Exercise Sheet 4

Theory of neural Dynamics and application to ML based on RC

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Problem: A

Simulate the LIF neuron model given in this lecture and show:

(1) The time series for the duration of T = 200 ms of both the membrane potential V and the input current $I(t) = Acos(\omega t)$, when for the first 50 ms the amplitude of the input current is A = 0, for the next 50 ms A = 100, the next 50 ms A = 200, and the last 50 ms A = 0. Assume that the frequency of the input current is ω = 0.25 Hz or any other number that you should indicate in your figures.

(2) From the results of your simulations, explain why you think the LIF is a Type I and not Type II neuron model.

Solution.

(1) Time series for ω = 0.25, 1.0 and 10.0:

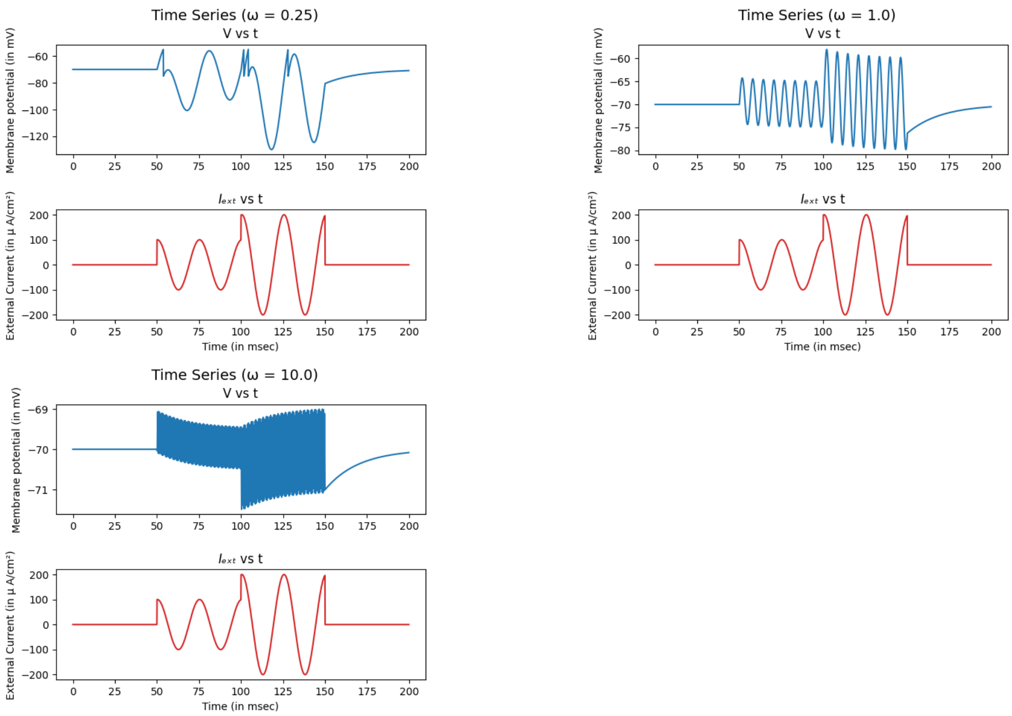


Figure 1: Time Series of membrane potential V and external current I_{ext} of the LIF neuron model for ω = 0.25, 1,0 and 10.0

(2) (a) **Type I neurons exhibit continuous firing as long as the input current remains above a specific threshold value:** We see this in part (1) where as long as the current has nonzero amplitude, the LIF neuron keeps firing.

(b) **Type I neurons respond to stimuli in a graded manner, with the membrane potential and firing rate changing proportionally to the intensity or duration of the input stimulus:** We see this phenomenon in all of the graphs in part (1) where upon the changing of the amplitude of the external current, we see a change in the membrane potential and firing rate.

(c) **Type I neurons do not display the characteristic spikes-and-quiescent pattern:** We see in part(1) that there in continuous spiking as long as the stimulus is above the threshold with no quiescence in between.

Therefore, we can conclude that the LIF is a Type I and not Type II neuron model.

Problem: B

Plot the v-nullcline and the w-nullcline of the Morris-Lecar neuron model and vary the input current and/or the maximum calcium conductance such that:

(1) The two nullclines intersect at only one fixed point.

(2) The two nullclines intersect at two fixed points.

(3) The two nullclines intersect at three fixed points.

Solution.

(1) The two nullclines intersect at only one fixed point:

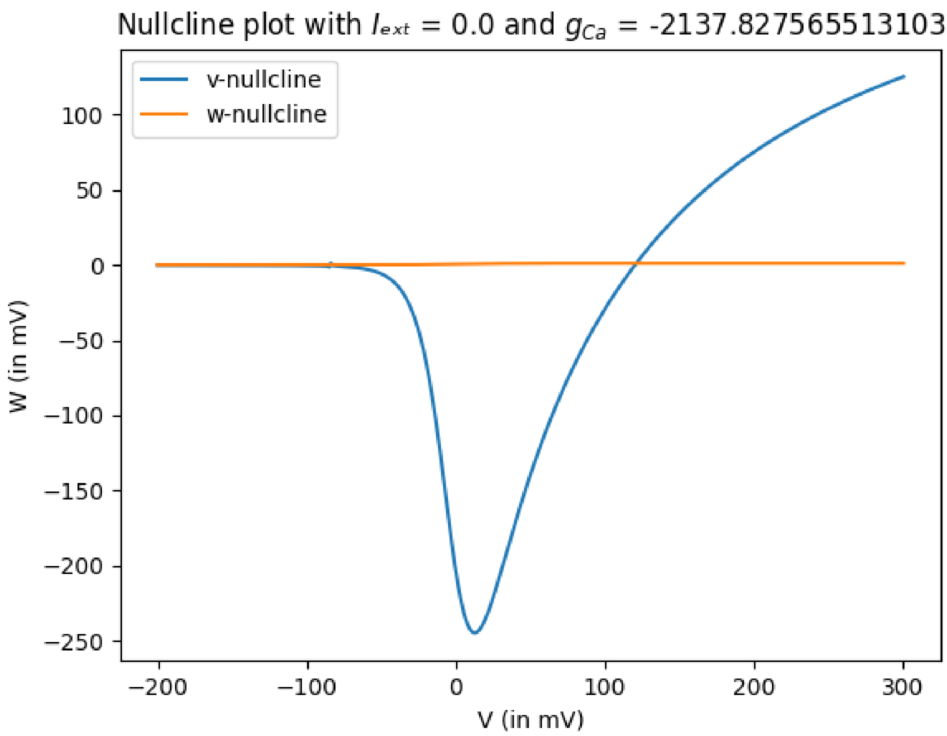


Figure 2: Plot of the v-nullcline and the w-nullcline of the Morris-Lecar neuron model with the two nullclines intersecting at only one fixed point. Note that the parameters for the graph are $I_{ext} = 0.0\ \mu A/cm^2$ and $g_{Ca} = -2137.827565513103\ \mu S/cm^2$.

(2) The two nullclines intersect at two fixed points:

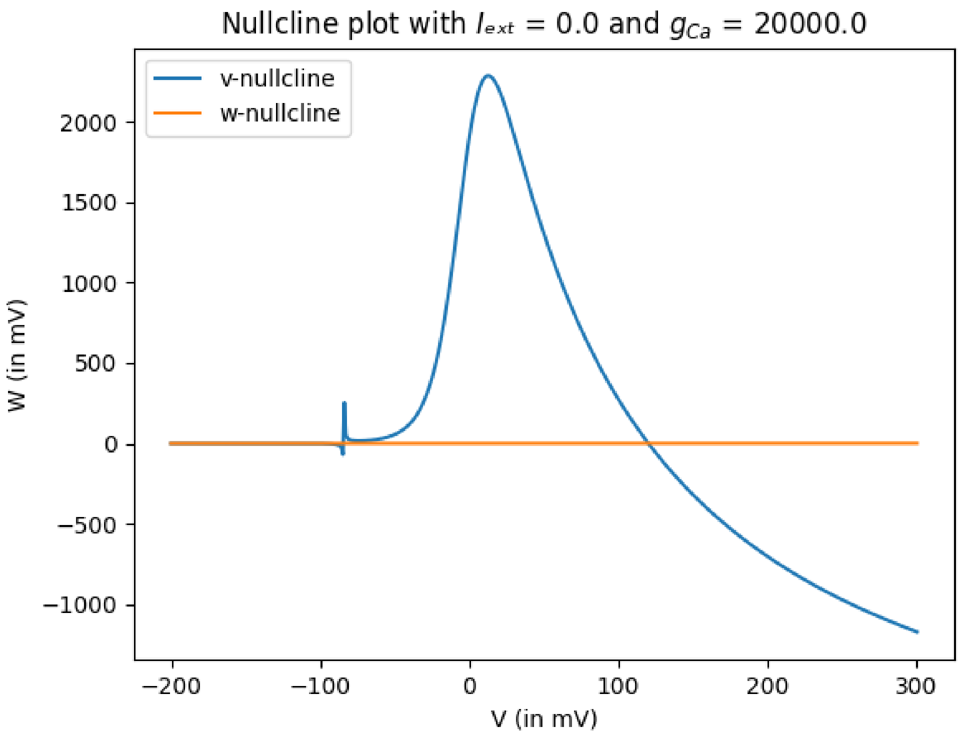


Figure 3: Plot of the v-nullcline and the w-nullcline of the Morris-Lecar neuron model with the two nullclines intersecting at two fixed points. Note that the parameters for the graph are $I_{ext} = 0.0\ \mu A/cm^2$ and $g_{Ca} = 20000.0\ \mu S/cm^2$.

(3) The two nullclines intersect at three fixed points:

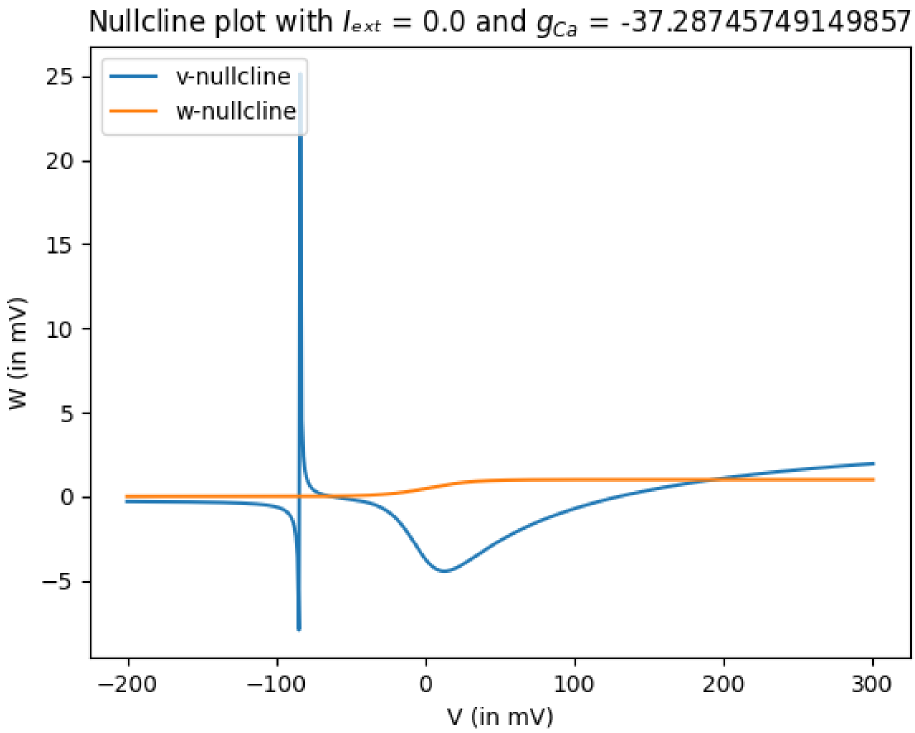


Figure 4: Plot of the v-nullcline and the w-nullcline of the Morris-Lecar neuron model with the two nullclines intersecting at three fixed points. Note that the parameters for the graph are $I_{ext} = 0.0\ \mu A/cm^2$ and $g_{Ca} = -37.28745749149857\ \mu S/cm^2$.

Problem: C

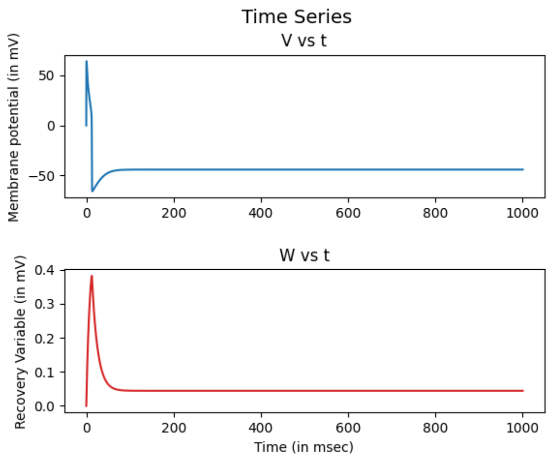
Show the time series and the corresponding phase portrait of the membrane potential and the recovery variables for a duration of T = 1000 ms when:

(1) The external current is $I_{ext} = 40$ nA

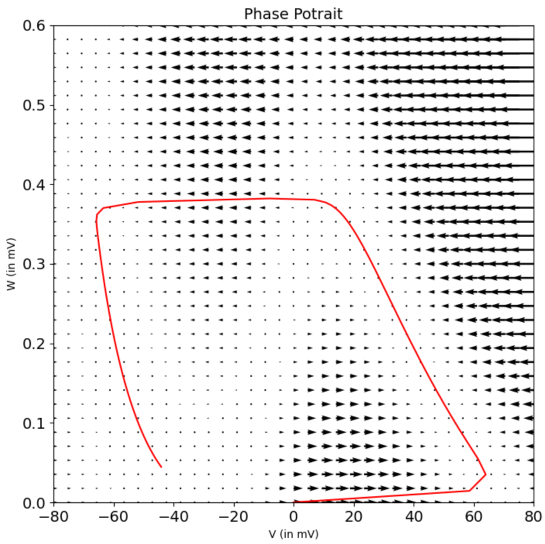
(2) The external current is $I_{ext} = 90$ nA

Solution.

(1) $I_{ext} = 40$ nA



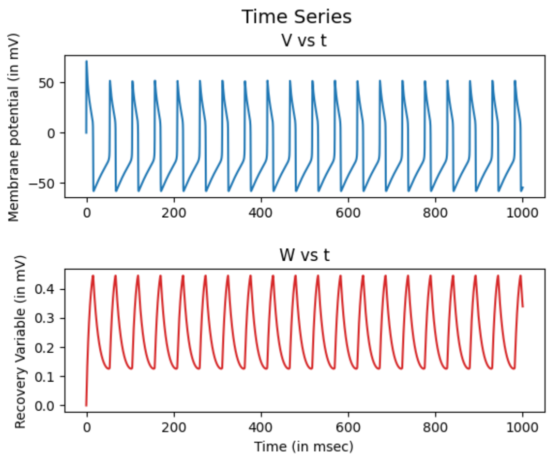
(a) Time Series of membrane potential V and recovery variable W



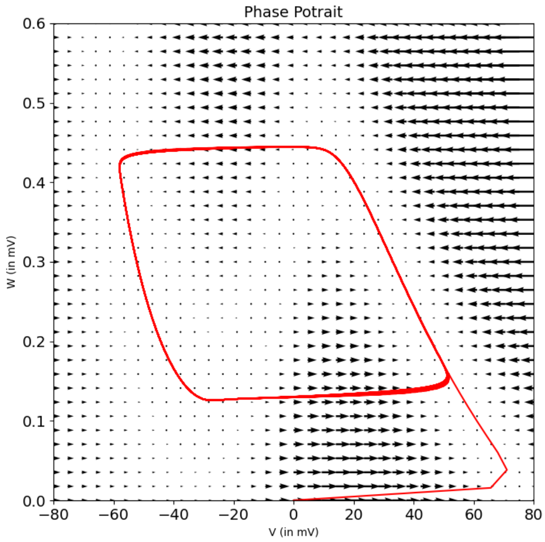
(b) Phase potrait of membrane potential V and recovery variable W

Figure 5: Time Series and Phase Portrait of the Morris-Lecar neuron model when the external current $I_{ext} = 40$ nA. Note that the membrane potential V and the recovery variable W are being intialized at 0.0 mV and 0.0 mV respectively.

(2) $I_{ext} = 90$ nA



(a) Time Series of membrane potential V and recovery variable W



(b) Phase potrait of membrane potential V and recovery variable W

Figure 6: Time Series and Phase Portrait of the Morris-Lecar neuron model when the external current $I_{ext} = 90$ nA. Note that the membrane potential V and the recovery variable W are being intialized at 0.0 mV and 0.0 mV respectively.