Dynamics of Crime Spread across a Network of Regions

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MAT482H1 S

Abstract

The spread of crime across different regions in an organized system is complex and dynamic that calls for a topical approach. In this paper, we formulate a compartment model to study the effect of crime spread across a network of regions. The purpose of this model is to find how a high level of movement between different regions affects the criminal population in these areas. Based on this movement, we then analyze how our chosen parameters influence the criminal population in each region. Consequently, this analysis allows us to make conclusions about long-term incarceration rates and crime policies.

Dynamics of Crime Spread across a Network of regions

Background

On October 24, 2017 CTV News' headlines claimed that in an upscale neighbourhood of Toronto, there had been over 160 break-ins in the past year. [1] On December 22, the Halton Police sent out a warning to all residents about a Break-and-Enter Spree taking place across the Greater Toronto Area. [2] A week before this warning, the Toronto Transit Commission (TTC) inaugurated phase one of its ambitious extension. [3] Phillips and Sandler's stochastic model claims that improving public transit systems has a tendency to increase the rate of crime. [4]

Based on these sources, the primary motivation of our model is to analyze the interaction of crime rates across the 5 regions of the Greater Toronto Area (GTA) [5], namely:

1. Toronto
2. Durham
3. Halton
4. Peel
5. York

Taking inspiration from McMillon, Simon et al. mechanistic model on the dynamics of crime spread, we shall attempt to build a mechanistic model that investigates how the extension of the TTC will affect the interaction of criminal populations between this network of 5 regions. [6]

Inspiration for the Model

To develop an appropriate model for a network of regions, such as the GTA, we took inspiration from the compartment model by Mcmillon [6]. The model uses the idea of treating crime as a disease to develop an initial model analogous to the SIS model of disease spread. The model is expanded to account for the complicated crime and punishment system in Chicago, and study the three strike policy. The main conclusion of their work was the calculation the reproductive rates, $\mathbf{R_o}$, and the conditions necessary for a low crime equilibrium. Using their model for crime spread in an isolated system, we developed a model to consider travel between populations.

To expand the initial model, we considered the effects of prisons on recidivism taking inspiration from the paper "The Effect of Prison Sentences on Recidivism" [8], which talks about whether imprisonment is the solution to reducing crime spread. The author looks at three schools of thought, namely:

1. Prisons are effective in crime control.
2. Prisons are 'schools of crime'.

3. 'Minimalist/interaction' position, dealing with the inmate interactions. The perspective discusses the how there is little change brought by prisons in the inmate behaviors, and that there is possibility of low risk inmates being influenced by the high risk inmates to maintain a life of crime

The conclusions made from the model is that imprisonment is not an effective way to reduce crime. Therefore, excessive incarceration can be very expensive and pointless. To ensure that prisons are not adversely affecting the inmates, prison officials should implement repeated widespread assessments of the convict's behavior. Finally, the primary justification of the prison should be to debilitate prisoners (especially the high risk convicted) in order for incarceration to be able to reduce crime committing tendencies of the criminal population. Using the finding, and suggesting of the paper, we expanded our model to consider scenarios we feel are important to the prison dynamics.

Model Approach and Formulation

The Compartment Model

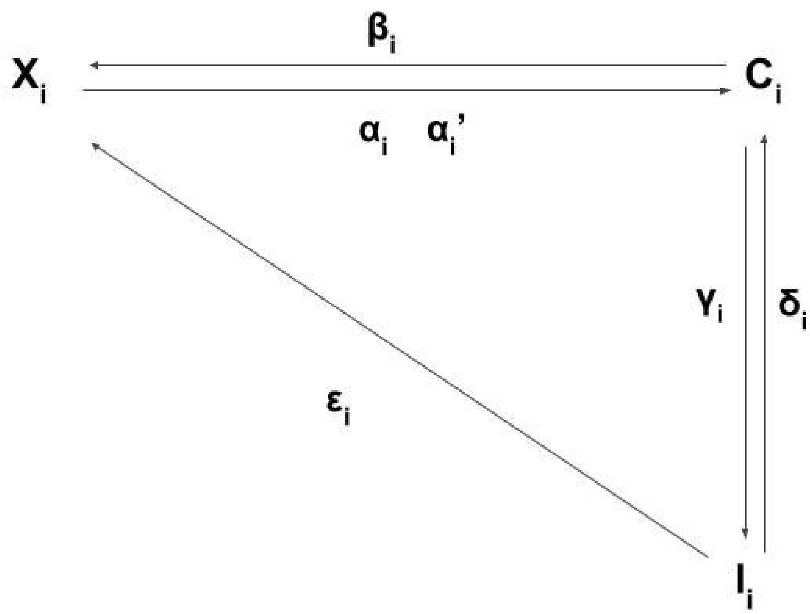
Our model approach is analogous to the Kermack-McKendrick Model that was discussed in lecture. We compartmentalized each region's (**indexed by i: $1 \leq i \leq 5$**) population into:

1. $\mathbf{X_i}$: This refers to the criminally-inactive population in region **i** as a function of time **t**.
2. $\mathbf{C_i}$: This refers to the criminally-active population in region **i** as a function of time **t**.
3. $\mathbf{I_i}$: This refers to the incarcerated population in region **i** as a function of time **t**.

To ensure the compartments interact with one another, we had to include specific parameters. The following list describes the parameters used in our model:

1. $\boldsymbol{\alpha_i}$: The contagion rate for region **i** at which the criminally-inactive population enters the criminally-active population.
2. $\boldsymbol{\alpha'_i}$: The natural propensity rate for region **i** at which the criminally-inactive population is motivated to join the criminally-active population.
3. $\boldsymbol{\beta_i}$: The rate for region **i** at which the criminally-active population returns to the criminally-inactive population.
4. $\boldsymbol{\gamma_i}$: The imprisonment rate for region **i**.
5. $\boldsymbol{\varepsilon_i}$: The rate for region **i** at which the prisoners, once released, return to the criminally-inactive population.
6. $\boldsymbol{\delta_i}$: The rate for region **i** at which the prisoners, once released, return to the criminally-active population.
7. $\boldsymbol{\theta_{ij}}$: The rate of movement from region i to region j.
8. $\boldsymbol{\theta_{ji}}$: The rate of movement from region j to region i.

The following flow chart describes how the parameters listed above facilitate interaction between the compartments:



The Equations

Based on this flow chart, we developed the following set of equations for each region **i**:

$$\frac{dX_i}{dt} = \beta_i C_i - \alpha_i X_i C_i - \alpha'_i X_i + \sum_j^5 \theta_{ji} X_j - \sum_j^5 \theta_{ij} X_i + \varepsilon_i I_i$$

$$\frac{dC_i}{dt} = -\beta_i C_i + \alpha_i X_i C_i + \alpha'_i X_i + \sum_j^5 \theta_{ij} C_j - \sum_j^5 \theta_{ji} C_i + \delta_i I_i - \gamma_i C_i$$

$$\frac{dI_i}{dt} = \gamma_i C_i - \delta_i I_i - \varepsilon_i I_i$$

Note that, these equations have been adapted from those developed in [6].

The first equation facilitates the dynamics of the criminally-inactive population in region **i**. The first term $\boldsymbol{\beta_i C_i}$ refers to the movement from $\mathbf{C_i}$ back to $\mathbf{X_i}$. This term allows those criminals who no longer exhibit criminal behavior to return to the non-criminal population. The second term $\boldsymbol{\alpha}_i\mathbf{X}_i\mathbf{C}_i$ captures the movement from $\mathbf{X}_i$ to $\mathbf{C}_i$. It is subtracted because this proportion of the regional population leaves $\mathbf{X}_i$. This term allows those non-criminals beginning to exhibit criminal behaviour to move into the criminal population. The third term $\boldsymbol{\alpha}'_i\mathbf{X}_i$ describes a movement from $\mathbf{X}_i$ to $\mathbf{C}_i$. It is subtracted because this proportion of the regional population leaves $\mathbf{X}_i$. This term allows those non-criminals This term allows those non-criminals who naturally tend towards crime to move into the criminal population. The fourth term $\sum_{j}^{5} \boldsymbol{\theta}_{ji}\mathbf{X}_j$ describes the movement of the non-criminal population of region **j** into region **i**. Similarly, the fifth term $\sum_{j}^{5} \boldsymbol{\theta}_{ij}\mathbf{X}_i$, describes the departure (subtraction) of the non-criminal population from region **i** and their movement into region **j.** Finally, the sixth term $\boldsymbol{\varepsilon}_i\mathbf{I}_i$ describes the movement of those prisoners, who once released, return to the non-criminal population.

The second equation facilitates the dynamics of the criminally-active population in region **i**. The initial three terms are exactly the same as those described above. However, there's a change in sign because of the nature of the second equation. The fourth term $\sum_{j}^{5} \boldsymbol{\theta}_{ji}\mathbf{C}_j$ describes the movement of the criminal population of region **j** into region **i**. Similarly, the fifth term $\sum_{j}^{5} \boldsymbol{\theta}_{ij}\mathbf{C}_i$, describes the departure (subtraction) of the criminal population from region **i** and their movement into region **j.** The sixth term $\boldsymbol{\delta}_i\mathbf{I}_i$ describes the movement of those prisoners, who once released, return to the criminal population. Finally, the seventh term $\boldsymbol{\gamma}_i\mathbf{C}_i$ describes those criminals that are caught and imprisoned.

The third equation facilitates the dynamics of the incarcerated population in region **i**. The first term is the same as the seventh term in the second equation. It's therefore added to the $\mathbf{I}_i$ compartment. Similarly, the last two terms of this equation are the same as the sixth term of the second equation and the last term of the first equation, respectively. These terms are subtracted because of the nature of this equation.

Note that, the summation of the three equations is 0 for each region **i**. This allows us to conclude that $X_i(t) + C_i(t) + I_i(t)$ is constant over time. That is to say, each regional population is conserved. Therefore, the total population across all the 5 regions is also conserved.

Assumptions

Given the preceding set of equations, our model makes the following assumptions:

1. The total population across the 5 regions is conserved.
2. The population in each compartment is homogeneous.
3. The justice system is perfect.
4. There is no movement from the Non-Criminal Population to the Incarcerated Population.
5. There are no obstructions within the transportation network.
6. Members of the Incarcerated Population cannot travel until released.

Analysis of the Model

To demonstrate the purposefulness of our model, we used our equations to simulate 2 different scenarios. This allowed us to make appropriate conclusions in the context of our objective.

Scenario 1

Our first scenario studies the effects of the movement between the regions. In this scenario, all parameters except for the travel rate are fixed. Note that, the parameters are independent in this scenario. To achieve this, we vary the travel rate ($\boldsymbol{\theta_{ij}}$ **and** $\boldsymbol{\theta_{ji}}$) for every possible combination of **j** and **i**. We then assume a step-by-step approach to see if the network structure affects how the criminally-active population behaves in the long-run.

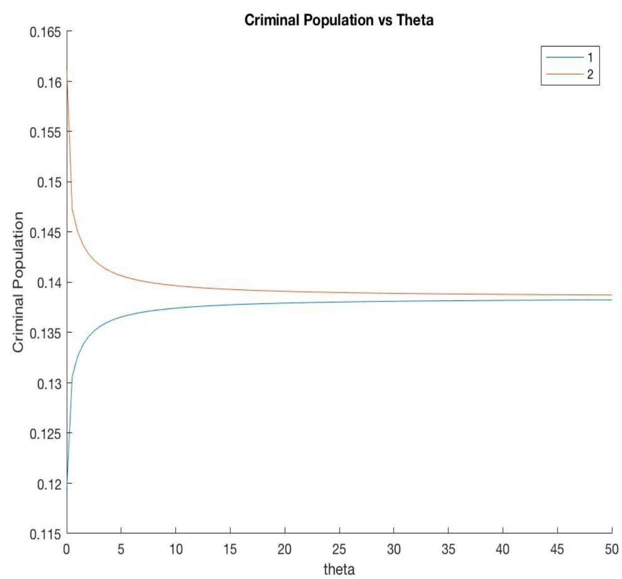
Scenario 2

For the second scenario, we analyse different parameters. In this scenario, the travel rate is fixed. However, depending on the case in consideration, different parameters are varied. To achieve this, we simulated a network of regions where one region is characterized by low crime. In addition to this, we also simulated a network of regions where one region is characterized by high crime. During our analysis, we noticed a possibility of the parameters $\boldsymbol{\varepsilon_i}$ and $\boldsymbol{\delta_i}$ being functions of $\boldsymbol{\gamma_i}$, due to the dynamics present in prisons. This allowed us to simulate a prison as a "school of crime".

Scenario 1

Step 1

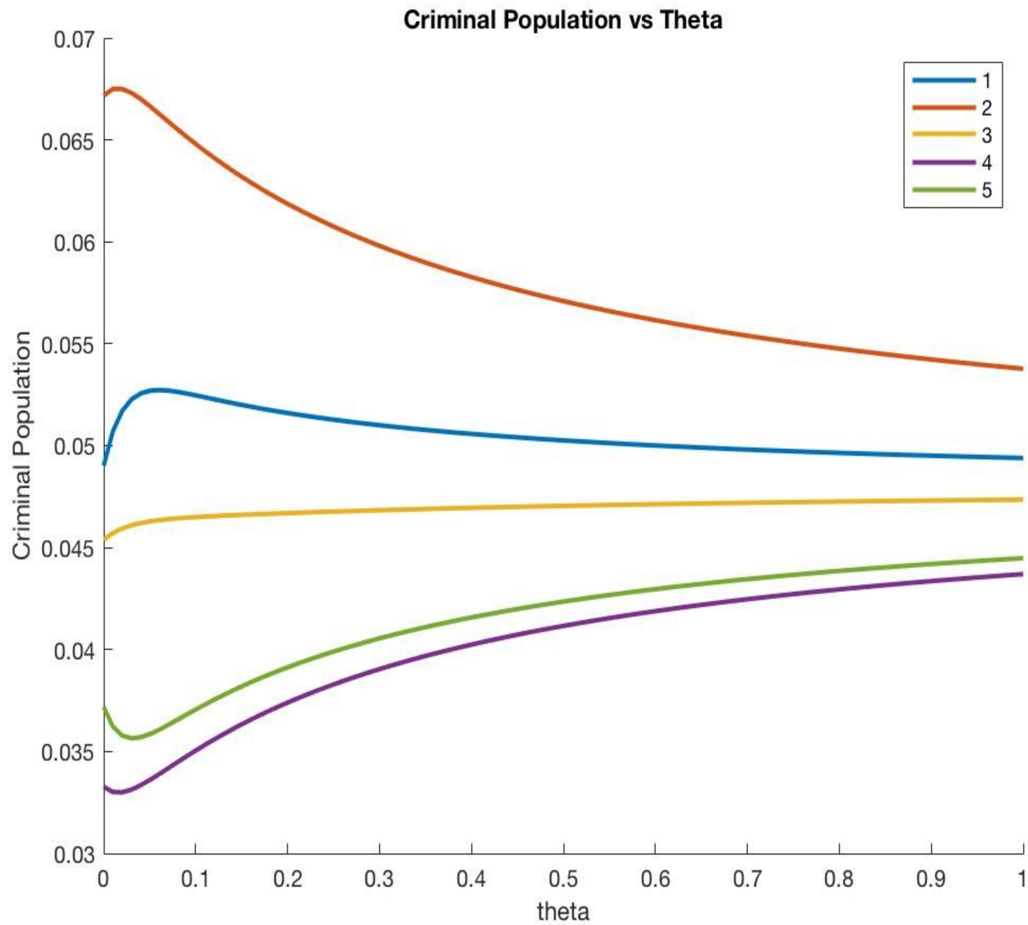
To understand the influence of the travel rate across our network of regions, we decided to begin by restricting movement to 2 regions. Based on our initial conditions, region 1 is a region of high-crime, and region 2 is a region of low-crime. The following figure was generated by plotting the long-run criminally-active populations for the two regions against an increasing travel rate (theta).



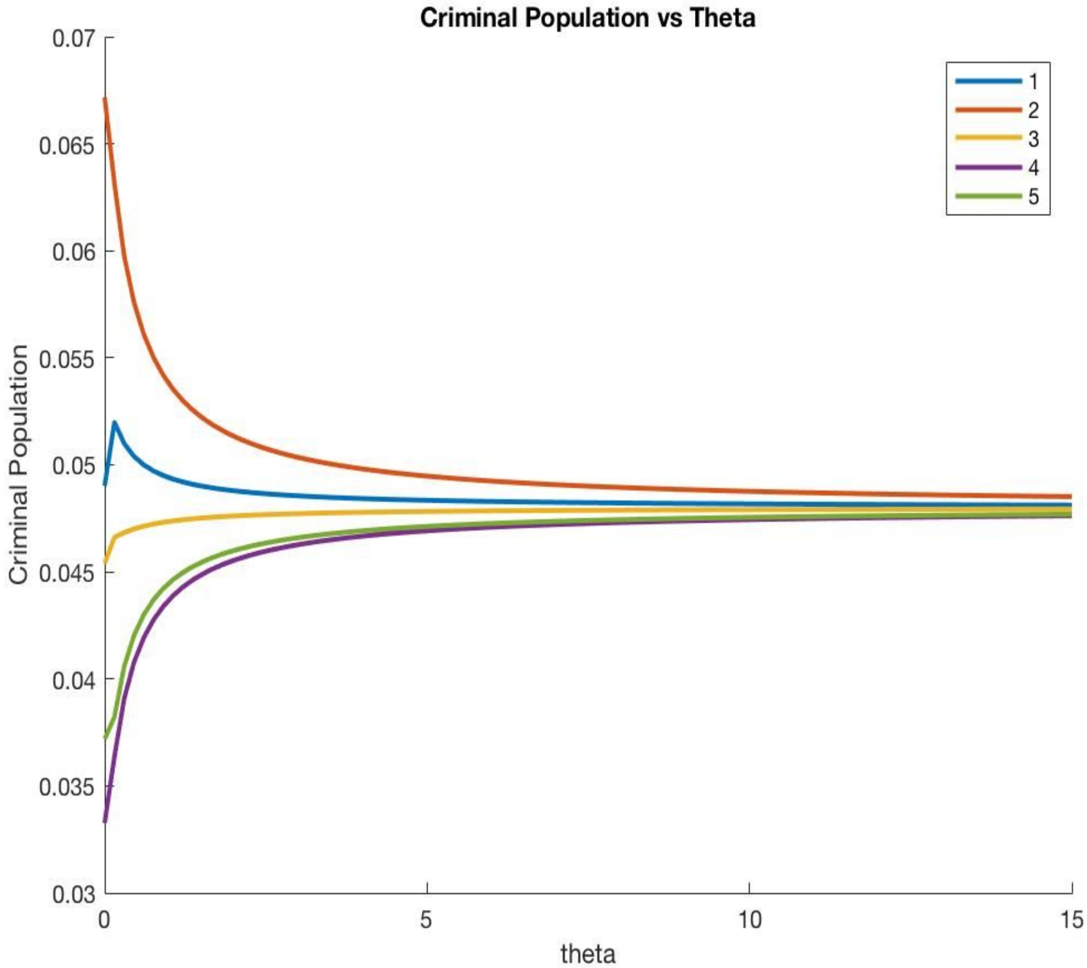
Step 2

We now extend the previous step to movement between a network of 5 regions. In this step, we set the initial conditions according to the current level of crime in each region. [9]] Region 1 and 2 are regions of relatively high crime. Region 4 and 5 are regions of relatively low crime. Region 3 lies between these two extremes.

When we vary the travel rate (theta) for small values in [0,1], we see a non-monotonic relation between theta and the steady-state criminal populations. In the following figure, a fall in the criminal population in region 4 and 5 corresponds to a rise in the criminal population for region 1 and 2.



To make appropriate conclusions, we ran our simulations for a large enough travel rate. In the following figure, we see that as the travel rate increases, the criminal populations eventually converge to a single criminal population composed of all criminally-active individuals across the 5 regions.



Step 3

In this step, we consider different network structures to see how the network structure affects the behaviour of the steady-state criminal population for each region when the travel rate is large enough. In the following subsections, the matrices introduce the network structure to our equations. The values in this matrix refer to the base travel rate i.e., for a fixed theta . Furthermore, the row indices of this matrix refer to region **j** and the column indices refer to region **i**. The purpose of including this matrix is to solidify the correctness of our approach. However, the next set of plots were generated by varying theta against the steady-state criminal populations for each region. Therefore, the non-zero values in the travel matrix will change for each value of theta.

Direct Transit

The direct transit network structure allows for movement to and from any region. This is the best representative of the GTA because it describes how all regions in the GTA are connected by one or the other form of transport. The previous two plots were generated using this network structure. Here's the base travel matrix for this network structure:

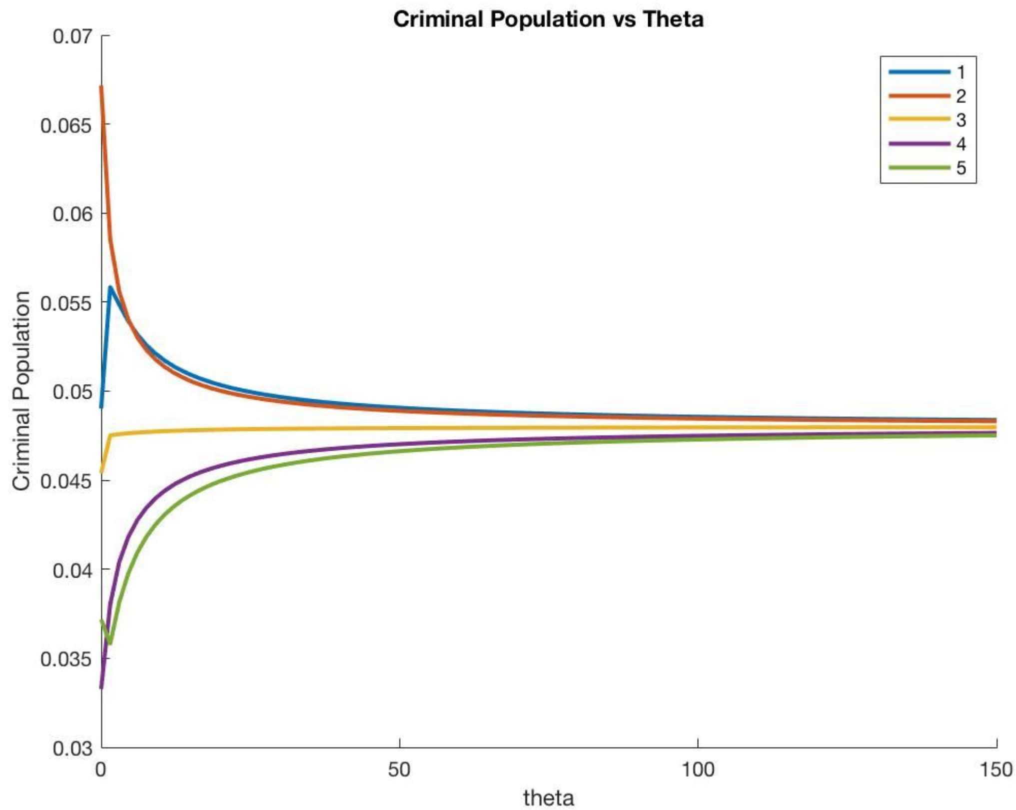
0	0.3000	0.3000	0.3000	0.3000
0.3000	0	0.3000	0.3000	0.3000
0.3000	0.3000	0	0.3000	0.3000
0.3000	0.3000	0.3000	0	0.3000
0.3000	0.3000	0.3000	0.3000	0

Movement between Neighbouring Regions

This network structure allows for sequential movement in the following manner: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 1$. Here's the base travel matrix for this network structure:

0	0.3000	0	0	0
0.3000	0	0.3000	0	0
0	0.3000	0	0.3000	0
0	0	0.3000	0	0.3000
0	0	0	0.3000	0

The following plot was generated for an increasing theta:



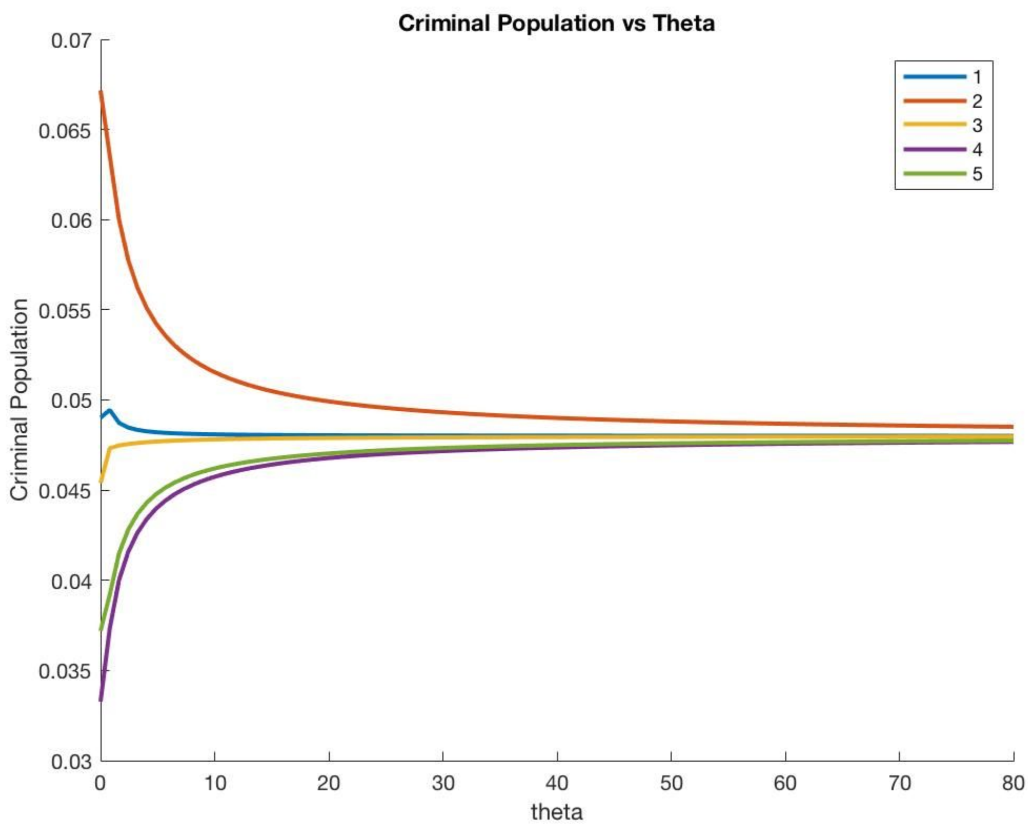
Initially, movement between neighbouring regions causes the criminal population in region 1 to increase. This is motivated by the decrease in the criminal population in region 5. The criminal population in 2 has a drastic decrease. As a result we see a slight increase in the criminal population in 3 till it stabilises to the constant value. However, notice the increase in the criminal population of region 4. Remember, region 4 was initially a region of low-crime. Therefore, it's highly possible that a proportion of the criminals from region 2 moved to region 4, as the travel rate increases. Notice that, for large enough theta, the criminal populations converge to a single criminal population composed of all criminally-active individuals across the 5 regions.

Movement between Toronto and Other Regions

Toronto is the hub of the GTA. Everyday over millions of people from different parts of the GTA commute to and from toronto. In the following matrix and plot, Toronto is region 1. Here’s the base travel matrix for this network structure:

0	0.3000	0.3000	0.3000	0.3000
0.3000	0	0	0	0
0.3000	0	0	0	0
0.3000	0	0	0	0
0.3000	0	0	0	0

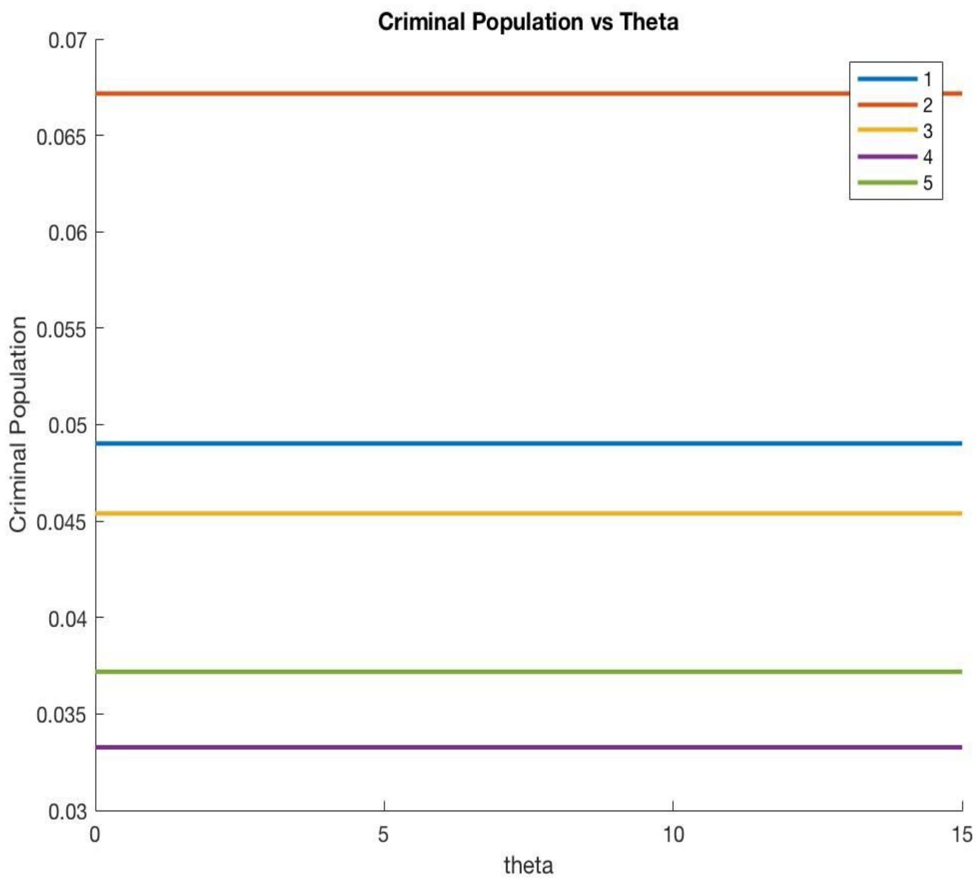
The following plot was generated for an increasing theta:



Given that region 1 refers to Toronto, it's a region of high-crime. In the preceding plot, notice there's an initial spike in the criminal population of Region 1. However, as theta increases, criminals move in and out of Toronto at a higher rate. This causes the population to eventually decrease to a constant level.

No Movement between Regions

This is a trivial network structure. The base travel matrix for this network structure is a 5x5 zero matrix, since we're considering a network of 5 regions. In the following plot, as the travel rate increases, the criminal population for each region is constant at its respective initial steady-state solution.



Conclusion

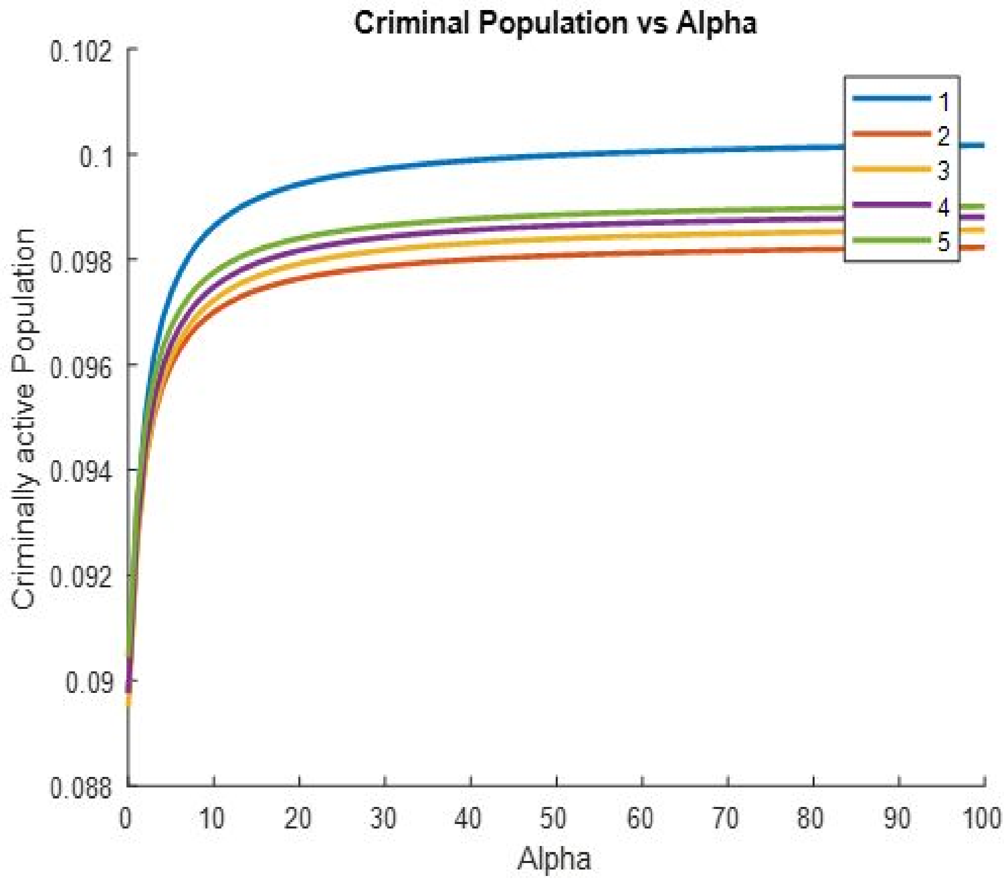
For a large enough theta, so long as there's movement within the network of regions, the network structure does not influence the behaviour of the criminal population in each region. This is because eventually the criminal populations converge to a single criminal population composed of all criminally-active individuals across the 5 regions. Furthermore, as long as there's movement within the network of regions, for smaller values of theta, there's a non-monotonic relation between the 5 regions.

Scenario 2

In the context of the GTA, and any urban city with a well-connected transport network, we expect to see a high travel rate between all regions. From now onwards we will only consider a high travel network, ($\boldsymbol{\theta}$ = 100).

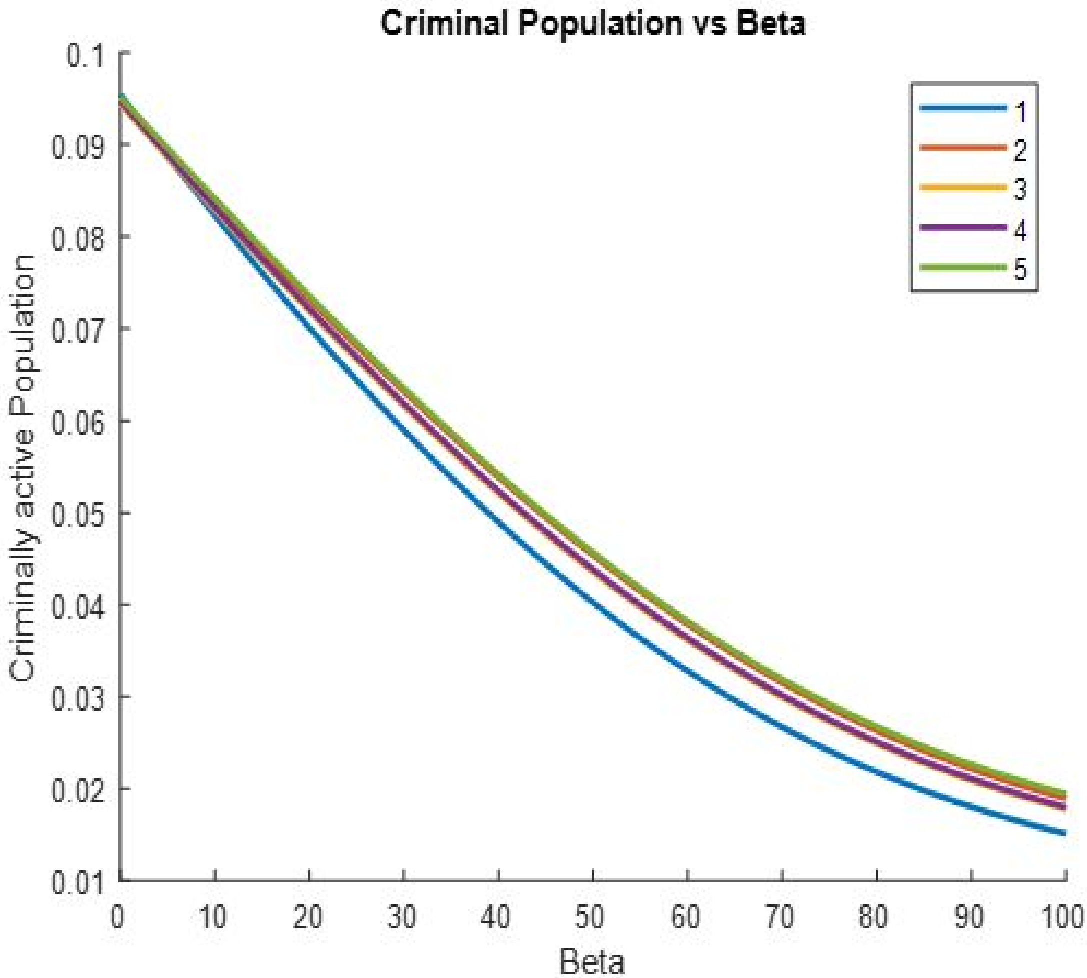
Case 1

In this case, we consider the effects of setting one region as a region of high-crime or low-crime. We studied the effects of what happens if the crime in region 1 increases. By increasing $\boldsymbol{\alpha_1}$ $\mathbf{(X_1 \Rightarrow C_1)}$ and plotting the steady state criminally active population as a fraction of the total population in the network we demonstrated that due to the high travel rate the steady state criminal activity in the other regions is pulled up by region 1. This can be seen in the following graph.



Case 2

Similarly, reducing the crime in region 1 by increasing $\boldsymbol{\beta_1(C_1 \Rightarrow X_1)}$, in the following graph, we see that region 1 pulls the criminally active population in all regions down.



Conclusion

When we vary other parameters to make region 1 high crime or low crime, the same results are obtained. From this we can conclude that in a high travel network of regions, it is very important to keep the criminal activity low. Furthermore, if we want to reduce crime in all the regions, it is enough to reduce the crime in one, since the high travel rate will pull down the crime in all regions. Extreme changes in one region results in extreme changes over all.

Expansion of the Model

The Equations for Parameter Relation

In this section, an important relations between the parameters $\boldsymbol{\gamma_i}$ ($\mathbf{C_i} \Rightarrow \mathbf{I_i}$), $\boldsymbol{\varepsilon_i}$ ($\mathbf{I_i} \Rightarrow \mathbf{X_i}$), and $\boldsymbol{\delta_i}$ ($\mathbf{I_i} \Rightarrow \mathbf{C_i}$) will be developed. We believe that $\boldsymbol{\varepsilon_i}$ and $\boldsymbol{\delta_i}$ can be written as functions of γ_i. The motivation behind this relation is the evidence that incarceration rate has an important effect on how prisons are run. There is evidence that prisons can often end up breeding crime, and it is more likely for an ex-convict to return to a life of crime. High incarceration rate also results in problems of overcrowding. In the previous model, we ignored the maximum capacity of prisons resulting in compartment $\mathbf{I}_i$ (incarcerated population) having an upper bound. Each region has a maximum capacity of prisoner that can be held, to account for this we believe increasing $\boldsymbol{\gamma}_i$ should result in an increase in atleast $\boldsymbol{\varepsilon}_i$, or $\boldsymbol{\delta}_i$. Furthermore, overcrowding results in greater release of prisoners, a burden on the justice system, and an increased interactions between the incarcerated population. Therefore, in changing $\gamma_{i,}$ we should expect a change in the parameters $\boldsymbol{\varepsilon}_i$, and $\boldsymbol{\delta}_i$.

We will attempt to model this relation between the parameters, and study the equations by varying the parameters for region 1 in a high travel network. This will enable us to study the effects on the steady state criminally active population in all regions as $\boldsymbol{\gamma}_1$ is varied. The proposed equations for this relation in region 1 are (since we get the same equations for every region we will only study changes in region 1 without loss of generality.)

$$\boldsymbol{\varepsilon_1 = \varepsilon_o + k_1 \gamma_1} \quad \mathbf{2(a)}$$

$$\boldsymbol{\delta_1 = \delta_o + k_2 \gamma_1} \quad \mathbf{2(b)}$$

$\mathbf{k}_1$ is the change in $\boldsymbol{\varepsilon}_1$ with respect to $\boldsymbol{\gamma}_1$, and $\boldsymbol{\varepsilon}_o$ is the initial value for $\boldsymbol{\varepsilon}_1$. Similarly, for $\boldsymbol{\delta}_1$ we have $\mathbf{k}_2$, which is the change in $\boldsymbol{\delta}_1$ with respect to γ_1, and $\boldsymbol{\delta}_o$ is the initial value for $\boldsymbol{\delta}_1$.

To study the new dynamics, we will study the 3 cases:

1. $\mathbf{k}_1$ and $\mathbf{k}_2 > 0$,
2. $\mathbf{k}_1 > \mathbf{0}$ and $\mathbf{k}_2 < 0$,
3. $\mathbf{k}_1 < \mathbf{0}$ and $\mathbf{k}_2 > 0$.

Note that when $\mathbf{k}_1$ and $\mathbf{k}_2$ are both **0**, we have no relation between the parameters (i.e. model 1). To study each case, we will be using the same values for all parameters, excluding γ_i **($\mathbf{C}_i$➡$\mathbf{I}_i$)**, ε_i **($\mathbf{I}_i$➡$\mathbf{X}_i$)**, and $\boldsymbol{\delta}_i$ **($\mathbf{I}_i$➡$\mathbf{C}_i$)**, to maintain consistency in the results.

The case where $\mathbf{k}_1$ and $\mathbf{k}_2$ are both less than 0 will not be studied, since we believe it is unreasonable to have less people leaving the prisons as the prisons get more crowded by the increase in $\boldsymbol{\gamma}_1$, due to the maximum prison capacity.

Case 1: k_1, k_2 > 0

This case is showing a scenario when there is overcrowding in the prisons of region 1, but the rehabilitation is neither weak, nor strong. This situation arises most commonly in prison dynamics, as suggested by previous studies, since it is easy to maintain. Here we have prisons causing little change to the behaviors of the prisoners, i.e. the prisoners leave with the same probability of engaging in criminal activity as they did when they arrived to prison. In this case the crime is being reduced by temporarily disabling the criminals from commiting a crime.

For the figure 2.1, we simulated the steady state criminal population by setting $\boldsymbol{\delta_o} = 3$, $\boldsymbol{\varepsilon_o} = 3$, $\mathbf{k_1} = 1$, and $\mathbf{k_2} = 6$. For figure 2.2 we set $\boldsymbol{\delta_o} = 3$, $\boldsymbol{\varepsilon_o} = 3$, $\mathbf{k_1} = 6$, and $\mathbf{k_2} = 1$. The choice of parameters is not important since a similar trend is observed. We see that as $\boldsymbol{\gamma_1}$ increases, for both examples, the criminal population being close to 10% of the total network population is brought down to a very low value. We have 2 parameters, namely $\boldsymbol{\delta_1}$ and $\boldsymbol{\gamma_1}$, that are decreasing the size of the criminally active population, and only one parameter, $\boldsymbol{\varepsilon_1}$, increasing the criminally active population. Ignoring the monetary costs of high incarceration rate, we can conclude that high incarceration is a good thing for $\mathbf{k_1}$, $\mathbf{k_2}$ > 0.

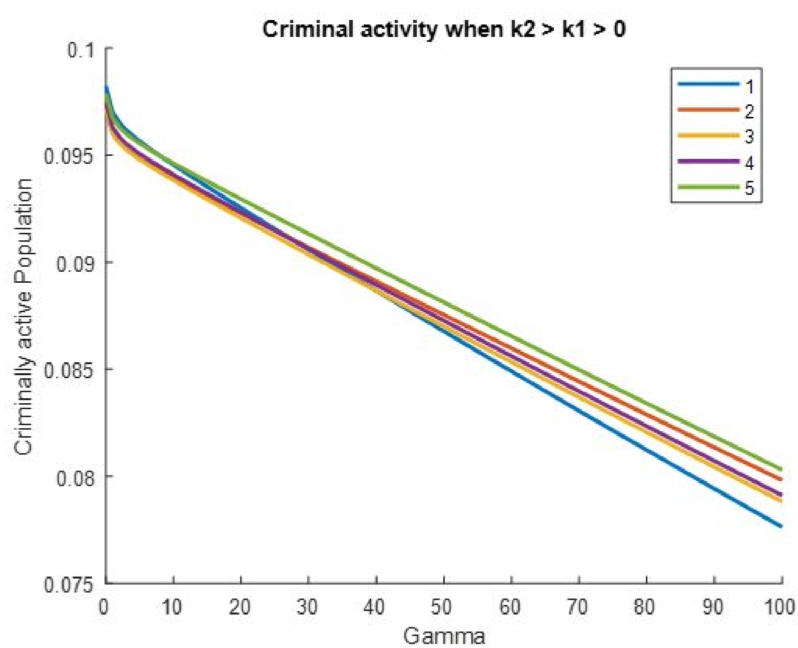


Fig. 2.1

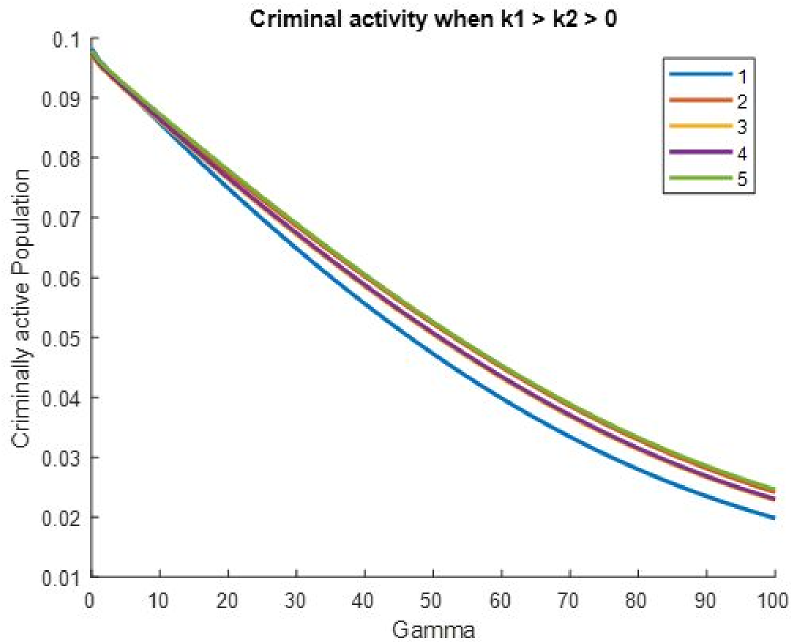


Fig 2.2

Case 2: $k_1 > 0$ and $k_2 < 0$

In this case, increase in $\boldsymbol{\gamma_1}$ decreases $\boldsymbol{\delta_1}$, and increases $\boldsymbol{\varepsilon_1}$, which is a result of when a city has a very strong rehabilitation system for newly released prisoners. This means that when a criminal is incarcerated, prison works the way we want it to by removing the criminal tendencies from the prisoners. From the previous studies conducted [6], we know that this is rarely the case.

Before simulating this case for region 1, we have to note that by making $\mathbf{k_2} < 0$, for a high enough $\boldsymbol{\gamma_1}$ our value of $\boldsymbol{\delta_1}$ becomes negative. To prevent that, we have to modify our equations so that we get:

$$\boldsymbol{\delta_1 = \begin{cases} \delta_o + k_2\gamma_1, & \text{if } \delta_o + k_2\gamma > 0 \\ 0, & \text{otherwise} \end{cases}}$$

Observing changes under several parameter conditions, the most interest result was for when $\boldsymbol{\delta_1}$ is driven down to the value of 0 by the increasing $\boldsymbol{\gamma_1}$**.** In the example shown in figure 2.3, we have set $\boldsymbol{\varepsilon_o} = 3$, $\boldsymbol{\delta_o} = 100$, $\mathbf{k_1} = 1$, and $\mathbf{k_2} = -5$. For $\boldsymbol{\gamma_1} < 20$, $\boldsymbol{\delta_1}$ is decreasing but not yet 0. As a result of decreasing $\boldsymbol{\delta_1}$, the decrease in the steady state criminally active population is at observed an increasing rate. When $\gamma_1 \geq 20$, $\boldsymbol{\delta_1}$ has a constant value of 0.

We can conclude that when rehabilitation is very strong, the criminally active population drops much quickly than compared to case 1.

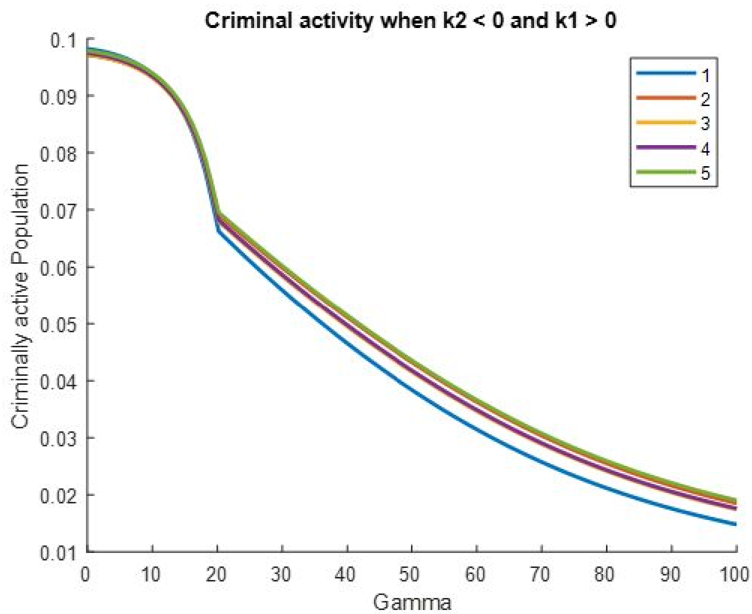


Figure 2.3

Case 3: $k_1 < 0$ and $k_2 > 0$

In this case, increase in $\boldsymbol{\gamma_1}$ increases $\boldsymbol{\delta_1}$, and decreases $\boldsymbol{\varepsilon_1}$, which occurs when prisons become 'schools of crime'. This is often the case when there is mass incarceration in the city, and the rehabilitation is extremely weak. The justice system does not fulfill its purpose, but instead it encourages crime, by the increased interactions between prisoners due to overcrowding causing the prisoner to become desensitized to criminal behavior, and by harsh practices, such as torture methods, of the prison guards causing psychological harm to the prisoners. As a result of this scenario, we will expect to see decrease in $\boldsymbol{\varepsilon_1}$ with respect to $\boldsymbol{\gamma_1}$.

As observed in case 2, we run into the problem of ε_1 becoming negative after a certain increase in $\boldsymbol{\gamma_1}$. We rewrite the equation for $\boldsymbol{\varepsilon_1}$ as:

$$\boldsymbol{\varepsilon = \begin{cases} \varepsilon_o + k_1\gamma, & \text{if } \varepsilon_o + k_1\gamma > 0 \\ 0, & \text{otherwise} \end{cases}}$$

We calculated the steady state criminal populations as we increased $\boldsymbol{\gamma_1}$, as shown in figure 2.4. The conditions set in figure 2.4 were $\boldsymbol{\varepsilon_o} = 50$, $\boldsymbol{\delta_o} = 1$, $\mathbf{k_1} = -1$, and $\mathbf{k_2} = 2$. We simulated the results for different parameter conditions and obtain the same trend. As observed in the graph, increasing $\boldsymbol{\gamma_1}$ for this relation causes an initial drop in the criminal population. After the criminal population reaches its minimum at $\boldsymbol{\gamma_1} = 24$ for all regions, the criminally active population started increasing again. For $\boldsymbol{\gamma_1} > 50$, there is no change in the criminally active population, since $\boldsymbol{\varepsilon_1} = 0$, and the increase in $\boldsymbol{\gamma_1}$ is being 'balanced out' by the increase in $\boldsymbol{\delta_1}$.

From this example, we can conclude that when the justice system is extremely weak, it is not always a good idea to incarcerate a lot of criminals. The optimal parameter conditions for $\boldsymbol{\gamma_1}$, $\boldsymbol{\varepsilon_1}$, and $\boldsymbol{\delta_1}$ are determined by all the other parameters in region 1, and the initial value conditions. To figure out the optimal values, we suggest that a plot be made for the steady state criminally active population against $\boldsymbol{\gamma_1}$, and the minimum value of $\boldsymbol{\gamma_1}$ be recorded. From the optimal value for $\boldsymbol{\gamma_1}$, values for $\boldsymbol{\varepsilon_1}$, and $\boldsymbol{\delta_1}$ can be calculated using the equations.

In the example in figure 2.4, the optimal $\boldsymbol{\gamma_1}$ is 24, which means that by plugging in the optimal $\boldsymbol{\gamma_1}$ in equations 2(a) and (b), we get $\boldsymbol{\varepsilon_1} = 26$, and $\boldsymbol{\delta_1} = 49$.

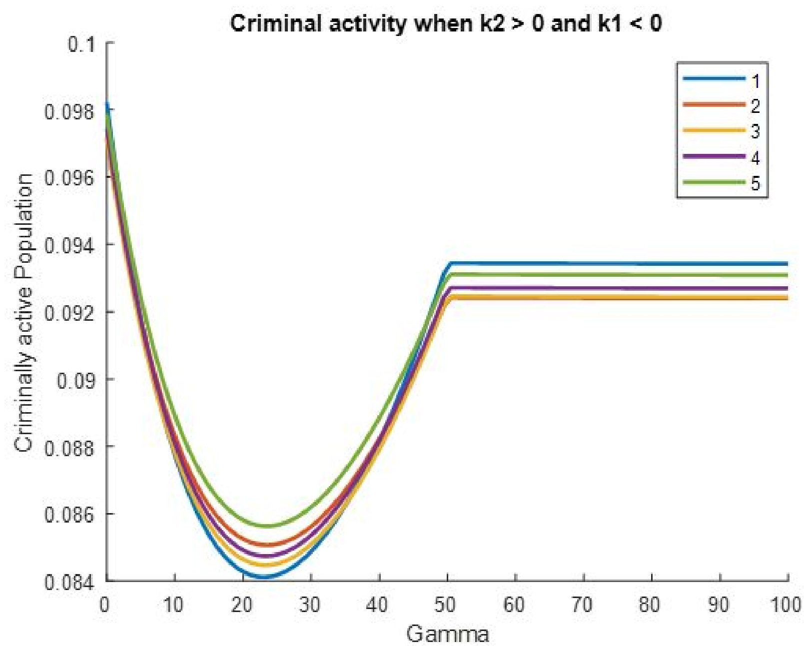


Figure 2.4

Summary and Results

To develop the compartment model for crime spread in a network of regions, we took inspiration from the dynamics provided by Mcmillon to extend the model to account for travel between a network of regions.

Scenario 1 and 2 *Parameters are independent of each other.*

- Introducing the travel parameter, $\boldsymbol{\theta_{ij}}$ for travel from region i to j, we decided to study the changes made to the isolated compartment model. We studied case 1 with 2 cities in the network. By varying $\boldsymbol{\theta = \theta_{12} = \theta_{21}}$ from 0 to 100, we see that the steady state criminally active populations for region 1 (initially high crime) and region 2 (initially low crime), converge to a single value. Therefore, as $\boldsymbol{\theta \rightarrow \infty}$ the regions populations become homogeneous.
- To extend the model to GTA (case 2), we varied $\boldsymbol{\theta}$ for different network structure against the criminally active populations. For small values of $\boldsymbol{\theta}$, we see different results for the different travel networks applied, but for large values $\boldsymbol{\theta}$ the same phenomenon (homogeneous compartments throughout the regions) is observed.
- To understand how to reduce crime in a high travel network, we set $\boldsymbol{\theta} = 100$ for the direct network (since all network structures are similar for $\boldsymbol{\theta} = 100$) we made region 1 high crime. We see that the steady state criminal population for all the other regions is pulled up by region 1. Similarly, by making region 1 low crime, we observe as the result of the high travel rate, that all the other regions also become low crime.

Expansion of the Model *Some parameters are dependent (namely* $\boldsymbol{\gamma_i}$, $\boldsymbol{\varepsilon_i}$, *and* $\boldsymbol{\delta_i}$)

In the previous model the variation of parameters was done under the assumption that the parameters are independent. Concluding, that this model does not encapture the full dynamics, we decided to study an important relation between $\boldsymbol{\gamma_i}$, $\boldsymbol{\varepsilon_i}$, and $\boldsymbol{\delta_i}$, parameters related to the justice system. To study this, we set the travel rate high, and varied the parameters according the equations:

$$\boldsymbol{\varepsilon_1 = \begin{cases} \varepsilon_0 + k_1\gamma_1, & \text{when } \varepsilon_o + k_1\gamma_1 > 0 \\ 0, & \text{otherwise} \end{cases}} \qquad \textbf{2(a)}$$

$$\boldsymbol{\delta_1 = \begin{cases} \delta_0 + k_2\gamma_1, & \text{when } \delta_0 + k_2\gamma_1 > 0 \\ 0, & \text{otherwise} \end{cases}} \qquad \textbf{2(b)}$$

It is important to note that these equations can be extended to all regions, however we study the relation for just region 1. We expect to see the same results for variation of these parameters in any region.

- Case 1: $\mathbf{k_1}$, $\mathbf{k_2} > 0$

  In this case we work under the assumption the prisoners leave with the same probability of returning to crime as they arrived to the prison, and that the rehabilitation in the city neither effective nor counter productive. We see that as $\boldsymbol{\gamma_1}$ increases the crime in all regions drop. This is a result of $\boldsymbol{\gamma_1}$ putting away criminals and the increasing $\boldsymbol{\varepsilon_1}$, and $\boldsymbol{\delta_1}$ 'cancel' out each other out.
- Case 2: $\mathbf{k_1} > 0$ and $\mathbf{k_2} < 0$

  Here we studied the scenario when the rehabilitation process is very strong, and the prison are successful in converting criminals to non criminals. Here we see that the decreasing $\boldsymbol{\delta_1}$ with increasing $\boldsymbol{\gamma_1}$, causes the criminally active population to drop at a very fast rate, once $\boldsymbol{\delta_1} = 0$, the rate of the decrease is slower. In this case we learn that proving rehabilitation of newly released prisoner is very important.
- Case 3: $\mathbf{k_2} > 0$ and $\mathbf{k_1} < 0$

  Here we are considering a scenario where prisons become 'schools of crime'. This can be explained by overcrowding in prisons causing an increase in crime, when the rehabilitation of the newly released prisoners is weak and the prison conditions are encouraging a life of crime rather than refraining from it. Here we see an initial drop in criminally active populations in all regions and but eventually an increase as $\boldsymbol{\gamma_1}$ becomes larger than its optimal value. When the value of $\boldsymbol{\varepsilon_1} = 0$, the criminally active population becomes constant. For this scenario it is important to maintain the optimal value of the parameters, which can be calculated by the simulation of the conditions in the city in question.

Assessment and Measures

1) For a high travel network of regions, the populations become homogeneous, therefore it is important to maintain the crime in **all** regions. An increase in crime in one region results in an increase in crime in all regions. Furthermore, to reduce crime new policies only need to be introduced in one region, since the high travel will reduce the crime in all. This method of crime control is more efficient.
2) In cities where the incarceration rate in high, it is important to be aware of the recidivism rate and the conditions in the prison. Due weak rehabilitation and poor prison conditions, high incarceration can lead to an increase in crime. In situations where there is high incarcerations it is

important to have a strong rehabilitation of newly released prisoners, and well maintained prisons.

3) We believe that it is often better use the criminal justice system to instill a fear of incarceration, rather than 'removing' the criminally active population, since the involvement of prisons can bring about several complications to be considered. This method of crime control will work to reduce the parameters $\boldsymbol{\alpha_i}$, $\boldsymbol{\alpha_i}'$, and $\boldsymbol{\beta_i}$, and always effective.

Questions

1) Why did you choose not to fit the model to actual data?
   We wanted to concentrate on the development of the model, and obtain results for certain cases we believe to be important. We wanted our model to be relevant to most urban cities, not limited to the GTA.

2) How do you think considering gangs recruitment would affect your model?
   There are existing compartment models on gang recruitment which focus on the contagious nature of criminal behavior. We believe our current model can be extended to add a new compartment, perhaps called G, for the gang population. In this case, the contagion rate will be different. Currently our model is assuming that all criminals, whether they belong to a gang or not, have the same contagious effect on the non-criminals.

3) What causes the criminally active population to move to criminally inactive without incarceration?
   This could occur if the criminals 'retire', before being caught. It is important to note that our equations for the compartments are with respect to time. If an individual is criminally active at one point, they do not have to be criminally active until they are incarcerated.

Contribution

Sneha Dasgupta: Initial Model, Network structures, Coding, Report and presentation writing
Heet Gala: Extension of the model to 5 cities, Coding, and Report writing
Saira Rizvi: Initial Model, Coding, Parameter variation and relations, and Report writing

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