Chapter 10

# Dynamics Of Networks

## Summary

- Representing Dynamic Topologies
- Analyzing Dynamic Networks
- Stream Graphs
- Random Models

## Reading

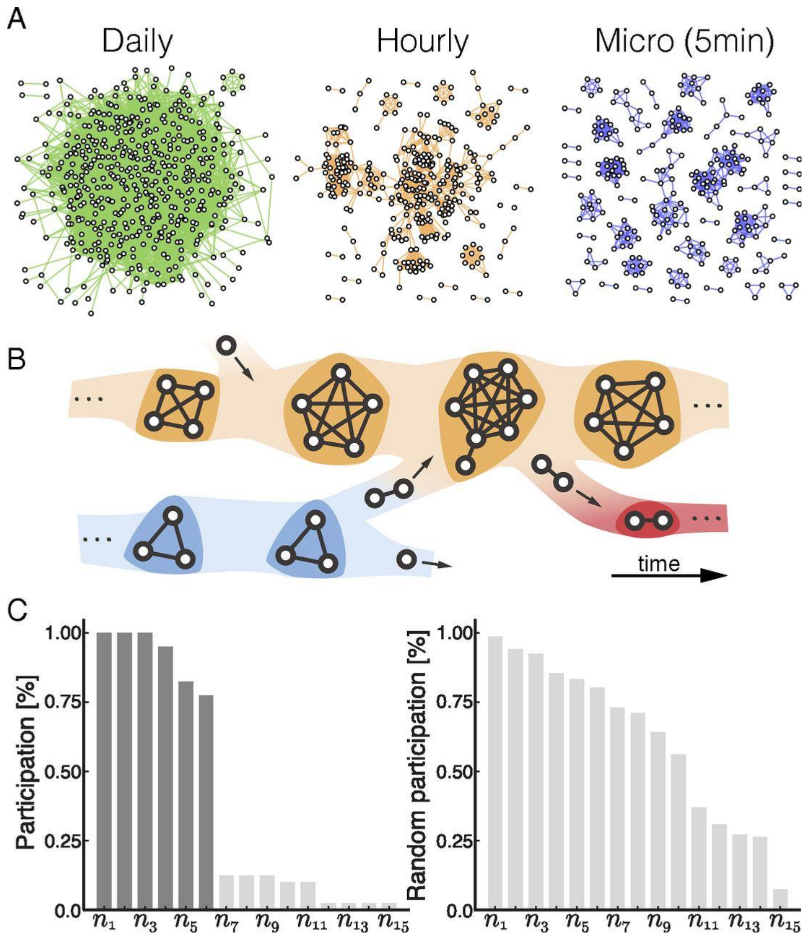
- “Temporal Networks” Holme et al.

# Representing Dynamic Topologies

# Why bother of time?

Most real world networks are dynamic

- Facebook friendship
  - People joining/leaving
  - Friend/Unfriend
- Twitter mention network
  - Each mention has a timestamp
  - Aggregated every day/month/year => still dynamic
- World Wide Web
- Urban networks
- Protein-protein interactions
- Brain networks
- ...



# Evolving Topologies

- Nodes can appear/disappear
- Edges can appear/disappear
- Nature of relations can change

How to represent those changes?

How to manipulate dynamic networks?

Three different levels of abstraction

**Semantic**

Relation
Vs.
Interactions

+

**Representation**

Interval Graphs
Graph Series
Link Streams

+

**Data Structure**

Interval List
Sequence of Graphs
Temporal edgelist

**Dynamic Networks**

General Paradigm

Semantic

# Relations Vs. Interactions

Topological perturbations may have different temporal scales depending on their intrinsic semantic value.

Two families:

- Relations (stable ties)
- Interactions (unstable ties)

## Relations

**01 Long term**

- Friend
- Colleague
- Family

**02 Short term**

- Collaboration in a project
- Same team in a game
- Attendees of a same class

## Interactions

**01 Instantaneous**

- Email
- Text message
- Co-authoring

**02 With Duration**

- Phone call
- Discussion
- Attendees of a same class

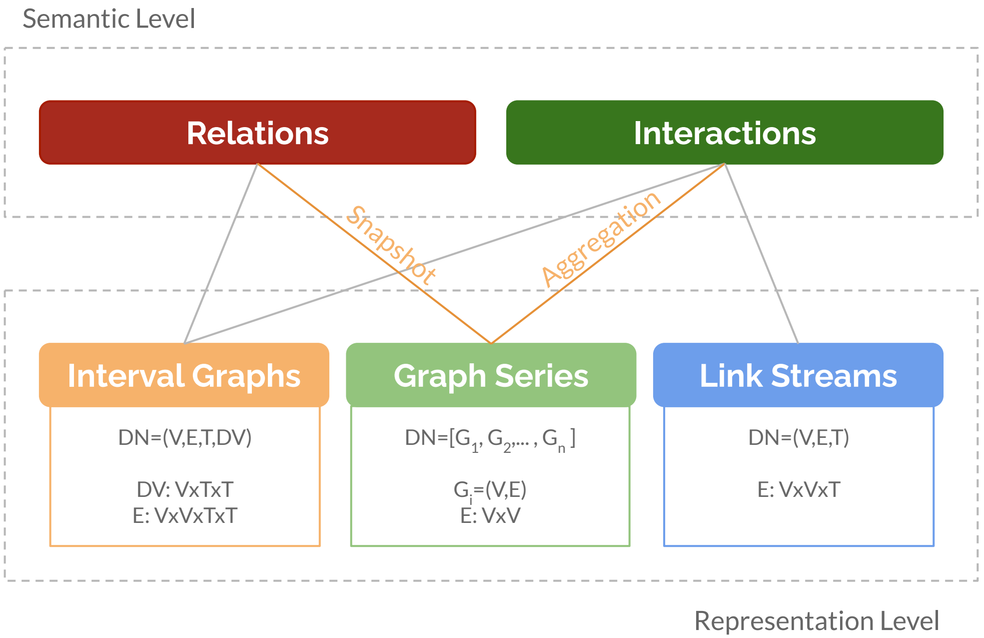
# Semantics and how to represent them

## Relations

The graph is more and more stable, until most observations are completely similar to previous/later ones (frequency faster than change rate)

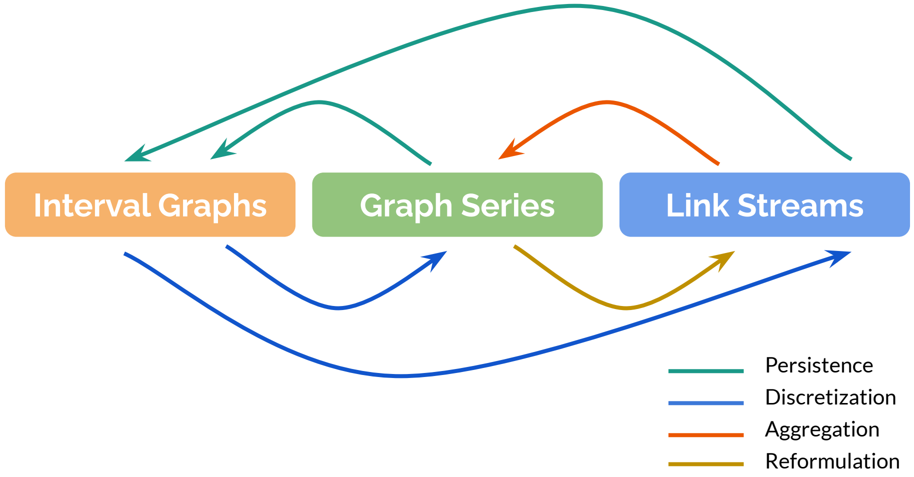
## Interactions

The graph is less and less stable, until each observation is a graph in itself, thus completely different from previous/later ones (frequency faster than observed events rate)



# Changing Representation

Alternative representations can be, to some extent, converted among them by applying appropriate data transformations



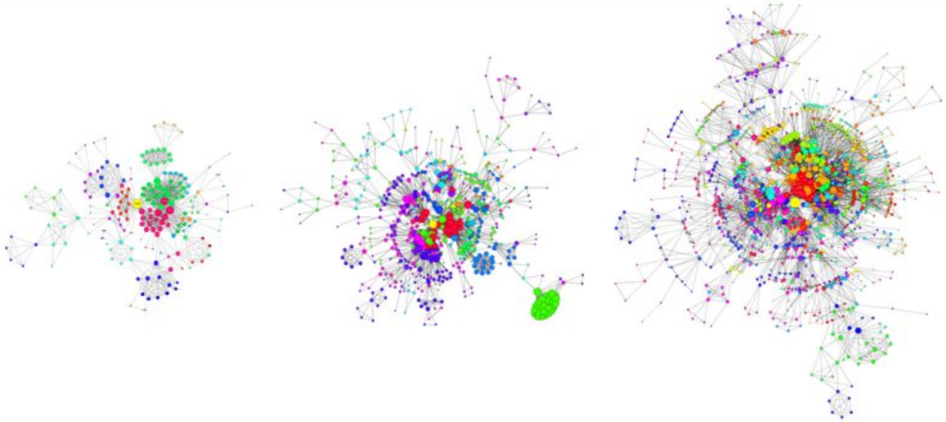
# Analyzing Dynamic Networks

A brief Introduction

# Unstable Snapshots

The evolution is represented as a series of a few snapshots

- Many changes between snapshots
  (Cannot be visualized as a “movie”)
- Each snapshot can be studied as a static graph
- Evolution of node properties can be studied “independently”
  (e.g., node i had low centrality in snapshot t and high centrality in snapshot t+n)



# Stable Network

Edges change (relatively) slowly

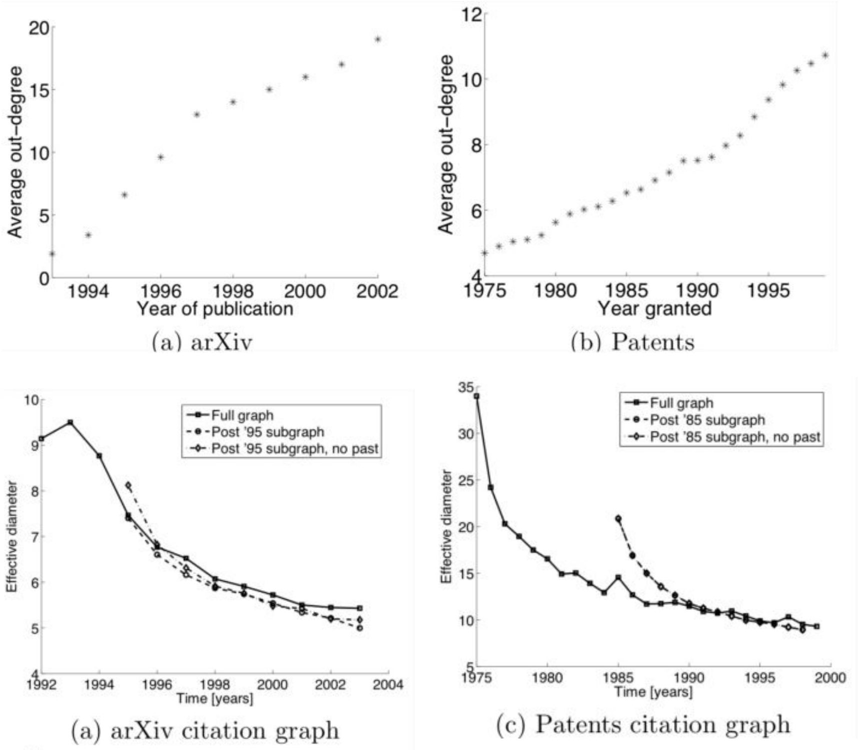
The network is well defined at any t

- Temporal network: nodes/edges described by (long lasting) intervals
- Enough snapshots to track nodes

A static analysis at every (relevant) t gives a dynamic vision

No formal distinction with previous case
(higher observation frequency)

Properties can be analyzed as time series




Leskovec, Jure, Jon Kleinberg, and Christos Faloutsos. "Graph evolution: Densification and shrinking diameters." ACM Transactions on Knowledge Discovery from Data. (2007)

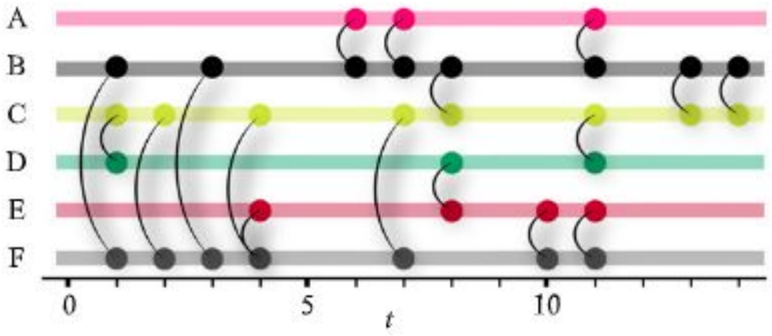
# Unstable Temporal Network

The network at a given t is not meaningful

How to analyze such a network?

Until recently, network was transformed using aggregation/sliding windows

- Information loss
- How to choose a proper aggregation window size?




Holme, Petter, and Jari Saramäki. Temporal networks. Physics reports 519.3 (2012)

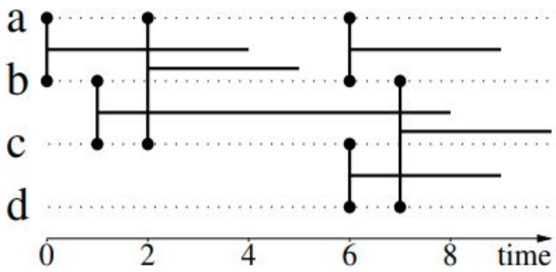
# Stream Graphs

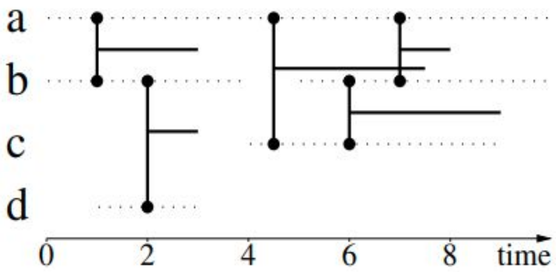
Network Properties, Centralities, Paths & Components

# Stream Graph

S = (T, V, W, E)

T: Possible Time
V: vertices
W: Vertices presence time
E: Edges presence time



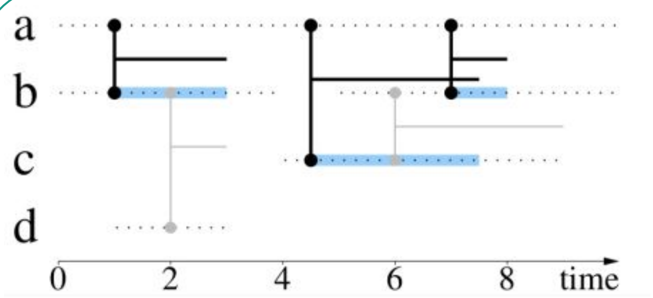




*Latapy, M., Viard, T., & Magnien, C. (2018). Stream graphs and link streams for the modeling of interactions over time. Social Network Analysis and Mining, 8(1), 61.*

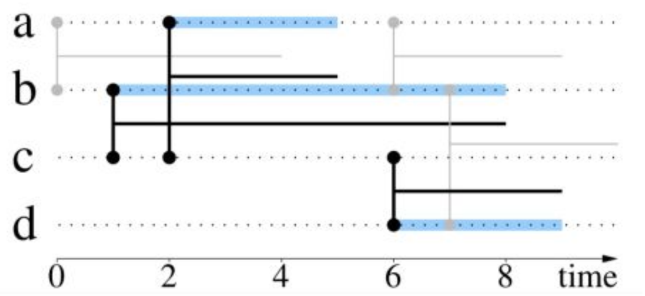
Stream Graph

# Indices

| | |
|---|---|
| Number of nodes | $n = \sum_{v \in V} n_v = \frac{\lvert W \rvert}{\lvert T \rvert}$ |
| Number of edges | $m = \sum_{uv \in V \otimes V} m_{uv} = \frac{\lvert E \rvert}{\lvert T \rvert}$ |
| Neighbors of a node | $N(v) = \{(t,u),(t,uv) \in E\}$ |
| Degree of a node | $d(v) = \frac{\lvert N(v) \rvert}{\lvert T \rvert} = \sum_{u \in V} \frac{\lvert T_{uv} \rvert}{\lvert T \rvert}$ |




$$N(a) = ([1,3] \cup [7,8]) \times \{b\} \cup [4.5, 7.5] \times \{c\}$$
$$d(a) = \frac{3}{10} + \frac{3}{10} = 0.6$$




$$N(c) = [2,5] \times \{a\} \cup [1,8] \times \{b\} \cup [6,9] \times \{d\}$$
$$d(c) = \frac{13}{10} = 1.3$$

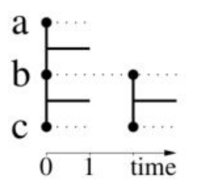
Stream Graph

# Indices

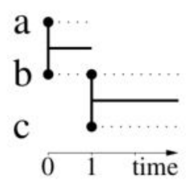
| | |
|---|---|
| Average Degree | $d(V) = \frac{1}{n} \cdot \sum_{v \in V} n_v \cdot d(v) = \sum_{v \in V} \frac{\lvert T_v \rvert}{\lvert W \rvert} \cdot d(v)$ |
| Clustering Coefficient | $\frac{\sum_{uw \in V \otimes V} \lvert T_{vu} \cap T_{vw} \cap T_{uw} \rvert}{\sum_{uw \in V \otimes V} \lvert T_{vu} \cap T_{vw} \rvert}$ |
| Density | $\delta(S) = \frac{\boxed{\sum_{uv \in V \otimes V} \lvert T_{uv} \rvert}}{\boxed{\sum_{uv \in V \otimes V} \lvert T_u \cap T_v \rvert}}$ |

Total Edge presence

Total overlapping time between each pair of nodes




n = 2
m=1
δ=0.75




n = 2
m=1
δ=1

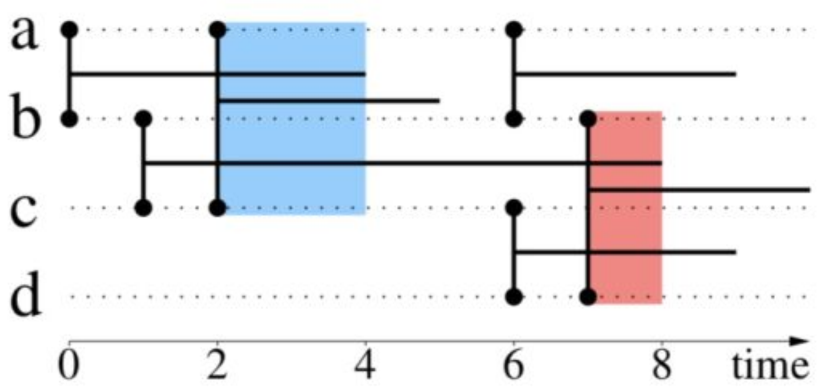
Stream Graph

# Cliques

Temporal cluster having density equal to 1

- All pairs of nodes in in a clique are linked together in S

A clique is maximal if there is no other clique in S that contains it

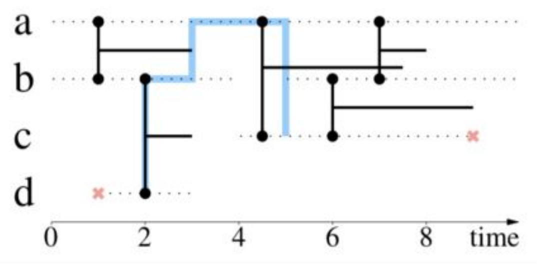


Stream Graph

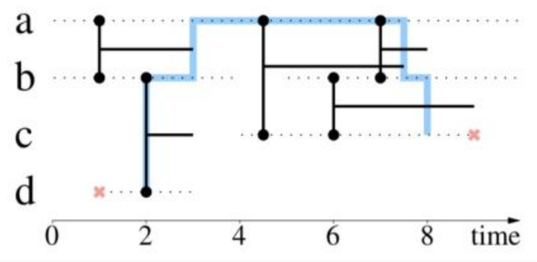
# Paths and Distances

A path in a stream graphs

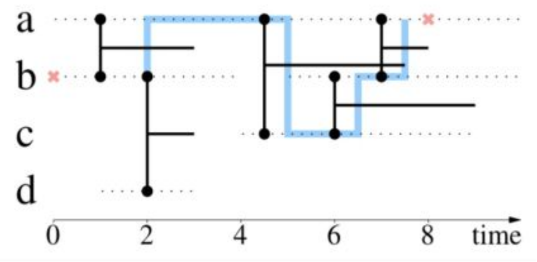
- Starts at a node and at a timestamp
- Ends at a node and at a timestamp
- Has a length
  (number of hops)
- Has a duration
  (duration from leaving node to reaching node)




**Path:** (d,1)(c,9)
**Length:** 3
**Duration:** 3




**Path:** (d,2)(c,8)
**Length:** 4
**Duration:** 6




**Path:** (b,0)(a,8)
**Length:** 4
**Duration:** 5

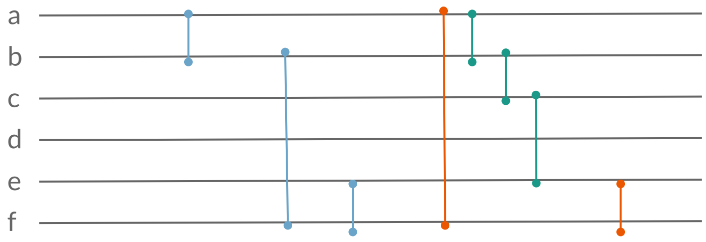
Stream Graph

# Paths and Distances

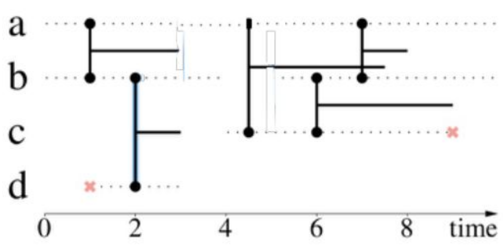
Several types of shortest paths in Stream graphs:

| Type | Description |
|---|---|
| Shortest path | Minimal length |
| Fastest path | Minimal Duration |
| Foremost path | First to reach |
| Fastest shortest paths | Minimum duration among minimum length |
| Shortest fastest paths | Minimum length among minimum duration |

From a to e (Foremost, Fastest, Shortest)




From (1,d) to (9,c)



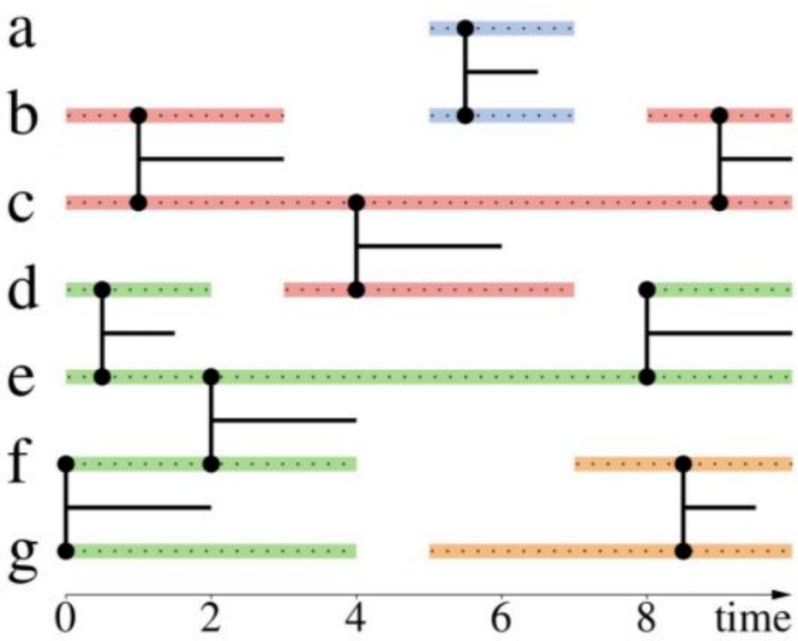

| | |
|---|---|
| Shortest path | (2.5, d, b) (3, b, a) (7, a, c) |
| Fastest path | (3, d, b) (3, b, a) (4.5, a, c) |
| Foremost path | (2, d, b) (2, b, a) (4.5, a, c) |
| Fastest shortest path | (3, d, b) (3, b, a) (4.5, a, c) |
| Shortest Fastest path from | (3, d, b) (3, b, a) (4.5, a, c) |

Stream Graph

# Connected Components

Weakly connected component:

- There is at least a non-temporally respecting path among each pair of node (component in the static, undirected, graph)



# Random Models for Dynamic Graphs

# Random Models

Random models for static graphs:

- ER, BA, WS, Configuration,...
- Each one preserving different characteristics

In dynamic networks, everything becomes more complicated...

Different Shuffling strategies can be leveraged and combined

**Snapshot shuffling**

Keep the order of snapshots, randomize network inside snapshot

**Sequence shuffling**

Keep each snapshot identical, but switch randomly their order

**Link shuffling**

Randomize aggregated graph, keep activation times

pick two node pairs activation time (u1,v1: t0, t1,...), (u2,v2: w0, w1, ...) and switch them

**Timeline shuffling**

Randomize nodes/edges activation time, conserve the aggregated graph

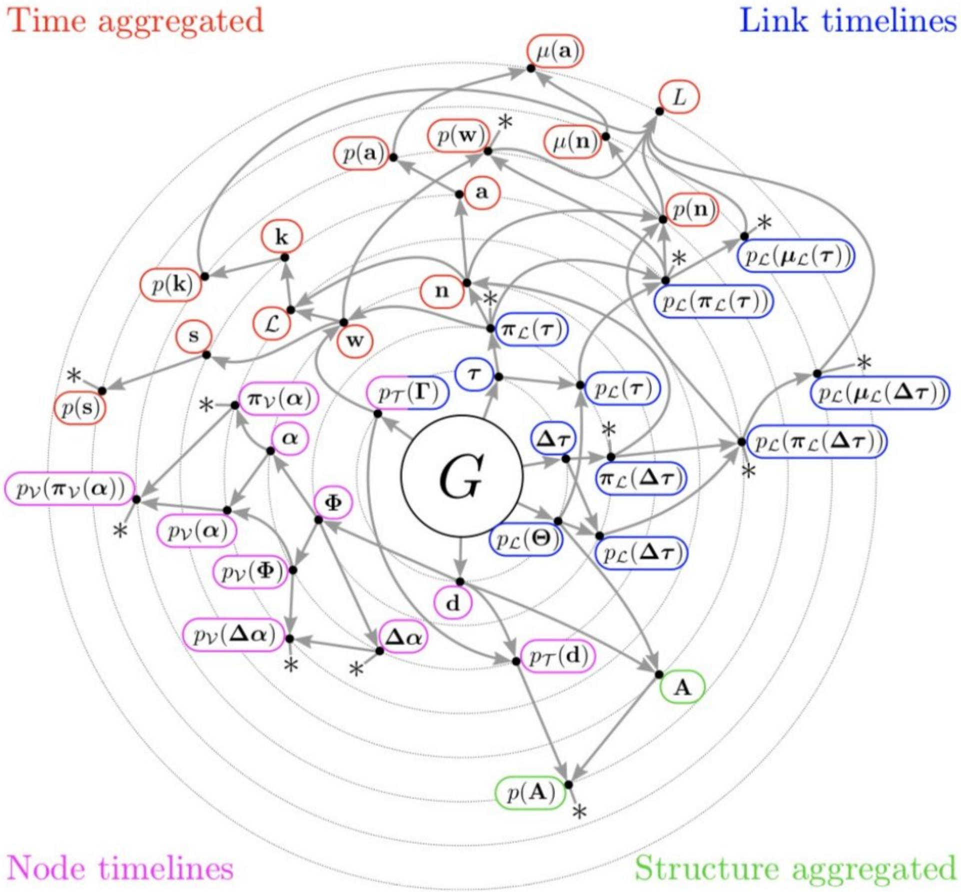
pick two edge observations (u1,v1,t1), (u2,v2,t2), switch t1 and t2

*Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." arXiv preprint arXiv:1806.04032 (2018).*

# Random Models

Note:

Different Shuffling strategies can be combined



# Activity Driven Model

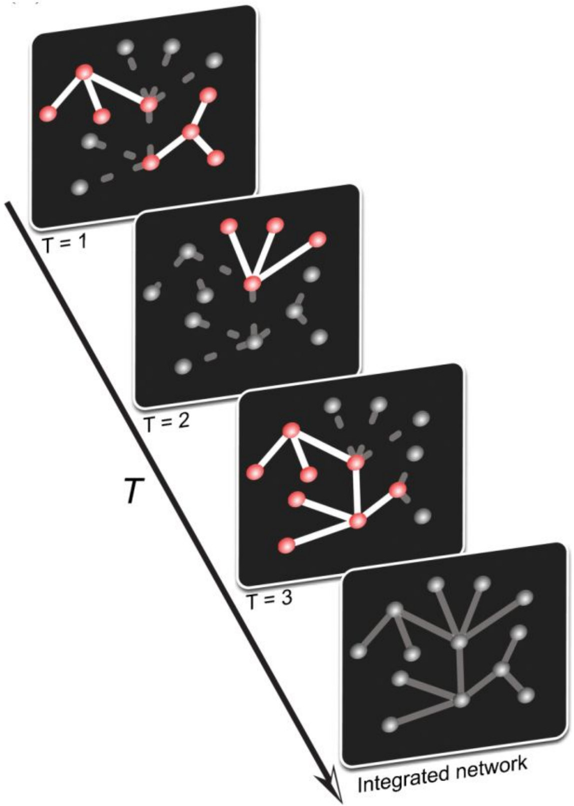
Agent based model of temporal interactions

General framework

- additional mechanisms can be added
- capable of simulating dynamical processes co-evolving with the contact dynamics

single a-priori assumption:

- agents have different activity potentials




*Perra, Nicola, et al. "Activity driven modeling of time varying networks." Scientific reports (2012).*

# Activity Driven Model

**Setup:**
N disconnected nodes, with pre-assigned activity rates

$$a_i = \eta x_i$$

where
- $x_i$ is the activity potential of node i (sampled from an arbitrary distribution F(x) and $x_i \in [\varepsilon, 1]$)
- $\eta$ is a scaling factor

Each time step t start with N disconnected nodes:

1. With probability $a_i$ node i is activated and connects to m other nodes randomly
2. With probability $1-a_i$ node i remains inactive (still can receive connections from other active nodes)

At the end of each time step we delete each link and start the loop over again

The structure of the actual network at each t will be a random network

The emerging degree distribution of the integrated network will follow the same scaling form as the pre-assigned activity distribution

# Memory and Social Reinforcement

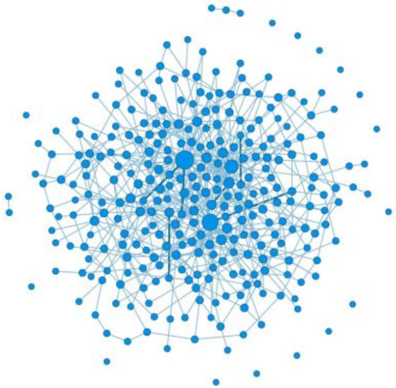
When a node is active it connects with probability

$$p(n) = c/(n + c)$$

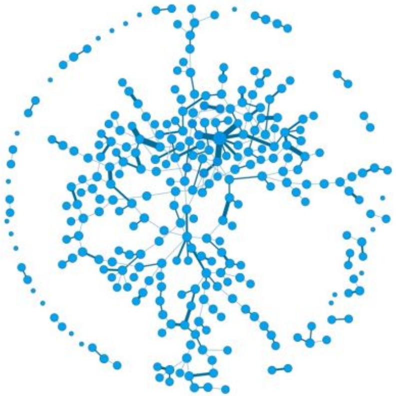
to a random node it has never connected before or with probability

$$1 - p(n)$$

to one of the n node who it has connected earlier



Without memory



With memory

After each iteration links are deleted but each node keeps remembering its previously connected egocentric network

*Karsai, Márton, Nicola Perra, and Alessandro Vespignani. "Time varying networks and the weakness of strong ties." Scientific reports 4 (2014): 4001.*

# Summarizing

# Time flies like an arrow…

Real world phenomena unfold through time

- Networks can be used to study them but increasing the complexity of the model

Different constraints require different modeling choices

- Relations or Interactions?
- Interval, graph series or link streams?

Static network analysis needs to be revised

- Degree, density, paths, components…
- What about centralities?

# Where next?

Two kind of dynamics:

- Dynamics of Networks
  (topological perturbations)
- Dynamics on Networks
  (diffusive phenomena: epidemics, opinion dynamics...)

Of course they can happen at the same time...

| Dynamics of Networks | Dynamics on Networks | Mixed Dynamics |
|---|---|---|
| **Assumption:** Topology evolution is faster than diffusive processes unfolding (if any) | **Assumption:** Diffusive processes unfolding is faster than topology evolution (if any) | **Assumption:** Diffusive processes unfolding and topology evolution have comparable rates |
| **Applications:**<br>- Link Prediction<br>- Dynamic Community Discovery<br>- ... | **Applications:**<br>- Epidemic spreading<br>- Opinion Dynamics<br>- ... | **Applications:**<br>- Diffusion shape topology<br>- Topology shape diffusion<br>- Feedback loops |

Chapter 9

# Conclusion

## Take Away Messages

1. Real phenomena evolve through time
2. Network modeling can fill such gap:
   a. Representation has to be enriched
   b. Indexes and concepts revised
   c. Null models reformulated

## Suggested Readings

- “Temporal Networks” Holme et al.
- “Stream graphs and link streams for the modeling of interactions over time” Latapy et al.

## What’s Next

Chapter 11:
**Link Prediction**