Homework 7: Due at class on April 23

1. The Euler characteristics in the lecture note is defined by

$$\chi(M) = \sum_{i\geq 0}(-1)^i \dim H_i(K;\mathbb{R}) \ .$$

Show that it is indeed equal to

$$\chi(M) = \sum_{i\geq 0}(-1)^i \dim C_i(K;\mathbb{R})$$

given a triangulation $|K| \to M$.

2. Show that the Euler characteristics of an odd-dimensional oriented closed manifold is zero.

3. Find the integer-valued homology group $H_\ell(\Sigma_g;\mathbb{Z})$ of a Riemann surface Σ_g of genus g. Compute their Euler characteristics.

4. Find both the integer-valued $H_\ell(M;\mathbb{Z})$ and the real-valued $H_\ell(M;\mathbb{R})$ homology groups of both $M = \mathbb{R}P^2$ and $M =$ Klein bottle. Compute their Euler characteristics.

5. Let us construct a 3-dimensional complex K from n tetrahedra $T_1, \cdots, T_n$ by the following two steps. First we arrange the tetrahedra in a cyclic pattern as in the figure, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1}, subscripts being taken mod n. Then we identify the bottom face of T_i with the top face of T_{i+1} for each i. Compute the homology groups of K.