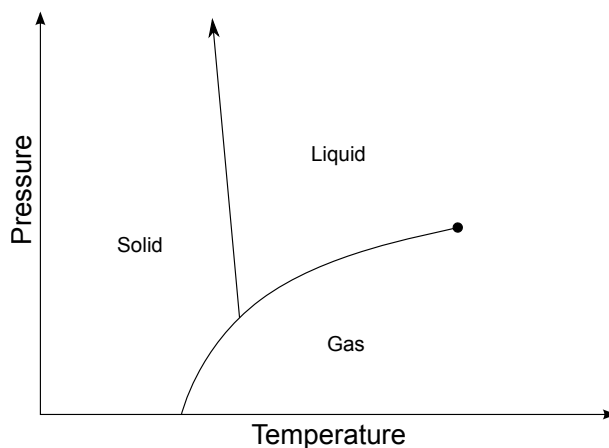# Homework 1: Due at class on Sep 23

1

The following figure is the phase diagram of H_2O. Where can you see a first-order and second-order phase transition? In addition, what is the order parameter?

2

- Derive the infinitesimal transformation (2.20) from the finite transformation (2.21) for SCT.

- Derive the conformal factor (2.22) of SCT.

- Using either explicit forms of the generators in Table 1 or (2.23), derive (2.24) in the lecture note.

3. Bonus problem (3pt)

The notations are the same as (1.3) and (1.4) of the lecture note.

3.1

Suppose that the free energy density $f := F/N$ obeys

$$f(t, h) = b^{-d} f(b^{y_t} t, b^{y_h} h) \ ,$$

under the scaling transformation $x \to x/b$. Show that the critical exponents can be expressed

$$\alpha = 2 - d/y_t$$
$$\beta = (d - y_h)/y_t$$
$$\gamma = (2y_h - d)/y_t$$
$$\delta = y_h/(d - y_h) \ .$$

3.2

Under the scaling transformation $x \to x/b$, the correlation length ξ should behave as $\xi \to \xi/b$. Show that $\nu = 1/y_t$. Using the fact that the susceptibility can be written in terms of the two-point correlation function of σ

$$\chi = \frac{1}{T} \int d^d \mathbf{r} \ G(\mathbf{r}) \ ,$$

express $\alpha, \beta, \gamma, \delta$ in terms of ν, η.