Packing Optimization: Algorithmic Strategies & Analysis

Introduction

Packing optimization is a classic problem in combinatorial optimization, with applications ranging from logistics and resource allocation to finance and computer science. This project explores and compares a variety of algorithmic strategies for solving the 0/1 knapsack problem, focusing on both exact and approximate methods. The goal is to analyze the strengths, weaknesses, and practical performance of each approach, while also highlighting the engineering and design decisions made throughout the development process. The following presentation provides a comprehensive overview of the project, including system architecture, dataset handling, algorithmic analysis, user interface, and key insights gained during implementation and testing.

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Identification of Work and Group Members

- **Project:** Packing Optimization Algorithms
- **Course:** DA 2024/25
- **Group Members:**

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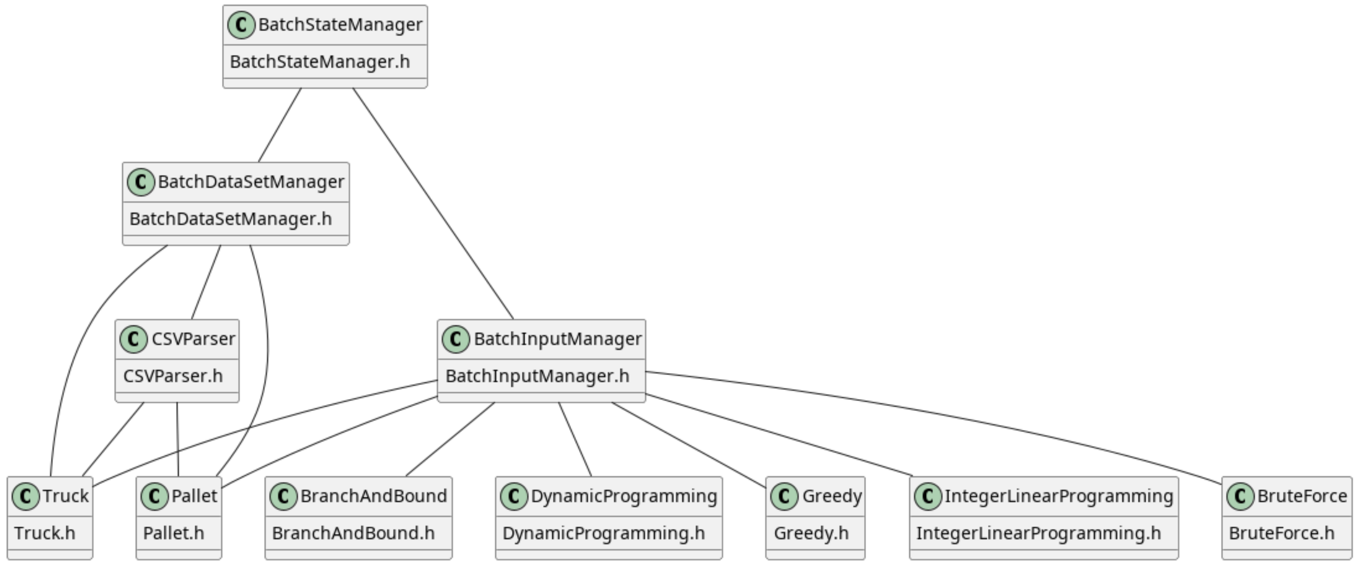
Class Diagram & File Involvement



- This diagram shows how the batch system manages dataset loading, parsing, and algorithm execution, and which files are involved in each class in a simplified way.

Reading the Dataset

- **CSVParser** loads pallets and truck data from CSV files.
- **BatchDataSetManager** uses `CSVParser` to populate `std::vector<Pallet>` and `Truck`.
- Datasets are selected and loaded interactively or via batch mode.

Implemented Functionalities & Algorithms

- **Brute Force (BF):** Exhaustively checks all possible combinations to guarantee the optimal solution. In case of ties (equal profit), it selects the solution with the lowest total weight, and if still tied, the fewest number of pallets—yielding the most real-world meaningful result. Impractical for large datasets due to exponential time.

- **Backtracking (BT):** Prunes branches exceeding constraints, exploring only feasible combinations. Faster than brute force, but still exponential in worst case.

- **Branch & Bound (BB):** Uses upper bounds to prune the search space. In practice, BB is the fastest and most robust approach for almost all real-world datasets, consistently outperforming other methods by solving most instances in microseconds. Only specially crafted pathological cases can slow it down significantly.

- **Dynamic Programming (DP):**

  - **DP Vector (Bottom-Up):** Fills a table for all capacities, reconstructs solution, ideal for dense problems. Returns the best-suited solution: among all optimal (max-profit) solutions, it chooses the one with the lowest total weight, and if still tied, the fewest pallets.

- **DP HashMap (Top-Down):** Memoized recursion, efficient for sparse problems, saves memory when few states are used. Also applies the same tie-breaking as DP Vector.
- **DP Optimized (2 Rows):** Uses only two rows for memory efficiency, computes only max profit (no reconstruction).

> **Note:** For both DP Vector and DP HashMap, the solution is not just any optimal one, but is selected according to a configurable sequence of tie-breaking criteria:
>
> 1. **Maximum profit** (always primary)
> 2. **Minimum total weight** (if the "draw condition" flag is enabled)
> 3. **Minimum number of pallets** (if the "draw condition" flag is enabled)
> 4. **Lexicographically smallest set of pallet IDs** (if the "lexicographical order" flag is enabled)
>
> By default, only profit is considered. Enabling the draw condition flag adds weight and count as secondary criteria, and enabling the lexicographical order flag adds a final, deterministic tie-breaker. This ensures that, even among all equally optimal solutions, the result is the most consistent and predictable for real-world use cases. Note that enabling lexicographical tie-breaking can significantly increase memory usage, especially for large problems, and should only be used when full optimality (including lexicographical order) is required.

- **Greedy Approximation:** Selects items by profit-to-weight ratio. Fastest, but not always optimal.

- **Integer Linear Programming (ILP):**

  - **Python (PuLP):** Flexible, slower, customizable.
  - **C++ (OR-Tools):** Fastest, robust, can be tuned for secondary objectives.

Note on Solution Meaningfulness & Trade-offs

- **DP (Vector/HashMap), Brute Force and Backtracking** always return not just any optimal solution, but the "best-fitted" (most meaningful) one: maximizing profit, then minimizing total weight, then minimizing the number of pallets, and—if there is still a tie—choosing the solution with the lexicographically smallest set of pallet IDs. This ensures that, even among all equally optimal solutions, the result is the most consistent and predictable for real-world use cases (e.g., always picking the same set of pallets for the same input).

- **Other algorithms (e.g., Branch & Bound, ILP)** may return any optimal solution, but are not guaranteed to find the best-fitted one according to these secondary criteria. This is because they prioritize speed and aggressive pruning, which can skip over equally optimal but more practical solutions.

- **Trade-off:** The enhanced tie-breaking in DP and BF adds a small computational overhead, but ensures the returned solution is not just mathematically optimal, but also practically best-suited for real applications. In contrast, faster algorithms may sacrifice this extra layer of optimality for performance.

- **Greedy:** The greedy algorithm is extremely fast and, in many cases, can produce solutions that are very close to optimal. However, it is not guaranteed to be optimal or to provide the best-fitted solution. Since greedy is a non-constant factor approximation for the 0/1 knapsack problem, its reliability cannot be blindly trusted—there are datasets where it performs well, but also cases where it can miss the optimal or most meaningful solution by a significant margin. Use with caution when solution quality is critical.

Strategy Analysis

- **Brute Force:** Always optimal, but only feasible for very small datasets.
- **Backtracking:** Prunes infeasible branches, much faster than BF for moderate datasets, but still exponential.
- **Branch & Bound:** Prunes aggressively using bounds, solves most datasets quickly, but can be trapped by specially crafted instances.
- **DP Vector:** Fast and reconstructs solutions for small/medium dense problems, but memory usage grows with capacity.
- **DP HashMap:** Best for sparse problems, saves memory, sometimes faster than vector.
- **DP Optimized:** Minimal memory, only max profit, not suitable for reconstructing solutions.
- **Greedy:** Extremely fast, but can miss optimal solutions, especially when high-value items have lower ratios.
- **ILP:** Always optimal, robust for all dataset sizes, C++ version is fastest, Python is more flexible.

Data Structures Used

- **Brute Force / Backtracking / Branch & Bound:** Arrays or vectors
- **DP Vector:** 2D array/vector (table)
- **DP HashMap:** Hash map (dictionary)
- **DP Optimized:** Two arrays/vectors
- **Greedy:** Array/vector
- **ILP:** Arrays, variables, constraints (solver-specific)

Time & Space Complexities

Strategy	Time Complexity	Space Complexity
Brute Force	O(2^n)	O(n)
Backtracking	O(2^n) (pruned)	O(n)
Branch & Bound	O(2^n) (pruned, fast)	O(n)
DP Vector	O(nW)	O(nW)
DP HashMap	O(#states)	O(#states)
DP Optimized	O(nW)	O(W)
Greedy	O(n log n)	O(n)
ILP (Python/C++)	Depends on solver	Depends on solver

- n = number of items, W = capacity, #states = number of unique (item, capacity) pairs used.

Comparative Analysis

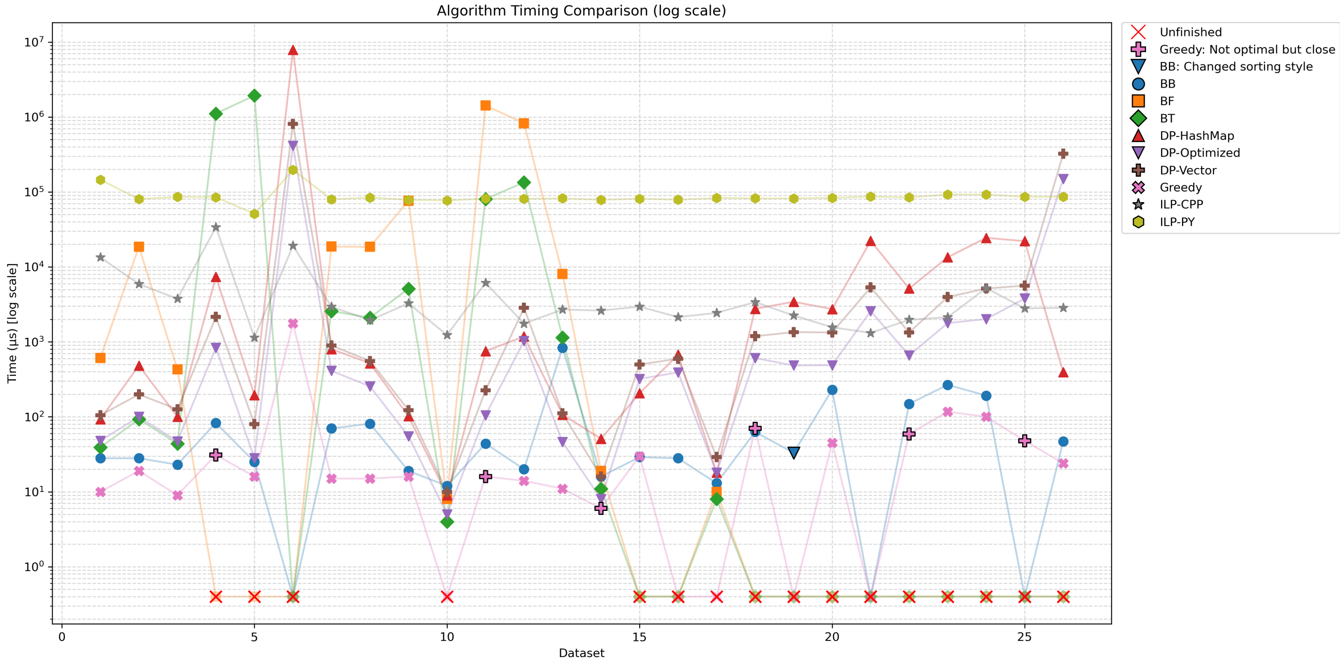


Figure: Timing comparison of all algorithms on all datasets. The y-axis is logarithmic, so each step represents a multiplication (e.g., 10, 100, 1000, ...). This allows both fast and slow algorithms to be compared on the same plot.

- **BF/BT:** Only for very small datasets. Exponential time.
- **BB:** Dominates in practice, solving almost all datasets in microseconds. Only fails to be fast on rare, specially crafted pathological cases (e.g., many similar ratios or ambiguous choices).
- **DP Vector:** Best for small/medium, dense problems. Memory grows with capacity.
- **DP HashMap:** Best for sparse, large-capacity problems. Saves memory, sometimes faster.
- **DP Optimized:** Minimal memory, but cannot reconstruct solution.
- **Greedy:** Fastest, and in practice often very close to optimal. Despite not having a constant-factor approximation guarantee, it frequently gives highly accurate results. However, it can also perform poorly—not only on classic traps (e.g., large high-value items with low ratio), but also on certain crafted or edge-case datasets where its heuristic fails to capture the optimal structure.
- **ILP:** Always optimal, robust for all sizes. C++ is fastest, Python is flexible.

Summary Table: Strengths & Weaknesses

Strategy	Strengths	Weaknesses
Brute Force	Always optimal	Exponential time, impractical
Backtracking	Prunes, faster than BF	Still exponential, times out
Branch & Bound	Fast, prunes aggressively	Can be trapped, not always meaningful
DP Vector	Fast, reconstructs solution	High memory for large W

Strategy	Strengths	Weaknesses
DP HashMap	Memory efficient for sparse	Slower for dense, hash overhead
DP Optimized	Minimal memory	No solution reconstruction
Greedy	Fastest, simple	Not always optimal
ILP	Always optimal, robust	Complex setup, can be overkill for small/medium problems

User Interface & Example of Use

- The application provides a clear command-line interface for interactive and batch use.
- **Dataset selection:**
  - Files should be named `Pallets_<X>.csv` and `TruckAndPallets_<X>.csv`.
  - Example prompt:

    ```
    Files should be in the format:
    Pallets_<X>.csv
    TruckAndPallets_<X>.csv

    <X> (empty line to exit):  01
    ```
- **Main menu:**
  - After dataset selection, the user sees:

    ```
    ----- PACKING OPTIMIZATION -----

    1: Run Algorithms
    2: Select Dataset
    3: Show Dataset
    4: Change Timeout

    Choose an option (empty line to exit): 1
    ```
- **Algorithm selection:**
  - User chooses from all implemented algorithms:

    ```
    1: BF
    2: BT
    3: BB
    ```

```
4: DP-VECTOR
5: DP-HASHMAP
6: DP-OPTIMIZED
7: GREEDY-APPROX
8: ILP-CPP
9: ILP-PY

Choose algorithm (empty line to exit): 1
```

- **Output:**
  - After running, a result file (e.g., `output/bf.txt`) is generated with the solution details, such as:

```
id, profit, weight
4, 8, 33
5, 9, 33
7, 7, 11
8, 2, 7
9, 3, 3

Total Weight: 87
Maximum Profit: 29

[BF] Execution time: 230 µs
```

- **Navigation:**
  - The user can travel through the batch menu, go back, change dataset, show dataset, set timeout limit, and more, making the interface flexible for experimentation and analysis.

Highlight: Custom Branch & Bound Pruning & Main Difficulties

Functionality to Highlight

- **Custom Branch & Bound (BB) Pruning:**
  - Developed a tailored pruning strategy in the BB algorithm that leverages upper bound estimation (fractional knapsack) and dynamic sorting (by value or ratio) to aggressively cut the search space.
  - In practice, this approach solves nearly all real-world datasets in microseconds, making BB the most robust and efficient method implemented.
  - The algorithm automatically retries with an alternative sorting strategy if a timeout is detected, further increasing its reliability.

Main Difficulties

- **Dataset Construction for Edge Cases:**

    - The greatest challenge was designing datasets that could actually make the custom BB pruning approach struggle or fail.
    - Most random or real datasets are solved instantly; only carefully crafted pathological cases (e.g., many items with similar profit/weight ratios or ambiguous optimal choices) can slow BB down.
    - This made empirical evaluation of worst-case performance and comparative analysis particularly challenging, but also highlighted the strength of the custom pruning logic.
- **Greedy Algorithm Limitations:**
    - It was surprisingly difficult to envision and create scenarios where the greedy approach would perform poorly, as it was not only close to optimal in many cases but actually produced the optimal solution in some datasets.
    - However, greedy is not a cheat code: it can still fail badly on certain crafted or subtle cases, and its lack of a constant-factor guarantee means it cannot be relied upon for all scenarios.

Conclusion

- No single strategy is best for all cases.
- Choice depends on dataset size, density, and need for reconstructing solutions.
- Hybrid approaches and careful selection of algorithms/data structures yield best practical results.
- The greedy approach, while often surprisingly close to optimal, and even optimal in some cases, can sometimes fail badly—not only on classic traps but also on less obvious, carefully constructed scenarios. This highlights the subtlety of greedy's limitations.