Assignment 5

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1. Use layering to get a factor f approximation algorithm for set cover, where f is the frequency of the most frequent element. Provide a tight example for this algorithm.

Solution:

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2. Let $G = (V, E)$ be an undirected graph with nonnegative edge costs. S, the senders and R, the receivers, are disjoint subsets of V. The problem is to find a minimum cost subgraph of G such that for every receiver r in R, there is at least one sender s in S such that there is a path connecting r to s in the subgraph. Give a factor 2 approximation algorithm that runs in polynomial time.

Solution:

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3. **(Bin Packing)** Consider a more restricted algorithm than First-Fit, called Next-Fit, which tries to pack the next item only in the most recently started bin. If it does not fit, it is packed in a new bin. Show that this algorithm also achieves factor 2. Give a factor 2 tight example.

Solution:

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4. (Hamilton cycle) Given an undirected complete graph, each edge is assigned with a nonnegative cost by the function c (eg. the cost for edge (u, v) is $c(u, v)$). Find a Hamilton cycle with the largest cost by the greedy approach, and prove the guarantee factor is 2.

Solution:

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5. Given a directed graph $G = (V, E)$, we need to pick a maximum cardinality set of edges from E so that the resulting subgraph is acyclic. Find a factor $\frac{1}{2}$ approximate algorithm for this problem.

Solution:

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6. (Knapsack) Given a set $S = \{a_1, ..., a_n\}$ of objects, with specified non-negative weights and profits, w_i, p_i respectively, and a "knapsack capacity" $B(w_i \leq B)$, find a subset of objects whose total weight is bounded by B and total profit is maximized.

1. Consider two types of greedy algorithms for the knapsack problem. Sort the objects by decreasing **ratio of profit to weight** or only by **profit**, and then greedily pick objects in this order. Show that these two algorithms can be made to perform arbitrarily badly.

2. Combining these two greedy algorithm, pick the more profitable solution in these two algorithms' results. Show that this algorithm achieves an approximation factor of 2.

Solution:

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7. (Maximum Cut) Given an undirected graph $G = (V, E)$, the *cardinality maximum cut problem* asks for a partition of V into sets S and $\bar{S}$ so that the number of edges running between these sets is maximized. Find a factor 2 approximation algorithm for this problem.

Solution:

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8. Consider the following problem: Given an undirected graph and compute the number of matchings (not the cardinality of a single matching, but the number of different ways of matching) in the graph . Show that if we have an α-approximation algorithm for it for some constant α, then we also have a PTAS.

Solution:

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9. Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Note, that edge lengths satisfy the triangle inequality. Give a factor $4/3$ approximation algorithm for the TSP in this special class of graphs.

Hint: A 2-matching in a graph G is a subset of edges M, such that every vertex is incident to exactly 2 edges of M. A minimum cost 2-matching can be computed in polynomial time.

Solution:

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