

**RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA, BHOPAL**

**New Scheme Based On AICTE Flexible Curricula**

**Mechanical Engineering, IV-Semester**

**ME402- INSTRUMENTATION & CONTROL**

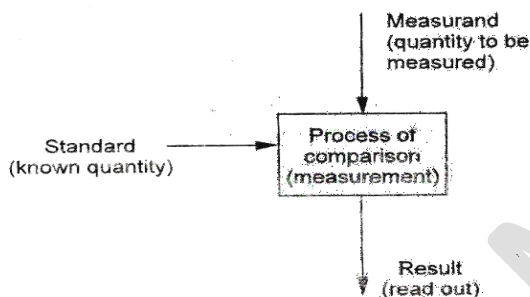
- [1] Introduction to instrument systems, classifications, functional elements of a measurement system, standards and calibration, static performance characteristics, measurement errors and uncertainties, analysis, sequential and random test, specifications of instrument static characteristics, data acquisition, reduction, data outlier detection,
- [2] Dynamic characteristics of the instruments, formulation of system equations, dynamic response, compensation, periodic input, harmonic signal non harmonic signal, Fourier transform, response to the transient input, response to random signal input, first and second order system compensation,
- [3] (a) Temperature measurements, thermometry based on thermal expansion, liquid in glass, bimetallic, electric resistance- thermometry, thermocouples, thermistors, detectors, (b) pressure and velocity measurements, barometer, manometer, dead weight tester, pressure gauges and transducers, dynamic measurements,(c) flow measurements, pressure differential meters, orifice meter, venturi meter, rota-meter,
- [4] strain gauges, strain and stress measurements, electrical circuits, compensations, motion force and torque measurements, displacement measurements, potentiometers, linear and rotary variable differential transformers, velocity measurements, electromagnetic technique, stroboscope, load cell, measurement of torque on rotating shaft, power estimation from rotating shaft.
- [5] Control systems, open loop and close loop control, mathematical modeling of dynamic systems – mechanical systems, electrical systems, fluid systems, thermal systems, transfer function, impulse response function, block diagrams of close loop systems, system modeling using software.

**BOOKS:**

- [1] Nakra B.C.Chaudhary K.K, Instrumentation measurement and analysisTata McGraw Hill, ISBN 0 07 451791 0
- [2] Richard S, Figiola & Donal E. Beasley, John Wiley, Theory and design of mechanical measurements.

## UNIT-1

**Measurement:** - The measurement of a given quantity is essentially an act or the result of comparison between the quantity (whose magnitude is unknown) & a predefined Standard. Since two quantities are compared, the result is expressed in numerical values.



**Figure.1 Fundamental measuring process**

These are two requirements which are to be satisfied to get good result from the measurement.

1. The standard must be accurately known and internationally accepted.
2. The apparatus and experimental procedure adopted for comparison must be provable.

### **Basic requirements of measurement:**

- The standard used for comparison purposes must be accurately defined & should be commonly accepted
- The apparatus used & the method adopted must be provable.

### **Measuring instrument:**

It may be defined as a device for determining the value or magnitude of a quantity or variable

**Instrumentation:**-The human senses cannot provide exact quantitative information about the knowledge of events occurring in our environments. The stringent requirements of precise and accurate measurements in the technological fields have, therefore, led to the development of mechanical aids called instruments.

The technology of using instruments to measure and control physical and chemical properties of materials is called instrumentation. In the measuring and controlling instruments are combined so that measurements provide impulses for remote automatic action, the result is called control system.

**Uses:**

- Study the function of different components and determine the cause of all functioning of the system, to formulate certain empirical relations.
- To test a product on materials for quality control.
- To discover effective components.
- To develop new theories.
- Monitor a data in the interest of health and safety.

**Methods of measurement:** Following are the different methods of measurement along with their details.

- **Direct Method:** Measurements are directly obtained. Ex: Vernier Caliper, Scales.
- **Indirect Method:** Obtained by measuring other quantities. Ex: Diameter measurement by using three wires.
- **Comparative Method:** It's compared with other known value. Ex: Comparators
- **Coincidence Method:** Measurements coincide with certain lines and signals.
- **Fundamental Method:** Measuring a quantity directly in related with the definition of that quantity.
- **Contact Method:** Sensor/Measuring tip touch the surface area. Ex: Vernier Caliper.
- **Transposition Method:** Quantity to be measured is first balanced by a known value and then balanced by another new known value. Ex: Determination of mass by balancing methods.
- **Complementary Method:** The value of quantity to be measured is combined with known value of the same quantity. Ex: Volume determination by liquid displacement.
- **Deflection Method:** The value to be measured is directly indicated by a deflection of pointer. Ex: Pressure Measurement.

**Standards:** The term standard is used to denote universally accepted specifications for devices, Component or processes which ensure conformity and interchangeability throughout a particular industry. A standard provides a reference for assigning a numerical value to a measured quantity. Each basic measurable quantity has associated with it an ultimate standard. Working standards, those used in conjunction with the various measurement making instruments.

The national institute of standards and technology (NIST) formerly called National Bureau of Standards (NBS), it was established by an act of congress in 1901, and the need for such body had been noted by the founders of the constitution. In order to maintain accuracy, standards in a vast industrial complex must be traceable to a single source, which may be national standards.

The following is the generalization of echelons of standards in the national measurement system.

- 1. Calibration standards:** Working standards of industrial or governmental laboratories.
- 2. Metrology standards:** Reference standards of industrial or Governmental laboratories.
- 3. National standards:** It includes prototype and natural phenomenon of SI (Systems International), the world wide system of weight and measures standards. Application of precise measurement has increased so much, that a single national laboratory to perform directly all the calibrations and standardization required by a large country with high technical development. It has led to the establishment of a considerable number of standardizing laboratories in industry and in various other areas. A standard provides a reference or datum for assigning a numerical value to a measured quantity.

**Type of Measurement:** The complexity of an instrument system depending upon measurement being made and upon the accuracy level to which the measurement is needed. Based upon the complexity of the measurement systems, the measurement are generally grouped into three categories.

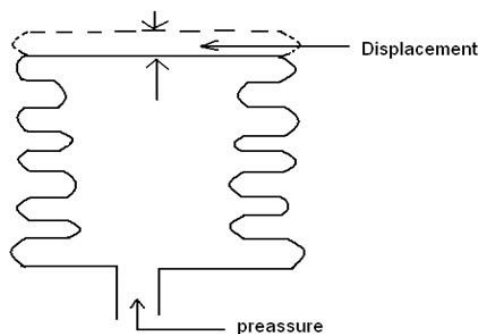
- i. Primary
- ii. Secondary
- iii. Tertiary.

**i. Primary:** In Primary Mode, The Sought Value Of A Physical Parameter Is Determined By Comparing It Directly With Reference Standards. The Requisite Information Is Obtainable Through Senses Of Sight And Touch. In the primary mode, the sought value of physical parameter is determined by comparing it directly with reference standards the required information is obtained to sense of side and touch.

Examples are:

- a) Matching of two lengths is determining the length of a object with ruler.
- b) Estimation the temperature difference between the components of the container by inserting fingers.
- c) Use of bean balance measure masses.
- d) Measurement of time by counting a number of strokes of a block.

**ii. Secondary:** - The Indirect Measurements Involving One Translation Are Called Secondary Measurements. The Conversion of Pressure Into Displacement By Bellows Is A Simple Example Of The Secondary Measurement.



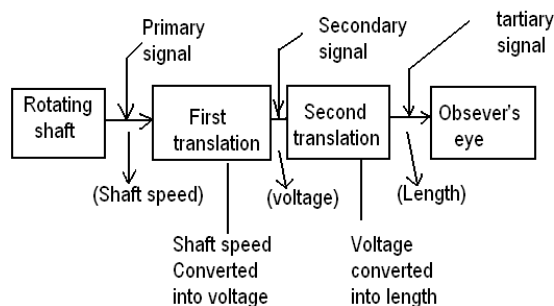
**Figure.2 Bellows convert pressure into displacement**

Examples are:

a) The convergent of pressure into displacement by means of are allows and the convergent of force into displacement.

b) Pressure measurement by manometer and the temperature measurement by mercury in glass tube thermometer.

iii. **Tertiary:** The Indirect Measurements Involving Two Conversions Are Called Tertiary Measurements. The Measurement Of The Speed Of A Rotating Shaft By Means Of An Electric Tachometer Is The Example Of The Tertiary Measurements.



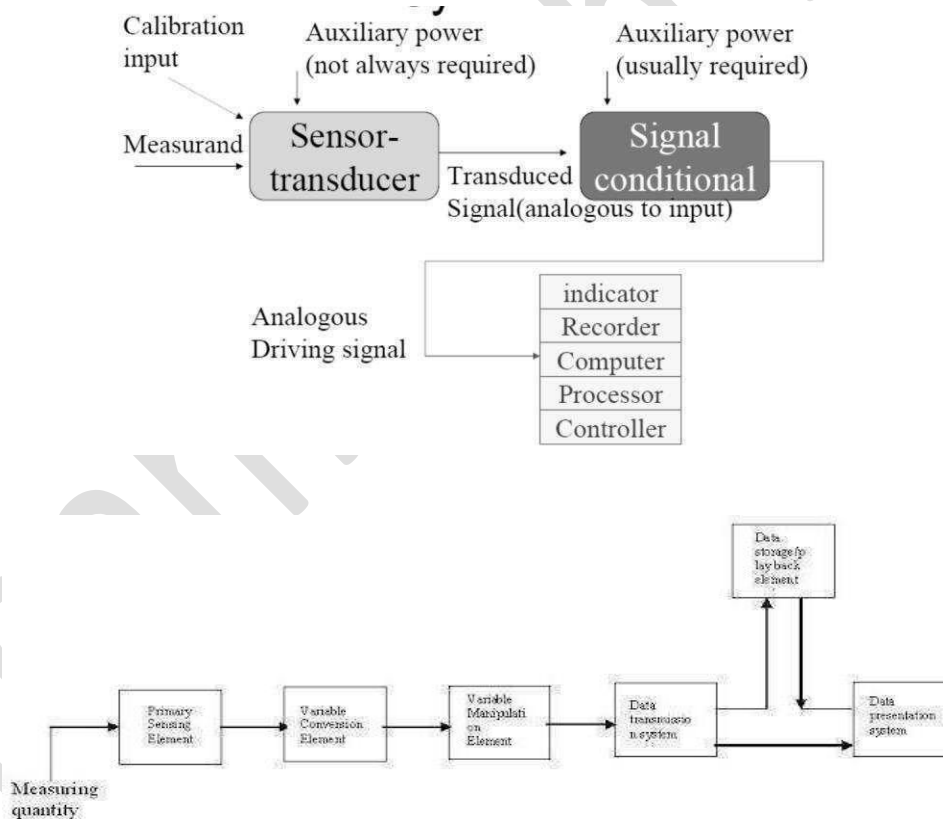
**Figure.3 Measurement of angular speed by an electric tachometer**

Examples: The measurement of static pressure by boundary tube pressure gauge is a typical example of tertiary measurement.

**Elements of a generalized measurement system:** To understand a measuring instrument or system, it is important to have a systematic organization and analysis of measurement systems. The operation of a measuring instrument or a system could be described in a generalized manner in terms of functional elements. Each functional element is made up of a component or groups of components which perform required and definite steps in the measurement. The functional elements do not provide the intricate details of the physical aspects of a specific instrument or a system. These may be taken as basic elements, whose scope is determined by their functioning rather than their construction.

The main functional elements of a measurement system are:

- Primary sensing element
- Variable conversion element
- Variable manipulation element
- Signal conditioning element
- Data transmission element
- Data presentation element.



**Figure.4 Generalized Measuring System**

**Primary sensing element:**

- The quantity under measurement makes its first contact with the primary sensing element of a measurement system.
- i.e., the measurand- (the unknown quantity which is to be measured) is first detected by primary sensor which gives the output in a different analogous form
- This output is then converted into an electrical signal by a transducer - (which converts energy from one form to another).
- The first stage of a measurement system is known as a detector Transducer stage Variable conversion element:
- The output of the primary sensing element may be electrical signal of any form; it may be voltage, a frequency or some other electrical Parameter. For the instrument to perform the desired function, it may be necessary to convert this output to some other suitable form.

#### **Variable manipulation element:**

- The function of this element is to manipulate the signal presented to it preserving the original nature of the signal.

It is not necessary that a variable manipulation element should follow the variable conversion element. Some non-linear processes like modulation, detection, sampling, filtering, chopping etc., are performed on the signal to bring it to the desired form to be accepted by the next stage of measurement system

- This process of conversion is called signal conditioning'
- The term signal conditioning includes many other functions in addition to Variable conversion & Variable manipulation

In fact the element that follows the primary sensing element in any instrument or measurement system is called signal conditioning element' When the elements of an instrument are actually physically separated, it becomes necessary to transmit data from one to another. The element that performs this function is called a data transmission element'. Example:

- Bourdon tube and bellows which transfer pressure into displacement.
- Proving ring and other elastic members which converts force into displacement.
- Rack and Pinion: It converts rotary to linear and vice versa.
- Thermo couple which converts information about temperature difference to information in the form of E.M.F.

### Data presentation element:

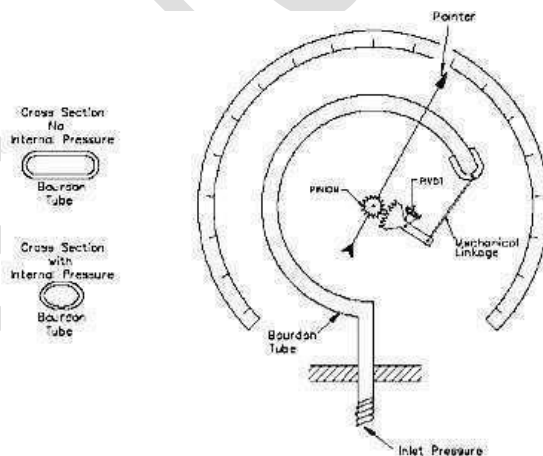
- The information about the quantity under measurement has to be conveyed to the personnel handling the instrument or the system for monitoring, control, or analysis purposes.
- This function is done by data presentation element

In case data is to be monitored, visual display devices are needed

- These devices may be analog or digital indicating instruments like ammeters, voltmeters etc

In case data is to be recorded, recorders like magnetic tapes, high speed camera & TV equipment, CRT, printers may be used. For control & analysis purpose microprocessor or computers may be used. The final stage in a measurement system is known as terminating stage'.

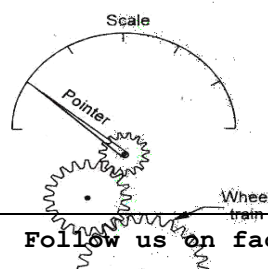
Let us consider an example here for the measurement of pressure using Bourdon tube. The pressure of fluid cannot be measured directly, hence Bourdon tube is used as the transducer to convert the property or signal of pressure into other property or signal that can be measured easily. The Bourdon tube is a thin tube with oval cross section and coiled into an arc with included angle less than 360 degree (see the fig). One end of this tube is connected to the inlet pressure and the other end, which is sealed, is connected to the pointer that moves on the angular scale. When the pressure is applied to the Bourdon tube the oval section tends to become circular, due to which the tube tends to uncoil and move the end connected to the pointer.



**Figure.5 Bourdon Tube Pressure gauge**

Here the Bourdon tube senses the pressure, and it acts as the transducer that detects the quantity to be measured.

Example:





**Figure.6 Dial Indicator**

**Noise:** In electronics, noise is a random fluctuation in an electrical signal, a characteristic of all electronic circuits. Noise generated by electronic devices varies greatly as it is produced by several different effects. Thermal noise is unavoidable at non-zero temperature, while other types depend mostly on device type (such as shot noise, which needs a steep potential barrier) or manufacturing quality and semiconductor defects, such as conductance fluctuations, including  $1/f$  noise.

Noise, or interference, can be defined as undesirable electrical signals, which distort or interfere with an original (or desired) signal. For examples, lightning (transient) and 50 or 60 Hz AC 'hum' (constant) from a general point of view, there must be three contributing factors before an electrical noise problem can exist.

- A source of electrical noise.
- A mechanism coupling the source to the affected circuit.
- A circuit conveying the sensitive communication signals.

Noise can be generated from within the system itself (internal noise) or from an outside source (external noise).

- Internal noise.
  - Thermal (electron movement).
  - Electrical design imperfections.
- External noise.
  - Natural origin (electrostatic interference).
  - Electromagnetic interference (EMI).
  - Radio frequency interference (RFI).
  - Cross talk.

**Thermal Noise:** Johnson–Nyquist noise (sometimes thermal, Johnson or Nyquist noise) is unavoidable, and generated by the random thermal motion of charge carriers (usually electrons), inside an electrical conductor, which happens regardless of any applied voltage. Thermal noise is approximately white, meaning that its power spectral density is nearly equal throughout the frequency spectrum. The amplitude of the signal has very

nearly a Gaussian probability density function. A communication system affected by thermal noise is often modeled as an additive white Gaussian noise (AWGN) channel.

**Shot Noise:** If electrons flow across a barrier, then they have discrete arrival times. Those discrete arrivals exhibit shot noise. The output of a shot noise generator is easily set by the current. Typically, the barrier in a diode is used.

**Flicker noise:** Flicker noise, also known as  $1/f$  noise, is a signal or process with a frequency spectrum that falls off steadily into the higher frequencies, with a pink spectrum. It occurs in almost all electronic devices, and results from a variety of effects, though always related to a direct current.

**Burst noise:** Burst noise consists of sudden step-like transitions between two or more levels (non-Gaussian), as high as several hundred microvolt's, at random and unpredictable times. Each shift in offset voltage or current lasts for several milliseconds, and the intervals between pulses tend to be in the audio range (less than 100 Hz), leading to the term *popcorn noise* for the popping or crackling sounds it produces in audio circuits.

**Transit-time noise:** If the time taken by the electrons from traveling from emitter to collector becomes comparable to the period of the signal being amplified, that is, at frequencies above VHF and beyond, so-called transit-time effect takes place and noise input admittance of the transistor increases. From the frequency at which this effect becomes significant it goes on increasing with frequency and quickly dominates over other terms.

**Types of Errors:** Basically there are three types of errors on the basis; they may arise from the source.

**a) Gross Errors:** This category of errors includes all the human mistakes while reading, recording and the readings. Mistakes in calculating the errors also come under this category. For example while taking the reading from the meter of the instrument he may read 21 as 31. All these types of error are come under this category. Gross errors can be avoided by using two suitable measures and they are written below:

1. A proper care should be taken in reading, recording the data. Also calculation of error should be done accurately.
2. By increasing the number of experimenters we can reduce the gross errors. If each experimenter takes different reading at different points, then by taking average of more readings we can reduce the gross errors.

**b) Systematic Errors:** In order to understand these kinds of errors, let us categorize the systematic errors as

**c) Instrumental Errors:** These errors may be due to wrong construction, calibration of the measuring instruments. These types of error may be arises due to friction or may be due to hysteresis. These types of errors also include the loading effect and misuse of the instruments. Misuse of the instruments results in the failure to adjust the zero of instruments. In order to

minimize the gross errors in measurement various correction factors must be applied and in extreme condition instrument must be re-calibrated carefully.

**d) Environmental Errors:** This type of error arises due to conditions external to instrument. External condition includes temperature, pressure, humidity or it may include external magnetic field. Following are the steps that one must follow in order to minimize the environmental errors:

- Try to maintain the temperature and humidity of the laboratory constant by making some arrangements.
- Ensure that there should not be any external magnetic or electrostatic field around the instrument.

**e) Observational Errors:** As the name suggests these types of errors are due wrong observations. The wrong observations may be due to PARALLAX. In order to minimize the PARALLAX error highly accurate meters are required, provided with mirrored scales.

**f) Random Errors:** After calculating all systematic errors, it is found that there are still some errors in measurement are left. These errors are known as random errors. Some of the reasons of the appearance of these errors are known but still some reasons are unknown. Hence we cannot fully eliminate these kinds of error.

#### Performance characteristics of a measuring instrument:-

1. Static characteristics
2. Dynamic characteristics

The performance characteristics of an instrument system is conclusion by low accurately the system measures the require input and how absolutely it reject the undesirable inputs.

$$\text{Error} = \text{measured value ( )} - \text{true value (( )}$$

#### 1. Static characteristics:

a) Range and span, b) Accuracy, error, correction, c) Calibration, d) Repeatability, e) Reproducibility, f) Precision, g) Sensitivity, h) Threshold, i) Resolution, j) Drift, k) Hysteresis, dead zone.

**a) Range and span:** The region between the limits with in which as instrument is designed to operate for measuring, indicating (or) recording a physical quantity is called the range of instrument. The range is expressed by standing the lower and upper values. Span represents the algebraic difference between the upper and lower range values of the instruments.

**Ex: -**

Range

Range 5 bar to 100 bar      Span=100-5=95 bar

Range 0 v to 75v              Span=75volts

**b) Accuracy, error, and correction:** No instrument gives an exact value of what is being measured; there is always some uncertainty in the measured values. This uncertainty express in terms of accuracy and error.

Accuracy of an indicated value (measured) may be defined as closeness to an accepted standard value (true value). The difference between measured value ( ) and true value ( ) of the quantity is expressed as instrument error.

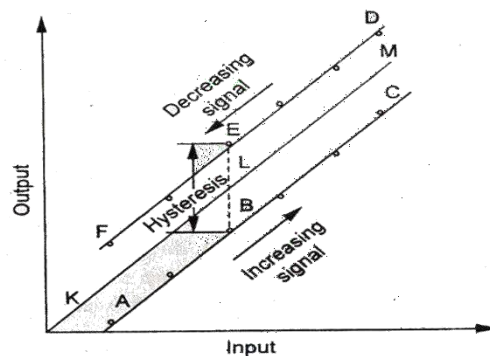
$$\text{Error} = \text{measured value ( )} - \text{true value ( )} -$$

Static correction is defined as -

$$= \text{true value ( )} - \text{measured value ( )}$$

**c) Calibration:** Calibration is the process of establishing the relationship between a measuring device and the units of measure. This is done by comparing a devise or the output of an instrument to a standard having known measurement characteristics. For example the length of a stick can be calibrated by comparing it to a standard that has a known length. Once the relationship of the stick to the standard is known the stick can be used to measure the length of other things.

The magnitude of the error and consequently the correction to be applied is determined by making a periodic comparison of the instrument with standards which are known to be constant. The entire procedure laid down for making, adjusting or checking a scale so that readings of an instrument or measurement system conform to an Accepted standard is called the calibration. The graphical representation of the calibration record is called calibration curve and this curve relates standard values of input or measurand to actual values of output throughout the operating range of the instrument. A comparison of the instrument reading may be made with



**Figure.7 Calibration curve**

- (i) a primary standard,
- (ii) a secondary standard of accuracy greater than the instrument to be calibrated,
- (iii) Known input source.

The following points and observations need consideration while calibrating an instrument:-

- (a) Calibration of the instrument is out with the instrument in the same (upright, horizontal etc.) and subjected same temperature and other environmental conditions under which it is to operate while in service.
- (b) The instrument is calibrated with values of the measuring impressed both in the increasing and in the decreasing order. The results are then expressed graphically, typically the output is plotted as the ordinate and the input or measuring as the abscissa.
- (c) Output readings for a series of impressed values going up the scale may not agree with the output readings for the same input values when going down.
- (d) Lines or curves plotted in the graphs may not close to form a loop.

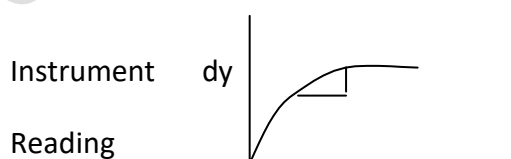
**d) Repeatability:** Repeatability describes the closeness of the output readings, when the same input is applied repeatability over a short period of time with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout.

**e) Reproducibility:** Reproducibility describes the closeness of output readings for the same input. When are changes in the method of measurement, observer, measuring instrument, and location, conditions of use and time of measurement

**f) Precision:** The instrument ability to reproduce a certain group of the readings with a given accuracy is known as precision i.e., if a no of measurements are made on the same true value then the degree of closeness of these measurements is called precision. It refers to the ability of an instrument to give its readings again and again in the same manner for constant input signals.

**g) Sensitivity:** Sensitivity of an instrument is the ratio of magnitude of response (output signal) to the magnitude of the quantity being measured (input signal) i.e.,

$$\text{Sensitivity} = \frac{\text{Change in the output signal}}{\text{Change in the input signal}}$$



### Measured quantity

**h) Threshold:** Threshold defines the minimum value of input which is necessary to cause detectable change from zero output. When the input to an instrument is gradually increased from zero, then the input must reach to a certain minimum value, so that the change in the output can be detected. The minimum value of input refers to threshold.

**i) Resolution:** It is defines as the increment in the input of the instrument for which input remains constant i.e., when the input given to the instrument is slowly increased for which the output remains same until the increment exceeds a different value.

**j) Drift:** The slow variation of the output signal of a measuring instrument is known as draft. The variation of the output signal is not due to any changes in the input quantity, but to the changes in the working conditions of the components inside the measuring instruments.

**k) Hysteresis, Dead zone:** Hysteresis is the maximum difference for the same measuring quantity (input signal) between the upscale and down scale reading during a full range measure in each direction.

Dead zone is the largest range through which an input signal can be varied without initiating any response from the indicating instrument it is due to the friction.

**Calibration:** It is very much essential to calibrate the instrument so as to maintain its accuracy. In case when the measuring and the sensing system are different it is very difficult to calibrate the system as an whole, so in that case we have to take into account the error producing properties of each component. Calibration is usually carried out by making adjustment such that when the instrument is having zero measured input then it should read out zero and when the instrument is measuring some dimension it should read it to its closest accurate value. It is very much important that calibration of any measuring system should be performed under the environmental conditions that are much closer to that under which the actual measurements are usually to be taken.

Calibration is the process of checking the dimension and tolerances of a gauge, or the accuracy of a measurement instrument by comparing it to the instrument/gauge that has been certified as a standard of known accuracy. Calibration of an instrument is done over a period of time, which is decided depending upon the usage of the instrument or on the materials of the parts from which it is made. The dimensions and the tolerances of the instrument/gauge are checked so that we can come to whether the instrument can be used again by calibrating it or is it wear out or deteriorated above the limit value. If it is so then it is thrown out or it is scrapped. If the gauge or the instrument is frequently used, then it will require more maintenance and frequent calibration. Calibration of instrument is done prior to its use and afterwards to verify that it is within the tolerance limit or not. Certification is given by making comparison between the

instrument/gauge with the reference standard whose calibration is traceable to National standard.

Stream Tech Notes

## UNIT-2

### Dynamic characteristics:

a) Speed of response and measuring lag, b) Fidelity and dynamic error, c) Over shoot, d) Dead time and dead zone, e) Frequency response.

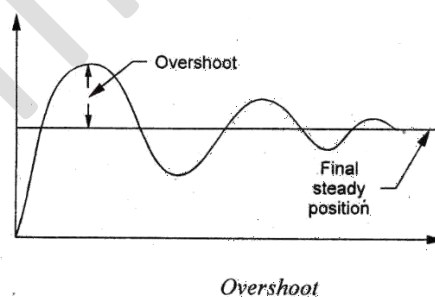
**a) Speed of response and measuring lag:** In a measuring instrument the speed of response (or) responsiveness is defined as the rapidity with which an instrument responds to a change in the value of the quantity being measured.

Measuring lag refers to delay in the response of an instrument to a change in the input signal. The lag is caused by conditions such as inertia, or resistance.

**b) Fidelity and dynamic errors:** Fidelity of an instrumentation system is defined as the degree of closeness with which the system indicates (or) records the signal which is upon it. It refers to the ability of the system to reproduce the output in the same form as the input. If the input is a sine wave then for 100% fidelity the output should also be a sine wave. The difference between the indicated quantity and the true value of the time quantity is the dynamic error. Here the static error of instrument is assumed to be zero.

**c) Over shoot:** Because of maximum and inertia. A moving part i.e., the pointer of the instrument does not immediately come to rest in the final deflected position. The pointer goes beyond the steady state i.e., it over shoots.

The over shoot is defined as the maximum amount by which the pointer moves beyond the steady state.

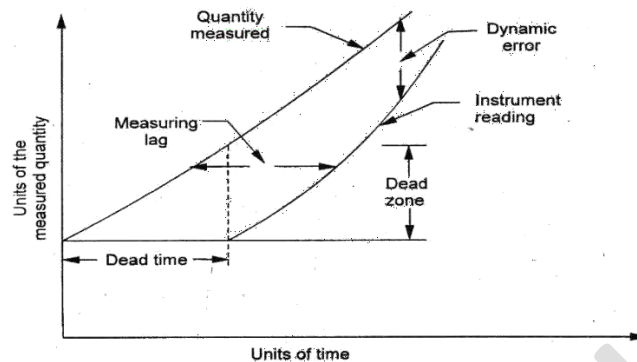


**Figure.1 Overshoot**

**d) dead time and dead zone:** Dead time is defined as the time required for an instrument to begin to respond to a change in the measured quantity it represent the time before the instrument begins to respond after the measured quantity has been altered.



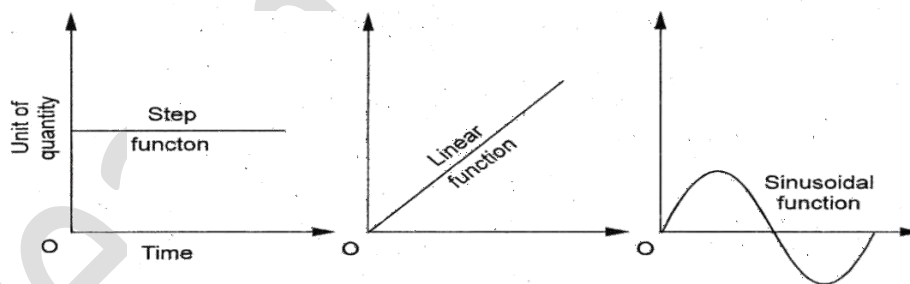
Dead zone define the largest change of the measured to which the instrument does not respond. Dead zone is the result as friction backlash in the instrument.



**Figure.2 Dynamic terms**

**e) Frequency response:** (The dynamic performance of both measuring and control system is determine by applying some known and predetermined input signal to its primary sensing element and them) Maximum frequency of the measured variable that an instrument is capable of following with error. The usual requirement is that the frequencies of the measured should not exceed 60% of the natural frequency measuring instrument.

**Standard test inputs:** The dynamic performance of both measuring and control system is determined by applying some known and predetermined input signal to its primary sensing element and then studying the behavior of the output signals.



**Figure.3 Standard input function**

The most common standard inputs used for dynamic analysis

1. Step functions
2. Linear (or) ramp functions
3. Sinusoidal (or) sine wave functions

**Static and dynamic response:** The static characteristics of measuring instruments are concerned only with the steady-state reading that the instrument settles down to, such as accuracy of the reading. The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets only apply when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected. In any linear, time-invariant measuring system, the following general relation can be written between input and output for time  $(t) > 0$ :

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i \quad (1)$$

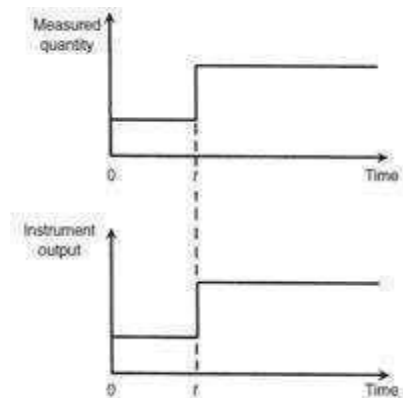
Where  $q_i$  is the measured quantity,  $q_o$  is the output reading, and  $a_0 \dots a_n$ ,  $b_0 \dots b_m$  are constants. If we limit consideration to that of step changes in the measured quantity only, then Equation (2) reduces to

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2)$$

### Zero-Order Instrument

$$a_0 q_o = b_0 q_i \quad \text{or} \quad q_o = b_0 q_i / a_0 = K q_i \quad (3)$$

If all the coefficients  $a_1 \dots a_n$  other than  $a_0$  in Equation (2) are assumed zero, then where  $K$  is a constant known as the instrument sensitivity as defined earlier. Any instrument that behaves according to Equation (3) is said to be of a zero-order type. Following a step change in the measured quantity at time  $t$ , the instrument output moves immediately to a new value at the same time instant  $t$ , as shown in Figure. A potentiometer, which measures motion is a good example of such an instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.



**First-Order Instrument:** If all the coefficients  $a_2 \dots a_n$  except for  $a_0$  and  $a_1$  are assumed zero in Equation

$$(2) \text{ then } a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (3)$$

Any instrument that behaves according to Equation (4) is known as a first-order instrument. If  $d/dt$  is replaced by the D operator in Equation (4), we get

$$a_1 D q_o + a_0 q_o = b_0 q_i \quad (4)$$

$$q_o = \frac{(b_0/a_0) q_i}{[1 + (a_1/a_0) D]} \quad (5)$$

Defining  $K \propto b_0/a_0$  as the static sensitivity and  $\tau \propto a_1/a_0$  as the time constant of the system,

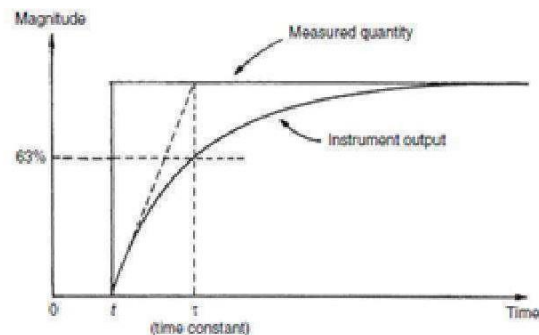
Equation (5) becomes  $q_o = \frac{K q_i}{1 + \tau D} \quad (6)$

**Second-Order Instrument:** If all coefficients  $a_3 \dots a_n$  other than  $a_0$ ,  $a_1$ , and  $a_2$  in Equation (2) are assumed zero, then we get

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

$$a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i$$

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2}$$



$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad (7) \quad \xi = a_1/2\sqrt{a_0 a_2}$$

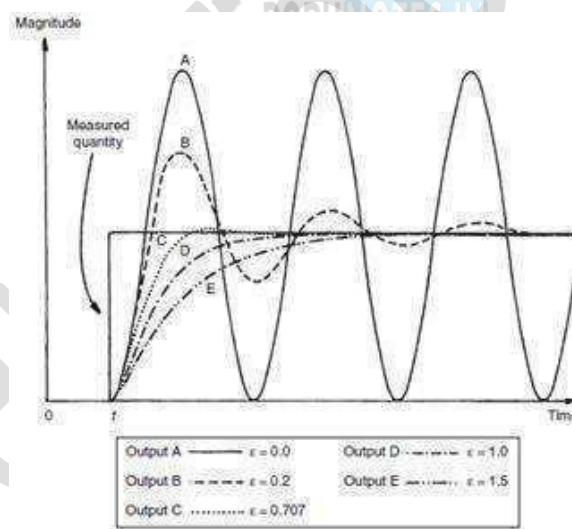
$$\xi = \frac{a_1}{2a_0 \sqrt{a_2/a_0}} = \frac{a_1 \omega}{2a_0} \quad (8)$$

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \quad (9)$$

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right)D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

$$\frac{q_o}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1}$$

This is the standard equation for a second-order system, and any instrument whose response can be described by it is known as a second-order instrument. If Equation (9) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter  $\xi$ . The output responses of a second-order instrument for various values of  $\xi$  following a step change in the value of the measured quantity at time  $t$  are shown in Figure.



**Figure.4 Second Order Response**

Commercial second-order instruments, of which the accelerometer is a common example, are generally designed to have a damping ratio ( $\xi$ ) somewhere in the range of 0.6–0.8.

## The Mean and Mode

The *sample mean* is the average and is computed as the sum of all the observed outcomes from the sample divided by the total number of events. We use  $\bar{x}$  as the symbol for the sample mean. In math terms,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x$$

.....where n are the sample size and the x corresponding to the observed valued.

### Example

Suppose you randomly sampled six acres in the Desolation Wilderness for a non-indigenous weed and came up with the following counts of this weed in this region:

34, 43, 81, 106, 106 and 115

We compute the sample mean by adding and dividing by the number of samples, 6.

$$\frac{34 + 43 + 81 + 106 + 106 + 115}{6} = 80.83$$

We can say that the sample mean of non-indigenous weed is 80.83.

The *mode* of a set of data is the number with the highest frequency. In the above example 106 is the mode, since it occurs twice and the rest of the outcomes occur only once.

The *population mean* is the average of the entire population and is usually impossible to compute. We use the Greek letter  $\mu$  for the population mean.

### Variance, Standard Deviation and Coefficient of Variation

The mean, mode, median, and trimmed mean do a nice job in telling where the center of the data set is, but often we are interested in more. For example, a pharmaceutical engineer develops a new drug that regulates iron in the blood. Suppose she finds out that the average sugar content after taking the medication is the optimal level. This does not mean that the drug is effective. There is a possibility that half of the patients have dangerously low sugar content while the other half has dangerously high content. Instead of the drug being an effective regulator, it is a deadly poison. What the pharmacist needs is a measure of how far the data is spread apart. This is what the variance and standard deviation do. First we show the formulas for these measurements. Then we will go through the steps on how to use the formulas.

We define the *variance* to be

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x - \bar{x})^2$$

.....and the *standard deviation* to be

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x - \bar{x})^2}$$

### Variance and Standard Deviation: Step by Step

1. Calculate the mean,  $\bar{x}$ .
2. Write a table that subtracts the mean from each observed value.
3. Square each of the differences.
4. Add this column.
5. Divide by  $n-1$  where  $n$  is the number of items in the sample this is the variance.
6. To get the *standard deviation* we take the square root of the variance.

### Example

The owner of the Chas Tahoe restaurant is interested in how much people spend at the restaurant. He examines 10 randomly selected receipts for parties of four and writes down the following data.

44, 50, 38, 96, 42, 47, 40, 39, 46, 50

He calculated the mean by adding and dividing by 10 to get

$$\bar{x} = 49.2$$

Below is the table for getting the standard deviation:

$x$	$x - 49.2$	$(x - 49.2)^2$
44	-5.2	27.04
50	0.8	0.64
38	11.2	125.44
96	46.8	2190.24

42	-7.2	51.84
47	-2.2	4.84
40	-9.2	84.64
39	-10.2	104.04
46	-3.2	10.24
50	0.8	0.64
<b>Total</b>		<b>2600.4</b>

Now

$$\frac{2600.4}{10 - 1} = 288.7$$

Hence the variance is 289 and the standard deviation is the square root of 289 = 17.

Since the standard deviation can be thought of measuring how far the data values lie from the mean, we take the mean and move one standard deviation in either direction. The mean for this example was about 49.2 and the standard deviation was 17. We have:

$$49.2 - 17 = 32.2;$$

$$49.2 + 17 = 66.2$$

What this means is that most of the patrons probably spend between \$32.20 and \$66.20.

The sample standard deviation will be denoted by  $s$  and the population standard deviation will be denoted by the Greek letter  $\sigma$ .

The sample variance will be denoted by  $s^2$  and the population variance will be denoted by  $\sigma^2$ .

The variance and standard deviation describe how spread out the data is. If the data all lies close to the mean, then the standard deviation will be small, while if the data is spread out over a large range of values,  $s$  will be large. Having outliers will increase the standard deviation.

One of the flaws involved with the standard deviation, is that it depends on the units that are used. One way of handling this difficulty, is called the *coefficient of variation* which is the standard deviation divided by the mean times 100%

$$CV = \frac{s}{x} 100\%$$

In the above example, it is

$$\frac{17}{49.2} \times 100\% = 34.6\%$$

This tells us that the standard deviation of the restaurant bills is 34.6% of the mean.

The following steps show how to calculate average deviation for the mean. If you want to calculate average deviation for the median, just replace any value for the mean with the value for the median.

The absolute deviation formula (i.e. the formula to calculate the distance for one point) is:

$$\text{Absolute deviation} = |x - \bar{x}|$$

Which leads to the average deviation formula:

$$Dx = (|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|) / N$$

Sample question: Find the average deviation of the following set of numbers: 3, 8, 8, 8, 8, 9, 9, 9, and 9.

Step 1: Find the mean:

$$(3 + 8 + 8 + 8 + 8 + 9 + 9 + 9 + 9) = 71.9 = 7.89.$$

Step 2: Find each individual absolute deviation using the formula  $|x - \bar{x}|$ .

$$|3 - 7.89| = 4.89$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|8 - 7.89| = 0.11$$

$$|9 - 7.89| = 1.11$$

$$|9 - 7.89| = 1.11$$

- $|9 - 7.89| = 1.11$

- $|9 - 7.89| = 1.11$

Step 3: Add up all of the values you found in Step 1.

$$4.89 + 0.11 + 0.11 + 0.11 + 0.11 + 1.11 + 1.11 + 1.11 + 1.11 = 9.77$$

Step 4: Divide by the number of items in your data set. There are 9 items, so:

$$9.77 / 9 = 1.09.$$

The average deviation is 1.09.

## Normal Distribution

A normal distribution, sometimes called the bell curve, is a distribution that occurs naturally in many situations. For example, the bell curve is seen in tests like the SAT and GRE. The bulk of students will score the average (C), while smaller numbers of students will score a B or D. An



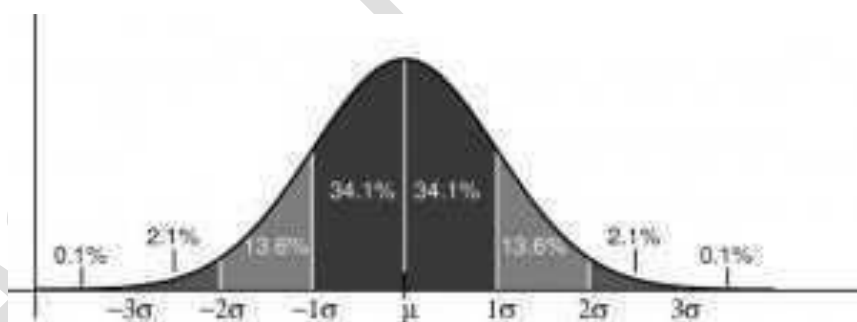
even smaller percentage of students score an F or an A. This creates a distribution that resembles a bell (hence the nickname). The bell curve is symmetrical. Half of the data will fall to the left of the mean; half will fall to the right.

Many groups follow this type of pattern. That's why it's widely used in business, statistics and in government bodies like the FDA:

- Heights of people.
- Measurement errors.
- Blood pressure.
- Points on a test.
- IQ scores.
- Salaries.

The empirical rule tells you what percentage of your data falls within a certain number of standard deviations from the mean:

- 68% of the data falls within one standard deviation of the mean.
- 95% of the data falls within two standard deviations of the mean.
- 99.7% of the data falls within three standard deviations of the mean.



The standard deviation controls the spread of the distribution. A smaller standard deviation means that the data is tightly clustered around the mean; the normal distribution will be taller. A larger standard deviation means that the data is spread out around the mean; the normal distribution will be flatter and wider.

### Properties of a normal distribution

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean,  $\mu$ ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

### Method of Least Squares

Suppose that we are given a data set of  $n$  observations from an experiment. Say that we are interested in fitting a straight line

$$y = ax + b$$

to the given data. Find the ' $n$ ' residuals  $e_i$  by:

$$e_i = y_i - (ax_i + b), \quad i = 1, 2, \dots, n \quad (2)$$

Now consider the sum of the squares of  $e_i$  i.e

$$\begin{aligned} E &= \sum_{i=1}^n e_i^2 \\ &= \sum_{i=1}^n [y_i - (ax_i + b)]^2 \quad (3) \end{aligned}$$

Note that  $E$  is a function of parameters  $a$  and  $b$ . We need to find  $a, b$  such that  $E$  is minimum. The necessary condition for  $E$  to be minimum is given by:

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = 0 \quad (4)$$

The condition  $\frac{\partial E}{\partial a} = 0$  yields:

$$\frac{\partial E}{\partial a} = \sum_{i=1}^n 2x_i[y_i - (ax_i + b)] = 0$$

$$a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i = \sum_{i=1}^n x_i y_i \quad (5)$$

i.e

Similarly the condition  $\frac{\partial E}{\partial b} = 0$  yields

$$a \sum_{i=1}^n x_i + nb = \sum_{i=1}^n y_i \quad (6)$$

Equations (5) and (6) are called as normal equations, which are to be solved to get desired values for a and b.

The expression for  $E$  i.e (3) can be re-written in a convenient way as follows:

$$E = \left( \sum_{i=1}^n y_i^2 - a \sum_{i=1}^n x_i y_i - b \sum_{i=1}^n y_i \right) \quad (7)$$

**Example:** Using the method of least squares, find an equation of the form

$y = ax + b$  That fits the following data:

x	0	1	2	3	4
y	1	5	10	22	38

Solution: Consider the normal equations of least square fit of a straight line i.e

$$a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i = \sum_{i=1}^n x_i y_i \quad (1)$$

$$a \sum_{i=1}^n x_i + nb = \sum_{i=1}^n y_i \quad (2)$$

Here  $n=5$ .

From the given data, we have,

x	Y	xy	x <sup>2</sup>
0	1	0	0
1	5	5	1
2	10	20	4
3	22	66	9
4	38	152	16

$$\sum_i x_i = 10 \quad \sum_i y_i = 76 \quad \sum_i x_i y_i = 243 \quad \sum_i x_i^2 = 30$$

Therefore the normal equations are given by:

$$30a + 10b = 243 \dots\dots\dots(3)$$

$$10a + 5b = 76 \dots\dots\dots(4)$$

On solving (3) and (4) we get

$$a = 9.1, b = -3 \dots\dots\dots(5)$$

Hence the required fit for the given data is

$$y = 9.1x - 3 \dots\dots\dots(6)$$

### Uncertainty Analysis

**Experimental uncertainty analysis** is a technique that analyses a *derived* quantity, based on the uncertainties in the experimentally *measured* quantities that are used in some form of mathematical relationship ("model") to calculate that derived quantity. The model used to convert the measurements into the derived quantity is usually based on fundamental principles of a science or engineering discipline.

The uncertainty has two components, namely, bias (related to *accuracy*) and the unavoidable random variation that occurs when making repeated measurements (related to *precision*). The measured quantities may have biases, and they certainly have random variation, so what needs to be addressed is how these are "propagated" into the uncertainty of the derived quantity. Uncertainty analysis is often called the "propagation of error."

It will be seen that this is a difficult and in fact sometimes intractable problem when handled in detail. Fortunately, approximate solutions are available that provide very useful results, and these approximations will be discussed in the context of a practical experimental example.

Stream Tech Notes

### UNIT-3

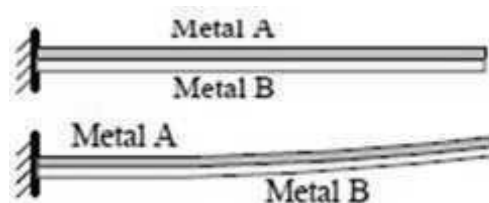
**Temperature measurement** Temperature is one of the most measured physical parameters in science and technology; typically for process thermal monitoring and control. There are many ways to measure temperature, using various principles. Four of the most common are:

1. Mechanical (liquid-in-glass thermometers, bimetallic strips, etc.)
2. Thermojunction (thermocouples)
3. Thermoresistive (RTDs and thermistors)
4. Radiative (infrared and optical pyrometers)

**Mechanical Temperature Measuring Devices:** A change in temperature causes some kind of mechanical motion, typically due to the fact that most materials expand with a rise in temperature. Mechanical thermometers can be constructed that use liquids, solids, or even gases as the temperature-sensitive material. The mechanical motion is read on a physical scale to infer the temperature.

#### **Bimetallic strip thermometer:**

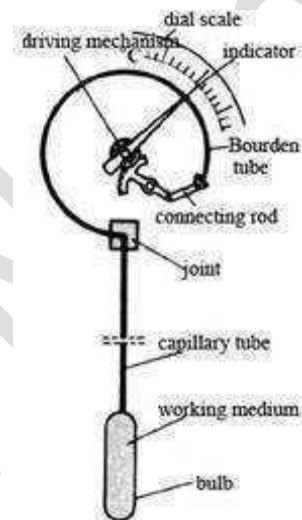
- Two dissimilar metals are bonded together into what is called a bimetallic strip, as sketched to the right.
- Suppose metal A has a smaller coefficient of thermal expansion than does metal B. As temperature increases, metal B expands more than does metal A, causing the bimetallic strip to curl upwards as sketched.
- One common application of bimetallic strips is in home thermostats, where a bimetallic strip is used as the arm of a switch between electrical contacts. As the room temperature changes, the bimetallic strip bends as discussed above. When the bimetallic strip bends far enough, it makes contact with electrical leads that turn the heat or air conditioning on or off.
- Another application is in circuit breakers. High temperature indicates over-current, which shuts off the circuit.
- Another common application is for use as oven, wood burner, or gas grill thermometers. These thermometers consist of a bimetallic strip wound up in a spiral, attached to a dial that is calibrated into a temperature scale.



**Figure.1 Bimetallic Strip**

## Pressure thermometer

- A pressure thermometer, while still considered mechanical, operates by the expansion of a gas instead of a liquid or solid. There are also pressure thermometers that use a liquid instead of a gas
- Suppose the gas inside the bulb and tube can be considered an ideal gas. The ideal gas law is  $PV = mRT$ , where  $P$  is the pressure,  $V$  is the volume of the gas,  $m$  is the mass of the gas,  $R$  is the gas constant for the specific gas (not the universal gas constant), and  $T$  is the absolute temperature of the gas.
- Specific gas constant  $R$  is a constant. The bulb and tube are of constant volume, so  $V$  is a constant. Also, the mass  $m$  of gas in the sealed bulb and tube must be constant (conservation of mass).
- A pressure thermometer therefore measures temperature indirectly by measuring pressure.
- The gage is a pressure gage, but is typically calibrated in units of temperature instead.
- A common application of this type of thermometer is measurement of outside temperature from the inside of a building. The bulb is placed outside, with the tube running through the wall into the inside.
- The gauge is on the inside. As  $T$  increases outside, the bulb temperature causes a corresponding increase in pressure, which is read as a temperature increase on the gauge.



## Thermocouples (Thermo-junctive temperature measuring devices)

Thomas Johan Seebeck discovered in 1821 that thermal energy can produce electric current. When two conductors made from dissimilar metals are connected forming two common junctions and the two junctions are exposed to two different temperatures, a net thermal emf is produced, the actual value being dependent on the materials used and the temperature difference between hot and cold junctions. The thermoelectric emf generated, in fact is due to the combination of two effects: Peltier effect and Thomson effect. A typical thermocouple junction is shown in fig. 5. The emf generated can be approximately expressed by the relationship:  $e_0 = C_1(T_1 - T_2) + C_2(T_1^2 - T_2^2)$  mV

Where,  $T_1$  and  $T_2$  are hot and cold junction temperatures in K.  $C_1$  and  $C_2$  are constants depending upon the materials. For Copper/Constantan thermocouple,  $C_1=62.1$  and  $C_2=0.045$ .

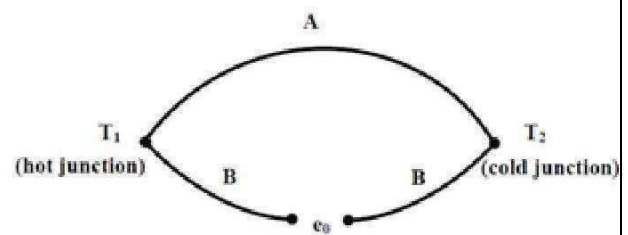


Figure.2 Thermocouple

Thermocouples are extensively used for measurement of temperature in industrial situations. The major reasons behind their popularity are:

- (i) They are rugged and readings are consistent
- (ii) They can measure over a wide range of temperature
- (iii) Their characteristics are almost linear with an accuracy of about 0.05%. However, the major shortcoming of thermocouples is low sensitivity compared to other temperature measuring devices (e.g. RTD, Thermistor).

### Thermocouple Materials

**Table-1 Thermocouple materials and Characteristics**

Type	Positive lead	Negative lead	Temperature range	Temperature coeff. variation $\mu\text{V}/^\circ\text{C}$	Most linear range and sensitivity in the range
R	Platinum-Rhodium (87% Pt, 13% Rh)	Platinum	0-1500°C	5.25-14.1	1100-1500°C 13.6-14.1 $\mu\text{V}/^\circ\text{C}$
S	Platinum-Rhodium (90% Pt, 10% Rh)	Platinum	0-1500°C	5.4-12.2	1100-1500°C 13.6-14.1 $\mu\text{V}/^\circ\text{C}$
K	Chromel (90%Ni, 10% Cr)	Alumel (Ni <sub>94</sub> Al <sub>2</sub> Mn <sub>3</sub> Si)	-200-1300°C	15.2-42.6	0-1000°C 38-42.9 $\mu\text{V}/^\circ\text{C}$
E	Chromel	Constantan (57%Cu, 43%Ni)	-200-1000°C	25.1-80.8	300-800°C 77.9-80.8 $\mu\text{V}/^\circ\text{C}$
T	Copper	Constantan	-200-350°C	15.8-61.8	nonlinear
J	Iron	Constantan	-150-750°C	21.8-64.6	100-500°C 54.4-55.9

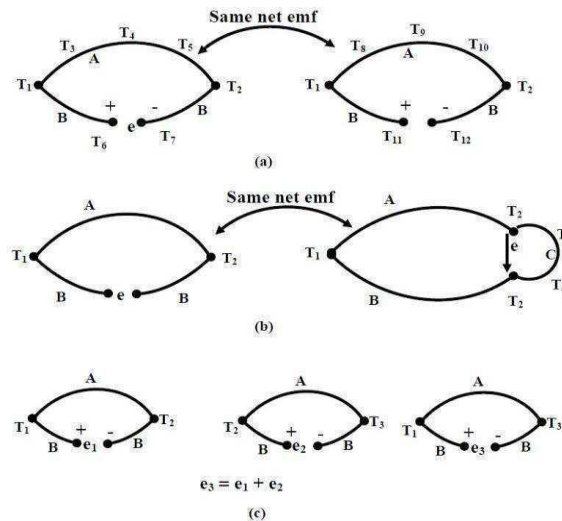
Theoretically, any pair of dissimilar materials can be used as a thermocouple. But in practice, only few materials have found applications for temperature measurement. The choice of materials is influenced by several factors, namely, sensitivity, stability in calibration, inertness in the operating atmosphere and reproducibility (i.e. the thermocouple can be replaced by a similar one without any recalibration). Table-I shows the common types of thermocouples, their types, composition, range, sensitivity etc. The upper range of the thermocouple is normally dependent on the atmosphere where it has been put. For example, the upper range of Chromel/ Alumel thermocouple can be increased in oxidizing atmosphere, while the upper range of Iron/ Constantan thermocouple can be increased in reducing atmosphere.

### Laws of Thermocouple



The Peltier and Thompson effects explain the basic principles of thermoelectric emf generation. But they are not sufficient for providing a suitable measuring technique at actual measuring situations. For this purpose, we have three laws of thermoelectric circuits that provide us useful practical tips for measurement of temperature. These laws are known as law of homogeneous circuit, law of intermediate metals and law of intermediate temperatures. These laws can be explained using figure

The first law can be explained using figure



(a) It says that the net thermo-emf generated is dependent on the materials and the temperatures of two junctions only, not on any intermediate temperature. According to the second law, if a third material is introduced at any point (thus forming two additional junctions)

It will not have any effect, if these two additional junctions remain at the same temperatures (figure b). This law makes it possible to insert a measuring device without altering the thermo-emf.

The third law is related to the calibration of the thermocouple. It says, if a thermocouple produces emf  $e_1$ , when its junctions are at  $T_1$  and  $T_2$ , and  $e_2$  when its junctions are at  $T_2$  and  $T_3$ ; then it will generate emf  $e_1 + e_2$  when the junction temperatures are at  $T_1$  and  $T_3$  (figure c).

The third law is particularly important from the point of view of reference junction compensation. The calibration chart of a thermocouple is prepared taking the cold or reference junction temperature as  $0^\circ\text{C}$ . But in actual measuring situation, seldom the reference junction temperature is kept at that temperature, it is normally kept at ambient temperature. The third law helps us to compute the actual temperature using the calibration chart.

### Thermo resistive temperature measuring devices

## Principle of operation

- A change in temperature causes the electrical resistance of a material to change.
- The resistance change is measured to infer the temperature change.
- There are two types of thermo resistive measuring devices: resistance temperature detectors and thermistors, both of which are described here.
- 

**Resistance temperature detectors:** A resistance temperature detector (abbreviated RTD) is basically either a long, small diameter metal wire (usually platinum) wound in a coil or an etched grid on a substrate, much like a strain gauge.

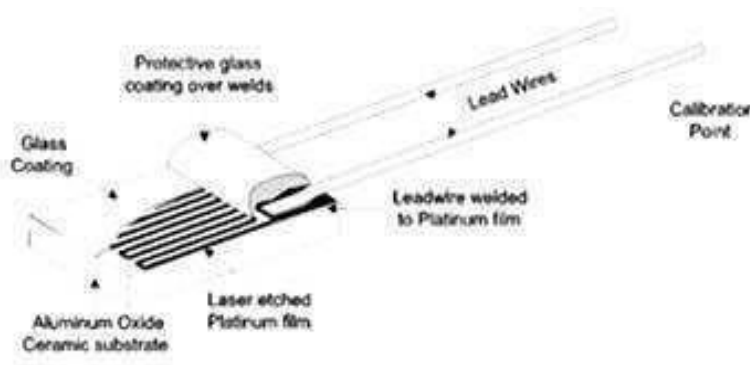
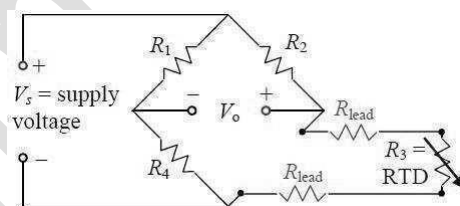


Figure.3 RTD

The resistance of an RTD increases with increasing temperature, just as the resistance of a strain gage increases with increasing strain. The resistance of the most common RTD is 100  $\Omega$  at 0°C.



If the temperature changes are large, or if precision is not critical, the RTD resistance can be measured directly to obtain the temperature. If the temperature changes are small, and/or high precision is needed, an electrical circuit is built to measure a change in resistance of the RTD, which is then used to calculate a change in temperature. One simple circuit is the quarter bridge Wheatstone bridge circuit, here called a two-wire RTD bridge circuit

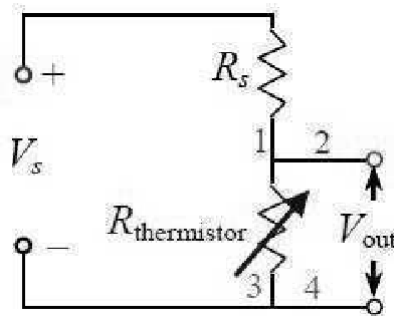
$R_{lead}$  represents the resistance of one of the wires (called lead wires) that run from the bridge to the RTD itself. Lead resistance is of little concern in strain gage circuits because

$R_{\text{lead}}$  remains constant at all times, and we can simply adjust one of the other resistors to zero the bridge.

For RTD circuits, however, some portions of the lead wires are exposed to changing temperatures. Since the resistance of metal wire changes with temperature,  $R_{\text{lead}}$  changes with  $T$  and this can cause errors in the measurement. This error can be non-trivial changes in lead resistance may be misinterpreted as changes in RTD resistance, and therefore give a false temperature measurement.

### Thermistors

A thermistor is similar to an RTD, but a semiconductor material is used instead of a metal. A thermistor is a solid state device. Resistance thermometry may be performed using thermistors. Thermistors are many times more sensitive than RTD's and hence are useful over limited ranges of temperature. They are small pieces of ceramic material made by sintering mixtures of metallic oxides of Manganese, Nickel, Cobalt, Copper and Iron etc.



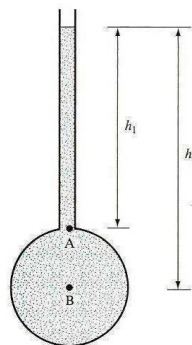
Resistance of a thermistor decreases non-linearly with temperature. Thermistors are extremely sensitive but over a narrow range of temperatures. A thermistor has larger sensitivity than does an RTD, but the resistance change with temperature is nonlinear, and therefore temperature must be calibrated with respect to resistance. Unlike RTDs, the resistance of a thermistor decreases with increasing temperature. The upper temperature limit of thermistors is typically lower than that of RTD. However, thermistors have greater sensitivity and are typically more accurate than RTDs or thermocouples. A simple voltage divider, where  $V_s$  is the supply voltage and  $R_s$  is a fixed (supply) resistor.  $R_s$  and  $V_s$  can be adjusted to obtain a desired range of output voltage  $V_{\text{out}}$  for a given range of temperature. If the proper value of  $R_s$  is used, the output voltage is nearly (but not exactly) linear with temperature. Some thermistors have 3 or 4 lead wires for convenience in wiring – two wires are connected to one side and two to the other side of the thermistor (labeled 1, 2 and 3, 4 above).

### Pressure Measurement

**Pressure Head:** Pressure in fluids may arise from many sources, for example pumps, gravity, momentum etc. Since  $p = \rho gh$ , a height of liquid column can be associated with the pressure  $p$  arising from such sources. This height,  $h$ , is known as the pressure head.

**Manometers:** A manometer (or liquid gauge) is a pressure measurement device which uses the relationship between pressure and head to give readings. In the following, we wish to measure the pressure of a fluid in a pipe.

**Piezometer:** This is the simplest gauge. A small vertical tube is connected to the pipe and its top is left open to the atmosphere, as shown.



**Figure.4 Piezometer**

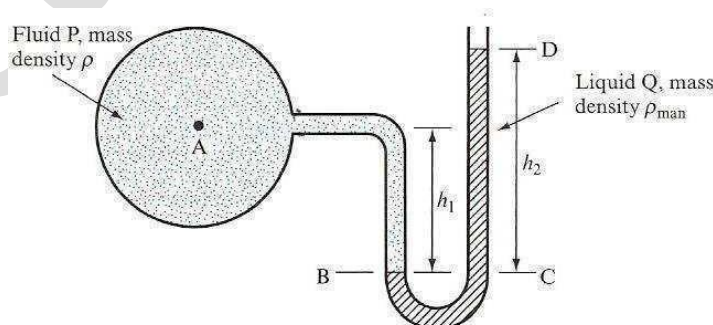
The pressure at A is equal to the pressure due to the column of liquid of height  $h_1$  :

$$p_A = \rho gh_1$$

Similarly,  $p_B = \rho gh_2$

The problem with this type of gauge is that for usual civil engineering applications the pressure is large (e.g.  $100 \text{ kN/m}^2$ ) and so the height of the column is impractical (e.g. 10 m). Also, obviously, such a gauge is useless for measuring gas pressures.

**U-tube Manometer:** To overcome the problems with the piezometer, the U-tube manometer seals the fluid by using a measuring (manometric) liquid:



**Figure.5 U-Tube Manometer**

Choosing the line  $BC$  as the interface between the measuring liquid and the fluid, we know:

$$\text{Pressure at } B, p_B = \text{Pressure at } C, p_C$$

For the left-hand side of the U-tube:

$$p_B = p_A + \rho g h_1$$

For the right hand side:

$$p_C = p_{man} g h_2$$

Where we have ignored atmospheric pressure and are thus dealing with gauge pressures.

Thus:  $p_B = p_C$

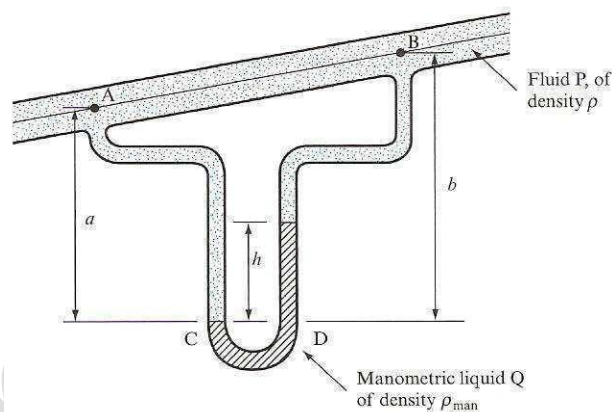
$$p_A + \rho g h_1 = p_{man} g h_2$$

And so:

$$p_A = p_{man} g h_2 - \rho g h_1$$

Notice that we have used the fact that in any continuous fluid, the pressure is the same at any horizontal level.

**Differential Manometer:** To measure the pressure difference between two points we use a u-tube as shown:



**Figure.6 Differential Manometer**

Using the same approach as before:

Pressure at C,  $p_C$  = Pressure at D,  $p_D$

$$p_A + \rho g a = p_B + \rho g (b - h) + p_{man} g h$$

Hence the pressure difference is:

$$p_A - p_B = \rho g (b - a) + h g (\rho_{man} - \rho)$$

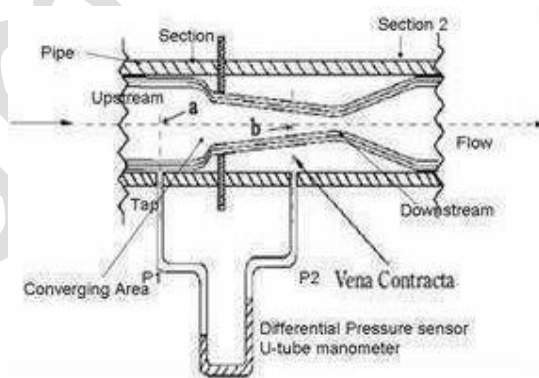
**Flow measurements:** The flow rate of a fluid flowing in a pipe under pressure is measured for a variety of applications, such as monitoring of pipe flow rate and control of industrial processes. Differential pressure flow meters, consisting of orifice, flow nozzle, and venturi meters, are widely used for pipe flow measurement and are the topic of this course. All

three of these meters use a constriction in the path of the pipe flow and measure the difference in pressure between the undisturbed flow and the flow through the constriction. That pressure difference can then be used to calculate the flow rate. Flow meter is a device that measures the rate of flow or quantity of a moving fluid in an open or closed conduit. Flow measuring devices are generally classified into four groups. They are

- 1. Mechanical type flow meters:** Fixed restriction variable head type flow meters using different sensors like orifice plate, venturi tube, flow nozzle, pitot tube, dall tube, quantity meters like positive displacement meters, mass flow meters etc. fall under mechanical type flow meters.
- 2. Inferential type flow meters:** Variable area flow meters (Rotameters), turbine flow meter, target flow meters etc.
- 3. Electrical type flow meters:** Electromagnetic flow meter, Ultrasonic flow meter, Laser doppler Anemometers etc. fall under electrical type flow meter.
- 4. Other flow meters:** Purge flow regulators, Flow meters for Solids flow measurement, Cross-correlation flow meter, Vortex shedding flow meters, flow switches etc.

**Orifice Flow Meter:** An Orifice flow meter is the most common head type flow measuring device. An orifice plate is inserted in the pipeline and the differential pressure across it is measured.

**Principle of Operation:** The orifice plate inserted in the pipeline causes an increase in flow velocity and a corresponding decrease in pressure. The flow pattern shows an effective decrease in cross section beyond the orifice plate, with a maximum velocity and minimum pressure at the vena contracta.

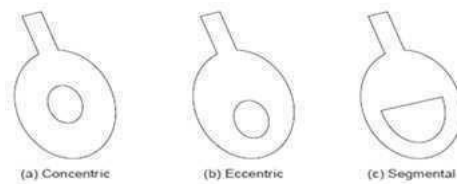


**Figure.7 Orifice Meter**

**The flow pattern and the sharp leading edge of the orifice plate which produces it are of major importance.** The sharp edge results in an almost pure line contact between the plate and the effective flow, with the negligible fluid-to-metal friction drag at the boundary.

**Types of Orifice Plates** The simplest form of orifice plate consists of a thin metal sheet, having in it a square edged or a sharp edged or round edged circular hole. There are three types of orifice plates namely

1. Concentric
2. Eccentric and
3. Segmental type.



**Figure.8 Types of Orifice Plates**

The concentric type is used for clean fluids. In metering dirty fluids, slurries and fluids containing solids, eccentric or segmental type is used in such a way that its lower edge coincides with the inside bottom of the pipe. This allows the solids to flow through without any obstruction. The orifice plate is inserted into the main pipeline between adjacent flanges, the outside diameters of the plate being turned to fit within the flange bolts. The flanges are either screwed or welded to the pipes.

### Applications

- The concentric orifice plate is used to measure flow rates of pure fluids and has a wide applicability as it has been standardized
- The eccentric and segmental orifice plates are used to measure flow rates of fluids containing suspended materials such as solids, oil mixed with water and wet steam.

### Advantages

- It is very cheap and easy method to measure flow rate
- It has predictable characteristics and occupies less space
- Can be used to measure flow rates in large pipes

### Limitations

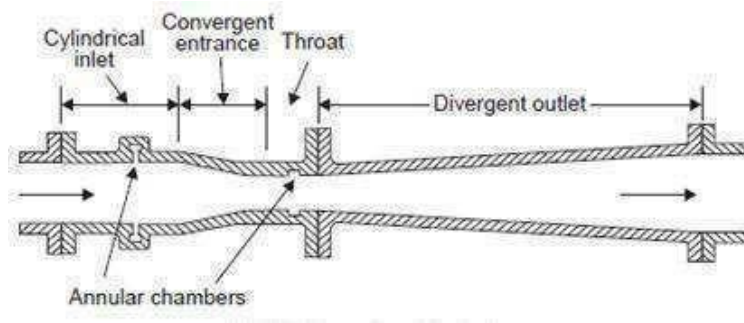
- The vena-contracta length depends on the roughness of the inner wall of the pipe and sharpness of the orifice plate. In certain case it becomes difficult to tap the minimum pressure due the above factor
- Pressure recovery at downstream is poor, that is, overall loss varies from 40 to 90% of the differential pressure.
- In the upstream straightening vanes are a must to obtain laminar flow conditions.
- The orifice plate gets corroded and due to this after sometime, inaccuracy occurs. The coefficient of discharge is low.

**Venturi Meter:** Venturi tubes are differential pressure producers, based on Bernoulli's Theorem. General performance and calculations are similar to those for orifice plates. In these devices, there is a continuous contact between the fluid flow and the surface of the primary device.

It consists of a cylindrical inlet section equal to the pipe diameter, a converging conical section in which the cross sectional area decreases causing the velocity to increase with a corresponding increase in the velocity head and a decrease in the pressure head; a



cylindrical throat section where the velocity is constant so that the decreased pressure head can be measured and a diverging recovery cone where the velocity decreases and almost all of the original pressure head is recovered. The unrecovered pressure head is commonly called as head loss.



**Figure.9 Long form Venturi**

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \frac{v_2^2}{2}$$

where

$p$  is pressure ( $\text{N/m}^2$ )

$v$  is velocity ( $\text{m/s}$ )

$\rho$  is the density of the liquid ( $\text{kg/m}^3$ ).

$$\therefore \dot{Q} = \frac{a_1 a_2}{\sqrt{(a_1^2 - a_2^2)}} \sqrt{\frac{2}{\rho} (p_1 - p_2)} \text{ m}^3/\text{s}$$

$$\dot{Q} = a_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

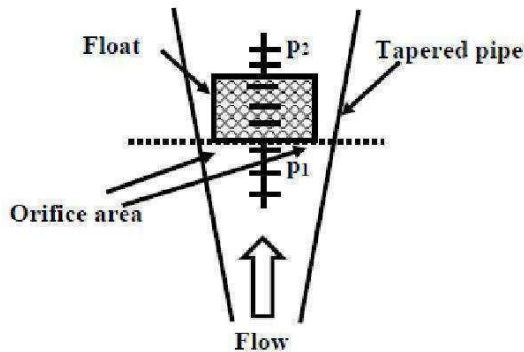
**Limitations:** This flow meter is limited to use on clean, non-corrosive liquids and gases, because it is impossible to clean out or flush out the pressure taps if they clog up with dirt or debris.

**Rotameter:** The orificemeter, Venturimeter and flow nozzle work on the principle of constant area variable pressure drop. Here the area of obstruction is constant, and the pressure drop changes with flow rate. On the other hand Rotameter works as a constant pressure drop variable area meter. It can be only be used in a vertical pipeline. Its accuracy is also less (2%) compared to other types of flow meters. But the major advantages of rotameter are, it is simple in construction, ready to install and the flow rate can be directly seen on a calibrated scale, without the help of any other device, e.g. differential pressure sensor etc. Moreover, it is useful for a wide range of variation of flow rates (10:1).

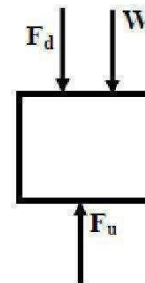
The basic construction of a rotameter is shown in figure. It consists of a vertical pipe, tapered downward. The flow passes from the bottom to the top. There is cylindrical type metallic float inside the tube. The fluid flows upward through the gap between the tube and the float.



As the float moves up or down there is a change in the gap, as a result changing the area of the orifice. In fact, the float settles down at a position, where the pressure drop across the orifice will create an upward thrust that will balance the downward force due to the gravity. The position of the float is calibrated with the flow rate.



**Figure.10 Rotameter**



**Figure.11 Force acting on float**

$\gamma_1$  = Specific weight of the float

$\gamma_2$  = specific weight of the fluid

$v$  = volume of the float

$A_f$  = Area of the float.

$A_t$  = Area of the tube at equilibrium (corresponding to the dotted line)

$$Q = \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2g}{\gamma_2} (p_1 - p_2)}$$

$F_d$  = Downward thrust on the float

$F_u$  = Upward thrust on the float

The major source of error in rotameter is due to the variation of density of the fluid. Besides, the presence of viscous force may also provide an additional force to the float.

### Applications:

- Can be used to measure flow rates of corrosive fluids
- Particularly useful to measure low flow rates

**Advantages:**

- Flow conditions are visible
- Flow rate is a linear function(uniform flow scales)
- Can be used to measure flow rates of liquids, gases and vapour
- By changing the float, tapered tube or both, the capacity of the rotameter can be changed.

**Limitations:**

- They should be installed vertically
- They cannot be used for measurements in moving objects
- The float will not be visible when coloured fluids are used, that is, when opaque fluid is used.
- For high pressure and temperature fluid flow measurements, they are expensive
- They cannot be used for fluids containing high percentage of solids in suspension.

## UNIT-4

**Theory of strain gauge:** The change in the value of resistance by straining the gauge may be partly explained by the normal dimensional behavior of elastic material. If a strip material is subjected to tension as shown in figure or in other words positively strained, its longitudinal dimension will increase while there will be a reduction in the lateral dimension. So when a gauge is subjected to a positive strain, its length increases while its area of cross section decreases as shown in figure.

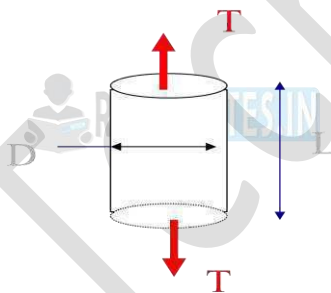
Since the resistance of a conductor is proportional to its length and inversely proportional to its area of cross section, the resistance of the gauge increases with positive strain. The change in the value of resistance of strained conductor is more than what can be accounted for an increase in resistance due to dimensional changes. the extra change in the value of resistivity of a change in the value of resistivity of a conductor when strained. This property, as described earlier is known as piezoresistive effect.

**Strain Defined:** Strain is defined as relative elongation in a particular direction

$$\epsilon_a = dL/L \text{ (axial strain)}$$

$$\epsilon_t = dD/D \text{ (transverse strain)}$$

$$\mu = \epsilon_t / \epsilon_a \text{ (Poisson's ratio)}$$



**Strain gauges:** The electrical resistance of a conductor changes when it is subjected to a mechanical deformation



Resistance =  $f(A...)$

Electrical 'esistance (' is a function of...

$\rho$                       Resistivity of the material (Ohms\*m)

L the length of the conductor (m)  
A the cross-sectional area of the conductor (m<sup>2</sup>)  
 $R = \rho * L/A$

Note R increases with:

- Increased material resistivity
- Increased length of conductor (wire)
- Decreased cross-sectional area (or diameter)
- Increased the temperatures

### Deriving the Gauge Factor (GF)

- We know that L and A both change as a wire is stretched it is reasonable to think that we can rewrite the equation

$$R = \rho * L/A$$

to relate strain to changes in resistance.

- Start with the differential:  $dR = d\rho * (L/A) + \rho * d(L/A)$

Expanding with the chain rule again one gets:

$$dR = d\rho * (L/A) + \rho/A * dL + \rho * L * (-1/A^2) * dA$$

- Divide left side by R and right side by equivalent ( $\rho * L/A$ ) to get:

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Substituting into the equation

$$A = \pi \left( \frac{D}{2} \right)^2, \text{ so } dA = \pi (2) \left( \frac{D}{2} \right) dD, \text{ or } \frac{dA}{A} = 2 \frac{dD}{D} = 2\varepsilon_t$$

$$\text{also, } \frac{dL}{L} = \varepsilon_a, \text{ so } \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2\varepsilon_t$$

Nothing the definition of Poisson's ratio

$$\frac{dR}{R} = \varepsilon_a (1 - 2\mu) + \frac{d\rho}{\rho}, \text{ or } GF \equiv \frac{dR/R}{\varepsilon_a} = 1 + 2\mu + \frac{1}{\varepsilon_a} \frac{d\rho}{\rho}$$

Hence, we define the Gauge Factor GF as:

$$GF = 1 + 2\mu$$

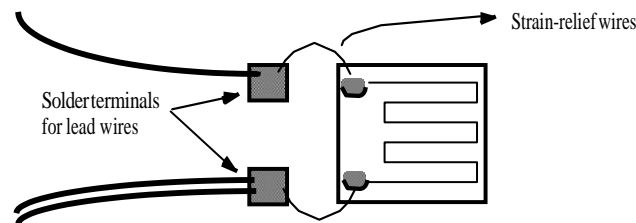
### Using Gauge Factors with Strain Gauges

$$GF = 1 + 2\mu$$

$$\varepsilon_a = \frac{1}{GF} \frac{\Delta R}{R}$$

In most applications DR and  $\epsilon$  are very small and so we use sensitive circuitry (amplified and filtered bridge circuit) contained within a strain-indicator box to read out directly in units of micro-strain. Hence this strain-indicator will require R (gauge nominal resistance) and GF (gauge factor)

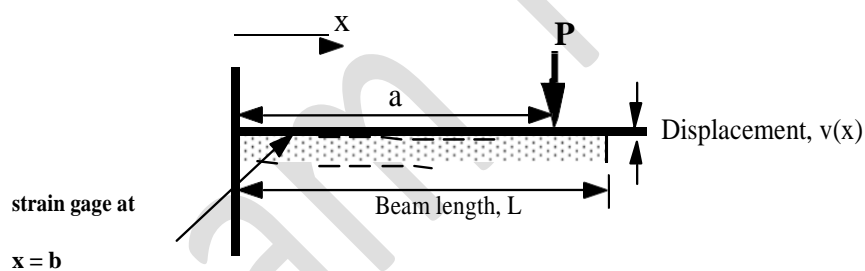
### Typical Strain Gauge:



### Steps for Installing Strain Gauges:

- Clean specimen – degreaser
- This are Chemically prepare gauge – Wet abrading with M-Prep Conditioner and Neutralizer
- Mount gauge and strain relief terminals on tape, align on specimen and apply adhesive
- Solder wire connections
- Test

### Beam Loading Example

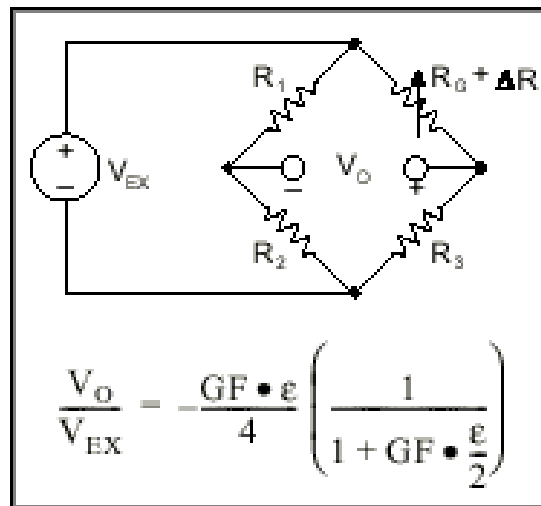


### Measuring Strain with a Bridge Circuit:

- A quarter-bridge circuit is one in which a simple Wheatstone bridge is used and one of the resistors is replaced with a strain gauge.
- $V_o$  may still be small such that amplification ( $Amp > 1.0$ ) is usually desirable

$$\epsilon \cong \frac{4}{Amp} \frac{V_o}{V_{ex}} \frac{1}{GF}$$

- Note:  $V_o$  and  $V_{ex}$  are also sometimes labeled as  $E_o$  and  $E_i$  (or  $E_{ex}$ )

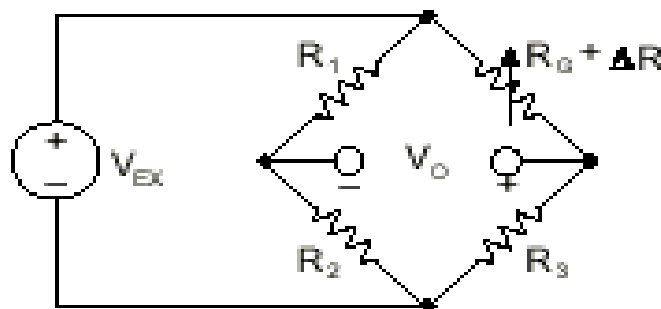


### Current (i) Limitations:

- In general gauges cannot handle large currents
- The current through the gage will be driven by the voltage potential across it.

Note: Text denotes the excitation voltage as  $V_i$ . It is also often labeled  $V_e$  or  $V_{ex}$ .

$$i_G = \frac{V_G}{R_G} = \frac{V_{EX}}{R_G + R_3}$$



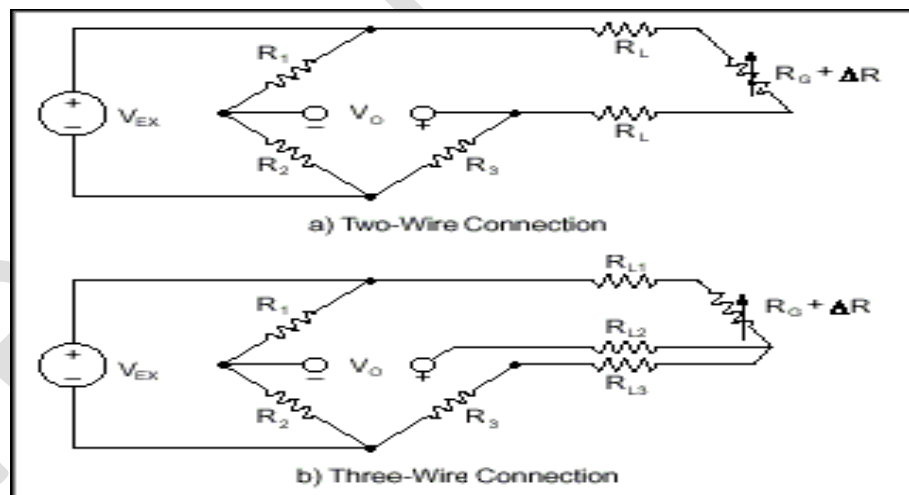
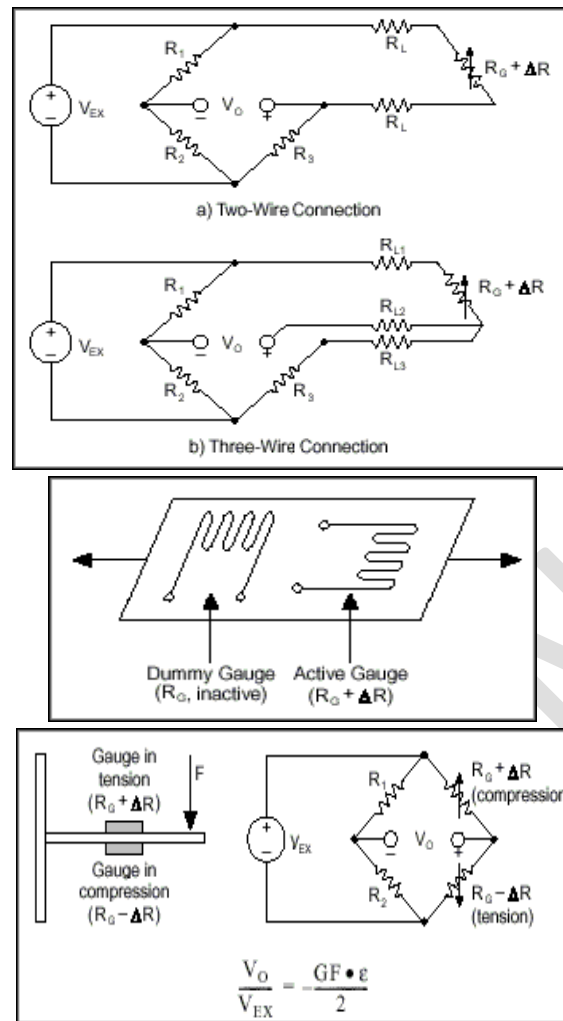
### Measuring Strain with a Strain-Indicator:

- First install a strain gauge
- Connect the wires from the strain gauge to the strain indicator.
- Apply loading conditions
- Read strain from strain indicator

Note that the indicator always displays 4 digits and reads in microstrain  
Thus, 0017 means 17 micro-inches / inch of strain.

### Strain gauge bridge enhancements:

- 3-wire combination addresses lead wire resistance
- Half-bridge– with a dummy gauge mounted transversely addresses gauge sensitivity to surface temperature
- Half bridge – amplification through use of dual gauges



### Theoretical Determination of Strain in a Loaded Cantilever Beam:

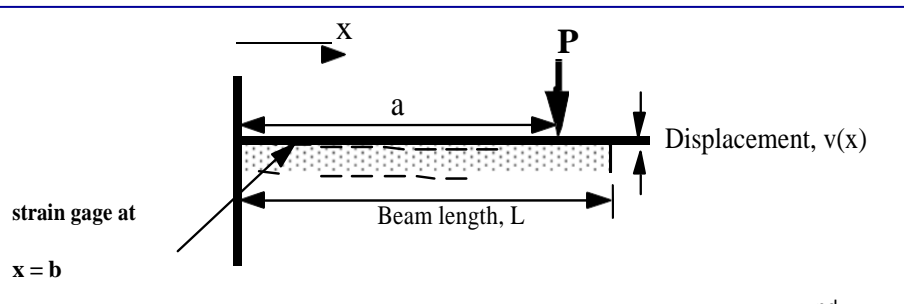
- You must either know the load  $P$  or the displacement ( $v$ )
- Determine displacement ( $v$ ) at  $x=a$
- to Know beam dimensions and material (and hence  $EI$ ) estimate the load  $P$

$$v = \frac{-Px^2(3a-x)}{6EI}, \text{ so } P = 3 \frac{vEI}{a^3}$$

- Calculate stress at location of gauge
- Calculate  $e$  from  $\sigma = \epsilon E$

$$\sigma = \frac{My}{I} = \frac{P \cdot b \cdot h / 2}{I}, \text{ where } h = \text{beam thickness}$$

**Strain Gauge Cantilever**



When the cantilever beam is “plucked” this will respond as a damped 2<sup>nd</sup> order system. The amplitude of vibration has the general form:

Where the damped frequency (what you measure) is related to the natural frequency ( $\omega_n$ ) by:

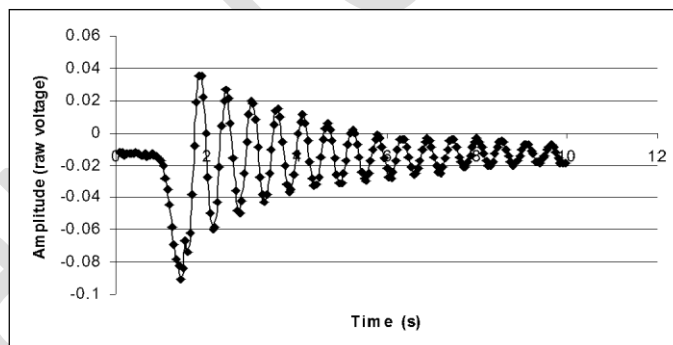
$$Y(t) = C e^{-\zeta \omega_n t} \sin(\omega_d t)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

The damping ratio (zeta) can be determined by plotting the natural log of the Amplitude or magnitude (M) vs time:

$$M(t) = C e^{-\zeta \omega_n t} \quad \text{so,} \quad \ln(M) = C_2 + (-\zeta \omega_n) \cdot t$$

So, the slope of the plot of  $\ln(M)$  vs.  $t$  is  $(-\zeta \omega_n)$



### Additional Considerations for natural frequency of “plucked” beams

- Note: Unless otherwise indicated, natural frequencies are expressed in terms of radians/sec.
- The natural frequency of a uniform beam is given by:

$$\omega_n = (1.875)^2 \sqrt{\frac{EI}{m' L^4}}$$

- $E$  is the modulus of elasticity,  $I$  is the moment of inertia about the centroid of the beam cross-section ( $bh^3/12$ ),  $m'$  is the mass per unit length of the beam (ie kg/m), and  $L$  is the cantilevered beam length



- If the beam is not uniform...

A mass at the end can be represented as an effective change in beam mass per unit length  
A hole in the end can be accounted for in a similar fashion...

### Types of Strain Gauges

1. Unbonded metal strain gauges
2. Bonded metal wire strain gauges
3. Bonded metal foil strain gauges
4. Vacuum deposited thin metal film strain gauges
5. Sputter deposited thin metal strain gauges
6. Bonded Semiconductor strain gauges.
7. diffused metal strain gauges.

**Unbonded Metal Strain Gauge:** An unbonded metal strain gauge is shown in Fig.1. This gauge consists of a wire stretched between two points in an insulating medium such as air. The wires are of copper nickel, chrome nickel or nickel iron alloys. The flexure element is connected via a rod to a diaphragm which is used for sensing of pressure. The wires are tensioned to avoid buckling when they experience a compressive force

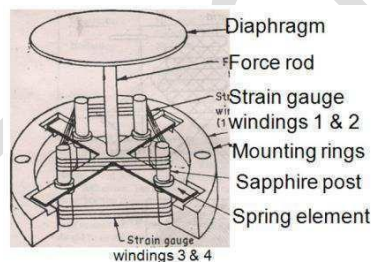


Fig.1

The unbonded metal wire gauges, used almost exclusively in transducer applications, employ preloaded

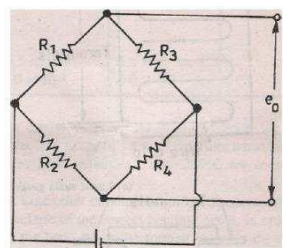


Fig.2

Resistance wires connected in a Wheatstone bridge as shown in fig.2 At initial preload, the strains and resistances of the four arms are nominally equal, with the result the output

voltage of the bridge,  $e_o = 0$ . Application of pressure produces a small displacement which is about 0.004 mm (full scale), the displacement increases tension in two wires and decreases it in the other two thereby increasing the resistance of two wires which are in tension and decreasing the resistance of the remaining two wires. This causes an unbalance of the bridge producing an output voltage which is proportional to the input displacement and hence to the applied pressure. Electric resistance of each arm is 120 to 1000, the input voltage to the bridge is 5 to 10 V, and the full scale output of the bridge is typically about 20 mV to 50 mV. Some of the unbonded metal wire gauges are shown in Fig. 3

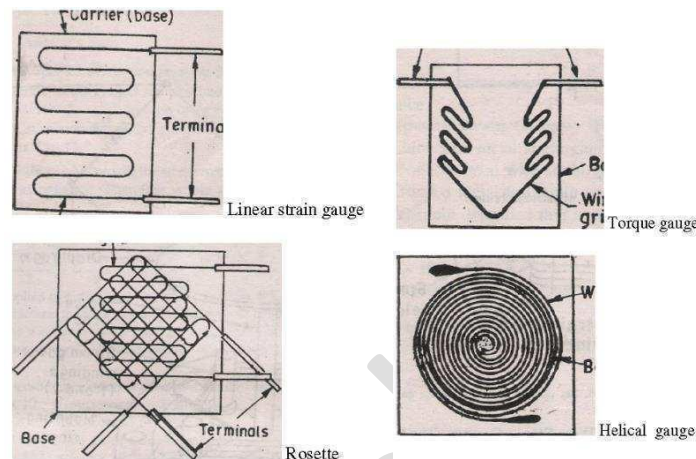


Fig.3

**Force:** Force is a measure of the interaction between bodies. Force takes a number of forms including short-range atomic forces, electromagnetic, and gravitational forces. Force is a vector quantity, with both direction and magnitude. If the forces acting on a body in equilibrium are summed around the periphery of the body then they add to zero. If there is any resultant force acting then the body is not in equilibrium and it will accelerate such that the rate of change of the body's momentum (velocity times mass) is equal to the force. If the body is held stationary in some way, then there will be reaction acting on the body from the support structure that is equal in magnitude and opposite in direction to the force imposed. Although the definition of force units (as given below) is based on acceleration of a free body, most force measurements are made on bodies in equilibrium, and are therefore measures of forces within a structure. Conceptually a structure can be 'cut' across any section and the forces acting within the body at that section are those which would act at the free surfaces if such a cut were made. This property is the basis of most force measurements – a physical support or link in a structure is replaced with a device that measures the forces acting at that point.

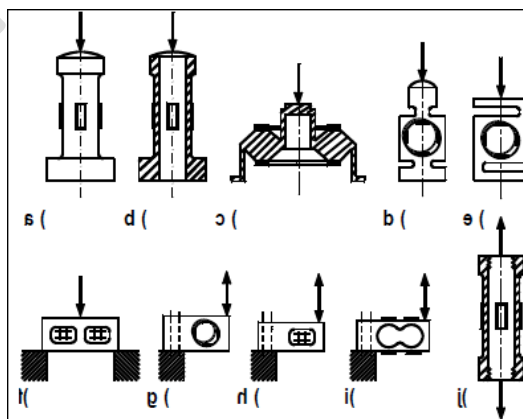
A force measurement system is made up of a transducer and associated instrumentation. The transducer is subjected to the force to be measured, and some resultant change in the element is measured by the associated instrumentation. The instrumentation may power the transducer in some way and also may process the output from the transducer before it is shown on an indicator to be read by the user. Strictly a transducer is a device that receives a physical stimulus and changes it into another measurable physical quantity through a

known relationship. In practice a force transducer is a chain of several transducers, for example the force may act upon a metal cylinder which is compressed by the force, the change in size alters the electrical resistance of a strain gauge bonded to the surface of the cylinder, and the instrumentation measures this change in resistance. In this guide the term force transducer will be used loosely to describe the part of the force measurement system which converts the applied force into an output which is measured by some associated instrumentation. For many types of force measurement system the term load cell is in common usage in place of force transducer. Also, the term device is from time to time used in place of transducer within the text of this guide to avoid distracting repetition of the word transducer. For the same reasons, the term measuring instrument will occasionally be used with the same meaning as force measurement system. As will be seen in the following sections, the instrumentation may be as simple as a dial gauge or as complex as a computer with associated analogue to digital converters and excitation circuitry. The indicated value is the output of the force measurement system, which may be in units of force or other units such as volts. If the indicated value is not in units of force, then the user may need to perform a calculation based on a calibration to obtain the calculated value.

**Strain gauge load cells** the most common type of force transducer, and one which is a clear example of an elastic device, is the strain gauge load cell,

The elastic element the shape of the elastic element used in load cells depends on a number of factors including the range of force to be measured, dimensional limits, and required performance and production costs. Figure 4 shows a selection of different elastic elements and gives their typical rated capacities. Each element is designed to measure the force acting along its principal axis, and not to be affected by other forces such as side loads. The arrows in the figure indicate the principal axis of each element.

The material used for the elastic element is usually tool steel, stainless steel, aluminium or beryllium copper, the aim being a material which exhibits a linear relationship between the stress (force applied) and strain (output) with low hysteresis and low creep in the working range. There also has to be high level of repeatability between force cycles to ensure that the load cell is a reliable measuring device. To achieve these characteristics it is usual to subject the material to a special heat treatment.



**Fig. 4 Typical elastic elements and their usual rated capacities**

### The electrical resistance strain gauge

In electrical terms, all **electrical resistance strain gauges** may be considered as a length of conducting material, like a wire, when a length of wire is subjected to a tension within its elastic limit, its length increases with corresponding decrease in its diameter and increase of its electrical resistance. If the conducting material is bonded to an elastic element under strain then the change in resistance may be measured, and used to calculate the force from the calibration of the device.

The most common materials used for the manufacture of strain gauges are copper-nickel, nickel-chromium, nickel-chromium-molybdenum and platinum-tungsten alloys. There are a variety of resistance strain gauges available for various applications, some of which are described below. Each strain gauge is designed to measure the strain along a clearly defined axis so that it can be properly aligned with the strain field.

The **foil strain gauge** is the most widely used type and several examples are shown in Figure 5. It has significant advantages over all other types of strain gauge and is employed in the majority of precision load cells.

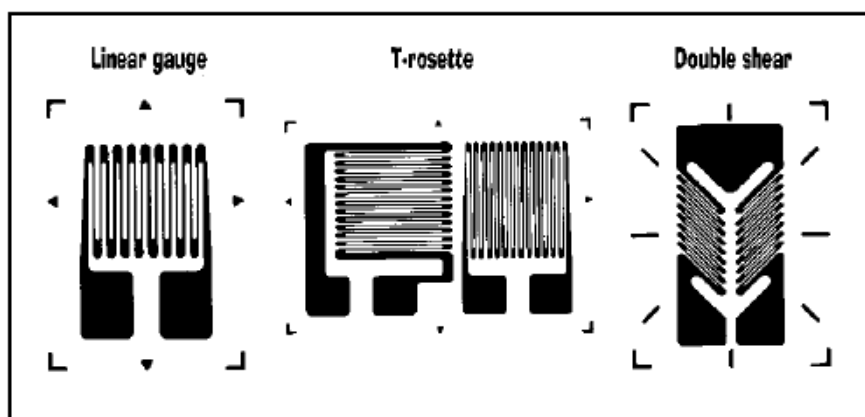


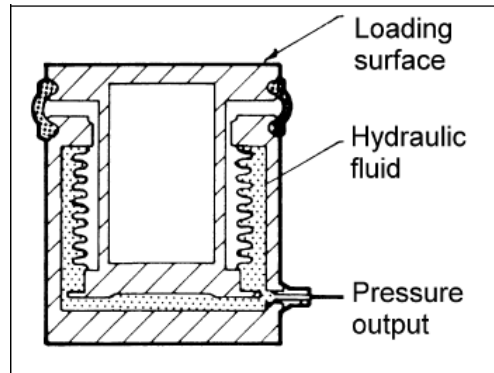
Fig 5 Typical metal foil strain gauges

### Measuring force through pressure

#### Hydraulic load cell

The hydraulic load cell is a device filled with a liquid (usually oil), which has a pre-load pressure. Application of the force to the loading member increases the fluid pressure, which is measured by a pressure transducer or displayed on a pressure gauge dial via a Bourdon tube.

When used with a pressure transducer, hydraulic load cells are inherently very stiff, deflecting only about 0.05 mm under full force conditions. Although capacities of up to 5 MN are available, most devices fall in to the range of 500 N to 200 kN. The pressure gauge used to monitor the force can be located several metres away from the device by the use of a special fluid-filled hose. In systems where more than one load cell is used a specially designed totaliser unit has to be employed.



**Fig. 6 An example of a hydraulic load cell**

Hydraulic load cells are self-contained and need no external power. They are inherently suitable for use in potentially explosive atmospheres and can be tension or compression devices. Uncertainties of around 0.25 % can be achieved with careful design and favourable application conditions. Uncertainties for total systems are more realistically 0.5 % - 1 %. The cells are sensitive to temperature changes and usually have facilities to adjust the zero output reading, the temperature coefficients are of the order of 0.02 % to 0.1 % per °C.

#### **Pneumatic load cell**

The operating principles of the pneumatic load cell are similar to those of the hydraulic load cell. The force is applied to one side of a piston or a diaphragm of flexible material and balanced by pneumatic pressure on the other side. This counteracting pressure is proportional to the force and is displayed on a pressure dial.

The sensing device consists of a chamber with a close-fitting cap. The air pressure is applied to the chamber and builds up until it is equal to the force on the cap. Any further increase in pressure will lift up the cap allowing the air to bleed around the edge until pressure equilibrium is achieved. At this equilibrium position the pressure in the chamber is an indication of the force on the cap and can be read by the pneumatic pressure dial gauge.

#### **Measurement of torque on rotating shafts**

For rotating motion, power is the product of torque and angular velocity

$$P = M \cdot \omega = M \cdot 2\pi \cdot n$$

Thus, to determine the power of the rotating motion the torque (M) and the revolution number (n) must be measured.

Measurement of the revolution number from the point of view of the measuring concept the instruments measuring the revolution number can be divided into three groups:

- speed indicators measuring the average revolution number,
- Tachometers measuring the momentary revolution number and
- Stroboscopes working on the principle of comparison.

a) Measurement of small revolution number can be performed simply with stopwatch and by counting revolutions with naked eye. When the mark on the rotating machine part gets to the marked position, we start the stopwatch and begin counting (with 0). Having measured the time (T) and the number of revolutions (N) the revolution number is simply  $n = N/T$ .

b) For higher speed of rotation a special counting device must be used. One of the simplest of these is the so-called jumping-figure speed counter. The rotating shaft of this device turns gears. One of them completes one revolution while the other rotates only  $1/10$ , and so on. Reading the numbers uniformly painted from 0-9 on the cylinder jacket we get the number of revolutions. Such a device is used in kilowatt-hour meters, water consumption, tape recorders, speedometers of cars etc.

c) Mechanical tachometers count the revolutions only for a fixed time, generally for 6 seconds. The time measuring device of the instrument connects its pointer for 6 seconds with that shaft of the instrument which joints the rotating machine part. After these six seconds there is no more connection which means at the same time the end of the measurement. A widely used example of this device is the Jacquet indicator. With pressing the starting button the instrument is zeroed and after releasing it the counting and the clockwork starts.

d) Electric tachometers operate with the same principle (counting the number of revolutions during some period of time), but the number of revolutions is measured in an optical way.

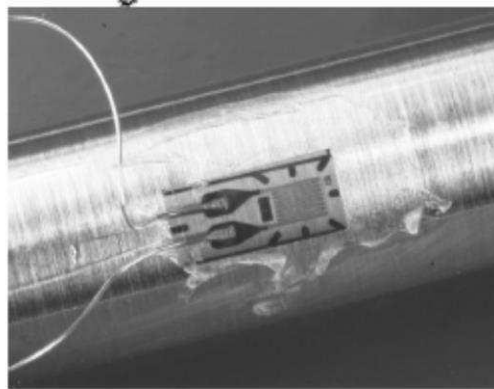
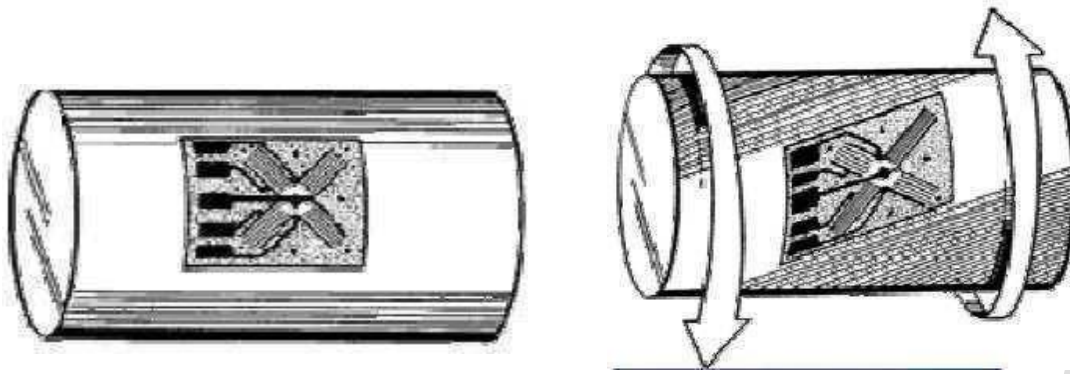
e) A stroboscope, also known as a strobe, is an instrument used to make a cyclically moving object appear to be slow-moving, or stationary. In its simplest form, a marker is placed to the rotating shaft and a lamp capable of emitting brief and rapid flashes of light is used. The frequency of the flash is adjusted so that it equals to the shaft's cyclic speed, at which point the object is seen to be either stationary or moving backward or forward, depending on the flash frequency.

**Measurement of torque** There are many ways of measuring torque, out of which the two most important ones are

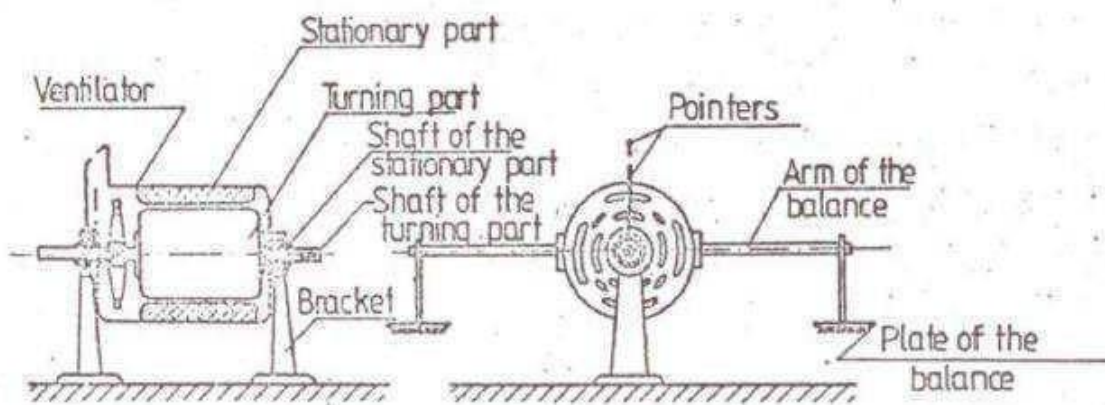
- Strain gauges and
- balancing motors

a) A strain gauge is a small electrical 'element' printed on a non-conductive substrate. The pattern of the element is arranged so that if the gauge is stretched (or compressed) in one direction (along operating axis of the gauge), the resistance of the element increases (or decreases) in relation to that stretch. A stretch perpendicular to the axis of the strain gauge has little effect on the resistance of the element. If a gauge is bonded to the shaft, with its axis aligned with the direction in which the shaft material stretches when a torque is applied, the strain gauge will also stretch and therefore the element will increase in resistance. By measuring the change of resistance, after appropriate calibration, one can measure the torque applied to the shaft.





b) Balancing machines (motor or generator) are special machines, whose housing is free to rotate and arms are mounted onto it.



## UNIT-5

**Introduction:** Automated control systems are becoming more common in new road vehicles. In general, automation is designed to assist with mechanical or electrical accomplishment of tasks (Wickens & Hollands, 2000). It involves actively selecting and transforming information, making decisions, and/or controlling processes (Lee & See, 2004). Automated vehicle control systems are intended to improve safety (crash avoidance and mitigation), comfort (decrease of driver's workload; improved driving comfort), traffic efficiency (road capacity usage; reduced congestion), and the environment (decreased traffic noise; reduced fuel consumption).

The automation of basic control functions (e.g., automatic transmission, anti-lock brakes and electronic stability control) has proven very effective, but the safety implications of more advanced systems may be unclear in some cases. It is controversial that system safety will always be enhanced by allocating functions to automatic devices rather than to the drivers. A potential concern may be the out-of-loop performance problems that have been widely documented as a potential negative consequence of automation (e.g., Weiner & Curry, 1980).

**System** – An interconnection of elements and devices for a desired purpose.

**Control System** – An interconnection of components forming a system configuration that will provide a desired response.

**Process** – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.

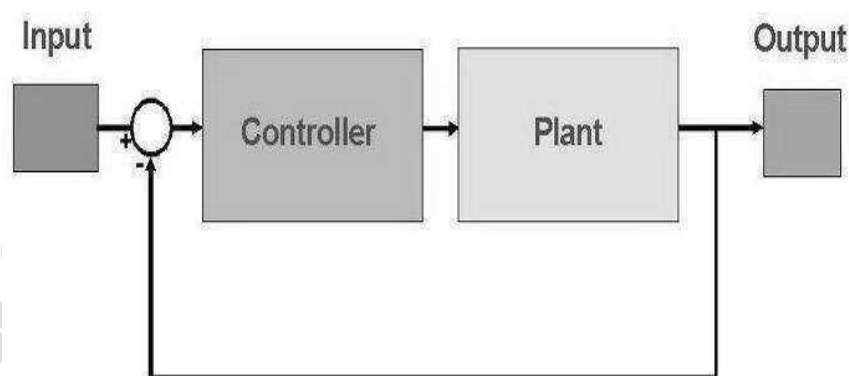


Fig.1

Control engineering is based on the foundations of feedback theory and linear system analysis, and it generates the concepts of network theory and communication theory.

Accordingly, control engineering is not limited to any engineering discipline but is applicable to aeronautical, chemical, mechanical, environmental, civil, and electrical engineering.

A control system is an interconnection of components forming a system configuration that will provide a desired system response. The basis for analysis of a system is the foundation



provided by linear system, which assumes a cause effect relationship for the components of a system.

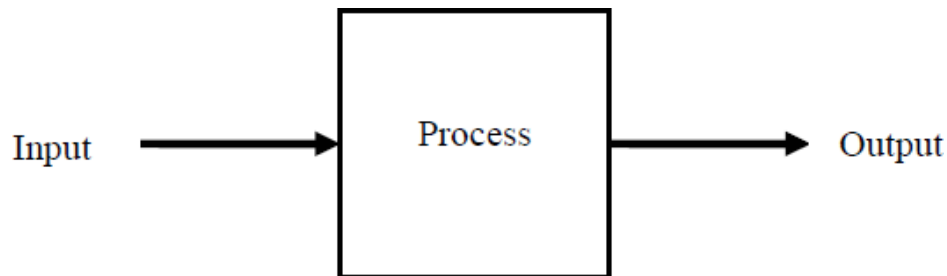


Fig.2: Basic Control system

An open-loop control system utilizes a controller or control actuator to obtain the desired response as shown in Figure. The open-loop control system utilizes an actuating device to control the process directly without using device. An example of an open-loop control system is an electric toaster.

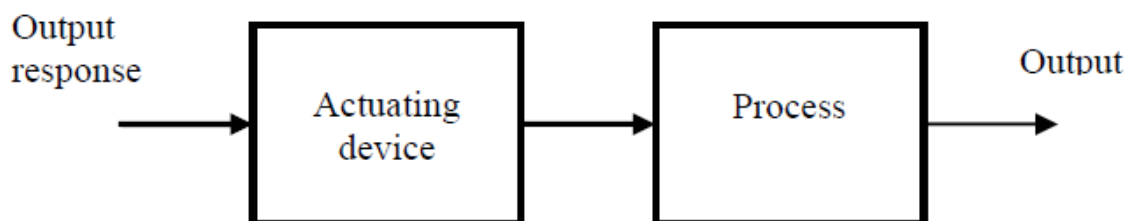


Fig.3: Open loop control system

A **closed-loop control system** utilizes an additional measure of the actual output to compare the actual output with the desired output response. The measure of the output is called the **feedback signal**. A feedback control system is a control system that tends to maintain a relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control. As the system is becoming more complex, the interrelationship of many controlled variables may be considered in the control scheme. An example of closed-loop control system is a person steering an automobile by looking at the auto's location on the road and making the appropriate adjustments.

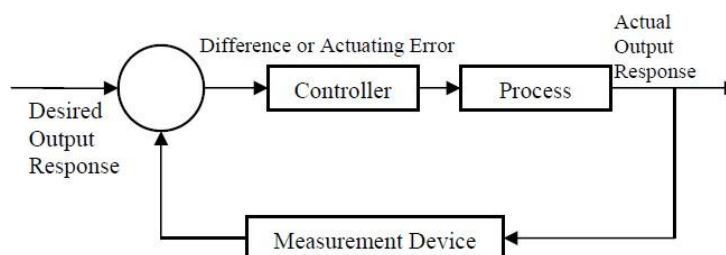
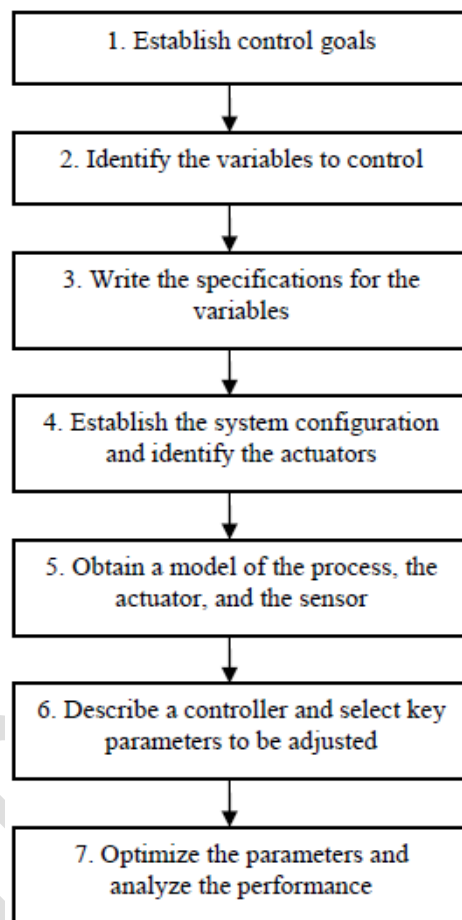


Fig.4: Close loop control system

## CONTROL SYSTEM DESIGN

- Variables to control are the quantities or conditions that are measured and controlled.
- Process is a natural, progressively continuing operation marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward certain result or end.
- A system is a combination of components that act together and perform a certain objective.



### Mathematical modelling of Dynamic Systems:

**Linear Systems:** For linear systems the principle of superposition is valid, and the response to a complex input can be calculated by summing up the responses to its components.

### Linear Time Invariant (LTI) Systems versus Linear Time Varying Systems

#### 1. Linear Time Invariant (LTI) Systems = systems:

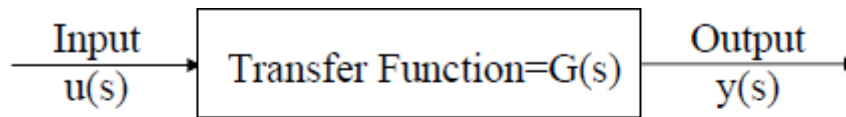
- represented by lumped components,
- described by linear differential equations
- Parameters of the equations are time invariant.

1. Systems with parameters that vary in time are called linear time varying systems.

### Transfer Function

For zero initial condition

$$\text{Transfer Function} = G(s) = \frac{L\{\text{output}\}}{L\{\text{input}\}}$$



$$G(s) = \frac{y(s)}{u(s)}$$

The transfer function of a system represents the link between the input to the system to the output of the system. The transfer function of a system  $G(s)$  is a complex function that describes system dynamics in s-domains opposed to the differential equations that describe system dynamics in time domain. The transfer function is independent of the input to the system and does not provide any information concerning the internal structure of the system.

Same transfer function can represent different systems. The transfer function permits to calculate the output or response for various inputs.

The transfer function can be calculated analytically starting from the physics equations or can be determined experimentally by measuring the output to various known inputs to the system.

### Impulse Response

The Laplace transform of an impulse function  $\delta(t)$  is given by  $L\{\delta(t)\} = 1$

The output of a system due to an impulse input  $u(s) = \delta(s) = 1$  is  $y(s) = G(s) \cdot u(s) = G(s)$

The impulse response of a system is identical to the transfer function of that system.

The inverse Laplace transforms of the impulse response  $G(s)$

$$L^{-1}\{G(s)\} = g(t)$$

The transfer function, that contains complete information about the dynamic characteristics of a system, can be obtained by applying an impulse input  $u(t) = \delta(t)$  and measuring system response  $y(t)$  which in this case is identical to  $g(t)$ . The transfer function will then be

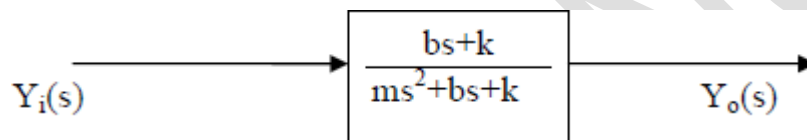
$$G(s) = L\{g(t)\}$$

**Block Diagrams:** Block diagrams are a graphical representation of the system model. The blocks represent physical or functional components of the system. Each block has inscribed the transfer function of that component relate the output of the component to its input.

Block diagrams consist of

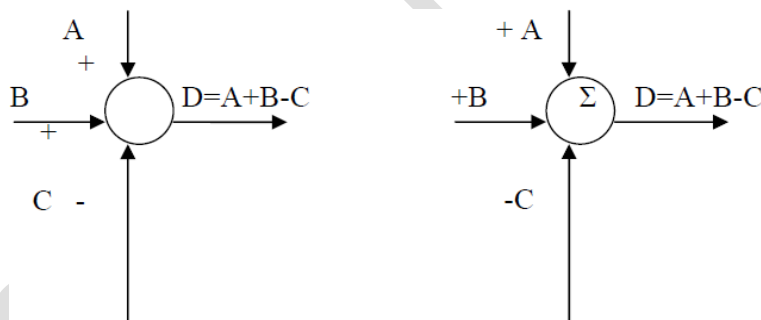
1. Blocks
2. Summation junctions
3. Paths
4. Branching points

### Block



Blocks represent physical or functional components in the system. In the block is inscribed the transfer function of that component of the system.

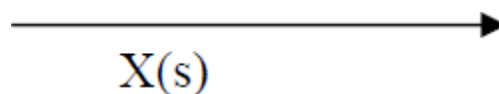
### Summation Junction



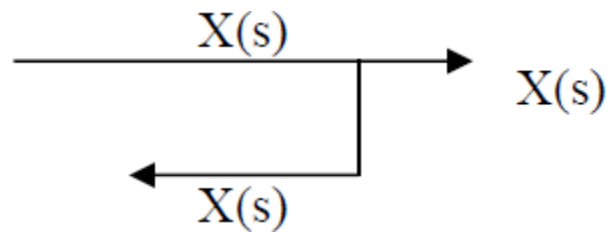
A summing junction results in the addition or subtraction of input signals for a single output.

### Path

Signal  $X(s)$  flows along the directed path:



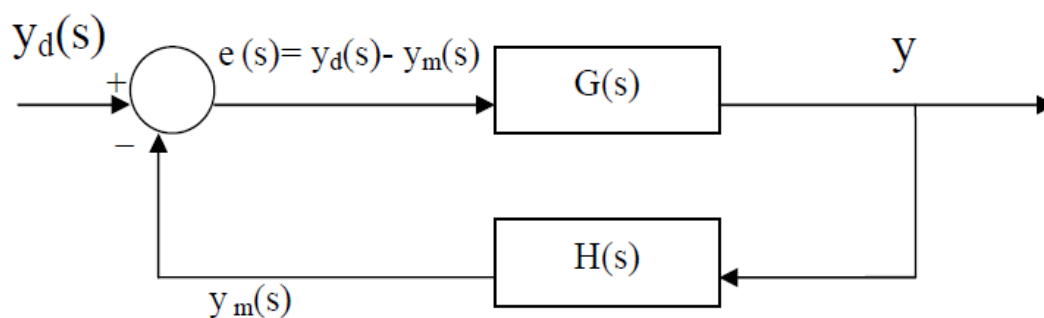
## Branching Point



At the branching point a signal splits into two signals of the same value.

## Block Diagram of a Closed Loop System

In a closed loop control system, also called feedback control system, the output variable  $y(s)$  is measured as  $y_m(s)$ , subtracted from its desired value  $y_d(s)$  to calculate the error  $e(s) = y_d(s) - y_m(s)$ .



$G(s)$  is feed forward transfer function (of the controller, actuator and the system)

$$G(s) = y(s) / e(s)$$

$H(s)$  is feedback transfer function (of the sensor)

$$H(s) = y_m(s) / y(s)$$

$G(s) H(s)$  = open loop transfer function

$$G(s) H(s) = [y_m(s) / y(s)] [y(s) / e(s)] = y_m(s) / e(s)$$

$y_d(s) / y(s)$  is closed loop transfer function obtained from the above equations by eliminating

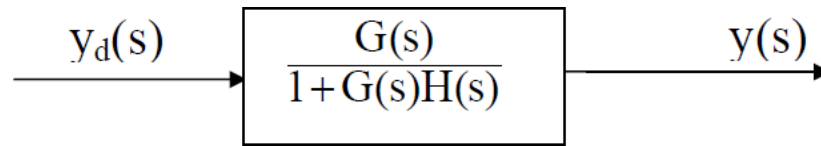
$e(s)$  and  $y_m(s)$

$$y(s) = G(s) e(s) = G(s) [y_d(s) - y_m(s)] = G(s) [y_d(s) - H(s)y(s)]$$

$$y(s) + G(s) H(s) y(s) = G(s) y_d(s)$$

$$y(s) / y_d(s) = G(s) / [1 + G(s) H(s)]$$

This equation gives the single block equivalent of the above closed loop system



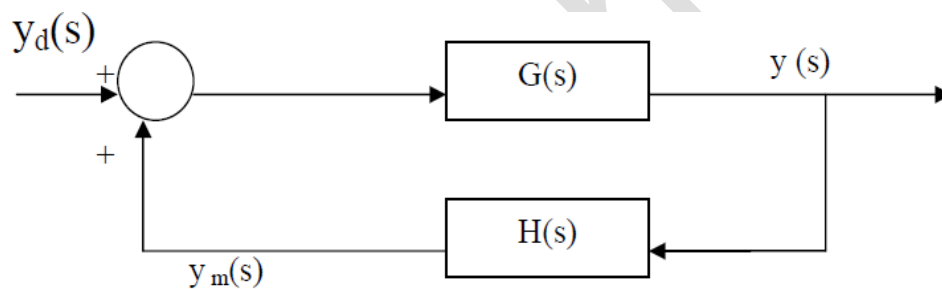
The transfer function represents the closed loop system dynamics with complex functions.

The output  $y(s)$  is given by

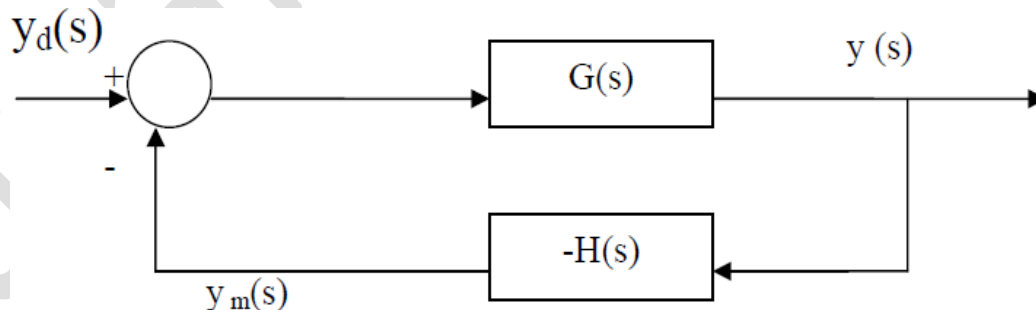
$$y(s) = G(s) / [1 + G(s) H(s)] y_d(s)$$

and depends on the closed loop transfer function and the desired value of the output  $y_d(s)$ , called also the input (to the closed loop system).

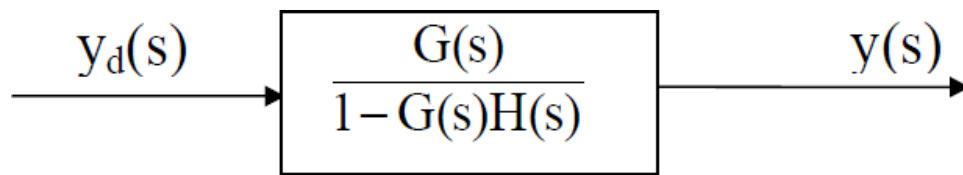
The following positive feedback block diagram



Is equivalent to negative feedback one if  $H(s)$  is replaced by  $-H(s)$ .



This is equivalent to the block.

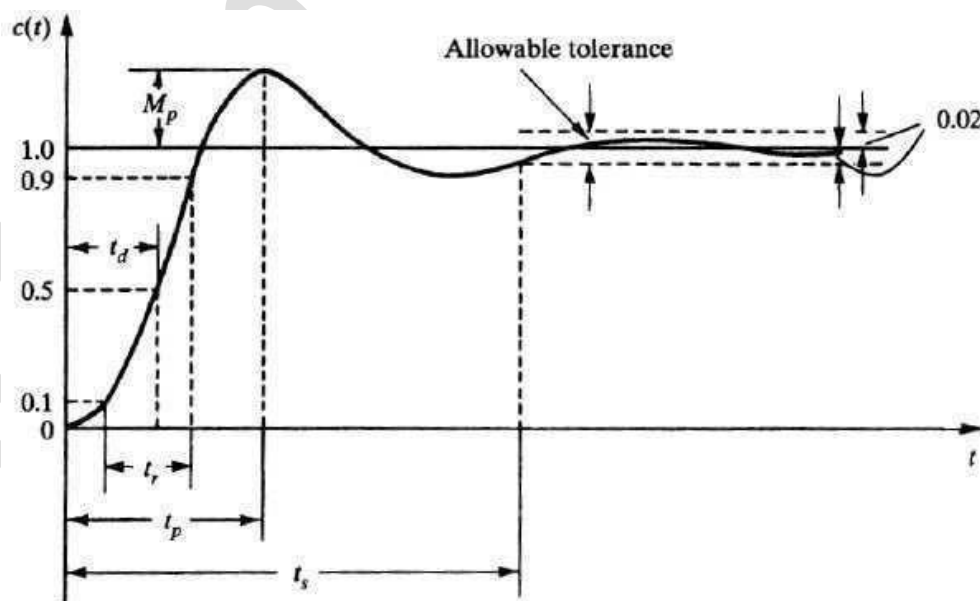


### Transient Response Specifications

Because systems that store energy cannot respond instantaneously, they exhibit a transient response when they are subjected to inputs or disturbances. Consequently, the transient response characteristics constitute one of the most important factors in system design.

In many practical cases, the desired performance characteristics of control systems can be given in terms of transient-response specifications. Frequently, such performance characteristics are specified in terms of the transient response to unit-step input, since such an input is easy to generate and is sufficiently drastic. (If the response of a linear system to a step input is known, it is mathematically possible to compute the system's response to any input). The transient response of a system to a unit step-input depends on initial conditions. For convenience in comparing the transient responses of various systems, it is common practice to use standard initial conditions: The system is at rest initially, with its output and all time derivatives thereof zero. Then the response characteristics can be easily compared.

The transient response of a practical control system often exhibits damped oscillations before reaching a steady state. In specifying the transient-response characteristics of a control system to a unit-step input, it is common to name the following:



Delay Time =  $T_d$

Risk Time =  $T_r$

Peak Time =  $T_p$

Maximum Overshoots =  $M_p$

Settling Time =  $T_s$

**Signal flow graph of control system** is further simplification of block diagram of control system. Here, the blocks of transfer function, summing symbols and take off points are eliminated by branches and nodes.

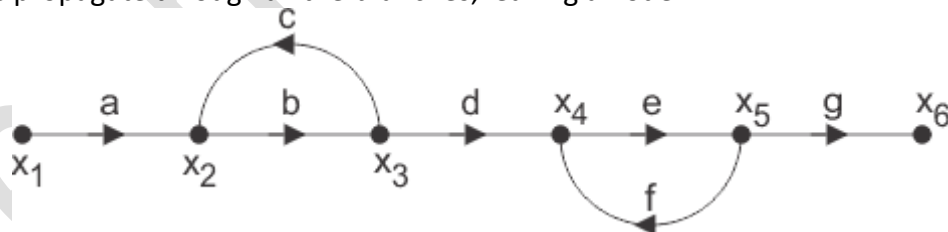
The transfer function is referred as transmittance in signal flow graph. Let us take an example of equation  $y = Kx$ . This equation can be represented with block diagram as below



The same equation can be represented by signal flow graph, where  $x$  is input variable node,  $y$  is output variable node and  $a$  is the transmittance of the branch connecting directly these two nodes.

#### Rules for Drawing Signal Flow Graph

1. The signal always travels along the branch towards the direction of indicated arrow in the branch.
2. The output signal of the branch is the product of transmittance and input signal of that branch.
3. Input signal at a node is summation of all the signals entering at that node.
4. Signals propagate through all the branches, leaving a node.



#### Simple Process of Calculating Expression of Transfer Function for Signal Flow Graph

- First, the input signal to be calculated at each node of the graph. The input signal to a node is summation of product of transmittance and the other end node variable of each of the branches arrowed towards the former node.
- Now by calculating input signal at all nodes will get numbers of equations which relating node variables and transmittance. More precisely, there will be one unique equation for each of the input variable node.



- By solving these equations we get, ultimate input and output of the entire signal flow graph of control system.
- Lastly by dividing inspiration of ultimate output to the expression of initial input we calculate the expiration of transfer function of that signal flow graph.

