

Homework 7: Due at class on April 18

1. The Euler characteristics in the lecture note is defined by

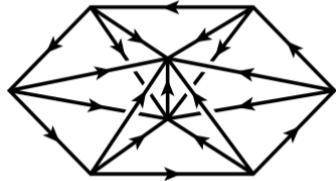
$$\chi(M) = \sum_{i \geq 0} (-1)^i \dim H_i(K; \mathbb{R}).$$

Show that it is indeed equal to

$$\chi(M) = \sum_{i \geq 0} (-1)^i \dim C_i(K; \mathbb{R})$$

given a triangulation $|K| \rightarrow M$.

2. Show that the Euler characteristics of an odd-dimensional oriented closed manifold is zero.
3. Find the integer-valued homology group $H_\ell(\Sigma_g; \mathbb{Z})$ of a Riemann surface Σ_g of genus g . Compute their Euler characteristics.
4. Find both the integer-valued $H_\ell(M; \mathbb{Z})$ and the real-valued $H_\ell(M; \mathbb{R})$ homology groups of both $M = \mathbb{RP}^2$ and $M = \text{Klein bottle}$. Compute their Euler characteristics.
5. Let us construct a 3-dimensional complex K from n tetrahedra T_1, \dots, T_n by the following two steps. First we arrange the tetrahedra in a cyclic pattern as in the figure, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1} , subscripts being taken mod n . Then we identify the bottom face of T_i with the top face of T_{i+1} for each i . Compute the homology groups of K .



6. (This is a bonus problem with extra 3 points which is NOT mandatory.) Let us define a manifold as

$$T_{-1} = \frac{S^2 \times I}{(x, 0) \sim (-x, 1)}$$

where the top and bottom of S^2 are identified by the antipodal map $x \mapsto -x$. Compute homology groups of T_{-1} .