Solution for fibonacci number:-

$$f(n) = f(n-1) + f(n-2) \quad \text{—— (1)}$$

Step 1:- put $f(n) = \alpha^n$ (constant $= \alpha$)

$$\alpha^n = \alpha^{n-1} + \alpha^{n-2}$$

Divide by α^{n-2}

$$\alpha^2 - \alpha - 1 = 0 \leftarrow \text{Characteristic eq}^n \text{ of recurrence}$$

roots of quadratic equation by $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\alpha = \frac{1 \pm \sqrt{5}}{2}$$

$$\alpha_1 = \frac{1-\sqrt{5}}{2} \qquad \alpha_2 = \frac{1+\sqrt{5}}{2}$$

Step 2:- $f(n) = c_1 \alpha_1^n + c_2 \alpha_2^n$ is a fibonacci sol^n

$$\begin{array}{cc} \uparrow & \downarrow \\ f(n-1) & f(n-2) \end{array}$$

$$f(n) = c_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + c_2 \left(\frac{1+\sqrt{5}}{2}\right)^n \quad \text{—— (11)}$$

Step 3:- No. of roots = No. of answers you have already

Here we have 2 roots α_1 and α_2

$f(0) = 0$ and $f(1) = 1$

$$f(0) = 0 = c_1 + c_2 \longrightarrow c_1 = -c_2 \quad \text{---} \quad (iii)$$

$$f(1) = 1 = c_1\left(\frac{1+\sqrt{5}}{2}\right)^{(1)} + c_2\left(\frac{1-\sqrt{5}}{2}\right)$$

from (iii),

$$1 = c_1\left(\frac{1+\sqrt{5}}{2}\right) - c_1\left(\frac{1-\sqrt{5}}{2}\right)$$

$c_1 = \frac{1}{\sqrt{5}}$ and $c_2 = \frac{-1}{\sqrt{5}}$

$$f(n) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\boxed{f(n) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]}$$

$\rightarrow$ Complexity $= O\left(\frac{1+\sqrt{5}}{2}\right)^n$

as $n \to \infty$ this will be close too (Hence, this will be ignored

$$T(n) = O(1.680)^n$$

Golden ratio

```
return (int)(Math.pow(((1+ Math.sqrt(5) /2), n) -
    Math.pow( 1 - Math.sqrt(5)) / 2), n)) /
        Math.sqrt(5))
```