Stimulus Identification from fMRI scans

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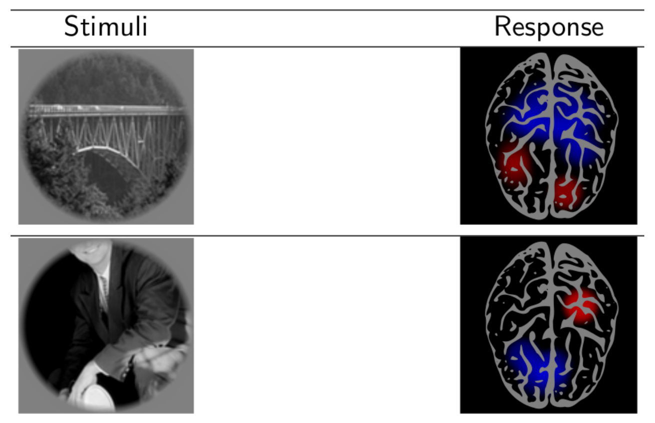
Setting

- Sequence of stimuli (pictures) shown at time $t = 1, \dots, T$
- Record subject's multivariate response $Y_t \in \mathbb{R}^p$
- Stimuli represented as *feature vector* $X_t \in \mathbb{R}^q$
- Linear model:

$$Y_{T \times p} = X_{T \times q} B_{q \times p} + E_{T \times p}$$

- E.g. Kay (2008)

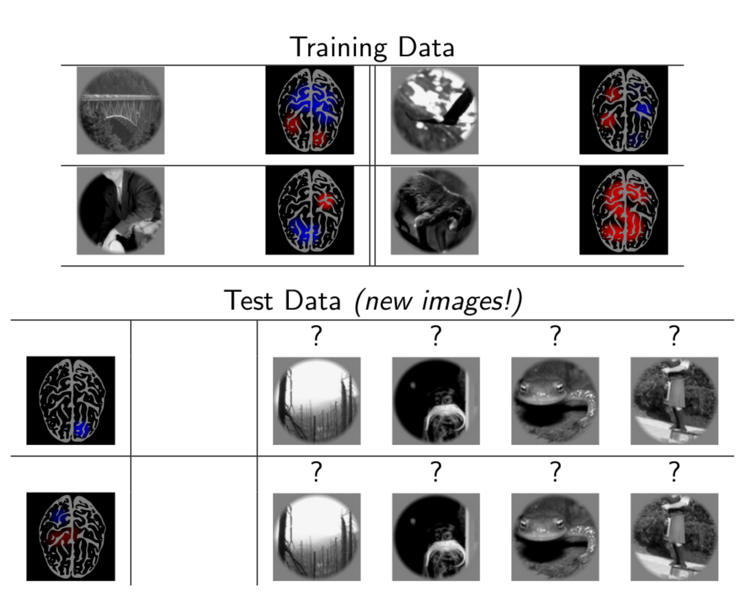


Identification

- Introduced in Kay (2008)
- Supervised learning task, validates the power of the linear model $Y = XB + E$
- Let S be a set of *new* stimuli (not in the training set) with features

$$\{x_1^{te}, \dots, x_\ell^{te}\}$$

- Scientist picks a stimulus i^* from S and measures the subject's reponse y^*
- Can the statistician *identify* the stimulus from y^*?



Objectives

- Develop new methodology for optimal identification

Key theoretical problems:

- Optimal identification given parameter estimates
- Estimation of model (e.g. linear model $Y = XB$)
- Estimation of noise $\Sigma_E = \text{Cov}(E)$

Optimality criteria:

- Bayes (average case) under parametric models
- Minimax regret under nonparametric models

We consider two methods: *maximum likelihood* and *empirical Bayes*, and compare to *Bayes risk*.

Maximum Likelihood

Procedure for identification of y^*, variants used in Kay (2008), Vu (2011)

- Obtain point estimates of coefficients B and noise covariance Σ_E
- E.g. B estimated using elastic net with CV (Zou 2005), shrinkage estimate for covariance

$$\hat{\Sigma}_E = \frac{1}{2}\hat{\text{Cov}}(Y - \hat{Y}) + \frac{1}{2}\text{diag}(\hat{\text{Cov}}(Y - \hat{Y}))$$

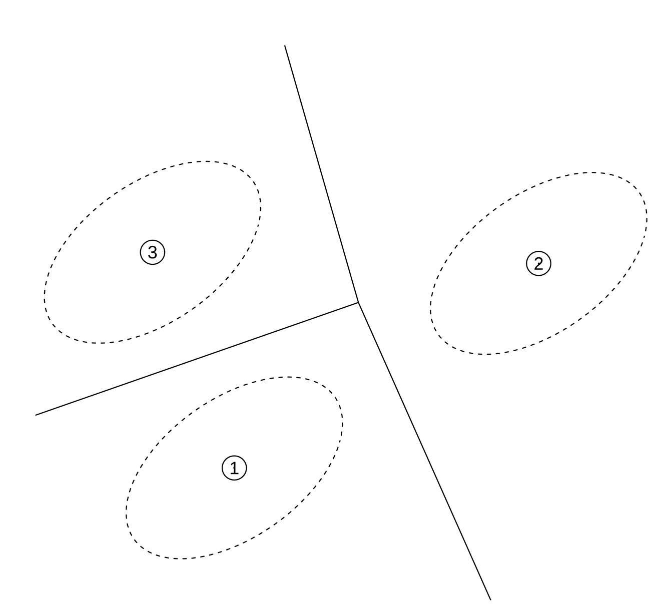
  where $\hat{\text{Cov}}$ denotes sample covariance
- Obtain predicted means for test stimuli

$$\hat{\mu}_i^{te} = (x_i^{te})^T B$$

- Identify the stimulus i^* by

$$i^* = \text{argmin}_i (\hat{\mu}_i^{te} - y^*)^T \hat{\Sigma}_E^{-1} (\hat{\mu}_i^{te} - y^*)$$

- Results in *linear decision boundaries*



What is Empirical Bayes?

- General approach for statistical problems (e.g. Efron 2004)
- Start with a hierarchical Bayes model with hyperparameters
- *Estimate* hyperparameters from data, e.g. maximizing marginal likelihood

Empirical Bayes

Model

- Noise $E_t \sim N(0, \Sigma_E)$ iid
- Coefficients $B_i \sim N(0, \sigma_i^2 I)$ for $i = 1, \dots, p$
- X non-random

Estimate hyperparameters

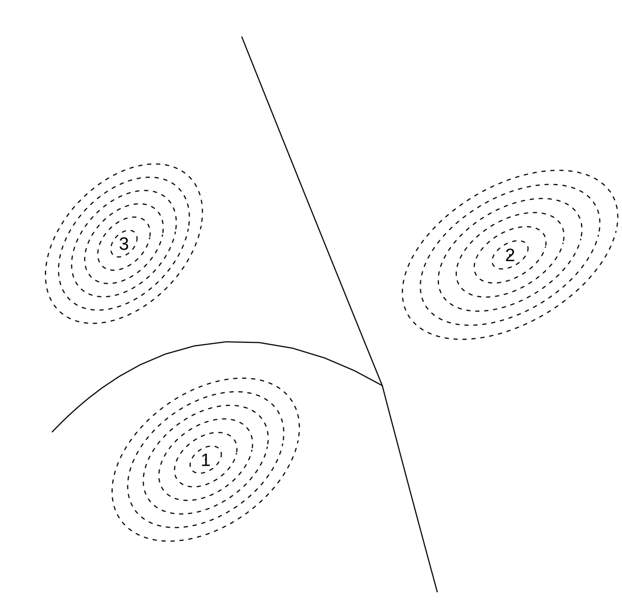
- Use *eigenprism* (Janson 2015) to estimate $\theta_i^2 = ||B_i||^2$ for $i = 1, \dots, p$
- Set $\sigma_i^2 = \hat{\theta}_i^2 / q$
- Estimate $\hat{B}$ as posterior mean given estimated σ_i^2
- Estimate Σ_E using residuals (same as in Maximum Likelihood)

Compute posterior

- Closed-form expressions for posteriors of B, μ_i^{te}
- Computational bottleneck: inverting the $pq \times pq$ covariance matrix of $\vec{B}$

Apply Bayes rule

- *Uncertainty* in B is reflected as *added noise*
- Result: posterior $\text{Cov}(y^* | i^*)$ varies, hence *quadratic boundaries*



Simulation Results

Ongoing Work

- Estimate noise covariance based on correlation structure of fMRI data (e.g. spatial correlation)
- Apply methods to data of Kay (2008)

References

Acknowledgements

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