Parametric RSA applied to Mixed-Gambles Task

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April 8, 2016

1 Introduction

Representational similarity analysis (Kriegeskorte et al.) is a distance-based approach for comparing how different subjects or different parts of the brain differ in perceiving a given fMRI task. Let $i = 1, \ldots, k$ index a collection of tasks, and let $r = 1, \ldots, n$ index a set of *representators* to be compared. A representator could be:

- The whole brain of a subject.
- A region of a subject's brain.
- A model-based prediction of the activity of a brain or region of interest.
- The subject's behaviorial response to the task (e.g. responses to survey questions)
- A manual labelling of the task into categories.

Each representor produces a representation of each stimulus, denoted $y^{(i,r)}$. The representation could be categorical, scalar-valued or multidimensional, or even non-numerical, e.g. text descriptions. Furthermore, the different representors could produce representations of different types. What is essential to the approach is a way to compute distances between representations coming from the same representor, i.e. a $k \times k$ distance matrix $d^{(r)}(i, j)$ of pairwise distances between stimuli, for $r = 1, \ldots, n$. Representational similarity analysis works by comparing these n distance matrices in order to draw conclusions about the nature of the representors.

Suppose that the representators all produce real, vector-valued representations, of possibly differing dimensionality v_r. Furthermore, suppose we choose the squared Euclidean distance

$$d^{(r)}(i,j) = \frac{1}{v_r} ||y_{(r,i)} - y_{(r,j)}||^2.$$

Here we divide by v_r so that in the commonly encountered case where the representation is obtained by sampling from a continuous function, the resulting distance is robust to fineness of discretization.

In many experimental designs, we have a parameterization of the stimuli, given by a vector x_i.

2 Data

We used the mixed-gambles data from Tom et al. 2007. The data consisted of 3 runs for each of 16 subjects. In each run, 16 different gambling tasks were presented. These tasks varied in the gain amount and loss amount.

Clusters were obtained using a parametric map from `http://neurovault.org/images/10680/`, then applying thresholding using FSL. A size threshold was applied, yielding 28 regions of interest.

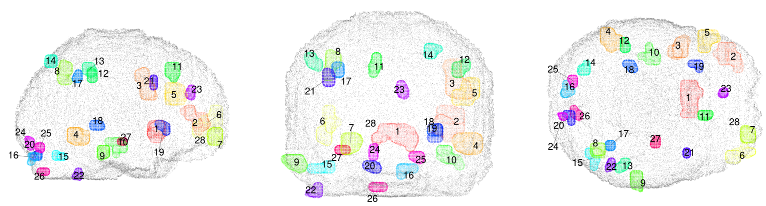

Figure 1: Regions of interest.

The scans were registered to a common template, then a standard linear model-based approach was used to extract an activity level per voxel per event per run. We extracted the regions of interest from the data. For a given region of interest, the data takes the form:

subject	run	gain	loss	voxel 1	voxel 2	...	voxel N
1	1	13	6.5	-222.8994	-373.85025	...	12.038
...	...	...	...	...	...	...	...
16	3	37	18.5	-136.89	-73.49	...	75.068146

where N is the number of voxels in the ROI.

3 Methods

We model the data using the following linear model. Let $s = 1, \ldots, 16$ index the subjects, and $e = 1, \ldots, 48$ index the events from all runs combined. Let $r = 1, \ldots, 28$ index the regions of interest, and $m = 1, \ldots, v_r$ the voxels in the ROI. Then the model for the activity of the mth voxel in the rth roi is

$$y_{s,e,r,m} = \langle \vec{x}_e, \beta_{s,r,m} \rangle + \mu_{s,r,m} + \epsilon_{s,e,r,m}$$

where $y_{s,e,r,m}$ is the activity of the mth voxel for the sth subject in the eth event (a scalar quantity), $\vec{x}_e$ is the 2x1 vector of parameters (gain and loss) for the eth event , and $\beta_{s,r,m}$ is a vector of coefficients specific to the subject and to the voxel. The coefficient vector $\beta_{s,r,m}$ is viewed as a *random effect* such to variation between subjects. $\mu_{s,r,m}$ is a subject-specific mean term for the voxel. Meanwhile, assume that the noise term $\epsilon_{s,e,r,m}$ is normal $N(0, \sigma^2_{s,r,m})$ with zero mean, and covariance specific to the subject and voxel. Assume that the noise terms for are correlated *within an event* but are independent *across events*. This assumption may not be realistic, since there is usually some autocorrelation across events.

$$\text{if } e \neq e' \text{ or } s \neq s', \text{ then } \mathrm{Cov}(e_{s,e,r,m}, e_{s',e',r,m}) = 0.$$

$$\mathrm{Cov}(e_{s,e,r,m}, e_{s,e,r',m'}) = \sigma_{r,m,r',m'}.$$

Write Σ_r for the $n_v \times n_v$ covariance matrix for voxels within a region of interest.

The parametric RSA approach is to define for each subject and ROI the feature-distance matrix $M^{(s,r)}$ with entries

$$M^{(s,r)}_{ij} = \frac{1}{v_r} \langle \beta_{s,r,i}, \beta_{s,r,j} \rangle.$$

The randomness of $\beta_{s,r,m}$ induces a random distribution for $M^{(s,r)}$ for a given randomly sampled subject. Let $F^{(r)}$ denote the distribution of the 2×2 matrix $M^{(s,r)}$.

We are interested in comparing feature-distance matrices across regions of interest. Suppose we wish to compare region r with region r'. The null hypothesis is that

$$H_0 : \mathbf{E}[M^{(s,r)}] = \mathbf{E}[M^{(s,r')}]$$

where the expectation is taken across the distribution of subjects.