

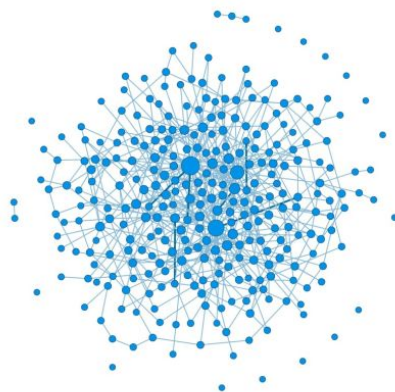
When a node is active it connects with probability

$$p(n) = c/(n + c)$$

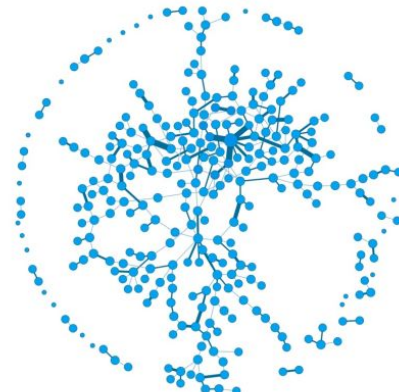
to a random node it has never connected before or with probability

$$1 - p(n)$$

to one of the n node who it has connected earlier



Without
memory



With
memory

After each iteration links are deleted but each node keeps remember to their previously connected egocentric network

Karsai, Márton, Nicola Perra, and Alessandro Vespignani. "Time varying networks and the weakness of strong ties." *Scientific reports* 4 (2014): 4001.

Summarizing



Time flies like an arrow...

Real world phenomena unfold through time

- Networks can be used to study them but increasing the complexity of the model

Different constraints require different modeling choices

- Relations or Interactions?
- Interval, graph series or link streams?

Static network analysis needs to be revised

- Degree, density, paths, components...
- What about centralities?



Where next?

Two kind of dynamics:

- **Dynamics of Networks**
(topological perturbations)
- **Dynamics on Networks**
(diffusive phenomena: epidemics, opinion dynamics...)

Of course they can happen at the same time...

Dynamics of Networks	Dynamics on Networks	Mixed Dynamics
<p>Assumption: Topology evolution is faster than diffusive processes unfolding (if any)</p> <p>Applications:</p> <ul style="list-style-type: none">- Link Prediction- Dynamic Community Discovery- ...	<p>Assumption: Diffusive processes unfolding is faster than topology evolution (if any)</p> <p>Applications:</p> <ul style="list-style-type: none">- Epidemic spreading- Opinion Dynamics- ...	<p>Assumption: Diffusive processes unfolding and topology evolution have comparable rates</p> <p>Applications:</p> <ul style="list-style-type: none">- Diffusion shape topology- Topology shape diffusion- Feedback loops

Chapter 9

Conclusion

Take Away Messages

1. Real phenomena evolve through time
2. Network modeling can fill such gap:
 - a. Representation has to be enriched
 - b. Indexes and concepts revised
 - c. Null models reformulated

Suggested Readings

- “Temporal Networks” Holme et al.
- “Stream graphs and link streams for the modeling of interactions over time” Latapy et al.

What's Next

Chapter 10:
Link Prediction

