Homework 4: Due at class on April 7

1 More on BRST

1.1 BRST charge

Using the explicit form of the BRST current

$$j_B = c(z)T^X(z) + :b(z)c(z)\partial c(z): + \frac{3}{2}:\partial^2 c(z): \ ,$$

express the BRST charge in terms of the X^μ Virasoro operators and the ghost oscillators as

$$Q_B = \sum_n c_n (L^X_{-n} - \delta_{n,0}) + \sum_{m,n} \frac{m-n}{2} : c_m c_n b_{-m-n} : \ .$$

Show that the OPE between two BRST currents is given by

$$j_B(z)j_B(w) = -\frac{c^X - 18}{2(z-w)^3} c\partial c(w) - \frac{c^X - 18}{4(z-w)^2} c\partial^2 c(w) - \frac{c^X - 26}{12(z-w)} c\partial^3 c(w) + \cdots ,$$

where c^X is the central charge of the X^μ bosonic string theory. Use this OPE to determine the anticommutator of the BRST charge with itself. For what value of c^X does this vanish?

1.2 Tj_B OPE

Show that the OPE between the total energy momentum tensor $T = T^X + T^g$ and j_B is given by

$$T(z)j_B(w) = \frac{c^X - 26}{2(z-w)^4} c(w) + \frac{j_B(w)}{(z-w)^2} + \frac{\partial j_B(w)}{z-w} + \cdots .$$

What does the result imply for j_B?

2 BRST quantization of closed string

Perform the BRST quantization at level 1 states of closed string. Show that the physical state takes the form

$$\epsilon_{\mu\nu} \alpha^u_{-1} \overline{\alpha}^\nu_{-1} |k, \downarrow, \downarrow\rangle \ , \tag{2.1}$$

where the polarization vectors ϵ are subject to

$$k \cdot \epsilon = 0 \tag{2.2}$$

$$\epsilon_{\mu\nu} \sim \epsilon_{\mu\nu} + \zeta_\mu k_\nu + k_\mu \tilde{\zeta}_\nu . \tag{2.3}$$

Note that the dilaton is the trace part of (2.1) so that the naive choice of the polarization vector for the dilaton is $\epsilon_{\mu\nu} = \eta_{\mu\nu}$. However, this choice of the polarization vector clearly does not satisfy $k \cdot \epsilon = 0$ in (2.2). Resolve this puzzle.