Estimating HRF and covariance structure

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January 19, 2015

Estimating HRF and amplitudes

- Use one block at a time
- Code the twelve stimuli as 1-12, the "null" signal as 13, and the 6 "calibration" signals as 14-19
- Transform stimuli assignments to a matrix S, dimension of S is $T \times K$, T is the time of time points, K the number of stimuli types

Estimating HRF and amplitudes

Transform estimated or assumed HRF $h = (h_1, ..., h_L)$ to matrix H(h). Dimension of H(h) is $T \times T$

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ \vdots \end{bmatrix} \rightarrow H(h) = \begin{bmatrix} h_1 & 0 & 0 & \cdots \\ h_2 & h_1 & 0 & \cdots \\ h_3 & h_2 & h_1 & \cdots \\ h_4 & h_3 & h_2 & \cdots \\ h_5 & h_4 & h_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Estimating amplitudes

Estimate the stimuli-specific amplitudes $\alpha = (\alpha_1, \dots, \alpha_K)$ by fitting the model

Estimating HRF

Suppose instead that α is known and h is unknown. Then let $t = S\alpha = (t_1, t_2, ...)$ and define

$$A(S\alpha) = \begin{bmatrix} t_1 & 0 & 0 & \cdots \\ t_2 & t_1 & 0 & \cdots \\ t_3 & t_2 & t_1 & \cdots \\ t_4 & t_3 & t_2 & \cdots \\ t_5 & t_4 & t_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Dimension of $A(S\alpha)$ is $T \times L$, where L is duration of HRF. Then fit

$$y \sim A(S\alpha)h + \text{const}$$

Fit h and α in alternating fashion until convergence.



Regularization

Define a penalty function P(h) by

$$P(h) = (h_1 - h_2)^2 + (h_2 - h_3)^2 + \cdots + (h_{L-1} - h_L)^2$$

Now, choosing $\lambda_h > 0$, fit

$$h = \operatorname{argmin} ||y - A(S\alpha)h + c||^2 + \lambda_h P(h)$$

Similarly, fit

$$\alpha = \operatorname{argmin} ||y - H(h)S\alpha + c||^2 + \lambda_{\alpha} ||\alpha||^2$$

Again, alternate until convergence.

Simulation

Compared fits with/without regularization.





Without regularization:

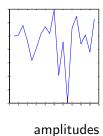


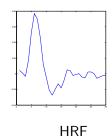


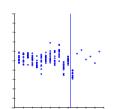
• With regularization:

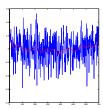
Left: amplitudes, right: HRF, blue: truth, red: estimate

Data: Block3









Covariance estimation

- Assume Gaussian process + noise model.
- That is, if r_j are residuals, for j = 1, ..., T,

$$r_j = \epsilon_j + \sum_{k=1}^K Z_k \cos(2\pi j/T) + \sum_{k=1}^K W_k \sin(2\pi j/T)$$

where $\epsilon_j \sim N(0, \sigma_0^2)$, Z_k , $W_k \sim N(0, \sigma_k^2)$ (all independent).

• Use the estimating equations

$$\mathbb{E}\left[\left(\frac{\sum r_j \cos(2\pi j/T)}{\sum \cos(2\pi j/T)^2}\right)^2\right] = \sigma_k^2 + \sigma_0^2$$

(replace cos by sin in the above.)

