The equations of active inference

Sanjeev Namjoshi and Fraser Patterson

ABOUT THIS BOOK

The active inference literature presents numerous equations used for modeling action and perception in continuous and discrete state-spaces. However, the proofs of these equations are either scattered in the literature (usually in supplemental material) or left as an exercise for the reader. The equations in the literature often use very different notational conventions making it difficult to make connections to other equations in the literature.

While preparing *Fundamentals of Active Inference*, the author (Sanjeev Namjoshi) derived many of these equations and consolidated those found in other papers and supplemental materials.[1] This book gathers all of these equations to provide a single compendium of proofs for every equation in the literature. This work was heavily edited and greatly improved by the work of Fraser Patterson who ensured the proofs were easy to follow and sound.

As this text is intended to serve as more of a reference than a textbook the reader is assumed to have basic familiarity with the active inference literature. Nevertheless, an overview of the basics is presented in the first chapter to orient the reader. The notational conventions are described in detail in the appendix.

---

[1]The vast majority of equations in *Fundamentals of Active Inference* are available in this text. The remaining proofs, largely for equations that are related to more conventional statistics or machine learning concepts, are presented in full on the author's website: https://www.sanjeevnamjoshi.com/faif/proofs.

CONTENTS

CHAPTER 1

FUNDAMENTALS

Nearly every equation in the active inference literature may be obtained by following the steps:

1. Define the probability distributions and factorizations of the generative model.

2. Use generative model factors to determine probabilistic form of variational free energy.

3. Replace probabilistic form of variational free energy with equations for corresponding probability distributions and simplify the equation.

4. Take the partial derivative of variational free energy with respect to a variable of interest.

5. Set this partial derivative equal to zero and solve for the variable of interest.

In this chapter we introduce the basic concepts at play in active inference and give a more detailed explanation of how these steps are applied.

1 Generative models

2 Variational free energy

3 Obtaining the update equations

BIBLIOGRAPHY

Black et al.: The Pricing of Options and Corporate Liabilities   **BlackScholes**

Fischer Black and Myron Scholes. "The Pricing of Options and Corporate Liabilities". In: *Journal of Political Economy* 81.3 (1973), pp. 637–654.

Merton: Theory of Rational Option Pricing   **Merton**

Robert Merton. "Theory of Rational Option Pricing". In: *The Bell Journal of Economics and Management Science* 4.1 (1973), pp. 141–183.

Hull: Options, Futures, and Other Derivatives   **Hull**

John Hull. *Options, Futures, and Other Derivatives.* 9th ed. Pearson, 2017.