Homework 2: Due at class on Sep 30

1

- Derive (3.13) from (3.11) in the lecture note.
- Write $l_0, \bar{l}_0$ in terms of polar coordinate as in (2.34).

2 Möbius transformation

Let us consider the Riemann sphere $S^2 = \mathbb{C} \cup \{\infty\}$. The action of $\mathrm{SL}(2,\mathbb{C})$ defined by

$$z \mapsto w = \frac{az+b}{cz+d} , \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C}) ,$$

maps the Riemann sphere onto itself. These transformations are called fractional linear transformations.

• Given three points z_1, z_2, z_3, find a fractional linear transformation which maps the points to $0, 1, \infty$.

• Given four points z_1, z_2, z_3, z_4, their **cross ratio** is defined by

$$[z_1, z_2, z_3, z_4] = \frac{(z_1 - z_3)(z_2 - z_4)}{(z_2 - z_3)(z_1 - z_4)} .$$

Show that the cross ratio is preserved by any fractional linear transformation

$$[z_1, z_2, z_3, z_4] = [w_1, w_2, w_3, w_4] .$$

3 Energy-momentum tensor

Let us consider the free boson on a d-dimensional flat space

$$S = \frac{1}{2} \int d^d x \, \partial_\mu \varphi \partial^\mu \varphi .$$

Derive the energy-momentum tensor by using the definition

$$T^{\mu\nu} = -\eta^{\mu\nu} \mathcal{L} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi .$$

Furthermore, let us consider the action of the free boson on a curved space-time where g is the metric on M

$$S = \frac{1}{2} \int_M d^d x \sqrt{g} \, g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi .$$

Find the explicit form of the following tensor

$$\widetilde{T}^{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}} .$$

4 Three-point function for primary fields

Derive the form of the three-point function for quasi-primary fields

$$\langle \phi_1(z_1)\phi_2(z_2)\phi_3(z_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{13}^{h_3+h_1-h_2}}\,.$$