Lecture 11

We are ready to learn **Heterotic string theories** [GHMR85b, GHMR85a, GHMR86]. The heterotic string is a hybrid construction of the left-moving sector of the 26-dimensional bosonic string and the right-moving sector of 10-dimensional superstring. The 16 extra bosons of the left-movers are compactified on particular 16-dimensional tori, leading to SO(32) or $E_8 \times E_8$. Since 16-dimensional tori have very special properties, we can also describe the left-movers in terms of 32 free fermions whose current algebra is associated to either SO(32) or $E_8 \times E_8$ with level $k = 1$. This hybridization of two different kinds of modes has been referred to as **heterosis**.

1 Bosonic construction

1.1 Toroidal compactifications

We have learnt the S^1 compactification so that we now generalize our analysis to the case of D-dimensions compactified on a torus T^D. The resulting theory is effectively $(26 - D)$-dimensional. The torus is defined by identifying points in the D-dimensional internal space as follows (compact dimensions are labeled with capital letters):

$$X^I \sim X^I + 2\pi e_i^I n^i = X^I + 2\pi W^I \ , \qquad \text{for} \quad n^i \in \mathbb{Z} \ . \tag{1.1}$$

The $\mathbf{e}_i = \{e_i^I\}$ $(i = 1 \cdots D)$ are D linear independent vectors called **vielbein** which generate a D-dimensional lattice Λ. In addition, the vielbein brings the metric into the standard Euclidean form:

$$G_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j = e_i^I e_j^J \delta_{IJ}, \qquad X^I \equiv e_i^I X^i \tag{1.2}$$

The torus on which we compactify is obtained by dividing $\mathbb{R}^D$ by Λ:

$$T^D = \frac{\mathbb{R}^D}{2\pi\Lambda} \ .$$

The momentum p^I conjugate to the coordinates X^I on the torus is quantized as $\mathbf{p} \cdot \mathbf{W} \in \mathbb{Z}$. Therefore, the momentum p takes its value on the dual lattice Λ^*

$$\Lambda^* \equiv \{e^{*Ii} m_i; \quad m_i \in \mathbb{Z}\}, \qquad G^{ij} = \mathbf{e}^{*i} \cdot \mathbf{e}^{*j} = e_I^{*i} e_J^{*j} \delta^{IJ}.$$

The condition which a closed string in the compact directions has to satisfy is

$$X^I(\sigma + 2\pi, \tau) = X^I(\sigma, \tau) + 2\pi W^I$$

so that W^I are analogues of winding number. We express the mode expansion for the compact direction as follows:

$$X^I(z) = x^I - i\sqrt{\frac{\alpha'}{2}} p_R^I \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m^I}{m z^m},$$

$$\overline{X}^I(\overline{z}) = \widetilde{x}^I - i\sqrt{\frac{\alpha'}{2}} p_L^I \ln \overline{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\widetilde{\alpha}_m^I}{m \overline{z}^m}.$$

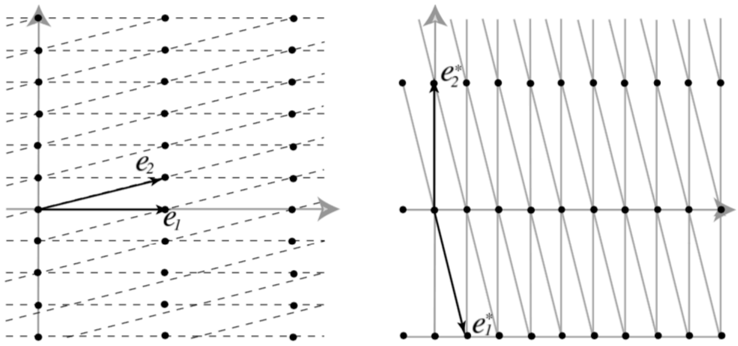


Figure 1: Lattice and its dual lattice

where the zero modes are

$$\mathbf{p}_L := p_L^I = \frac{1}{\sqrt{2}}\left[\sqrt{\alpha'}p^I + \frac{W^I}{\sqrt{\alpha'}}\right] = \frac{1}{\sqrt{2}}\left[\sqrt{\alpha'}e^{*Ii}m_i + \frac{e_i^I}{\sqrt{\alpha'}}n^i\right] \ ,$$
$$\mathbf{p}_R := p_R^I = \frac{1}{\sqrt{2}}\left[\sqrt{\alpha'}p^I - \frac{W^I}{\sqrt{\alpha'}}\right] = \frac{1}{\sqrt{2}}\left[\sqrt{\alpha'}e^{*Ii}m_i - \frac{e_i^I}{\sqrt{\alpha'}}n^i\right] \ . \qquad (1.3)$$

The mass formula and the level matching condition are now

$$\begin{aligned}\alpha' M^2 &= 2(N + \widetilde{N} - 2) + (\alpha' p_I p^I + \frac{1}{\alpha'}W_I W^I) \\ &= 2(N + \widetilde{N} - 2) + (\alpha' m_i m_j G^{ij} + \frac{1}{\alpha'}n^i n^j G_{ij}) \\ N - \widetilde{N} &= p_I W^I = m_i n^i \end{aligned} \qquad (1.4)$$

As we have seen before, the expressions for p_L and p_R suggest **T-duality** between the winding number W^I and the momentum p^I. In fact, **T-duality** is equivalence between a pair of compactification lattices $\mathbf{e}_i$ and $\mathbf{e}'_i$ that are related as $\sqrt{\alpha''}\mathbf{e}'_i = \frac{\mathbf{e}^{*i}}{\sqrt{\alpha''}}$. These two compactifications give the same spectrum since their allowed values of the momenta are related as

$$\mathbf{p}_L \leftrightarrow \mathbf{p}'_L; \qquad \mathbf{p}_R \leftrightarrow -\mathbf{p}'_R \qquad (1.5)$$

by interchanging the labels m_i and n^i.

Now let us combine the zero modes into the $(D+D)$-dimensional vectors $\mathbf{P} = (\mathbf{p}_L, \mathbf{p}_R)$. This construction treats Λ and Λ^* on equal footing as

$$\mathbf{P} = \mathbf{E}^{*i}m_i + \mathbf{E}_j n^j,$$

where

$$\mathbf{E}_j = \frac{1}{\sqrt{\alpha'}}\left(\mathbf{e}_j, -\mathbf{e}_j\right) \ , \quad \mathbf{E}^{*i} = \sqrt{\alpha'}\left(\mathbf{e}^{*i}, \mathbf{e}^{*i}\right).$$

Note that the length of the lattice is normalized by the string length $\sqrt{\alpha'} = \ell_s$. Hence $\mathbf{P}$ takes value in a $(D+D)$-dimensional lattice $\Gamma_{D,D}$ spanned by $\{\mathbf{E}^{*i}\}$ and $\{\mathbf{E}_j\}$ that satisfies the following properties:

- **Lorentzian** if the signature of the metric G is $((+1)^D, (-1)^D)$,
- **integral** if $v \cdot w \in \mathbb{Z}$ for all $v, w \in \Gamma_{D,D}$,
- **even** if $\Gamma_{D,D}$ is integral and v^2 is even for all $v \in \Gamma_{D,D}$,
- **self-dual** if $\Gamma_{D,D} = (\Gamma_{D,D})^*$,
- **unimodular** if $\mathrm{Vol}(\Gamma_{D,D}) = |\det G| = 1$.

In fact, the metric of this lattice is defined by

$$\mathbf{P} \cdot \mathbf{P}' = (\mathbf{p}_L \cdot \mathbf{p}_L - \mathbf{p}_R \cdot \mathbf{p}_R) = m_i n'^i + m'_i n^i$$

so that it is Lorentzian. Because of $\mathbf{P} \cdot \mathbf{P} \in 2\mathbb{Z}$, it is even. The self-dual property will be shown in Homework. The unimodular property $\mathrm{Vol}(\Gamma_{D,D}) = \mathrm{Vol}(\Gamma_{D,D}) = 1$ immediately follows from the self-dual property. The lattice $\Gamma_{D,D}$ in the torus compactification of the string is called **Narain lattice**.

The partition function of the bosonic string compactified on a torus T^D is easy to write down:

$$Z^{\mathrm{bos}}_{\Gamma_{D,D}} = \frac{1}{\tau_2^{(24-D)/2}|\eta(q)|^{48}} \sum_{(\mathbf{p}_R,\mathbf{p}_L)\in\Gamma_{D,D}} q^{\frac{1}{2}\mathbf{p}_R^2}\overline{q}^{\frac{1}{2}\mathbf{p}_L^2}$$

where $|\eta(q)|^{48}$ is the bosonic oscillator contribution and $\tau_2^{(24-D)/2}$ comes from the integral of non-compact momenta. This is easy to generalize to the type II string compactified on T^D

$$Z^{\text{Type II}}_{\Gamma_{D,D}} = \frac{1}{\tau_2^{(8-D)/2}|\eta(q)|^{24}} \frac{1}{4}\Big| - \vartheta_2^4(\tau) + \vartheta_3^4(\tau) - \vartheta_4^4(\tau)\Big|^2 \sum_{(\mathbf{p}_R,\mathbf{p}_L)\in\Gamma_{D,D}} q^{\frac{1}{2}\mathbf{p}_R^2}\overline{q}^{\frac{1}{2}\mathbf{p}_L^2}$$

which vanishes by virtue of the Jacobi-Riemann identity.

1.2 Heterotic strings

After our discussion of toroidal compactifications, we are now prepared to introduce the ten-dimensional heterotic string. As mentioned in the beginning, Heterotic string is a combination of the left-moving sector of the 26-dimensional bosonic string combined with the right-moving sector of the 10-dimensional superstring. The left-moving bosonic string is compactified on a 16-dimensional torus so that the momenta of the additional chiral bosons $X^I(\overline{z})$ takes value on 16-dimensional lattice Γ_{16}, *i.e* $\mathbf{p}_L \in \Gamma_{16}$. Hence, the partition function of Heterotic string can be written as

$$Z^{\mathrm{het}}(\tau) = \frac{1}{\tau_2^4\eta(q)^{12}\eta(\overline{q})^{24}}\Big(- \vartheta_2^4(\tau) + \vartheta_3^4(\tau) - \vartheta_4^4(\tau)\Big) \sum_{\mathbf{p}_L\in\Gamma_{16}} \overline{q}^{\frac{1}{2}\mathbf{p}_L^2} \tag{1.6}$$

Here $\eta(q)^8\eta(\overline{q})^{24}$ is the bosonic oscillator contribution, the τ_2^4 factor arises from the zero modes of the uncompactified transverse coordinates and $\vartheta_i^4/\eta(q)^4$ comes from the world-sheet fermions. The most interesting part of this partition function is the lattice sum

$$P(\tau) := \sum_{\mathbf{p}_L \in \Gamma_{16}} \overline{q}^{\frac{1}{2}\mathbf{p}_L^2}$$

Since the partition function (1.6) should be invariant under the modular transformation $\mathrm{SL}(2,\mathbb{Z})$, the modular transformation of η and ϑ_i tell us that

$$T : P(\tau+1) = P(\tau)\ , \qquad S : P(-1/\tau) = \tau^8 P(\tau)\ .$$

The invariance under T-transformation clearly demands that $\mathbf{p}_L^2 \in 2\mathbb{Z}$ so that Γ_{16} must be **even**. For the S-transformation, we make use of the Poisson resummation formula

$$\sum_{\mathbf{p}\in\Lambda} e^{-\pi\alpha(\mathbf{p}+\mathbf{x})^2+2\pi i\mathbf{y}\cdot(\mathbf{p}+\mathbf{x})} = \frac{1}{\mathrm{Vol}(\Lambda)\alpha^{\dim\Lambda/2}} \sum_{\mathbf{q}\in\Lambda^*} e^{-2\pi i\mathbf{q}\cdot\mathbf{x}-\frac{\pi}{\alpha}(\mathbf{y}+\mathbf{q})^2}$$

which amounts to

$$P(-1/\tau) = \frac{\tau^8}{\mathrm{Vol}(\Lambda)} \sum_{\mathbf{p}_L\in(\Gamma_{16})^*} \overline{q}^{\frac{1}{2}\mathbf{p}_L^2}\ .$$

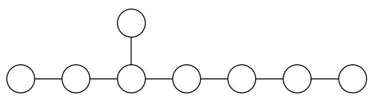
This requires that the lattice Γ_{16} is **self-dual**, *i.e.* $(\Gamma_{16})^* = \Gamma_{16}$ so that $\mathrm{Vol}(\Lambda) = 1$.

It turns out that there are only two even self-dual Euclidean lattices in 16 dimensions

- the root lattice of $E_8 \times E_8$
- the weight lattice of $\mathrm{Spin}(32)/\mathbb{Z}_2$

The metric G_{ij} of the root lattice of E_8 is the Cartan matrix of E_8 [1]:

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}.$$



Let us investigate the on-shell spectrum of the heterotic string more carefully. As usual, there is the tachyonic vacuum of the bosonic string. At the massless level we have oscillator excitations $\widetilde{\alpha}^{\mu}_{-1}|0\rangle$, $\widetilde{\alpha}^{I}_{-1}|0\rangle$ in left-moving sector. The former transform like space-time vectors whereas the internal oscillator excitations correspond to the left-moving part of the Abelian $\mathrm{U}(1)^{16}$ gauge boson. They build the **Cartan subalgebra** of $E_8 \times E_8$ or $SO(32)$. Both the root lattice of $E_8 \times E_8$ and the weight lattice of $\mathrm{Spin}(32)/\mathbb{Z}_2$ contain 480 vectors of $(\text{length})^2 = 2$ and generate the 496-dimensional

[1] Unfortunately, we do not have time to talk about exceptional Lie algebra or a classification of semi-simple Lie algebras [Kir08]. In particular, if you want to get some intuition of weight and root lattices, see [Kir08, Fig 7.3, Fig 8.1, Fig 8.2] for A_2. If you want to understand the structure E_8 related to string theory, we refer to [GSW87, §6].

non-Abelian gauge bosons of these groups. Remarkably, although Heterotic strings are closed strings, gauge fields show up thanks to the extra 16-dimensional tori! This can be also understood as novel **stringy effect** and gauge groups are restricted only to either $E_8 \times E_8$ or SO(32) in order for the theory to be consistent. Moreover, we have seen that SO(32) gauge group appears in Type I string theory. As we will see in the subsequent lecture, this is not coincident because Type I and Heterotic SO(32) are related by **S-duality**.

As a result, the massless spectra of Heterotic string are as follows

- Gravitons, B-fields, dilaton in 10D

$$\psi^{\mu}_{-\frac{1}{2}}|0\rangle_{\rm NS} \otimes \widetilde{\alpha}^{\nu}_{-1}|0\rangle$$

- their supersymmetric partners, gravitino and dilatino

$$|\mathbf{s}\rangle_{\rm R} \otimes \widetilde{\alpha}^{\nu}_{-1}|0\rangle$$

- 496 gauge bosons of $E_8 \times E_8$ or SO(32)

$$\psi^{\mu}_{-\frac{1}{2}}|0\rangle_{\rm NS} \otimes \widetilde{\alpha}^{I}_{-1}|0\rangle \ , \qquad \psi^{\mu}_{-\frac{1}{2}}|0\rangle_{\rm NS} \otimes |\mathbf{p}_L^2 = 2\rangle$$

- 496 supersymmetric partners, gaugini

$$|\mathbf{s}\rangle_{\rm R} \otimes \widetilde{\alpha}^{I}_{-1}|0\rangle \ , \qquad |\mathbf{s}\rangle_{\rm R} \otimes |\mathbf{p}_L^2 = 2\rangle$$

Indeed, Heterotic string theory is 10D $\mathcal{N} = 1$ supergravity coupled to 10D $\mathcal{N} = 1$ $E_8 \times E_8$ or SO(32) super-Yang-Mills theory so that it has 16 real supersymmetric charges.

2 Fermionic construction

The 16 bosonic fields compactified on the self-dual lattice can be described by fermionic fields, which is called **fermionization**. Therefore, we will describe fermionic construction of Heterotic string theory next.

The world-sheet action of Heterotic string theory is given by

$$\begin{aligned} S^{\rm m} &= \frac{1}{4\pi}\int d^2z \left(\frac{2}{\alpha'}\partial X^{\mu}\overline{\partial}X_{\mu} + \psi^{\mu}\overline{\partial}\psi_{\mu} + \widetilde{\lambda}^{A}\partial\widetilde{\lambda}_{A}\right) \\ S^{\rm gh} &= \frac{1}{2\pi}\int d^2z \left(b\overline{\partial}c + \overline{b}\partial\overline{c} + \beta\overline{\partial}\gamma\right) \end{aligned} \tag{2.1}$$

where μ are 10-dimensional indices and the right-moving sector is supersymmetric. In order for the theory to be Weyl-anomaly free, the central charge

$$c^{\rm tot} = c^X + c^{\psi} + c^{bc} + c^{\beta\gamma} + c^{\lambda} = 10 + \frac{5}{2} - 26 + \frac{11}{2} + c^{\lambda} = c^{\lambda} - 8$$

should vanish. Since each Majorana-Weyl anti-chiral fermion $\widetilde{\lambda}^A$ contributes $\frac{1}{4}$ to the central charge, we need 32 left-moving fermions $\widetilde{\lambda}^A$ in the action.

It turns out that there are two possible boundary conditions on the left-moving fermions $\widetilde{\lambda}_A$ which give rise to fully consistent string theories. If we impose the same boundary condition to all, it leads to SO(32) gauge group. On the other hand, if we impose one boundary condition to a half and the other boundary condition to the other half, we obtain $E_8 \times E_8$ gauge group.

2.1 Heterotic SO(32) (HO)

For Heterotic SO(32), we impose the same boundary condition to all the left-moving fermions as

$$\begin{aligned} \widetilde{\lambda}^A(t,\sigma+2\pi) &= +\widetilde{\lambda}^A(t,\sigma) && \text{R: periodic on cylinder} \\ \widetilde{\lambda}^A(t,\sigma+2\pi) &= -\widetilde{\lambda}^A(t,\sigma) && \text{NS: anti-periodic on cylinder} \end{aligned} \tag{2.2}$$

so that there is a global symmetry SO(32) that rotates $\widetilde{\lambda}^A$ $(A = 1, \cdots, 32)$. In order for the theory to be consistent, we have to impose GSO projection on the left-moving sector. In HO theory, we pick only states with odd fermionic number in NS sector and those with even fermionic number

$$P^{\rm HO}_{\rm NS} := \frac{1-(-1)^F}{2}$$

whereas we keep only the states with even fermion number

$$P^{\rm HO}_{\rm R} := \frac{1+(-1)^F}{2} \ .$$

In addition, we have to impose the level matching condition

$$N - a = \widetilde{N} - \widetilde{a} \tag{2.3}$$

where the normal ordering constants in the left-moving sector are

$$\widetilde{a}_{\rm NS} = \frac{8}{24} + \frac{32}{48} = 1 \ , \qquad \widetilde{a}_{\rm R} = \frac{8}{24} - \frac{32}{24} = -1 \ .$$

Here the first term comes from the left-moving bosonic field $\overline{X}^i$ and the second term comes from $\widetilde{\lambda}^A$. Hence, the R sector contains only massive states. Contrary to the supersymmetric right-mover, the Tachyon state $|0\rangle_{\rm NS}$ in the NS sector is preserved under the GSO projection. However, there is no corresponding state in the right-moving sector so that it does not obey the level matching condition (2.3). As a result, the left-moving Tachyon is not included in the spectrum. Then, the first excited states after the GSO projection in the NS sector are

$$\begin{aligned} \alpha^i_{-1}|0\rangle_{\rm NS} \ , &\qquad (\mathbf{8_v}, \mathbf{1}) \\ \widetilde{\lambda}^A_{-1/2}\widetilde{\lambda}^B_{-1/2}|0\rangle_{\rm NS} \ , &\qquad (\mathbf{1}, \mathbf{adj}) \end{aligned}$$

where the bold letters are the representations of $\mathrm{SO}(8) \times \mathrm{SO}(32)$. The adjoint representation **adj** of SO(32) is the antisymmetric tensor with dimension $32 \times 31/2 = \mathbf{496}$. The following table shows the massless spectrum of HO where the first row represent the 10d $\mathcal{N} = 1$ supergravity multiplet whereas the second row shows $\mathcal{N} = 1$ gauge multiplet in the adjoint of SO(32) as we have seen in the bosonic construction.

Left\Right	$\mathbf{8_v}$	$\mathbf{8_c}$
$(\mathbf{8_v}, \mathbf{1})$	$\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}$ $\phi \quad B_{\mu\nu} \quad G_{\mu\nu}$	$\mathbf{8_s} \oplus \mathbf{56_c}$ $\lambda^+ \quad \psi_m^-$
$(\mathbf{1}, \mathbf{496})$	SO(32) gauge boson $A^\mu_{[A,B]}$	SO(32) gaugini $\eta_{[A,B]}$

2.2 Heterotic $E_8 \times E_8$ (HE)

The second heterotic string theory is obtained by dividing the $\widetilde{\lambda}^A$ into two sets of 16 with independent boundary conditions,

$$\widetilde{\lambda}^A(t, \sigma + 2\pi) = \begin{cases} \epsilon_1 \widetilde{\lambda}^A(t, \sigma) & A = 1, \cdots, 16 \\ \epsilon_2 \widetilde{\lambda}^A(t, \sigma) & A = 17, \cdots, 32 \end{cases}$$

where $\epsilon_i = \pm 1$. Therefore, in the left-moving sector, we need to take the following boundary conditions into account

$$(\mathrm{NS}_1, \mathrm{NS}_2) \ , \quad (\mathrm{R}_1, \mathrm{NS}_2) \ , \quad (\mathrm{NS}_1, \mathrm{R}_2) \ , \quad (\mathrm{R}_1, \mathrm{R}_2) \ .$$

Consequently, the global symmetry is broken to $\mathrm{SO}(16)_1 \times \mathrm{SO}(16)_2$. The GSO projection is imposed to the two sets of left-movers independently:

$$P^{\mathrm{HE}}_{\mathrm{NS}_i} := \frac{1 - (-1)^F}{2} \qquad P^{\mathrm{HE}}_{\mathrm{R}_i} := \frac{1 + (-1)^F}{2} \ .$$

We also apply for the level-matching condition (2.3). The normal ordering constant in each boundary condition is

$$\widetilde{a}_{\mathrm{NS}_1, \mathrm{NS}_2} = 1 \ , \qquad \widetilde{a}_{\mathrm{R}_1, \mathrm{NS}_2} = \widetilde{a}_{\mathrm{NS}_1, \mathrm{R}_2} = \frac{8}{24} + \frac{16}{48} - \frac{16}{24} = 0 \ , \qquad \widetilde{a}_{\mathrm{R}_1, \mathrm{R}_2} = -1 \ .$$

Again, $(\mathrm{R}_1, \mathrm{R}_2)$ boundary condition has only massive states. Although the Tachyon state $|0\rangle_{\mathrm{NS}_1, \mathrm{NS}_2}$ in the NS sector is preserved under the GSO projection, it does not obey the level-matching condition (2.3) so that it is not present in the spectrum. Then, the massless states are

$$\begin{aligned} \alpha^i_{-1}|0\rangle_{\mathrm{NS}_1, \mathrm{NS}_2} \ , \qquad & (\mathbf{8_v}, \mathbf{1}) \\ \widetilde{\lambda}^A_{-1/2}\widetilde{\lambda}^B_{-1/2}|0\rangle_{\mathrm{NS}_1, \mathrm{NS}_2} \ , \qquad & (\mathbf{1}, \mathbf{adj}, \mathbf{1}) \text{ or } (\mathbf{1}, \mathbf{1}, \mathbf{adj}) \end{aligned} \tag{2.4}$$

where the bold letters are the representations of $\mathrm{SO}(8) \times \mathrm{SO}(16)_1 \times \mathrm{SO}(16)_2$. Note that the GSO projection requires either $1 \le A, B \le 16$ or $17 \le A, B \le 32$ in (2.4). The adjoint representation of SO(16) is of $16 \times 15/2 = \mathbf{120}$ dimension.

In $(\mathrm{R}_1, \mathrm{NS}_2)$ and $(\mathrm{NS}_1, \mathrm{R}_2)$, the ground states are massless since the normal ordering constant is zero. Since the 16 $\widetilde{\lambda}_0^A$ zero modes form 8 raising and 8 lowering operators

$$\widetilde{\lambda}_0^{K\pm} = 2^{-1/2}(\widetilde{\lambda}_0^{2K-1} \pm i\widetilde{\lambda}_0^{2K}) , \qquad K = 1, \cdots, 8 \text{ or } K = 9, \cdots, 16 ,$$

the $2^8 = \mathbf{256}$-dimensional spinor representation of SO(16) becomes massless. However, the GSO projection picks positive chirality $\mathbf{128}$ out of $\mathbf{256} = \mathbf{128} + \mathbf{128}'$ in the Ramond sector. Hence, the ground states are $(\mathbf{1}, \mathbf{128}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{128})$ under $\mathrm{SO}(8) \times \mathrm{SO}(16)_1 \times \mathrm{SO}(16)_2$ in $(\mathrm{R}_1, \mathrm{NS}_2)$ and $(\mathrm{NS}_1, \mathrm{R}_2)$, respectively .

All in all, the left-moving massless states form the representations of $\mathrm{SO}(8) \times \mathrm{SO}(16)_1 \times \mathrm{SO}(16)_2$

$$(\mathbf{8_v}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{120}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{120}) + (\mathbf{1}, \mathbf{128}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{128})$$

This spectrum strongly suggests that gauge symmetry is enhanced $\mathrm{SO}(16) \to E_8$ because E_8 has dimension $\mathbf{120} + \mathbf{128} = \mathbf{248}$ which is also the dimension of the adjoint representation E_8. In fact, E_8 has an SO(16) subgroup under which the E_8 adjoint $\mathbf{248}$ transforms as $\mathbf{120} + \mathbf{128}$. Hence, the massless spectrum is the 10d $\mathcal{N} = 1$ supergravity multiplet plus an $\mathcal{N} = 1$ $E_8 \times E_8$ gauge multiplet. Even in fermionic construction, we have reproduce the 496-dimensional adjoint representations of both SO(32) and $E_8 \times E_8$ gauge groups.

2.3 No D-branes in Heterotic strings

We have seen that D-branes are charged to RR fields in Type II theories. However, there is no RR field in Heterotic string theories because there is only world-sheet supersymmetry in the right-moving sector. In other words, although the RR $(p+1)$-form field strength G in Type II theories can be expressed as

$$G = \overline{\psi}^L \Gamma^{\mu_1 \cdots \mu_{p+1}} \psi^R ,$$

there is no ψ^L in Heterotic string theories. Hence, there is no D-brane in Heterotic string theories. Consequently, Heteroting string theories are the theories of closed strings[2]. However, apart from the fundamental strings, there are extended objects, **NS5-branes or Heterotic fivebranes**, in Heterotic string theories and they are magnetically charged under the B-field.

[2]However Polchinski argues in [Pol06] that there exist open Heterotic strings.

References

[GHMR85a] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm. Heterotic String Theory. 1. The Free Heterotic String. *Nucl. Phys.*, B256:253, 1985.

[GHMR85b] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm. The Heterotic String. *Phys. Rev. Lett.*, 54:502–505, 1985.

[GHMR86] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm. Heterotic String Theory. 2. The Interacting Heterotic String. *Nucl. Phys.*, B267:75–124, 1986.

[GSW87] M. B. Green, J. H. Schwarz, and E. Witten. *Superstring Theory Vol. 1,2*. Cambridge University Press, 1987.

[Kir08] A. Kirillov. *An introduction to Lie groups and Lie algebras*, volume 113. Cambridge University Press, 2008.

[Pol06] J. Polchinski. Open heterotic strings. *JHEP*, 09:082, 2006, `arXiv:hep-th/0510033`.