Matrix Computation Homework 1

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November 26, 2025

Problem 1. (Eigenvalue)

1)

Solution.

Let λ be an eigenvalue of the matrix A, and let v be the corresponding eigenvector. Then, we have:

$$Av = \lambda v$$

Since,

$$P(A) = A^3 - 2A^2 - A + 2I = 0$$

any eigenvalue λ of A must satisfy the corresponding scalar polynomial equation $P(\lambda) = 0$.

Substituting λ into the polynomial:

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$

Then we can get that:

$$\begin{aligned} \lambda^2(\lambda - 2) - 1(\lambda - 2) = 0 \\ (\lambda^2 - 1)(\lambda - 2) = 0 \\ (\lambda - 1)(\lambda + 1)(\lambda - 2) = 0 \end{aligned}$$

Thus,we can get that

$$\lambda \in \{1, -1, 2\}$$

Then, we can get the λ pairs:

λ_1	-1	-1	-1	-1	-1	-1	1	1	1	2
λ_2	-1	-1	-1	1	1	2	1	1	2	2
λ_3	-1	1	2	1	2	2	1	2	2	2

Table 1: λ pairs

2)

Solution.

Since the eigenvalues of A are:

$$\lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 \in \{-1, 1, 2\}$$

we can get that the eigenvalues of $(A + I)$ are:

$$\lambda_1' = 1 + 1 = 2, \quad \lambda_2' = -1 + 1 = 0, \quad \lambda_3' \in \{0, 2, 3\}$$

Thus,

$$det(A + I) = \lambda_1' \cdot \lambda_2' \cdot \lambda_3' = 0$$

Problem 2. (Eigenvector and similarity)