Humble opinions for graduate study

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Abstract

As a faculty member with six years of experience at Fudan University, I have had the fortunate opportunity to observe and interact with students here. Gradually, I have come to understand the characteristics of Fudan students and the university system. While there have been instances of minor conflicts and disagreements with students, I have learned valuable lessons from these experiences. Therefore, drawing from my personal experiences, I present my humble opinions and advice to students who are pursuing graduate studies and research in theoretical (mathematical) physics. My goal in writing this is to assist students in having a more fulfilling research experience in graduate school, and to mitigate any potential misunderstandings that may arise due to a lack of communication. As a supervisor, I also write this as a reminder to myself.

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1 Prologue

If you are an undergraduate student in physics, I suppose that you are currently learning fundamental concepts in the field and finding it intriguing to observe how physics explains the behavior of Nature. As you progress in your studies, you may become captivated by the elegant and fascinating aspects of physics. As you approach graduation, you may wonder what path to take to make original contributions through research activity.

For graduate students, I presume that you have established a strong foundation in established physical concepts and wish to further your studies by engaging in research. The transition from learning to research can be challenging and uncertain for some students. It is normal for graduate students to experience a mixture of excitement and apprehension as they embark on this new chapter in their academic journey.

From my viewpoint, there is a gap between the learning and research aspects of the field, and some students are not fully aware of this difference. To facilitate this transition and support students in their research endeavors, I believe it is important to have a comprehensive understanding of the shift from learning to research. This can help undergraduate students make informed decisions about their future paths, and allow graduate students to embark on their research with confidence.

Having worked here, I have come to understand the characteristics and tendencies of Fudan students. It is unwise to demonstrate old farts "today's youths are..." by shunting the young off, but it is rather important to engage in constructive communication with the young. The advantage of the old is their wealth of experience. Therefore, I put forth my humble thoughts, drawn from my personal experiences and observations, hoping that they will be helpful to students who aspire to conduct valuable research during graduate studies.

There are many excellent resources available for guidance on graduate studies, including articles by renowned scholars [Ste87, Hue87, Ati08, Wei03]. Additionally, you may find valuable insights in Terry Tao's blog post "Career Advice" and 于品老师的在北大数院2023年开学典礼上的发言.

This article focuses specifically on graduate studies in theoretical and mathematical physics, with some aspects that can also be applied to mathematics. However, it is important to note that this article is based on my personal experiences and reflects my individual perspective. Graduate students have varied personalities and their goals in their studies

will depend on the individual. Please take this article as a personal viewpoint, and not as universal advice.

Disclaimer: All the pictures here are stolen from the Internet, which violates Copyright! But please don't accuse me! Also, I improve my phrasing by ChatGPT.

2 Before going into research

2.1 Building up basics

A solid grasp of the foundational subjects in physics is essential for conducting successful research in graduate studies. Thus, during undergraduate study, the first priority is to lay the foundations of basic physics. Building a strong foundation in physics can be achieved through basic methods such as attending lectures, taking thorough notes, reading textbooks, and solving exercises and homework assignments.

When you read a textbook, it is important to read not just what is written but what it means and what the author wants to express. During class, it can be sometimes more important to understand the key points and find relations with other fields, rather than mechanically taking notes from the blackboard or thinking about the derivation of each equation. (Remember that I do not mean to deny the importance of taking notes, and deriving equations.)

The following list includes examples of standard textbooks for reference, but it is unnecessary to read all of them. You may find alternative textbooks that better suit your learning style. To check your understanding, I would like to assess if you are able to answer the following questions without relying on any external sources.

Classical mechanics: For example, the standard textbooks would be [LL13, Gol11, Arn74].

- Describe both Lagrangian and Hamiltonian formalisms and how they can be used to derive the equations of motion for a system.
- What is the conserved quantity when a system has rotational symmetry?

Electrodynamics: For example, the standard textbooks would be [Jac99].

- Can you write down Maxwell equations? Which symmetry do Maxwell equations enjoy?
- Describe the gauge transformation and its consequences in electrodynamics.

Statistical mechanics: For example, the standard textbooks would be [Pat16].

- Derive the equipartition theorem and use it to determine the temperature dependence of specific heat.
- What are the definitions of the microcanonical, canonical, and grand canonical ensembles? How are they different?

Quantum mechanics: For example, the standard textbooks would be [SC95, Sak06].

- Can you explain how quantum mechanics is able to explain phenomena that classical mechanics cannot?
- Can you write the Hamiltonian of harmonic oscillator and quantize it? Can you find the full spectrum of the theory?

General relativity: For example, the standard textbooks would be [Dir96, Wal10, Car19].

- What are the main principles based on which Einstein constructed general relativity?
- Can you write down the Einstein equations? Can you derive it from the Einstein-Hilbert action?

Mathematical methods in physics:

- Did you learn linear algebra, calculus, complex analysis, Fourier transformation, and special functions? Can you explain one physical phenomenon using the above mathematical subject?
- Find the eigenvalues and eigenvectors of the following matrix and explain the connection to Fourier transformation.

$$\begin{pmatrix} 2 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & -1 & 2 & -1 & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & -1 & 2 & -1 \\ & & & & & -1 & 2 \end{pmatrix}$$

Quantum field theory: For example, the standard textbooks would be [PS95, Sch13].

- Can you write down the Dirac equation? Can you explain why the Dirac equation can be regarded as the "square root" of the Klain-Gordon equation?
- What is the definition of the β-function? Why do we need to consider the β-function in the microscopic world?

2.2 Asking questions and beyond

During a lecture, it is important to actively participate and ask questions. The advantage of attending class is the opportunity to interact directly with the lecturer, ask questions, and gain a deeper understanding of the content. If you find yourself struggling to understand a certain concept, it is likely that other students may also be stumbling at the same point.

Asking questions not only helps to clarify the concept, but it also contributes to a lively and engaging classroom atmosphere.

However, it is important to note that not all questions are created equal. For instance, asking about the derivation of an identity without being aware of an error in your calculation can be considered a lackluster or boring question. (See §4.3.) On the other hand, a sharp question that gets to the heart of the concept can lead to a more profound understanding. An example of a particularly insightful question I received during a quantum field theory course was the following:

> In the canonical quantization, we take equal-time
>
> $$[\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})$$
> $$[\phi(t,\boldsymbol{x}),\phi(t,\boldsymbol{y})] = 0 = [\pi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})]\ .$$
>
> If we take a Lorentz boost, the spatial slice will change so that the meaning of "equal-time" changes. Is the canonical quantization independent of a choice of a frame of reference?

When this question was asked, I was thrilled as it is a very intrinsic question, that shows the student is thinking critically about the content. (I leave it as an exercise to find the answer to this question here.) However, if you keep asking naive questions as if you were ordering a food delivery (外卖), thinking that if you order a question, the supervisor or the lecturer will deliver the answer, you will never acquire the ability to understand a concept and subject on your own. Therefore, as you learn more, it is important to ask deeper and more meaningful questions. To increase the level of your questioning, it is advisable that once you come up with a question, you ask the question to yourself. Can you find an answer by yourself? Is there a simple answer to your question? This process will sharpen your ability to find a good question.

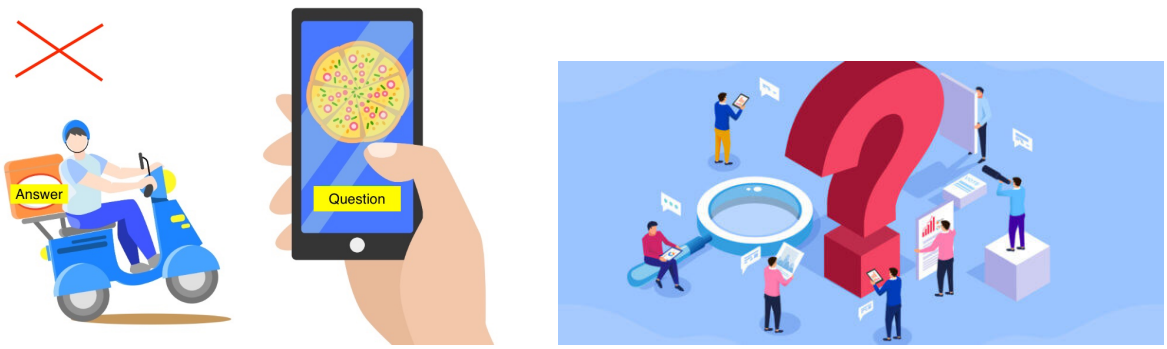

Figure 1: Instead of keep asking naive questions, it is recommended to examine your questions from multiple angles. To find a way out, it is important to confront the question and stay with it.

Moreover, I wish students to appreciate the situation that they do not understand. During the learning process (and moreover research), there will be many concepts and ideas that

may be difficult to grasp. It is natural to feel uncomfortable when faced with what you do not understand, and some may be tempted to quickly forget about it. However, when it comes to understanding difficult concepts, it is important to not dismiss what you do not understand. Instead, keep it in the back of your mind and allow it to slowly grow over time. Through this process, you may be able to verbalize what you do not understand, and come into contact with new knowledge or different perspectives that can lead to a clearer understanding. Regardless of the outcome, the process of nurturing what you do not understand in your mind will increase your sensitivity to the subject and develop the ability to make breakthroughs in your understanding. This is an essential characteristic of researchers in theoretical physics and mathematics, who accept that you may not understand or be able to solve a problem and instead choose to stay with it and face it for a while. I think that this is the most important process for finding a way out.

2.3 Finding your interest

Because research involves lots of difficulties, it would not last unless you are truly interested in the research field. Therefore, it is important to clarify your specific interests within theoretical physics. Theoretical physics is vast, and your interests may be found in unexpected places. So keep looking for what truly excites your passion for research. (兴趣是最好的老师.)

Figure 2: Make full use of the library, as it offers a priceless opportunity to discover a wealth of valuable books.

To this end, I recommend that students visit the library and peruse its collection of books. The math department library (光华楼12楼), in particular, has an extensive selection of valuable books. (Sorry about the unfortunate situation of the library at Physics Department :(.) It is a valuable and precious experience for students to see many interesting books in front of them at the library and realize that there are such deep theories, so physics is so broad, and so much you do not know. In this way, students can understand the connections between different fields in theoretical physics. By asking yourself which book you want to read and understand, you can uncover your interests.

In AIP Oral History and AMS Notices, you can find insightful interviews with renowned physicists and mathematicians, including Weinberg, Witten, and many others. These interviews offer a precious opportunity to gain an understanding of their research motivations and the atmosphere in which they work, without getting bogged down in technical details. Additionally, one can observe the development of their career and research.

Once you narrow down your interest, you can explore research papers on arXiv. Although it will be challenging to comprehend technical details, it is still valuable to identify papers that you would like to understand. Start by highlighting keywords and concepts and looking into their definitions and explanations. This is the first step toward research and will help you chart your potential research path in the future. It is good to aspire to the standards of researchers you admire and papers you want to understand, and use this as a guiding principle when selecting your graduate studies.

2.4 Choosing supervisor and research group

After you find your interests, I would suggest students to carefully consider their goals and expectations for graduate school before selecting their advisor. While compatibility between individuals is an important factor, it is also crucial to consider your interest and the advisor's approach to instruction and academic rigor. In the end, it is ultimately up to the student to make an informed decision.

At Fudan University, some students select an advisor during their freshman or sophomore year without fully comprehending the advisor's research focus. It is not easy for undergraduate students to accurately predict their future research direction, and it could be dangerous to blindly fix your interest and direction too early. It is important to be open-mind and flexible. Don't hesitate to change your research area and advisor if you find that it is not a good fit during your undergraduate or master's studies.

Research and supervising experience: Assessing the research and supervising experience of a potential mentor or advisor is crucial. Review their previous accomplishments in the field and check for recent activity at arXiv, inspire, and Google Scholar. (My record can be found at arXiv, inspire, and Google Scholar.) Examine their research networks, and assess the publications and citation records of students they have supervised. (See my website.) While numbers can vary greatly depending on the field and research style, it is important to base judgments on objective data.

Group atmosphere. Opinions of students and others: Prior to selecting a research group, it is important to contact current students and postdocs in the group. It is also a good idea to ask other faculty members, especially those who have no direct stake in the group. These conversations can provide valuable insights and help you envision yourself working within the group. Remember that the ability to perform research can be different from the ability to supervise students. Also, even if an advisor is active in research, s/he may have

too many students to provide sufficient guidance and support to their students due to the large group size.

Research strategy: It is important to consider the consistency and alignment of a potential mentor or advisor's research strategy with your own goals and future projects. Assess the potential for future growth and development in the field, and imagine how it relates to your own research aspirations, and decide if it aligns with your desired direction.

3 Attitude toward research

3.1 Difference between research and coursework

You learn an established theory in the coursework while you have to establish a new result in research. Therefore, research is not a simple extension of the coursework. As explained below, to establish new results, you must examine numerous examples, perform extensive computations, and thoroughly scrutinize the results. It is best to view coursework and research as distinct entities. Actually, the transition from passive learning to active research requires a drastic change in mindset, as research demands a level of creativity and dedicated engagement.

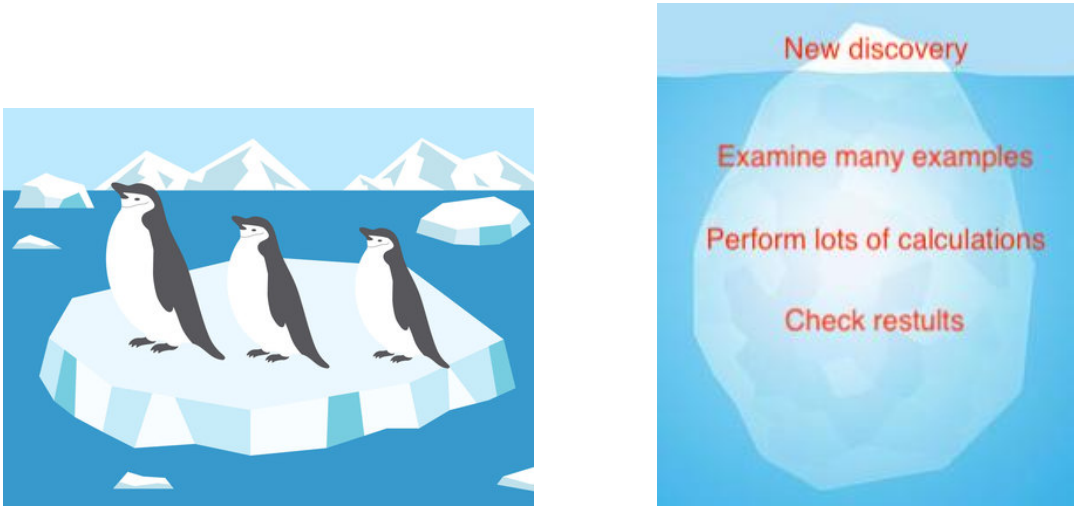

Figure 3: In coursework, you only see the tip of the iceberg, but there is a vast amount of hard work hidden beneath the surface. This is what research demands.

Conducting research in theoretical (mathematical) physics requires a significant investment of time and patience. To solve problems in theoretical (mathematical) physics, it is important to be "slow", "steady" and "calm". (研究要踏踏实实，兢兢业业.) I want to emphasize that we should never be in a hurry to solve or think about problems. It is not surprising to take years to get satisfactory results in this field, making it a challenging and

often frustrating process. It is crucial to be optimistic and to maintain a strong-minded and persistent attitude, not giving up easily when obstacles arise.

3.2 Overcoming psychological problem

It is common to experience periods of stress or intellectual insecurity once you start your research, particularly during the first year or two.

> The first year or two of research is the most difficult. There is so much to learn. One struggles unsuccessfully with small problems and one has serious doubts about one's ability to prove anything interesting. I went through such a period in my second year of research, and Jean-Pierre Serre, perhaps the outstanding mathematician of my generation, told me that he too had contemplated giving up at one stage.
>
> Only the mediocre are supremely confident of their ability. The better you are, the higher the standards you set yourself –you can see beyond your immediate reach. – Sir Michael Atiyah [Ati08]

I highly recommend that you start by reading Dorsa Amir's blog post on maintaining mental health during graduate study. As noted, the transition from coursework to research often requires a shift in mindset and can be challenging, even from a psychological perspective. However, with some simple strategies, it is possible to minimize these difficulties and maintain a healthy outlook during this period of transition.

Step-by-step: It is more likely that students who are ambitious, proud, serious, and motivated, have a mental problem such as impostor syndrome, or symptoms of depression. In fact, from my observation, these students have set overly high (research) goals, and they feel they are not meeting their own ideals, thinking "I should be able to do more but I'm not!". Setting lofty goals may be simply daunting, but progress can be achieved only through incremental steps. By consistently making small but meaningful efforts, individuals can gradually surpass their previous limitations and attain personal growth and development. The key is to approach the goal through a series of manageable, deliberate steps rather than trying to reach it all at once. (千里之行，始于足下.) (君子之道，辟如行远必自迩，辟如登高必自卑.)

Regular and healthy life: Leading a consistent lifestyle and getting sufficient sleep can positively impact your mental health. Exercise and hobbies can enhance both mental and physical well-being. To maintain mental well-being, it is recommended to avoid comparing yourself to others, and to steer clear of overly competitive people. If experiencing symptoms of depression, seek support from a trusted individual.

Changing environment: Unfortunately, I have noticed that some students are negatively influenced by the grade-supremacy assessment system and 内卷文化, leading to a narrow-minded focus on superficial aspects. The current Chinese educational system, which prioritizes grades and emphasizes competitiveness, can limit the perspectives of students. （人比人，气死人） This causes them to neglect important learning processes and opportunities, and it can hinder creative and critical thinking skills. The narrow perspective harms their research abilities and overall growth. If you feel that you are being negatively affected by this type of culture, I would suggest considering studying abroad. Exposure to different cultures and perspectives can broaden your outlook and help you develop a more well-rounded perspective.

It is not rare to feel overwhelmed by the demands of research, and many people experience difficulties along the way. However, with perseverance, dedication, and a strong work ethic, you can overcome these challenges and produce meaningful and impactful research. Don't be discouraged, and keep in mind that many successful researchers have faced similar difficulties. Take it one step at a time, and seek help and support when you need it.

3.3 To level up yourself

It is true that talent plays a significant role in subjects like mathematics and theoretical physics. However, it is important to note that even those without exceptional abilities can still make valuable contributions through research. With persistence and effort, one can continue to develop their skills and tap into one's full potential. (熟能生巧，勤能补拙.) Even with my little (miserably tiny) talent, I have somehow managed to write papers. So don't worry! With continued efforts to enhance your skills, there is potential for you to do much better research than I do.

Learn from failures and mistakes: It is recommended to focus on your weaknesses and areas of improvement, rather than solely highlighting strengths. This can be accomplished by examining failures and mistakes, analyzing the reasons behind them, and using those experiences to learn and grow. Research always involves making mistakes, and learning from them is essential for personal growth and improvement. While it may be painful to confront failures and weaknesses, it is the most effective (in my opinion, the only) way to progress and develop as a researcher. Remember that there is significant potential for growth in areas where you are currently not competent, instead of simply building upon existing strengths. Confront weaknesses and failures instead of small achievements. （做人要敢于面对现实，敢于挑战新事物，敢于承认错误。）

Draw pictures and connect dots: Developing the ability to visualize concepts, organize information, and understand the connections between them is crucial in the study of theoretical physics. Crafting a well-structured and comprehensive map of what you have studied, with the ability to zoom in and out, enables you to gain a broad understanding of the big picture while still being able to retain the technical details. It is essential to always assess

Figure 4: Cultivate the huge wasteland (what you cannot do) instead of decorate your small garden (what you can do).

the role and importance of concepts and how they connect to others. Through this process, one can find insightful cross-disciplinary research questions and write well-structured papers. So illustrate pictures and concepts in your mind!

From my observations and experience, not so many students at Fudan are aware of the importance of this skill. (只见树木，不见森林.) It is not just about applying the method taught in class or by the advisor to a specific problem, but rather, it involves constructing a map of theoretical physics within one's mind and continuously developing it.

Imitate how the smart think: It is important to remember that there will always be people who are smarter than you. (Otherwise, you should worry about your overconfidence.) Therefore, it is beneficial to observe how they think and attempt to emulate their thought process. This does *not* mean copying their research topics or ideas, but rather understanding how they arrive at them. For example, if someone asks a thoughtful question, try to understand how they came up with it. If someone finds a good research problem, try to understand their thought process for finding it. By observing smart people and mimicking their way of thinking, you can improve your own ability.

Overall, it is recommended to spend time with excellent individuals who are passionate about their research and careers. As mentioned in [Hue87], the energy and enthusiasm of

these excellent individuals can be contagious and serve as inspiration.

3.4 Long-term perspectives

As graduate school is not only a place to train researchers, it is important to recognize that becoming a researcher is not the only career path available. It is ideal to dedicate yourself to research in graduate study and assess if research is a suitable path for you. It is important to maintain a healthy and positive outlook and not to view research as the sole definite path. If it turns out that research is not the right fit, there are other options available. The experience gained from intensively engaging research in graduate school will still prove valuable, regardless of the career path ultimately chosen.

> Maldacena remembers when he wasn't sure whether he should try going into a research career in physics and hopes that other students in his position will not let that fear keep them from trying it. "Maybe they will find that they are better than they expected," he says. "Or maybe they will love it more than they expected." – Taking a Risk on Theoretical Physics, Symmetry Magazine

I think that even if you could put everything of yourself in desperation (say, 1X hours a day without a single day off) into research, it would last at most for two or three years, but there is a high risk of burnout. Rather, to sustain a long-term research career, it is necessary to maintain a professional attitude and keep working consistently. It is also important for a long-term commitment to be deeply passionate about theoretical physics. While dedication and hard work are essential, luck also plays a significant role - finding good problems, having a supportive supervisor, getting favorable opportunities, and so on. It can also be helpful to maintain mental health to set a career deadline, for example age 32, to evaluate your research career and consider alternative options after that. The outcome of your career decision and the rewards you reap from your decision are entirely in your control. It is crucial to make a thoughtful career choice by taking into account your personal values and beliefs.

As you go on a research career, you will learn valuable lessons that are applicable to any professional career. These universal truths include the importance of continuous self-improvement, perseverance, and determination. In particular, I highly recommend watching the "Principles For Success" by Ray Dalio. This video offers a comprehensive overview of the universal principles that underpin a successful professional career. By incorporating these principles into your own career development, you can enhance the possibility to achieve your goals and to overcome obstacles.

4 Practical advice for research

4.1 Starting with a specific problem

In undergraduate studies, the coursework mostly covers subjects established in the early 20th century. Given that it is now 202X, there is a significant gap between the coursework

and the current research frontier. However, if you keep learning established theories, you will never get to a point of research. Therefore, a student needs to move from learning an established theory and realistically, has to focus on one specific problem.

> But what I am saying is that when you get started, to really become a mathematician, the key step is to realize that at some point you have to stop reading books. You have to think on your own. You have to become your own authority. There is not an authority to which you have to refer. At some point you have to realize that whether something is written in a book or not doesn't matter. – Alain Connes [Zal09]

Before starting any research, it is advisable to have a discussion with your supervisor about the topic and problem. By considering factors such as mutual interests, current research advancements, feasibility, and difficulty, you can identify a specific research problem that is appropriate for both you and your supervisor.

It is not necessary to understand all in the research area. Instead, it is crucial to acknowledge what you don't understand and set it aside. Start with problems that you are capable of tackling. Don't be discouraged by what you don't know, as you can fill in the gaps as you progress. As you continue with your research, you will gradually gain an understanding of the related areas.

Balancing both sides of research is vital for a graduate student. On one hand, it is important to focus on a specific research problem, but on the other hand, it is necessary to keep learning about the research background in the field. Achieving a good balance between these two aspects is key to success in graduate studies. However, the primary motivation should always be to solve a research problem and produce new results, as this is the job of a researcher.

4.2 Addressing problem

If a problem can be solved easily after a brief attempt, the problem is not so meaningful. Rather, an important problem is often difficult, and the essence of research lies in seeking a solution to a difficult problem. Hence, allocate plenty of time and apply your wisdom, experiencing the joys and pains of thinking. Simple observations are often overlooked, yet they can be surprisingly straightforward when solved. In any case, merely having fixed preconceptions and observing the problem from a distance is inadequate. Solutions are often found through repeated trial and error. To effectively tackle the problem, it is important to "move the problem around" by

- revisiting the problem to see if the problem was properly formulated
- generalizing, specializing, or simplifying the problem
- considering similar problems in other settings
- finding a toy model or examples
- examining literature

- seeking the opinions of other experts

If trying various approaches does not work, it may be better to set the problem aside for a while.

4.3 Critical thinking

Research requires the ability to think critically and independently. I encourage the student to question assumptions, processes, derivations and results by considering alternative perspectives and verifications.

During research, after completing a calculation, derivation or task, it is essential to critically evaluate the results obtained. Ask yourself

- if the results make sense,
- if there are any errors in the derivation,
- if there is a way to verify the results,
- if they are consistent with existing literature.

Being able to critically assess the validity of your results is an important skill for a researcher. This includes the ability to pinpoint where mistakes may exist and to rectify any errors in the calculation or derivation. Being able to identify and address discrepancies in results is a critical component of the research. *Unfortunately, there are often instances where students hastily report results to their supervisor without a simple check, even if the results are apparently implausible. It is a common problem among a segment of Chinese students.* Hence, I strongly encourage students to take the time to reflect on their research process and critically evaluate their results. This is a fundamental aspect of conducting research, as it develops critical thinking and independence. The ongoing process of self-reflection and evaluation is essential to continuously improving these abilities.

4.4 Researcher's life

Physics is a field that never stops growing, and it is crucial to be continuously learning and updating knowledge. I encourage students to be prepared to keep learning throughout their careers. (Read Terrence Tao's blog post.)

For instance, it is common to check new papers on arXiv every day for interesting advancements. Attending seminars is another effective way to stay updated on recent developments. Nowadays, many online talks are accessible so that students can efficiently learn about new papers by participating in related seminars. Additionally, discussions and interactions with experts and fellow researchers prove to be both inspiring and beneficial for enhancing research and comprehension.

> Why is there such a big expansion from the informal discussion to the talk to the paper? One-on-one, people use wide channels of communication that go far

> beyond formal mathematical language. They use gestures, they draw pictures and diagrams, they make sound effects and use body language. Communication is more likely to be two-way, so that people can concentrate on what needs the most attention. With these channels of communication, they are in a much better position to convey what's going on, not just in their logical and linguistic facilities, but in their other mental facilities as well. – William P. Thurston [Thu94].

While reviewing new papers and attending seminars, it is also important to look for potential research topics of interest. Ultimately, to become an independent researcher, it is eventually important to find a suitable research problem and solve it by yourself. Research in theoretical physics is a constantly evolving field, and new discoveries and theories are being made all the time. To find a worthwhile research topic, it is important to be open-minded and curious to explore new ideas and possibilities. We need to keep learning and updating our knowledge.

Also, if you write a paper, you may receive an invitation to give a seminar presentation. Giving a clear presentation is critical, and nurturing good presentation skills is therefore indispensable. For preparation for a talk, I refer to David Tong's slides. Modern theoretical physics is very much a collaborative activity rather than an individual one. In this way, you may be able to find an opportunity for collaboration with other researchers.

4.5 Final academic year

In the final year of a master's or PhD program, submitting a thesis is a crucial requirement. At Fudan University, the deadline for submitting the thesis is at the end of March. To ensure that you have sufficient time to complete your thesis, it is recommended that you begin writing at the start of January at the latest. Writing a thesis can be a time-consuming process, so it is essential to allocate ample time for this task.（撰写毕业论文一定要按部就班。）

In addition to writing your thesis, you will need to prepare for the thesis defense, which is typically held at the end of May. During the defense, you will need to present your work to the committee members for 30 minutes (for a master's thesis) or 40 minutes (for a PhD thesis). It's recommended that you consult with your supervisor to organize external committee members who are professors eligible to supervise master's and PhD students. It is also advisable to practice your presentation before the defense to ensure that you are well-prepared. It is essential to keep in mind that not all of the committee members may be familiar with your area of expertise. Therefore, it is crucial to make your presentation accessible to non-experts.

As you near the end of your PhD program, it's important to consider your future career plans. If you wish to continue in research, applying for a postdoctoral position is a common route. To do so, you'll need to prepare a comprehensive application package that includes your CV, research statement, and letters of recommendation. In some cases, a cover letter may also be necessary. To help ensure that your application is strong and stands out, consider the following practical tips:

CV

- Make sure your CV is concise, clearly written and well-organized.
- Highlight your academic background, research experience, publications, presentations, awards, and skills relevant to the postdoc position you are applying for.

Research statement

- Clearly articulate your research experience, interests, goals, and future plans.
- Showcase your ability to perform independent research and your potential to make a significant contribution to the field.
- Emphasize your commitment to the field and your enthusiasm for the specific research project you will be working on as a postdoc.

Letters of Recommendation

- Choose referees who are familiar with your research experience and can speak to your ability to perform independent research.
- Provide referees with detailed information about the postdoc position you are applying for, your CV, and a brief description of your research experience.
- Give referees *enough time!* to write a well-thought-out letter of recommendation.

Cover letter

- Use the cover letter to introduce yourself and explain why you are interested in the postdoc position.
- Highlight your relevant experience, skills, and achievements, and explain how they make you a strong candidate for the position.

Remember that these documents are critical components of your postdoc application, so take the time to carefully prepare and tailor each one to the specific position you are applying for. Typically, the deadline for applications is at the end of November; however, it is important to note that some of the most prestigious universities require submissions earlier.

5 Specific to my group

First of all, if you want to choose me as your supervisor, please reach out to me as soon as possible. This is especially important for students outside of Fudan. Before accepting you into my supervision, I would like to conduct a brief interview to assess your background. Please make sure to read this entire article prior to contacting me. I also created a short video for a potential student so you may refer to it.

I prefer that students have a strong foundation rather than a vague "knowledge" of the research frontier. Essential qualities such as the ability to concentrate deeply, think critically, have a passion for research, and persevere through challenging projects are far more valuable

than having a superficial knowledge base. (This cannot be judged from grade assessment.) In fact, many students in my group have undergraduate degrees in fields other than physics. Due to the nature of my research, I want to invite one student (at most two students) with a good foundation and high motivation each year.

5.1 Performing research under my supervision

My research interest lies at the interface of geometry, algebra, quantum field theory and string theory. Recently, I have been working on the exact results of supersymmetric theories of various dimensions. Using the exact results, I have been trying to understand their physical meanings in dualities and string theory on one side, and their mathematical interpretations on the other side. In particular, BPS sectors in supersymmetric theories have profound and intriguing interpretations in geometry and algebra, which has been fascinating me. I wish to explore more about this fruitful interaction between physics and mathematics through supersymmetric theories.

Nevertheless, students may find it challenging to enter the field due to the increasing technicality and complexity of formal aspects of supersymmetric theories and related mathematical physics (as demonstrated by big review sets such as [Tes16, P$^+$17]). However, as an advisor, it is my responsibility to break a difficult research problem into smaller, feasible ones and provide students with a clear research direction on a step-by-step basis. Once a research goal is established and the key concepts are understood, I believe that it is manageable for the student to obtain the background knowledge required.

One effective and efficient way to start research under my supervision is through computations of exact partition functions and spectra of BPS states. By learning computational methods and starting with simple examples, students can gradually increase the complexity and make new discoveries in supersymmetric observables. For this, programming skills in Mathematica will be helpful.

If you want to make a contribution to research more conceptually, you need to have a stronger background in the research field. Nonetheless, computations and extensive examination of examples are also an unavoidable part of the research process, as outlined in §3.1. As in §4.1, I will discuss potential research topics with the student, taking into account their aspirations and academic background, and provide suggestions for research problems. It is uncertain that the student is best suited for the research problem until they try. It is a continuous process of trial and error while taking into account the student's potential reaction to the research challenge. The outcome of the research is never certain, but that is how the research under my supervision proceeds.

While we perform research, rather than simply providing students with answers, I encourage them to think critically and solve problems on their own. So I suggest the student to revisit §3.3, §4.2, §4.3 when encountering difficulties in research. Actually, this is the main purpose to write this article. (授人以鱼不如授人以渔.) It is also noteworthy that it usually takes at least one year to finish one research project.

5.2 Reading seminar

As mentioned, my research lies at the interface between physics and mathematics. Students can learn physics from the coursework, but they are often unfamiliar with reading math books. Thus, in my group, we pick one math book (mostly classic literature), and read it through for an entire year by organizing a meeting every week.

2020 §2–4 of Costello's book [Cos22] for mathematical foundation of QFT. The meetings lasted only for one semester.

2022 §7–23 of Fulton and Harris [FH91] to study representation theory of Lie algebras.

2023 Milnor and Stasheff [MS74] to study characteristic classes.

2024 Milnor, Lectures on the h-cobordism theorem; Gompf-Stipsicz, 4-manifolds and Kirby Calculus [GS99]

2025 Gompf-Stipsicz [GS99] to study 4-manifolds; Categorical Aspects of Topological Quantum Field Theories, Bruce H. Bartlett [Bar05]

2026 Kodaira, Complex Manifolds and Deformation of Complex Structures [Kod12]

For preparation for the reading seminar, I request that students thoroughly read websites of Kawahigashi and Ochiai.[1] As instructed on the websites, students are expected to give a presentation without looking at notes and the book during the presentation. This does not mean memorizing every word. Rather, by having a comprehensive understanding of the main concepts and their interrelations, one can reproduce the contents of the book naturally. Also, I ask the students to pay attention to the following:

- Make a strict distinction between what you understand and what you do not understand.
- When encountering confusion, return to the definition and scrutinize the meaning of a problem.
- Distinguish between the trivial and non-trivial parts of an argument or claim.
- Create examples once you come across a new concept and theorem. It is important to find the "simplest but non-trivial example".
- Make your hands dirty by using pen and papers frequently

This seminar series also serves to improve presentation skills whose importance is emphasized in §4.4. (台上一个小时，台下十年功！) The most common mistake students make is to attempt to present all the information they have learned. If you prepare a lot for a presentation, it is tempting to talk about all of what you prepared and understood. However,

[1] In fact, these websites are very famous among mathematicians and theoretical physicists in Japan. I ask students to translate it by using DeepL or Google Translate.

trying to cover too much material can obscure key concepts, waste time on unimportant details, and make the presentation dull. To avoid mistakes of this kind, the student must judiciously determine the significance of different elements, prioritize the most important part and deliver clear and concise explanations.

While students may struggle at first, they will be eventually able to give a good presentation with practice. (In fact, my students become able to give presentations without the need to refer to their notes.) This reading seminar involves careful and thorough examinations of mathematical concepts on a step-by-step basis, and the students are eventually familiar with the mathematical concepts and their connections to physics. I have found this reading seminar to be highly effective in promoting student understanding and will continue this tradition in my group.

5.3 Inspired from seminal papers

As mentioned in §4.1, it is vital to take the balance between tackling the research problem and acquiring the necessary background knowledge. As the students become familiar with advanced topics such as supersymmetry and string theory, I suggest that they thoroughly read a seminal paper in the field.

Groundbreaking research papers presenting significant results feature a meticulously organized structure and hold tremendous potential for driving future advancements and cross-disciplinary applications. These thought-provoking and insightful works can be considered seminal pieces of scholarship, comparable to impeccable works of art.

It is recommended to thoroughly read such a seminal paper, carefully analyzing the details, reading between the lines, and evaluating their underlying implications. Ensure a clear and comprehensive understanding from beginning to end. This will enhance your skills and capabilities.

For reference, I list examples of seminal papers relevant to my research. (I intentionally choose classic papers related more to quantum field theory here.) These are simply some examples, and you may find others by yourself.

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5.4 Strictness and gentleness

While it may be tempting for students to seek out a "gentle" supervisor, I would like to encourage them to consider the long-term implications of this decision. The terms "strict" and "gentle" do not necessarily refer to the personality or character of the advisor, but rather the level of effort and achievement expected of the student and the level of guidance provided in response to that effort.

It is important to note that, in many cases, it is easier for a supervisor to adopt a "gentle" approach, where standards for students are lowered and minimal guidance is provided. Unfortunately, this type of approach is not conducive to the development of a student's full potential and can lead to a lack of academic rigor.

As a responsible supervisor, it is my duty to set rigorous standards for my students and provide the necessary guidance to help them reach those standards. This approach may be more challenging for both the student and the advisor, but ultimately it results in a more meaningful and productive educational experience.

While I am aware that I am definitely classified as a "strict" faculty member (so be prepared!), I believe that this approach is in the best interest of my students and their academic development. I strive to provide the guidance and support necessary to help my students reach their full potential.

5.5 To be an independent researcher

As researchers, both students and faculty members should strive to maintain an equal and respectful relationship when it comes to conducting research. It is important to remember that the advisor, like yourself, is also a human being who should not be viewed as an

infallible authority. In the field of research, where many questions remain unanswered and the solutions are unknown, it is common that even the advisor does not know the answers. While I may have more experience and may have expectations or aspirations for your research, it is not always the case that my guidance will be correct. In particular, research *often* does not progress as planned, so it is not always the fault of the student for research progress. Also, there may be simple misunderstandings or errors that are overlooked between the advisor and students.

It is important to actively voice any concerns or objections you may have with your advisor's ideas or explanations, and to engage in constructive, scientific and logical dialogue if you are dissatisfied with their policies. As a responsible advisor, I should take into consideration any reasonable opinions and respond accordingly, or incorporate any objections. It is not beneficial for the learning process for students to simply accept everything that is said by their advisor. The ideal scenario is for both parties to work collaboratively and cooperatively, building mutual understanding and trust as they tackle difficult problems. This type of scientific discussion is crucial for building trust among researchers, which is a vital aspect of training and performing research. It is important for students not to be content with simply being a student, but to strive to become independent researchers.

I wish to remind you that climbing a mountain is done with your own steps, journeys are undertaken through your own initiative, and research is also something you must engage in independently. The role of a mentor is not to walk the path for you, but to accurately assess your abilities and provide guidance on the next steps you should take, as well as what you are capable of achieving. This role parallels that of a mountain guide or a navigator—helping you steer clear of dangerous routes, precarious cliffs, and unproductive dead ends.

5.6 Dreams

One of my dreams as a supervisor is that one of my students will write a single-authored paper in the future. While it can be challenging for a student to find a problem, solve it and write the results independently, it is an important step in their development as a researcher. While I provide guidance and support in the research process, at some point, it is necessary for the student to take ownership of the project and work independently. I believe that even a slight generalization of a well-understood problem, such as extending from SU(2) to SU(N), can provide a valuable learning experience for the student. I hope that one day, a student in my group will have the courage to take on this challenge and complete a research project entirely on their own.

My second dream is to collaborate with a mathematics student on a rigorous paper in mathematics. As a researcher at the intersection of mathematics and physics, I am able to provide potential research topics that align with both of our interests. With a mathematics student, we can delve into the literature and discuss the most promising direction for our collaboration. I see this as an opportunity for mutual learning and growth, as I hope to deepen my understanding of mathematics while guiding the student through the research process. Although I have not yet had the experience of supervising a mathematics student, I am eager to take on this new challenge and broaden the scope of my research. While

success is not guaranteed, I am enthusiastic about this potential opportunity to collaborate on research in mathematics with a student.

6 Epilogue

Undergraduate students may be pondering about the type of research they wish to undertake. Meanwhile, master's and doctoral students may be concerned about the accomplishment of their research. This article is written to help students understand and prepare better for research in theoretical physics by broadening perspectives. There have been instances where students who expressed interest in joining my research group lacked a proper understanding of my research field and limited knowledge of the work involved. I write this article to serve as a bridge to improve communication with my students and their understanding of research.

Several of my students have left my group without experiencing so much growth. While my first PhD student was able to thrive under my rigorous supervision, I acknowledge that I could have done more to further develop his and their potential. As in §3.3, I have introspected on my shortcomings as a supervisor and candidly articulated areas for improvement. The purpose of this article is also to facilitate my growth as an advisor, enabling me to impart knowledge beyond the realm of research and broaden my scope as a supervisor.

My research lies in the field of mathematical physics, a unique field that lies at the intersection of physics and mathematics. It may be an unfamiliar field to many students in both physics and math departments (or physicists and mathematicians). Hence, I have provided a brief guide on how to get started and prepare for this field. Indeed, the fascinating and valuable concepts and results are hidden precisely where different disciplines intertwine. The field of mathematical physics studies the interaction of mathematics, a discipline that encompasses distillations of (abstract) concepts, languages, and techniques on a rigorous footing, and physics, a field that describes how Nature behaves. I hope that this article will inspire more students to pursue their future in this fertile and exciting field.

I am aware that research is not an easy journey. There is always something new to learn, and there are numerous uncertainties. (I have never felt that research is easy. Even writing a single paper is a formidable task for me.) I may appear (very) strict with my students, but my intention is to raise their levels and standards so that we can tackle challenging projects together. One may think that research in mathematical physics requires a strong background in both physics and mathematics. However, I believe that with a clear focus on the research goal and a proper grasp of the key points, even students can delve deep into research. As their advisor, it is my duty to guide them in this process and I hope that they will give their best effort, producing high-quality results.

愿你前程一路繁花相送，阳光照亮你的前路，前途无限，未来可期。

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