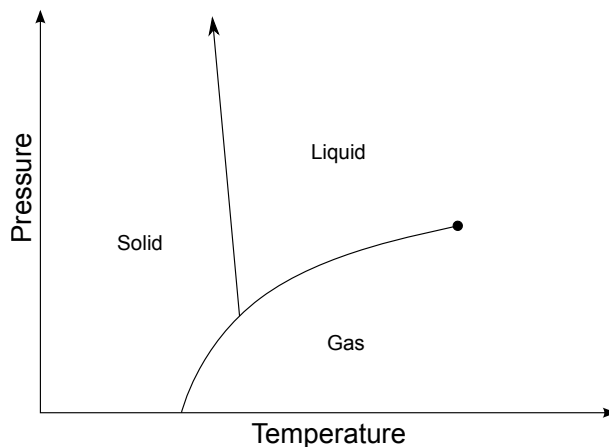


Homework 1: Due at class on Sep 21

1

The following figure is the phase diagram of H_2O . Where can you see a first-order and second-order phase transition? In addition, what is the order parameter?



2

2.1

Suppose that the free energy density $f := F/N$ obeys

$$f(t, h) = b^{-d} f(b^{y_t} t, b^{y_h} h) ,$$

under the scaling transformation $x \rightarrow x/b$. Show that the critical exponents can be expressed

$$\begin{aligned}\alpha &= 2 - d/y_t \\ \beta &= (d - y_h)/y_t \\ \gamma &= (2y_h - d)/y_t \\ \delta &= y_h/(d - y_h) .\end{aligned}$$

2.2

Under the scaling transformation $x \rightarrow x/b$, the correlation length ξ should behave as $\xi \rightarrow \xi/b$. Show that $\nu = 1/y_t$. Using the fact that the susceptibility can be written in terms of the two-point correlation function of σ

$$\chi = \frac{1}{T} \int d^d \mathbf{r} G(\mathbf{r}) ,$$

express $\alpha, \beta, \gamma, \delta$ in terms of ν, η .

3

Using (2.23), derive (2.25) in the lecture note.