Homework 5: Due at class on Nov 23

1 Derivation

Derive the equation (7.36) and (7.37) of the lecture note.

2 Bhabha scattering

In this probelm, we consider the process of electron-positron ($e^+e^- \to e^+e^-$) scattering in QED.

2.1

Compute the amplitude for this process by summing two Feynman diagrams. Your answer will depend on the polarizations of the initial and final electrons and positrons. Note that, unlike the $e^+e^- \to \mu^+\mu^-$ process considered in class, your answer will be the sum of two Feynman diagrams. The overall sign is not too important, but make sure to get the relative sign between these two Feynman diagrams correct!

2.2

Imagine that we do not measure the polarizations of the initial or final states. Compute the scattering probability $\frac{1}{4}\sum|\mathcal{M}|^2$, where we average over initial polarizations and sum over final polarizations. Write your answer as a function of the momenta of the incoming and outgoing particles. You may make the approximation where $E_{cm} \gg m$, so that we can just ignore the electron mass.

2.3

Now compute the differential cross section in centre of mass frame, and write it as a function of the Mandelstam variables as

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left(u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right)$$

Take the reference frame as in Peskin-Schroeder above Eqn(5.11), write this formula as a function of E_{cm} and θ. Note that the result diverges at $\theta \to 0$. Can you explain why?

3 Integral

Perform the following integral and show the identity

$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-\Delta+i\epsilon]^n} = (-1)^n \frac{i}{16\pi^2}\frac{1}{(n-1)(n-2)}\frac{1}{\Delta^{n-2}}\,,$$

in the following two ways, and check both give the same answer:

- Start with the calculation of the dk^0 integral via Cauchy's theorem and then integrate over d^3k.
- Alternatively the k^0 integration can be done via a Wick rotation and afterwards substituting $k^0 = ik_E^0, (\boldsymbol{k} \equiv \boldsymbol{k}_E)$. Then the integral can be done in the four-dimensional Euclidean space (with $k^2 = -((k_E^0)^2 + \boldsymbol{k}^2))$ For that purpose you will need the integral over the four-dimensional solid angle $\int d\Omega_4 = 2\pi^2$.

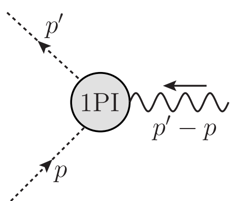
4 1-loop correction in scalar QED

The Lagrangian of the scalar QED is given by

$$\mathcal{L}_{\text{scalar}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi \tag{4.1}$$

where the covariant derivative is expressed by $D_\mu = \partial_\mu + ieA_\mu$. Find the diagrams which contribute to the 1PI diagrams of the scalar vertex in the order $\mathcal{O}(e^3)$ and write down the integral expression for

$$-ieK^\mu(p, p') \equiv$$



$$= -ie(p + p')^\mu + e^3 K^\mu_{\text{1-loop}}(p, p') + \mathcal{O}(e^5)\ . \tag{4.2}$$

Perform momentum integration by using the Pauli-Villars regularization.