2d (0,2) dualities from 4d

Based on work with Jiaqun Jiang and Jiahao Zheng [arXiv:2407.17350]

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Basics of 2d (0,2) supermmetric theories

- **Supersymmetry:** Two right-moving supercharges Q_+, $\tilde{Q}_+$ with algebra

$$\{Q_+, \tilde{Q}_+\} = 2P_{++}.$$

  There are no left-moving supercharges.

- **Multiplets:**
  - **Chiral multiplet:** (ϕ, ψ_+) with constraint $\bar{D}_+\Phi = 0$.
  - **Fermi multiplet:** (ψ_-, F) satisfying $\bar{D}_+\Gamma = E(\Phi)$.
  - **Vector multiplet:** gauge field (A_μ, λ_-) in the adjoint representation.

- **Interactions:**
  - **E-term:** holomorphic constraint in Fermi multiplet, $E_a(\Phi)$.
  - **J-term:** superpotential-like interaction, $\int d\theta^+ \Gamma_a J^a(\Phi)$.
  - Supersymmetry requires $\sum_a E_a J^a = 0$.

- **Key features:**
  - Right-moving supersymmetry $\Rightarrow$ heterotic structure.
  - Left- and right-moving central charges (c_L, c_R) can differ.
  - Rich IR dynamics and dualities: chiral algebras, mirror symmetry, modular forms, etc.

Tools to study 2d (0,2) theories

Elliptic genus $$\mathcal{I}^{(0,2)_{\mathrm{R}}}(q,z) = \mathrm{Tr}_{\mathrm{R}}(-1)^F q^{H_L} \prod_a z_a^{f_a}$$

Chiral

$$\mathcal{I}_{\text{chi}}^{(0,2)_{\mathrm{NS}}}(\tau,u) = \prod_{w\in\lambda} \frac{\eta(q)}{\vartheta_4(q^{\frac{r-1}{2}} z^w)}$$

Fermi

$$\mathcal{I}_{\text{fer}}^{(0,2)_{\mathrm{NS}}}(\tau,u) = \prod_{w\in\lambda} \frac{\vartheta_4(q^{\frac{r}{2}} z^w)}{\eta(q)}$$

Vector

$$\mathcal{I}_{\text{vec}}^{(0,2)_{\mathrm{R|NS}}}(q,z) = \frac{\eta(q)^{2\,\mathrm{rk}G}}{|W_G|} \prod_{\alpha\in\Delta} i\frac{\vartheta_1(z^\alpha)}{\eta(q)} .$$

$$\mathcal{I}^{(0,2)_{\mathrm{R|NS}}} = \int_{\mathrm{JK}} \prod_{\text{gauge}} \frac{dz}{2\pi i z} \mathcal{I}_{\text{vec}}^{(0,2)_{\mathrm{R|NS}}}(q,z) \prod_{\text{matter}} \mathcal{I}_{\text{chi}}^{(0,2)_{\mathrm{R|NS}}}(q,z) \mathcal{I}_{\text{fer}}^{(0,2)_{\mathrm{R|NS}}}(q,z)$$

't Hooft anomaly: $\mathrm{Tr}\,\gamma^3 f^a f^b = k_F \delta^{ab}$

central change: $c_R = 3\,\mathrm{Tr}(\gamma^3 R^2)$

$U(1)_R$-charge: c-extremization

1. The enengy spectrum ≥ 0
2. normalizable (compact) vacuum

This assumption fails in the theory of our class

Gravitational anomaly: $c_R - c_L = \mathrm{Tr}(\gamma^3)$

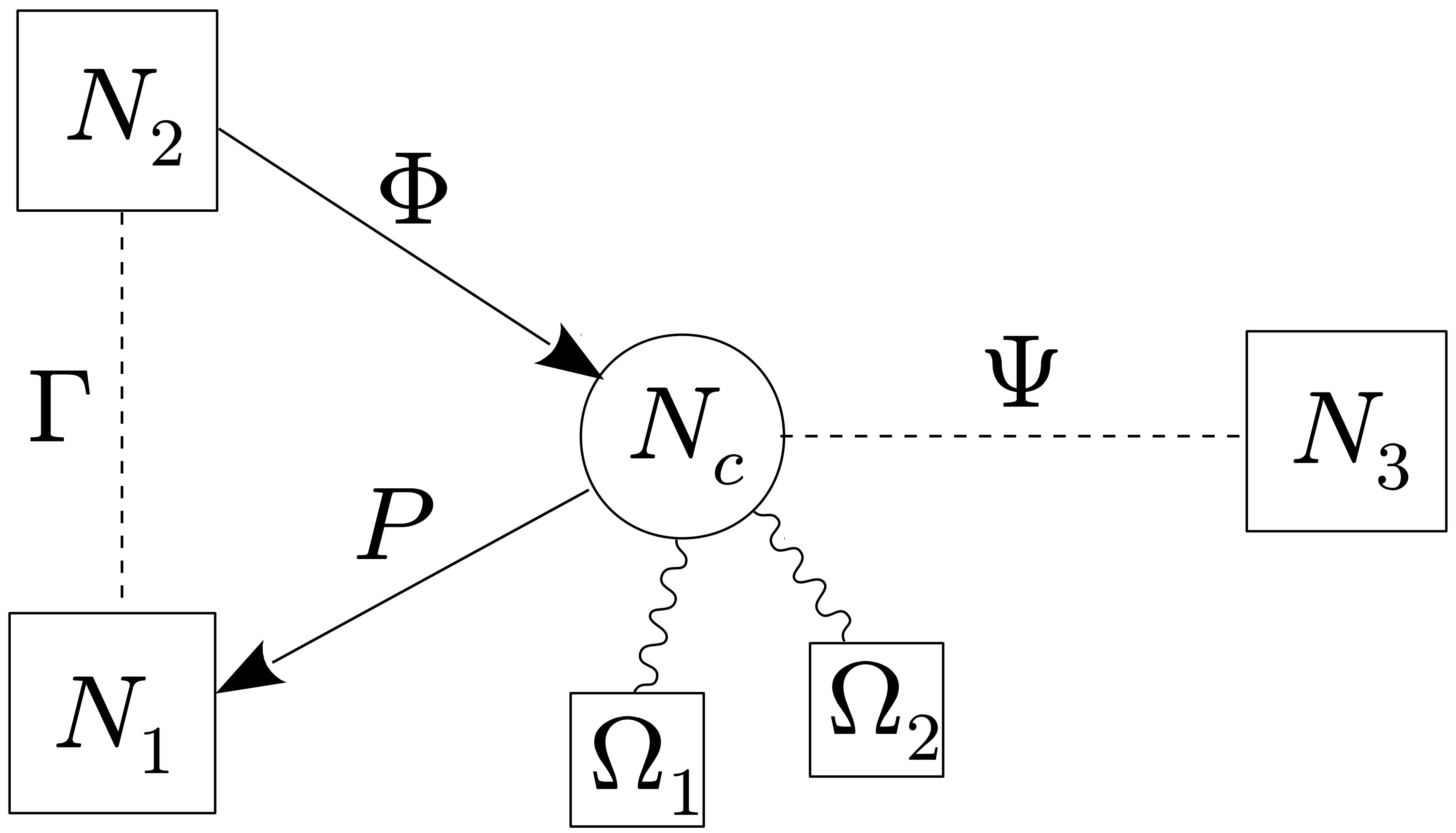
2d (0,2) triality

Gadde-Gukov-Putrov

- Triality: IR equivalence of three 2d $\mathcal{N}=(0,2)$ gauge theories
- Gauge group is a unitary gauge group
- Matter contents

	Φ	Ψ	P	Γ	Ω
$U(N_c)$	$\square$	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$	det
$SU(N_1)$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\overline{\square}$	$\mathbf{1}$
$SU(N_2)$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
$SU(N_3)$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$

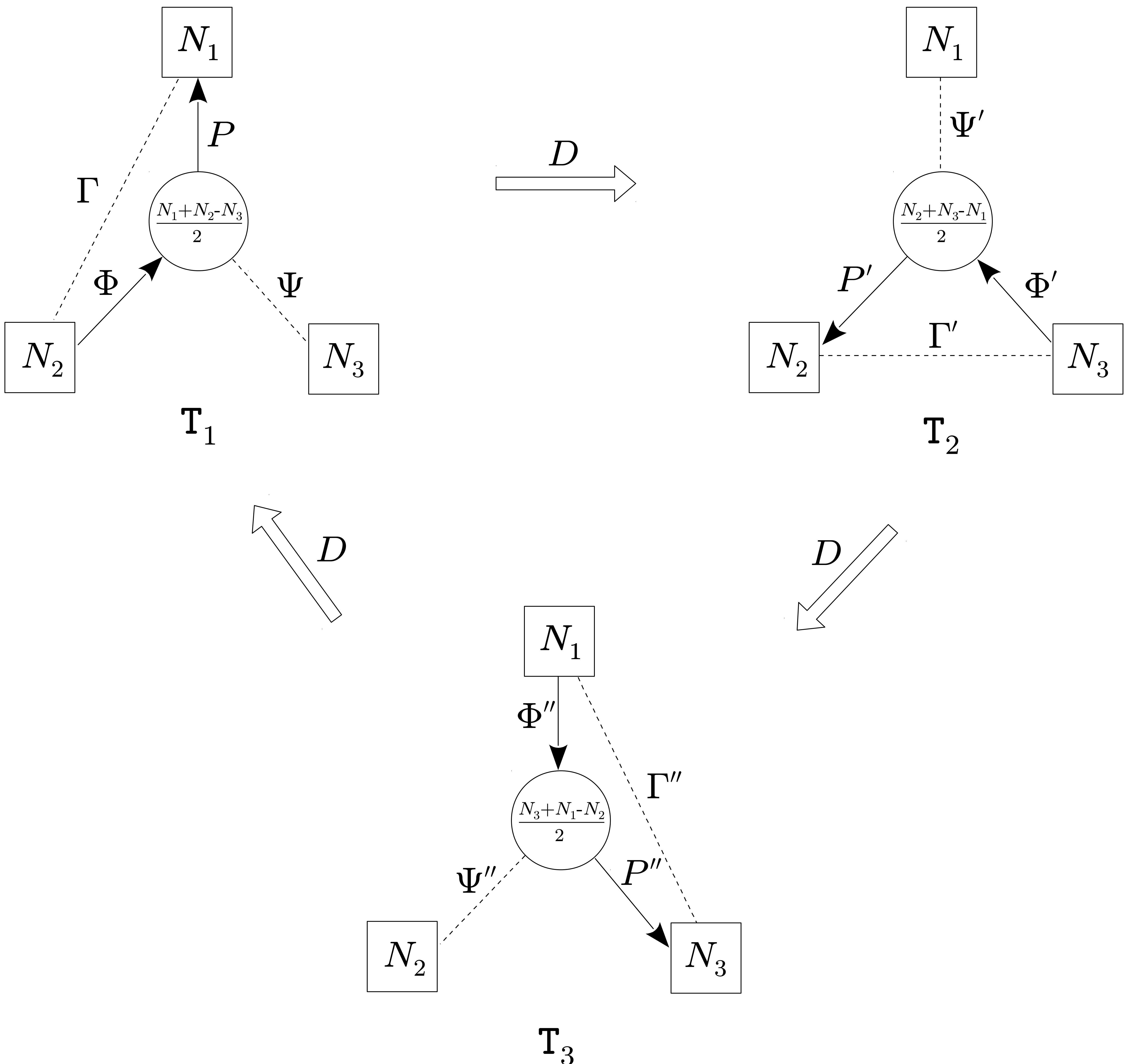
$$N_c = (N_1 + N_2 - N_3)/2$$



- J-term: $\mathcal{L}_J = \int d\theta^+ \,\mathrm{Tr}(\Gamma\Phi P)\big|_{\bar{\theta}^+=0}$
- Triality under permutation of (N_1, N_2, N_3)

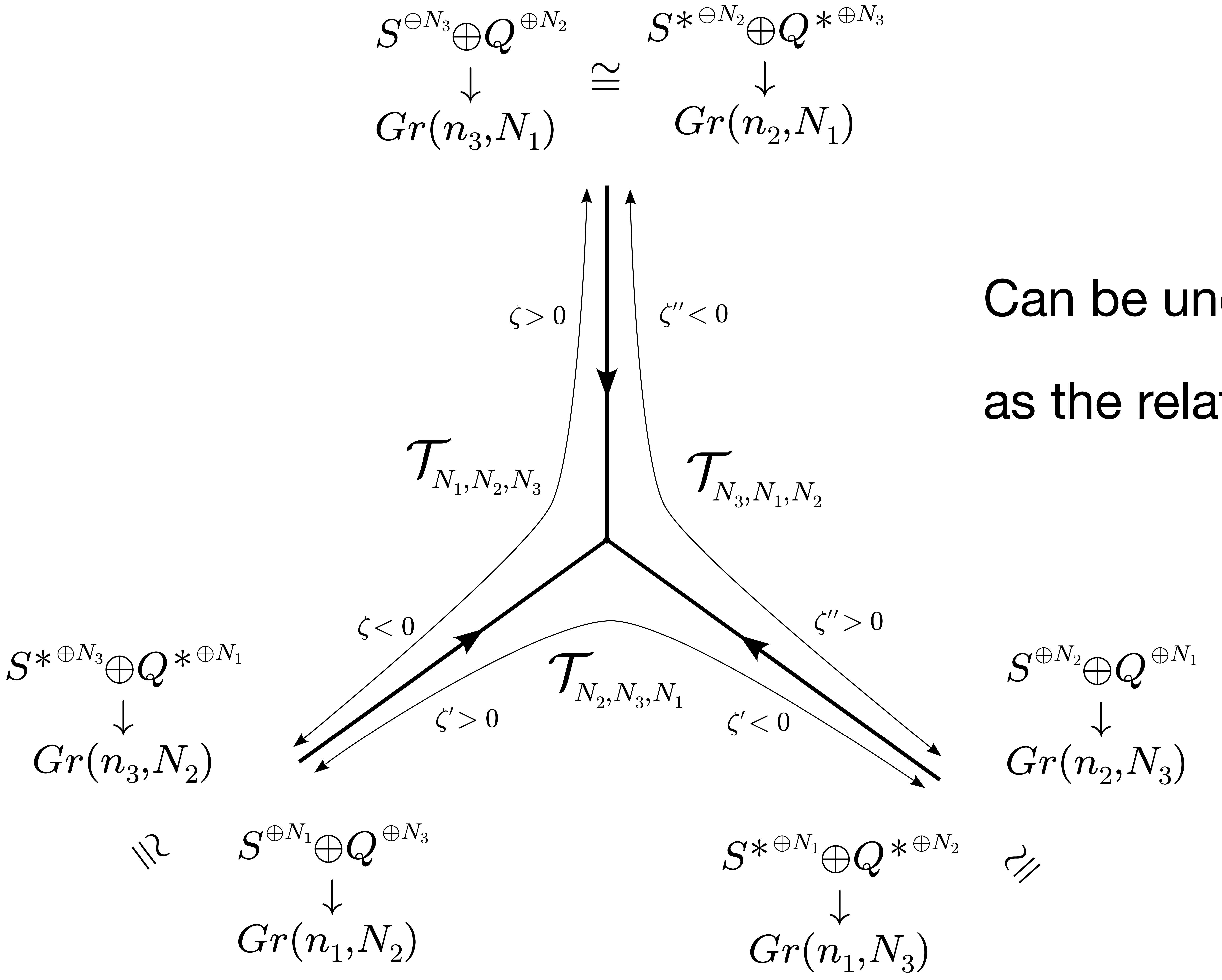
2d (0,2) triality

Gadde-Gukov-Putrov



2d (0,2) triality

Gadde-Gukov-Putrov



Can be understood

as the relation of the IR geometry

Twisted compactification from 4d N=1 to 2d (0,2)

Closset

Gadde-Razamat-Willett

- Start with a 4d $\mathcal{N}=1$ supersymmetric gauge theory on

$$M_4 = \mathbb{R}^{1,1} \times S^2.$$

- To preserve supersymmetry on the curved space S^2, perform a topological twist. The spin connection on S^2 is twisted by the $U(1)_R$ symmetry:

$$U(1)_{\text{twist}} = U(1)_{\text{spin}}(S^2) + U(1)_R.$$

- The $U(1)_R$ charge must be an integer to define the twist globally on S^2.
- After reduction (shrinking S^2), we obtain a 2d $\mathcal{N}=(0,2)$ theory on $\mathbb{R}^{1,1}$.
- The field content depends on the R-charges:

$$4\text{d } \mathcal{N}=1 \text{ vector multiplet } \longrightarrow \text{ 2d (0,2) vector multiplet}$$

$$4\text{d } \mathcal{N}=1 \text{ chiral multiplet } \longrightarrow \begin{cases} (1-r)\ \text{(0,2) chiral for } r<1, \\ (r-1)\ \text{(0,2) Fermi for } r>1. \end{cases}$$

Twisted compactification from 4d N=2 to 2d (0,2)

Gadde-Razamat-Willett

Cecotti-Song-Yan-Vafa

N-Pan-Zheng

- 4d $\mathcal{N}=2$ SCFTs have $SU(2)_R \times U(1)_r$ symmetry.
- We take the diagonal subgroup of $U(1)_R \times U(1)_r$ and twist it with $U(1)_{\text{spin}}(S^2)$ to obtain a 2d (0,2) theory.
- To perform a well-defined compactification on S^2, we further twist by a $U(1)_f$ flavor symmetry.

		Spacetime		R-sym		Flavor			
		$SU(2)_1$	$SU(2)_2$	$SU(2)_R$	$U(1)_r$	$U(1)_f$	$U(1)_{T^2}$	$U(1)_{S^2}$	$U(1)^{(0,2)}$
SUSY	Q^1_-	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	0	-1	-1	0
	Q^1_+	$\frac{1}{2}$	0	$\frac{1}{2}$	1	0	1	1	2
	Q^2_-	$-\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	-1	-1	-1
	Q^2_+	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	1	1	1
	$\tilde{Q}^1_{\dot{-}}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	-1	0	-1	1	1
	$\tilde{Q}^1_{\dot{+}}$	0	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	1	-1	-1
	$\tilde{Q}^2_{\dot{-}}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	0	-1	1	0
	$\tilde{Q}^2_{\dot{+}}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	-1	0	1	-1	-2
1/2-Hypers	q	0	0	$\frac{1}{2}$	0	1	0	0	0
	$\tilde{q}$	0	0	$\frac{1}{2}$	0	-1	0	0	1
Adj	Φ	0	0	0	2	0	0	0	1

Twisted compactification from 4d N=2 to 2d (0,2)

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- 4d $\mathcal{N}=2$ SCFTs have $SU(2)_R \times U(1)_r$ symmetry.
- We take the diagonal subgroup of $U(1)_R \times U(1)_r$ and twist it with $U(1)_{\text{spin}}(S^2)$ to obtain a 2d (0,2) theory.
- To perform a well-defined compactification on S^2, we further twist by a $U(1)_f$ flavor symmetry.
- Under this twisted compactification:

$$\begin{aligned} \text{4d } \mathcal{N}=2 \text{ vector multiplet} &\longrightarrow \text{2d (0,2) vector multiplet,} \\ \text{4d } \mathcal{N}=2 \text{ hypermultiplet} &\longrightarrow \text{2d (0,2) chiral multiplet.} \end{aligned}$$

- Vanishing of the 4d β-function is equivalent to the gauge anomaly-free condition in 2d:

$$\beta = 0 \quad \longleftrightarrow \quad \mathrm{Tr}\big(\gamma_3 G^2\big) = 0 \ .$$

- The integrand of the 2d (0,2) elliptic genus matches that of the Schur index of the original 4d $\mathcal{N}=2$ SCFT.
- The (0,2) reduction of 4d $\mathcal{N}=2$ SCFTs leads to 2d $\mathcal{N}=(0,2)$ gauge theories dual to Landau–Ginzburg models.

Reduction of Seiberg duality

Gadde-Razamat-Willett

Electric side

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$	
q	$\square$	$\overline{\square}$	1	1	$\frac{N_f-N_c}{N_f}$	$\longrightarrow r_a$
$\tilde{q}$	$\overline{\square}$	1	$\square$	-1	$\frac{N_f-N_c}{N_f}$	$\longrightarrow \tilde{r}_a$
λ	adj	1	1	0	1	

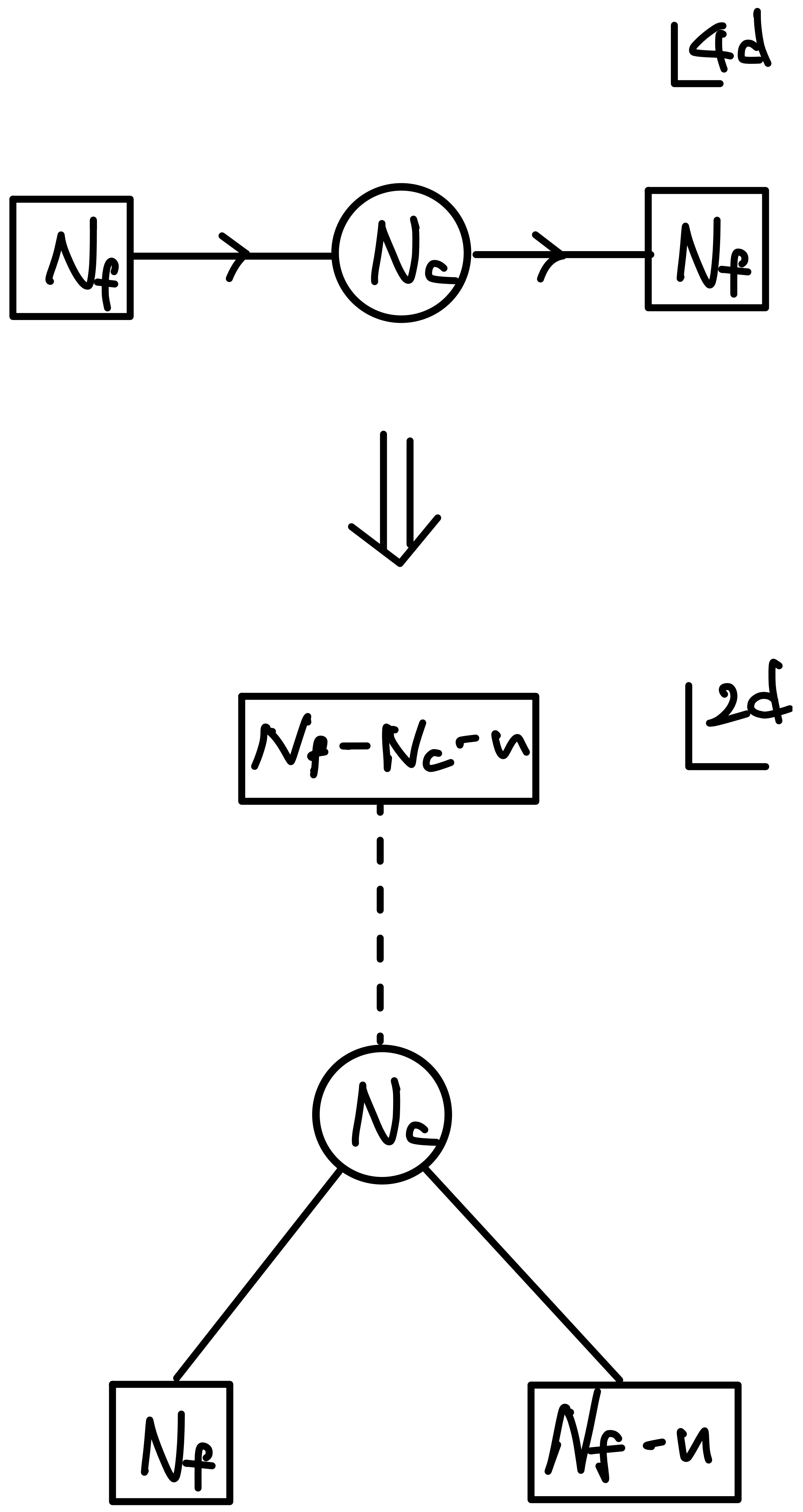
- **Mixed gauge anomaly (4d) $\rightarrow$ constraint for R-charges:**

$$\mathrm{Tr}\,(R\,GG) = \sum_{a=1}^{N_f}(r_a-1)\,T_{\square} + \sum_{a=1}^{N_f}(\tilde{r}_a-1)\,T_{\overline{\square}} + 1\cdot T_{\text{adj}} = 0.$$

$$\Longrightarrow \sum_{a=1}^{N_f}(r_a+\tilde{r}_a) \;=\; 2\,(N_f-N_c).$$

- reassign the R-charges

$$r_a = (0,\ldots,0)$$
$$\tilde{r}_a = (\underbrace{2,\ldots,2}_{N_f-N_c-n},\underbrace{1,\ldots,1}_{2n},\underbrace{0,\ldots,0}_{N_c-n})$$

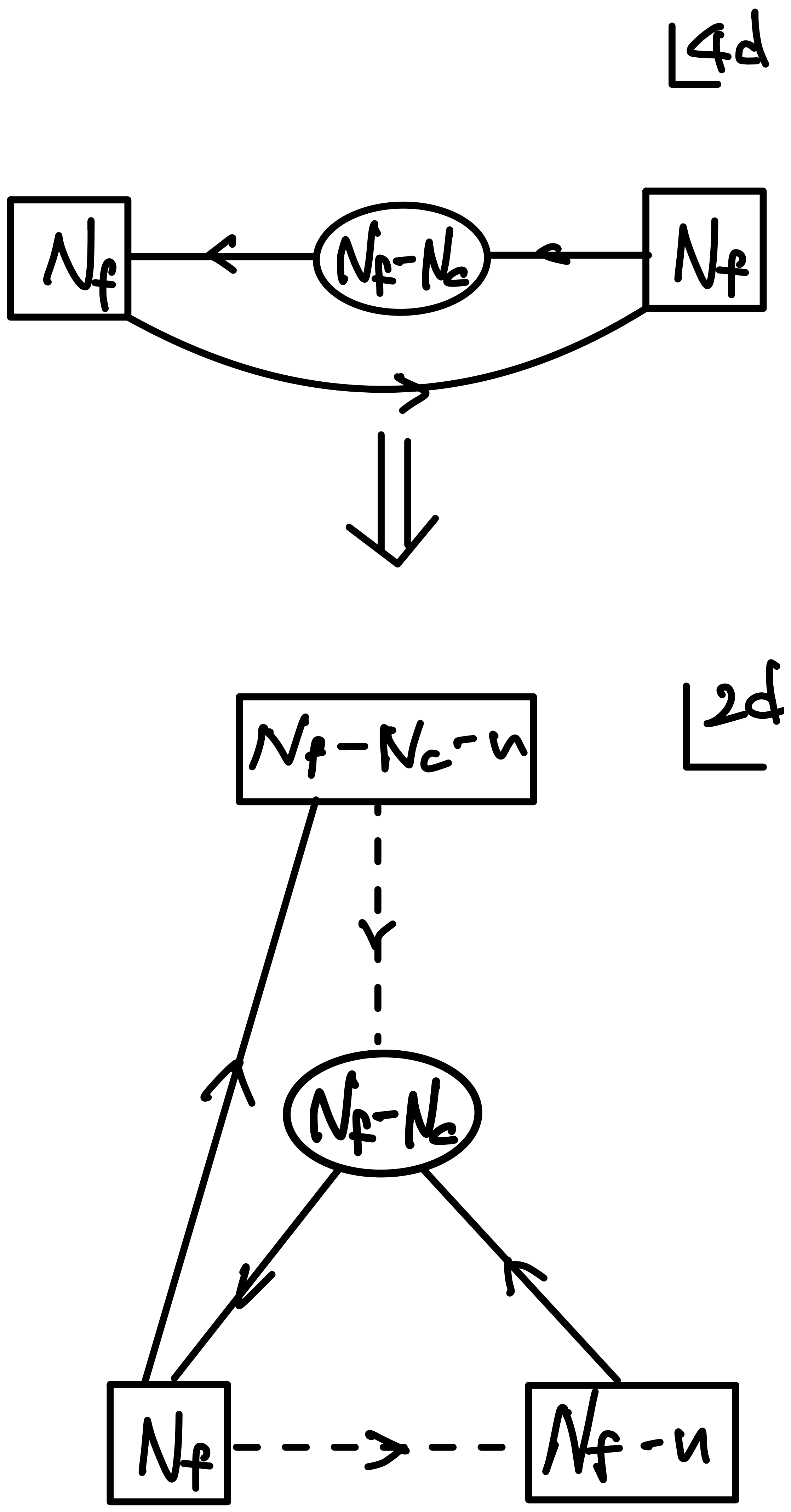


Reduction of Seiberg duality

Gadde-Razamat-Willett

Magnetic side

	$SU(N_f-N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$	
$\tilde{Q}$	$\overline{\square}$	1	$\square$	$\frac{-N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$	$\longrightarrow \tilde{r}_a$
Q	$\square$	$\overline{\square}$	1	$\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$	$\longrightarrow r_a$
Λ	adj	1	1	0	1	
M	1	$\square$	$\overline{\square}$	0	$2\frac{N_f-N_c}{N_c}$	$\longrightarrow 2-r_a-\tilde{r}_a$



- **Mixed gauge anomaly (4d) $\rightarrow$ constraint for R-charges:**

$$\mathrm{Tr}\,(R\,GG) = \sum_{a=1}^{N_f}(r_a-1)\,T_{\square} + \sum_{a=1}^{N_f}(\tilde{r}_a-1)\,T_{\overline{\square}} + 1\cdot T_{\mathrm{adj}} = 0.$$

$$\Longrightarrow \sum_{a=1}^{N_f}(r_a+\tilde{r}_a) \;=\; 2N_c.$$

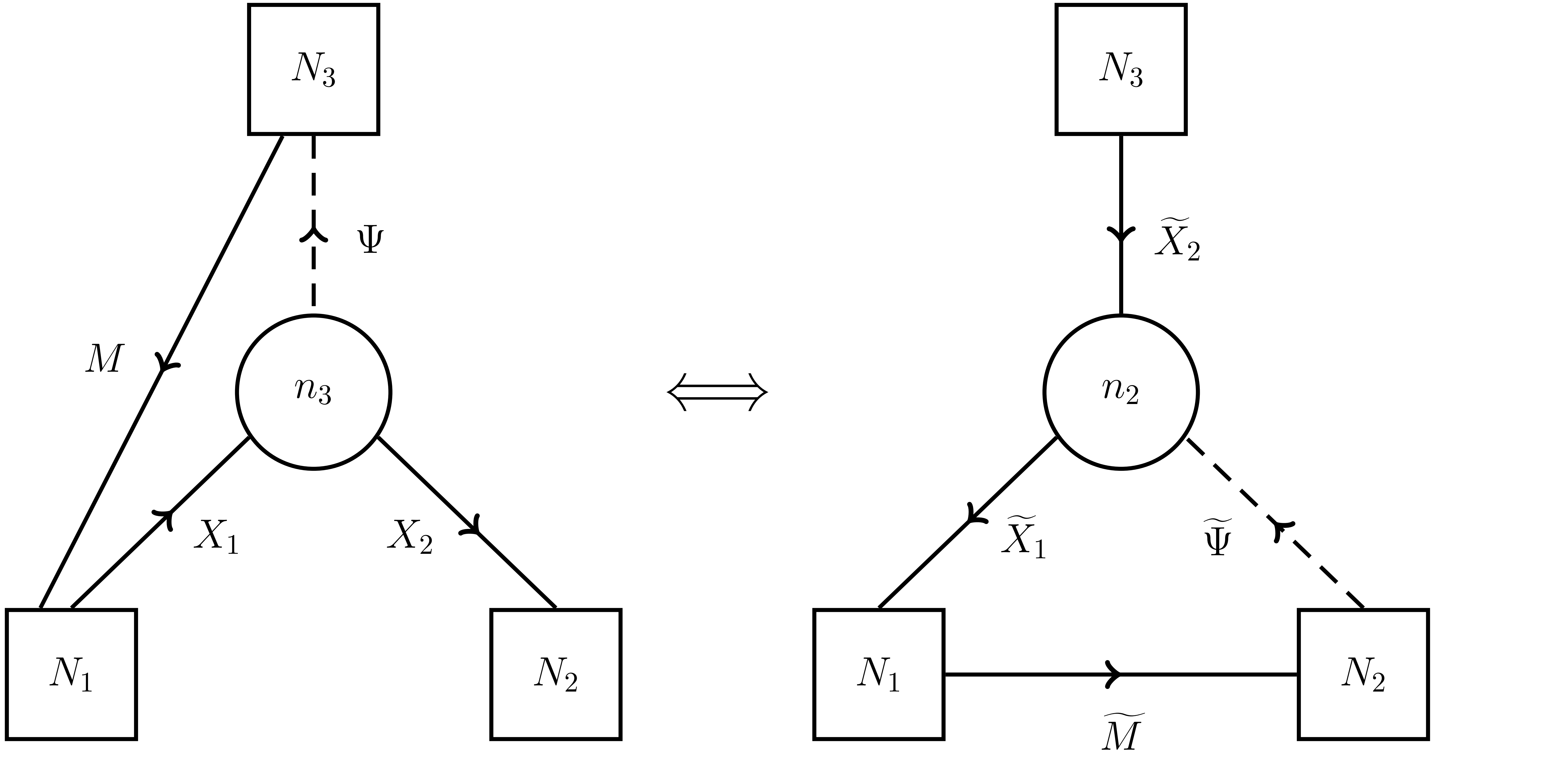
- reassign the R-charges

$$\tilde{r}_a = (0,\ldots,0)$$
$$r_a = (\underbrace{2,\ldots,2}_{N_c-n},\underbrace{1,\ldots,1}_{2n},\underbrace{0,\ldots,0}_{N_f-N_c-n})$$

Reduction of Seiberg duality

Gadde-Razamat-Willett

Swapping Fermi Meson and relabelling, we obtain 2d (0,2) duality



$$n_3 = \frac{N_1+N_2-N_3}{2} \quad \text{and} \quad n_2 = \frac{N_1+N_3-N_2}{2} \quad (N_1 \geq N_2 + N_3)$$

Unitary vs Special Unitary

- The rank condition $N_1 \geq N_2 + N_3$ for $\mathrm{SU}(N_c)$ theory is incompatible with the triality constraint $N_i + N_{i+1} \geq N_{i+2}$
- The Fayet–Iliopoulos (FI) term, essential in the $\mathrm{U}(N_c)$ case for connecting different geometric phases, cannot be introduced for $\mathrm{SU}(N_c)$.
- As a result, the two geometries cannot be smoothly connected through an $\mathrm{SU}(N_c)$ gauge theory.
- This explains why the triality that holds for $\mathrm{U}(N_c)$ fails when naively replacing the gauge group by $\mathrm{SU}(N_c)$.
- With the SU gauge groups, the dual pair flows to the non-linear sigma model with the same target manifold

$$\begin{array}{ccc} \det(S) \oplus S^{\oplus N_2} \oplus Q^{\oplus N_3} & & \det(Q^*) \oplus Q^{*\oplus N_2} \oplus S^{*\oplus N_3} \\ \downarrow & \cong & \downarrow \\ \mathrm{Gr}(n_3, N_1) & & \mathrm{Gr}(n_2, N_1) \end{array}$$

Unitary vs Special Unitary

- All chiral multiplets parametrize some non-compact directions, violating a key assumption underlying c-extremization.
- A non-holomorphic current appears for the flavor symmetry associated with these non-compact directions. Such a current cannot mix with the R-symmetry current, breaking a fundamental requirement of the c-extremization principle.
- Consequently, the naive application of c-extremization is invalid in this context. In such cases, the chiral multiplets have $r = 0$ and the Fermi multiplets have $r = 1$.
- This is in contrast to the 2d $(0, 2)$ triality: due to the compactness of the Grassmannian target space and the presence of the J-term superpotential constraint, c-extremization must be applied.
- The central charges for the dual theories are given by

$$c_L = 2\,(n_3^2 + N_1 N_3 + 1), \qquad c_R = 3\,(n_3^2 + N_1 N_3 + 1)\ .$$

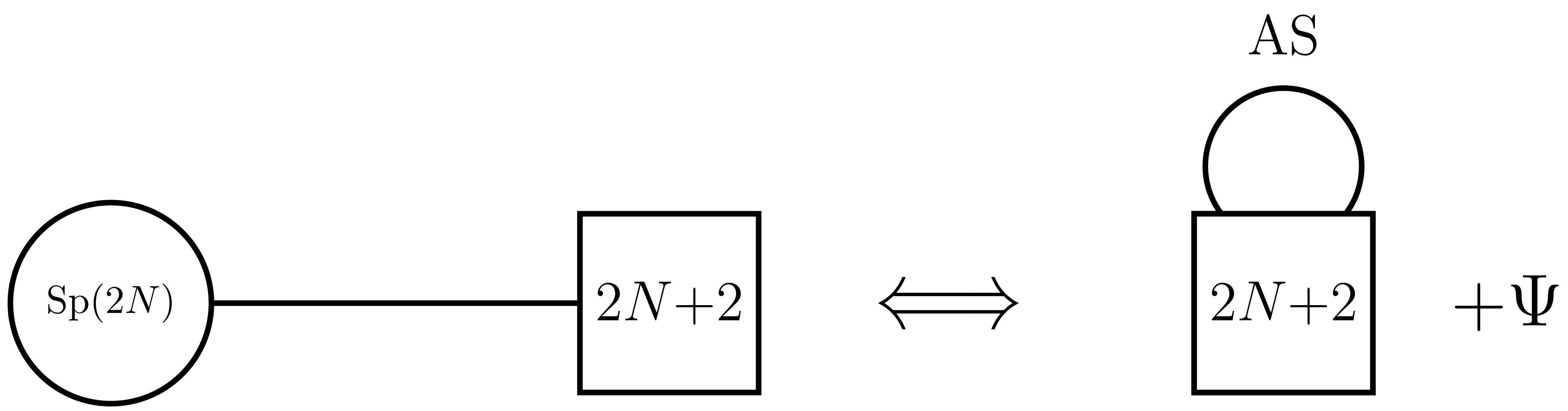
- The elliptic genera also match between the dual theories.

(0,2) Seiberg-like duality to new triality

Gadde-Razamat-Willett

Sacchi

- The twisted compactification on S^2 is applied to 4d $\mathcal{N}=1$ dualities with Sp$(2N)$ gauge groups.
- It leads to a 2d $\mathcal{N}=(0,2)$ duality between Sp$(2N)$ SQCD and a Landau–Ginzburg model



(0,2) Seiberg-like duality to new triality

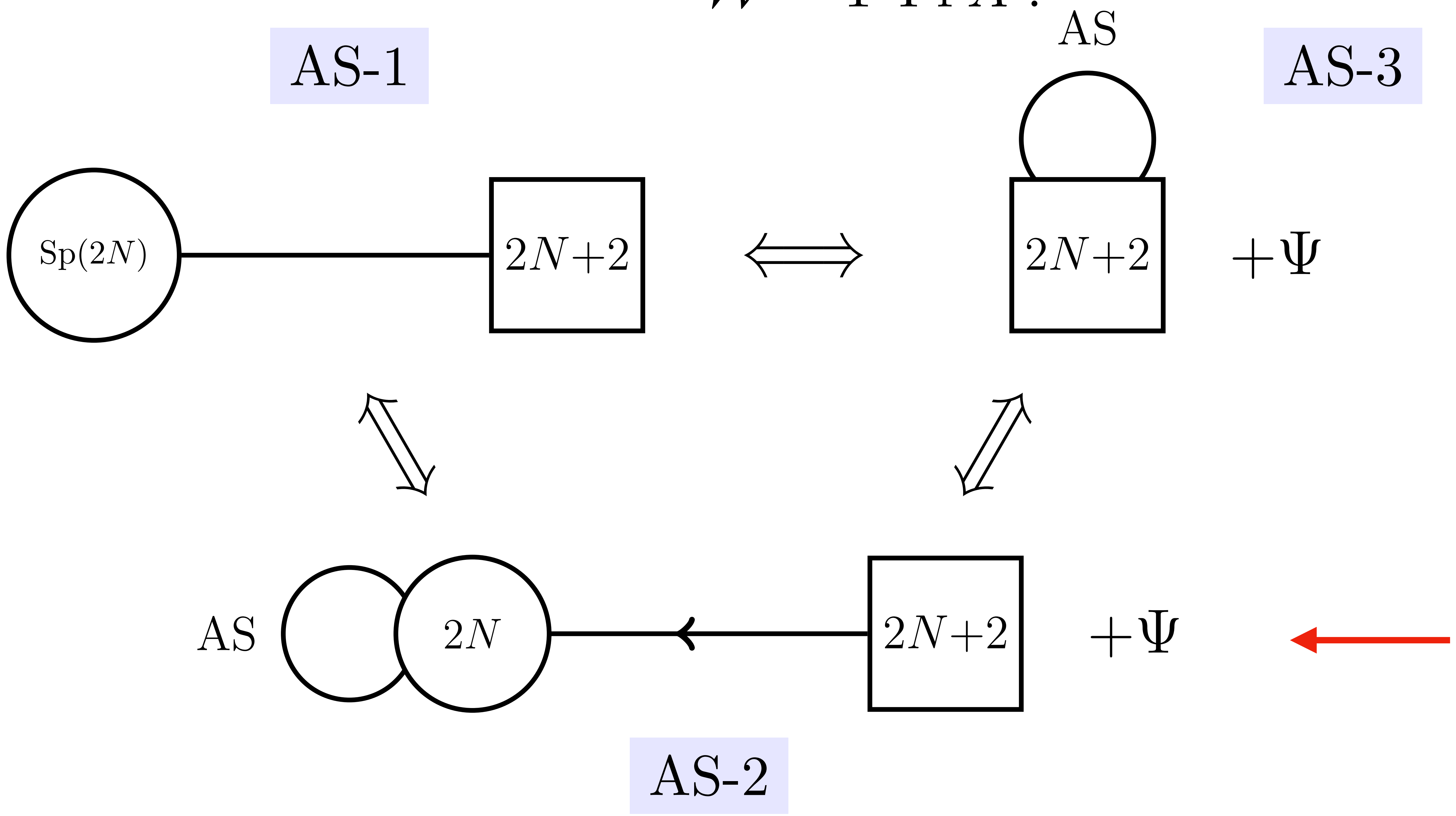
AS-1. Sp$(2N)$ gauge theory with $2N+2$ fundamental chirals Z and no superpotential.

AS-2. SU$(2N)$ gauge theory with one anti-symmetric chiral X and $2N+2$ fundamental chirals Y. Additionally, there is a neutral Fermi multiplet Ψ, forming a superpotential

$$\mathcal{W} = \Psi \ \mathrm{Pf}\, X \ .$$

AS-3. LG model of one Fermi Ψ and $(N+1)(2N+1)$ chirals, forming an anti-symmetric $(2N+2)\times(2N+2)$ matrix A with a superpotential

$$\mathcal{W} = \Psi \ \mathrm{Pf}\, A \ .$$



Twisted comp'n of 4d N=2
SU(2N)+1AS+(2N+2)F

(0,2) Seiberg-like duality to new triality

Sym-1. SO(N) gauge theory with $N-2$ fundamental chirals Y. Additionally, there is a gauge neutral chiral X and Fermi Ψ, forming a superpotential

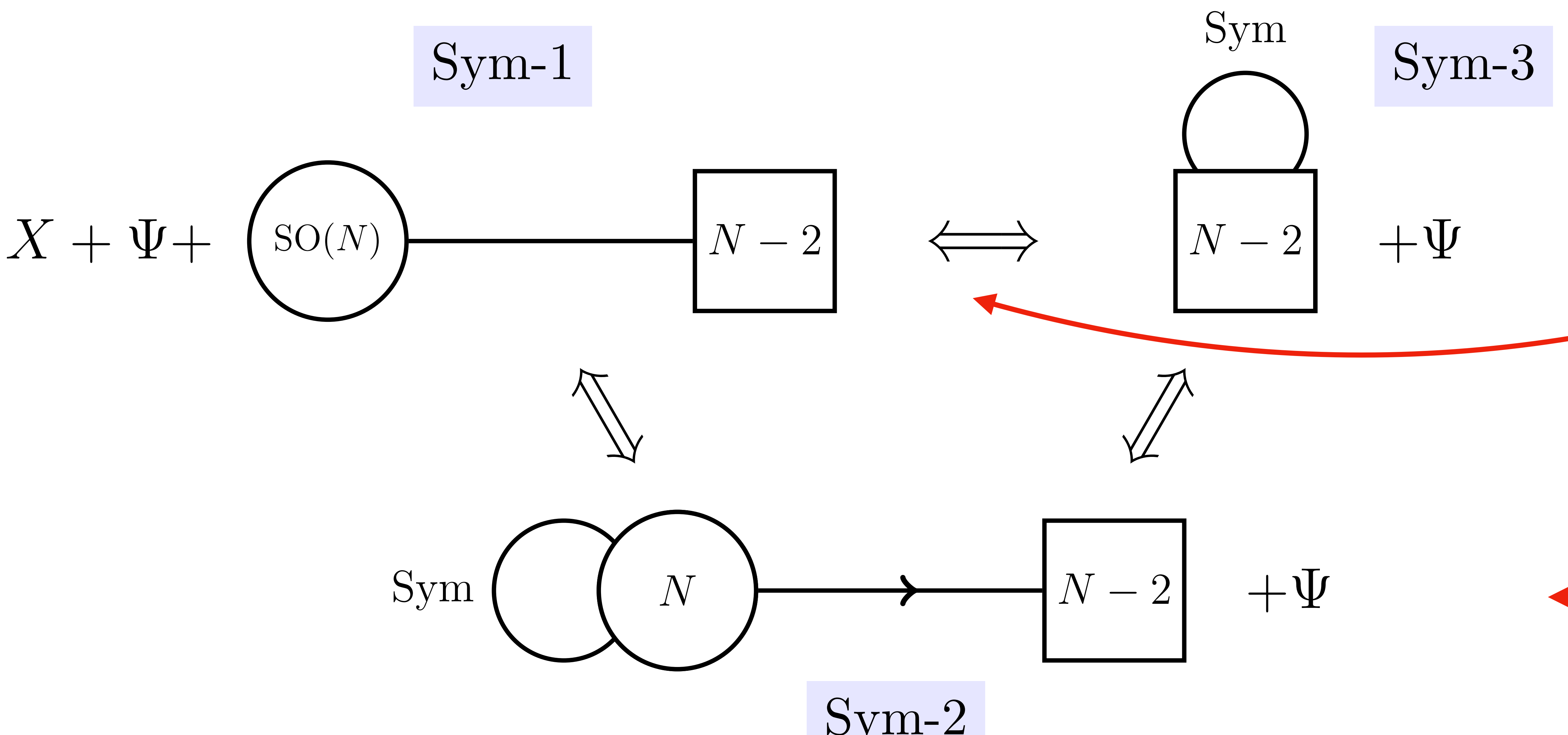
$$\mathcal{W} = \Psi(X^2 + \det A) \ , \qquad A_{ij} := \eta^{\alpha\beta} Y_{\alpha,i} Y_{\beta,j} \ .$$

Sym-2. SU(N) gauge theory with one symmetric chiral Z and $N-2$ fundamental chirals W. Furthermore, there is a neutral Fermi multiplet Ψ, which forms a superpotential

$$\mathcal{W} = \Psi \det A \ , \qquad A_{ij} = Z_{\alpha\beta} W_i^{\alpha} W_j^{\beta} \ .$$

Sym-3. LG model of one Fermi Ψ and $(N-2)(N-1)/2$ chirals, forming a symmetric $(N-2)\times(N-2)$ matrix A with a superpotential

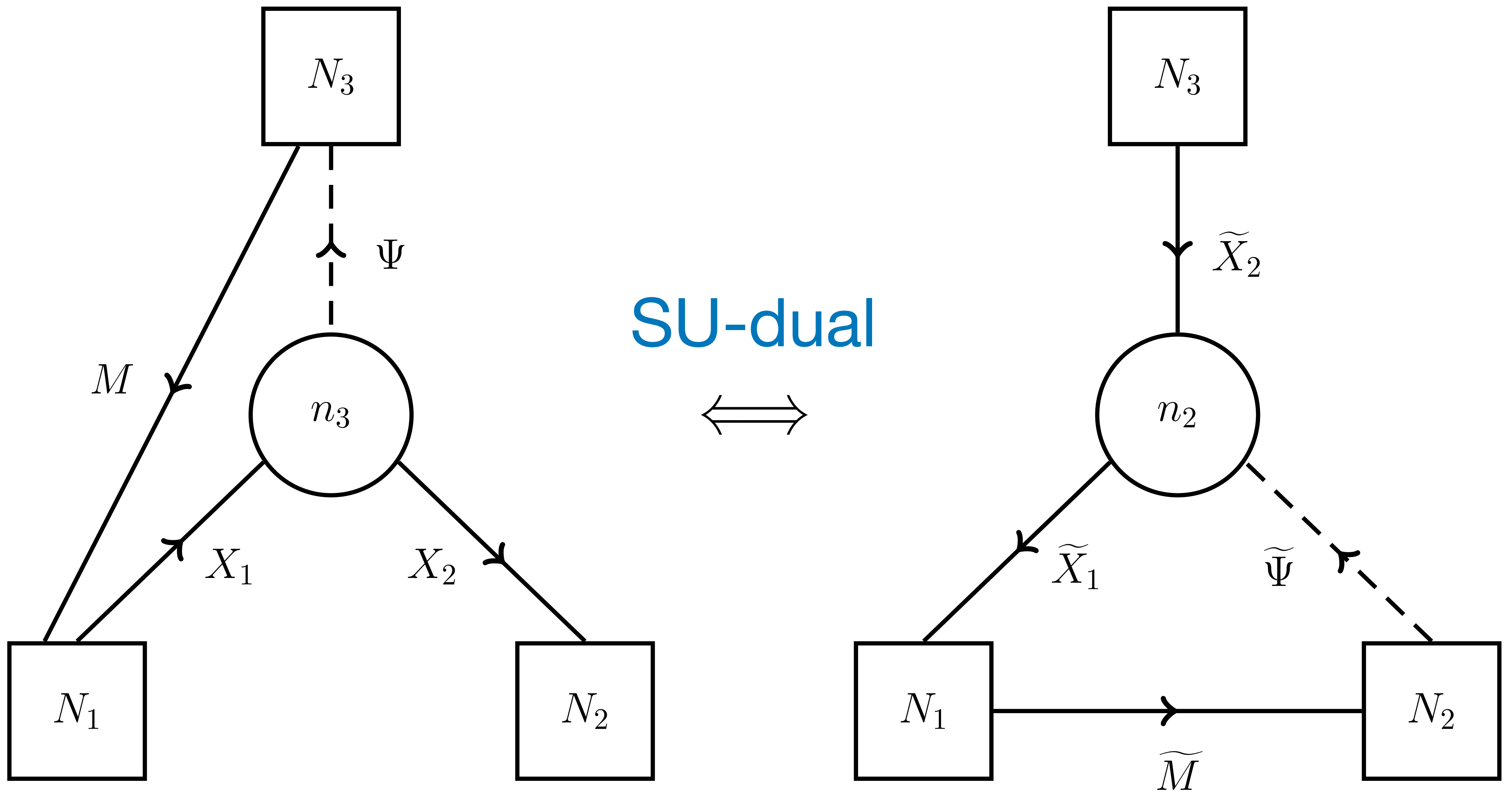
$$\mathcal{W} = \Psi \det A \ .$$

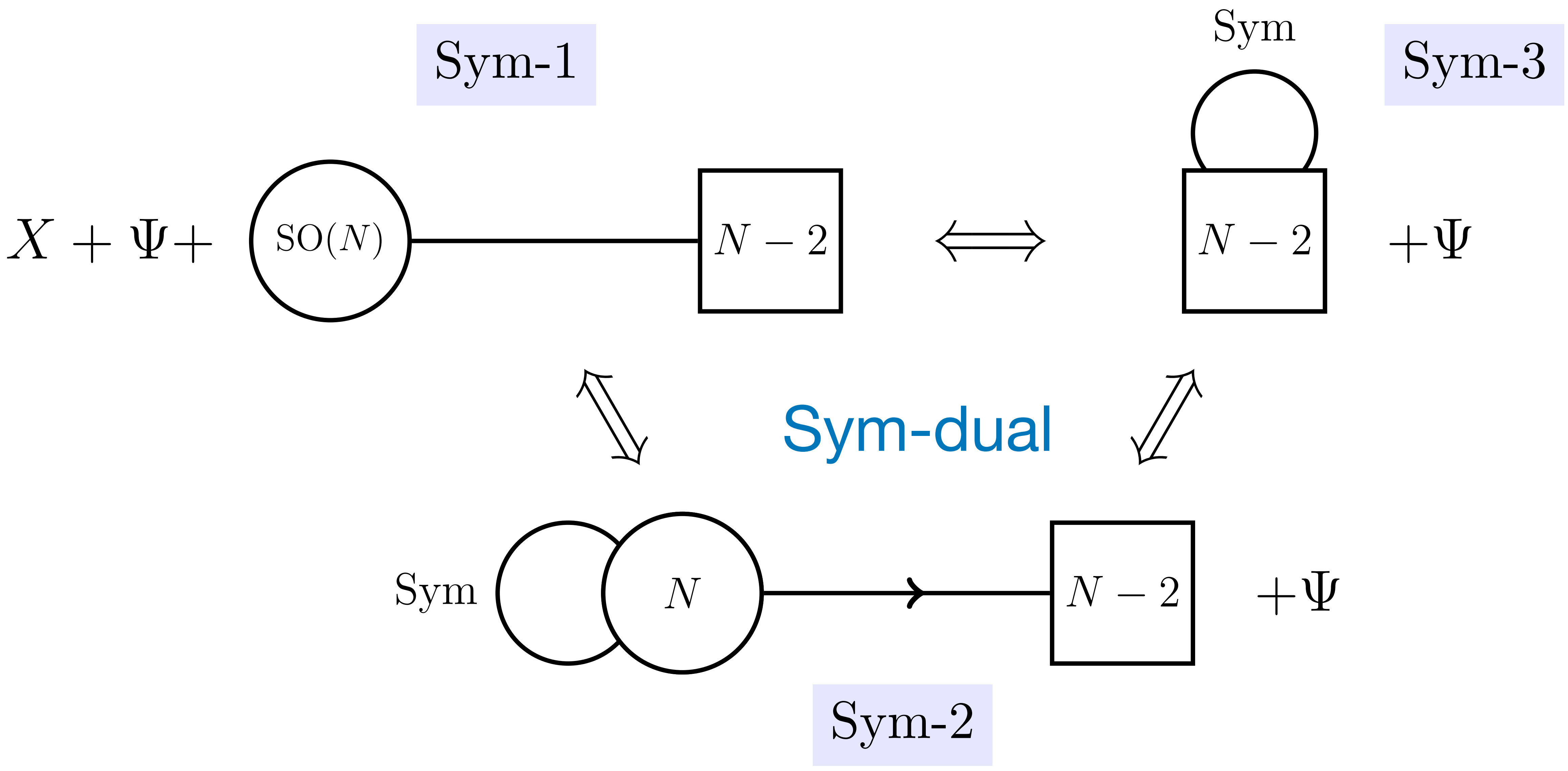


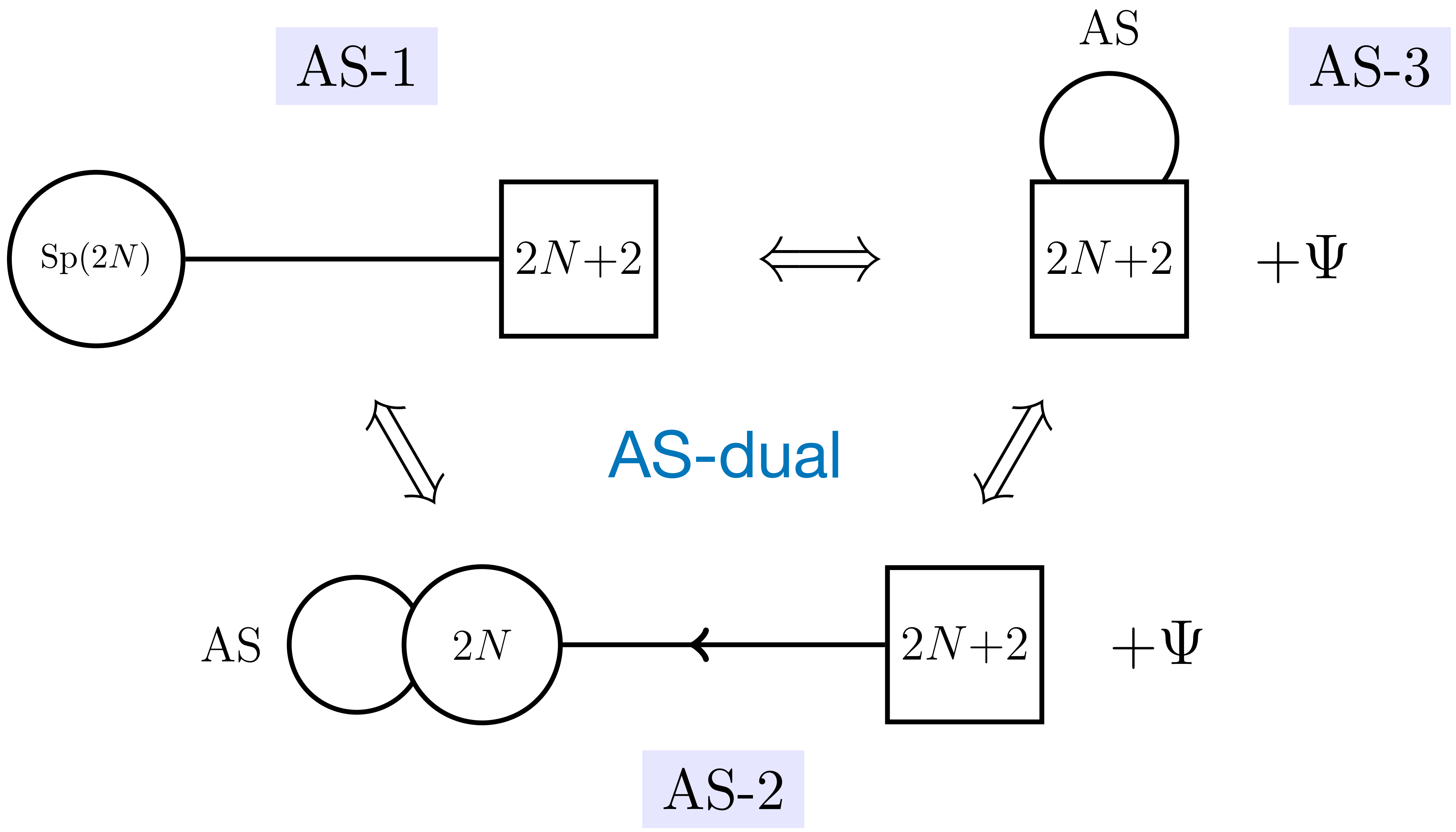
Twisted comp'n of 4d N=1
Intriligator-Seiberg duality

Twisted comp'n of 4d N=2
SU(N)+1Sym+(2N-2)F

Towards dualities from dualities

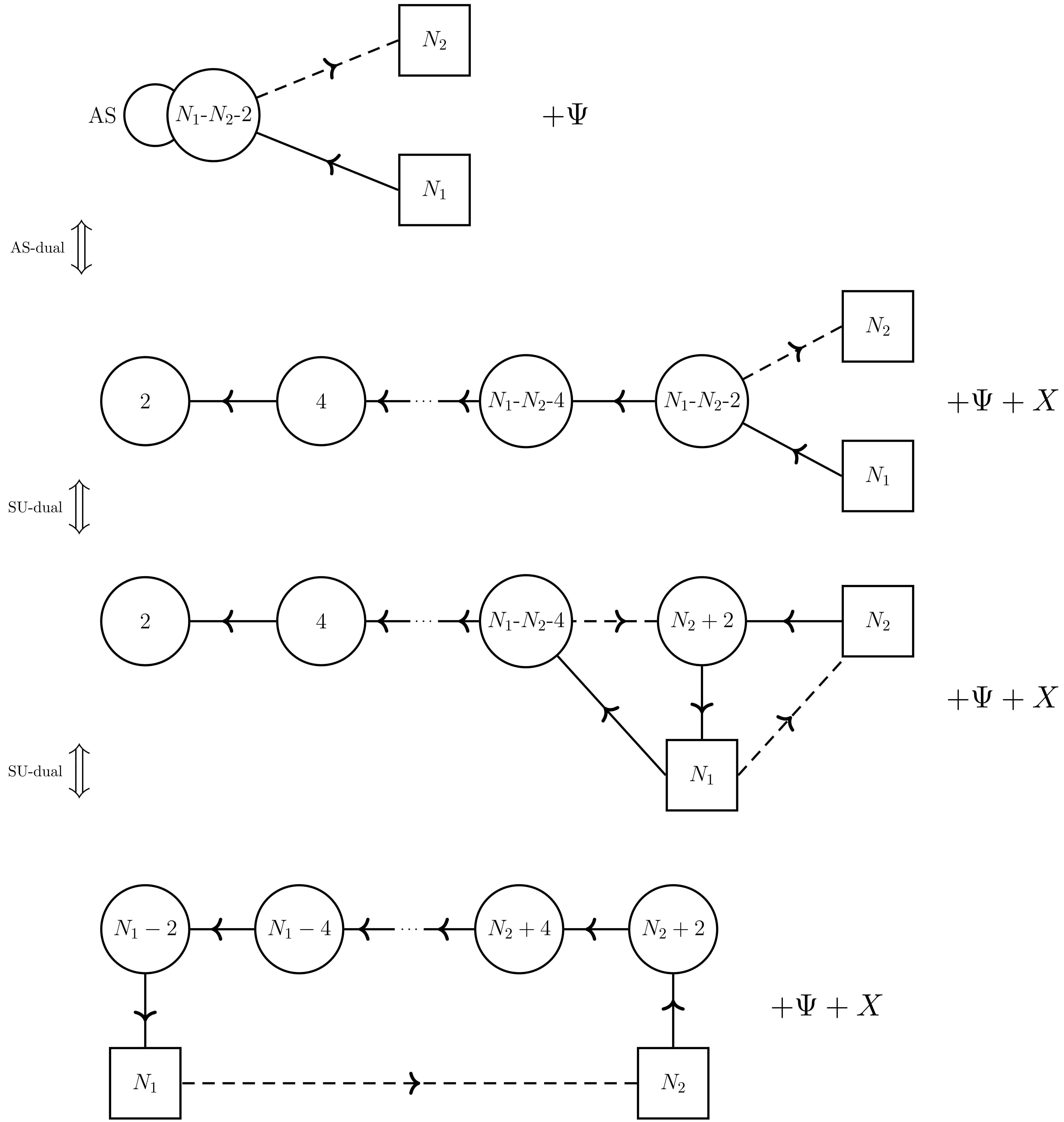






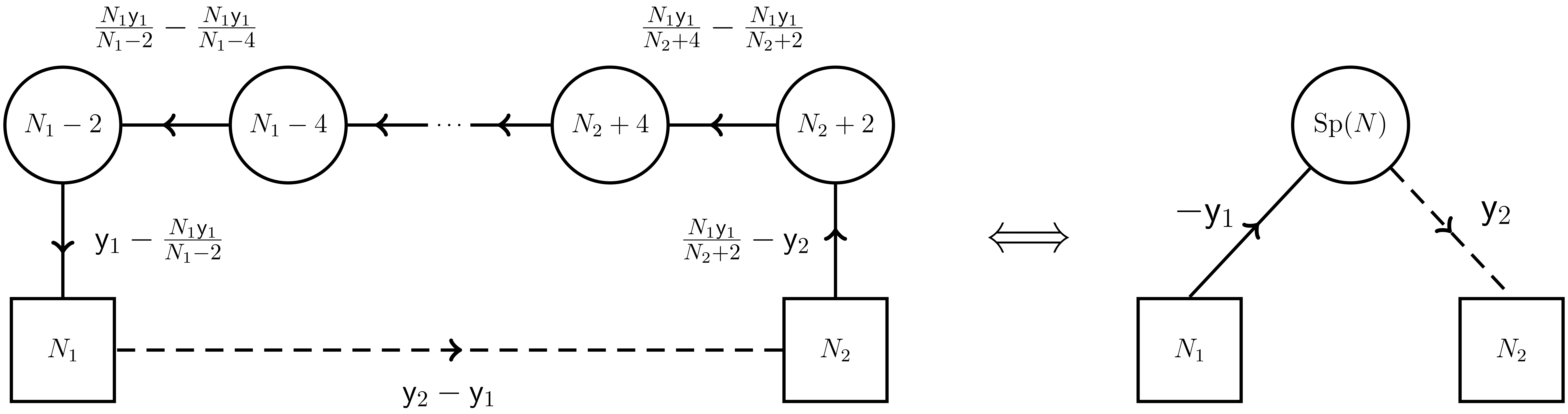
Basic building dualities

A chain of dualities



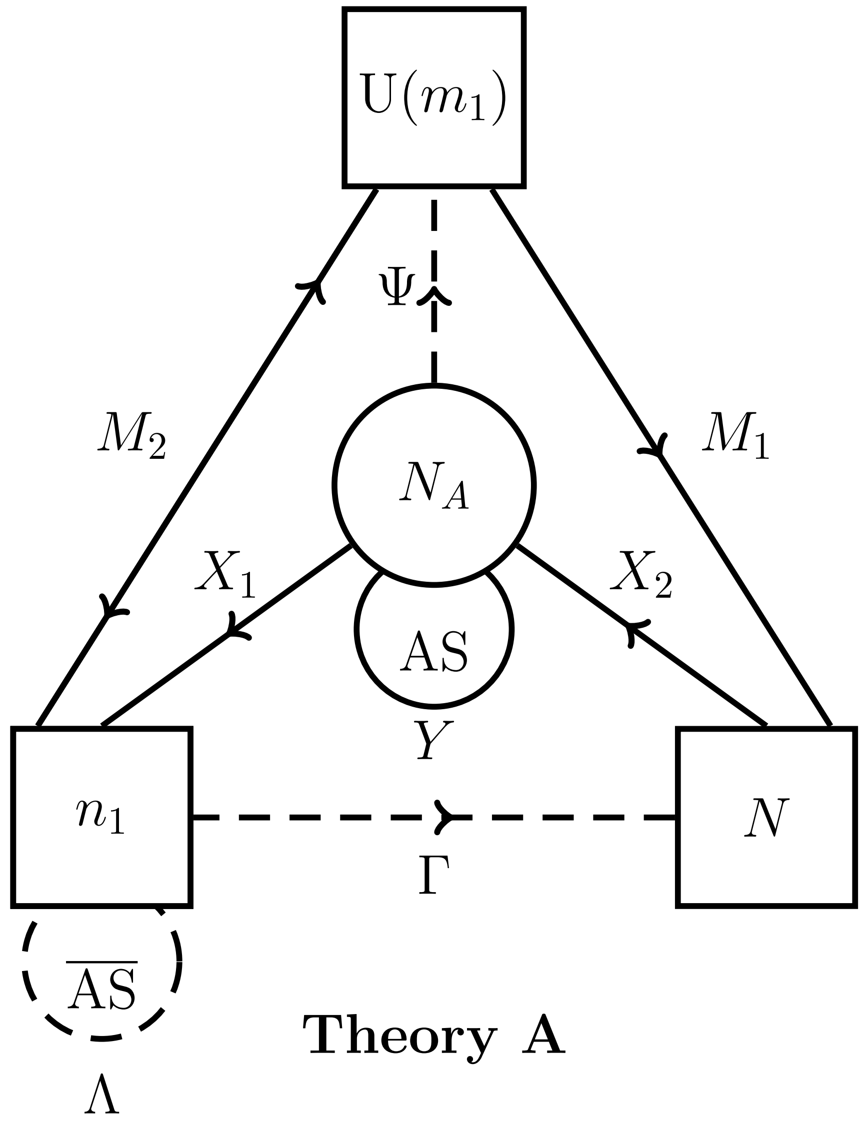
Duality between SU quiver gauge theory and Sp gauge theory

After Higssing, we obtain the duality

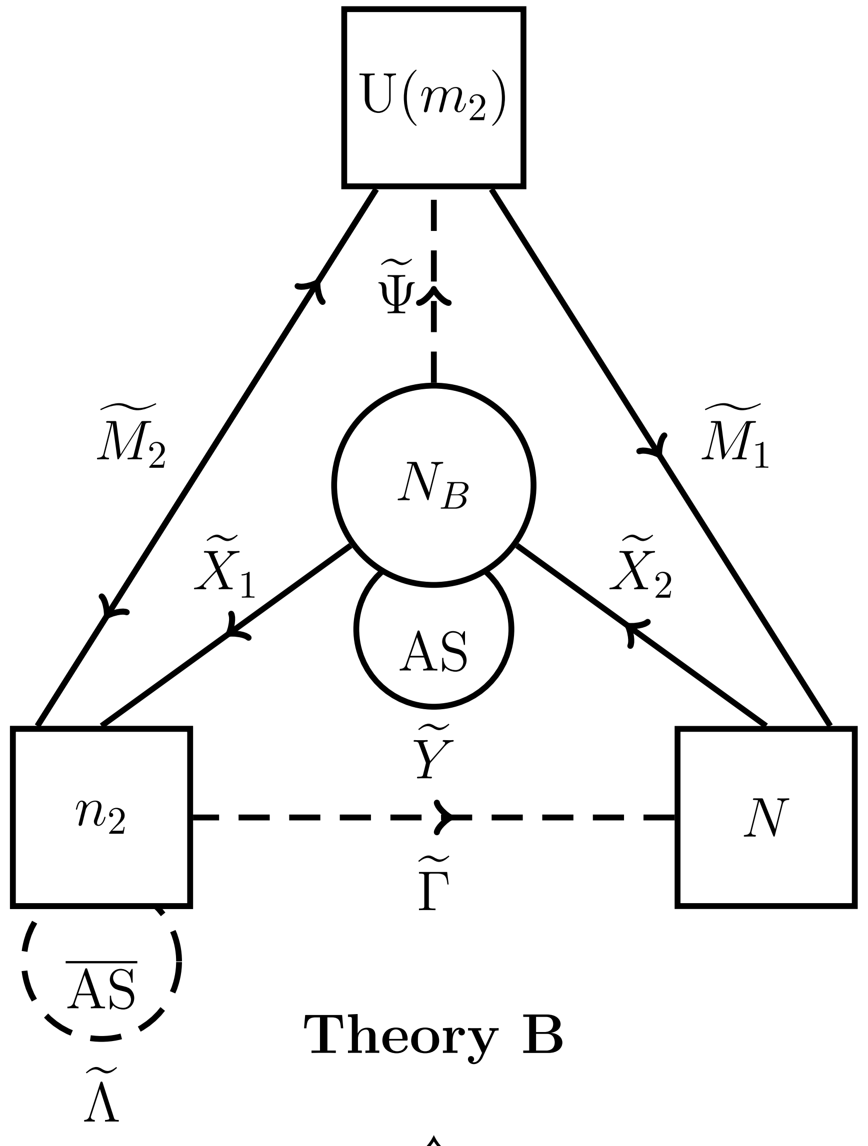


Duality between SU gauge theory with AS matter

$$N_A = N + n_1 - m_1 - 2$$



Theory A



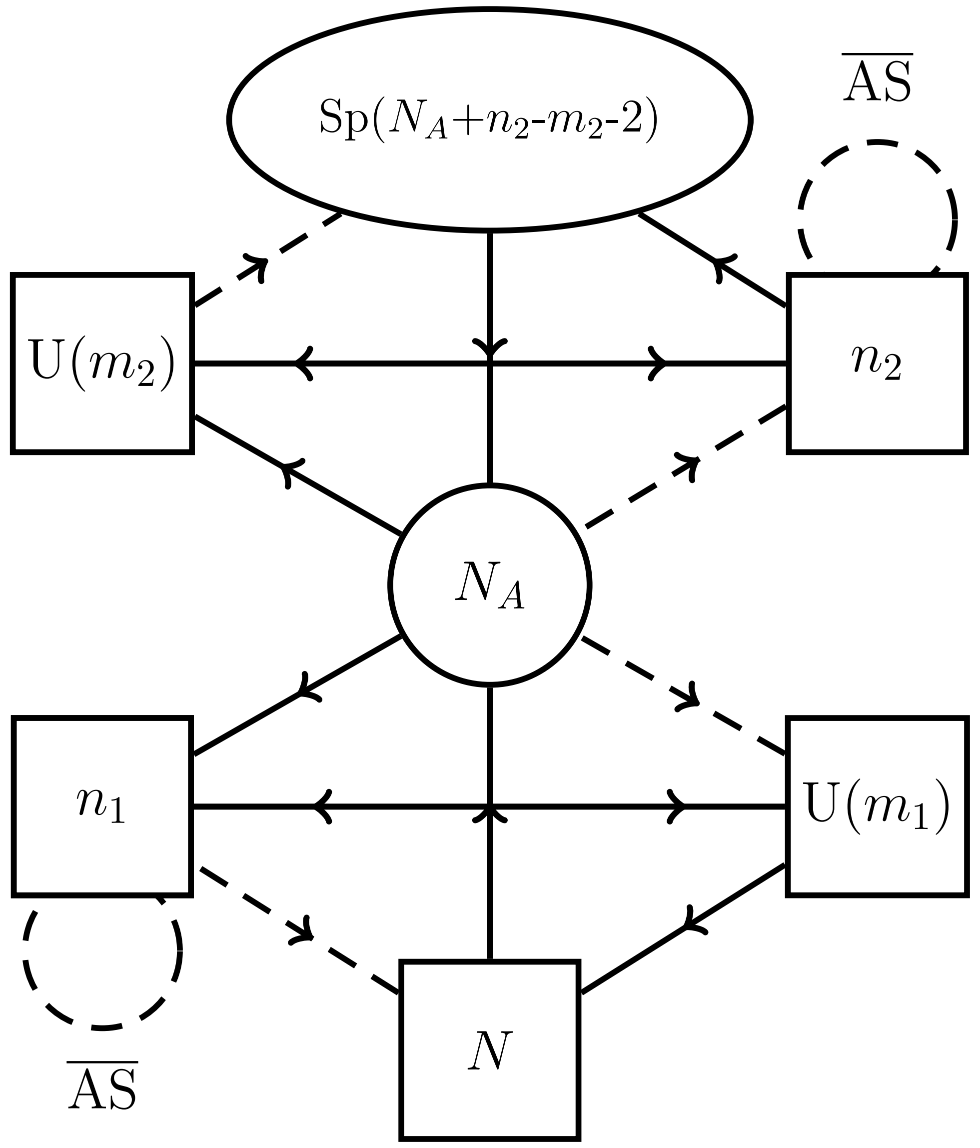
Theory B

$$N_B = N + n_2 - m_2 - 2$$

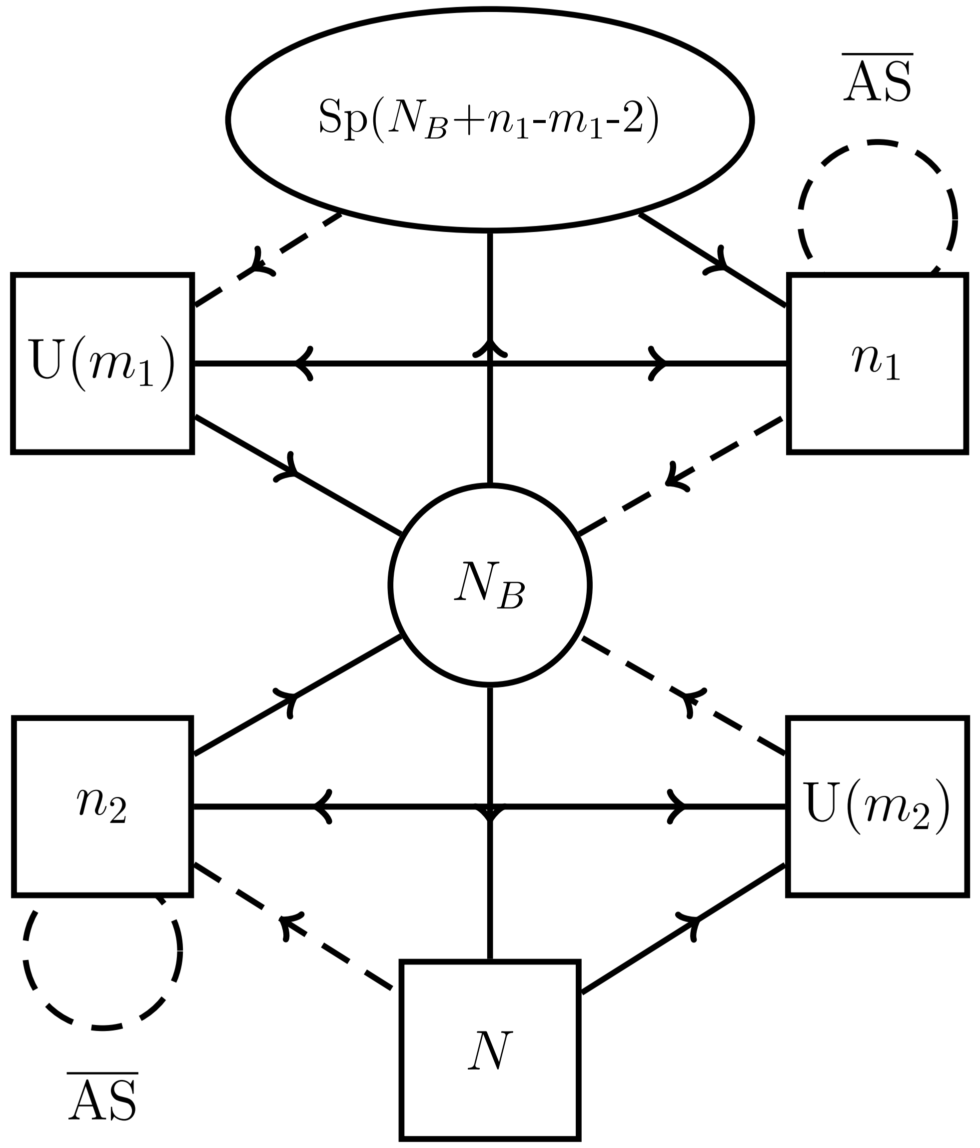
$\Downarrow$ AS-dual

$$m_{1,2} = 0 \text{ or } 1$$

$\Uparrow$ AS-dual

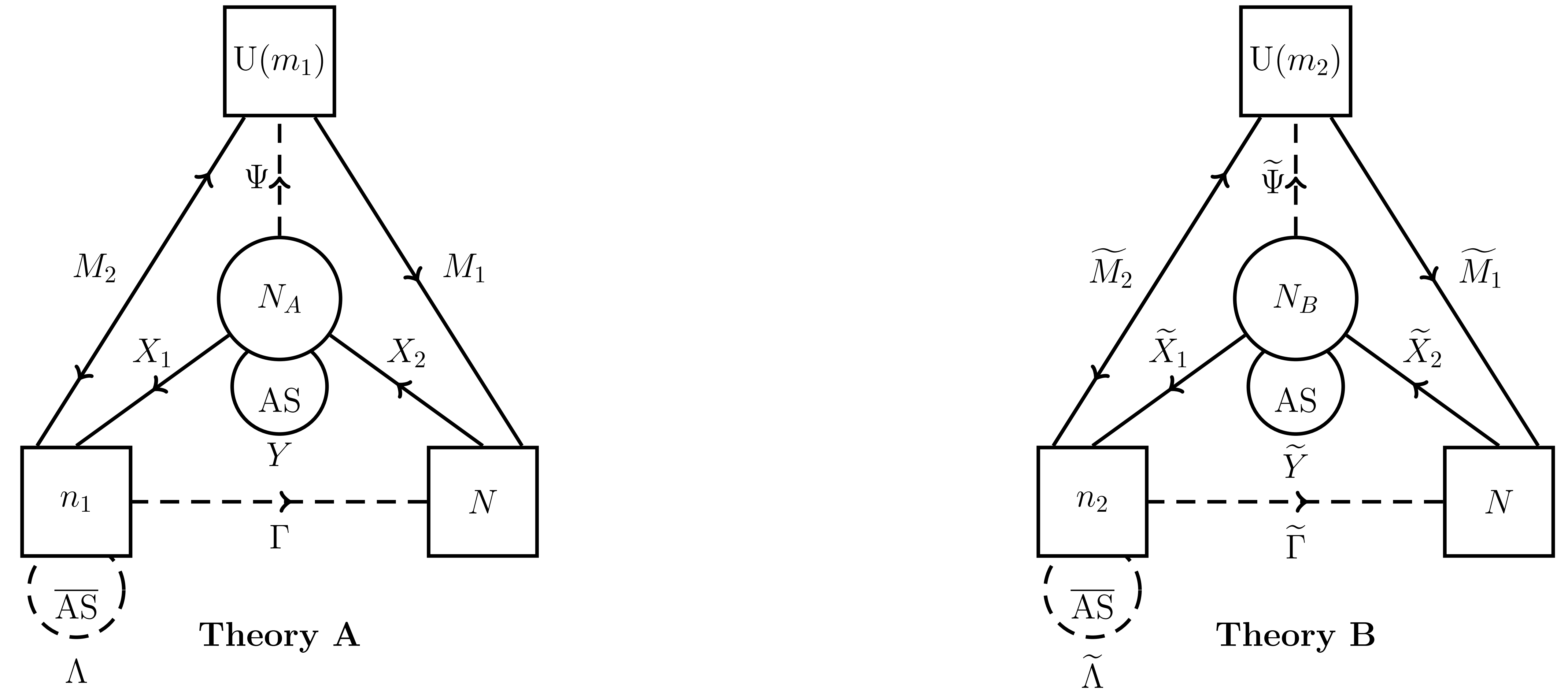


$\xRightarrow{\text{SU-dual}}$



Duality between SU gauge theory with AS matter

$$N_A = N + n_1 - m_1 - 2$$

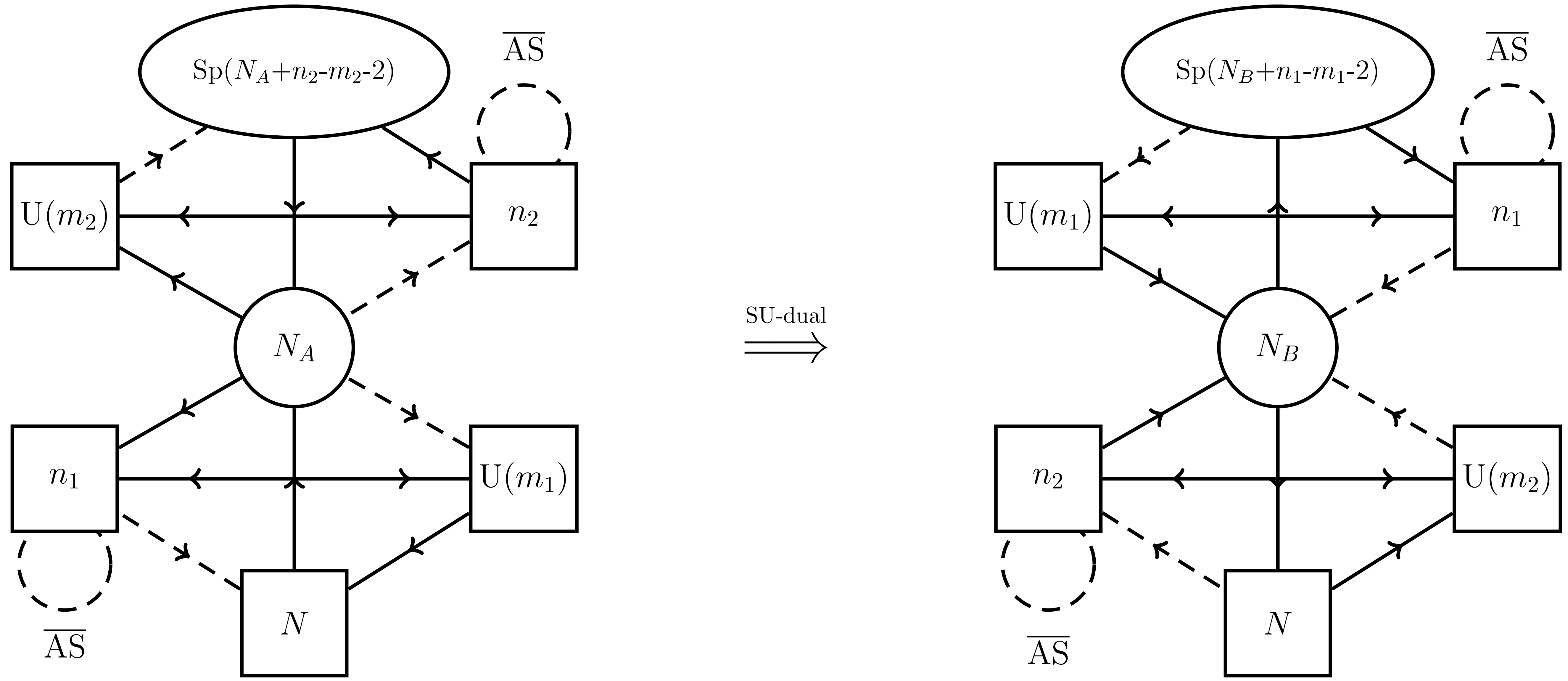


$$N_B = N + n_2 - m_2 - 2$$

AS-dual

Generalization of Amariti-Glorioso-Mantegazza-Morgante-Zanetti

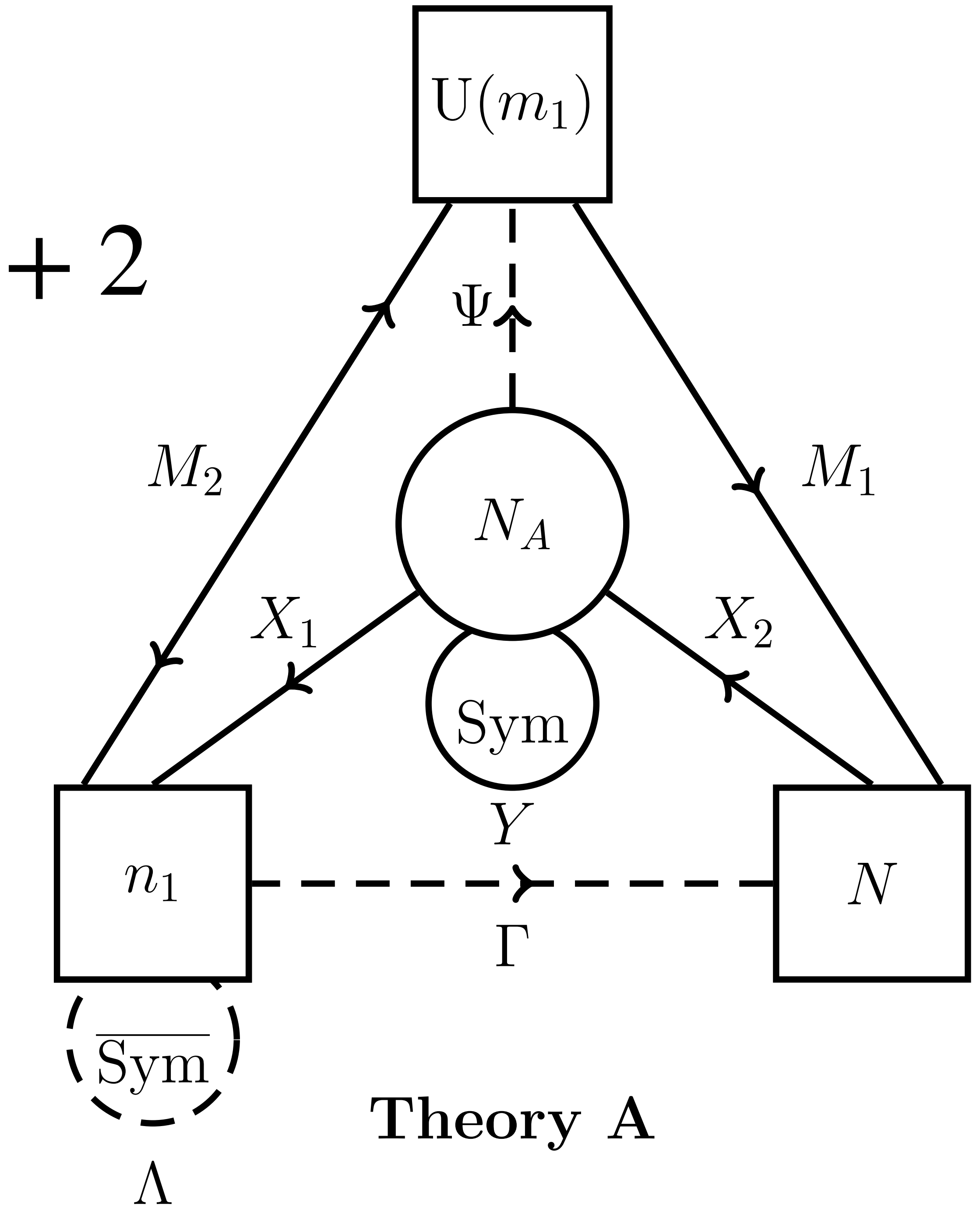
AS-dual



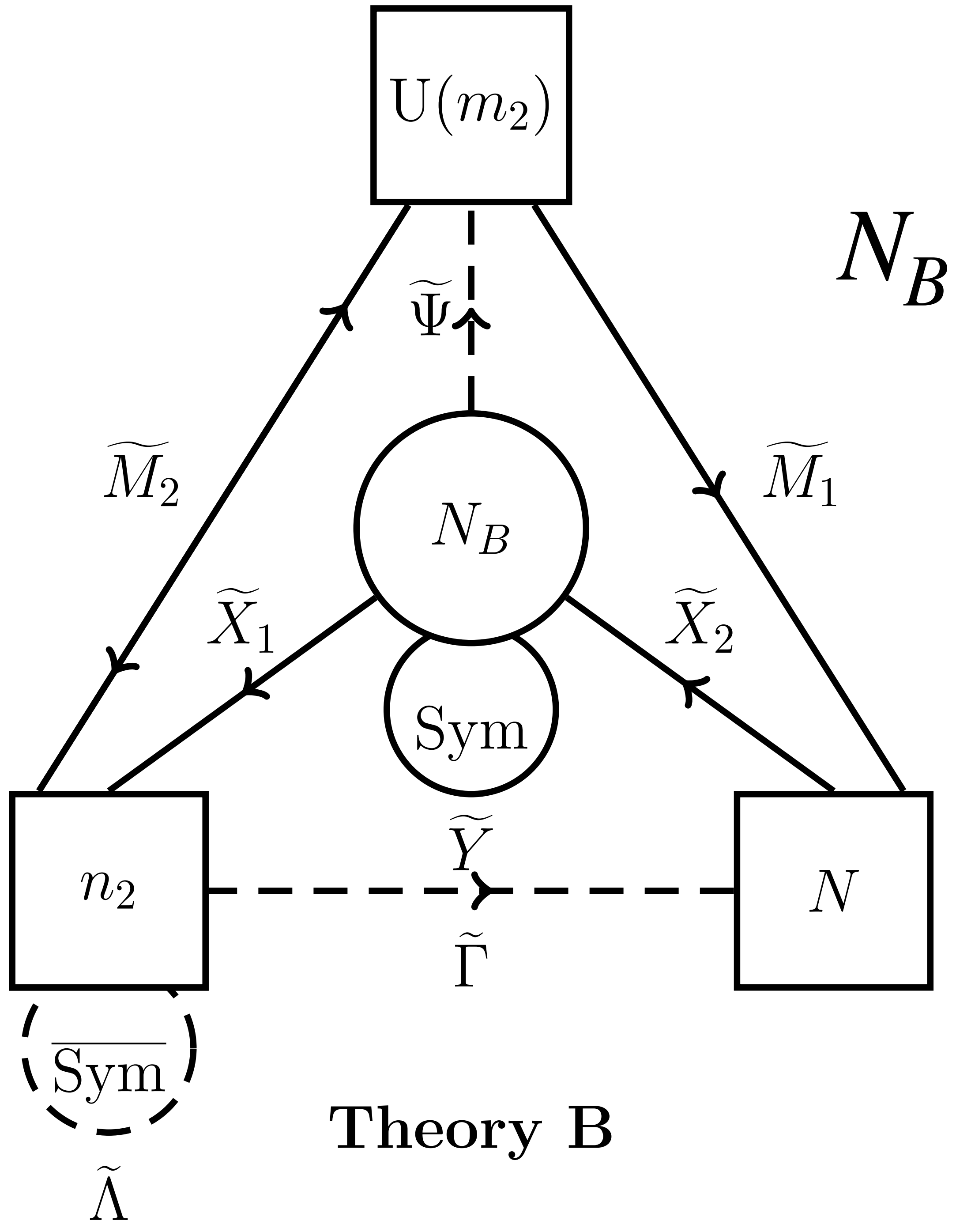
Duality between SU gauge theory with Sym matter

A similar chain of dualities leads to duality with symmetric mattter

$N_A = N + n_1 - m_1 + 2$

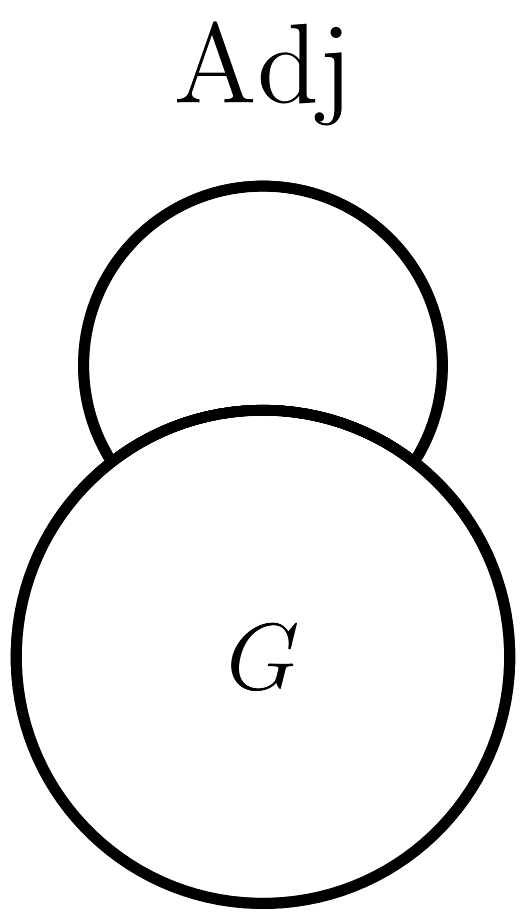


$m_{1,2} = 0$ or 1



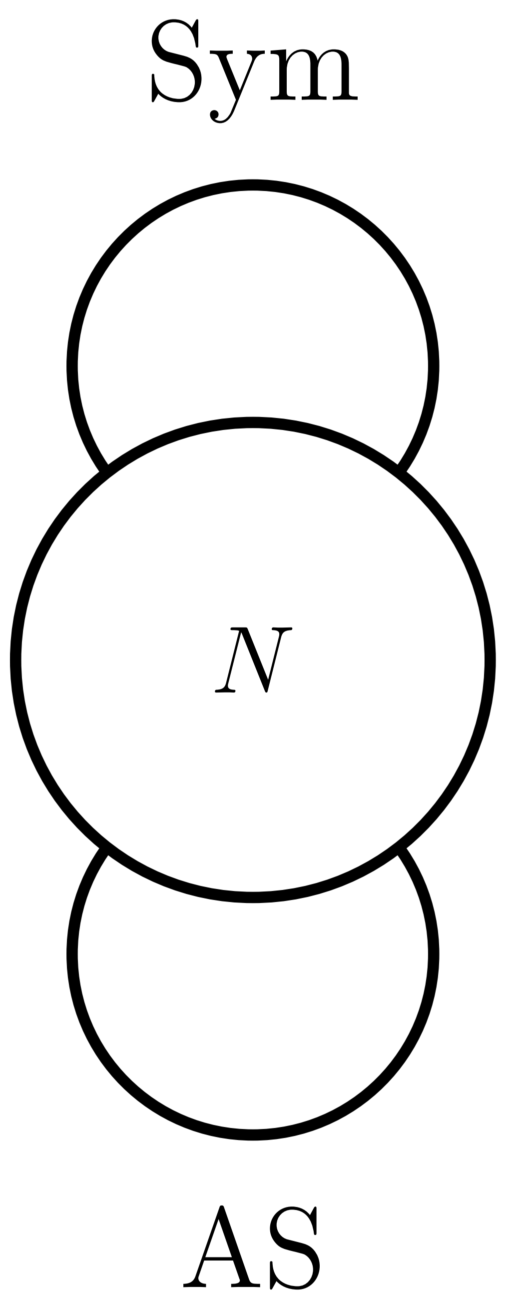
$N_B = N + n_2 - m_2 + 2$

Gauge/Landau-Ginzburg duality



$\Longleftrightarrow$ rk G chiral multiplets corresponding to $\mathrm{Tr}(\phi^i)$

$+\Psi_1^{(0)} + \Psi_2^{(0)}$
$+\Psi_3^{(0)} + Z^{(0)}$



$\Longleftrightarrow$ $\left(\frac{3N}{2}\right)$ chirals $+\left(\frac{3N}{2}+1\right)$ Fermi's with superpotential

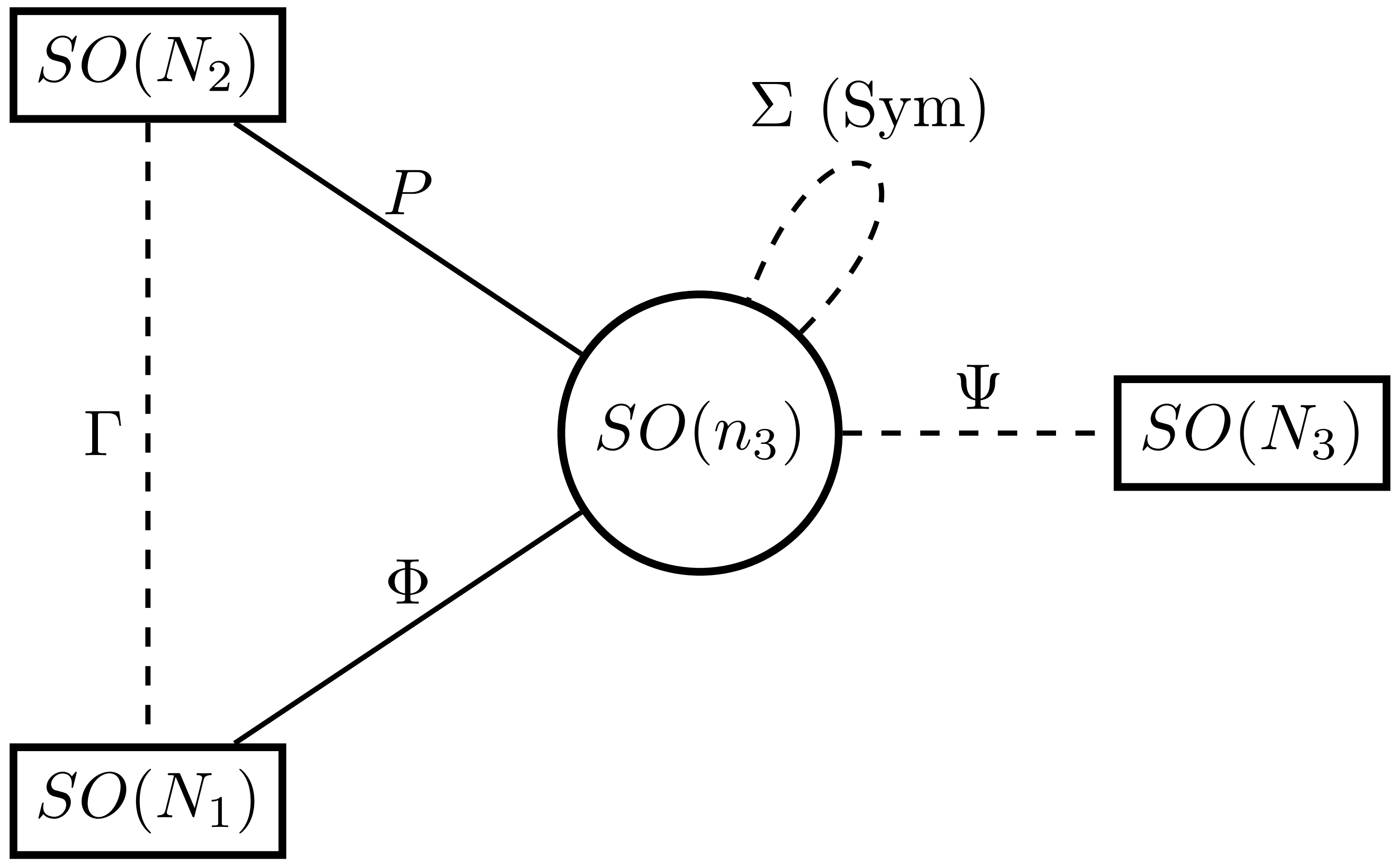
(0,1) triality

Gukov-Putrov-Pei

- Consider a family of 2d $\mathcal{N}=(0,1)$ gauge theories with $SO(n_3)$ gauge group and $SO(N_1)\times SO(N_2)\times SO(N_3)$ flavor symmetry.

- The matter content consists of scalar and Fermi multiplets in the representations summarized below:

type	symbol	$SO(n_3)$	$SO(N_1)$	$SO(N_2)$	$SO(N_3)$
scalar	Φ	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
scalar	P	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$
Fermi	Ψ	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\square$
Fermi	Γ	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
Fermi	Σ	Sym	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$



- The superpotential is given by

$$\int d^2x\, d\theta^+ \left[\sum_{\alpha,\beta=1}^{n_3} \Sigma^{\alpha\beta} \left(A\sum_{i=1}^{N_1} \Phi_i^\alpha \Phi_i^\beta + B\sum_{\ell=1}^{N_2} P_\ell^\alpha P_\ell^\beta - C\,\delta^{\alpha\beta} \right) + \sum_{i,\ell,\alpha} \Gamma_{i,\ell} \Phi_i^\alpha P_\ell^\alpha \right].$$

- The $(0,1)$ triality is permutation of (N_1, N_2, N_3). The IR dynamics depends only on the relative signs of A, B, C, giving rise to a triality among three dual descriptions.

(0,1) elliptic genus

Gukov-Putrov-Pei
J. Yagi
Bao-Yamazaki-Zhou

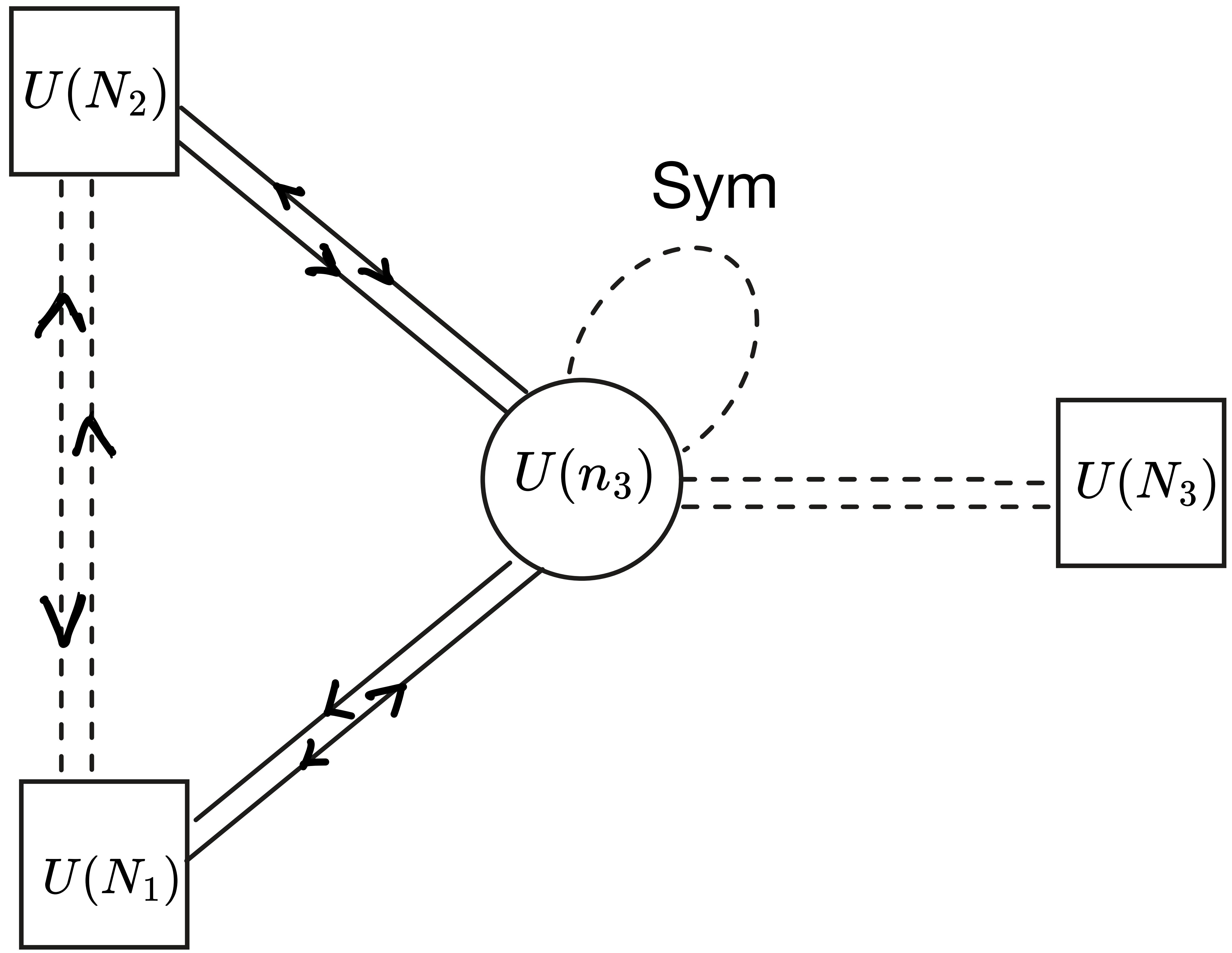
Jeffrey-Kirwan residue method can be applied to (0,1) elliptic genus

$$
\begin{aligned}
I =& \frac{1}{2^{n_3/2-1}\,(n_3/2)!}\sum_{u^*}\mathrm{Res}_{u^*}(\eta)\left(\prod_{\alpha=1}^{n_3/2}\mathrm{d}u_\alpha\right)\left(\frac{2\pi\eta(\tau)^2}{\mathrm{i}}\right)^{n_3/2}\left(\prod_{\alpha<\beta}^{n_3/2}\frac{\mathrm{i}\theta_1(u_\alpha+u_\beta)}{\eta(\tau)}\frac{\mathrm{i}\theta_1(u_\alpha-u_\beta)}{\eta(\tau)}\right)\\
&\left(\prod_{\alpha>\beta}^{n_3/2}\frac{\mathrm{i}\theta_1(u_\alpha+u_\beta)}{\eta(\tau)}\frac{\mathrm{i}\theta_1(u_\alpha-u_\beta)}{\eta(\tau)}\right)\left(\prod_{\alpha=1}^{n_3/2}\frac{\mathrm{i}\theta_1(2u_\alpha)}{\eta(\tau)}\right)\\
&\left(\prod_{\alpha=1}^{n_3/2}\prod_{a=1}^{N_3/2}\frac{\mathrm{i}\theta_1(u_\alpha+\nu_a)}{\eta(\tau)}\frac{\mathrm{i}\theta_1(u_\alpha-\nu_a)}{\eta(\tau)}\right)\left(\prod_{i=1}^{N_1/2}\prod_{r=1}^{N_2/2}\frac{\mathrm{i}\theta_1(\lambda_i-\mu_r)}{\eta(\tau)}\frac{\mathrm{i}\theta_1(\lambda_i+\mu_r)}{\eta(\tau)}\right)\\
&\left(\prod_{\alpha=1}^{n_3/2}\prod_{i=1}^{N_1/2}\frac{\mathrm{i}\eta(\tau)}{\theta_1(u_\alpha+\lambda_i)}\frac{\mathrm{i}\eta(\tau)}{\theta_1(u_\alpha-\lambda_i)}\right)\left(\prod_{\alpha=1}^{n_3/2}\prod_{r=1}^{N_2/2}\frac{\mathrm{i}\eta(\tau)}{\theta_1(u_\alpha+\mu_r)}\frac{\mathrm{i}\eta(\tau)}{\theta_1(u_\alpha-\mu_r)}\right).
\end{aligned}
$$

New (0,2) triality

Elliptic genus can be interpreted as that of the following (0,2) theory

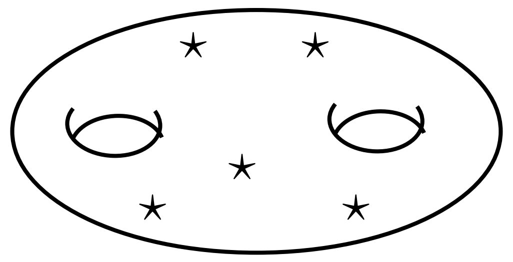
The permutation of (N_1, N_2, N_3) leads to a new (0,2) triality



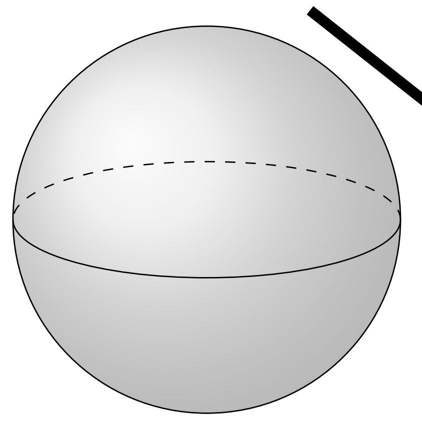
Other stories…

N-Pan-Zheng
Jiang-N-Zheng

6d N=(2,0) SCFT



4d N=2 SCFT of class S $SU(2)_R \times U(1)_r$



Top'l twist with $U(1)_R$

$U(1)_R \times U(1)_r$

Top'l twist with $U(1)_r$

2d N=(2,2) theory

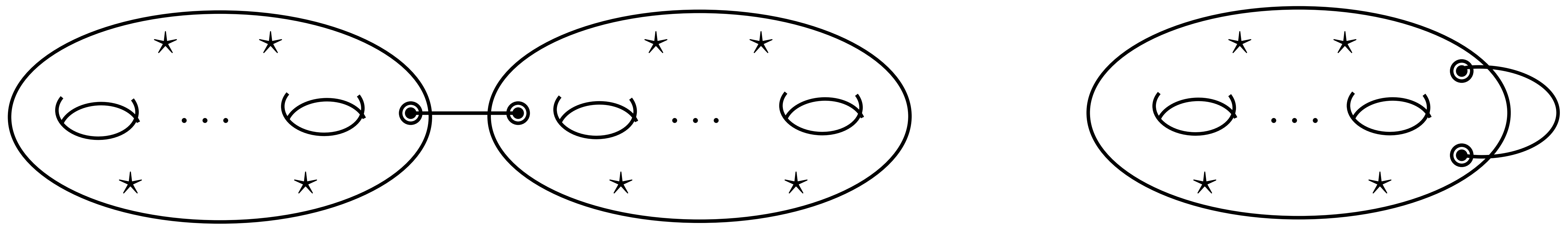
2d N=(0,2) theory

2d N=(0,4) theory

EG is surprisingly simple and independent of frames

$$\mathcal{I} = \prod_{i=1}^{g-1} \mathcal{I}_{\text{genus}} \prod_{i=1}^{n} \mathcal{I}_{\text{puncture}}$$

They exhibit TOFT structure.



Open problems

- It is natural to ask whether the relation between elliptic genera and chiral algebra characters extends to these new $(0,2)$ reductions.
- Brane constructions: Seek string or brane setups realizing the new $(0,2)$ dualities and trialities; such constructions could clarify their structure and generalizations.
- Integrable structures: Explore connections between these dualities and 2d integrable systems.
- Further studies in 2d supersymmetry:
  - Discovery of new $\mathcal{N} = (2,2)$ and $\mathcal{N} = (0,2)$ non-Abelian dualities
  - Supersymmetry enhancement from $(0,2)$ to $(2,2)$
  - Construction of non-Lagrangian 2d theories
  - Supersymmetric boundary conditions in $(2,2)$ and $(0,2)$ theories

We hope this work motivates further exploration of the vast landscape of 2d supersymmetric theories.