GRChombo: An adaptable numerical relativity code for fundamental physics

Tomas Andrade[1], **Llibert Areste Salo**[2], **Josu C. Aurrekoetxea**[3], **Jamie Bamber**[4], **Katy Clough**[4], **Robin Croft**[5], **Eloy de Jong**[3], **Amelia Drew**[5], **Alejandro Duran**[6], **Pedro G. Ferreira**[4], **Pau Figueras**[2], **Hal Finkel**[7], **Tiago França**[2], **Bo-Xuan Ge**[3], **Chenxia Gu**[2], **Thomas Helfer**[8], **Juha Jäykkä**[5], **Cristian Joana**[9], **Markus Kunesch**[5], **Kacper Kornet**[5], **Eugene A. Lim**[3], **Francesco Muia**[5], **Zainab Nazari**[10, 11], **Miren Radia**[5], **Justin Ripley**[5], **Paul Shellard**[5], **Ulrich Sperhake**[5, 12], **Dina Traykova**[4], **Saran Tunyasuvunakool**[5], **Zipeng Wang**[8], **James Y. Widdicombe**[3], and **Kaze Wong**[8]

1 Departament de Fisica Quantica i Astrofisica, Institut de Ciencies del Cosmos, Universitat de Barcelona, Marti i Franques 1, 08028 Barcelona, Spain **2** School of Mathematical Sciences, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom **3** Theoretical Particle Physics and Cosmology, King's College London, Strand, London, WC2R 2LS, United Kingdom **4** Astrophysics, Oxford University, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, United Kingdom **5** Department of Applied Mathematics and Theoretical Physics (DAMTP), University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, United Kingdom **6** Intel Iberia, Torre Picasso Plaza Pablo Ruiz Picasso 1 Madrid, 28020 Spain **7** Argonne National Laboratory (ANL), 9700 S. Cass Avenue, Argonne, IL 60439-4815, United States **8** Henry A. Rowland Department of Physics & Astronomy, Johns Hopkins University, 3701 San Martin Drive, Baltimore, Maryland (MD) 21218, United States **9** Cosmology, Universe and Relativity at Louvain (CURL), Institut de Recherche en Mathematique et Physique, University of Louvain, 2 Chemin du Cyclotron, 1348 Louvain-la-Neuve, Belgium **10** Department of Physics, Bogazici University, 34342 Bebek, 80820 Istanbul, Turkey **11** HECAP Section, Abdus Salam International Centre for Theoretical Physics (ICTP), 34151, Trieste, Italy **12** California Institute of Technology, Pasadena, California 91125, USA

Summary

The 2015 detection of gravitational waves (GWs) from a binary black hole merger (B. P. Abbott & others, 2016) was a breakthrough moment for science. More detections have since been made by the Advanced LIGO/Virgo network (Aasi & others, 2015; B. P. Abbott & others, 2018; R. Abbott & others, 2020; Acernese & others, 2015) and future ground and space based detectors (Amaro-Seoane & others, 2017; Hu & Wu, 2017; Luo & others, 2016; Saleem & others, 2021; Somiya, 2012) will further expand our reach.

Strong gravity regimes are described by the *Einstein Field Equation* (EFE) of General Relativity (Einstein, 1916)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \ , \tag{1}$$

where $g_{\mu\nu}$ is the gravitational metric describing spacetime distances, R and $R_{\mu\nu}$ are related to its second derivatives in space and time, and $T_{\mu\nu}$ is the stress-energy tensor of any matter or fields present. Analytic solutions to the EFE only exist where there is a high degree of symmetry; in general the equations must be solved numerically. The need for observational predictions has thus led to the development of *numerical relativity* (NR), methods for numerically solving the above expression, typically utilising high-performance computing (HPC) resources. Expanding out the tensorial notation above, the EFE is a set of coupled, nonlinear second-order partial differential equations for $g_{\mu\nu}$, which describes the curvature of spacetime

in the presence of stress-energy $T_{\mu\nu}$, that is, schematically the equation we are trying to solve has the form:

$$\partial_t \partial_t g_{\mu\nu} \sim \partial_x \partial_x g_{\mu\nu} + \partial_y \partial_y g_{\mu\nu} + \partial_z \partial_z g_{\mu\nu} + \text{nonlinear cross terms} + 8\pi G T_{\mu\nu} \ , \quad (2)$$

where the indices μ, ν run over the spacetime indices – in 4 dimensions, t, x, y, z. Given that $g_{\mu\nu}$ is symmetric in its indices, this gives a set of ten coupled nonlinear partial differential equations, sourced by the stress-energy of any matter or fields present in the spacetime.

One common approach to NR is to specify an initial spatial distribution for the metric and matter fields (subject to certain constraints), and then solve a time evolution for all metric and matter quantities, thus populating their values throughout the four-dimensional spacetime. The canonical example of this is the simulation of two black holes in orbit around each other, which permits extraction of the gravitational wave signal produced during the merger. Such numerical results have been instrumental in discovering signals in the noisy LIGO/VIRGO detector data, as well as confirming the predictions of GR to a high precision in the strong field regime (B. P. Abbott & others, 2016; R. Abbott & others, 2020).

GRChombo is an open-source code for performing such NR time evolutions, built on top of the publicly available Chombo software (Adams & others, 2015) for the solution of PDEs. Whilst GRChombo uses standard techniques in NR, it focusses on applications in theoretical physics where adaptability, both in terms of grid structure, and in terms of code modification, are key drivers.

Key features of GRChombo

Since its initial announcement in 2015 (Clough et al., 2015), the GRChombo code has become a fully mature, open-source NR resource.

The key features of GRChombo are as follows:

- BSSN/CCZ4 formalism with moving punctures: GRChombo evolves the Einstein equation in the BSSN (Baumgarte & Shapiro, 1999; Nakamura et al., 1987; Shibata & Nakamura, 1995) or CCZ4 (Alic et al., 2012; Gundlach et al., 2005) formalism with conformal factor $\chi = \det(\gamma_{ij})^{-1/3}$, where γ_{ij} is the induced metric on the spatial hyperslices. Singularities of black holes are managed using the moving puncture gauge conditions (Baker et al., 2006; Campanelli et al., 2006), and Kreiss-Oliger dissipation (Kreiss et al., 1973) is used to control high-frequency noise, both from truncation and the interpolation associated with regridding.

- Boundary Conditions: The code implements periodic, Sommerfeld (radiative), extrapolating and reflective boundary conditions.

- Initial Conditions: The current examples provide analytic or semi-analytic initial data for black hole binaries, Kerr black holes and scalar matter. The code also incorporates a standalone version of the TwoPunctures code (Ansorg et al., 2004) for accurate binary BH data of arbitrary spins (up to the usual limit for Bowen-York data of around $a/M = 0.9$ for the dimensionless spin parameter), masses and momenta.

- Diagnostics: GRChombo has routines for finding black hole horizons, calculating spacetime masses, angular momenta, densities, fluxes and extracting gravitational waves.

- C++ class structure: GRChombo is written in the C++ language, and makes heavy use of object oriented programming (OOP) and templating.

- Parallelism: GRChombo uses hybrid OpenMP/MPI parallelism with explicit vectorisation of the evolution equations via intrinsics, and is AVX-512 compliant. Our code demonstrates efficient strong scaling up to several thousand CPU-cores for a typical BH binary problem, and further for larger problem sizes.

- Adaptive Mesh Refinement: The underlying Chombo code provides Berger-Oliger style (M. J. Berger & Oliger, 1984) AMR with block-structured Berger-Rigoutsos grid generation (M. Berger & Rigoutsos, 1991). The tagging of refinement regions is fully flexible and can be based on truncation error or other user-defined measures.

The code continues to be actively developed with a number of ongoing projects to add new features.

Statement of Need

Several 3+1D NR codes using the moving puncture formulation already exist and are under active development. The Einstein Toolkit (http://einsteintoolkit.org/), with its related Cactus (http://cactuscode.org) (Loffler & others, 2012; Schnetter et al., 2004), and Kranc (http://kranccode.org) (Husa et al., 2006) infrastructure used by LEAN (Sperhake, 2007; Zilhao et al., 2010) and Canuda (https://bitbucket.org/canuda) (Witek et al., 2019). Other notable but non-public codes include BAM (Bruegmann et al., 2008; Marronetti et al., 2007), AMSS-NCKU (Galaviz et al., 2010), PAMR/AMRD and HAD (East et al., 2012; Neilsen et al., 2007). Codes such as SPeC (Pfeiffer et al., 2003) and bamps (Hilditch et al., 2016) implement the generalised harmonic formulation of the Einstein equations using a pseudospectral method, and discontinuous Galerkin methods are used in SpECTRE (https://spectre-code.org) (Deppe et al., 2021; Kidder & others, 2017) (see also (Cao et al., 2018)). NRPy (http://astro.phys.wvu.edu/bhathome) (Ruchlin et al., 2018) is a code aimed for use on non-HPC systems, which generate C code from Python, and uses adapted coordinate systems to minimise computational costs. CosmoGRaPH (https://cwru-pat.github.io/cosmograph) (Mertens et al., 2016) and GRAMSES (Barrera-Hinojosa & Li, 2020) are among several NR codes targeted at cosmological applications (see (Adamek et al., 2020) for a comparison) and which also employ particle methods. Simflowny (https://bitbucket.org/iac3/simflowny/wiki/Home) (Palenzuela et al., 2018), like CosmoGRaPH, is based on the SAMRAI infrastructure, and has targeted fluid and MHD applications. GRAthena++ (Daszuta et al., 2021) makes use of oct-tree AMR to maximise scaling.

While GRChombo is not the only open-source NR code, its unique features (detailed above) have made it one of the premier codes for numerical relativity, especially in the study of fundamental physics beyond standard binary mergers. In particular, GRChombo's highly flexible adaptive mesh refinement scheme allows for complicated "many-boxes-in-many-boxes" topology , enabling users to simulate non-trivial systems, such as ring-like configurations Helfer, Aurrekoetxea, et al. (2019) and inhomogeneous cosmological spacetimes (Aurrekoetxea, Clough, et al., 2020; Clough et al., 2017; Clough, Flauger, et al., 2018; Joana & Clesse, 2021). Nevertheless, with its efficient scalability and AMR capabilities, it can also play a leading role in the continuing efforts to simulate "standard" binary mergers to the required sensitivities required for the upcoming LISA space mission (Radia et al., 2021). Finally, GRChombo's object-oriented and template-based code can be rapidly modified for non-standard problems such as higher-dimensional spacetimes (Andrade et al., 2020; Bantilan et al., 2019; Figueras et al., 2016, 2017), modified gravity systems (Figueras & França, 2020) and additional fundamental fields (Alexandre & Clough, 2018; Bamber et al., 2021; Clough, Dietrich, et al., 2018; Clough et al., 2019; Dietrich et al., 2019; Helfer et al., 2017; Helfer, Lim, et al., 2019; Muia et al., 2019; Nazari et al., 2021; Widdicombe et al., 2020).

Key research projects using GRChombo

The wide range of fundamental physics problems for which the code has been used so far includes:

- the simulation of inhomogeneous pre-inflationary spacetimes, bubble collisions and preheating in early universe cosmology (Aurrekoetxea, Clough, et al., 2020; Clough et al.,

2017; Clough, Flauger, et al., 2018; Joana & Clesse, 2021).
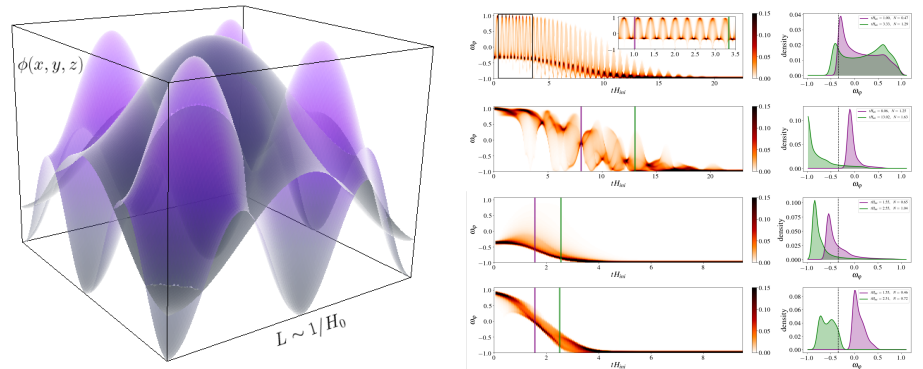
Images of Imhomogeneous inflaton field in (Aurrekoetxea, Clough, et al., 2020) and evolution of the equation of state and density in (Joana & Clesse, 2021).

- the study of modified gravity, and violation of cosmic censorship (Andrade et al., 2020; Bantilan et al., 2019; Figueras et al., 2016, 2017; Figueras & França, 2020).
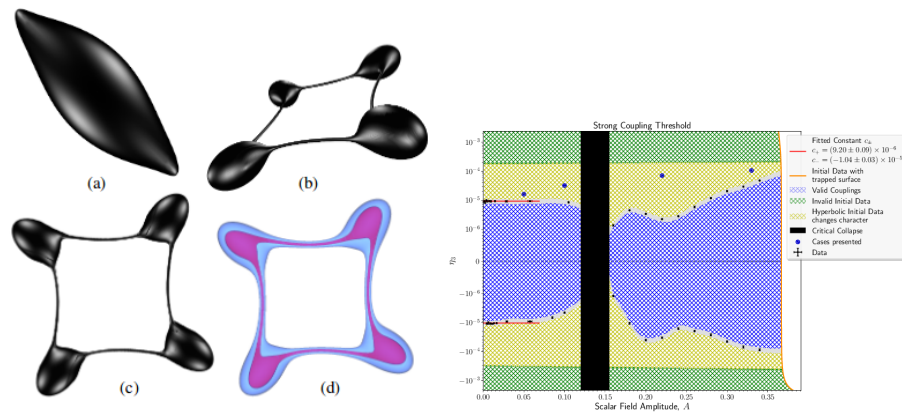

Images of testing cosmic censorship with higher-dimensional black rings in (Figueras et al., 2017) and mapping regions of validity for modified gravity in (Figueras & França, 2020).

- the formation, collapse and collisions of exotic compact objects (ECOs) and dark matter stars (Clough, Dietrich, et al., 2018; Dietrich et al., 2019; Helfer et al., 2017; Helfer, Lim, et al., 2019; Muia et al., 2019; Nazari et al., 2021; Widdicombe et al., 2018, 2020).
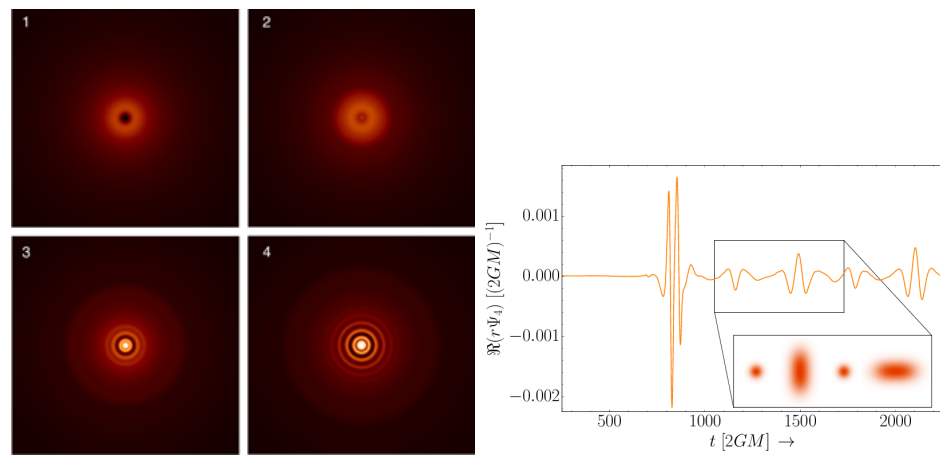
Images of axion star collapse from (Helfer et al., 2017) and GW signals from an ECO collision (Helfer, Lim, et al., 2019).

- gravitational wave emission from cosmic string collapse (Aurrekoetxea, Helfer, et al., 2020; Helfer, Aurrekoetxea, et al., 2019) and scalar radiation from global cosmic(/axion) strings (Drew & Shellard, 2019).
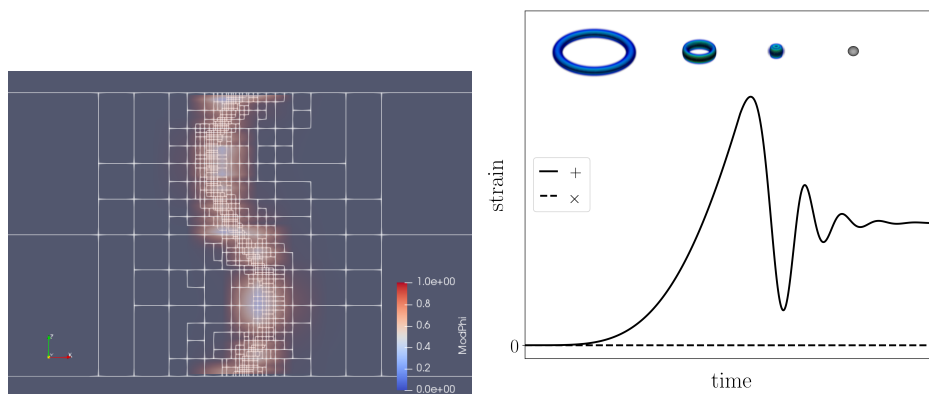
Images of global axion strings from (Drew & Shellard, 2019) and the GW signal from cosmic string loop collapse in (Aurrekoetxea, Helfer, et al., 2020).

- the study of light bosonic dark matter and neutrino-like particles in black holes environments (Alexandre & Clough, 2018; Bamber et al., 2021; Clough et al., 2019; Traykova et al., 2021).
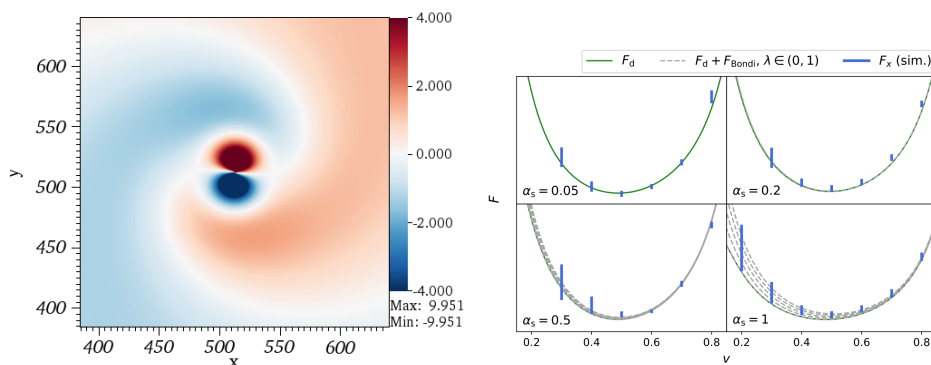
Images of scalar field accretion around a spinning BH from (Bamber et al., 2021), and the relativistic scaling of dynamical friction from (Traykova et al., 2021).

- the study of gravitational recoil in unequal mass binaries (Radia et al., 2021).
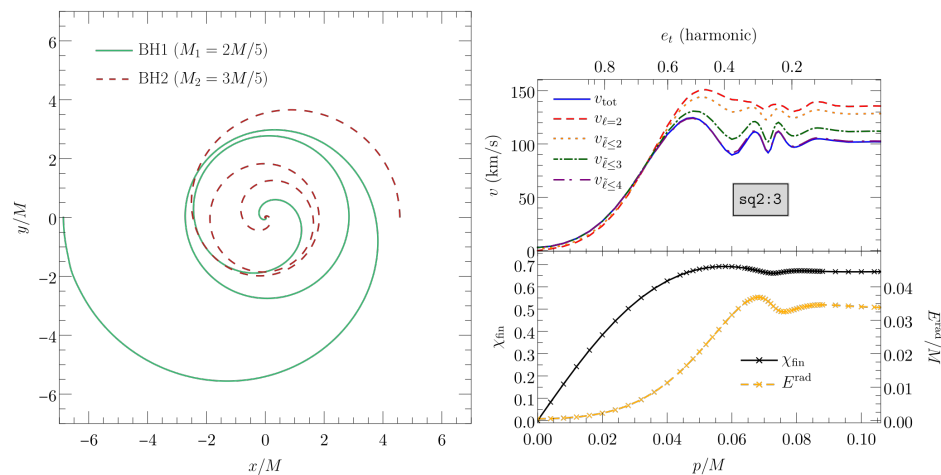
Images of the BH trajectories and recoil velocities, spin and radiated energy in ([Radia et al., 2021](#)).

Acknowledgements

References

Aasi, J., & others. (2015). Advanced LIGO. *Class. Quant. Grav.*, *32*, 074001. https://doi.org/10.1088/0264-9381/32/7/074001

Abbott, B. P., & others. (2016). Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.*, *116*(6), 061102. https://doi.org/10.1103/PhysRevLett.116.061102

Abbott, B. P., & others. (2018). Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO, Advanced Virgo and KAGRA. *Living Rev. Rel.*, *21*(1), 3. https://doi.org/10.1007/s41114-018-0012-9

Abbott, R., & others. (2020). *GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run.* http://arxiv.org/abs/2010.14527

Acernese, F., & others. (2015). Advanced Virgo: a second-generation interferometric gravitational wave detector. *Class. Quant. Grav.*, *32*(2), 024001. https://doi.org/10.1088/0264-9381/32/2/024001

Adamek, J., Barrera-Hinojosa, C., Bruni, M., Li, B., Macpherson, H. J., & Mertens, J. B. (2020). *Numerical solutions to Einstein's equations in a shearing-dust universe: a code comparison.* https://doi.org/10.1088/1361-6382/ab939b

Adams, M., & others. (2015). *Chombo software package for AMR applications - design document.*

Alexandre, J., & Clough, K. (2018). Black hole interference patterns in flavor oscillations. *Phys. Rev. D*, *98*(4), 043004. https://doi.org/10.1103/PhysRevD.98.043004

Alic, D., Bona-Casas, C., Bona, C., Rezzolla, L., & Palenzuela, C. (2012). Conformal and covariant formulation of the Z4 system with constraint-violation damping. *Phys. Rev. D*, *85*, 064040. https://doi.org/10.1103/PhysRevD.85.064040

Amaro-Seoane, P., & others. (2017). *Laser Interferometer Space Antenna.* http://arxiv.org/abs/1702.00786

Andrade, T., Figueras, P., & Sperhake, U. (2020). *Evidence for violations of Weak Cosmic Censorship in black hole collisions in higher dimensions.* http://arxiv.org/abs/2011.03049

Ansorg, M., Bruegmann, B., & Tichy, W. (2004). A single-domain spectral method for black hole puncture data. *Phys. Rev. D*, *70*, 064011. https://doi.org/10.1103/PhysRevD.70.064011

Aurrekoetxea, J. C., Clough, K., Flauger, R., & Lim, E. A. (2020). The effects of potential shape on inhomogeneous inflation. *JCAP*, *05*, 030. https://doi.org/10.1088/1475-7516/2020/05/030

Aurrekoetxea, J. C., Helfer, T., & Lim, E. A. (2020). Coherent gravitational waveforms and memory from cosmic string loops. *Class. Quant. Grav.*, *37*(20), 204001. https://doi.org/10.1088/1361-6382/aba28b

Baker, J. G., Centrella, J., Choi, D.-I., Koppitz, M., & Meter, J. van. (2006). Gravitational wave extraction from an inspiraling configuration of merging black holes. *Phys. Rev. Lett.*, *96*, 111102. https://doi.org/10.1103/PhysRevLett.96.111102

Bamber, J., Clough, K., Ferreira, P. G., Hui, L., & Lagos, M. (2021). Growth of accretion driven scalar hair around Kerr black holes. *Phys. Rev. D*, *103*(4), 044059. https://doi.org/10.1103/PhysRevD.103.044059

Bantilan, H., Figueras, P., Kunesch, M., & Panosso Macedo, R. (2019). End point of nonaxisymmetric black hole instabilities in higher dimensions. *Phys. Rev. D*, *100*(8), 086014.

https://doi.org/10.1103/PhysRevD.100.086014

Barrera-Hinojosa, C., & Li, B. (2020). GRAMSES: a new route to general relativistic N-body simulations in cosmology. Part I. Methodology and code description. *JCAP*, *01*, 007. https://doi.org/10.1088/1475-7516/2020/01/007

Baumgarte, T. W., & Shapiro, S. L. (1999). On the numerical integration of Einstein's field equations. *Phys. Rev. D*, *59*, 024007. https://doi.org/10.1103/PhysRevD.59.024007

Berger, M. J., & Oliger, J. (1984). Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations. *J. Comput. Phys.*, *53*, 484. https://doi.org/10.1016/0021-9991(84)90073-1

Berger, M., & Rigoutsos, I. (1991). An algorithm for point clustering and grid generation. *IEEE Transactions on Systems, Man, and Cybernetics*, *21*(5), 1278–1286. https://doi.org/10.1109/21.120081

Bruegmann, B., Gonzalez, J. A., Hannam, M., Husa, S., Sperhake, U., & Tichy, W. (2008). Calibration of moving puncture simulations. *Phys. Rev. D*, *77*, 024027. https://doi.org/10.1103/PhysRevD.77.024027

Campanelli, M., Lousto, C. O., Marronetti, P., & Zlochower, Y. (2006). Accurate evolutions of orbiting black-hole binaries without excision. *Phys. Rev. Lett.*, *96*, 111101. https://doi.org/10.1103/PhysRevLett.96.111101

Cao, Z., Fu, P., Ji, L.-W., & Xia, Y. (2018). Application of local discontinuous Galerkin method to Einstein equations. *Int. J. Mod. Phys. D*, *28*(01), 1950014. https://doi.org/10.1142/S0218271819500147

Clough, K., Dietrich, T., & Niemeyer, J. C. (2018). Axion star collisions with black holes and neutron stars in full 3D numerical relativity. *Phys. Rev. D*, *98*(8), 083020. https://doi.org/10.1103/PhysRevD.98.083020

Clough, K., Ferreira, P. G., & Lagos, M. (2019). Growth of massive scalar hair around a Schwarzschild black hole. *Phys. Rev. D*, *100*(6), 063014. https://doi.org/10.1103/PhysRevD.100.063014

Clough, K., Figueras, P., Finkel, H., Kunesch, M., Lim, E. A., & Tunyasuvunakool, S. (2015). GRChombo : Numerical relativity with adaptive mesh refinement. *Class. Quant. Grav.*, *32*, 24. https://doi.org/10.1088/0264-9381/32/24/245011

Clough, K., Flauger, R., & Lim, E. A. (2018). Robustness of inflation to large tensor perturbations. *JCAP*, *05*, 065. https://doi.org/10.1088/1475-7516/2018/05/065

Clough, K., Lim, E. A., DiNunno, B. S., Fischler, W., Flauger, R., & Paban, S. (2017). Robustness of inflation to inhomogeneous initial conditions. *JCAP*, *09*, 025. https://doi.org/10.1088/1475-7516/2017/09/025

Daszuta, B., Zappa, F., Cook, W., Radice, D., Bernuzzi, S., & Morozova, V. (2021). *GRAthena++: puncture evolutions on vertex-centered oct-tree AMR.* http://arxiv.org/abs/2101.08289

Deppe, N., Throwe, W., Kidder, L. E., Fischer, N. L., Armaza, C., Bonilla, G. S., Hébert, F., Kumar, P., Lovelace, G., Moxon, J., O'Shea, E., Pfeiffer, H. P., Scheel, M. A., Teukolsky, S. A., Anantpurkar, I., Boyle, M., Foucart, F., Giesler, M., Iozzo, D. A. B., … Wlodarczyk, T. (2021). *SpECTRE* (Version 2021.05.03) [Computer software]. Zenodo. https://doi.org/10.5281/zenodo.4734670

Dietrich, T., Ossokine, S., & Clough, K. (2019). Full 3D numerical relativity simulations of neutron star–boson star collisions with BAM. *Class. Quant. Grav.*, *36*(2), 025002. https://doi.org/10.1088/1361-6382/aaf43e

Drew, A., & Shellard, E. P. S. (2019). *Radiation from Global Topological Strings using Adaptive Mesh Refinement: Methodology and Massless Modes*. http://arxiv.org/abs/1910.01718

East, W. E., Pretorius, F., & Stephens, B. C. (2012). Hydrodynamics in full general relativity with conservative AMR. *Phys. Rev. D*, *85*, 124010. https://doi.org/10.1103/PhysRevD.85.124010

Einstein, A. (1916). The Foundation of the General Theory of Relativity. *Annalen Phys.*, *49*(7), 769–822. https://doi.org/10.1002/andp.200590044

Figueras, P., & França, T. (2020). Gravitational collapse in cubic Horndeski theories. *Class. Quant. Grav.*, *37*(22), 225009. https://doi.org/10.1088/1361-6382/abb693

Figueras, P., Kunesch, M., Lehner, L., & Tunyasuvunakool, S. (2017). End Point of the Ultraspinning Instability and Violation of Cosmic Censorship. *Phys. Rev. Lett.*, *118*(15), 151103. https://doi.org/10.1103/PhysRevLett.118.151103

Figueras, P., Kunesch, M., & Tunyasuvunakool, S. (2016). End Point of Black Ring Instabilities and the Weak Cosmic Censorship Conjecture. *Phys. Rev. Lett.*, *116*(7), 071102. https://doi.org/10.1103/PhysRevLett.116.071102

Galaviz, P., Bruegmann, B., & Cao, Z. (2010). Numerical evolution of multiple black holes with accurate initial data. *Phys. Rev. D*, *82*, 024005. https://doi.org/10.1103/PhysRevD.82.024005

Gundlach, C., Martin-Garcia, J. M., Calabrese, G., & Hinder, I. (2005). Constraint damping in the Z4 formulation and harmonic gauge. *Class. Quant. Grav.*, *22*, 3767–3774. https://doi.org/10.1088/0264-9381/22/17/025

Helfer, T., Aurrekoetxea, J. C., & Lim, E. A. (2019). Cosmic string loop collapse in full general relativity. *Phys. Rev. D*, *99*(10), 104028. https://doi.org/10.1103/PhysRevD.99.104028

Helfer, T., Lim, E. A., Garcia, M. A. G., & Amin, M. A. (2019). Gravitational wave emission from collisions of compact scalar solitons. *Phys. Rev. D*, *99*(4), 044046. https://doi.org/10.1103/PhysRevD.99.044046

Helfer, T., Marsh, D. J. E., Clough, K., Fairbairn, M., Lim, E. A., & Becerril, R. (2017). Black hole formation from axion stars. *JCAP*, *03*, 055. https://doi.org/10.1088/1475-7516/2017/03/055

Hilditch, D., Weyhausen, A., & Brügmann, B. (2016). Pseudospectral method for gravitational wave collapse. *Phys. Rev. D*, *93*(6), 063006. https://doi.org/10.1103/PhysRevD.93.063006

Hu, W.-R., & Wu, Y.-L. (2017). The Taiji Program in Space for gravitational wave physics and the nature of gravity. *Natl. Sci. Rev.*, *4*(5), 685–686. https://doi.org/10.1093/nsr/nwx116

Husa, S., Hinder, I., & Lechner, C. (2006). Kranc: A Mathematica application to generate numerical codes for tensorial evolution equations. *Comput. Phys. Commun.*, *174*, 983–1004. https://doi.org/10.1016/j.cpc.2006.02.002

Joana, C., & Clesse, S. (2021). Inhomogeneous preinflation across Hubble scales in full general relativity. *Phys. Rev. D*, *103*(8), 083501. https://doi.org/10.1103/PhysRevD.103.083501

Kidder, L. E., & others. (2017). SpECTRE: A Task-based Discontinuous Galerkin Code for Relativistic Astrophysics. *J. Comput. Phys.*, *335*, 84–114. https://doi.org/10.1016/j.jcp.2016.12.059

Kreiss, H. O., Oliger, J., Committee, G. A. R. Programme. J. O., Scientific Unions, I. C. of, & Organization, W. M. (1973). *Methods for the approximate solution of time dependent*

problems. International Council of Scientific Unions, World Meteorological Organization. https://books.google.co.uk/books?id=OxMZAQAAIAAJ

Loffler, F., & others. (2012). The Einstein Toolkit: A Community Computational Infrastructure for Relativistic Astrophysics. *Class. Quant. Grav.*, *29*, 115001. https://doi.org/10.1088/0264-9381/29/11/115001

Luo, J., & others. (2016). TianQin: a space-borne gravitational wave detector. *Class. Quant. Grav.*, *33*(3), 035010. https://doi.org/10.1088/0264-9381/33/3/035010

Marronetti, P., Tichy, W., Bruegmann, B., Gonzalez, J., Hannam, M., Husa, S., & Sperhake, U. (2007). Binary black holes on a budget: Simulations using workstations. *Class. Quant. Grav.*, *24*, S43–S58. https://doi.org/10.1088/0264-9381/24/12/S05

Mertens, J. B., Giblin, J. T., & Starkman, G. D. (2016). Integration of inhomogeneous cosmological spacetimes in the BSSN formalism. *Phys. Rev. D*, *93*(12), 124059. https://doi.org/10.1103/PhysRevD.93.124059

Muia, F., Cicoli, M., Clough, K., Pedro, F., Quevedo, F., & Vacca, G. P. (2019). The fate of dense scalar stars. *JCAP*, *07*, 044. https://doi.org/10.1088/1475-7516/2019/07/044

Nakamura, T., Oohara, K., & Kojima, Y. (1987). General Relativistic Collapse to Black Holes and Gravitational Waves from Black Holes. *Prog. Theor. Phys. Suppl.*, *90*, 1–218. https://doi.org/10.1143/PTPS.90.1

Nazari, Z., Cicoli, M., Clough, K., & Muia, F. (2021). Oscillon collapse to black holes. *JCAP*, *05*, 027. https://doi.org/10.1088/1475-7516/2021/05/027

Neilsen, D., Hirschmann, E. W., Anderson, M., & Liebling, S. L. (2007). Adaptive Mesh Refinement and Relativistic MHD. *11th Marcel Grossmann Meeting on General Relativity*, 1579–1581. https://doi.org/10.1142/9789812834300_0200

Palenzuela, C., Miñano, B., Viganò, D., Arbona, A., Bona-Casas, C., Rigo, A., Bezares, M., Bona, C., & Massó, J. (2018). A Simflowny-based finite-difference code for high-performance computing in numerical relativity. *Class. Quant. Grav.*, *35*(18), 185007. https://doi.org/10.1088/1361-6382/aad7f6

Pfeiffer, H. P., Kidder, L. E., Scheel, M. A., & Teukolsky, S. A. (2003). A multidomain spectral method for solving elliptic equations. *Comput. Phys. Commun.*, *152*, 253–273. https://doi.org/10.1016/S0010-4655(02)00847-0

Radia, M., Sperhake, U., Berti, E., & Croft, R. (2021). Anomalies in the gravitational recoil of eccentric black-hole mergers with unequal mass ratios. *Phys. Rev. D*, *103*(10), 104006. https://doi.org/10.1103/PhysRevD.103.104006

Ruchlin, I., Etienne, Z. B., & Baumgarte, T. W. (2018). SENR/NRPy+: Numerical relativity in singular curvilinear coordinate systems. *Phys. Rev. D*, *97*(6), 064036. https://doi.org/10.1103/PhysRevD.97.064036

Saleem, M., & others. (2021). *The Science Case for LIGO-India*. http://arxiv.org/abs/2105.01716

Schnetter, E., Hawley, S. H., & Hawke, I. (2004). Evolutions in 3-D numerical relativity using fixed mesh refinement. *Class. Quant. Grav.*, *21*, 1465–1488. https://doi.org/10.1088/0264-9381/21/6/014

Shibata, M., & Nakamura, T. (1995). Evolution of three-dimensional gravitational waves: Harmonic slicing case. *Phys. Rev. D*, *52*, 5428–5444. https://doi.org/10.1103/PhysRevD.52.5428

Somiya, K. (2012). Detector configuration of KAGRA: The Japanese cryogenic gravitational-wave detector. *Class. Quant. Grav.*, *29*, 124007. https://doi.org/10.1088/0264-9381/29/12/124007

Sperhake, U. (2007). Binary black-hole evolutions of excision and puncture data. *Phys. Rev. D*, *76*, 104015. https://doi.org/10.1103/PhysRevD.76.104015

Traykova, D., Berti, E., Clough, K., Ferreira, P. G., Helfer, T., & Hui, L. (2021). *Dynamical friction from scalar dark matter in the relativistic regime*. http://arxiv.org/abs/InPrep

Widdicombe, J. Y., Helfer, T., & Lim, E. A. (2020). Black hole formation in relativistic oscillaton collisions. *JCAP*, *01*, 027. https://doi.org/10.1088/1475-7516/2020/01/027

Widdicombe, J. Y., Helfer, T., Marsh, D. J. E., & Lim, E. A. (2018). Formation of relativistic axion stars. *JCAP*, *10*, 005. https://doi.org/10.1088/1475-7516/2018/10/005

Witek, H., Gualtieri, L., Pani, P., & Sotiriou, T. P. (2019). Black holes and binary mergers in scalar Gauss-Bonnet gravity: scalar field dynamics. *Phys. Rev. D*, *99*(6), 064035. https://doi.org/10.1103/PhysRevD.99.064035

Zilhao, M., Witek, H., Sperhake, U., Cardoso, V., Gualtieri, L., Herdeiro, C., & Nerozzi, A. (2010). Numerical relativity for D dimensional axially symmetric space-times: formalism and code tests. *Phys. Rev. D*, *81*, 084052. https://doi.org/10.1103/PhysRevD.81.084052