Exercise Sheet 1

Theory of neural Dynamics and application to ML based on RC

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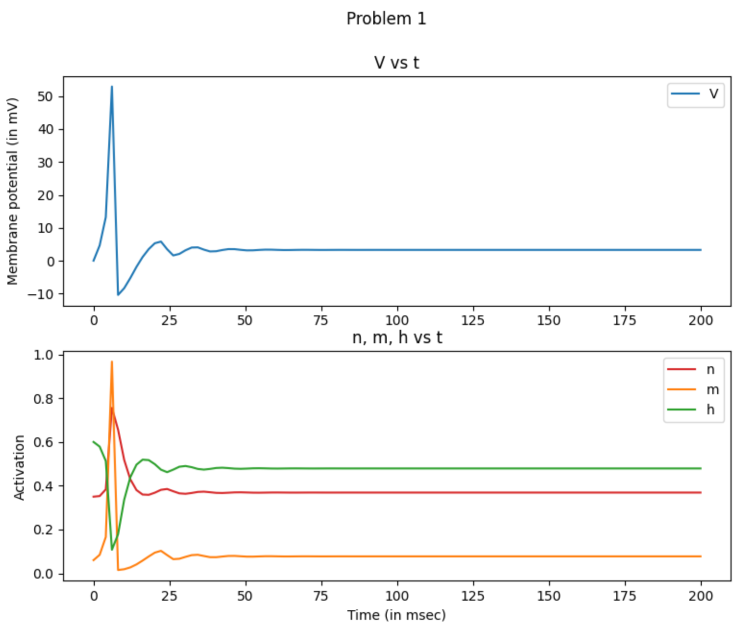
Part 1: the Hodgkin-Huxley neuron model and its simulations

Problem 1

By fixing the external stimulus at Iext = 5.2 and the initial conditions at V (0) = 0, n(0) = 0.35, m(0) = 0.06, and h(0) = 0.6, use an ODE integrator package in Python of your choice (e.g., solve ivp, odeint, ode, all in module scipy.integrate) to simulate the Hodgkin-Huxley neuron model in eq:1 with the parameter values given above. Plot (using, e.g., Matplotlib) the time series of the membrane potential (V(t)) and each gating variable (m(t),h(t),n(t)) for a duration of T = 200 milliseconds. Label the axes and provide a legend for each of your plots. Your submission should include the Python code used to simulate the model and generate the plots. Briefly (not more than 10 sentences) discuss the results and any observations you made.

Solution.

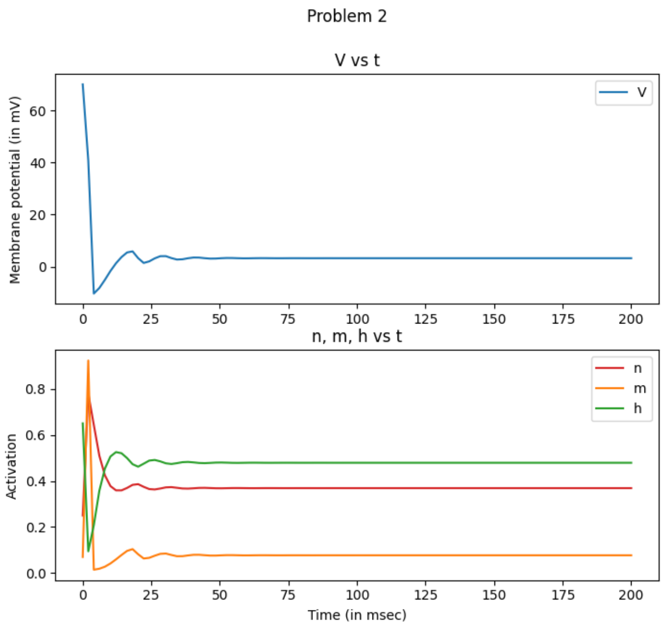


In all the graphs, (V vs t and n.m.h vs t), we see a spike (upward in case of V, n and m and downward in case of h) from the initial state.
In V vs t and m vs t, we then see the graphs descend to a trough to a value less than the initial condition after which the they rebound to a constant value very close to the initial condition bar a few minor oscillations.
In n vs t and h vs t, there is no significant trough after the initial spike. It should also be noted that n vs t and h vs t are mirror images of each other
Furthermore, we can deduce from the spike and the oscillations that V and m are 'faster' than n and h with respect to t.

Problem 2

Repeat Problem 1 with new initial conditions given by V (0) = 70, n(0) = 0.25, m(0) = 0.07, and h(0) = 0.65. Note the appropriateness of initial conditions for the gating variables (recall from the lecture that the gating variable probability measures.)

Solution.



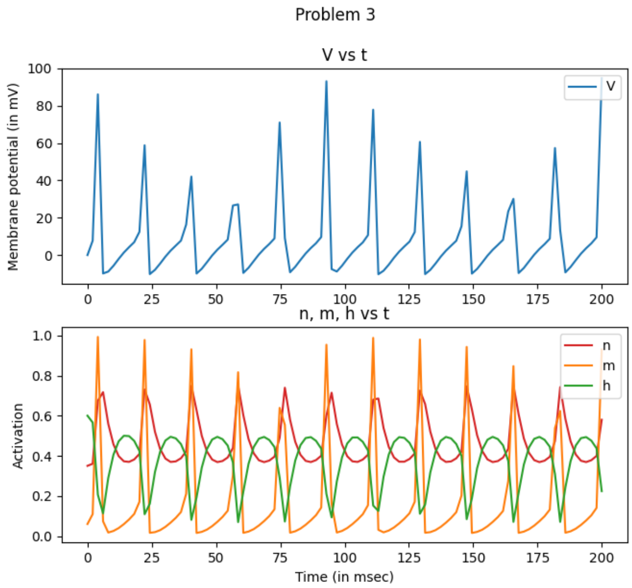
There is very little change in the nature of the graph in Problem 2 compared to the graphs in Problem 1. The only difference is the fact that all the graphs seem to have 'shifted' left with respect to t.

This can be attributed to the different initial conditions. These starting values in Problem 2 appear to have been achieved during or near the spike in Problem 1.

Problem 3

Repeat Problem 1 with a new external stimulus given by Iext = 6.8 and new initial conditions given by V (0) = 0, n(0) = 0.35, m(0) = 0.06, and h(0) = 0.6.

Solution.



With a new external current, the graphs almost completely change. instead of a single large peak and trough, there are a periodic peaks and troughs of varying but comparable magnitude. The magnitudes taken together are also periodic in nature.

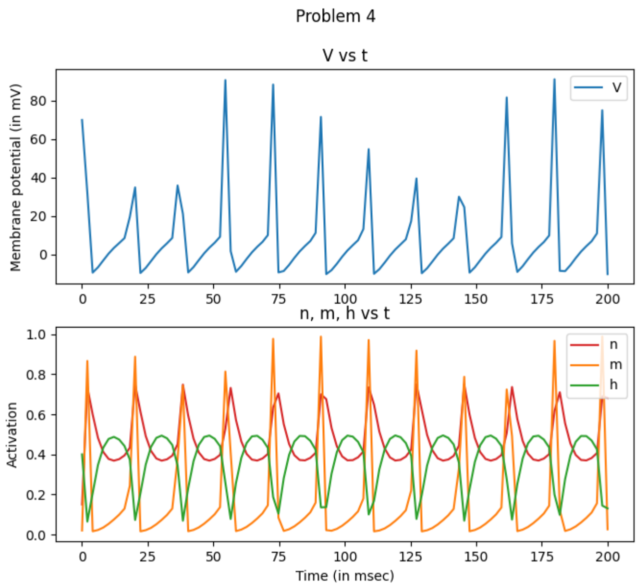
However, a few fundamental things about the nature of the graphs remain unchanged.

The graphs n vs t and h vs t are still mirror images of each other and V and m are still 'faster' than n and h with respect to t.

Problem 4

Repeat Problem 3 with new initial conditions given by V (0) = 70, n(0) = 0.15, m(0) = 0.02, and h(0) = 0.4.

Solution.



Like Problem 2, the only difference in the graph in Problem 4 compared to the one in Problem 3 is that fact that the new initial conditions cause the graph to 'shift' a little to the left. It appears that the new initial conditions in Problem 4 seems to have been achieved during or near the first burst or spike in problem 3.

Part 2: Timescale separation and dimension reduction

Problem 1

Consider the following set of coupled ODEs, each characterized by the timescales ε_1 and ε_2,

$$\begin{cases} \frac{dx}{dt} = \varepsilon_1^{-1}[-x(t) + y(t) + I(t)] \\ \frac{dy}{dt} = [-y(t) + x(t)^2 + A]/\varepsilon_2 \end{cases} \tag{3}$$

(a) If ε_1 ______ ε_2, then the system can be reduced to:

$$\varepsilon_1(\frac{dx}{dt}) = [-x(t) + x(t)^2 + I(t) + A] \tag{4}$$

(b) If ε_2 ______ ε_1, then the system can be reduced to:

$$\frac{dy}{dt} = \frac{1}{\varepsilon_2}[-y(t) + [y(t) + I(t)]^2 + A] \tag{5}$$

(c) If ε_1 ______ ε_2, then none of the ODEs in (a) and (b) can be correct.

Solution.

(a) If $\varepsilon_1 \underline{\gg} \varepsilon_2$,
$\frac{dy}{dt} = [-y(t) + x(t)^2 + A]/\varepsilon_2$ can be reduced to $-y(t) + x(t)^2 + A = 0$.
So, $y(t)$ can be expressed as $x(t) + A$.
Substituting $y(t)$ in $\frac{dx}{dt} = \varepsilon_1^{-1}[-x(t) + y(t) + I(t)]$, we get:
$\varepsilon_1(\frac{dx}{dt}) = [-x(t) + x(t)^2 + I(t) + A]$, which is equation (4).

Therefore, the solution is:

If$\varepsilon_1 \underline{\gg} \varepsilon_2$, then the system can be reduced to:

$$\varepsilon_1(\frac{dx}{dt}) = [-x(t) + x(t)^2 + I(t) + A] \tag{4}$$

(b) If $\varepsilon_2 \underline{\gg} \varepsilon_1$,
$\frac{dx}{dt} = \varepsilon_1^{-1}[-x(t) + y(t) + I(t)]$ can be reduced to $-x(t) + y(t) + I(t) = 0$.
So, $x(t)$ can be expressed as $y(t) + I(t)$.
Substituting $x(t)$ in $\frac{dy}{dt} = [-y(t) + x(t)^2 + A]/\varepsilon_2$, we get:
$\frac{dy}{dt} = \frac{1}{\varepsilon_2}[-y(t) + [y(t) + I(t)]^2 + A]$, which is equation (5).

Therefore, the solution is:

If$\varepsilon_2 \underline{\gg} \varepsilon_1$, then the system can be reduced to:

$$\frac{dy}{dt} = \frac{1}{\varepsilon_2}[-y(t) + [y(t) + I(t)]^2 + A] \tag{5}$$

(c) Ifε_1 and ε_2 are comparable in magnitude, then none of the reductions in (a) and (b) can be done.

Therefore, the solution is:

$\boxed{\text{If}\varepsilon_2 \approxeq \varepsilon_1\text{, then none of the ODEs in (a) and (b) can be correct.}}$

Problem 2

A channel with gating variable $R(t)$, given by $\frac{dR}{dt} = \varepsilon_1^{-1}[-R(t) + R_0(V)]$ influences the voltage $V(t)$ given by $\varepsilon_2 \frac{dV}{dt} = -[V(t) - V_0] + R(t)^2 A$.
A reduction of dimension

(a) is possible, and the result is $\varepsilon_1 \frac{dR}{dt} = -R(t) + R_0[V_0 + R(t)^2 A]$ if ε_2 ______ ε_1.

(b) is impossible if ε_1 ______ ε_2.

(c) is possible, and the result is $\varepsilon_2 \frac{dV}{dt} = -V(t) + V_0 + R_0(V)^2 A$ if ε_2 ______ ε_1.

Solution.

(a) If $\varepsilon_1 >> \varepsilon_2$,
$\varepsilon_2 \frac{dV}{dt} = -[V(t) - V_0] + R(t)^2 A$ can be written as $V(t) = R(t)^2 A + V_0$.
Substituting, we get,
$\frac{dR}{dt} = \varepsilon_1^{-1}[-R(t) + R_0(R(t)^2 A + V_0))]$.

Therefore, the solution is:

A reduction of dimension is possible, and the result is $\varepsilon_1 \frac{dR}{dt} = -R(t) + R_0[V_0 + R(t)^2 A]$ if $\varepsilon_2 \underline{\ll} \varepsilon_1$.

(b) If ε_1 and ε_2 are comparable in magnitude, a reduction of dimension will not be possible.

Therefore, the solution is:

A reduction of dimension is impossible if $\varepsilon_1 \underline{\approx} \varepsilon_2$.

(c) If $\varepsilon_1 \ll \varepsilon_2$,
$\frac{dR}{dt} = \varepsilon_1^{-1}[-R(t) + R_0(V)]$ can be written as $0 = -R(t) + R_0(V)$.
This gives us $R(t) = R_0(V)$.
Substituting, we get,
$\varepsilon_2 \frac{dV}{dt} = -V(t) + V_0 + R_0(V)^2 A$

Therefore, the solution is:

A reduction of dimension is possible, and the result is $\varepsilon_2 \frac{dV}{dt} = -V(t) + V_0 + R_0(V)^2 A$ if $\varepsilon_2 \underline{\gg} \varepsilon_1$.