

## Homework 7: Due at class on April 19

1. Show that the Euler characteristics is equal to

$$\chi(M) = \sum_{i \geq 0} (-1)^i \dim C_i(K; \mathbb{R})$$

given a triangulation  $|K| \rightarrow M$ .

2. Show that the Euler characteristics of an odd-dimensional oriented closed manifold is zero.
3. Find the integer-valued homology group  $H_\ell(\Sigma_g; \mathbb{Z})$  of a Riemann surface  $\Sigma_g$  of genus  $g$ .
4. Find both the integer-valued  $H_\ell(\mathbb{R}P^2; \mathbb{Z})$  and the real-valued  $H_\ell(\mathbb{R}P^2; \mathbb{R})$  homology groups of  $\mathbb{R}P^2$ . Does the Poincaré duality hold?
5. Let us construct a 3-dimensional complex  $K$  from  $n$  tetrahedra  $T_1, \dots, T_n$  by the following two steps. First we arrange the tetrahedra in a cyclic pattern as in the figure, so that each  $T_i$  shares a common vertical face with its two neighbors  $T_{i-1}$  and  $T_{i+1}$ , subscripts being taken mod  $n$ . Then we identify the bottom face of  $T_i$  with the top face of  $T_{i+1}$  for each  $i$ . Compute the homology groups of  $K$ .

