Homework 4: Due at class on April 2

1 bc ghost CFT

1.1 energy-momentum tensor

Given the bc ghost action (Euclidian signature)

$$S_{\text{gh}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{h}\, b^{ab} \nabla_a c_b \,,$$

calculate the stress tensor for the bc ghosts by

$$T_{ab} = -\frac{4\pi}{\sqrt{h}} \frac{\delta S}{\delta h^{ab}}$$

Note that the covariant derivative ∇^α contains the Christoffel symbol and b_{ab} is symmetric traceless. Show that it becomes

$$T^{\text{gh}}(z) = -2 : b(z)\partial c(z) : + : c(z)\partial b(z) : \tag{1.1}$$

in the conformally flat metric.

1.2 TT OPE

Using the energy-momentum (1.1), derive the TT OPE in the bc ghost CFT

$$T(z)\,T(w) = \frac{-13}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \cdots$$

1.3 BRST charge

Using the explicit form of the BRST current

$$j_B = c(z)T^X(z) + : b(z)c(z)\partial c(z) : + \frac{3}{2} : \partial^2 c(z) : \,,$$

express the BRST charge in terms of the X^μ Virasoro operators and the ghost oscillators as

$$Q_B = \sum_n c_n (L^X_{-n} - \delta_{n,0}) + \sum_{m,n} \frac{m-n}{2} : c_m c_n b_{-m-n} : \,.$$

Show that the OPE between two BRST currents is given by

$$j_B(z)j_B(w) = -\frac{c^X - 18}{2(z-w)^3} c\partial c(w) - \frac{c^X - 18}{4(z-w)^2} c\partial^2 c(w) - \frac{c^X - 26}{12(z-w)} c\partial^3 c(w) + \cdots \,,$$

where c^X is the central charge of the X^μ bosonic string theory. Use this OPE to determine the anticommutator of the BRST charge with itself. For what value of c^X does this vanish?

1.4 Tj_B OPE

Show that the OPE between the total energy momentum tensor $T = T^X + T^{gh}$ and j_B is given by

$$T(z)j_B(w) = \frac{c^X - 26}{2(z-w)^4}c(w) + \frac{j_B(w)}{(z-w)^2} + \frac{\partial j_B(w)}{z-w} + \cdots .$$

What does the result imply for j_B?

2 $\beta\gamma$ ghost CFT

Now let us consider the same action as the bc ghost system

$$S = \frac{1}{2\pi} \int d^2z\, \beta \bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} ,$$

but now β and γ are bosonic fields. Hence, their OPEs are (pay attention to sign)

$$\gamma(z)\beta(w) = -\beta(z)\gamma(w) = \frac{1}{z-w} + \cdots$$

If β and γ are primary fields of weights $(\lambda, 0)$ and $(1-\lambda, 0)$ respectively, the form of the stress energy tensor (holomorphic part) is

$$T(z) =: (\partial\beta)\gamma : -\lambda\,\partial : \beta\gamma :$$

Calculate the the TT OPE to determine the central charge of the CFT in terms of λ.