RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA, BHOPAL

New Scheme Based On AICTE Flexible Curricula

B. Tech. First Year

Branch- Common to All Disciplines

BT204	**Basic Civil Engineering & Mechanics**	**3L-0T-2P**	**4 Credits**

Course Contents:

Unit I Building Materials & Construction

Stones, bricks, cement, lime, timber-types, properties, test & uses, laboratory tests concrete and mortar Materials: Workability, Strength properties of Concrete, Nominal proportion of Concrete preparation of concrete, compaction, curing.

Elements of Building Construction, Foundations conventional spread footings, RCC footings, brick masonry walls, plastering and pointing, floors, roofs, Doors, windows, lintels, staircases – types and their suitability

Unit II Surveying & Positioning:

Introduction to surveying Instruments – levels, thedolites, plane tables and related devices. Electronic surveying instruments etc. Measurement of distances – conventional and EDM methods, measurement of directions by different methods, measurement of elevations by different methods. Reciprocal leveling.

Unit III Mapping & sensing:

Mapping details and contouring, Profile Cross sectioning and measurement of areas, volumes, application of measurements in quantity computations, Survey stations, Introduction of remote sensing and its applications.

Engineering Mechanics

Unit IV

Forces and Equilibrium: Graphical and Analytical Treatment of Concurrent and non-concurrent Co- planner forces, free Diagram, Force Diagram and Bow's notations, Application of Equilibrium Concepts: Analysis of plane Trusses: Method of joints, Method of Sections. Frictional force in equilibrium problems

Unit – V

Centre of Gravity and moment of Inertia: Centroid and Centre of Gravity, Moment Inertia of Area and Mass, Radius of Gyration, Introduction to product of Inertia and Principle Axes. Support Reactions, Shear force and bending moment Diagram for Cantilever & simply supported beam with concentrated, distributed load and Couple.

Reference Books:

1. S. Ramamrutam & R.Narayanan; Basic Civil Engineering, Dhanpat Rai Pub.
2. Prasad I.B., Applied Mechanics, Khanna Publication.
3. Punmia, B.C., Surveying, Standard book depot.
4. Shesha Prakash and Mogaveer; Elements of Civil Engg & Engg. Mechanics; PHI
5. S.P,Timoshenko, Mechanics of stricture, East West press Pvt.Ltd.
6. Surveying by Duggal – Tata McGraw Hill New Delhi.
7. Building Construction by S.C. Rangwala- Charotar publications House, Anand.
8. Building Construction by Grucharan Singh- Standard Book House, New Delhi
9. Global Positioning System Principles and application- Gopi, TMH
10. R.C. Hibbler – Engineering Mechanics: Statics & Dynamics.
11. A. Boresi & Schmidt- Engineering Mechines- statics dynamics, Thomson' Books
12. R.K. Rajput, Engineering Mechanics S.Chand & Co.

Unit: 01

BUILDING MATERIALS:

1. Bricks: Manufacturing, field and laboratory test, Engineering properties.

2. Cement: Types, physical properties, laboratory tests

3. Concrete and Mortar Materials: Workability, Strength Properties of Concrete, Nominal proportion of Concrete, Preparation of Concrete, Compaction Curving.

Mortar: Properties and Uses.

Bricks:

Properties:

All properties of brick are affected by raw material composition and the manufacturing process. Most manufacturers blend different clays to achieve the desired properties of the raw materials and of the fired brick. This improves the overall quality of the finished product. The quality control during the manufacturing process permits the manufacturer to limit variations due to processing and to produce a more uniform product. The most important properties of brick are 1) durability, 2) color, 3) texture, 4) size variation, 5) compressive strength and 6) absorption.

Durability

The durability of brick depends upon achieving incipient fusion and partial vitrification during firing. Because compressive strength and absorption values are also related to the firing temperatures, these properties, together with saturation coefficient, are currently taken as predictors of durability in brick specifications. However, because of differences in raw materials and manufacturing methods, a single set of values of compressive strength and absorption will not reliably indicate the degree of firing.

Color

The color of fired clay depends upon its chemical composition, the firing temperatures and the method of firing control. Of all the oxides commonly found in clays, iron probably has the greatest effect on color. Regardless of its natural color, clay containing iron in practically any form will exhibit a shade of red when exposed to an oxidizing fire because of the formation of ferrous oxide. When fired in a reducing atmosphere, the same clay will assume a dark (or black) hue. Creating a reducing atmosphere in the kiln is known as flashing or reduction firing. Given the same raw material and manufacturing method, darker colors are associated with higher firing temperatures, lower absorption values and higher compressive strength values. However, for products made from different raw materials, there is no direct relationship between strength and color or absorption and color.

Texture, Coatings and Glazes

Many brick have smooth or sand-finished textures produced by the dies or molds used in forming. Smooth textures, commonly referred to as a die skin, results from pressure exerted by the steel die as the clay passes through it in the extrusion process. Most extruded brick have the die skin removed and the surface further treated to produce other textures using devices that cut, scratch, roll, brush or otherwise roughen the surface as the clay column leaves the die (see **Photo 6**).

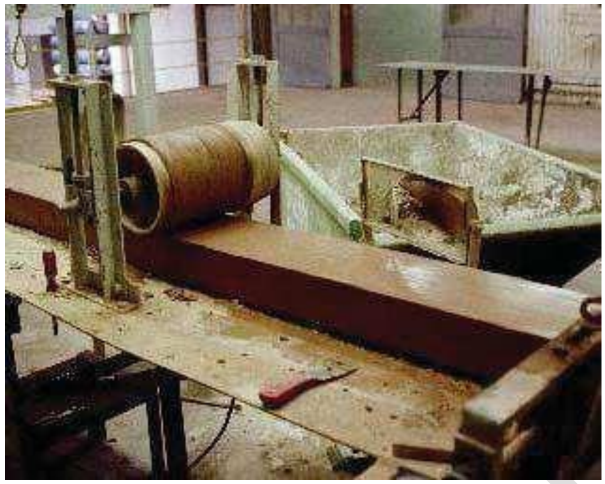


Figure: Some Brick Textures are applied by Passing under a Roller after Extrusion

Size Variation

Because clays shrink during both drying and firing, allowances are made in the forming process to achieve the desired size of the finished brick. Both drying shrinkage and firing shrinkage vary for different clays, usually falling within the following ranges:

- **Drying shrinkage: 2 to 4 percent**
- **Firing shrinkage: 2.5 to 4 percent**

Firing shrinkage increases with higher temperatures, which produce darker shades. When a wide range of colors is desired, some variation between the sizes of the dark and light units is inevitable. To obtain products of uniform size, manufacturers control factors contributing to shrinkage. Because of normal variations in raw materials and temperature variations within kilns, absolute uniformity is impossible. Consequently, specifications for brick allow size variations.

Compressive Strength and Absorption

Both compressive strength and absorption are affected by properties of the clay, method of manufacture and degree of firing. For a given clay and method of manufacture, higher compressive strength values and lower absorption values are associated with higher firing temperatures. Although absorption and compressive strength can be controlled by manufacturing and firing methods, these properties depend largely upon the properties of the raw materials.

Rail kilns

In modern brickworks, this is usually done in a continuously fired tunnel kiln, in which the bricks are fired as they move slowly through the kiln on conveyors, rails, or kiln cars, which achieves a more consistent brick product. The bricks often have lime, ash, and organic matter added, which accelerates the burning process.

Bull's Trench Kilns

In India, brick making is typically a manual process. The most common type of brick kiln in use there is the Bull's Trench Kiln (BTK), based on a design developed by British engineer W. Bull in the late 19th century.

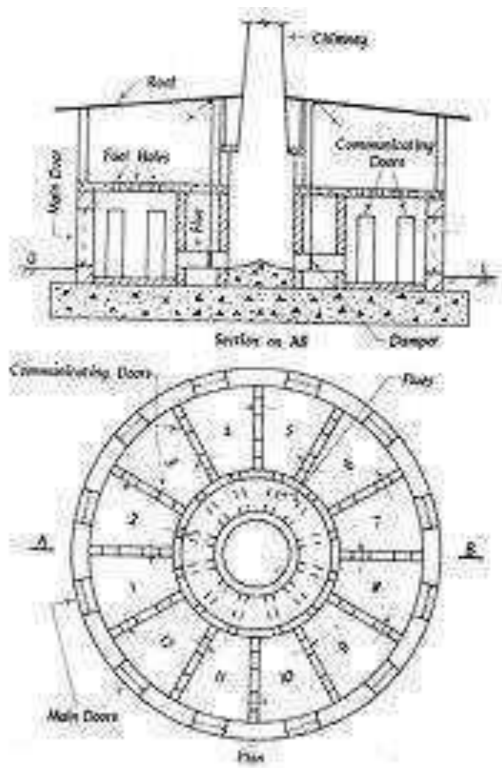


Figure 2: Bull's Trench Kiln

An oval or circular trench is dug, 6–9 meters wide, 2-2.5 meters deep, and 100–150 meters in circumference. A tall exhaust chimney is constructed in the centre. Half or more of the trench is filled with "green" (unfired) bricks which are stacked in an open lattice pattern to allow airflow. The lattice is capped with a roofing layer of finished brick. In operation, new green bricks, along with roofing bricks, are stacked at one end of the brick pile; cooled finished bricks are removed from the other end for transport to their destinations. In the middle, the brick workers create a firing zone by dropping fuel (coal, wood, oil, debris, and so on) through access holes in the roof above the trench. The advantage of the BTK design is a much greater energy efficiency compared with clamp or scove kilns. Sheet metal or boards are used to route the airflow through the brick lattice so that fresh air flows first through the recently burned bricks, heating the air, then through the active burning zone. The air continues through the green brick zone (pre-heating and drying the bricks), and finally out the chimney, where the rising gases create suction which pulls air through the system. The reuse of heated air yields savings in fuel cost.

As with the rail process above, the BTK process is continuous. A half dozen laborers working around the clock can fire approximately 15,000–25,000 bricks a day. Unlike the rail process, in the BTK process the bricks do not move. Instead, the locations at which the bricks are loaded, fired, and unloaded gradually rotate through the trench.

Use

Bricks are used for building, block paving and pavement. Bricks in the metallurgy and glass industries are often used for lining furnaces, in particular refractory bricks such as silica, magnesia, chamotte and neutral (chromo magnesite) refractory bricks. This type of brick must have good thermal shock resistance, refractoriness under load, high melting point, and satisfactory porosity. Engineering bricks are used where strength, low water porosity or acid (flue gas) resistance are needed.

Cement

Cement is the product obtained by burning a well-proportioned mixture of silicious, argillaceous and calcareous materials and crushing the same into a fine powder. In the most general sense of the word, cement is a binder, a substance that sets and hardens

independently, and can bind other materials together. The word "cement" traces to the Romans, who used the term opus caementicium to describe masonry resembling modern concrete that was made from crushed rock with burnt lime as binder. The volcanic ash and pulverized brick additives that were added to the burnt lime to obtain a hydraulic binder were later referred to as cementum, cimentum, cäment, and cement.

Cements used in construction can be characterized as being either hydraulic or non-hydraulic. Hydraulic cements (*e.g.,* Portland cement) harden because of hydration, a chemical reaction between the anhydrous cement powder and water. Thus, they can harden underwater or when constantly exposed to wet weather. The chemical reaction results in hydrates that are not very water-soluble and so are quite durable in water. Non-hydraulic cements do not harden underwater; for example, slaked limes harden by reaction with atmospheric carbon dioxide.

The most important uses of cement are as an ingredient in the production of mortar in masonry, and of concrete, a combination of cement and an aggregate to form a strong building material. In 1824, Joseph Aspdin patented a material, which he called Portland cement, because the render made from it was in color similar to the prestigious Portland stone.

Types of cement

1. **Ordinary Portland cement (OPC – IS 269): Cement is made by heating limestone (calcium carbonate) with small quantities of other materials (such as clay) to 1450 °C in a kiln, in a process known as calcination, whereby a molecule of carbon dioxide is liberated from the calcium carbonate to form calcium oxide, or quicklime, which is then blended with the other materials that have been included in the mix. The resulting hard substance, called 'clinker', is then ground with a small amount of gypsum into a powder to make 'Ordinary Portland Cement', the most commonly used type of cement (often referred to as OPC). Portland cement is a basic ingredient of concrete, mortar and most non-specialty grout. The most common use for Portland cement is in the production of concrete. Concrete is a composite material consisting of aggregate (gravel and sand), cement, and water. As a construction material, concrete can be cast in almost any shape desired, and once hardened, can become a structural (load bearing) element. Portland cement may be grey or white.**

The Portland cement consists of the following components:

1) **Lime 60% to 67%.**
2) **Silica 17% to 25%.**
3) **Alumina 03% to 08%.**
4) **Iron oxide 0.50% to 06%.**
5) **Magnesia 0.10% to 04%.**
6) **Soda and Potash 0.20% to 01%.**

7) **Sulphur Trioxide 01% to 2.75%.**
8) **Free Lime 0% to 1%.**

Cement is a complicated mixture of chemical compounds. Ordinary Portland cement contains the following compounds:

1) **Tri calcium silicate $(CaO)_3 SiO_2$ denoted by C_3S.**
2) **Di calcium silicate $(CaO)_2 SiO_2$ denoted by C_2S.**
3) **Tri calcium Aluminate $(CaO)_3 Al_2O_3$ denoted by C_3A.**
4) **Tetra calcium Alumino ferrite $(CaO)_4 Al_2O_3 Fe_2O_3$ denoted by C_4AF.**

About 70% to 80% of cement is contributed by C_3S and C_2S which are responsible for the strength of cement. These compounds provide to cement the property to resist the attacks of acids and alkalis and make the cement durable. Tri calcium silicate has the property of hydrating rapidly and is responsible to provide not only early strength but also ultimate strength. Di calcium silicate has the property of hydrating less rapidly and provides the strength after duration of 7 days. Tri calcium Aluminate gets hydrated rapidly and is also responsible to provide early strength but is found to slightly retard the ultimate strength. The compound is susceptible to be attacked by alkalis and salts. Tetra calcium Alumino ferrite appears to be redundant not known to provide any strength and is at times considered as an undesirable compound which gets into cement.

2. **Rapid hardening Portland cement**

This type of cement is manufactured in a manner similar to that of ordinary Portland cement, but it differs in respect of chemical composition, degree of burning and grinding. This contains greater lime content than ordinary Portland cement. This requires a better control of burning the ingredients over a longer period of time. The clinker is ground to a much finer extent than the ordinary Portland cement. The quick rate of gain of strength is due to a higher content of Tri calcium silicate and finer grinding of clinker. For the same water cement ratio, it gains strength in 3 days what the ordinary Portland cement gains in 7 days; and it gains a strength in 7 days what the ordinary cement gains in 28 days. The main advantage in using rapid hardening Portland cement is that the shuttering may be struck considerably earlier, thus saving considerable time and expenses. This cement is particularly satisfactory under conditions of frost. This cement can also be used for road work, so that the road can be open to traffic with least delay.

3. **Low Heat Portland cement**

During the process of setting and hardening of cement, an appreciable rise of temperature occurs. The heat generated in ordinary cement at the end of 3 days may be at the order of 80 calories per gram of cement. In massive constructions like abutments, retaining walls, dams etc. the rate at which the heat can be lost at the surface is lower than at which the heat is initially generated. The temperature rise is an important factor, because the shrinking which

may take place during the cooling processes introduces tensile stresses in concrete. This will therefore affect the water tightness of the structure and may lead to deterioration. In low heat cement the heat involved is less. As per the code the heat of hydration for low heat cement shall not exceed 65 calories per gram of cement at 7 days, 75 calories per gram of cement at 28 days. The strength of this cement is a little lower than that of ordinary cement during the first two or three months, but its strength after this period is practically the same as that of ordinary cement. This cement offers better resistance to chemical deterioration than ordinary Portland cement.

Low heat cement is obtained by increasing Di calcium silicate content, decreasing Tri calcium silicate content and considerably bringing down Tri calcium Aluminate. The lesser rate of gain of strength in the initial period is due to the reduction of Tri calcium silicate and Tri calcium Aluminate. However, in order the rate of gain of strength may be ensured sufficiently, the IS code recommends a specific surface requirement of 3200 cm^2/gm.

4. **Portland Blast Furnace cement (IS 455)**

This is the cement obtained by inter grinding Portland cement clinker with blast furnace slag. The blast furnace slag content shall not exceed 65%. The property of cement is not detracted by the above blending. On the contrary, the blending provides some advantages. The granulated slags possess latent hydraulic properties which are activated to a great extent when the slag is catalyzed and inter ground with Portland cement clinker. This cement can be used at all places where OPC is used. In addition due to low heat evolution it can be used in huge mass concrete structures like dams, retaining walls, foundations and abutment of bridges. This cement has a good Sulphate resistance and is very suitable for use in construction in sea water.

5. **Portland Pozzolana cement (IS 1489)**

This cement is made either by inter grinding Portland clinker and pozzolana or uniform blending Portland cement and fine pozzolana. Inter grinding does not present any difficulty whereas, blending tends to produce a non-uniform product. The IS code has suggested that the latter method may be confined to factories where intimate blending can be guaranteed through mechanical means.

The pozzolana content varies from 10% to 25% by weight of cement. Pozzolanas do not possess cementing value themselves but have the property of combining with lime to produce a stable lime Pozzolana compound which possesses cementing property. Since the free lime which is readily attacked chemically is removed, the pozzolana concretes have a greater resistance to chemical agencies. They can also resist attack by sea water better than OPC since pozzolana cement has a lower heat evolution and is properly used in the construction dams. The pozzolana used in the manufacture of cement in India consists of burnt clay of shale or flyash. The pozzolana must satisfy very strictly certain requirements in

order to use it successfully. The following table shows the compressive strengths reached by ordinary Portland cement and Portland pozzolana cement.

Age in days	Compressive strength in N/mm^2.	
	Ordinary Portland cement	Portland Pozzolana cement
1	8	8
3	19	16
7	26	25
14	31	30
15	38	38

6. **White cement:**

White cement has practically the same composition and has the same strength as OPC. To obtain the white color the iron oxide content is reduced considerably (about 1%). The raw materials for this cement are lime stone, and china clay with low iron content. Sometimes cryolite (Sodium alumino fluoride) is added as a flux to aid in burning. The cement is burnt in rotary kiln with coal as fuel. This cement is used as a base for colored cement. It is used for interior and exterior decorative works, facing works, flooring works, ornamental concrete products, paths of garden, swimming pools etc.

7. **Colored cement:**

Colored cement is obtained by adding 5% to 10% of ground pigments to OPC. The following table gives the pigments generally used to get different colors.

Desired color	Pigment used
Green	Chromium oxide or greenish blue ultra-marine
Blue	Ultra marine
Blue black	Carbon black
Black	Black oxide of Manganese
Red	Red oxide of iron.

8. **Sulphate resisting cement:**

This cement is similar to OPC, but in this case it contains greater proportion of Tri calcium silicate and Di calcium silicate and a low content of Tri calcium Aluminate and tetra calcium Alumino ferrite. The chemical combination of Tri calcium Aluminate and tetra calcium Alumino ferrite renders the cement resistant to Sulphate. This type of cement is particularly useful in the airid western regions of India where the water deposits a large amount of sulphates in the soil.

9. Hydrophobic cement:

This is special type cement manufactured by the Associated Cement Company (ACC) which is claimed to have the property of repelling water. Hence the cement stored or transported does not spoiled even during the monsoons. This cement is made by adding water repellant chemicals to ordinary Portland clinkers in the process of grinding. The chemicals added form a molecular coating over every grain of cement and thus prevent any absorption of moisture from air by the cement. In concreting process, in the mixer, the coating gets removed by abrasion.

10. Oil well cement:

After the oil well is dug, it is necessary to fill the clearance between the steel lining and the wall of the well. It is necessary to grout the porous strata so that the wall of the well is prevented from collapse. It is also necessary to keep the ground waters out of the shaft of the well. Oil wells are dug to great depths (1000 m to 5000 m) and at high temperatures. Thus in such conditions the cement used for sealing the oil wells be in such a state, the slurry if pumped into the well casing will remain in the fluid state for a few hours before it can harden. For such a condition ACC have manufactured the oil well cement which is designed to be best suited for sealing the deep oil wells at temperature not exceeding 100 °C.

11. Air entrant cement:

In this case indigenously air entraining agents like glues, resins, sodium salts etc. are added while grinding the clinker. This cement provides greater workability of concrete at lesser water cement ratio. This cement increases the frost resisting capacity of concrete.

Physical requirements for cement (IS 269, 455, 1489)

The following table gives the physical requirements for cement.

Type of cement	Ordinary Portland cement	Rapid hardening PC	Low heat PC	Blast furnace slag cement	Portland pozzolana cement
Fineness-Residue by weight on 90µ IS sieve minimum specific surface cm^2/gm	≤ 10% 2250	≤ 5% 3250	--- 3200	≤ 10% 2250	≤ 5% 3000
Soundness expansion (Le Chatelier)	≤ 10 mm	≤ 10 mm	≤ 10 mm	≤ 10 mm	≤ 10 mm
Setting time Initial Final	≤ 30 min ≤ 600 min	≤ 30 min ≤ 600 min	≤ 60 min ≤ 600 min	≤ 30 min ≤ 600 min	≤ 30 min ≤ 600 min

Minimum compressive strength (N/mm^2) 1 day 3 days 7 days 14 days 28 days	 --- 16 22 --- ---	 16 28 --- --- ---	 --- 10 16 --- 35	 --- 16 22 --- ---	 --- --- 18 25 ---
			Heat of hydration not to exceed 65cal/gm at 7 days and 75cal/gm at 28 days		Average drying shrinkage of mortar is not to exceed 0.15%.

Chemical requirements for cement (IS 269, 455, 1489)

The following table gives the physical requirements for cement.

Type of cement	Ordinary and rapid hardening Portland cement	Low heat PC	Blast furnace slag cement	Portland pozzolana cement
	Maximum	Permissible value		
Insoluble residue	15%	15%	1.5%	x+{1.5(100-x)/100} x=declared percentage of pozzolana in the cement.
Magnesia (MgO)	6%	6%	8%	5%
Sulphor as SO_3	2.75%	2.75%	3%	2.75%
Sulphor as sulphide	--	--	1.5%	--
Oxide of Manganese (Mn_2O_3)	--	--	2%	--
Ignition loss	4%	4%	4%	5%

Role of various ingredients of cement
The various ingredients in cement have their characteristic functions.

Silica provides strength to cement. Excess silica may enhance the strength but increases the setting time.
Lime also provides strength to cement. Excess of lime is resulting in objectionable expansion. Insufficient lime reduces the strength and decreases the setting time.
Alumina makes the cement quick setting. It brings down the clinking temperature acting as a flux. Excess of this ingredient lowers the strength.
Iron oxide provides colour to the cement. This also imparts strength and hardness to the cement.
Magnesia when present in small extent provides hardness and colour. Excess of this ingredient seriously affect the soundness of cement.
Alkalies in excess are disadvantageous as they may cause alkali aggregate reaction, efflorescence and staining of concrete and brickwork. Generally alkalies present in the raw materials get removed by the flue gases in the burning process.

Harmful ingredients in cement:

i. Excess of Alumina reduces the strength.
ii. Magnesia content should not exceed 8%. This will affect soundness of cement.
iii. Excess of lime produces objectionable expansion leading to disintegration.
iv. Excess of alkalies produces alkali aggregate reaction, efflorescence.

Manufacturing of cement:
The three distinct operations involved in the manufacture of cement are:\

(i) Mixing of raw materials.
(ii) Burning.
(iii) Grinding.

Mixing of raw materials: The raw materials of cement are lime stone and clay are mixed in dry condition or wet condition. Accordingly the process of mixing is called dry process or wet process.
Dry process: This is an outdated process. In this process lime stone and clay are broken into pieces of nearly 25 mm size by crushers. An air draft is passed over the crushed materials. Now these are ground to a fine powder in the grinding mills. Each ingredient is separately stored in separate hoppers.
The powdered materials are now mixed in the required proportion. The raw mix so obtained is stored in the storage tank. From the storage tank the raw mix is passed on to the granulator consisting to a sloping rotating drum. Water about 15% is sprinkled and powdered material to become nodules. These are now burnt in the kiln.
Wet process: The raw materials viz. limestone and clay are taken in the right proportion and crushed to a fine powder in jaw crushers. The crushed materials in the proper proportionare conveyed into the ball mill. Water is added and the mixture is ground to form a slurry. The slurry is conveyed into the correcting silos in which the chemical composition of the slurry is examined. Now the slurry is passed on to the rotary kiln.
Burning: The slurry is burnt in a rotary kiln. The rotary kiln consists of a long cylinder 50 m to 100 m long supported on roller bearings provided over masonry pillars at intervals of 15 m to

20 m. The inner surface of the cylinder is lined with fire bricks. The cylinder is placed at a slope of 1 in 30. The cylinder rotates at1 revolution per minute. The heating is done by injecting pulverized coal using hot air blast from the lower end, while the slurry from hopper is supplied at the upper end. The slurry soon changes into nodules which roll down the kiln reaching hotter zones. As the nodules are heated to a very high temperature (1600°C), they are changed into hard clinkers. The white hot clinkers discharged out of the kiln are cooled in coolers by passing cold air.

Grinding: The clinkers are now ground in the ball mills and finally in the Tube mills. About 2% to 4% of gypsum is added to control the setting time of cement. After completion of grinding the cement is sieved and packed in bags.

Testing of Portland cement:

Field tests for cement:

When cement is received at a construction site, a few field tests can be made to ascertain its quality. They are

1) **Cement should be uniform in colour. Portland cement should be greenish grey in colour.**
2) **When a small sample of cement is rubbed between fingers, it should feel smooth.**
3) **The cement mass must feel cool when touched by hand.**
4) **There should be no lumps in the cement. Lumps indicate cement that has already set.**

Figure 4: The cement (left) and clinker (right)

Laboratory tests for cement: To ascertain the qualities of cement correctly, a number of tests are performed in the laboratory as suggested by IS codes. They are;

1) **Sampling: A sample meant for testing shall be drawn from at least 12 different bags or barrels or containers or from 12 different positions in heap if the cement is loose. The final sample shall weigh at least 5 kg. The sample is placed in a dry metal can closed with tight cover to exclude external air and moisture.**
2) **Chemical Composition: The IS specified tests are;**
   i. ***Ignition loss*: One gram of sample is heated in a porcelain crucible continuously for a period of 1 hour at 900°C to 1000°C. If platinum crucible is used, it is enough to heat the sample for 15 minutes. After heating the sample, it is cooled and again weighed. The percentage loss of weight should not exceed 4%.**
   ii. ***Insoluble residue*: one gram of cement is weighed and put in a 40 cc of water and stirred. Then 10 cc of hydrochloric acid of specific gravity 1.18 should be**

added and stirred. Any lumps present shall be carefully broken and the mixture is boiled for 10 minutes. The mixture, now, shall be filtered. Using hot water the container should be rinsed 5times and the filter should be washed 10 times. Now the residue left on the filter should be washed in a beaker containing 30 cc of hot distilled water. This is boiled for a duration of 10 minutes with 30 cc of sodium carbonate solution of twice the normal strength. The solution obtained is again filtered through the same filter paper. The filter paper is now washed 5 times with water. Now it is washed with hydrochloric acid solution of twice the normal strength and then with water so that it is free from chlorides. The filter paper is dried, then ignited and weighed. The insoluble residue shall not exceed 1.5%.

iii. *Lime and alumina*: The ratio of percentage alumina to the percentage of iron oxide shall not be less than 0.66. The percentage of lime to silica. Alumina and iron oxide when determined by the formula in percentage

$$(CaO - 0.7SO_2) / (2.8SiO_2 + 1.2\ Al_2O_3 + 0.65Fe_2O_3)$$

Should not exceed 1.02 nor be less than 0.65.

iv. *Magnesia*: Free magnesia shall not be present at more than 5 percent.

v. *Sulphur*: This shall not exceed 2.75%.

3) Fineness: Sieve test: 100 grams of cement is weighed accurately and placed on IS sieve No. 9. Any air set lumps are broken using fingers. Now gentle sieving is done continuously for fifteen minutes. The residue left is weighed. This shall not exceed 10% by weight of the sample.

Air permeability test: In this test specific surface in square cm per gram of cement is measured. The permissible values as given below:

Sl. No.	Type of cement	Specific surface area requirement in cm^2/gm
1	Ordinary Portland cement	2250
2	Portland blast furnace slag cement	2250
3	Rapid hardening Portland cement	3250
4	Low heat Portland cement	3200
5	Portland pozzolana cement	3000

Figure 3: Vicat Apparatus with plunger and needle.

4) **Consistency: This is a test conducted to estimate the quantity of water to be mixed with cement to form a paste of normal consistency for use in other tests. 300 grams of cement is mixed with 25% of water and a paste is prepared. The paste obtained should be filled in the mould of the Vicat's apparatus. The interval of time between the instant of adding water to the dry cement and the instant of common cement of filling the mould is called the time of gauging. The time of gauging must be 4±0.25 minutes.**
   The plunger, of diameter 10 mm is lowered gently on the paste in the mould. Now the settlement of plunger is noted. If the settlement is between 5 and 7 mm from the bottom of the mould, the amount of water added is correct. If this condition is not satisfied, the test must be repeated again by changing the percentage of water until the stipulated extent of penetration of the plunger is reached.

5) **Test for setting time: The object of this test is to make a distinction between normal setting and quick setting types of cement and also to detect deterioration due to storage.**
   Weigh 300 grams of cement and add the percentage of water required to have normal consistency. The paste obtained is filled in the mould. In the Vicat's apparatus, attach the needle of square section i.e. 1 mm x 1 mm to the Vicat's rod. Lower the system so that the needle just touches the surface of the paste and gently release. Find out if the needle pierces into the paste fully. If it does, again observe whether the needle pierces fully into the paste. This is repeated till the needle does not pierce into the paste completely. Now the interval of time between this instant and the instant at which water was added to the cement is called the initial setting time.

Now change the needle to the third one which has a projecting sharp point in the centre with an annular attachment. Now release the needle as before on the same paste. The needle as well as the attachment will make their impressions on the paste. Repeat this process till only the needle makes the impression but not the attachment. The interval of time between this instant and the instant at which water was added to the cement is called the final setting time.

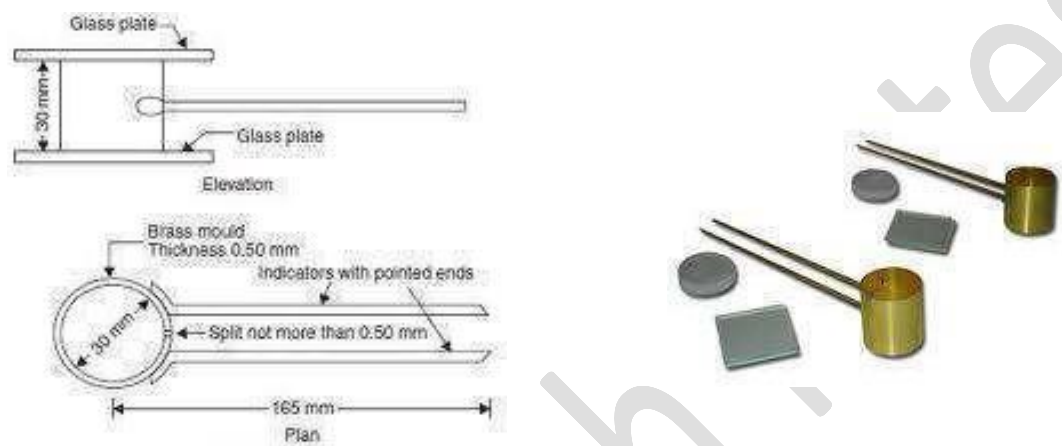


Figure 4: Le Chatelier apparatus

6) Test for Soundness: If excess of lime is present during the manufacture of cement, it is liable to remain uncombined and over burnt in the kiln. Such over burnt lime particles slake at very slow rate and expand. If after setting of cement is completed, the over burnt lime expand, it will lead to the disruption of cement mass. The object of the soundness test is to detect the presence of over-burnt uncombined lime. The cement is tested by using Le Chatelier apparatus as shown in Figure 4.

   This apparatus consists of a small split cylinder of brass 0.5 mm thick. This forms a mould 30 mm internal diameter and 30 mm in height. On each side of the split an indicator is attached having pointed end. The distance between the centre of cylinder and the pointed end of each indicator is 165 mm. The mould is placed on a glass plate and is then filled with cement paste. The cement paste should be formed by gauging with 0.78 of the water required to give a paste of normal consistency. After filling the mould, another glass plate is placed on the top and is immediately submerged in water at a temperature of 27°C to 32°C for 24 hours. After this interval of time, the distance between the ends of the pointers is measured. Now the mould is again submerged and brought to boiling point in 25 to 30 minutes and boiled for duration of one hour. Then the mould is taken out of water and the distance between the ends of the pointers is measured. The difference between this measurement and the original measurement should not exceed 10 mm.

7) Tensile strength: The tensile strength of cement is estimated by determining the tensile strength of cement sand mortar. For conducting the test briquettes of standard size are prepared. The briquettes are 2.54 cm thick and will have a minimum sectional area 2.54 cm x 2.54 cm. The cement mortar is made by mixing one part of cement to

three parts of Leighton Buzzard sand. The percentage of water to be used for making the mix is 0.2Pa + 2.5 where Pa is the percentage of water for normal consistency of cement. At least 12 briquettes are to be made. After the mould is made a small heap of paste is placed on the top and is lightly beaten with the standard spatula until on the surface water appears. Now the same is repeated on the other face of the briquette. The briquettes are kept in a damp tank, the relative humidity in the tank being 90%. Now six briquettes are tested after 3 days and the other six after 7 days in a testing machine to destruction. The load must be increased at the rate of 3.5 N/mm^2 of section for each minute.

Stress at failure = Load at failure/Minimum sectional area of briquette.

The following are the requirements for good cement.

Sl. No.	Type of cement	Tensile strength in N/mm^2.	
1	Portland cement	3 days Not less than 2.0	7 days Not less than 2.5
2	Rapid hardening cement	1 days Not less than 2.0	3 days Not less than 3.0

8) Compressive strength: This is a test to study the quality of cement about compressive strength. The compressive strength is studied by determining the compressive strength of cement sand mortar.

180 grams of cement should be mixed with 3 times its weight of standard Leighton Buzzard sand for one minute and then mixed with water whose percentage is given by

$P = (Pa/4) + 3.5$

where Pa is the percentage of water for normal consistency. The mixing should be done for 3 minutes. And in this duration the mixture must reach a uniform color. The mould shall be of 7.06 cm side. The mortar is filled in the mould and the compaction is done by using a standard vibration machine for 2 minutes. After this the cubes are kept at a temperature for about 27°C in a chamber for 24 hours in which the relative humidity is 90%. After this the cubes are removed from the moulds and are kept under fresh water till they are tested.

3 cubes are tested at the end of three days and 3 more may be tested at the end of 7 days. The strength requirements are given below.

After 3 days: 11.5 N/mm^2 for ordinary cement.

21.0 N/mm^2 for Rapid hardening cement.

After 7 days: 17.5 N/mm^2 for ordinary cement.

UNIT-02

SURVEYING AND POSITIONING

1. Linear measurements: Chain and Tape Surveying, Errors, Obstacles, Booking and Plotting, Calculation of Areas.
2. Angular Measurements: Bearing, Prismatic Compass, Local Attraction, Bowditch's Rule of correction, traverse open and closed, plotting of traverse, accuracy and precision.
3. Levelling: Types of Levels, Levelling Staff, Measurements, recording, curvature and Refraction correction, reciprocal levelling, sensitivity of level.
4. Contours: Properties, uses, plotting of contours, measurement of drainage and volume of reservoir.
5. Measurement of area by Planimeter.

Surveying is the science of making measurements to determine the positions of points or stations, above, on or beneath the surface of the earth. The process of surveying is carried out in land, water and also in space. The measurements involved in surveying are mainly distances (both horizontal and vertical) and directions. The data procurement phase in surveying is called field work and the analysis of data is called as office computations.

Classification of surveys:

A. Classification based on the location of survey as:

a. Land surveys.
b. Hydrographic surveys.
c. Astronomical surveys.

B. Classification based on the purpose of survey as:

a. Topographical survey.
b. Land survey.
c. Engineering survey.
d. Geodetic survey.

C. Classification based on the instruments or method employed as:

a. Chain and tape survey.
b. Compass and Theodolite survey.
c. Plane table survey.
d. Triangulation survey.
e. Tacheometric survey.
f. Hydrographical survey.
g. Photographical and aerial survey.

h. Astronomical survey.

Geodetic survey:

In this survey the area to be surveyed is considerably large to include the curvature of the earth. Geodetic surveying involves extremely accurate measurements of distances and angles. Adequate recognition is given to the spheroidal shape of the earth and in the computations provisions are made for convergency of the true meridians, and for correcting the lengths of base lines to the equivalent length projected on the mean sea level.

Plane surveying:

This branch of surveying considers the surface of the earth under survey to be plane. Curvature of earth is ignored and all calculations are made by using formulae of plane trigonometry. All meridians are taken as parallel. All plumb lines are taken as parallel. Plane surveying principles are followed for small areas.

Topographical survey:

This is a survey undertaken to establish on a map the topography or the natural features of the area, like rivers, canals, lakes, roads, railways, towns, etc.

Land surveys:

This is a survey done to fix property lines, calculation of land areas and transfer of real property from one owner to the other.

Engineering Surveys:

These consists of operations of obtaining data required to plan and design an engineering project and providing the proper position and dimensional control at the site so that the building or the project or the highway etc. is constructed in the proper place and as designed.

Basic principle of surveying:

The following two basic principles should be considered while determining relative position of points on the surface of earth:-

1. Determining suitable method for locating a point: - it is always practicable to select two points in the field to measure the distance between them. These can be represented on paper by two points placed in a convenient position.

2. Working from whole to the part: - in surveying an area, it is essential to establish first of all a system of control points with great precision. Minor control points can then be established by less precise method and the details can be located afterwards by method of triangulation or traversing between control points.

Chain surveying

Chain surveying is a method of surveying in which only linear measurements are directly made in the field. The main instruments used are chain, tape, offset rods, cross staff, optical square. This is the simplest method of surveying which is resorted in the following cases:

1. For small areas.
2. To prepare large scale maps and to locate boundaries very accurately.
3. The site is an open ground without complicated undulating profiles, obstacles etc.
4. The ground is fairly level.

Principle of chain surveying:

The plot is divided into a number of well conditioned (nearly equilateral) triangles. This triangle is surveyed. The area within each primary triangle can be divided into minor or secondary triangles which are all surveyed for their exact location within each primary triangle. This process is based on working from whole to part and the accumulation of errors is avoided.

Instruments used in chain surveying:

The chain: A chain is a unit of length. It measures 66 feet, or 22 yards, or 100 links, or 4 rods (20.1168 m). In 1620, the clergyman Edmund Gunter developed a method of surveying land accurately with low technology equipment, using what became known as Gunter's chain; this was 66 feet long and from the practice of using his chain, the word transferred to the actual measured unit. His chain had 100 links, and the link is used as a subdivision of the chain as a unit of length. The chain also survives as the length of a cricket pitch, being the distance between the wickets. The chain is composed of one hundred links, connected each to each by two rings, and furnished with a tally mark at the end of every ten links. A link in measurement includes a ring at each end, and is seven and ninety two one hundredths inches long. In all the chains which we make the rings are oval and are sawed and well closed, the ends of the wore forming the hook being also filed and bent close to the link, to avoid kinking. The oval rings are about one third stronger than round ones.

Handles - The handles are of brass and form part of the end links, to which they are connected by a short link and jam nuts, by which the length of the chain is adjusted.

Tallies - The tallies are of brass, and have one, two, three or four notches, as they mark ten, twenty, thirty or forty links from either end. The fiftieth link is marked by a rounded tally to distinguish it from the others.

Following are the various types of chain in common use:

1) Metric chains
2) Gunter` s chain or surveyors chain
3) Engineers chain
4) Revenue chain

5) Steel band or Band chain

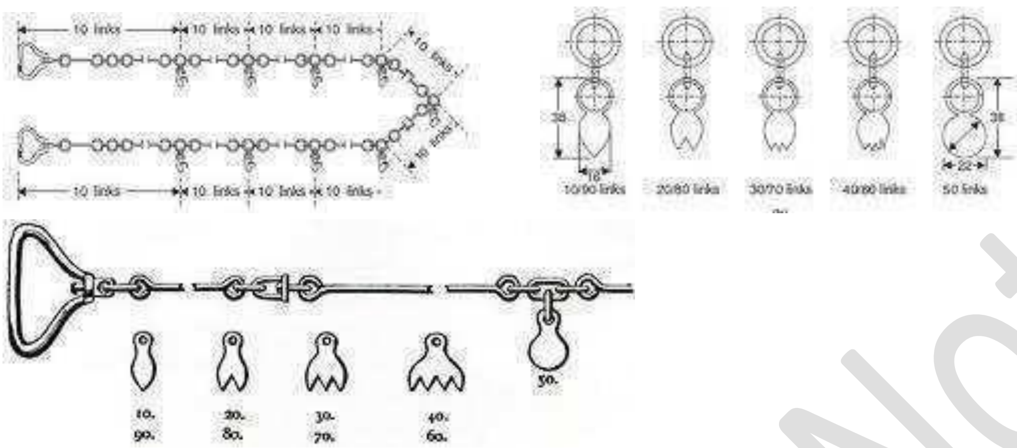


Figure 1: Chains

Metric chain: Metric chains are made in lengths 20m and 30m. Tallies are fixed at every five-meter length and brass rings are provided at every meter length except where tallies are attached

Engineers' Chains - Engineers' chains differ from surveyors' chains, in that a link including a ring at each end is one foot long, and the wire is of steel Nos. 8, 10 and 12. They are either fifty or one hundred feet long, and are furnished with swivel handles and tallies like those just described.

Tapes

Tapes are used in surveying to measure horizontal, vertical, and slope distances. They may be made of a ribbon or a band of steel, an alloy of steel, cloth reinforced with metal or synthetic materials. Tapes are issued in various lengths and widths and graduated in a variety of ways.

The following are the various types of tapes

i. Cloth tape
ii. Metallic tape
iii. Steel tape
iv. Invar tape

Among the above, metallic tapes are widely used in surveying. A metallic tape is made of varnished strip of waterproof line interwoven with small brass, copper or bronze wires. These are light in weight and flexible and are made 2m, 5m 10m, 20m, 30m, and 50m.

Metallic Tapes: A metallic tape is made of high-grade synthetic material with strong metallic. Strands (bronze-brass- copper wire) woven in the warped face of the tape and coated with a tough plastic for durability. Standard lengths are 50 and 100 ft. Metallic tapes are generally used for rough measurements, such as cross-sectional work, road-work slope staking, side shots in topographic surveys, and many others in the same category. Nonmetallic tapes woven from synthetic yarn, such as nylon, and coated with plastic are available; some surveyors prefer to use tapes of this type. Nonmetallic tapes are of special value to power and utility field personnel, especially when they are working in the vicinity of high-voltage circuits.

Steel Tapes

For direct linear measurements of ordinary or more accurate precision, a steel tape is required. The most commonly used length is 100 ft, but tapes are also available in 50-, 200-, 300-, and 500-ft lengths.

Various types of surveying tapes are shown in figure 2. View A shows a metallic tape; view B, a steel tape on an open reel; view C, a steel tape or, a closed reel. View D shows a special type of low-expansion steel tape used in high-order work; it is generally called an Invar tape or Lovar tape.

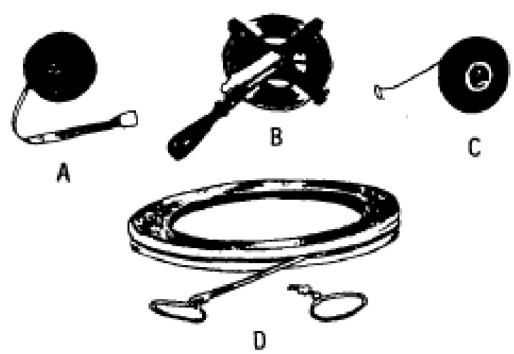


Figure 2: Surveying tapes.

Invar Tapes

Nickel-steel alloy tapes, known as Invar, Nilvar, or Lovar, have a coefficient of thermal expansion of about one-tenth to one-thirtieth (as low as 0.0000002 per 10 F) that of steel. These tapes are used primarily in high-precision taping. These tapes must be handled in exactly the same manner as other precise surveying instruments.

Arrows

Arrows are made of good quality hardened steel wire of 4 mm diameter. The arrows are made 400 mm in length, are pointed at one and the other end is bent into a loop or circle. Figure 3 shows the details of arrow.

Ranging rods

Ranging rods are used to range some intermediate points in the survey line. The length of the ranging rod is either 2m or 3m (Refer Figure 4). They are shod at bottom with a heavy iron point. Ranging rods are divided into equal parts 0.2m long and they are painted alternately

black and white or red and white or red, white and black. When they are at considerable distance, red and white or white and yellow flags about 25 cm square should be fastened at the top.

Figure 3: Arrows

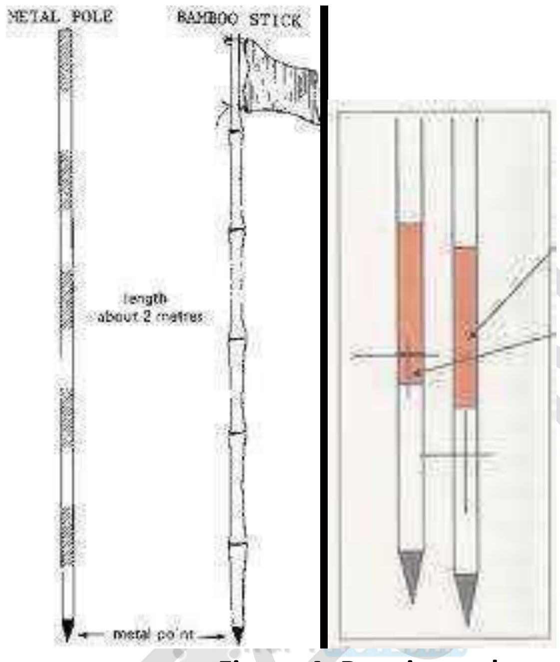


Figure 4: Ranging rods

Figure 5: Plumb Bob

Plumb-bob

A plumb-bob or a plummet is a weight, usually with a pointed tip on the bottom that is suspended from a string and used as a vertical reference line, or plumb-line. It is essentially the y-axis equivalent of a "water level". They are used with a variety of instruments (including levels, theodolites, and steel tapes) to set the instrument exactly over a fixed survey marker, or to transcribe positions onto the ground for placing a marker (Refer Figure 5).

Pegs

These are rods made from hard timber and tapered at one end, generally 25mm or 30mm square and 150mm long wooden pegs are used to mark the position of the station on.

Cross staff

The simplest instrument used for setting out a right angle. The common forms of cross staff are shown in Figure 6.



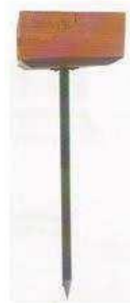


Figure 6: Metal cross staff Optical Square Wooden cross staff.

Calculations of Field Area: By this method of survey, the field as divided in to right angled triangles and trapezoids are calculated as under:

Area of right -angled triangle = ½ base x Height.

Area of trapezoid= sum of parallel sides/2 x Height.

Add the areas of all the triangles & trapezoids and sum is equal to the total of a field.

The computations for area should be written in a tabular from as given below.

Sl. No.	Figure	Chainage in m.	Base in M.	Offset in M.	Mean offset in m.	Area in m^2 = Col 4 x Col 6

Ranging out Survey Line

In measuring the length of a survey line called chain line, it is necessary that the chain should be laid out on the ground in a straight line between the end stations.

Ranging: “The process of establishing intermediate point on a straight line between two end points is known as ranging”. Ranging must be done before a survey line is chained. It may be necessary to establish a number of intermediate points prior to chaining when chain line is much longer. Ranging may be done by direct observation by the naked eye or by line ranger or by Theodolite. Generally, ranging is done by naked eye with the help of three ranging rods.

Ranging is of two kinds:

1. Direct Ranging
2. Indirect or reciprocal ranging

1. Direct Ranging: When intermediate ranging rods are fixed on a straight line by direct observation from end stations, the process is known as direct ranging. Direct ranging is possible when the end stations are intervisible.

Assume that A and B two end stations of chain line (Refer Figure 7), where two ranging rods are already fixed. Suppose it is required to fix a ranging rod at the intermediate point P on the chain line in such a way that the points A, P & B are in same straight line. The surveyor stands about two meters behind the ranging rod at A by looking towards line AB. The assistant holds ranging rod at P vertically at arms length the rod should be held tightly by the thumb and forefinger. Now the surveyor direct the assistant to move the ranging rod to the left or right until the three ranging rods come exactly the same straight line. The ranging will be perfect, when the three ranging rods coincide and appear as a single rod. When the surveyor is satisfied that the ranging is prefect, he signals the assistant to fix the ranging rod on the ground. By following the same procedure, the other ranging rods may be fixed on the line.

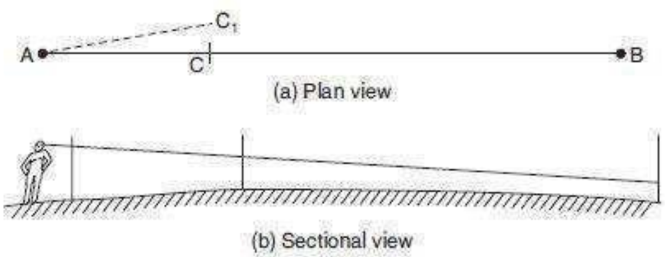


Figure 7: Direct Ranging

2. Indirect or Reciprocal Ranging: Indirect ranging is used when the end stations are not intervisible due to high ground or a hill or if the ends are too long. In such cases, intermediate points can be fixed on the survey line by a process known as reciprocal ranging.

Let A & B be the two stations with rising ground or a hill (Refer Figure 8). Let two chainmen with ranging rods take up positions at M and P, such that, chainmen at M1 can see both rods at P1 and B and the chainmen at P1 can see the ranging rods at M1 and A. The chainmen at P1 directs the chainmen at M1 to shift the ranging rod at M2 in line with A and then chainman at M2 directs the chainmen at P1 to shift the ranging rod to P2 in line with B, by successively directing each other to be in line with the end points. Their positions will be changed until finally they are both in line with A & B exactly on line AB. Now the four ranging rods at A M P & B are on same straight line. This method may also be used in ranging a line across a valley or river.

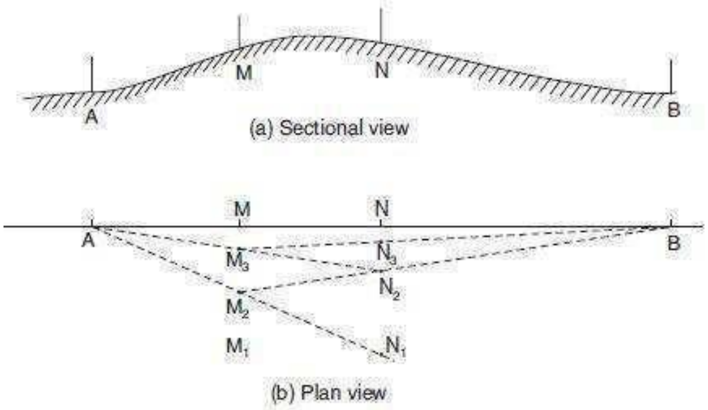


Figure 8: Indirect or Reciprocal Ranging

Survey Station

Survey stations are of two kinds

1. **Main Stations**
2. **Subsidiary or tie**

Main Stations: Main stations are the end of the lines, which command the boundaries of the survey, and the lines joining the main stations re called the main survey line or the chain lines.

Subsidiary or the tie stations: Subsidiary or the tie stations are the point selected on the main survey lines, where it is necessary to locate the interior detail such as fences, hedges, building etc.

Tie or subsidiary lines: A tie line joints two fixed points on the main survey lines. It helps to checking the accuracy of surveying and to locate the interior details. The position of each tie line should be close to some features, such as paths, building etc.

Base Lines: It is main and longest line, which passes approximately through the centre of the field. All the other measurements to show the details of the work are taken with respect of this line.

Check Line: A check line also termed as a proof line is a line joining the apex of a triangle to some fixed points on any two sides of a triangle. A check line is measured to check the accuracy of the framework. The length of a check line, as measured on the ground should agree with its length on the plan.

Offsets:

These are the lateral measurements from the base line to fix the positions of the different objects of the work with respect to base line. These are generally set at right angle offsets. It can also be drawn with the help of a tape. There are two kinds of offsets:

1) Perpendicular offsets, and
2) Oblique offsets.

The measurements are taken at right angle to the survey line called perpendicular or right angled offsets. The measurements which are not made at right angles to the survey line are called oblique offsets or tie line offsets.

Plane Table Surveying

A plane table is a device used in surveying and related disciplines to provide a solid and level surface on which to make field drawings, charts and maps. It is a graphical method of surveying in which field work and plotting are done simultaneously in the field. It is very effective method of surveying for preparing small or medium size topographical plans. It is not as accurate as the other survey methods and results. It is particularly adopting in small mapping. Plane table surveying is used for locating the field computation of area of field.

Merits:

1. It is one of the most rapid methods of surveying.
2. Field notes are not required, and thus the possibility of mistakes in booking is eliminated.
3. Measuring of lines and angles is mostly dispensed with since they are obtained graphically.
4. Since the maps are plotted in the field, there is no chance of omitting necessary measurements.
5. The surveyor is fully confident about the true representation of the area since he can always compare his work with actual features on the ground and cannot, therefore, outlook any essential detail.
6. The surveyor can check the accuracy of his work more frequently and from any position he may desire, thus eliminating all the error at the spot.
7. It is particularly suitable for filling in details in hilly areas and in magnetic areas where chain and compass surveys are not suitable.
8. Contours and other irregular objects may be accurately represented on the map since the tract is in view.
9. It is less costly than Theodolite survey.
10. No great skill is required in making a satisfactory map and the work can be entrusted even to a subordinate.

Demerits:

a. Plane Table Essentially a tropical instruments.
b. It is not suitable to work in wet climate.

c. There are several accessories to be carried out and therefore they are likely to be lost.
d. It is not suitable for accurate work.

Plane table construction

A plane table consists of a smooth table surface mounted on the tripod. The mount allows the table to be leveled. The connection between the table top and the base permits one to level the table precisely, using bubble levels, in a horizontal plane. The base, a tripod, is designed to support the table over a specific point on land. By adjusting the length of the legs, one can bring the table level regardless of the roughness of the terrain.

Parts of plane Table:

Plane table essentially consists of Drawing board mounted on tripod and Alidade.

1. Drawing board mounted on tripod: A sheet of drawing paper, called plane table sheet is fastened to the board. Board is made up of well seasoned wood such as teak of size 40x30 to 75x60cm. It had plane and smooth top. It is mounted on a tripod in manner that it can be leveled. Leveling up of the table is done by shifting the legs of tripod. Some tripod provided with leveling screw or by ball and socket head for accurate leveling.

2. Alidade: Alidade consists of two vertical sight vane fitted at end the end of straightedge. The straight edge ruler usually made of brass or teak wood graduated beloved edge. One of the sight veins is provided with narrow slit and the other with a central vertical wire or hair. Beveled working edge alidade is called fiducial edge.

Accessories:

a. A trough campus for marking the direction magnetic meridian on paper.
b. Sprit level for leveling the table.
c. Forked plumb for centering the table.
d. Water proof cover to protect the sheet from rain.

Centering: It is the process of keeping the table over the station that the point on the paper representing the station being occupied is vertically over the point on the ground. It is done by forked plumb bob.

Orientation: When the table has to be set up at more than one station it is necessary that it is be oriented so that the lines on the paper remain parallel to the lie which they represent on the ground. So orientation is "the process of keeping the table to the position which is occupied at the first station".

Orientation is done by two methods:

a. By use of the magnetic needle.
b. Orientation by back sighting.

a. **Orientation by the magnetic needle:** To orient the table at any subsequent station, the through compass(or circular box compass) is placed along the line representing the magnetic meridian which has been drawn on the paper at the first station, and the board is then turned until the ends of the needle o\are opposite the zeros of the scale. The board is then clamped in position. It is suitable for rough small scale mapping.

b. **Orientation by back sighting:** This is the most accurate method of orientation and is always be preferred. Suppose a table is set up over station Q on the line PQ which ahs been previously drowned as PQ from station p. The alidade is placed along the line QP and board then turned until the line of sight bisects the ranging rod at P. Board is then properly clamped.

Figure 9: Plane table mounted on a tripod and Telescopic Alidade

Use of a plane table

In use, a plane table is set over a point and brought to precise horizontal level. A drawing sheet is attached to the surface and an alidade is used to sight objects of interest. The alidade, in modern examples of the instrument a rule with a telescopic sight, can then be used to construct a line on the drawing that is in the direction of the object of interest.

By using the alidade as a surveying level, information on the topography of the site can be directly recorded on the drawing as elevations. Distances to the objects can be measured directly or by the use of stadia marks in the telescope of the alidade.

Methods of Plane Table

1. Radiation
2. Intersection
3. Traversing
4. Resection

1. **Radiation:** This method is useful in surveying small areas which can be commanded from one station. From a station, the suitable is selected. Rays are drawn to various objects. The distance of the object from the station are measured and marked off on the ray.

2. Intersection: In this method, the positions of the object on the plan are fixed by the intersection of rays drawn from two instrument stations. The line joining these instrument stations are called baseline.
3. Traversing: This method is used for running survey lines for close or open traverse. This is the main method of plane table and is similar to compass or theodolite traversing. This method consists in running a traverse with a plane table; locating details by taking offsets in usual manner.
4. Resection: This method is used for establishing instrument station on a plan with reference to two points already plotted on the plan. The procedure adopted is as follows:
   a. Select the traverse stations say A, B, C etc.
   b. Set up the table over one of them say A. select the point A suitably on the sheet. Level and centre the table over A.
   c. Mark the direction of magnetic meridian on the top corner to the sheet by means of trough compass.
   d. With the alidade touching A, sight B and draw the ray.
   e. Measure the distance AB and scale off AB, thus fixing the position of B on the sheet which represents their station B on the ground.
   f. Shift the table and set up at B with b over B and orient it by placing. The alidade along BA, turning the table until the line of sight strikes A, and then clamp it.
   g. With the alidade touching B sight C and draw a ray.
   h. Measure the line BC and cut off BC to scale. Proceed similarly at other stations, in each case orienting by back side before taking forward sight until all the remaining stations are plotted.

Electronic Distance Measuring Instruments (EDM)

In surveying, the standard measurement device for many years remained the steel tape measure. Newer electronic measuring devices, however, have begun to take the place of the tape. In surveying applications, surveyors can take electronic distance measurements from helicopters covering distances and terrain that would have been near impossible with older methods. Laser distance meters in carpentry can measure any of the things carpenters used to use a tape measure for, and can send the measurements directly to a computer removing the possibility of forgotten or transposed numbers.

A distance measuring instrument is provided for simple field wise mapping of an area. The instrument comprises an electronic distance meter, a unit for determining a vertical angle for aligning the instrument with a measuring point, and a unit for obtaining a horizontal angle for

the alignment of the instrument with a measuring point. The horizontal angle unit comprises a terrestrial magnetic-field detector which comprises at least two detector units fixedly mounted in the instrument. The units are directed in mutually different directions so that at least two components of the magnetic flux in an instrument-based coordinate system are obtained by the units. A calculating unit is arranged to convert the coordinates of the components of the terrestrial magnetic field from the instrument-based coordinate system to an earth-based coordinate system. The vertical angle obtained from the vertical angle unit is used for this conversion. The calculating unit is arranged to calculate the direction of the terrestrial magnetic field in a horizontal plane and determine the horizontal angle relative to a reference direction.

Figure 10: EDM

Figure 11: Total station

Total station

A total station is an electronic/optical instrument used in modern surveying. The total station is an electronic theodolite (transit) integrated with an electronic distance meter (EDM) to read slope distances from the instrument to a particular point ref Figure 11).

Robotic total stations allow the operator to control the instrument from a distance via remote control. This eliminates the need for an assistant staff member as the operator holds the reflector and controls the total station from the observed point.

Coordinate measurement: Coordinates of an unknown point relative to a known coordinate can be determined using the total station as long as a direct line of sight can be established between the two points. Angles and distances are measured from the total station to points under survey, and the coordinates (X, Y, and Z or easting, northing and elevation) of surveyed points relative to the total station position are calculated using trigonometry and triangulation. To determine an absolute location a Total Station requires line of sight

observations and must be set up over a known point or with line of sight to 2 or more points with known location.

For this reason, some total stations also have a Global Navigation Satellite System receiver and do not require a direct line of sight to determine coordinates. However, GNSS measurements may require longer occupation periods and offer relatively poor accuracy in the vertical axis.

Angle measurement: Most modern total station instruments measure angles by means of electro-optical scanning of extremely precise digital bar-codes etched on rotating glass cylinders or discs within the instrument. The best quality total stations are capable of measuring angles to 0.5 arc-second. Inexpensive "construction grade" total stations can generally measure angles to 5 or 10 arc-seconds.

Distance measurement: Measurement of distance is accomplished with a modulated microwave or infrared carrier signal, generated by a small solid-state emitter within the instrument's optical path, and reflected by a prism reflector or the object under survey. The modulation pattern in the returning signal is read and interpreted by the computer in the total station. The distance is determined by emitting and receiving multiple frequencies, and determining the integer number of wavelengths to the target for each frequency. Most total stations use purpose-built glass corner cube prism reflectors for the EDM signal. A typical total station can measure distances with an accuracy of about 1.5 mm + 2 parts per million over a distance of up to 1,500 m. Reflector less total stations can measure distances to any object that is reasonably light in color, up to a few hundred meters.

Data processing: Some models include internal electronic data storage to record distance, horizontal angle, and vertical angle measured, while other models are equipped to write these measurements to an external data collector, such as a hand-held computer.

When data is downloaded from a total station onto a computer, application software can be used to compute results and generate a map of the surveyed area. The new generation of total stations can also show the map on the touch-screen of the instrument right after measuring the points.

Applications:

a. Total stations are mainly used by land surveyors and Civil Engineers, either to record features as in Topographic Surveying or to set out features (such as roads, houses or boundaries).
b. They are also used by archaeologists to record excavations and by police, crime scene investigators, private accident reconstructions and insurance companies to take measurements of scenes.
c. Mining: Total stations are the primary survey instrument used in mining surveying.

d. A total station is used to record the absolute location of the tunnel walls (stopes), ceilings (backs), and floors as the drifts of an underground mine are driven. The recorded data are then downloaded into a CAD program, and compared to the designed layout of the tunnel. The survey party installs control stations at regular intervals. These are small steel plugs installed in pairs in holes drilled into walls or the back. For wall stations, two plugs are installed in opposite walls, forming a line perpendicular to the drift. For back stations, two plugs are installed in the back, forming a line parallel to the drift. A set of plugs can be used to locate the total station set up in a drift or tunnel by processing measurements to the plugs by intersection and resection.

COMPASS SURVEY

Figure 12: Surveyors Compass

Figure 13: Prismatic Compass

A compass is a navigational instrument that shows directions in a frame of reference that is stationary relative to the surface of the earth. The frame of reference defines the four *cardinal directions* (or *points*) – north, south, east, and west. Intermediate directions are also defined. North corresponds to zero degrees, and the angles increase clockwise, so east is 90 degrees, south is 180, and west is 270.

1. Compass survey is a method of surveying by taking bearings and linear distances to produce plan.
2. Bearing is measured using prismatic compass, while the linear distance is measured using measuring tape.
3. Bearing in compass surveying means angle is made by chain line or survey line by referring it to magnetic meridian or magnetic north.

DEFINITION OF FEW COMPASS TERMS

MERIDIAN – it is the fixed direction in which the bearings of survey lines are expressed

BEARING – it is horizontal angle between the reference meridian and the survey line measured in clockwise or anticlockwise direction

TRUE MERIDIAN – The true meridian passing through a point on the earth surface is the line in which a plane passing through and the north and south poles, intersects the surface of the earth.

TRUE BEARING – The horizontal angle measured clockwise between the true meridian and the line is called true bearing of the line.

MAGNETIC MERIDIAN – the direction indicated by a freely suspended and balanced magnetic needle unaffected by local attractive forces

MAGNETIC BEARING – The horizontal angle which a line makes with the magnetic meridian.

DESIGNATION OF BEARINGS

The bearing is expressed in the following two ways:

1. Whole circle bearings
2. Quadrant bearings

WHOLE CIRCLE BEARING

The angle thus measured between the reference meridian and the line. It will have values between 0º and 360º

QUADRANTAL BEARING

The horizontal angle which a line makes with the north or south direction of the meridian whichever is nearer the line measured in the clockwise or counter clockwise direction. It will have value up to 90º.

TYPES OF COMPASS

There are two forms of compass in common use

1. The prismatic compass
2. The surveyor's compass

The prismatic compass is very valuable instrument and is commonly used for rough surveys where speed and not the accuracy is main consideration. The surveyor's compass was formerly much used for land surveys but now-a-days, it is little used.

Differences between Prismatic compass and Surveyors compass:

Prismatic Compass (Refer Figure 13): The sighting of an object and reading of the bearing are done simultaneously. The graduated ring remains stationary as it is attached to magnetic needle. While the compass needle and the eye sight vane can be rotated. The graduations are made in such a way that 0 or 360 is at the south, 180 at north, 90 at south and 270 at east. Sighting of the object and the taking of reading is done simultaneously. Prismatic compass can be used without a tripod.

Surveyor's Compass (Refer Figure 14): An object is sighted first and the bearing is then read by going vertically over the middle point. The graduated ring being attached to the compass moves with sights. But needle remains stationary when box is rotated. The graduations are made as 00 at north & south 900 at east and west. The east and west positions are interchanged in order to read the bearing in quadrantal bearing system. Sighting the object is done first. Then the reading is to be taken with naked eye by looking above the needle point Surveyor's compass cannot be used without a tripod.

LOCAL ATTRACTION

Detection of local attraction: Local attraction at a place can be detected by observing bearings. If the fore and back bearings of the line differ exactly by 180°, there is no local attraction at either station provided instrumental and observational errors are eliminated. But if this difference is not equal to 180°, then local attraction exists there either at one or at both ends of the line. The list of materials which cause local attraction are:

(i) Magnetic rock or iron ore,

(ii) Steel structures, iron poles, rails, electric poles and wires,

(iii) Key bunch, knife, iron buttons, steel rimmed spectacles, and

(iv) Chain, arrows, hammer, clearing axe etc.

Surveyor is expected to take care to avoid local attractions listed in (iii) and (iv) above.

Correcting Observed Bearings:

If local attraction is detected in a compass survey observed bearings may be corrected by any one of the following two methods:

Method I: It may be noted that the included angle is not influenced by local attraction as both readings are equally affected. Hence, first calculate included angles at each station, commencing from the unaffected line and using included angles, the corrected bearings of all lines may be calculated.

Method II: In this method, errors due to local attraction at each of the affected station is found starting from the bearing of a unaffected local attraction, the bearing of the successive lines are adjusted.

LEVELING

Leveling is the operation required in the determination or the comparison of heights of points on the surface of the earth. Leveling also is the procedure used when one is determining differences in elevation between points that are remote from each other. An elevation is a vertical distance above or below a reference datum. In land surveying, the reference datum is mean sea level (MSL).

Parts of Level (Refer Figure 15)

Objective lens - compound lens used to magnify object

Reticle - a pair of perpendicular reference lines that defines the line of sight, commonly called *cross hairs*

Negative lens - located between objective lens and reticle is used to bring object into focus on reticle plane

Eyepiece lens - Used to focus cross hairs (i.e. bring focus on cross hair plane)

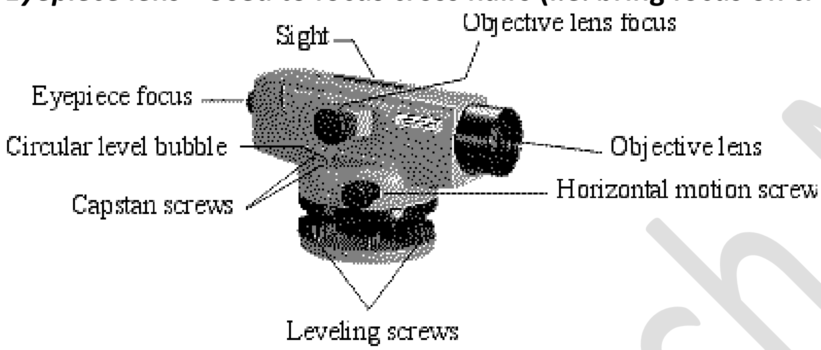


Figure 15: Parts of Level

Important: Lack of proper focus in object and eyepiece results in *parallax*, apparent shifting of object caused by motion of eye. Focus eyepiece and then object - check for parallax. Discuss this with your lab instructor.

Leveling screws - used to bring *level* instrument. Once leveled, the instrument's line of sight with scribe out a horizontal plane.

Circular level bubble - (aka bull's eye bubble) - 10' bubble used to bring automatic compensator into working range.

Level Vials: Level vials are used to orient surveying instruments with respect to the direction of gravity. Tube level vial is manufactured so that the inside of the glass lies along a sphere. The radius of curvature of this sphere determines the vial *sensitivity*. Longer radii are more sensitive and precise than shorter radii vials. Alternatively, it takes more time to center a bubble in a sensitive vial.

Rise and Fall Method

For the same set up of an instrument, Staff reading is more at a lower point and less for a higher point. Thus, staff readings provide information regarding relative rise and fall of terrain points. This provides the basics behind rise and fall method for finding out elevation of unknown points.

With reference to Table when the instrument is at I_1, the staff reading at A (2.365m) is more than that at S_1 which indicates that there is a rise from station A to S_1 and accordingly the difference between them (1.130m) is entered under the rise column in Table 1. To find the elevation of S_1 (101.130m), the rise (1.130m) has been added to the elevation of A (100.0m). For instrument set up at I_2 , S_1 has been treated as a point of known elevation and considered for back sight (having reading 0.685m) . Foresight is taken at S_2 and read as 3.570m i.e., S_2 is at lower than S_1. Thus, there is a fall from S_1 and S_2 and there difference (2.885m) is entered under the fall column in Table 13.1. To find the elevation of S_2 (98.245m), the fall (2.885m) has been subtracted from the elevation of S_1 (101.130m). In this way, elevations of points are calculated by Rise and fall method.

Level book note for Rise and Fall method

	Staff Reading		Difference in Elevation		Elevation	
Points	B.S (m)	F.S.(m)	Rise (m)	Fall (m)	R.L (m)	Remark
A	2.365				100.000	B.M.
S_1	0.685	1.235	1.130		101.130	T.P.$_1$
S_2	1.745	3.570		2.885	98.245	T.P. $_2$
B		2.340		0.595	97.650	

Arithmetic Check for Reduction of Level

In case of Rise and Fall method for Reduction of level, following arithmetic checks are applied to verify calculations.

$\sum$ B.S. - $\sum$ F.S. = $\sum$ Rise - $\sum$ Fall = Last R.L. - First R.L.

With reference to Table 13.1:

$\sum$ B.S. - $\sum$ F.S. = 4.795 - 7.145 = - 2.350

$\sum$ Rise - $\sum$ Fall. = 1.130 - 3.480 = - 2.350

Last R.L. - First R.L. = 97.650 - 100.000 = -2.350

Height of Instrument Method

In any particular set up of an instrument height of instrument, which is the elevation of the line of sight, is constant. The elevation of unknown points can be obtained by subtracting the staff readings at the desired points from the height of instrument. This is the basic behind the height of instrument method for reduction of level.

With reference to Table 2, when the instrument is at I_1, the staff reading observed at A is 2.365m. The elevation of the line of sight i.e., the height of instrument is 102.365m obtained by adding the elevation of A (100.0m) with the staff reading observed at A (2.365m). The elevation of S_1 (101.130m) is determined by subtracting its foresight reading (1.235m) from the height of instrument (102.365m) when the instrument is at I_1. Next, the instrument is set up at I_2. S_1 is considered as a point of known elevation and back sight reading (0.685m) is taken. The height of the instrument (101.815 m) is then calculated by adding back sight reading (0.685m) with the elevation (R.L.) of point S_1 (101.130m). Foresight is taken at S_2 and its elevation (98.245m) is determined by subtracting the foresight (3.570m) from the height of the instrument (101.815 m). In this way, elevations of points are calculated by Height of instrument method.

Level book note for Height of instrument method

	Staff Reading		Height of Instrument (m)	R.L. (m)	Remarks
Points	B.S (m)	F.S.(m)			
A	2.365		102.365	100.000	B.M.
S_1	0.685	1.235	101.815	101.130	T.P.$_1$
S_2	1.745	3.570		98.245	T.P.$_2$
B		2.340		97.650	

Arithmetic Check for Reduction of Level

In case of Height of instrument method for Reduction of level, following arithmetic checks are applied to verify calculations.

∑ B.S. - ∑ F.S. = Last R.L. - First R.L.

With reference to Table 13.2:

∑ B.S. - ∑ F.S. = 4.795 - 7.145 = - 2.350

Last R.L. - First R.L. = 97.650 - 100.000 = -2.350

Example 2: Data from a differential leveling have been found in the order of B.S., F.S ......etc. starting with the initial reading on B.M. (elevation 150.485 m) are as follows : 1.205, 1.860, 0.125, 1.915, 0.395, 2.615, 0.880, 1.760, 1.960, 0.920, 2.595, 0.915, 2.255, 0.515, 2.305, 1.170. The final reading closes on B.M. Put the data in a complete field note form and carryout reduction of level by Rise and fall method. All units are in meters.

Solution :

Level book note for Rise and Fall method

B.S. (m)	F.S. (m)	Rise (m)	Fall (m)	Elevation (m)	Remark
1.205				150.485	B.M.
0.125	1.860		0.655	149.830	
0.395	1.915		1.7290	148.040	

0.880	2.615		2.220	145.820	
1.960	1.760		0.880	144.940	
2.595	0.920	1.040		145.980	
2.255	0.915	1.680		147.660	
2.305	0.515	1.740		149.450	
	1.170	1.135		150.535	B.M.

Arithmetic Check for Reduction of Level

∑ B.S. = 11.720 m; ∑ F.S. = 11.670 m

Therefore ∑ B.S - ∑ F.S. = 0.050 m

∑ Rise = 5.595 m; ∑ Fall = 5.545 m

Therefore ∑ Rise - ∑ Fall = 0.050 m

Last R.L. - First R.L. = 150.535 - 150.485 = 0.050 m.

∑ B.S - ∑ F.S. = ∑ Rise - ∑ Fall = Last R.L. - First R.L.

Example 3: Data from a differential leveling have been found in the order of B.S., F.S ..... etc. starting with the initial reading on B.M. (elevation 150.485 m) are as follows : 1.205, 1.860, 0.125, 1.915, 0.395, 2.615, 0.880, 1.760, 1.960, 0.920, 2.595, 0.915, 2.255, 0.515, 2.305, 1.170. The final reading closes on B.M. Put the data in a complete field note form and carry out reduction of level by Height of instrument method. All units are in meters.

Level book note for Height of instrument method

	Staff Reading		Height of Instrument (m)	R.L. (m)	Remarks
Points	B.S (m)	F.S.(m)			
A	1.205		151.690	150.485	B.M.
S_1	0.125	1.860	149.955	149.830	T.P.$_1$
S_2	0.395	1.915	148.435	148.040	T.P.$_2$
B	0.880	2.615	146.700	145.820	
	1.960	1.760	146.900	144.940	
	2.595	0.920	148.575	145.980	
	2.255	0.915	149.915	147.660	
	2.305	0.515	151.705	149.400	
		1.170		150.535	

Arithmetic Check for Reduction of Level

∑ B.S. = 11.720 m; ∑ F.S. = 11.670 m

Therefore ∑ B.S - ∑ F.S. = 0.050 m

Last R.L. - First R.L. = 150.535 - 150.485 = 0.050 m.

∑ B.S - ∑ F.S. = Last R.L. - First R.L.

Example 4: A Leveling work is carried out from the point A to B and the readings are listed below. The instrument is shifted after 6th and 13th reading. Calculate the level difference between A and B. The RL of bench mark is 47.195, the first reading is taken to BM, the second reading is at A and the last reading is at B.

By Height of Instrument method:

Level book note for Height of instrument method

Staff Reading			Height of Instrument (m)	R.L. (m)	Remarks
B.S (m)	I.S.(m)	F.S.(m)			
1.575			48.770	47.195	BM
	1.250			47.520	A
	0.850			47.920	
	1.330			47.440	
	1.580			47.190	
2.550		1.450	49.870	47.320	CP1
	1.980			47.890	
	1.760			48.110	
	1.710			48.160	
	1.840			48.030	
	1.920			47.950	
1.830		3.250	48.450	46.620	CP2
		2.260		46.190	B

Level difference between A and B = 47.195 – 46.190 = 1.005.

Arithmetic Check for Reduction of Level

∑ B.S. = 5.955 m; ∑ F.S. = 6.960 m

Therefore ∑ B.S - ∑ F.S. = -1.005 m

Last R.L. - First R.L. = 46.190 – 47.195 = -1.005 m.

∑ B.S - ∑ F.S. = Last R.L. - First R.L.

By Rise and fall method:

Level book note for Rise and Fall method

Staff Reading			Rise	Fall	R.L. (m)	Remarks
B.S (m)	I.S.(m)	F.S.(m)				
1.575					47.195	BM
	1.250		0.325		47.520	A
	0.850		0.400		47.920	
	1.330			0.480	47.440	
	1.580			0.250	47.190	
2.550		1.450	0.130		47.320	CP1
	1.980		0.570		47.890	
	1.760		0.220		48.110	
	1.710		0.050		48.160	
	1.840			0.130	48.030	
	1.920			0.080	47.950	
1.830		3.250		1.330	46.620	CP2
		2.260		0.430	46.190	B

Level difference between A and B = 47.195 – 46.190 = 1.005.

Arithmetic Check for Reduction of Level
∑ B.S. = 5.955 m; ∑ F.S. = 6.960 m
Therefore ∑ B.S - ∑ F.S. = -1.005 m
∑ Rise = 1.695 m; ∑ Fall = 2.700 m
Therefore ∑ Rise - ∑ Fall = -1.005 m

Last R.L. - First R.L. = 46.190 – 47.195 = -1.005 m.
∑ B.S - ∑ F.S. = ∑ Rise - ∑ Fall = Last R.L. - First R.L.

Example 5: The following consecutive readings were taken with a level and a 4.0 m staff on a continuously sloping ground at common interval of 30 m. 0.780, 1.535, 1.955, 2.430, 2.985, 3.480, 1.155, 1.960, 2.365, 3.640, 0.935, 1.045, 1.630 and 2.545. The reduced level of first point A was 180.750. Rule out a page of level field book and enter the above readings. Calculate the reduced levels of all the points and apply proper check for calculation. (June 2013)
Solution:

By Rise and fall method:

Level book note for Rise and Fall method

Staff Reading			Rise	Fall	R.L. (m)	Remarks
B.S (m)	I.S.(m)	F.S.(m)				
0.780					180.750	A
	1.535			0.755	179.995	
	1.955			0.420	179.575	
	2.430			0.475	179.100	
	2.985			0.555	178.545	
1.155		3.480		0.495	178.050	
	1.960			0.805	177.245	
	2.365			0.405	176.840	
0.935		3.640		1.275	175.565	
	1.045			0.110	175.455	
	1.630			0.585	174.870	
		2.545		0.915	173.955	

Arithmetic Check for Reduction of Level

∑ B.S. = 2.870 m; ∑ F.S. = 9.665 m

Therefore ∑ B.S - ∑ F.S. = -6.795 m

∑ Rise = 0.000 m; ∑ Fall = 6.795 m

Therefore ∑ Rise - ∑ Fall = -6.795 m

Last R.L. - First R.L. = 173.955 – 180.750 = -6.795 m.

∑ B.S - ∑ F.S. = ∑ Rise - ∑ Fall = Last R.L. - First R.L.

By Height of Instrument method:

Level book note for Height of instrument method

Staff Reading			Height of Instrument (m)	R.L. (m)	Remarks
B.S (m)	I.S.(m)	F.S.(m)			
0.780			181.530	180.750	A
	1.535			179.995	
	1.955			179.575	
	2.430			179.100	
	2.985			178.545	

1.155		3.480	179.205	178.050	
	1.960			177.245	
	2.365			176.840	
0.935		3.640	176.500	175.565	
	1.045			175.455	
	1.630			174.870	
		2.545		173.955	

Arithmetic Check for Reduction of Level

∑ B.S. = 2.870 m; ∑ F.S. = 9.665 m

Therefore ∑ B.S - ∑ F.S. = -6.795 m

Last R.L. - First R.L. = 173.955 – 180.750 = -6.795 m.

∑ B.S - ∑ F.S. = ∑ Rise - ∑ Fall = Last R.L. - First R.L.

Reciprocal Leveling: To find accurate relative elevations of two widely separated intervisible points (between which levels cannot be set), reciprocal leveling is being used.

To find the difference in elevation between two points, say X and Y, a level is set up at L near X and readings (X_1 and Y_1) are observed with staff on both X and Y respectively. The level is then set up near Y and staff readings (Y_2 and X_2) are taken respectively to the near and distant points. If the differences in the set of observations are not same, then the observations are fraught with errors. The errors may arise out of the curvature of the earth or intervening atmosphere (associated with variation in temperature and refraction) or instrument (due to error in collimation) or any combination of these.

The true difference in elevation and errors associated with observation, if any, can be found as follows:

Let the true difference in elevation between the points be rh and the total error be e. Assuming, no error on observation of staff near the level (as the distance is very small)

Then,

rh = $X_1 \sim (Y_1 - e)$ [From first set of observation]

and

rh = $(X_2 - e) \sim Y_2$ [From second set of observation]

$\Delta h = [(X_1 \sim Y_1) + (X_2 \sim Y_2)]/2$

Or Δh = [(First difference in observation) + (Second difference in observation)]/2

Thus, the true difference in elevation between any two points can be obtained by taking the mean of the two differences in observation.

Thus, total error in observations can be obtained by taking the difference of the two differences in observation.

Total error e = $[(X_1 \sim Y_1) - (X_2 \sim Y_2)]/2$

Or e = [(First difference in observation) - (Second difference in observation)]/2

The total error consist of error due to curvature of the earth, atmospheric errors (due to temperature and refraction) and instrumental errors (due to error in collimation) etc.

Theodolite

A theodolite is a precision instrument for measuring angles in the horizontal and vertical planes. Theodolites are used mainly for surveying applications, and have been adapted for specialized purposes in fields like metrology and rocket launch technology. A modern theodolite consists of a movable telescope mounted within two perpendicular axes—the horizontal or trunnion axis, and the vertical axis. When the telescope is pointed at a target object, the angle of each of these axes can be measured with great precision, typically two seconds of arc. The most important instrument for exact survey work and many types are available to meet varying requirements of accuracy and precision, with direct readings of the circle ranging from 5 min to 0.1 sec.

CLASSIFICATION:

A. Theodolite may be classified into transit and non-transit theodolite.

Transit theodolite: A theodolite is said to be transit one when its telescope can be revolved through 180° in a vertical plane about its horizontal axis, thus directing the telescope in exactly opposite direction.

Non-transit theodolite: A theodolite is said to be a non-transit one when its telescope cannot be revolved through 180° in a vertical plane about its horizontal axis.

B. Based on the type the theodolite is divided into three types based on angles, which are vernier, optical and electronic.

Vernier Theodolite: Uses vernier scale

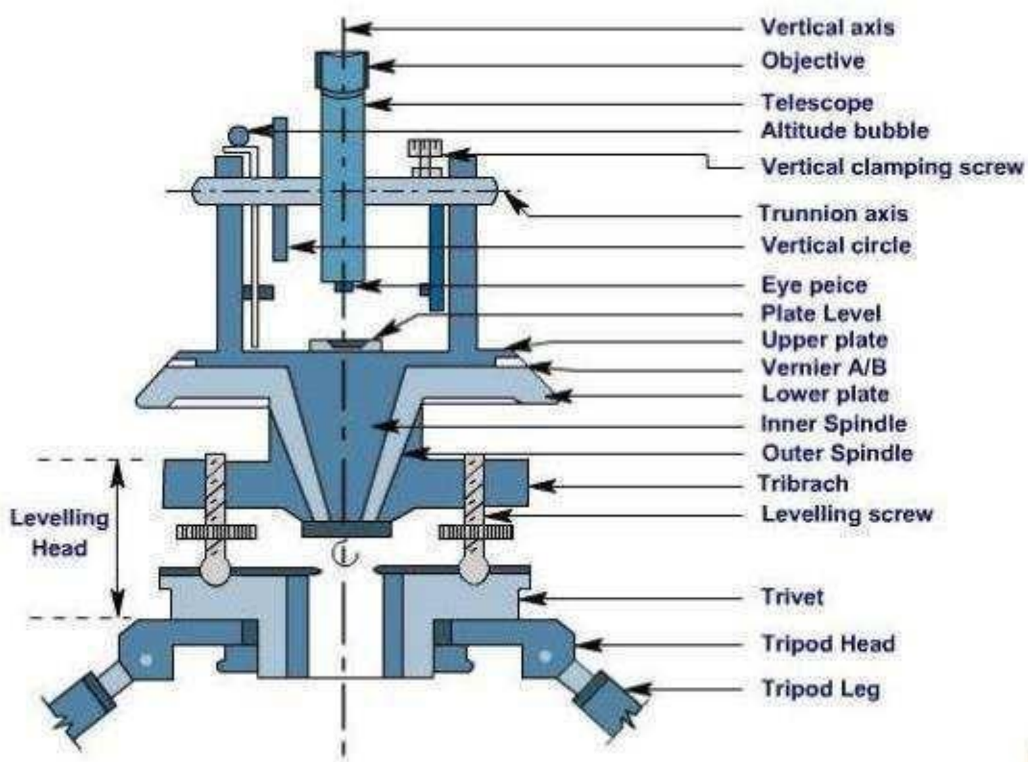
Optical Theodolite: Uses optical with horizontal and vertical circles made from transparent glasses and graduated scale.

Electronic Theodolite: Has a screen with digits for angles on front and back of the instrument.

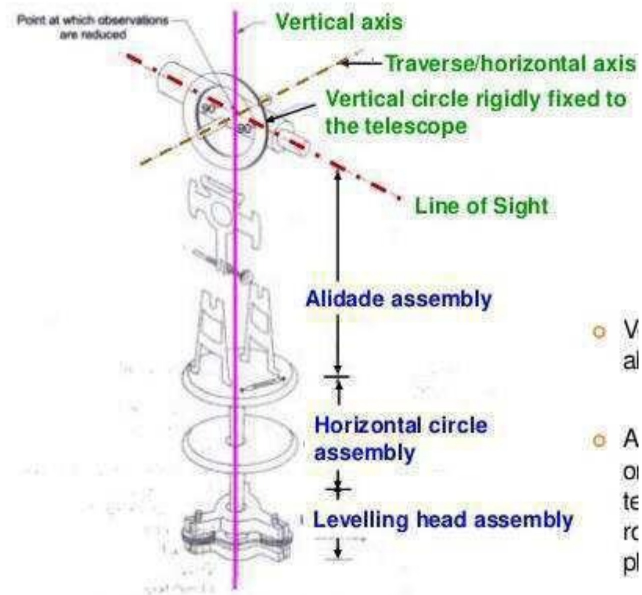
The face of the current observation (telescope position) is the side on which the vertical circle is, when viewed from the eyepiece, which is either face left or face right. The telescope has its own clamp and tangent screws.

A theodolite is mounted on its tripod head by means of a forced centering plate or tribrach containing four thumbscrews, or in modern theodolites, three for rapid leveling. Before use, a theodolite must be precisely placed vertical above the point to be measured using a plumb bob, optical plummet or laser plummet. The instrument is then set level using levelling footscrews and circular and more precise tubular spirit bubbles.

Both axes of a theodolite are equipped with graduated circles that can be read through magnifying lenses. The vertical circle which 'transits' about the horizontal axis should read 90° (100 grad) when the sight axis is horizontal, or 270° when the instrument is in its second position, that is, "turned over" or "plunged". Half of the difference between the two positions is called the "index error".



Sectional view of theodolite



Three assemblies of theodolite View of theodolite

ADJUSTMENT OF A THEODOLITE

The adjustments of a theodolite are of two kinds: Permanent adjustment and Temporary adjustment

PERMANENT ADJUSTMENT: The permanent adjustment are made to establish the fixed relationships between the fundamental lines of the instrument, and once made, they last for long time. They are essential for the accuracy of observations.

TEMPORARY ADJUSTMENT: The temporary adjustments are made at each set up of the instrument before starting taking observations with the instrument. There are three temporary adjustment of a theodolite.

a. **Centering**
b. **Levelling**
c. **Focusing**

a. CENTERING: Centering means bringing the vertical axis of the theodolite immediately over a station mark. The station mark should be represented by well-defined point such as end of a nail driven on the top of a peg or the intersection points of a cross marked at the surface below the instrument etc.

b. LEVELLING: Having centered and approximately leveled the instrument, it is accurately leveled with reference to the plate levels by means of foot-screws so that the vertical axis is made truly vertical.

c. FOCUSSING: This is done in two steps focusing of the eye-piece distinct vision of the cross-hairs at diaphragm and focusing the object-glass for bringing the image of the object into the plane of the diaphragm.

Unit 03

Mapping details and contouring, Profile Cross sectioning and measurement of areas, volumes, application of measurements in quantity computations, Survey stations, Introduction of remote sensing and its applications.

Mapping & Sensing

A contour line is an imaginary line which connects points of equal elevation. Such lines are drawn on the plan of an area after establishing reduced levels of several points in the area. The contour lines in an area are drawn keeping difference in elevation of between two consecutive lines constant. For example, Fig. 1 shows contours in an area with contour interval of 1 m. On contour lines the level of lines is also written. For example, the line of intersection of the water surface of a still lake or pond with the surrounding ground represents a contour line. It facilitates depiction of the relief of terrain in a two dimensional plan or map. The process of tracing contour lines on the surface of earth is called contouring. A contour map gives the idea of the altitudes of the surface features as well as their relative positions in a plan.

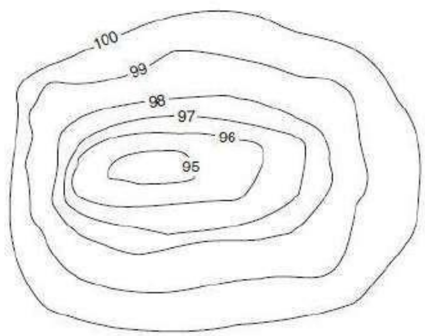


Figure 1: Contours of an area.

Contour interval:

"The vertical distance between any two successive contours on a given map is called the contour interval". Contour intervals usually vary from 25 to 250 cm in engineering work. In rough country, the vertical distance between contours is kept greater while in flat areas 25 to 50 cm contour intervals are used.

The contours have the following characteristics:

1. Contour lines must close, not necessarily in the limits of the plan.
2. Widely spaced contour indicates flat surface.
3. Closely spaced contour indicates steep ground.
4. Equally spaced contour indicates uniform slope.
5. Irregular contours indicate uneven surface.
6. Approximately concentric closed contours with decreasing values towards centre (Fig. 1) indicate a pond.
7. Approximately concentric closed contours with increasing values towards centre indicate hills.
8. Contour lines with U-shape with convexity towards lower ground indicate ridge (Fig. 2).
9. Contour lines with V-shaped with convexity towards higher ground indicate valley (Fig. 3).
10. Contour lines generally do not meet or intersect each other.
11. If contour lines are meeting in some portion, it shows existence of a vertical cliff (Fig. 4).

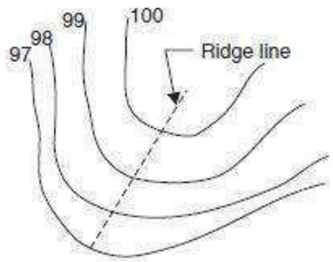


Figure 2: Contours indicating ridge.

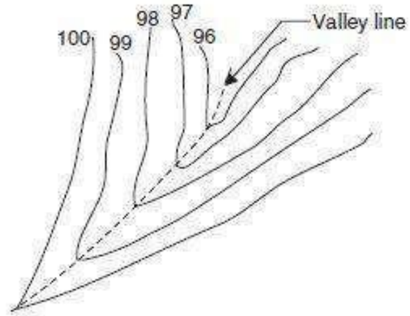


Figure 3: Contours indicating valley.

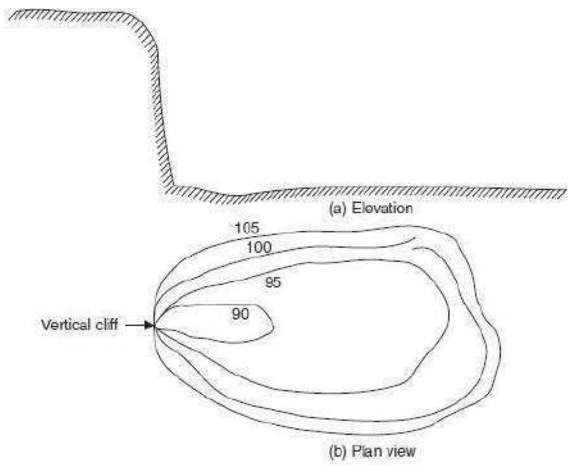


Figure 4: Contours indicating vertical cliff.

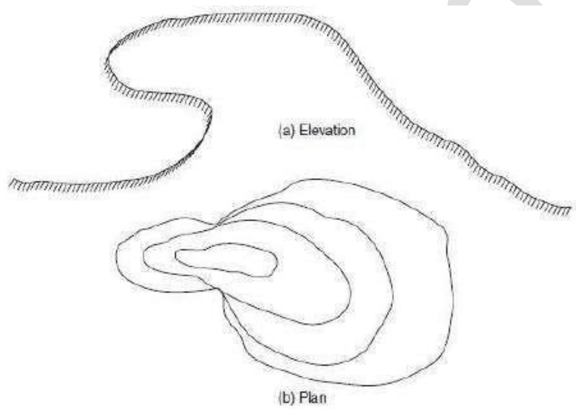
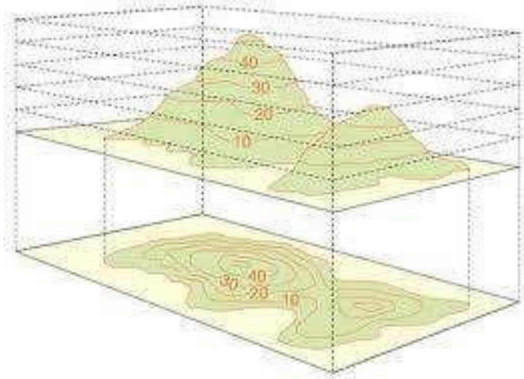


Figure 5: Contours indicating overhanging cliff.

Uses of Contour Maps:

Contour maps are extremely useful for various engineering works:

1. A civil engineer studies the contours and finds out the nature of the ground to identify.
2. Suitable site for the project works to be taken up.
3. By drawing the section in the plan, it is possible to find out profile of the ground along that line. It helps in finding out depth of cutting and filling, if formation level of road/railway is decided.
4. Cost estimates can be made with the help of the contour maps.
5. Intervisibility of any two points can be found by drawing profile of the ground along that line.
6. The routes of the railway, road, canal or sewer lines can be decided so as to minimize and balance earthworks.
7. Catchment area and hence quantity of water flow at any point of nalla or river can be found. This study is very important in locating bunds, dams and also to find out flood levels.
8. Contours may be used to determine area of the catchments and the capacity of the reservoir.
9. In agricultural work, contours maps are useful as guide lines in planning land improvement project. The tile drainage system can be conveniently planned whit contour maps.



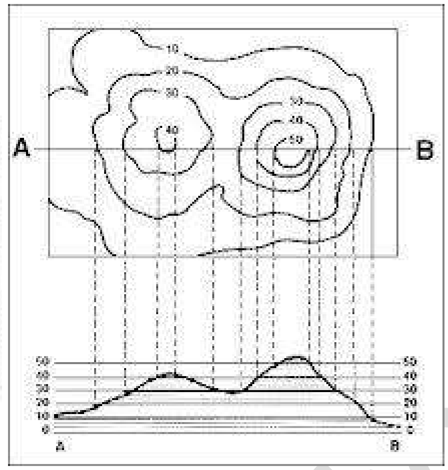


Figure 6: Uses of Contours.

Characteristic of Contour Lines:

1. All points on a contour line have same elevation.
2. Contour line close to each other on s plan view; represent very steep ground. Contour lines for apart indicate relatively flat land.
3. On uniform slopes the contour lines are spaced uniformly .along plane surfaces these lines are straight and parallel to one another.
4. Contour lines Crosse ridge lines or valley lines at right angles valley contour are convex towards the stream.
5. Contour lines cannot and anywhere, but close on themselves. Either within or outside the limits of map they cannot merge or cross one another.
6. A series of closed contour on the map indicate a depression or a summit, depending whether the successes contour have lower or higher values inside
7. At ridge line the contour lines form carves of U shape .At Valley lines they farm sharp curves of shape

REMOTE SENSING

Remote Sensing is the collection of information relating to objects without being in physical contact with them. Thus our eyes and ears are remote sensors, and the same is true for cameras and microphones and for many instruments used for all kinds of applications Or, said another way: Remote sensing is the process of acquiring data/information about objects/substances not in direct contact with the sensor, by gathering its inputs using electromagnetic radiation or acoustical waves that emanate from the targets of interest. An aerial photograph is a common example of a remotely sensed (by camera and film, or now digital) product.

Introduction: The sun is a source of energy or radiation, which provides a very convenient source of energy for remote sensing. The sun's energy is either reflected, as it is for visible wavelengths, or absorbed and then reemitted, as it is for thermal infrared wavelengths.

There are two main types of remote sensing: Passive remote sensing and Active remote sensing.

Passive remote sensing: Passive sensors detect natural radiation that is emitted or reflected by the object or surrounding area being observed. Reflected sunlight is the most Passive 1-Passive sensors detect natural radiation that is emitted or reflected by the object or surrounding area being observed. Reflected sunlight is the most common source of radiation measured by passive sensors. Examples of passive remote sensors include film photography, infrared, and radiometers.

Active remote sensing: It is on the other hand, emits energy in order to scan objects and areas whereupon a sensor then detects and measures the radiation that is reflected or backscattered from the target. RADAR is an example of active remote sensing where the time delay between emission and return is measured, establishing the location, height, speeds and direction of an object.

Overview

Remote sensing makes it possible to collect data on dangerous or inaccessible areas. Remote sensing applications include monitoring deforestation in areas such as the Amazon Basin, the effects of climate change on glaciers and Arctic and Antarctic regions, and depth sounding of coastal and ocean depths. Military collection during the cold war made use of stand-off collection of data about dangerous border areas. Remote sensing also replaces costly and slow data collection on the ground, ensuring in the process that areas or objects are not disturbed.

Principles and Process of Remote Sensing

Remote sensing actually done from satellites as Landsat or airplane or on the ground. To repeat the essence of the definition above, remote sensing uses instruments that house sensors to view the spectral, spatial and radiometric relations of observable objects and materials at a distance. Most sensing modes are based on sampling of photons corresponding frequency in the electromagnetic (EM) spectrum.

In much of remote sensing, the process involves an interaction between incident radiation and the targets of interest. This is exemplified by the use of imaging systems where the following seven elements are involved. Note, however that remote sensing also involves the sensing of emitted energy and the use of non-emitted sensors.

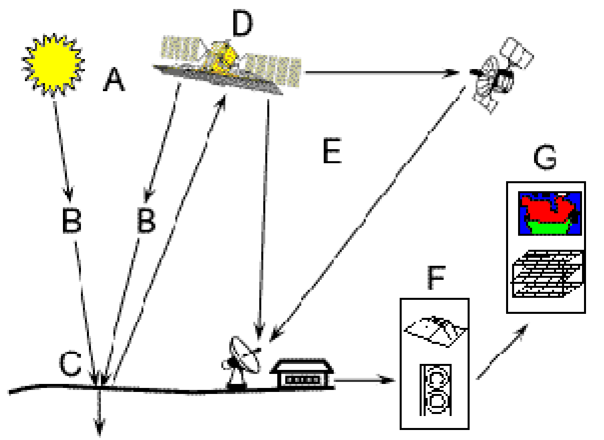


Figure 7: Principles of remote sensing.

i. Energy Source or Illumination (A) - The first requirement for remote sensing is to have an energy source which illuminates or provides electromagnetic energy to the target of interest.

ii. Radiation and the Atmosphere (B) - As the energy travels from its source to the target, it will come in contact with and interact with the atmosphere it passes through. This interaction may take place a second time as the energy travels from the target to the sensor.

iii. Interaction with the Target (C) - Once the energy makes its way to the target through the atmosphere, it interacts with the target depending on the properties of both the target and the radiation.

iv. Recording of Energy by the Sensor (D) - After the energy has been scattered by, or emitted from the target, we require a sensor (remote - not in contact with the target) to collect and record the electromagnetic radiation.

v. Transmission, Reception, and Processing (E) - The energy recorded by the sensor has to be transmitted, often in electronic form, to a receiving and processing station where the data are processed.

vi. Interpretation and Analysis (F) - The processed image is interpreted, visually and/or digitally or electronically, to extract information about the target, which was illuminated.

vii. Application (G) - The final element of the remote sensing process is achieved when we apply the information we have been able to extract from the imagery about the target in order to better understand it, reveal some new information, or assist in solving a particular problem.

Applications of Remote Sensing

There are probably hundreds of applications - these are typical:

1. Meteorology - Study of atmospheric temperature, pressure, water vapour, and wind velocity.
2. Oceanography: Measuring sea surface temperature, mapping ocean currents, and wave energy spectra and depth sounding of coastal and ocean depths Glaciology- Measuring ice cap volumes, ice stream velocity, and sea ice distribution. (Glacial)
3. Geology- Identification of rock type, mapping faults and structure.
4. Geodesy- Measuring the figure of the Earth and its gravity field.
5. Topography and cartography - Improving digital elevation models.
6. Agriculture Monitoring the biomass of land vegetation
7. Forest- monitoring the health of crops, mapping soil moisture
8. Botany- forecasting crop yields.
9. Hydrology- Assessing water resources from snow, rainfall and underground aquifers.
10. Disaster warning and assessment - Monitoring of floods and landslides, monitoring volcanic activity, assessing damage zones from natural disasters.
11. Planning applications - Mapping ecological zones, monitoring deforestation, monitoring urban land use.
12. Oil and mineral exploration- Locating natural oil seeps and slicks, mapping geological structures, monitoring oil field subsidence.
13. Military- developing precise maps for planning, monitoring military infrastructure, monitoring ship and troop movements
14. Urban- determining the status of a growing crop
15. Climate- the effects of climate change on glaciers and Arctic and Antarctic Regions
16. Sea- Monitoring the extent of flooding
17. Space program- is the backbone of the space program

Profile Cross sectioning

Survey Station: Survey stations are of two kinds

Main Stations, Subsidiary or tie

Main Stations: Main stations are the end of the lines, which command the boundaries of the survey, and the lines joining the main stations re called the main survey line or the chain lines.

Subsidiary or the tie stations: Subsidiary or the tie stations are the point selected on the main survey lines, where it is necessary to locate the interior detail such as fences, hedges, building etc.

Tie or subsidiary lines: A tie line joints two fixed points on the main survey lines. It helps to checking the accuracy of surveying and to locate the interior details. The position of each tie line should be close to some features, such as paths, building etc.

Base Lines: It is main and longest line, which passes approximately through the centre of the field. All the other measurements to show the details of the work are taken with respect of this line.

Check Line: A check line also termed as a proof line is a line joining the apex of a triangle to some fixed points on any two sides of a triangle. A check line is measured to check the accuracy of the framework. The length of a check line, as measured on the ground should agree with its length on the plan.

Offsets: These are the lateral measurements from the base line to fix the positions of the different objects of the work with respect to base line. These are generally set at right angle offsets. It can also be drawn with the help of a tape.

There are two kinds of offsets:

Perpendicular offsets, and Oblique offsets.

The measurements are taken at right angle to the survey line called perpendicular or right angled offsets. The measurements which are not made at right angles to the survey line are called oblique offsets or tie line offsets.

Procedure in chain survey:

1. Reconnaissance: The preliminary inspection of the area to be surveyed is called reconnaissance. The surveyor inspects the area to be surveyed, survey or prepares index sketch or key plan.
2. Marking Station: Surveyor fixes up the required no stations at places from where maximum possible stations are possible.
3. Then he selects the way for passing the main line, which should be horizontal and clean as possible and should pass approximately through the centre of work.
4. Then ranging roads are fixed on the stations.
5. After fixing the stations, chaining could be started.
6. Make ranging wherever necessary.
7. Measure the change and offset.
8. Enter in the field the book.

Cross Staff Survey

This type of survey is undertaken to locate boundaries of a field or a field or plat and determinations of its area.

Principle: The principle of the survey is to divide the given area in to number of right angled triangles and trapezoids and to calculate and plot the areas of triangles and trapezoids. Instruments/Material Required: Two Chains, arrows taps, ranging rods, cross-staff and drawing material

Procedure: Two chains are usually provided one for measuring distance along the chain line and other for measuring the offsets. The cross staff is used to set out the perpendicular directions for offsets. In this survey, the base line runs through the center of the area, so that the offsets are left or right side of base line are fairly equal. To check accuracy length of the boundary lines may also be measured. After the field work is over, the survey is plotted to a suitable scale.

Measurement of areas

1. Calculations of Field Area:

By this method of survey, the field as divided in to right angled triangles and trapezoids are calculated as under:

Area of right -angled triangle = ½ base x Height.

Area of trapezoid= sum of parallel sides/2 x Height.

Add the areas of all the triangles & trapezoids and sum is equal to the total of a field.

The computations for area should be written in a tabular from as given below.

Sl. No.	Figure	Chainage in m.	Base in M.	Offset in M.	Mean offset in m.	Area in m^2 = Col 4 x Col 6

Problem 1:

The following are the details of chain survey. Plot the area for the given details and calculate the area.

65.4 m F
52.4 m 18.50 m E
D 13.50 m 42.00 m
28.00 m 22.00 m C
B 14.5 m 12.00 m
0.0 m A

Solution: Plot the area for the given data.

Tabular column:

Sl. No.	Figure	Chainage in m.	Base in M.	Offset in M.	Mean offset in m.	Area in m^2 = Col 4 x Col 6
1.	A12B	0.0 to 12.0	12.0	0.0 and 14.5	7.25	87.00
2.	A28C	12.0 to 28.0	16.0	0.0 and 22.0	11.0	176.00
3.	12BD42	12.0 to 42.0	30.0	14.5 and 13.5	14.0	420.00
4.	28CE52.4	28.0 to 52.40	24.4	22.0 and 18.5	20.25	494.10
5.	D42F	42.0 to 65.4	23.4	13.5 and 0.0	6.75	157.95
6.	52.4EF	52.4 to 65.4	13.0	18.5 and 0.0	9.25	120.25
	Total area in m^2 (ABDFECA)					1455.30

Problem 2:

The following are the details of chain survey. Plot the area for the given details and calculate the area.

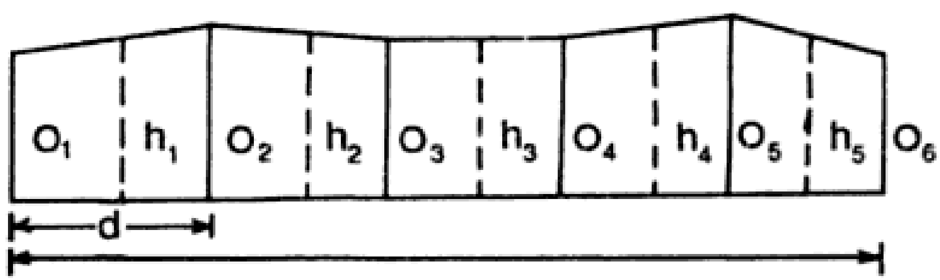
115.4 m G
102.0 m 18.50 m F
E 10.0 m 82.00 m
68.00 m 12.00 m D
C 23.5 m 42.00 m
28.00 m 14.00 m B
0.0 m A

Solution: Plot the area for the given data.

Tabular column:

Sl. No.	Figure	Chainage in m.	Base in M.	Offset in M.	Mean offset in m.	Area in m^2 = Col 4 x Col 6
1.	A42C	0.0 to 42.0	42.0	0.0 and 23.5	11.75	493.50
2.	A28B	0.0 to 28.0	28.0	0.0 and 14.0	7.0	196.00
3.	28BD68	28.0 to 68.0	40.0	14.0 and 12.0	13.0	520.00
4.	42CE82	42.0 to 82.0	40.0	23.5 and 10.0	16.75	670.00
5.	68DF102	68.0 to 102	34.0	12.0 and 18.5	15.25	518.50
6.	E82G	82.0 to 115.4	33.4	18.5 and 0.0	9.25	308.95
	Total area in m^2 (ABDFGECA)					2706.95

2. **Measurement of areas by mid ordinate method:**



L = Length of base line

Let $O_1, O_2, O_3, \ldots, O_n$ = Ordinates at equal intervals.

d = Common distance between ordinates.

L = Length of Base line = d * (n - 1)

$h_1, h_2, h_3, \ldots, h_n$ = Mid-ordinates

n = Number of ordinates.

$$h_1 = (O_1 + O_2)/2$$

$$h_2 = (O_2 + O_3)/2$$

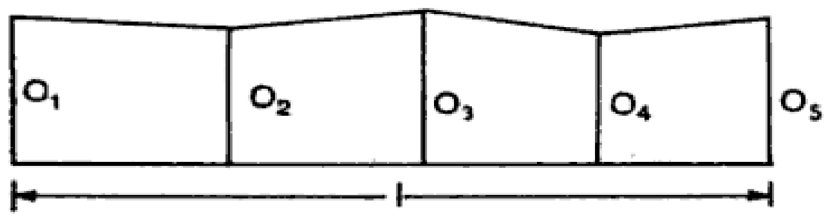
$$h_3 = (O_3 + O_4)/2$$

$$\ldots\ldots\ldots\ldots$$

$$h_{n-1} = (O_{n-1} + O_n)/2$$

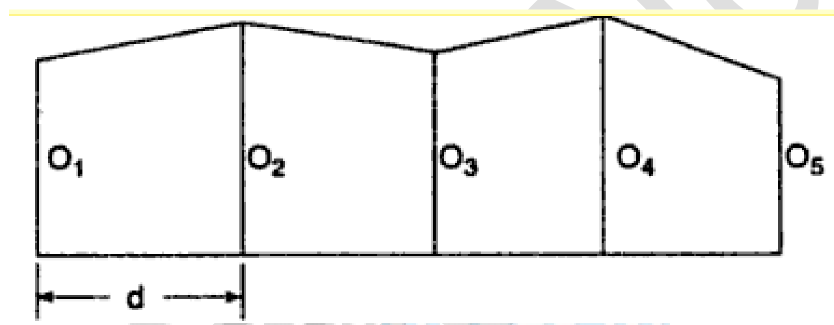
$$\text{Area} = A = d * (h_1 + h_2 + h_3 + \ldots\ldots + h_{n-1})$$

3. **Measurement of areas by average ordinate method:**



Let $O_1, O_2, O_3, \ldots, O_n$ = Ordinates at equal intervals.
d = Common distance between ordinates.
L = Length of Base line = d * (n - 1)
n = Number of ordinates.
Area = A = L * ($O_1 + O_2 + O_3 + \ldots\ldots + O_n$)/n

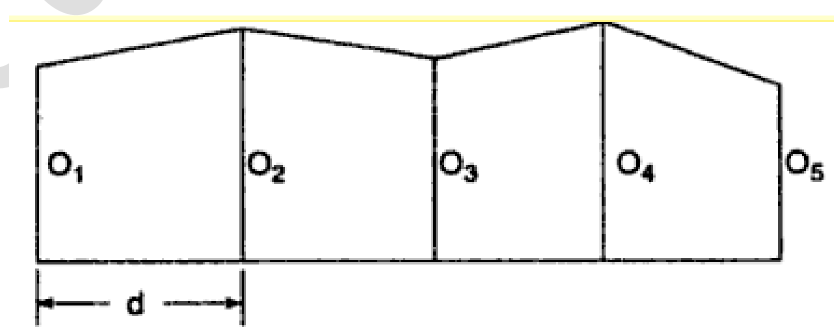
4. **Measurement of areas by Trapezoidal rule.**



Let $O_1, O_2, O_3, \ldots, O_n$ = Ordinates at equal intervals.
d = Common distance between ordinates.
n = Number of ordinates.

Area = A = d * [($O_1 + O_n$)/2 + ($O_2 + \ldots\ldots + O_{n-1}$)]

5. **Measurement of areas by Simpson's rule.**
In this rule, the boundaries between the end of ordinates are assumed to form an arc of parabola. Hence Simpson's rule is some times called as parabolic rule.



Let $O_1, O_2, O_3, \ldots, O_n$ = Ordinates at equal intervals.
d = Common distance between ordinates.
n = Number of ordinates.

Area = A

$= (d/3) * [(O_1 + O_n)/2 + 2*(O_2 + O_4 + O_6 + \ldots\ldots) + 4 *(O_3 + O_5 + O_7 + \ldots\ldots)]$

Problem 01: The following perpendicular offsets were taken from chain line to an irregular boundary:

Chainage	**0.0**	**10.0**	**25.0**	**42.0**	**60.0**	**75.0 m**
Offset	**15.5**	**26.2**	**31.8**	**25.6**	**29.0**	**31.5**

Calculate the area between the chain line, the boundary and the end offsets.

1. **Area by mid ordinate method:**

n = 06.

d = 10.0 m.

$h_1 = (O_1 + O_2)/2 = (15.5 + 26.2)/2 = 20.85$

$h_2 = (O_2 + O_3)/2 = (26.2 + 31.8)/2 = 29.00$

$h_3 = (O_3 + O_4)/2 = (31.8 + 25.6)/2 = 28.70$

$h_4 = (O_4 + O_5)/2 = (25.6 + 29.0)/2 = 27.30$

$h_5 = (O_5 + O_6)/2 = (29.0 + 31.5)/2 = 30.25$

Area = $A = d * (h_1 + h_2 + h_3 + \ldots\ldots + h_{n-1})$

$= 10.0 * (20.85 + 29.0 + 28.7 + 27.3 + 30.25)$

$= 1361.00\ m^2$.

2. **Area by average ordinate method:**

n = 06.

d = 10.0 m.

L = Length of Base line = d * (n - 1) = 10 * (6-1) = 50.0 m.

Area = $L * (O_1 + O_2 + O_3 + \ldots\ldots + O_n)/n$

$= 50.0 * (15.5 + 26.2 + 31.8 + 25.6 + 29.0 + 31.5)/6$

$= 1330.00\ m^2$.

3. **Area by Trapezoidal rule:**

n = 06.

d = 10.0 m.

Area = $d * [(O_1 + O_n)/2 + (O_2 + \ldots\ldots + O_{n-1})]$

$= 10.0 * [(15.5 + 31.5)/2 + 26.2 + 31.8 + 25.6 + 29.0]$

$= 1361.00\ m^2$.

4. **Area by Simpson's rule:**

n = 06.

d = 10.0 m.

Area = $(d/3) * [(O_1 + O_n)/2 + 2*(O_2 + O_4 + O_6 + \ldots\ldots) + 4 *(O_3 + O_5 + O_7 + \ldots\ldots)]$

$= (10.0/3) * [(15.5 + 31.5)/2 + 2 * (26.2 + 25.6) + 4 * (31.8 + 29.0)]$

$= 1234.33\ m^2$.

Problem 02:

The following perpendicular offsets were taken from a chain line to a hedge:

Chainage:	**0.0**	**15.0**	**30.0**	**45.0**	**60.0**	**75.0**	**90.0**	**105.0**	**120.0**	**135.0 m.**
Offset:	**7.6**	**8.5**	**10.7**	**12.8**	**10.6**	**9.5**	**8.3**	**7.9**	**6.4**	**4.4 m.**

Calculate the area by mid ordinate, average ordinate, Trapezoidal rule and Simpson's rule.

1. **Area by mid ordinate method:**
   n = 10.
   d = 15.0 m.

$h_1 = (O_1 + O_2)/2 = (7.6 + 8.5)/2 = 8.05$

$h_2 = (O_2 + O_3)/2 = (8.5 + 10.7)/2 = 9.60$

$h_3 = (O_3 + O_4)/2 = (10.7 + 12.8)/2 = 11.75$

$h_4 = (O_4 + O_5)/2 = (12.8 + 10.6)/2 = 11.70$

$h_5 = (O_5 + O_6)/2 = (10.6 + 9.5)/2 = 10.05$

$h_6 = (O_6 + O_7)/2 = (9.5 + 8.3)/2 = 8.90$

$h_7 = (O_7 + O_8)/2 = (8.3 + 7.9)/2 = 8.10$

$h_8 = (O_8 + O_9)/2 = (7.9 + 6.4)/2 = 7.15$

$h_9 = (O_9 + O_{10})/2 = (6.4 + 4.4)/2 = 5.40$

Area = $A = d * (h_1 + h_2 + h_3 + \ldots\ldots + h_{n-1})$
= 15.0 * (8.05 + 9.60 + 11.75 + 11.70 + 10.05 + 8.90 + 8.10 + 7.15 + 5.40)
= 1210.50 m^2.

2. **Area by average ordinate method:**
   n = 10.
   d = 15.0 m.
   L = Length of Base line = d * (n - 1) = 15 * (10-1) = 135.0 m.
   Area = $L * (O_1 + O_2 + O_3 + \ldots\ldots + O_n)/n$
   = 135.0 * (7.6 + 8.5 + 10.7 + 12.8 + 10.6 + 9.5 + 8.3 + 7.9 + 6.4 + 4.4)/10
   = 1170.45 m^2.
3. **Area by Trapezoidal rule:**
   n = 10.
   d = 15.0 m.
   Area = $d * [(O_1 + O_n)/2 + (O_2 + \ldots\ldots + O_{n-1})]$
   = 15.0 * [(7.6 + 4.4)/2 + 8.5 + 10.7 + 12.8 + 10.6 + 9.5 + 8.3 + 7.9 + 6.4]
   = 1210.50 m^2.
4. **Area by Simpson's rule:**
   n = 10.
   d = 15.0 m.

Area = $(d/3) * [(O_1 + O_n)/2 + 2*(O_2 + O_4 + O_6 + ...) + 4 *(O_3 + O_5 + O_7 + ...)]$
$= (15.0/3) * [(7.6 + 4.4)/2 + 2 * (8.5 + 12.8 + 9.5 + 7.9) +$
$4 * (10.7 + 10.6 + 8.3 + 6.4)]$
$= 1137.00\ m^2$.

Measurement of Volume

1. **Volume by Trapezoidal rule:**

Let $A_1, A_2, A_3, ..., A_n$ = Areas at equal intervals.
d = Common distance between areas.
n = Number of areas.

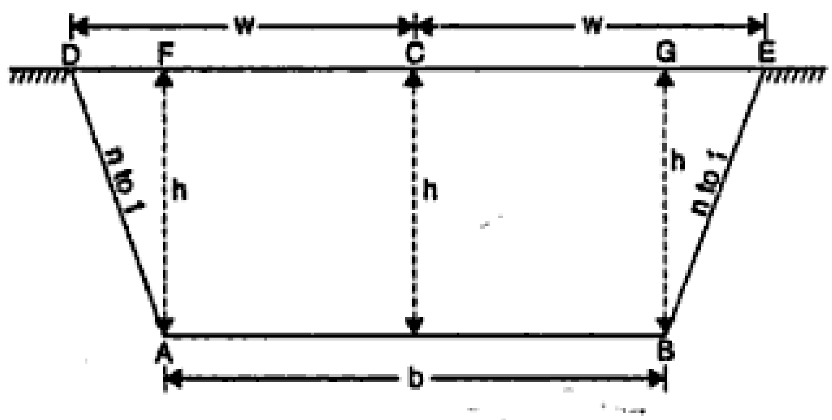
Volume = V = $d * [(A_1 + A_n)/2 + (A_2 + ... + A_{n-1})]$

2. **Volume by Simpson's rule:**

Let $A_1, A_2, A_3, ..., A_n$ = Areas at equal intervals.
d = Common distance between areas.
n = Number of areas.

Area = A
$= (d/3) * [(A_1 + A_n)/2 + 2*(A_2 + A_4 + A_6 + ...) + 4 *(A_3 + A_5 + A_7 + ...)]$

Area of trapezium:



Area of trapezium = $b * h + 2((1/2) \times n * h * h) = (b + n*h)h$
$= b*h + n*h^2$.

Where b = Top width of embankment or Bottom width of cutting
n = side slope as 1 in n or n to 1

Problem 01:

A railway embankment is 12 m wide. The ground is level in the direction transeverse to the centre line. Calculate the volume contained in a 120 m length by trapezoidal and prismoidal rule, if the side slope is 1.5:1. The centre height at 20 m interval are 3.7 m, 2.6 m, 4.0 m, 3.4 m, 2.8 m, 3.0 m, 2.2 m.

Solution:

If h is the height of embankment, then the area is given by the equation
$A = 12 \times h + 2((1/2) \times 1.5h \times h) = (b + nh)h$

$= 12h + 1.5h^2$.

Point No.	Chainages	Centre height	Area in m^2.
0	0.0	3.70	54.6675
1	20.0	2.60	36.2700
2	40.0	4.00	60.0000
3	60.0	3.40	49.4700
4	80.0	2.80	39.4800
5	100.0	3.00	42.7500
6	120.0	2.20	30.0300

Volume by trapezoidal formula

Volume = (20)[{54.6675 + 30.0300}/2 +{36.27 +60.00 + 49.47 + 39.48 + 42.75 }]
= 5406.375 m^3.
Volume by Prismoidal formula
Volume = (20/3)[{54.6675 + 30.0300} + 2 x {36.27 + 49.47 + 42.75 } + 4 x {60.00 + 39.48 }]
= 4648.325 m^3.

Problem 02:
A railway embankment 400 m long is 12 m wide at the formation level and has the side slope 2:1. The ground levels across the centre line are as under:

Chainage:	**0.0**	**100.0**	**200.0**	**300.0**	**400.0 m.**
Offset:	**203.6**	**204.2**	**205.7**	**206.1**	**207.3 m.**

The formation level at zero chainage is 207.00 and the embankment has a rising gradient of 1:100. The ground is level across the centre line. Calculate the volume of earthwork.

Solution:

If h is the height of embankment, then the area is given by the equation
$A = 12 \times h + 2((1/2) \times 1.5h \times h) = (b + sh)h$
$= 12h + 1.5h^2$.
In this problem, side slope n = 2, b = 12.0 m.
Formation level at 0.0 m chain age is 207.0.
The longitudinal gradient is 1 in 100.
Therefore formation level at 100.0 m chain age is 207.0 + 1.0 = 208.00. The results are shown in the following table.

Chainage in m.	Existing ground level (GL)	Formation level (FL)	Depth of filling (FL – GL)	Area in m^2. ($b*h + n*h^2$)
0.0	203.6	207.0	3.4	63.92
100.0	204.2	208.0	3.8	74.48
200.0	205.7	209.0	3.3	61.38
300.0	206.1	210.0	3.9	77.22
400.0	207.3	211.0	3.7	71.78

Volume by trapezoidal formula

Volume = (100)[{63.92 + 71.78}/2 + {74.48 +61.38 + 77.22 }]
= 28093.00 m^3.

Volume by Prismoidal formula

Volume = (100/3)[{63.92 + 71.78} + 2 x {74.48 + 77.22 } + 4 x {61.38 }]
= 20559.00 m^3.

Problem 03:

The areas within the contour line at the site of reservoir and the face of the proposed Dam are as follows:

Contour	Area in m^2.	Contour	Area in m^2.
101	1000	106	1350000
102	12800	107	1985000
103	95200	108	2286000
104	147600	109	2512000
105	872500	110	3517000

Taking 101 as the bottom level of the reservoir and 110 as the top level, calculate the capacity of the reservoir.

Solution:

Volume by trapezoidal formula

Volume = (1.0)[{1000 + 3517000}/2 + {12800+95200+ 147600 + 872500 + 1350000 + 1985000 + 2286000 + 2512000}]
= 1,10,20,100.00 m^3.

Volume by Prismoidal formula

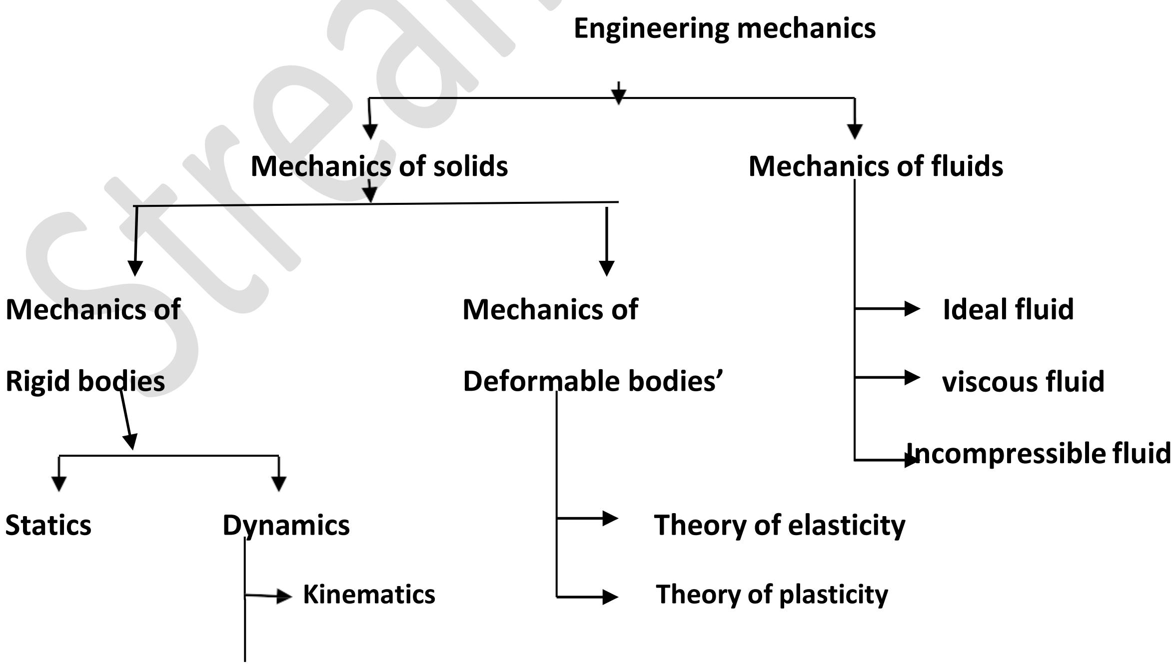
Volume = (100/3)[{ 1000 + 3517000} + 2 x {12800+ 147600 + 1350000 + 2286000} + 4 x {95200+ 872500 + 1985000 + 2512000}]
= 1,04,03,533.33 m^3.

UNIT-I

FORCES AND EQUILIBRIUM: Graphical and Analytical Treatment of concurrent and non-concurrent Coplanar forces, force diagrams and Bow's notations, application to simple engineering structures and Components, method of joints, method of sections for forces in members of plan frames and trusses.

The branch of physical science that deals with the state of rest or the state of motion of the bodies under the action of different forces is termed as Mechanics.

Classification of Engineering Mechanics:
Depending upon the body to which the mechanics is applied, the engineering mechanics
Is classified as
(a) Mechanics of Solids, and
(b) Mechanics of Fluids.
The mechanics of solids is further classified as mechanics of rigid bodies and mechanics of deformable bodies. The bodies which will not deform or the body in which deformation can be neglected in the analysis are called as rigid bodies.
The mechanics of the rigid bodies dealing with the bodies at rest is termed as Statics and that dealing with bodies in motion is called Dynamics.
The dynamics dealing with the problems without referring to the forces causing the motion of the body is termed as Kinematics and if it deals with the forces causing motion also, is called Kinetics.
If the internal stresses developed in a body are to be studied, the deformation of the body should be considered. This field of mechanics is called Mechanics of Deformable Bodies/ Strength of Materials/Solid Mechanics. This field may be further divided into Theory of Elasticity and Theory of Plasticity.
Liquid and gases deform continuously with application of very small shear forces. Such materials are called Fluids. The mechanics dealing with behavior of such materials is called Fluid Mechanics. The classification of mechanics is summarized below in flow chart.



→ **Kinetics**

Basic idealization in Mechanics:

The following are the terms basic to study mechanics, which should be understood clearly:

Mass: The quantity of the matter possessed by a body is called mass. The mass of a body will not change unless the body is damaged and part of it is physically separated. When a body is taken out in a spacecraft, the mass will not change but its weight may change due to change in gravitational force. Even the body may become weightless when gravitational force vanishes but the mass remain the same.

Continuum: A body consists of several matters. It is a well-known fact that each particle can be subdivided into molecules, atoms and electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as continuum.

Rigid Body: In physics, a rigid body is an idealization of a solid body in which deformation is neglected. In other words, the distance between any two given points of a rigid body remains constant in time regardless of external forces exerted on it. Even though such an object cannot physically exist due to relativity, objects can normally be assumed to be perfectly rigid if they are not moving near the speed of light. In Figure 1 points *A* and *B* are the original position in a body. Many engineering problems can be solved satisfactorily by assuming bodies rigid.

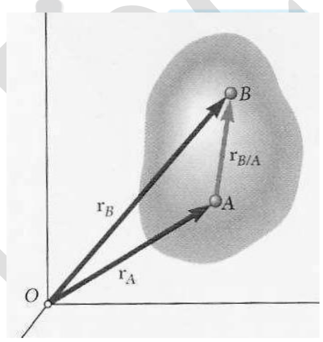


Figure 1: Rigid body concept.

Particle

It is an object that has mass but no dimensions. Examples of such situations are

— A bomber airplane is a particle for a gunner operating from the ground.

— A ship in mid sea is a particle in the study of its relative motion from a control tower.

— In the study of movement of the earth in celestial sphere, earth is treated as a particle.

Law of Transmissibility of Force: According to this law the conditions of equilibrium or conditions of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action.

Let P be the force acting on a rigid body at point *A* as shown in Figure 2 below. According to the law of transmissibility of force, this force has the same effect on the state of body as the force P applied at point *B*.

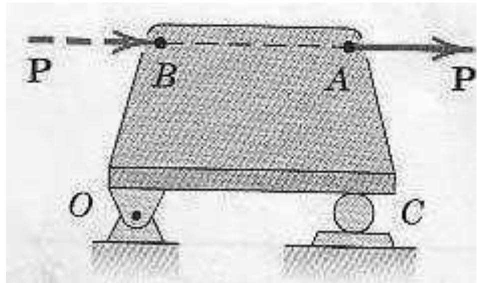


Figure 2: Principle of Transmissibility of forces.

Composition of Forces: The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

Resultant Force: It is possible to find a single force which will have the same effect as that of a number of forces acting on a body. Such a single force is called resultant force. The process of finding out the resultant force is called composition of forces.

Graphical methods:

Parallelogram Law of Forces:

The parallelogram law of forces enables us to determine the single force called resultant which can replace the two forces acting at a point with the same effect as that of the two forces. This law was formulated based on experimental results. This law states that if two forces acting simultaneously on a body at a point are presented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.

In Figure 4 the force *P* and force *Q* are acting on a body at point O. Then to get resultant of these forces parallelogram O*ACB* is constructed such that *OA* is equal to P and *OB* is equal to Q. Then according to this law, the diagonal *OC* represents the resultant in the direction and magnitude. Thus the resultant of the forces *P* and *Q* on the body is equal to units corresponding to *OC* in the direction α to *P*.

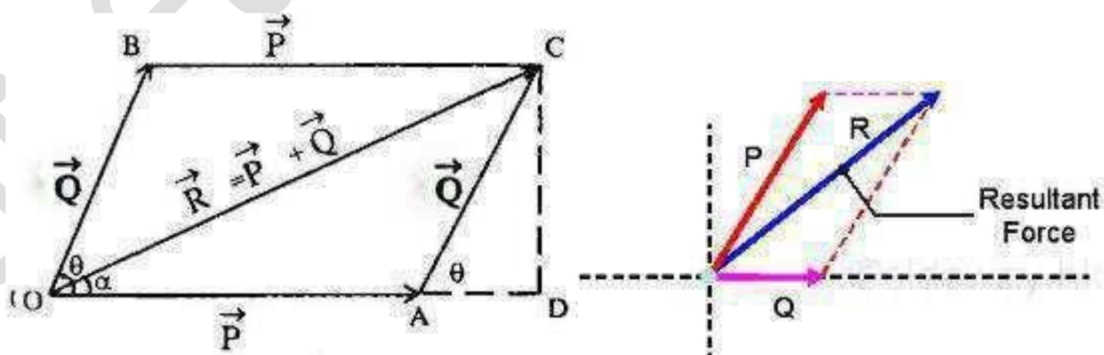


Figure 4: Parallelogram Law of Forces.

From Figure 4, $\sum F_x = P + Q \cos\theta$ $\sum F_y = Q \sin\theta$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

Derived laws:

Referring to Figure 5, we can get the resultant *R* by constructing the triangle. Line is drawn to represent F1 and *line* to represent *F2*. Then *the closing side of triangle* should represent the resultant of F1 and F2. Then we have derived triangle law of forces from fundamental law parallelogram law of forces. The Triangle Law of Forces may be stated as If two forces acting on a body are represented one after another by the sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last point.

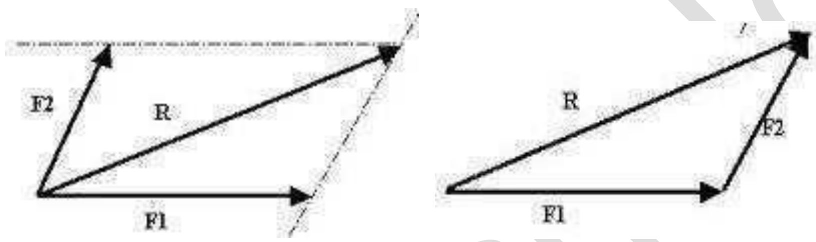


Figure 5: Triangle Law of Forces.

If more than two concurrent forces are acting on a body, two forces at a time can be combined by triangle law of forces and finally resultant of all the forces acting on the body may be obtained.

A system of four concurrent forces acting on a body is shown in Figure 6. *AB* represents *P* and *BC* represents *Q*, *CD* represents *S and DE* represents *T*. Hence according to triangle law of forces *AC* represents the resultant of *P* and *Q*, *AD* represents the resultant of P, Q and S. *AE* represents the resultant of *P, Q, S and T*. Thus resultant *R* is represented by the closing line of the polygon *ABCDE* in the direction *AE.* Thus we have derived polygon of law of forces and it may be stated as '*If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in a order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken from first point to last point.*

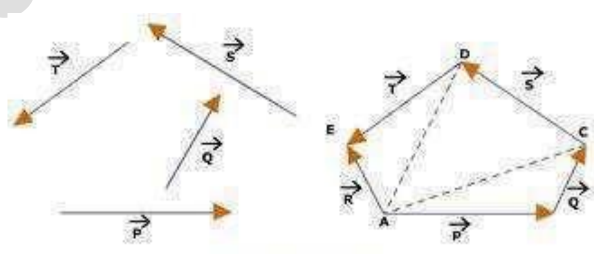


Figure 6: Triangle Law of Forces.

Resolving Forces, Calculating Resultants

Resolving forces refers to the process of finding two or more forces which, when combined, will

Produce a force with the same magnitude and direction as the original. The most common use of the process is finding the components of the original force in the Cartesian coordinate directions x and y. Breaking down a force into its Cartesian coordinate components (e.g., Fx, Fy) and using Cartesian components to determine the force and direction of a resultant force are common tasks when solving statics problems. These will be demonstrated here using a two-dimensional problem involving coplanar forces.

Example:

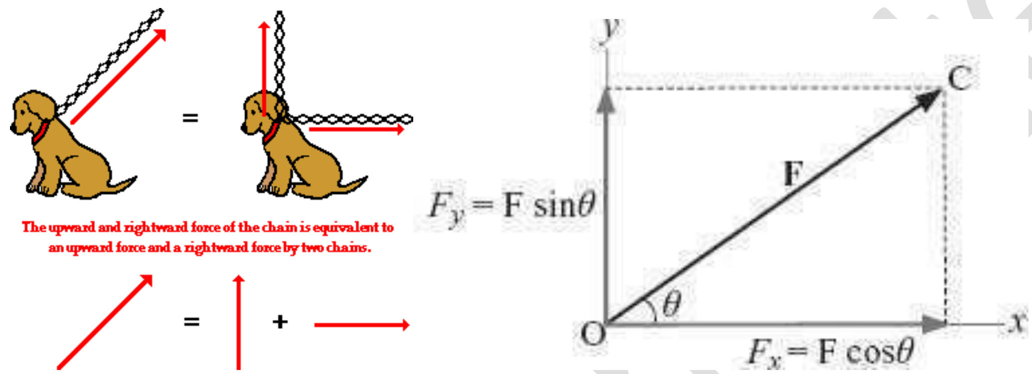


Figure 7: Rectangular components of Forces.

If the force F is resolved into two components which are perpendicular to each other, then these are called *rectangular components F_x* and F_y (Figure 7)

Units:

Length (*L*), Mass (*M*) and Time (*S*) are the fundamental units in mechanics. The units of all other quantities may be expressed in terms of these basic units. The three commonly used systems in engineering are

— Meter-Kilogram—Second (MKS) system

— Centimeter—Gram—Second (CGS) system, and

— Foot—Pound—Second (FPS) system.

The units of length, mass and time used in the system are used to name the systems. Using these basic units, the units for other quantities can be found. For example, in MKS the units for the various quantities are as shown below:

Area Square meter m^2, Volume Cubic meter m^3, Velocity Meter per second m/sec, Acceleration Meter per second per second m/sec^2.

Unit of Force: Presently the whole world is in the process of switching over to SI *system of units.* SI stands for System International units or International System of units. As in MKS system, in SI system also the fundamental units are meter for length, kilogram for mass and second for time. The difference between MKS and SI system arise mainly in selecting the unit of force.

We know that Force = Mass × Acceleration

In SI system unit of force is defined as that force which causes 1 kg mass to move with an acceleration of 1m/sec^2 and is termed as 1 Newton. Unit of force can be derived as

Unit of Force = kg × m/sec^2 = kg – m/sec^2 = 1N.
The weight of i kg mass is mg which is equal to 9.81 x 1 kg m/s2 = 9.81 N.
The prefixes used in SI system when quantities are too big or too small are shown in Table 1.1.

Table 1.1: Prefixes and Symbols of Multiplying Factors in SI

Multiplying Factor	***Prefix***	***Symbol***
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{0}	--	--
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Characteristics of force:

From Newton's first law, we defined the force as the agency which tries to change state of stress or state of uniform motion of the body. From Newton's second law of motion we arrived at practical definition of unit force as the force required producing unit acceleration in a body of unit mass. Thus 1 Newton is the force required to produce an acceleration of 1 m/sec^2 in a body of 1 kg mass. It may be noted that a force is completely specified only when the following four characteristics are specified:

— Magnitude
— Point of application

— Line of action, and
— Direction
In the example shown below;
Magnitude of the three forces is 100 N.
Point of application is A.
Line of action is a line passing through A and along the force.
Direction is 30° with horizontal.

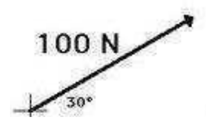


Figure 7: Characteristics of Forces.

In Figure 8 a ladder is kept against a wall and horizontal floor. At point *C*, a person weighing 800 N is standing. The force applied by the person on the ladder has the following characteristics:
— Magnitude is 800 N
— The point of application is at *C* which is x m from *A* along the ladder.
— The line of action is vertical, and
— The direction is downward.

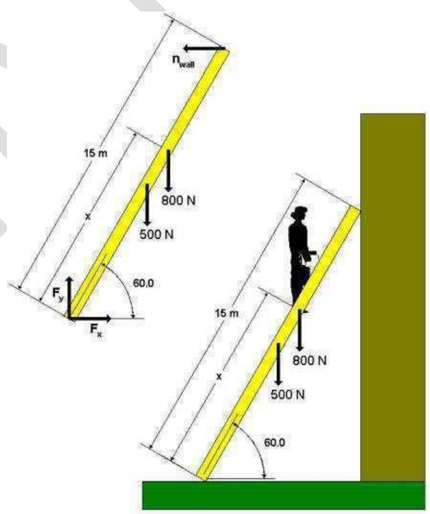
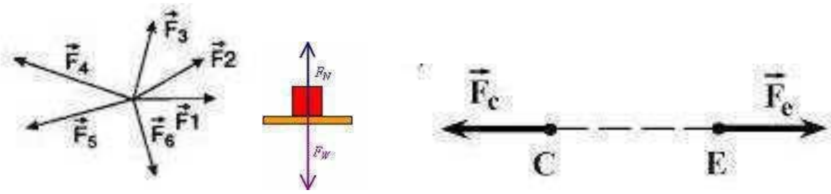


Figure 8: Forces in Ladder.

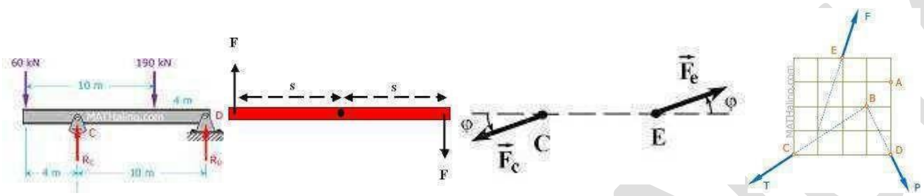
System of Forces:

When several forces act simultaneously on a body, they constitute a *system of forces.* If all the forces in a system do not lie in a single plane they constitute the *system of forces in space.* If all the forces in a system lie in a single plane, it is called a *coplanar force system.* If the line of action of all the forces in a system passes through a single point, it is called a *concurrent force system.* In a *system of parallel forces* all the

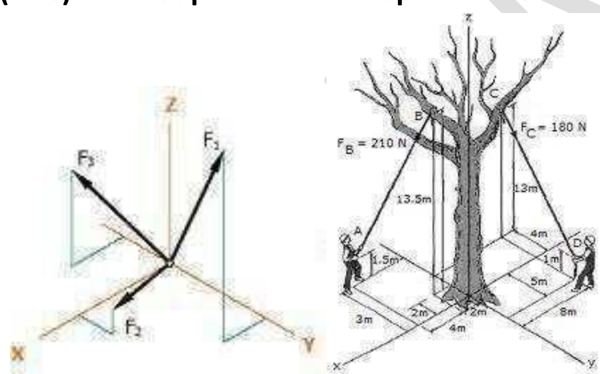
forces are parallel to each other. If the line of action of all the forces lies along a single line then it is called a *collinear force system.* Various system of forces, their characteristics and examples are given in Table 1.2 and shown in Fig. 9



Concurrent forces. **Collinear Forces**



Parallel forces (Like) **Couple** **Coplanar Non concurrent forces.**



Non coplanar concurrent forces **Non coplanar non concurrent forces**

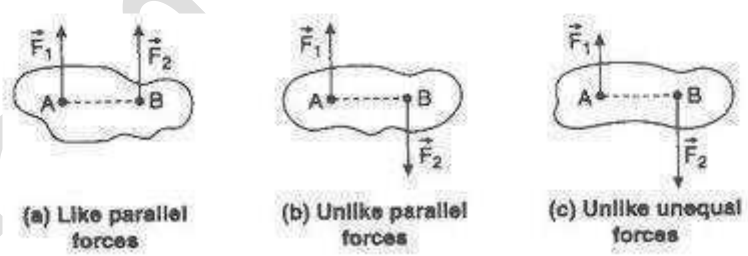


Figure 9: Types of Forces.

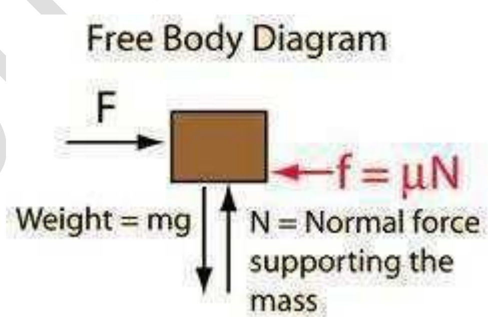
Table 1.2: System of Forces

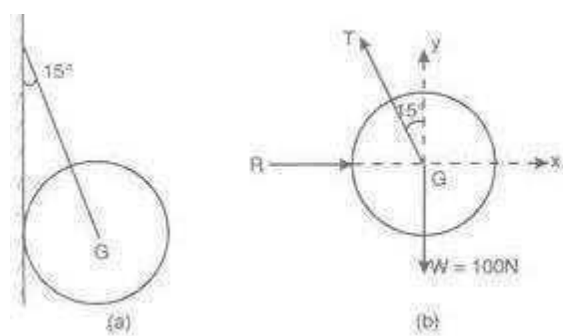
Force System	*Characteristics*	*Examples*
Coplanar parallel forces	**All forces are parallel to each other and lie in a single plane.**	**Forces on a rope in a tug of war.**
Coplanar like parallel forces	**All forces are parallel to each other, lie in a single plane and are**	**System of forces acting on a beam subjected to vertical**

	acting in the same direction.	loads (including reactions).
Coplanar concurrent forces	All forces lie in the same plane, but their lines of action pass through a single point.	Weight of a stationary train on a rail when the track is straight.
Collinear forces	Line of action of all the forces act along the same line.	Forces on a rod resting against a wall.
Coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane.	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity.
Non-coplanar parallel forces	All the forces are parallel to each other, but not in same plane.	The weight of benches in a classroom.
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point.	A tripod carrying a camera.
Non-coplanar non-concurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point.	Forces acting on a moving bus.

Free Body Diagram:

A free-body diagram is a sketch of an object of interest with all the surrounding objects stripped away and all of the forces acting on the body shown. A free body diagram, sometimes called a force diagram, is a pictorial device, often a rough working sketch, used by engineers and physicists to analyze the forces and moments acting on a body. The body itself may consist of multiple components, an automobile for example, or just a part of a component, a short section of a beam for example, anything in fact that may be considered to act as a single body, if only for a moment. A whole series of such diagrams may be necessary to analyze forces in a complex problem. The free body in a free body diagram is not free of constraints, it is just that the constraints have been replaced by arrows representing the forces and moments they generate.





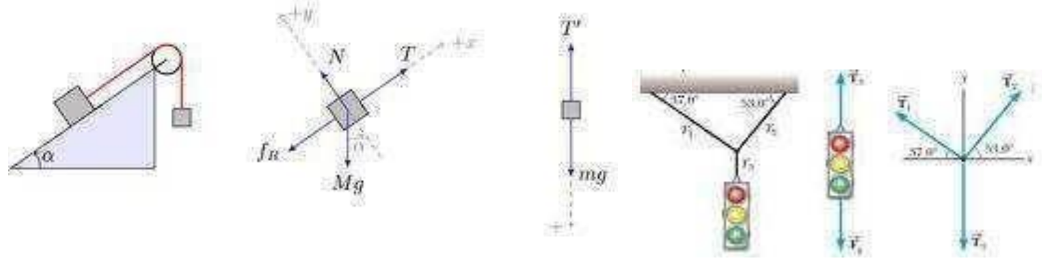


Figure 10: Examples for Free Body diagram.

Resultant of concurrent forces:

The resultant force can be obtained by resolving the force in to rectangular components along x and y axis.

The magnitude of resultant force is given by ‘ $=\sqrt{\{(\sum Fx)^2 + (\sum Fy)^2\}}$

The inclination of resultant force with x axis is given by the equation

$$\Theta x = \tan^{-1}[\sum Fy/\sum Fx]$$

Where ∑ Fx = Algebraic sum of x components of all forces.

∑Fy = Algebraic sum of y components of all forces. (Refer Figure 11)

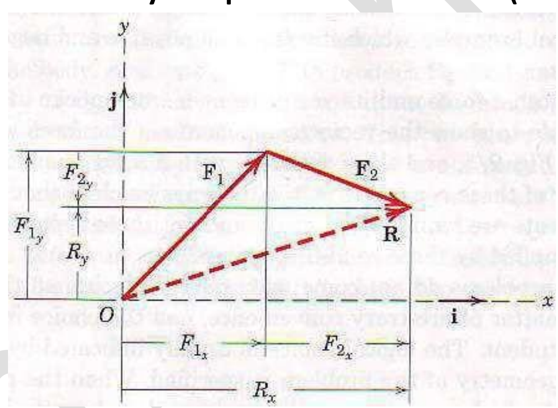


Figure: Resultant of concurrent force systems.

Equilibrant of concurrent forces:

Mathematically E = - R

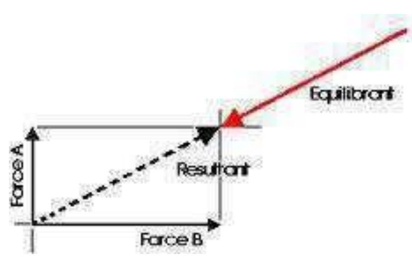


Figure: Equilibrant of concurrent force systems.

Moment of a Force

The Moment of a force is the turning effect about a pivot point. To develop a moment, the force must act upon the body to attempt to rotate it. A moment is can occur when forces are equal and opposite but not directly in line with each other.

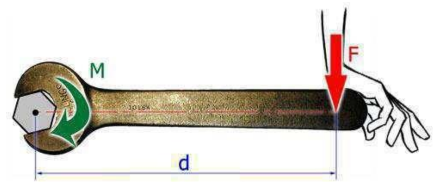


Figure: Moment of a force.

The Moment of force acting about a point or axis is found by multiplying the Force (F) by the perpendicular distance from the axis (d), called the *lever arm.*

Moment = Force x Perpendicular Distance

M = F x d, Unit is Newton meter (Nm).

Force Couples

Two equal forces of opposite direction, with a distance d between them will cause a moment, where; a special case of moments is a couple. A couple consists of two parallel forces that are equal in magnitude, opposite in sense and do not share a line of action. It does not produce any translation, only rotation. The resultant *force* of a couple is zero, but it produces a pure moment. A tap wrench is an example of a couple. The two hand forces are equal but opposite direction. Taking moments about the centre (both clockwise); Moment = F * d + F * d = 2Fd

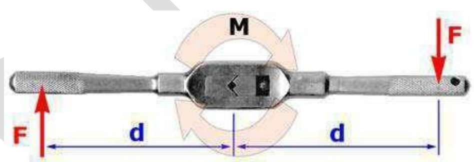


Figure: Couples.

The moment caused by a couple = the force * the distance between them.

Principle of Moments *(Varignon's Theorem)* The Principle of Moments, also known as Varignon's Theorem, states that the moment of any force is equal to the algebraic sum of the moments of the components of that force.

Proof of Varignon's theorem:

Let us consider any point 'O' lying in the plane of the forces, as a moment center.

Now, Moment of force P about O will be $Pd = P\ (OA \cos\theta) = OA\ (P \cos\theta) = OA \times P_x$ … (1)

Moment of force P1 about O.

$P_1 d_1 = P_1\ (OA \cos\theta_1) = OA\ (P_1 \cos\theta_1) = OA \times P_{x1}$ …(2)

Moment of force P2 about O.

$P_2 d_2 = P_2\ (OA \cos\theta_2) = OA\ (P_2 \cos\theta_2) = OA \times P_{x2}$ …(3)

Adding eq. (2) and (3)

P1d1 + P2d2 = OA (Px1 + Px2)

But Px = Px1 + Px2

The sum of x-components of the forces P1 and P2 = x-components of the resultant P.

∴ OA × Px = P1d1 + P2d2

From eq. (1) we see that OA × Px = Pd

∴ Pd = P1d1 + P2d2 Hence proved

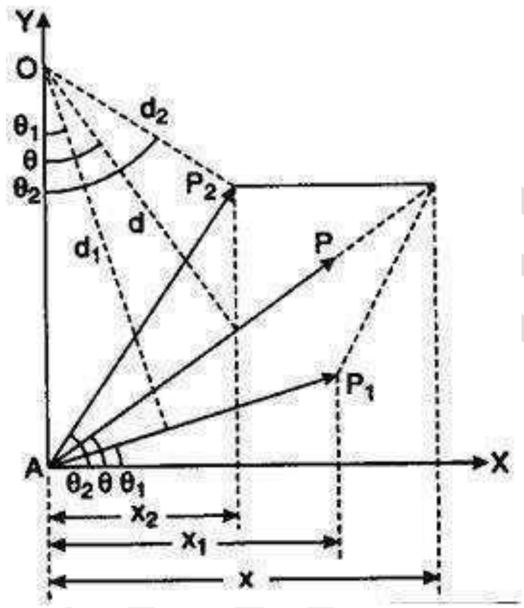


Figure: Varignon's theorem of moments.

Resultant of non-concurrent forces:

The resultant force can be obtained by resolving the force in to rectangular components along x and y axis.

The magnitude of resultant force is given by ' =√ {(∑ Fx)^2 + (∑ Fy)^2}

The inclination of resultant force with x axis is given by the equation

$$\Theta x = \tan^{-1}[\sum Fy/\sum Fx]$$

Where ∑ Fx = Algebraic sum of x components of all forces.

∑ Fy = Algebraic sum of y components of all forces.

The point of application of resultant force is obtained by applying the principle of moments.

Equilibrium of concurrent forces:

In the equilibrium of concurrent force system the resultant force is equal to zero.

Resultant force = R = 0;

∑ Fx = 0

∑ Fy = 0

Where ∑ Fx = Algebraic sum of x components of all forces.

∑ Fy = Algebraic sum of y components of all forces.

The above conditions are also called as static equilibrium conditions for concurrent force system.

Equilibrium of non-concurrent forces:

In the equilibrium of non-concurrent force system the resultant force and resultant moment are equal to zero.

Resultant force = R = 0;

$\sum F_x = 0$

$\sum F_y = 0$

Resultant moment = 0;

$\sum M_A = 0$

Where $\sum F_x$ **= Algebraic sum of x components of all forces.**

$\sum F_y$ **= Algebraic sum of y components of all forces.**

$\sum M_A$ **= Algebraic sum of moments of all forces with respect to any point A.**

The above conditions are also called as static equilibrium conditions for non-concurrent force system.

Problems on Resultant of Coplanar concurrent forces:

Problem 01: Determine the resultant force of the force system as shown in figure 1(P: 01).

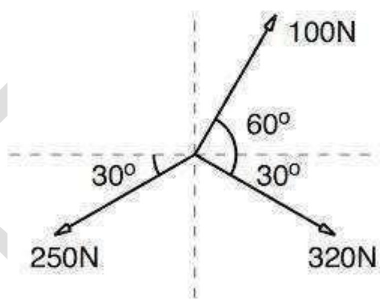


Figure: (P: 01)

Solution:

$\sum F_x$ = ˊx

= 100 Cos 60 – 250 Cos 30 + 320 Cos 30

= 50.00 – 216.51 + 277.13

= 110.62 N

$\sum F_y$ = ˊy

= 100 Sin 60 - 250 Sin 30 – 320 Sin 30

= 86.60 - 125.00 – 160.00

= - 198.40 N

$R = \sqrt{110.62^2 + 198.40^2}$ = 227.15 N

$\Theta_x = \tan^{-1} 198.40 / 110.62$ = 60.95°

Θ_x = 60.95°

R = 227.15 N

Problem 02: Determine the resultant force of the force system as shown in figure 2(P: 02).

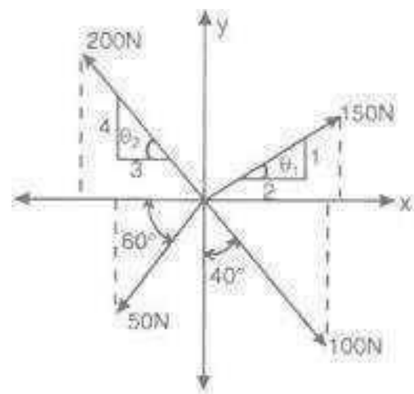


Figure (P: 02)

Solution:

$\Theta_1 = \tan^{-1} 1/2 = 26.57°$ $\quad\Theta_2 = \tan^{-1} 4/3 = 53.13°$

$\sum F_x =$ ʼx

= 150 Cos 26.57 – 200 Cos 53.13 – 50 Cos 60 + 100 Cos 50

= 134.16 – 120.00 – 25.00 + 64.28

= 53.44 N

$\sum F_y = 0$

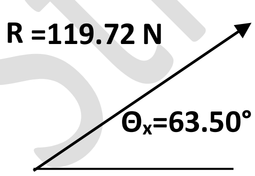
= 150 Sin 26.57 + 200 Sin 53.13 – 50 Sin 60 - 100 Sin 50

= 67.09 + 160.00 – 43.30 - 76.60

= 107.19 N

$R = \sqrt{53.44^2 + 107.19^2} = 119.77$ N

$\Theta_x = \tan^{-1} 107.19/53.44 = 63.50°$



Problem 03: Determine the magnitude and direction of missing force P of the force system as shown in figure 3 (P: 03).

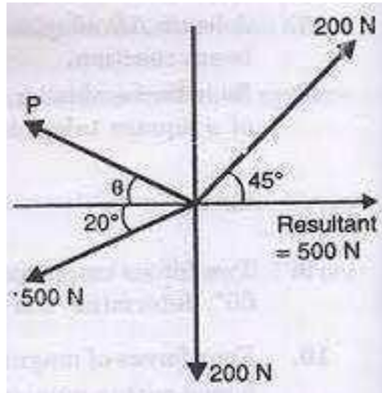


Figure (P: 03)

Solution:

∑ Fx = ʻx = 500N

= 200 Cos 45 – 500 Cos 20 – P Cos θ

P Cos θ = -328.424

∑ Fy = ʻy = 0

= 200 Sin 45 - P Sin θ – 500 Sin 20 = 0

P Sin θ = -29.588

P Sin θ/ P Cos θ = Tan θ = -29.588 /-328.424 = 0.09

Θ = 5.14°

P = - 329.754 N

Θ = 5.14°

P = 329.754 N

Problems on Resultant of Coplanar non concurrent forces:

Problem 04: Determine the resultant force of the force system as shown in figure (P: 04).

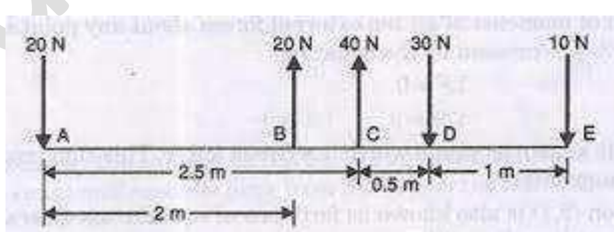


Figure (P: 04)

Solution:

∑ Fx = ʻx

= 0

∑ Fy = ʻy

= -20 + 20 + 40 – 30 - 10 = 0

R = 0 N

Taking moment about the point A, we get

20 * 0 – 20 * 2.0 – 40 * 2.5 + 30 * 3.0 + 10 * 4.0 = -10.0 N-m

The system does not have any resultant force i.e. = r = 0.0 but has a resultant moment of magnitude 10 N. m anticlockwise.

Problem 05: Determine the resultant force of the force system as shown in figure (P: 05).

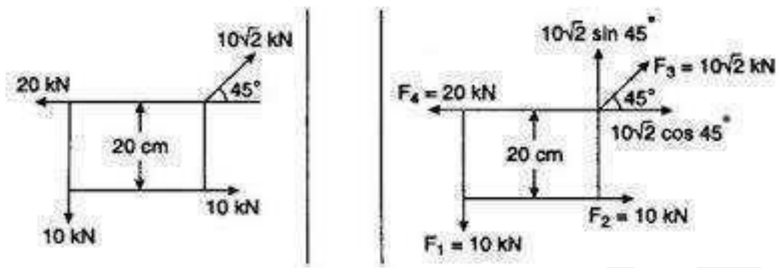


Figure (P: 05)

Solution:

∑ Fx = ˊx

= -20 + 10√2 Cos 45 + 10 = 0.0

∑ Fy = ˊy

= 10√2 Sin 45 – 10 = 0.0

R = 0.0 N

Taking moment about the point where 10√2 is acting, we get

- 10 * 0.2 + 10 * 0.3 = - 1.0 N-m

The system does not have any resultant force i.e. = R = 0.0 but has a resultant moment of magnitude 1.0 N. m anticlockwise.

Problem 06: Determine the resultant force of the force system as shown in figure (P: 06).

Solution:

$\Theta_1 = \tan^{-1} 1/1$ = 45.0° $\Theta_2 = \tan^{-1} 3/4$ = 36.86° $\Theta_3 = \tan^{-1} 1/2$ = 26.57°

∑ Fx = ˊx

= 2.0 Cos 45 + 5 cos 36.86 – 1.5 cos 26.56

= 1.41 + 4 – 1.34 = 4.07 kN

∑ Fy = ˊy

= 2.0 Sin 45 - 5 Sin 36.86 – 1.5 Sin 26.56

= 1.41 - 3.00 – 0.67 = - 2.26 kN

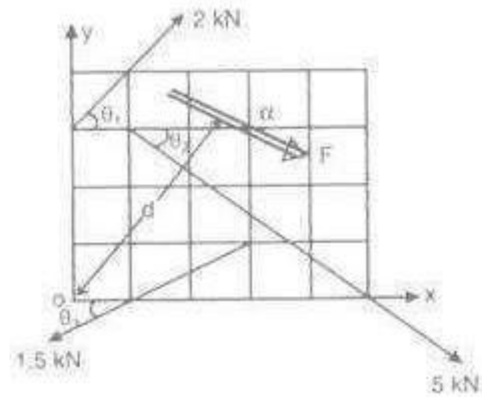


Figure (P: 06)

R = $\sqrt{4.07^2 + 2.26^2}$ = 4.66 kN

Θ_x = $\tan^{-1} 2.26 / 4.07$ = 29.04°

Θ_x=29.04°

R =4.66 KN

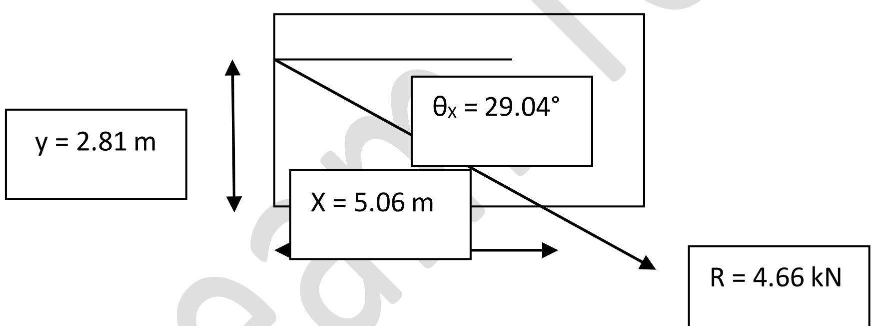
Assuming the grid size as 1.0 m x 1.0 m.

Taking moment about the point O, we get

-2Cos 45*3 + 5Cos 36.86*3 + 5Sin 36.86*1 – 1.5 Cos 26.56*1 + 1.5 Sin 26.56*3 = 11.42 kN.m.

Rx * y = 4.07*7; y = 2.81 m.

Ry * x = 2.26*7; x = 5.06 m.



Problem 07: Determine the magnitude of force F and location of force F from a for the force system as shown in figure 7(P: 07).

Solution:

∑ Fx = ʼx = 0

= 0.0

∑ Fy = ʼy = 600 N

= 100 + F + 300 = 400 + F

F = 200 N

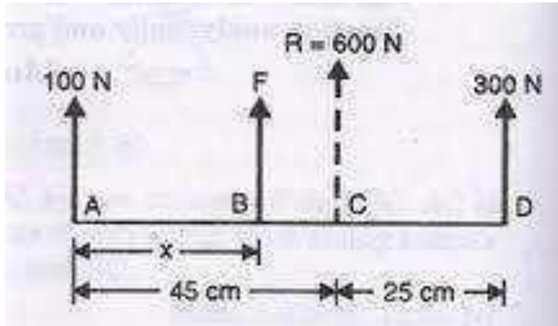


Figure (P: 07)

Applying varignon's theorem with respect to point A, we get

F*x + 300 * 70 = R * 45 = 600 * 45 N. cm.

X = 30.0 cm.

Problem 07: Determine the resultant force of the force system as shown in figure 7(P: 07). Diameter of circle is 2.0 m.

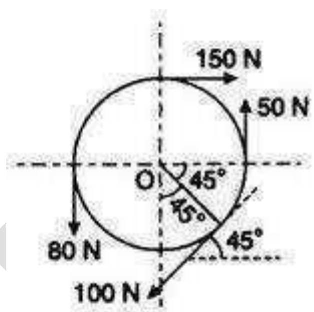


Figure (P: 07)

Solution:

$\sum F_x$ = ˂x

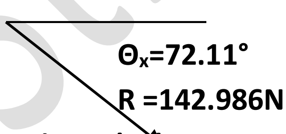
= 150 – 100 Cos 45 = 150 – 106.07 = 43.93 N

$\sum F_y$ = ˂y

= 50 – 80 – 100 Sin 45 = 50 – 80 – 106.07 = - 136.07 N

R = $\sqrt{43.93^2 + 136.07^2}$ = 142.986 N

Θ_x = $\tan^{-1} 136.07/43.93$ = 72.11°



Applying Varignon's theorem with respect to point O, we get

150 * 1 – 50 * 1 + 100 * 1 – 80 * 1 = 120.0 N m.

Rx * y = 43.93*y = 120.0; y = 2.73 m.

Ry * x = 136.07*x 120.0; x = 0.88 m.

Problems on Equilibrium of Coplanar concurrent forces:

Problem 08: Determine the directions of forces 20 kN and 30 kN with respect to the horizontal of the force system in equilibrium as shown in figure 08 (P: 08).

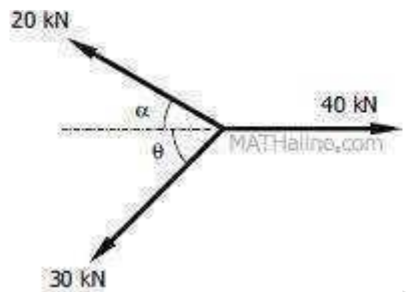


Figure (P: 08)

Solution:

$\sum Fx = 0$

$= 40 - 20 \cos\alpha - 30 \cos\theta = 0$

$\cos\alpha = 1.5 \cos\theta - 2$

$\sum FY = 0$

$= 20 \sin\alpha - 30 \sin\theta = 0$

$\sin\alpha = 1.5 \sin\theta$

$\cos^2\alpha + \sin^2\alpha = 1.5^2 \cos^2\theta + 4 + 2*2*1.5* \cos\theta + 1.5^2 \sin^2\theta$

Solving $\theta = (28.96)°$

$\alpha = (46.57)°$

Problem 09: Determine the magnitudes of P and F forces of the force system in equilibrium as shown in figure 09 (P: 09).

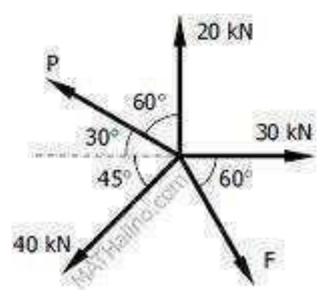


Figure (P: 09).

Solution:

$\sum Fx = 0$

$= 30 - P \cos 30 - 40 \cos 45 + F \cos 60 = 0$

$\sum Fy = 0$

$= 20 + P \sin 30 - 40 \sin 45 - F \sin 60 = 0$

Solving F = 1.31 kN.
P = 21.33 kN.

Problems on strings:

Problem 10: Determine the magnitudes of θ with vertical for the string CD and tension in all segments of the string as shown in figure 10 (P: 10).

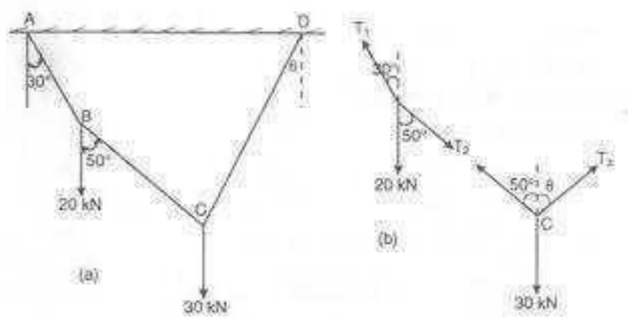


Figure (P: 10).

Solution:

From the free body of joint B,

T_{AB}/Sin 50 = 20/Sin 160 = T_{BC}/Sin 150

T_{AB} = 44.79 N.

T_{BC} = 29.24 N.

From the free body of joint C,

29.24/Sin (180-θ) = 30/Sin (50+θ) = T_{CD}/Sin 130

29.24 Sin (50+θ) = 30 Sin (180-θ)

22.4 cos θ + 18.79 Sin θ = 30 cos θ

18.79 Sin θ = 7.6 Cos θ

Θ = 22.02°

T_{CD} = 59.74 N.

Problem 11: Determine the magnitudes of tension in all segments of the string as shown in figure (P: 11) when the weight of the block is 300 N.

Solution:

From the free body of joint,

T_1/Sin 135 = (T_1 = 300)/Sin 135 = T_2/Sin 90

T_2 = 424.26 N.

T_3 = 300.00 N.

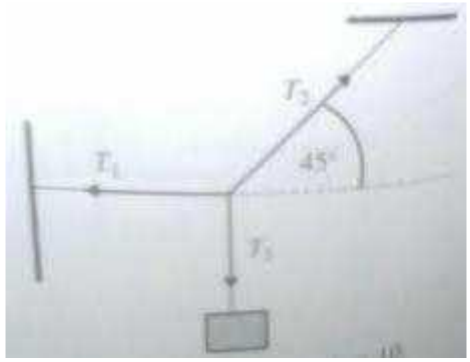


Figure (P: 11).

Problem 12: Determine the magnitudes of tension in all segments of the string and the weight W for the string system as shown in figure 12 (P: 12).

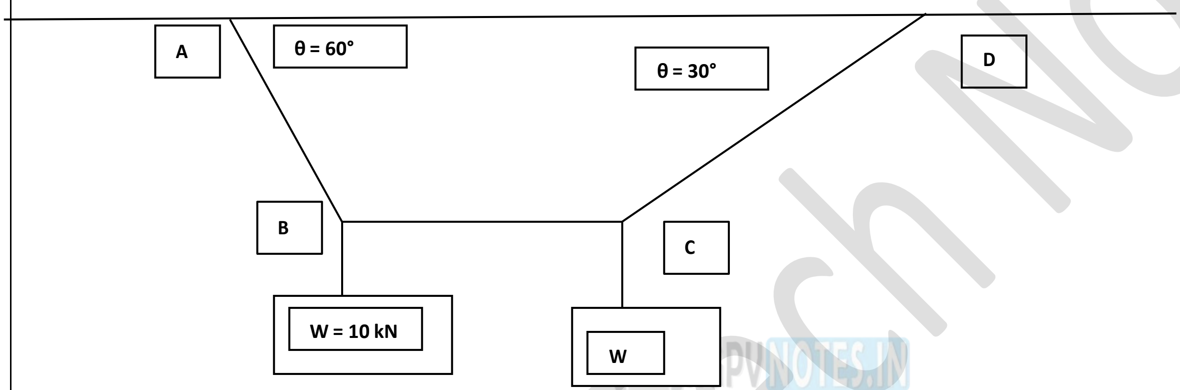


Figure 12 (P: 12).

Solution:

From the free body of joint B,

$T_{AB}/\text{Sin } 90 = 10/\text{Sin } 150 = T_{BC}/\text{Sin } 150$

$T_{CD} = 11.54$ kN.

$T_{BC} = 5.77$ kN.

From the free body of joint C,

$T_{BC}/\text{Sin } 120 = W/\text{Sin } 150 = T_{CD}/\text{Sin } 90$

$W = 3.33$ kN.

$T_{CD} = 6.67$ kN.

Problems on Rollers/Cylinders:

Note 01: When a circular surface come in contact with a plane surface, the reaction force at this contact point is perpendicular the plane surface as shown in figure. In Figure 17 (a) the plane surface is horizontal and the reaction R_A is perpendicular to plane (vertical) and in figure (b) the reaction R_A is also perpendicular the plane surface.

Note 02: When two circular surfaces come in contact with each other, the reaction force at this contact point is always along the line connecting their centre. In Figure 17 (d) the reaction R_{AB}at the point B is along O_1O_2 which is the line connecting the centres of each cylinders.

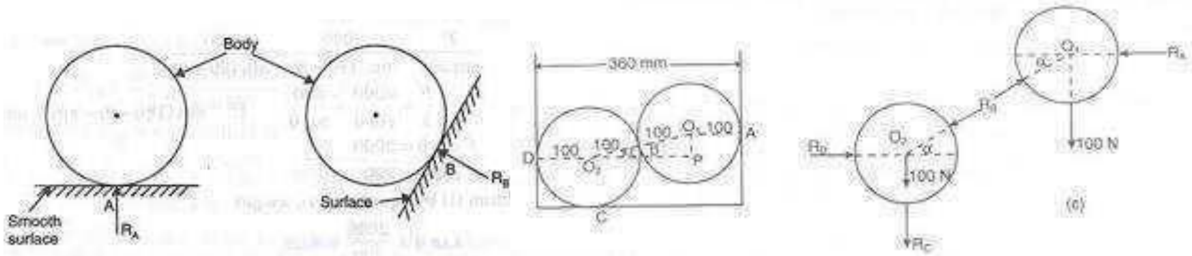


Figure 17 (a) (b) (c) (d)

Problem 13: Determine the reactions at the contact surfaces A and B for the cylinder of weight 1000 N resting as shown in figure 13 (P: 13). In the problem α = 70° and β = 40°.

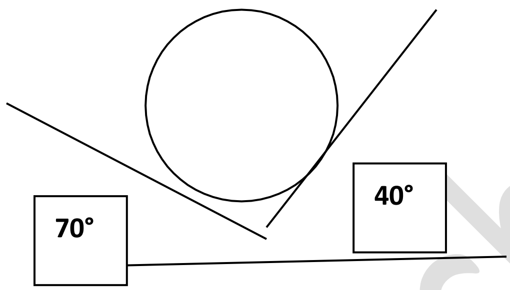
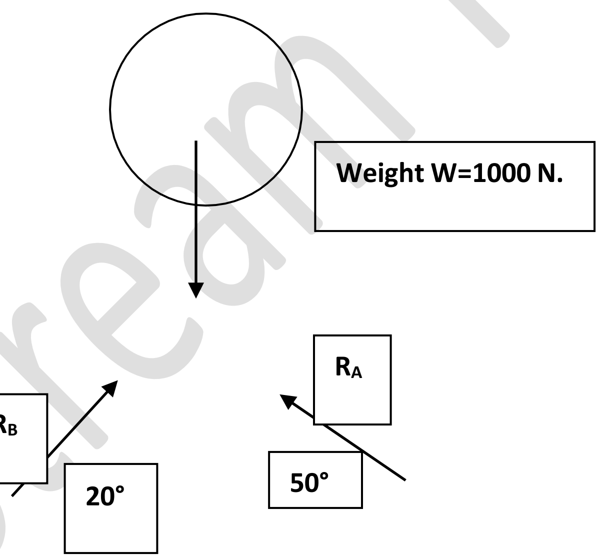


Figure 13 (P: 13).

The FBD of the cylinder are as given below



R_A/Sin (110) = R_B/Sin (140) = 1000/Sin 110

R_A = 1000 N.

R_B = 684.04 N

Problem 14: Determine the reactions at the contact surfaces A, B, C and D for the cylinder of weight 2.0 kN and 5.0 kN resting as shown in figure 14 (P: 14).

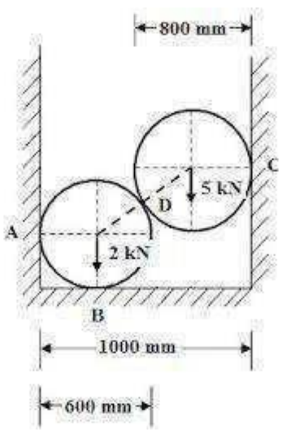


Figure 14 (P: 14).

Solution:

The inclination of line connecting centre (θ) can be calculated as given below.

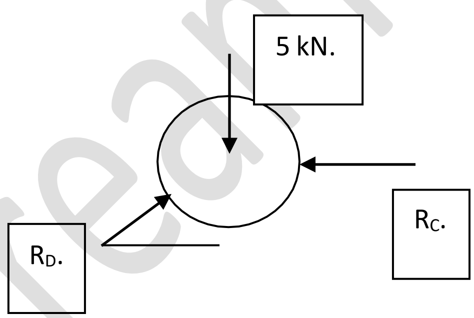
Length of line O1O2 is given by Radius of 1 + Radius of 2 = 400 + 400 = 800 mm.

Length of horizontal line = 100 – 400 – 400 = 200.00 mm.

Cos θ = 200/800

Θ = 75.52°

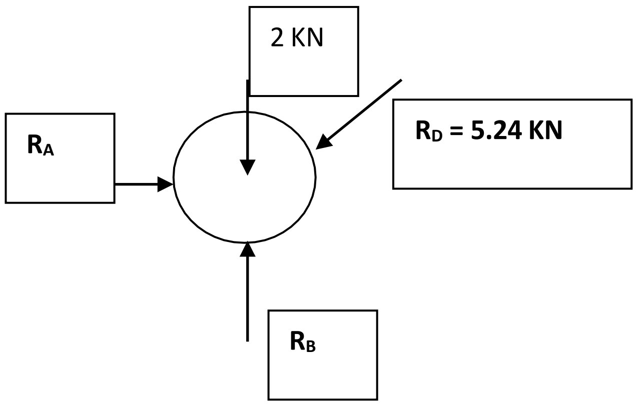
From the free body of top cylinder,



R_D/Sin 90 = 5/Sin 107.48 = R_C/Sin 162.52

R_C = 1.57 kN.

R_D = 5.24 kN.



R_D - 2 – 5.24 Sin 72.52 = 0

R_D = 7.0 KN.

R_A - 5.24 Cos 72.52 = 0

R_A = 1.57 KN.

Bow's notation:

Adding and subtracting two vectors (Graphical Method): When we add two vectors A and B by graphical method to get A + B, we take vector A, put the tail of B on the head of A. Then we draw a vector from the tail of A to the head of B. That vector represents the resultant R(Figure 4).

Let us try to understand that it is indeed meaningful to add two vectors like this. Imagine the following situations. Suppose when we hit a ball, we can give it velocity. Now imagine a ball is moving with velocity and you hit it an additional velocity. From experience you know that the ball will now start moving in a direction different from that of. This final direction is the direction of and the magnitude of velocity now is going to be given by the length of.

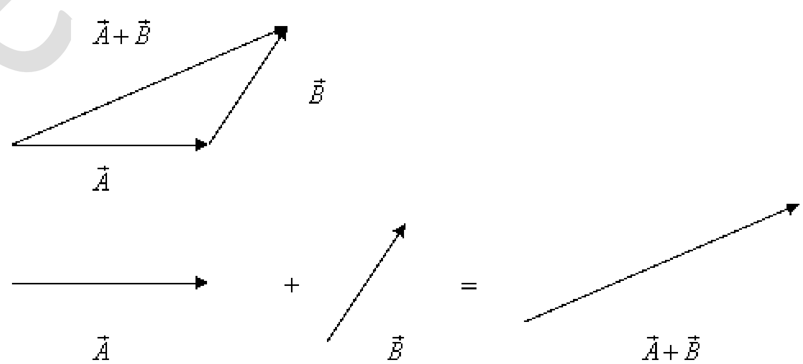


Figure: Adding two vectors.

Trusses

Having set up the basics for studying equilibrium of bodies, we are now ready to discuss the trusses that are used in making stable load-bearing structures. The examples of these are the sides of the bridges or tall TV towers or towers that carry electricity wires. Schematic diagram of a structure on the side of a bridge is drawn in figure 18.

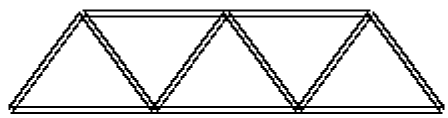


Figure 18: Truss

The structure shown in figure 18 is essentially a two-dimensional structure. This is known as a plane truss. On the other hand, a microwave or mobile phone tower is a three-dimensional structure. Thus there are two categories of trusses - Plane trusses like on the sides of a bridge and space trusses like the TV towers. In this course, we will be concentrating on plane trusses in which the basic elements are stuck together in a plane.

Now we are ready to build a truss and analyze it. We are going to build it by adding more and more of triangles together. As you can see, when we add these triangles, the member of joints j and the number of members (rods) m are related as follows:

$$m = 2j - 3$$

This makes a truss statically determinate. This is easily understood as follows. First consider the entire truss as one system. If it is to be statically determinate, there should be only three unknown forces on it because for forces in a plane there are three equilibrium conditions. Fixing one of its ends a pin joint and putting the other one on a roller does that (roller also gives the additional advantage that it can help in adjusting any change in the length of a member due to deformations). If we wish to determine these external forces and the force in each member of the truss, the total number of unknowns becomes $m + 3$. We solve for these unknowns by writing equilibrium conditions for each pin; there will be $2j$ such equations. For the system to be determinate we should have $m + 3 = 2j$, which is the condition given above. If we add any more members, these are redundant. On the other hand, less number of members will make the truss unstable and it will collapse when loaded. This will happen because the truss will not be able to provide the required number of forces for all equilibrium conditions to be satisfied. Statically determinate trusses are known as simple trusses.

We now wish to obtain the forces generated in various arms of a truss when it is loaded externally. This is done under the following assumptions:

1. If the middle line of the members of a truss meet at a point that point is taken as a pin joint. This is a very god assumption because as we have seen earlier while introducing a truss (triangle with pin joint), the load is transferred on to other member of the trusses so that forces remain essentially collinear with the member.
2. All external loads are applied on pin connections.
3. All members' weight is equally divided on connecting pins.

There are two methods of determining forces in the members of a truss - Method of joints and method of sections.

Method of joints: In method of joints, we look at the equilibrium of the pin at the joints. Since the forces are concurrent at the pin, there is no moment equation and only two equations for equilibrium viz. ∑ Fx = 0 and ∑ Fy = 0. Therefore we start our analysis at a point where one known load and at most two unknown forces are there. The weight of each member is divided into two halves and that is supported by each pin. The method of joints consists of satisfying the equilibrium equations for forces acting on each joint.

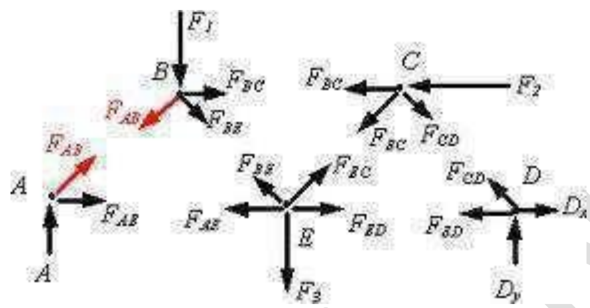


Figure 19 (a): FBD of joints in method of joints.

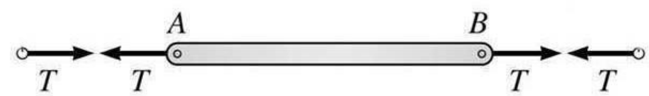


Figure 19 (b): Tension member of truss.

Figure 19 (c): Compression member of truss.

Procedure for analysis - the following is a procedure for analyzing a truss using the method of joints:

1. **If possible, determine the support reactions**
2. **Draw the free body diagram for each joint. In general, assume all the force member reactions are tension (this is not a rule, however, it is helpful in keeping track of tension and compression members).**
3. **Write the equations of equilibrium for each joint,**
   ∑ Fx = 0 and ∑ Fy = 0
4. **If possible, begin solving the equilibrium equations at a joint where only two unknown reactions exist. Work your way from joint to joint, selecting the new joint using the criterion of two unknown reactions.**
5. **Solve the joint equations of equilibrium simultaneously, typically using a computer or an advanced calculator.**

Method of sections: As the name suggests in method of sections we make sections through a truss and then calculate the force in the members of the truss though which the cut is made. For example, if I take the

problem we just solved in the method of joints and make a section S_1, S_2 (see figure 9), we will be able to determine the forces in members BC, BE and FE by considering the equilibrium of the portion to the left or the right of the section.

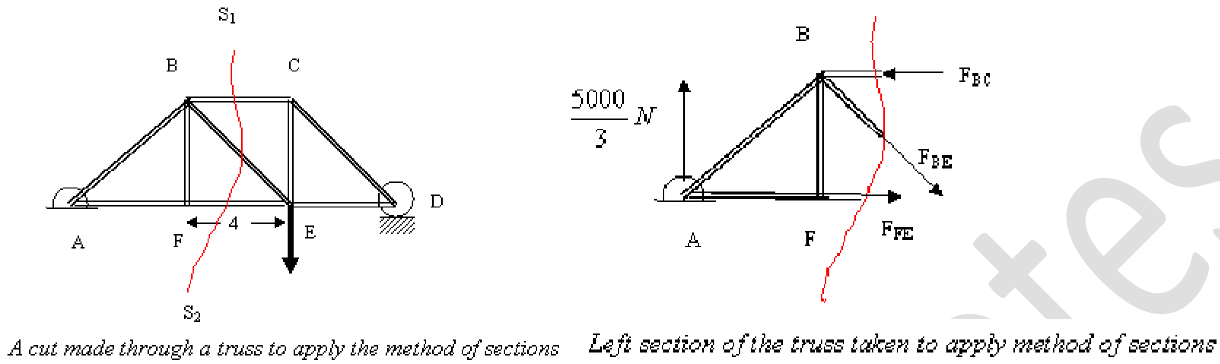


Figure 20: method of sections.

Since this entire section is in equilibrium, $\sum Fx = 0$, $\sum Fy = 0$and $\sum M = 0$. Notice that we are now using all three equations for equilibrium since the forces in individual members are not concurrent. The direction of force in each member, one can pretty much guess by inspection. Thus the force in the section of members BE must be pointing down because there is no other member that can give a downward force to counterbalance 5000/3 *N* reaction at A. This clearly tells us that *F BE* is tensile. Similarly, to counter the torque about B generated by 5000/3 *N* force at A, the force on FE should also be from F to E. Thus this force is also tensile. If we next consider the balance of torque about A, 5000/3 N and F_{FE} do not give any torque about A. So to counter torque generated by F_{BE} , the force on BC must act towards B, thereby making the force compressive.

After this illustration let me put down the steps that are taken to solve for forces in members of a truss by method of sections:

1. **Make a cut to divide the truss into section, passing the cut through members where the force is needed.**
2. **Make the cut through three member of a truss because with three equilibrium equations viz$\sum Fx = 0$, $\sum Fy = 0$and $\sum M = 0$. we can solve for a maximum of three forces.**
3. **Apply equilibrium conditions and solve for the desired forces.**

In applying method of sections, ingenuity lies in making a proper. The method after a way of directly calculating desired force circumventing the hard work involved in applying the method of joints where one must solve for each joint.

Unit V:

Centre of Gravity and moment of Inertia: Centroid and Centre of Gravity, Moment Inertia of Area and Mass, Radius of Gyration, Introduction to product of Inertia and Principle Axes. Support Reactions, Shear force and bending moment Diagram for Cantilever & simply supported beam with concentrated, distributed load and Couple. Centre of Gravity

A single point where the entire weight or mass of a body is concentrated is known as centre of gravity of that body. Centre of gravity is generally denoted by 'G'. In general when a rigid body lies in a field of force acts on each particle of the body. We equivalently represent the system of forces by single force acting at a specific point. This point is known as centre of gravity.

Centroid

Centroid is another term to centre of gravity. It is the Centroid of plane geometrical figures like rectangle, triangle, trapezoid, circle etc. the word Centroid is used when there are only geometrical figures instead of weight or mass. Therefore, centre of gravity of plane geometrical figure is termed as Centroid or centre of area. Two distances are required for each area in evaluation of centre of gravity. One is from reference x axis and the other is from reference Y axis. These distances are denoted as x' and y'. Centroid can be determined by the method of integration.

CENROIDS OF AREAS BY FIRST MOMENT OF AREA

Consider the following lamina. Let's assume that it has been exposed to gravitational field. Obviously every single element will experience a gravitational force towards the centre of earth. Further let's assume the body has practical dimensions, and then we can easily conclude that all elementary forces will be unidirectional and parallel.

Consider G to be the Centroid of the irregular lamina. As shown in first figure we can easily represent the net force passing through the single point G. We can also divide the entire region into let's say n small elements. Let's say the coordinates to be (x_1,y_1), (x_2,y_2), (x_3,y_3) (x_n , y_n) as shown in figure . Let W_1, W_2, W_3,… , W_n be the elementary forces acting on the elementary elements. Clearly,

$$W = W_1 + W_2 + W_3 + \dots + W_n$$

Let's consider plate of uniform thickness and a homogenous density. Now weight of small element is directly proportional to its thickness, area and density as:

$$W = \Upsilon\, t\, dA.$$

Where ϒ is the density per unit volume, t is the thickness; dA is the area of the small element. So we can replace W with this relationship in the expression we obtained in the prior topic. Therefore we get:

Centroid of area:

$$x_c = W_1x_1 + W_2x_2 + W_3x_3 + \;\; W_nx_n / W ,$$

$$y_c = W_1y_1 + W_2y_2 + W_3y_3 + W_ny_n / W$$ **Substituting for weights we get,**

$$x_c = A_1x_1 + A_2x_2 + A_3x_3 + \;\; A_nx_n / A ,$$

$$Yc = A_1y_1 + A_2y_2 + A_3y_3 + \;\; A_ny_n / A$$

Where x,y are the coordinate of the small element and dA the elemental area. Also A (total area of the plate).(x_c ,$y_{c,}$) is called the Centroid of area of the lamina. If the surface is homogenous we conclude that it is the same as centre of gravity.

Thus it follows from the above discussion that Centroid of a area can be determined by dividing first moment of the area with the area itself. If the first moment of area with respect to an axis is zero, it indicates that the point lies on that axis itself. So we can conclude that the first moment about the axis will be zero about the axis of symmetry. Further Centroid also lies on the axis of symmetry. If a body has more than one axis of symmetry then Centroid will lie on the point of intersection of the axes. For some type of surfaces of bodies there lies a probability that the centre of gravity may lie outside the body. Secondly centre of gravity represents the entire lamina; therefore we can replace the entire body by the single point with a force acting on it when needed. From the above discussion we can draw the following differences between centre of gravity and Centroid;

1. The term centre of gravity applies to bodies with weight, and Centroid applies to lines, plane areas and volumes.
2. Centre of gravity of a body is a point through which the resultant gravitational force (weight) acts for any orientation of the body whereas Centroid is a point in a line plane area volume such that the moment of area about any axis through that point is zero.

OBTAINING CENTROIDS BY INTEGRATION

The general expression of Centroid of a body is given by:

$$x_c= \int x\, dA/A, \quad y_c=\int y\, dA/A$$

Where dA is the area of element.

We divide the area into thin rectangular strips or sectors. For rectangle it is pre-known that its centre of gravity lies at the centre of the rectangle.

CENTROID OF SOME STANDARD GEOMETRIC FIGURES:

Following results are obtained by integration which will be explained later. Results for symmetrical objects like square, circle, cylinder, rectangle, ring etc. are omitted. For such cases Centroid can be pre-assumed to be the geometric centre of the body.

1. Centroid of Rectangle of breadth b and depth d:

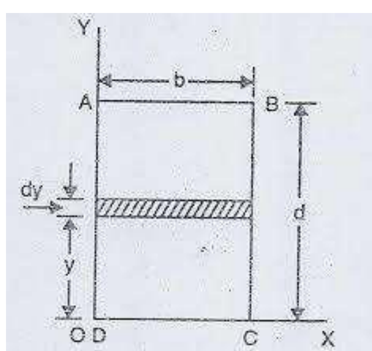


Figure1: Centroid of Rectangle

Area of the element dA = b*dy

From the method of integration we know that Ay' = ∫y dA

(b*d) y' = $\int_0^y y * dA$ **=** $\int_0^d y * b * dy$ **= (b/2) [y^2] = (b/2) [d^2 – 0]**

Y' = d/2

Similarly x' = b/2

3. Centroid of Triangle of breadth b and height h:

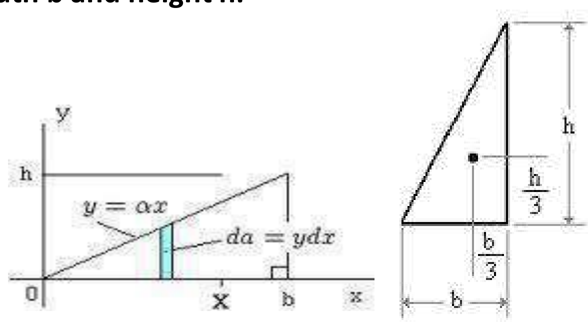


Figure2: Centroid of Triangle

Area of the element dA = y * dx

From similar triangles y/h = x/b

Therefore y = h/b * x When x = 0, y = 0 and when x = b, y = h. It satisfies all values of x.

Substituting above in the area equation we get dA = y * dx = (h/b * x) dx.

From the method of integration we know that Ax' = ∫x dA = $\int_0^x xdA$ **=** $\int_0^b x * (h/b * x)dx$

(b*h/2) x' = (h/b) (x^3/3) with limits 0 to b.

Therefore x' = 2*b/3

Similarly y' = h/3.

3. Centroid of quarter circle of radius R:

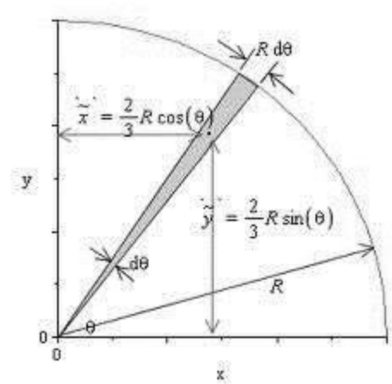


Figure3: Centroid of Quarter Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{\pi/2} (\frac{2}{3}) R\cos\theta\, R * R\, d\theta/2$

$= 2R^3/3 \int_0^{\pi/2} \cos\theta\, d\theta$

$= 2R^3/3\ [\sin\theta]$ with the limits 0 to π/2.

Therefore x' = (4*R)/ (3*π)

From the method of integration we know that Ay' = ∫x dA = $\int_0^{\pi/2} (\frac{2}{3}) R\, Sin\,\theta\, R * R\, d\theta/2$

$= 2R^3/3 \int_0^{\pi/2} Sin\,\theta\, d\theta$

$= 2R^3/3\ [Sin\,\theta]$ with the limits 0 to π/2.

Therefore y' = (4*R)/ (3*π)

4. Centroid of semi-circle of radius R:

(Semi-circle with symmetry about vertical axis)

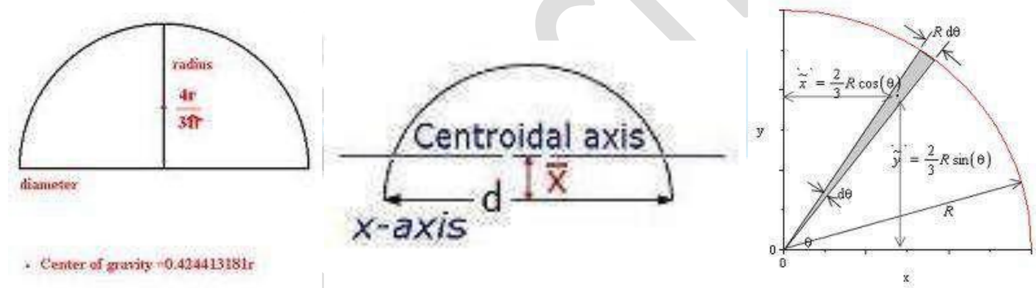


Figure4: Centroid of Semi Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{\pi} (\frac{2}{3}) R\cos\theta\, R * R\, d\theta/2$

$= 2R^3/3 \int_0^{\pi} \cos\theta\, d\theta$

$= 2R^3/3\ [\sin\theta]$ with the limits 0 to π.

Therefore x' = 0

From the method of integration we know that Ay' = ∫x dA = $\int_0^{\pi} (\frac{2}{3}) R\, Sin\,\theta\, R * R\, d\theta/2$

$= 2R^3/3 \int_0^{\pi} Sin\,\theta\, d\theta$

$= 2R^3/3\ [Sin\,\theta]$ with the limits 0 to π.

Therefore y' = (4*R)/ (3*π) = 0.424413181 R

5. Centroid of semi-circle of radius R:

(Semi-circle with symmetry about horizontal axis)

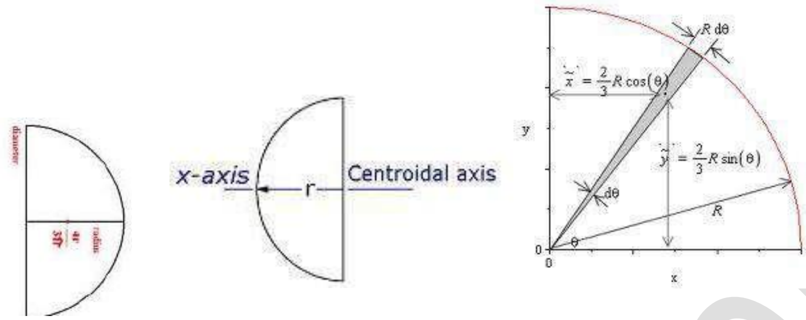


Figure5: Centroid of Semi circle with symmetry about horizontal axis

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_{-\pi/2}^{\pi/2} (\frac{2}{3}) R\cos\theta\ R * R\ d\theta/2$

= 2R³/3 $\int_{-\pi/2}^{\pi/2} \cos\theta\ d\theta$

= 2R³/3 [sin θ] with the limits - π/2 to +π/2.

Therefore x' = (4*R)/ (3*π) = 0.424413181 R

From the method of integration we know that Ay' = ∫x dA = $\int_{0}^{\pi} (\frac{2}{3}) R\ Sin\ \theta\ R * R\ d\theta/2$

= 2R³/3 $\int_{0}^{\pi} Sin\,\theta\ d\theta$

= 2R³/3 [Sin θ] with the limits - π/2 to +π/2.

Therefore y' = 0

Note: Centroid of a semi-circle is always at a distance of (4*R)/ (3*π) from the diameter.

6. Centroid of circle of radius R:

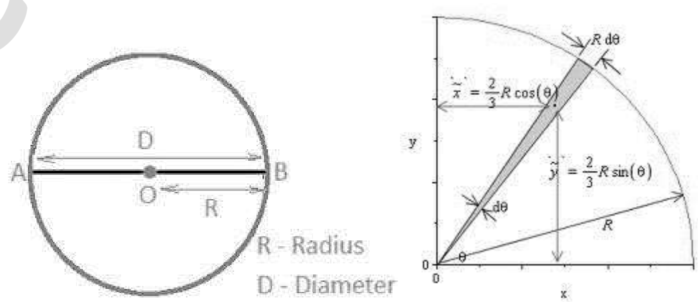


Figure6: Centroid of Circle

Area of the element dA = R*Rdθ/2

From the method of integration we know that Ax' = ∫x dA = $\int_0^{2\pi} (\frac{2}{3}) R\cos\theta\, R * R\, d\theta/2$

$$= 2R^3/3 \int_0^{2\pi} \cos\theta\, d\theta$$

$$= 2R^3/3\, [\sin\theta] \text{ with the limits 0 to } 2\pi.$$

Therefore x' = 0

From the method of integration we know that Ay' = ∫x dA = $\int_0^{2\pi} (\frac{2}{3}) R\, Sin\,\theta\, R * R\, d\theta/2$

$$= 2R^3/3 \int_0^{2\pi} Sin\,\theta\, d\theta$$

$$= 2R^3/3\, [Sin\,\theta] \text{ with the limits 0 to } 2\pi.$$

Therefore y' = 0

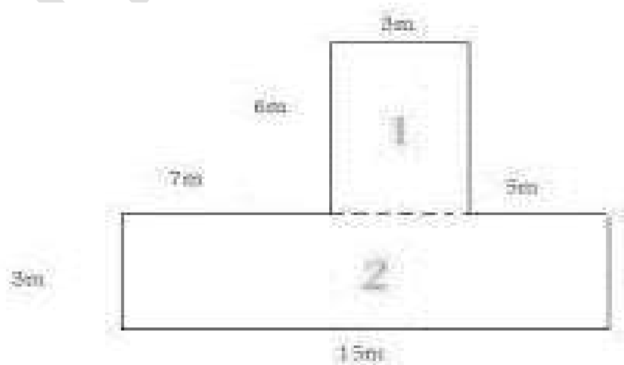
Centroid of composite figure:

Composite figure is the combination of regular figure. Divide the composite figure into number of parts such that centroid of each part is known. Determine the coordinates of centroid of each part from the given XX and YY axis. Let the coordinates of each part be (x_1, y_1), (x_2, y_2)…

Also calculate the area of each part as A_1, A_2, A_3…

Then $x' = \frac{A1*x1+A2*x2+A3*x3+\cdots}{A1+A2+A3+\cdots}$

$y' = \frac{A1*y1+A2*y2+A3*y3+\cdots}{A1+A2+A3+\cdots}$



N the above composite figure it is divided in to two parts 1

In the above composite figure it is divided in to two parts 1 and 2.

Moment of Inertia:

The second moment of area, also known as moment of inertia of plane area, area moment of inertia, or second area moment, is a geometrical property of an area which reflects how its points are distributed with

regard to an arbitrary axis. The second moment of area is typically denoted with either I for an axis that lies in the plane or with J for an axis perpendicular to the plane. Its unit of dimension is length to fourth power, L^4.

In the field of structural engineering, the second moment of area of the cross-section of a beam is an important property used in the calculation of the beam's deflection and the calculation of stress caused by a moment applied to the beam.

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

Where x and y are the distance to some reference plane.

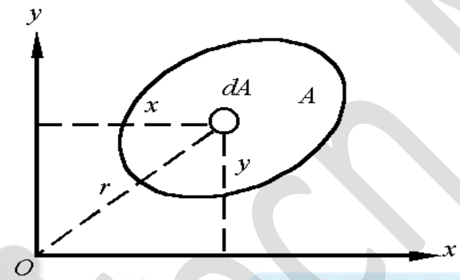


Figure7: Moment of Inertia

The polar second moment of area, $I = \int_A r^2 \, \mathrm{d}A$, where r is the distance to some reference axis. In each case the integral is over all the infinitesimal elements of area, dA, in some two-dimensional cross-section.

Perpendicular Axis Theorem

The moment of inertia (MI) of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.

That means the Moment of Inertia $I_{zz} = I_{xx}+I_{yy}$

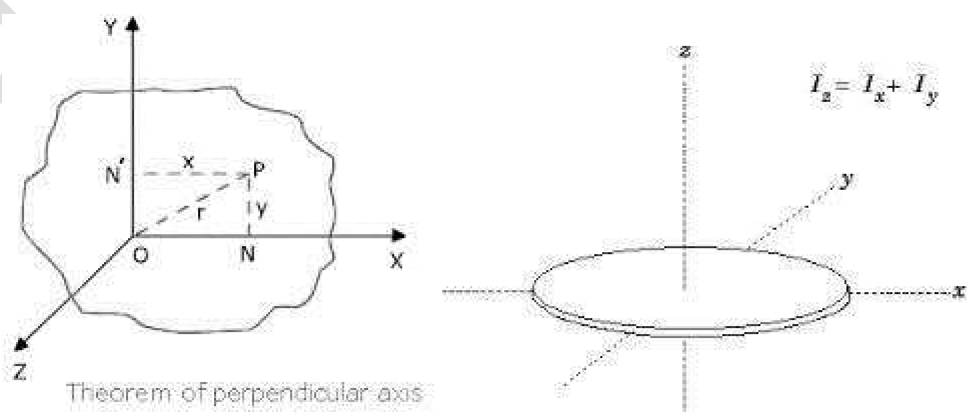


Figure8: Perpendicular Axis Theorem

Parallel Axis Theorem:

The moment of area of an object about any axis parallel to the centroid axis is the sum of MI about its centroid axis and the product of area with the square of distance of from the reference axis.

Essentially, $I_{XX} = I_{X'X'} + A\, y_c^2$

Where A is the cross-sectional area.

Yc is the perpendicular distance between the centroid axis and the parallel axis.

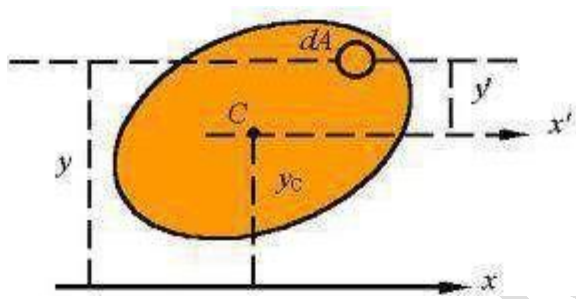


Figure9: Parallel Axis Theorem

$$
\begin{aligned}
I_x &= \int y^2 dA \\
&= \int (\bar{y} + y_c)^2 dA \\
&= \int \bar{y}^2 dA + 2\int y_c\,\bar{y}\,dA + \int y_c^2\,dA \\
&= \bar{y}^2 A + 2\bar{y}\underbrace{\int y_c\,dA}_{0} + I_{x_c} \\
&= \bar{y}^2 A + I_{x_c}
\end{aligned}
$$

Radius of gyration

In structural engineering, the two-dimensional radius of gyration is used to describe the distribution of cross sectional area in a column around its centroidal axis. The radius of gyration is given by the following formula

$K = \sqrt[2]{I/A}$ $K_{xx} = \sqrt[2]{Ixx/A}$ $K_{yy} = \sqrt[2]{Iyy/}$ $K_{zz} = \sqrt[2]{Izz/A}$

Where I is the second moment of area and A is the total cross-sectional area.

Moment of Inertia of regular geometrical figures.

1. Moment of inertia of semi-circle of radius R:

(Semi-circle with symmetry about horizontal axis)

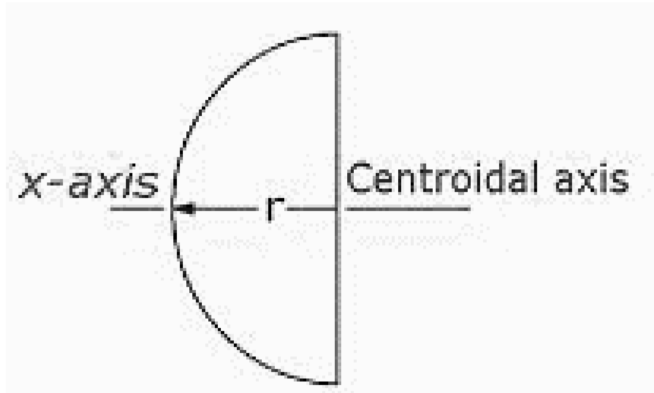


Figure9: Moment of inertia of semi-circle of radius R

$I_{xx} = I_{yy} = \pi R^4/8$, $I_{y'y'} = 0.11 R^4$.

2. Moment of inertia of quarter circle of radius R:

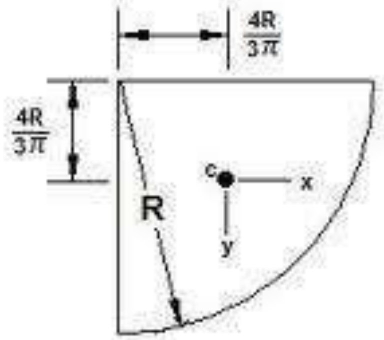
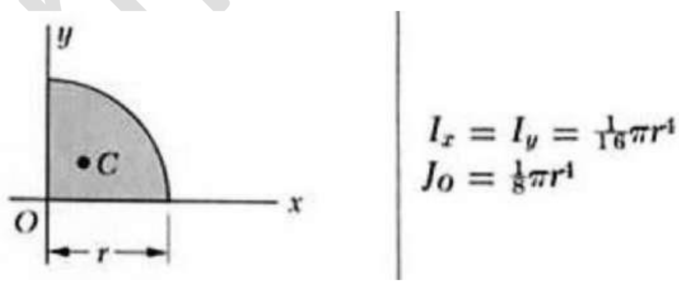


Figure10: Moment of inertia of quarter circle of radius R

$I_{xx} = I_{yy} = \pi R^4/16$, $I_{y'y'} = 0.05488 R^4. = 0.055 R^4$.



4. Moment of inertia of circle of radius R:

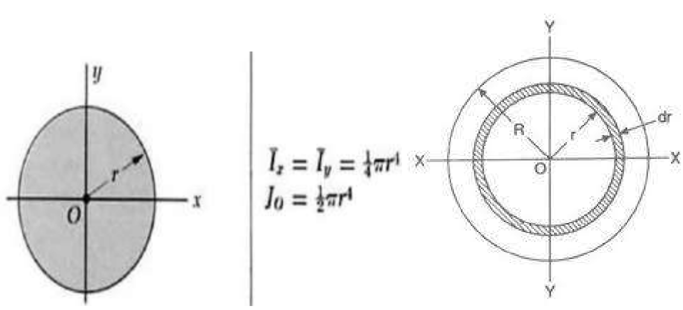


Figure11: Moment of inertia of circle of radius R

Area of Element dA = 2π r dr.

$I_{zz} = \int z^2 dA = \int z^2(2\pi\, r\, dr) = \pi R^4/2$

$I_{zz} = I_{xx} + I_{yy}$ (by perpendicular axis theorem)

We know that for a circle $I_{xx} = I_{yy}$

Therefore $I_{xx} = I_{yy} = I_{zz}/2 = \pi R^4/4$

5. Moment of inertia of rectangle of breadth b and depth d:

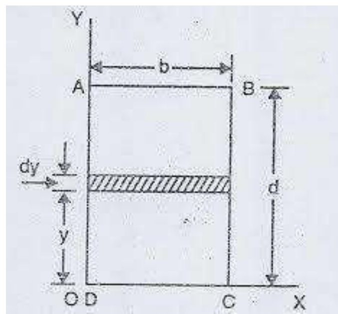


Figure12: Moment of inertia of rectangle

Area of Element dA = b *dy.

$I_{zz} = \int y^2 dA = \int_0^d bx^2\, dy \quad = bd^3/12$

Similarly $I_{yy} = \int x^2 dA = b^3 d/12$

6. Moment of inertia of triangle of breadth b and height h:

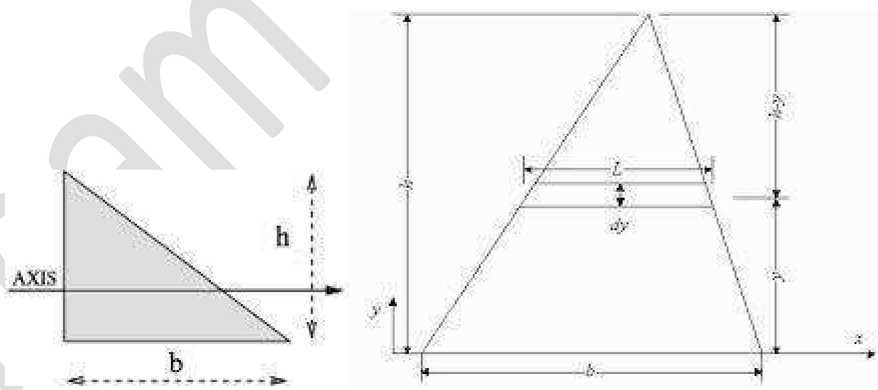


Figure13: Moment of inertia of triangle

From similar triangles; L/b = (h-y)/h

L = b (h-y)/h

Area of Element dA = L *dy = b (h-y)/h dy.

$I_{xx} = \int y^2 dA = \int_0^h y^2 b\,(h-y)/h\, dy \qquad = bh^3/12$

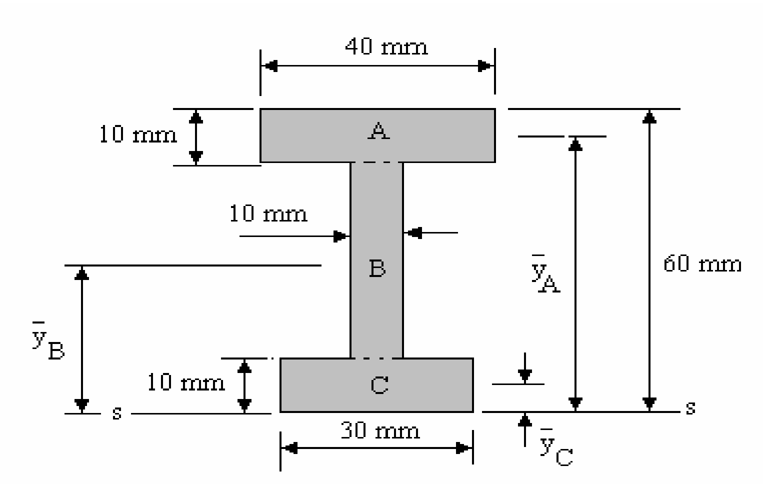
$I_{xc} = I_{xx} + A y_c^2 = I_{xx} + (bh/2)(h/3)^2 = bh^3/36$

Similarly $I_{yy} = \int x^2 dA = \int_0^b x^2 h\,(b-x)/b\,dy$ $= b^3h/12$

$I_{yc} = I_{yy} + Ay_c^2 = I_{xx} + (bh/2)(b/3)^2 = b^3h/36$

MOMENT OF INERTIA:

Calculate the 1st. moment of area for the shape shown about the axis s-s and find the position of the Centroid.



SOLUTION:

The shape is not symmetrical so the centroid is not half way between the top and bottom edges. First determine the distance from the axis s-s to the centre of each part A, B and C. A systematic tabular method is recommended.

Part	Area	Centroid (y)	A*y
A	400	55	22000
B	400	30	12000
C	300	5	1500
Total	1100		35500

The total first moment of area is 35,500 mm^3.

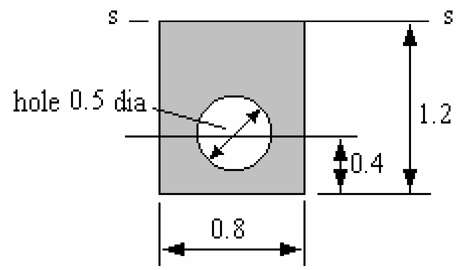
This must also be given by A y' for the whole section hence

Y' = 35 500/1100 = 32.27 mm.

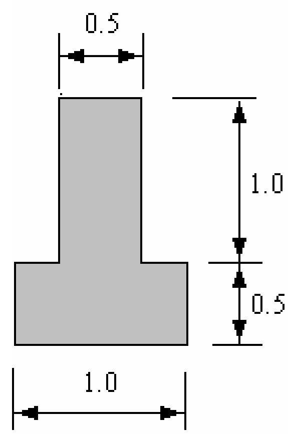
The centroid is 32.77 mm from the bottom edge.

1. Find the distance of the centroid from the axis s – s. All dimensions are in meters.

(0.549 m).



2. Find the distance of the Centroid from the bottom edge. All dimensions are in meters. (0.625 m)



Product of Inertia:

Relative to two rectangular axes, the sum of the products formed by multiplying the mass (or, sometimes, the area) of each element of a figure by the product of the coordinates corresponding to those axis. gure by the product of the coordinates corresponding to those axes.

Product of inertia

The product of inertia of area *A* relative to the indicated *XY* rectangular axes is $I_{XY} = \int xy\, dA$ (see illustration). The product of inertia of themass contained in volume *V* relative to the *XY* axes is $I_{XY} = \int xy\rho\, dV$—similarly for I_{YZ} and I_{ZX}.

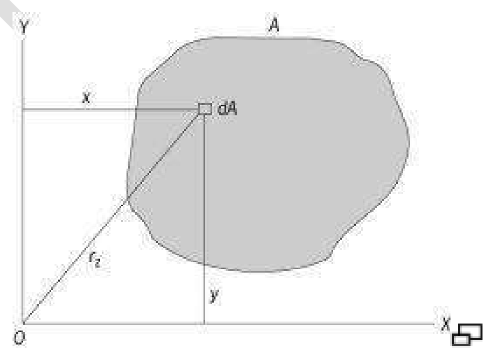


Figure13: Product of Inertia

Product of inertia of an area:

Relative to principal axes of inertia, the product of inertia of a figure is zero. If a figure is mirror symmetrical about a *YZ* plane, $I_{ZX} = I_{XY} = 0$.*See*

Moment of inertia Principal Moments of Inertia:

One of the major interests in the moment of inertia of area A is determining the orientation of the orthogonal axes passing a pole on the area with maximum or minimum moment of inertia about the axes.

Product of Inertia: Similar to the moment of inertia, a product of inertia can also be obtained from an integral over an area by multiplying the product of the coordinates x and y about the reference coordinate

$$\Delta I_{xy} = xy\Delta A$$

$$\Rightarrow \sum \Delta I_{xy} = \sum xy\Delta A$$

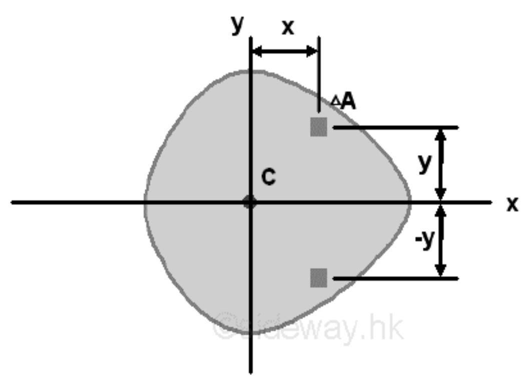
$$\Rightarrow \lim_{\Delta A \to 0} \sum \Delta I_{xy} = \lim_{\Delta A \to 0} \sum xy\Delta A$$

$$\Rightarrow \sum dI_{xy} = \sum xydA$$

$$I_{xy} = \int xydA$$

When considering the second moment of an area as the effect of the first moment acting on the same reference axis, the product moment of an area can be considered as the cross effect of the first moment acting on the orthogonal axis through a origin O at the specified orientation with respect to the area A.

Unlike the moment of inertia, although the elemental area is positive, the product of inertia can be positive, negative, or zero because the value of the coordinates x and y can be positive, negative, or zero. Similar to the first moment of an area about the the axis of symmetry, when one or both of the coordinate axes, x and y are the axis of symmetry of the area A, the integral, the product of inertia I_{xy} about the coordinate axes is zero. For example, a symmetrical area,



Although the area A is not symmetrical about axis y, however since the area is symmetrical about axis x, for any elemental area at a distance y above the axis x, there is always an elemental area below the axis x at the same mirror location of distance -y below the axis x. Therefore the product of inertia of a paired elemental area will cancel out each other and becomes zero, and the integral will reduces to zero also. Imply

$$I_{xy} = \int xydA$$

$$dA = dxdy = dydx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2}\int_{-y}^{y} xydydx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2}\left[\frac{1}{2}xy^2\right]_{-y}^{y} dx$$

$$\Rightarrow I_{xy} = \int_{-x_1}^{x_2} 0dx = 0$$

Since the product of inertia of a symmetrical area about one or two axes of symmetry must be zero, the product of inertia of an area with respect to axes can be used to test the dissymmetry or imbalance of the area about x and y axes because when the product of inertia about x and y axes is not equal to zero, the area is not symmetrical about both x and y axes. But when the product of inertia about x and y axes is equal to zero, the area may be not symmetrical about x and y axes.

Example of Product Moment of Inertia of a Right Angle Triangle

Product Moment of Inertia of a Right Angle Triangle by Double Integration

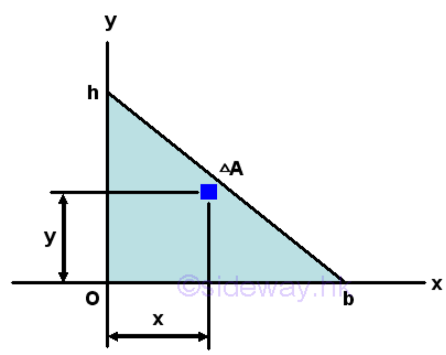


Figure14: Product Moment of Inertia of a Right Angle Triangle

The product moment of an area A of a right angle triangle about the axes xy is

$$\begin{cases} dA = dydx \\ \frac{y}{h} = 1 - \frac{x}{b} \Rightarrow y = h\left(1 - \frac{x}{b}\right) \end{cases}$$

$$I_{xy} = \int xydA$$

$$\Rightarrow I_{xy} = \int_0^b \int_0^y xydydx = \int_0^b \left[\frac{1}{2}xy^2\right]_0^y dx = \int_0^b \frac{1}{2}xy^2 dx$$

$$\Rightarrow I_{xy} = \int_0^b \frac{1}{2}x\left(h\left(1-\frac{x}{b}\right)\right)^2 dx = \frac{1}{2}h^2\int_0^b\left(x - \frac{2x^2}{b} + \frac{x^3}{b^2}\right)dx$$

$$\Rightarrow I_{xy} = \frac{1}{2}h^2\left[\frac{x^2}{2} - \frac{2x^3}{3b} + \frac{x^4}{4b^2}\right]_0^b = \frac{1}{2}h^2\left(\frac{b^2}{2} - \frac{2b^3}{3b} + \frac{b^4}{4b^2}\right)$$

$$\Rightarrow I_{xy} = \frac{1}{2}h^2b^2\left(\frac{6}{12} - \frac{8}{12} + \frac{3}{12}\right) = \frac{1}{24}h^2b^2$$

Support Reactions:

If a support prevents translation of a body in a given direction, a force is developed on the body in that direction. The three common types of connections which join a built structure to its foundation are roller, pinned and fixed. A fourth type, not often found in building structures, is known as a simple support. This is often idealized as a frictionless surface. All of these supports can be located anywhere along a structural element. They are found at the ends, at midpoints, or at any other intermediate points. The type of support connection determines the type of load that the support can resist. The support type also has a great effect on the load bearing capacity of each element, and therefore the system.

Roller Supports: Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped at any angle. The resulting reaction force is always a single force that is perpendicular to, and away from, the surface. Roller supports are commonly located at one end of long bridges. This allows the bridge structure to expand and contract with temperature changes. A roller support cannot provide resistance to lateral forces.

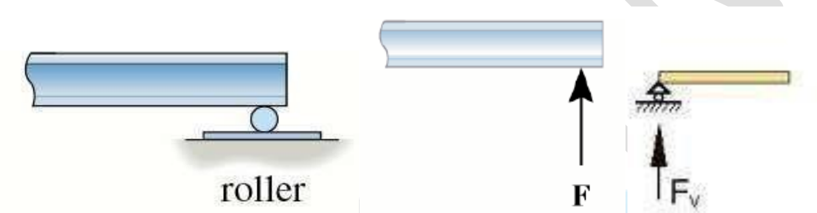


Figure15: Roller Support

The connection point on the bar cannot move downward.

Pinned support: A pinned support can resist both vertical and horizontal forces but not a moment. They allow the rotation, but not to translate in any direction. The knee can be idealized as a connection which allows rotation in only one direction and provides resistance to lateral movement.

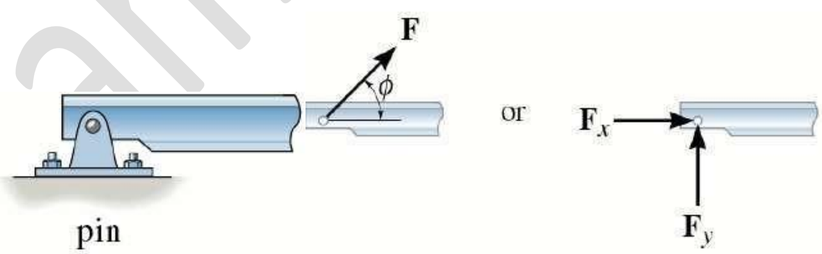


Figure16: Roller Support

The joint cannot move in vertical and horizontal directions.

FIXED SUPPORTS: Fixed support can resist vertical and horizontal forces as well as a moment. Since they restrain both rotation and translation, they are also known as rigid supports. A flagpole set into a concrete base is a good example of this kind of support. The representation of fixed supports always includes two forces (horizontal and vertical) and a moment.

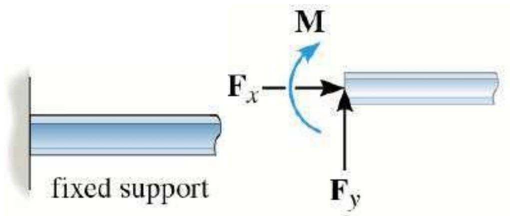


Figure17: Fixed Support

The support prevents translation in vertical and horizontal directions and also rotation, Hence a couple moments is developed on the body in that direction as well.

Types of beams:

<u>Simply supported beam:</u> The beam with one end hinged and the other end roller is called simply supported beam.

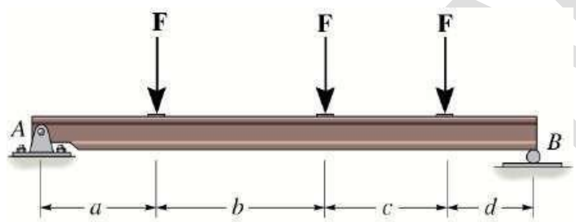


Figure18: Simply Supported Beam

<u>Cantilever beam</u>: The beam with one end fixed and the other end free is called Cantilever beam.

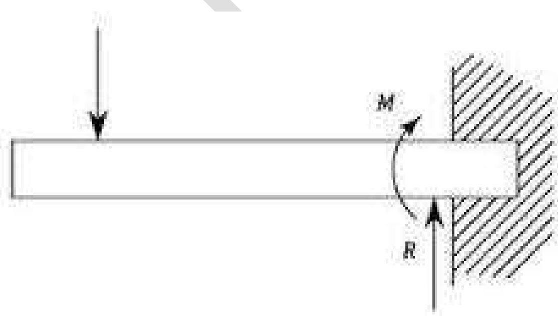


Figure19: Cantilever Beam

<u>Determinate and indeterminate beams</u>: If a beam has equal number of support reactions and static equilibrium conditions, then it is called as statically determinate beam. If a beam has more number of support reactions than the numbers of static equilibrium conditions, then it is called as statically indeterminate beam.

<u>Dynamics</u>

For dynamics as the mathematical analysis of the motion of bodies as a result of impressed forces, see analytical dynamics.

Kinematics is the branch of mechanics that deals with *motion* without regard to forces or energy.

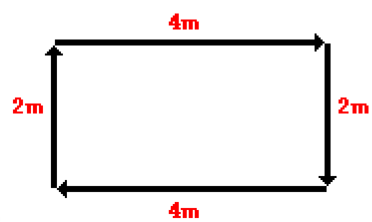
Kinetics is the branch of mechanics that deals with *motion* with regard to forces or energy.

Dynamics is a branch of applied mathematics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to *kinematics*, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.

Distance and displacement are two quantities that may seem to mean the same thing yet have distinctly different definitions and meanings.

- **Distance is a scalar quantity that refers to "how much ground an object has covered" during its motion.**
- **Displacement is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.**

To test your understanding of this distinction, consider the motion depicted in the diagram below. A man walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North.



Even though the man has walked a total distance of 12 meters, his displacement is 0 meters.

Velocity is a vector quantity that is defined as the rate at which an object changes its position.
Acceleration is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity.
Newton's First Law of Motion:

Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces. They have been expressed in several different ways, over nearly three centuries,[1] and can be summarized as follows.

First law when viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force.

Second law: The vector sum of the external forces F on an object is equal to the mass *m* of that object multiplied by the acceleration vector a of the object: F = *m*a.

Third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

I. **Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.**

II. The relationship between an object's mass *m*, its acceleration a, and the applied force *F* is *F = ma*. Acceleration and force are vectors (as indicated by their symbols being displayed in slant bold font); in this law the direction of the force vector is the same as the direction of the acceleration vector.

III. For every action there is an equal and opposite reaction.

Newton's first law

The first law states that if the net force (the vector sum of all forces acting on an object) is zero, then the velocity of the object is constant.

The first law can be stated mathematically as

$$\Sigma F = 0, dv/dt = 0$$

Consequently,

- An object that is at rest will stay at rest unless an external force acts upon it.
- An object that is in motion will not change its velocity unless an external force acts upon it.

This is known as *uniform motion*. An object *continues* to do whatever it happens to be doing unless a force is exerted upon it. If it is at rest, it continues in a state of rest. If an object is moving, it continues to move without turning or changing its speed. This is evident in space probes that continually move in outer space. Changes in motion must be imposed against the tendency of an object to retain its state of motion. In the absence of net forces, a moving object tends to move along a straight line path indefinitely.

Newton's second law

The second law states that the net force on an object is equal to the rate of change (that is, the *derivative*) of its momentum in an inertial reference frame:

$$\Sigma F = m, dv/dt = ma,$$

Where F is the net force applied, *m* is the mass of the body, and a is the body's acceleration. Thus, the net force applied to a body produces a proportional acceleration. In other words, if a body is accelerating, then there is a force on it.

Consistent with the first law, the time derivative of the momentum is non-zero when the momentum changes direction, even if there is no change in its magnitude; such is the case with uniform circular motion. The relationship also implies the conservation of momentum: when the net force on the body is zero, the momentum of the body is constant. Any net force is equal to the rate of change of the momentum.

Newton's third law

The third law states that all forces between two objects exist in equal magnitude and opposite direction: if one object *A* exerts a force F_A on a second object *B*, then *B* simultaneously exerts a force F_B on *A*, and the two forces are equal and opposite: $F_A = -F_B$. The third law means that all forces are *interactions* between different bodies, and thus that there is no such thing as a unidirectional force or a force that acts on only one body. This law is sometimes referred to as the *action-reaction law*, with F_A called the "action" and F_B the "reaction". The action and the reaction are simultaneous, and it does not matter which is called the *action* and which is called *reaction*; both forces are part of a single interaction, and neither force exists without the other.

The two forces in Newton's third law are of the same type (e.g., if the road exerts a forward frictional force on an accelerating car's tires, then it is also a frictional force that Newton's third law predicts for the tires pushing backward on the road).

From a conceptual standpoint, Newton's third law is seen when a person walks: they push against the floor, and the floor pushes against the person. Similarly, the tires of a car push against the road while the road pushes back on the tires—the tires and road simultaneously push against each other. In swimming, a person interacts with the water, pushing the water backward, while the water simultaneously pushes the person forward—both the person and the water push against each other. The reaction forces account for the motion in these examples. These forces depend on friction; a person or car on ice, for example, may be unable to exert the action force to produce the needed reaction force.

Newton's law of universal gravitation

Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This is a general physical law derived from empirical observations by what Isaac Newton called induction.

Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them

$F = (G m_1 m_2)/r^2$

Where:

- F is the force between the masses;
- G is the gravitational constant (6.674×10^{-11} N · (m/kg)^2);
- m_1 is the first mass;
- m_2 is the second mass;

r is the distance between the centers of the masses.

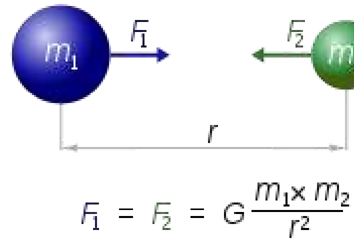


Figure20: Newton's law of universal gravitation

Assuming SI units, *F* is measured in Newton (N), m_1 and m_2 in kilograms (kg), *r* in meters (m), and the constant *G* is approximately equal to 6.674×10^{-11} N m^2 kg^{-2}.

Motion: **Motion is a change in position of an object with respect to time. Motion is typically described in terms of displacement, distance (scalar), velocity, acceleration, time and speed. Motion of a body is observed by attaching a frame of reference to an observer and measuring the change in position of the body relative to that frame.**

If the position of a body is not changing with the time with respect to a given frame of reference the body is said to be *at rest*, *motionless*, *immobile*, *stationary*, or to have constant (time-invariant) position. An object's motion cannot change unless it is acted upon by a force, as described by Newton's first law. Momentum is a quantity which is used for measuring motion of an object. An object's momentum is directly related to the object's mass and velocity, and the total momentum of all objects in an isolated system (one not affected by external forces) does not change with time, as described by the law of conservation of momentum.

As there is no absolute frame of reference, *absolute motion* cannot be determined. Thus, everything in the universe can be considered to be moving.

Main types of simple motion

There are two types of basic motion: translation and rotation. Translation means motion along a path. Rotation means motion around a fixed axis. An axis is the centre around which something rotates. As we have mentioned before, each type of motion is controlled by a different type of force. Translation is defined by the net force (sum of different forces) acting on an object. Rotation is defined by torque. Torque is a force which causes the rotation of an object.

1. **Linear motion or translatory motion is the most basic of all motions. Linear motion is the type of motion in which all parts of an object move in the same direction and each part moves an equal distance. Linear motion is measured by speed and direction. Distance travelled by an object per unit of time is called velocity. The 2 types are-**

a) **Rectilinear motion -motion of a body in a straight line-- eg: a car moving on the road, a coconut falling down from the tree, a child moving down in a slide.**
b) **Curvilinear translatory motion-motion of a body along a curved path--eg: Man running in a 400 m race along the circular path.**

2. **Rotary motion or circular motion is motion in a circle. This type of motion is the starting point of many mechanisms. Example: a spinning wheel. The types are**

a) **Rotatary motion-- part of the body occupies a particular position at a time. eg: rotation of the earth on its axis, rotation of the blades of a fan, movement of the hands of a clock.**
b) **Revolution- motion of the whole body in a circle around a central fixed point. eg: movement of the earth around the sun.**

3. **Reciprocating motion or oscillatory motion is back and forth motion. Example: motion of a swing, movement of the 'bob' of a pendulum in a clock.**
4. **Periodic motion the bodies occupy a particular position at regular intervals.**

eg; position of minute hand in a clock once in every 60 minutes, hour hand in a clock once in every 12 hours., position of planets in their orbits around the sun. non periodic motion-vibratory motion of the drum

5. Irregular or random motion is motion which has no obvious pattern to its movement. Example: a flying bee, movement of football players in the field.

Very often, objects move by complicated motion. Complicated motion can be broken down into simpler types of motion. An example of complicated motion is a flying Frisbee. The movement of a Frisbee consists of a linear motion and a rotary motion.