Homework 1: Due at class on Sep 22

1

If an action is Weyl invariant, then show (without relying on an explicit form of action) that its stress-energy tensor is traceless.

2

Using the mode expansions in the lecture note, derive from the canonical commutation relation for X^μ and Π_μ

$$\begin{aligned} &[X^\mu(\sigma,\tau),\Pi_\nu(\sigma',\tau)] = i\delta(\sigma-\sigma')\,\delta^\mu{}_\nu &&, \\ &[X^\mu(\sigma,\tau),X^\nu(\sigma',\tau)] = [\Pi_\mu(\sigma,\tau),\Pi_\nu(\sigma',\tau)] = 0 &&. \end{aligned}$$

commutation relations for the Fourier modes x^μ, p^μ, α_n^μ and $\widetilde{\alpha}_n^\mu$

$$[x^\mu,p_\nu] = i\delta^\mu{}_\nu \quad \text{and} \quad [\alpha_n^\mu,\alpha_m^\nu] = [\widetilde{\alpha}_n^\mu,\tilde{\alpha}_m^\nu] = n\,\eta^{\mu\nu}\delta_{n+m,\,0}\ ,$$

3

Problem 2.3 in Becker-Becker-Schwarz.