Assignment 3

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1. Prove or disprove the following statement. If all capacities in a network are distinct, then there exists a unique flow function that gives the maximum flow.

Solution:

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2. An edge of a flow network is called **critical** if decreasing the capacity of this edge results in a decrease in the maximum flow value. Present an efficient algorithm that, given an s-t network G finds any critical edge in a network(assuming one exists).

Solution:

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3. Let $G = (V, E)$ be an undirected weighted graph with two distinguished vertices $s, t \in V$. Give an efficient algorithm to find a minimum weight cut that separates s from t.

Solution:

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4. You are given a matrix with fractional elements between 0 and 1. The sum of all numbers in each row and in each column is integer. Prove that we can always round each element to 0 or 1 so that the sum of each row and each column remains unchanged and design a polynomial time algorithm to find such a rounding result.

Solution:

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5. Suppose that, in addition to edge capacities, a flow network has **vertex capacities**. That is each vertex has a limit on how much flow can pass though. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G. How many vertices and edges does G' have?

Solution:

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6. Consider a bipartite graph $G = (X \cup Y, E)$ with parts X and Y. Each part contains $2k$ vertices (i.e. $|X| = |Y| = 2k$). Suppose that $deg(u) \geq k$ for every $u \in X \cup Y$. Prove that G has a perfect matching.

Solution:

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7. You are designing a experiment in which you want to measure certain properties $p_1, \ldots, p_n$ of a yeast culture. You have a set of tools $t_1, \ldots, t_m$ that can each measure a subset S_i of the properties. For example, tool t_i measures S_i may equal $\{p_7, p_8\}$. To be sure that your results are not due to noise or other artifact, you must measure every property at least k times using k different tools.

- Give a polynomial-time algorithm that decides whether the tools you have are sufficient to measure the desired properties the desired number of times.
- Suppose each tool t_i comes from manufacturer M_i and we have the additional constraint that the tools to test any property p_i can't all come from the same manufacturer. Give a polynomial-time algorithm to solve this problem.

Solution:

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8. Consider a flow network $G = (V, E)$ with positive edge capacities $\{c(e)\}$. Let $f : E \to \mathbb{R}_{\geq 0}$ be a maximum flow in G, and G_f be the residual graph. Denote by S the set of nodes reachable from s in G_f and by T the set of nodes from which t is reachable in G_f. Prove that $V = S \cup T$ if and only if G has a **unique** s-t minimum cut.

Solution:

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