1.

let $h_{lpf}(t)=\frac{\sin(w_0 t)}{\pi t} \Longleftrightarrow H_{lpf}(w)=\begin{cases}1, & \text{for } |w|<w_c \\ 0, & \text{otherwise}\end{cases}$

then $H_{hpf}(w)=\begin{cases}1, & \text{for } |w|>w_0 \\ 0, & \text{otherwise}\end{cases} = 1-H_{lpf}(w)$

$\Rightarrow h_{hpf}(t)=F^{-1}\{1-H_{lpf}(w)\}=\delta(t)-h_{lpf}(t)$

$h_{hpf}[n]=F^{-1}\{1-H_{lpf}(e^{jw})\}=\delta[n]-h_{lpf}[n]$

$= \delta[n]-\frac{\sin(w_0 n)}{\pi n}$

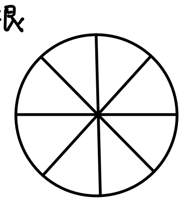
2.

(a)

$\because e^{-j2\pi n/N}$ 是 $(e^{-j2\pi})^N=1$ 的根

$\therefore$ 其將平均分散在單位圓上

如右圖所示



可發現把所有根加總

會互相抵銷

$\Rightarrow \sum_{n=0}^{N-1} e^{-j2\pi n/N}=0$ #

(b)

$$F_n=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & e^{-2\pi j\frac{1}{n}} & e^{-2\pi j\frac{2}{n}} & & & e^{-2\pi j\frac{n-1}{n}} \\ 1 & e^{-2\pi j\frac{2}{n}} & e^{-2\pi j\frac{4}{n}} & \ddots & & e^{-2\pi j\frac{2(n-1)}{n}} \\ \vdots & \vdots & & & \ddots & \vdots \\ 1 & e^{-2\pi j\frac{n-1}{n}} & e^{-2\pi j\frac{2(n-1)}{n}} & & & e^{-2\pi j\frac{(n-1)(n-1)}{n}} \end{bmatrix}$$

$x[n]$ 的 DFT $\Rightarrow F_n\begin{bmatrix}\vdots\end{bmatrix}$

由(a)可知根加總為0

$\Rightarrow$ 除第一列為N，其餘皆0

則 $X_k=\begin{cases}N, & k=0 \\ 0, & \text{otherwise}\end{cases}$ #

3.

(a) $x[n] = [2, 0, 1, 0]$

$y[n] = [1, -1, 0, 0]$

$z[n] = x[n] \otimes y[n]$

$$\equiv \sum_{m=0}^{3} y[m]\, x[(n-m) \bmod 4]$$

$$z[0] = \sum_{m=0}^{3} y[m]\, x[(0-m) \bmod 4]$$

$$= 2 + 0 + 0 + 0 = 2$$

$$z[1] = \sum_{m=0}^{3} y[m]\, x[(1-m) \bmod 4]$$

$$= -2 + 0 + 0 + 0 = -2$$

$$z[2] = \sum_{m=0}^{3} y[m]\, x[(2-m) \bmod 4]$$

$$= 0 + 0 + 1 + 0 = 1$$

$$z[3] = \sum_{m=0}^{3} y[m]\, x[(n-m) \bmod 4]$$

$$= 0 + 0 - 1 + 0 = -1$$

$$\Rightarrow z[n] = x[n] \otimes y[n] = [2, -2, 1, -1]$$

(b)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-2\pi j \frac{1}{4}} & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{3}{4}} \\ 1 & e^{-2\pi j \frac{2}{4}} & e^{-2\pi j \frac{4}{4}} & e^{-2\pi j \frac{6}{4}} \\ 1 & e^{-2\pi j \frac{3}{4}} & e^{-2\pi j \frac{6}{4}} & e^{-2\pi j \frac{9}{4}} \end{bmatrix}$$

$$F_4 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \quad F_4 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+j \\ 2 \\ 1-j \end{bmatrix}$$

$\Rightarrow X[k] = [3, 1, 3, 1]$ $\quad \Rightarrow Y[k] = [0, 1+j, 2, 1-j]$ #

(c) $Z[k] = X[k]Y[k] = [0, 1+j, 6, 1-j]$ #

(d)

$$F_4^{-1} = \frac{F_4^*}{4} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{2\pi j \frac{1}{4}} & e^{2\pi j \frac{2}{4}} & e^{2\pi j \frac{3}{4}} \\ 1 & e^{2\pi j \frac{2}{4}} & e^{2\pi j \frac{4}{4}} & e^{2\pi j \frac{6}{4}} \\ 1 & e^{2\pi j \frac{3}{4}} & e^{2\pi j \frac{6}{4}} & e^{2\pi j \frac{9}{4}} \end{bmatrix}$$

$$F_4^{-1} \begin{bmatrix} 0 \\ 1+j \\ 6 \\ 1-j \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$

4.

$$V[k] = \sum_{n=0}^{2N-1} v[n]\, e^{-j \frac{2\pi k}{2N} n}$$

$$= \sum_{n=0}^{N-1} v[2n]\, e^{-j \frac{2\pi k}{2N} \cdot 2n} + \sum_{n=0}^{N-1} v[2n+1]\, e^{-j \frac{2\pi k}{2n} (2n+1)}$$

$$= \sum_{n=0}^{N-1} g[n]\, e^{-j \frac{2\pi k}{N} n} + \sum_{n=0}^{N-1} h[n]\, e^{-j \frac{2\pi k}{N} n}\, e^{-j \frac{\pi k}{N}}$$

$$= G[k \bmod N] + e^{-j \frac{\pi k}{N}} H[k \bmod N]$$

$$\Rightarrow f[k] = e^{-\frac{j\pi}{N} k}$$ #