Homework 9: Due at class on May 12

1 Contribution from symplectic representation

Consider

$$P\Lambda^T P = \pm\Lambda \tag{1.1}$$

for $P = i\begin{pmatrix} 0 & -\mathbb{1}_{k\times k} \\ \mathbb{1}_{k\times k} & 0 \end{pmatrix}$ where $n = 2k$. Derive the dimension(degrees of freedom) of Λ for $+$ and $-$, respectively, and show that $\mathrm{tr}[\Omega_\Lambda] = -n$.

2 Orientation flip in superstring

Consider the orientation flip operator Ω for world-sheet fermions of closed string.

- Define the action of the orientation operator on the fermions $\psi^\mu(t,\sigma)$, $\overline{\psi}^\mu(t,\sigma)$ in NS- and R-sector properly, and derive the action on their modes. (Hint: the orientation flip is define for c as $\Omega : c(t,\sigma) \to -\bar{c}(t, 2\pi - \sigma)$. The minus sign in from of $\bar{c}$ is coming from relative phase of overall coefficient of the mode expansion in cylinder: $c = i\sum c_n e^{in(it+\sigma)}$ and $\bar{c} = -i\sum \bar{c}_n e^{in(it+\sigma)}$.
- Consider IIB RR-fields as in Prob. 2 of Homework 8, and show that only the 2-form RR-field survives under the Ω projection $\frac{1+\Omega}{2}$. (Hint: you have to consider field strength of n-form RR-fields because RR-fields themselves are not physical degrees of freedom. Note that RR-fields are real valued object so its complex conjugate is itself. Also note that the complex conjugate of fermions gives minus sign $(\psi^\dagger\chi)^\dagger = -\chi^\dagger\psi$ due to their statistics. You can assume that gamma matrices are invariant under Ω. This is because zero modes are identical in L and R so Γ is actually sum of L and R, i.e. $\Gamma = \Gamma^L + \Gamma^R$, which is manifestly invariant under Ω.)

3 SO(32) in Type I

In the lecture, we evaluate the cylinder, Klein bottle and Möbius strip amplitude in unoriented bosonic string. Evaluate the cylinder, Klein bottle and Möbius strip amplitude in the RR-sector of superstring theory. Show the divergence is absent only when the gauge group is SO(32).