Homework 2: Due at class on March 23

1 Renormalization of ϕ^3-theory

Let us consider ϕ^3-theory in 6d spacetime with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3$$

1. Determine the superficial divergent amplitudes. Write the renormalized Lagrangian density and derive the Feynman rules.
2. Calculate the propagator correction at one-loop order and determine δ_Z and δ_m.
3. Calculate the vertex correction and find δ_g.

2 β, γ for Yukawa theory

Let us consider the pseudoscalar Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - ig\bar{\psi}\gamma^5\psi\phi - \frac{\lambda}{4!}\phi^4$$

where ϕ is a real scalar field and ψ is a Dirac fermion.

2.1

Compute the one-loop contributions to β_m, β_M and γ_ψ, γ_ϕ.

2.2

Compute the Callan-Symanzik β functions for λ and g :

$$\beta_\lambda(\lambda, g), \quad \beta_g(\lambda, g)$$

to leading order in coupling constants, assuming that λ and g^2 are of the same order. Sketch the coupling constant flows in the λ-g plane.