1.

$$H_{hp}(e^{jw}) = \begin{cases} 1, & -\pi < w < -w_c \lor w_c < w < \pi \\ 0, & \text{otherwise} \end{cases}$$

$$h_{hp}[n] = \frac{1}{2\pi}\int_{-\pi}^{-w_c} e^{jwn}\,dw + \frac{1}{2\pi}\int_{w_c}^{\pi} e^{jwn}\,dw$$

$$= \frac{1}{2\pi jn} e^{jwn}\Big|_{-\pi}^{-w_c} + \frac{1}{2\pi jn} e^{jwn}\Big|_{w_c}^{\pi}$$

$$= \frac{1}{2\pi jn} \cdot -2j\sin(w_c n) + 0$$

$$= -\frac{\sin(w_c n)}{\pi n} \quad \#$$

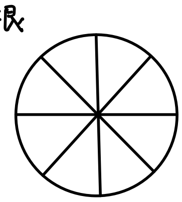
2.

(a)

$\because$ $e^{-j2\pi n/N}$ 是 $(e^{-j2\pi})^N = 1$ 的根

$\therefore$ 其將平均分散在單位圓上

如右圖所示



可發現把所有根加總

會互相抵銷

$$\Rightarrow \sum_{n=0}^{N-1} e^{-j2\pi n/N} = 0 \quad \#$$

(b)

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & e^{-2\pi j\frac{1}{n}} & e^{-2\pi j\frac{2}{n}} & & & e^{-2\pi j\frac{n-1}{n}} \\ 1 & e^{-2\pi j\frac{2}{n}} & e^{-2\pi j\frac{4}{n}} & \ddots & & e^{-2\pi j\frac{2(n-1)}{n}} \\ \vdots & \vdots & & & \ddots & \vdots \\ 1 & e^{-2\pi j\frac{n-1}{n}} & e^{-2\pi j\frac{2(n-1)}{n}} & & & e^{-2\pi j\frac{(n-1)(n-1)}{n}} \end{bmatrix}$$

$x[n]$ 的 DFT $\Rightarrow F_n \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

由 (a) 可知根加總為 0

$\Rightarrow$ 除第一列為 N，其餘皆 0

則 $X_k = \begin{cases} N, & k=0 \\ 0, & \text{otherwise} \end{cases} \quad \#$

3.

(a) $x[n] = [2, 0, 1, 0]$

$y[n] = [1, -1, 0, 0]$

令 $z[n] = x[n] \circledast y[n]$

$$\equiv \sum_{m=0}^{3} y[m]\, x[(n-m) \bmod 4]$$

$$z[0] = \sum_{m=0}^{3} y[m]\, x[(0-m) \bmod 4]$$

$$= 2 + 0 + 0 + 0 = 2$$

$$z[1] = \sum_{m=0}^{3} y[m]\, x[(1-m) \bmod 4]$$

$$= -2 + 0 + 0 + 0 = -2$$

$$z[2] = \sum_{m=0}^{3} y[m]\, x[(2-m) \bmod 4]$$

$$= 0 + 0 + 1 + 0 = 1$$

$$z[3] = \sum_{m=0}^{3} y[m]\, x[(n-m) \bmod 4]$$

$$= 0 + 0 - 1 + 0 = -1$$

$\Rightarrow z[n] = x[n] \circledast y[n] = [2, -2, 1, -1]$ #

(b)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-2\pi j \frac{1}{4}} & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{3}{4}} \\ 1 & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{4}{4}} & e^{-2\pi j \frac{6}{4}} \\ 1 & e^{-2\pi j \frac{3}{4}} & e^{-2\pi j \frac{6}{4}} & e^{-2\pi j \frac{9}{4}} \end{bmatrix}$$

$$F_4 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \quad F_4 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+j \\ 2 \\ 1-j \end{bmatrix}$$

$\Rightarrow X[k] = [3, 1, 3, 1]$ $\quad \Rightarrow Y[k] = [0, 1+j, 2, 1-j]$ #

(c) $Z[k] = X[k]Y[k] = [0, 1+j, 6, 1-j]$ #

(d)

$$F_4^{-1} = \frac{F_4^{*}}{4} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{2\pi j \frac{1}{4}} & e^{2\pi j \frac{2}{4}} & e^{2\pi j \frac{3}{4}} \\ 1 & e^{2\pi j \frac{2}{4}} & e^{2\pi j \frac{4}{4}} & e^{2\pi j \frac{6}{4}} \\ 1 & e^{2\pi j \frac{3}{4}} & e^{2\pi j \frac{6}{4}} & e^{2\pi j \frac{9}{4}} \end{bmatrix}$$

$$F_4^{-1} \begin{bmatrix} 0 \\ 1+j \\ 6 \\ 1-j \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$ #

4.

$$V[k] = \sum_{n=0}^{2N-1} v[n]\, e^{-j\frac{2\pi k}{2N} n}$$

$$= \sum_{n=0}^{N-1} v[2n]\, e^{-j\frac{2\pi k}{2N} \cdot 2n} + \sum_{n=0}^{N-1} v[2n+1]\, e^{-j\frac{2\pi k}{2n}(2n+1)}$$

$$= \sum_{n=0}^{N-1} g[n]\, e^{-j\frac{2\pi k}{N} n} + \sum_{n=0}^{N-1} h[n]\, e^{-j\frac{2\pi k}{N} n}\, e^{-j\frac{\pi k}{N}}$$

$$= G[k \bmod N] + e^{-j\frac{\pi k}{N}} H[k \bmod N]$$

$\Rightarrow f[k] = e^{-\frac{j\pi}{N} k}$ #