Assignment 4

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1. Transform following problems into linear programming

(1)

$$\begin{aligned} \text{maximize} \quad & 5x + 2y \\ \text{subject to} \quad & 0 \leq x \leq 20 \\ & |x - y| \leq 5 \end{aligned}$$

(2)

$$\begin{aligned} \text{maximize} \quad & min(x1, x2, x3) \\ \text{subject to} \quad & x1 + x2 + x3 = 15 \end{aligned}$$

Solution:

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2. Given a graph G, each vertex v_i has a profit p_i and each edge e_{ij} has a cost c_{ij}. Define the profit of a cycle is the total profits of all the vertices in the cycle, and the cost of a cycle is the total costs of all the edges in the cycle. We need to find a simple cycle in G which contains a given vertex v_0, and maximize the profit of it within the cost bound B. Write the linear programming formulation of this problem.

Solution:

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3. Consider the following optimization problem

$$\begin{array}{ll} \text{minimize} & f_0(\mathbf{x}) \\ \text{subject to } f_i(\mathbf{x}) \leq 0, & i = 1, \ldots, m \end{array}$$

with variable $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{R}^n$. Note that the program may not be linear. The Lagrangian $L : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}$ associated with the program is defined as

$$L(\mathbf{x}, \lambda) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x})$$

where $\lambda = (\lambda_1, \ldots, \lambda_m) \in \mathbf{R}^m$

Define the Lagrange dual function $g : \mathbf{R}^m \to \mathbf{R}$ as the minimum value of the Lagrangian over x : for $\lambda \in \mathbf{R}^m$,

$$g(\lambda) = \inf_{\mathbf{x}} L(\mathbf{x}, \lambda)$$

We write $\lambda \geq 0$ if $\lambda_i \geq 0$ for all $1 \leq i \leq m$ and let p^* be the optimal value of original program.

Show that:

$$g(\lambda) \leq p^*, \text{ for every } \lambda \geq 0$$

Solution:

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4. Consider the following optimization problem

$$\begin{aligned}&\text{maximize } g(\lambda)\\&\text{subject to } \lambda \geq 0\end{aligned}$$

Show that if the program in previous exercise is a linear program, then this program is its dual program.

Solution:

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5. Prove the complementary slackness property of linear programs.

Solution:

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6. Write the dual problem of:

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & A_1 x \le b_1 \\ & A_2 x \ge b_2 \\ & A_3 x = b_3 \end{array}$$

Solution:

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7. Prove the König theorem: Let G be bipartite, then cardinality of maximum matching = cardinality of minimum vertex cover.

Solution:

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8. Show that the dual of the dual of a linear program is the primal linear program.
Solution:

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