Lecture 12

1 Supergravity

String theory includes massless states as well as massive states. However, in a low-energy(IR) region, we do not see the stringy massive states because the mass $M^2 \sim \frac{1}{\alpha'} \sim \frac{1}{l_s^2}$ is assumed to be very heavy. Therefore, in the IR limit, we can describe the theory by so called **effective theory**, which only contains the lightest particles(states). The effective theory, of course, does not contain all the information of the original theory, however, it does give us some information of the original theory.

For the bosonic string theory the lightest particles are the massless states (except non-physical tachyon), and we saw that the effective theory is (Lecture 4)

$$S_{\text{eff}} = \frac{1}{2\kappa_{26}^2} \int d^{26}X \sqrt{-G} e^{-2\Phi} \left[R - \frac{1}{12} H_{\mu\lambda\rho} H^{\mu\lambda\rho} + 4\nabla^\lambda \Phi \nabla_\lambda \Phi \right] ,$$

where $2\kappa_{26}^2$ is a gravitational coupling and it is related to the Newton constant as $2\kappa_{26}^2 = 16\pi G_N$. What we will see below is supersymmetric versions of this, that are effective theories of IIA/IIB superstring theory and called type **IIA/IIB SUGRA** (SUpersymmetric GRAvity).

1.1 Local SUSY

Before going to the details of the IIA/IIB SUGRA let us see general features of SUGRA.

First of all, in order to define spinors in a curved space we need a vielbein $e^a_M(x^P)$, which transform a local coordinate x^M into a tangent space coordinate (tangent vector) x^a and vice versa (see also Fig. 1):

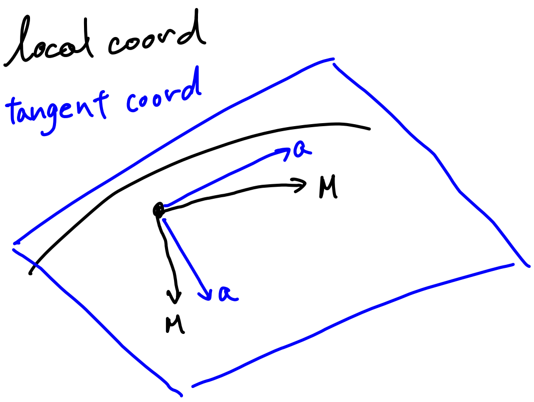

Figure 1: Local coordinate and tangent coordinate.

$$x^a = e^a_M x^M , \quad x^M = e^M_a x^a ,$$

which is defined through space-time metric G_{MN} by

$$G_{MN}(x^P) = \eta_{ab} e^a_M(x^P) e^b_N(x^P) \ .$$

Now we can use spinor representations and gamma matrices thanks to the vielbein:

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab} \ .$$

In the case that a theory includes gravity translation invariance is promoted to general coordinate transformation invariance (the symmetry is localized). Similarly, global Lorentz symmetry is promoted to local Lorentz symmetry (this is done by the vielbein):

$$\Lambda^a{}_b e^M_a(x^P) e^b_N(x^P) \equiv \Lambda^M_N(x^P) \ .$$

Gravity is nothing but a gauge field for general coordinate transformation. In general, gauge field has gauge degrees of freedom

$$\delta A_M = D_M \lambda \ ,$$

and for gravity it is

$$\delta e^a_M = D_M \lambda^a \ ,$$

where λ^a is a vector gauge transformation parameter, and D_M is a covariant derivative.

Since the translation is promoted to local one supersymmetry, whose square is roughly translation, should also be promoted to local supersymmetry. Correspondingly, there must be a gauge field that transforms with a spinor gauge parameter ξ^α as follows.

$$\delta \psi^\alpha_M = D_M \xi^\alpha \ ,$$

where ψ^α_M is called Rarita-Schwinger field or **gravitino**.

SUGRA theory always includes the two gauge fields, e^a_M and ψ^α_M, and their action is given by

$$S = \frac{1}{2\kappa_D^2} \int d^D x \ e \ \left[R - 2i\psi_M \Gamma^{MNP} D_N \psi_P \right] \ ,$$

where $e = \det e^a_M$, and we omitted the spinor index α. The SUSY transformations for the fields are

$$\delta_\epsilon e^a_M = i\bar{\epsilon}\Gamma^a \psi_m \ , \qquad \delta_\epsilon \psi_M = D_M \epsilon \ .$$

1.2 11d SUGRA

Supersymmetry puts a strong constraint on the dimension of the theory. If we limit ourselves to consider fields up to spin-2, then, it is known that the highest dimension is 11. This is roughly because the D.O.F of fermions grows exponentially: $2^{[D/2]}$, on the other hand, that of bosons grows as a power of D: $\frac{(D-1)(D-2)}{2} - 1$ (graviton). Therefore, to balance fermions and bosons we cannot go arbitrary higher.

Although the existence of fermions is crucial, what we need, to see the connections to string theories, is the bosonic part.

Let us see the action of the 11d SUGRA.

$$2\kappa_{11}^2 S_{11} = \int d^{11}x\sqrt{-G}\left[R - \frac{1}{2}K_{(4)}^2\right] - \frac{1}{6}\int d^{11}x\ M_{(3)} \wedge K_{(4)} \wedge K_{(4)}\ ,$$

where $K_4 = dM_3$ is a field strength of a rank 3 anti-symmetric tensor M_3, and $K_{(4)}^2 = K_{(4)} \wedge *K_{(4)}$. The 11d SUGRA consists of three fields; one is the graviton G_{MN} (44 states), the other boson is rank 3 anti-symmetric tensor $M_{(3)}$ (84 states), and the gravitino ψ_M (128 states). We can see that the numbers of fermions and bosons are balanced. Note that there is only one parameter κ_{11}, which can be written in terms of Planck length l_p: $\frac{1}{2\kappa_{11}^2} = \frac{2\pi}{(2\pi l_p)^9}$.

You may wonder why we looked into such a non-interesting theory in a sense that what we want is 10 dimension, rather than 11 dimension. One reason is that the interesting 10d SUGRA can be derived by dimensional reduction from the 11 SUGRA. Other reason, which is rather surprising, will be clear later.

1.3 10d Type IIA SUGRA

Let us first write down the action S_{A}, which is a sum of following three terms.

$$\begin{aligned}
S_{\mathrm{A,NS}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}H_{(3)}^2\right]\ ,\\
S_{\mathrm{A,R}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left[-\frac{1}{2}G_{(2)}^2 - \frac{1}{2}\widetilde{G}_{(4)}^2\right]\ ,\\
S_{\mathrm{A,CS}} &= -\frac{1}{4\kappa_{10}^2}\int B_{(2)} \wedge G_{(4)} \wedge G_{(4)}\ ,
\end{aligned}$$

where $H_{(3)} = dB_{(2)}$, $G_{(2)} = dC_{(1)}$, $\widetilde{G}_{(4)} = G_{(4)} - C_{(1)} \wedge H_{(3)}$, $G_{(4)} = dC_{(3)}$, and $2\kappa_{10}^2 = (2\pi l_s)^8/2\pi$.

The IIA fields are coming from reduction of 11d SUGRA fields (the compactified direction is denoted as θ):

$$\begin{aligned}
&G_{MN} \Rightarrow G_{\mu\nu}\ ,\quad G_{\mu\theta} \Rightarrow C_1\ ,\quad G_{\theta\theta} \Rightarrow \Phi\ ,\\
&M_{\mu\nu\theta} \Rightarrow B_{\mu\nu}\ ,\quad M_{\mu\nu\rho} \Rightarrow C_{(3)}\ .
\end{aligned}$$

In order to see the concrete reduction let us substitute following expression:

$$\begin{aligned} ds_{11}^2 &= G_{MN}dx^M dx^N = G_{\mu\nu}dx^\mu dx^\nu + l_p^2 e^{2\sigma}(d\theta + C_{(1)})^2 \ , \\ M_{(3)} &= C_{(3)} + B_{(2)} \wedge d\theta \ , \quad K_{(4)} = \widetilde{G}_{(4)} + H_{(3)} \wedge (d\theta + C_{(1)}) \ , \end{aligned}$$

into the action, and then, the action becomes

$$\begin{aligned} S_{11} \Rightarrow & \frac{2\pi l_p}{2\kappa_{11}^2} \int d^{10}x\sqrt{-G} \left[e^\sigma R - \frac{1}{2}e^{3\sigma}G_{(2)}^2 - \frac{1}{2}e^{-\sigma}H_{(3)}^2 - \frac{1}{2}e^\sigma \widetilde{G}_{(4)}^2 \right] \\ & - \frac{2\pi l_p}{4\kappa_{11}^2} \int d^{11}x \ B_{(2)} \wedge G_{(4)} \wedge G_{(4)} \ . \end{aligned} \tag{1.1}$$

Furthermore, we rescale the metric $G_{\mu\nu} = e^{-\sigma}G_{s,\mu\nu}$:

$$\Rightarrow \frac{2\pi l_p}{2\kappa_{11}^2} \int d^{10}x\sqrt{-G_s} \left[e^{-3\sigma} \left(R + 9\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}H_{(3)}^2 \right) - \frac{1}{2}G_{(2)}^2 - \frac{1}{2}\widetilde{G}_{(4)}^2 \right] + \cdots \ .$$

Therefore, we can identify $3\sigma = 2\Phi$.

Let us define $R = l_p e^\sigma$, which is a radius of the compactified circle. Then, we have following relation.

$$e^{3\sigma} = e^{2\Phi} \quad \Rightarrow \quad \left(\frac{R}{l_p} \right)^3 = g_s^2 \ .$$

Furthermore, we compare the coupling constant of 11d SUGRA with reduction (1.1) and IIA SUGRA:

$$\frac{2\pi R}{2\kappa_{11}^2} = \frac{e^{-2\Phi}}{2\kappa_{10}^2} \quad \Rightarrow \quad \frac{2\pi(2\pi R)}{(2\pi l_p)^9} = \frac{2\pi}{g_s^2(2\pi l_p)^8} \quad \Rightarrow \quad \frac{R}{l_p^3} = \frac{1}{l_s^2} \ .$$

Combining the two relation we have $R = g_s l_s$.

1.4 10d Type IIB SUGRA

Let us first write down the action $S_{\rm B}$, which is a sum of following three terms.

$$\begin{aligned} S_{\rm B,NS} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}e^{-2\Phi} \left[R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}H_{(3)}^2 \right] \ , \\ S_{\rm B,R} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G} \left[-\frac{1}{2}G_{(1)}^2 - \frac{1}{2}\widetilde{G}_{(3)}^2 - \frac{1}{4}\widetilde{G}_{(5)}^2 \right] \ , \\ S_{\rm B,CS} &= -\frac{1}{4\kappa_{10}^2} \int C_{(4)} \wedge H_{(3)} \wedge G_{(3)} \ , \end{aligned}$$

where $H_{(3)} = dB_{(2)}$, $G_{(1)} = dC_{(0)}$, $G_{(3)} = dC_{(2)}$, $G_{(5)} = dC_{(4)}$, $\widetilde{G}_{(3)} = G_{(3)} - C_{(0)}H_{(3)}$, and $\widetilde{G}_{(5)} = G_{(5)} - \frac{1}{2}C_{(2)} \wedge H_{(3)} + \frac{1}{2}B_{(2)} \wedge G_{(3)}$.

As we saw in the string theory analysis the string spectrum includes self-dual 4-form. The self-dual condition, in the language here, is $*\widetilde{G}_{(5)} = \widetilde{G}_{(5)}$, which is different from E.O.M $d * \widetilde{G}_{(5)} = 0$ or Bianchi id $d\widetilde{G}_{(5)} = 0$. Hence, the condition must be imposed by hand. This is problematic when you quantize the theory, however, it is not problem for our purpose.

2 String dualities

In the previous lectures we saw that there exist so called T-duality, which relates IIA, IIB, and also type I superstring theories. Although it exchanges the space-time radius $R \leftrightarrow \widetilde{R} = \alpha'/R$ it does not affect coupling constant g_s.

Our argument is based on perturbation theory, which means that the string coupling g_s is small, otherwise we cannot discuss "1 string" states. Therefore, we do not know what happen in a strong coupling region $g_s >> 1$. Since the electro-magnetic duality inverse the coupling we may wonder the same thing could happen in the string theory.

2.1 S-duality of type IIB SUGRA

Let us write the action so that we can see the hidden symmetry of the action. We use $G_{E,\mu\nu} = e^{-\Phi/2} G_{\mu\nu}$, $\tau = C_{(0)} + ie^{-\Phi}$,

$$\mathbb{M} = \frac{1}{\operatorname{Im}\tau} \begin{pmatrix} |\tau|^2 & -\operatorname{Re}\tau \\ -\operatorname{Re}\tau & 1 \end{pmatrix} , \qquad \mathbb{F}_{(3)} = \begin{pmatrix} H_{(3)} \\ G_{(3)} \end{pmatrix} ,$$

and the action becomes

$$S_{\text{B}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G_E} \left[R_E - \frac{\partial_\mu \tau \partial^\mu \overline{\tau}}{2(\operatorname{Im}\tau)^2} - \frac{1}{2} \mathbb{F}_{(3)} \cdot \mathbb{M} \cdot \mathbb{F}_{(3)} - \frac{1}{4} \widetilde{G}_{(5)}^2 \right] - \frac{1}{4\kappa_{10}^2} \int C_{(4)} \wedge \mathbb{F}_{(3)}^T \wedge \epsilon \mathbb{F}_{(3)} ,$$

where $\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. This action is invariant under the $SL(2,\mathbb{R})$ transformation:

$$\tau' = \frac{a\tau + b}{c\tau + d} , \quad \mathbb{M}' = (\Lambda^{-1})^T \mathbb{M} \Lambda^{-1} , \quad \mathbb{F}'_{(3)} = \Lambda \mathbb{F}_{(3)} , \quad \Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix} .$$

G_E and $C_{(4)}$ are invariant under the transformation.

Let us consider consider D-branes coupled to the RR-fields. D3-brane is invariant because $C_{(4)}$ is invariant. 5-branes (NS5 and D5) are magnetically coupled to 2-form fields ($B_{(2)}$ and $C_{(2)}$), therefore,

$$\mathbb{F}'_{(3)} = \Lambda \mathbb{F}_{(3)} \quad \Rightarrow \quad d\mathbb{F}'_{(3)} = \begin{pmatrix} J'_{\text{NS5}} \\ J'_{\text{D5}} \end{pmatrix} = \Lambda \begin{pmatrix} J_{\text{NS5}} \\ J_{\text{D5}} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \text{NS5}' \\ \text{D5}' \end{pmatrix} = \Lambda \begin{pmatrix} \text{NS5} \\ \text{D5} \end{pmatrix} .$$

Strings (F1 and D1) are electrically coupled to 2-form fields, hence, have a following action

$$S = \int_{\text{F1}} B_{(2)} + \int_{\text{D1}} C_{(2)} + \cdots .$$

this should be invariant, therefore,

$$\begin{pmatrix} \text{F1} & \text{D1} \end{pmatrix} \begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \text{F1}' & \text{D1}' \end{pmatrix} = \begin{pmatrix} \text{F1} & \text{D1} \end{pmatrix} \Lambda^{-1} \quad \Leftarrow \quad \begin{pmatrix} B'_{(2)} \\ C'_{(2)} \end{pmatrix} = \Lambda \begin{pmatrix} B_{(2)} \\ C_{(2)} \end{pmatrix} .$$

Due to the Dirac quantization condition, the electric and the magnetic charges must be integers, and hence, the true symmetry is $SL(2,\mathbb{Z})$.

In the case of $\Lambda = S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ fields transform $\tau \leftrightarrow -1/\tau$, F1 $\leftrightarrow$ D1, and NS5 $\leftrightarrow$ D5. Especially, if $\langle C_0 \rangle = 0$ $g_s \leftrightarrow 1/g_s$. Therefore, this is a strong-weak duality ! and typically called **S-duality**.

Other elements of $SL(2,\mathbb{Z})$ lead infinitely many bound states of F1 and D1: (p,q)-string, and those of NS5 and D5: (p,q) 5-branes.

The argument here implies that IIB superstring theory has also $SL(2,\mathbb{Z})$ symmetry, and there many evidences for it but there is no proof so far.

2.2 M-theory

Since the IIB theory has $SL(2,\mathbb{Z})$ symmetry, we may expect the similar thing happens in IIA as well. However, we immediately notice, e.g. that there is no partner for $B_{(2)}$ field, etc.

For IIA superstring theory, rather surprising phenomenon happens in strong coupling region. As we saw in the dimensional reduction of 11d SUGRA we have $R = g_s l_s$. This means that when we consider strong coupling region another spacetime direction emerges (de-compactification) !! So far, this deduction is purely from SUGRA analysis, however, if we assume that the 11d SUGRA is an effective theory of something like "string" theory, then, further miraculous coincidence happens.

Witten proposed such a "string"-like theory and put a name, **M-theory**. According to him the M stands for Magic, Mysterious, or Membrane. Some people also include Matrix, etc. so why not come up with your own M !

Since the 11d SUGRA has 3-form anti-symmetric fields there must be corresponding objects that coupled to the field electrically and magnetically. They are called **M2-brane** and **M5-brane**, which are $(1+2)$- and $(1+5)$-dimensional objects, respectively. Let us assume that they have following tensions and charges

$$T_{M2} = \mu_{M2} = \frac{2\pi}{(2\pi l_p)^3} \ , \qquad T_{M5} = \mu_{M5} = \frac{2\pi}{(2\pi l_p)^6} \ .$$

When we put this M-theory on a circle S^1, then, it should reduce to the IIA superstring theory. Let us see this by comparing the branes and their tensions (see Table 1). You should confirm that the tensions perfectly agree.

Note that M-theory is NOT even defined in a sense that we do not know how to quantize the M-branes. However, the existence of such a theory tells us a lot. Especially, even though no effective theory of M5-branes in flat space is known, M5-branes wrapped on Riemann manifolds leads amazing relation between a d-dim topological theory and $6-d$ supersymmetric gauge theories (typical one is M5 wrapped on Riemann surface and the relation is called AGT), which is an ongoing, hot research topic.

Table 1: Wrapped or unwrapped M-branes and corresponding string and D-branes, with their tensions.

Dimension	0	1	2	4	5	6
M on S^1	KK-mom.	M2/S^1	M2	M5/S^1	M5	KK-mono.
	$\frac{1}{R}$	$\frac{2\pi \cdot 2\pi R}{(2\pi l_p)^3}$	$\frac{2\pi}{(2\pi l_p)^3}$	$\frac{2\pi \cdot 2\pi R}{(2\pi l_p)^6}$	$\frac{2\pi}{(2\pi l_p)^6}$	$\frac{2\pi (2\pi R)^2}{(2\pi l_p)^9}$
IIA	D0	F1	D2	D4	NS5	D6
	$\frac{2\pi}{g_s(2\pi l_s)}$	$\frac{2\pi}{(2\pi l_s)^2}$	$\frac{2\pi}{g_s(2\pi l_s)^3}$	$\frac{2\pi}{g_s(2\pi l_s)^5}$	$\frac{2\pi}{g_s^2(2\pi l_s)^6}$	$\frac{2\pi}{g_s(2\pi l_s)^7}$

3 D-brane dynamics

There is so called Dirac-Born-Infeld action, which is an effective action for D-branes. It describes motion of the D-branes as well as gauge field living on their world-volume.

Let us consider a simple set up (see Fig. 2). The space-time is $\mathbb{R}_t \times \mathbb{R} \times S^1$ and

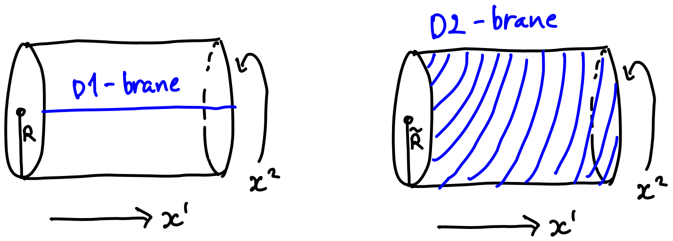

Figure 2: D1-brane and its T-dual D2-brane.

the metric is $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$. D1-brane locates at X_2 ($\sim X_2 + 2\pi R$), and D2-brane has Wilson line A_2 ($\sim A_2 + \frac{1}{\tilde{R}}$). Note that here A_2 is not a 2-form but $A_{\mu=2}$. T-duality relates these quantities (Lecture 9):

$$X_2 = 2\pi\alpha' A_2 \ .$$

Now consider vibrating D1-brane $X^2 = X^2(x^1)$ (see Fig. 3). It maps to a field strength

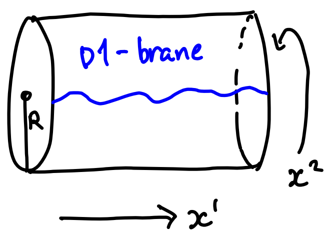

Figure 3: Vibrating D1-brane.

$F_{12} = \partial_1 X_2(x^1) \neq 0$ on D2-brane. Suppose the vibrating D1-brane is described by

Nambu-Goto action

$$
\begin{aligned}
S_{\mathrm{D1}} &= -T_{\mathrm{D1}} \int d^2x \sqrt{-\det\left(G_{\mu\nu}\frac{\partial X^\mu}{\partial x^a}\frac{\partial X^\nu}{\partial x^b}\right)} \quad \text{with} \quad X^0 = x^0 \ , \quad X^1 = x^1 \ , \\
&= -T_{\mathrm{D1}} \int dx^0 dx^1 \sqrt{1 + \left(\frac{\partial X^2}{\partial x^1}\right)^2} \ .
\end{aligned}
$$

This expression seems to coincide with

$$
\begin{aligned}
S_{\mathrm{D2}} &= -T_{\mathrm{D2}} \int d^3x \sqrt{-\det\left(G_{\mu\nu}\frac{\partial X^\mu}{\partial x^a}\frac{\partial X^\nu}{\partial x^b} + 2\pi\alpha' F_{ab}\right)} \\
&= -T_{\mathrm{D2}} \cdot 2\pi\widetilde{R} \cdot \int dx^0 dx^1 \sqrt{1 + (2\pi\alpha' F_{12})^2} \ .
\end{aligned}
$$

From the dimension analysis D-brane tension should be the following form.

$$
T_{\mathrm{D}p} \sim \frac{\text{mass}}{p\text{-dim vol}} \quad \Rightarrow \quad T_{\mathrm{D}p} \sim \frac{1}{l_s^{p+1}} \ .
$$

From the argument above in order for the two action to coincide we need $T_{\mathrm{D1}} = 2\pi\widetilde{R}T_{\mathrm{D2}}$. On the other hand, we do not want $T_{\mathrm{D}p}$ to depend on R because $T_{\mathrm{D}p}$ should be independent of space-time geometry. Note that D-brane effective theory is supposed to reproduce open string amplitude, whose leading contribution is the disk amplitude $\sim e^{-\langle\Phi\rangle}$. Thus, we propose

$$
S_{\mathrm{D}p} = -T_{\mathrm{D}p} \int d^{p+1}x e^{-\Phi(X)} \sqrt{-\det\left(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi\alpha' F_{ab}\right)} \ .
$$

Then, the ration of effective tension is

$$
\frac{T^{\mathrm{eff}}_{\mathrm{D1}}}{T^{\mathrm{eff}}_{\mathrm{D2}}} = \frac{T_{\mathrm{D1}}e^{-\Phi}}{T_{\mathrm{D2}}e^{-\widetilde{\Phi}}} = \frac{T_{\mathrm{D1}}}{T_{\mathrm{D2}}} \cdot \frac{\widetilde{R}}{l_s} = 2\pi\widetilde{R} \quad \Rightarrow \quad T_{\mathrm{D1}} = 2\pi l_s \cdot T_{\mathrm{D2}} \ .
$$

Note that the dilation field transforms under T-duality as $e^{-\widetilde{\Phi}} = e^{-\Phi}\frac{\widetilde{R}}{l_s}$. For D-branes in superstring theory the correct normalization is $T_{\mathrm{D}p} = \frac{2\pi}{(2\pi l_s)^{p+1}}$.

We heavily used the fact that Dp-branes couples to C_{p+1}. Let us consider a concrete coupling. It should be the following form.

$$
S_{\mathrm{D}p} = \cdots + \mu_{\mathrm{D}p} \cdot \int d^{p+1}x e^{-\Phi} C_{\mu_1\cdots\mu_{p+1}}(X) \frac{\partial X^{\mu_1}}{\partial x^1} \cdots \frac{\partial X^{\mu_{p+1}}}{\partial x^{p+1}} = \cdots + \mu_{\mathrm{D}p} \cdot \int_{\mathrm{D}p} e^{-\Phi} C_{(p+1)} \ .
$$

By considering T-duality we can conclude that $\mu_{\mathrm{D}p} = \frac{2\pi}{(2\pi l_s)^{p+1}} = T_{\mathrm{D}p}$ (up to a constant). Note that we have to use proper normalization of the RR-fields, depending on the context, which either string analysis or SUGRA analysis. The convention above is for string analysis. For SUGRA analysis it is (called canonically normalized one)

$$S_{\mathrm{D}p} = \cdots + \mu_{\mathrm{D}p} \cdot \int_{\mathrm{D}p} C_{(p+1)} \ .$$

Let us again vibrating D1-brane with the RR-coupling. T-duality connects the following two expression.

$$\begin{aligned} S_{\mathrm{D1}} &= \cdots + \mu_{\mathrm{D1}} \cdot \int dx^0 dx^1 e^{-\Phi} \left(C_{01} + C_{02} \frac{\partial X^2}{x^1} \right) \ , \\ S_{\mathrm{D2}} &= \cdots + \mu_{\mathrm{D2}} \cdot \int dx^0 dx^1 dx^2 e^{-\widetilde{\Phi}} \left(\widetilde{C}_{012} + \widetilde{C}_0 \cdot s\pi\alpha' F_{12} \right) \ , \end{aligned}$$

where $C_{01} \leftrightarrow \widetilde{C}_{012}$, $C_{02} \leftrightarrow \widetilde{C}_0$, and $X^2 \leftrightarrow 2\pi\alpha' A_2$. This can be understood as follows (see Fig.). The vibrating D1-brane consists of straight D1-brane along x^1 and local

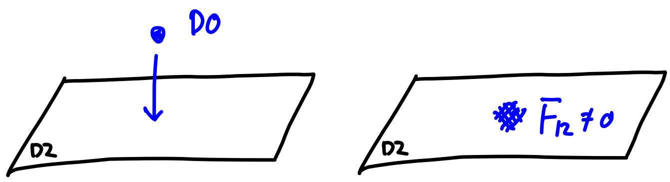

Figure 4: D0-D0 bound state.

vibration along x^2. After T-duality along x^2 the vibration part becomes D0-brane and gives $F_{12} \neq 0$. Generalization of the RR-coupling is

$$S_{\mathrm{D}p} = \cdots + \mu_{\mathrm{D}p} \cdot \int C_{RR} \wedge \exp(2\pi\alpha' F_{(2)}) \ ,$$

where $C_{RR} = \sum_n C_{(n)}$.

Finally, full general form of Dp-brane action is given as follows.

$$\begin{aligned} S_{\mathrm{D}p} = -T_{\mathrm{D}p} \int d^{p+1}x e^{-\Phi(X)} \sqrt{-\det\left(G_{ab} + 2\pi\alpha' F_{ab} - B_{ab}\right)} \\ + \mu_{\mathrm{D}p} \cdot \int C_{RR} \wedge \exp(2\pi\alpha' F_{(2)} - B_{(2)}) \ , \end{aligned}$$

where $G_{ab} = G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$ and $B_{ab} = B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$.

Dirac quantization condition

Let us consider a $(p+1)$-form RR field. The electrically coupled object is Dp-brane and the magnetic one is D$(6-p)$-brane. The magnetic charge of the D$(6-p)$-brane is

$$\mu_{6-p} = \frac{1}{2\kappa_{10}^2} \int_{B^{p+3}} dF_{(p+2)} = \frac{1}{2\kappa_{10}^2} \int_{\partial B^{p+3}} F_{(p+2)} \ .$$

Now consider Dp-brane moving around D$(6-p)$-brane.

$$S \sim \mu_p \int_M C_{(p+1)} = \mu_p \int_{\partial^{-1}M} F_{(p+2)} \ ,$$

where $\partial^{-1}M$ is manifolds whose boundary is M. The choice of $\partial^{-1}M$ is arbitrary, and does not affect the result. Consider $\partial^{-1}M_N - \partial^{-1}M_S$ surround the D$(6-p)$-brane.

$$\mu_p \int_{\partial^{-1}M_N} F_{(p+2)} - \mu_p \int_{\partial^{-1}M_S} F_{(p+2)} = \mu_p \mu_{6-p} 2\kappa_{10}^2 \in 2\pi\mathbb{Z} \ .$$

The values $\mu_p = \frac{2\pi}{(2\pi l_s)^{p+1}}$, $2\kappa_{10}^2 = \frac{(2\pi l_s)^8}{2\pi}$ satisfy the quantization condition.

Assume the condition works also for F1-string and NS5-brane,

$$T_{\mathrm{F1}} \cdot T_{\mathrm{NS5}} \cdot 2\kappa_{10}^2 g_s^2 = 2\pi \ ,$$

where we are in the string frame. Since $T_{\mathrm{F1}} = \frac{2\pi}{(2\pi l_s)^2}$,

$$T_{\mathrm{NS5}} = \frac{2\pi}{T_{\mathrm{F1}} \cdot 2\kappa_{10}^2 g_s^2} = \frac{2\pi}{(2\pi l_s)^6 g_s^2} \ .$$