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Teaching Power System Analysis Courses using MATPOWER

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Abstract— **One of the major constraints faced in the teaching of power system analysis courses has been non-availability of practical power system for demonstration. This paper focuses on a new and efficient method to the teaching of power system analysis courses to the upper-division undergraduate students. The intent is to present MATPOWER to students as a good instructional tool to complement the teaching of power system analysis courses in the class. This would certainly facilitate the teaching and understanding of the subject matter better. The features and the relative merits which make the package preferred to some of the commercially available software packages, in certain applications, are highlighted in the paper. With self study in mind, the paper is written to simplify the daunting task of carrying out power flow analyses especially on large power networks. One illustrative example is examined in the paper. The power flow solution by the Newton-Raphson method is demonstrated using the conventional approach prior to solving it using the MATPOWER package. The result obtained using the package is guaranteed to be accurate and reasonably fast.**

Keywords— MATLAB, MATPOWER, Jacobian matrix, power flow, power system analysis course, teaching

I. INTRODUCTION

One of the major constraints faced in the teaching of power system analysis courses especially at undergraduate level is the non-availability of real-life power system for demonstration in the laboratory. This makes certain aspects of the course uninteresting to the students since it is full of complex mathematics that may be extremely laborious, error-prone, and time-consuming to solve manually. However, with the advent of personal computers, the story is different today. These computers have become so powerful and advanced that they can easily be employed to carry out some simulations in the analysis of large interconnected power networks. The ability of these modern computers to provide useful information and react to responses has been responsible for its integration into power engineering curriculum [1].

By and large, a number of commercial software packages have been developed for solving power system analysis problems. For example, PSS/E, ETAP, PowerWorld, ERACS, MATLAB/Simulink, PSCAD, etc, are all graphical user interface enabled and user-friendly software for carrying out time-domain computer simulations of power systems. However, these packages require good modelling and simulation knowledge, and may also be difficult to use for complex and large power systems. In addition, none of these packages allows the user to add new algorithms to it or even change the source code [2]. Other simulation tools that are used for power system analysis include UWPFLOW [3], Power System Toolbox (PST) [4], Power System Analysis Toolbox (PSAT) [5], MATPOWER [6], Voltage Stability Toolbox (VST) [7], to mention but a few. Even though these software use MATLAB as the platform on which they run, one good thing about them is that they are all free open source packages and their codes could be modified [2]. This is particularly important for researchers and students who are interested in developing and testing new, unconventional algorithms [8]. For a list of these packages and applications of each, interested reader could consult [5, 9].

Apart from these, Saadat [1] has done a marvellous job in facilitating the understanding of power system analysis course to the students. He developed a number of M-files functions that could be run in MATLAB environment to ease the task of computer simulations for the students without having to do detailed programming. In the book, several examples are illustrated for a better understanding. However, MATPOWER is more flexible and capable of computing and displaying more detailed results, especially for contingency studies. Other advantages of MATPOWER include simplicity and robustness. It can also be used to verify analytical methods developed by students to validate their mathematical models quickly.

Since power flow analysis is a basic tool of many power system courses [10], the example examined in the literature is limited to power flow studies. This paper assumes that the targeted audience is already accustomed to MATLAB; else a very good introduction to MATLAB from electrical engineering point of view is available in [11]. The rest part of this literature is structured as follows: Section II is dedicated to the definition as well as the features of MATPOWER package. An illustrative example of a 5-bus power system is treated using both the conventional approach adopting Newton-Raphson method and the MATPOWER package in Section III. The power flow results are presented and discussed in Section IV, while Section V concludes the paper.

II. RUNNING MATPOWER

MATPOWER is an open source simulation tool for solving power flow and optimal power flow problems in power system analysis. The package, whose code is written in MATLAB M-files, is specifically targeted at researchers and educators in the field of power engineering [12]. MATPOWER is designed to give the best performance while keeping the code simple and easy to modify by the user. It could be downloaded free of charge from [13].

To install MATPOWER on a personal computer, MATLAB version 6 or a later version with Optimisation Toolbox is needed [12]. The downloaded file requires to be unzipped and placed in a location on MATLAB path. For example, an installation of 3.2 version of MATPOWER is demonstrated as follows: **C:\ProgramFiles\MATLAB\R2006a\matpower3.2**. To run a programme in MATPOWER, start up MATLAB and type the desired M-file name at the command window prompt, then press the enter key. For example, typing:

» runpf ('case9')

and thereafter press the enter key at the command window prompt of MATLAB would run a power flow of 9-bus power system already pre-stored in MATPOWER. This file could then be saved in another name and modified to the taste of the user as appropriate.

To carry out a load flow analysis, MATPOWER has five solvers to do this. These solvers are Newton, XB fast-decoupled, BX fast-decoupled, Gauss-Seidel, and DC power flow; which can be accessed via the runpf function. The power flow default solver uses the Newton's method. To use any of the rest solvers, it is necessary that the PF_ALG option must be set explicitly. Interested reader could consult [12] for detail explanation of each of these solvers and their settings.

III. POWER FLOW CASE STUDY

A. The Problem

An example from [1], (example 7.9 on page 295) solved using MATLAB is chosen to demonstrate in this study. Figure 1 shows the one-line diagram of a simple 5-bus power system with generator at buses 1, 2 and 3. Bus 1, with its voltage set at $1.06\angle 0^o$ pu, is taken as the slack bus. Voltage magnitude and real power generation at buses 2 and 3 are 1.045 pu, 40 MW, and 1.030 pu, 30 MW, respectively. The load MW and MVar values are shown on the diagram. One-half of the line capacitive susceptance and line impedances are presented in per unit on a 100-MVA base in Table 1. It is required to obtain the power flow solution for the system.

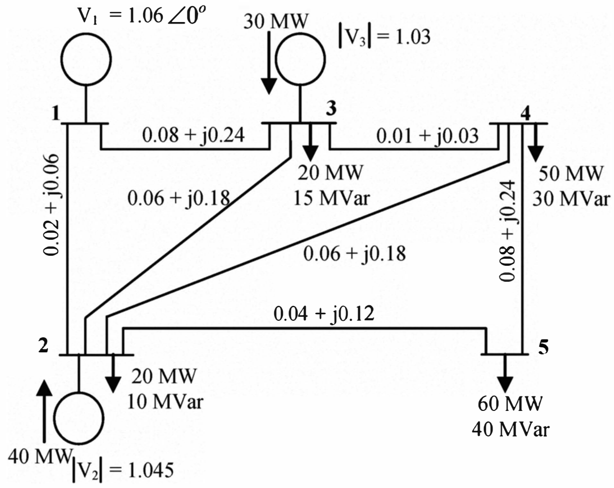

Fig. 1 One-line Diagram of a 5-bus System

TABLE 1 CAPACITIVE SUSCEPTANCE OF THE SYSTEM

Line	½ B
1 – 2	0.030
1 – 3	0.025
2 – 3	0.020
2 – 4	0.020
2 – 5	0.015
3 – 4	0.010
4 – 5	0.025

B. The Manual Solution

Solving the problem manually from the first principles is necessary to enable the students grasp the fundamental concept of power flow studies. The steps involved are as follow.

The line impedances and their equivalent admittances are presented:

$$Z_{12} = 0.02 + j0.06 \Rightarrow y_{12} = 5 - j15$$

$$Z_{13} = 0.08 + j0.24 \Rightarrow y_{13} = 1.25 - j3.75$$

$$Z_{23} = 0.06 + j0.18 \Rightarrow y_{23} = 1.67 - j5.0$$

$$Z_{24} = 0.06 + j0.18 \Rightarrow y_{24} = 1.67 - j5.0$$

$$Z_{25} = 0.04 + j0.12 \Rightarrow y_{25} = 2.5 - j7.5$$

$$Z_{34} = 0.01 + j0.03 \Rightarrow y_{34} = 10 - j30$$

$$Z_{45} = 0.08 + j0.24 \Rightarrow y_{45} = 1.25 - j3.75$$

Therefore the bus admittance matrix is formed as:

$$Y_{bus} = \begin{bmatrix} 6.25 - j18.75 & -5 + j15 & -1.25 + j3.75 & 0 & 0 \\ -5 + j15 & 10.84 - j32.5 & -1.67 + j5 & -1.67 + j5 & -2.5 + j7.5 \\ -1.25 + j3.75 & -1.67 + j5 & 12.92 - j38.75 & -10 + j30 & 0 \\ 0 & -1.67 + j5 & -10 + j30 & 12.92 - j38.75 & -1.25 + j3.75 \\ 0 & -2.5 + j7.5 & 0 & -1.25 + j3.75 & 3.75 - j11.25 \end{bmatrix}$$

Where $Y_{ii} = \sum_{j=1}^{n} y_{ij}$ $\quad i \neq j$ and n is number of bus.

Converting the bus admittance matrix to polar form with the angles in radians yields:

$$Y_{bus} = \begin{bmatrix} 19.76\angle-1.25 & 15.81\angle1.89 & 3.95\angle1.89 & 0 & 0 \\ 15.81\angle1.89 & 34.26\angle-1.25 & 5.27\angle1.89 & 5.27\angle1.89 & 7.91\angle1.89 \\ 3.95\angle1.89 & 5.27\angle1.89 & 40.85\angle-1.25 & 31.62\angle1.89 & 0 \\ 0 & 5.27\angle1.89 & 31.62\angle1.89 & 40.85\angle-1.25 & 3.95\angle1.89 \\ 0 & 7.91\angle1.89 & 0 & 3.95\angle1.89 & 11.86\angle-1.25 \end{bmatrix}$$

Generally, the complex power at bus i of any system is

$$P_i - jQ_i = |V_i|\angle-\delta_i \sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j \tag{1}$$

Separating the real and imaginary parts of eqn (1) yields

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{2}$$

and

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \tag{3}$$

Based on these two general equations, the expressions for the known real power at buses 2, 3, 4 and 5, and that of the reactive power at buses 4 and 5 in our power flow problem are respectively given below:

$$P_2 = 0.2 + |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2|Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) + |V_2||V_4||Y_{24}|\cos(\theta_{24} - \delta_2 + \delta_4) + |V_2||V_5||Y_{25}|\cos(\theta_{25} - \delta_2 + \delta_5) \tag{4}$$

$$P_3 = 0.2 + |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2|Y_{33}|\cos\theta_{33} + |V_3||V_4||Y_{34}|\cos(\theta_{34} - \delta_3 + \delta_4) + |V_3||V_5||Y_{35}|\cos(\theta_{35} - \delta_3 + \delta_5) \tag{5}$$

$$P_4 = |V_4||V_1||Y_{41}|\cos(\theta_{41} - \delta_4 + \delta_1) + |V_4||V_2||Y_{42}|\ldots \cos(\theta_{42} - \delta_4 + \delta_2) + |V_4||V_3||Y_{43}|\cos(\theta_{43} - \delta_4 + \delta_3) + |V_4|^2|Y_{44}| + \cos\theta_{44} + |V_4||V_5||Y_{45}|\cos(\theta_{45} - \delta_4 + \delta_5) \tag{6}$$

$$P_5 = |V_5||V_1||Y_{51}|\cos(\theta_{51} - \delta_5 + \delta_1) + |V_5||V_2||Y_{52}|\cos(\theta_{52} - \delta_5 + \delta_2) + |V_5||V_3||Y_{53}|\cos(\theta_{53} - \delta_5 + \delta_3) + |V_5||V_4||Y_{54}|\cos(\theta_{54} - \delta_5 + \delta_4) + |V_5|^2|Y_{55}|\cos\theta_{55} \tag{7}$$

$$Q_4 = -|V_4||V_1||Y_{41}|\sin(\theta_{41} - \delta_4 + \delta_1) - |V_4||V_2||Y_{42}|\sin(\theta_{42} - \delta_4 + \delta_2) - |V_4||V_3||Y_{43}|\sin(\theta_{43} - \delta_4 + \delta_3) - |V_4|^2|Y_{44}|\sin\theta_{44} - |V_4||V_5||Y_{45}|\sin(\theta_{45} - \delta_4 + \delta_5) \tag{8}$$

$$Q_5 = -|V_5||V_1||Y_{51}|\sin(\theta_{51} - \delta_5 + \delta_1) - |V_5||V_2||Y_{52}|\sin(\theta_{52} - \delta_5 + \delta_2) - |V_5||V_3||Y_{53}|\sin(\theta_{53} - \delta_5 + \delta_3) - |V_5||V_4||Y_{54}|\sin(\theta_{54} - \delta_5 + \delta_4) - |V_5|^2|Y_{55}|\sin\theta_{55} \tag{9}$$

To obtain the elements of the Jacobian matrix, the derivatives of each of the above equations are taken with respect to the unknown variables δ_2, δ_3, δ_4, δ_5, $|V_4|$ and $|V_5|$ as presented in the following equations.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) + |V_2||V_4||Y_{24}|\sin(\theta_{24} - \delta_2 + \delta_4) + |V_2||V_5||Y_{25}|\sin(\theta_{25} - \delta_2 + \delta_5) \tag{10}$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \tag{11}$$

$$\frac{\partial P_2}{\partial \delta_4} = -|V_2||V_4||Y_{24}|\sin(\theta_{24} - \delta_2 + \delta_4) \tag{12}$$

$$\frac{\partial P_2}{\partial \delta_5} = -|V_2||V_5||Y_{25}|\sin(\theta_{25} - \delta_2 + \delta_5) \tag{13}$$

$$\frac{\partial P_2}{\partial |V_4|} = |V_2||Y_{24}|\cos(\theta_{24} - \delta_2 + \delta_4) \tag{14}$$

$$\frac{\partial P_2}{\partial |V_5|} = |V_2||Y_{25}|\cos(\theta_{25} - \delta_2 + \delta_5) \tag{15}$$

In the same vein, the derivatives of P_3

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) \tag{16}$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_4||Y_{34}|\sin(\theta_{34} - \delta_3 + \delta_4) + |V_3||V_5||Y_{35}|\sin(\theta_{35} - \delta_3 + \delta_5) \tag{17}$$

$$\frac{\partial P_3}{\partial \delta_4} = -|V_3||V_4||Y_{34}|\sin(\theta_{34} - \delta_3 + \delta_4) \tag{18}$$

$$\frac{\partial P_3}{\partial \delta_5} = -|V_3||V_5||Y_{35}|\sin(\theta_{35} - \delta_3 + \delta_5) \tag{19}$$

$$\frac{\partial P_3}{\partial |V_4|} = |V_3||Y_{34}|\cos(\theta_{34} - \delta_3 + \delta_4) \tag{20}$$

$$\frac{\partial P_3}{\partial |V_5|} = |V_3||Y_{35}|\cos(\theta_{35} - \delta_3 + \delta_5) \tag{21}$$

For P_4, the derivatives are

$$\frac{\partial P_4}{\partial \delta_2} = -|V_4||V_2||Y_{42}|\sin(\theta_{42} - \delta_4 + \delta_2) \tag{22}$$

$$\frac{\partial P_4}{\partial \delta_3} = -|V_4||V_3||Y_{43}|\sin(\theta_{43} - \delta_4 + \delta_3) \quad (23)$$

$$\frac{\partial P_4}{\partial \delta_4} = |V_4||V_1||Y_{41}|\sin(\theta_{41} - \delta_4 + \delta_1) + |V_4||V_2||Y_{42}| \sin(\theta_{42} - \delta_4 + \delta_2) + |V_4||V_3||Y_{43}|\sin(\theta_{43} - \delta_4 + \delta_3) + |V_4||V_5||Y_{45}|\sin(\theta_{45} - \delta_4 + \delta_5) \quad (24)$$

$$\frac{\partial P_4}{\partial \delta_5} = -|V_4||V_5||Y_{45}|\sin(\theta_{45} - \delta_4 + \delta_5) \quad (25)$$

$$\frac{\partial P_4}{\partial |V_4|} = |V_1||Y_{41}|\cos(\theta_{41} - \delta_4 + \delta_1) + |V_2||Y_{42}|\cos(\theta_{42} - \delta_4 + \delta_2) + |V_3||Y_{43}|\cos(\theta_{43} - \delta_4 + \delta_3) + 2|V_4||Y_{44}| \cos\theta_{44} + |V_5||Y_{45}|\cos(\theta_{45} - \delta_4 + \delta_5) \quad (26)$$

$$\frac{\partial P_4}{\partial |V_5|} = |V_4||Y_{45}|\cos(\theta_{45} - \delta_4 + \delta_5) \quad (27)$$

Similarly for P_5

$$\frac{\partial P_5}{\partial \delta_2} = -|V_5||V_2||Y_{52}|\sin(\theta_{52} - \delta_5 + \delta_2) \quad (28)$$

$$\frac{\partial P_5}{\partial \delta_3} = -|V_5||V_3||Y_{53}|\sin(\theta_{53} - \delta_5 + \delta_3) \quad (29)$$

$$\frac{\partial P_5}{\partial \delta_4} = -|V_5||V_4||Y_{54}|\sin(\theta_{54} - \delta_5 + \delta_4) \quad (30)$$

$$\frac{\partial P_5}{\partial \delta_5} = |V_5||V_1||Y_{51}|\sin(\theta_{51} - \delta_5 + \delta_1) + |V_5||V_2||Y_{52}| \sin(\theta_{52} - \delta_5 + \delta_2) + |V_5||V_3||Y_{53}|\sin(\theta_{53} - \delta_5 + \delta_3) + |V_5||V_4||Y_{54}|\sin(\theta_{54} - \delta_5 + \delta_4) \quad (31)$$

$$\frac{\partial P_5}{\partial |V_4|} = |V_5||Y_{54}|\cos(\theta_{54} - \delta_5 + \delta_4) \quad (32)$$

$$\frac{\partial P_5}{\partial |V_5|} = |V_1||Y_{51}|\cos(\theta_{51} - \delta_5 + \delta_1) + |V_2||Y_{52}| \cos(\theta_{52} - \delta_5 + \delta_2) + |V_3||Y_{53}|\cos(\theta_{53} - \delta_5 + \delta_3) + |V_4||Y_{54}|\cos(\theta_{54} - \delta_5 + \delta_4) + 2|V_5||Y_{55}|\cos\theta_{55} \quad (33)$$

The derivatives of Q_4 are

$$\frac{\partial Q_4}{\partial \delta_2} = -|V_4||V_2||Y_{42}|\cos(\theta_{42} - \delta_4 + \delta_2) \quad (34)$$

$$\frac{\partial Q_4}{\partial \delta_3} = -|V_4||V_3||Y_{43}|\cos(\theta_{43} - \delta_4 + \delta_3) \quad (35)$$

$$\frac{\partial Q_4}{\partial \delta_4} = |V_4||V_1||Y_{41}|\cos(\theta_{41} - \delta_4 + \delta_1) + |V_4||V_2||Y_{42}| \cos(\theta_{42} - \delta_4 + \delta_2) + |V_4||V_3||Y_{43}|\cos(\theta_{43} - \delta_4 + \delta_3) + |V_4||V_5||Y_{45}|\cos(\theta_{45} - \delta_4 + \delta_5) \quad (36)$$

$$\frac{\partial Q_4}{\partial \delta_5} = -|V_4||V_5||Y_{45}|\cos(\theta_{45} - \delta_4 + \delta_5) \quad (37)$$

$$\frac{\partial Q_4}{\partial |V_4|} = -|V_1||Y_{41}|\sin(\theta_{41} - \delta_4 + \delta_1) - |V_2||Y_{42}| \sin(\theta_{42} - \delta_4 + \delta_2) - |V_3||Y_{43}|\sin(\theta_{43} - \delta_4 + \delta_3) - 2|V_4||Y_{44}|\sin\theta_{44} - |V_5||Y_{45}|\sin(\theta_{45} - \delta_4 + \delta_5) \quad (38)$$

$$\frac{\partial Q_4}{\partial |V_5|} = -|V_4||Y_{45}|\sin(\theta_{45} - \delta_4 + \delta_5) \quad (39)$$

Finally for Q_5

$$\frac{\partial Q_5}{\partial \delta_2} = -|V_5||V_2||Y_{52}|\cos(\theta_{52} - \delta_5 + \delta_2) \quad (40)$$

$$\frac{\partial Q_5}{\partial \delta_3} = -|V_5||V_3||Y_{53}|\cos(\theta_{53} - \delta_5 + \delta_3) \quad (41)$$

$$\frac{\partial Q_5}{\partial \delta_4} = -|V_5||V_4||Y_{54}|\cos(\theta_{54} - \delta_5 + \delta_4) \quad (42)$$

$$\frac{\partial Q_5}{\partial \delta_5} = |V_5||V_1||Y_{51}|\cos(\theta_{51} - \delta_5 + \delta_1) + |V_5||V_2||Y_{52}| \cos(\theta_{52} - \delta_5 + \delta_2) + |V_5||V_3||Y_{53}|\cos(\theta_{53} - \delta_5 + \delta_3) + |V_5||V_4||Y_{54}|\cos(\theta_{54} - \delta_5 + \delta_4) \quad (43)$$

$$\frac{\partial Q_5}{\partial |V_4|} = -|V_5||Y_{54}|\sin(\theta_{54} - \delta_5 + \delta_4) \quad (44)$$

$$\frac{\partial Q_5}{\partial |V_5|} = -|V_1||Y_{51}|\sin(\theta_{51} - \delta_5 + \delta_1) - |V_2||Y_{52}| \sin(\theta_{52} - \delta_5 + \delta_2) - |V_3||Y_{53}|\sin(\theta_{53} - \delta_5 + \delta_3) - |V_4| |Y_{54}|\sin(\theta_{54} - \delta_5 + \delta_4) - 2|V_5||Y_{55}|\sin\theta_{55} \quad (45)$$

Starting with an initial estimate of $\left|V_4^{(0)}\right| = 1$, $\delta_4^{(0)} = 0°$; $\left|V_5^{(0)}\right| = 1$, $\delta_5^{(0)} = 0°$; and $\delta_2^{(0)} = \delta_3^{(0)} = 0°$. Meanwhile, the load and generation expressed in per units are $P_2^{sch} = 0.4$, $P_3^{sch} = 0.3$, $P_4^{sch} = -0.5$, $P_5^{sch} = -0.6$, $Q_4^{sch} = -0.3$, $Q_5^{sch} = -0.4$. The power residuals are computed as follows, where $P_2^{(0)}$, $P_3^{(0)}$, $P_4^{(0)}$, $P_5^{(0)}$, $Q_4^{(0)}$, $Q_5^{(0)}$ are calculated from eqns (4) to (9) respectively.

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = 0.4 - 0.389 = 0.011$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 0.3 - 0.312 = -0.012$$
$$\Delta P_4^{(0)} = P_4^{sch} - P_4^{(0)} = -0.5 - (-0.31) = -0.190$$
$$\Delta P_5^{(0)} = P_5^{sch} - P_5^{(0)} = -0.6 - (-0.094) = -0.506$$
$$\Delta Q_4^{(0)} = Q_4^{sch} - Q_4^{(0)} = -0.3 - (-1.137) = 0.837$$
$$\Delta Q_5^{(0)} = Q_5^{sch} - Q_5^{(0)} = -0.4 - (-0.344) = -0.056$$

$$\begin{bmatrix} 0.011 \\ -0.012 \\ -0.190 \\ -0.506 \\ 0.837 \\ -0.056 \end{bmatrix} = \begin{bmatrix} 35.09 & -5.39 & -5.23 & -7.85 & -1.73 & -2.59 \\ -5.39 & 40.40 & -30.90 & 0 & -10.22 & 0 \\ -5.23 & -30.92 & 39.9 & -3.75 & 12.57 & -1.24 \\ -7.85 & 0 & -3.75 & 11.60 & -1.24 & 3.65 \\ 1.73 & 10.22 & -13.2 & -3.75 & 37.63 & -3.75 \\ 2.59 & 0 & 1.24 & -3.83 & -3.75 & 10.91 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\delta_4^{(0)} \\ \Delta\delta_5^{(0)} \\ \Delta|V_4^{(0)}| \\ \Delta|V_5^{(0)}| \end{bmatrix}$$

The solution of the above matrix with the new bus voltages in the first iteration give

$$\Delta\delta_2^{(0)} = -0.0312 \qquad \delta_2^{(1)} = 0 + (-0.0312) = -0.0312$$
$$\Delta\delta_3^{(0)} = -0.0413 \qquad \delta_3^{(1)} = 0 + (-0.0413) = -0.0413$$
$$\Delta\delta_4^{(0)} = -0.0538 \qquad \delta_4^{(1)} = 0 + (-0.0538) = -0.0538$$
$$\Delta\delta_5^{(0)} = -0.0764 \qquad \delta_5^{(1)} = 0 + (-0.0764) = -0.0764$$
$$\Delta|V_4^{(0)}| = 0.0173 \qquad |V_4^{(1)}| = 1 + 0.0173 = 1.0173$$
$$\Delta|V_5^{(0)}| = -0.0125 \qquad |V_5^{(1)}| = 1 + (-0.0125) = 0.9875$$

The above steps were repeated until the solution converged in 3 iterations with a maximum power mismatch of 2.5×10^{-4} with $V_4 = 1.021 \angle -3.255^o$, $V_5 = 0.992 \angle -4.407^o$, $\delta_2 = \angle -1.822^o$, $\delta_3 = \angle -2.670^o$

The slack bus real and reactive powers and the reactive power at buses2 and 3 are:

$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3| |Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3) + |V_1||V_4||Y_{14}|\cos(\theta_{14} - \delta_1 + \delta_4) + |V_1||V_5||Y_{15}|\cos(\theta_{15} - \delta_1 + \delta_5) \qquad (46)$$

$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3| |Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3) - |V_1||V_4||Y_{14}|\sin(\theta_{14} - \delta_1 + \delta_4) - |V_1||V_5||Y_{15}|\sin(\theta_{15} - \delta_1 + \delta_5) \qquad (47)$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2|Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) - |V_2||V_4||Y_{24}|\sin(\theta_{24} - \delta_2 + \delta_4) - |V_2||V_5||Y_{25}|\sin(\theta_{25} - \delta_2 + \delta_5) - 0.1 \qquad (48)$$

$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33} - |V_3||V_4||Y_{34}|\sin(\theta_{34} - \delta_3 + \delta_4) - |V_3||V_5||Y_{35}|\sin(\theta_{35} - \delta_3 + \delta_5) - 0.15 \qquad (49)$$

Upon substitutions of the calculated values, we have

$P_1 = 0.8316$ p.u.

$Q_1 = 0.0807$ p.u.

$Q_2 = 0.4081$ p.u.

$Q_3 = 0.2592$ p.u.

The total system loss $P_L = (P_1 + P_2 + P_3) - (P_4 + P_5 + 0.4)$ p.u.

$= (0.8316 + 0.4 + 0.3) - (0.5 + 0.6 + 0.4)$

$= 0.0316$ p.u. $= 3.16$ MW

C. The MATPOWER Solution

Solving the power flow problem using MATPOWER requires the system data to be entered as shown in Fig. 2. In entering the bus data, *type 1*, *type 2*, *type 3* and *type 4* are used for the voltage-controlled buses, the load buses, the slack bus, and the isolated buses respectively. It is assumed in this example that the voltage level of the line is 132 kV, though any voltage level could be entered as the base kV.

It should be noted that the total line charging susceptance are entered in the branch data even though one-half line charging suseptance are supplied. The *status 1* in the generator and branch data denotes that both devices are in service. *Status 0* means the devices are out of service, while *status 2* indicates that the devices are isolated from the rest power system. The power flow of system under consideration is run by typing at the MATLAB command prompt

» runpf ('Example7')

and striking the enter key. The result which converged in 0.02 seconds is presented in Fig. 3.

```
Function [baseMVA, bus, gen, branch] = Example7
%   Power flow data for 5 bus, 3 generator case.
%             MATPOWER
%%     ----- Power Flow Data  -----%%
%% system MVA base
baseMVA = 100;
%% bus data
%  bus_i type Pd  Qd  Gs  Bs area Vm  Va  basekV  zone Vmax  Vmin
bus = [
     1    3   0   0   0   0  1    1   0  132    1    1.06   0.94;
     2    2  20  10   0   0  1    1   0  132    1    1.06   0.94;
     3    2  20  15   0   0  1    1   0  132    1    1.06   0.94;
     4    1  50  30   0   0  1    1   0  132    1    1.06   0.94;
     5    1  60  40   0   0  1    1   0  132    1    1.06   0.94;
];
%% generator data
%  bus Pg  Qg  Qmax  Qmin  Vg   mBase status Pmax  Pmin
gen = [
    1   0   0   50    10   1.06   100    1   100   10;
    2  40  30   50    10   1.045  100    1   100   10;
    3  30  10   40    10   1.03   100    1   100   10;
];
%% branch data
%  The MVA limit of the branches are not given, so each was set to 150
%  fbus tbus  r     x      b     rateA rateB  rateC ratio angle status
branch = [
     1    2  0.02  0.06  0.060  150    0     0     0     0     1;
     1    3  0.08  0.24  0.050  150    0     0     0     0     1;
     2    3  0.06  0.18  0.040  150    0     0     0     0     1;
     2    4  0.06  0.18  0.040  150    0     0     0     0     1;
     2    5  0.04  0.12  0.030  150    0     0     0     0     1;
     3    4  0.01  0.03  0.020  150    0     0     0     0     1;
     4    5  0.08  0.24  0.050  150    0     0     0     0     1;
];
return;
```

Fig. 2 MATPOWER Power Flow Solution

Newton's method power flow converged in 3 iterations.
Converged in 0.02 seconds.

Bus Data

Bus #	Voltage Mag(pu)	Voltage Ang(deg)	Generation P (MW)	Generation Q (MVAr)	Load P (MW)	Load Q (MVAr)
1	1.060	0.000	83.05	7.27	-	-
2	1.045	-1.782	40.00	41.81	20.00	10.00
3	1.030	-2.664	30.00	24.15	20.00	15.00
4	1.019	-3.243	-	-	50.00	30.00
5	0.990	-4.405	-	-	60.00	40.00
		Total:	153.05	73.23	150.00	95.00

Branch Data

Brnch #	From Bus	To Bus	From Bus Injection P (MW)	From Bus Injection Q (MVAr)	To Bus Injection P (MW)	To Bus Injection Q (MVAr)	Loss (I^2 * Z) P (MW)	Loss (I^2 * Z) Q (MVAr)
1	1	2	59.90	4.06	-59.25	-8.76	0.648	1.95
2	1	3	23.15	3.22	-22.74	-7.45	0.407	1.22
3	2	3	10.91	2.96	-10.83	-7.02	0.080	0.24
4	2	4	18.22	7.24	-17.99	-10.81	0.231	0.69
5	2	5	50.12	30.37	-48.83	-29.59	1.295	3.89
6	3	4	43.58	23.63	-43.34	-25.02	0.236	0.71
7	4	5	11.33	5.83	-11.17	-10.41	0.154	0.46
						Total:	3.053	9.16

Fig. 3 MATPOWER Power Flow Results

| System Summary |

How many?		How much?	P (MW)	Q (MVAr)
Buses	5	Total Gen Capacity	300.0	30.0 to 140.0
Generators	3	On-line Capacity	300.0	30.0 to 140.0
Committed Gens	3	Generation (actual)	153.1	73.2
Loads	4	Load	150.0	95.0
Fixed	4	Fixed	150.0	95.0
Dispatchable	0	Dispatchable	0.0 of 0.0	0.0
Shunts	0	Shunt (inj)	0.0	0.0
Branches	7	Losses (I^2 * Z)	3.05	9.16
Transformers	0	Branch Charging (inj)	-	30.9
Inter-ties	0	Total Inter-tie Flow	0.0	0.0
Areas	1			

	Minimum	Maximum
Voltage Magnitude	0.990 p.u. @ bus 5	1.060 p.u. @ bus 1
Voltage Angle	-4.41 deg @ bus 5	0.00 deg @ bus 1
P Losses (I^2*R)	-	1.30 MW @ line 2-5
Q Losses (I^2*X)	-	3.89 MVAr @ line 2-5

Fig. 4. System Result Summary

The result of the power flow problem obtained from MATPOWER package is exactly as that obtained in MATLAB using Sadaat's approach [1]. This fact confirms the accuracy of the MATPOWER package. However, comparing the result obtained from Sub-section III (B) shows a slight difference of about 1 % error margin. This is definitely the result of accumulated round-off of figures prior to the final answer. Another beauty of MATPOWER is that it has the capability to compute and display some other details of the systems that are not available in [1]. These details, tagged System Summary are presented in Fig. 4. In the said figure, all the power flow results of the system under consideration could be viewed at a glance. These details, such as, the minimum and the maximum voltage magnitude and their respective buses, the maximum active and reactive power losses and the branches where they can be found, etc. All these are interesting features that could be of significant value during a contingency analysis of the power system.

V. CONCLUSION

This paper has presented a new open-source MATPOWER as an efficient tool to facilitate the teaching of power system analysis courses at undergraduate level. The MATLAB-based package has some features that make it suitable for both research and educational purposes since its source code can be modified to the taste of the user. Even though the package can only handle power flow and optimal power flow at present, concerted efforts are on in extending the applications to other areas of power engineering. The result of the power flow problem obtained with MATPOWER was compared with that of Saadat [1] and the two are exactly the same. However, the additional desirable feature of the former is that it is simple and robust. MATPOWER is also good for carrying out line outage contingency simulations of power systems quickly.

As a matter of fact, this is not to say that using software packages for simulation of power systems does not have some disadvantages as pointed out in [14]. However, as far as the authors of this paper are concerned, the advantages outweigh the disadvantages, which make computer simulations indispensable in the teaching of power system analysis courses.

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