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Let $u = x - 26$ & $v = y - 17$
where 26 and 17 are assumed means of x and y series respectively;
here $n = 10$

Mean of $x = \bar{x} = \frac{\Sigma x}{n} = \frac{256}{10} = 25.6$

Mean of $Y = \bar{Y} = \frac{\Sigma Y}{n} = \frac{172}{10} = 17.2$

x	$u = x - 26$	u^2	Y	$V = Y - 17$	V^2	uv
25	-1	1	18	1	1	-1
22	-4	16	15	-2	4	-16
28	2	4	26	3	9	18
26	0	0	17	0	0	0
35	9	81	22	5	25	~~45~~
20	-6	36	14	-3	9	18
22	-4	16	16	-1	1	4
40	14	196	21	4	16	56
20	-6	36	15	-2	4	12
18	-8	64	14	-3	9	24
Σx 256	-4	450	172	2	78	~~340~~ 172

y on $x \Rightarrow b_{yx} = \dfrac{\Sigma uv - \frac{\Sigma u \cdot \Sigma v}{n}}{\left\{\Sigma u^2 - \frac{(\Sigma u)^2}{n}\right\}}$

$$= \frac{\frac{340}{172} - \frac{(-4)\times 2}{10}}{450 - \frac{16}{10}}$$

$$= \frac{\cancel{170.8}}{\cancel{448.4}} \quad \frac{172.8}{448.4}.$$

$$byx = \underline{\underline{\cancel{0.460}}} \quad \underline{\underline{0.385}}$$

$\Rightarrow$ x on y $= bxy = \dfrac{\Sigma uv - \dfrac{\Sigma u \cdot \Sigma v}{n}}{\Sigma v^2 - \dfrac{(\Sigma v)^2}{n}}$

$$= \frac{172 + \frac{8}{10}}{78 - 0.4}$$

$$= \underline{\underline{2.226}}$$

$\Rightarrow$ Regression line of y on x:

$$y - \bar{y} = byx(x - \bar{x})$$

$$y - 17.2 = 0.385(x - 25.6)$$

$$0.385x - y = \cancel{27.1456} = 0$$

$$y = 0.385x + 7.34 \quad — \text{①}$$

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Regression line of x on y:

$$x - \bar{x} = bxy(y - \bar{y}$$

$$x - 25.6 = 2.226(y - 17.2)$$

$$x - 25.6 = 2.226y - 38.28$$

$$x = 2.226y - 12.68 \quad \text{--- (2)}$$

When age of wife is 19

$$x = 42.294 - 12.68$$

Age of husband $x = \underline{29.614} \simeq 30$

Age of wife when age of husband is 30

$$y = 0.385x + 7.34$$

$$= 0.385 \times 30 + 7.34$$

$$= 18.89$$

$$\simeq 19$$

Coefficient of Correlation $= \sqrt{byx \times bxy}$

$$= \sqrt{0.384 \times 2.23}$$

$$= \underline{0.925}$$

Qu: Calculate the Coefficient of regression line and find the two line of regression from the following data.

x : 78 89 97 69 59 79 68 61

y : 125 137 156 112 107 136 123 108

$\Sigma x = \frac{600}{8} = 75$

$\Sigma y = \frac{1004}{8} = 125.5$

x	u = x − $\bar{x}$	u^2	y	V = y − $\bar{y}$	v^2	uv
78	9	81	125	13	169	117
89	20	400	137	25	625	500
97	28	784	156	44	1936	1232
69	0	0	112	0	0	0
59	-10	100	107	-5	25	50
79	10	100	136	24	576	240
68	-1	1	123	11	121	-11
61	-8	64	108	-4	16	32
Total 600	48	1530	1004	108	3468	2160

y on x $b_{yx} = \dfrac{\Sigma uv - \dfrac{\Sigma u \cdot \Sigma v}{n}}{\left\{\Sigma u^2 - \dfrac{(\Sigma u)^2}{n}\right\}}$

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$$= \frac{2160 - \frac{48 \times 108}{8}}{1530 - \frac{2304}{8}}$$

$$= \frac{1512}{1242} = \underline{\underline{1.2173}}$$

$$\boxed{byx = 1.2173}$$

x on y $bxy = \dfrac{\Sigma uv - \frac{\Sigma u \cdot \Sigma v}{n}}{\left\{\Sigma v^2 - \frac{(\Sigma v)^2}{n}\right\}}$

$$= \frac{2160 - \frac{48 \times 108}{8}}{3468 - \frac{11664}{8}}$$

$$= \frac{1512}{2010} = \underline{\underline{0.7522}}$$

$$r = \sqrt{bxy \times byx}$$

$$= \sqrt{0.7522 \times 1.2173}$$

$$= \underline{\underline{0.956}}$$

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Ques. Calculation check

The following data represent rainfall x and yield of paddy per hectare y in a particular area. Find the line of regression of x on y

x : 113 102 95 120 140 130 125

y : 1.8 1.5 1.3 1.9 1.1 2.0 1.7

x	$u = x - \bar{x}$	u^2	y	$v = y - \bar{y}$	v^2	uv
113	−7	49	1.8	−0.1	0.01	0.7
102	−18	324	1.5	−0.4	0.16	7.2
95	−25	625	1.3	−0.6	0.36	9
100	0	0	1.9	0	0	0
140	20	400	1.1	−0.8	0.64	−16
130	10	100	2.0	0.1	0.01	1
125	5	25	1.7	−0.2	0.04	−1
Total 825	−15	1523	11.3	−2	1.22	0.9

y on x $b_{yx} = \dfrac{\Sigma uv - \dfrac{\Sigma u \cdot \Sigma v}{n}}{\Sigma u^2 - \dfrac{(\Sigma u)^2}{n}}$

$= \dfrac{0.9 - \dfrac{15 \times 2}{7}}{1523 - \dfrac{225}{7}}$

$= \dfrac{-3.385}{1490.85}$

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$= -2.270 \times 10^{-3}$

$byx = -0.002270$

~~bxy~~ = x on y $bxy = \dfrac{\Sigma uv - \dfrac{\Sigma u \Sigma v}{n}}{\Sigma u^2 - \dfrac{(\Sigma u)^2}{n}}$

$= \dfrac{0.9 - \dfrac{30}{7}}{1.22 - \dfrac{4}{7}} = -\dfrac{3.385}{0.6485}$

$bxy = -5.2197$

$r = \sqrt{-0.002270 \times -5.2197}$

$r = -0.10885$

19.69

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CHI - SQUARE (x^2) Test

Defination: If $O_1, O_2, \ldots, O_n$ be a set of observed frequencies and $E_1, E_2, \ldots, E_n$ be the corresponding set of expected frequencies then CHI-SQUARE is defined by the relation

$$x^2 = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \ldots + \frac{(O_n - E_n)^2}{E_n}$$

with $(n-1)$ degree of freedom

$\{ \Sigma O_i = \Sigma E_i = n$ (The Total frequencies).

CHI-SQUARE Distribution:

If $x_1, x_2 \ldots x_n$ be n independent normal variate betwee with mean 0 & std deviation unity. Then $x_1^2 + x_2^2 + \ldots + x_n^2$ is a random variate having CHI-Square distribution

i.e $Y = Y_0 \cdot e^{-(x)^2/2} \{x^2\}^{(v-1)/2}$

where $V = n-1$

$x^2 =$ CHI-SQUARE.

f x^2 >, so it is rejected

se: accpt.

Que:1 In Experiment's on pea breeding the following frequency of seeds were obtained

Round & Yellow	wrinkled & yellow	Round & green	wrinkled & green	Total
315	101	108	32	556

Theory Predict's that the frequencies should be in preparations: 9:3:3:1. Examine the Corresponding between theory and Experiments.

Sol: The Corresponding frequencies are

$$\frac{9}{16} \times 556 = 312.75$$

$$\frac{3}{16} \times 556 = 104.25$$

$$\frac{3}{16} \times 556 = 104.25$$

$$\frac{1}{16} \times 556 = 34.75$$

$$\chi^2 = \frac{(315-313)^2}{313} + \frac{(101-104)^2}{104} + \frac{(108-104)^2}{104} + \frac{(32-35)^2}{35}$$

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$$\chi^2 = \frac{4}{313} + \frac{9}{104} + \frac{16}{104} + \frac{9}{35}$$

$$= 0.012 + 0.086 + 0.15 + 0.25$$

$$= 0.51 \text{ (approx)}$$

Degree of freedom $= V - 1$ $V \rightarrow$ Total No of entries

$= 4 - 1$

$= 3$

$\chi^2_{0.05} = 7.818$

at 3

Hence, the Calculate value of CHI-SQUARE is much less than CHI-SQUARE 0.5

There is a very high degree of Agreement b/w ~~theorey~~ theory & experiment.

Que:2 A Set of five similar coins is tossed 320 times and the result is

No of heads :	0	1	2	3	4	5
frequency :	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.

~~Qut.~~
Sol: For $\nu = 5$ we have $x^2_{0.05} = 11.07$

p: Probability of getting head $= \frac{1}{2}$

q: Probability of getting tail $= \frac{1}{2}$

hence the Theoretical frequency of getting 0, 1, 2, 3, 4, 5 heads are the successive terms of the binomial Expansion ~~320~~

$320(P+q)^n$

$= 320\{p^5 + 5p^4q + 10p^3 \cdot q^2 + 10p^2q^3 + 5pq^4 + q^5\}$

$= 320\left\{\frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32}\right\}$

$= 10 + 50 + 100 + 100 + 50 + 10$

$=$

Thus the Theoritical frequencies are

10, 50, 100, 100, 50, 10

Hence

$X^2 = \frac{(6-10)^2}{10} + \frac{(27-50)^2}{50} + \frac{(72-100)^2}{100} +$

$\frac{(112-100)^2}{100} + \frac{(71-50)^2}{50} + \frac{(32-10)^2}{10}$

$=$

$= \frac{16}{10} + \frac{529}{50} + \frac{784}{100} + \frac{144}{100} + \frac{441}{50} + \frac{484}{10}$

$= 1.6 + 10.58 + 7.84 + 1.44 + 8.82 + 48.4$

$= \underline{\underline{78.68}}$

Qu:3 Fit a Poisson distribution to the following data and test for its goodness of fit at least of significant 0.05.

x :	0	1	2	3	4
f :	419	352	154	58	19

$P(x) = \frac{1002 \times e^{-m} \cdot m^{x}}{\lfloor x}$

$\text{mean} = \frac{\sum f(x)}{f} = \frac{352 + 308 + 174 + 76}{1002}$

$= \frac{910}{1002} = 0.9$

$P(0) = \frac{1002 \times e^{-0.9} \cdot 0.9^{0}}{\lfloor 0} = 407.38$

$P(1) = \frac{1002 \times e^{-0.9} \cdot (0.9)^{1}}{1} = 366.64$

$P(2) = \frac{1002 \times e^{-0.9} (0.9)^{2}}{\lfloor 2} = 164.99$

$$P(3) = \frac{1002 \times e^{-0.9} \cdot (0.9)^3}{\underline{|3}} = 49.49.$$

$$P(4) = \frac{1002 \times e^{-0.9} (0.9)^4}{\underline{|4}} = 11.13.$$

Theoretical frequencies

407.38 , 366.64 , 164.99 , 49.49 , 11.13

Hence

$$X2 = \frac{(419-407.38)^2}{407.38} + \frac{(352-366.64)^2}{366.64} +$$

$$\frac{(19-11.13)^2}{11.13} + \frac{(154-164.99)^2}{164.99} + \frac{(58-49.49)^2}{49.49}$$

$$X2 = 0.331 + 0.584 + 0.732 + 1.463 + 5.564$$

$$X2 = \underline{8.674}$$

Que:4 In a Sample Survey of Public Openion Answers to the question.
1. Do you drink
2. Are you in favour of local option on Sale of Liquor are tabulated below

	Yes	NO	Total
Yes	56	31	87
NO	18	6	24
Total	74	37	111

Can you peform proper whether or not the local option on the Sale of liquor is dependent on induisable drink, Given that the value of χ^2 at 5% level of Significat is 3.841

Sol: (I) Null hypothesis H_0 — The option on the Sale of liquor is not dependent with the induisable drinking.

(II) Calculation of expected / Theoretical frequencies f_e

The expected frequencies Corresponds to the Theoretical frequencies are calculated as follow:

Cell 11

$$f_{e_{11}} = \frac{87 \times 74}{111} = 58$$

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Cell 12 is –

$$fe_{12} = \frac{87 \times 37}{111} = 29$$

Cell 21 is

$$fe_{21} = \frac{74 \times 24}{111} = 16$$

Cell 22

$$fe_{22} = \frac{24 \times 37}{111} = 8.$$

III) Calculation of χ^2 Statistic:

$$\chi^2 = \Sigma \frac{(fo - fe)^2}{fe}$$

fo	fe	$(fo-fe)^2/fe = \chi^2$
56	58	0.068
31	29	0.137
18	16	0.250
6	8	0.5
		$\Sigma\chi^2 = 0.955$

also d.o.f = deg (v) = (m–1) (n–1)
= (2–1) (2–1)
= 1

m → no. of row.
n → no of col.

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(IV) The tabulated value of χ^2 at 5% label of Significant and 1 degree of freedom.

$$\chi^2_{0.05;1} = 3.841$$

(V) Decision :
Clearly Calculated value of $\chi^2 = 0.955$ is less than tabulated value i.e 3.841

The Null hypothesis is ~~expected~~. accepted.

Liquor is not dependent with the indusible drinking

Que:5 50 Students selected at random from 500 Students Enrolled in a Computer program were classified according to age and grade point. given the following data :

grade Points	Age (in years) 20 & under	21-30	above 30	Total
up to 5.0	3	5	2	10
5.1 to 7.5	8	~~7~~	5	20
7.6 to 10	4	8	8	20
Total	15	20	15	N = 50

F - distribution

Let $x_1, x_2 \dots x_n$ and $y_1, y_2 \dots y_n$ be the values of 2 independent random sample be drawn from the random & normal population σ^2 & having equal variance

Let $\bar{x}_1$, and $\bar{x}_2$ be the sample means and

$$S_1^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_i - \bar{x})^2$$

$$S_2^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_i - \bar{y})^2$$

be the sample variance then we define F

$$\boxed{F = \frac{S_1^2}{S_2^2}} \quad ; \quad S_1^2 > S_2^2.$$

with $\left.\begin{matrix} \nu_1 = n_1 - 1 \\ \nu_2 = n_2 - 1 \end{matrix}\right\}$ degree of freedom

Que: Random sample are drawn from two population and the following result

Sample x :	20	16	26	27	23	22	18	24	25	19
Sample Y :	27	33	42	35	32	34	38	28	41	43
							39		37	

Find the Variance of two binomial and these whether the two sample has same variance
(given that $F_{0.05}$ for 11 and 9 d.o.f is 3.112.

Sol: Given that $n_1 = 10$ and $n_2 = 12$ with d.o.f $V = n_1 - 1 = 9$ and $V = n_2 - 1 = 11$

S Step 1: Null hypothesis H_0: Let $\sigma_1^2 = \sigma_2^2$
i.e the two samples have the same variance.

Step 2: Calculation of F-Statistic

$\bar{x} = 22$
$\bar{y} = 35.75$

we have to find s_1^2 and s_2^2

Sample X | Sample Y

x	$x - \bar{x}$	$(x-\bar{x})^2$	y	$y - \bar{y}$	$(y-\bar{y})^2$
20	-2	4	27	-8.75	76.56
16	-6	36	33	-2.75	7.56
26	4	16	42	6.25	39.06
27	5	25	35	-0.75	0.5625
23	1	1	32	-3.75	14.062
22	0	0	34	-1.75	3.0625
18	-4	16	38	2.25	5.0625
24	2	4	28	-7.75	60.0625
25	3	9	41	5.25	27.5625 ~~11.819~~
19	-3	9	43	7.25	52.5625
			39	3.3	10.89
$\Sigma = 220$	$\Sigma = 0$	$\Sigma = 120$	37	1.3	1.69

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$$\Sigma(y-\bar{y})^2 = 298.69$$

$$S_1^2 = \frac{120}{9} = 13.33$$

$$S_2^2 = \frac{298.69}{11} = 27.15$$

$$\Rightarrow S_2^2 > S_1^2$$

$$F = \frac{S_2^2}{S_1^2} = \frac{27.15}{13.33} = 2.03$$

Hence calculated value of F is 2.03

Step 3: The tabulated value of F at 5% level of significance for the degree of freedom at 11 & 9 is 3.112 i.e.

$$F_{0.05} = 3.112$$

Step 4: Decision

Calculated value of F is less than tabulated value of F

$$F_{cal} < F_{Tab.}$$

The Null hypothesis H_0 is accepted.

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Que: Student's t - distribution

Ans 1 Consider a Small Sample of Size n drawn from a normal population with mean μ and S.D σ if $\bar{x}$ and σ_s be the Sample mean and std deviation

Then the Statistic "t" is defined as

$$\boxed{t = \frac{\bar{x} - \mu}{\sigma} \cdot \sqrt{n}}$$ Or

$$\boxed{t = \frac{\bar{x} - \mu}{\sigma_s} \sqrt{n-1}}$$

where $\nu = n-1$ is degree of freedom

The 9 item of a Sample have the following Value

45, 47, 50, 52, 48, 47, 49, 53, 51

does the mean of these differ Significance from the assumend mean 47.5.

We will find the mean and S.D of Sample as follows:

x	$d = x - 48$	d^2
45	-3	9
47	-1	1
50	2	4
52	4	16
48	0	0
47	-1	1
49	1	1
53	5	25
51	3	9
$\Sigma x = 442$	Σd 10	Σd^2 66

$$\bar{x} = \text{mean} = 48 + \frac{\Sigma d}{9}$$

$$= 48 + \frac{10}{9}$$

$$= 49.11$$

$$\sigma_s^2 = \frac{\Sigma d^2}{9} - \left(\frac{\Sigma d}{9}\right)^2$$

$$= \frac{66}{9} - 1.23$$

$$= 7.33 - 1.23$$

$$\sigma_s^2 = 6.10.$$

$$\sigma_s = \underline{\underline{2.469}}$$

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$$t = \frac{\bar{x} - \mu}{\sigma_s} \sqrt{n-1}$$

$$= \frac{49.11 - 47.5}{2.469} \times \sqrt{8}$$

$$= 0.652 \times \sqrt{8}$$

$$= 1.844$$

degree of freedom $= n - 1 = 9 - 1 = 8$

$t_{0.05} = 2.31$

Calculated value of t is less than $t_{0.05}$

i.e $t < t_{0.05}$

The value of t is not significant.

Qu: The average number of articles produced by two machines per day are 200 & 250 with std deviation 20 & 25 respectively on the basis of record of 25 days production. Can you regard both the machines equally efficient at 1% level of significance.

Given $t_{0.01;48} = 2.58$

Sol: Given that $n_1 = 25$, $\bar{x}_1 = 200$, S.d. $\sigma_1 = 20$

$n_2 = 25$; $\bar{x}_2 = 250$; S.D $\sigma_2 = 25$

$$S^2 = \frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2 - 2}$$

$$= \frac{25 \times 400 + 25 \times 625}{48}$$

$$= 533.85$$

$$S = 23.105$$

Degree of freedom $(v) = n_1 + n_2 - 2 = 48$.

Step I Null hypothesis H_0 – Both the machines are equally efficient.

$$H_0 : u_1 = u_2$$

Step II Calculation of 't' statistics

$$|t| = \frac{\bar{x}_1 - \bar{x}_2}{S} \cdot \sqrt{\frac{n_1 \cdot n_2}{n_1 + n_2}}$$

$$|t| = \frac{200 - 250}{23.105} \sqrt{\frac{25 \times 25}{50}} = \cancel{189.17 \times 1.1767}$$

$$|t| = 7.65$$

Fisher - Z - distribution.

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$Z = \frac{1}{2}\log_e F$ in the F - dis

i.e $\log_e F = 2Z$

$F = e^{2Z}$. in the F-dis we get Fisher - Z distribution

Test whether the two sets of observation

I : 17 27 18 25 27 29 27 23 17
II : 16 16 20 16 20 17 15 21

Indicate sample drawn from the same universe (the value of Z at 5% level for 8 and 7 degree of freedom is 0.6575

Sol: Given $n_1 = 9$; $n_2 = 8$, $\bar{x} = 23.33$

with dof $v_1 = n_1 - 1 = 8$ $\bar{Y} = 17.62$
$v_2 = n_2 - 1 = 7$

First observation | Second observation

x	$x-\bar{x}$	$(x-\bar{x})^2$	Y	$(Y-\bar{Y})$	$(Y-\bar{Y})^2$
17	-6.33	40.06	16	-1.62	2.629
27	3.67	13.46	16	-1.62	2.629
18	-5.33	28.40	20	2.38	5.664
25	1.67	2.78	16	-1.62	2.621
27	3.67	13.46	20	2.38	5.664
29	5.67	32.148	17	0.62	0.384

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27	3.67	13.46	15	-2.62	6.86
23	-0.33	0.108	21	3.38	11.42
17	-6.33	40.068			
$\Sigma x =$		183.94.			37.879

Calculation of Z - Statistics

we have

$$S_1^2 = \frac{\Sigma (x-\bar{x})^2}{n_1 - 1} = \frac{183.94}{8}$$

$$= 22.9925$$

$$\cancel{S_1} = \cancel{4.79}$$

$$S_2^2 = \cancel{37.} \frac{\Sigma (y-\bar{y})^2}{n_2 - 1}$$

$$= 5.41$$

$$S_1^2 > S_2^2$$

$$Z = \frac{1}{2} \log_e \left(\frac{22.99}{5.41} \right)$$

$$= \frac{1.4468}{2} = 0.7234$$

The tabulated of Z at 5% level of significance with dof 8 & 7 is 0.6575.

$Z_{0.05}$ ~~te~~ 0.6575

∴ Calculated value of z is 0.7236

i.e $Z_{calculated} = 0.7236$

$0.7236 \not< tabulated\ 0.6575$

∴ The null hypothesis is rejected.

The two variance are not same.

Curve Fitting

Method of Least Square.

Que: If ~~R~~ P is the Pull required to lift a load W, by means of pulley block find a linear law of the form

$$P = mW + C \quad \text{—— (I)}$$

Connecting P & W using the following data.

P : 12 15 21 25

W : 50 70 100 120

Sol: The Normal eq^n of (I) are

$$\Sigma P = m\Sigma W + \Sigma C$$

i.e $\Sigma P = m\Sigma W + 4C$ —— (1)

$\Sigma PW = m\Sigma W^2 + C\Sigma W$ —— (2)

P	W	PW	W^2
12	50	600	2500
15	70	1050	4900
21	100	2100	10000
25	120	3000	14,400
73	340	6750	31800

eq (1)

$$73 = m(340) + 4c \quad \text{(iii)}$$

$$73 = 340m + 4c$$

eq (2)

$$\cancel{65}$$

$$6750 = 31800m + 340c.$$

$$m = \frac{109}{580} \qquad c = \frac{66}{29}$$

$$m = 0.187$$

$$c = 2.27$$

$$P = 0.1870\,w + 2.2785$$

when $w = 150$ kg

$$P = 28.05 + 2.2785$$

$$= 30.3285$$

Que: Fit a second degree parabola to the following data

x:	0	1	2	3	4
y:	1	1.8	1.3	2.5	6.3

$$y = a + bx + cx^2 \quad \text{—— (I)}$$

Normal eq^n of (I) are

$$\Sigma y = \Sigma a + b\Sigma x + c\Sigma x^2 \quad \text{—— (1)}$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{—— (2)}$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{—— (3)}$$

	x	y	x^2	xy	x^3	x^2y	x^4
	0	1	0	0	0	0	0
	1	1.8	1	1.8	1	1.8	1
	2	1.3	4	2.6	8	5.2	16
	3	2.5	9	7.5	27	22.5	81
	4	6.3	16	25.2	64	100.8	256
$\Sigma =$	10	12.9	30	37.1	100	130.3	354

Now substitute the following in eq (1), (2), (3)

$$12.9 = 5a + 10b + 30c$$
$$37.1 = 10a + 30b + 100c$$
$$130.3 = 30a + 100b + 354c$$

$$a = \frac{71}{50}, \quad b = \frac{-107}{100}, \quad c = \frac{11}{20}$$

$a = 1.42$
$b = -1.07$
$c = 0.55$

$$y = 1.42 - 1.07x + 0.55x^2$$

$$\boxed{y = 1.42 - 1.07x + 0.55x^2}$$

Que: Fit a second degree parabola to the following data.

x :	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y :	1.1	1.3	1.6	2.0	2.7	3.4	4.1

$$y = a + bx + cx^2 \quad \text{---} \; \text{I}$$

Normal equation of I are.

$$\Sigma y = \Sigma a + b\Sigma x + c\Sigma x^2 \quad \text{---} \; (1)$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{---} \; (2)$$
$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{---} \; (3)$$

from Table,

$$16.2 = 7a + 17.5b + 50.75c \quad \text{---} \; (4)$$
$$47.65 = 17.5a + 50.75b + 157.966c \quad \text{---} \; (5)$$
$$154.475 = 50.75a + 157.966b + 538.78c \quad \text{---} \; (6)$$

x	y	x^2	xy	x^3	x^2y	x^4
1.0	1.1	1	1.1	1	1.1	1
1.5	1.3	2.25	1.95	3.375	2.925	5.06
2.0	1.6	4.0	3.2	4.096	6.4	6.55
2.5	2.0	6.25	5	15.625	12.5	39.06
3.0	2.7	9	8.1	27	24.3	81
3.5	3.4	12.25	11.9	42.87	41.65	150.06
4.0	4.1	16	16.4	64	65.6	256
17.5	16.2	50.75	47.65	157.966	154.475	538.73

$a = 0.381$

$b = 0.305$

$c = 0.16$

Substitute a b c in equation I

$$\boxed{y = 0.381 + 0.305x + 0.16x^2}$$

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Fitting of other Curve's

Que: A $Y = ax^b$

$\log_{10} Y = \log_{10} a + b \log_{10} x$

i.e $Y = A + bX$ where $X = \log_{10} x$, $Y = \log_{10} Y$

$\Sigma Y = nA + b\Sigma X$ — ①
$\Sigma XY = A\Sigma X + b\Sigma X^2$ — ②

Que:

V:	350	400	500	600
t:	61	26	7	26

Fitted the Curve $V = at^b$.

(B) $Y = a \cdot e^{bx}$

$\log_{10} Y = \log_{10} a + bx \cdot \log_{10} e$

$Y = A + Bx$ where $Y = \log_{10} Y$

$\Sigma Y = nA + B\Sigma x$ — ① $A = \log_{10} a$
$\Sigma xy = A\Sigma x + B\Sigma x^2$ — ② $B = b \cdot \log_{10} e$

Que:

V :	350	400	500	600
t :	61	26	7	26

Curve = $V = at^b$

1: $\log_{10} V = \log_{10} a + b \log_{10} t$

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Let $T = \log_{10} t$ & $A = \log_{10} a$

$V = \log_{10} v$

$\therefore V = A + bT$ —— (I)

$\Sigma V = nA + b\Sigma T$ —— (1)

$\Sigma VT = A\Sigma T + b\Sigma T^2$ —— (2)

	V	T	T^2	VT
	350	61	3721	21350
	400	26	676	10400
	500	7	49	3500
	600	26	676	15600
Total	1850	120	5122	50850

Substituting Values

$1850 = 4A + 120b$ —— (3)

$50850 = 120A + 5122b$ —— (4)

$A = 554.15$

$b = -3.055$

$V = 554.15 \cancel{+} - 3.055T$

$$\boxed{V = 554.15 - 3.055T}$$

Bivariate Distribution

Let x_1, x_2 be a random variable with the real plane as the sample space i.e x_1 and x_2 can be take real values from $-\infty$ to ∞ then x_1 x_2 has a continuous probability distribution. If there is a density function $f(x_1, x_2)$ such that

$$Pr\{a_1 < x_1 < a_2 ; b_1 < x_1 < b_2\}$$

$$= \int_{b_1}^{b_2} \int_{a_1}^{a_2} f(x_1, x_2)\, dx_2 \cdot dx_1$$

with $f(x_1, x_2) \geq 0$

and

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2)\, dx_1\, dx_2 = 1$$

also.

$$\frac{\partial}{\partial x_2} \cdot \frac{\partial}{\partial x_1} f(x_1, x_2) = f(x_1, x_2)$$

and $f(-\infty, \infty) = 1$

Conditional Probability Density:

The Conditional probability Density of x_1 gives that x_2 assume the value of. $f(x_1/x_2) = \frac{f(x_1, x_2)}{f_2(x_2)}$ if $f_2(x_2) \neq 0$

Ex: given the Joint density

$$f(x_1, x_2) = \begin{cases} \frac{k}{(1+x_1+x_2)^3}, & \text{for } x_1>0, x_2>0 \\ 0, & \text{Otherwise} \end{cases}$$

Que: Find k and the marginal density of x_1 & x_2, find the Conditional density of x_1 given that x_2 assume the value of x_2. Check wether the random variable are independent

Sol: W.K.T.

$$\int_0^\infty \int_0^\infty \frac{k}{(1+x_1+x_2)^3} dx_1 . dx_2 = 1$$

$$k \int_0^\infty \int_0^\infty (1+x_1+x_2)^{-3} dx_1 dx_2$$

$$\frac{k}{-2} \int_0^\infty \frac{(1+x_1+x_2)^{-2+}}{1+\cancel{x}_0} dx_2$$

$$\frac{k}{\cancel{2}} \int_0^\infty \frac{k}{2y^2+4y+2} = \frac{k}{2} = 1$$

$$\boxed{k = 2}$$

Then the marginal density of x_1 is

$$f_1(x_1) = \int_0^\infty f(x_1, x_2)\, dx_2$$

$$= \int_0^\infty \frac{k}{(1+x_1+x_2)^3}\, dx_2$$

$$= \int_0^\infty 2(1+x_1+x_2)^{-3}\, dx_2$$

$$= \frac{2}{-2}\left[1+x_1+x_2\right)^{-2}$$

$$= \Big[-(1+x_1+x_2)^{-2}\Big]_0^\infty$$

$$= (1+x_1)^{-2} = \frac{1}{(1+x_1)^2}$$

for $x_1 > 0$

$$f_2(x_2) = \int_0^\infty f(x_1, x_2)\, dx_1$$

$$= \frac{1}{(1+x_2)^2} \quad ; \quad x_2 > 0$$

The Conditional density of x_1 gives x_2 assume The value x_2 is $f(x_1/x_2) = \dfrac{f(x_1, x_2)}{f_2(x_2)}$

$$= \frac{\frac{2}{(1+x_1+x_2)^3}}{\frac{1}{(1+x_2)^2}}$$

$$= \frac{2(1+x_2)^2}{(1+x_1+x_2)^3}.$$

Test of Significance for large Sample:

We know that the binomial distribution tends to normal for large n. Suppose we wish to test the hypothesis that the probability of success in such trial is p. Suppose it to be true then mean (μ), and standard deviation (σ) of the sampling distribution of no. of success are

$$\mu = np$$
$$\sigma = \sqrt{npq}.$$

respectively.

For a normal distribution only 5% of the number lie outside $\mu \pm 1.96\sigma$ which only 1% of the numbers lie outside 2.58σ.

If x be the observed number of success in the sample z is the std no. curve

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

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Thus, we have the following test of Significance

1. if $|Z| < 1.96$
difference between the observed and expected number of success is not significant.

2. if $|Z| > 1.96$
difference is significant at 5% level of significance.

3. if $|Z| > 2.58$ / (3)
difference is significant at 1% level of significance.

Que: A Coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbaised at 5% level of significance

Sol: Suppose the Coin is unbiased

Then the probability of getting the head at toss $= \frac{1}{2}$

Expected No of Success $= \frac{1}{2} \times 400 = 200$
(i.e calculated).

and the observed value of Success $= 216$

Thus the excess of observed value over expected value

$$x - np = 216 - 200 = 16$$

$$\boxed{x - np = 16}$$

also S.D of Simple Sampling $= \sqrt{npq}$

$$= \sqrt{400 \times \frac{1}{2} \times \frac{1}{2}} = 10$$

$$Z = \frac{x - np}{\sigma} = \frac{16}{10} = 1.6$$

→ The null hypothesis is not Significant

Que: A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times on the assumption of random throwing. Do the data indicate a unbaised die.

Suppose the die is unbaised

Then the Prob of Success (~~Calculated~~) $= \frac{2}{6} = \frac{1}{3}$

Expected Success (calculated) $= \frac{1}{3} \times 9000$

$= 3000$

$1 - \frac{1}{3}$

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Observed = 3240

Thus the excess of observed value over expected value

$$x - np = \cancel{96}\ 3240 - 3000$$
$$= 240$$

also S.D of Sampling $= \sqrt{npq}$

$$= \sqrt{9000 \times \frac{\cancel{2}}{\cancel{6}\,3} \times \frac{\cancel{4}\,2}{\cancel{6}\,3}}$$
$$= \sqrt{2000}$$
$$= \cancel{89.6}\ 44.72$$

$$Z = \frac{x - np}{\sigma} = \frac{240}{44.72}$$

$$Z = 5.36$$

Thus, Null hypothesis is Significant at 1 % level of Significance.

$$\frac{16 \times 15 \times 14}{3 \times 2 \times 1}$$

Conditional Probability.

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Que: 1 Find the Probability of a throwing a Sum of 7 in a single throw with two dice

Sol:
$$n(S) = 36$$
$$n(E) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$
$$n(E) = 6$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6} \;//$$

Que: 2 From a bag containing 5 white, 7 red, and 4 black balls a man draw 3 at random. find the probability of being all white

$$n(S) = 5+7+4 = 16$$
$$n(E) = 5$$
$$P(E) = {}^{16}C_3 \times \left(\frac{5}{16}\right)^3 \times \frac{7}{16}^0 \times \frac{4}{16}^0$$

$$P(\text{success}) = \frac{5}{16} \qquad = \cancel{\frac{16 \times 15 \times 14}{3 \times 2 \times 1}} \times \cancel{\frac{125}{16 \times 16 \times 16}}$$

$$P(\text{failure}) = \frac{11}{16} \qquad = \cancel{\frac{35 \times 125}{16 \times 16}} \qquad {}^{16}C_3 \times \left(\frac{5}{16}\right)^3 \times \left(\frac{15}{16}\right)^{13}$$

$$= \frac{16 \times 15 \times 14}{3 \times 2 \times 1} \times 0.489$$

$$= \cancel{17.0} \qquad = 35 \times 0.489 \times 0.007$$

$$= 0.131$$

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Sol: Total No of balls = 5 + 7 + 4 = 16

The total No of ways in which 3 balls can be drawn = $^{16}C_3$ = 560

The Sample Space n(S) = 560.

Let E be the event of all three balls are white = $5C_3$ = 10 = n(E) = 100

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{560} = 0.017$$

Que 3 from a pack of 52 cards two cards are drawn at random. find the probability of following events:

(1) both cards are Spade.
(2) One card is of Spade and one card of diamond.

Total Cards = 52

Total No. of ways in which 2 cards can be drawn = $52C_2$ = $\frac{52 \times 51}{2 \times 1}$ = 1326

The Sample Space n(S) = 1326.

Let E_1 be the event of 2 cards or Spade = $13C_2$ = $\frac{13 \times 12}{2 \times 1}$ = 78 = n(E)

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$P(\text{both cards Spade}) = \frac{78}{1326} = \underline{\underline{0.058}}$

Let E_2 be the event of 1 card of Spade & 1 of diamond $= \cancel{13} \; \cancel{20} {}^{\cancel{2}}C_2 \times \cancel{\left(\frac{1}{13}\right)}$

$= {}^{13}C_1 \times {}^{13}C_1 = 13 \times 13 = 169$

$P(1 \to \text{Spade and } 1 \to \text{diamond}) = \frac{169}{1326}$

$= \underline{\underline{0.1274}}$

Properties:

I. If E is an event and $\bar{E}$ is its Complementary event then

$$P(E) + P(\bar{E}) = 1$$

II. If E_1 and E_2 are any two events then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

III. If E_1 and E_2 are mutually exclusive then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2).$$

Ques: Two cards are drawn at random from a pack of cards. Find the probability that both these cards are of black colour or both are aces.

Sample Space $= {}^{52}C_2$

$= 1326.$

E_1 (both cards are black) $= \cancel{13}\,{}^{26}C_2$

$= \frac{26 \times 25}{2 \times 1} = 325.$

E_2 (both are aces) $= {}^{4}C_2 = \frac{4 \times 3}{2 \times 1} = 6$

$P(E_1) = \frac{325}{1326} = 0.245$

$P(E_2) = \frac{6}{1326} = 0.0045.$

$n(E_1 \cap E_2) = 1$

$P(E_1 \cap E_2) = \frac{1}{1326} = 0.00075$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.245 + 0.0045 - 0.0075$

$= 0.24$

$$= \frac{330}{1326} = \frac{165}{663} = \frac{55}{221}.$$

Ans: Conditional Probability.

When the happening of an event E_1 depend upon the happening of another event E_2. Then the probability of event is called Conditional Probability. It is denoted by

$$P(E_1 / E_2) -$$

i.e $P(E_1 \cap E_2) = P(E_1) . P(E_2 / E_1)$

Que: If A & B are two events $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

Then Evaluate (I) $P(A/B)$ (II) $P(B/A)$ (III) $P(A \cup B)$

(I) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{4} \times 3 = \dfrac{3}{4}$

(II) $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1}{4} \times \dfrac{2}{1} = \dfrac{1}{2}$

(III) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{6 + 4 - 3}{12} = \frac{7}{12} //.$$