Duality of 3d N=4 theories with 1-form symmetry

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Joint work with M. Sperling, H. E. Wang, Z. Zhong

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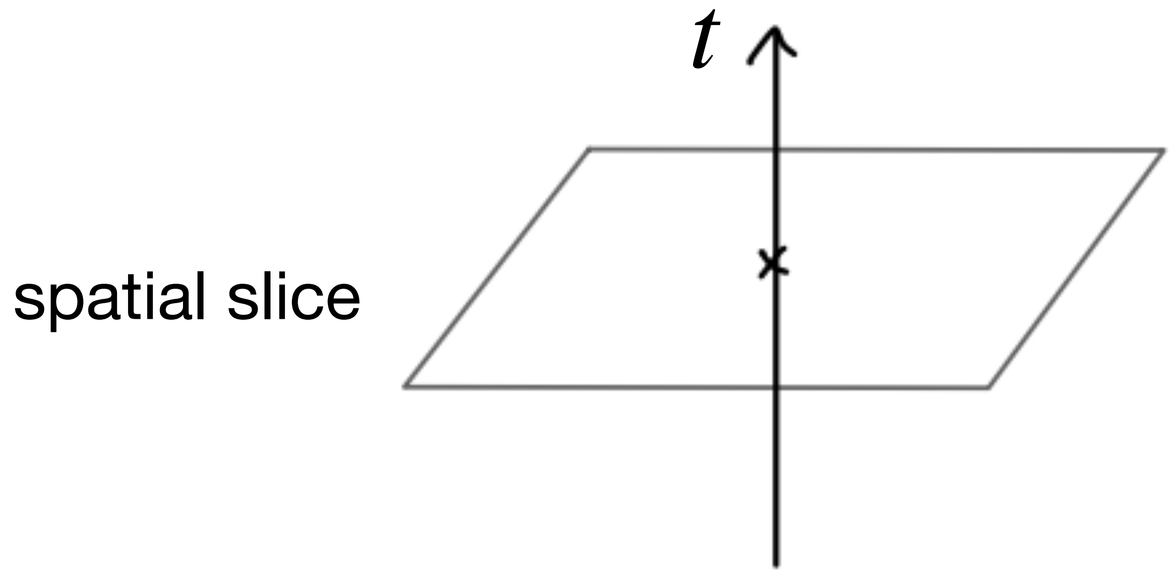
Higher-form Symmetry

Symmetry in Field Theory

- Everything starts from Noether's theorem

Continuous symmetry $\longrightarrow$ Conserved current $\partial_\mu j^\mu = 0$

$\downarrow$



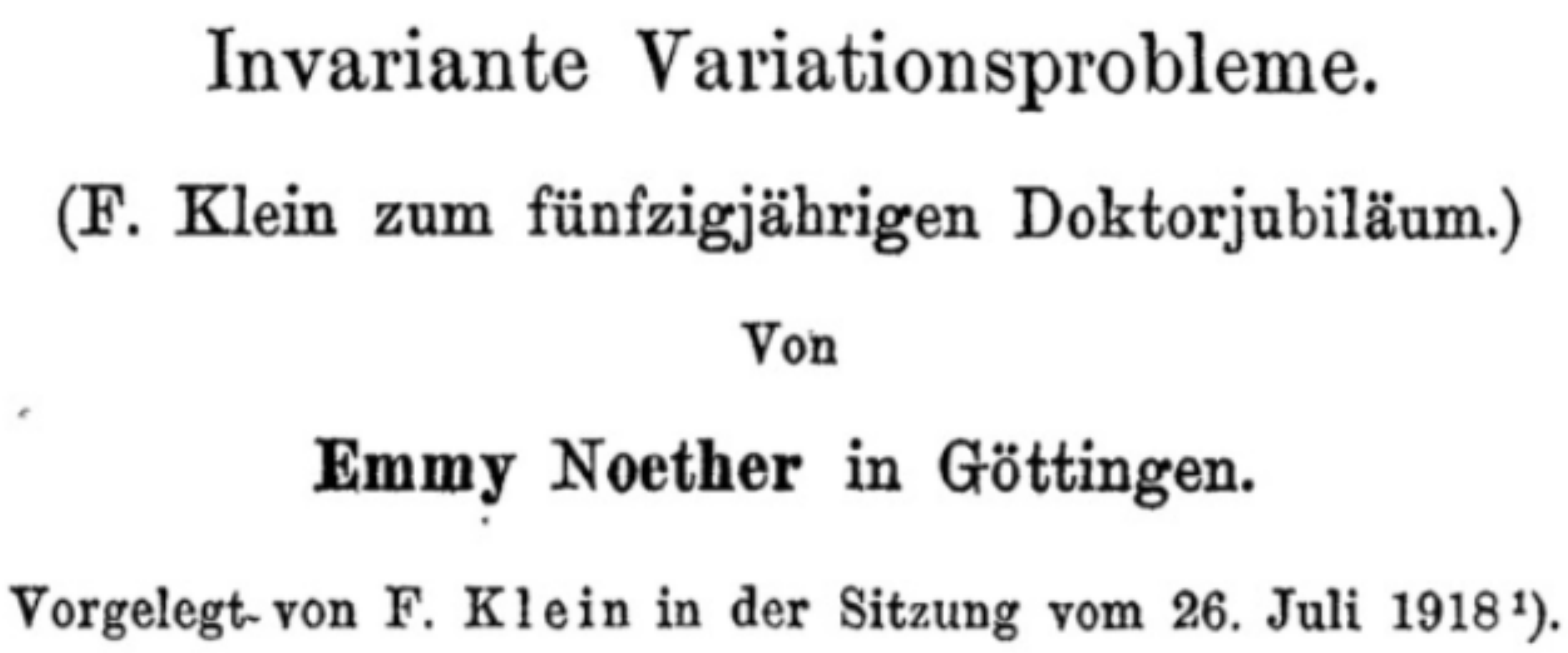
Conserved charge $Q = \int d^{d-1}x \, j^0 \qquad \frac{dQ}{dt} = 0$

- Example

Time translation symmetry $\longrightarrow$ E

Spatial translational symmetry $\longrightarrow$ P

Rotational symmetry $\longrightarrow$ L



Invariante Variationsprobleme.

(F. Klein zum fünfzigjährigen Doktorjubiläum.)

Von

Emmy Noether in Göttingen.

Vorgelegt von F. Klein in der Sitzung vom 26. Juli 1918 [1]).

Symmetry in Quantum Field Theory

- Consider a correlation function of local operators

$$\langle \Phi^{i_1}(y_1)\Phi^{i_2}(y_2)\cdots\Phi^{i_n}(y_n)\rangle = \int \mathscr{D}\Phi \; \Phi^{i_1}(y_1)\Phi^{i_2}(y_2)\cdots\Phi^{i_n}(y_n)e^{iS}$$

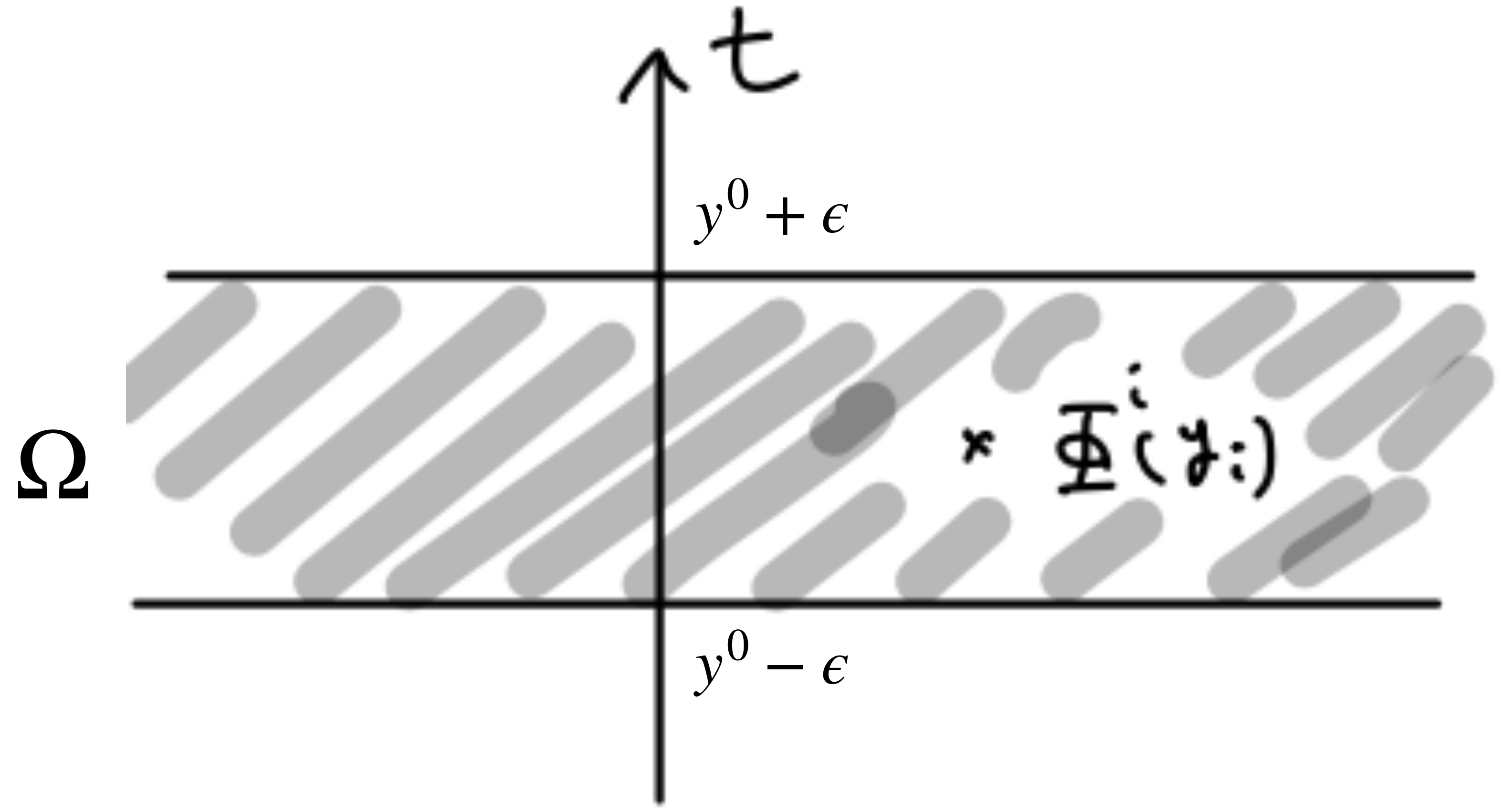
- Under the symmetry variation $\Phi^i \to \Phi^i + \epsilon S^i{}_j\Phi^j \longrightarrow \delta S = -\int \epsilon(x)\partial_\mu j^\mu$

Ward-Takahashi identity: Quantum version of Noether's theorem

$$i\langle \partial_\mu j^\mu(x)\Phi^{i_1}(y_1)\Phi^{i_2}(y_2)\cdots\rangle = \sum_k \delta^{(4)}(x-y_k)S^{i_k}{}_{j_k}\langle \Phi^{i_1}(y_1)\Phi^{i_2}(y_2)\cdots\Phi^{j_k}(y_k)\cdots\rangle$$

- Integrating this identity over Ω, symmetry transformation by conserved charge Q

$$i\langle [Q,\Phi^i(y)]\cdots\rangle = S^i{}_j\langle \Phi^j(y)\cdots\rangle$$

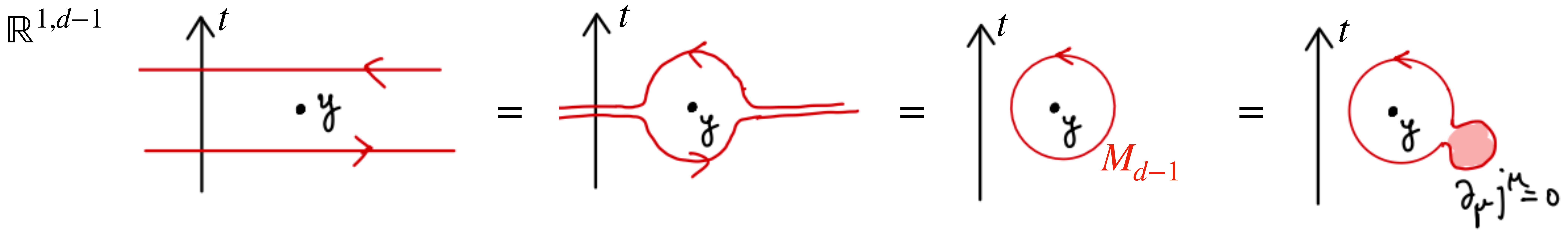


Higher-form symmetry

Gaiotto-Kapustin-Seiberg-Willett

- Recall that symmetry transformation by conserved charge Q

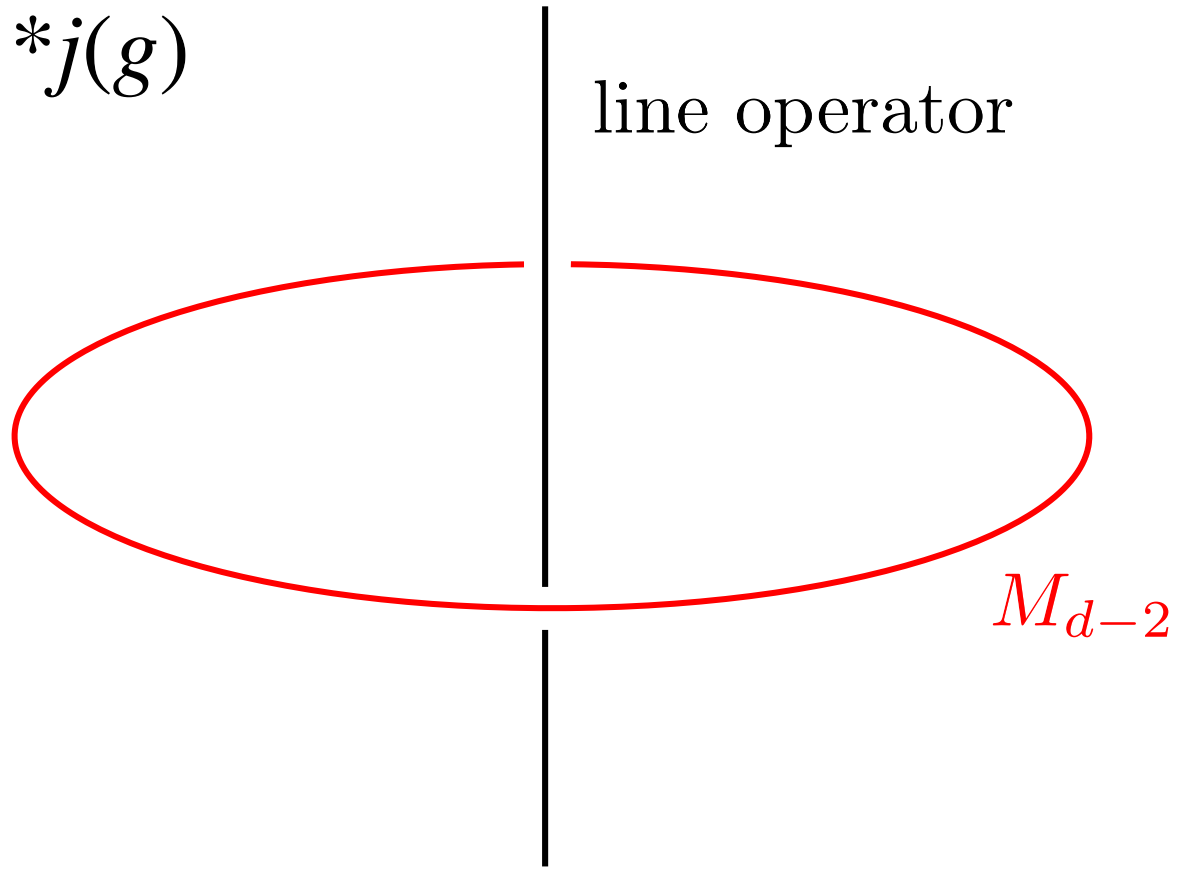
$$i\langle [Q, \Phi^i(y)] \cdots \rangle = S^i{}_j \langle \Phi^j(y) \cdots \rangle$$



- Symmetry transformation is given by linking with topological object $U_g(M_{d-1})$

$$\langle U_g(M_{d-1}) \Phi^i(y) \cdots \rangle = R(g)^i{}_j \langle \Phi^j(y) \cdots \rangle \qquad R(g)^i{}_j = e^{i \int_{M_{d-1}} {}^* j(g)}$$

- This symmetry action can be generalized for extended operator



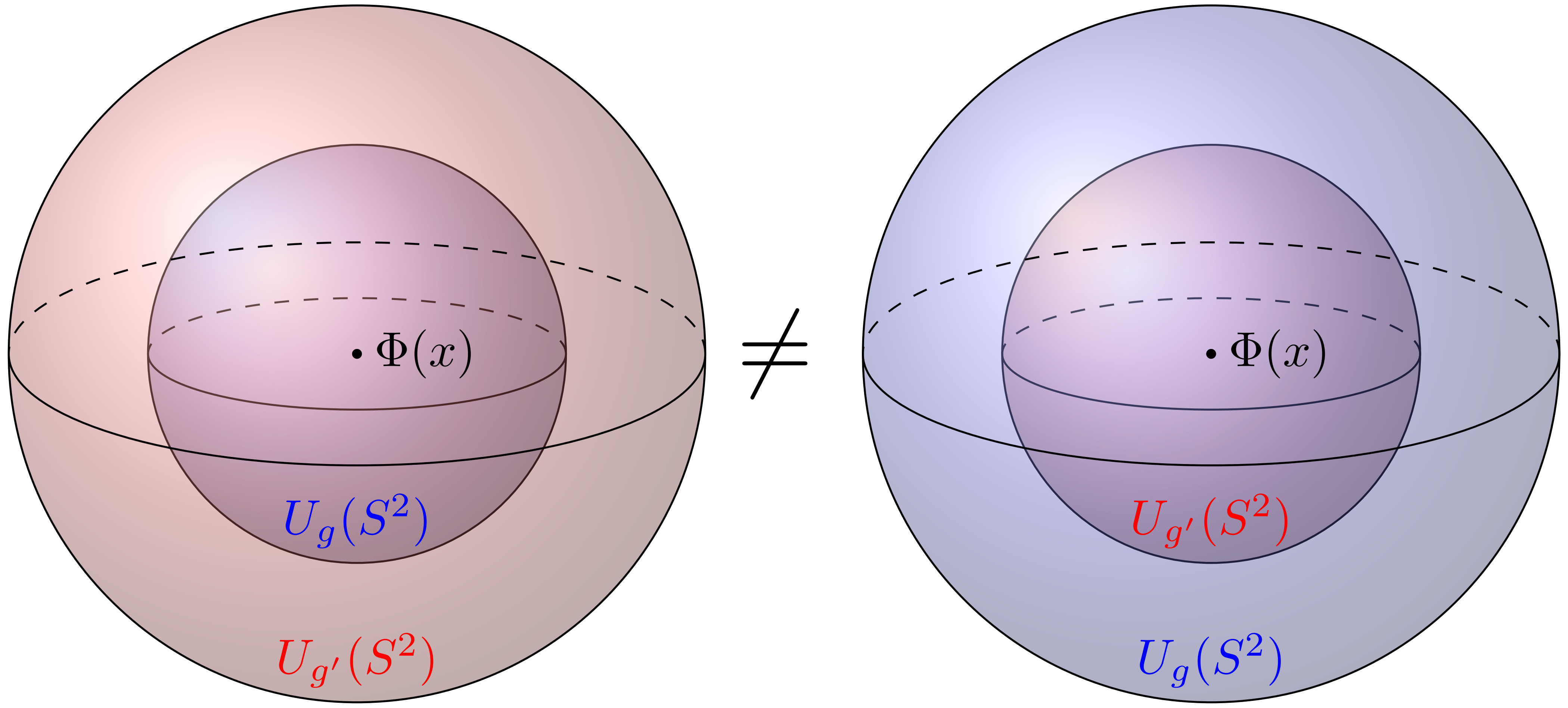
$$\langle U_g(M_{d-2}) L \rangle = e^{i \, \mathrm{Link}(M_{d-2}, L)} \langle L \rangle$$

- $\boldsymbol{p}$-form symmetry is a "topological operator $U_g(M_{d-p-1})$" acting on $\boldsymbol{p}$-dim'l objects

Higher-form symmetry

- This formalism naturally incorporates discrete symmetry (cf. Noether theorem)

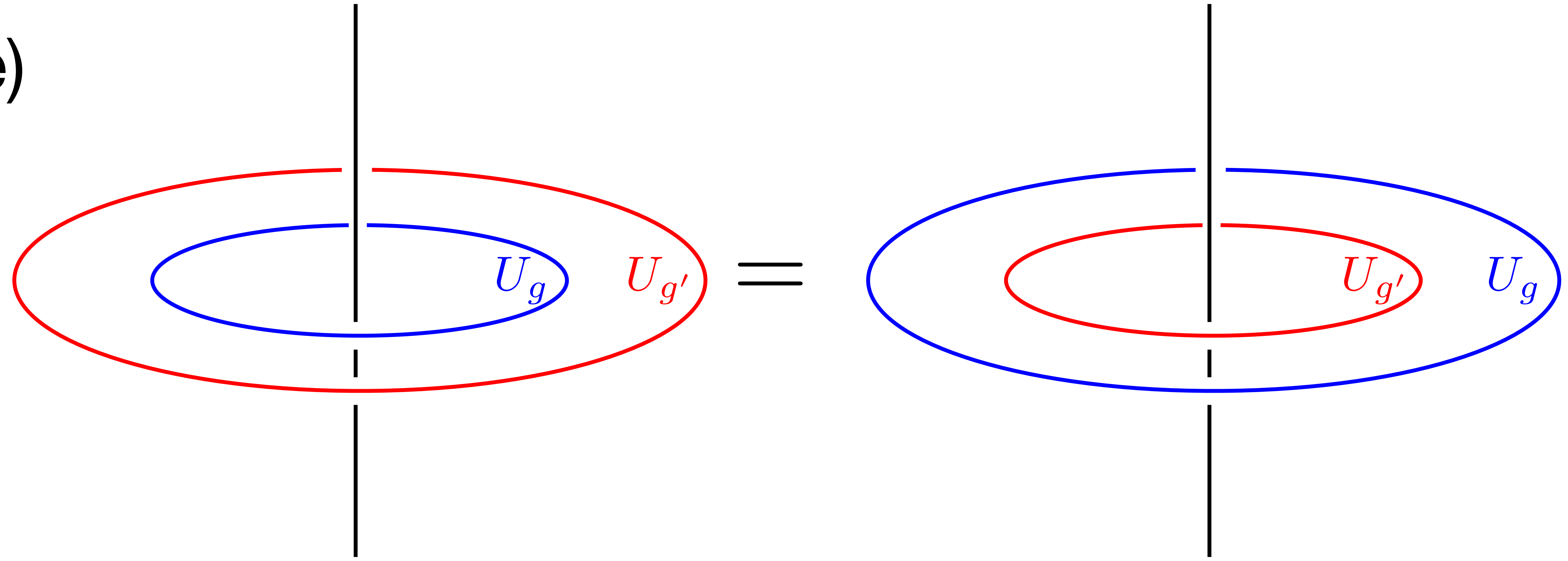
- 0-form symmetry can be non-Abelian (non-commutative)



In general,

$$U_{g'}(M_{d-1})U_g(M'_{d-1}) \neq U_g(M_{d-1})U_{g'}(M'_{d-1})$$

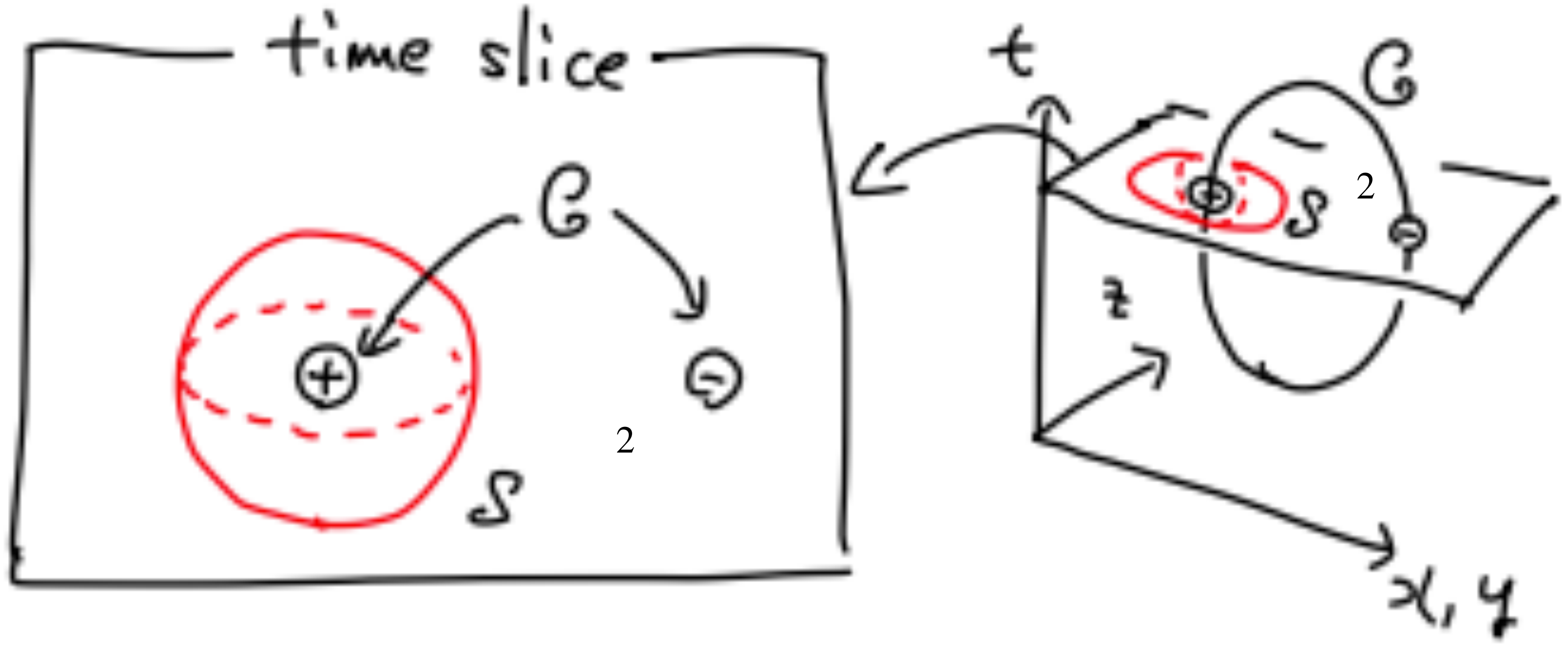
- Higher-form symmetry is Abelian (commutative)



Main message

Symmetry = Topological operator

1-form symmetry in gauge theory



- 1-form symmetry of Maxwell theory

  Electric 1-form $U(1)^{(1)}_e$: $\partial^\mu F_{\mu\nu} = 0$ by e.o.m.

  Magnetic 1-form $U(1)^{(1)}_m$: $\partial^\mu ({}^*F)_{\mu\nu} = 0$ by Bianchi

  Wilson loop is charged under $U(1)^{(1)}_e$: $\langle U_{e^{i\alpha}}(S^2) W(c) \rangle = e^{i\alpha \int_{S^2} {}^*F} \langle W(c) \rangle$

  't Hooft loop is charged under $U(1)^{(1)}_m$: $\langle U_{e^{i\alpha}}(S^2) H(c) \rangle = e^{i\alpha \int_{S^2} F} \langle H(c) \rangle$

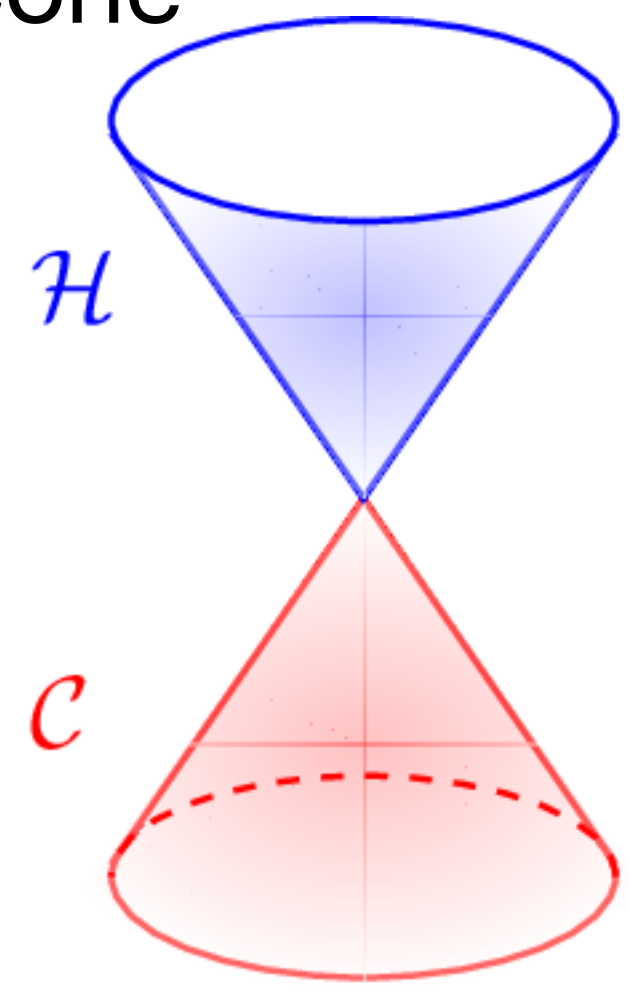
- SU(N) Yang-Mills has 1-form symmetry: the center $\mathbb{Z}^{(1)}_N$ of SU(N)

$$\begin{pmatrix} e^{2\pi i k/N} & 0 & \cdots & 0 \\ 0 & e^{2\pi i k/N} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{2\pi i k/N} \end{pmatrix} \qquad k = 1, \ldots, N$$

3d Mirror Symmetry

3d N=4 theories and mirror symmetry

hyper-Kahler cone



3d supersymmetric theories with eight supercharges

Intriligator-Seiberg

If it has Lagrangian description,

Vector multiplets → Coulomb branch

Hyper multiplets → Higgs branch

Manifest in brane constructions in Type IIB string theory: D3-D5-NS5 (with O-plane)

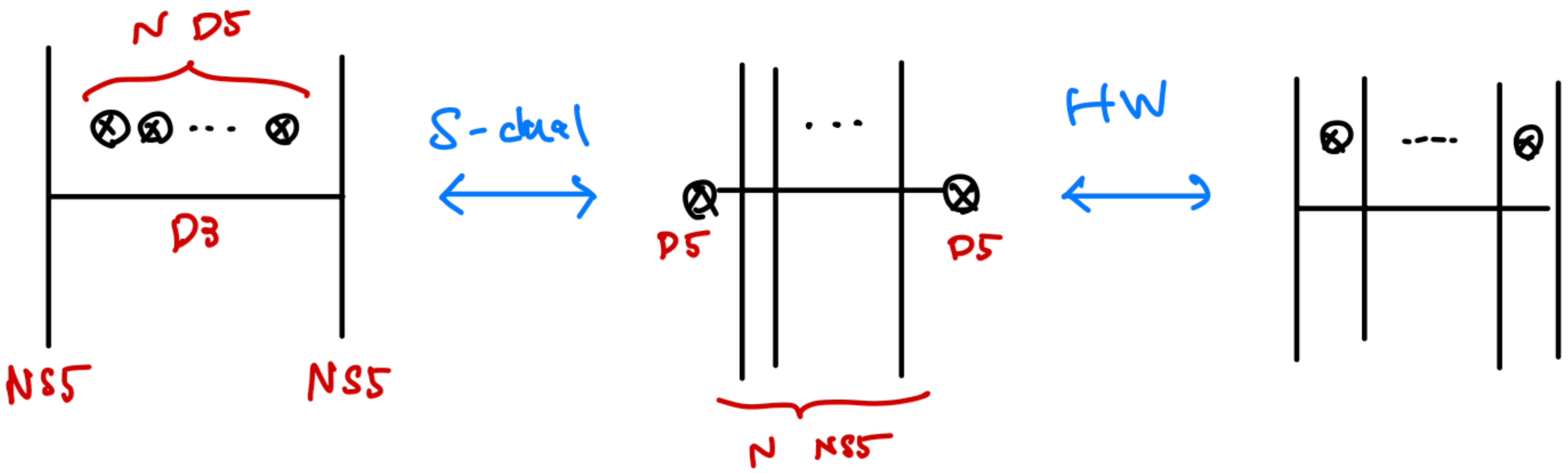
Hanany-Witten

3d mirror symmetry: Instrumental IR duality of 3d N=4 theories

$SU(2)_H$	$\longleftrightarrow$	$SU(2)_C$
Higgs	$\longleftrightarrow$	Coulomb
FI	$\longleftrightarrow$	masses
electric particles	$\longleftrightarrow$	vortex particles
Wilson lines	$\longleftrightarrow$	vortex lines

IIB construction and Hanany-Witten move

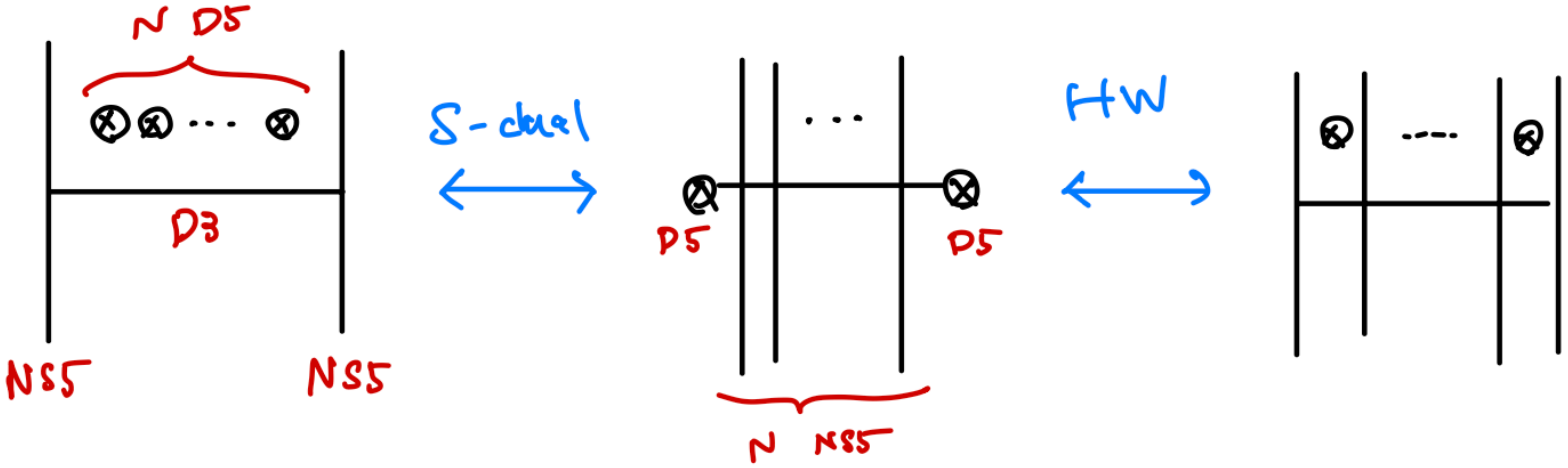
Baby example: Abelian mirror pair

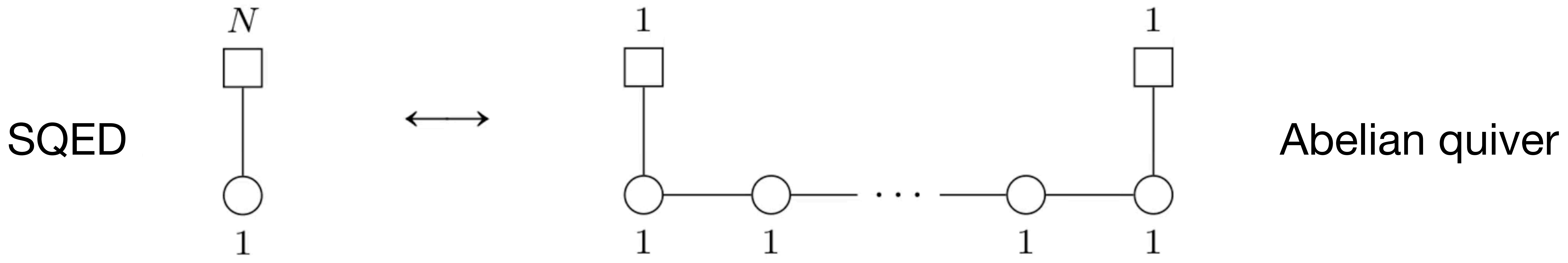


IIB	0	1	2	3	4	5	6	7	8	9
NS5	×	×	×	×	×	×				
D3	×	×	×				×			
D5	×	×	×					×	×	×
	$\leftarrow \mathbb{R}^{1,2} \rightarrow$			$\underbrace{\leftarrow \mathbb{R}^3_{3,4,5} \rightarrow}_{\circlearrowleft SU(2)_C}$				$\underbrace{\leftarrow \mathbb{R}^3_{7,8,9} \rightarrow}_{\circlearrowleft SU(2)_H}$		

IIB construction and Hanany-Witten move

Baby example: Abelian mirror pair





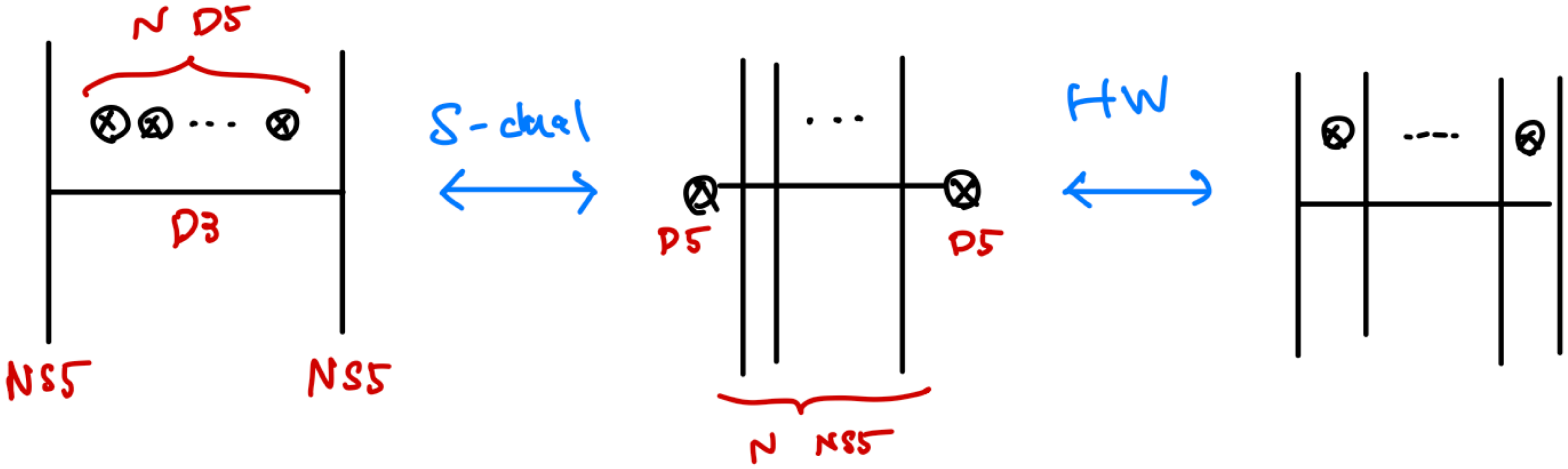
- $G_f = \mathrm{PSU}(N)$
- $G_t = \mathrm{U}(1)$

- $G_f = \mathrm{U}(1)$
- $G_t = \mathrm{PSU}(N)$

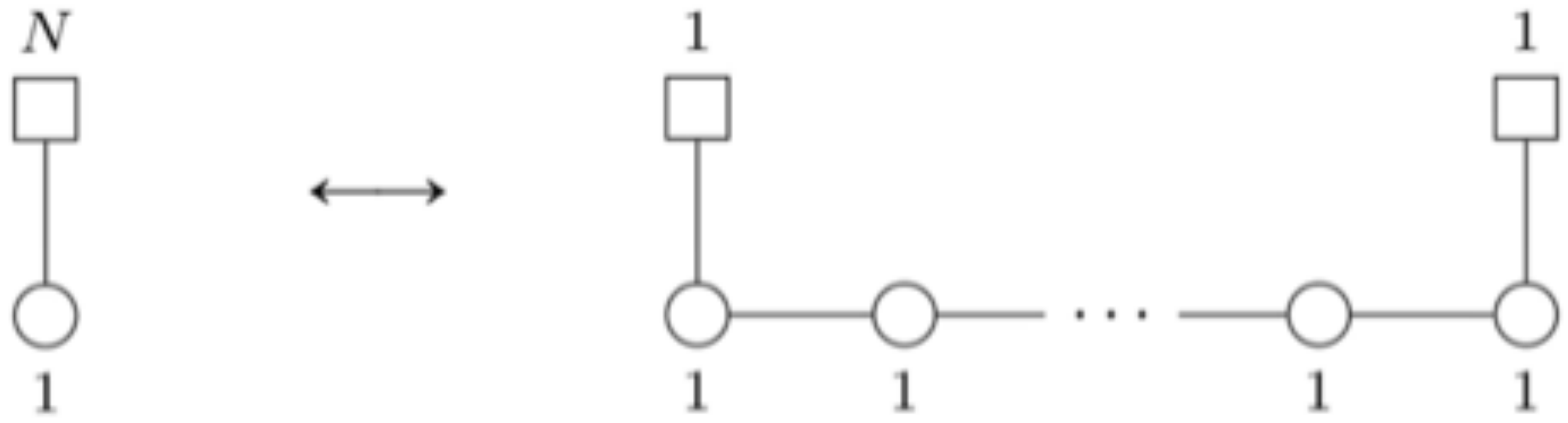
IIB construction and Hanany-Witten move

2

Baby example: Abelian mirror pair



SQED



Abelian quiver

Higgs: minimal nilpotent orbit of type A_{N-1}

Coulomb: ALE space $\mathbb{C}^2/\mathbb{Z}_N$

Coulomb: minimal nilpotent orbit of type A_{N-1}

Higgs: ALE space $\mathbb{C}^2/\mathbb{Z}_N$

Line defects in mirror symmetry

Natural mirror symmetry between Wilson lines and vortex lines   Assel-Gomis

Wilson lines

via holonomy $\sim P\exp\int\rho\left(A_t\right)\mathrm{d}t$

coupling 1d SQM w/ fermionic hypers

vortex lines

via singular solutions to BPS eqs

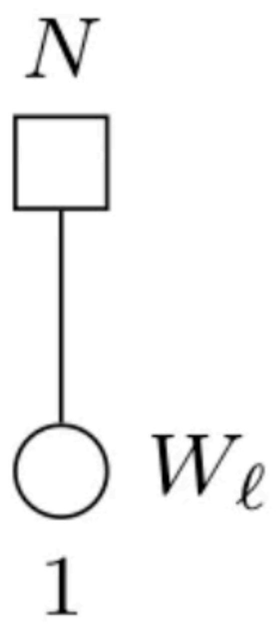
coupling 1d SQM w/ bosonic chirals

For mirror symmetry

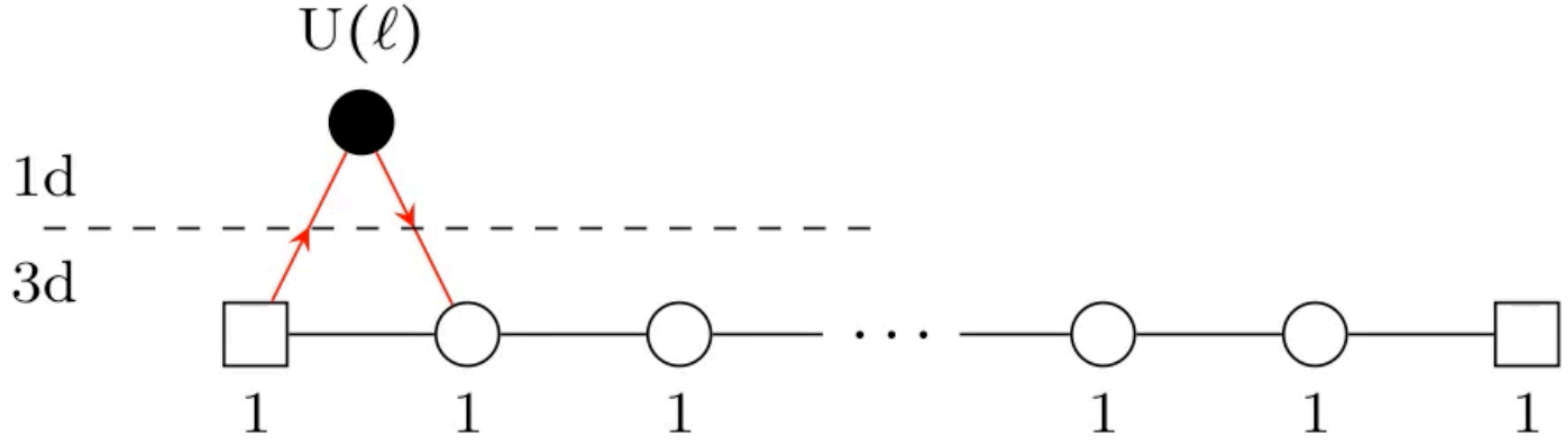
non-trivial: flavor vortex lines $\longleftrightarrow$ gauge Wilson lines

Line defects in mirror symmetry

Very basic example: Abelian mirror pair

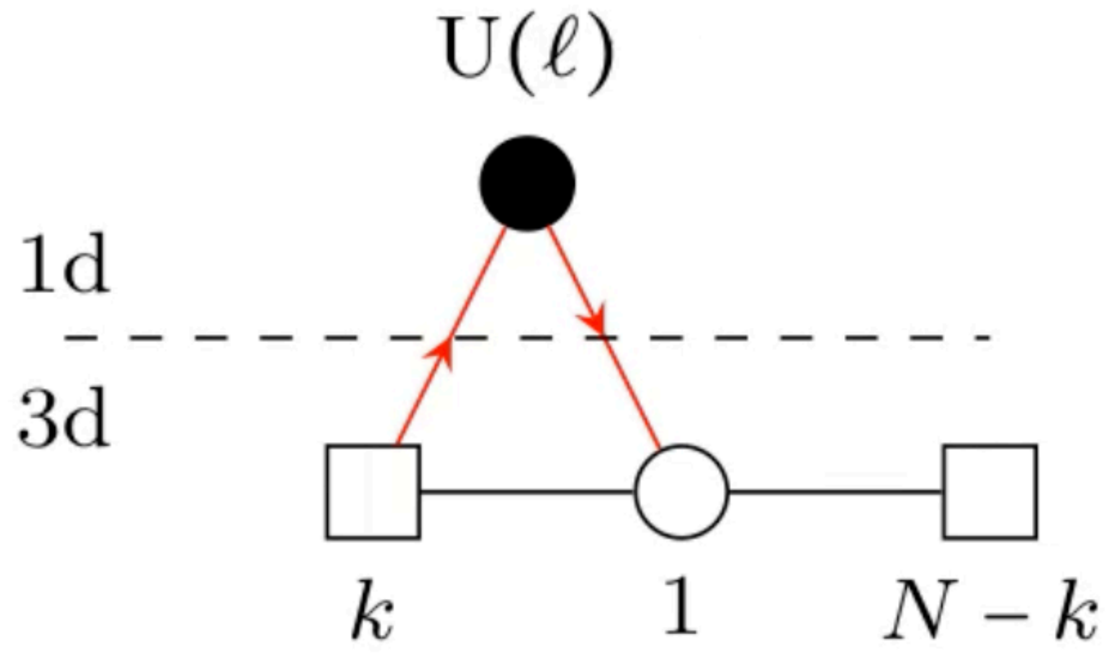


$\longleftrightarrow$

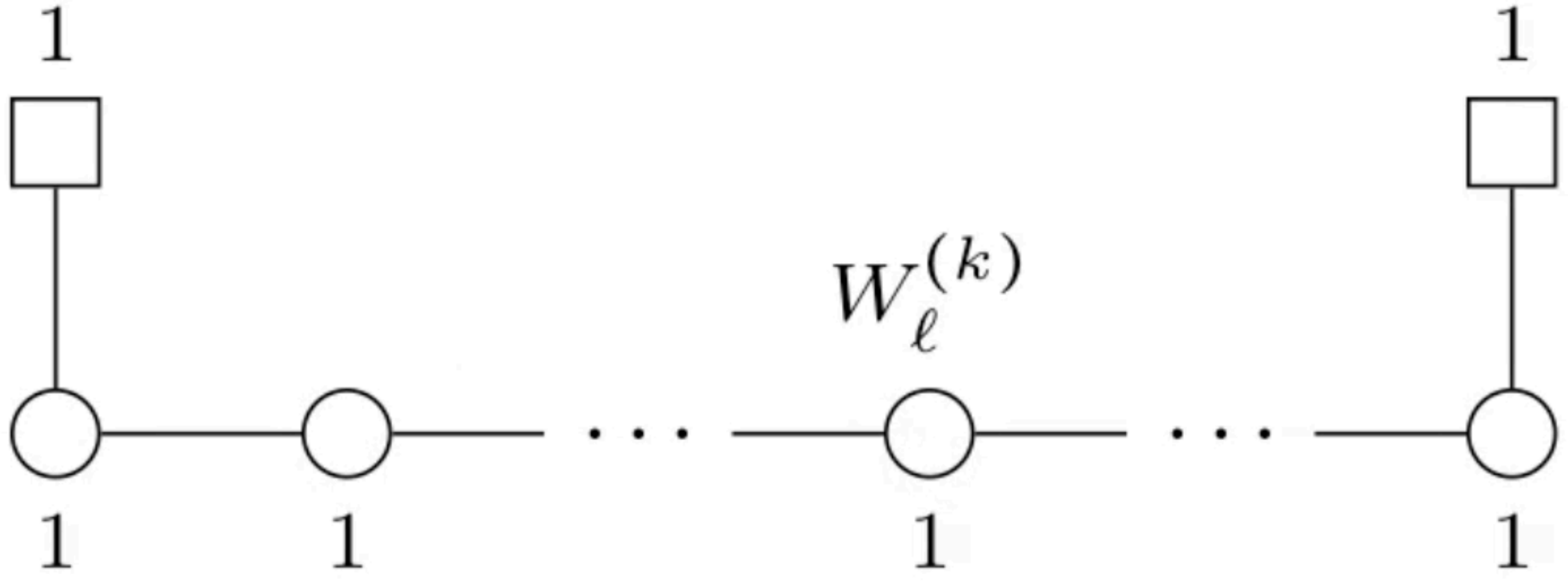


lattice of inequivalent Wilson lines : $\mathbb{Z}$

lattice of inequivalent Vortex lines : $\mathbb{Z}$



$\longleftrightarrow$



lattice of inequivalent Vortex lines : $\dfrac{\mathbb{Z}^N}{\mathbb{Z}(1,\ldots,1)}$

lattice of inequivalent Wilson lines : $\dfrac{\mathbb{Z}^N}{\mathbb{Z}(1,\ldots,1)}$

Symmetries for 3d N=4 theories

Generalized (higher-form) symmetries

- 0-form global symmetries G

  Flavor symmetry (Higgs branch isometry)

  Topological symmetry (Coulomb branch isometry)

- 1-form symmetries $\Gamma^{(1)}$: restrict to discrete groups $\Gamma \cong \prod \mathbb{Z}_{k_I}$

  Gauging a discrete 0 -form symmetry → discrete 1 -form symmetry

  As 0 -form symmetry comes in two "types" → 1-form comes in two types

- 2-group structure   Tachikawa, Benini-Cordova-Hsin

  Sometimes 1-form and 0-form symmetry interact

3d mirror symmetry with 1-form symmetry

Our motivation is to find 3d mirror pair with non-trivial 1-form symmetries

Symmetry analysis

- 0-form symmetries:

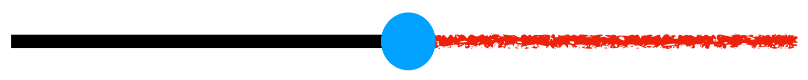
    Hilbert series, supersymmetric index

    G_t : balanced set of nodes and monopole generators

    G_f : gauge invariant operators

- 1-form and 2-group symmetries

    Line defects and their equivalence classes

1-form  Center sym of flavor algebra

Local op

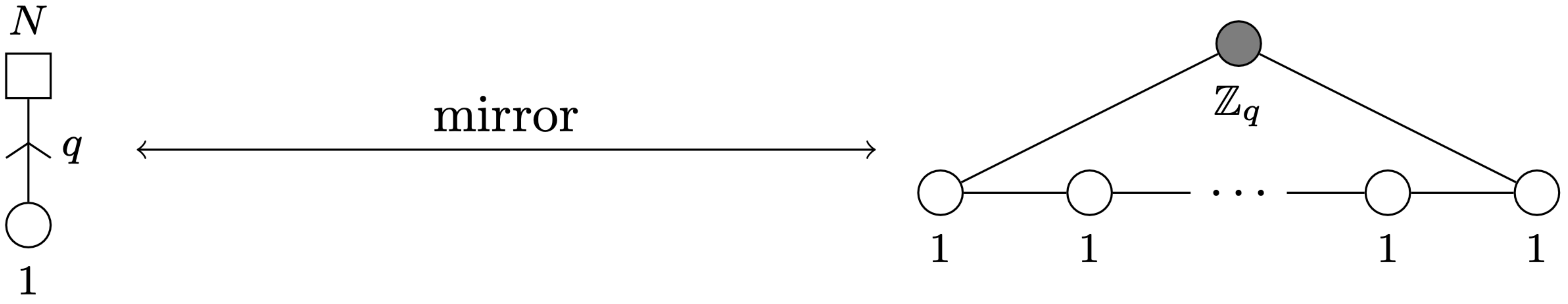
Abelian mirror symmetry with 1-form symmetry

Baby example: Abelian mirror pair

SN-Sperling-Wang-Zhong

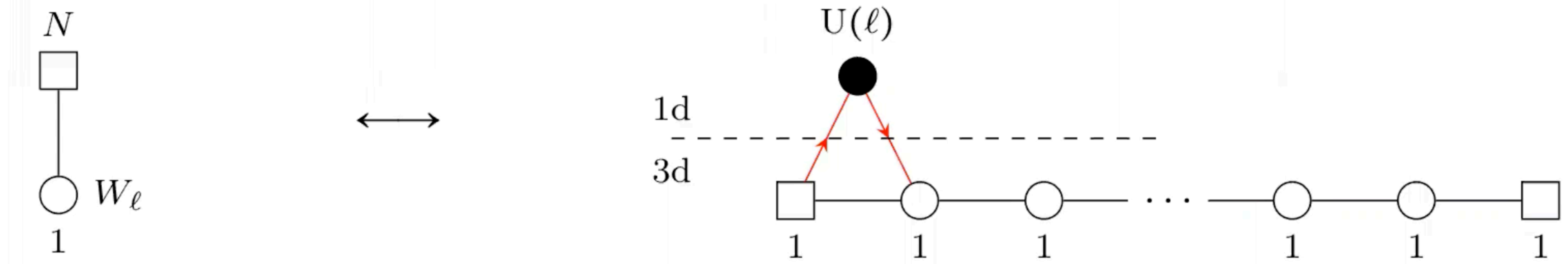
SQED with N hypers of charge q

gauge $\mathbb{Z}_q \subset \mathrm{U}(1)_f$



gauge $\mathbb{Z}_q \subset \mathrm{U}(1)_t$

1-form symmetry



Wilson line with charge q

Flavor vortex line with charge q can end

2-group structure

Short exact sequence: $0 \to \Gamma^{[1]} \to \mathcal{E} \to \mathcal{Z} = \mathbb{Z}_N \to 0$ splits iff $\gcd(q, N) = 1$

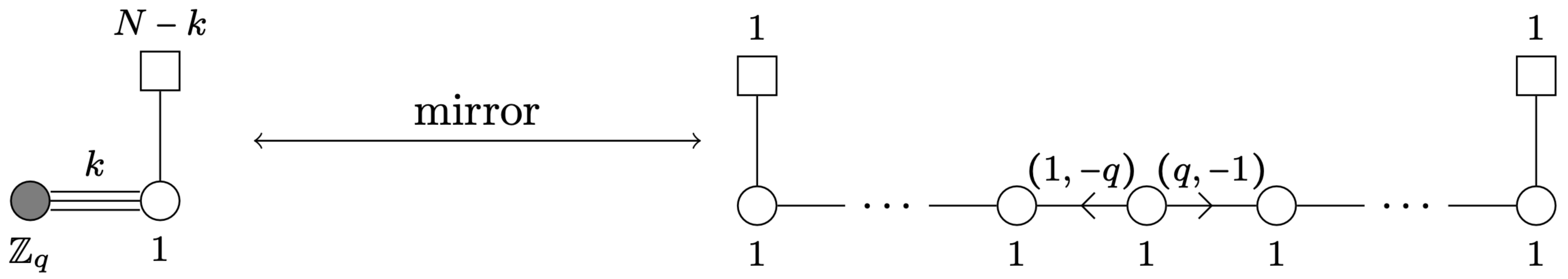
- If $\gcd(q, N) > 1$: *potential* 2-group structure between $\Gamma^{[1]}$ and G_f

Abelian mirror symmetry with 1-form symmetry

Baby example: Abelian mirror pair

SQED with N hypers of charge q

gauge $\mathbb{Z}_q \subset \mathrm{U}(1)_f$

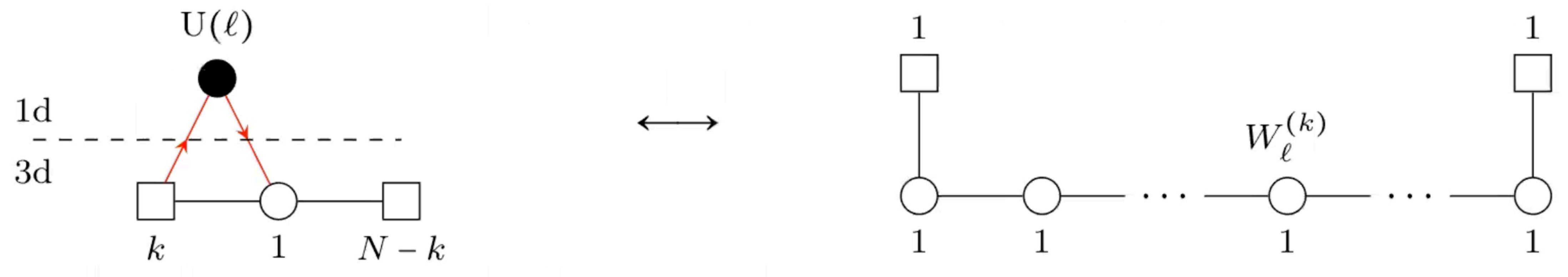


0-form symmetry

$$\mathrm{PSU}(N) \to \frac{\mathrm{SU}(k)\times \mathrm{U}(1)\times \mathrm{SU}(N-k)}{\mathbb{Z}_k \times \mathbb{Z}_{N-k}}$$

$\mathbb{Z}_k$ generated by $(e^{\frac{2\pi i}{k}}, e^{-\frac{2\pi i q}{k}}, 1)$ and $\mathbb{Z}_{N-k}$ generated by $(1, e^{\frac{2\pi i q}{N-k}}, e^{\frac{2\pi i}{N-k}})$

1-form symmetry



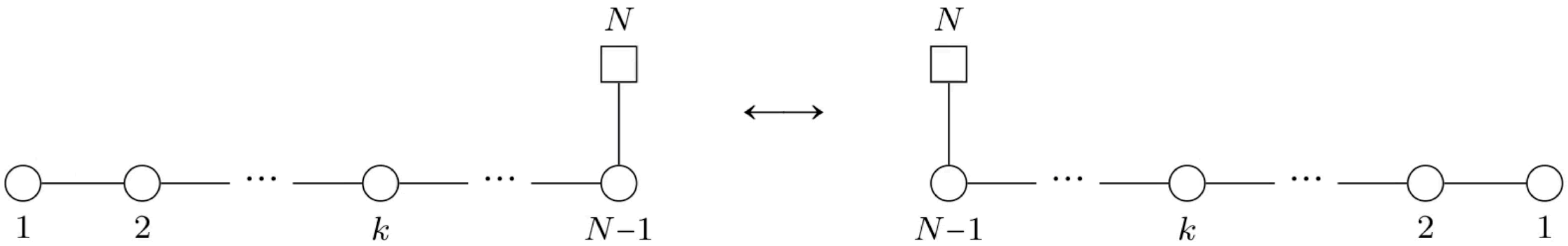
Flavor vortex line with charge q can end

Wilson line with charge q

Non-abelian mirror symmetry with 1-form symmetry

Story is analogous for SQCD and its mirror

More interesting example is $T[SU(N)]$ theory: Self-Mirror Gaiotto-Witten

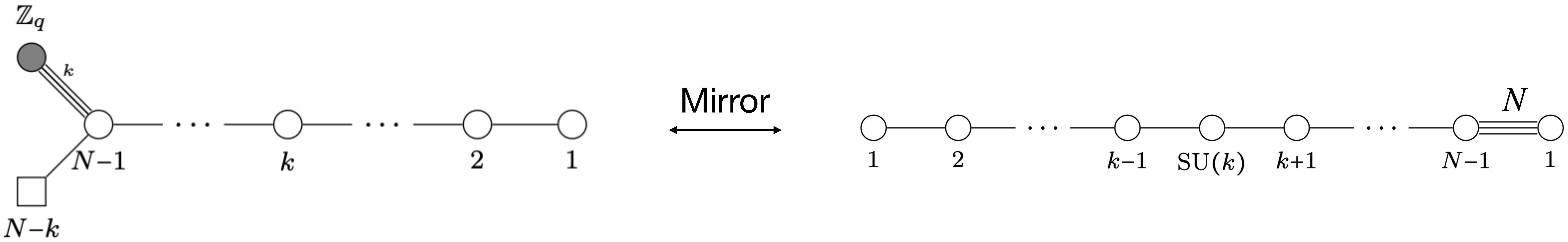


All other $T_\rho^\sigma[\mathrm{SU}(N)]$ obtained via Higgs/Coulomb branch Higgsings

Global symmetry $\mathrm{PSU}(N)_t \times \mathrm{PSU}(N)_f$

Question: what happens if a $\mathbb{Z}_q \subset \mathrm{PSU}(N)_{t,f}$ is gauged?

$T[SU(N)]$ theory with 1-form symmetry



0-form symmetry: $\mathrm{PSU}(N) \to \dfrac{\mathrm{SU}(k)\times \mathrm{U}(1)\times \mathrm{SU}(N-k)}{\mathbb{Z}_k \times \mathbb{Z}_{N-k}}$

$\mathbb{Z}_k$ generated by $(e^{\frac{2\pi \mathrm{i}}{k}},1,1)$ and $\mathbb{Z}_{N-k}$ generated by $(1, e^{\frac{2\pi \mathrm{i} q}{N-k}}, e^{\frac{2\pi \mathrm{i}}{N-k}})$

1-form symmetry: $\mathbb{Z}_q$ by construction

2-group symmetry:

$$0 \to \Gamma^{(1)} = \mathbb{Z}_q \to \mathcal{E} = \mathbb{Z}_{q\cdot N} \to \mathcal{Z} = \mathbb{Z}_N \to 0$$

which does not split whenever $\gcd(N,q) > 1$

Brief summary

- 3d mirror symmetry with 1-form symmetry can be obtained by gauging discrete 0 -form symmetries

Related work also by Bhardwaj-Bullimore-Ferrari-Schafer-Nameki

- We study many examples: $T_\rho^\sigma[\mathrm{SU}(N)]$

  D-type quivers

  C-type quivers

  Orthosymplectic quivers

- We explicitly check Hilbert series of Higgs and Coulomb branches

- We also analyze 0-form, 1-form, and 2-group structure

Magnetic quivers

Higgs branch

Misbelief

Higgs branch with 8 SUSY: Easy, classical, the same under dimensional reduction/lift

Higgs branches are hard!

Misbelief

Higgs branch with 8 SUSY: Easy, ~~classical, the same under dimensional~~ reduction/lift

Higgs branch can drastically change at certain points of parameter (moduli) space

This is because extra massless degree of freedom show up at strong coupling

6d : physics of tensionless strings

5d : physics of massless gauge instantons

4d : physics of non-Lagrangian theories

Very difficult to capture

Higgs branch of 6d N=(1,0) theory at finite and infinite-coupling

Higgs branch of 5d N=1 theory at infinite-coupling

Higgs branch of 4d N=2 non-Lagrangian theory: Argyres-Douglas, Minahan-Nemechansky

Magnetic quiver

Rely on understand of 3d N=4 Coulomb branches of Lagrangian theories.

Many quantum corrections, but under control!

Characterization of moduli space by physical quantities

Dimension

Global symmetry

Chiral ring relation

Hilbert series: monopole formula

Cabrera-Hanany-Yagi

Cabrera-Hanany-Sperling

Many others

Magnetic quiver technique

Use brane to identify 3d N=4 theory

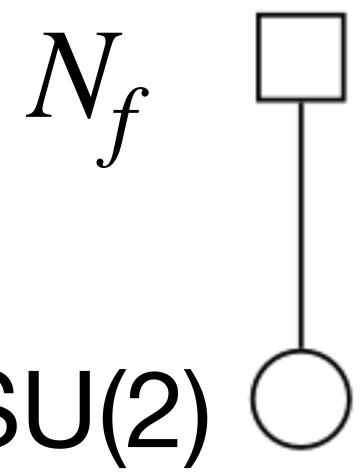
Higgs branch of difficult theory $=$ Coulomb branch of 3d N=4 theory

e.g. 6d N=(1,0) theory

5d N=1 theory at infinite-coupling

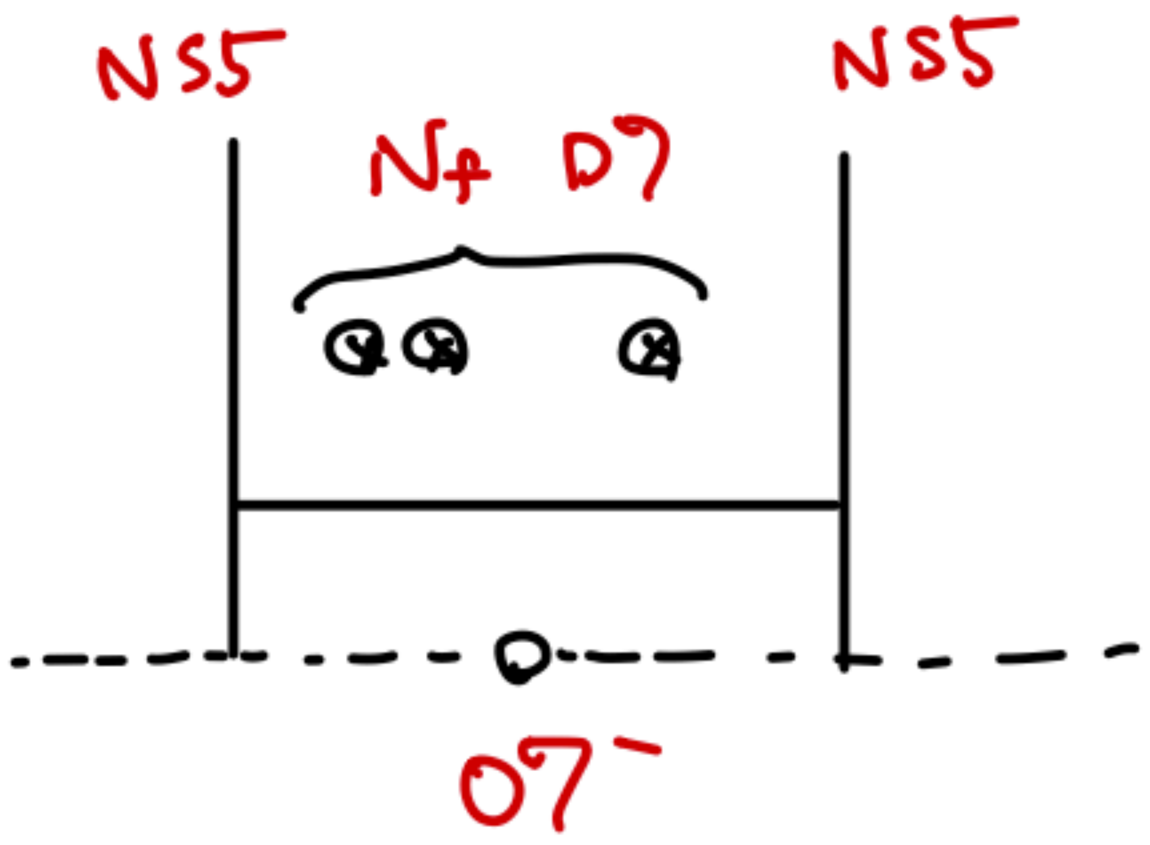
Magnetic quiver

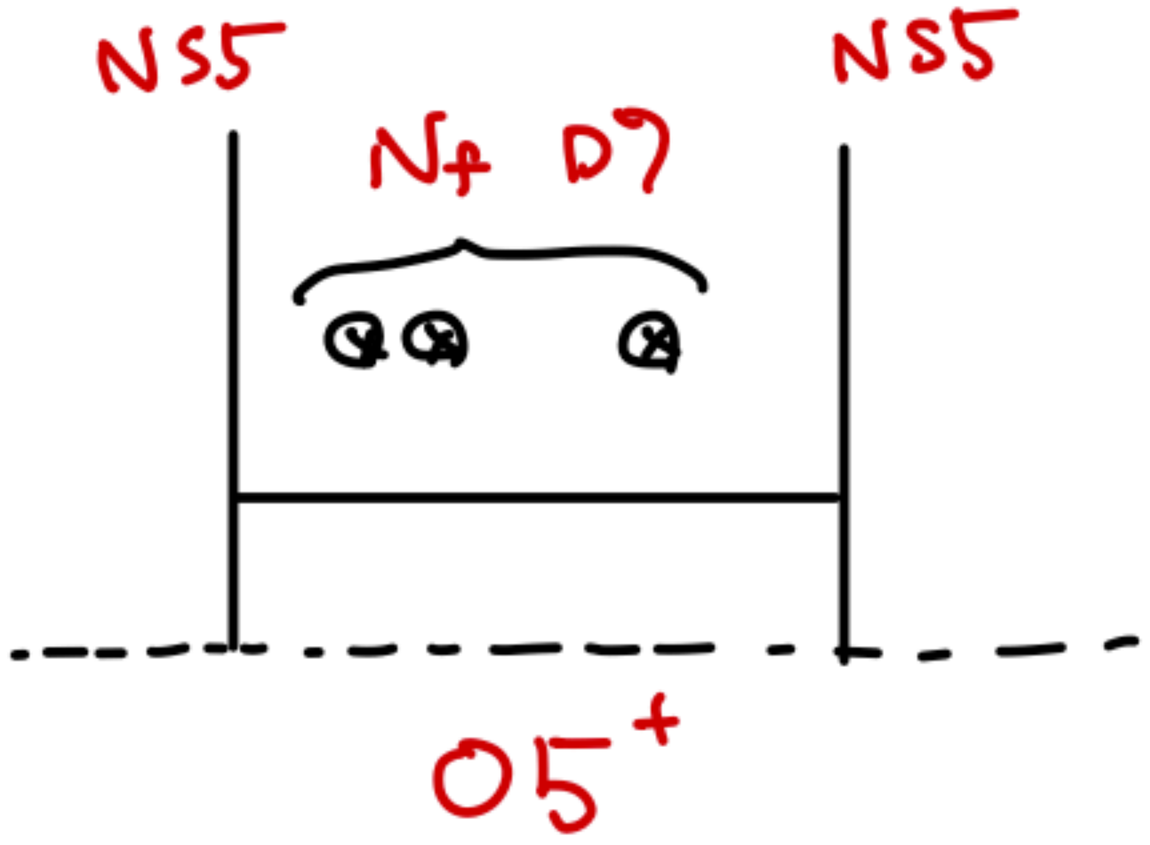
Consider 5d N=1 SU(2) gauge theory with N_f flavors at infinite-coupling



Well-known to have E_{N_f+1} global symmetry

There are 2 brane constructions: $O7^-$ or. $O5^+$





Type IIB	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS5	×	×	×	×	×	×				
D5/O5	×	×	×	×	×		×			
D7/O7	×	×	×	×	×			×	×	×

Magnetic quiver

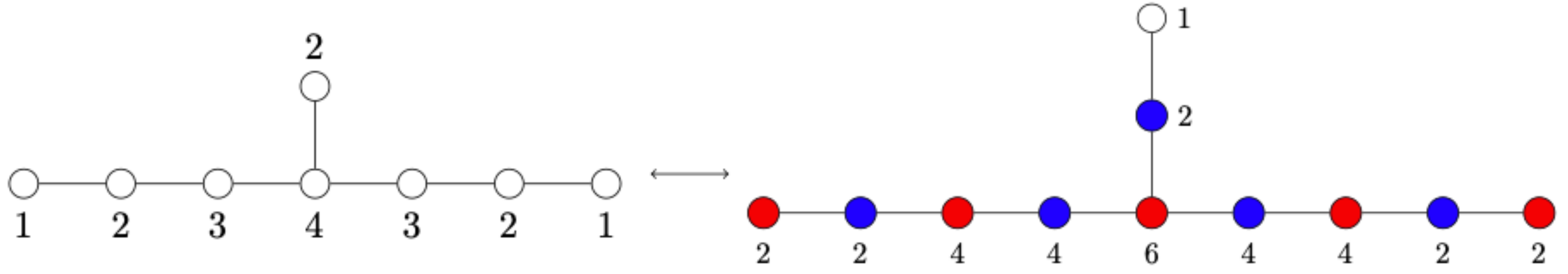
Correspondingly, there are 2 magnetic quivers Bourget-Grimminger-Hanany-Sperling-Zhong

For $O7^-$ construction, one can use Sen's decomposition: $O7^- \rightarrow$ 2 D7

$\longrightarrow$ 3d N=4 unitary quiver theory

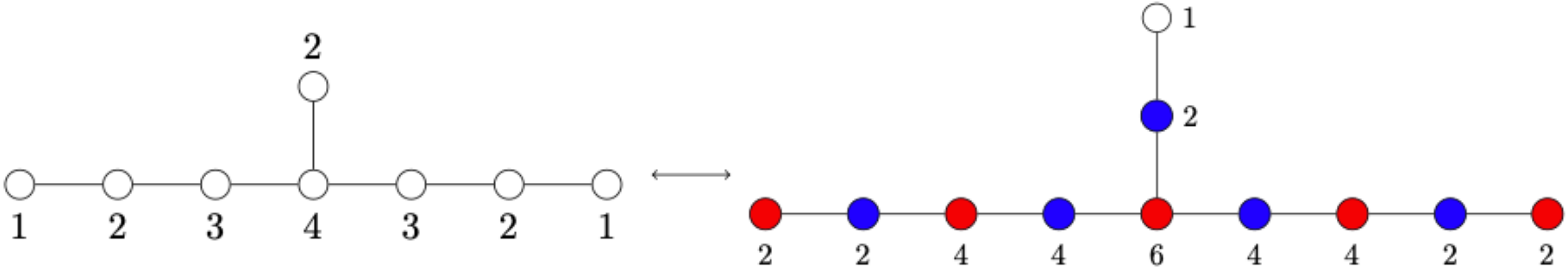
For $O5^+$ construction $\longrightarrow$ 3d N=4 orthosymplectic quiver theory

$N_f = 6$ Case



The same Coulomb branch: the E_7 minimal nilpotent orbit $\overline{\mathcal{O}}^{\mathfrak{e}_7}_{\text{min}}$

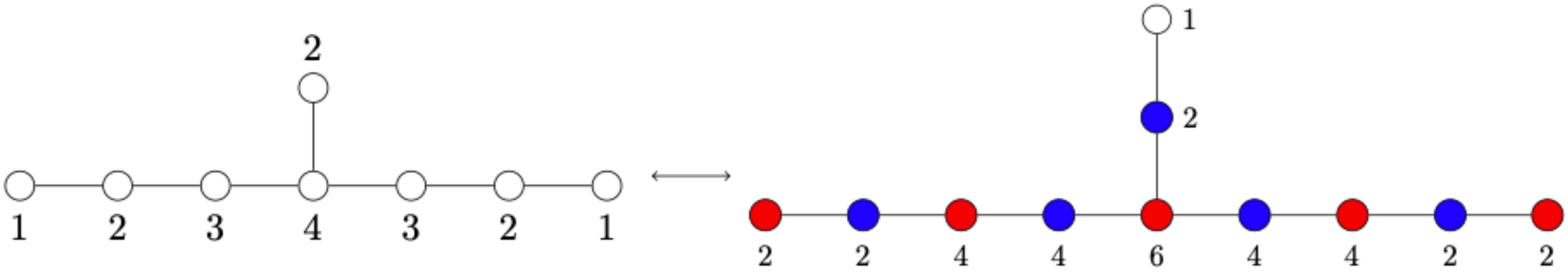
Magnetic quiver



Not only the same Coulomb branch: the E_7 minimal nilpotent orbit $\overline{\mathcal{O}}^{\mathfrak{e}_7}_{\text{min}}$

But also Higgs branch Hilbert series agree!

Magnetic quiver



Not only the same Coulomb branch: the E_7 minimal nilpotent orbit $\overline{\mathcal{O}}^{\mathfrak{e}_7}_{\text{min}}$

But also Higgs branch Hilbert series agree!

SN-Sperling-Wang-Zhong

⟶ In fact, they are dual to each other

superconformal indices agree

Line operator spectrum agree

In fact, orthosymplectic quiver has manifest $\mathbb{Z}_2$ 1-form symmetry

We can identify the corresponding $\mathbb{Z}_2$ 1-form symmetry in unitary quiver

Conclusion

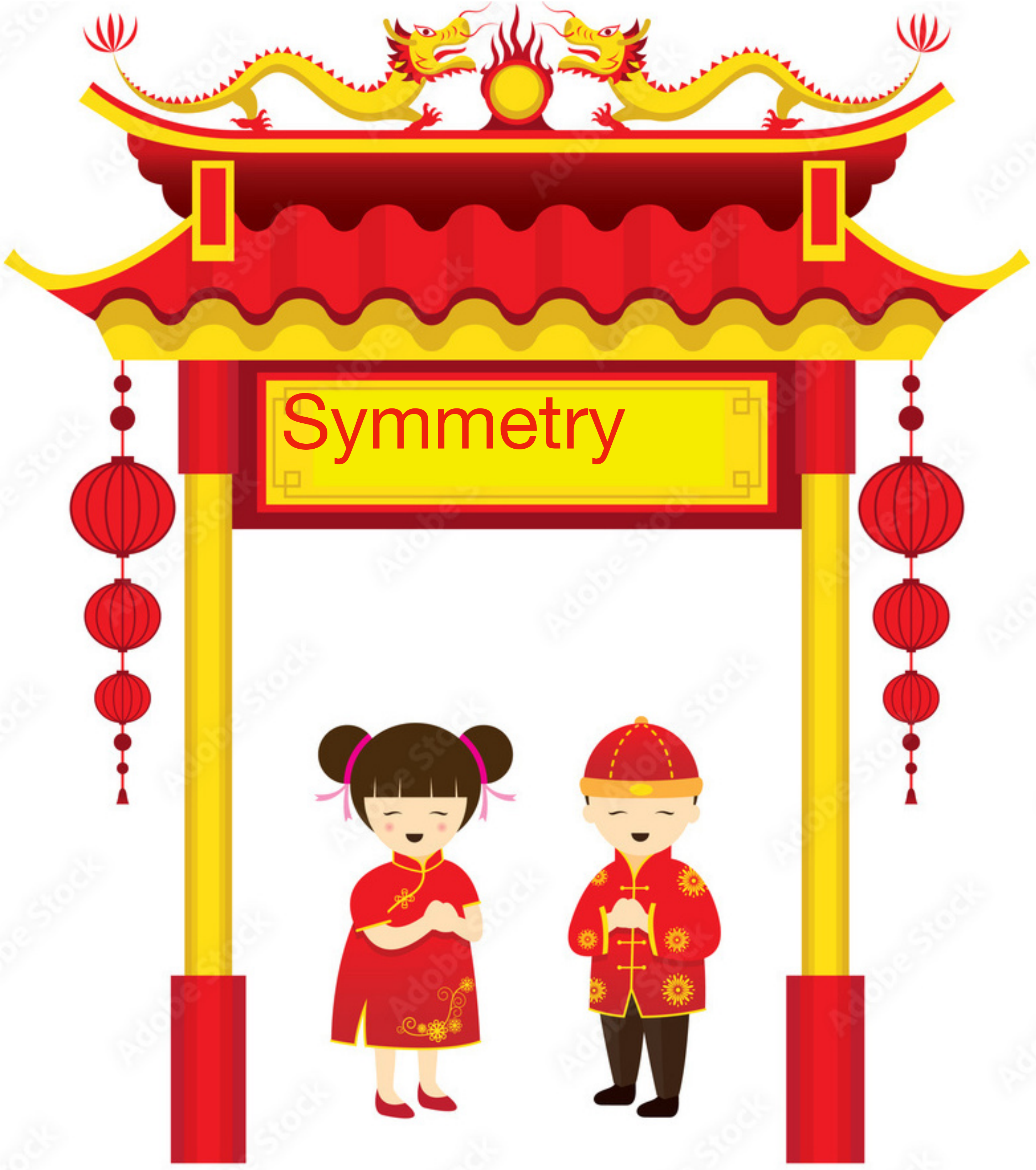
We study 3d mirror symmetry with 1-form symmetry

Study of magnetic quiver leads to new 3d N=4 duality

We just witness merely the tip of gigantic iceberg.

It's very active area of research.

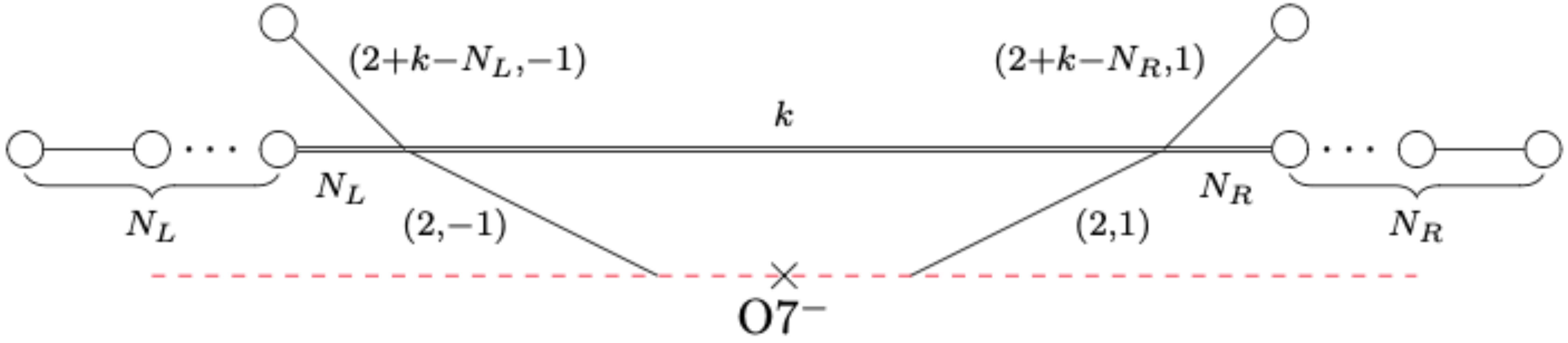
You are encouraged to explore this direction
and contribute to its progress!



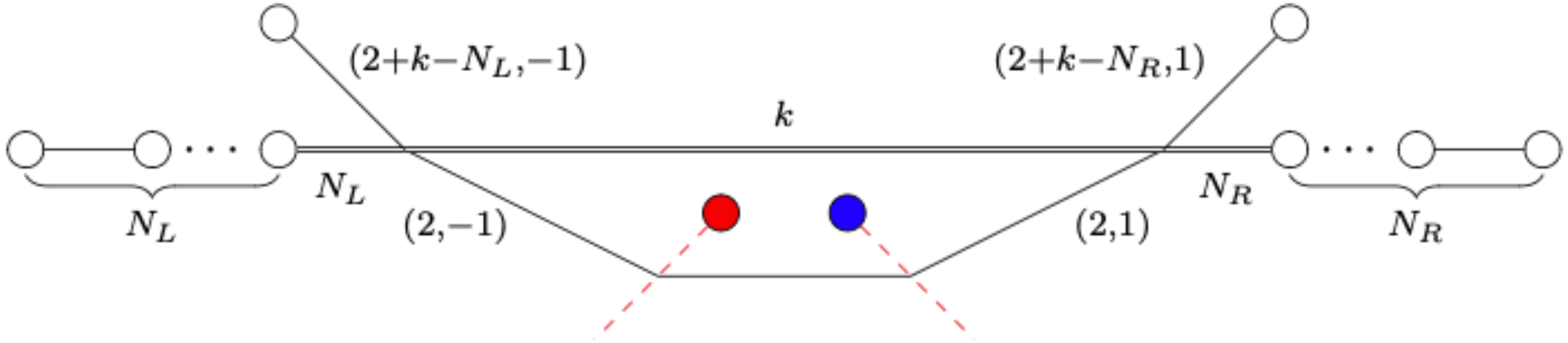
5-brane with O7^{-}

5-brane web for Sp(k) with $N_f = N_L + N_R$ fundamental flavours

Figures from [arXiv:2004.04082]
Bourget-Grimminger-Hanany-Sperling-Zhong

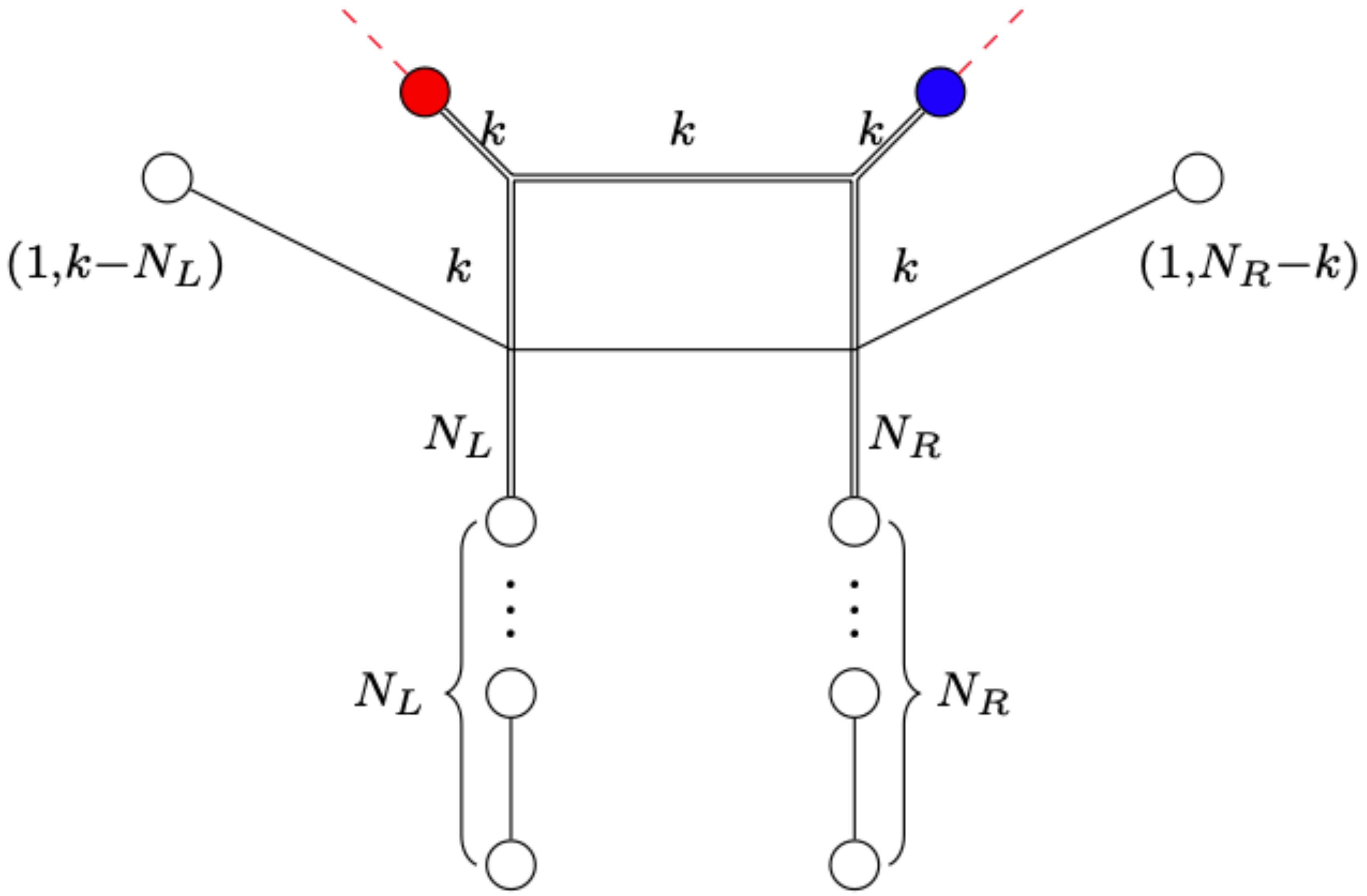


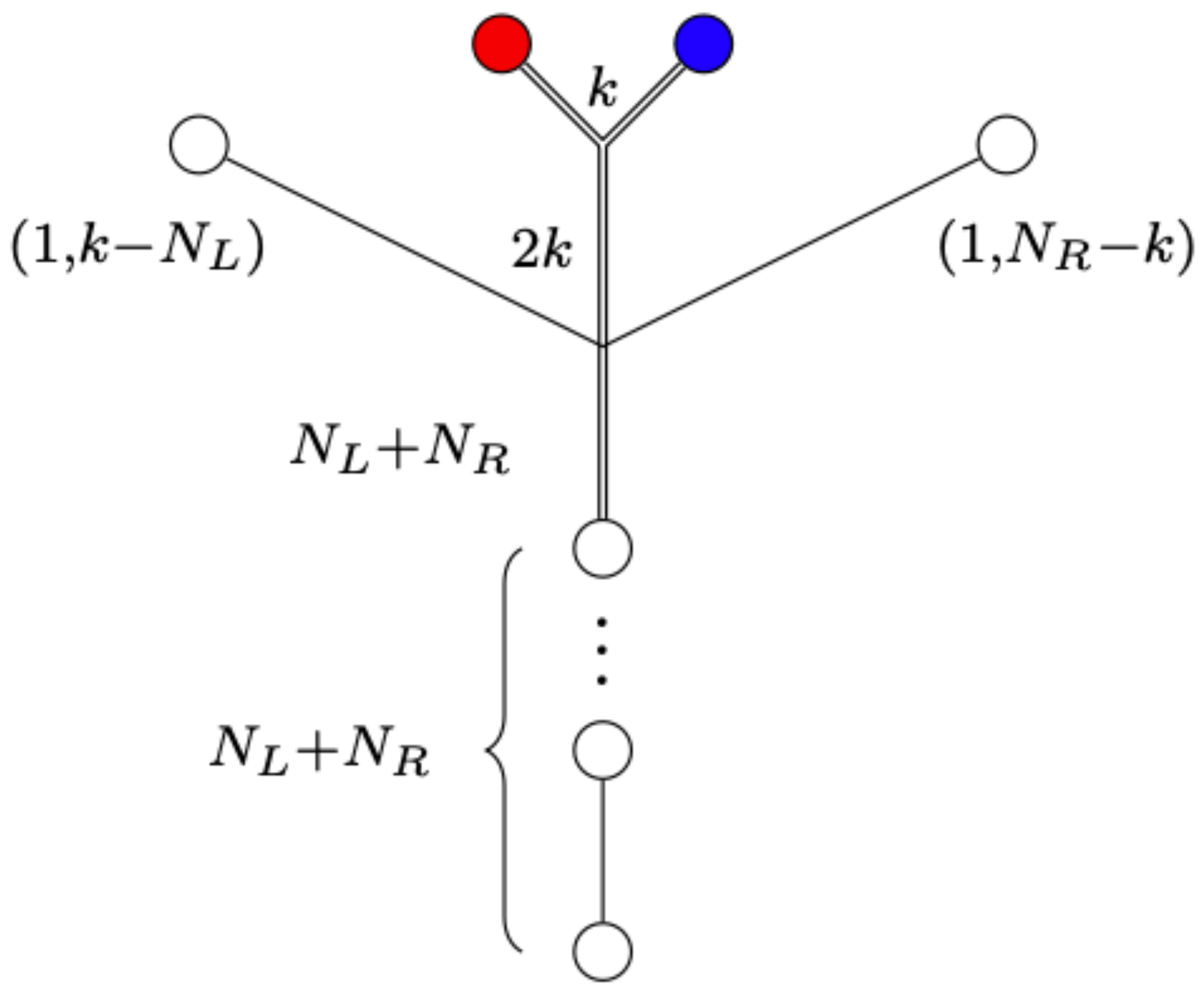
Sen's decomposition: O7^{-} resolved to [1,1] 7-brane + [1,-1] 7-brane



5-brane with O7^{-}

After Hanany-Witten, take infinite-coupling limit





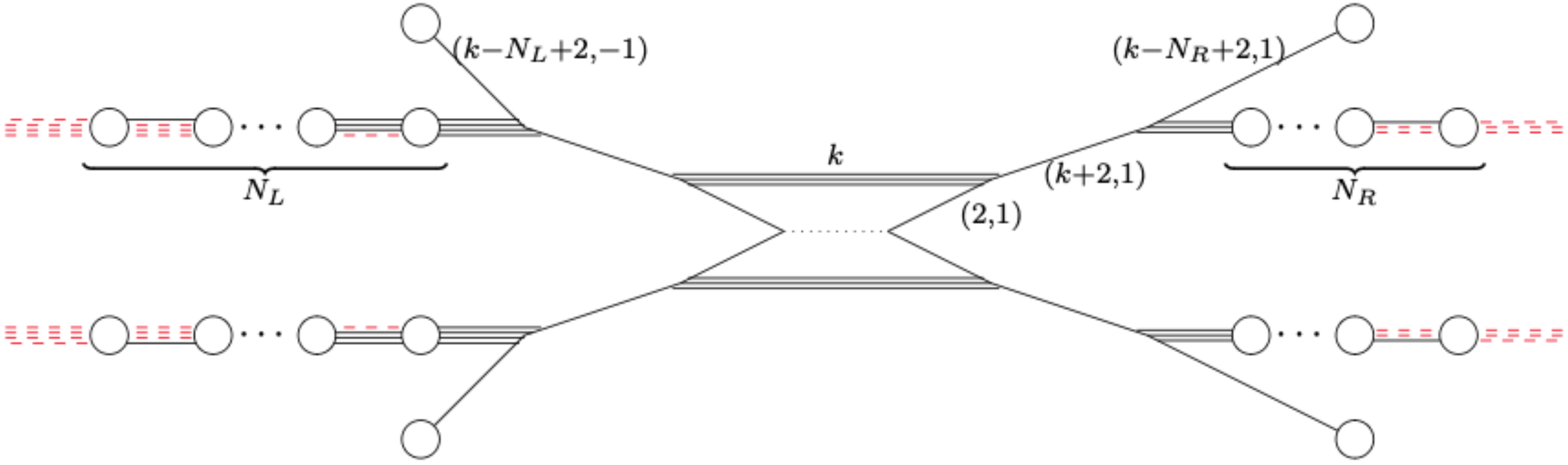
electric theory: SU(2) with N_f=6

magnetic gauge nodes
$\longleftrightarrow$ maximal subdivision

magnetic hypermultiplets
$\longleftrightarrow$ intersection number

5-brane with O5$^+$

5-brane web for Sp(k) with $N_f = N_L + N_R$ fundamental flavours



Higgs branch phase at infinite-coupling limit