Lecture 10

1 Type I superstring theory

What we will learn:

- Type I superstring theory.
- In type I theory, we only have D1-, D5-, and D9-, branes.
- $O9^-$-plane is needed for type I to be consistent theory.
- T-duality of type I theory.

So far, we have learnt two kinds of superstring theory, which are type IIA and IIB. Both of them are closed superstring theory. Note that we also learnt about D-branes in type II theories, which leads open string. However, note that we are considering a situation in which there exist D-branes, so it is not a theory.

Actually, an open string cannot exist in type II theories (without D-branes), due to supersymmetry as follows. After GSO projection an open string can have one of the following massless states:

$$P_{\mathrm{GSO}}: \quad \mathrm{NS}+,\ \mathrm{R}+ = \mathbf{8}_v + \mathbf{8}_s \ , \tag{1}$$

$$\widetilde{P}_{\mathrm{GSO}}: \quad \mathrm{NS}+,\ \mathrm{R}- = \mathbf{8}_v + \mathbf{8}_c \ , \tag{2}$$

where $\mathbf{8}_v$ is a space-time vector field and $\mathbf{8}_{s,c}$ are gauginos. $\mathcal{N}=2$ supersymmetry requires for $\mathbf{8}_v$ to have two superpartners as like for a graviton to have two gravitinos. However, as you see, the gauge field has only one superpertner, and hence, cannot exist in type II theories.

The existence of (parallel) D-branes, as you might notice, breaks the half of supersymmetries, and hence, an open string can exist in such a situation.

The world-sheets(WS) we have considered are oriented ones. However, it is quite natural to consider processes in Fig. 1. Therefore, we will consider so called **unoriented string**

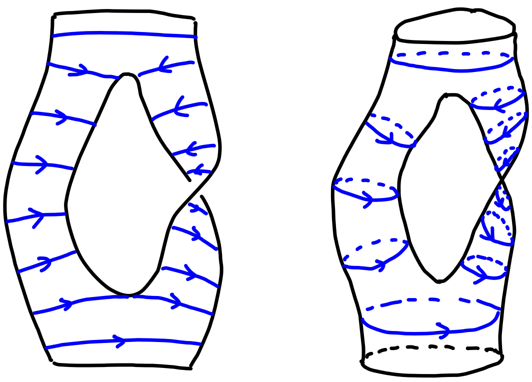

Fig. 1: Unoriented processes. The left is open string one, and the right is closed string one.

in this lecture, which leads the type I superstring theory. Argument below is almost in bosonic string, for simplicity, but it is straightfoward to apply for superstring.

1.1 Orientation flip operation

As the name indicates the orientation flip is nothing but an exchange of left-/right-mover for closed string and a reversal of the direction for open string.

Closed string

Let us define an orientation flip operator Ω for bosonic string, which flips the orientation of an closed string:

$$\Omega : \begin{cases} X(t,\sigma) & \leftrightarrow & X(t,-\sigma) \\ b(t,\sigma) & \leftrightarrow & \bar{b}(t,-\sigma) \\ c(t,\sigma) & \leftrightarrow & -\bar{c}(t,-\sigma) \end{cases} , \tag{3}$$

where t is the Eculideanized WS time, not τ (though there is no special meaning on it at this point). Or, the oprerator acts on the modes as

$$\Omega : \begin{cases} \alpha_n^\mu & \leftrightarrow & \widetilde{\alpha}_n^\mu \\ b_n & \leftrightarrow & \bar{b}_n \\ c_n & \leftrightarrow & \bar{c}_n \end{cases} , \tag{4}$$

where the modes are

$$X(t,\sigma) = x^\mu - i\alpha' p^\mu t + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^\mu e^{n(-t+i\sigma)} + \widetilde{\alpha}_n^\mu e^{n(-t-i\sigma)} \right) , \tag{5}$$

$$b(t,\sigma) = -\sum_{n\in\mathbb{Z}} b_n e^{n(-t+i\sigma)} , \quad \bar{b}(t,\sigma) = -\sum_{n\in\mathbb{Z}} \bar{b}_n e^{n(-t-i\sigma)} , \tag{6}$$

$$c(t,\sigma) = i\sum_{n\in\mathbb{Z}} c_n e^{n(-t+i\sigma)} , \quad \bar{c}(t,\sigma) = -i\sum_{n\in\mathbb{Z}} \bar{c}_n e^{n(-t-i\sigma)} , \tag{7}$$

where we can also use $w = -it - \sigma \leftarrow \tau - \sigma$ ($z = e^{iw}$).

We keep only Ω invariant states:

OK	Tachyon	$\vert k\rangle$
OK	Dilaton	$\alpha_{-1}\cdot\widetilde{\alpha}_{-1}\vert k\rangle$
NG	B-field	$\alpha_{-1}^{[\mu}\widetilde{\alpha}_{-1}^{\nu]}\vert k\rangle$
OK	Graviton	$\left(\alpha_{-1}^{(\mu}\widetilde{\alpha}_{-1}^{\nu)} - \frac{\delta^{\mu\nu}}{D}\alpha_{-1}\cdot\widetilde{\alpha}_{-1}\right)\vert k\rangle$
	$\vdots$	

No B-field means that a closed string, which is electrically coupled to B-fields, is not a stable object and should decay.

Open string

Let us put a D25-brane and consider Neumann boundary conditions. Then similarly, we define the orientation flip operator for open string:

$$\Omega : \begin{cases} X(t,\sigma) & \leftrightarrow & X(t,\pi-\sigma) \\ b(t,\sigma) & \leftrightarrow & \bar{b}(t,\pi-\sigma) \\ c(t,\sigma) & \leftrightarrow & -\bar{c}(t,\pi-\sigma) \end{cases} , \tag{8}$$

or, for the modes expansion

$$\Omega : \begin{cases} \alpha_n^\mu & \leftrightarrow & (-1)^n \alpha_n^\mu \\ b_n & \leftrightarrow & (-1)^n b_n \\ c_n & \leftrightarrow & (-1)^n c_n \end{cases} , \tag{9}$$

where the modes are

$$X(t,\sigma) = x^\mu - 2i\alpha' p^\mu t + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha_n^\mu}{n} \left(e^{n(-t+i\sigma)} + e^{n(-t-i\sigma)}\right) , \tag{10}$$

$$b(t,\sigma) = -\sum_{n\in\mathbb{Z}} b_n e^{n(-t+i\sigma)} , \quad \bar{b}(t,\sigma) = -\sum_{n\in\mathbb{Z}} b_n e^{n(-t-i\sigma)} , \tag{11}$$

$$c(t,\sigma) = i\sum_{n\in\mathbb{Z}} c_n e^{n(-t+i\sigma)} , \quad \bar{c}(t,\sigma) = -i\sum_{n\in\mathbb{Z}} c_n e^{n(-t-i\sigma)} . \tag{12}$$

Naively, the massless vector field state is Ω variant. However, we can keep the state by using Chan-Paton factor:

$$|\Phi;\Lambda\rangle \equiv |\Phi; ij\rangle \Lambda_{ij} . \tag{13}$$

When the orientation flips, the Chan-Paton indices are exchanged, so

$$\Omega|\Phi;\Lambda\rangle = |\Omega\Phi;\Lambda^T\rangle . \tag{14}$$

Therefore, Ω invariante states are

$$\begin{array}{lll} \text{Tachyon} & |k;\Lambda\rangle & \Lambda^T = \Lambda \\ \text{Vector} & \alpha_{-1}^\mu|k;\Lambda\rangle & \Lambda^T = -\Lambda \\ \vdots & & \end{array}$$

The $n\times n$ hermitian, anti-symmetric matrix forms a $SO(n)$ algebra. Therefore, the vector field is identified with a $SO(n)$ gauge field.

When we flip the orientation there could be a shuflle of the Chan-Paton index because the D-branes are coincident. Let us denote this as follows.

$$\Omega|\Phi; ij\rangle = |\Omega\Phi; kl\rangle P_{kj} P_{il}^{-1} \quad (P \in U(n)) , \tag{15}$$

$$\Omega|\Phi;\Lambda\rangle = |\Omega\Phi; P\Lambda^T P^{-1}\rangle . \tag{16}$$

There is a natural constraint $\Omega^2 = 1$, and equivalence relation: $\Lambda \sim \widetilde{\Lambda} = U\Lambda U^{-1}$ ($U \in U(n)$). The constraint leads

$$\Omega^2|\Phi;\Lambda\rangle = \Omega|\Omega\Phi; P\Lambda^T P^{-1}\rangle = |\Phi; P(P\Lambda^T P^{-1})^T P^{-1}\rangle , \tag{17}$$

$$\therefore \qquad \Lambda = PP^{-T}\Lambda(PP^{-T})^{-1} . \tag{18}$$

For general Λ to satisfy the relation above we need $PP^{-T} = 1$. Since the order of the index is artificial, physics is invariant under the re-labelling, $\Lambda \sim \widetilde{\Lambda} = U\Lambda U^{-1}$, and this leads the equivalence class for P as well:

$$P\Lambda^T P^{-1} = P(U^{-1}\widetilde{\Lambda}U)^T P^{-1} = PU^T\widetilde{\Lambda}^T U^{-T} P^{-1} = U^{-1}(UPU^T)\widetilde{\Lambda}^T(UPU^T)^{-1}U . \tag{19}$$

Thus, $\widetilde{P} \sim UPU^T$. Subject to the constraint and the equivalence class, we have two physically inequivalent choice for P:

- $P = 1 \qquad \Lambda^T = -\Lambda \qquad \cdots\ SO(n)$ gauge symmetry.
- $P = i \begin{pmatrix} 0 & -\mathbf{1}_{k\times k} \\ \mathbf{1}_{k\times k} & 0 \end{pmatrix} \qquad (n = 2k)$
  $\Lambda^T = \begin{pmatrix} 0 & -\mathbf{1}_{k\times k} \\ \mathbf{1}_{k\times k} & 0 \end{pmatrix} \Lambda \begin{pmatrix} 0 & -\mathbf{1}_{k\times k} \\ \mathbf{1}_{k\times k} & 0 \end{pmatrix} \qquad \cdots\ Sp(n)$ gauge symmetry.

Unoriented superstring spectrum

The orientation flip is a swap of left- and right-mover, therefore, IIB theory has the world-sheet parity Ω because it is a non-chiral theory. The flip projection eliminates B-field [2] in NS-NS sector, as well as half of NS-R R-NS sector $\mathbf{8}_c + \mathbf{56}_s$ (only diagonal part survive). Supersymmetry requires that the number of bosons and fermions are the same, which implies that [0] and $[4]_+$ are eliminated and only the second rank anti-symmetric field [2] survives in R-R sector. The remainig states are

$$\begin{aligned} [0] + [2] + (2) + \mathbf{8}_c + \mathbf{56} &= \mathbf{1} + \mathbf{28} + \mathbf{35} + \mathbf{8}_c + \mathbf{56}_s \\ &= \Phi + C_{\mu\nu} + G_{\mu\nu} + \lambda^- + \psi_\mu^+ \ . \end{aligned} \tag{20}$$

Absence of the C and $C^+_{\mu\nu\rho\sigma}$ means that there is no D(-1)-, D3-, D7-branes in type I theory. **Only D1-, D5-, D9-branes exist in type I theory**, as the first two D-branes are electrically and magnetically coupled to $C_{\mu\nu}$, respectively. D9 is space-filling and non-dynamical, and is necessary to have an open string. Actually, this type I closed superstring theory is inconsistent as we will see, and it necessarily includes open string $\mathbf{8}_v + \mathbf{8}_s$ (choice of the GSO projection follows from IIB). Therefore, the spectrum of type I theory (unoriented, open plus closed supersting) is as follows.

$$[0] + [2] + (2) + \mathbf{8}_c + \mathbf{56} + (\mathbf{8}_v + \mathbf{8}_s)_{SO(n) \text{ or } Sp(n)} \ . \tag{21}$$

Anomaly or tadpole cancellation argument, which we will see in the following subsection, shows that **only** $SO(32)$ **is consistent**.

1.2 Amplitude of Type I theory

As we saw in the previous subsection unoriented open string has a $SO(n)$ or $Sp(n)$ massless gauge field. However, all of them are anomalous, which means that the theories are inconsistent, except $SO(32)$ as we will see below. There are a few ways to see the anomaly. We utilize vacuum amplitudes.

First, let us consider "oriented" open string amplitude, whose WS is cylinder(=annulus), see Fig. 2. It is clear from the picture that

- The range is $0 \leq \mathrm{Re}\, w \leq \pi$, the period is $w \sim w + 2\pi i l$.
- There is a real modulus l; the amplitude needs a b zero mode insertion.
- There is a real isometry, shift of $\mathrm{Im}\, w$; the amplitude needs a c zero mode insertion.

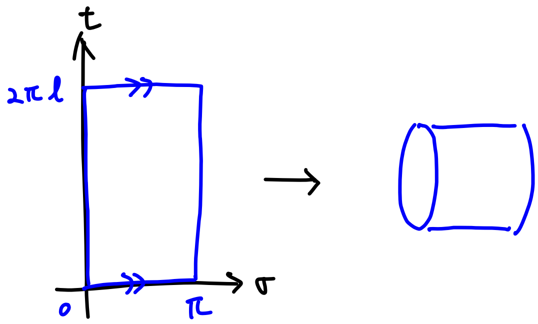

Fig. 2: Cylinder.

The cylinder partition function is

$$A_{0,C} = \int \frac{dl}{2l} \langle b_0 c_0 \rangle_{\text{gh}} \langle 1 \rangle_{\text{mat}} \ , \tag{22}$$

where

$$b_0 = \frac{1}{2\pi} \int_0^\pi \left[dw\ b(w) + d\overline{w}\ \overline{b}(\overline{w}) \right] \ , \tag{23}$$

$$c_0 = \frac{i}{2\pi} \int_0^\pi \left[dw\ c(w) - d\overline{w}\ \overline{c}(\overline{w}) \right] \ . \tag{24}$$

Assume that there are n D25- (or D9- for superstring) branes so that all of the boundary condition is Neumann. Using operator formalism we can derive each contributions as follows.

$$\begin{aligned} \langle 1 \rangle_{\text{mat}} &= n^2 \ \text{tr} \left[q^{L_0 - \frac{c}{24}} \right] \qquad (q = e^{-2\pi l} \ , \ L_0 = \alpha' p^2 + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n) \\ &= n^2 \cdot \frac{i V_{26}}{(2\pi)^{26}} (2 l \alpha')^{-13} \cdot \eta(il)^{-26} \ . \end{aligned} \tag{25}$$

$$\langle b_0 c_0 \rangle_{\text{gh}} = \text{tr} \left[(-1)^F b_0 c_0 q^{L_0 - \frac{c}{24}} \right] = \eta(il)^2 \qquad (L_0 = \sum_{n \in \mathbb{Z}} n :b_{-n} c_n: -1) \ . \tag{26}$$

Therefore, the amplitude is

$$A_{0,C} = n^2 \cdot \frac{i V_{26}}{(2\pi l_s)^{26}} \int_0^\infty \frac{dl}{2l} \frac{1}{(2l)^{13} \eta(il)^{24}} \ , \tag{27}$$

where $l_s = \sqrt{\alpha'}$ is the string length. Let us look into the physical information that can be read off from the amplitude.

- UV divergence: there is a UV divergence from $l \to 0$, as opposed to the closed string case.
- Open string short 1-loop = closed string long propagation (see Fig. 3).
  This is justified by rewriting the amplitude. Using $\eta(il) = l^{-\frac{1}{2}} \eta(il^{-1})$ we have

$$A_{0,C} = \frac{n^2}{2^{14}} \cdot \frac{i V_{26}}{(2\pi l_s)^{26}} \int_0^\infty ds\ \eta(is)^{-24} \ , \tag{28}$$

$$\text{where} \quad \eta(is)^{-24} = q^{-1} + 24 + \cdots \equiv \sum_{N=0}^{\infty} \mathcal{N}_N q^{N-1} \quad (q = e^{-2\pi s}) \ . \tag{29}$$

Compare with the torus partition function (exercise).

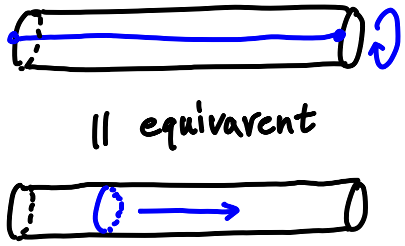

Fig. 3: Pictorical "proof" of the equivalence between open string 1-loop and closed string propagation.

- The UV divergence $l \to 0$ is replaced by IR divergence $s \to \infty$ of a closed string propagation, which can be understood particle propagations (sum of lines) as follows (see also Fig. 4).

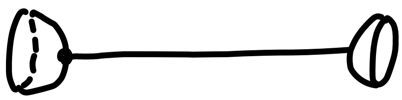

Fig. 4: Intermediate propagation is replaced by particles(lines).

$$\int_0^\infty ds \sum_{N=0}^{\infty} \mathcal{N}_N e^{-2\pi s(N-1)} \sim \sum_i \int_0^\infty ds\ e^{-s(k^2+m_i^2)}\bigg|_{k=0} = \sum_i \frac{1}{k^2+m_i^2}\bigg|_{k=0} . \tag{30}$$

We can see that the IR divergence is from massless particle propagation (graviton etc.), which is absorbed or emitted from D25-branes.

- In conclusion, the divergence is due to the existence of the D25-branes, which has definite tension (this is why they emit graviton/dilaton). Can this not be eliminated ?? $\to$ unoriented string.

Klein bottle amplitude

Next, let us consider a Klein bottle for WS and compute the amplitude on it. As the Klein bottle can be realized by orientation flip operator as in Fig. 5 it should be

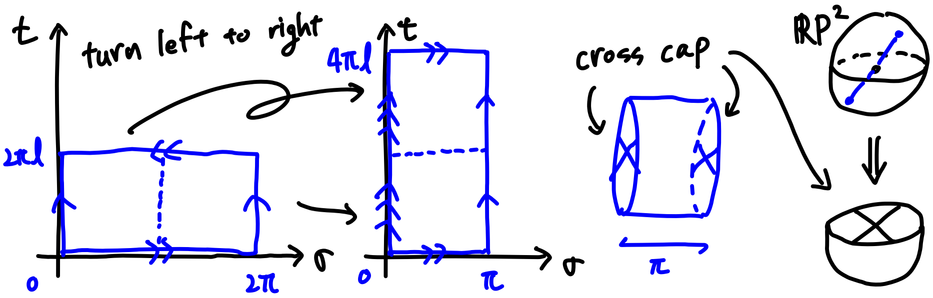

Fig. 5: Klein bottle. It can be described by a cylinder with cross cap boundary on both ends.

$$A_{0,K} = \frac{1}{2}\int \frac{dl}{2l}\mathrm{tr}\left[\Omega(-1)^F \frac{1}{2}(b_0+\bar{b}_0)\frac{1}{2}(c_0+\bar{c}_0)q^{L_0+\bar{L}_0-\frac{c}{12}}\right] \qquad (q=e^{-2\pi l}) , \tag{31}$$

where the factor of $\frac{1}{2}$ is from the projection operator $\frac{1+\Omega}{2}$, or you can also regard it as an additional gauge redundancy $w \to \overline{w}$.

One can rewrite the amplitude as follows.

$$A_{0,K} = \frac{1}{2} \int \frac{dl}{2l} \mathrm{tr}\left[(-1)^F b_0 c_0 q^{2L_0 - \frac{c}{12}}\right] \ , \tag{32}$$

where $c = c_{\mathrm{mat}} + c_{\mathrm{gs}} = 0$, and

$$L_0 = \frac{\alpha'}{4} p^2 + \sum_{n=1}^{\infty} (\alpha_{-n} \cdot \alpha_n + n b_{-n} c_n + n c_{-n} b_n) - 1 \ . \tag{33}$$

Thus, the result is

$$A_{0,K} = \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{dl}{4l} \frac{1}{l^{13} \eta(2il)^{24}} \ , \tag{34}$$

$$= 2^{13} \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{ds}{4} \eta(is)^{-24} \ . \tag{35}$$

Möbius strip amplitude

Finally, let us consider a Möbius strip for WS (see Fig. 6). It has

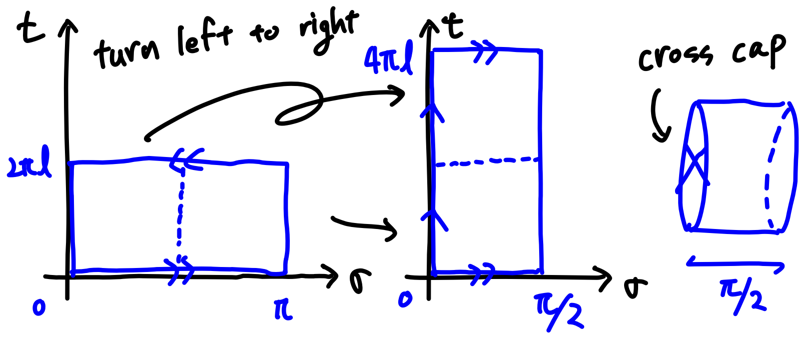

Fig. 6: Möbius strip.

- The range is $0 \le \sigma \le \pi$, the period is $2\pi l$, coming together with the orientation flip: $(t, \sigma) \sim (t + 2\pi l, \pi - \sigma)$.
- There is a real modulus l; the amplitude needs a b zero mode insertion.
- There is a real isometry, shift of t; the amplitude needs a c zero mode insertion.

The Möbius strip amplitude is

$$A_{0,M} = \frac{1}{2} \int \frac{dl}{2l} \mathrm{tr}\left[\Omega (-1)^F b_0 c_0 q^{L_0 - \frac{c}{12}}\right] \qquad (q = e^{-2\pi l}) \ . \tag{36}$$

The trace is over Hilbert space of the matter and ghost sectors on the strip as well as the Chan-Paton index, which gives in total n^2 degeneracy for each state. We can divide the effect of Ω into two parts as follows.

$$\Omega |\Phi; \Lambda\rangle = |\Omega\Phi; P\Lambda^T P\rangle \equiv \Omega_\Phi \cdot \Omega_\Lambda |\Phi; \Lambda\rangle \ . \tag{37}$$

Let us see the Ω_Λ, which is defined as

$$\Omega_\Lambda = \frac{P\Lambda^T P}{\Lambda} \ . \tag{38}$$

In the case of $SO(n)$, which means $P = 1$,

$$\Omega_{\Lambda,SO} = \frac{\Lambda^T}{\Lambda} = \begin{cases} +1 & \text{(for symmetric } \Lambda) \\ -1 & \text{(for anti-symmetric } \Lambda) \end{cases} \ , \tag{39}$$

Therefore,

$$\mathrm{tr}_{\Lambda,SO}\left[\Omega\right] = \frac{n(n+1)}{2} - \frac{n(n-1)}{2} = n \ . \tag{40}$$

In the case of $Sp(n)$ (exercise)

$$\mathrm{tr}_{\Lambda,Sp}\left[\Omega\right] = -n \ . \tag{41}$$

The matter and ghost part is

$$\begin{aligned} \mathrm{tr}_\Phi\left[\Omega(-1)^F b_0 c_0 q^{L_0 - \frac{c}{12}}\right] &= \frac{iV_{26}}{(2\pi l_s)^{26}} \frac{1}{(2l)^{13}} \cdot q^{-1} \prod_{n=1}^{\infty} \frac{(1-(-q)^n)^2}{(1-(-q)^n)^{26}} \\ &= \frac{iV_{26}}{(2\pi l_s)^{26}} \frac{1}{(2l)^{13}} \cdot \frac{-1}{\eta(il+\frac{1}{2})^{24}} \ . \end{aligned} \tag{42}$$

Therefore, the Möbius strim amplitude is

$$A_{0,M} = \pm n \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{dl}{4l} \frac{1}{(2l)^{13}} \cdot \frac{-1}{\eta(il+\frac{1}{2})^{24}} \ , \tag{43}$$

$$= \mp n \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{ds}{2} \eta(is+\frac{1}{2})^{-24} \ , \tag{44}$$

where we used $\sqrt{2l}\eta(il+\frac{1}{2}) = \eta(\frac{i}{4l}+\frac{1}{2})$.

To sum up, three amplitudes are (introduced additional $\frac{1}{2}$ factor for Cylinder as an unoriented amplitude)

$$A_{0,C} = \frac{n^2}{2^{13}} \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int_0^\infty \frac{ds}{4} \eta(is)^{-24} \ , \tag{45}$$

$$A_{0,K} = 2^{13} \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{ds}{4} \eta(is)^{-24} \ , \tag{46}$$

$$A_{0,M} = \mp 2n \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int \frac{ds}{4} \eta(is+\frac{1}{2})^{-24} \ , \tag{47}$$

where

$$\eta(is)^{-24} = q^{-1} + 24 + \mathcal{O}(q) \qquad (q = e^{-2\pi s}) \ , \tag{48}$$

$$\eta(is+\frac{1}{2})^{-24} = -q^{-1} + 24 + \mathcal{O}(q) \ . \tag{49}$$

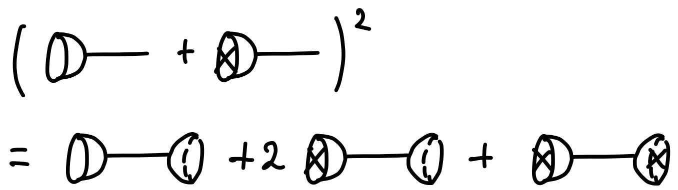

Fig. 7: Pictorical expression for the unoriented open string amplitude.

As we saw in oriented string case the massless states lead IR singularity. On the other hand, in the unoriented open string case we have

$$\frac{1}{2^{13}}\left[n \mp 2^{13}\right]^2 \cdot \frac{iV_{26}}{(2\pi l_s)^{26}} \int_0^\infty \frac{ds}{4} \cdot 24 \ , \tag{50}$$

which vanish for $SO(2^{13} = 8192)$. This cancellation can be illustrated as like Fig. 7. The cross cap shows another object (other than D-brane) that absorb and emit gravitons etc., which is called O-plane. In this situation it should be space-filling, hence, it is $\mathrm{O}25^{\pm}$-plane ($+$ for Sp and $-$ for SO). $\mathrm{O}25^{\pm}$-plane has $\pm 2^{13}$ times that of D25-brane, and so single $\mathrm{O}25^{-}$-plane cancel tension of 2^{13} D25-branes.

Although our discussion was in bosonic string, parallel argument perfectly works for superstring and it leads that **the IR divergence vanishes for** $SO(32)$. This means that

$$\text{Type I} = \text{Type IIB} + 32 \text{ D9-branes} + \text{O9}^{-}\text{-plane} \ . \tag{51}$$

Note that in the superstring case D-branes and O-plane have RR-charge in addition to tension, which has relations

$$T_{\mathrm{O9}^{\pm}} = \pm 32 \cdot T_{\mathrm{D9}} \quad (\text{tension}) \ , \quad Q_{\mathrm{O9}^{\pm}} = \pm 32 \cdot Q_{\mathrm{D9}} \quad (\text{RR-charge}) \ . \tag{52}$$

1.3 T-duality of type I theory

Let us recall T-duality. Consider X^i is S^1 compactified and T-duality acts as follows:

$$T_i: \quad X^i(z,\overline{z}) = X^i(z) + \overline{X}^i(\overline{z}) \quad \to \quad X'^i(z,\overline{z}) = X^i(z) - \overline{X}^i(\overline{z}) \ . \tag{53}$$

On the other hand, the orientation flip acts as follows:

$$\Omega: \quad X^i(z,\overline{z}) = X^i(z) + \overline{X}^i(\overline{z}) \quad \to \quad X^i(\overline{z},z) = \overline{X}^i(z) + X^i(\overline{z}) \ . \tag{54}$$

Therefore, in the T-dual coordinate X' the orientation flip acts as

$$\Omega: \quad X'^i(z,\overline{z}) \quad \to \quad -X'^i(\overline{z},z) \ . \tag{55}$$

This is understood as space-time **oribfold** as well as world-sheet orientation flip (see Fig. 8), which is called **orientifold**. Therefore, the dual space is not S^1 but $S^1/\mathbb{Z}_2$ with radius $\widetilde{R} = \frac{\alpha'}{R}$. Note that there are two fixed points, where O-planes sit and causes the ST reversal and the orientaiton flip.

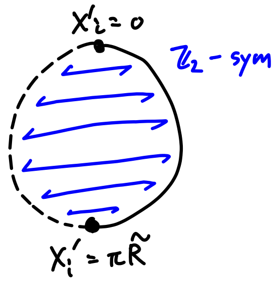

Fig. 8: $\mathbb{Z}_2$ orbifold of S^1.

Let us consider type I superstring theory with X^9 compactified on S^1 and take T-duality along the S^1. With a proper Wilson line

$$A_9 = i \begin{pmatrix} & -a_1 & & & \\ a_1 & & & & \\ & & & -a_2 & \\ & & a_2 & & \\ & & & & \ddots \end{pmatrix} \tag{56}$$

D8-branes sit at different points in $\mathbb{Z}_2$ symmetric way (see Fig. 9). Note that an O9$^-$-plane

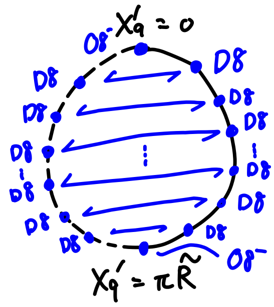

Fig. 9: T-dual of type I superstring theory.

splits into two O8$^-$-plane. Accordingly, tension and RR-charge reduce by 2.

In the end, T-dual of type I on S^1 is

$$\text{Type IIA on } S^1/\mathbb{Z}_2 \quad \text{with} \quad 2\ \text{O8}^-\text{-plane} \ + 32\ \text{D8-branes} \ . \tag{57}$$

Of course, one can consider further T-dualities along other directions. Each T-duality doubles the number of O-planes, and hence, reduces the tension and the RR-charges. Namely, we have following relations:

$$T_{\text{O}p^\pm} = \pm 2^{p-4} \cdot T_{\text{D}p} \quad (\text{tension}) \ , \quad Q_{\text{O}p^\pm} = \pm 2^{p-4} \cdot Q_{\text{D}p} \quad (\text{RR-charge}) \ . \tag{58}$$