Homework 11: Due at class on May 26

1 Supersymmetric transformation in SUGRA

Show the action

$$S = \frac{1}{2\kappa_D^2} \int d^D x \, e \, \left[R - 2i\psi_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho \right] \, , \tag{1.1}$$

is invariant under the supersymmetric transformation:

$$\delta\psi_\mu^\alpha = D_\mu \epsilon^\alpha \, , \qquad \delta_\epsilon e_\mu^a = i\overline{\epsilon}\Gamma^a \psi_\mu \, . \tag{1.2}$$

2 S-duality in IIB SUGRA

Show that the actions (12.16) and (13.3) are equivalent. Show that the action (13.3) is invariant under $\mathrm{SL}(2,\mathbb{R})$-transformations.

3 S-duality between Type I and Heterotic SO(32)

In the lecture, the low-energy effective action of Type I string is given by (13.4). Also, the low-energy effective action of Heterotic $\mathrm{SO}(32)$ is given by (13.5).

(i) Explain why the Yang-Mills action S_{YM} in Type I (13.4) has $e^{-\Phi}$ whereas S_{YM} in Heterotic $\mathrm{SO}(32)$ (13.5) has $e^{-2\Phi}$.

(ii) Show that Type I (13.4) and Heterotic SO(32) (13.5) actions are related by the following the field definitions

$$\begin{aligned} G^I_{\mu\nu} &\leftrightarrow e^{-\Phi^H} G^H_{\mu\nu} \, , \qquad & \Phi^I &\leftrightarrow -\Phi^H \\ \widetilde{G}^I_{(3)} &\leftrightarrow \widetilde{H}^H_{(3)} \, , \qquad & A^I &\leftrightarrow A^H \, . \end{aligned}$$

(The equation numbers are all in the lecture note.)