Homework 7: Due at class on April 21

1. Show that $\partial_{n-1} \cdot \partial_n = 0$

2. Show that the Euler characteristics is equal to

$$\chi(X) = \sum_{i \geq 0} (-1)^i \dim C_i(K, \mathbb{R})$$

given a triangulation $|K| \to X$.

3. Show that the Euler characteristics of an odd-dimensional compact oriented closed manifold is zero.

4. Find the integer-valued homology group $H_\ell(\Sigma_g, \mathbb{Z})$ of a Riemann surface Σ_g of genus g.

5. Find both the integer-valued $H_\ell(\mathbb{R}P^2, \mathbb{Z})$ and the real-valued $H_\ell(\mathbb{R}P^2, \mathbb{R})$ homology groups of $\mathbb{R}P^2$. Does the Poincaré duality hold?