Homework 13: Due at class on Dec 21

1 Derivations

1.1

Derive (9.54).

1.2

Derive (9.59) from (9.57) by the coordinate transformation (9.58).

1.3

Derive all the steps in (9.63).

2 Killing vectors in AdS3

Killing vector fields are the infinitesimal generators of isometries of a Riemannian manifold. In terms of the global AdS_3 coordinate

$$ds^2 = R^2 \left(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2\right) \ ,$$

they can be written as

$$\begin{aligned}
\zeta_{-1} &= \frac{1}{2}\left[\tanh(\rho) e^{-i(t+\phi)}\partial_t + \coth(\rho) e^{-i(t+\phi)}\partial_\phi + i e^{-i(t+\phi)}\partial_\rho\right] \\
\zeta_0 &= \frac{1}{2}\left(\partial_t + \partial_\phi\right) \\
\zeta_1 &= \frac{1}{2}\left[\tanh(\rho) e^{i(t+\phi)}\partial_t + \coth(\rho) e^{i(t+\phi)}\partial_\phi - i e^{i(t+\phi)}\partial_\rho\right]
\end{aligned} \ .$$

Find the commutation relation $[\zeta_i, \zeta_j]$ and the corresponding Lie algebra.