

Supporting Data – Equations for SD representations

We can cast the dynamics of the system as a system of differential equations for the changes in distribution of biomass amongst the populations. These equations are solved at each step by a fourth order Runge-Kutta scheme that calculates the biomass distribution for each species.

So we denote our types by

P – plants, from germination onwards,

F – fruit, which are available for consumption,

S – viable seeds, which have passed through a herbivore,

H – herbivores, which eat the plants and fruit, and

C – carnivores, which eat the juvenile herbivores,

and we take the notation $N_A(t)$ to denote the total biomass of things of type A at the time t , $N_A(t, x)$ indicates the total biomass of the members of type A which have a size x , and $\hat{N}_A(t, x)$ denotes the proportion of the population that has a size x at t . Thus we have

$$N_A(t) = \int_0^\infty N_A(t, x) dx$$

and

$$\hat{N}_A(t, x) = \frac{N_A(t, x)}{N_A(t)}.$$

Clearly for positive values of $N_A(t)$, $\hat{N}_A(t, x) \in [0, 1]$, for non-positive values of $N_A(t)$ we will take $\hat{N}_A(t, 0) = 1$.

So for plants, we also make the following assumptions:

- Plants produce fruit continuously (without seasonal variation), and their mortality is uniformly distributed with respect to size and distributed evenly through the year,
- plants grow slowly enough that it is possible for all the adult herbivores to die before a plant fruits — this means that a near complete collapse of the plant population can trigger a collapse in the herbivore population
- fruit has a probability of spoiling, and the associated seeds will spoil
- seeds that have passed through the juveniles will germinate

The change in plant biomass for a given size due solely to growth is

$$dN_P(t)|_{\text{growth}}(t, x) = \left(1 - \frac{N_P(t)}{K_P}\right) \int_0^\infty \Gamma_P(x, y) N_P(t, y) dy \quad (1)$$

where $\Gamma_P(a, b)$ is the probability that a plant of size b will grow to size a . The germination of seeds contributes

$$dN_P|_{\text{germination}}(t, x) = \left(1 - \frac{N_P(t)}{K_P}\right) N_S(t) \Gamma_S(x) \quad (2)$$

where $\Gamma_S(x)$ is the probability of a seed growing to size x . We will also make germination success dependent on the density of plants relative to the carrying capacity, K_P — for the sake of the argument, germination might only occur in full sunlight.

The change in biomass in plants due to predation by herbivores is

$$dN_P|_{\text{predation}}(t, x) = - \int_0^\infty N_P(t, x) N_H(t, y) E_{HP}(y, x) dy \quad (3)$$

where $E_{HP}(y, x)$ is the probability that a plant of size x will be eaten by a herbivore of size y .

The mortality is given by

$$dN_P|_{\text{mortality}}(t, x) = -\Omega_P N_P(t, x) \quad (4)$$

Thus, the collective change in biomass can be written

$$\begin{aligned} dN_P(t, x) = & \left(1 - \frac{N_P(t)}{K_P}\right) \left[\int_0^\infty N_S(t) \Gamma_S(y) dy + \int_0^\infty \Gamma_P(x, y) N_P(t, y) dy \right] \\ & - N_P(t, x) \left[\Omega_P + \int_0^\infty N_H(t, y) E_{HP}(x, y) dy \right]. \end{aligned}$$

We'll take the production of fruit to be related to the surface area of a minimal bounding volume of a plant, so we include a quadratic term in x with the assumption that volume is proportional to mass, so

$$dN_F|_{\text{production}}(t) = \int_0^\infty \rho_P f_P x^{\frac{2}{3}} N_P(t, x) dx, \quad (5)$$

where $f_P x^{\frac{2}{3}}$ is the (“instantaneous”) likelihood that a plant with a mass of x will fruit and we assume that ρ_P , the mean size of fruit, to be similar across all sizes of plant. This is clearly a little silly, since newly germinated plants are unlikely to produce fruit that may be larger than they are, but in a spirit of blindly hacking through the undergrowth, we'll forge ahead.

A decrease in the population of fruit may be the result of “spoilage” and by the predation of the (juvenile) herbivores, so

$$dN_F|_{\text{consumption}}(t) = -N_F(t) \left(\kappa_F + \int_0^\infty N_H(t, y) E_{HF}(y) dy \right) \quad (6)$$

where $\kappa_F N_F$ is a “spoilage” term that is constant across the population. Putting the equations together, we get

$$dN_F(t) = \int_0^\infty \rho_P f_P x^{\frac{2}{3}} N_P(t, x) dx - N_F(t) \left(\kappa_F + \int_0^\infty N_H(t, y) E_{HF}(y) dy \right) \quad (7)$$

Seeds are fairly simple,

$$dN_S(t) = N_F(t) \int_0^\infty N_H(t, y) E_{HF}(y) dy - \kappa_S - \left(1 - \frac{N_P(t)}{K_P}\right) N_S \int_0^\infty (t) \Gamma_S(x) dx \quad (8)$$

The any change in the biomass of herbivores is determined by birth, growth, natural mortality, starvation and the effect of predation; the only real difference between adults and juveniles is their size and the functions E_{HF} and E_{HP} . Thus we get the following component equations:

$$dN_H|_{birth}(t, x) = N_H(t, x) f_H(x) \quad (9)$$

$$dN_H|_{growth}(t, x) = N_H(t, x) \left(\Gamma_H(x) \int_0^\infty N_P(t, w) E_{HP}(x, w) dw + \min\left(0, \frac{N_F - \omega_{HF}(x) N_H}{N_F}\right) + \Gamma_H(x) N_F(t) E_{HF}(x) - M_H \right) \quad (10)$$

$$dN_H|_{mortality}(t, x) = -N_H(t, x) \left(\Omega_H(x) + \int_0^\infty N_C(y) E_{CH}(y, x) dy \right) \quad (11)$$

where $\Omega_H(x)$ is the probability of natural mortality for a herbivore, and f_H is the expected reproductive contribution from a herbivore with a size of x , $\Gamma_H(x)$ is the growth coefficient for herbivores of size x and M_H is a maintenance rate per unit of biomass. Combining these components, we get the equation

$$dN_H(t, x) = N_H(t, x) \left[f_H(x) + \Gamma_H(x) \int_0^\infty N_P(t, w) E_{HP}(x, w) dw + \min\left(0, \frac{N_F - \omega_{HF}(x) N_H}{N_F}\right) + \Gamma_H(x) N_F(t) E_{HF}(x) - \left(M_H + \int_0^\infty N_C(y) E_{CH}(y, x) dy \right) \right] \quad (12)$$

The population of carnivores is largely similar to that of herbivores with the exception that they prey on the herbivores and do not suffer from predation themselves. growth, birth, and their ability to catch juvenile herbivores.

$$dN_C(t, x) = N_C(t, x) \left[(f_C(x) + \Gamma_C(x) \int_0^\infty N_H(t, w) E_{CH}(x, w) dw + \min\left(0, \int_0^\infty \frac{N_H(y) - \omega_{CH}(x) N_H(y)}{N_H(y)} dy\right) - (M_C + \Omega_C(x))) \right] \quad (13)$$

where $\omega_{AB}(x) \geq 1$ indicates how much an animal with a mass, x , needs to eat relative to the mass of the prey¹. The predation functions ω_{HP} , ω_{HF} , E_{CH} , E_{HP} and E_{HF} are constructed so that the preying species observes the conditions on the system: $E_{CH}(x, y)$, for example, is constructed so that for small x the carnivores do not prey on herbivores of any size, and for larger x they prey only upon herbivores with small values of y . Similarly, $E_{HF}(x)$ is only non-zero for small values of x , and E_{HP} is non-zero where E_{HF} is zero. The implication of this is that $\int_0^\infty \omega_{HP}(x) \omega_{HF}(x) dx = 0$ and $\int_0^\infty E_{HP}(x) E_{HF}(x) dx = 0$.

¹This relationship is likely to only hold in very specific circumstances.