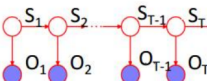PNLP quiz 4)
Q1)

- Given HMM parameters $p(S_1), p(S_t|S_{t-1}), p(O_t|S_t)$ & observation sequence $\{O_t\}_{t=1}^{T}$

find probability of observed sequence

$$p(\{O_t\}_{t=1}^{T}) = \sum_{S_1,\ldots,S_T} p(\{O_t\}_{t=1}^{T}, \{S_t\}_{t=1}^{T})$$

$$= \sum_{S_1,\ldots,S_T} p(S_1) \prod_{t=2}^{T} p(S_t|S_{t-1}) \prod_{t=1}^{T} p(O_t|S_t)$$

Q2)

1. P(X4=3|X3=2)=p23=2/3. (give one mark if answer is correct else 0)

2. P(X3=1|X2=1)=p11=1/4.. (give one mark if answer is correct else 0)

3. P(X0=1,X1=2)=P(X0=1)P(X1=2|X0=1)
   =1/3· p12
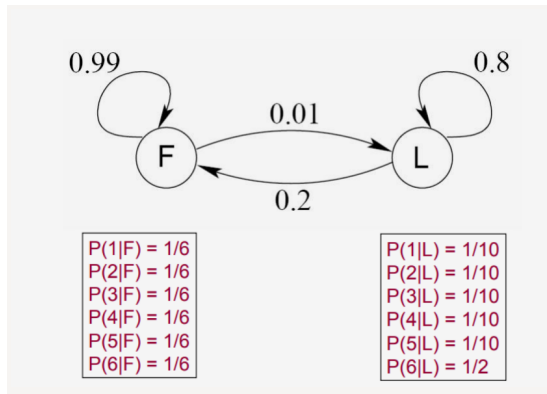   =(1/3)·(1/2)=⅙ (give one mark if answer is correct else 0)

4.

$$P(X_0 = 1, X_1 = 2, X_2 = 3)$$
$$= P(X_0 = 1)P(X_1 = 2|X_0 = 1)P(X_2 = 3|X_1 = 2, X_0 = 1)$$
$$= P(X_0 = 1)P(X_1 = 2|X_0 = 1)P(X_2 = 3|X_1 = 2) \quad \text{(by Markov property)}$$
$$= \frac{1}{3} \cdot p_{12} \cdot p_{23}$$
$$= \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3}$$
$$= \frac{1}{9}.$$

(give 2 marks if answer is correct else 0)

Q3.)

Reference solution:

Note*: State transition probabilities are different in the reference solution,


```
        0.99                              0.8

                    0.01
          ( F )  ─────────────►  ( L )

                    0.2

  P(1|F) = 1/6                P(1|L) = 1/10
  P(2|F) = 1/6                P(2|L) = 1/10
  P(3|F) = 1/6                P(3|L) = 1/10
  P(4|F) = 1/6                P(4|L) = 1/10
  P(5|F) = 1/6                P(5|L) = 1/10
  P(6|F) = 1/6                P(6|L) = 1/2
```

Part 2)

	6	2	6
F	(1/6)×(1/2) = 1/12	(1/6)×max{(1/12)×0.99, (1/4)×0.2} = 0.01375	(1/6)×max{0.01375×0.99, 0.02×0.2} = 0.00226875
L	(1/2)×(1/2) = 1/4	(1/10)×max{(1/12)×0.01, (1/4)×0.8} = 0.02	(1/2)×max{0.01375×0.01, 0.02×0.8} = 0.08

Most probable die sequence: L L L

part1)Need to take summation values in place of max()