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New Scheme Based On AICTE Flexible Curricula

B. Tech. First Year

Branch- Common to All Disciplines

BT201	**Engineering Physics**	**2L-1T-2P**	**4 Credits**

Course Contents:

Module 1: Wave nature of particles and the Schrodinger equation (8 lectures)

Introduction to Quantum mechanics, Wave nature of Particles, operators ,Time-dependent and time-independent Schrodinger equation for wavefunction, Application: Particle in a One dimensional Box, Born interpretation, Free-particle wavefunction and wave-packets, v_g and v_p relation Uncertainty principle.

Module 2: Wave optics (8 lectures)

Huygens' principle, superposition of waves and interference of light by wave front splitting and amplitude splitting; Young's double slit experiment, Newton's rings, Michelson interferometer, Mach-Zehnder interferometer.

Farunhofer diffraction from a single slit and a circular aperture, the Rayleigh criterion for limit of resolution and its application to vision; Diffraction gratings and their resolving power.

Module 3: Introduction to solids (8 lectures)

Free electron theory of metals, Fermi level of Intrinsic and extrinsic, density of states, Bloch's theorem for particles in a periodic potential, Kronig-Penney model(no derivation) and origin of energy bands. V-I characteristics of PN junction, Zener diode, Solar Cell, Hall Effec .

Module 4: Lasers (8 lectures)

Einstein's theory of matter radiation interaction and A and B coefficients; amplification of light by population inversion, different types of lasers: gas lasers (He-Ne, CO_2), solid-state lasers(ruby, Neodymium),Properties of laser beams: mono-chromaticity, coherence, directionality and brightness, laser speckles, applications of lasers in science, engineering and medicine. Introduction to Optical fiber, acceptance angle and cone, Numerical aperture, V number, attenuation.

Module 5: Electrostatics in vacuum (8 lectures)

Calculation of electric field and electrostatic potential for a charge distribution; Electric displacement, Basic Introduction to Dielectrics, Gradient, Divergence and curl,Stokes' theorem, Gauss Theorem, Continuity equation for current densities; Maxwell's equation in vacuum and non-conducting medium; Poynting vector.

List of Experiment

1. To determine the dispersive power of prism.
2. To determine the λ of sodium light with the help of newton's Ring.
3. Resolving Power of Telescope.
4. YDSE (Young's double slit Experiment).
5. To determine the frequency of AC mains supply.
6. V-I Characteristics of P-N junction diode.
7. To determine the λ of diode loses by single slit diffraction.
8. To determine the plank's constant with the help of photocell.
9. Hall's effect experiment.
10. Calibration of ammeter by using reference zener diode.

11. To study the effect of temperature on reverse saturation current in P-N junction diode and to determine the energy band gap.
12. To determine the ʌ of sodium by using plane diffraction grating.
13. To determine the prominent lines of mercury source by plane diffraction grating.
14. To determine the numerical aperture of an optical fiber.
15. To determine ʌ of given laser by plane diffraction grating.

Suggested Reference Books

1. A. Ghatak, Optics.
2. O. Svelto, Principles of Lasers.
3. David Griffiths, Introduction to Electrodynamics.
4. D.J. Griffiths, Quantum Mechanics.
5. Halliday & Resnick, Physics.

UNIT-1
QUANTUM MECHANICS

Q.1 What do you understand by matter waves? **(Feb10)**
Or
Discuss the concept of de-Broglie matter waves. **(June 15)**

Ans: The optical phenomenon, such as interference, diffraction and polarization of light could be explained by wave theory of light; whereas photoelectric effect or Compton Effect of light could only be explained if we consider the light as particle. Hence light shows itself wave nature at one end while particle nature on the other hand.

This nature of light is known as dual nature and the property is known as wave particle duality.

In 1924 Louis de-Broglie proposed that the matter also possess dual character like light. His concept about the dual nature of matter was based on the following facts:

(i) Matter and light both are forms of energy and each of them can be transformed into the other.

(ii) Both are governed by the space time symmetries of the theory of the relativity.

According to Louis de-Broglie, a moving particle is surrounded by a wave whose wavelength depends upon the mass of the particle and its velocity. These waves associated with the matter particles are known as matter waves or de-Broglie waves.

De-Broglie provided a connection between, the wavelength of matter waves and momentum of the particle i.e. $\lambda = \frac{h}{p}$ (1)

Properties of Matter-waves

1. Matter-waves are associated with any moving body and their wavelength is given by $\lambda = \frac{h}{mv}$
2. The wavelength of matter-waves is inversely proportional to the velocity of the body. Hence, a body at rest has an infinite wavelength whereas the one traveling with a high velocity has a lower wavelength.
3. Wavelength of matter-waves depends on the mass of the body and decreases with increase in mass. Because of this, the wave-like behavior of heavier objects is not very evident whereas the wave nature of subatomic particles can be observed experimentally.
4. Amplitude of the matter-waves at a particular space and time depends on the probability of finding the particle at that space and time.
5. Unlike other waves, there is no physical quantity that varies periodically in the case of matter waves.
6. Matter waves are represented by a wave packet made up of a group of waves of slightly differing wavelengths. Hence, we talk of group velocity of matter waves rather than the phase velocity.
7. Matter-waves show similar properties as other waves such as interference and diffraction.

Q.2 Define group velocity and particle velocity in reference of a group wave. **(Dec 10)**
Or
Define particle velocity, group velocity and phase velocity. **(Dec 11,14, 15)**

Ans: **Group velocity:** When two or more than two waves travel in a medium then the superposition of these waves result in the formation of a wave packet. The velocity with which a wave packet advances in medium is called group velocity. The group velocity of the wave packet is given by-

$v_g = \frac{d\omega}{dk}$

Phase velocity or wave velocity: - when a monochromatic wave (wave of a single frequency or wavelength) travels through a medium, the velocity with which the wave advances in the forward direction of the medium is called wave or phase velocity. The phase velocity of the matter wave is given by- $v_p = \frac{\omega}{k} = \frac{2\pi u}{\frac{2\pi}{\lambda}} = \nu\lambda$

Particle velocity: Particle velocity is the velocity of the material particle moving in medium or space for a matter wave group velocity equals particle velocity. It is denoted by v.

Q.3 Derive mathematical expression for group and phase velocity. **(Dec 15)**

Ans: *(Include the definitions of phase and group velocities as above)*

Expression for group velocity: - When two or more than two waves travel in a medium then the superposition of these waves result in the formation of a wave packet. The velocity with which a wave packet advances in medium is called group velocity. Let us consider a wave group which consists of two harmonic waves of equal amplitude but slightly different frequencies ω_1 & ω_2 and propagation constants k_1 & k_2. Their separate displacements are given by-

$y_1 = a\sin(\omega_1 t - k_1 x)$ (1)

And $y_2 = a\sin(\omega_2 t - k_2 x)$ (2)

Their superposition gives-

$$y = y_1 + y_2$$

Or

$$Y = a[\sin(\omega_1 t - k_1 x) + (\omega_2 t - k_2 x)]$$

$$Y = 2a\sin\left[\frac{\omega_1 + \omega_2}{2}t - \frac{k_1 + k_2}{2}x\right]\cos\left[\frac{\omega_1 - \omega_2}{2}t - \frac{k_1 - k_2}{2}x\right]$$

As $\omega_1 \sim \omega_2 \sim \omega$ *and* $k_1 \sim k_2 \sim k$

Therefore $\frac{\omega 1 + \omega 2}{2} = \omega$ and $\frac{k1 + k2}{2} = k,$ $\omega_1 - \omega_2 = \Delta\omega$ *and*

$$k_1 - k_2 = \Delta k$$

Hence $Y = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]\sin(\omega t - kx)$ (3)

This represents a wave system with frequency ω and propagation constant k but with amplitude given by

$A = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$ (4)

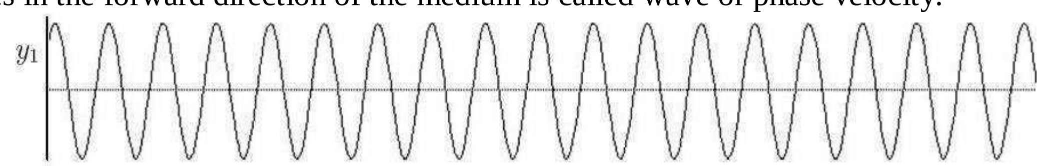
The wave system of the equation 3 can be represented as

The velocity of above wave system is called group velocity and represented by

$$v_g = \frac{\Delta\omega}{\Delta k}$$

If the frequency interval between the harmonics of the wave packet is infinitely small then $v_g = \frac{d\omega}{dk}$ (5)

Expression for Phase velocity or wave velocity: - when a monochromatic wave (wave of a single frequency or wavelength) travels through a medium, the velocity with which the wave advances in the forward direction of the medium is called wave or phase velocity.



	A plane harmonic wave travelling along +ve x-direction is given by- $$y = a\sin(\omega t - kx) \qquad (1)$$ Where „a" is amplitude, $\omega(=2\pi u)$ is the angular frequency and $k(=\frac{2\pi}{\lambda})$ is the propagation constant of the wave. The quantity $(\omega t - kx)$ in the wave equation is called the phase of the wave. The planes of constant phase are defined by- $$(\omega t - kx) = constant$$ Differentiating above wrt time, we get- $$\omega - k\frac{dx}{dt} = 0 \qquad (2)$$ Or $$\frac{dx}{dt} = v_p = \frac{\omega}{k} = \frac{2\pi u}{\frac{2\pi}{\lambda}} = v\lambda \qquad (3)$$ Thus velocity with which planes of constant phase advances through the medium is equal to the wave velocity.
Q.4	Show that the phase velocity of matter waves exceeds the velocity of light.
Ans:	The velocity with which planes of constant phase advances through the medium equal to the wave velocity or phase velocity and is given as $v_p = \frac{\omega}{k} = \frac{2\pi u}{\frac{2\pi}{\lambda}} = v\lambda$. As we know that $\omega = 2\pi\nu = \frac{2\pi mc^2}{h}$ By the theory of special relativity $m = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ Therefore $\omega = \frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$ And $k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h}$ $$k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$ As $v_p = \frac{\omega}{k}$ Or $v_p = \frac{\frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}}{\frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}}$ Or $v_p = \frac{c^2}{v}$ Since no material particle can travel faster than the velocity of light, therefore phase velocity of matter wave is greater than the velocity of light.
Q.5	Give the relation between group velocity and particle velocity. **(Dec 10, 14, 15, 17)** Or Shoe that the group velocity of matter waves equals particle velocity. Or Establish relation between particle and group velocities.
Ans:	**Proof group velocity equals particle velocity-** Group velocity of a matter wave is given as $v_g = \frac{d\omega}{dk}$ or $v_g = \frac{\frac{d\omega}{dv}}{\frac{dk}{dv}}$ $$\frac{d\omega}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 c^2}{h}\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]$$

	$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h} [-\frac{1}{2}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}(-\frac{2v}{c^2})]$ $\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h} [(1-\frac{v^2}{c^2})^{-\frac{3}{2}}]$ Now $\frac{dk}{dv} = \frac{d}{dv}[\frac{2\pi m_0 v}{h}(1-\frac{v^2}{c^2})^{-\frac{1}{2}}]$ Or $\frac{dk}{dv} = [\frac{2\pi m_0}{h}\{v(-\frac{1}{2}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}(-\frac{2v}{c^2})) + (1-\frac{v^2}{c^2})^{-\frac{1}{2}}\}]$ $\frac{dk}{dv} = [\frac{2\pi m_0}{h}\{((1-\frac{v^2}{c^2})^{-\frac{3}{2}}(\frac{v^2}{c^2})) + (1-\frac{v^2}{c^2})^{-\frac{1}{2}}\}]$ $\frac{dk}{dv} = [\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}\{(\frac{v^2}{c^2}) + (1-\frac{v^2}{c^2})\}]$ $\frac{dk}{dv} = [\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}]$ Therefore $v_g = \frac{\frac{2\pi m_0 v}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}}{\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}}$ or $v_g = v$
Q.6	Show that for non-relativistic free particle the phase velocity is half of the group velocity.**(Jan 16, Jun 13 Dec 15)**
Ans:	As we know that for a non relativistic free particle its total energy is $E = \frac{1}{2}mv^2$ also $v_g = v$ Therefore $E = \frac{1}{2}mv_g^2$ as $E = h\nu$ Hence $h\nu = \frac{1}{2}mv_g^2$ For phase velocity of matter waves $v_p = \nu\lambda$ or $h\frac{v_p}{\lambda} = \frac{1}{2}mv_g^2$ According to de-Broglie‟s hypothesis $\lambda = \frac{h}{mv}$ or $\lambda = \frac{h}{mv_g}$ Therefore $v_p = \frac{1}{2}v_g$
Q.7	Show that group velocity is less than the phase velocity in dispersive medium. **(Jun 14)**
Ans:	**Relation between Phase and Group velocity:** The phase velocity of a wave is given by $v_p = \frac{\omega}{k}$ and $v_g = \frac{d\omega}{dk}$

$$v_g = \frac{d}{dk}(k v_p)$$

Or $$v_g = k v_p + k\frac{dv_p}{dk}$$

As $$k = \frac{2\pi}{\lambda}$$

$$v_g = k v_p + \frac{2\pi}{\lambda}\frac{dv_p}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$v_g = k v_p + \frac{1}{\lambda}\frac{dv_p}{d\left(\frac{1}{\lambda}\right)}$$

Therefore $$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

This is the desired relation between group velocity v_g phase velocities v_p **.**

Q.8 Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)**

Or

Prove that electrons do not exist inside the nucleus using uncertainty principle. **(Apr 10)**

Ans: **Heisenberg's Uncertainty Principle:** According to Heisenberg‟s uncertainty principle "it is impossible to determine simultaneously the position and momentum of any particle with any desired accuracy, rather the product of uncertainties in the position and momentum is always greater than or equal to h/4π, where h is Planck‟s constant.

Mathematically $\Delta x \Delta p \geq \frac{h}{4\pi}$ Where Δx Uncertainty in the position

Δp Uncertainty in the momentum

Like position and momentum, other two pairs of conjugate physical quantities are *time* and *energy*, and orbital angular momentum and angular position, the Heisenberg‟s uncertainty principle follows for these two pairs also and can be written as $\Delta E \Delta t \geq \frac{h}{4\pi}$ and $\Delta L \Delta \geq \frac{h}{4\pi}$

Where ΔE uncertainty in the Energy

Δt Uncertainty in the time

ΔL Uncertainty in the orbital angular momentum

Δ Uncertainty in the angular position

Non Existence of Electrons in the Nucleus

The size of a typical nucleus is of the order of 10^{-14}m. If any particle is to exist within nucleus then the uncertainty in the position of the particle will be-

$$\Delta x = 2 \text{ X } 10^{-14} m$$

Then according the uncertainty principle the uncertainty in the momentum will be given as

$$\Delta p = \frac{h}{2 \text{ X } 10^{-14} m} = \frac{6.6 \text{ X } 10^{-34}}{2 \text{ X } 10^{-14} m} = 3.31 \text{ X } 10^{-20}\ Kg\ m/sec$$

Thus the magnitude of the momentum of the particle must be at least of this order. Then the relativistic energy of electron in the nucleus will be

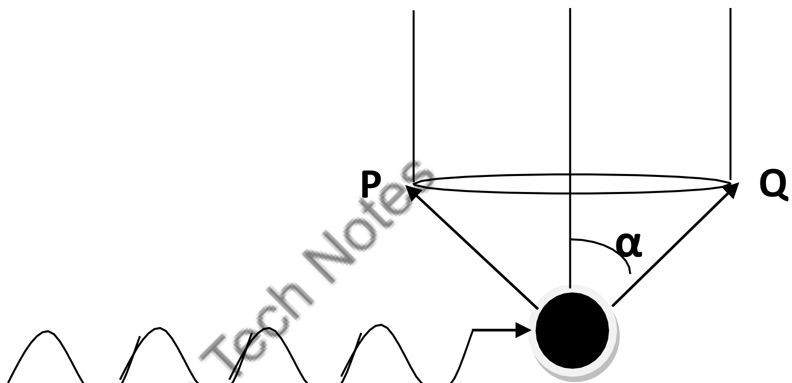
$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

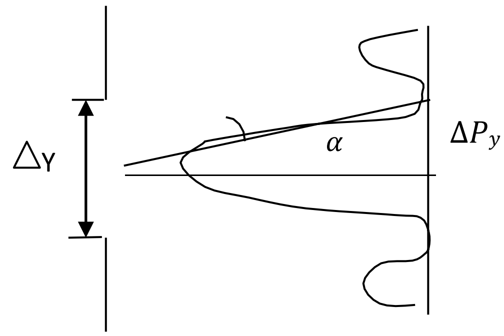
$$= \sqrt{(9.1 \text{ X } 10^{-31})^2 \text{X } (3 \text{ X } 10^8)^4 + (3.31 \text{ X } 10^{-20})^2 \text{X } (3 \text{ X } 10^8)^2}$$

$$= \sqrt{(6.7 \text{ X } 10^{-27}) + (98.6 \text{ X } 10^{-24})}$$

$$= \sqrt{(9860 + 0.67) \text{X } 10^{-26}}$$

$$= 99.3 \text{ X } 10^{-13}\ Joules$$

	$= 62\ MeV.$ Thus if electron resides insides the nucleus, it must possess energy of the order of 62 MeV while the electrons emitted in beta decay have only the energies of the order of 3 MeV. The big mismatch in two values confirms that electron does not reside inside the nucleus.
Q.9	Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)** Or Derive Heisenberg principle from hypothetical gamma ray microscope. **(June 13)**
Ans.	**Determination of position of electron by gamma ray microscope** Consider a free electron beneath the center of the microscope's lens The circular lens forms a cone of angle 2α from the electron. The electron is then illuminated from the left by gamma rays--high energy light which has the shortest wavelength. According to a principle of wave optics, the microscope can resolve objects to a size of Δx, which is related to the wavelength λ of the gamma ray, by the expression: $\Delta x = \lambda / (2\sin A)$  To be observed by the microscope, the gamma ray must be scattered into any angle within the cone of angle 2α. In quantum mechanics, the gamma ray carries momentum, as if it were a particle. The total momentum p is related to the wavelength by the formula p = h / λ, where h is Planck's constant. In the extreme case of diffraction of the gamma ray to the right edge of the lens, i.e. at the point Q then the momentum in the x direction would be (h/λ) sinα , while at another extreme at point P the momentum in the x direction would be (-h/λ) sinα The total change in the momentum of the photon will be $\Delta p = (h/\lambda)\sin\alpha - (-(h/\lambda)\sin\alpha)$ $\Delta p = (2h/\lambda)\sin\alpha$ $\Delta x\Delta p = h$ We obtain a reciprocal relationship between the minimum uncertainty in the measured position, Δx, of the electron along the x axis and the uncertainty in its momentum, Δp, in the x direction this is in accordance with the Heisenberg uncertainty principle.
Q.10	Explain the diffraction of electron by a single slit to illustrate Heisenberg uncertainty principle. **(Dec 14)** Or Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)**
Ans:	**Diffraction of electron by single slit** Let us consider a narrow beam of electrons of momentum *p* is travelling in +*x* direction and

	passing through a slit of width Δy. Since electrons are passing through the width Δy therefore the uncertainty in the position of the electrons will be equal to Δy . A diffraction pattern as shown in the figure will be formed by electron beam after passing through the slit.  The first minimum of Fraunhoffer diffraction is given as $d \sin\alpha = n\lambda$. For the first order minima $n = 1$. The slit width is Δy therefore $\Delta y \sin\alpha = \lambda$ or $\Delta y = \frac{\lambda}{\sin\alpha}$ 1 The moving electrons initially have no component of momentum along y- direction because they are moving along x-axis. But after diffraction from slit the electrons have a component of momentum $p \sin\alpha$ along y-direction. Now as electron end up anywhere between $-\alpha$ to α, the y-component of momentum may lie somewhere between $p \sin\alpha$ to $-p \sin\alpha$. Hence the uncertainty in the y-component of the momentum will be $\Delta p = p\sin\alpha - (-p\sin\alpha) = 2p\sin\alpha$ Or $\Delta p = \frac{2h}{\lambda}\sin\alpha$ 2 Taking the product of equation 1 and 2 we get $\Delta y.\Delta p = \frac{2h}{\lambda}\sin\alpha . \frac{\lambda}{\sin\alpha}$ $\Delta y.\Delta p = 2h$ The above relation is in agreement with the uncertainty principle.
Q.11	Explain Heisenberg uncertainty principle and give elementary proof for it. **(Dec 12)**
Ans:	**Heisenberg's Uncertainty Principle:** According to Heisenberg"s uncertainty principle "it is impossible to determine simultaneously the position and momentum of any particle with any desired accuracy, rather the product of uncertainties in the position and momentum is always greater than or equal to h/4π, where h is Planck"s constant. Mathematically $\Delta x \Delta p \geq \frac{h}{4\pi}$ Where Δx Uncertainty in the position Δp Uncertainty in the momentum **Proof:** It is possible to prove the Heisenberg"s principle by using the fact that a moving particle is associated by a group of waves and the group velocity equals particle velocity. Since particle is considered as group of waves this implies that the particle cannot be considered as a localized entity. It indicates that there is always a limit to the accuracy with which one can measure its particle properties. Let us consider a particle of mass *m* moving with the velocity *v*, the particle can be shown as surrounded by de-Broglie waves as shown in the figure. Formation of above wave packet can be explained by considering two waves of angular

	frequencies ω_1 and ω_2 and propagation constants k_1 and k_2. These waves can be represented as $f_1 = A\sin(\omega_1 t - k_1 x)$ (1) $f_2 = A\sin(\omega_2 t - k_2 x)$ (2) Superposition of these two results in $f = f_1 + f_2$ (3) $f = A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$ As $\omega_1 \sim \omega_2 = \omega$ and $k_1 \sim k_2 = k$ therefore $\frac{\omega 1+\omega 2}{2} = \omega$ and $\frac{k_1+k_2}{2} = k$ Also $\omega_1 - \omega_2 = \Delta\omega$ and $k_1 - k_2 = \Delta k$ Hence $f = 2A\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)\sin(\omega t - kx)$ (4) The resultant wave packet travels with velocity v_g. The position of the particle is not certain rather it is somewhere between one node and next node. The error in the measurement of the position of the particle is therefore equal to the distance between these two nodes. A node is formed when $\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) = 0$. This is possible when $\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$ thus if x_1 and x_2 represents the position of the two successive nodes, then at any instant t, we get $\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_1\right) = (2n+1)\frac{\pi}{2}$ (5) $\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_2\right) = (2n+3)\frac{\pi}{2}$ (6) On subtracting equation 5 from equation 6 we get $\frac{k}{2}(x_2 - x_2) = \pi$ $x_2 - x_1 = \frac{2\pi}{k}$ therefore $\Delta x = \frac{2\pi}{\Delta k} = \frac{2\pi}{\Delta\lambda} = \Delta\lambda$ $\Delta x = \frac{h}{\Delta p}$ or $\Delta x \Delta p = h$ More precise calculations show that $\Delta x \Delta p \geq \frac{h}{4\pi}$ **Energy & time uncertainty relation** As we know $t = \frac{x}{v_g}$ and $\Delta t = \frac{\Delta x}{v_g}$ (8) also $E = \frac{1}{2}mv^2$ or $2E = mv^2$ on differentiating wrt to v we get $\frac{dE}{dv} = 2mv$ or $dE = mvdv$ or $\Delta E = mv\Delta v$ (9) Using equation 8 and 9 we get $\Delta E \Delta t \geq \frac{\Delta x}{v_g} mv\Delta v$ or $\Delta E \Delta t \geq \Delta x \Delta p$ Therefore $\Delta E \Delta t = h$
Q.12	What do you understand by wavefunction, discuss its physical significance? **(Dec14)** **Or** Discuss the concept of wavefunction associated with particle. Give examples of admissible wave function. Why derivative of wave function should be continuous everywhere? **(Jun 13)** **Or** Discuss the concept of probability density. Also define the wave function ψ. **(Jun 15)**

Ans: Variations of the „Ψ" forms the matter waves. So it converts the particle and its associated wave statistically. The wave function or complex displacement Ψ is a complex quantity and we cannot measure it. The matter wave can be represented by wave function. This wave function is used to identify the state of a particle in an atomic structure. It tells us where the particle is likely to be not where it is. The probability of finding a particle in a particular volume element dτ is given by,

$$P(r)\,d\tau = |\Psi^*\Psi|\,d\tau$$

Where Ψ^* is called the complex conjugate of Ψ.

Being a complex function, it does not have a direct physical meaning, but when we multiply this with its complex conjugate, the product $|\Psi|^2$ has physical meaning. [We will speak normally the intensity of light at a point rather than the amplitude of light at a point since intensity (square of amplitude) is a measurable and real quantity].

A physically acceptable wave function must possess the following properties:

i) „Ψ" must be single valued everywhere inside a wave packet.

ii) „Ψ" must be finite since it tells us about the probability.

iii) „Ψ" must be continuous i.e. the derivative of the „Ψ" should not vanish at the boundaries of wave packet. As „Ψ" is related to a real particle, it cannot have a discontinuity at any boundary where potential changes.

iv) $\iiint |\Psi|^2\, d\tau = 1$ when the particle presence is certain in the space. Ψ Satisfying above requirement is said to be normalized.

Q.13 What is meant by operators? Obtain an operator for the energy "E" and momentum "P". **(Dec 14, 15)**

Ans: In quantum mechanics operators are the mathematical operations, corresponding to physical observables. When these operators operate on a wave function then they give the Eigen value of corresponding physical observable.

Operator of energy

Let us consider a matter wave travelling in the positive *x* direction it could be represented by

$$f(x,t) = Ae^{-i(\omega t - kx)} \qquad 1$$

Differentiating equation (1) with respect to time

$$\frac{\partial f}{\partial t} = -i\omega A e^{-i(\omega t - kx)}$$

As $\omega = 2\pi v$ and $E = hv$

Therefore

$$\frac{\partial f}{\partial t} = -i\frac{2\pi E}{h} A e^{-i(\omega t - kx)}$$

	$$\frac{\partial \psi}{\partial t} = -\mathrm{i}\frac{2\pi E}{h}\psi$$ $$\mathrm{i}\frac{h}{2\pi}\frac{\partial \psi}{\partial t} = E\psi$$ Or $$E\psi = \mathrm{i}\hbar\frac{\partial \psi}{\partial t}$$ Therefore $E = \mathrm{i}\hbar\frac{\partial}{\partial t}$ is called operator of energy **Operator of momentum** Let us consider a matter wave travelling in the positive x direction it could be represented by $\psi(x,t) = A\mathrm{e}^{-\mathrm{i}(\omega t - kx)}$ 1 Differentiating equation (1) with respect to space $$\frac{\partial \psi}{\partial x} = \mathrm{i}kA\mathrm{e}^{-\mathrm{i}(\omega t - kx)}$$ As $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{mv} = \frac{h}{P}$ Therefore $$\frac{\partial \psi}{\partial x} = \mathrm{i}\frac{2\pi P}{h}A\mathrm{e}^{-\mathrm{i}(\omega t - kx)}$$ $$\frac{\partial \psi}{\partial x} = \mathrm{i}\frac{2\pi P}{h}\psi$$ $$-\mathrm{i}\frac{h}{2\pi}\frac{\partial \psi}{\partial x} = P\psi$$ Or $$P\psi = -\mathrm{i}\hbar\frac{\partial \psi}{\partial x}$$ Therefore $P = -\mathrm{i}\hbar\frac{\partial}{\partial x}$ is called operator of momentum.
Q.14	Derive Schrodinger"s time independent wave equation. **(Feb 10, Dec 11, 12, 13, 14 Apr 10, Jun 15, Dec 17)**
Ans:	A matter wave travelling in the positive x direction with angular frequency 'ω' and wave number 'k' can be represented by $$\psi(x,t) = A\mathrm{e}^{-\mathrm{i}(\omega t - kx)} \quad (1)$$ Differentiating equation (1) with respect to space $$\frac{\partial \psi}{\partial x} = \mathrm{i}kA\mathrm{e}^{-\mathrm{i}(\omega t - kx)}$$ Differentiating again with respect to space $$\frac{\partial^2 \psi}{\partial x^2} = \mathrm{i}^2k^2A\mathrm{e}^{-\mathrm{i}(\omega t - kx)} \quad (2)$$

	As $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{mv}$ Therefore equation (2) can be written as $$\frac{\partial^2 \boldsymbol{f}}{\partial x^2} = \mathrm{i}^2 \frac{4\pi^2 m^2 v^2}{h^2} \boldsymbol{f}$$ Or $$\frac{\partial^2 \mathrm{f}}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2} \mathrm{f} = 0 \qquad (3)$$ Total energy of a particle can be written as $$E = V + \frac{1}{2} mv^2$$ Or $$2m(E - V) = m^2 v^2 \qquad (4)$$ By using equations (3) and (4) we can write $$\frac{\partial^2 \boldsymbol{f}}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\boldsymbol{f} = 0$$ The equation above is known as time independent wave equation it can also be written as $$\frac{\partial^2 \boldsymbol{f}}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\boldsymbol{f} = 0$$
Q.15	Derive Schrodinger"s time dependent wave equation.
Ans:	A matter wave travelling in the positive x direction with angular frequency 'ω' and wave number '$\boldsymbol{k}$' can be represented by $\boldsymbol{f}(x, t) = A\mathrm{e}^{-\mathrm{i}(\omega t - kx)}$ (1) The time independent wave equation for particle above can be written as $$\frac{\partial^2 \boldsymbol{f}}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\boldsymbol{f} = 0$$ Rearranging the terms in equation above $$\frac{\partial^2 \boldsymbol{f}}{\partial x^2} = -\frac{2m}{\hbar^2}(E - V)\boldsymbol{f}$$ $$-\frac{\hbar^2}{2m}\frac{\partial^2 \boldsymbol{f}}{\partial x^2} = E\boldsymbol{f} - V\boldsymbol{f}$$ $$-\frac{\hbar^2}{2m}\frac{\partial^2 \boldsymbol{f}}{\partial x^2} + V\boldsymbol{f} = E\boldsymbol{f}$$ Using the operator of energy equation above can be written as: $$-\frac{\hbar^2}{2m}\frac{\partial^2 \boldsymbol{f}}{\partial x^2} + V\boldsymbol{f} = \mathrm{i}\hbar \frac{\partial \boldsymbol{f}}{\partial t}$$ Above form of the equation is known as time dependent Schrodinger wave equation.

Q.16	Obtain energy level for a particle trapped in infinitely deep square potential well. **(Jun 14)** **Or** Find the energy Eigen values and corresponding de-Broglie"s wavelength associated with the lowest three energy state of particle enclosed in one dimensional infinite potential well. **(Jun 13)** **Or** Obtain expression of energy levels for particle trapped in one dimensional square with infinitely deep potential well. **(Dec 13)** **Or** Obtain expression for Eigen function of particle in one dimensional potential well of infinite height. **(Dec 11)** **OR** Obtain wave function expression for a particle trapped in infinitely deep square potential well. **(Jun 14)**
Ans:	**Particle in One Dimensional Box** Let us consider a particle moving inside a box. The dimension of box is „L". The description of potential inside the box ix $V = 0 \text{ for } 0 < x < L$ $V = \infty \text{ for } x \le 0 \text{ and } x \ge L$ Since potential has infinite value at $x \le 0$ and $x \ge L$ i.e. particle cannot exist on the boundary of the box and also outside of the box. Therefore the waves function $\psi = 0$ at the boundary of the box. Since the $V = 0$ inside the box therefore the time independent Schrödinger equation inside the box will be $\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0$ 1 Or $\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0$ 2 Where $K = \sqrt{\frac{2mE}{\hbar}}$ The general solution for the above equation will be $\psi = A \sin Kx + B \cos Kx$ 3 Using boundary conditions $\psi = 0$ at $x = 0$ we get $0 = A \sin 0 + B$ or $B = 0$ Equation 3 now becomes $\psi = A \sin Kx$ 4 $\psi = 0$ At $x = L$ therefore equation 4 now is $0 = A \sin KL$ Since $A \neq 0$ Therefore $0 = \sin KL$ or $KL = n\pi$ here $n=1, 2, 3....$ Now equation 4 can be rewritten as $\psi = A \sin \frac{n\pi}{L} x$ 5 The energy Eigen value will be $E = \frac{K^2\hbar^2}{2m}$ Or $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$ 6

From the equation above it is evident that the energy levels of the particle inside a box are quantized.

Normalized Wave Function:

On applying the condition of normalization over the wave function

$$\int_0^L [f]^2 dx = 1$$

$$A^2 \int_0^L \sin^2 \left(\frac{n\pi x}{L}\right) dx = 1$$

Or

$$\frac{A^2}{2} \int_0^L \left[1 - \cos\left(\frac{2\pi n x}{L}\right)\right] dx = 1$$

$\Longrightarrow$

$$\frac{A^2}{2}[L] = 1 \text{ Or } A = \sqrt{\frac{2}{L}}$$

By placing the value of A in equation 5 we get normalized wave function/Eigen function.

$$f = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$$

Q.17 Obtain energy level for a free particle.

Or

Obtain the solution of Schrodinger wave equation for a free particle.

Ans: The time independent wave equation for particle above can be written as

$$\frac{\partial^2 f}{\partial x^2} + \frac{2m}{\hbar^2}(E - V) f = 0 \quad (1)$$

For a free particle value of V can conveniently be considered as zero, hence the time independent wave equation for a free particle will be

$$\frac{\partial^2 f}{\partial x^2} + \frac{2mE}{\hbar^2} f = 0$$

Or

$$\frac{\partial^2 f}{\partial x^2} + K^2 f = 0 \quad (2)$$

Where

$$K = \sqrt{\frac{2mE}{\hbar}} \quad (3)$$

As the particle is free therefore no boundary condition can be applied on the general solution of the equation given by

$$f = A \sin Kx + B \cos Kx$$

Therefore using equation (3) energy Eigen values of the free particle can be given as-

$$E = \frac{\hbar^2 K^2}{2m} \quad (4)$$

From equation above it can be seen that the energy levels of a free particle are continuous.

UNIT-2
WAVE OPTICS

Q.1 Explain Huygens's wave theory of light.

Ans: According to Huygens's principal, each point in a source of light emits waves in all directions in hypothetical medium ether. To explain the wave theory of light Huygens proposed the concept of wavefront which is defined as an imaginary surface in the forward direction of light propagation at which all the points oscillates in the same phase.

Shape of wavefront depends upon the shape of light source, based on this wavefronts are divided as

1. ***Spherical wavefront:*** If the source of light is a point source then the shape of wavefront in such case will be spherical.
2. ***Cylindrical wavefront:*** If the source of light is a line source then the shape of wavefront in such case will be cylindrical.
3. ***Plane wavefront:*** If the source of light is at very large distance then the shape of wavefront in such case will be plane.

Huygens' principle of light propagation is stated in two parts:

1. Every point on the given wavefront called 'primary wavefront' acts as a fresh source of new disturbance, called 'secondary wavelets' that travel in all directions with the velocity of light in the medium.
2. The envelope or the locus of these wavelets in the forward direction gives the position of the new wavefront at any subsequent time.

Q.2 Explain the principle of superposition of light?

Ans: According to the principle of superposition when two or more than two waves are travelling in a medium then the resultant displacement of a particle of medium is the algebraic sum of displacements produced by individual waves. For example if 'n' waves are travelling in a medium and each one of them produces displacement Y1, Y2, Y3, Y4,……Yn, then the resultant displacement of the particle will be given as:

$$Y = Y1 + Y2 + Y3 + Y4 + \dots + Yn$$

Q.3 Explain the term interference and give the analytical treatment of interference. **(Feb 10)**

Ans: **Interference**: When two or more than two waves travel in a medium then the modification in the intensity distribution in the region of superposition is called as interference. At the points where waves superpose with the same phase the intensity is maximum; and is called as constructive interference. While the points where waves superpose with the opposite phase the intensity is minimum; and is called as destructive interference.

Types of Interference: Interference can be classified on the basis of the way interference is produced. The interference is classified as:

Interference by Division of wavefront: When the incident wavefront is divided into two parts by the phenomenon of reflection refraction. When these two divided parts reunite then the interference obtained is called interference by division of wavefront. The examples are Fresnel bi prism, Lloyd's mirror.

Interference by Division of amplitude: When the incident amplitude is divided into two parts by the phenomenon of reflection refraction. When these two divided amplitudes reunite then the interference obtained is called interference by division of amplitude. The examples are interference in thin films, Newton's ring, and Michelson interferometer.

Analytical treatment of interference: Let us consider two plane waves in a medium in the same direction. The displacement of individual waves is given by y_1& y_2, while their amplitudes are a_1& a_2, the angular frequency of these waves is 'ω'. Mathematically the waves can be represented as

$$y_1 = a_1 \sin \omega t \qquad 1$$

$$y_2 = a_2 \sin(\omega t + \varphi) \qquad 2$$

Where φ is the phase difference between two waves.

According to the principle of superposition the resultant displacement at any point will be
$Y = y_1 + y_2$ 3
Therefore $Y = a_1 \sin \omega t + a_2 \sin(\omega t + \varphi)$ 4
Or $Y = a_1 \sin \omega t + a_2 \sin \omega t\, cos\varphi + a_2 \cos \omega t\, sin\varphi$ 5
$Y = (a_1 + a_2 cos\varphi) \sin \omega t + a_2 sin\varphi \cos \omega t$ 6
Let $(a_1 + a_2 cos\varphi) = Acos\theta$ 7
And $a_2 sin\varphi = Asin\theta$ 8
The equation 6 can be rewritten as-$Y = Acos\theta \sin \omega t + Asin\theta \cos \omega t$
Or $Y = A \sin(\omega t + \theta)$ 9
The equation 9 represents a simple harmonic plane wave with the amplitude . The amplitude can be calculated by squaring and adding the equations 6 & 7
$A^2 = a_1^2 + a_2^2 + 2a_1a_2cos\varphi$ 10

From the equation above it is evident that the resultant amplitude depends on the phase difference between two waves. Let us consider the extremes that is the condition of constructive and destructive interference

Condition for Maxima or constructive interference: If the$\varphi = 2n\pi$ where $n = 0,1,2,3 \dots \dots$ then $A^2 = (a_1 + a_2)^2$ if $a_1 = a_2 = a$ then $A^2 = I = 4a^2$

Condition for Minima or destructive interference: If the$\varphi = (2n + 1)\pi$ where $n = 0,1,2,3 \dots \dots$ then $A^2 = (a_1 - a_2)^2$ if $a_1 = a_2 = a$ then $A^2 = I = 0$

Q.4 What is interference of light? Discuss the basic conditions for obtaining sustained interference pattern. **(Dec 06, Jun 09)**

Ans: **Interference**: When two or more than two waves travel in a medium then the modification in the intensity distribution in the region of superposition is called as interference. At the points where waves superpose with the same phase the intensity is maximum; and is called as constructive interference. While the points where waves superpose with the opposite phase the intensity is minimum; and is called as destructive interference.

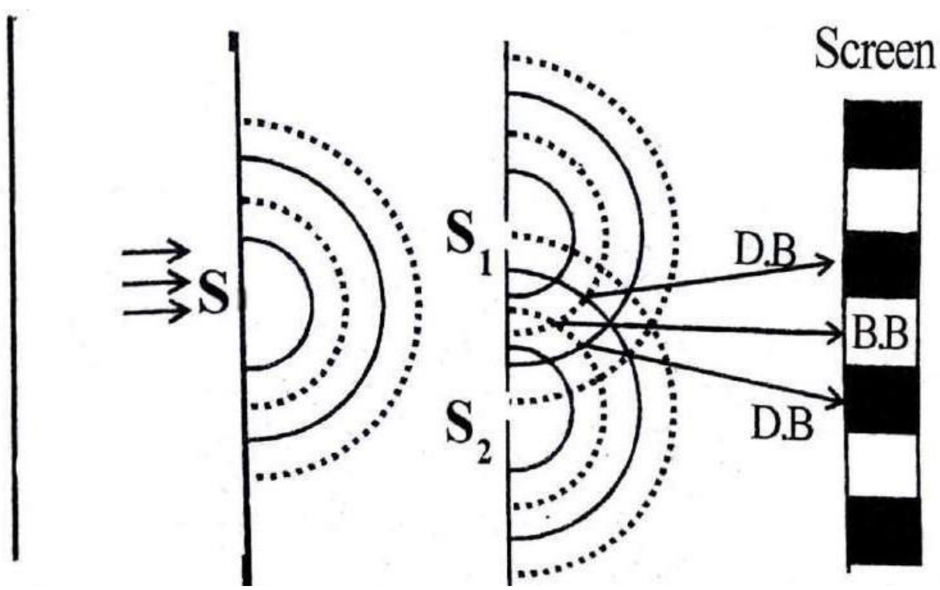
Conditions for sustained interference:

1. **Coherent**: Waves superimposing upon each other must be coherent to produce sustained interference i.e. the phase difference between these waves should remain constant with respect to time.
2. **Monochromatic**: Waves superimposing upon each other must be monochromatic to produce sustained interference i.e. waves superimposing upon each other should have same frequency or same wavelength.
3. **Amplitude:** Waves superimposing upon each other should have same amplitude otherwise destructive interference will have some light and it result in poor contrast.
4. **State of polarization:** Waves superimposing upon each other have same state of polarization to obtain sustained interference.

Q.5 Explain Young's double slit experiment and find the expression for the fringe width.
Or
Explain Young's double slit experiment and show that the separation between two bright or dark fringes is same and it does not depends upon the order of the fringe.
Or
Explain Young's double slit experiment and show that the separation between two bright or dark fringes is directly proportional to the wavelength of source.

Ans: **Formation of fringes:** Light from a narrow adjustable slit S is allowed to fall on double slit assembly. As a result the incident wave front is divided into two parts. When these two parts reunite it creates interference pattern as shown in the figure.

In the figure two kinds of lines represent opposite phases. Where similar lies crosses each other constructive interference pattern and where different kind of lines crosses each other we obtain destructive interference.



Determination of fringe width in case of Interference by Young's double slit: Let us consider a monochromatic source of light. The light from the source is incident on Fresnel bi prism and is divided in to two coherent sources S1 & S2. The light from these sources interferes and an interference pattern is obtained on the screen as shown.

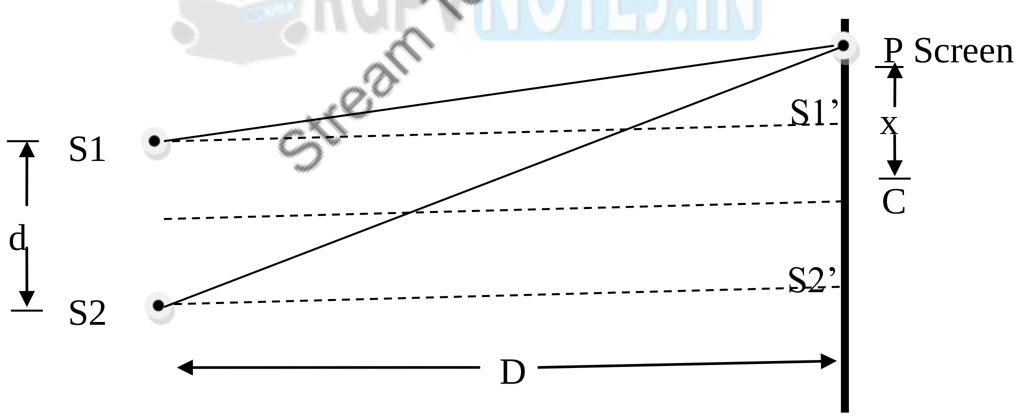


Figure 1: Interference by two virtual sources

To find out the fringe width we proceed with the determination of condition of constructive and destructive interference. Let us consider a point 'P' at the screen. The path taken by the wave from source S1 to the point P is S1P, while the path for wave from S2 will be S2P. The path difference at point P between two waves will be- S2P – S1P. Let the separation between two virtual sources be d, while the distance between the sources to screen is D.
The distance of the point P form the centre of screen is x.
In ΔS1S1'P according to Pythagoras theorem

$$S1P^2 = S1S1'^2 + S1'P^2 \qquad 1$$

$$S2P^2 = S2S2'^2 + S2'P^2 \qquad 2$$

Since as shown in figure 1

$$S1S1' = S2S2' = D$$

$S1P = x - \frac{d}{2}$ and $S2P = x + \frac{d}{2}$ Hence equations 1 and 2 can be rewritten as- $S1P^2 = D^2 + (x - \frac{d}{2})^2$ and $S2P^2 = D^2 + (x + \frac{d}{2})^2$

Therefore $S2P^2 - S1P^2 = 2xd$ *and* $S2P - S1P = \frac{2xd}{2D}$

The path difference between two waves reaching at point P is-

$$S2P - S1P = \frac{xd}{D}$$

Condition of constructive Interference or Maxima- For constructive interference the path difference between two waves must be equal to integer multiple of wavelength 'λ' i.e. $\frac{xd}{D} = n\lambda$ or $x = \frac{n\lambda D}{d}$ for nth bright fringe we can say $x_n = \frac{n\lambda D}{d}$ where x_n stands for the position of nth bright fringe.

Condition of destructive Interference or Minima- For destructive interference the path difference between two waves must be equal to odd multiple of half of wavelength (λ/2) i.e. $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$ or $x = \frac{(2n+1)\lambda D}{2d}$ for nth bright fringe we can say $x_n = \frac{(2n+1)\lambda D}{2d}$ where x_n stands for the position of nth dark fringe.

Fringe width: The distance between two consecutive dark and bright fringe is denoted by $\bar{X}$

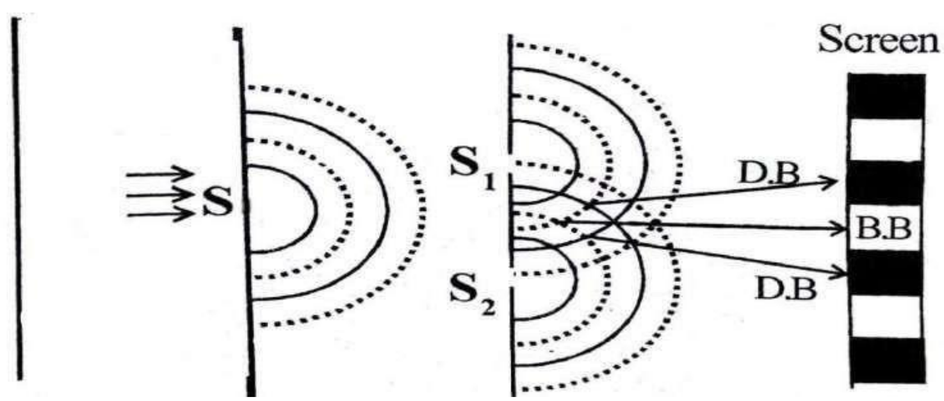
The position of nth bright fringe is given as: $x_n = \frac{n\lambda D}{d}$ while the position of (n+1)th bright fringe will be given as $x_{n+1} = \frac{(n+1)\lambda D}{d}$ the distance between two will be $x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$ we can say that the distance between two bright fringes does not depends upon the order of fringes.

The position of nth dark fringe is given as: $x_n = \frac{(2n+1)\lambda D}{2d}$ while the position of (n+1)th dark fringe will be given as $x_n = \frac{(2n+3)\lambda D}{2d}$ the distance between two will be $x_{n+1} - x_n = \frac{(2n+1)\lambda D}{d} - \frac{(2n+3)\lambda D}{2d} = \frac{\lambda D}{d}$ we can say that the distance between two dark fringes does not depends upon the order of fringes.

Q.6 Explain the formation of fringes in Young's double slit experiment. How wavelength of light source is determined using Young's double slit method.

Ans: **Formation of fringes:** Light from a narrow adjustable slit S is allowed to fall on double slit assembly. As a result the incident wave front is divided into two parts. When these two parts reunite it creates interference pattern as shown in the figure.

In the figure two kinds of lines represent opposite phases. Where similar lies crosses each other constructive interference pattern and where different kind of lines crosses each other we obtain destructive interference.



Determination of wavelength by Young's double slit method: The monochromatic source single slit, double slit and eye piece are arranged on the optical bench. Following Procedure is adopted for obtaining the fringes with good contrast.

- The optical bench should be made parallel using spirit level.
- Widen the slit and set the cross wire of eye-piece vertical.

Following measurements are now made:

1. **Measurement of fringe width (β):** Adjust the vertical cross wire of eye-piece on a bright fringe. Take reading. Then eye-piece is moved laterally so that vertical crosswire coincides with the successive bright fringe and corresponding readings are noted. The difference of these readings gives $\bar{X}$
2. **Measurement of D:** Take the reading of position of the slit and eyepiece. The difference between these readings gives **D**.
3. **Distance between two slits (d):** It is measured with the help of eye-piece by illuminating two slits one after another.
   Using the formula below wave length of light source can be determined

$$\lambda = \frac{\beta d}{D}$$

Q.7 Explain the formation of Newton's Ring in reflected light. Prove that in reflected light $D_n \alpha \sqrt{n}$ and $D_n' \alpha \sqrt{(2n+1)}$ where D_n and D_n' are diameter of dark and bright rings respectively. **(Dec 14)**

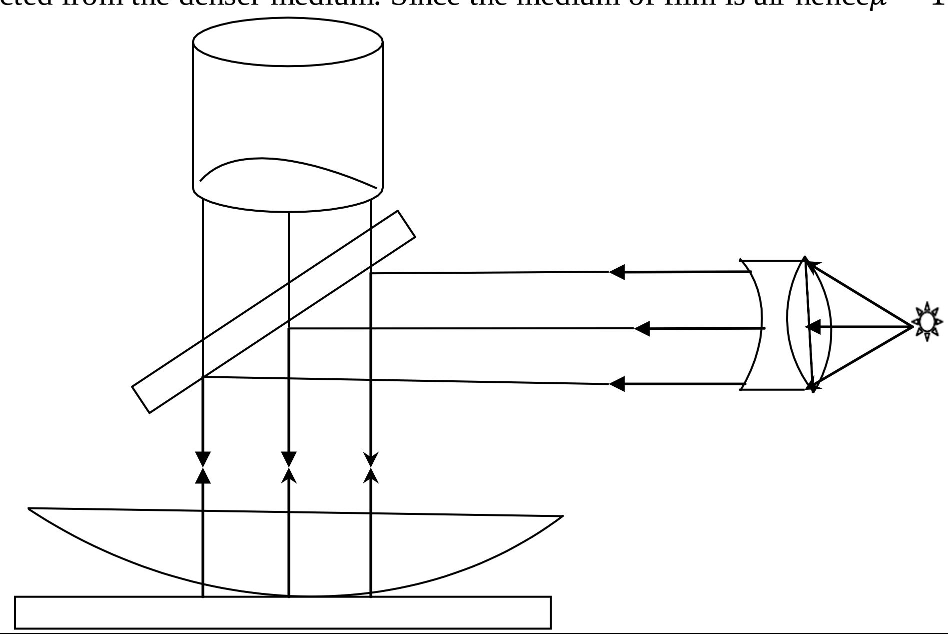
Or

Explain the mathematical treatment of Newton's ring **(Jun 16)**

Ans: **Experimental arrangement & Formation of Newton's Rings:** The experimental arrangement is shown in the figure below. A large Plano-convex lens is placed over a glass plate with its convex surface in contact with the glass plate. Then an air film of gradually increasing thickness is formed between the glass plate and Plano- convex lens. The light from a monochromatic source is allowed to fall normally over this assembly using a plane glass plate inclined at angle of 45^0.

The light rays reflected from the bottom of Plano convex lens and upper surface of glass plate superimpose on each other to produce interference pattern. The interference pattern is in the form of concentric bright and dark circles. These concentric bright and dark circles are called as Newton's rings. The fringes obtained are circular because the thickness of air film is constant in a circle around the point of contact. The obtained interference pattern is observed using a travelling microscope. The fringes obtained are called the fringes of equal thickness.

Theory: The path difference between two rays in case of Newton's rings will be $2\mu t \cos r + \frac{\lambda}{2}$ .The additional path difference of $\frac{\lambda}{2}$ is due to the fact that the ray reflected from the top surface of glass plate is reflected from the denser medium. Since the medium of film is air hence $\mu = 1$.



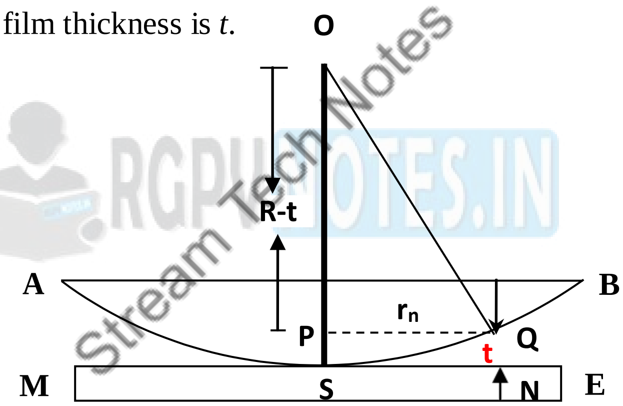
Also for normal incidence the value of $r = 0$ as a consequence the effective path difference is given by $2t + \frac{\lambda}{2}$

Condition of Maxima: For maxima the path difference must be an integral multiple of λ i.e. $2t + \frac{\lambda}{2} = n\lambda$or $2t = (2n - 1)\frac{\lambda}{2}$. Since at the point of contact the thickness of film is zero and the effective path difference is $\frac{\lambda}{2}$. Therefore the centre of Newton's rings is dark.

Condition of Minima: For minima the path difference must be an odd multiple of $\frac{\lambda}{2}$ i.e. $2t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$ or $2t = n\lambda$

Newton's Rings in Transmitted light: The effective path difference between to interfering rays in the transmitted part of Newton's rings is$2t$. Therefore the condition of maxima is $2t = n\lambda$ and the condition of minima is $2t = (2n + 1)\frac{\lambda}{2}$ hence we can say that the interference pattern in the reflected and transmitted system is complimentary to each other. The centre in the transmitted system will be bright because the effective path difference will be zero at the point of contact.

Diameters of dark Newton's rings are proportional to the square root of natural numbers: Refer to the figure below let ASB be the lens placed on the glass plate ME, the point of contact being S. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of Newton's ring where the film thickness is t.



In triangle OPQ, $OP = R - t$ and $OQ = R = OS$, $PS = t = QN$ and $PQ = r_n = SN$ using Pythagoras theorem for triangle OPQ $\quad OQ^2 = OP^2 + PQ^2$

Or

$$R^2 = (R - t)^2 + {r_n}^2$$
$$R^2 = R^2 + t^2 - 2Rt + {r_n}^2$$

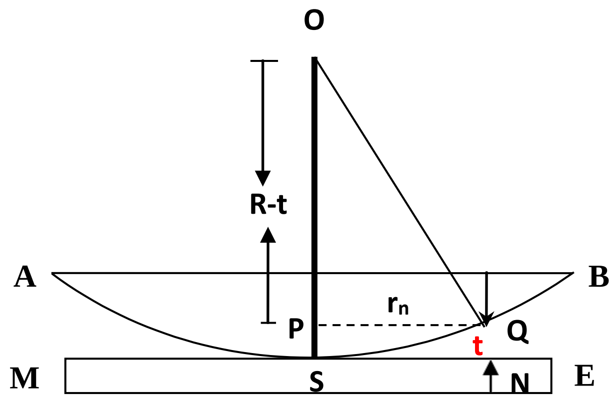
In the above equation t^2 can be neglected because t is very small.

Therefore ${r_n}^2 = 2Rt$

For dark rings in case of Newton's rings $2t = n\lambda$ where n=0, 1, 2…

Hence ${r_n}^2 = n\lambda R$ or ${D_n}^2 = 4n\lambda R$ where ${D_n}^2$ is diameter of nth dark ring. Or $D_n = 2\sqrt{n\lambda R}$ the diameter of dark rings is proportional to the square root f natural numbers.

Diameters of bright Newton's rings are proportional to the square root of odd natural numbers: Refer to the figure below let ASB be the lens placed on the glass plate ME, the point of contact being S. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of Newton's bright ring where the film thickness is t.



In triangle OPQ, $OP = R - t$ and $OQ = R = OS$, $PS = t = QN$ and $PQ = r_n = SN$ using Pythagoras theorem for triangle OPQ $OQ^2 = OP^2 + PQ^2$

$$OQ^2 = OP^2 + PQ^2$$

Or
$$R^2 = (R-t)^2 + r_n{}^2$$
$$R^2 = R^2 + t^2 - 2Rt + r_n{}^2$$

In the above equation t^2 can be neglected because t is very small.
Therefore $r_n^{\ 2} = 2Rt$. For bright rings in case of Newton rings $2t = (2n+1)\frac{\lambda}{2}$ where n=0, 1, 2….
Hence $r_n^{\ 2} = (2n+1)\frac{\lambda}{2}R$ or $D_n^{\ 2} = 4(2n+1)\frac{\lambda}{2}R$ where $D_n^{\ 2}$ is diameter of nth bright ring.
Or $D_n = 2\sqrt{(2n+1)\frac{\lambda}{2}R}$ the diameter of dark rings is proportional to the square root of odd natural numbers.

Q. 8 Describe and explain the formation of Newton "ring in reflected monochromatic light. Explain why Newton's rings are circular? How Wavelength of given Monochromatic light is calculated? **(Jun 08)**

Or

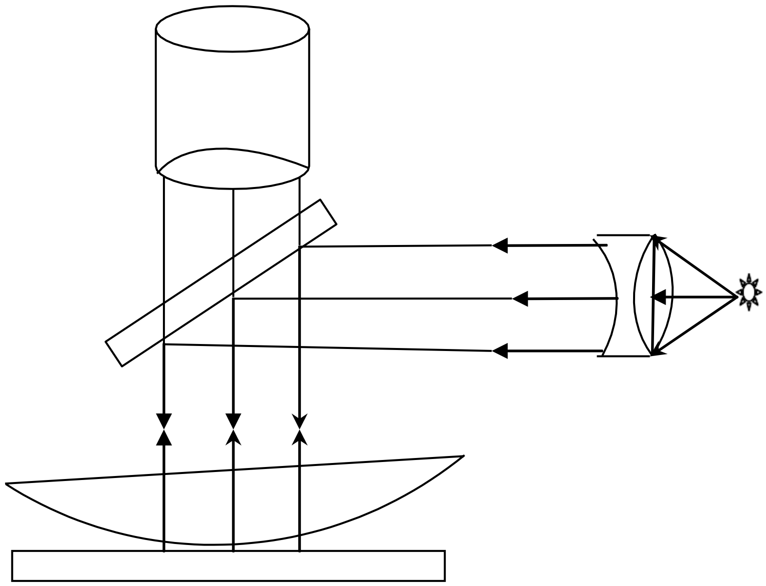
How are the circular fringes obtained in Newton's rings experiment? Why these fringes are called the fringes of equal thickness? Why the central fringe is a dark spot when examined in reflected light? **(Apr 09)**

Ans: **Experimental arrangement:** The experimental arrangement is shown in the figure below. A large Plano-convex lens is placed over a glass plate with its convex surface in contact with the glass plate. Then an air film of gradually increasing thickness is formed between the glass plate and Plano- convex lens. The light from a monochromatic source is allowed to fall normally over this assembly using a plane glass plate inclined at angle of 45^0.
The light rays reflected from the bottom of Plano convex lens and upper surface of glass plate superimpose on each other to produce interference pattern. The interference pattern is in the form of concentric bright and dark circles. These concentric bright and dark circles are called as Newton's rings. The fringes obtained are circular because the thickness of air film is constant in a circle around the point of contact. The obtained interference pattern is observed using a travelling microscope. The fringes obtained are called the fringes of equal thickness.
The path difference in the reflected part of Newton's ring experiment is given as

$$2t + \frac{\lambda}{2}$$

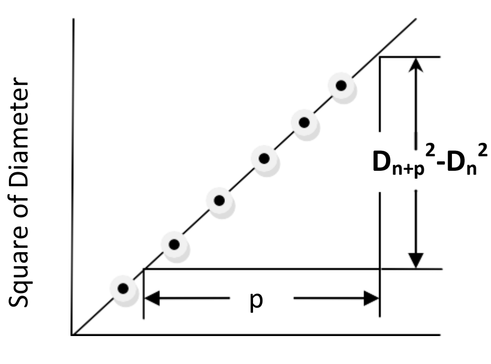
At the point of contact of glass plate and plano convex lens, thickness of air film is zero, therefore the effective path difference at the center is $\frac{\lambda}{2}$ which satisfy the condition of destructive interference. Hence the central fringe is dark in Newton's ring experiment.
Theory & Formula: As the square of diameter of nth dark ring is $D_n{}^2 = 4n\lambda R$ and that of

$(n+p)^{th}$ dark ring will be $D_{n+p}{}^2 = 4(n+p)\lambda R$ where p is any integer. Therefore the formula of wavelength will be $\lambda = \frac{D_{n+p}{}^2 - D_n{}^2}{4pR}$.



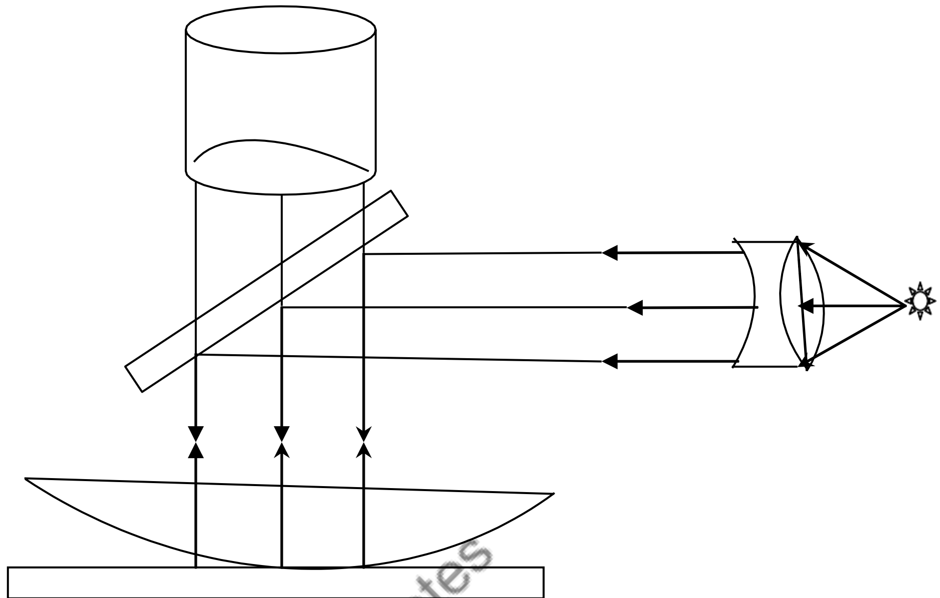
Procedure for determination of diameter of dark rings:

- The cross wire of microscope is focused, and then the crosswire is moved to extreme left to the circular interference pattern.
- Crosswire is tangentially adjusted to any dark ring (say 15^{th}) at its outer edge and the reading of microscope is noted.
- Crosswire is moved towards right and the reading is recorded, when crosswire becomes tangential to the outer edge of every alternate dark ring. The procedure is continued till the center of interference pattern.
- On reaching the center crosswire is moved towards the right and the reading of alternate dark rings are noted till 15^{th} dark ring. The subtraction of the reading of the left and right provides the diameter of the ring.
- The graph is plotted between D^2 and the number of ring n. The graph is a straight line with the slope $\frac{D_{n+p}{}^2 - D_n{}^2}{p}$. The radius of curvature of lens is measured and using the formula $\lambda = \frac{D_{n+p}{}^2 - D_n{}^2}{4pR}$ the wavelength of light can be found.



Q.9 How Newton's ring can be used to determine the refractive index of liquid? Derive the necessary formula. **(Jan 16)**

Ans: **Theory:** The path difference between two rays in case of Newton's rings will be $2\mu t\cos r+\frac{\lambda}{2}$ .The additional path difference of $\frac{\lambda}{2}$ is due to the fact that the ray reflected from the top surface of glass plate is reflected from the denser medium. Since the medium of film is air hence $\mu=1$.



Also for normal incidence the value of $r=0$ as a consequence the effective path difference is given by $2t+\frac{\lambda}{2}$

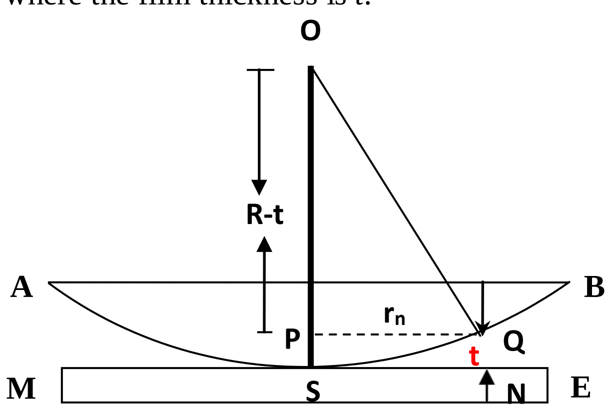
Condition of Minima in case of air film: For minima the path difference must be an odd multiple of $\frac{\lambda}{2}$ i.e. $2t+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$ or $2t=n\lambda$

Condition of Minima in case of liquid film: If the liquid having refractive index μ is placed between the plano convex lens and glass plate then for minima we can write

$$2\mu t+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2} \qquad \text{Or} \qquad 2t=\frac{n\lambda}{\mu}$$

Formula for refractive index of liquid:

Refer to the figure below let ASB be the lens placed on the glass plate ME, the point of contact being S. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of Newton's ring where the film thickness is *t*.



In triangle OPQ, $OP = R - t$ and $OQ = R = OS$, $PS = t = QN$ and $PQ = r_n = SN$ using Pythagoras theorem for triangle OPQ $OQ^2 = OP^2 + PQ^2$

Or $$R^2 = (R - t)^2 + r_n{}^2$$
$$R^2 = R^2 + t^2 - 2Rt + r_n{}^2$$

In the above equation t^2 can be neglected because t is very small.

Therefore $r_n{}^2 = 2Rt$ or $D_n{}^2 = 4(2Rt)$

In case of air film we can write $2t = n\lambda$ therefore $D_n{}^2 = 4n\lambda R$

In case of liquid film we can write $2t = \frac{n\lambda}{\mu}$ therefore $D'_n{}^2 = \frac{4n\lambda R}{\mu}$

By taking the ratio of diameter of n^{th} dark ring in presence of air film and in presence of liquid film we get refractive index of liquid

$$\frac{D_n{}^2}{D'_n{}^2} = \mu$$

Q.10 Describe the construction and working of Michelson Interferometer. How it can be used for measuring wavelength of monochromatic light. **(Jul 06, Jun 08)**

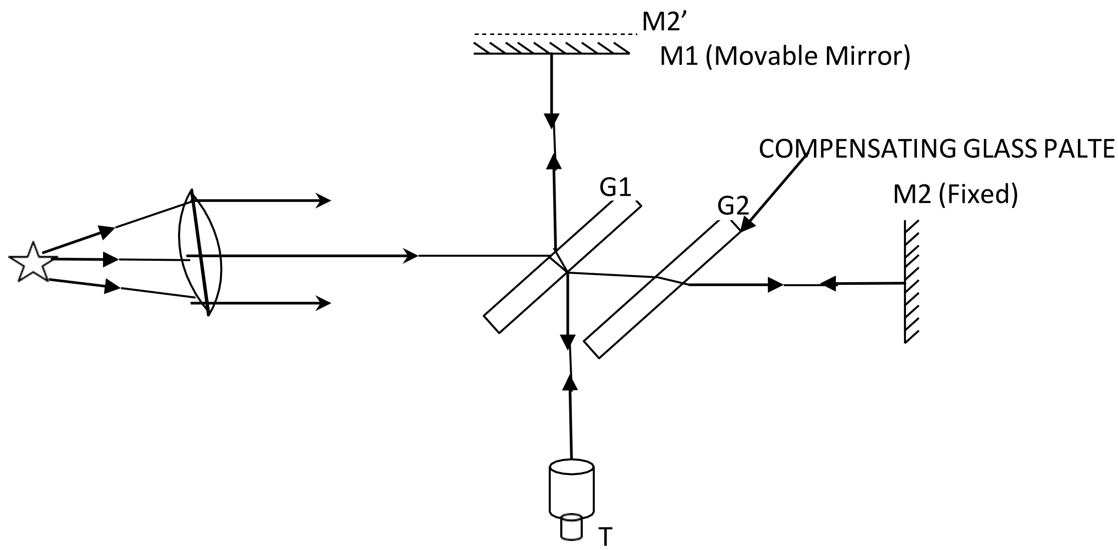
Ans: **Michelson Interferometer:** In Michelson's interferometer light from an extended source is divided into two coherent beams by partial reflection and partial refraction at semi-silvered glass plate. These beams are made to reunite at the same glass plate after reflection from two perpendicular placed mirrors.

Construction:

- The Michelson interferometer consists of two plane mirror M1 and M2 which are held perpendicular to each other as shown in figure.
- M2 is fixed while M1 can be moved backward and forward using micrometer screw capable of a measurement up to 10^{-5}cms.
- The mirrors M1 and M2 are provided with spring and screw arrangement at their backs. With the help of these screws the tilt of mirror can be adjusted.
- G1 and G2 are two identical glass plates i.e. having same thickness and the same refractive index. These glass plates are inclined at 45^0 to horizontal.
- The rear surface of plate G1 is partially polished. T is the telescope which receives the reflected light from the mirrors M1 and M2.

Working:

- A source of monochromatic light S is placed at the focus of a convex lens L, to obtain parallel light beam.
- The light from source is incident upon semi-silvered glass plate. The part of incident intensity is reflected towards mirror M1 and part of incident intensity is refracted towards mirror M2.
- Since the beams are incident normally on mirror M1 and mirror M2, therefore after reflection from the mirrors the beams retrace the same path.
- The reflected beams superpose on each other at glass plate to produce interference pattern. The produced interference pattern is observed by telescope T.



Function of Plate G2:

- In absence of plate G2 ray 1 travels through plate G1 twice while ray 2 does not travel through glass (G1) at all.
- Hence in absence of plate G2 the path of rays 1 and are not equal in glass.
- To equalize these paths plate G2 of same thickness and material as that of G1 is introduced in the path of ray no.2. Because of this nature plate G2 is called compensating glass plate.

Theory: The path difference between two rays in case of Michelson interferometer depends upon the distances of mirror M1 and M2 from the glass plate G1. The path difference can be altered by moving the mirror M1 in axial direction. The generalized expression for the path difference between two interfering rays is $2\mu t\cos r + \frac{\lambda}{2}$

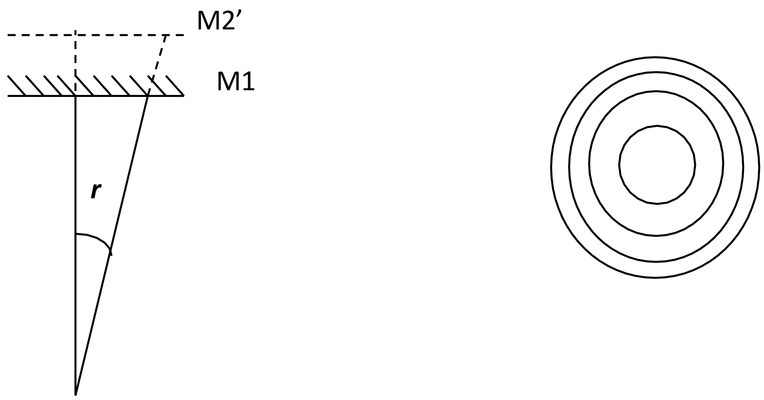
Condition of maxima: For maxima the path difference must be integral multiple of $\boldsymbol{\lambda}$i.e. $2\mu t\cos r + \frac{\lambda}{2} = n\lambda$

Condition of minima: For minima the path difference must be odd multiple of $\frac{\boldsymbol{\lambda}}{\boldsymbol{2}}$i.e. $2\mu t\cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

Types of fringes:

Circular fringes: The path difference between two rays in case of Michelson interferometer is given by $2\mu t\cos r + \frac{\lambda}{2}$ For a given wavelength λ and for air film $\mu = 1$, when the two mirrors are exactly perpendicular to each other them *'t'* is also constant. In this situation path difference depends on the *'r'*. When both mirrors are exactly perpendicular to each other then the image of mirror M2 will be exactly parallel to the mirror M1and the value of *'r'* remains constant for a circular geometry; hence the circular fringes are obtained. The fringes are called the fringes of equal inclination.

Localized fringes: When the mirrors M1 and M2 are not exactly perpendicular to each other, then the air film between the image M2' and M1 is wedge shape. Since light is incident on the film at different angles, curved fringes with convexity towards the edge of wedge are seen as shown in figure 'a' and 'b'. When M1 and M2' intersect in the middle straight fringes are obtained as shown in figure 'c'.



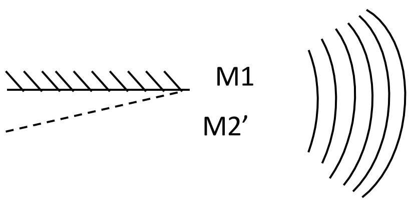


Figure: a

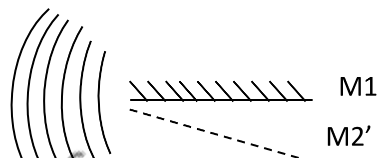


Figure: b

Figure: c

Applications of Michelson Interferometer:

Determination of wavelength:

- The Michelson interferometer is adjusted for circular fringes.
- Vertical crosswire of telescope is adjusted at the central fringe.
- The position $'x_1'$of mirror M1 is noted on the graduated scale.
- Looking through the eye-piece of the telescope, the mirror M1 is moved using micrometer screw, and the number of fringes crossing the field of view is counted.
- When ‘N’ number of fringes crosses the field of view then the position of $'x_2'$ of the Mirror M1 is noted.
- The displacement $'x'$ of the mirror M1 is obtained by subtracting $'x_1'$ from $'x_2'$ i.e. $x = x_1 - x_2$.
- By using the formula $\lambda = \frac{2x}{N}$ the wavelength of light is obtained.

Q.11 What is diffraction differentiate between Fresnel and Fraunhoffer class of diffraction? **(Feb 10)**

Ans: **Diffraction:** The phenomenon of bending of light at the edges of obstacles is known as diffraction. Diffraction is divided into two classes Fresnel diffraction and Fraunhoffer diffraction.

Difference between Fresnel and Fraunhoffer diffraction:

Fresnel diffraction	Fraunhoffer diffraction
1. In the Fresnel class diffraction the distance between the source and obstacle is finite	1. In the Fraunhoffer class diffraction the distance between the source and obstacle is infinite
2. The wavefronts can be spherical or cylindrical.	The wavefronts here are plane wavefronts only.
3. No lenses are required to produce Fresnel diffraction.	Lenses are required to produce diffraction pattern inside laboratory.
4. The image obtained at the screen is image of obstacle.	4. The image obtained at the screen is image of source.

Q.12 Explain the conditions for maxima and minim for diffraction at single slit. **(Jun 10)**

Or

Derive an expression for intensity distribution due to Fraunhoffer diffraction at single slit.**(Jun 14)**

Or

Obtain an expression for maxima and minima due to diffraction of light by single slit. **(Dec 13)**

Or

Show that the intensity at first maxima is 5% of the intensity at zero order maxima in case of diffraction at single slit. **(Jun 14)**

Ans: **Diffraction at single slit:** Let us consider a narrow slit with its width 'd'. The light from a monochromatic source 'S' falls on this slit through a convex lens. The incident wavefront is diffracted by and angle θ and is focused at screen at point P.

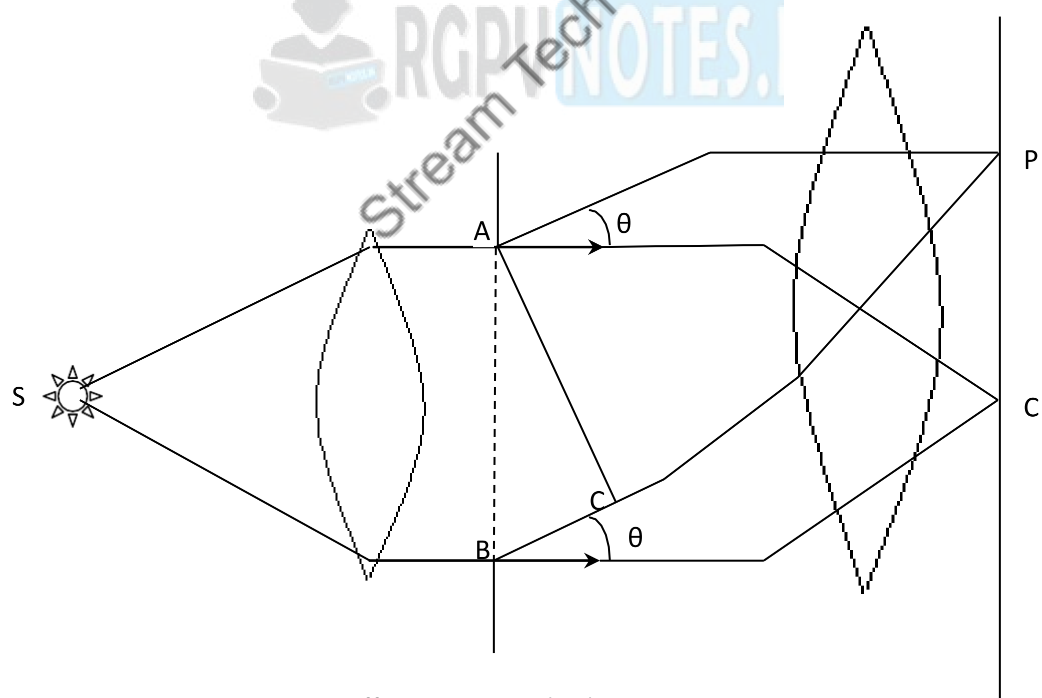


Figure 1: Diffraction at single slit

The path difference between two rays of parallel wavefront from the point A and point B will be equal to BC.

In triangle ABC $\sin\theta = \frac{BC}{AB}$ or $BC = AB\sin\theta$ since $AB = e$ therefore $BC = e\sin\theta$.

As $Phase\ difference = \frac{2\pi}{\lambda} * pathdifference$ therefore the phase difference between two rays from point A and B will be $\emptyset = \frac{2\pi}{\lambda} * e\sin\theta$ where $\emptyset = Phase\ difference$.

Since between A and B there will be large number of secondary sources generating the light wave. If we consider the 'n' number of sources between A to B, then the phase difference between two consecutive sources will be

$$\emptyset_n = \frac{2\pi}{\lambda} * d\sin\theta \quad 1$$

To find out the resultant amplitude at point P by 'n' number of sources we use the vectorial addition by polygon method. As there are 'n' vectors and the phase difference between two consecutive vectors is $\emptyset_n$. We draw a polygon with its side length representing the amplitude 'A' of individual source. As the slit width is small therefore the amplitude of all sources can be considered as same.

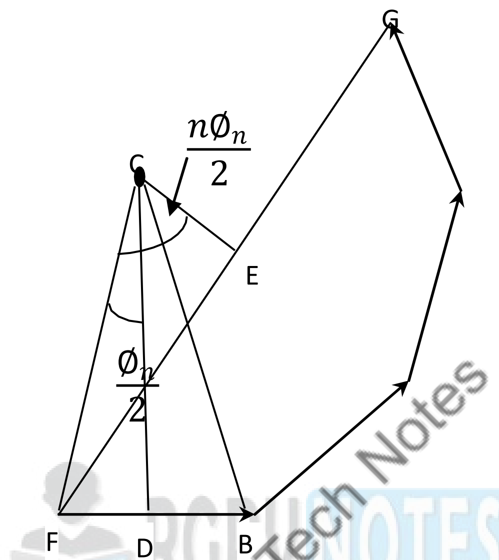


Figure 2: Polygon of vectors

According to polygon addition of vectors each vector subtends an angle at the centre of polygon, equal to the angle between two vectors, as shown in figure 2.

In figure 2 $FB = A$ the amplitude of individual vector, CD is perpendicular bisector at FB.

Angle $FCB = \emptyset_n$ and $FCD = \frac{\emptyset_n}{2}$

$FD = DB = \frac{A}{2}$. Angle $FCE = \frac{n\emptyset_n}{2}$.

In triangle FCD $\sin\frac{\emptyset_n}{2} = \frac{FD}{FC}$ or

$$\sin\frac{\emptyset_n}{2} = \frac{R}{2FC} \quad 2$$

As FG represents resultant amplitude and $FG = R$ since CE is perpendicular bisector at FG therefore $FE = EG = \frac{R}{2}$

Now in triangle FCE

$$\sin\frac{n\emptyset_n}{2} = \frac{FE}{FC}$$

Or

$$\sin\frac{n\emptyset_n}{2} = \frac{R}{2FC} \quad 3$$

On dividing equation 3 by equation 2 we get

$$\frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}} = \frac{\frac{R}{2FC}}{\frac{A}{2FC}}$$

Or

$$\frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}} = \frac{R}{A}$$

Therefore

$$R = A\frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}}$$

by substituting the value of $\emptyset_n$ from equation 1 we get

$$R = A\frac{\sin(\frac{\pi}{\lambda} e \sin\theta)}{\sin(\frac{\pi}{n\lambda} e \sin\theta)} \quad 4$$

let $\frac{\pi}{\lambda} e \sin\theta = \alpha$ then

$$R = A\frac{\sin\alpha}{\sin\frac{\alpha}{n}}$$

as n is very large therefore $\frac{\alpha}{n}$ will be very small and hence $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$

The equation 4 reduces to

$$R = nA\frac{\sin\alpha}{\alpha} \quad 5$$

The equation 5 gives the resultant amplitude at point P. The resultant intensity at point P will be

$$I = n^2A^2(\frac{\sin\alpha}{\alpha})^2 \quad 6$$

Principle Maxima: As $\alpha = 0$ the term $(\frac{\sin\alpha}{\alpha}) = 1$, therefore the intensity $I = n^2A^2$ or $I = I_{max}$.

As $\alpha = 0$ indicates $\theta = 0$ i.e. the intensity is maximum at the centre of the screen.

Secondary Maxima: For other maxima apart from centre $\alpha = (2m+1)\frac{\pi}{2}$ where m= 1, 2, 3…

For first maxima m=1 and $I \cong \frac{I_{max}}{22}$; for second maxima m=2 and $I \cong \frac{I_{max}}{62}$ as we can see that the intensity at secondary maxima decreases.

Condition of Minima: Minimum intensity is obtained when $\sin\alpha = 0$ but $\alpha \neq 0$. I.e. $\alpha = \pm m\pi$ where m=1, 2, 3…. In this situation I=0 because the term $(\frac{\sin\alpha}{\alpha}) = 0$,

As $\alpha = \frac{\pi}{\lambda} e \sin\theta$

therefore the condition of minima can also be written as

$$e \sin\theta = \pm m\lambda$$

Q.13 Give the construction and theory of plane transmission grating. Obtain an expression for resolving power of grating. **(Dec 12)**

Or

Obtain expression for intensity distribution of Fraunhoffer diffraction due to N slits. Also draw the graph of intensity variation due to interference term and diffraction term. **(Apr 10)**

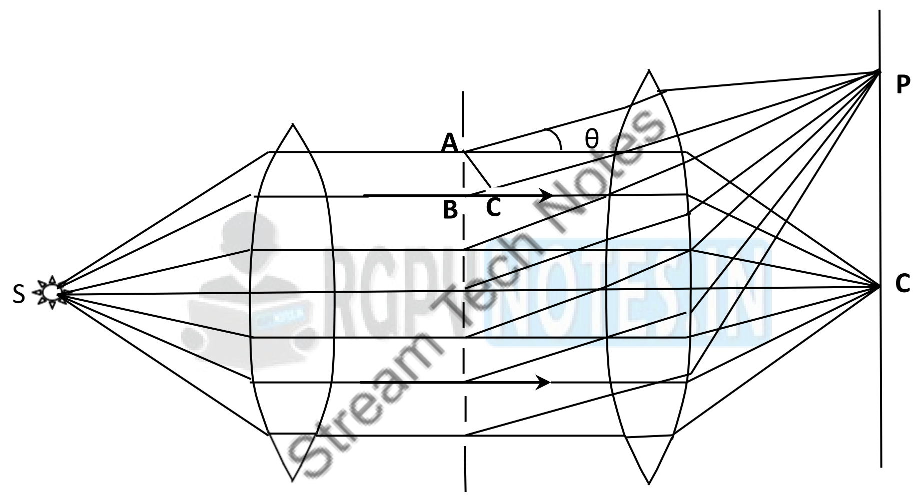
Or

Give the theory of plane transmission grating with the help of neat diagram. **(Dec 11)**

Ans: **Diffraction due to N-slits or diffraction grating:**

- A plane diffraction grating is an arrangement consisting of a large number of close, parallel, straight, transparent and equidistant (d) slits. The slit width (e) is same for all the slits.
- A grating is made by drawing a series of very fine equidistant and parallel lines on optically plane glass plate by means of a fine diamond pen.

Theory of Plane Transmission Grating: Let us consider 'n' parallel slits with slit width 'e' and the separation between the slits equal to 'd'. The light from a monochromatic source 'S' falls on this slit assembly through a convex lens. The incident wavefront is diffracted by an angle θ and is focused at screen at point P.



The path difference between two rays of parallel wavefront from the point A and point B will be equal to BC.

In triangle ABC $\sin\theta = \frac{BC}{AB}$ or $BC = AB\sin\theta$ since $AB = e + d$ Therefore

$BC = (e + d)\sin\theta$ 1

As $Phase\ difference = \frac{2\pi}{\lambda} * pathdifference$ therefore the phase difference between two rays from point A and B will be $\emptyset = \frac{2\pi}{\lambda} * (e + d)\sin\theta$ 2 where

$\emptyset = Phase\ difference.$

We are having 'n' vectors with phase difference $\emptyset$and superimposing each other at point P, the amplitude of individual vector $R = nA(\frac{\sin\alpha}{\alpha})$, here 'n' is number of secondary sources within single slit and *'A'* is amplitude of each vector within single slit.

Using the polygon addition of 'n' vectors, the resultant at point P can be written as

$I_p = R_N^2 = N^2R^2\left(\frac{Sin\ N\frac{\phi}{2}}{N\ Sin\ \frac{\phi}{2}}\right)^2$ 3

let $\frac{\phi}{2} = \beta$ now equation 3 can be rewritten as-

$$I_p = R_N^2 = N^2R^2\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 \qquad 4$$

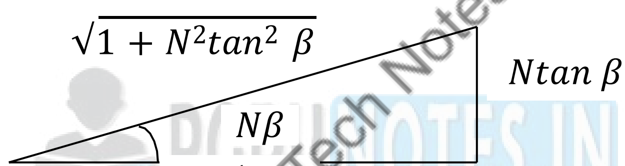
Condition of Maxima: For intensity to be maximum $\beta = \pm n\pi$ where n=1, 2, 3…… when $\beta = \pm n\pi$; $\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{0}{0}$ therefore by applying L-Hospitals rule $\left(\frac{\frac{d}{d\beta}(Sin\ N\beta)}{\frac{d}{d\beta}(N\ Sin\ \beta)}\right) = \frac{N\ Cos\ N\beta}{N\ Cos\ \beta} = 1$ when $\beta = \pm n\pi$ and the intensity is maximum. i.e. $I_p = R_N^2 = N^2R^2$ is condition of maxima. By placing the value of β in $\beta = \pm n\pi$ we get $(e + d)\sin\theta = \pm n\lambda$ this equation is known as grating equation.

Condition of Minima: For intensity to be minimum i.e. I=0 we must have $Sin\ \beta = 0$ but $Sin\ N\beta \neq 0$ i.e. $\beta = \pm m\pi$ where m=1, 2, 3……N-1 . This indicates that there are N-1 minimas between two consecutive maximas, this suggest that there must be N-2 secondary maximas between two principle maximas.

Intensity of secondary maxima: To find the intensity of secondary maxima we differentiate the intensity with respect to β and equate it equal to zero. i.e. differentiate the equation 4 wrt to β and equate it equal to zero. $\frac{dI}{d\beta} = 2N^2R^2\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 \frac{N\sin\beta\ N\cos N\beta - \sin N\beta\ N\cos\beta}{N^2\sin^2\beta} = 0$

From above we get $\frac{N\sin\beta\ N\cos N\beta - \sin N\beta\ N\cos\beta}{N^2\sin^2\beta} = 0$ or $N\tan\beta = \tan N\beta \qquad 5$

To find the intensity we derive the bracket $\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2$ under the light of equation 5. Let us consider a triangle as shown below



Now using the triangle above we can write $\sin N\beta = \frac{Ntan\ \beta}{\sqrt{1+N^2tan^2\beta}}$ or

$$\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{N^2tan^2\ \beta}{1+N^2tan^2\ \beta}\frac{1}{N^2sin^2\ \beta}$$

On simplifying the above equation we get

$$\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{N^2}{1+(N^2-1)sin^2\ \beta}$$

Therefore the intensity at secondary maxima can be written as

$$I_s = N^2R^2\frac{N^2}{1+(N^2-1)sin^2\ \beta}$$

Hence the intensity of the secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)sin^2\ \beta}$ whereas the intensity ofprinciple maxima is proportional to N^2. Therefore

$$\frac{I_s}{I_p} = \frac{Intensity\ of\ secondary\ maxima}{Intensity\ of\ principal\ maxima} = \frac{1}{1+(N^2-1)sin^2\ \beta}$$

This indicates that greater the value of N weaker is the intensity of secondary maxima.

Dispersive power of a Grating: The dispersive power of a grating is defined as the rate of change of the angle of diffraction with the wavelength of the light. It is expressed as $\frac{d\theta}{d\lambda}$. As grating equation is $(e+d)\sin\theta = \pm n\lambda$ differentiating this with respect to λ we get $(e + d\cos\theta\ d\theta d\lambda = n$ or $d\theta d\lambda = ne + d\cos\theta$ the equation obtained is dispersive power of grating. This implies that the dispersive power is

1. Directly proportional to the order of spectrum ‘n’.
2. Inversely proportional to the grating element (e + d)

3. Inversely proportional to $\cos\theta$.

Angular Half Width: The n^{th} order principle maximum is obtained in the direction θ_n given by
$(e+d)\sin\theta_n = \pm n\lambda$ 1

Let the first minimum adjacent to the n^{th} maximum be obtained in the direction $\theta_n + d\theta_n$ where $d\theta_n$ is called the angular half width of n^{th} maximum. The minima's are obtained in the directions given by
$N(e+d)\sin\theta_n + d\theta_n = \pm m\lambda$ 2

where N= total number of slits and m= integer except 0, N, 2N,……. nN. This gives $m = nN + 1$. Equation 2 now can be rewritten as

$$N(e+d)\sin(\theta_n + d\theta_n) = \pm(nN+1)\lambda$$

Or $N(e+d)\,(\sin\theta_n\cos d\theta_n + \sin d\theta_n\cos\theta_n) = \pm(nN+1)\lambda$

Since $d\theta_n$ is small therefore $\sin d\theta_n = d\theta_n$ and $\cos d\theta_n = 1$

The equation above reduces to

$$N(e+d)\sin\theta_n + N(e+d)(d\theta_n\cos\theta_n) = \pm(nN\lambda + \lambda) \quad 3$$

using equation 1, equation 3 can be rearranged as –

$$d\theta_n = \frac{\lambda}{N(e+d)\cos\theta_n}$$

Using equation 1, above equation can be rearranged as

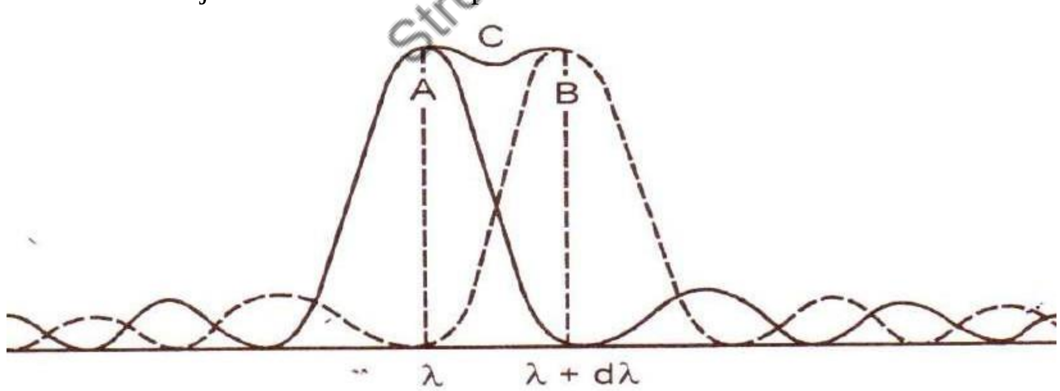
$$d\theta_n = \frac{\lambda}{Nn\cot\theta_n}$$

The expression above is known as angular half width.

Resolving power of Optical Instruments The ability of an optical instrument to resolve the images of two nearby points is termed as its resolving power.

Rayleigh's Criterion of Resolution:

Two spectral lines of equal intensities are said to be resolved if the principle maximum of diffraction pattern of one falls on the first minimum of the diffraction pattern of the other. Suppose two wavelengths λ and $\lambda + d\lambda$ are incident on an optical instrument then they will be said to resolved just if the diffraction pattern is as shown below.



Resolving Power of Diffraction Grating

The resolving power of a diffraction grating represents its ability to form separate lines for wavelengths very close together. It is given by $\frac{\lambda}{d\lambda}$. Let a parallel beam of wavelength λ and $\lambda + d\lambda$ is incident normally on a diffraction grating. The n^{th} principal maximum is formed in the direction θ_n then $(e+d)\sin\theta_n = n\lambda$ 1

The first minimum adjacent to n^{th} principal maximum be formed in the direction $\theta_n + d\theta_n$, then we have- $(e+d)\sin(\theta_n + d\theta_n) = n\lambda + \frac{\lambda}{N}$ 2

Because there are $(N-1)$ minimas between two principal maximas. According to Rayleigh's criterion the n^{th} maxima due to wavelength $\lambda + d\lambda$ must be formed in the direction $\theta_n + d\theta_n$

i.e we have

$$(e+d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \qquad 3$$

By equations 2 & 3 we can write $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}Or\frac{\lambda}{d\lambda} = nN$this is the desired expression for resolving power of grating.Thus the resolving power of grating is equal to the product of total number of rulings on the grating and the order of spectrum.

Notes Introduction to Solids

Q.1 Give the free electron model of solids and explain its limitations. (April10)

Ans: **Free Electron Model of Metals**

1. Loosely bound electrons in a metal are free to roam around
2. These conduction electrons can be treated as a perfect gas obeying FD statistics.
3. We neglect interactions with atomic ions and self-interactions, but the electrons are still bound to the solid

Free electron model successfully predicts temperature dependence of electrical conductivity of metals.

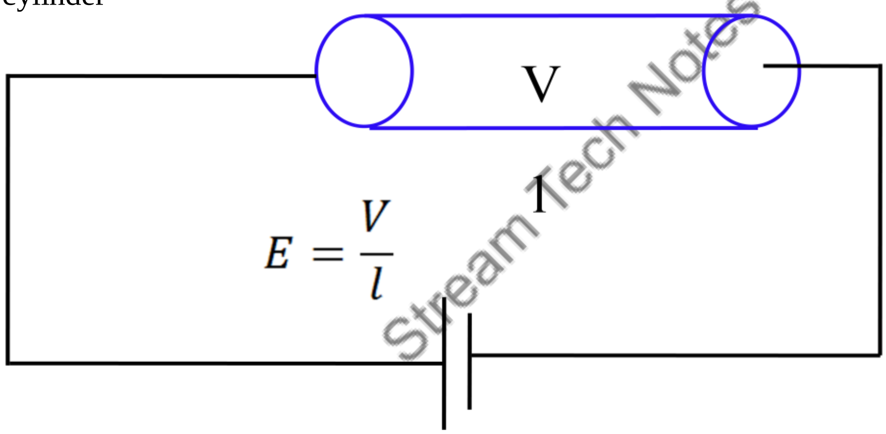
Derivation for Conductivity of Metal: Mean kinetic energy electron gas at temperature T is given as

$$\frac{1}{2}m(C)^2 = \frac{3}{2}KT$$

Hence $C^2 = \frac{3KT}{m}$

$$\text{or } C = \sqrt{\frac{3KT}{m}} \qquad (1)$$

Let us consider a metallic cylinder of length l an electric field is applied across the length of cylinder



If 'R' is the resistance of metallic cylinder considered then according to ohms law

$$V = IR \qquad (2)$$

Also

$$R = \rho\frac{l}{A} \qquad (3)$$

Therefore $V = IR$ Or $V = I\rho\frac{l}{A}$

$$\frac{V}{l\rho} = \frac{I}{A}$$

$$J = \sigma E \qquad (4)$$

As the electric field intensity is E, therefore the force experienced by the electron of material

	$$F = eE$$ Hence acceleration produced in the electron will be $$a = \frac{eE}{m}$$ It implies that the drift velocity of electron will be $v_d = ar$, where r is mean free time between two successive collisions when electron is accelerated under the influence of applied filed. $$v_d = \frac{eE}{m} r$$ Mean free time r can be given as $r = \frac{\lambda}{C}$ Hence $$v_d = \frac{eE\,\lambda}{m\,\bar{C}} \qquad (5)$$ As we know that $J = nev_d$ (6) Using equation (1) (4) (5) and (6) we can write $$\sigma = \frac{ne^2\lambda}{\sqrt{3mKT}}$$ or $$\rho = \frac{\sqrt{3mKT}}{ne^2\lambda}$$
Q.2	Calculate the density of states in the energy range E and E+dE using free electron model.
Ans:	If N is the total number of states between the energy states E and E + dE , then the density of states will be given as $$D(E) = \frac{dN}{dE}$$ Allowed states of energy according to the solution of Schrodinger equation $$E = \frac{({n_x}^2 + {n_y}^2 + {n_z}^2)\pi^2\hbar^2}{2mL^2}$$ Or $${n_x}^2 + {n_y}^2 + {n_z}^2 = \frac{2mL^2}{\pi^2\hbar^2}E$$ Equation above represents a sphere in n-space with radius $$R = \sqrt{\frac{2mL^2}{\pi^2\hbar^2}E}$$ As n_x, n_y and n_z can have only positive integer non zero values, therefore allowed states of

energy will be only one octant of sphere Therefore

$$N = \frac{1}{8} \cdot \frac{4}{3}\pi \left(\frac{2mL^2}{\pi^2\hbar^2}E\right)^{3/2}$$

Or

$$N = \frac{L^3}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$$

In the equation above $L^3 = V$, hence $N = \frac{V}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$

As each states can accommodate two electrons one with spin up and other with spin down, therefore total number of states will be

$$N = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$$

The equation above gives the total available states of energy for electron in volume V.

Q.3 Explain the band theory of solids.

Or

Differentiate insulator, conductor and semiconductors on the basis of band theory of solids.

Ans: **Band Theory of Solids**

The material in atomic state possesses discrete energy levels as in gaseous state. In gaseous state the atoms are much far away from each other and they do not influence the energy levels of the other atom. When two atoms are brought closer than their valance electrons interact with each other and significant changes in the energy levels of valance electron is observed. In a solid a large number electrons are very close packed together and the outer most energy level of individual is splitted into various energy levels very closely spaced, these closely spaced energy levels forms a virtual continuum and is called as band of energy. The electrons can occupy these bands obeying Pauli's exclusion principle and as consequence some of energy states cannot be occupied by the electrons resulting in the formation of forbidden energy band. The lower band of energy is called as valance band while the upper energy band is called conduction band.

The formation of bands is illustrated in the following diagrams. In the figure 'a' individual atom is shown. In the figure 'b' the energy levels of the individual atom is shown on graph. The length of horizontal lines parallel to x- axis indicates the circumference of the energy level. While the position of horizontal line on y-axis gives the energy value of an energy level.

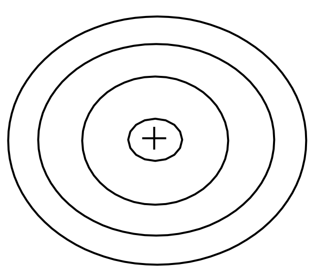


Figure: a

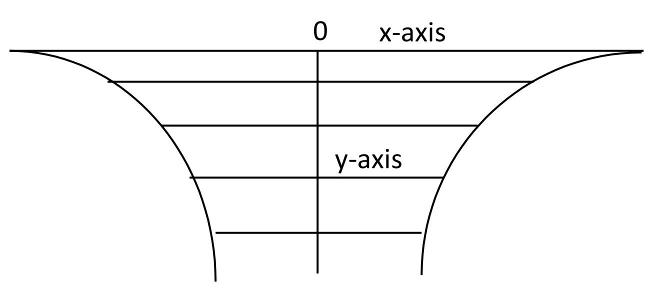


Figure: b

In the figure c two atoms are shown closer and the overlapping of valence energy levels is observed.

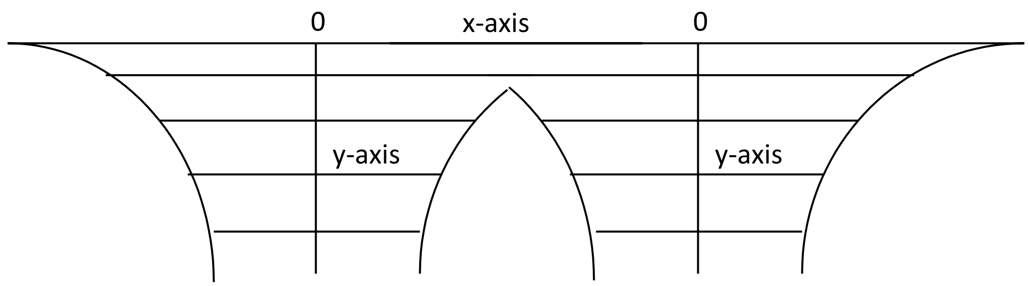


Figure: c

In the last figure n numbers of atoms are shown closer and the formation of bands is also shown

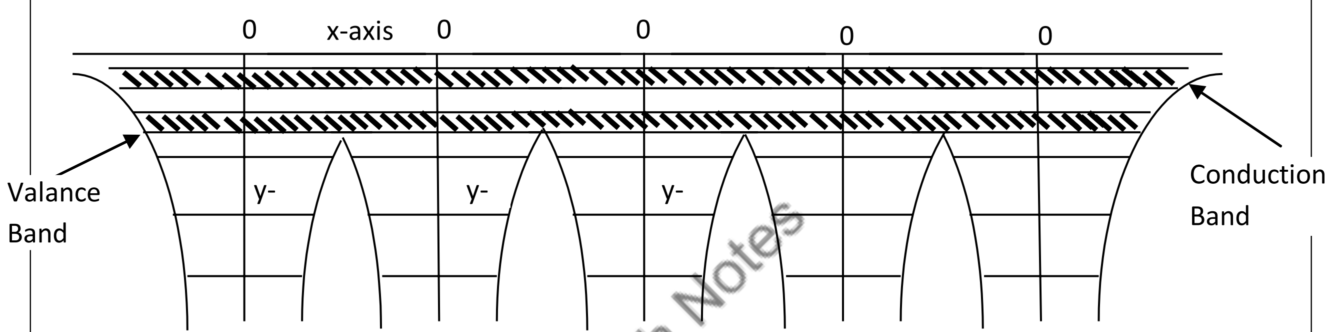
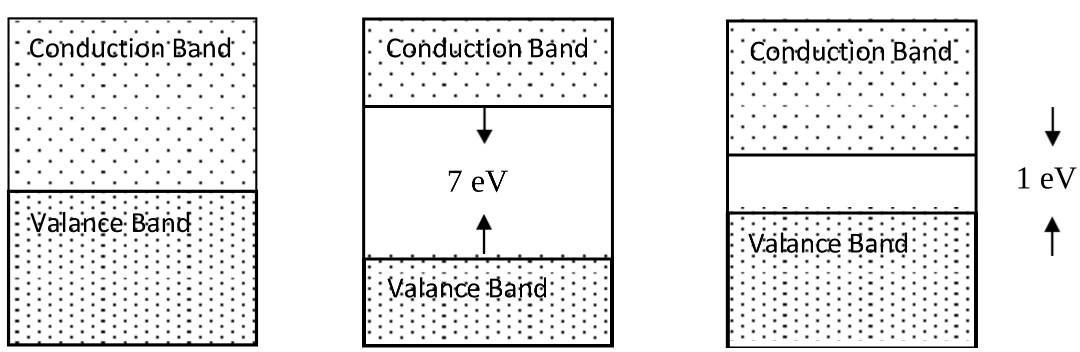


Figure: d

In the figure'd' formation of bands for a solid is depicted. In these bands where electron can freely move is known as **conduction band,** valance electrons attached to the atom forms **valance band,** whereas energy bands which cannot be occupied by electrons is known as **forbidden energy band.**

On the basis of band theory **conductors** are the solids in which valance band and conduction bands overlap each other, whereas in **insulators** separation between valance and conduction band is large and is typically of the order of 7 eV. **Semiconductors** are the solids who have a moderate energy gap between valance and conduction band typically of the order of 1 eV.

Energy band diagram for conductor, insulator and semiconductor is shown in the figures below.



Conductor Insulator Semiconductor

Q. 4 Define semiconductor. Distinguish between intrinsic and extrinsic semiconductor. **((Feb10)**

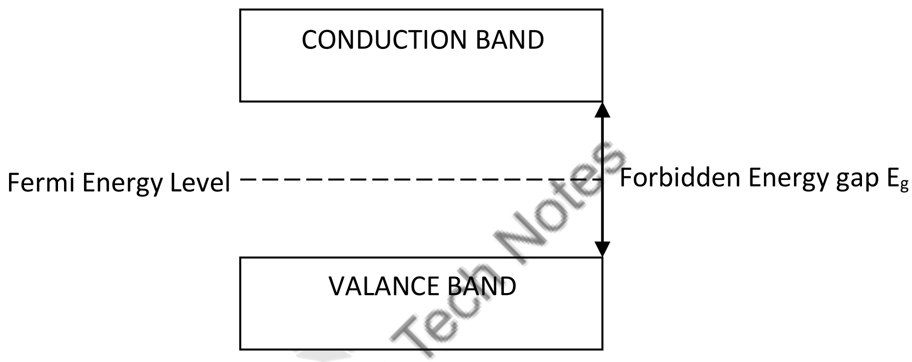
Or

Define n-type and p-type extrinsic semiconductors. **(Jun15)**

Ans: **Intrinsic Semiconductors**

The intrinsic semiconductors are the semiconductors are elemental semiconductors or also called as pure semiconductors. These semiconductors are made up of one kind of atoms only. The examples of intrinsic semiconductors are Silicon (Si) and Germanium (Ge). The intrinsic semiconductors have several limitations when practical applications are considered, because in intrinsic semiconductors the energy gap cannot be changed as per requirement.

At T=0 K the intrinsic semiconductor behaves as insulators. Because no electron exists in the conduction band. But at ordinary temperatures due to thermal agitations the covalent bonds are broken and some electrons move to conduction band. The number of electrons and holes are equal inside an intrinsic semiconductor. The Fermi energy level inside an intrinsic semiconductor lies in the middle of energy gap. The energy band diagram for intrinsic semiconductor is shown below.

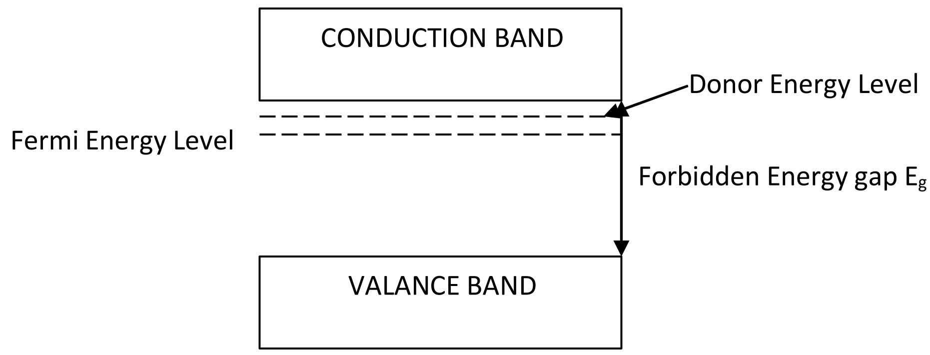


Extrinsic Semiconductors

The energy gap and conductivity of intrinsic semiconductors cannot be tailored as per the requirement of application. This problem can be overcome by doping suitable impurity in the intrinsic semiconductors. Thus obtained semiconductors are called as extrinsic semiconductors. Depending upon the type of impurity added the extrinsic semiconductors can be classified as N-type and P-type semiconductors.

N-type Semiconductors

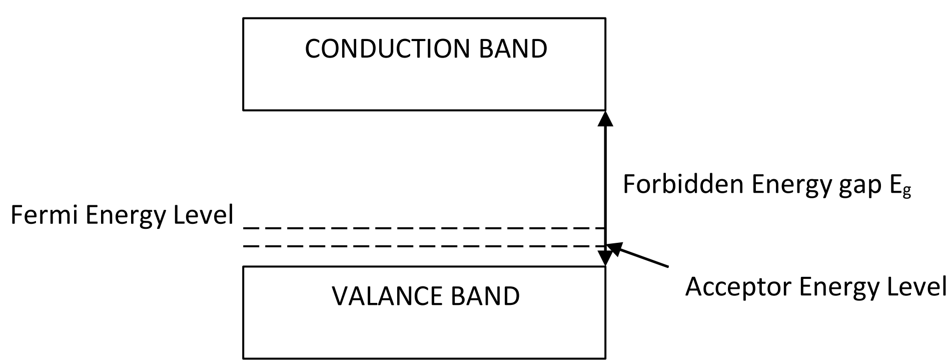
When intrinsic semiconductors either Si or Ge are doped with pentavalent material, then the obtained semiconductor material is called N-type semiconductor. The Si or Ge are tetravalent materials hence the four electrons in the outer shell of these materials forms the covalent bond with another Si or Ge atoms to complete their octet. When a pentavalent impurity Like Arsenic (As) is added to the Si or Ge then the four out of five valance electrons are shared by the host atoms(Si or Ge) while the fifth electrons of the impurity is loosely bound to its parent atom. These loosely bound electrons give rise to new energy levels which exists in the energy gap just below the conduction band and are called as donor levels. At ordinary temperature all the electrons of the donor level move to the conduction band. Thus the electrons become majority charge carriers as compared holes. Since the number of carriers are more in conduction band as compared to holes in valance band the Fermi level in N-type semiconductor shifts towards conduction band. The band diagram for N-type semiconductor is shown in figure below



Band diagram of N-type semiconductor

P-type Semiconductors

When intrinsic semiconductors either Si or Ge are doped with trivalent material, then the obtained semiconductor material is called P-type semiconductor. The Si or Ge are tetravalent materials hence the four electrons in the outer shell of these materials forms the covalent bond with another Si or Ge atoms to complete their octet. When a trivalent impurity Like Arsenic (Al) is added to the Si or Ge then the three valance electrons of impurity atom is shared by the host atoms (Si or Ge) and one of the electrons of host atom remain unshared. This result in the deficiency of an electron which tries to capture the electron from nearby covalent bond. Thus electron deficiency exists in valance band which give rise to new energy levels which exists in the energy gap just above the conduction band and are called as acceptor levels. Thus the holes become majority charge carriers as compared electrons. Since the numbers of carriers are more in valance band as compared to electrons in conduction band the Fermi level in P-type semiconductor shifts towards valance band. The band diagram for P-type semiconductor is shown in figure below



Band diagram of P-type semiconductor

Q.5	What is Fermi energy level? **Or** Explain the effect of temperature on Fermi Dirac distribution function. **(Jun14)**
Ans:	**Fermi Dirac distribution function & Fermi energy level**

The Fermi Dirac distribution function gives the probability of finding an electron in energy state E for a given temperature t. mathematically it is expressed as

$$F(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$$

Where E_F is Fermi energy level and K is Boltzmann's constant.

Let $T = 0\ Kelvin$ and $E > E_F$ then the value of Fermi Dirac distribution function will be $F(E) = 0$. This implies that all the energy levels above the Fermi energy levels are empty at $T = 0\ Kelvin$.

If $T = 0\ Kelvin$ and $E < E_F$ then the value of Fermi Dirac distribution function will be $F(E) = 1$. This implies that all the energy levels below the Fermi energy levels are completely occupied by electrons at $T = 0\ Kelvin$ or we can say that Fermi energy level is the highest possible level of energy for electron at $T = 0\ Kelvin$.

For the temperature above 0 $Kelvin$ and $E = E_F$ the value of Fermi Dirac distribution function will be $F(E) = \frac{1}{2}$ i.e. we can say that for the temperatures above 0 $Kelvin$ the Fermi level is the level of energy for which the probability of occupancy of electron is $\frac{1}{2}$.

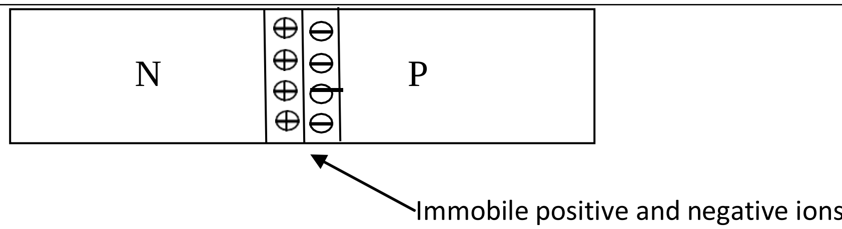
Variation of Fermi Dirac function with temperature is shown in figure below. This figure clearly indicates that conduction band is completely empty at T=0K whereas with the increase in the temperature more energy states in the conduction band is occupied indicated by the increased value of F(E).

Variation of Fermi Function with Temperature

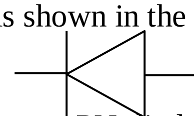
Q.6 What is P-N junction diode? Explain the characteristics of P-N junction diode under reverse and forward bias. **(Dec10, 11)**

Ans: **PN Junction diode** When a slab of intrinsic semiconducting material is doped with trivalent impurity at one end while other end of the slab is doped with pentavalent impurity then a PN junction diode is formed.

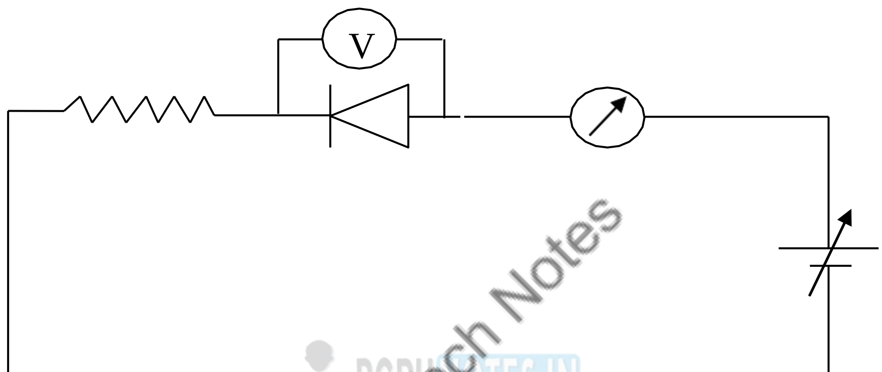
PN Junction under no bias When the junction is formed the flow of electrons starts across the junction due to concentration gradient. As the density of electron is more on N-side as compared to P-side. The electrons and holes recombine across the junction; as a result the ions are uncovered on both P and N-side in a small region around the junction. On N-side +ve ions are uncovered while on P-side –ve ions are uncovered these uncovered ions sets a potential barrier across the junction and prevents the flow of electrons and holes. Hence no current flows across the junction under no bias condition. Figure below represents the diagram of PN junction.



The symbol of PN junction diode is shown in the diagram below.



Forward biased PN junction When a PN diode is connected in a circuit such that its P side is connected to the positive terminal of battery and N side is connected to negative of battery. Then the diode is said to be under forward bias.



The negative potential at the N side forces the electrons to cross the junction. Initially when the value of applied potential is less than the potential barrier no current flows across the junction as the potential applied becomes greater than potential barrier electrons starts to cross the junction further increase in applied potential forces more electrons to cross the junction and a current in the circuit increases rapidly while the voltage across the diode does not change significantly as shown in the curve below.

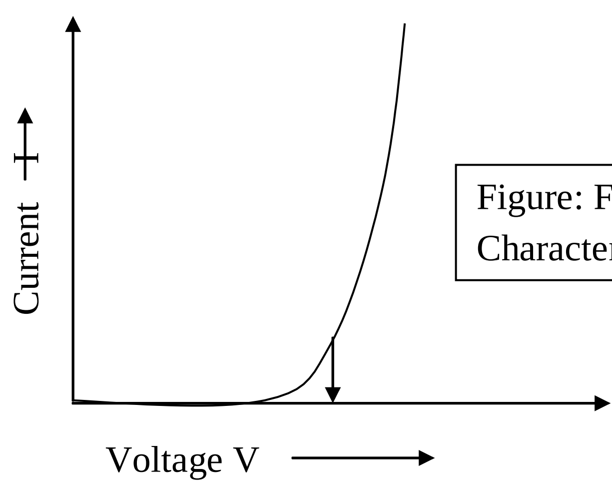
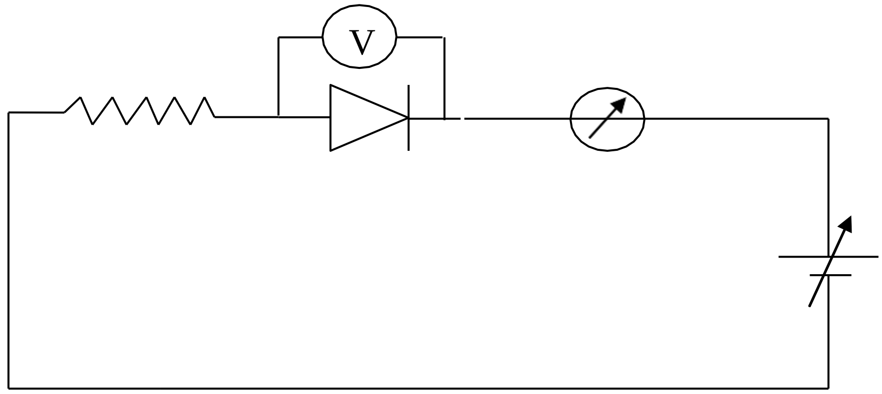


Figure: Forward Biased Diode Characteristics

Reverse biased PN junction When a PN diode is connected in a circuit such that its P side is connected to the negative terminal of battery and N side is connected to positive of battery. Then the diode is said to be under forward bias.



The positive potential at the N side forces the electrons to move away from the junction. Similarly holes also moves away from the junction as consequence majority charge carriers do not contribute to the current in the circuit. But the minority electrons from P side and minority holes from N side moves towards junction and a small current flows across the junction due small number of charge carriers. Further increase in the applied potential does not result in the increase of current. The current is called as reverse saturation current and it depends upon the junction temperature rather than applied potential.

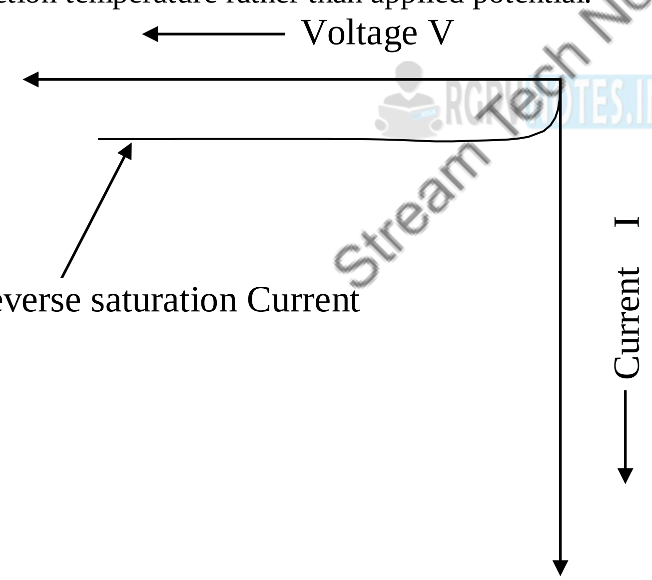
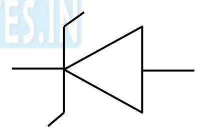
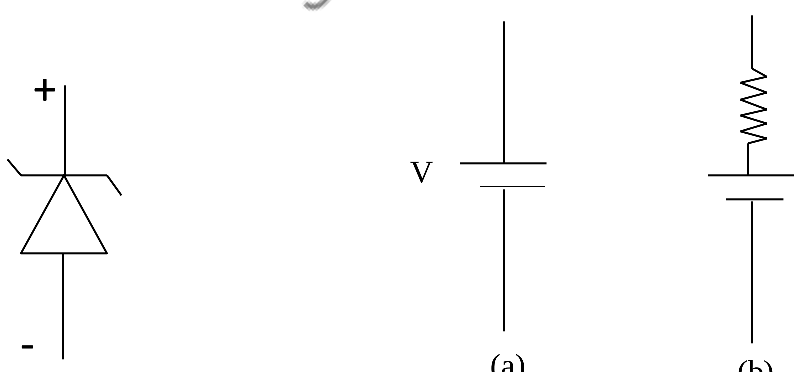
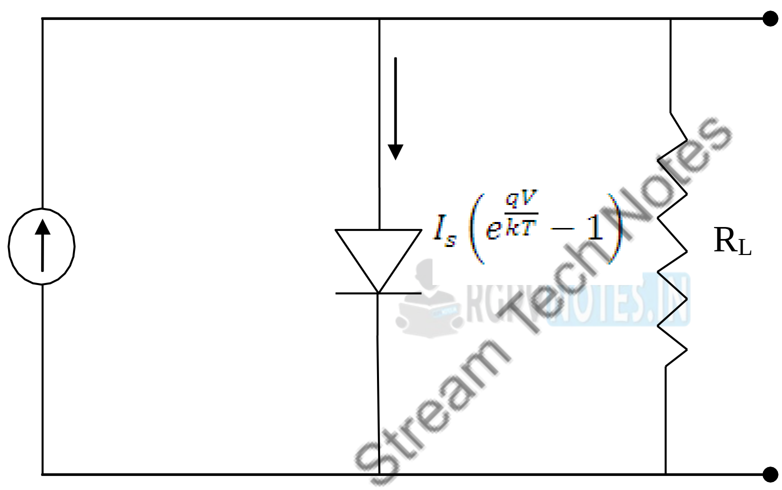
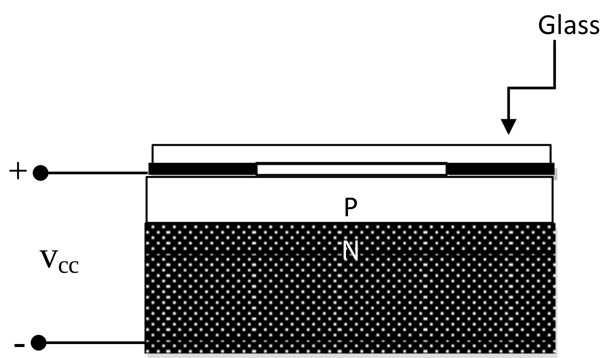


Figure: Reverse Biased Diode Characteristics

Reverse bias and Breakdown When a diode is connected in the reverse bias then the majority charge carriers moves away from the junction and the immobile ions on P and N side gets uncovered resulting in the widening of depletion layer and minority charge carriers contribute to small current. When the applied potential is increased it leads to abrupt increase of reverse current this is called as breakdown. Two kind of mechanism are responsible for the abrupt current change

1. **Avalanche breakdown** This kind of break down occurs when the impurity concentration is lower. The increase in the reverse applied potential does not leads to increase in the current but increase in the potential results in the increase in the kinetic energy of

	electron, when electron acquires the kinetic energy of the order of the strength of covalent bond then this electron breaks the covalent bond of the atom resulting in the electron hole pair. Thus produced electrons get accelerated and break another covalent bond and the process continues in the generation of large number of current carriers and large current starts to flow across the junction. The avalanche breakdown results in the damage of diode. The breakdown curve in this case is gradual near breakdown voltage. 2. **Zener breakdown** This kind of break down occurs when the impurity concentration is higher. The increase in the reverse applied potential does not leads to increase in the current but increase in the potential results in the widening of depletion layer. Thus a large electric field is set across junction, when the strength of internal field is of the order of the strength of covalent bond then this field breaks the covalent bond of the atoms resulting in the generation of large number of electron hole pairs and large current starts to flow across the junction. The zener breakdown does not damage the diode. When reverse potential is removed then the diode acquires its original state. The breakdown curve in this case is sharp near breakdown voltage.
Q.7	What is Zener diode? Draw the equivalent circuit of an ideal and actual Zener diode. What are its uses? **(Jun 12, 14, 15)**
Ans:	**Zener Diode:** Zener diode is made by heavily doped P and N type semiconductors and the surface are of the junction is also increased to avoid the increase in junction temperature in case of break down. When the break down occurs then the zener curve suggests that the voltage becomes constant and increase in the applied potential results in the increase in the reverse current. This property of the zener diode is used for the voltage regulator applications. The symbol of zener diode is shown in the figure below.  **Equivalent circuit for ideal and actual zener diode**  Reverse biased zener diode Equivalent circuit (a) Ideal (b) Actual For an ideal zener diode in the reverse bias the voltage across the diode will remain constant irrespective of the current through the diode, however for practical applications it is limited by zener impedance. A zener diode can be used as voltage regulator, voltage clipper, wave shaper and rectifier. ***(Note that V-I characteristics of zener diode are similar to P-N junction diode***

	except in the reverse bias where zener breakdown takes place unlike avalanche break down in P-N junction diode)
Q.8	Discuss the basic operation and characteristics of a solar cell with necessary diagram. **(Dec13)** **or** Explain V-I characteristics of a photovoltaic cell. **(Jun16)** **Or** Write a short note on solar cell **(Jun13,10)** **Or** Describe with the diagram the basic operation and characteristics of a solar cell including the short circuit current and open circuit voltage. Derive the expression for the maximum power delivered to the load, fill factor and efficiency of the solar cell.
Ans:	**Solar cell:** A solar cell converts the optical energy into the electrical energy it is a PN junction diode. When the light is incident on the PN junction the flow of electron and hole pair results in the photocurrent. The equivalent circuit for solar cell is given in the diagram below  The construction of solar cell is given in the diagram below  **Working;** when a photon collides with the valance electron either in P-type material or N-type material, it imparts sufficient energy to the electron to leave its parent atom. As a result, free electrons and holes are generated on each side of junction. In P-type material electrons are

minority carriers and similarly holes are minority in N-type materials. These holes and electrons move towards junction without applied bias. The result is increase in minority carriers flow.

Characteristics of solar cell: Let us consider a solar cell with a resistive load R. when the light is incident on the PN junction it produces photocurrent I_L. This current produces a voltage drop across the solar cell and the PN junction becomes effectively forward bias. The current I_F due to forward bias is into opposite direction to photocurrent I_L. Therefore the net current is given by

$$I = I_L - I_F = I_L - I_S\,(e^{\frac{qV}{kT}} - 1) \qquad 1$$

Where I_S is reverse saturation current.

Under open circuit condition I=0. Therefore,

$$0 = I_L - I_F = I_L - I_S\,(e^{\frac{qV_{OC}}{kT}} - 1)$$

Or

$$\frac{I_L}{I_S} = (e^{\frac{qV_{OC}}{kT}} - 1)$$

$$\frac{I_L}{I_S} + 1 = (e^{\frac{qV_{OC}}{kT}})$$

On taking log
$$\log_e\left(\frac{I_L}{I_S} + 1\right) = \frac{qV_{OC}}{kT}$$

Or
$$V_{OC} = \frac{kT}{q}\log_e\left(\frac{I_L}{I_S} + 1\right)$$

The power delivered to load will be $P = IV = I_L V - I_S\,(e^{\frac{qV_{OC}}{kT}} - 1)\,V$

For maximum power $\frac{dP}{dV} = 0 = I_L - I_S\,(e^{\frac{qV_m}{kT}} - 1) - I_S V_m \frac{q}{kT}(e^{\frac{qV_m}{kT}} - 1)$

$$\frac{I_L + I_S}{[1 + (\frac{qV_m}{kT})]} = I_S\,(e^{\frac{qV_m}{kT}}) \qquad 2$$

Using equation 1 for maximum current I_m we get

$$I_m = I_L - I_S\,(e^{\frac{qV_m}{kT}} - 1)$$

Or

$$I_m = (I_L + I_S) - I_S\,(e^{\frac{qV_m}{kT}}) \qquad 3$$

Using equation 2 and 3 we get

$$I_m = (I_L + I_S)\left[\frac{V_m}{V_m + (\frac{kT}{q})}\right]$$

Hence the maximum power delivered by solar cell will be

$$P_m = V_m I_m = (I_L + I_S)\left[\frac{V_m^2}{V_m + (\frac{kT}{q})}\right]$$

Fill Factor: the ratio of maximum obtainable power to the product of the open-circuit voltage and short-circuit current

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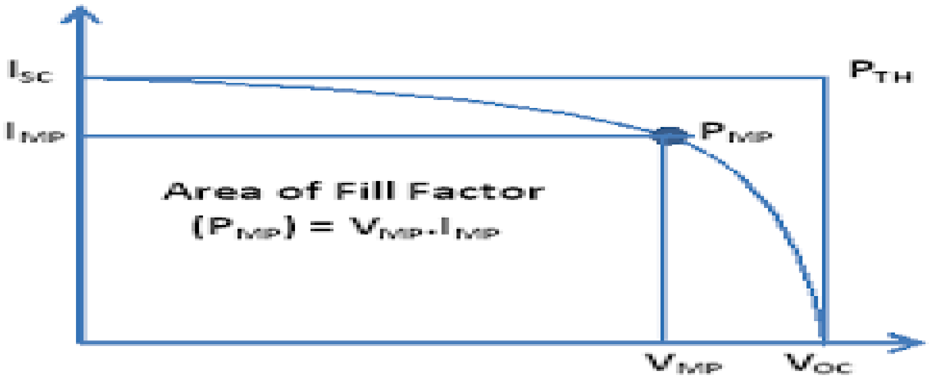
<tr><td></td><td>

$$FF = \frac{P_{max}}{I_{sc}V_{oc}}$$



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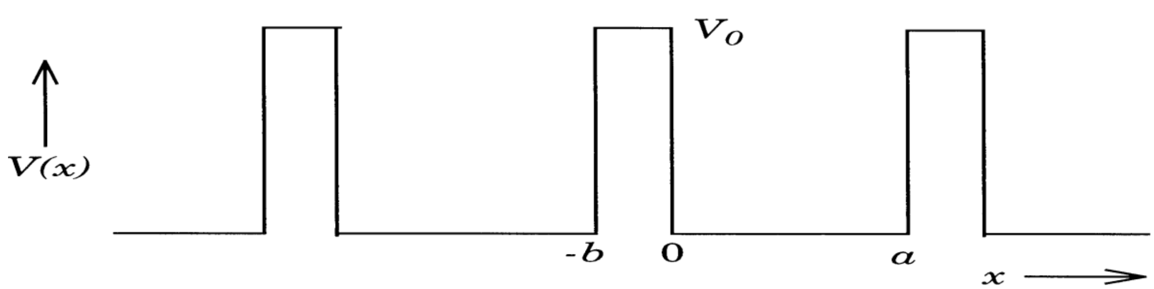
<tr><td>Q.9</td><td>What is Kronig-Penny model of solids and show that it leads to energy band structure of solids. (Dec10)
Or
Discuss the salient features of Kronig-Penny model. (Jun11)
Or
Draw periodic potential observed by an electron, moving in one dimensional crystal lattice. Discuss Kronig-Penny model proposed for periodic potential. Write Schrodinger wave equation for such potential and discuss its solution. (Dec12)
Or
Describe the behavior of electron in periodic potential using final expression of Kronig-Penny model. (Dec13)
Or
Describe the behavior of electron in periodic potential using final expression of Kronig-Penny model and explain the formation of energy bands. (Jun14)
Or
Find the effective mass of electron on the basis of Kronig-Penny Model. (Dec15)</td></tr>
<tr><td>Ans:</td><td>Kronig Penny Model
The Kronig Peny model considers that electrons inside solid moves in a periodic potential due to rest electrons and ion cores. An approximate picture inside solid is shown in the figure below.



The picture above is one-dimensional periodic potential of electron inside solid. Even though the model is one-dimensional, it is the periodicity of the potential that is the crucial property that yields electronic band structure. The mathematical form of the repeating unit of the potential is

$$V(x) = V_0 \text{ for } -b < x < 0$$
</td></tr>
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$$V(x) = 0 \text{ for } 0 < x < a$$

As shown in figure 1 the potentail has a period of $c = a + b$. The Schrodinger equation for this model will be

$$\frac{d^2}{dx^2} + \frac{2mE}{\hbar^2} = 0 \text{ for } 0 < x < a \qquad 1$$

$$\frac{d^2}{dx^2} + \frac{2m(E - V_0)}{\hbar^2} = 0 \text{ for } -b < x < 0 \qquad 2$$

The solution satisfying above equations will be obtained by using Bloch Theorem and will be

$$P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos Ka \qquad 3$$

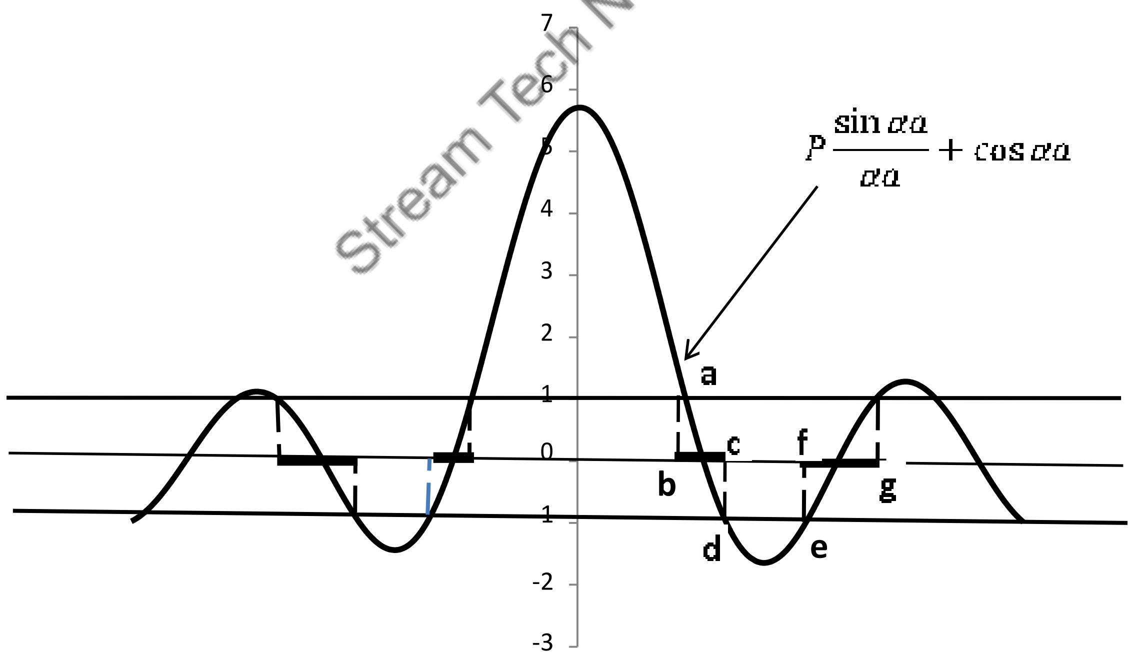
Where

$$P = \frac{maV_0 b}{\hbar^2}$$

and

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

In equation 3 there are two variables α and K. The right hand side of equation can have values only between +1 and -1. This implies that the equation 3 will be valid only when left hand side of equation will have values between +1 and -1. To find the allowed values of αa we plot left hand side of the equation with respect to αa. In the plot below we have drawn two lines parallel to the x-axis at y-values +1 and -1.



In the above plot the thick line from point 'b' to 'c' shows the allowed values of energy for electrons. Because the term $P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a$ is between +1 to -1, hence the continuous allowed values indicate the formation of band. While the values of αa between points 'c' to 'f' indicates the forbidden band because the term $P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a$ having the values less than -1.

- The solution also suggests that the width of allowed band increases with increase in the value of αa.

- For P=0 the width of forbidden band becomes zero and the electron is a free electro.
- For $P = \infty$ the energy band reduces to the energy levels.
- At $Ka = \pm n\pi$ where $n= 1, 2, 3$.....the energy curve shows the discontinuity.
- The region between $\frac{\pi}{a}$ to $-\frac{\pi}{a}$ is called the first Brillouin zone.

Effective Mass of electron

As $E = \frac{P^2}{2m}$ also $P = \hbar k$ by de-Broglie's hypothesis.

Therefore $E = \frac{\hbar^2 k^2}{2m}$, thus electron energy is parabolic, with wave vector k. the electron mass is inversely related to the curvature (second derivative) of the E-k relationship, since $\frac{d^2E}{dk^2} = \frac{\hbar^2}{m^*}$

$$m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

From the equation above it is evident that the effective mass m^* is inversely proportional to the $\frac{d^2E}{dk^2}$. Following is concluded for the effective mass

- If the curvature E vs k is large the value of effective mass m^* will be small.
- If the curvature E vs k is small the value of effective mass m^* will be larger.

Q.11 What is Hall effect? Give an elementary theory of Hall effect. **(Dec11, 12, Jun 12, Jan16)**

Or

What are the potential applications of Hall effect? **(Jun11)**

Or

Show that Hall coefficient is independent of the applied magnetic field and is inversely proportional to density of electronic charge. **(Jun12, 14)**

Or

Describe the Hall effect and Hall coefficient in detail. **(Jun10)**

Or

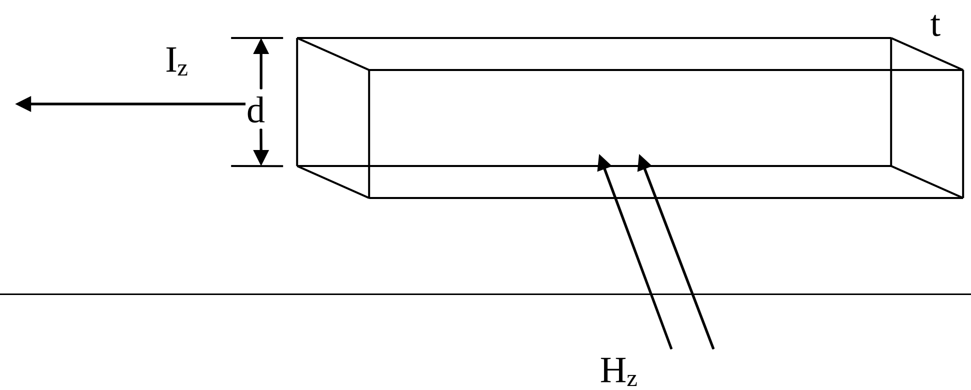
Deduce an expression for Hall effect of a solid and describe a method for its determination experimentally. What important information are obtained from its measurement? **(Dec12)**

Ans: **Hall Effect:**

When a specimen (metal or semiconductor) carrying current I is placed in transverse magnetic field. Then a potential is induced in the specimen in the perpendicular direction to both the current and the magnetic field. This phenomenon is called as Hall Effect.

Derivation

Let us consider an n-type material placed in an electric field in positive x-direction and a magnetic field is applied normal to the field in z-direction. Then the transverse magnetic field will exert Lorentz force on the electrons and electrons will accumulate at one side of specimen giving rise to potential.



When the value of force due to generated field equals the Lorentz force then the induced potential acquires the equilibrium in this condition.

Hall force = Lorentz force

$$F_H = F_L$$

$$-qE_H = v_x B_z q$$

Where v_x is drift velocity of electrons.

Or

$$E_H = -v_x B_z$$

As $j_x = Nv_x q$ we get $j_x = -N\frac{E_H}{B_z}q = -N\frac{V_H}{dB_z}q$

Or $V_H = -\frac{j_x d B_z}{Nq} = -\frac{I B_z d}{NqA}$

Since t is thickness $V_H = -\frac{I B_z}{Nqt}$

The expression above is called as hall voltage. The value of Hall field per unit magnetic field and per unit current density is called as hall coefficient and is denoted by R_H.

$$R_H = -\frac{E_H}{j_x B_z} = -\frac{V_h/d}{j_x B_z}$$

$$R_H = -\frac{1}{Nq}$$

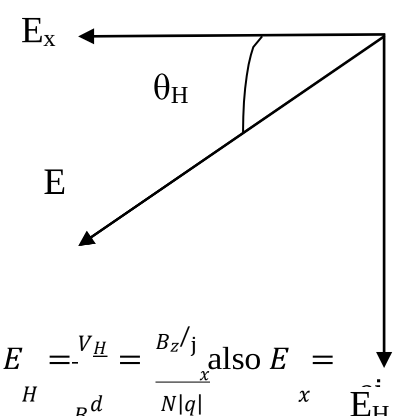
In terms of Hall coefficient the Hall voltage is given by

$$V_H = R_H \frac{I B_z}{t}$$

Since the electron mobility is given as $\mu_n = \frac{\sigma}{N|q|}$ therefore

$$\mu_n = |R_H|\sigma$$

The direction of Hall field is the direction the net field in the semiconductor. The net field in the semiconductor is the vector sum of applied field E_x and Hall field E_H as shown in figure below



$\tan_H = \frac{E_H}{E_x}$ as $E_H = \frac{V_H}{d} = \frac{B_z/j_x}{N|q|}$ also $E_x =$

Therefore $\tan_H = \frac{B_z d}{N|q|\sigma}$ or $\tan_H = \sigma R_H B_z$

Since $\sigma R_H = \mu_n$ therefore $\tan_H = \mu_n B_z$

Hence $_H = tan^{-1}(\mu_n B_z)$ this is known as direction of Hall field.

	Applications: 1. Hall effect can be used for the measurement of the strength of magnetic field 2. It is used for the determination of carrier concentration of semiconductors 3. The sign of Hall coefficient tells about the predominant charge carriers in a semiconductor 4. Hall effect can be used for the amplification of weak signals

UNIT-4
LASERS

Q.1 What is a Laser? Give major properties of laser light. **(Jun 16)**
Or
Explain in brief the characteristics of a laser beam. **(Jun 15, Dec 12)**
Or
How does laser differs from ordinary light? **(Dec 12)**

Ans: **Laser Characteristics**

The word LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. In a laser light amplification occur due to stimulated emission. Laser light differs from ordinary light sources in following respects

1. **Coherent:** Laser posses' high degree of coherence in comparison to ordinary light sources. In other words the phase relationship remains constant temporally and spatially for longer span of time and space.
2. **Monochromatic:** Lasers posses' high degree of monochromaticity i.e. the line width for a laser is very less in comparison to ordinary light source. In ordinary light source the value of $\Delta\lambda$ ranges from 100 to 1000 A^0 while for a laser the value of $\Delta\lambda$ remains within few angstroms
3. **Intensity:** The intensity of laser beam is extremely high in comparison to ordinary light.
4. **Directionality:** Lasers are highly directional and show least divergence upon travelling distances. This occurs because of photon selection by optical resonator.

Q.2 What are Einstein's coefficients? Derive Einstein's relation. **(Jun 13)**
Or
Obtain the relation between the transition probabilities of Einstein's A and B coefficients. **(Dec 10 & 13)**
Or
Establish the relationship between Einstein's coefficients A and B. **(Jun 10)**
Or
Explain the three quantum processes of interaction of radiation with the matter and derive the relationship between Einstein's coefficients.

Ans: **Einstein coefficients** are mathematical quantities which are a measure of the probability of absorption or emission of light by an atom or molecule. The Einstein A coefficient is related to the rate of spontaneous emission of light and the Einstein B coefficients are related to the absorption and stimulated emission of light.

Interaction of radiation with matter and Einstein's Coefficients

Three quantum processes when light interacts with matter are

1. Absorption: Consider an atomic system with energy levels E_1 and E_2. Electron is present in the ground state. The schematic of absorption process is presented in the figure 1

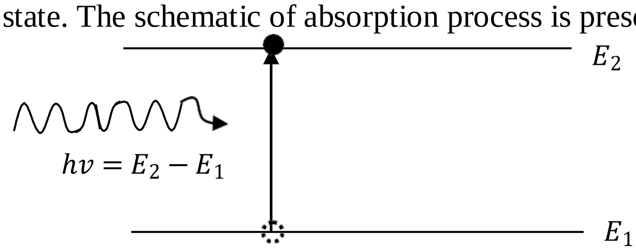


Figure 1: Electron gets excited by absorption of incident photon

When a photon having energy equal to $h\nu = E_2 - E_1$ is incident on such a system then electron absorbs this photon and gets excited to upper energy level.

The number of photons absorbed can be expressed as-

$$N_a = B_{12} N_1 u(\nu) \qquad (1)$$

Here N_a- No. of photons absorbed, B_{12} Einstein's coefficient, N_1 is the number of electrons in the energy state E_1 and $u(v)$ is the density of electromagnetic radiation.

2. Spontaneous Emission: Consider an atomic system with energy levels E_1 and E_2. Electron is present in the excited state. The schematic of spontaneous emission is presented in the figure 2

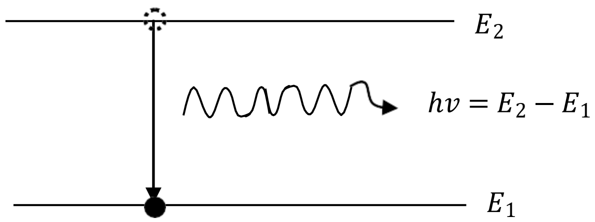


Figure 2: Electron making downward transition spontaneously

In this system the electron is in the excited state. This electron makes a downward transition when its life time is completed in the excited state. In this process it emits a photon of energy $hv = E_2 - E_1$. The number of photons emitted by the process of spontaneous emission can be expressed as

$$N_{sp} = A_{21}N_2 \qquad (2)$$

Here N_{sp} - No. of photons emitted spontaneously, A_{21} Einstein's coefficient, N_2 is the number of electrons in the energy state E_2.

3. Stimulated Emission: Consider an atomic system with two energy levels E_1 and E_2 as shown in the figure 2. In this system the electron is in the excited state and a photon is incident on such a system having energy equal to $hv = E_2 - E_1$. Thus electron is forced for a premature downward transition. In this process two photons are obtained incident and emitted. These photons are identical in all respects.

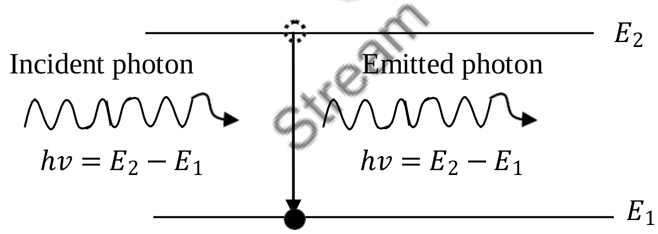


Figure 3: Electron making downward transition under the influence of incident photon

The number of photons emitted by the process of stimulated emission can be expressed as

$$N_{st} = B_{21}N_2u(v) \qquad (3)$$

Here N_{st} - No. of photons emitted by stimulated emission, B_{21} Einstein's coefficient, N_2 is the number of electrons in the energy state E_2 and $u(v)$ is the density of electromagnetic radiation.

To make the process of stimulated emission dominant the number of electrons in the excited state should be more than in the lower lasing energy level. The density of electromagnetic radiation should also be higher. Both facts are established by the ration of equation 3 to the equation 1.

Under the condition of equilibrium the number of photons must equal the number of photons emitted. We can write

Number of photons absorbed= Number of photons emitted by spontaneous emission + Number of photons emitted by stimulated emission

Thus

$$B_{12}N_1u(v) = A_{21}N_2 + B_{21}N_2u(v) \ (4)$$

Upon rearranging the terms we can write

$$u(v)(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

Or

$$u(v) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$u(v) = \frac{A_{21}}{B_{21}}\frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1}$$

Under the condition of equilibrium it can be shown that

$$B_{12} = B_{21}$$

Hence $u(v) = \frac{A_{21}}{B_{21}}\frac{1}{\frac{N_1}{N_2}-1}$; As $\frac{N_1}{N_2} = e^{\frac{hv}{kT}}$

Therefore $u(v) = \frac{A_{21}}{B_{21}}\frac{1}{e^{\frac{hv}{kT}}-1}$

According to the theory of black body radiation the density of electromagnetic radiation is given as

$u(v) = \frac{8\pi hv^3}{c^3}\frac{1}{e^{\frac{hv}{kT}}-1}dv$

Therefore the relation between the Einstein's coefficient can be written as

$$\frac{A_{21}}{B_{21}} = \frac{8\pi hv^3}{c^3}dv$$

It shows that the probability of spontaneous emission increases with the energy difference between two states.

Q.3 What is meant by stimulated emission? Explain the basic condition in which stimulated condition dominates. **(Dec 14)**

Or

What do you understand by spontaneous and stimulated emission? **(Dec 15)**

Ans: **1. Spontaneous Emission:** Consider an atomic system with energy levels E_1 and E_2. Electron is present in the excited state. The schematic of spontaneous emission is presented in the figure 2

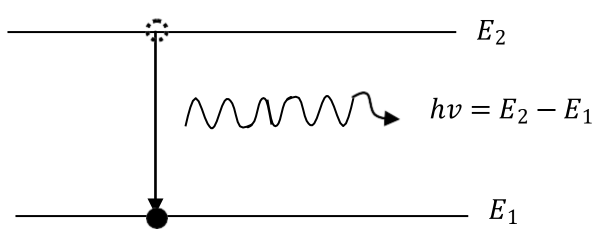


Figure 2: Electron making downward transition spontaneously

In this system the electron is in the excited state. This electron makes a downward transition when its life time is completed in the excited state. In this process it emits a photon of energy $hv = E_2 - E_1$. The number of photons emitted by the process of spontaneous emission can be expressed as

$$N_{sp} = A_{21}N_2 \qquad (2)$$

Here N_{sp}- No. of photons emitted spontaneously, A_{21} Einstein's coefficient, N_2 is the number of electrons in the energy state E_2.

2. Stimulated Emission: Consider an atomic system with two energy levels E_1 and E_2 as shown in the figure 2. In this system the electron is in the excited state and a photon is incident on such a system having energy equal to $hv = E_2 - E_1$. Thus electron is forced for a premature downward transition. In this process two photons are obtained incident and emitted. These photons are identical in all respects.

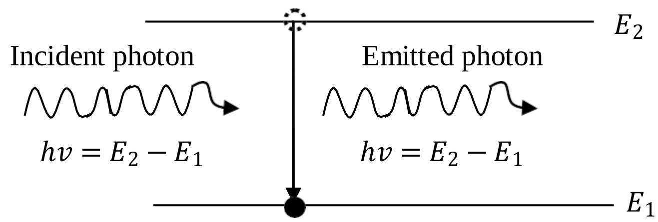


Figure 3: Electron making downward transition under the influence of incident photon

The number of photons emitted by the process of stimulated emission can be expressed as

$$N_{st} = B_{21}N_2u(v) \qquad (3)$$

Here N_{st}- No. of photons emitted by stimulated emission, B_{21} Einstein's coefficient, N_2 is the number of electrons in the energy state E_2 and $u(v)$ is the density of electromagnetic radiation. Stimulated emission will dominate spontaneous emission when $N_2 > N_1$.

Q.4 What is population inversion in lasers and how it is achieved? **(Jun 11 Dec 11 & 15)**

Or

State necessary condition for strong stimulated emission. **(Jun 13)**

Or

Define population inversion and show that it is necessary for amplification of light in active medium. **(Apr 10 Jan 16)**

Ans: **Population inversion:** The number of atoms per unit volume occupying a certain energy state is called population. In any atomic system ordinarily the number of electrons in the lower energy state (N_1) is more in comparison to the population of excited state (N_2). For achieving the light amplification an artificial situation is created in which the number of electrons at the excited state is more than the number of electrons in lower energy state. This situation is called as population inversion.

In any atomic system number of photons absorbed may be written as:

$$N_a = B_{12}N_1u(v)$$

It means absorption is proportional to the N_1, whereas number of photons emitted by stimulated emission may be written as:

$$N_{st} = B_{21}N_2u(v)$$

Equation above indicates that stimulated emission is directly proportional to the N_2 i.e. the population of excited state. It clearly establishes that if the number of electrons at excited state N_2 is more than the number of electrons at ground state N_1, then probability of stimulated emission will be more in comparison to the absorption and the light amplification will take place.

Q.5 What are the principle pumping schemes? **(Dec 11 & 15)**

Or

Explain three and four level pumping scheme and give reason which one is more efficient.

Ans: **Three level** pumping schemes in laser essentially refer to the lasing mechanism in which population inversion is achieved between an excited and the ground energy state. Since population inversion in such system is achieved between excited and the ground energy state therefore more pumping energy is required as naturally more number of electrons stay in the ground energy state. The schematic diagram of a three level energy system is shown in the figure below.

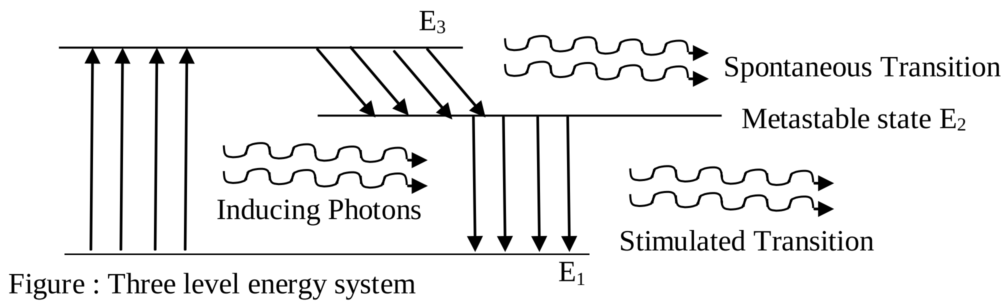


Figure : Three level energy system

Mechanism: In three level systems initially electrons are excited to the energy level E_3 by means of pumping. Energy level E_3 is ordinary excited state and electrons rapidly decay to energy level E_2 which is Metastable state. Here electrons stay longer than usual and situation of population inversion is achieved between ground state (E_1) and energy level E_2.

Now photons having energy equal to $hv = E_2 - E_1$ is incident over this system to initiate stimulated emission and light amplification.

Four level pumping schemes in laser essentially refer to the lasing mechanism in which population inversion is achieved between two excited states. Since population inversion in such system is achieved between two excited states therefore less pumping energy is required as naturally more number of electrons stay in the ground energy state.

The schematic diagram of a four level energy system is shown in the figure below.

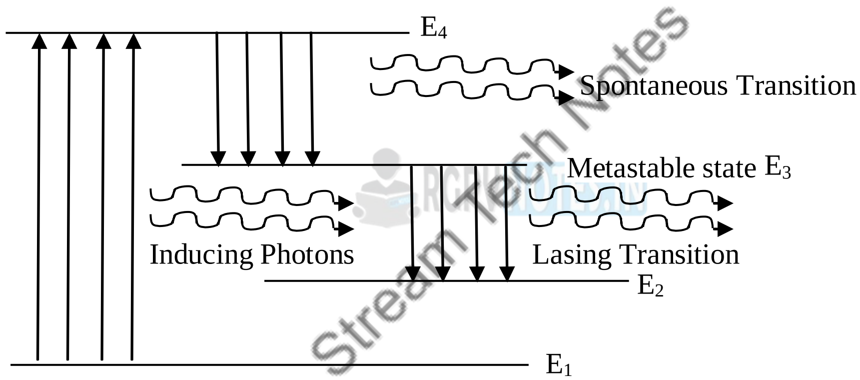


Figure: Four level energy system

Mechanism: In four level system initially electrons are excited to the higher levels of energy by means of pumping. This higher level of energy level is ordinary excited state and electrons rapidly decay to lower excited energy level E_3 which is Metastable state. Here electrons stay longer than usual and situation of population inversion is achieved between upper excited state (E_3) and lower excited state (E_2).

Now photons having energy equal to $hv = E_3 - E_2$ are incident over this system to initiate stimulated emission and light amplification.

Since in four level energy system population inversion is achieved between two excited states, therefore it requires less pumping. Whereas in three energy level system lower lasing level is ground state, therefore more than 50% of electrons need to be excited. As a result more pumping is required for three energy level systems in comparison to four level energy systems. Hence four energy level laser systems are better.

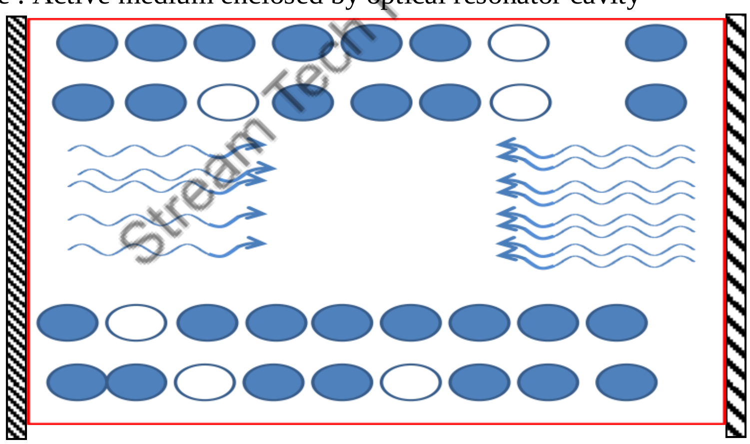
Q.6 Explain the different components of laser.

Or

Write short note on optical resonators. **(Jun 10)**

Ans: **Components of Laser**

Active Medium: The active medium is a collection of atoms or molecules, which can be excited to achieve population inversion situation for amplification of light by stimulated emission. The active medium can be in any state of matter: solid, liquid, gas or plasma. The active medium determines

	the emitted wavelengths possible by lasing transition. **Pumping:** Pumping is an energy source working on an active medium for achieving population inversion. Various pumping mechanisms are: i. Optical Pumping ii. Electric Discharge iii. Inelastic Atomic Collisions iv. Direct Conversion v. Chemical reaction **Optical Resonator:** Optical resonator is a pair of mirror facing each other. The active medium is enclosed in this cavity. In optical resonator one of the mirrors is partially polished while other mirror is completely polished. Optical cavity ensures the availability of photons for stimulated emission and contributes for light amplification by optical feedback. A schematic diagram of optical resonator is given in the figure below.  Figure : Active medium enclosed by optical resonator cavity  **Action of Optical resonator:** In the above figure an active medium enclosed between two parallel mirrors is shown. In the active medium hollow spheres present electrons in the ground state while filled sphere presents electrons in the excited state. **Step1:** Some of the electrons from the excited makes down ward transition spontaneously resulting in the emission of photons. **Step2:** Out of the emitted photons, The photons traveling in the axial direction is reflected back into active medium. **Step3:** These reflected photons initiates the process of stimulated emission into active medium and more photons are obtained. **Step4:** After multiple reflections a strong laser beam emerges out from partially reflective end of the optical resonator.
Q.7	Describe the construction and working of one laser with help of diagram. **(Jun10)** **Or** Describe the construction and working of ruby laser with help of a necessary diagram. **(Jun12 Dec15)** **Or**

Explain the construction and working of ruby laser with necessary energy level diagram. **(Jun13 Dec14)**

Or

Explain how the population inversion and pumping scheme processes are realized in Ruby laser? **(Dec11)**

Or

Describe the construction and working of a solid state laser with help of a necessary diagram.

Or

Describe the construction and working of a three energy level laser with help of a necessary diagram.

Ans: **Construction:** Ruby laser is three energy level laser. It consists of a cylindrical ruby rod. The ends of ruby rod are polished to act as optical resonator. This ruby rod is wrapped with helical shaped xenon lamp.

Xenon lamp acts as optical pumping source. The active medium in ruby laser is Cr^{+} ions in the host material Al_2O_3. Construction of Ruby laser is shown in the figure 1.

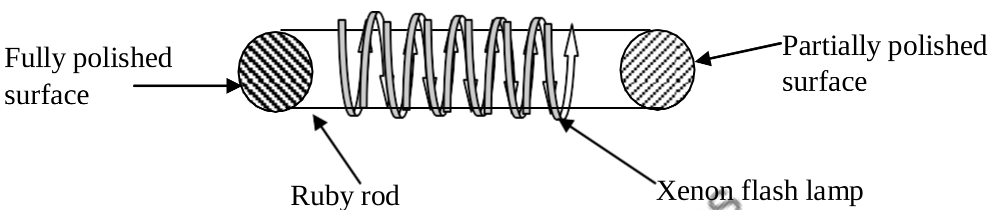
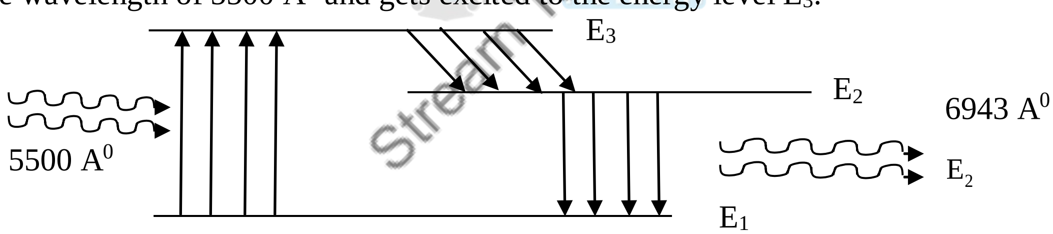


Figure 1: Ruby Laser

Working: The energy level diagram for Cr^{+} ion is shown in figure 2. Initially most of the electrons occupy ground state. When the light from xenon lamp falls on ruby rod then the electron absorbs the wavelength of 5500 A^0 and gets excited to the energy level E_3.



Energy level E_3 is an ordinary excited state and the electrons from this excited state rapidly decay to the Metastable energy state E_2. As E_2 is Metastable state electrons start to accumulate at this level and a situation of population inversion is achieved between the E_2 and ground state. Photons emitted due to spontaneous decay and reflected back by polished ends initiates the stimulated emission. Thus emitted photons reflect back and forth between two polished ends and when the beam of photons become sufficiently large it emerges out from the partially polished end in the form of pulse of wavelength 6943 A^0.

Limitations: Ruby laser is a three level laser and has very low efficiency. The output is in the form of pulse. As xenon flash lamp is used for optical pumping and only small fraction of spectrum is used for excitation.

Applications: Ruby laser is used in creating 3-D images using holography it is also used in medical filed for tattoo removal.

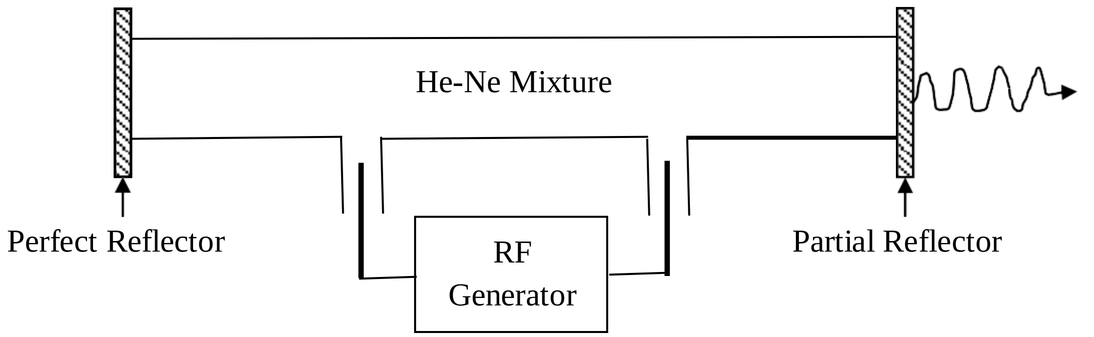
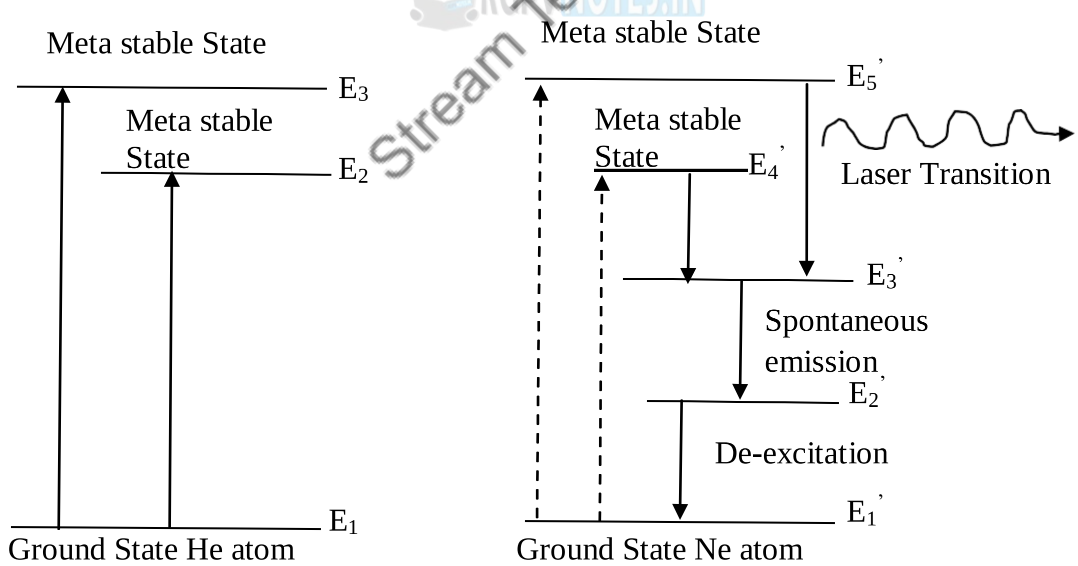
Q.8 Explain with the help of a neat diagram the principle and working of He-Ne laser. **(Jan 16)**

Or

Explain construction and working of He-Ne laser with neat diagrams. **(Dec12 & 13)**

Or

Explain construction and working of He-Ne laser with the help of energy level diagrams. **(Feb10, Apr10,Jun11, 14 & 16)**

	Or Explain construction and working of any gaseous laser. **Or** Explain construction and working of any four level laser.
Ans:	Helium Neon laser is a four level gas laser. In this laser lasing transition occur between energy levels of Neon atom. **Construction:** The schematic diagram of He-Ne laser is shown in the figure 1. It consists of a long discharge tube with diameter 1 cm and length about 50 cm. Discharge tube is filled with mixture of Helium and Neon in ratio of 10:1. Both ends of discharge tube is equipped with mirrors to act as optical resonator.  **Working:** The energy level diagram for Helium and Neon is shown in the figure 2. When the current is passed through the mixture of He and Ne. The helium atoms are excited to the nearest excited state. When these excited helium atoms collides with the neon atoms then they transfer their energy to neon atoms. Thus neon atoms are raised to energy level E_5.  As a result population inversion is achieved between E_5' and E_3'. The transition between two emits light of wavelength 6328 A^0. Helium Neon laser operates in CW mode. The energy level E_2' is depopulated by collision with the walls of discharge tube, so that laser operation continues without interruption.
Q.9	Explain with the help of a neat diagram the principle and working of CO_2 laser. **Or** Explain construction and working of CO_2 laser with neat diagrams. **Or**

Explain construction and working of CO_2 laser with the help of energy level diagrams.

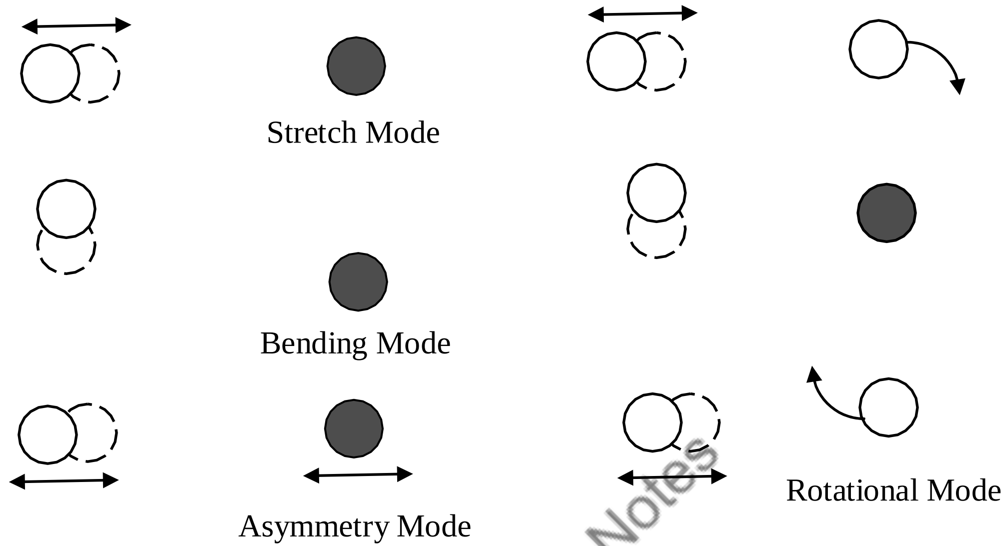
Or

Explain construction and working of any gaseous laser.

Or

Explain construction and working of any four level laser.

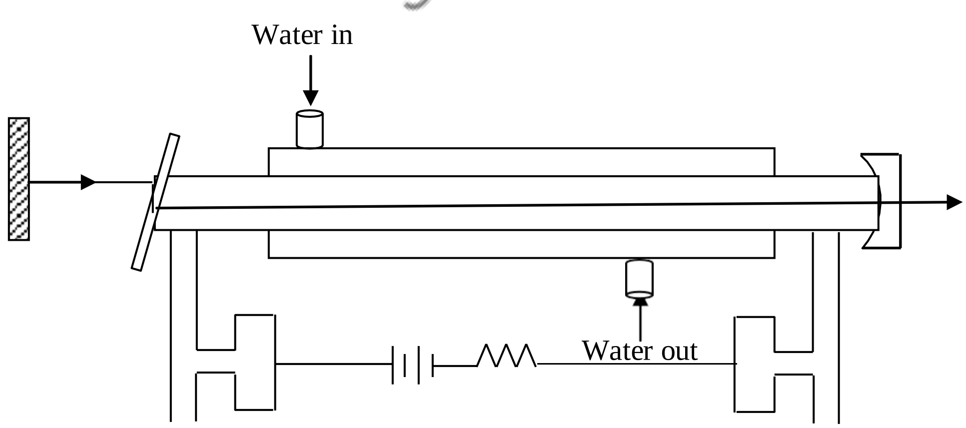
Ans: **Construction:** CO_2 is a four level molecular laser, which depends upon the transition between the different vibrational states of CO_2 molecule. Different vibrational states of CO_2 molecule are presented in the figure 1.



In the figure 1 we can see the carbon atom at the center and two oxygen atoms are attached at both sides of carbon atom. Hence carbon dioxide molecule can possess three independent modes of vibration namely: Stretch, asymmetric and bending mode.

Figure 2 shows schematic diagram of CO_2 laser. It consists of a discharge tube having cross sectional area about 1.5 mm^2 and length about 260 mm. The discharge tube is filled with CO_2, N_2 and He in the ration 1:2:3. CO_2 molecules serves as the active medium for lasing transition.

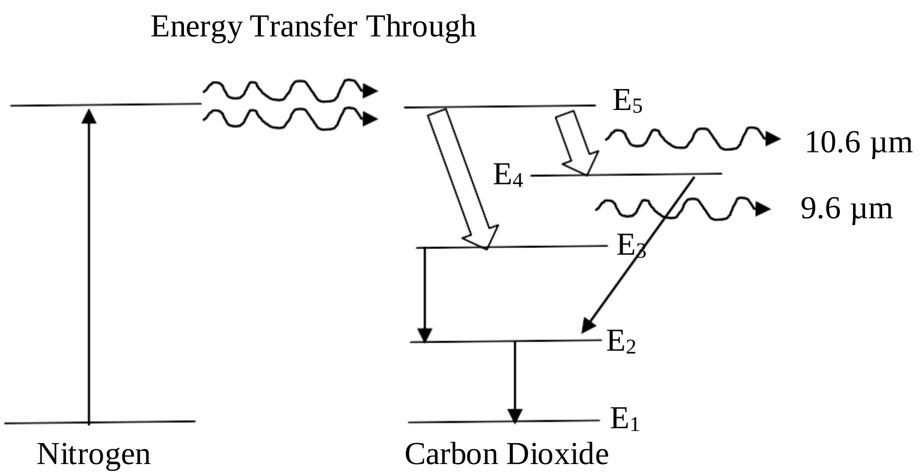
A high voltage DC source is connected across discharge tube.



Working: Different vibrational states of CO_2 molecule and lowest excited state of N_2 is shown in the figure 3.

When current is passed through discharge tube, Nitrogen gets excited to the first excited state. Now Nitrogen transfers its energy to CO_2 molecules through inelastic collisions. Thus CO_2 atoms are raised to the energy level E_5, which is a metastable state. As a result situation of population inversion is achieved between E_5 and E_4, also between E_5 and E_3.

Initial spontaneous transition initiates stimulated emission and light amplification occur by multiple oscillation between optical resonator. The lasing transition corresponds to the wavelengths 9.6 μm and 10.6 μm.

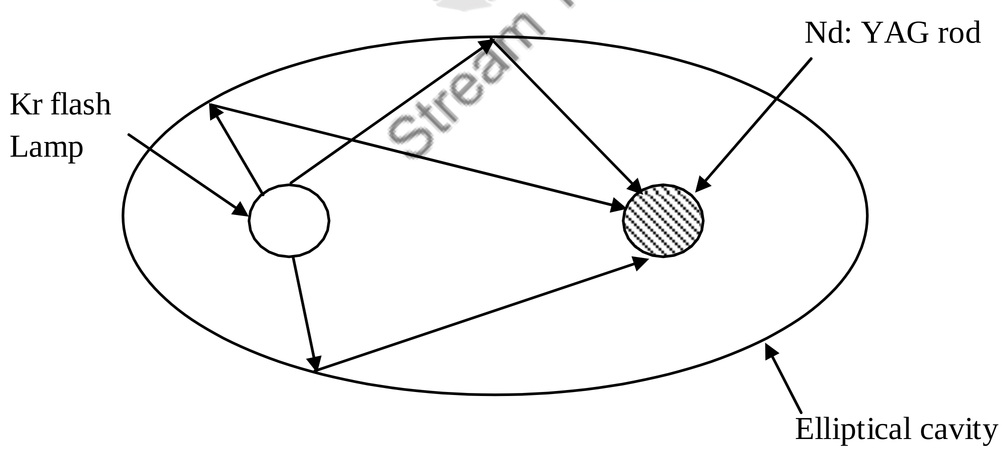


Applications: carbon dioxide laser is used in industry for cutting and welding, it also used in surgery to seal small blood vessels.

Q.10 Describe construction and working of Ruby laser with the help of necessary diagrams.
Or
Explain the working of any solid state laser
Or
Explain the working of four level solid state laser

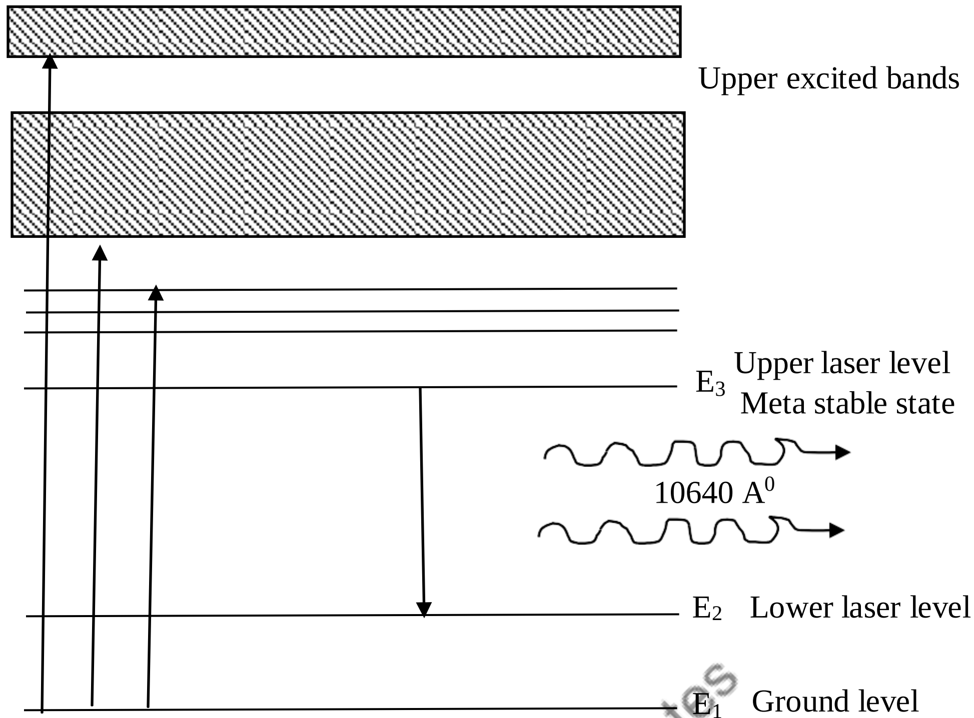
Ans: Nd: YAG laser is a four level solid state laser. Here Nd stands for Neodymium and YAG stands for Yttrium aluminum garnet ($Y_3Al_5O_{15}$). In this laser Neodymium ions acts as active medium.

Construction: Construction of Nd: YAG laser is shown in the figure 1. In this laser Nd: YAG rod and krypton arc lamp are placed in an elliptical cavity. Krypton lamp provides the optical pumping for excitation of Neodymium ions. Elliptical cavity ensures that all emitted radiations form Krypton lamp falls on Nd: YAG rod. Thus optical pumping efficiency of Nd: YAG laser is far better than Ruby laser.

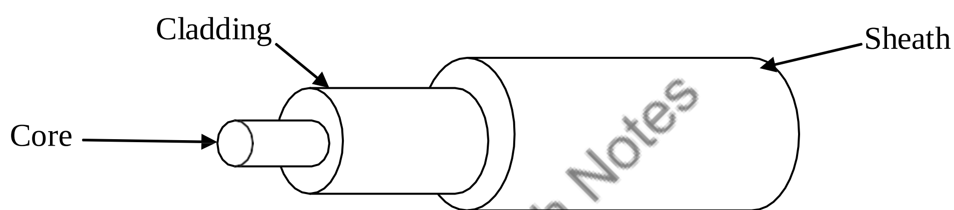


Working: Energy levels of Nd^{3+} ions are shown in the figure 2.

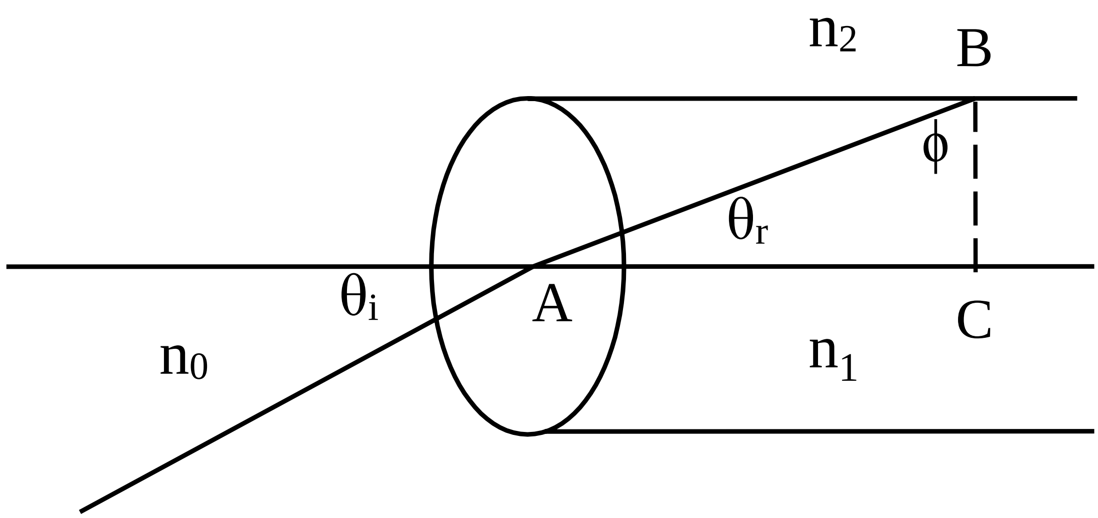
When the light from Kr lamp falls on Nd: YAG rod. Then electrons of Nd^{3+} ions absorbs the energy spectrum 5500 A^0 to 8000 A^0 and gets excited to the upper energy bands. From these energy bands electrons rapidly decays to energy levels E_3 which is a meta stable state. As a result population inversion is achieved between E_3 and E_2. Transition between E_3 and E_2 emits light in the infrared region having wavelength 10640 A^0. Energy level in Nd^{3+} ions is far above from ground state and hence can't be populated by thermal excitations. Nd: YAG laser can be operated in CW and pulsed mode


Upper excited bands
E_3 Upper laser level
Meta stable state
10640 A^0
E_2 Lower laser level
E_1 Ground level

UNIT-4
Fiber Optics

Q.11	What is an optical fiber? **(Jun12, Dec10, Jan16)** **Or** Describe the structure of a typical optical fiber used in practice. **(Dec10)** **Or** Describe the construction of an optical fiber. **(Feb10, Apr10, Dec14)**
Ans:	An optical fiber is thin transparent wire made up of glass or plastic and is used to guide the light waves by means of total internal reflection with minimum losses. **Construction:** Typical structure of an optical fiber is shown in figure below. Optical fiber consists of three sections. Innermost part of an optical fiber is called as **core** its refractive index is shown as n_1. Outer layer surrounding the core is called as **cladding** its refractive index is shown by n_2. Outermost layer of an optical fiber is a protective layer of plastic which is termed as sheath and it protects the core and cladding of optical fiber. For an Optical Fiber $n_1 > n_2$. 
Q.12	Explain the principle of propagation of light waves in the optical fiber. What are acceptance angle, acceptance cone and numerical aperture? **(Feb10, Apr10,Jun10, Dec14)** **Or** With the help of Ray diagram show how optical fibers can guide light waves? Derive an expression for acceptance angle of optical fiber. What is meant by acceptance cone? **(Jun13)** **Or** Derive expression for numerical aperture of a step index fiber. **(Jun11,16,Dec14)** **Or** Define acceptance angle and acceptance cone in fiber optics. Derive expression for acceptance angle. **(Dec10,11)** **Or** Explain how glass fiber guides light from one end to other. Define acceptance angle of an optical fiber. **(Jun12)**
Ans:	Light ray travel the length of optical fiber by phenomenon of total internal reflection. When a light ray travels from denser to the rarer medium then it moves away from the normal. When the angle of incidence in the denser medium is greater than a particular value then total light is reflected back into the denser medium. This angle of incidence is known as critical angle and the phenomenon is known as total internal reflection. In an optical fiber refractive index of core (n_1) is greater than the refractive index of cladding (n_2). Maximum value of the launching angle for the light in an optical fiber which satisfies the condition of total internal reflection is known as **acceptance angle**. Three dimensional structure around the fiber launch end made by **acceptance angle** is termed as **acceptance cone**. Derivation of Acceptance angle: A ray of light incident from the outer medium of refractive index n_0 at the launching end of the fiber (core of RI n_1)by making an angle θ_i with the axis of the fiber

which refract through angle θ_r. Angle made by refracted ray at the core-cladding interface is ϕ .



For outer medium (n_0) and core (n_1) interface
According to Snell's law

$$n_0 \, Sin\theta_i = n_1 \, Sin\theta_r$$

Or $$Sin\theta_i = (n_1/n_0) \, Sin\theta_r \quad (1)$$

→ $$Sin\theta_i = (n_1/n_0) \, Sin(90-\phi)$$

$$Sin\theta_i = (n_1/n_0) \, Cos\phi \quad (2)$$

When $\theta_i=\theta_{max}$ **Then** $\phi=\phi_c$
Therefore equation (2) can be rewritten as

$$Sin\theta_{max} = (n_1/n_0) \, Cos\phi_c \quad (3)$$

According to the Condition of Total Internal Reflection

$$Sin\phi_c = (n_2/n_1) \quad (4)$$

Using equations (3) and (4) we can write

$$Sin_{\,max} = \frac{n_1}{n_0}\sqrt{\left(1-\frac{n_2^2}{n_1^2}\right)}$$

This is known as numerical aperture (N.A.) of the fiber and θ_{max} is called the acceptance angle of the fiber. Acceptance angle is the maximum angle made by incident ray with the axis of fiber within which light ray can propagate through fiber.Hence **N.A.** = $\frac{\sqrt{n_1^2-n_2^2}}{n_0}$

Acceptance angle $\theta_m = n^{-1} \frac{\sqrt{n_1^2-n_2^2}}{n_0}$

If outer medium is air or vacuum than $n_0 \approx 1$

N.A. = $\sqrt{n_1^2 - n_2^2}$

Acceptance angle $\theta_m = n^{-1}(\sqrt{n_1^2 - n_2^2})$

Q.13 Explain the types of fibers and their index profiles. **(Jan16,Jun11,15,Dec15)**

Ans: **Modes of Propagation:** In an optical fiber light travels through total internal reflection. During the travel light can take different paths inside the optical fibers. These different paths are known as the modes of propagation.

Based on the differences in the structure of core and the modes of propagation, optical fibers are classified into three categories:

1. **Single mode step index optical fiber:** As the name suggests this type of fiber supports only one mode of propagation and the refractive index of the core remains uniform. In Single mode fibers intermodal dispersion does not take place, however at the same time it has difficulty of coupling of the light into the fiber.
2. **Multimode step index optical fiber:** As the name suggests this type of fiber can support multiple modes of propagation and the refractive index of the core remains uniform. In these fibers intermodal dispersion occurs and may cause signal distortion.
3. **Multimode graded index optical fiber:** As the name suggests this type of fiber can support multiple modes of propagation and the refractive index of the core gradually decreases from the center of the core towards the outer of the core. In these fibers intermodal dispersion does occur, however it is less in comparison to the **Multimode step index optical fiber.**

   Index profiles for step index and graded index fibers is shown in the figures 1 and respectively

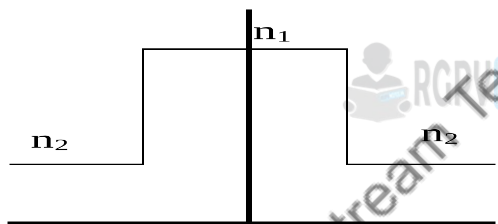


Fig 1: Index Profile of Step Index Fiber

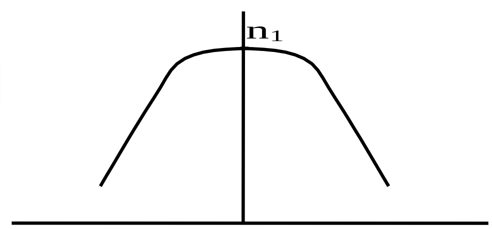
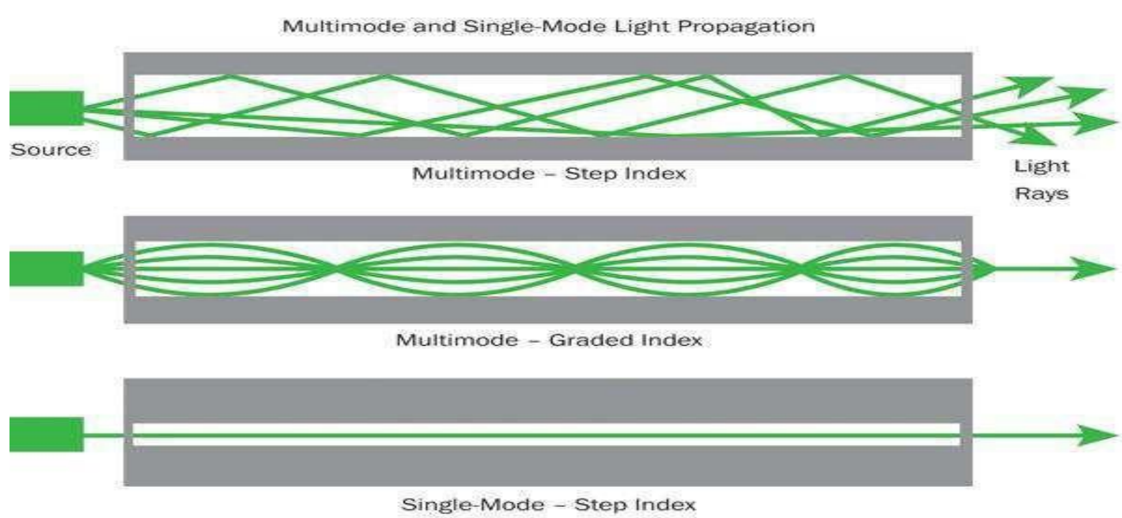


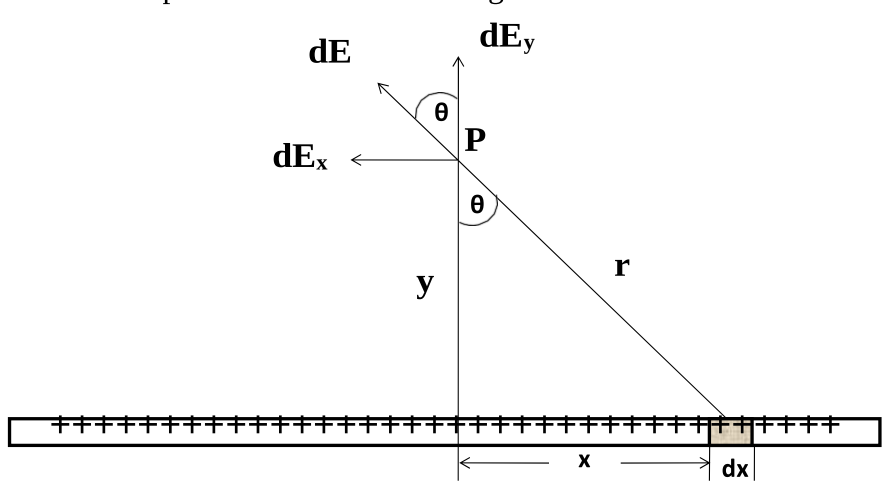
Fig 2: Index Profile of Graded Index Fiber

Figure below represents the modes of propagation in three types of the fibers.



Q.14	What is V-number or normalized frequency of an optical fiber? **(Jun10, Dec12)**
Ans:	V-number or normalized frequency gives the idea about the cutoff frequency and the number of modes supported by an optical fiber. V-number for an optical fiber is given as: $$V = \frac{2\pi a}{\lambda}\sqrt{(n_1^2 - n_2^2)}$$ Where $n_1 - Refractive\ index\ of\ core$ $n_2 - Refractive\ index\ of\ clading$ $a - Core\ Radius$ $\lambda - Wavelength$ If the value of V<2.405 then the fiber is said single mode optical fiber whereas for V>2.405 multi mode optical fiber. $Number\ of\ modes\ supported\ by\ step\ index\ Fiber\ is\ given\ as = \frac{V^2}{2}$ $Number\ of\ modes\ supported\ by\ graded\ index\ fiber\ is\ given\ as = \frac{V^2}{4}$
Q.15	What is attenuation of signal in an optical fiber?
Ans:	**Attenuation** is the rate at which the signal light decreases in intensity during the transmission of light through optical fiber. Attenuation coefficient α is given as :

ELECTROSTATISTICS IN VACUUM

Q.1	Explain the Coulomb's law of Electro statistics.
Ans:	Coulomb proposed the laws for interaction between static electric charges. Which are as: **First Law:** Like Charges repel each other and unlike charges attract each other **Second Law:** Magnitude of the force exerted by the charges on each other is inversely proportional to the square of the distance between them and is directly proportional of the product of their charges $F \propto \frac{1}{r^2}$ and $F \propto q_1 q_2$ $F = K\frac{q_1 q_2}{r^2}$ $\text{Where } K = \frac{1}{4\pi\epsilon_0\epsilon_r}$ $\text{For air } \epsilon_r = 1$
Q.2	Explain the terms: Electric Field Intensity, Electric Lines of Forces and Electric Flux.
Ans:	Electric Force experienced by unit positive charge in the electric filed is known as **Electric Field Intensity** and is denoted by **E.** Expression for electric filed intensity is $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ Path along which a unit positive charge tend to move when free to do so in the electric filed is known as **Electric Lines of Forces.** Number of electric lines of forces through a specified area I called as **Electric Flux** it is denoted by Greek letter ϕ. Mathematically electric flux through a surface S is given as $= \int E.ds$
Q.3	Calculate the electric field intensity due to an infinite line charge.
Ans:	Let us consider a line of infinite length having charge per unit length λ. Now we consider a point P at Y distance from line and consider a small length element dx having charge $dq=\lambda dx$, which is at a distance r from point P as shown in the figure  The x and y component of the force dE will be $dE_x = -dE\ Sin\ \ and\ dE_y = dE\ Cos$ As x components at P will cancel out each other therefore net filed will be due to only y-component.

Therefore

$$E=\int_{x=-\infty}^{x=+\infty} dE_y=\int_{x=-\infty}^{x=+\infty} dE\,Cos$$

Or $E=\int_{x=0}^{x=+\infty} 2dE\,Cos$

$$E=\int_{x=0}^{x=+\infty}\frac{2dq\,Cos}{4\pi\epsilon_0 r^2} \qquad (1)$$

As $dq=\lambda dx$ therefore

$$E=\int_{x=0}^{x=+\infty}\frac{2\lambda dx\,Cos}{4\pi\epsilon_0 r^2}$$

Or

$$E=\frac{\lambda}{2\pi\epsilon_0}\int_0^{\infty}\frac{dx}{r^2}Cos \qquad (2)$$

From the figure above we can write

$$\frac{x}{y}=tan \qquad (3)$$

Or

$$dx=y\,sec^2 d \qquad (4)$$

Also

$$r^2=x^2+y^2 \qquad (5)$$

Placing the value of **dx** and **r^2** in equation (2) we get

$$E=\frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}}\frac{y\,sec^2 d}{x^2+y^2}Cos \qquad (6)$$

Using equation (3), above equation can be rewritten as

$$E=\frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}}\frac{y\,sec^2 d}{y^2 tan^2+y^2}Cos$$

Or

$$E=\frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}}\frac{y\,sec^2 Cos\,d}{y^2 sec^2}$$

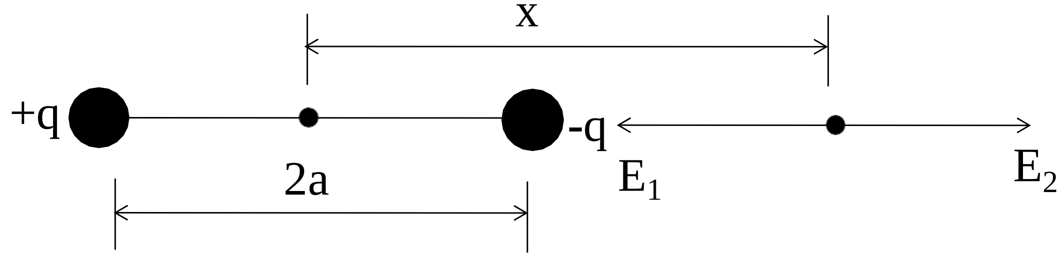
$$E=\frac{\lambda}{2\pi\epsilon_0}\int_0^{\frac{\pi}{2}}\frac{Cos}{y}\,d$$

$$E=\frac{\lambda}{2\pi\epsilon_0 y}\left[Sin\right]_0^{\frac{\pi}{2}}$$

$$E=\frac{\lambda}{2\pi\epsilon_0 y}$$

Q.4	Calculate the electric field intensity due to a dipole at axial position.

Ans: Let us consider a dipole of separated by length ***2a***. Now we consider a point P at ***x*** distance from the center of dipole as shown in the figure



Charge at P will experience two forces E_1 and E_2 in opposite directions. Where

$$E_1 = \frac{q}{4\pi\epsilon_0 (x-a)^2} \quad \text{and} \quad E_2 = \frac{q}{4\pi\epsilon_0 (x+a)^2}$$

The resultant electric field intensity will be

$$E = E_1 - E_2$$

Or

$$E = \frac{q}{4\pi\epsilon_0 (x-a)^2} - \frac{q}{4\pi\epsilon_0 (x+a)^2}$$

$$\Rightarrow \quad E = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right)$$

$$\Rightarrow \quad E = \frac{q}{4\pi\epsilon_0}\left(\frac{(x+a)^2 - (x-a)^2}{(x-a)^2 \times (x+a)^2}\right)$$

$$\Rightarrow \quad E = \frac{q}{4\pi\epsilon_0}\left(\frac{4xa}{(x^2-a^2)^2}\right)$$

If $x>>a$ then

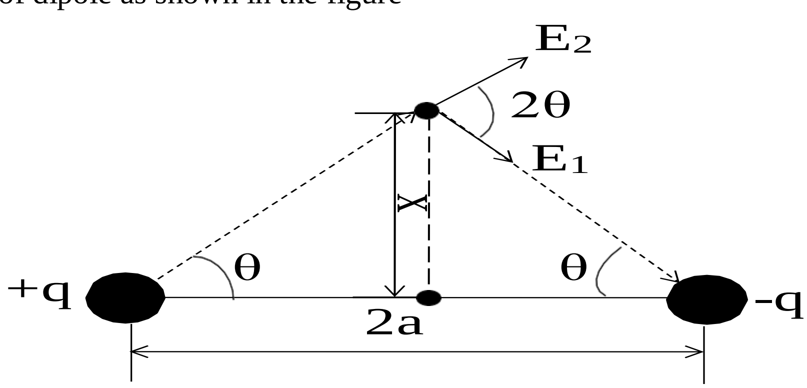
$$E = \frac{q}{4\pi\epsilon_0}\left(\frac{4xa}{x^4}\right)$$

$$\Rightarrow \quad E = \frac{4qa}{4\pi\epsilon_0 x^3}$$

$$\Rightarrow \quad E = \frac{2\times 2qa}{4\pi\epsilon_0 x^3} \quad \text{or} \quad E = \frac{2p}{4\pi\epsilon_0 x^3}$$

Q.5	Calculate the electric field intensity due to a dipole at perpendicular bisector of dipole.

Ans: Let us consider a dipole of separated by length ***2a***. Now we consider a point P at ***y*** distance from the center of dipole as shown in the figure



Electric filed at point P will be E_1 and E_2 at an angle 2θ, The magnitude of both the forces are equal i.e. we can write

$$E_1 = E_2 = \frac{q}{4\pi\epsilon_0 (x^2 + a^2)}$$

therefore the resultant electric filed intensity can be resolved as

$$E^2 = \left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2 + \left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2 + 2\left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2 Cos\, 2$$

$$E^2 = 2\left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2 (1 + Cos\, 2)$$

$$E^2 = 2\left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2 (1 + 2\, Cos^2 - 1)$$

$$E^2 = 4\, Cos^2 \left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)^2$$

$$E = 2\, Cos \left(\frac{q}{4\pi\epsilon_0 (x^2 + a^2)}\right)$$

$$As \quad Cos = \left(\frac{a}{\sqrt{(x^2 + a^2)}}\right)$$

Therefore

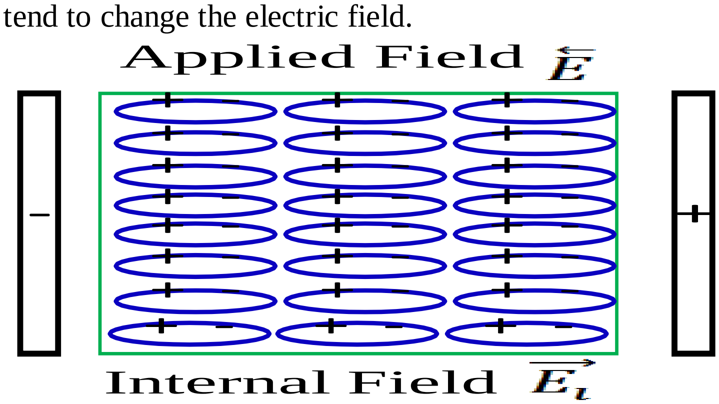
$$E = 2\left(\frac{a}{\sqrt{(x^2 + a^2)}}\right) \frac{q}{4\pi\epsilon_0 (x^2 + a^2)}$$

$$E = \left(\frac{2aq}{4\pi\epsilon_0 (x^2 + a^2)^{3/2}}\right)$$

$$E = \left(\frac{P}{4\pi\epsilon_0 (x^2 + a^2)^{3/2}}\right)$$

Q.6 Define the term Dielectrics, Dielectric Constant, Electric Polarization (P) and Displacement Current (D), also derive the relation between D, E and P.

Ans: **Dielectrics** materials are basically non conducting materials, used for storage of static charge. These materials does not allow the flow of electricity through them when placed in the electric field, but they tend to change the electric field.



Dielectric Constant or Relative Permitivity

The dielectric constant is the ratio of the permittivity of a substance to the permittivity of free space

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Dielectric constant can also be expressed as ratio of the capacitance of a capacitor with an insulating material between its plates, to its capacitance in case of vacuum between its plates.

$$\epsilon_r = \frac{C}{C_0}$$

Electric Polarization

It is defined as induced dipole moment per unit volume and is denoted by P. By definition we can write

$$P = \frac{ql}{V}$$

or

$$P = \frac{ql}{Al}$$

$$P = \frac{q}{A} = \sigma_p$$

Equation above suggests that electric polarization numerically equals to the induced surface charge density.

Relation between D , E and P

As shown in the figure above when a dielectric is subjected to an external field then an internal filed is produced in the dielectric which is in the opposite direction of the applied field. Therefore the net electric field E can be given as

$$E = E_a - E_i$$

Where E_a is applied external field and E_i is induced internal electric field.

$$E = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

Where σ and σ_p are free and induced surface charge densities respectively.

$$\epsilon_0 E = \sigma - \sigma_p$$

or

$$\epsilon_0 E = D - P$$

Here D is called **Displacement Vector** its magnitude equals to the surface density of free charges σ. Idea of displacement current is introduced to justify the flow of electric current in capacitive circuit between the plates of capacitor where no charges are present.

Q.7 Explain the terms gradient, divergence and curl. Also give their physical significance.

Ans: **Gradient:** Gradient of a scalar filed is a vector quantity, whose magnitude gives the maximum space rate variation of that scalar quantity at that point. It tends to point in the direction of greatest change of scalar field. Mathematically it can be represented as

$$grad\, f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k = \nabla f$$

Here $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

Physical significance: Gradient of potential field gives the electric field intensity i.e.

$$\bar{E} = \frac{\partial V}{\partial x}$$

Divergence: The divergence of a vector field at a point is a scalar quantity of magnitude equal to the flux of that vector field diverging out per unit volume. It tells us about the presence of source and sink within the volume under consideration. It is mathematically represented as

$$\mathrm{div}(A) = \nabla . A$$

$$\mathrm{div}(A) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

Physical Significance: If div (A) = 0; then no source or sink is present in the volume under consideration it is also said as solenoid vector function. When div (A) >0 it indicates the presence of source and in case div (A) < 0 it indicates the presence of sink.

Here $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

Curl: Curl of a vector field at point is a vector quantity whose magnitude is equal to the maximum value of line integral of that vector per unit area along the boundary of a small elementary area around that point. It is given as

$$curl\ A = \nabla \times A$$

$$curl\ A = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$$

Physical Significance: Curl of a vector field tells about the rotation of the vector field. If for a vector A value of $curl\ A = \nabla \times A = 0$ then the vector is said to be irrotational.

Q.8 Explain the Gauss' divergence theorem and Stokes' theorem.

Ans: **Gauss's divergence theorem:** This theorem states that the flux of a vector field $\bar{F}$ over any closed surface S is equal to the volume integral of the divergence of vector field over the volume enclosed by the surface S i.e.

$$\bar{F}.\bar{dS} = \quad div\ \bar{F}\bar{dV} = \quad (\nabla.\bar{F})\ \bar{dV}$$

Divergence of any vector filed is defined as flux per unit volume. Mathematically we can write divergence of any vector field as

$$div\ \bar{F} = \frac{\bar{F}.\bar{dS}}{\bar{dV}}$$

Hence

$$\bar{F}.\bar{dS} = \quad div\ \bar{F}\bar{dV}$$

Stokes' theorem.

Stokes' theorem states that the net circulation of vector $\bar{F}$ over some open surface $\bar{S}$ equals to the line integral of $\bar{F}$ along the closed contour C which bounds $\bar{S}$ thus

$$(\nabla \times \bar{F}).\bar{dS} = \quad \bar{F}.\bar{dl}$$

In a conservative field

$$\bar{F}.\bar{dl} = 0$$

By Stokes' theorem we can write

$$(\nabla \times \bar{F}).\bar{dS} = 0$$

Or

$$\nabla \times \bar{F} = 0$$

Thus curl of a conservative filed is zero.

Q.9 Give the continuity equation of current densities. Explain its physical significance.

Ans **Continuity equation**

Let us consider a volume *V* bounded by a surface *S*. A net charge *Q* exists within this region. If a net current *I* flows across the surface out of this region, from the principle of conservation of charge this current can be equated to the time rate of decrease of charge within this volume. Similarly, if a net current flows into the region, the charge in the volume must increase at a rate equal to the current. Thus we can write

$$I = -\frac{dQ}{dt}$$

$$J.ds = -\frac{d}{dt}\rho.dV$$

On Applying Gauss' divergence theorem we can write,

$$\nabla.J dV = -\frac{d}{dt}\quad \rho.dV$$

$$\nabla.J + \frac{\partial \rho}{\partial t} = 0$$

Equation above is known as equation of continuity.

Physical Significance

Current is the movement of charge. The **continuity equation** says that if charge is moving out of a differential volume (i.e. divergence of **current** density is positive) then the amount of charge within that volume is going to decrease, so the rate of change of charge density is negative.

Q.10 Derive the Maxwell's equation for free space.

Ans: **Maxwell's first equation**

Consider a surface S bounding total charge q. Then using Gauss' law the amount of flux from the surface S can be written as

$$\bar{E}.\bar{dS} = \frac{q}{\epsilon_0}$$

Or

$$\epsilon_0\bar{E}.\bar{dS} = q$$

Or

$$\bar{D}.\bar{dS} = q$$

Or

$$\bar{D}.\bar{dS} = \quad \rho\, dV$$

Using Gauss's divergence theorem

$$\nabla . \bar{D} dV = \rho \, dV$$

On comparing both sides we can write

$$\nabla . \bar{D} = \rho$$

The expression above is Maxwell's first equation.

Maxwell's second equation

As we know that isolated magnetic poles does not exist. They always exist in pairs. As a consequence magnetic lines of forces entering any arbitrary closed surface are exactly the same as leaving it. Thus flux of magnetic induction B across any closed surface is always zero.

$$\bar{B} . d\bar{S} = 0$$

By Gauss's divergence theorem

$$\nabla . \bar{B} dV = 0$$

Or

$$\nabla . \bar{B} = 0$$

Equation above is Maxwell's second equation.

Maxwell's third equation

According to Faraday's law of electromagnetic induction, the induced emf in a closed loop equals negative rate of change of magnetic flux

$$e = -\frac{\partial}{\partial t}$$

As

$$= B . dS$$

Hence we can write

$$e = \frac{\partial}{\partial t} B . dS$$

By definition of emf it said that emf equals the work done in carrying a unit charge around a closed loop therefore

$$e = \int E . dl$$

Or

$$\int E . dl = \frac{\partial}{\partial t} B . dS$$

Using Stoke's theorem

$$\nabla \times E . dS = \frac{\partial}{\partial t} B . dS$$

Or

$$\nabla \times E . dS + \frac{\partial}{\partial t} B . dS = 0$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

Or

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is Maxwell's third equation.

Maxwell's fourth equation

According to Ampere's law

$$H.dl = I$$

As

$$I = \quad J.ds$$

Therefore

$$H.dl = \quad J.ds$$

Using Stokes, Theorem

$$\nabla \times H.dS = \quad J.ds$$

$\Rightarrow$
$$\nabla \times H - \mathrm{j} = 0$$

Taking the divergence of the above equation

$$\nabla.(\nabla \times H) - \nabla.J = 0$$

Since divergence of curl of any vector is always zero

Therefore $\nabla.J = 0$

This is the case for steady fields, so equation above need to be changed for general cases.

According to Gauss' Law

$$\nabla.E = \frac{\rho}{\epsilon_0}$$

Differentiating the equation wrt Time

$$\nabla.\epsilon_0 \frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$$

Adding $\nabla.J$ to both sides of above equation we get

$$\nabla.\mathrm{J} + \nabla.\epsilon_0 \frac{\partial E}{\partial t} = \nabla.\mathrm{J} + \frac{\partial \rho}{\partial t}$$

RHS of the above equation is continuity equation. Therefore

$$\nabla.\mathrm{J} + \nabla.\frac{\partial D}{\partial t} = 0$$

Here $\epsilon_0 E = D$

So the $\nabla.(\nabla \times H) - \nabla.J = 0$ will be rewritten as

$$\nabla.(\nabla \times H) - \nabla.\mathrm{J} + \nabla.\frac{\partial D}{\partial t} = 0$$

Or

$$\nabla \times H = \mathrm{J} + \frac{\partial D}{\partial t}$$

Equation above is Maxwell's fourth equation.

Q.11 Write Maxwell's equations in integral form.

Ans: **Maxwell's equation in integral form.**

1. **Maxwell's first equation in integral form**

$$\nabla . D = \rho$$

Integrating above over an arbitrary volume

$$\nabla . D \; dV = \quad \rho \; dV$$

Using Gauss's divergence theorem

$$D . dS = \quad \rho \; dV$$

Physical significance: It signifies that the net outward flux of electric displacement vector equals total charge within the volume

2. **Maxwell's second equation in integral form**

$$\nabla . B = 0$$

Integrating above over an arbitrary volume

$$\nabla . B \; dV = 0$$

Using Gauss's divergence theorem

$$B . dS = 0$$

Physical significance: It signifies that the magnetic induction flux through any closed surface is equal to zero.

3. **Maxwell's third equation in integral form**

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Integrating over a surface S bounded by a curve C

$$\nabla \times E \; . dS = - \quad \frac{\partial B}{\partial t} \; . dS$$

Using Stokes' theorem

$$E . dl = -\frac{\partial}{\partial t} \quad B \; . dS$$

The value of emf around a closed loop equals to negative rate of change of magnetic flux linked with the path.

4. **Maxwell's fourth equation in integral form**

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Integrating over a surface S bounded by a curve C

$$\nabla \times H \; . dS = \quad (J + \frac{\partial D}{\partial t}) \; . dS$$

Using Stokes' theorem

$$H . dl = \quad (J + \frac{\partial D}{\partial t}) \; . dS$$

5. The value of mmf around a closed loop equals to conduction current plus of total displacement current linked with the path.

