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New Scheme Based On AICTE Flexible Curricula

Mechanical Engineering, VI-Semester

ME- 602 Machine Component and Design

Unit 1: **Introduction to stress in machine component:**

Stress concentration and fatigue: causes of stress concentration; stress concentration in tension, bending and torsion; reduction of stress concentration, theoretical stress concentration factor, notch sensitivity, fatigue stress concentration factor, cyclic loading, endurance limit, S-N Curve, loading factor, size factor, surface factor. Design consideration for fatigue, Goodman and modified Goodman's diagram, Soderberg equation, Gerber parabola, design for finite life, cumulative fatigue damage factor.

Unit 2: Shafts:

Design of shaft under combined bending, twisting and axial loading; shock and fatigue factors, design for rigidity; Design of shaft subjected to dynamic load; Design of keys and shaft couplings.

Unit 3: **Springs:**

Design of helical compression and tension springs, consideration of dimensional and functional constraints, leaf springs and torsion springs; fatigue loading of springs, surge in spring; special springs, Power Screws: design of power screw and power nut, differential and compound screw, design of simple screw jack.

Unit 4 : Brakes & Clutches:

Materials for friction surface, uniform pressure and uniform wear theories, Design of friction clutches: Disk, plate clutches, cone & centrifugal clutches. Design of brakes: Rope, band & block brake, Internal expending brakes, Disk brakes.

Unit 5: Journal Bearing:

Types of lubrication, viscosity, hydrodynamic theory, design factors, temperature and viscosity considerations, Reynold's equation, stable and unstable operation, heat dissipation and thermal equilibrium, boundary lubrication, dimensionless numbers, Design of journal bearings, Rolling-element Bearings: Types of rolling contact bearing, bearing friction and power loss, bearing life; Radial, thrust & axial loads; Static & dynamic load capacities; Selection of ball and roller bearings; lubrication and sealing.

References:

1. Shingley J.E; Machine Design; TMH
2. Sharma and Purohit; Design of Machine elements; PHI
3. Wentzell Timothy H; Machine Design; Cengage learning
4. Mubeen; Machine Design; Khanna Publisher
5. Ganesh Babu K and Srithar k; Design of Machine Elements; TMH
6. Sharma & Agrawal; Machine Design; Kataria & sons
7. Maleev; Machnine Design;

List of Experiment (Pl. expand it):
1. Design considerations for fatigue.
2. Design criteria and procedure for springs.
3. Design of shaft.

4. Design of keys.
5. Design of couplings.
6. Design of leaf spring for a given load.
7. Design of power screw and nut.
8. Design of Centrifugal clutch.
9. Design of disc brake.
10. Design considerations for roller bearings.

ME-602 Machine Component and Design

Syllabus

Unit 1:

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Subject Notes

Definition

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time-consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Machine Design Considerations

Following are the general considerations in designing a machine component:

1. **Type of load and stresses caused by the load - T**he load, on a machine component, may act in several ways due to which the internal stresses are set up.
2. **The motion of the parts or kinematics of the machine -** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be:
   a) The rectilinear motion which includes unidirectional and reciprocating motions.
   b) The curvilinear motion which includes rotary, oscillatory and simple harmonic.
   c) Constant velocity.
   d) Constant or variable acceleration.
3. **Selection of materials -** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behavior under working conditions. Some of the important characteristics of materials are strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc.
4. **Form and size of the parts -** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.
5. **Frictional resistance and lubrication -** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. **Convenient and economical features -** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take-up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. **Use of standard parts -** The use of standard parts is closely related to cost because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts, and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers, and taps and also to decrease the number of wrenches required.
8. **Safety of operation -** Some machines are dangerous to operate, especially those which are speeded up to ensure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with the operation of the machine.
9. **Workshop facilities -** A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.
10. **Number of machines to be manufactured -** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for a small number of the product will not permit any undue expense in the workshop processes so that the designer should restrict his specification to standard parts as much as possible.
11. **Cost of construction -** The cost of construction of an article is the most important consideration involved in the design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further consideration. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.
12. **Assembling -** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Machine Design Procedure

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

1. **Recognition of need.** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.
2. **Synthesis (Mechanisms).** Select the possible mechanism or group of mechanisms which will give the desired motion.
3. **Analysis of forces.** Find the forces acting on each member of the machine and the energy transmitted by each member.
4. **Material selection.** Select the material best suited for each member of the machine.

5. **Design of elements (Size and Stresses).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.
6. **Modification.** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.
7. **Detailed drawing.** Draw the detailed drawing of each component and the assembly of the machine with the complete specification for the manufacturing processes suggested.
8. **Production.** The component, as per the drawing, is manufactured in the workshop.

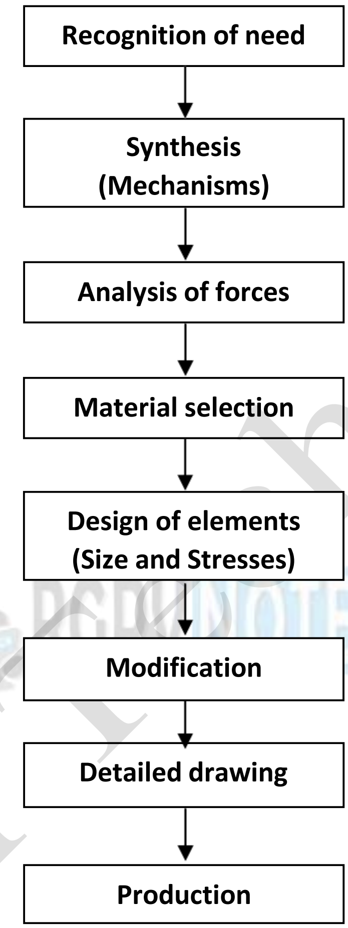


Fig. 1.1 Machine Design Procedure

Material selection

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc. The metals may be further classified as:

(a) Ferrous metals, and (b) Non-ferrous metals.

The **ferrous metals** are those which have the iron as their main constituents, such as cast iron, wrought iron, and steel.

The **non-ferrous** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties.

Physical Properties of Metals

The physical properties of the metals include luster, color, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. **Strength.** It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.
2. **Stiffness.** It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.
3. **Elasticity.** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.
4. **Plasticity.** It is the property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.
5. **Ductility.** It is the property of a material enabling it to be drawn into a wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminum, nickel, zinc, tin and lead.
6. **Brittleness.** It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.
7. **Malleability.** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper, and aluminum.
8. **Toughness.** It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed to the point of fracture. This property is desirable in parts subjected to shock and impact loads.
9. **Machinability.** It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.
10. **Resilience.** It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.
11. **Creep.** When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers, and turbines.
12. **Fatigue.** When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which is usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.
13. **Hardness.** It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are

dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

i. Brinell hardness test,
ii. Rockwell hardness test,
iii. Vickers hardness (also called Diamond Pyramid) test, and
iv. Shore scleroscope

Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulfur, manganese, and phosphorus on steel.

1. Manganese (Mn) – Manganese improves hardenability, ductility and wears resistance. Mn eliminates the formation of harmful iron sulphides, increasing strength at high temperatures.

2. Nickel (Ni) – Nickel increases strength, impact strength and toughness, impart corrosion resistance in combination with other elements.

3. Chromium (Cr) – Chromium improves hardenability, strength and wear resistance, sharply increases corrosion resistance at high concentrations (> 12%).

4. Tungsten (W) – Tungsten increases hardness particularly at elevated temperatures due to stable carbides, refines grain size.

5. Vanadium (V) – Vanadium increases strength, hardness, creep resistance and impact resistance due to the formation of hard vanadium carbides, limits grain size.

6. Molybdenum (Mo) – Molybdenum increases hardenability and strength particularly at high temperatures and under dynamic conditions.

7. Silicon (Si) – Silicon improves strength, elasticity, acid resistance and promotes large grain sizes, which cause increasing magnetic permeability.

8. Titanium (Ti) – Titanium improves strength and corrosion resistance, limits austenite grain size.

9. Cobalt (Co) – Cobalt improves strength at high temperatures and magnetic permeability.

10. Zirconium (Zr) – Zirconium increases strength and limits grain sizes.

11. Boron (B) – Boron highly effective hardenability agent, improves deformability and machinability.

12. Copper (Cu) – Copper improves corrosion resistance.

13. Aluminium (Al) – Aluminium acts as a deoxidizer, limits austenite grain growth.

Modes of failure –

It is the basic manner or mechanism of the failure or damage process. Failure is not performing as intended, may occur at any stage/action during manufacture or in-service due to one or more causes.

Some basic knowledge of modes of failure is needed for:

i. Modern design methods e.g. latest Standards aim at designing against each of the modes of failure which may be feasible for a particular component or equipment and service conditions.
ii. Manufacture and testing where differences may be required for virtually the same product when different materials or service conditions apply e.g. low temperature (brittle fracture), extensive cyclic loading (fatigue), or seawater (corrosion).
iii. Operation of equipment to be within design limits, but if these are to be exceeded the likelihood of failure by different modes can be seriously increased.
iv. Risk management of critical equipment where assessments are made of the hazards, feasible failure modes, the likelihood of failure & consequences e.g. FMEA or FMECA (Failure Modes Effects & Consequence Analysis).
v. In-service inspection to assess acceptability, avoidance or rectification of any degradation.
vi. Failure analysis to help determine and identify probable failure causes, methods of avoiding repetition and possible improvements by innovation.
vii. Training of technologists and key technical people on the "why" of various practices aimed at achieving safety of people and plant and protection of the environment through prevention of failures.

Various Modes of Failure are: -

1. Creep

a) Distortion **S**

b) Cracking **U**
c) Rupture (through wall) **U**
d) Creep-fatigue **U**
e) Creep buckling

2. **Brittle Fracture** U
a) Ferritic steel at notches at low temperature
b) Low ductility Material: Cast iron, glass etc

3. **Fatigue U**
a) Mechanical fatigue (high/low cycle)
b) Thermal fatigue
c) See also corrosion fatigue

4. **Excessive Deformation (elastic or plastic) S or U**
a) Thermal distortion by overheating
b) Overloading e.g. by over pressurization, overload, (earthquakes, earth settlement, and wind).
c) Causing leakage at mechanical joints.
d) Ratcheting incremental collapse – progressive (plastic deformation)
e) Mechanical gouging

5. **Ductile Fracture U**
a) Rupture following excessive plastic deformation. (Includes ductile tearing).
b) Lamellar tearing

6. **8.Instability (Buckling) U**
a) Elastic
b) Plastic or elastic – plastic
c) Creep buckling
d) Overturning

7. **Sustained Load Cracking U**
a) In 6000 series aluminum alloys

8. **Combinations of Above S or U**
a) Mixed modes e.g. corrosion & fatigue leading to brittle fracture; or corrosion-creep-fatigue.

U = Ultimate or Strength limit state
S = Serviceability limit state

Theories of failure: -

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uniaxial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding i.e. when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

1. **Maximum Principal or Normal Stress Theory (Rankine's Theory)**

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well-defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.S.) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \sigma_{t1} / \text{F. S.}, \text{ for ductile materials}$$

$$\sigma_{t1} = \sigma_u / \text{F. S.}, \text{ for brittle materials}$$

Where σ_{yt} = Yield point stress in tension as determined from simple tension test, and
σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials.

However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

2. **Maximum Shear Stress Theory (Guest's or Tresca's Theory)**

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt} / \text{F.S.}$$

Where τ_{max} = Maximum shear stress in a bi-axial stress system,
τ_{yt} = Shear stress at yield point as determined from simple tension test, and
F.S. = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation **(i)** may be written as

$$\tau_{max} = \sigma_{yt} / 2 \text{ X F.S.}$$

This theory is mostly used for designing members of ductile materials.

3. **Maximum Principal Strain Theory (Saint Venant's Theory)**

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (i.e. strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = (\sigma_{t1} / E) – (\sigma_{t2} / m. E)$$

∴ According to the above theory

$$\varepsilon_{max} = (\sigma_{t1} / E) – (\sigma_{t2} / m. E) = \varepsilon = \sigma_{yt} / E \text{ X F.S.}$$

Where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,
ε = Strain at yield point as determined from simple tension test,
1/m = Poisson's ratio,
E = Young's modulus, and
F.S. = Factor of safety.

From equation **(i),** we may write that

$$\sigma_{t1} – \sigma_{t2} / m = \sigma_{yt} / \text{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

4. **Maximum Strain Energy Theory (Haigh's Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (i.e. strain energy at the yield point) per unit volume as determined from simple tension test.
We know that strain energy per unit volume in a bi-axial stress system,
$$U_1 = 1/2E\ [(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1} \text{ X } \sigma_{t2} / m]$$
And limiting strain energy per unit volume for yielding as determined from simple tension test,
$$U_2 = 1/2E\ (\sigma_{yt} / F.S.)^2$$
According to the above theory, U1 = U2
$$1/2E\ [(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1} \text{ X } \sigma_{t2} / m] = 1/2E\ (\sigma_{yt} / F.S.)^2$$
$$\text{Or } (\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1} \text{ X } \sigma_{t2} / m = (\sigma_{yt} / F.S.)^2$$
This theory may be used for ductile materials.

5. **Maximum Distortion Energy Theory (Hencky and Von Mises Theory)**
According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e. distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as
$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\ \sigma_{t1} \text{ X } \sigma_{t2} = (\sigma_{yt} / F.S.)^2$$
This theory is mostly used for ductile materials in place of maximum strain energy theory.

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called **stress concentration.**

It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 1.2. A little consideration will show that the nominal stress in the right and left-hand sides will be uniform but in the region where the cross section is changing, a redistribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

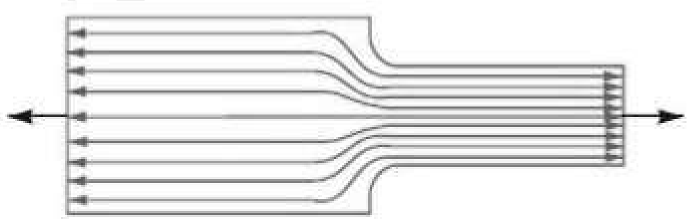


Fig. 1.2 Stress Concentration

Theoretical or Form Stress Concentration Factor

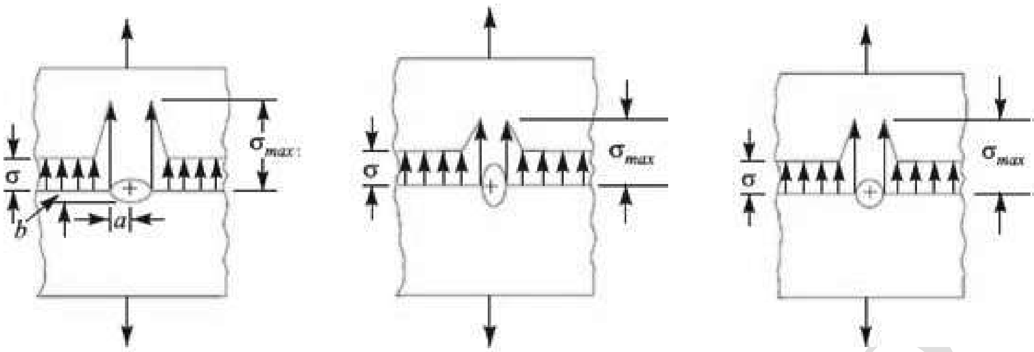
The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon the net area.
Mathematically, theoretical or form stress concentration factor,
K_t = Maximum stress / Nominal stress
The value of K_t depends upon the material and geometry of the part.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig. 1.3. We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by
$$\sigma_{max} = \sigma\ (\ 1 + 2a/b)$$
And the theoretical stress concentration factor,

$$K_t = \sigma_{max} / \sigma = (1 + 2a/b)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Fig. 1.3 (b)] and the increase in stress is small. When the hole is circular as shown in Fig. 1.3 (c), then a/b = 1 and the maximum stress is three times the nominal value.



a/b = 2, $\sigma_{max} = 5\sigma$ (a)   a/b = 1/2, $\sigma_{max} = 2\sigma$ (b)   a/b = 1, $\sigma_{max} = 3\sigma$ (c)

Fig. 1.3 Stress concentrations due to holes

The stress concentration in the notched tension member, as shown in Fig. 1.4, is influenced by the depth of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma (1 + 2a/r)$$

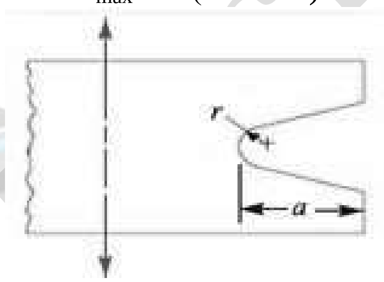


Fig. 1.4 Stress concentrations due to notches

Factors to be considered while designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches, and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.
4. The parts should be protected from the corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the surface of the part increase its fatigue strength.

Causes of stress concentration

1. The abrupt change in cross-section of machine member e.g. stepped shaft, key way’s, oil groove.
2. The concentrated load applied on small area – examples of this are
i) Contact between wheel & rail.
ii) Contact between ball & race.
iii) Contact between gear teeth.
3. Variation in properties of material From point to point - examples of this are
i) Internal cavities or blowholes.
ii) Cavities in welding.
iii) Nonmetallic inclusions.

Notch sensitivity:

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term **notch sensitivity** is applied to this behavior. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$q = (K_f - 1) / (K_t - 1)$

Or $K_f = 1 + q (K_t - 1)$ [For tensile or bending stress]

And $K_{fs} = 1 + q (K_{ts} - 1)$ [For shear stress]

Where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Fatigue stress concentration factor:

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

K_f = Endurance limit without stress concentration / Endurance limit with stress concentration

Factors affecting endurance limit:

1. Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load

$\sigma_{eb} = \sigma_e \times K_b = \sigma_e$ $(\because K_b = 1)$

Endurance limit for reversed axial load,

$\sigma_{ea} = \sigma_e \times K_a$

And endurance limit for reversed torsional or shear load,

$\tau_e = \sigma_e \times K_s$

2. Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 1.5 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

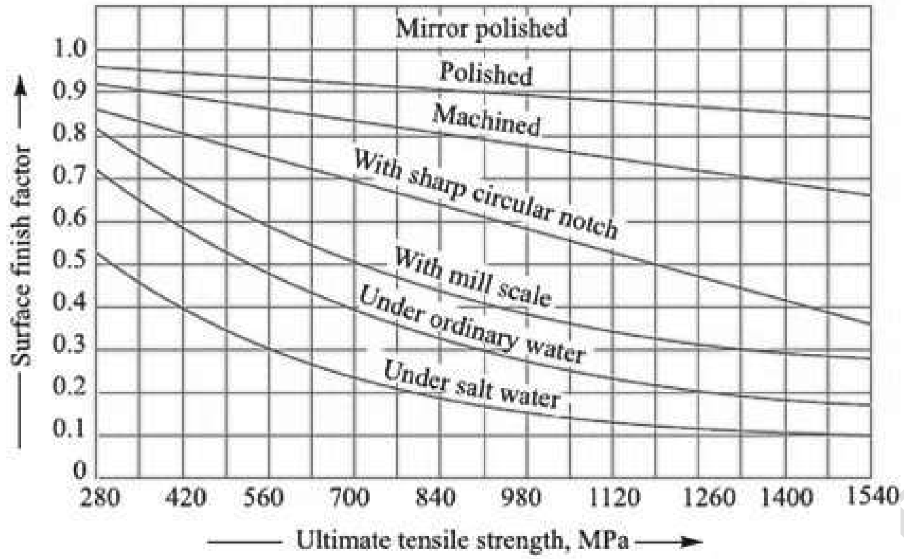


Fig. 1.5 Surface finish factor for various surface conditions

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finishes factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$\sigma_{e1} = \sigma_{eb} \times K_{sur} = \sigma_e \times K_b \times K_{sur} = \sigma_e \times K_{sur}$ (∵ $K_b = 1$) (For reversed bending load)

$\sigma_{e1} = \sigma_{ea} \times K_{sur} = \sigma_e \times K_a \times K_{sur}$ (For reversed axial load)

$\sigma_{e1} = \tau_e \times K_{sur} = \sigma_e \times K_s \times K_{sur}$ (For reversed torsional or shear load)

3. Size Factor

A little consideration will show that if the size of the standard specimen is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$\sigma_{e2} = \sigma_{e1} \times K_{sz}$ (Considering surface finish factor also)

$\sigma_{e2} = \sigma_{eb} \times K_{sur} \times K_{sz} = \sigma_e \times K_b \times K_{sur} \times K_{sz} = \sigma_e \times K_{sur} \times K_{sz}$ (∵ $Kb = 1$)

$\sigma_{e2} = \sigma_{ea} \times K_{sur} \times K_{sz} = \sigma_e \times K_a \times K_{sur} \times K_{sz}$ (For reversed axial load)

$\sigma_{e2} = \tau_e \times K_{sur} \times K_{sz} = \sigma_e \times K_s \times K_{sur} \times K_{sz}$ (For reversed torsional or shear load)

4. Material Variables

Different materials have different static properties and so also different fatigue properties. Generally, it is seen that the materials which have better static properties are also good under fatigue loading. No exact relationship between fatigue strength and ultimate tensile strength for various materials exists. But for preliminary design, particularly when data are not available, following relations are quite useful.

σ_u = ultimate tensile strength

σ_e = fatigue strength in fully reversed bending load or rotating bending, polished and smooth specimen

σ_{ea} = fatigue strength in fully reversed axial load

τ_e = fatigue strength in fully reversed shearing stress

Then

$\sigma_e = 0.5\ \sigma_u$, for steels

$\sigma_e = 0.4\ \sigma_u$, for cast iron

$\sigma_e = 0.3\ \sigma_u$, for non-ferrous and alloys

$\sigma_{ea} = 0.86\ \sigma_e$

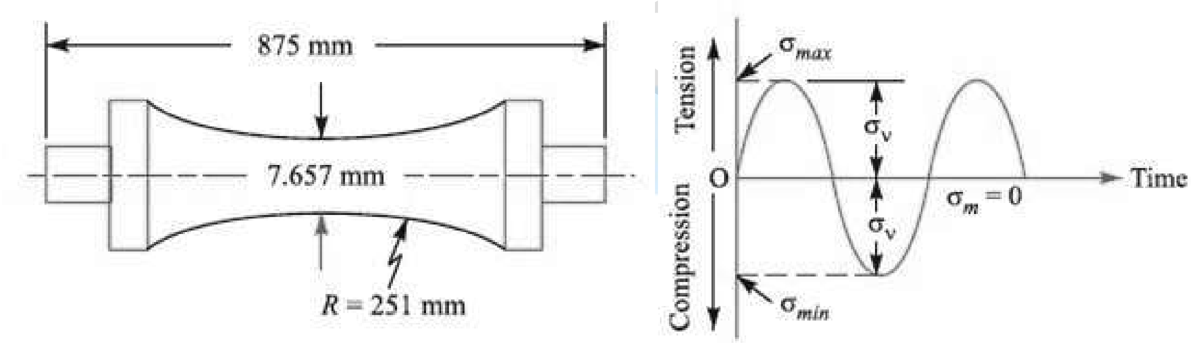
$\tau_e = 0.5\ \sigma_e$, for ductile materials

$\tau_e = 0.2\ \sigma_u$, for non-ferrous metals and alloys
$\tau_e = 0.8\ \sigma_u$, for cast iron

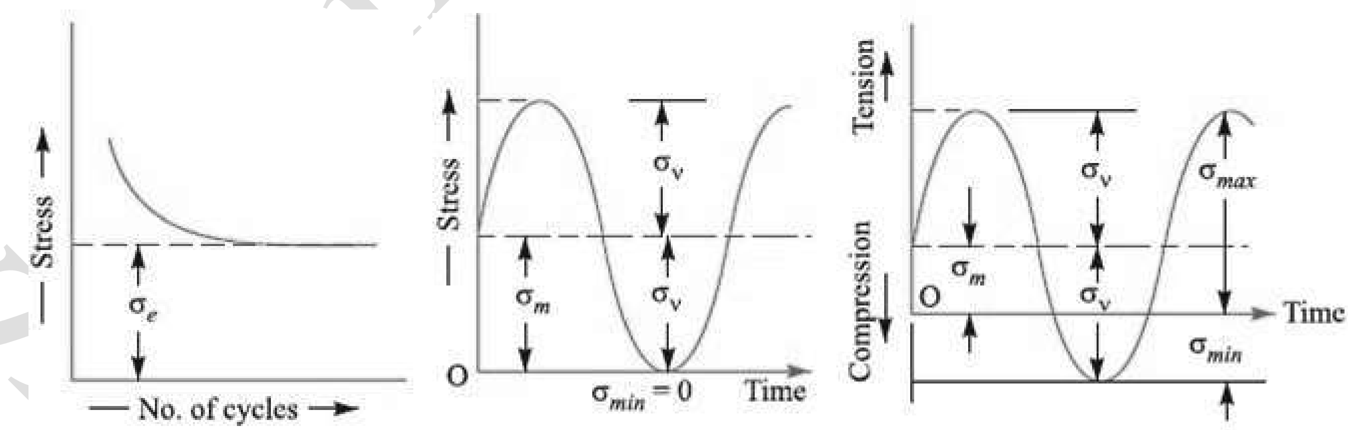
Theoretical stress concentration factor:
The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon the net area.
Mathematically, theoretical or form stress concentration factor,
K_t = Maximum stress / Nominal stress
The value of Kt depends upon the material and geometry of the part.

Cyclic loading/Fatigue
Structures and Machine Elements are subjected repeated loads, called cyclic loads, and the resulting cyclic stresses can lead to microscopic physical damage to the materials involved. Even at stresses well below the material's ultimate strength, this damage can accumulate with continued cycling until it develops into a crack or other damage that leads to failure of the component. The process of accumulating damage and finally to failure due to cyclic loading is called fatigue.

Fatigue and Endurance Limit
It has been found experimentally that when a material is subjected to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which is usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is affected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard Specimen (b) Completely reversed stresses



(c) Endurance or Fatigue Limit (d) Repeated Stresses (e) Fluctuating Stresses

Fig. 1.6 Fatigue and Endurance Limit

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 1.6 (a) is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibers varies from maximum compressive to maximum tensile while the bending stress at the lower fibers varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle.

This is represented by a time-stress diagram as shown in Fig. 1.6 (b). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in a stress-cycle curve as shown in Fig. 1.6 (c). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 1.6 (c), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as **endurance** or **fatigue limit** (σ_e). It is defined as the maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for an infinite number of cycles (usually 107 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term **endurance strength** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 1.6 (b). In actual practice, many machine members undergo a different range of stress than the completely reversed stress.

The stress **versus** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 1.6 (e). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 1.6 (e):

1. Mean or average stress,

$\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$

2. Reversed stress component or alternating or variable stress,

$\sigma_v = (\sigma_{max} - \sigma_{min}) / 2$

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 1.7 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice:

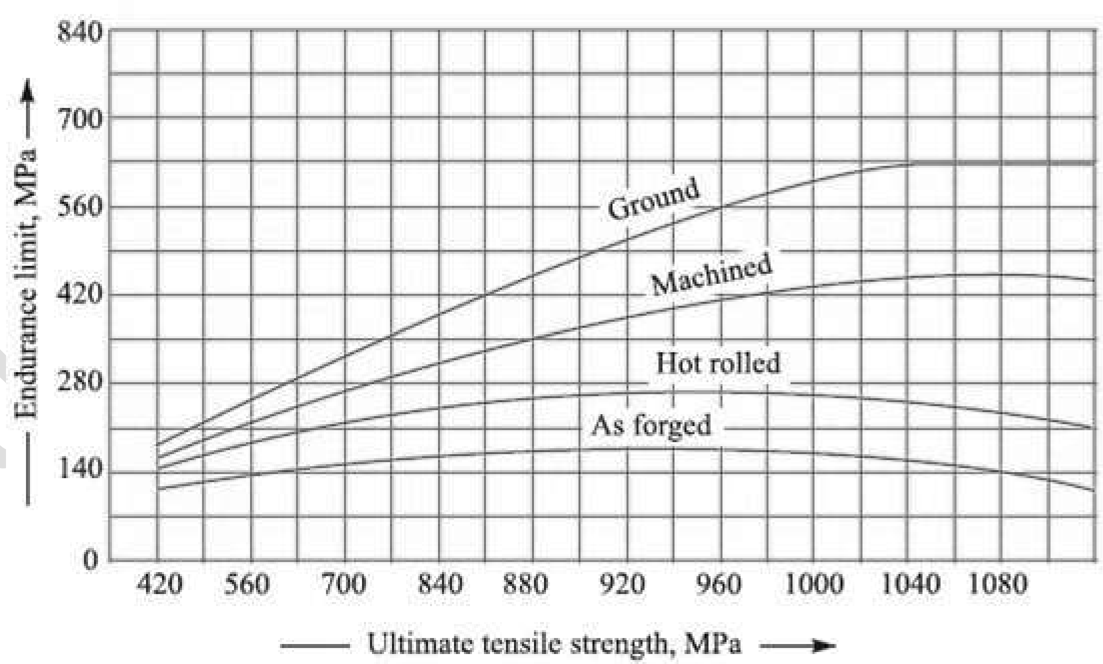


Fig. 1.7 Endurance limit of steel corresponding to ultimate tensile strength

For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, σe = 0.3 σu

Design consideration for fatigue:

Gerber Parabola: -

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Fig. 1.8. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$. A parabolic curve is drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig., but because of scattering in the test points, a straight line relationship (i.e. Goodman line and Soderberg line) is usually preferred in designing machine parts.

According to Gerber, variable stress,

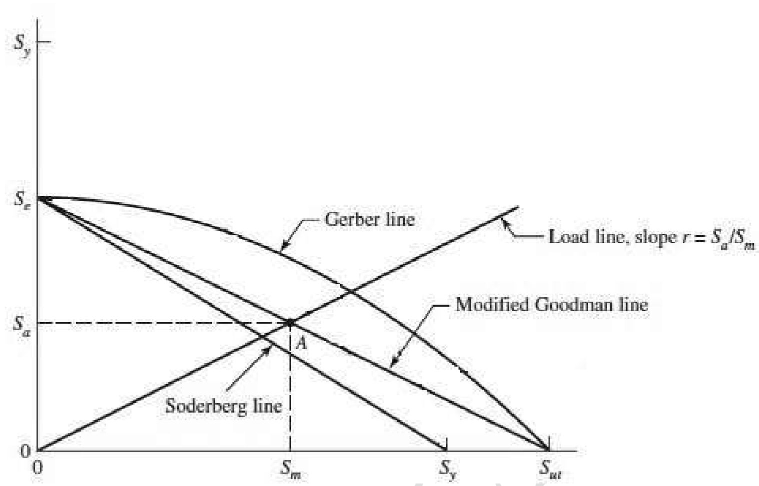


Fig. 1.8 Gerber Method

$\sigma_v = \sigma_e$ [1/F.S. – $(\sigma_m/\sigma_m)^2$ x F.S.]

Where F.S. = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_m = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the above equation may be written as

1/F.S. = $(\sigma_m/\sigma_m)^2$ x F.S. + (σ_v x K_f)/ σ_e

Goodman Line: -

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line AB in Fig. 1.9, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

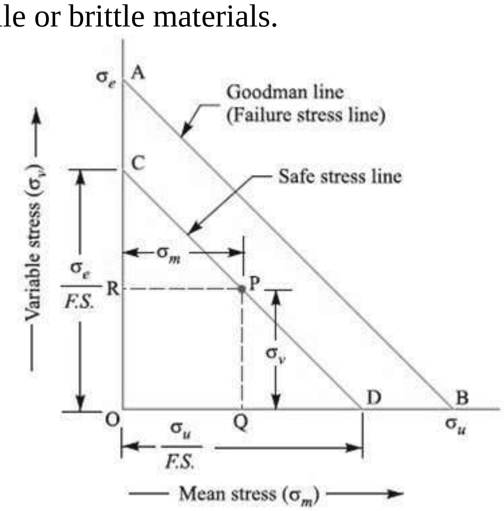


Fig. 1.9 Goodman's Method

In Fig. 1.9, line AB connecting σ_e and σ_u is called **Goodman's failure stress line.** If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD.

Now from similar triangles COD and PQD,

PQ/CO = QD/OD = (OD – OQ)/OD = 1 – OQ/OD (∵QD = OD – OQ)

σ_v / σ_e/F.S. = 1 - σ_m / σ_u/F.S.

$\sigma_v = \sigma_e$/F.S. [1 - σ_m / σ_u/F.S.] = σ_e [1/F.S. - σ_m / σ_u]

1/F.S. = $\sigma_m / \sigma_u + \sigma_v / \sigma_e$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machines and structural parts that are subjected to fatigue loads contain regions of high-stress concentration, therefore equation (i) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The above equation may now be written as

1/F.S. = $\sigma_m / \sigma_u + \sigma_v$ x K_f / σ_e

Where F.S. = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the above equation may be written as

1/F.S. = $\sigma_m / \sigma_u + \sigma_v$ x K_f / σ_{eb} x K_{sur} x K_{sz} = $\sigma_m / \sigma_u + \sigma_v$ x K_f / σ_e x K_b x K_{sur} x K_{sz}

1/F.S. = $\sigma_m / \sigma_u + \sigma_v$ x K_f / σ_e x K_{sur} x K_{sz} (∵σ_e x $K_b = \sigma_e$ and K_b = 1)

Where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

Soderberg Line: -

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line AB in Fig. 1.10, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.

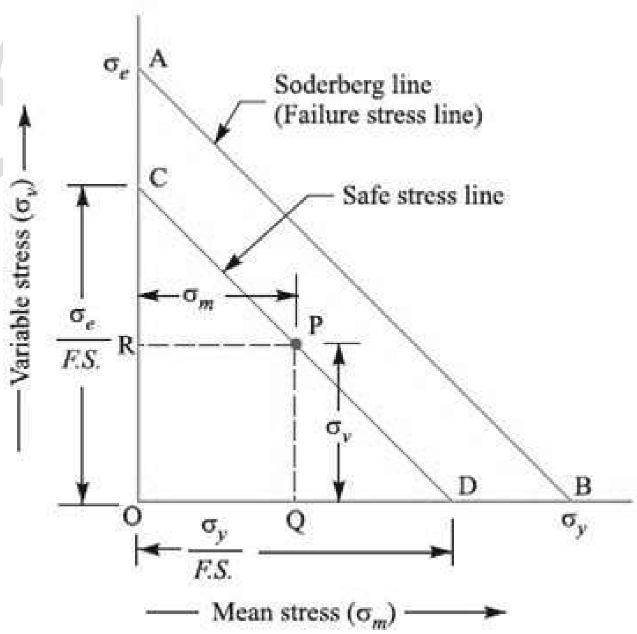


Fig. 1.10 Soderberg Method

The line AB connecting σ_e and σ_y, as shown in Fig. 1.10, is called Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the endurance limit and yield strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD. Now from similar triangles COD and PQD,

PQ/CO = QD/OD = (OD – OQ)/OD = 1 – OQ/OD (∵QD = OD – OQ)

σ_v / σ_e/F.S. = 1 - σ_m / σ_y/F.S.

$\sigma_v = \sigma_e$/F.S. [1 - σ_m / σ_y/F.S.] = σ_e [1/F.S. - σ_m / σ_y]

1/F.S. = $\sigma_m / \sigma_y + \sigma_v / \sigma_e$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v). Thus the above equation may be written as

1/F.S. = $\sigma_m / \sigma_y + \sigma_v \times K_f / \sigma_e$

Considering the load factor, surface finish factor and size factor, the above equation may be written as

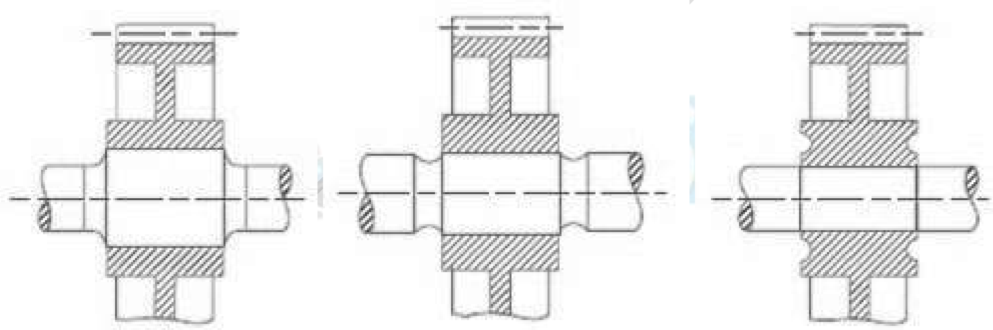
1/F.S. = $\sigma_m / \sigma_u + \sigma_v \times K_f / \sigma_{eb} \times K_{sur} \times K_{sz}$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Factors to be considered while designing machine parts to avoid fatigue failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches, and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.
4. The parts should be protected from the corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the surface of the part increase its fatigue strength.



(a) (b) (c)

Fig. 1.11 Methods of reducing stress concentration of a press fit

Design for finite life

S-N Curve:

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction.

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. For the rotating-beam test, a constant bending load is applied, and the number of revolutions (stress reversals) of the beam required for failure is recorded. The first test is made at a stress that is somewhat of the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an S-N diagram (Fig. 1.12 a).

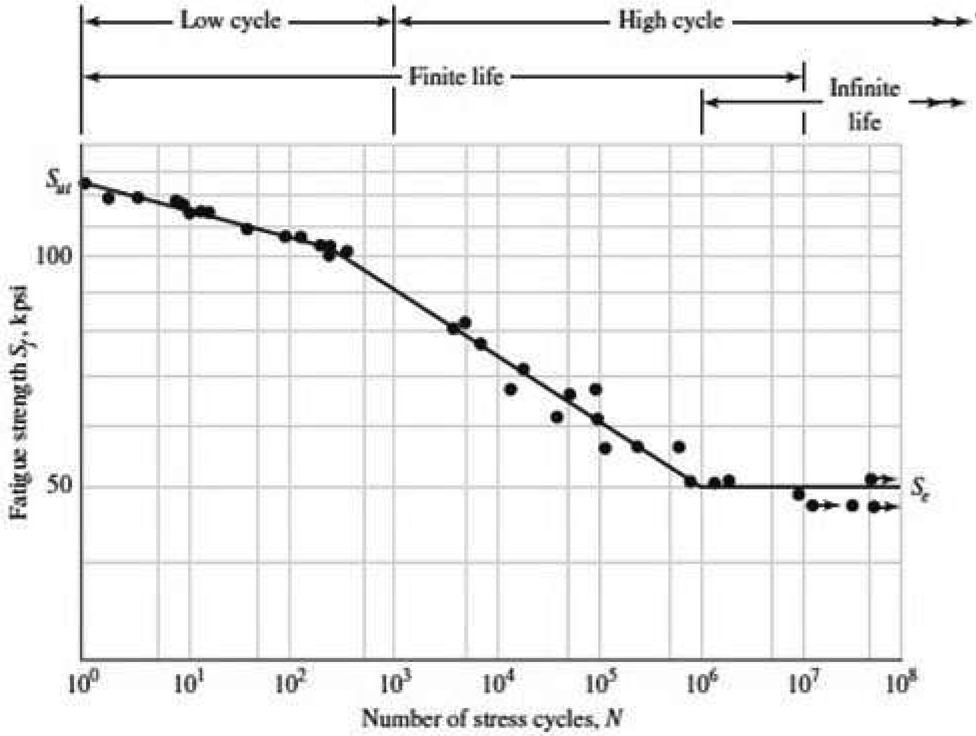


Fig. 1.12 **(a)** An S-N diagram plotted from the results of completely reversed axial fatigue tests

The ordinate of the S-N diagram is called the fatigue strength S_f; a statement of this strength value must always be accompanied by a statement of the number of cycles N to which it corresponds.

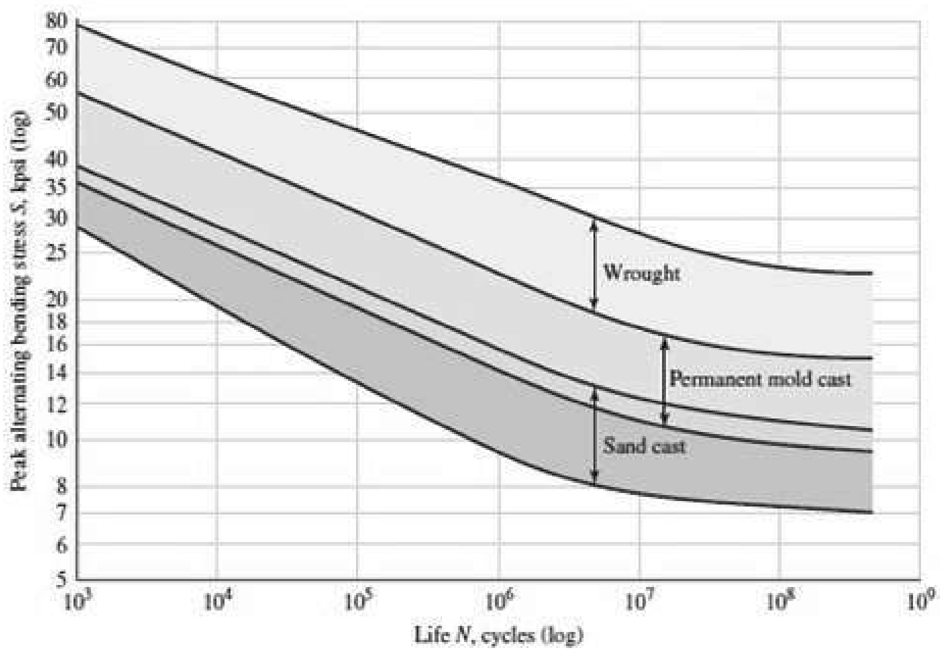


Fig. 1.12 **(b)** S-N bands for representative aluminum alloys, excluding wrought alloys

Soon we shall learn that S-N diagrams can be determined either for a test specimen or for an actual mechanical element. Even when the material of the test specimen and that of the mechanical element are identical, there will be significant differences between the diagrams for the two.

The S-N diagram is usually obtained by completely reversed stress cycles, in which the stress level alternates between equal magnitudes of tension and compression. We note that a stress cycle (N = 1) constitutes a single application and removal of a load and then another application and removal of the load in the opposite direction. Thus N = 1/2 means the load is applied once and then removed, which is the case with the simple tension test.

The body of knowledge available on fatigue failure from N = 1 to N = 1000 cycles is generally classified as low-cycle fatigue, as indicated in Fig. 1.12 (a). High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than 10^3 cycles.
We also distinguish a finite-life region and an infinite-life region in Fig. a. The boundary between these regions cannot be clearly defined except for a specific material; but it lies somewhere between 10^6 and 10^7 cycles for steels, as shown in Fig. 1.12 (a).
Because of this necessity for testing, it would really be unnecessary for us to proceed any further in the study of fatigue failure except for one important reason: the desire to know why fatigue failures occur so that the most effective method or methods can be used to improve fatigue strength.

Cumulative fatigue damage

In certain applications, the mechanical component is subjected to different stress levels for different parts of the work cycle. The life of such a component is determined by Miner's equation. Suppose that a component is subjected to completely reversed stresses (σ_1) for (n_1) cycles, (σ_2) for (n_2) cycles, and so on. Let N_1 be the number os stress cycles before fatigue failure if only the alternating stress (σ_1) is acting. One stress cycle will consume ($1/N_1$) of the fatigue life and since there are n1 such cycles at this stress level. The proportionate damage of fatigue life will be [($1/N_1$) n_1) or (n_1/N_1). Similarly, the proportionate damage at stress level (σ_2) will be (n_2/N_2). Adding these quantities, we get,

$$(n_1/ N_1) + (n_2/ N_2) + \ldots\ldots. + (n_x/ N_x) = 1$$

The above equation is known as Miner's equation. Sometimes, the number of cycles n_1, n_2,…. at stress levels σ_1, σ_2,… etc. Let N be the total life of the component. Then,

$$n_1 = \alpha_1 N$$

$$n_2 = \alpha_2 N$$

Substituting these values in Miner's equation,

$$(\alpha_1/ N_1) + (\alpha_2/ N_2) + \ldots.. + (\alpha_x/ N_x) = 1/N$$

Also,

$$\alpha_1 + \alpha_2 + \alpha_3 + \ldots.. + \alpha_x = 1$$

With the help of the above equations, the life of the component subjected to different stress levels can be determined.

Department of Mechanical Engineering

ME-602 Machine Component and Design

Unit – II
Shafts

Syllabus :

Design of shaft under combined bending, twisting and axial loading; shock and fatigue factors, design for rigidity; Design of shaft subjected to dynamic load; Design of keys and shaft couplings.

Subject Notes

Introduction: -

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

Material Used for Shafts

The material used for shafts should have the following properties:

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view:

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength

2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft May be obtained by using the torsion equation. We know that for round solid shaft, polar moment of Inertia

$$J = \pi/32 * d^4$$

$$T/J = r/r$$

From above two equations we can write

$$T = \frac{\pi}{16} * r * d^3$$

If shaft is hollow than

$$T = \frac{\pi}{16} * r * \frac{\{d_o^4 - d_i^4\}}{d_o}$$

where *T* = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

of Machine Design

r = Distance from neutral axis to the outer most fiber

= *d* / 2; where *d* is the diameter of the shaft.

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fiber

For solid shaft $$M = \frac{\pi}{32} * \sigma_b * d^3$$

For hollow shaft $$M = \frac{\pi}{32} * \sigma_b * \frac{\{d_o^4 - d_i^4\}}{d_o}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine’s theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b= Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$r_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4r^2}$$

$$r_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by *Te*. equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces
The same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max})
Equal to the allowable shear stress (τ) for the material, the equation may be written as

$$T_e = \frac{\pi}{16} * r * d^3$$

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$(\sigma_b)_{max} = \frac{1}{2}(\sigma_b) + \frac{1}{2}\sqrt{\sigma_b^2 + 4r^2}$$

$$(\sigma_b)_{max} = \frac{32}{\pi d^3}[\frac{1}{2}M + \frac{1}{2}\sqrt{M^2 + T^2}]$$

The expression $[\frac{1}{2}M + \frac{1}{2}\sqrt{M^2 + T^2}]$ is known as **equivalent bending moment** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress** (σ_b) **as the actual bending moment.** By limiting the maximum normal stress $(\sigma_b)_{max}$ equal to the allowable bending stress (σ_b) then the equation may be written as

$$M_e = \frac{\pi}{32} * \sigma_b * d^3$$

Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors Must be taken into account for the computed twisting moment (T) and bending moment (M). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment

$$T_e = \sqrt{(K_m M)^2 + (K_T T)^2}$$

K_m = Combined shock and fatigue factor for bending, and
K_t = Combined shock and fatigue factor for torsion.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per meter length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per meter length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

2. *Lateral rigidity.* It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam.

Types of Keys

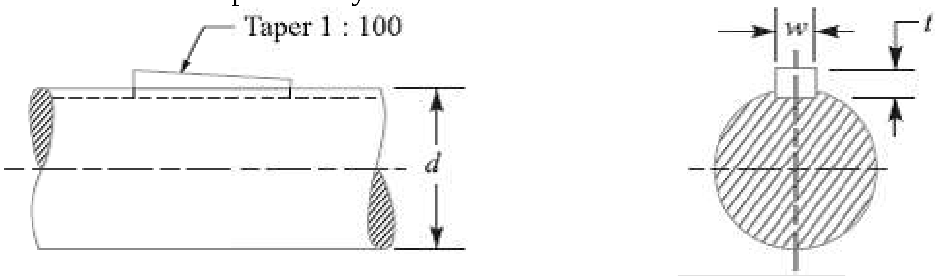
The following types of keys are important from the subject point of view:
1. Sunk keys,
2. Saddle keys,
3. Tangent keys,

4. Round keys, and
5. Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:

1. Rectangular sunk key. A rectangular sunk key is shown in Fig. The usual proportions of this key are:
Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$
Where d = Diameter of the shaft or diameter of the hole in the hub.
The key has taper 1 in 100 on the top side only.

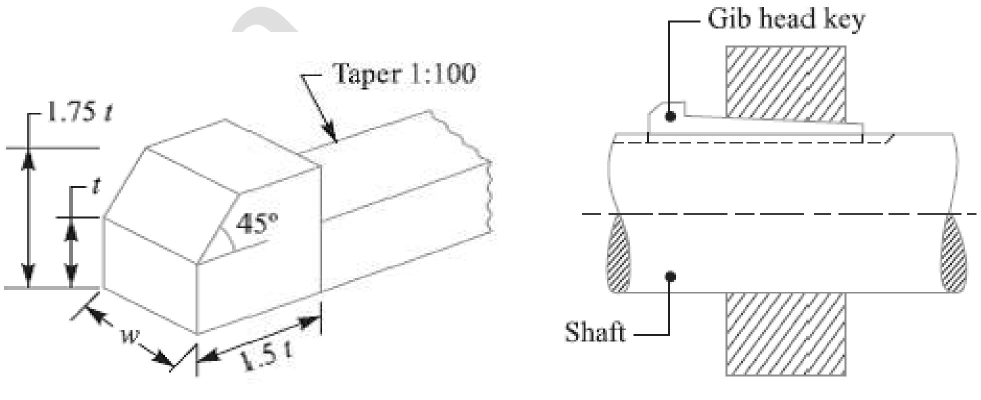


Rectangular sunk key

2. Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.
$w = t = d / 4$

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as **gib head.** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. (a) and its use in shown in Fig. (b).

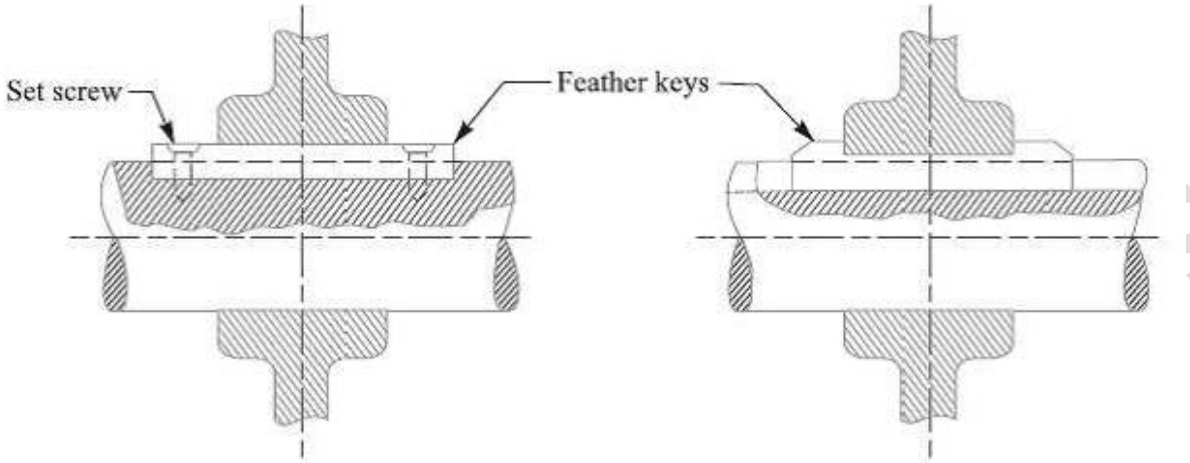


(a) (b)

Gib-head key

The usual proportions of the gib head key are:
Width, $w = d / 4$;
and thickness at large end, $t = 2w / 3 = d / 6$

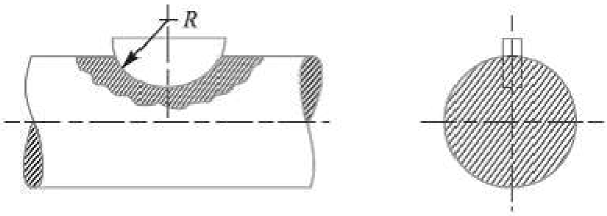
5. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



(a) b)

Feather Key

6. Woodruff key. The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



Woodruff key

The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

The disadvantages are:

1. The depth of the keyway weakens the shaft.

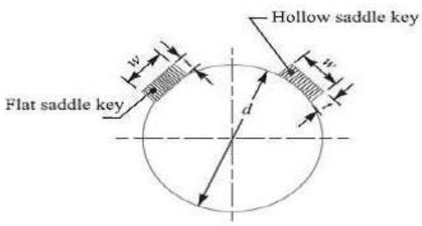
2. It cannot be used as a feather.

Saddle keys

The saddle keys are of the following two types:

1. Flat saddle key, and **2.** Hollow saddle key

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. It is likely to slip round the shaft under load. Therefore, it is used for comparatively light loads.



T = w/3 = d/12

Saddle key

A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

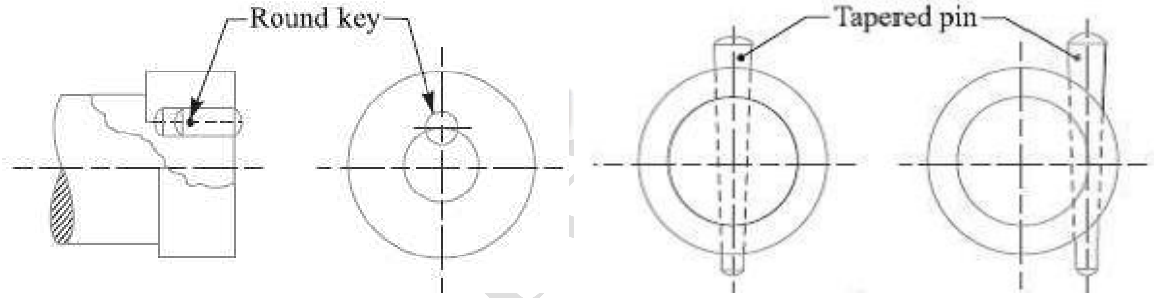
Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy-duty shafts.

Tangent Key

Round Keys

The round keys, as shown in Fig. (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives. Sometimes the tapered pin, as shown in Fig. (b) Is held in place by the friction between the pin and the reamed tapered holes.

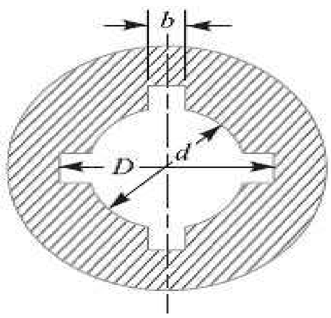


(a) (b)

Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Fig. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



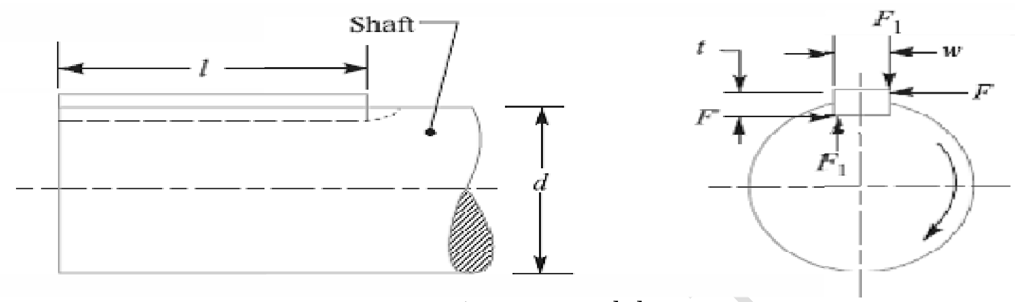
D = 1.25d and b = 0.25D

Splines

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.
2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.
The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.
The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig.
In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.



Forces acting on a sunk key

Strength of a Sunk Key
A key connecting the shaft and hub is shown in Fig.
Let T = Torque transmitted by the shaft,
F = Tangential force acting at the circumference of the shaft,
d = Diameter of shaft,
l = Length of key,
w = Width of key.
t = Thickness of key, and
τ and σ_c = Shear and crushing stresses for the material of key.
A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.
Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,
F = Area resisting shearing × Shear stress = $l \times w \times \tau$
∴ Torque transmitted by the shaft,
$T = F \times d/2 = l \times w \times \tau \times d/2$
Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,
F = Area resisting crushing × Crushing stress = $l \times (t/2)\, \sigma_c$
∴ Torque transmitted by the shaft,
$T = F \times d/2 = l \times (t/2)\, \sigma_c \times d/2$
The key is equally strong in shearing and crushing, if
$l \times w \times \tau \times d/2 = l \times (t/2)\, \sigma_c \times d/2$
$w/t = \sigma_c/2\tau$
The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore, from above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing.
In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.
We know that the shearing strength of key,
$T = l \times w \times \tau \times d/2$
and torsional shear strength of the shaft,
$T = \pi/16 \times \tau_1 \times d^3$
On equating above equations, we get

$l \times w \times \tau \times d/2 = \pi/16 \times \tau_1 \times d^3$

$l = \pi/8 \times (\tau_1 \times d^2)/w \times \tau = \pi d/2 \times (\tau_1/\tau) = 1.571d \times (\tau_1/\tau)$ ... (Taking $w = d/4$)

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$\therefore l = 1.571\ d$ ... [From above equation]

Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$e = 1 - 0.2\ (w/d) - 1.1\ (h/d)$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = Thickness of Key (t) / 2

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation:

$k_\theta = 1 + 0.4\ (w/d) + 0.7\ (h/d)$

where k_θ = Reduction factor for angular twist.

Shaft Coupling

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements:

1. It should be easy to connect or disconnect.

2. It should transmit the full power from one shaft to the other shaft without losses.

3. It should hold the shafts in perfect alignment.

4. It should reduce the transmission of shock loads from one shaft to another shaft.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

***Rigid coupling*.** It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(a) Sleeve or muff coupling.

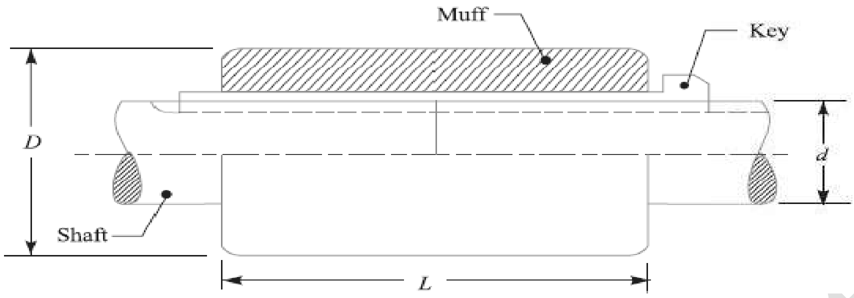
(b) Clamp or split-muff or compression coupling, and

(c) Flange coupling.

Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head

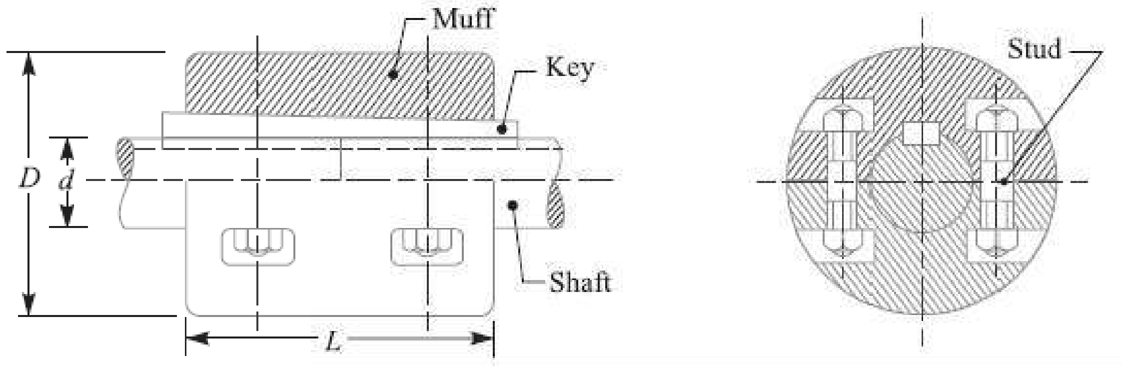
key, as shown in Fig. 13.10. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows:
Outer diameter of the sleeve, $D = 2d + 13$ mm
And length of the sleeve, $L = 3.5\, d$



Sleeve and muff Coupling

Clamp or Compression Coupling

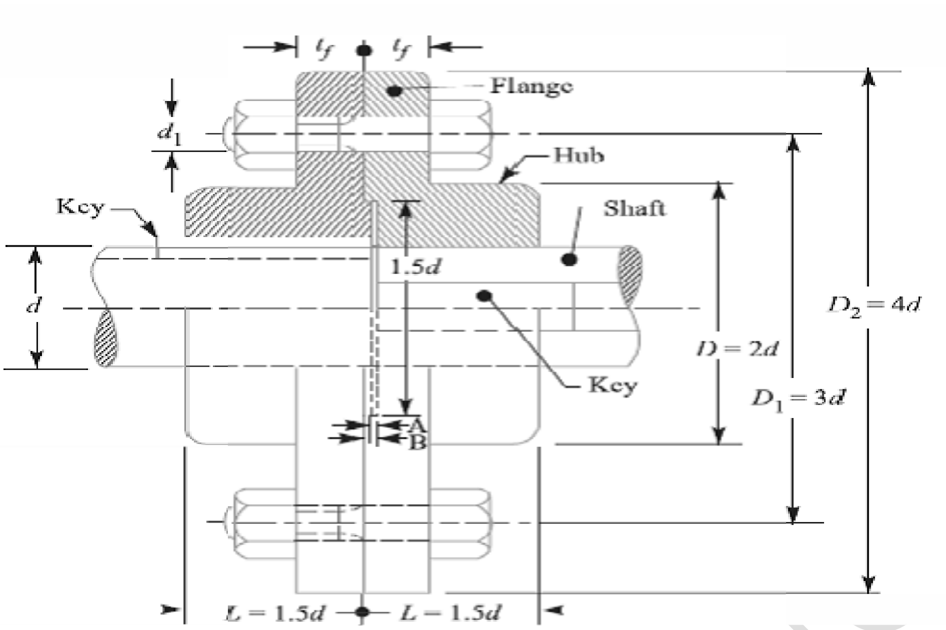
It is also known as ***split muff coupling*.** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupl ng are:
Diameter of the muff or sleeve, $D = 2d + 13$ mm
Length of the muff or sleeve, $L = 3.5\, d$
where d = Diameter of the shaft.



Clamp or Compression Coupling

Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angl to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess This helps to bring the shafts into line and to maintain alignment.



Flange Coupling

The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting.

keys

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view:

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys, and
5. Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:

1. Rectangular sunk key. A rectangular sunk key is shown in Fig. The usual proportions of this key are:
Width of key, w = d / 4; and thickness of key, t = 2w / 3 = d / 6
Where d = Diameter of the shaft or diameter of the hole in the hub.
The key has taper 1 in 100 on the top side only.

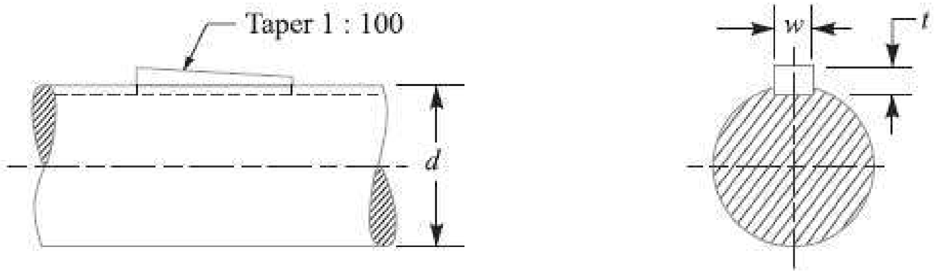
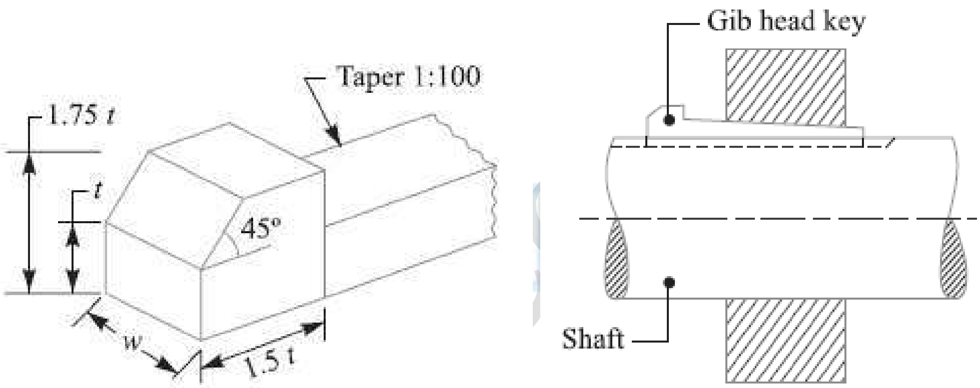


Fig. 2.20 Rectangular sunk key

2. Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.
w = t = d / 4

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as **gib head.** It is usually provided to facilitate the removal of the key. A gib head key is shown in Fig. 2.21 (a) and its use is shown in Fig. 2.21 (b).

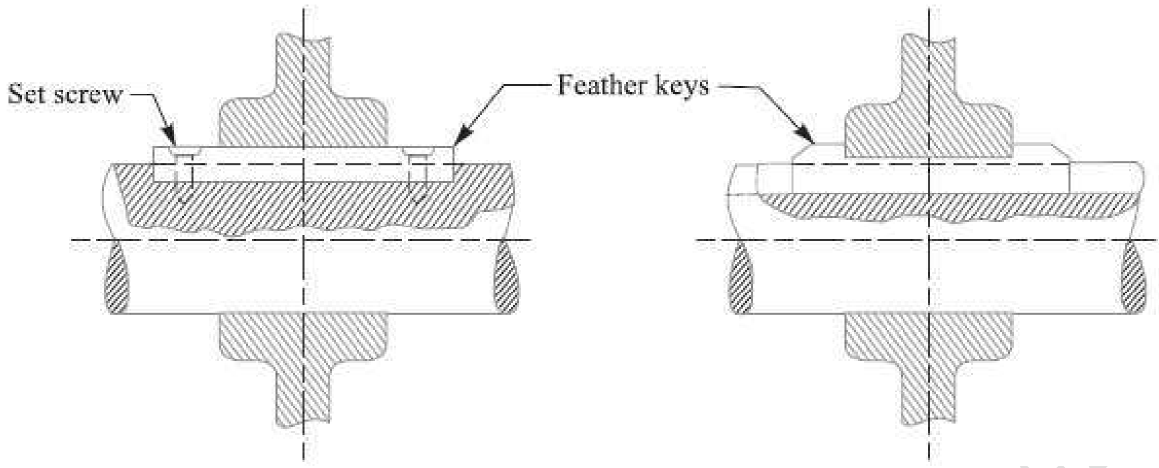


(a) (b)

Fig. 2.21 Gib-head key

The usual proportions of the gib head key are:
Width, w = d / 4;
and thickness at large end, t = 2w / 3 = d / 6

5. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key is a sliding fit in the keyway of the moving piece.



(a) (b)

Fig. 2.22 Feather Key

The feather key may be screwed to the shaft as shown in Fig. 2.22 (a) or it may have double gib heads as shown in Fig. 2.22 (b). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

The following table shows the proportions of standard parallel, tapered and gib head keys, according to IS: 2292 and 2293-1974 (Reaffirmed 1992).

Proportions of standard parallel tapered and gib head keys

aft diameter (mm) up to including	Key cross-section		aft diameter (mm) up to including	Key cross-section	
	Width (mm)	Thickness (mm)		Width (mm)	Thickness (mm)
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

6. Woodruff key. The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having a segmental cross-section in front view as shown in Fig. 2.23. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

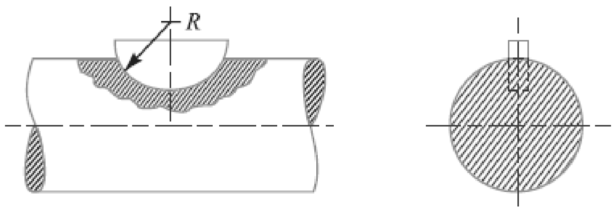
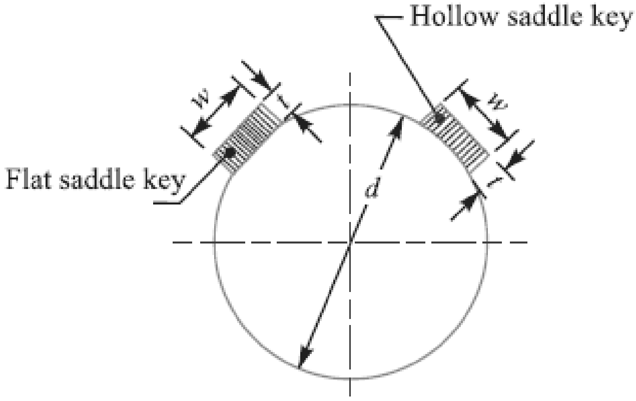


Fig. 2.23 Woodruff key

The main advantages of a woodruff key are as follows:
1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.
The disadvantages are:
1. The depth of the keyway weakens the shaft.
2. It cannot be used as a feather.

Saddle keys
The saddle keys are of the following two types:
1. Flat saddle key, and **2.** Hollow saddle key

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. 2.24. It is likely to slip around the shaft under load. Therefore it is used for comparatively light loads.



T = w/3 = d/12
Fig. 2.24 Saddle key

A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys
The tangent keys are fitted in the pair at right angles as shown in Fig. 2.25. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

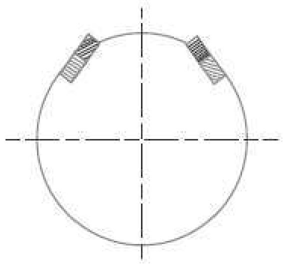
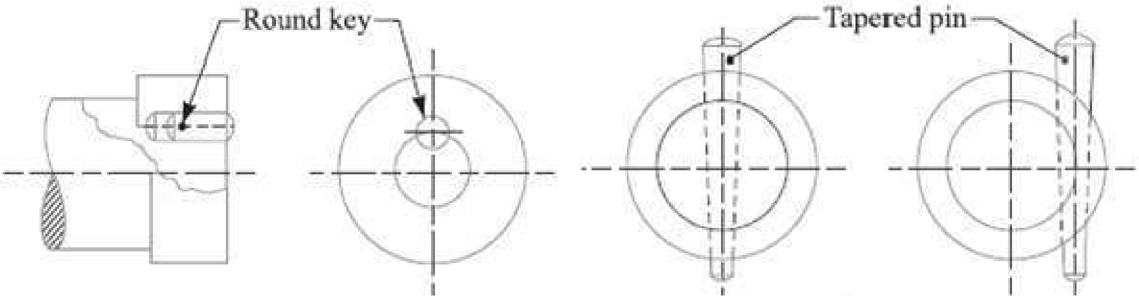


Fig. 2.25 Tangent Key

Round Keys
The round keys, as shown in Fig. 2.26 (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives. Sometimes the tapered pin, as shown in Fig. 2.26 (b) is held in place by the friction between the pin and the reamed tapered holes.



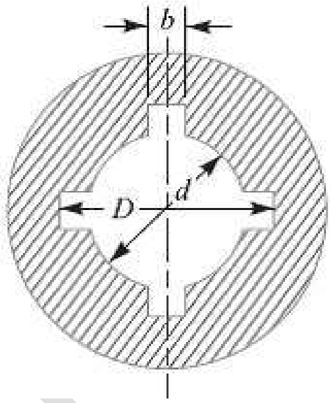
(a) (b)

Fig. 2.26 Round keys

Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Fig. 2.27 These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement, as well as positive drive, is obtained.



D = 1.25d and b = 0.25D

Fig. 2.27 Splines

Forces acting on a Sunk Key

When a key is used for transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1. Forces (F_1) due to fit of the key in its keyway, as in a tight-fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub. The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 2.28.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of the key is uniform.

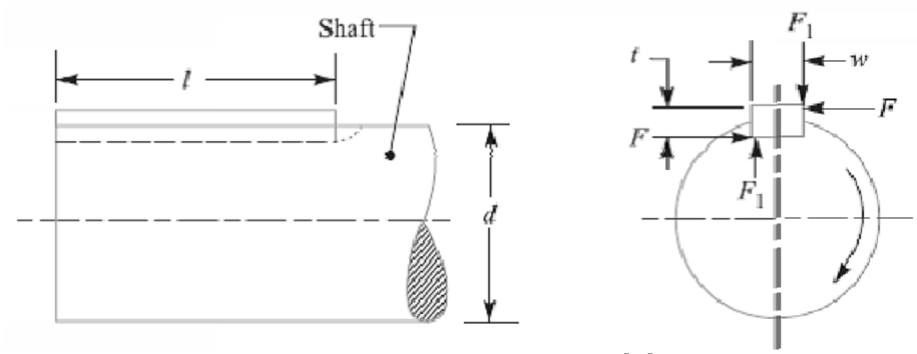


Fig. 2.28 Forces acting on a sunk key

Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 2.28

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of the shaft,

l = Length of the key,

w = Width of the key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = $l \times w \times \tau$

∴ Torque transmitted by the shaft,

$T = F \times d/2 = l \times w \times \tau \times d/2$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress = $l \times (t/2)\, \sigma_c$

∴ Torque transmitted by the shaft,

$T = F \times d/2 = l \times (t/2)\, \sigma_c \times d/2$

The key is equally strong in shearing and crushing, if

$l \times w \times \tau \times d/2 = l \times (t/2)\, \sigma_c \times d/2$

$w/t = \sigma_c/2\tau$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$T = l \times w \times \tau \times d/2$

And torsional shear strength of the shaft,

$T = \pi/16 \times \tau_1 \times d^3$

On equating above equations, we get

$l \times w \times \tau \times d/2 = \pi/16 \times \tau_1 \times d^3$

$l = \pi/8 \times (\tau_1 \times d^2)/w \times \tau = \pi\, d/2 \times (\tau_1/\tau) = 1.571d \times (\tau_1/\tau)$ ... (Taking w = d/4)

When the key material is same as that of the shaft, then $\tau = \tau_1$.

∴ $l = 1.571\, d$ ... [From above equation]

Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. In other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$e = 1 – 0.2 (w/d) – 1.1 (h/d)$

Where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = Thickness of Key (t) / 2

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation:

$k_\theta = 1 + 0.4 (w/d) + 0.7 (h/d)$

Where k_θ = Reduction factor for an angular twist.

Machine Component and Design (ME -602)
Subject Notes
Unit-III

Syllabus : Design of helical compression and tension springs, consideration of dimensional and functional constraints, leaf springs and torsion springs; fatigue loading of springs, surge in spring; special springs, Power Screws: design of power screw and power nut, differential and compound screw, design of simple screw jack.

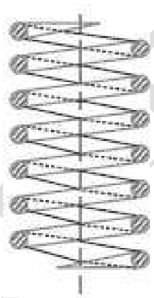
Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:
1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, aircraft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring-loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

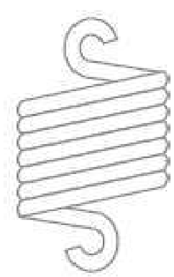
Types of Spring

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are **compression helical spring** as shown in Fig. 4.1 (a), and **tension helical spring** as shown in Fig. 4.1 (b).



(a) Compression Helical Spring



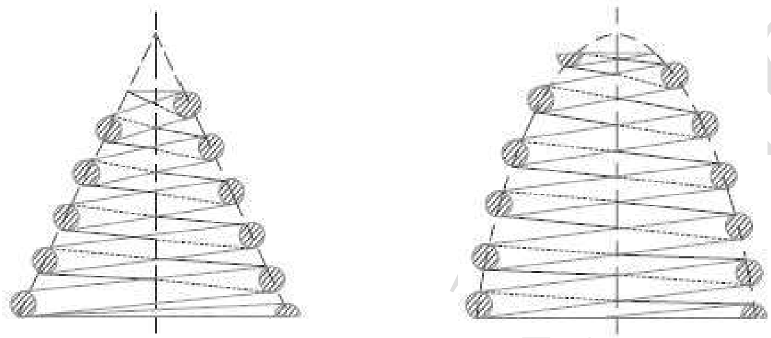
(b) Tension Helical Spring

Fig. 4.1 Helical springs

The helical springs are said to be **closely coiled** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small; it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.
In **open coiled helical springs,** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs is limited, therefore our discussion shall confine to closely coiled helical springs only.
The helical springs have the following advantages:
(a) These are easy to manufacture.

(b) These are available in wide range.
(c) These are reliable.
(d) These have constant spring rate.
(e) Their performance can be predicted more accurately.
(f) Their characteristics can be varied by changing dimensions.

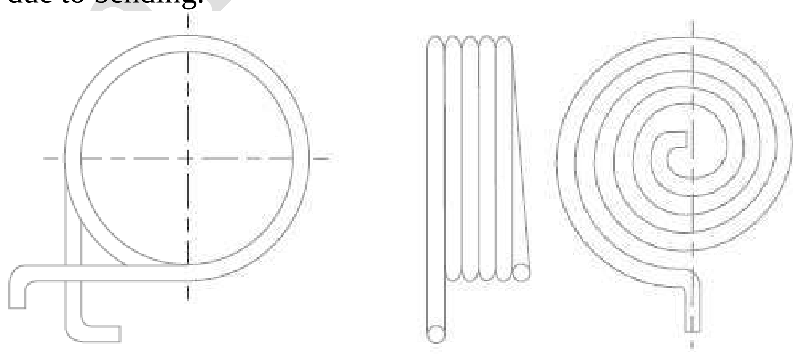
2. Conical and volute springs. The conical and volute springs, as shown in Fig. 4.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 4.2 (a) is wound with a uniform pitch whereas the volute springs, as shown in Fig. 4.2 (b) are wound in the form of a paraboloid with constant pitch and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilized in vibration problems where springs are used to support a body that has a varying mass.



(a) Conical Spring (b) Volute Spring
Fig. 4.2 Conical and Volute Spring

The major stresses produced in conical and volute springs are also shear stresses due to twisting.

3. Torsion springs. These springs may be of the **helical** or **spiral** type as shown in Fig. 4.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of the flat strip are used in watches and clocks. The major stresses produced in torsion springs are tensile and compressive due to bending.



(a) Helical torsion spring (b) Spiral torsion spring.
Fig. 4.3 Torsion springs

4. Laminated or leaf springs. The laminated or leaf spring (also known as **flat spring** or **carriage spring**) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. 4.4 These are mostly used in automobiles. The major stresses produced in leaf springs are tensile and compressive stresses.

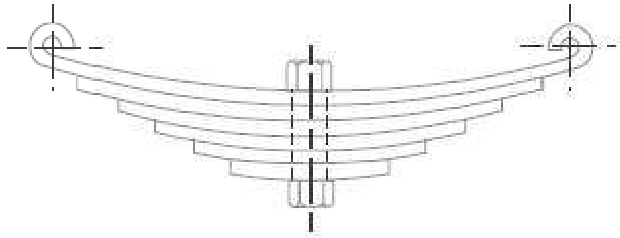


Fig. 4.4 Laminated or leaf springs

5. Disc or Belleville springs. These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 4.5. These springs are used in applications where high spring rates and compact spring units are required. The major stresses produced in disc or Belleville springs are tensile and compressive stresses.

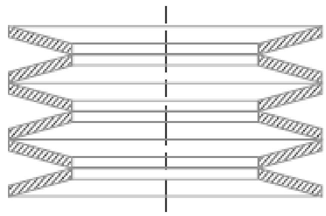


Fig. 4.5 Disc or Belleville springs

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends on the service for which they are used i.e. severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with the only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 percent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance, and corrosion resistance.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size; smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.** The solid length of a spring is the product of a total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'. d$
Where n' = a total number of coils, and
d = Diameter of the wire.

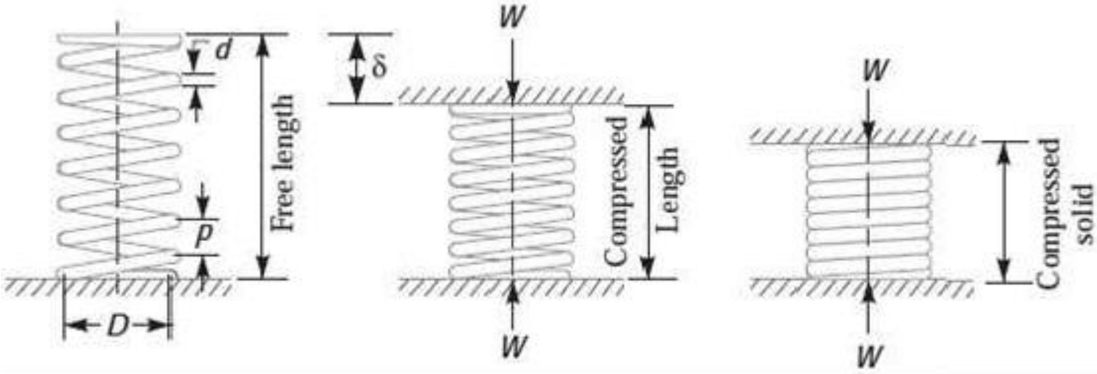


Fig. 4.6 Compression spring nomenclature

2. Free length. The free length of a compression spring, as shown in Fig. 4.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).
Mathematically,
Free length of the spring,
L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)
$= n'. d + \delta_{max} + 0.15\ \delta_{max}$
The following relation may also be used to find the free length of the spring, i.e.
$L_F = n'. d + \delta_{max} + (n' - 1) \times 1$ mm
In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,
Spring index, $C = D / d$
Where D = Mean diameter of the coil, and
d = Diameter of the wire.

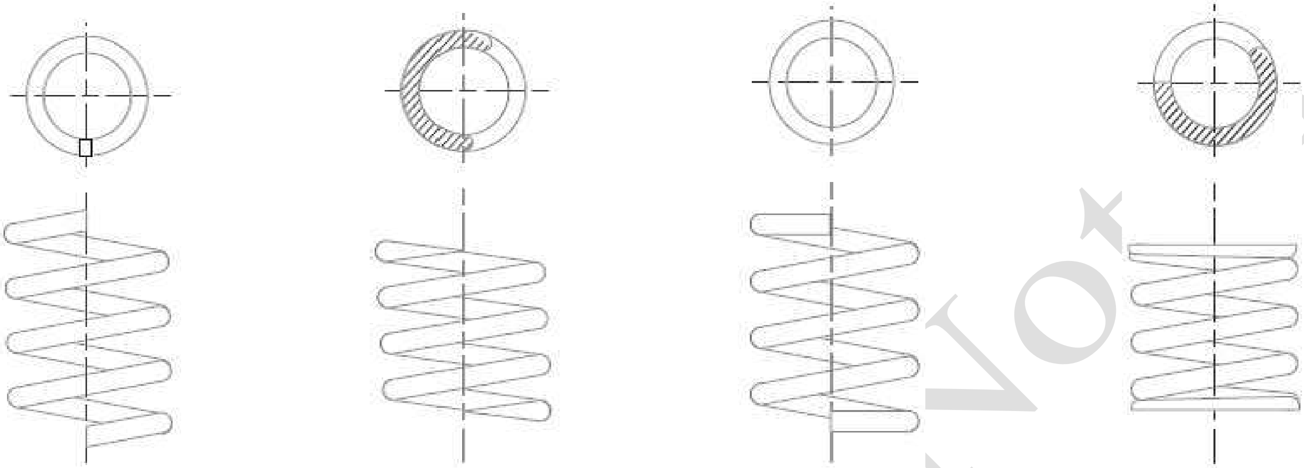
4. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,
Spring rate, $k = W / \delta$
Where W = Load, and
δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in the uncompressed state. Mathematically,
Pitch of the coil, p = Free length / $(n' - 1)$
The pitch of the coil may also be obtained by using the following relation, i.e.
Pitch of the coil, $p = (L_F - L_S) / n' + d$
Where L_F = Free length of the spring,
L_S = Solid length of the spring,
n' = Total number of coils, and
d = Diameter of the wire.
In choosing the pitch of the coils, the following points should be noted:
(a) The pitch of the coils should be such that if the spring is accidentally or carelessly compressed, the stress does not increase the yield point stress in torsion.

(b) The spring should not close up before the maximum service load is reached.

End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig. 4.7.



(a) Plain Ends (b) Ground Ends (c) Squared Ends (d) Squared and Ground Ends

Fig. 4.7 End connections for compression helical spring

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as **inactive coils**. The turns which impart spring action are known as **active turns.** As the load increases, the number of inactive coils also increases due to the seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.

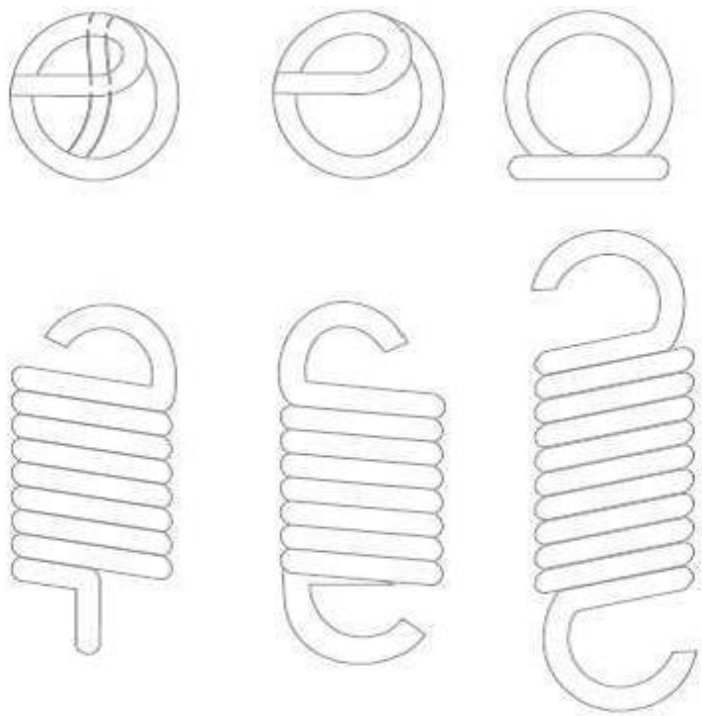
Total number of turns, solid length and free length for different types of end connections

S. No	Type of end	Total number of turns (n')	Solid length	Free length
01.	Plain Ends	n	(n + 1) d	p x n +d
02.	Ground End	n	n x d	p x n
03.	Squared Ends	n + 2	(n + 3) d	p x n + 3d
04.	Squared and Ground Ends	n + 2	(n + 2) d	p x n + 2d

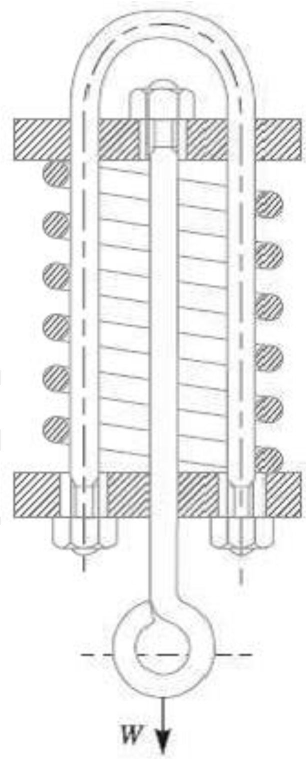
Where n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

End Connections for Tension Helical Springs

The tensile springs are provided with hooks or loops as shown in Fig. 4.8. These loops may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.
The main disadvantage of tension spring is the failure of the spring when the wire breaks. A compression spring used for carrying a tensile load is shown in Fig. 4.9.



End connection for tension helical springs
Fig. 4.8

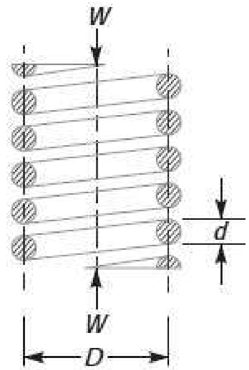


Compression spring for carrying tensile load
Fig. 4.9

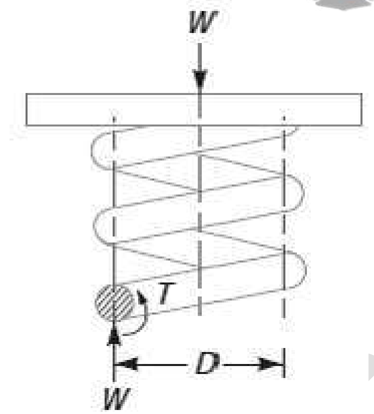
Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 4.10 (a).

Let D = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ = Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ = Deflection of the spring, as a result of an axial load W.



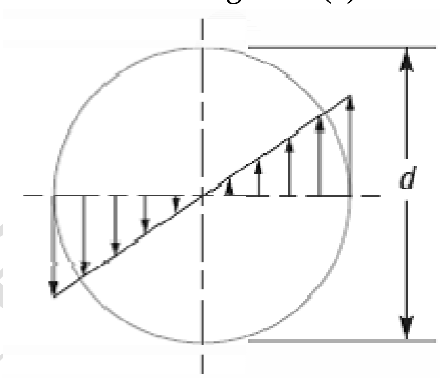
(a) Axially loaded helical spring



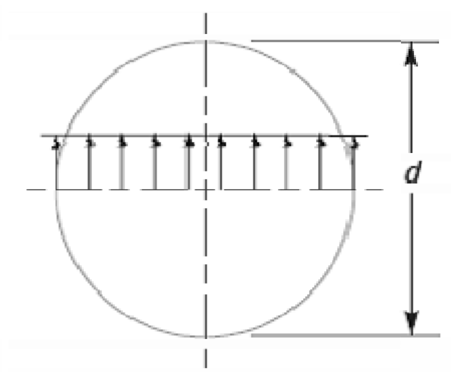
(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear

Fig. 4.10

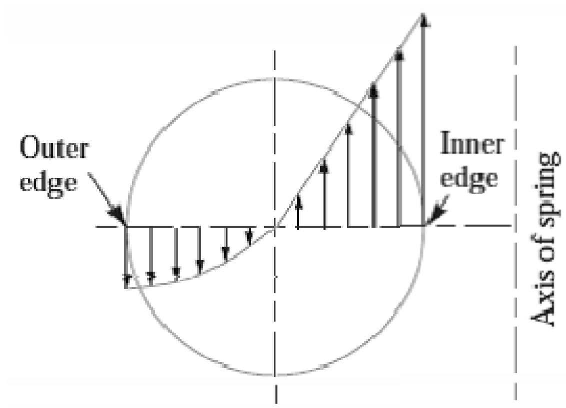
Now consider a part of the compression spring as shown in Fig. 4.10 (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.
A little consideration will show that part of the spring, as shown in Fig. 4.10 (b), is in equilibrium with the action of two forces W and the twisting moment T. We know that the twisting moment,
$T = W \times D/2 = \pi/16 \times \tau_1 \times d^3$
$\therefore \tau_1 = 8W.D / \pi d^3$
The torsional shear stress diagram is shown in Fig. (a).
In addition to the torsional shear stress ($\tau1$) induced in the wire, the following stresses also act on the wire:
1. Direct shear stress due to the load W, and
2. Stress due to the curvature of wire.
We know that direct shear stress due to the load W,
τ_2 = Load / Cross-sectional area of the wire
$\tau_2 = W / (\pi /4 \times d^2) = 4W /\pi. d^2$
The direct shear stress diagram is shown in Fig. 4.11 (b) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 4.11 (c).



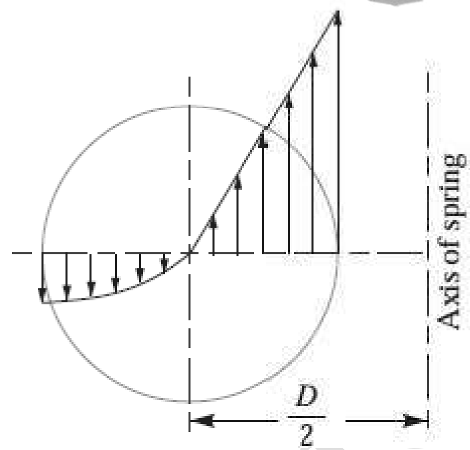
(a) Torsional shear stress diagram



(b) Direct shear stress diagram



(c) Resultant torsional shear and direct shear stress diagram



(d) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 4.11 Superposition of stresses in a helical spring

We know that the resultant shear stress induced in the wire,

$\tau = \tau_1 \pm \tau_2 = 8W.D / \pi\ d^3 \pm 4W / \pi.\ d^2$

The **positive** sign is used for the inner edge of the wire and **negative** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$= 8W.D / \pi\ d^3 + 4W / \pi.\ d^2 = 8W.D / \pi\ d^3\ (1 + d/2D)$

$= 8W.D / \pi\ d^3\ (1 + 1/2C)$ ... (Substituting D/d = C)

$= K_S \times 8W.D / \pi\ d^3$

Where K_S = Shear stress factor = (1 + 1/2C)

From the above equation, it can be observed that the effect of direct shear [$8W.D / \pi\ d^3 \times 1/2C$] is appreciable for springs of small spring index C. Also, we have neglected the effect of wire curvature in above equation. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear, and curvature shear stress is shown in Fig. 23.11 (d).

∴ Maximum shear stress induced in the wire,

$\tau = K \times 8W.D / \pi\ d^3 = K \times 8W.C / \pi\ d^2$

Where $K = (4C - 1) / (4C - 4) - 0.615/C$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 4.12

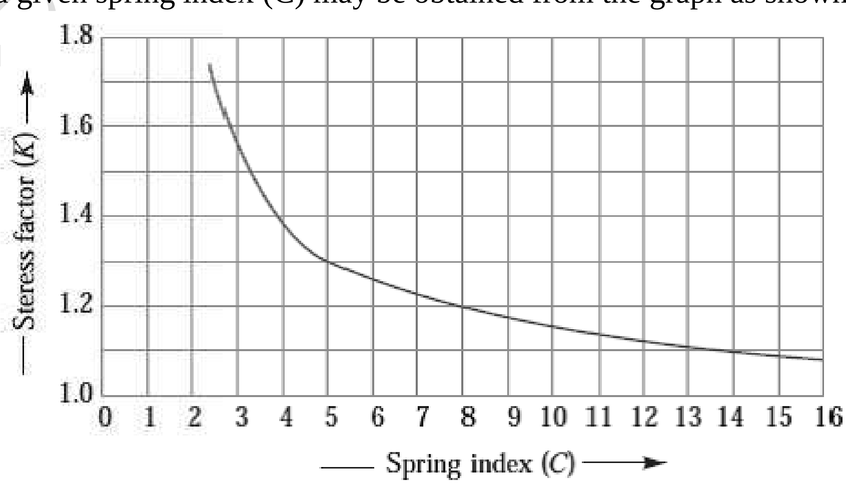


Fig. 4.12 Wahl's stress factor for helical springs

We see from Fig. 4.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that
Total active length of the wire,
l = Length of one coil × No. of active coils = $\pi D \times n$
Let θ = Angular deflection of the wire when acted upon by the torque T.
∴ Axial deflection of the spring,
$\delta = \theta \times D/2$
We also know that
$T/J = \tau/D/2 = G.\ \theta/l$
Now considering
$T/J = G.\ \theta/l$
$\theta = T.\ l / G.\ J$
Where J = Polar moment of inertia of the spring wire
$= \pi/32 \times d^4$, d being the diameter of spring wire.
And G = Modulus of rigidity for the material of the spring wire.
Now substituting the values of l and J in the above equation, we have
$\theta = T.\ l / G.\ J = (W \text{ x } D/2)\ \pi.\ D.\ n / \pi/32 \text{ x } d^4 \text{ x } G = 16\ W.\ D^2.\ n / G.\ d^4$
Substituting this value of θ in the equation of deflection, we have
$\delta = 16\ W.\ D^2.\ n / G.\ d^4 \times D/2 = 8\ W.\ D^3\ .\ n / G.\ d^4 = 8\ W.\ C^3\ .\ n / G.\ d$ $(C = D/d)$
And the stiffness of the spring or spring rate,
$W / \delta = G.\ d^4 / 8\ D^3.\ n = G.\ d / 8\ C^3.\ n$

Eccentric Loading of Spring

Sometimes, the load on the springs does not coincide with the axis of the spring, i.e. the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor D/(2e + D), where D is the mean diameter of the spring.

Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig. 4.13. The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, i.e.
$W_{cr} = k \times K_B \times L_F$
Where k = Spring rate or stiffness of the spring = W/δ,
L_F = Free length of the spring, and
K_B = Buckling factor depending upon the ratio L_F/D.
The buckling factor (KB) for the hinged end and built-in end springs may be taken from the following table.

Fixed End Guided End

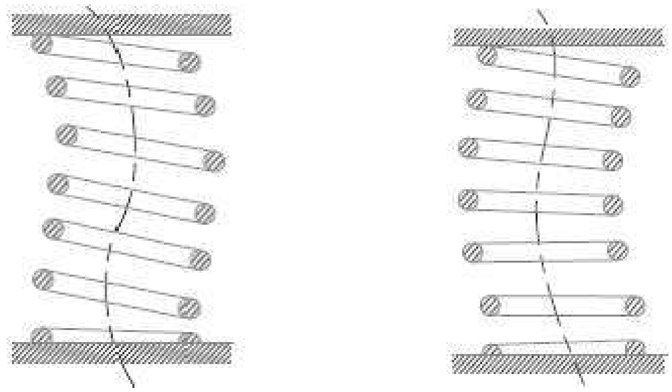


Fig. 4.13 Buckling of compression springs

Values of buckling factor (K_B)

LF/D	Hinged end spring	Built-in end spring	LF / D	Hinged end spring	Built-in end spring
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a **hinged end spring** is one which is supported on pivots at both ends as in case of springs having plain ends whereas a **built-in end spring** is one in which a squared and ground end spring is compressed between two rigid and parallel flat plates.

In order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for an increase in spring diameter during compression.

Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up the whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called a **surge.**

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order.

The natural frequency for springs clamped between two plates is given by

$f_n = d \sqrt{(6G.\ g/\rho)} / 2\pi D^2.\ n$ cycles/sec

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.
The surge in springs may be eliminated by using the following methods:
1. By using friction dampers on the center coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having a pitch of the coils near the ends different than at the center to have different natural frequencies.

Energy Stored in Helical Springs of Circular Wire
We know that the springs are used for storing energy which is equal to the work done on it by some external load.
Let W = Load applied in the spring, and
δ = Deflection produced in the spring due to the load W.
Assuming that the load is applied gradually, the energy stored in the spring is,
$U = \frac{1}{2} W \delta$
We have already discussed that the maximum shear stress induced in the spring wire,
$\tau = K \times 8W.D / \pi d^3$
$W = \tau \pi d^3 / K \times 8D$
We know that deflection of the spring,
$\delta = 8\ W.\ D^3.\ n / G.\ d^4 = 8\ \tau\ \pi\ d^3 \times D^3 .\ n / K \times 8D \times G.\ d^4 = \pi\ \tau \times D^2 .\ n / K.\ d.\ G$
Substituting the values of W and δ in the above equation of U, we have
$U = \frac{1}{2} \times (\tau \pi d^3 / K \times 8D) \times (\pi \tau \times D^2 .\ n / K.\ d.\ G)$
$U = \tau^2/4K^2G \times (\pi D n) \times (\pi/4 \times d^2)$
$U = \tau^2/4K^2G \times V$
Where V = Volume of the spring wire
= Length of spring wire × Cross-sectional area of spring wire
$= (\pi D n) \times (\pi/4 \times d^2)$

Stress and Deflection in Helical Springs of Non-circular Wire
The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However, these springs have the following main disadvantages:
1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming a helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favorable as for circular wires.
But this effect is negligible where loading is of static nature.
For springs made of rectangular wire, as shown in Fig. 4.14, the maximum shear stress is given by
$\tau = K \times W.\ D\ (1.5t + 0.9b) / b^2 .\ t^2$

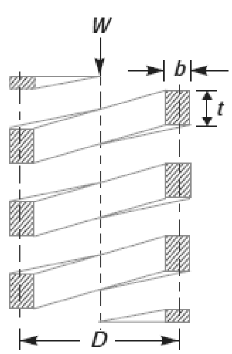


Fig. 4.14 Spring of rectangular wire

This expression is applicable when the longer side (i.e. t > b) is parallel to the axis of the spring. But when the shorter side (i.e. t < b) is parallel to the axis of the spring, the maximum shear stress,

$\tau = K \times W.\ D\ (1.5b + 0.9t) / b^2.\ t^2$

And deflection of the spring,

$\delta = 2.45\ W.\ D^3.\ n / G.\ b^3\ (t - 0.56b)$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$\tau = K\ 2.4W.\ D / b^3$

And deflection of the spring,

$\delta = 5.568\ W.\ D^3.\ n / G.\ b^4 = 5.568\ W.\ C^3.\ n / G.\ b$ (C = D/b)

Where b = Side of the square

Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 4.15.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to $\tau_e / 2$ and the variable shear stress is also equal to $\tau_e / 2$. A line drawn from A to B (the yield point in shear, τ_y) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength (τ_y), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below:

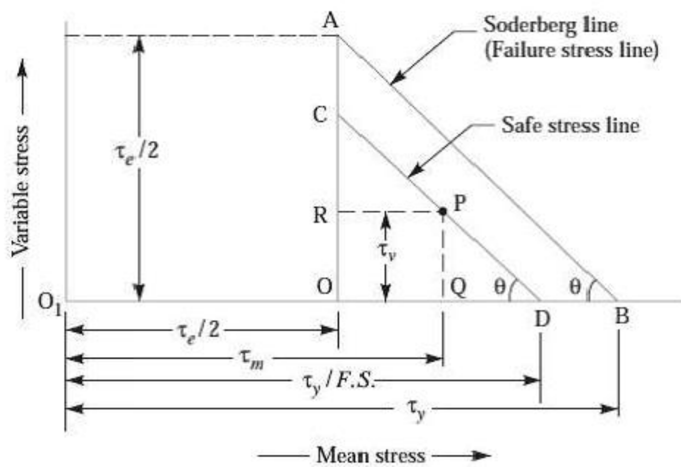


Fig. 4.15 Modified Soderberg method for helical springs

From similar triangles PQD and AOB, we have

PQ/QD = OA/OB

$PQ / (O_1D - O_1Q) = OA / (O_1B - O_1O)$

$\tau_V / (\tau_Y/F.S. - \tau_m) = \tau_e/2 / (\tau_Y - \tau_e/2) = \tau_e / (2\tau_Y - \tau_e)$

$2\ \tau_v\ .\ \tau_y - \tau_v.\ \tau_e = \tau_e.\ \tau_Y/F.S. - \tau_m.\ \tau_e$

$\tau_e.\ \tau_Y/F.S. = 2\ \tau_v\ .\ \tau_y - \tau_v.\ \tau_e + \tau_m.\ \tau_e$

Dividing both sides by $\tau_e.\ \tau_y$ and rearranging, we have

$1 / F.S. = (\tau_m - \tau_v) / \tau_y + 2\tau_v / \tau_e$

Springs in Series

Consider two springs connected in series as shown in Fig. 4.16.

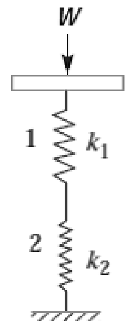


Fig. 4.16 Springs in series

Let W = Load carried by the springs,
δ_1 = Deflection of spring 1,
δ_2 = Deflection of spring 2,
k_1 = Stiffness of spring 1 = W / δ_1, and
k_2 = Stiffness of spring 2 = W / δ_2
A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.
∴ Total deflection of the springs,
$\delta = \delta_1 + \delta_2$
$W / k = W / k_2 + W / k_2$
$1 / k = 1 / k_2 + 1 / k_2$
Where k = Combined stiffness of the springs.

Springs in Parallel
Consider two springs connected in parallel as shown in Fig. 4.17

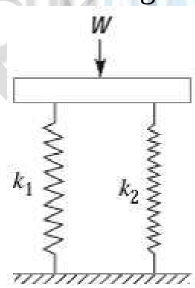


Fig. 4.17 Springs in parallel

Let W = Load carried by the springs,
W_1 = Load shared by spring 1,
W_2 = Load shared by spring 2,
k_1 = Stiffness of spring 1, and
k_2 = Stiffness of spring 2.
A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.
We know that $W = W_1 + W_2$
Or $\delta.\ k = \delta.k_1 + \delta.k_2$
∴ $k = k_1 + k_2$
Where k = Combined stiffness of the springs, and
δ = Deflection produced

Concentric or Composite Springs
A concentric or composite spring is used for one of the following purposes:
1. To obtain greater spring force within a given space.

2. To ensure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 4.18 (a) and are compressed equally. Such springs are used in automobile clutches; valve springs in aircraft, heavy-duty diesel engines, and rail-road car suspension systems.

Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 4.18 (b). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (K), it is desirable to have the same spring index (C).

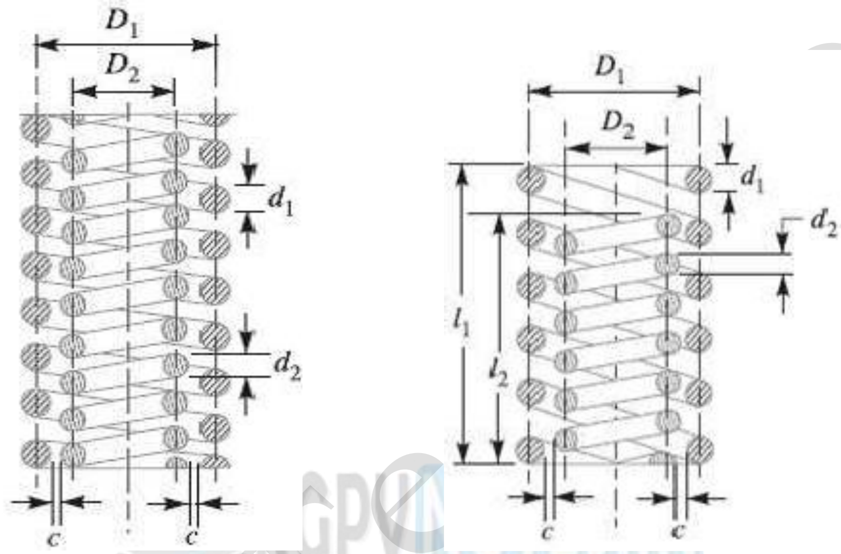


Fig. 4.18 Concentric springs

Consider a concentric spring as shown in Fig. (a).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by innerspring,

d_1 = Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of innerspring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of innerspring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of innerspring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of innerspring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, i.e.

$\tau_1 = \tau_2$

$8W_1. D_1. K_1 / \pi d_1^3 = 8W_2. D_2. K_2 / \pi d_2^3$

When stress factor, $K_1 = K_2$, then

$W_1. D_1 / d_1^3 = W_2. D_2 / d_2^3$

If both the springs are effective throughout their working range, then their free length and deflection are equal, i.e.

$\delta_1 = \delta_2$

$8W_1. D_1^3. n_1 / G. d_1^4 = 8W_2. D_2^3. n_2 / G. d_2^4$
$W_1. D_1^3. n_1 / d_1^4 = W_2. D_2^3. n_2 / d_2^4$
When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, i.e. $n_1.d_1 = n_2.d_2$
∴ above equation of deflection may be written as
$W_1. D_1^3 / d_1^5 = W_2. D_2^3 / d_2^5$
Now dividing above equation by equation of shear stress, we have
$D_1^2 / d_1^2 = D_2^2 / d_2^2$
$D_1 / d_1 = D_2 /d_2 = C$, Spring Index
i.e. the springs should be designed in such a way that the spring index for both the springs is same.
From equation of shear stress and above equation, we have
$W_1 / d_1^2 = W_2/ d_2^2$
$W_1 / W_2 = d_1 / d_2$
From Fig. 4.18 (a), we find that the radial clearance between the two springs,
$c = (D_1/2 – D_2/2) – (d_1/2+d_2/2)$
Usually, the radial clearance between the two springs is taken as $(d_1 – d_2)/2$
$(D_1/2 – D_2/2) – (d_1/2+d_2/2) = (d_1 – d_2)/2$
$(D_1 – D_2)/2 = d_1$
From above equation, we find that
$D_1 = C.d_1$, and $D_2 = C.d_2$
Substituting the values of D_1 and D_2 in above equation, we have
$(C.d_1 - C.d_2) / 2 = d_1$
$C.d_1 – 2d_1 = C.d_2$
$d_1 (C – 2) = C.d_2$
$d_1 / d_2 = C / (C – 2)$

Helical Torsion Springs
The helical torsion springs as shown in Fig. 4.19, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas, in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as indoor hinges, brush holders in electric motors, automobile starters etc.

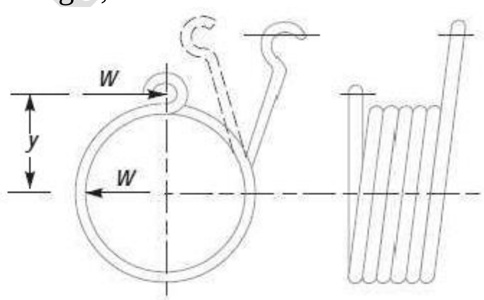


Fig. 4.19 Helical Torsion Spring
A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is
$\sigma_b = K \times 32 M / \pi. d^3 = K \times 32 W. y / \pi. d^3$
Where K = Wahl's stress factor = $4C^2 – C – 1 / 4C^2 - 4C$
C = Spring index,
M = Bending moment = W × y,
W = Load acting on the spring,
y = Distance of load from the spring axis, and
d = Diameter of spring wire.

And total angle of twist or angular deflection,
$\theta = M. l / E. I = M. \pi. D. n/ E. (\pi d^4/64)$
$\theta = 64 M. D. n / E. d^4$
Where l = Length of the wire = π.D.n,
E = Young's modulus,
I = Moment of inertia = $(\pi / 64) \times d^4$
D = Diameter of the spring, and
n = Number of turns.
And deflection,
$\delta = \theta \times y = (64 M. D. n / E. d^4) \times y$
When the spring is made of rectangular wire having width b and thickness t, then
$\sigma_b = K \times 6 M / t. b^2 = K \times 6 W. y / t. b^2$
Where $K = 3C^2 – C – 0.8 / 3C^2 - 3C$
Angular deflection, $\theta = 12 \pi. M. D. n/ E. t. b^3$
And
$\delta = \theta \times y = (12 \pi. M. D. n/ E. t. b^3) \times y$
In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have
$\sigma_b = K \times 6 M / b^3 = K \times 6 W. y / b^3$
Angular deflection, $\theta = 12 \pi. M. D. n/ E. b^4$
And
$\delta = \theta \times y = (12 \pi. M. D. n/ E. b^4) \times y$

Flat Spiral Spring
A flat spring is a long thin strip of the elastic material wound like a spiral as shown in Fig. 4.20. These springs are frequently used in watches and gramophones etc.
When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of a number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilized in any useful way while the spirals open out slowly. Usually, the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending.

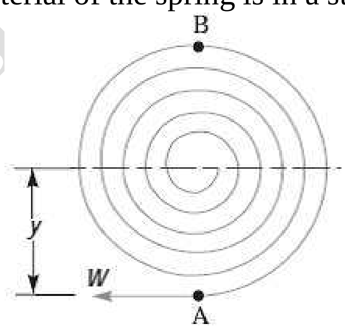


Fig. 4.20 Flat Spiral Spring

Let W = Force applied at the outer end A of the spring,
y = Distance of centre of gravity of the spring from A,
l = Length of strip forming the spring,
b = Width of strip,
t = Thickness of strip,
I = Moment of inertia of the spring section = $b.t^3/12$, and
Z = Section modulus of the spring section = $b.t^2/6$
When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$M = W \times y$

The greatest bending moment occurs in the spring at B which is at a maximum distance from the application of W.

∴ Bending moment at B,

$M_B = M_{max} = W \times 2y = 2W.y = 2M$

∴ Maximum bending stress induced in the spring material,

$\sigma_b = M_{max} / Z = 2W.y / b.t^2/6 = 12\ W.y / b.t^2 = 12\ M / b.t^2$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$\theta = M.\ l / E.\ I = 12\ M.\ l / E.\ b.t^3$

And the deflection,

$\delta = \theta \times y = M.\ l.\ y / E.\ I = (12\ M.\ l / E.\ b.t^3) \times y = (12\ W.\ y\ l / E.\ b.t^3) \times y$

$\delta = 12\ W.\ l.\ y^2 / E.\ b.t^3$

$\delta = \sigma_b.\ y / E.\ t$ $\quad (\sigma_b = 12\ W.y / b.t^2)$

The strain energy stored in the spring

$= ½ \times M.\ \theta = ½ \times M.\ M.\ l / E.\ I = ½ \times M^2.\ l / E.\ I$

$= ½ \times W^2.\ y^2.\ l / E.\ (b.t^3/12) = 6\ W^2.\ y^2.\ l / E.\ b.\ t^3$

(Multiplying the numerator and denominator by 24 bt)

$= (\sigma_b^2/24\ E) \times b.\ t.\ l = (\sigma_b^2/24\ E) \times$ Volume of the spring

Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

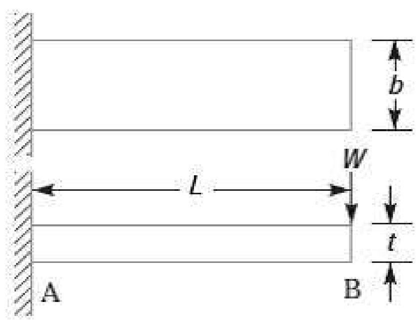


Fig. 4.21 Flat spring (cantilever type)

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. 4.21. This plate may be used as a flat spring.

Let t = Thickness of plate,

b = Width of the plate, and

L = Length of plate or distance of the load W from the cantilever end.

We know that the maximum bending moment at the cantilever end A,

$M = W.L$

And section modulus, $Z = I/y = (b.\ t^3/12) / (t/2) = 1/6 \times b.\ t^2$

∴ Bending stress in such a spring

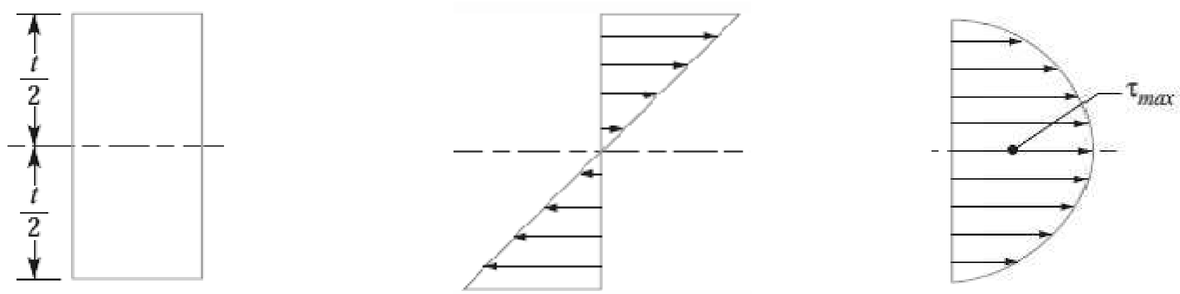
$\sigma = M/Z = W.L / (1/6 \times b.\ t^2) = 6\ W.L / b.\ t^2$

We know that the maximum deflection of a cantilever with a concentrated load at the free end is given by

$\delta = W.L^3/3\ E.I = W.L^3/3\ E.\ (b.\ t^3/12) = 4\ W.L^3/ E.\ b.\ t^3$

$\delta = 2\ \sigma.L^2/ 3\ E.\ t$ $\quad (\sigma = 6\ W.L / b.\ t^2)$

It may be noted that due to bending moment, top fibers will be in tension and the bottom fibers are in compression, but the shear stress is zero at the extreme fibers and maximum at the center, as shown in Fig. Hence for analysis, both stresses need not be taken into account simultaneously. We shall consider the bending stress only.



(a) Cross-section of plate (b) Bending stress diagram (c) Shear stress diagram

Fig. 4.22

If the spring is not of cantilever type but it is like a simply supported beam, with length 2L and load 2W in the center, as shown in Fig. 4.22,

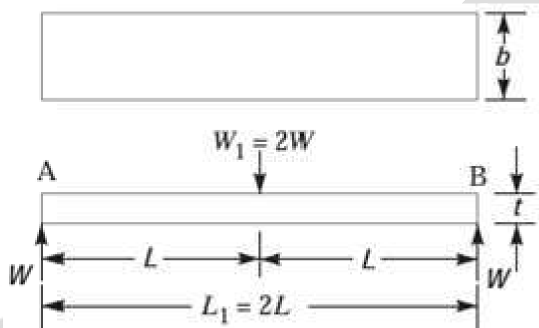


Fig. 4.22

Then Maximum bending moment in the centre,

$M = W.L$

Section modulus, $Z = b.t^2 / 6$

∴ Bending stress, $\sigma = M/Z = W.L / b.t^2 / 6 = 6\ W.L / b.t^2$

We know that maximum deflection of a simply supported beam loaded in the centre is given by

$\delta = W_1.\ L_1^{\ 3} / 48\ E.I = 2W.\ (2L)^{3} / 48\ E.I$ ... (In this case, $W_1 = 2W$, and $L_1 = 2L$)

$\delta = W.L^3 / 3\ E.I$

From above we see that a spring such as automobile spring (semi-elliptical spring) with length 2L and loaded in the centre by a load 2W, may be treated as a double cantilever.

If the plate of the cantilever is cut into a series of n strips of width b and these are placed as shown in Fig. 4.22, the above equations, may be written as

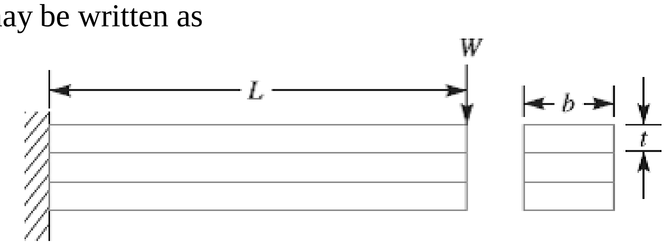


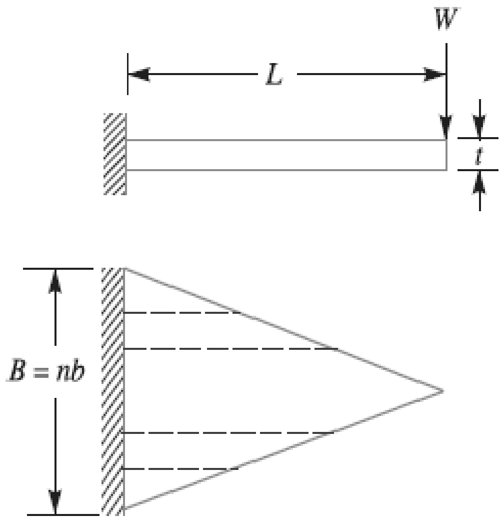
Fig. 4.23

$\sigma = 6\ W.L^3/n.b.t^2$

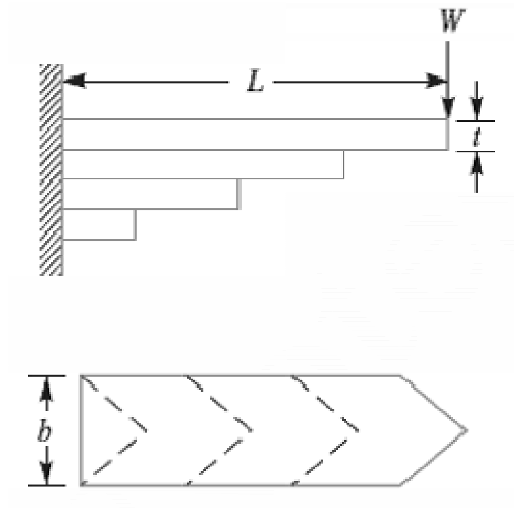
And $\delta = 4\ W.L^3 /\ n.\ E.\ b.\ t^3 = 2\ \sigma.L^2 /\ 3\ E.\ t$

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring has a maximum at the support.

If a triangular plate is used as shown in Fig. 4.24 (a), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 4.24 (b) to form a graduated or laminated leaf spring, then



(a)



(b)

Fig. 4.24 Laminated leaf spring

$\sigma = 6\ W.L/n.b.t^2$

And $\delta = 6\ W.L^3/n.E.b.t^3 = \sigma.L^2/E.\ t$

Where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from above equations, that for the same deflection, the stress in the uniform cross-section leaves (i.e. full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes F and G are used to indicate the full length (or uniform cross section) and graduated leaves, then

$\sigma_F = 3/2\ \sigma_G$

$6\ W_F.L/n_F.b.t^2 = 3/2\ (6\ W_G.L/n_G.b.t^2)$

$W_F\ /\ n_F = 3/2\ (W_G\ /\ n_G)$

$W_F\ /\ W_G = 3/2\ (n_F\ /\ n_G)$

Adding 1 to both sides, we have

$W_F\ /\ W_G + 1 = 3\ n_F\ /\ 2\ n_G + 1$

Or

$(W_F + W_G)/\ W_G = (3\ n_F + 2\ n_G)/\ 2\ n_G$

$W_G = 2\ n_G\ (W_F + W_G)/\ (3\ n_F + 2\ n_G) = 2\ n_G\ .\ W/\ (3\ n_F + 2\ n_G)$

Where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From above equation, we may write

$W_G\ /\ W_F = 2/3\ (n_G\ /\ n_F)$

Adding 1 to both sides, we have

$W_G\ /\ W_F + 1 = 2n_G\ /\ 3n_F + 1$

Or

$(W_G + W_F)/\ W_F = (2n_G + 3n_F)/\ 3n_F$

$W_F = 3n_F\ (W_G + W_F)/\ (2n_G + 3n_F) = 3n_F.\ W\ /\ (2n_G + 3n_F)$

∴ Bending stress for full length leaves,
$\sigma_F = 6\ W_F.L/n_F.b.t^2 = 6\ L/n_F.b.t^2 \times 3n_F.\ W / (2n_G + 3n_F) = 18\ W.L / b.t^2\ (2n_G + 3n_F)$
Since
$\sigma_F = 3/2\ \sigma_G$
Therefore
$\sigma_G = 2/3\ \sigma_F = 2/3\ [18\ W.L / b.t^2\ (2n_G + 3n_F)] = 12\ W.L / b.t^2\ (2n_G + 3n_F)$
The deflection in full length and graduated leaves is given by equation above, i.e.
$\delta = 2\ \sigma_F.L^2/ 3\ E.\ t = 2\ .L^2/ 3\ E.\ t\ [18\ W.L / b.t^2\ (2n_G + 3n_F)] = 12\ W.L^3 / E.b.t^3\ (2n_G + 3n_F)$

Construction of Leaf Spring

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 4.25.

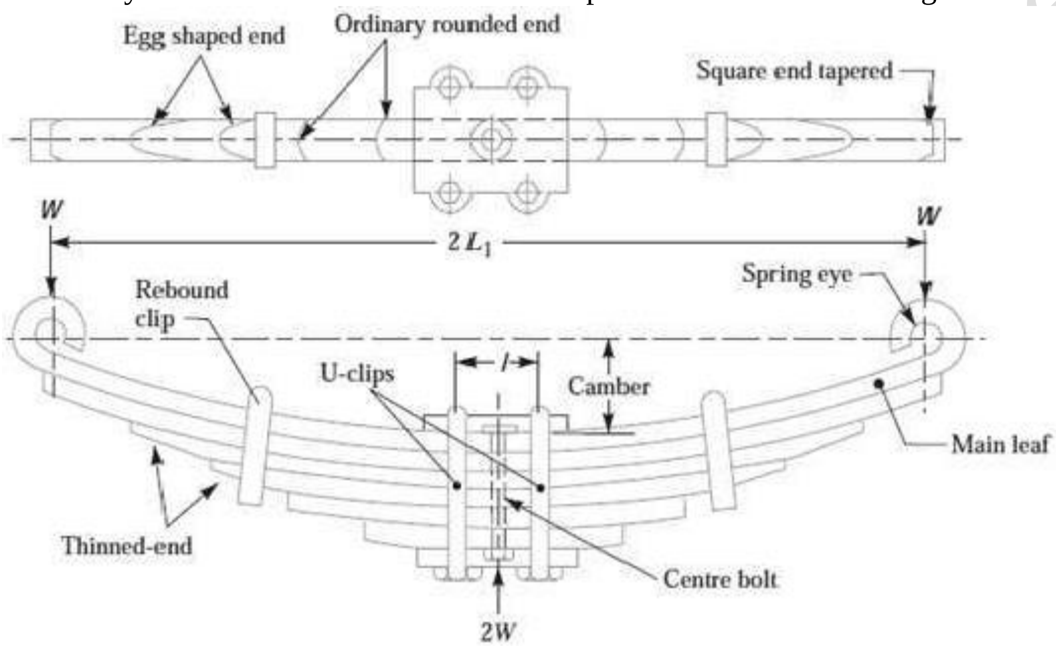


Fig. 4.25 Semi-elliptical leaf spring

The longest leaf known as a main leaf or master leaf has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its supports. Usually, the eyes, through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction material such as bronze or rubber. The other leaves of the spring are known as graduated leaves. In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms as shown in Fig. Since the master leaf has to withstand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full-length leaves and the rest graduated leaves as shown in Fig. 4.25.
Rebound clips are located at intermediate positions in the length of the spring so that the graduated leaves also share the stresses induced in the full-length leaves when the spring rebounds.

Equalized Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full-length leaves is 50% greater than the stress in the graduated leaves. In order to utilize the material to the best advantage, all the leaves should be equally stressed. This condition may be obtained in the following two ways:
1. By making the full-length leaves of smaller thickness than the graduated leaves. In this way, the full-length leaves will induce smaller bending stress due to a small distance from the neutral axis to the edge of the leaf.
2. By giving a greater radius of curvature to the full-length leaves than graduated leaves, as shown in Fig., before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the

leaves. This initial gap, as shown by C in Fig. 4.26, is called **nip.** When the central bolt, holding the various leaves together, is tightened, the full-length leaf will bend back as shown dotted in Fig. and have an initial stress in a direction opposite to that of the normal load. The graduated leaves will have an initial stress in the same direction as that of the normal load. When the load is gradually applied to the spring, the full-length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full-length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full-length leaves may have the lower stress. This is desirable in automobile springs in which full-length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip C.

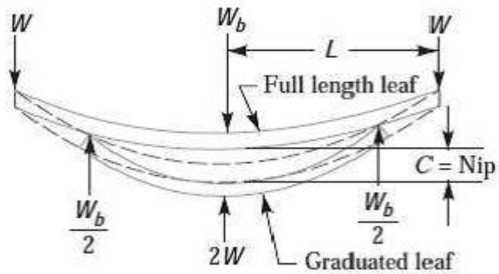


Fig. 4.26

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap C. In other words,

$\delta_G = \delta_F + C$

$C = \delta_F - \delta_G = 6\ W_G.L^3/\ n_G.E.B.t^3 - 4\ W_G.L^3/\ n_G.E.B.t^3$

Since the stresses are equal, therefore

$\sigma_G = \sigma_F$

$6\ W_F.L/n_F.b.t^2 = 6\ W_G.L/n_G.b.t^2$

$W_F\ /\ n_F = W_G\ /\ n_G$

$W_G = (n_G\ /\ n_F)\ W_F$

$W_G = (n_G\ /\ n)\ W$

And $W_F = (n_G\ /n_F)\ W_G$

Substituting the values of W_G and W_F in above equation of C, we have

$C = 6\ W.L^3/\ n.E.B.t^3 - 4\ W.L^3/\ n.E.B.t^3 = 2\ W.L^3/\ n.E.B.t^3$

The load on the clip bolts (W_b) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$\therefore C = \delta_F + \delta_G$

$2\ W.L^3/\ n.E.B.t^3 = 4\ L^3/\ n_F.E.B.t^3\ (W_b/2) + 6\ L^3/\ n_F.E.B.t^3\ (W_b/2)$

$W/n = W_b\ /\ n_F + 3\ W_b\ /\ 2\ n_F = (2\ n_G.\ W_b + 3\ n_F.\ W_b)\ /2\ n_F.\ n_G = W_b\ (2\ n_G + 3\ n_F)\ /\ 2n_F.\ n_G$

$W_b = 2n_F.\ n_G.\ W/\ 2\ (2\ n_G + 3\ n_F)$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load **minus** the initial stress.

Power screws

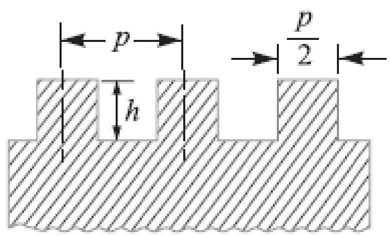
The power screws (also known as translation screws) are used to convert rotary motion into translator motion. For example, in the case of the lead screw of the lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of the screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others, the nut rotates while the screw moves axially with no rotation.

Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws:

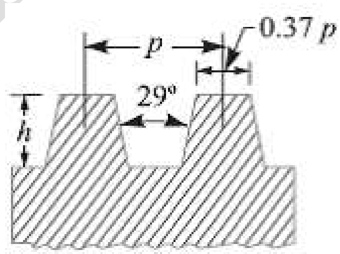
1. Square thread. A square thread, as shown in Fig. 2.81 is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a single point tool and it cannot be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices.



h = 0.5p

Fig. 2.81 Square Thread

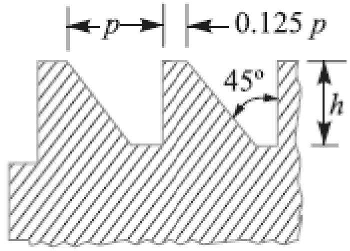
2. Acme or trapezoidal thread. An acme or trapezoidal thread, as shown in Fig. 2.82, is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduces some bursting pressure on the nut but increases its area in shear. It is used where a split nut is required and where provision is made to take up wears as in the lead screw of a lathe. Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread.



h = 0.5p + 0.25 mm

Fig. 2.82 Acme Thread

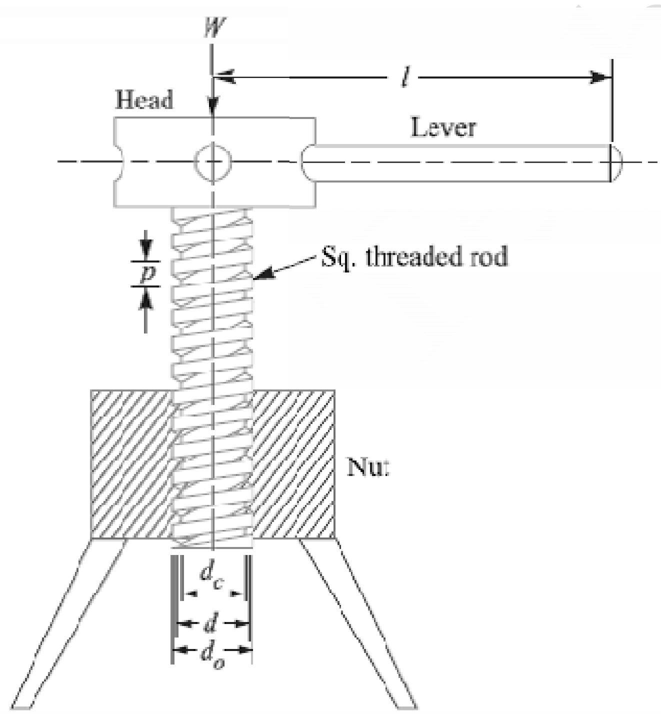
3. Buttress thread. A buttress thread, as shown in Fig. 2.83, is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.



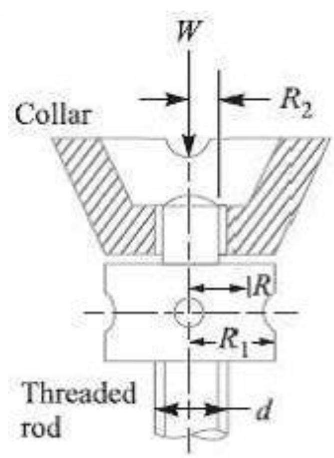
h = 0.75p
Fig. 2.83 Buttress Thread

Multiple Threads
The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such types of threads are usually found in high-speed actuators.

Torque Required to Raise Load by Square Threaded Screws
The torque required to raise a load by means of the square-threaded screw may be determined by considering a screw jack as shown in Fig. 2.84 (a). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.



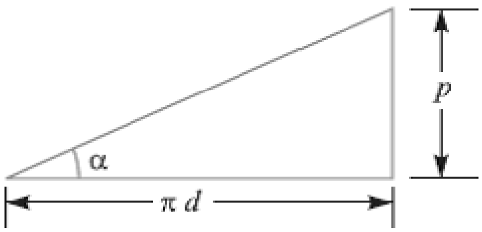
(a) Screw Jack

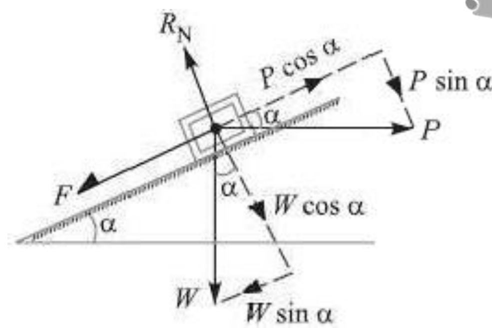


(b) Thrust Collar

Fig. 2.84

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 2.85 (a).





(a) Development of screw (b) Forces acting on the screw

Fig. 2.85

Let p = Pitch of the screw,
d = Mean diameter of the screw,
α = Helix angle,
P = Effort applied at the circumference of the screw to lift the load,
W = Load to be lifted, and
μ = Coefficient of friction, between the screw and nut = tan φ, where φ is the friction angle.
From the geometry of the Fig. 2.85 (a), we find that
$\tan \alpha = p / \pi d$
Since the principle, on which screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 2.85 (b).
Since the load is being lifted, therefore the force of friction (F = μ. RN) will act downwards. All the forces acting on the body are shown in Fig. 2.85 (b).
Resolving the forces along the plane,
$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . RN$
And resolving the forces perpendicular to the plane,
$RN = P \sin \alpha + W \cos \alpha$
Substituting this value of RN in the above equation, we have
$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$
$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$
Or $P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$
Or $P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$
$P = W (\sin \alpha + \mu \cos \alpha) / (\cos \alpha - \mu \sin \alpha)$
Substituting the value of $\mu = \tan \varphi$ in the above equation, we get
Or $P = W (\sin \alpha + \tan \varphi, \cos \alpha) / (\cos \alpha - \tan \varphi \sin \alpha)$
Multiplying the numerator and denominator by cos φ, we have
$P = W \times (\sin \alpha \cos \varphi + \sin \varphi \cos \alpha) / \cos \alpha \cos \varphi - \sin \alpha \sin \varphi)$
$P = W \times \sin (\alpha + \varphi) / \cos (\alpha + \varphi) = \tan (\alpha + \varphi)$
∴ Torque required overcoming friction between the screw and nut,
$T_1 = P \times d/2 = W \tan (\alpha + \varphi)\, d/2$
When the axial load is taken up by a thrust collar as shown in Fig (b), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,
$T_2 = 2/3 \times \mu_1 \times W [(R_1^3 - R_2^3) / (R_1^2 - R_2^2)]$......................................(Assuming uniform pressure conditions)
$T_2 = \mu_1 \times W [(R_1 + R_2) / 2] = \mu_1 \times W \times R$ .... (Assuming uniform wear conditions)
Where R_1 and R_2 = Outside and inside radii of collar,
R = Mean radius of collar = $(R_1 + R_2)/ 2$
And μ_1 = Coefficient of friction for the collar.
∴ Total torque required to overcome friction (i.e. to rotate the screw),
$T = T_1 + T_2$

If an effort P1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, i.e.
$T = P \times d/2 = P_1 \times l$

Torque Required to Lower Load by Square Threaded Screws
A little consideration will show that when the load is being lowered, the force of friction ($F = \mu . RN$) will act upwards. All the forces acting on the body are shown in Fig. 2.86.
Resolving the forces along the plane,

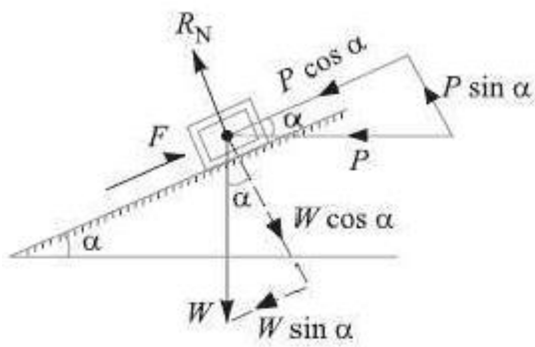


Fig. 2.86

$P \cos \alpha = F - W \sin \alpha = \mu RN - W \sin \alpha$
And resolving the forces perpendicular to the plane,
$RN = W \cos \alpha - P \sin \alpha$
Substituting this value of RN in above equation, we have,
$P \cos \alpha = \mu (W \cos \alpha - P \sin \alpha) - W \sin \alpha = \mu W \cos \alpha - \mu P \sin \alpha - W \sin \alpha$
Or $P \cos \alpha + \mu P \sin \alpha = \mu W \cos \alpha - W \sin \alpha$
$P (\cos \alpha + \mu \sin \alpha) = W (\mu \cos \alpha - \sin \alpha)$
$P = W (\mu \cos \alpha - \sin \alpha) / (\cos \alpha + \mu \sin \alpha)$
Substituting the value of $\mu = \tan \varphi$ in the above equation, we have
$P = W (\tan \varphi \cos \alpha - \sin \alpha) / (\cos \alpha + \tan \varphi \sin \alpha)$
Multiplying the numerator and denominator by $\cos \varphi$, we have
$P = W (\sin \varphi \cos \alpha - \sin \alpha \cos \varphi) / (\cos \alpha \cos \varphi + \sin \alpha \sin \varphi)$
$P = W \text{ x } \sin (\alpha - \varphi) / \cos (\alpha - \varphi) = \tan (\alpha - \varphi)$
∴ Torque required overcoming friction between the screw and nut,
$T_1 = P \text{ x } d/2 = W \tan (\alpha - \varphi) \, d/2$

Efficiency of Square Threaded Screws
The efficiency of square threaded screws may be defined as the ratio between the ideal efforts (i.e. the effort required to move the load, neglecting friction) to the actual effort (i.e. the effort required to move the load taking friction into account).
We have seen that the effort applied at the circumference of the screw to lift the load is
$P = W \tan (\alpha + \varphi)$
Where W = Load to be lifted,
α = Helix angle,
φ = Angle of friction, and
μ = Coefficient of friction between the screw and nut = $\tan \varphi$.
If there would have been no friction between the screw and the nut then φ will be equal to zero.
The value of effort P0 necessary to raise the load will then be given by the equation,
$P_0 = W \tan \alpha$
∴ Efficiency,
η = Ideal effort / Actual effort = $P_0/P = W \tan \alpha / W \tan (\alpha + \varphi)$
This shows that the efficiency of a screw jack is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

η = Torque required moving the load, neglecting friction/ Torque required to move the load, including screw and collar friction

$\eta = T_0/T = (P_0 \text{ x } d/2) / (P_0 \text{ x } d/2 + \mu_1. WR)$

Maximum Efficiency of a Square Threaded Screw

We have seen above that the efficiency of a square threaded screw,

$\eta = \tan \alpha / \tan (\alpha + \varphi) = (\sin \alpha / \cos \alpha) / (\sin (\alpha + \varphi) / \cos (\alpha + \varphi))$

$\eta = \sin \alpha \text{ x } \cos (\alpha + \varphi) / \cos \alpha \text{ x } \sin (\alpha + \varphi)$

Multiplying the numerator and denominator by 2, we have

$\eta = 2 \text{ x } \sin \alpha \text{ x } \cos (\alpha + \varphi) / 2 \text{ x } \cos \alpha \text{ x } \sin (\alpha + \varphi)$

$\eta = \sin (2\alpha + \varphi) - \sin \varphi / \sin (2\alpha + \varphi) + \sin \varphi$

[2 sin A cos B sin (A+B) + sin (A-B) & 2 cos A cos B sin (A+B) - sin (A-B)]

The efficiency given by equation **(ii)** will be maximum when $\sin (2\alpha + \varphi)$ is maximum, i.e. when

$\sin (2\alpha + \varphi) = 1$ or when $2\alpha + \varphi = 90°$

$\therefore 2\alpha = 90° – \varphi$ or $\alpha = 45° – \varphi / 2$

Substituting the value of 2α in above equation, we have maximum efficiency,

$\eta_{max} = \sin (90° - \varphi + \varphi) - \sin \varphi / \sin (90° - \varphi + \varphi) + \sin \varphi = \sin 90° - \sin \varphi / \sin 90° + \sin \varphi$

$\eta_{max} = 1 - \sin \varphi / 1 + \sin \varphi$

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ME-602 Machine Component and Design

Unit – IV
Brakes & Clutches

Syllabus :
Materials for friction surface, uniform pressure and uniform wear theories, Design of friction clutches: Disc,plate clutches, cone & centrifugal clutches. Design of brakes: Rope, band & block brake, Internal expending brakes, Disk brakes

Subject Notes
Clutch Introduction

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

Types of Clutches
Following are the two main types of clutches commonly used in engineering practice:
1. Positive clutches, and 2. Friction clutches.

Positive Clutches
The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch*.**

Friction Clutches
A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces.

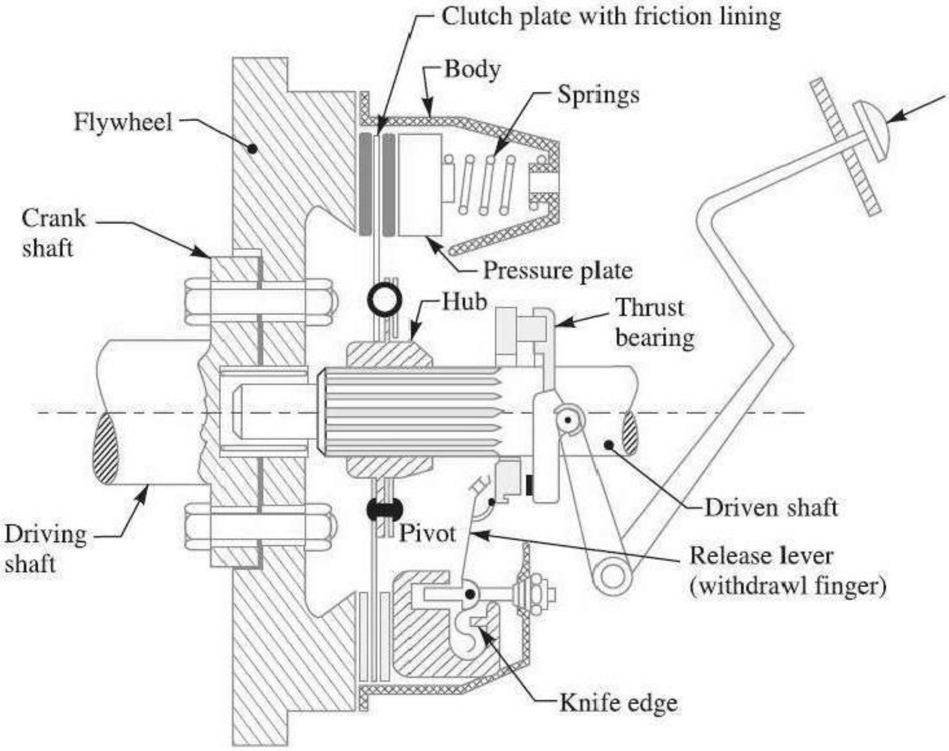
Material for Friction Surfaces
1. It should have a high and uniform coefficient of friction.
2. It should not be affected by moisture and oil.
3. It should have the ability to withstand high temperatures caused by slippage.
4. It should have high heat conductivity.
5. It should have high resistance to wear and scoring.

Considerations in Designing a Friction Clutch
The following considerations must be kept in mind while designing a friction clutch.
1. The suitable material forming the contact surfaces should be selected.
2. The moving parts of the clutch should have low weight in order to minimize the inertia load, especially in high speed service.
3. The clutch should not require any external force to maintain contact of the friction surfaces.
4. The provision for taking up wear of the contact surfaces must be provided.
5. The clutch should have provision for facilitating repairs.
6. The clutch should have provision for carrying away the heat generated at the contact

surfaces.
7. The projecting parts of the clutch should be covered by guard.

Types of Friction Clutches
Though there are many types of friction clutches, yet the following are important from the subject point of view:
1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

Single Disc or Plate Clutch



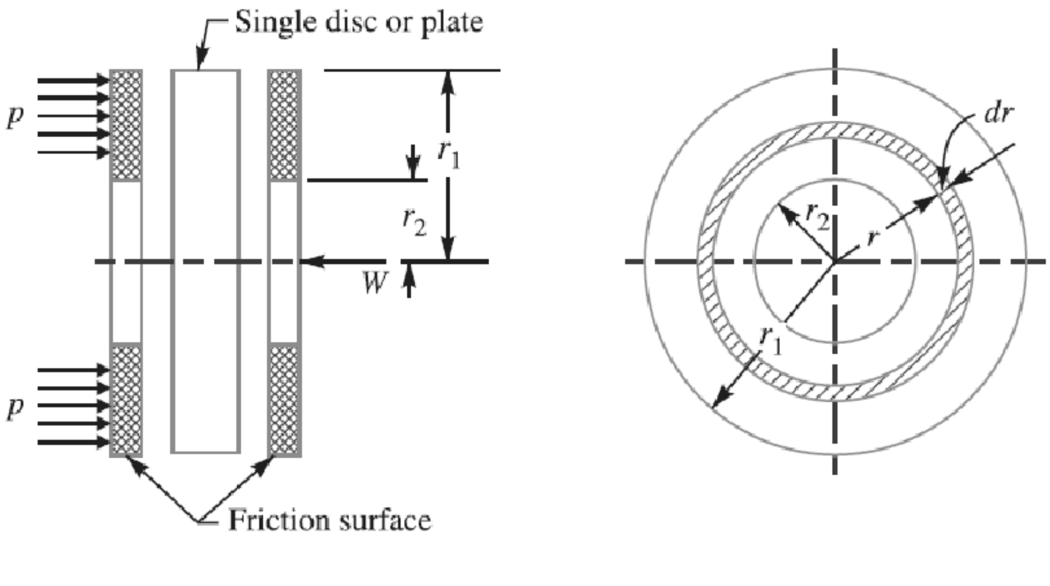
A single disc or plate clutch, as shown in Fig., consists of a clutch plate whose both sides are faced with a frictional material It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and th pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.
The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional

force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Design of a Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (W) as shown in Fig. 24.3 (a).



(a) (b)

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces,

r = Mean radius of the friction face, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 24.3 (b).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform axial wear.

1. ***Considering uniform pressure.*** When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (a), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

where W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\,\mu.\,W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\,.W.R$$

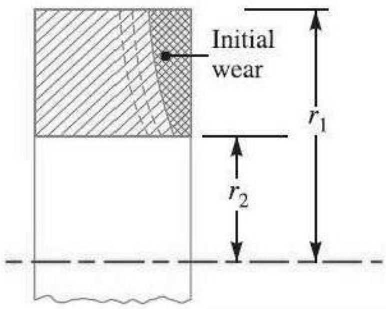
where $R = \dfrac{2}{3}\left[\dfrac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$ = Mean radius of the friction surface.

nsidering uniform axial wear. The basic principle in designing machine parts that are ...) wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear $\propto$ Work of friction $\propto p.V$

or $\quad p.V = K$ (a constant) or $p = K/V \quad$ ...(*i*)

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product $p.V$ is constant over the entire surface. After this, the wear will be uniform as shown in Fig.



Uniform axial wear.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r = C \text{ (a constant) or } p = C/r \qquad ...(ii)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\,C\,dr = 2\pi\,C\,[r]_{r_2}^{r_1} = 2\pi\,C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because p = C/r)$$

$\therefore$ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\,C.r.dr = 2\pi\,\mu\,C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

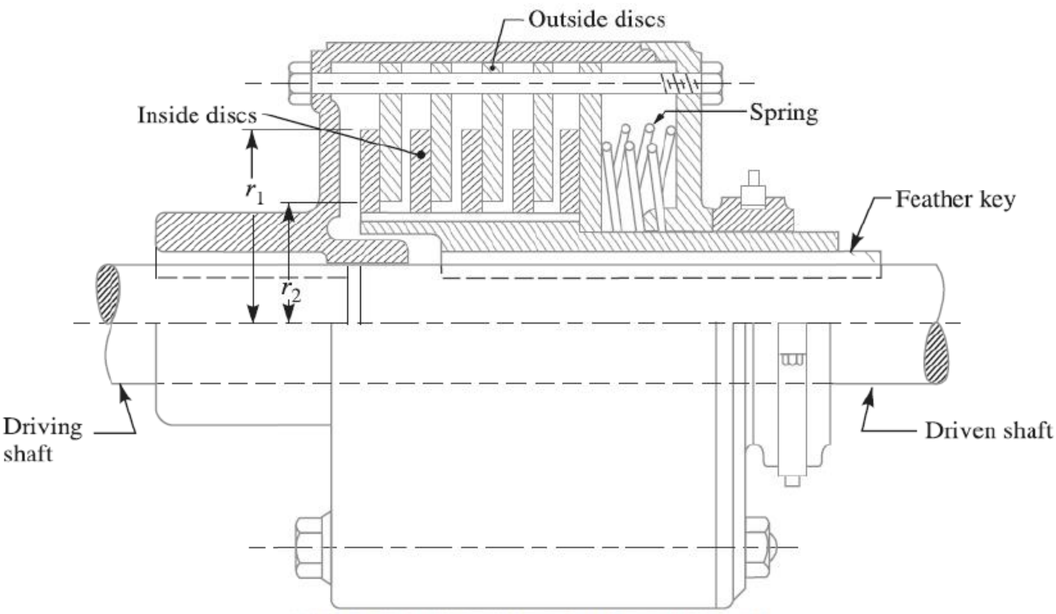
$$= 2\pi\mu\,.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

Multiple Disc Clutch

A multiple disc clutch, as shown in Fig., may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.



Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

$\therefore$ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

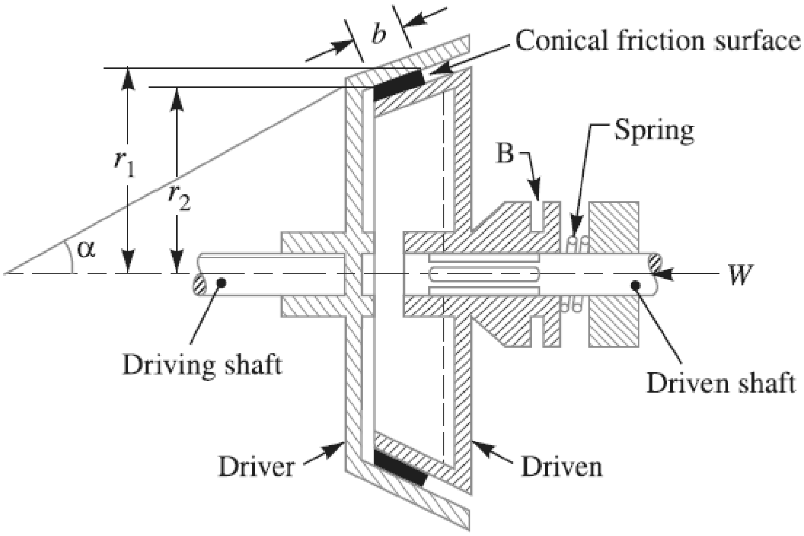
where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{... (For uniform wear)}$$

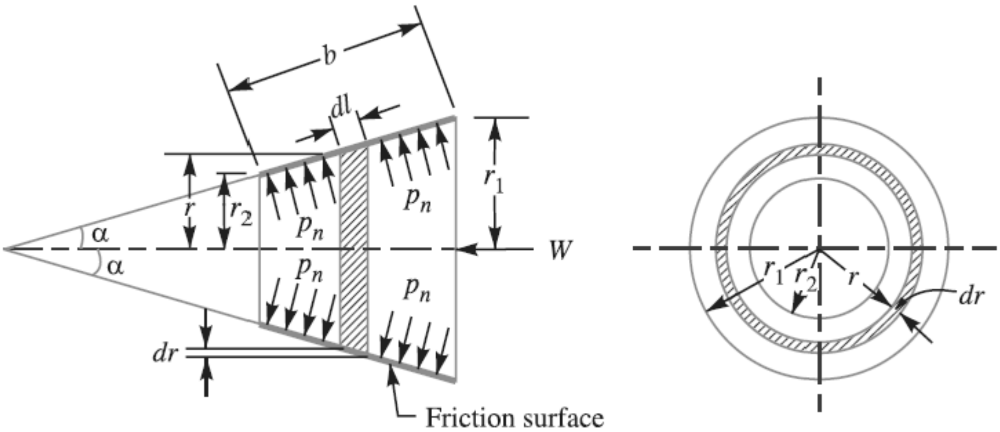
Cone Clutch

A cone clutch, as shown in Fig., was extensively used in automobiles, but now-a-days it has been replaced completely by the disc clutch. It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some m terial like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.



Design of a Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Fig.. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.



Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between the contact surfaces),

r_1 = Outer radius of friction surface,

r_2 = Inner radius of friction surface,

R = Mean radius of friction surface = $\frac{r_1 + r_2}{2}$,

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

Consider a small ring of radius r and thickness dr as shown in Fig. . Let dl is the length of ring of the friction surface, such that,

$$dl = dr \operatorname{cosec} \alpha$$

$$\therefore \quad \text{Area of ring} = 2\pi r.\, dl = 2\pi\, r.dr \operatorname{cosec} \alpha$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

1. ***Considering uniform pressure***

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin \alpha = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha \times \sin \alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\ p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n\, [(r_1)^2 - (r_2)^2]$$

and

$$p_n = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]} \qquad \ldots(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha \times r$$

$$= 2\pi\, \mu.p_n \operatorname{cosec} \alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

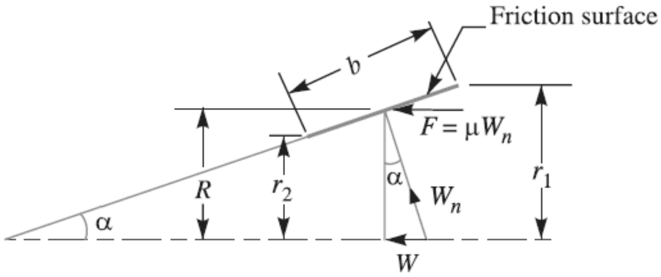
$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n. \operatorname{cosec} \alpha.r^2\ dr = 2\pi\ \mu.p_n \operatorname{cosec} \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.p_n \operatorname{cosec} \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

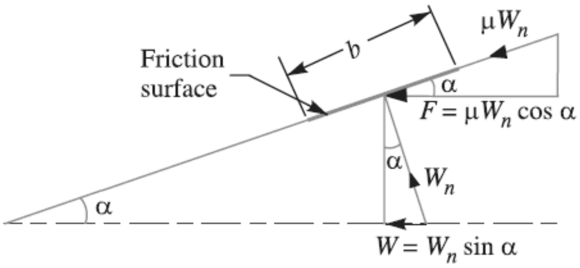
Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\ \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad ...(ii)$$



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

2. ***Considering uniform wear***

In Fig. let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \qquad p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C/r$$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\,\alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \times 2\pi\, r.dr\ \text{cosec}\,\alpha \times \sin\alpha$$

$$= 2\pi \times p_r.r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \qquad ...\left(\because p_r = \frac{C}{r}\right)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\,[r]_{r_2}^{r_1} = 2\pi\ C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\,(r_1 - r_2)} \qquad ...(iii)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\, r.dr\ \text{cosec}\,\alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu . p_r \times 2\pi\, r.dr \text{ cosec } \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi\, r.dr \text{ cosec } \alpha \times r = 2\pi\, \mu . C \text{ cosec } \alpha \times r\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu . C \text{ cosec } \alpha \times r\, dr = 2\pi \mu . C \text{ cosec } \alpha \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu . C \text{ cosec } \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2} \right]$$

Substituting the value of C from equation ***(iii)***, we have

$$T = 2\pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)} \times \text{cosec } \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2} \right]$$

$$= \mu . W \text{ cosec } \alpha \left[\frac{r_1 + r_2}{2} \right] = \mu\, WR \text{ cosec } \alpha \qquad \text{...}(iv)$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface.

Since the normal force acting on the friction surface, $W_n = W \text{ cosec } \alpha$, therefore the equation ***(iv)*** may be written as

$$T = \mu\, W_n R \qquad \text{...}(v)$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 24.8 (*a*) and (*b*) respectively.

From Fig. (*a*), we find that

$$r_1 - r_2 = b \sin \alpha \quad \text{and} \quad R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R$$

$\therefore$ From equation ***(i)***, normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi\,(r_1 + r_2)\,(r_1 - r_2)} = \frac{W}{2\,\pi\, R.b \sin \alpha}$$

or $W = p_n \times 2\pi\, R.b \sin \alpha = W_n \sin \alpha$

where W_n = Normal load acting on the friction surface $= p_n \times 2\pi R.b$

Now the equation ***(iv)*** may be written as

$$T = \mu\,(p_n \times 2\pi\, R.\, b \sin \alpha)\, R \text{ cosec } \alpha = 2\pi\, \mu . p_n\, R^2 . b$$

The following points may be noted for a cone clutch :

1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

2. If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu . W_n \cos \alpha$) acts on the clutch which resists the engagement, and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu . W_n \cos\alpha = W_n . \sin\alpha + \mu\, W_n \cos\alpha$$
$$= W_n (\sin\alpha + \mu\cos\alpha)$$

It has been found experimentally that the term ($\mu\, W_n . \cos\alpha$) is only 25 percent effective.

$$\therefore \quad W_e = W_n \sin\alpha + 0.25\,\mu\, W_n \cos\alpha = W_n (\sin\alpha + 0.25\,\mu\cos\alpha)$$

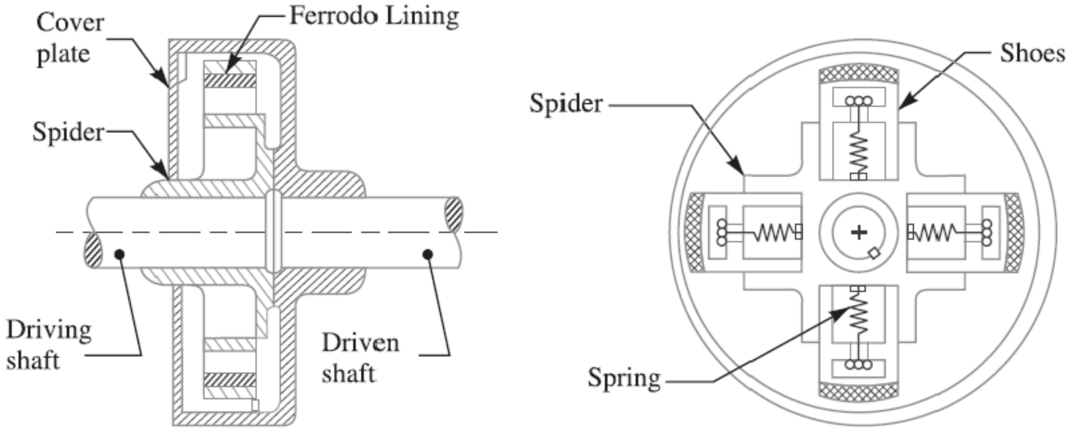
3. Under steady operation of the clutch, a decrease in the semi-cone angle (α) increases the torque produced by the clutch (T) and reduces the axial force (W). During engaging period, the axial force required for engaging the clutch (W_e) increases under the influence of friction as the angle α decreases. The value of α can not be decreased much because smaller semi-cone angle (α) requires larger axial force for its disengagement.

If the clutch is to be designed for free disengagement, the value of $\tan\alpha$ must be greater than μ. In case the value of $\tan\alpha$ is less than μ, the clutch will not disengage itself and axial force required to disengage the clutch is given by

$$W_d = W_n (\mu\cos\alpha - \sin\alpha)$$

Centrifugal Clutch

The centrifugal clutches are usually incorporated int the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.



Design of a Centrifugal Clutch

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for the design of a centrifugal clutch.

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig.

Let m = Mass of each shoe,
n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

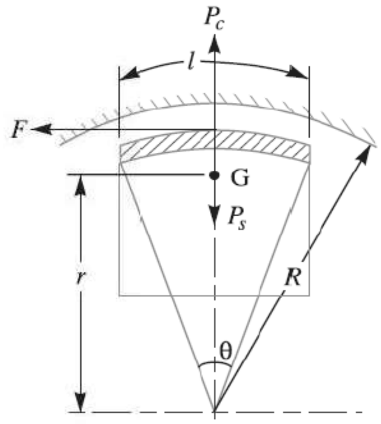
N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad / s $= 2\pi N / 60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed,



$$P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\,r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\,m.\omega^2.r$$

∴ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\,m.\omega^2.r = \frac{7}{16}\,m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. ***Size of the shoes***

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ = Angle subtended by the shoes at the centre of the spider in radians, and

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm^2.

We know that $\theta = \dfrac{l}{R}$ or $l = \theta.R = \dfrac{\pi}{3}R$ ...(Assuming $\theta = 60° = \pi/3$ rad)

$\therefore$ Area of contact of the shoe

$$= l \cdot b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

3. *Dimensions of the spring*

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.

BRAKES

Introduction

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The design or capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absor ed.

The major functional difference between a clutch and a brake is that a clutch i used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

Types of Brakes

The brakes, according to the means used for transforming the energy by the braking element,are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

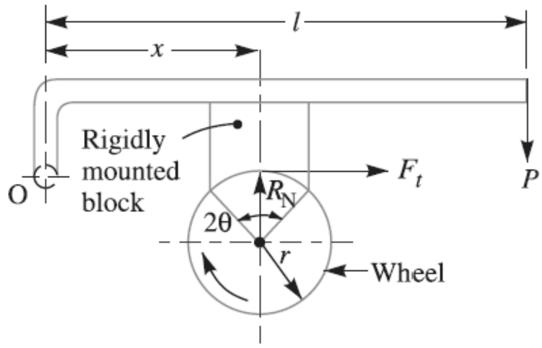
The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel. The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

(a) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes.*** According to the shape of the friction element, these brakes may be block or shoe brakes and band brakes.

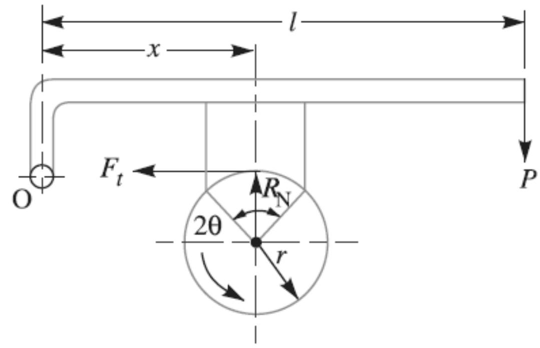
(b) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig.The other end of the lever is pivoted on a fixed fulcrum *O*.



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N \quad \text{...}(i)$$

and the braking torque, $T_B = F_t \cdot r = \mu\, R_N \cdot r$ ...(*ii*)

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x = P \times l \qquad \text{or} \qquad R_N = \frac{P \times l}{x}$$

$\therefore$ Braking torque, $T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 25.1 (*b*), then the braking torque is same, *i.e.*

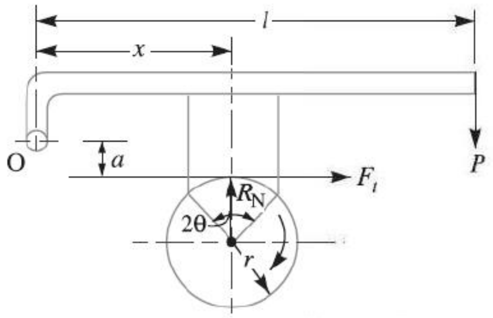
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then for equilibrium, taking moments about the fulcrum *O*,

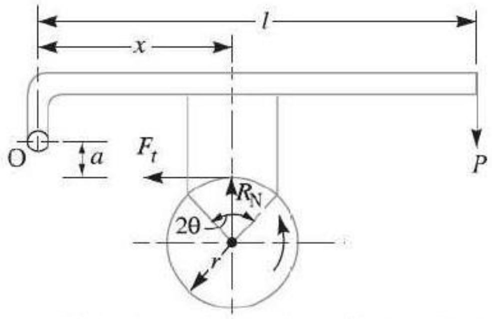
$$R_N \times x + F_t \times a = P.l$$

or $\quad R_N \times x + \mu\, R_N \times a = P.l \quad$ or $\quad R_N = \frac{P.l}{x + \mu.a}$

and braking torque, $\quad T_B = \mu\, R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

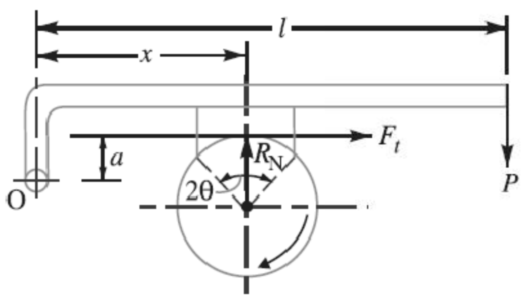
When the brake wheel rotates anticlockwise, as shown in Fig. (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad ...(i)$$

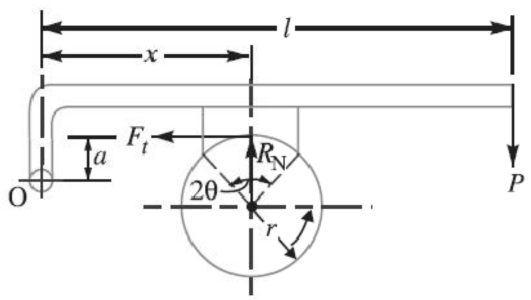
or $\quad R_N(x - \mu.a) = P.l \quad$ or $\quad R_N = \frac{P.l}{x - \mu.a}$

and braking torque, $\quad T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(ii)$$

or $$R_N(x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l$$

or $$R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

From above we see that when the brake wheel rotates anticlockwise in case 2 [Fig. (*b*)] and when it rotates clockwise in case 3 [Fig. (*a*)], the equations ***(i)*** and ***(ii)*** are same, *i.e.*

$R_N \times x = P.l + \mu.R_N.a$

From this we see that the moment of frictional force ($\mu.\ R_N.a$) adds to the moment of force ($P.l$). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be **self energizing brakes.**

When the frictional force is great enough to apply the brake with no external force, then the brake is said to be **self-locking brake.** From the above expression, we see that if $x \leq \mu.a$, then P will be negative or equal to zero. This means no external force is needed to apply the brake and hence the brake is self locking. Therefore the condition for the brake to be self locking is $x \leq \mu.a$ The self-locking brake is used only in back-stop applications.The brake should be self-energizing and not the self-locking.In order to avoid self-locking and to prevent the brake from grabbing, x is kept greater than $\mu.a$.If A_b is the projected bearing area of the block or shoe, then the bearing pressure on the shoe,

$P_b = R_N / A_b$

We know that A_b = Width of shoe × Projected length of shoe = $w\ (2r \sin \theta)$

When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (*R*N) and produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake is used.

Pivoted Block or Shoe Brake

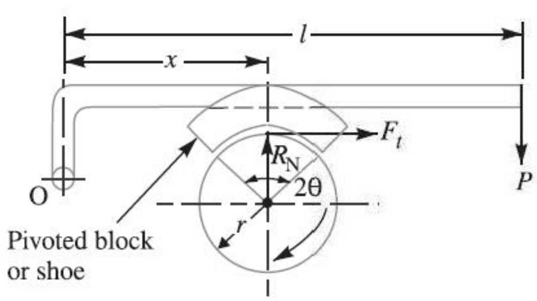
We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever as shown in Fig. 25.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

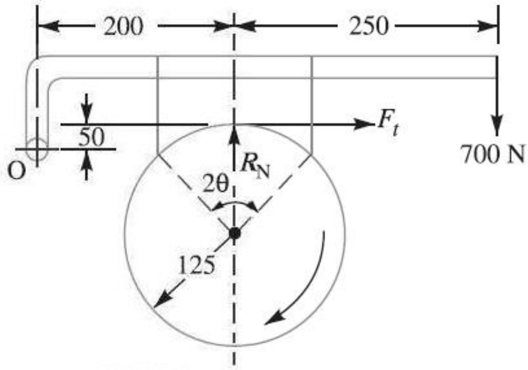
$$T_B = F_t \times r = \mu'.R_N.\ r$$

where μ' = Equivalent coefficient of friction $= \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.





All dimensions in mm.

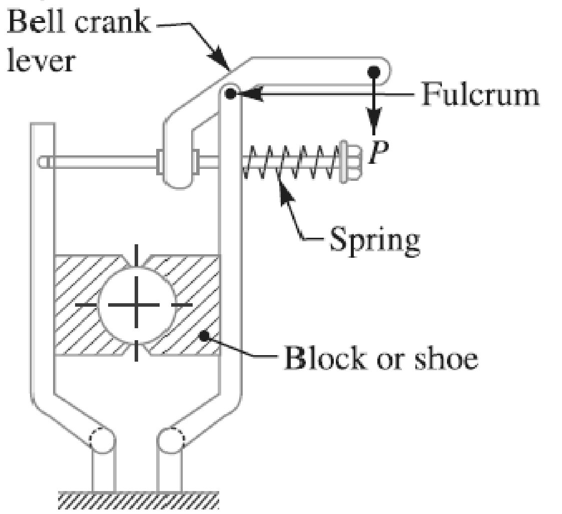
Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, and additional load is thrown on the shaft bearings due to the normal force (*RN*). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake as shown in Fig., is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force *P* is applied to the bell crank lever, the spring is compressed

and the brake is released. This type of brake is often used on electric cranes and the force *P* is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load. In a double block brake, the braking action is doubled by the use of two blocks and the two blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$T_B = (F_{t1} + F_{t2})\ r$

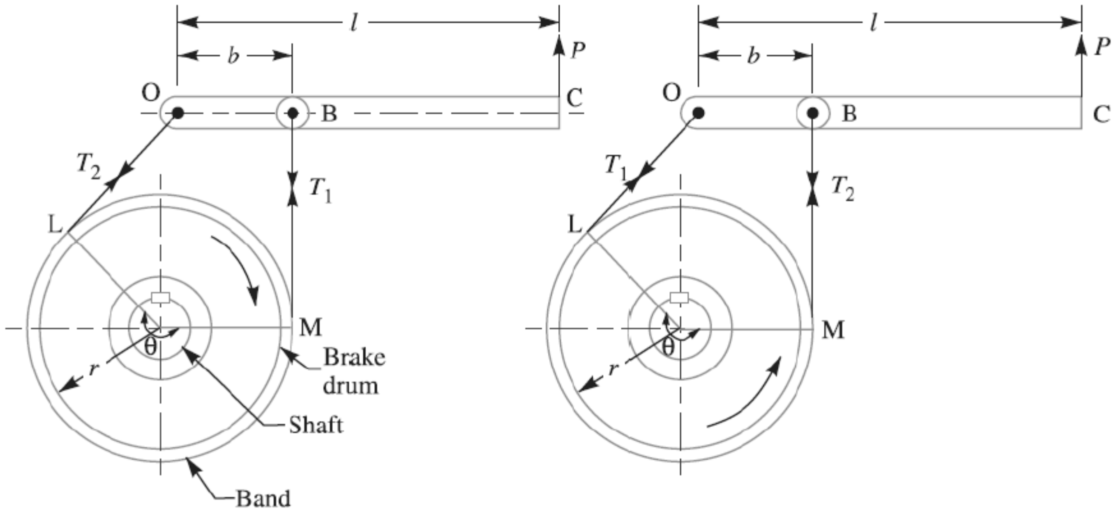
where, F_{t1} and F_{t2} are the braking forces on the two blocks.



Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance *b* from the fulcrum.

When a force *P* is applied to the lever at *C*, the lever turns about the fulcrum pin *O* and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force *P* on the lever at *C* may be determined as discussed below :



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum $= r + t/2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{...(Considering thickness of band)}$$

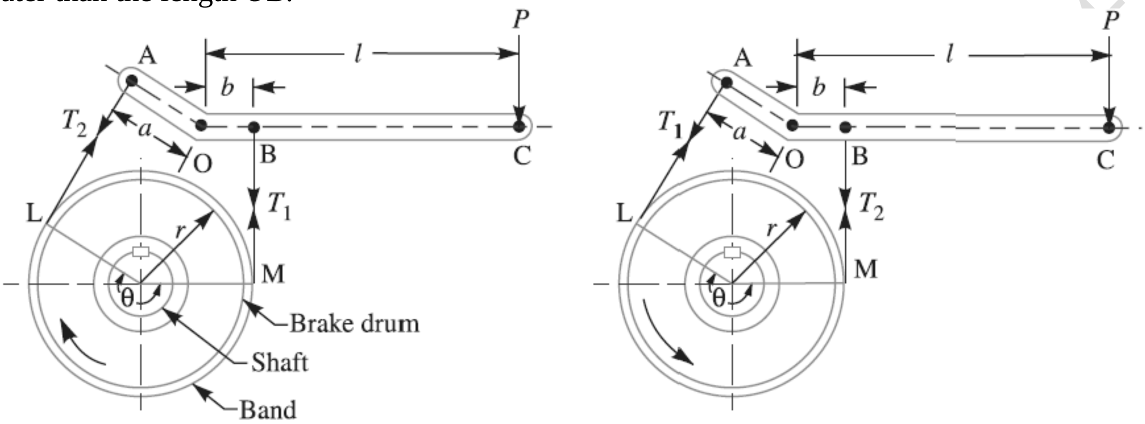
Now considering the equilibrium of the lever *OBC*. It may be noted that when the drum rotates in the clockwise direction as shown in Fig. (*a*), the end of the band attached to the fulcrum *O* will be slack with tension *T*2 and end of the band attached to *B* will be tight with tension *T*1. On the other hand, when the drum rotates in the anticlockwise direction as shown in Fig. (*b*), the tensions in the band will reverse, *i.e*. the end of the band attached to the fulcrum *O* will be tight with tension *T*1 and the end of the band attached to *B* will be slack with tension *T*2. Now taking moments about the fulcrum *O*, we have

$P.l = T1.b$ ...(for clockwise rotation of the drum)
and $P.l = T2.b$ ...(for anticlockwise rotation of the drum)

where l = Length of the lever from the fulcrum (OC), and
b = Perpendicular distance from O to the line of action of T_1 or T_2.

Differential Band Brake

In a differential band brake, as shown in Fig, the ends of the band are joined at A and B to a lever AOC pivoted on a fixed pin or fulcrum O. It may be noted that for the band to tighten, the length OA must be greater than the length OB.



(*a*) Clockwise rotation of the drum. (*b*) Anticlockwise rotation of the drum.

The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now considering the equilibrium of the lever AOC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. *a*, the end of the band attached to A will be slackwith tension $T2$ and end of the band attached to B will be tight with tension $T1$. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. *b*, the end of the band attached to A will be tight with tension $T1$ and end of the band attached to B will be slack with tension $T2$. Now taking moments about the fulcrum O, we have

$$P.l + T_1.b = T_2.a \quad \text{...(for clockwise rotation of the drum)}$$

or $$P.l = T_2.a - T_1.b \quad \text{...}(i)$$

and $$P.l + T_2.b = T_1.a \quad \text{...(for anticlockwise rotation of the drum)}$$

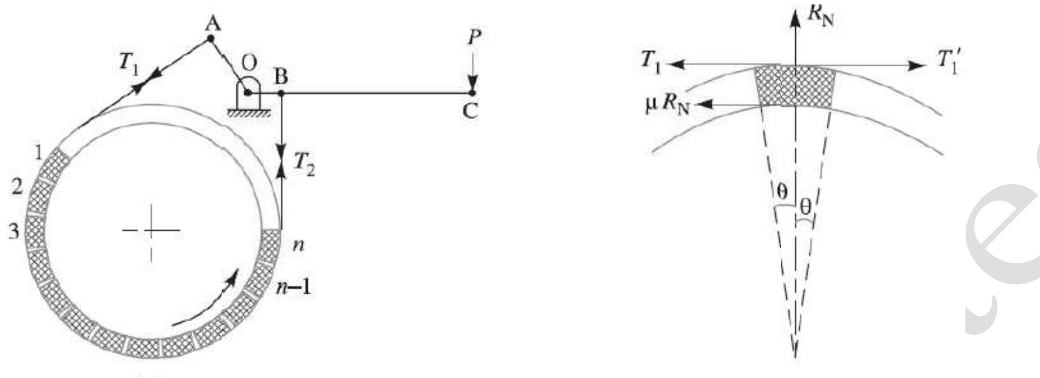
or $$P.l = T_1.a - T_2.b \quad \text{...}(ii)$$

We have discussed in block brakes, that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations ***(i)*** and ***(ii)*** that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment $P.l$) for the clockwise and anticlockwise rotation of the drum respectively. We have also discussed that when the force P is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self-locking is
$T_2.a \leq T_1.b$ or $T_2/T_1 \leq b / a$

and for anticlockwise rotation of the drum, the condition for self-locking is
$T_1.a \leq T_2.b$ or $T_1/T_2 \leq b / a$

Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. *a*. The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks, each subtending an angle 2 θ at the centre and the drum rotates in anticlockwise direction.



(*a*) (*b*)

Let T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

T_2', T_3' etc. = Tensions in the band between the second and third block, between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. *b*. This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1') \sin \theta = R_N \qquad \text{...}(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos \theta = \mu.R_N \qquad \text{...}(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1')\cos\theta}{(T_1 + T_1')\sin\theta} = \frac{\mu.R_N}{R_N}$$

or
$$(T_1 - T_1') = \mu \tan\theta\,(T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

Similarly it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = \ldots = \frac{T_{n-1}}{T_2} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \ldots \times \frac{T_{n-1}}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n \qquad \ldots(iii)$$

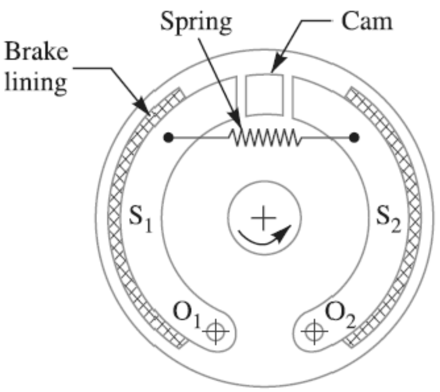
Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$

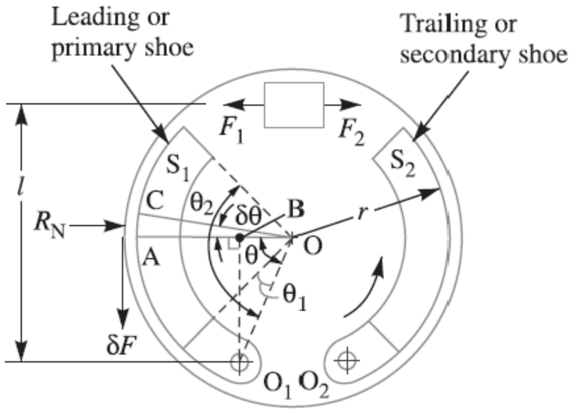
$$= (T_1 - T_2)\, r \qquad \ldots\text{(Neglecting thickness of band)}$$

Internal Expanding Brake

An internal expanding brake consists of two shoes *S*1 and *S*2 as shown in Fig. *a* The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum *O*1 and *O*2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. *a*. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. b. It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading* or *primary shoe*** while the right hand shoe is known as ***trailing* or *secondary shoe*.**

Let *r* = Internal radius of the wheel rim.

b = Width of the brake lining.

*p*1 = Maximum intensity of normal pressure,

*p*N = Normal pressure,

*F*1 = Force exerted by the cam on the leading shoe, and

*F*2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining *AC* subtending an angle δθ at the centre. Let *OA* makes an angle θ with OO_1 as shown in Fig.*b*. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at *A* will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from *O*1 to *OA*, *i.e.* O_1B. From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at *A*, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

$\therefore$ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$
$$= p_N (b \, . \, r \, . \, \delta\theta) = p_1 \sin\theta \, (b \, . \, r \, . \, \delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu . \delta R_N = \mu \, p_1 \sin\theta \, (b \, . \, r \, . \, \delta\theta)$$

$\therefore$ Braking torque due to the element about *O*,

$$\delta T_B = \delta F \, . \, r = \mu \, p_1 \sin\theta \, (b \, . \, r \, . \, \delta\theta) \, r = \mu \, p_1 \, b \, r^2 (\sin\theta . \delta\theta)$$

and total braking torque about *O* for whole of one shoe,

$$T_B = \mu \, p_1 \, b \, r^2 \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \mu \, p_1 \, b \, r^2 \, [-\cos\theta]_{\theta_1}^{\theta_2}$$
$$= \mu \, p_1 \, b \, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1 B = \delta R_N (OO_1 \sin\theta)$$
$$= p_1 \sin\theta \, (b \, . \, r \, . \, \delta\theta) \, (OO_1 \sin\theta) = p_1 \sin^2\theta \, (b \, . \, r \, . \, \delta\theta) \, OO_1$$

Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta \, (b \, . \, r \, \delta\theta) \, OO_1 = p_1 \, . \, b \, . \, r \, . \, OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta$$

$$= p_1 . b . r . OO_1 \int_{\theta_1}^{\theta_2} \tfrac{1}{2} (1 - \cos 2\theta) \, d\theta \qquad \ldots \left[\because \sin^2\theta = \tfrac{1}{2} (1 - \cos 2\theta)\right]$$

$$= \tfrac{1}{2} \, p_1 \, . \, b \, . \, r \, . \, OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \tfrac{1}{2} \, p_1 \, . \, b \, . \, r \, . \, OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \tfrac{1}{2} \, p_1 \, . \, b \, . \, r \, . \, OO_1 \, [(\theta_2 - \theta_1) + \tfrac{1}{2} (\sin 2\theta_1 - \sin 2\theta_2)]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F \, (r - OO_1 \cos\theta) \qquad \ldots (\because AB = r - OO_1 \cos\theta)$$
$$= \mu \, . \, p_1 \sin\theta \, (b \, . \, r \, . \, \delta\theta) \, (r - OO_1 \cos\theta)$$
$$= \mu \, . \, p_1 \, . \, b \, . \, r \, (r \sin\theta - OO_1 \sin\theta \cos\theta) \, \delta\theta$$
$$= \mu \, . \, p_1 \, . \, b \, . \, r \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta\right) \delta\theta \qquad \ldots (\because 2 \sin\theta \cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu \cdot p_1 \cdot b \cdot r \int_{\theta_1}^{\theta_2} \left(r \sin\theta - \frac{OO_1}{2} \sin 2\theta \right) d\theta$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta + \frac{OO_1}{4} \cos 2\theta \right]_{\theta_1}^{\theta_2}$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta_2 + \frac{OO_1}{4} \cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4} \cos 2\theta_1 \right]$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

If $M_F > M_N$, then the brake becomes self locking.

Disc Brake

Main Components of Disc brake

1. **Wheel Hub:** The disc rotor is attached to the wheel hub and it rotates with it. The wheel of the vehicle is bolted to the wheel hub.

2. **Caliper Assembly**

   The caliper assembly consist of

   (i) **Brake pad:** It makes contact with the rotor disc and due to the friction between the brake pad and rotor disc the vehicle speed reduces and it stops.

   (ii) **Caliper bracket**

   (iii) **Caliper frame**

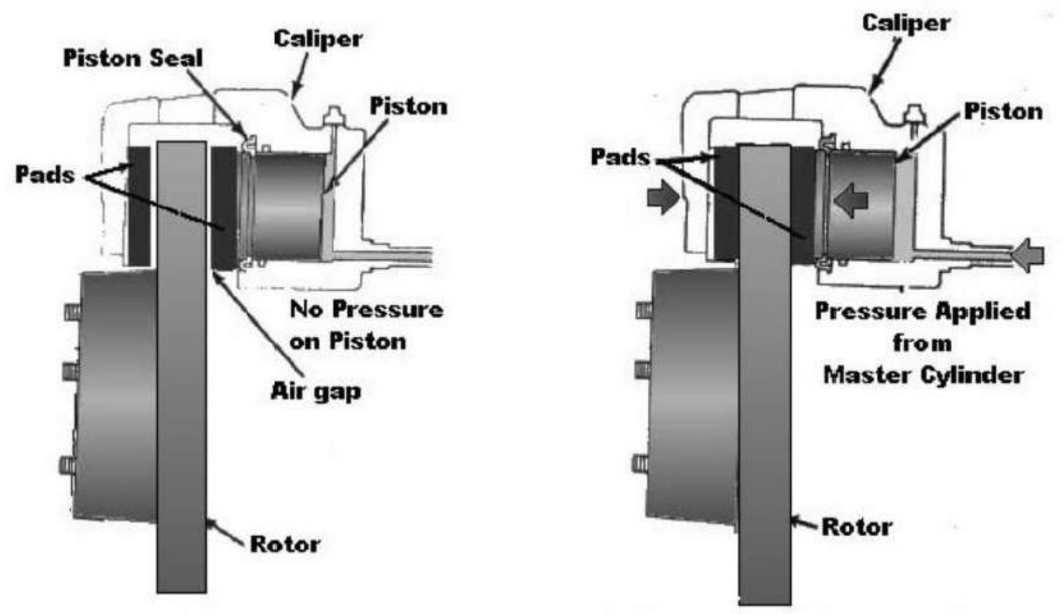
   (iv) **Piston:** It applies the brake force on the brake pads when brake lever is pressed.

   (v) **Slider pin:** It is the sliding pin which slides in the hole when brake is applied.

   (vi) **Dust boots:** It prevents the entry of dust into the caliper pin or slider pin hole.

3. **Disc Rotor:** It is the rotating part of disc brake. When brakes are applied, a lot of heat is generated which can decrease the braking efficiency, so the rotor has drilled vent holes on it which dissipates the heat.

Working



- **(i)** When brake pedal is pressed, the high pressure fluid from the master cylinder pushes the piston outward.
- **(ii)** The piston pushes the brake pad against the rotating disc.
- **(iii)** As the inner brake pad touches rotor, the fluid pressure exerts further force and the caliper moves inward and pulls the outward brake pad towards the rotating disc and it t uches the disc.
- **(iv)** Now both the brake pads are pushes the rotating disc, a large amount of friction is generated in between the pads and rotating disc and slows down the vehicle and finally let it stop.
- **(v)** When brake pad is released, the piston moves inward, the brake pad away from the rotating disc. And the vehicle again starts to move.

Advantages

1.It is lighter than drum brakes.

2. It has better cooling (because the braking surface is directly exposed to the air)

3. It offers better resistance to fade.

4. It provides uniform pressure distribution

5. By design they are self-adjusting brakes.

Disadvantages

1. It is costlier than drum brakes.
2. Higher pedal pressure is required for stopping the vehicle. This brake system is installed with vacuum booster.
3. No servo action is present.
4. It is difficult to attach a suitable parking attachment.

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Department of Mechanical Engineering

ME-602 Machine Component and Design

Unit – V
Journal Bearing

RGPV Syllabus :

Types of lubrication, viscosity, hydrodynamic theory, design factors, temperature and viscosity considerations, Reynold's equation, stable and unstable operation, heat dissipation and thermal equilibrium, boundary lubrication, dimensionless numbers, Design of journal bearings, Rolling-element Bearings: Types of rolling contact bearing, bearing friction and power loss, bearing life; Radial, thrust & axial loads; Static & dynamic load capacities; Selection of ball and roller bearings; lubrication and sealing.

Subject Notes

Introduction

A bearing is a machine element which supports another moving machine element (known as a journal). It permits a relative motion between the contact surfaces of the members while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as a lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicone oils, greases etc., may be used.

Reynold's equation:

The theoretical analysis of hydrodynamic lubrication is traced to Osborne Reynolds's study of the laboratory investigation of railroad bearings in England by Beauchamp Tower during the early 1880s (Fig. 5.1). The oil hole was drilled to test the effect of adding an oiler at this point. The tower was surprised to discover that when the test device was operated without the oiler installed, oil flowed out of the hole! He tried to block this flow by pounding cork and wooden stoppers into the hole, but the hydrodynamic pressure forced them out. At this point, Tower connected a pressure gage to the oil hole and subsequently made experimental measurements of the oil film pressures at various locations. He then discovered that the summation of the local hydrodynamic pressure time differential projected bearing area was equal to the load supported by the bearing.

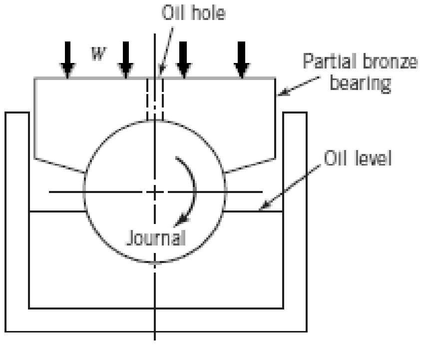


Fig. 5.1 Schematic representation of Beauchamp Tower's experiment

Reynolds's theoretical analysis led to his fundamental equation of hydrodynamic lubrication. The following derivation of the Reynolds equation applies to one-dimensional flow between flat plates. This analysis can also be applied to journal bearings because the journal radius is so large in comparison to oil film thickness. The assumed one-dimensional flow amounts to neglecting bearing side leakage and is approximately valid for bearings with L/D ratios greater than about 1.5. The derivation begins with the equation for the equilibrium of forces in the x-direction acting on the fluid element shown in Fig.

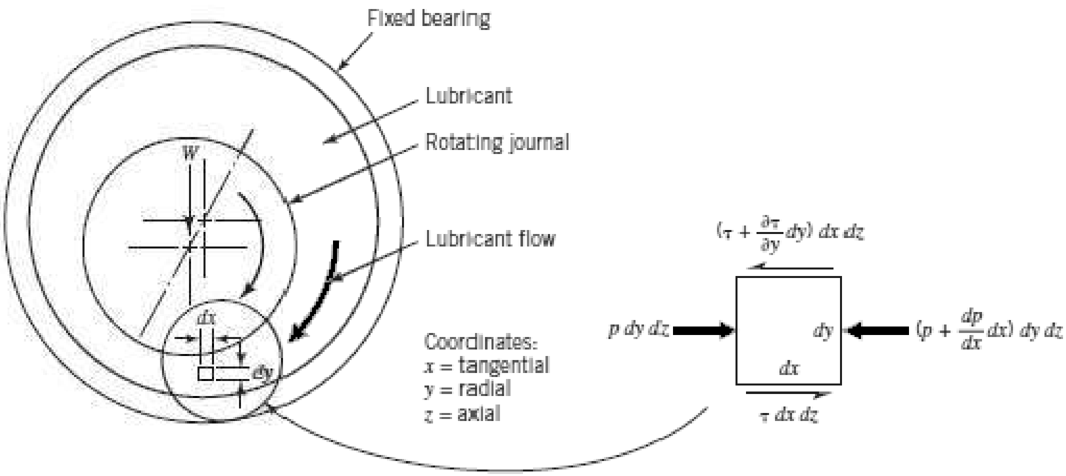


Fig. 5.2

The pressure and viscous forces acting on an element of lubricant, for simplicity, only X components are shown

pdy. dz + τ. dx. dz – [p + (dp / dx) dx] dy. dz – [τ + (δτ / δy) dy] dx. dz = 0 (a)

Above equation is Reynold's equation for one-dimensional flow. Summarizing the assumptions that were made: The fluid is Newtonian, incompressible, of constant viscosity, and experiences no inertial or gravitational forces; the fluid experiences laminar flow, with no slip at the boundary surfaces; the film is so thin that (1) it experiences negligible pressure variation over its thickness, and (2) the journal radius can be considered infinite in comparison.

$d / dx\, (h^3 / \mu .\ dp / dx) = 6U\, dh / dx$ ....(i)

When fluid flow in the z-direction is included (i.e., axial flow and end leakage), a similar development gives the Reynolds equation for two-dimensional flow:

$\delta / \delta x\, (h^3 / \mu .\ \delta p / \delta x) + \delta / \delta z\, (h^3 / \mu .\ \delta p / \delta z) = 6U\, \delta h / \delta x$ ....(ii)

Modern bearings tend to be shorter than those used a few decades ago. Ratios of length to diameter (L/D) are commonly in the range of 0.25 to 0.75. This results in the flow in the z-direction (and the end leakage) being a major portion of the total. For these short bearings, Ocvirk proposed neglecting the x term in the Reynolds equation, giving

$\delta / \delta z (h^3 / \mu . \delta p / \delta z) = 6U \delta h / \delta x$ ….(iii)

Unlike Equations (i), and (ii), Eq. (iii) can be readily integrated and thus used for design and analysis purposes. The procedure is commonly known as Ocvirk's short bearing approximation.

Heat dissipation and thermal equilibrium:

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$Q_g = \mu . W . V$ N-m/s or J/s or watts

Where μ = Coefficient of friction,

W = Load on the bearing in N,

= Pressure on the bearing in N/mm2 × Projected area of the bearing in mm2 = p (l × d),

V = Rubbing velocity in m/s = π d N/60, d is in meters, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size, and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$Q_d = C . A (t_b - t_a)$ J/s or W (1 J/s = 1 W)

Where C = Heat dissipation coefficient in W/m^2/°C,

A = Projected area of the bearing in m^2 = l × d,

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

The value of C has been determined experimentally by O. Lasche. The values depend on the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m^2/°C), for journal bearings, may be taken as follows:

For unventilated bearings (Still air)

= 140 to 420 W/m^2/°C

For well-ventilated bearings

= 490 to 1400 W/m^2/°C

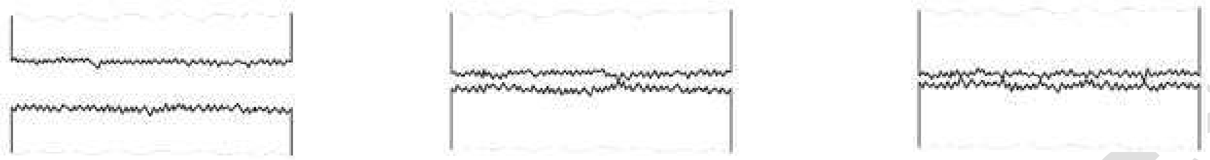
It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t0) and the temperature of the outside air (t_a). In other words,

$t_b - t_a = ½ (t_0 - t_a)$

Types of Lubrication:

Lubrication is commonly classified according to the degree with which the lubricant separates the sliding surfaces. Fig. 5.3 illustrates three basic cases.

1. In hydrodynamic lubrication, the surfaces are completely separated by the lubricant film. The load tending to bring the surfaces together is supported entirely by fluid pressure generated by the relative motion of the surfaces (as journal rotation). Surface wear does not occur, and friction losses originate only within the lubricant film. Typical film thicknesses at the thinnest point (designated h_0) are 0.008 to 0.020 mm (0.0003 to 0.0008 in.). Typical values of coefficient of friction (f) are 0.002 to 0.010.



(a) Hydrodynamic Lubrication (b) Mixed-Film Lubrication (c) Boundary Lubrication

Fig. 5.3

2. In mixed-film lubrication, the surface peaks are intermittently in contact, and there is partial hydrodynamic support. With proper design, surface wear can be mild. Coefficients of friction commonly range from 0.004 to 0.10.

3. In boundary lubrication, surface contact is continuous and extensive, but the lubricant is continuously "smeared" over the surfaces and provides a continuously renewed adsorbed sur ace film that reduces friction and wears. Typical values of f are 0.05 to 0.20.

The most desirable type of lubrication is obviously hydrodynamic, and this is treated in more detail beginning with the next section.

Complete surface separation (as in Figure 5.3 (a)) can also be achieved by hydrostatic lubrication. A highly pressurized fluid such as air, oil, or water is introduced into the load-bearing area. Since the fluid is pressurized by external means, full surface separation can be obtained whether or not there is relative motion between the surfaces. The principal advantage is extremely low friction at all times, including during starting and low-speed operation. Disadvantages are the cost, complication, and bulk of the external source of fluid pressurization. Hydrostatic lubrication is used only for specialized applications.

Boundary lubrication

"Boundary lubrication is lubrication by a liquid under conditions where the solid surfaces are so close together that appreciable contact between opposing asperities is possible. The friction and wear in boundary lubrication are determined predominantly by the interaction between the solids and between the solids and the liquid. The bulk flow properties of the liquid play little or no part in the friction and wear behavior. "

Characteristics required for Thin Film Lubrication:

1. Required long chain molecules, with an active end group, which by attaching itself to the solid surface builds a surface layer. A number of layers reduces lubrication in friction coefficient.

2. It should be dissolvable in mineral/lubricating oils.

3. Temperature stability: It is important because the increase in operating temperature may cause a reduction in a molecular attraction that may lead to detachment of boundary additives from surface.

In sliding contact under air or water, the protective oxide is torn away, exposing the pure metal of both surfaces. These may be weld toge her before oxygen can reform the protective layer. Therefore, boundary lubricants are required when metals are covered with a natural protective layer of oxide.

Dimensionless numbers:

The coefficient of friction in the design of bearings is of great importance because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e.

(i) ZN/p

(ii) d/c and

(iii) l/d

Therefore the coefficient of friction may be expressed as

$\mu = \varphi$ [ZN/p, d/c, l/d]

Where μ = Coefficient of friction,

φ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

N = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2,

= Load on the journal $\div$ l $\times$ d

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p will be termed as bearing characteristic number and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 5.4. The factor ZN/p helps to predict the performance of a bearing.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) is so low, or the pressure (p) is so great that their combination ZN / p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication -exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts.

It may be noted that the part PQ of the curve represents stable operating conditions since, from any point of stability, a decrease in viscosity (Z) will reduce ZN / p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z).

From Fig., we see that the minimum amount of friction occurs at A and at this point the value of ZN / p is known as bearing modulus which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear, and heating. In order to prevent such conditions, the bearing should be designed for a value of ZN / p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of ZN / p = 15 K may be used.

From above, it is concluded that when the value of ZN / p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN / p is less than K, then the oil film will rupture and there is a metal to metal contact.

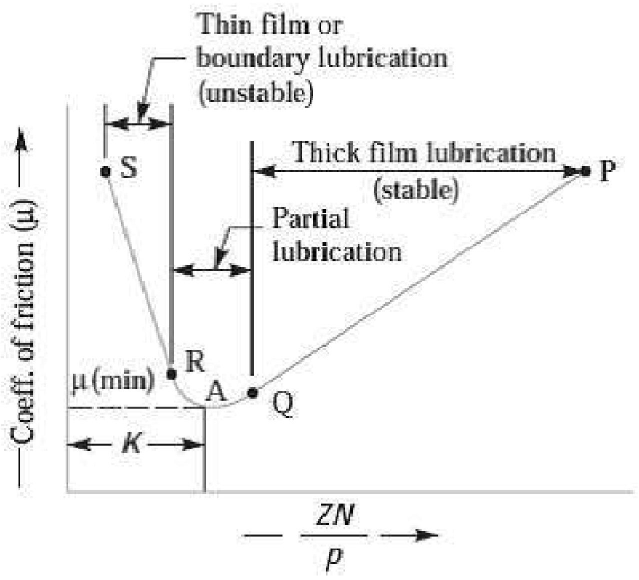


Fig. 5.4 Variation of coefficient of friction with ZN/p

Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well-lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data may be used.

Coefficient of friction,

$\mu = 33/10^8$ (ZN/p) (d/c) + k ... (when Z is in kg / m-s and p is in N / mm^2)

Where Z, N, p, d, and c have usual meanings as discussed in the previously, and

k = Factor to correct for end leakage.

It depends upon the ratio of length to the diameter of the bearing (i.e. l/d).

= 0.002 for l / d ratios of 0.75 to 2.8.

Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins is known as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e.

Critical pressure or minimum operating pressure,

$p = (ZN / 4.75 \times 10^6) (d / c)^2 \times [l / (d + l)]$ ...(when Z is in kg / m-s)

Design of journal bearings

Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

Sommerfeld number = $(ZN / p) (d / c)^2$

For design purposes, its value is taken as follows:

$(ZN / p) (d / c)^2 = 14.3 \times 10^6$ ... (when Z is in kg / m-s and p is in N / mm^2)

Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.
1. Determine the bearing length by choosing a ratio of l / d from the table.
2. Check the bearing pressure, p = W / l. d from the table for probable satisfactory value.
3. Assume a lubricant from the table and its operating temperature (t_0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high-temperature installations such as steam turbines.
4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in the table, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c / d from the table.
6. Determine the coefficient of friction (μ).
7. Determine the heat generated.
8. Determine the heat dissipated.
9. Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

Rolling-element Bearings

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called anti-friction bearings.

Advantages and Disadvantages of Rolling Contact Bearings over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. The accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

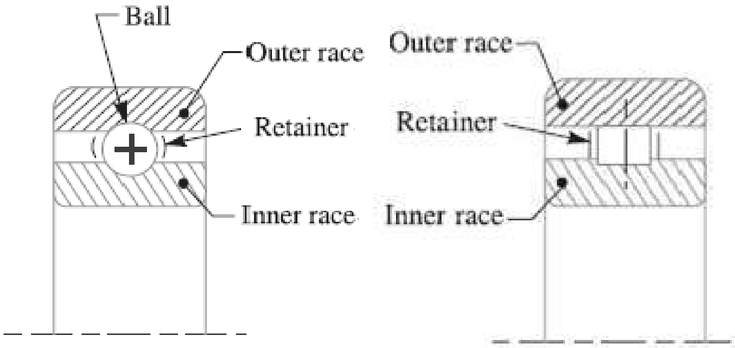
Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

Types of rolling contact bearing:

Following are the two types of rolling contact bearings:

1. Ball bearings; and 2. Roller bearings



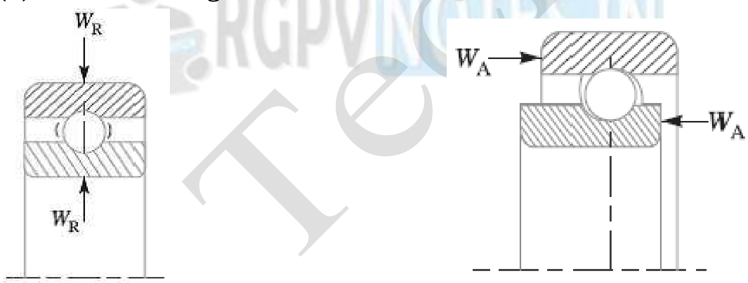
(a) Ball Bearing (b) Roller Bearing

Fig. 5.5 Ball and roller bearings

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 5.5. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and are usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as:

(a) Radial bearings, and (b) Thrust bearings.



(a) Radial Bearing (b) Thrust Bearing

Fig. 5.6 Radial and thrust ball bearings

The radial and thrust ball bearings are shown in Fig. 5.6 (a) and 5.6 (b) respectively. When a ball bearing supports only a radial load (W_R), the plane of rotation of the ball is normal to the center line of the bearing, as shown in Fig. 5.6 (a). The action of thrust load (W_A) is to shift the plane of rotation of the balls, as shown in Fig. 5.6. The radial and thrust loads both may be carried simultaneously.

Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. Single row deep groove bearing. A single row deep groove bearing is shown in Fig. 5.7. During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races. The races are then centered and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

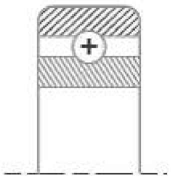


Fig. 5.7 Single Row Deep-Groove Bearing

2. Filling notch bearing. A filling notch bearing is shown in Fig. 5.8. These bearings have notches in the inner and outer races which permit more balls to be inserted than in deep groove ball bearings. The notches do not extend to the bottom of the raceway and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains a larger number of balls than a corresponding un-notched one, therefore it has a larger bearing load capacity.

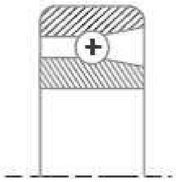


Fig. 5.8 Filling Notch Bearing

3. Angular contact bearing. An angular contact bearing is shown in Fig. 5.9. These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

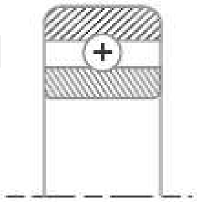


Fig. 5.9 Angular Contact Bearing

4. Double row bearing. A double row bearing is shown in Fig. 5.10. These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

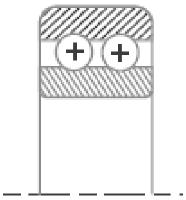


Fig. 5.10 Double Row Bearing

5. Self-aligning bearing. A self-aligning bearing is shown in Fig. 5.11. These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing is too small to

accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings:
(a) Externally self-aligning bearing, and (b) Internally self-aligning bearing

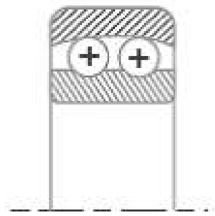
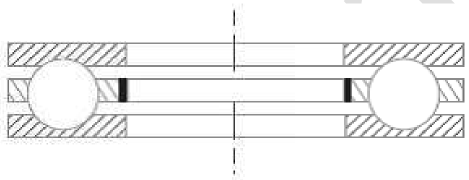


Fig. 5.11 Self-aligning Bearing

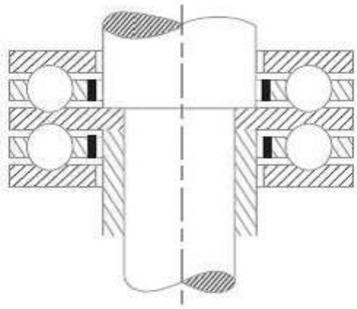
In an externally self-aligning bearing, the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in housing, as shown in Fig. 5.11. In case of internally self-aligning bearing, the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

Thrust Ball Bearings

The thrust ball bearings are used for carrying thrust loads exclusively and at speeds below 2000 r.p.m. At high speeds, centrifugal force causes the balls to be forced out of the races. Therefore at high speeds, it is recommended that angular contact ball bearings should be used in place of thrust ball bearings.



(a) Single Direction Thrust Ball Bearing



(b) Double Direction Thrust Ball Bearing

Fig. 5.12 Thrust ball bearing

A thrust ball bearing may be a single direction, flat face as shown in Fig. 5.12 (a) or a double direction with a flat face as shown in Fig. 5.12 (b).

Types of Roller Bearings

Following are the principal types of roller bearings:

1. Cylindrical roller bearings. A cylindrical roller bearing is shown in Fig. 5.13. These bearings have short rollers guided in a cage. These bearings are relatively rigid against the radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such types of bearings are used in high-speed service.

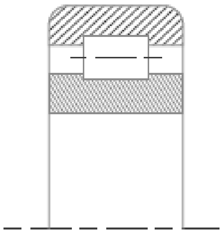


Fig. 5.13 Cylinder Roller Bearing

2. Spherical roller bearings. A spherical roller bearing is shown in Fig. 5.14. These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of a sphere. These bearings can normally tolerate angular misalignment in the order of ± 1½°and when used with a double row of rollers, these can carry thrust loads in either direction.

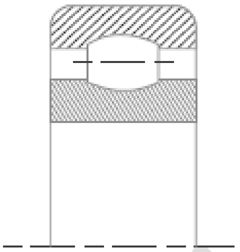


Fig. 5.14 Spherical Roller Bearing

3. Needle roller bearings. A needle roller bearing is shown in Fig. 5.15. These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy-duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

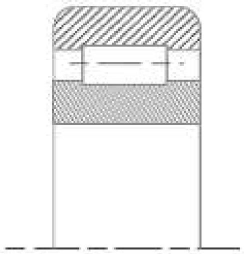


Fig. 5.15 Needle Roller Bearing

4. Tapered roller bearings. A tapered roller bearing is shown in Fig. 5.16. The rollers and raceways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

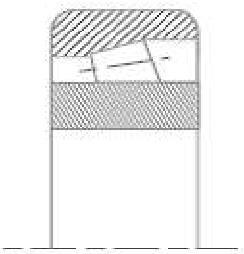


Fig. 5.16 Tapered Roller Bearing

Bearing friction and power loss:

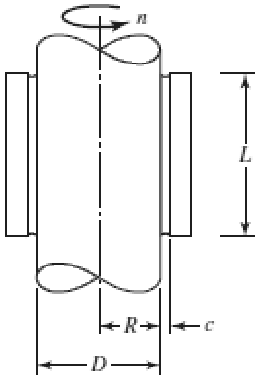
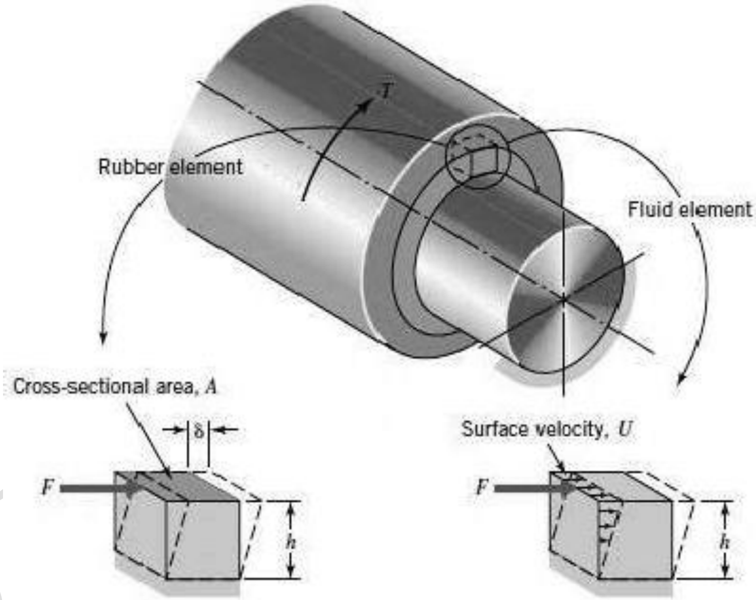


Fig. 5.17 Unloaded journal bearing used for Petroff's analysis

The original analysis of viscous friction drag in what is now known as a hydrodynamic bearing (above Fig. 5.17) is credited to Petroff and was published in 1883. It applies to the simplified "ideal" case of no eccentricity between bearing and journal, and hence, no "wedging action" and no ability of the oil film to support a load, and no lubricant flow in the axial direction.



$\delta = Fh/AG$ $\qquad$ $F = Fh/A\mu$

Where G = Shear Modulus $\qquad$ Where μ = Absolute Viscosity

Fig. 5.18 Analogy between shear modulus of elasticity (of a solid) and viscosity (of a fluid)

With reference to above Fig. 5.18 (c), an expression for viscous friction drag torque is derived by considering the entire cylindrical oil film as the "liquid block" acted upon by force F. Solving the equation given in the figure for F gives

$F = \mu A U/h$

Where

F = friction torque/shaft radius = T f /R
A = 2pRL
U = 2pRn (where n is in revolutions per second)
h = c where, c = radial clearance = (bearing diameter – shaft diameter) b/2
Substituting and solving for friction torque gives
$T_f = 4\pi\ 2\mu.\ n.\ L.\ R^3 / c$
If a small radial load W is applied to the shaft, the frictional drag force can be considered equal to the product f_W, with friction torque expressed as
T_f = f W R = f (DLP) R
Where P is the radial load per unit of projected bearing area. The imposition of load W will, of course, cause the shaft to become somewhat eccentric in the bearing. If the effect of this on Eq. b can be considered negligible, Eq. b, and c can be equated to give
f = 2π2 (μ n/p) (R/c)
This is the well-known Petroff equation. It provides a quick and simple means of obtaining reasonable estimates of coefficients of friction of lightly loaded bearings. Note that Petroff's equation identifies two very important bearing parameters. The significance of μ n/p and the ratio R/c is on the order of 500 to 1000 and is the inverse of the clearance ratio.

Bearing life:
The life of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements.
The rating life of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 percent of a group of bearings will complete or exceed before the first evidence of fatigue develops (i.e. only 10 percent of a group of bearings fail due to fatigue).
The term minimum life is also used to denote the rating life. It has been found that the life which 50 percent of a group of bearings will complete or exceed is approximately 5 times the life which 90 percent of the bearings will complete or exceed. In other words, we may say that the average life of a bearing is 5 times the rating life (or minimum life). It may be noted that the longest life of a single bearing is seldom longer than the 4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

Static load capacities:
Basic Static Load Rating of Rolling Contact Bearings
The load carried by a non-rotating bearing is called a static load. The **basic static load rating** is defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which corresponds to a total permanent deformation of the ball (or roller) and race, at the most heavily stressed contact, equal to 0.0001 times the ball (or roller) diameter.
In single row angular contact ball bearings, the basic static load relates to the radial component of the load, which causes a purely radial displacement of the bearing rings in relation to each other.
According to IS: 3823–1984, the basic static load rating (C_0) in Newton for the ball and roller bearings may be obtained as discussed below:

1. For radial ball bearings, the basic static radial load rating (C_0) is given by

$$C_0 = f_0.i.Z.D^2 \cos \alpha$$

Where i = Number of rows of balls in any one bearing,

Z = Number of ball per row,

D = Diameter of balls, in mm,

α = Nominal angle of contact i.e. the nominal angle between the line of action of the ball load and a plane perpendicular to the axis of bearing, and

f_0 = A factor depending upon the type of bearing.

The value of factor (f_0) for bearings made of hardened steel is taken as follows:

f_0 = 3.33, for self-aligning ball bearings

= 12.3, for radial contact and angular contact groove ball bearings.

2. For radial roller bearings, the basic static radial load rating is given by

$$C_0 = f_0.i.Z.l_e.D \cos \alpha$$

Where i = Number of rows of rollers in the bearing,

Z = Number of rollers per row,

l_e = Effective length of contact between one roller and that ring (or washer)

Where the contact is the shortest (in mm). It is equal to the overall length of roller minus roller chamfers or grinding undercuts,

D = Diameter of the roller in mm. It is the mean diameter in case of tapered rollers,

α = Nominal angle of contact. It is the angle between the line of action of the roller resultant load and a plane perpendicular to the axis of the bearing, and

f_0 = 21.6, for bearings made of hardened steel.

3. For thrust ball bearings, the basic static axial load rating is given by

$$C_0 = f_0.Z.D^2 \sin \alpha$$

Where Z = Number of balls carrying thrust in one direction, and

f_0 = 49, for bearings made of hardened steel.

4. For thrust roller bearings, the basic static axial load rating is given by

$$C_0 = f_0.Z.l_e.D.\sin \alpha$$

Where Z = Number of rollers carrying thrust in one direction, and

f_0 = 98.1, for bearings made of hardened steel.

Static Equivalent Load for Rolling Contact Bearings

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that which occurs under the actual conditions of loading.

The static equivalent radial load (W_{0R}) for radial or roller bearings under combined radial and axial or thrust loads is given by the greater magnitude of those obtained by the following two equations, i.e.

1. $W_{0R} = X_0.W_R + Y_0.W_A$; and **2.** $W_{0R} = W_R$

Where W_R = Radial load,

W_A = Axial or thrust load,
X_0 = Radial load factor, and
Y_0 = Axial or thrust load factor.

According to IS: 3824 – 1984, the values of X0 and Y0 for different bearings are given in the following table:

S. No.	Type of Bearing	Single Row Bearing		Double Row Bearing	
		X_0	Y_0	X_0	Y_0
01.	Radial contact groove ball bearings	0.60	0.50	0.60	0.50
02.	Self-aligning ball or roller bearings and tapered bearing	0.50	0.22 cot θ	1	0.44 cot θ
03.	Angular contact groove bearings:				
	α = 15°	0.50	0.46	1	0.92
	α = 20°	0.50	0.42	1	0.84
	α = 25°	0.50	0.38	1	0.76
	α = 30°	0.50	0.33	1	0.66
	α = 35°	0.50	0.29	1	0.58
	α = 40°	0.50	0.26	1	0.52
	α = 45°	0.50	0.22	1	0.44

Dynamic load capacities:

Basic Dynamic Load Rating of Rolling Contact Bearings

The basic dynamic load rating is defined as the constant stationary radial load (in case of radial ball or roller bearings) or constant axial load (in case of thrust ball or roller bearings) which a group of apparently identical bearings with stationary outer ring can endure for a rating life of one million revolutions (which is equivalent to 500 hours of operation at 33.3 r.p.m.) with only 10 percent failure.

The basic dynamic load rating (C) in Newton for the ball and roller bearings may be obtained as discussed below :

1. According to IS: 3824 (Part 1)– 1983, the basic dynamic radial load rating for radial and angular contact ball bearings, except the filling slot type, with balls not larger than 25.4 mm in diameter, is given by

$$C = f_c\,(i \cos \alpha)^{0.7}\, Z^{2/3}.\, D^{1.8}$$

And for balls larger than 25.4 mm in diameter,

$$C = 3.647\, f_c\,(i \cos \alpha)^{0.7}\, Z^{2/3}.\, D^{1.4}$$

Where f_c = A factor, depending upon the geometry of the bearing components, the accuracy of manufacture and the material used.

2. According to IS: 3824 (Part 2)–1983, the basic dynamic radial load rating for radial roller bearings is given by

$$C = f_c\,(i.l_e \cos \alpha)^{7/9}\, Z^{3/4}.\, D^{29/27}$$

3. According to IS: 3824 (Part 3)–1983, the basic dynamic axial load rating for single row, single or double direction thrust ball bearings is given as follows:

(a) For balls not larger than 25.4 mm in diameter and $\alpha = 90^\circ$,

$$C = f_c \cdot Z^{2/3} \cdot D^{1.8}$$

(b) For balls not larger than 25.4 mm in diameter and $\alpha \neq 90^\circ$,

$$C = f_c (\cos\alpha)^{0.7} \tan\alpha \cdot Z^{2/3} \cdot D^{1.8}$$

(c) For balls larger than 25.4 mm in diameter and $\alpha = 90^\circ$

$$C = 3.647 f_c \cdot Z^{2/3} \cdot D^{1.4}$$

(d) For balls larger than 25.4 mm in diameter and $\alpha \neq 90^\circ$,

$$C = 3.647 f_c (\cos\alpha)^{0.7} \tan\alpha \cdot Z^{2/3} \cdot D^{1.4}$$

4. According to IS: 3824 (Part 4)–1983, the basic dynamic axial load rating for single row, single or double direction thrust roller bearings is given by

$$C = f_c \cdot l_e^{7/9} \cdot Z^{3/4} \cdot D^{29/27} \quad \ldots \text{(when } \alpha = 90^\circ\text{)}$$

$$= f_c (l_e \cos\alpha)^{7/9} \tan\alpha \cdot Z^{3/4} \cdot D^{29/27} \quad \ldots \text{(when } \alpha \neq 90^\circ\text{)}$$

Dynamic Equivalent Load for Rolling Contact Bearings

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$W = X \cdot V \cdot W_R + Y \cdot W_A$

Where V = A rotation factor,

= 1, for all types of bearings, when the inner race is rotating,

= 1, for self-aligning bearings, when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

Selection of ball and roller bearings:

In order to select a most suitable ball bearing, first of all, the basic dynamic radial load is calculated. It is then multiplied by the service factor (K_S) to get the design basic dynamic radial load capacity. The service factor for the ball bearings is shown in the following table.

S. No.	Type of service	Service factor (KS) for radial ball bearings
01.	Uniform and steady load	1.0
02.	Light shock load	1.5
03.	Moderate shock load	2.0
04.	Heavy shock load	2.5
05.	Extreme shock load	3.0

Lubrication and sealing:

The ball and roller bearings are lubricated for the following purposes:

1. To reduce friction and wear between the sliding parts of the bearing,
2. To prevent rusting or corrosion of the bearing surfaces,
3. To protect the bearing surfaces from water, dirt etc., and
4. To dissipate the heat.

In general, oil or light grease is used for lubricating ball and roller bearings. Only pure mineral oil or a calcium-base grease should be used. If there is a possibility of moisture contact, then potassium or sodium-base greases may be used. Another additional advantage of the grease is that it forms a seal to keep out dirt or any other foreign substance. It may be noted that too much oil or grease cause the temperature of the bearing to rise due to churning. The temperature should be kept below 90ºC and in no case, a bearing should operate above 150° C.

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