How many neurons does it take to classify a lightbulb?

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(Joint work with Yuval Benjamini)

Overview

Background and motivation

- Information theory and network information theory
- Entropy, conditional entropy, and mutual information
- Studying the neural code
- Functional fMRI study of face recognition

Questions

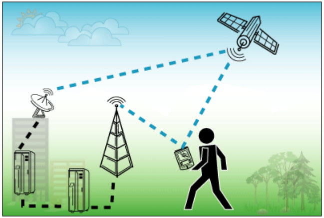
- Can random stimuli samples be used to estimate mutual information?
- Can we obtain mutual information from the Bayes error?
- Can we obtain mutual information from observed classification rates?

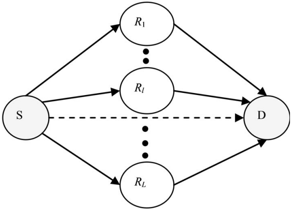
Methods

- Using Fano's inequality
- Using low-SNR universality

Results

The complexity of modern communications system is made possible by Shannon's theory of information.





A information-processing network can be analyzed in terms of interactions between its components (which are viewed as random variables.) Image credit

CartouCHe, Aziz et al. 2011

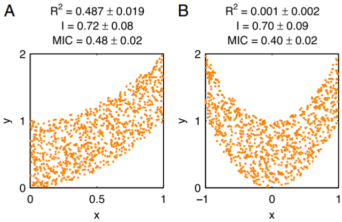
Entropy and mutual information

Let X and Y be real-valued random variables with a joint density $p(x, y)$ with respect to a measure μ, and marginals $p_X(x)$ wrt μ_X and $p_Y(y)$ wrt μ_Y. The entropy, conditional entropy, and mutual information are nonlinear measures of spread, conditional spread, and dependence.

Quantity	Definition	Linear analogue
Entropy	$H(X) = -\int \log p(x) p(x) \mu_X(dx)$	$\mathrm{Var}(X)$
Conditional entropy	$H(X\|Y) = \mathbf{E}[H(X\|Y)]$	$\mathbf{E}[\mathrm{Var}(X\|Y)]$
Mutual information	$I(X; Y) = H(X) - H(X\|Y)$	$\mathrm{Cor}(X, Y)$

The above definition includes both *differential* entropy and *discrete* entropy. Information theorists tend to use log base 2, we will use natural logs in this talk.

Properties of mutual information



- Nonnegative: $I(X;Y) \geq 0$.
- Symmetric: $I(X;Y) = I(Y;X)$
- Bijection-invariant: $I(\phi(X);\psi(Y)) = I(\psi(Y);\phi(X))$.
- Additivity. If $(X_1, Y_1) \perp (X_2, Y_2)$, then

$$I((X_1, X_2);(Y_1, Y2)) = I(X_1;Y_1) + I(X_2;Y_2)$$

- Relation to KL divergence $\mathbb{D}$.

$$\mathbb{D}(p(x,y)||p(x)p(y)) = I(X;Y)$$

Image credit Kinney et al. 2014

References

- Cover and Thomas. Elements of information theory.
- Muirhead. Aspects of multivariate statistical theory.
- van der Vaart. Asymptotic statistics.