Exercises in Rosen Book (Discrete Maths), Exercises 1.1: Propositional Logic

Andreas Hansen

May 2025

Section 1: Logic and Proofs

Section 1.1: Propositional Logic

Exc 1.1: Which of these sentences are propositions? What are the truth values of those that are propositions?

(a) Boston is the capital of Massachusetts.

(b) Miami is the capital of Florida.

(c) $2 + 3 = 5$

(d) $5 + 7 = 10$

(e) $x + 2 = 11$

(f) Answer this question.

Solution:

Propositions must have clearly defined truth values, so a proposition must be a declarative sentence with no free variables:
a): This is a proposition that is true.
b): This is a proposition that is false.
c): This is a proposition that is true.
d): This is a proposition that is false.
e): This is not a proposition; its truth value depends on the value of x.
f): This is not a proposition; it's a command.

Exc 1.2: Which of these sentences are propositions? What are the truth values of those that are propositions?

(a) Do not pass go.

(b) What time is it?

(c) There are no black flies in Maine.

(d) $4 + x = 5$

(e) The moon is made of green cheese.

(f) $2^n \geq 100$

Solution:

Propositions must have clearly defined truth values, so a proposition must be a declarative sentence with no free variables:
a): This is not a proposition; it's a command.
b): This is not a proposition; it's a question.
c): This is a proposition that is false.
d): This is not a proposition; its truth value depends on the value of x.
e): This is a proposition that is false.
f): This is not a proposition; its truth value depends on the value of n.

Exc 1.3: What is the negation of each of these propositions?

(a) Linda is younger than Sanjay.

(b) Mei makes more money than Isabella.

(c) Moshe is taller than Monica.

(d) Abby is richer than Ricardo.

Solution:

a): Linda is not younger than Sanjay
b): Mei does not make more money than Isabella
c): Moshe is not taller than Monica
d): Abby is not rucher than Ricardo

Exc 1.4: What is the negation of each of these propositions?

(a) Janice has more Facebook friends than Juan.

(b) Quincy is smarter than Venkat.

(c) Zelda drives more miles to school than Paola.

(d) Briana sleeps longer than Gloria.

Solution:

a): Janice does not have more Facebook friends than Juan.
b): Quincy is not smarter than Venkat.
c): Zelda does not drive more miles to school than Paola.
d): Briana does not sleep longer than Gloria.

Exc 1.5: What is the negation of each of these propositions?

(a) Mei has an MP3 player.

(b) There is no pollution in New Jersey.

(c) $2 + 1 = 3$

(d) The summer in Maine is hot and sunny.

Solution:

a): Mei does not have a MP3 player.
b): There is pollution in New Jersey.
c): $2 + 1 \neq 3$
d): The summer in Maine is not hot or not sunny
This last one is a bit more tricky, but if we say: $P =$ The summer is hot and $Q =$ The summer is sunny
Then the original statement is a conjuction between P and Q:

$$P \wedge Q$$

And now we have to make a negation on a conjuction: "**The negation of a conjunction is logically equivalent to the disjunction of the negation of the statements making up the conjunction. To negate an "and" statement, negate each part and change the "and" to "or".** (DeMorgan's Laws)"

$$\neg (P \wedge Q) \equiv \neg P \vee \neg Q \quad , \quad \text{(Logic notation)}$$

Can also be written as:

$$\sim (P \wedge Q) \equiv \overline{P} \vee \overline{Q} \quad , \quad \text{(Mix of older notation and Boolean Algebra)}$$

or:

$$\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q} \quad , \quad \text{(Full Boolean Algebra)}$$

or many other combinations (e.g. in Programming they use ! for negations)
The verbal negation is then: *The summer in Maine is not hot or not sunny*

Exc 1.6: What is the negation of each of these propositions?

(a) Jennifer and Teja are friends.

(b) There are 13 items in a baker's dozen.

(c) Abby sent more than 100 text messages yesterday

(d) 121 is a perfect square.

Solution:

a): Jennifer and Teja are not friends.
b): There are not 13 items in a baker's dozen.
Alt: The number of items in a baker's dozen is not equal to 13.
c): Abby sent fewer than 101 text messages yesterday.
Alt: Abby sent at most 100 text messages yesterday.
d): 121 is not a perfect square.

Exc 1.7: What is the negation of each of these propositions?

(a) Steve has more than 100 GB free disk space on his laptop.

(b) Zach blocks e-mails and texts from Jennifer.

(c) $7 \cdot 11 \cdot 13 = 999$

(d) Diane rode her bicycle 100 miles on Sunday.

Solution:

a): Steve has at most 100 GB free disk space on this laptop.
Alt: Steve has 100 GB or less free disk space on his laptop.
Alt2: Steve does not have more than 100 GB free disk space on his laptop
b): Zach does not block emails or does not block texts from Jeniffer.
Alt: Zach doesn't block both emails and texts from Jeniffer (natural language)
Same as in **Exercise 1.5 d)**
P = Zach blocks emails from Jennifer and Q = Zach blocks texts from Jennifer. So original statement is a conjunction between P and Q written as $P \wedge Q$.
The negation of the conjuction is (via DeMorgan´s Laws);

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

So verbal negation becomes: *Zach does not block emails or does not block texts from Jeniffer*
c): $7 \cdot 11 \cdot 13 \neq 999$
d): Diane did not ride her bicycle 100 miles on Sunday.
Alt: Diane rode less than or more than 100 miles on Sunday

Exc 1.8: Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

(a) Smartphone B has the most RAM of these three smartphones.

(b) Smartphone C has more ROM or a higher resolution camera than Smartphone B

(c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

(d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera

(e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Solution:

a): True, because 288 MB > 256 MB (compared with Model A) and 288 MB > 128 MB (compared with Model C)
b): True, because 5 MP > 4 MP and only one of the conditions needs to be met because of the word *or*.
c): False, because Smartphone B's camera resolution (4 MP) is not higher than A's (8 MP). Since this is a conjunction ('and'), all conditions must be true—but the camera comparison fails.
d): False, because the hypothesis of this conditional statement is true and the conclusion is false.
e): False, because the first part of this biconditional statement is false and the second part is true.
This deals with a biconditinal statement, so we have: P = Smartphone A has more RAM than Smartphone B Q = Smartphone B has more RAM than Smartphone A, then we have the bicondition:

$$P \leftrightarrow Q \quad , \quad \text{(biconditional)}$$

Check first statement P: 256 MB $\not>$ 288 MB (compared with B), so A has NOT more ram and the first statement is false.
Check second statement Q: 288 MB > 256 MB (compared with A), so B has more RAM than A and the second statement is true.

Exc 1.9: Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

(a) Quixote Media had the largest annual revenue

(b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

(c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.

(d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

(e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

Solution: