Homework 10: Due at class on May 17

1. Show that Euler characteristics of a compact Lie group is zero.

2. Write down the definitions of the following Lie algebras: $\mathfrak{gl}(n,\mathbb{C})$, $\mathfrak{sl}(n,\mathbb{C})$, $\mathfrak{su}(n)$, $\mathfrak{sp}(n,\mathbb{R})$ and $\mathfrak{so}(n,\mathbb{R})$.

3. Show that there are matrices $A, B \in \mathfrak{gl}(n,\mathbb{C})$ such that

$$e^A e^B \neq e^{A+B} \ .$$

Modify this equation in such a way that the equality holds for those matrices A, B.

4. If $f : M \hookrightarrow N$ is an embedding, the quotient bundle $f^*TN/TM = NM$ is a vector bundle over M called **the normal bundle** of M where f^*TN is the pullback bundle. (See Definition 10.9 in the lecture note.) Show that the normal bundle is trivial for all spheres $S^n \subset \mathbb{R}^{n+1}$.

5. Show that there is a line (rank one vector) bundle L over S^2 such that $TS^2 \oplus L$ is trivial where TS^2 is the tangent bundle of S^2. Hint: The tangent bundle of the flat space is trivial $T\mathbb{R}^n \cong \mathbb{R}^{2n}$.