Homework 3: Due at class on March 31

1 bc ghost

1.1 Energy-momentum tensor

Given the bc ghost action (Euclidian signature)

$$S_g = \frac{1}{2\pi}\int d^2\sigma\sqrt{h}\, b^{ab}\nabla_a c_b\ ,$$

calculate the stress tensor for the bc ghosts by

$$T_{ab} = -\frac{4\pi}{\sqrt{h}}\frac{\delta S}{\delta h^{ab}}$$

Note that the covariant derivative ∇^α contains the Christoffel symbol and b_{ab} is symmetric traceless. Show that it becomes

$$T^g(z) = -2 : b(z)\partial c(z) : + : c(z)\partial b(z) : \tag{1.1}$$

in the conformally flat metric.

1.2 Ghost TT OPE

Compute the TT OPE in the bc ghost CFT, and find the central charge.

2 BRST transformation

2.1

Show that the action

$$S_X + S_g = \frac{1}{2\pi}\int d^2z\left(\frac{1}{\alpha'}\partial X^\mu\bar\partial X_\mu + b\bar\partial c + \bar b\partial\bar c\right).$$

is invariant under the BRST transformations:

$$\begin{aligned}
&\delta_B X^\mu = i\epsilon(c\partial + \bar c\bar\partial)X^\mu \\
&\delta_B c = i\epsilon c\partial c \quad \delta_B\bar c = i\epsilon\bar c\bar\partial\bar c, \\
&\delta_B b = i\epsilon\left(T^X + T^g\right) \quad \delta_B\bar b = i\epsilon\left(\overline{T}^X + \overline{T}^g\right)
\end{aligned}\tag{2.1}$$

2.2

Using the explicit form of the BRST current,

$$j_B = c(z)T^X(z) + : b(z)c(z)\partial c(z) : + \frac{3}{2} : \partial^2 c(z) : \ .$$

show that the BRST transformations of the fields are given in (2.1).

3 Bonus problem (Due at the end of May)

Apply the perturbation theory to the action

$$S_{\text{closed}} = \frac{1}{4\pi\alpha'} \int d^2\sigma \left(\sqrt{h} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + i\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) + \alpha' \sqrt{h} R^{(2)} \Phi(X) \right) .$$

derive the β-functions

$$\begin{aligned}
\beta[G_{\mu\nu}] &= \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \Phi - \frac{\alpha'}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} + \mathcal{O}(\alpha'^2) , \\
\beta[B_{\mu\nu}] &= -\frac{\alpha'}{2} \nabla^\lambda H_{\lambda\mu\nu} + \alpha' \nabla^\lambda \Phi H_{\lambda\mu\nu} + \mathcal{O}(\alpha'^2) , \\
\beta[\Phi] &= \frac{D-26}{6} - \frac{\alpha'}{2} \nabla^2 \Phi + \alpha' \nabla^\lambda \Phi \nabla_\lambda \Phi - \frac{\alpha'}{24} H_{\mu\lambda\rho} H^{\mu\lambda\rho} + \mathcal{O}(\alpha'^2) .
\end{aligned}$$