- Random variable : If a real variable 'x' be associated with the outcome of a random experiment then since the value taken depends on chance is called random variable.

- Discrete Variate : If a random variable 'x' take a finite set of value then it is known as discrete variate

- Continuous Variate : If a random variable 'x' assumes an infinite number of uncountable values, It is called as Continuous variate.

Que:1 If a die is tossed thrice, Success is getting on 1 or 6. Find the mean and variance of the number of success.

Sol: Probability of success $= \frac{2}{6} = \frac{1}{3}$

Probability of failure $= \frac{4}{6} = \frac{2}{3}$

Probability of No Success $= 3C_0 \times \left(\frac{2}{3}\right)^3$

$= \frac{3}{3} \times \frac{8}{27}$

$= \frac{8}{27}$

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Probability of One success and 2 failure

$= {}^nC_r \; p^{n-r} \; q^r$

$= 3C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^2$

$= 3 \times \frac{1}{3} \times \frac{4}{9} = \frac{4}{9}$

Probability of two success and 1 failure

$= 3C_2 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^1$

$= \frac{3 \times 2}{2 \times 1} \times \frac{1}{9} \times \frac{2}{3}$

$= \frac{2}{9}$

Probability of 3 Success =

$3C_3 \times \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

x_i	:	0	1	2	3
P_i	:	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

Mean (m) $= \sum p_i x_i$

$= \left[\frac{4}{9} + \frac{4}{9} + \frac{1}{9}\right] = \underline{\underline{1}}$

also, $\sum p_i x_i^2 = \left[\frac{4}{9} + \frac{8}{9} + \frac{9}{27}\right] = \underline{\underline{\frac{5}{3}}}$

Variance $(\sigma^2) = \sum p_i x_i^2 - (m)^2$

$= \frac{5}{3} - 1$

$= \underline{\underline{\frac{2}{3}}}$

:2 The Probability density function of a variate x is

x :	0	1	2	3	4	5	6
$P(x)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

find (i) $P(x \leq 4)$; $P(x \geq 5)$; $P(3 < x < 6)$

(ii) what will be the minimum value of k if $P(x \leq 2) > 3$

$\because \sum_{i=0}^{6} p_i = 1$

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$49k = 1$

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$$\boxed{k = \frac{1}{49}}$$

(i) ⓐ $P(x \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$= k + 3k + 5k + 7k + 9k$

$= 25k \quad \left[\because k = \frac{1}{49}\right]$

$= \frac{25}{49}$

(b) $P(x \geq 5) = P(5) + P(6)$

$= 11k + 13k$

$= 24k \quad \left[\because k = \frac{1}{49}\right]$

$= \frac{24}{49}$

ⓒ $P(3 < x < 6) = P(4) + P(5)$

$= 9k + 11k$

$= 20k \quad \left[k = \frac{1}{49}\right]$

$= \frac{20}{49}$

(ii) $P(x \leq 2) > 3$

$P(0) + P(1) + P(2)$

$k + 3k + 5k > 3$

$9k > 3$

$k > \frac{3}{9} \quad \boxed{k > \frac{1}{3}}$

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Que:3 A random variable 'x' has a following Probability

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) Value of k
(ii) $p(x<6)$; $P(x \geq 6)$
(iii) $P(0<x<6)$

Sol:(i) $\because \sum_{i=0}^{7} P_i = 1$

$$k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$
$$10k^2+9k-1=0$$
$$k = \frac{1}{10}, -1$$
$$\therefore \boxed{k = \frac{1}{10}}$$

(ii) $P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$
$$= k+2k+2k+3k+k^2$$
$$= k(k+8)$$
$$= 0.81$$

$P(x \geq 6) = P(6)+P(7)$
$$= 2k^2+7k^2+k$$
$$= 9k^2+k$$
$$= 0.19$$

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(iii) $P(0 < x < 6) = P(1) + P(2) + P(3) + P(4) + P(5)$
$= 2K + 2K + 3K + K^2 + K$
$= K(K + 8)$
$= \underline{0.81}$

Distribution function

if $F(x) = P(x \le x) = \int_{-\infty}^{\infty} f(x)\,dx$

then $F(x)$ is defined as the Cumulative distribution function.

Properties of the Distribution function.

(i) If $F(x) = f(x) \geq 0$; $F(x)$ is a increasing function.

(ii) $F(-\infty) = 0$
(iii) $F(\infty) = 1$
(iv) $P(a \le x \le b) = \int_{a}^{b} f(x)\,dx$.

Que:1 Is the function define as follows:

$$f(x) = \begin{cases} e^{-x} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$

a distribution function.

(ii) Determine the Prob. that the variate having this density in the interval (1, 2)

(iii) Also find the Cummulative Probability function F(2).

Sol: $\because \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$

$$= 0 + \int_{0}^{\infty} e^{-x}\,dx$$

$$= \left| -e^{-x} \right|_{0}^{\infty}$$

$$= -e^{-\infty} + e^{0}$$

$$= 1$$

$\therefore$ It is a distribution function.

(ii) $P(1, 2) = \int_{1}^{2} f(x)\,dx$

$$= \int_{1}^{2} e^{-x} = \left| -e^{-x} \right|_{1}^{2}$$

$$= -e^{-2} + e^{-1}$$

(iii) $F(2) = \int_{-\infty}^{2} f(x)\,dx$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= 0 + \left[-e^{-x} \right]_{0}^{2}$$

$$= -e^{-2} + 1$$

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Que: 2 'X' is a Continuous random variable with Probability density function given by

$$f(x) = \begin{cases} kx & ; \quad 0 \le x \le 2 \\ 2k & ; \quad 2 \le x \le 4 \\ -kx + 6k & ; \quad 4 \le x \le 6 \end{cases}$$

Find k and mean value of x

Sol: W.K.T $\int_0^6 f(x)\,dx = 1$

$$\int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx = 1$$

$$\int_0^2 kx + \int_2^4 2k + \int_4^6 -kx + 6k = 1$$

$$\left[\frac{kx^2}{2}\right]_0^2 + \left[2kx\right]_2^4 + \left[\frac{-kx^2}{2} + 6kx\right]_4^6 = 1$$

$$2k + [8k - 4k] + [-18k + 36k + 8k - 24k = 1$$

$$8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

Mean of x $= \int_0^6 x \cdot f(x)\,dx$

$$= \int_0^2 kx^2\,dx + \int_2^4 2kx + \int_4^6 -kx^2+6kx$$

$$= \frac{1}{8}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{4}\left[\frac{x^2}{2}\right]_2^4 + \frac{1}{8}\left[\frac{-x^3}{3} + 3x^2\right]_4^6$$

$$= \frac{1}{3} + \frac{6}{4} + \frac{1}{8}\left[36 - 48 + \frac{64}{3}\right]$$

$$= \frac{22}{12} + \frac{1}{8}\left[\frac{28}{3}\right]$$

$$= \frac{22}{12} + \frac{14}{12} = \frac{36}{12} = 3$$

Mean of x is 3

Que:3 A Variate x has the probability distribution

x :	-3	6	9
$P(x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(x)$ and $E(x^2)$ hence Evaluate $E(2x+1)^2$.

$$E(x) = \sum x \cdot P(x)$$

$$= -\frac{1}{2} + 3 + 3 = \frac{11}{2}$$

$$E(x^2) = \Sigma x^2 . p(x)$$

$$= -\frac{9}{6} + \frac{36}{2} + \frac{81}{3}$$

$$= \frac{93}{2}$$

$$E(2x+1)^2 = E[4x^2 + 1 + 4x]$$

$$= 4E(x^2) + 4E(x) + 1$$

$$= 4 \times \frac{93}{2} + 4 \times \frac{11}{2} + 1$$

$$= 209$$

Que:4 A random variable x has the P.d.f

$$f(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ a & ; \quad 1 \le x \le 2 \\ -ax + 3a & ; \quad 2 \le x \le 3 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Find (i) Value of a

(ii) $P(x \le 1.5)$

Sol: W.K.T $\int_0^3 f(x)\,dx = 1$

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$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 -ax+3a\,dx = 1$$

$$\left|\frac{ax^2}{2}\right|_0^1 + \left|ax\right|_1^2 + \left|-\frac{ax^2}{2}+3ax\right|_2^3 = 1$$

$$\frac{a}{2} + a + \left[-\frac{9a}{2} + 9a + 2a - 6a\right] = 1$$

$$\frac{3a - 9a + 18a + 4a - 12a}{2} = 1$$

$$2a = 1$$

$$\boxed{a = \frac{1}{2}}$$

(ii) $P(x \leq 1.5)$

$$\int_0^1 ax\,dx + \int_1^{1.5} a\,dx$$

$$\left|\frac{ax^2}{2}\right|_0^1 + \left|ax\right|_1^{1.5}$$

$$\frac{a}{2} + \frac{a}{2} = a =$$

$$= \frac{1}{2}$$

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Que: 5 Find the Value of k for P.d.f

$$f(x) = \begin{cases} kx^2 & : 0 \le x \le 3 \\ 0 & : \text{otherwise} \end{cases}$$

and compute $P(1 \le x \le 2)$ also find the distribution function.

Sol: W.K.T $\int_0^3 kx^2 \, dx = 1$

$$\left[\frac{kx^3}{3}\right]_0^3 = 1$$

$$9k = 1 \qquad \boxed{k = \frac{1}{9}}$$

$$P(1 \le x \le 2) = \int_1^2 kx^2 = \left[\frac{kx^3}{3}\right]_1^2$$

$$= \frac{1}{9}\left[\frac{8}{3} - \frac{1}{3}\right] = \cancel{7} \; \frac{7}{27} //$$

Que:6 A frequency distribution f^n is defined by

$$f(x) = \begin{cases} x^3 & ; \ 0 \le x \le 1 \\ 3(2-x)^3 & ; \ 1 \le x \le 2 \end{cases}$$

PT $f(x)$ is a P.d.f & find ~~an~~. S.D.

Sol: To prove : $f(x)$ is P.d.f

i.e $\int_0^2 f(x)\,dx = 1$

$$\int_0^1 x^3\,dx + \int_1^2 3(2-x)^3\,dx = 1$$

$$\left|\frac{x^4}{4}\right|_0^1 + \left[\frac{-3}{4}(2-x)^4\right]_1^2 = 1$$

$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

Hence Proved

Now Standard deviation $= \int_0^2 x^2 f(x).dx$

$$= \int_0^1 x^2 . x^3 dx + 3\int_1^2 x^2(2-x)^3\,dx$$

$$= \int_0^1 x^5\,dx + \int_1^2 24x^2 + 3x^5 + 36x^3 - 18x^4\,dx$$

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$$\left[\frac{x^6}{6}\right]_0^1 + \left[8x^3 + \frac{x^6}{2} + 9x^4 - \frac{18x^5}{5}\right]_1^2$$

$$\frac{1}{6} + \left[64 + 32 + 144 - \frac{576}{5}\right] - \left[8 + \frac{1}{2} + 9 - \frac{18}{5}\right]$$

$$\frac{1}{6} + \frac{624}{5} - \frac{139}{10}$$

$$\frac{3170}{30} = \frac{317}{3}$$

Que:7 Find the value of K so that the following function represents p.d.f

$$f(x) = \begin{cases} 0 & ; \quad x \le -1 \\ K(x+1) & ; \quad -1 < x < 3 \\ 4K & ; \quad 3 \le x \le 4 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Also find Median

Sol: W.K.T $\int_{-\infty}^{4} f(x) = 1$

$$\int_{-1}^{3} kx + k + \int_{3}^{4} 4k = 1$$

$$\left[\frac{kx^2}{2} + kx\right]_{-1}^{3} + \left[4kx\right]_{3}^{4} = 1$$

$$\frac{9k}{2} + 3k - \frac{k}{2} + k + 16k - 12k = 1$$

$$12k = 1$$

$$\boxed{k = \frac{1}{12}}$$

By defination of Median

$$\int_{-\infty}^{m} f(x)\,dx = \frac{1}{2},$$

$$\int_{-1}^{3} kx + k \quad dx + \int_{3}^{m} 4k\,dx = \frac{1}{2}$$

$$\left|\frac{kx^2}{2} + kx\right|_{-1}^{3} + \left|4kx\right|_{3}^{m} = \frac{1}{2}$$

$$\frac{9k}{2} + 3k - \frac{k}{2} + k + 4mk - 12mk = \frac{1}{2}$$

$$\cancel{\frac{9}{2}} \times \cancel{\frac{1}{12}} \cancel{+3}$$

$$8k - 8mk = \frac{1}{2}$$

$$8k[1-m] = \frac{1}{2}$$

$$1 - m = \frac{1}{2 \times 8k} \qquad \left[k = \frac{1}{12}\right]$$

$$1 - m = \frac{1 \times \cancel{12}^{\cancel{6}\,3}}{\cancel{2 \times 8}\,4}$$

$$1 - m = \frac{3}{4}$$

$$m = 1 - \frac{3}{4}$$

$$\boxed{m = \frac{1}{4}}$$

Que: 8 If the function $f(x)$ is defined as

$$f(x) = \begin{cases} 0 & ; \; x < 2 \\ \frac{1}{18}(3+2x) & ; \; 2 \le x \le 4 \\ 0 & ; \; x > 4 \end{cases}$$

To Prove - $f(x)$ is a Pdf
also find $P(2 \le x \le 3)$.

Sol: W.K.T $\int_2^4 f(x) = 1$

$$\frac{1}{18}\int_2^4 3 + 2x = 1$$

$$\left[3x + x^2\right]_2^4 = 18$$

$$\frac{12 + 16 - [6 + 4]}{18} = 1$$

$$\frac{18}{18}$$

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$\therefore$ It is a Pdf

Now $P(2 \le x \le 3) = \int_2^3 f(x)\,dx$

$= \frac{1}{18}\int_2^3 3 + 2x \; dx$

$= \frac{1}{18}\left[3x + x^2\right]_2^3$

$= \frac{1}{18}\left[[9+9] - (6+4)\right]$

$= \frac{1}{18}[18 - 10] = \frac{8}{18}$

$= \frac{4}{9}$

Que:9 A random variable 'x' has a following probability function.

$X = x$:	0	1	2	3
$P(x)$:	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

WKT for P.d.f

$F(x) = P(X \le x) = \sum_{X \le x_i} P(x_i)$

Putting $X = 0, 1, 2, 3 \ldots$

1. $F(0) = P(x \leq 0) = P(0) = 0$

2. $F(1) = P(x \leq 1) = P(0) + P(1) = \frac{1}{5}$

3. $F(2) = P(x \leq 2) = P(0) + P(1) + P(2)$

$= \frac{3}{5}$

4. $F(3) = P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= 0 + \frac{1}{5} + \frac{2}{5} + \frac{2}{5} = 1$

Que: 10 Find the mean, Variance & Standard deviation for the discrete random Variate 'X' having the following probability mass function.

Sol: $P(x) = \begin{cases} 1/3 & x = -2 \\ 1/3 & x = -1 \\ 1/3 & x = 1 \\ 0 & \text{Otherwise} \end{cases}$

Sol.

$X = x$	:	-2	-1	1	Otherwise
$P(x)$	:	$1/3$	$1/3$	$1/3$	0

Mean $E(x) = \sum x_i E(x_i$

$= -\frac{2}{3} - \frac{1}{3} + \frac{1}{3} = -\frac{2}{3}$

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S.D $E(x^2) = E(x^2) = x_i^2 (E x_i) - x^2$

$= \frac{4}{3} + \frac{1}{3} + \frac{1}{3} = 2$

Variance $= E(x^2) - [E(x)]^2$

$= 2 - \left(\frac{2}{3}\right)^2$

$= 2 - \frac{4}{9} = \frac{14}{9}$

Standard deviation $= \sqrt{\text{Variance}}$

$= \frac{\sqrt{14}}{3}$

Binomial distribution

$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$P(r) = (p+q)^n$

where $p + q = 1$

$${}^{n}C_{r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

Que:1 Find the mean & variance of binomial distribution $r = 0, 1, 2, 3, \ldots n$.

Sol: For mean we fill find the first moment about origin

Proof: $\mu_1' = r \cdot P(r)$

$= \sum r \cdot {}^nC_r \cdot p^r q^{n-r}$

$= \sum r \dfrac{\lfloor n}{\lfloor n-r \lfloor r} p^r q^{n-r}$

$= \sum r \dfrac{n \lfloor n-1}{r \lfloor r-1 \lfloor (n-1)-(r-1)} \cdot p \cdot p^{r-1} q^{(n-1)-(r-1)}$

$\mu_1' = np \sum {}^{n-1}C_{r-2} \, p^{r-1} q^{(n-1)-(r-1)}$

$= np (p+q)^{n-1}$

$= np (1)^{n-1} \quad [(1)^n = 1]$

$= np$

$\therefore \boxed{\text{mean} = np}$

Similarly, we can show the 2nd moment about origin

$\mu_2' = r^2 \cdot P(r)$

$= npq + n^2 p^2$

Que: 1 Find the mean & variance of binomial distribution $r = 0, 1, 2, 3, \ldots n$.

Sol: For mean we fill find the first moment about origin

Proof: $\mu_1' = r \cdot p(r)$

$$= \Sigma r \, {}^nC_r \, p^r q^{n-r}$$

$$= \Sigma r \frac{\lfloor n}{\lfloor n-r \lfloor r} p^r q^{n-r}$$

$$= \Sigma r \frac{n \lfloor n-1}{r \lfloor r-1 \lfloor (n-1)-(r-1)} \cdot p \cdot p^{r-1} q^{(n-1)-(r-1)}$$

$$\mu_1' = np \, \Sigma \, {}^{n-1}C_{r-2} \, p^{r-1} q^{(n-1)-(r-1)}$$

$$= np \, (p+q)^{n-1}$$

$$= np \, (1)^{n-1} \quad [(1)^n = 1]$$

$$= np$$

$\therefore$ $\boxed{\text{mean} = np}$

Similarly, we can show the 2nd moment about origin

$$\mu_2' = r^2 \cdot p(r)$$

$$= npq + n^2p^2$$

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$$\text{Variance} = \mu_2' - (\mu_1)^2$$

$$= npq - n^2p^2 - n^2p^2$$

$$= \underline{npq}$$

Que:2 In a binomial distribution the mean & S.D are 12 and 2 respectively. Find n & P.

$$m = np = 12$$

$$S.D = \sqrt{npq} = 2$$

$$q = \frac{4}{12} = \frac{1}{3} \qquad \boxed{P = \frac{1}{3}}$$

$$P + q = 1 \qquad \boxed{P = \frac{2}{3}}$$

$$n = \frac{12 \times 3}{2} = 18 \qquad \boxed{n = 18}$$

Que:3 Using the binomial distribution find the probability of rolling

Atmost 2.

Sol: $n = 5 \qquad P = \frac{1}{6} \qquad q = \frac{5}{6}$

$P(\text{atmost } 2) = P(0) + P(1) + P(2)$

$$= {}^nC_r \, p^r \, q^{n-r}$$

$$= 5c_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 + 5c_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4 + 5c_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

$$= \left(\frac{5}{6}\right)^5 + \cancel{5} \times \left(\frac{5}{6}\right)^5 + \frac{5 \times 4}{2 \times 1} \times \left(\frac{5}{6}\right)^3 \times \frac{1}{36}$$

$$= \left(\frac{5}{6}\right)^3 \left[1 + 1 + \frac{\cancel{10}\,5}{\cancel{36}\,18}\right]$$

$$= \frac{125}{216} \times 2.27$$

$$= \underline{\underline{1.3136}}$$

Que: 4 A Coin is tossed 4 times. What is the Probability of getting
1. two heads
2. atleast two heads.

Sol: $n = 4$, $P = \frac{1}{2}$, $q = \frac{1}{2}$

$$P(r) = {}^nC_r \, p^r \, q^{n-r}$$

$$P(2 \text{ heads}) = 4C_2 \, p^2 \, q^2$$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{1}{4} \times \frac{1}{4} = \underline{\underline{\frac{3}{8}}}$$

$P(\text{atleast two heads}) = P(2) + P(3) + P(4)$
$= \frac{3}{8} + \frac{1}{4} + \frac{1}{16} = \frac{11}{16}$

Ans:5 A cubical die is thrown in set of 8. The occurance of 5 or 6 is called a success in what proportion of the set you expect 3 success.

Sol: $n = 8$ $\quad p = \frac{2}{6} = \frac{1}{3}$ $\quad q = \frac{2}{3}$

$P(3) = 8c_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{8-3}$

$= \underline{\underline{0.2781}}$

Ans:6 The 10 percent of screws produces in a certain factory turn out to be defective. find Probability that in a sample of 10 screw of random exactly 2 will be defective.

Sol: $P = \frac{1}{10}$ $\quad q = \frac{9}{10}$ $\quad n = 10$

$P(2) = 10c_2 (P)^2 (q)^8$

$= \frac{10 \times 9}{2 \times 1} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^8$

$= \underline{\underline{0.1937}}$

Que. 7 The probability of entering Students in CA with graduate is 0.5. Determine the probability that out of 10 Students (i) none (ii) one (iii) at least 1 will graduate.

Sol: $n = 10$ $P_1 = 0.5$

$q = 1 - P = 0.5$

$P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$

$P(0) = {}^nC_0 \cdot p^0 \, q^n$

$= \frac{\lfloor 10}{\lfloor 0 \lfloor 10} \cdot (0.5)^0 \, (0.5)^{10}$

$= 0.0009765$

$P(1) = {}^nC_1 \, p^1 \, q^9$

$= 10 \times 0.5 \times (0.5)^9$

$= 0.009765$

$P(\text{at least one}) = P(1) + P(2) + \cdots + P(10)$

$= 1 - P(0)$

$= 0.9990235$

Que: 8 Find the binomial distribution whose mean $m = 4$ and Variance is 3 also find its mode.

Sol mean $= np = 4$

Variance $= npq = 3$

$q = \frac{3}{4}$

$p + q = 1$

$p = 1 - \frac{3}{4}$

$\Rightarrow p = \frac{1}{4}$

$n = 4 \times 4$

$n = 16$

$$\text{Mode} = \text{Integral part of } (np + p)$$
$$= \text{Integral part of } \left(\frac{16}{4} + \frac{1}{4}\right)$$
$$= \text{integral part of } \left[\frac{17}{4}\right]$$
$$= 4$$

$$\boxed{\text{Mode} = 4}$$

Que: 9 ~~with~~ Fit a Binomial distribution to the following data and compare a theoratical frequencies with actual ones.

(1)

x :	0	1	2	3	4	5
f :	2	14	20	34	22	8

Sol: $n = 5$

$$m = \frac{\Sigma f(x)}{\Sigma f}$$

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$$= \frac{2\times0+14\times1+20\times2+34\times3+22\times4+8\times5}{2+14+20+34+22+8}$$

$$= \frac{284}{100}$$

$$= 2.84$$

$$\text{Mean} = np = 2.84$$
$$5p = 2.84$$
$$p = \frac{2.84}{5}$$
$$p = 0.568$$

$$q = 1 - p$$
$$= 0.432$$

$$\text{Fit} = (p+q)^n$$

$$= (0.568 + 0.432)^5$$

or $P(r) = {}^nC_r \, p^r \, q^{n-r}$

Que: 10 A bag contain 3 red and 4 black balls, one ball is drawn and then replaced in the bag and the process is repeated. Getting a red ball in draw is considered a success. Find the distribution of capital X, where X denote the number of success in 3 drawn, Assuming that in each draw each ball is exactly like to be selected.

Sol: Total balls $= 7$

red balls $= \frac{3}{7}$

black balls $= 4/7$

$P(r) = {}^nC_r \, p^r \, q^{n-r}$ $\boxed{n = 3}$

$P(0) = {}^3C_0 \left(\frac{3}{7}\right)^0 \left(\frac{4}{7}\right)^3 = \frac{64}{343}$

$= \left(\frac{4}{7}\right)^3$

$P(1) = {}^3C_1 \left(\frac{3}{7}\right)^1 \left(\frac{4}{7}\right)^2 = \frac{9 \times 16}{343} = \frac{14}{34}$

$P(2) = {}^3C_2 \left(\frac{3}{7}\right)^2 \left(\frac{4}{7}\right)^1 = \frac{27 \times 4}{343} = \frac{10}{34}$

$P(3) = {}^3C_3 \left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^0 = \frac{27}{343}$

Que: 11 Out of 800 families with 4 Children each How many families would be expected to have (i) Two boys & Two girls.
(ii) Atleast one boy.
(iii) No girl
(iv) at most two girls
Assume equal Probabilities for boy & girls.

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Que : 12 The probability that a bomb drop from a plane will strike the target is 1/5, If 6 bomb are droped find the probability that (i) exactly two will strike the target.
(ii) atleast two will strike the target

Sol: $P = \frac{1}{5}$ $q = 1 - P = \frac{4}{5}$ $n = 6$

$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$P(2) = {}^{6}C_{2} \left(\frac{1}{5}\right)^{2} \left(\frac{4}{5}\right)^{4}$

$= \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \times \frac{1}{\cancel{25}\,5} \times \frac{256}{625}$

$= \frac{768}{3125} = 0.2457$

$P(\text{at least } 2) = 1 - \{P(0) + P(1)\}$

$= 1 - \left\{ {}^{6}C_{0} \left(\frac{1}{5}\right)^{0} \left(\frac{4}{5}\right)^{6} + {}^{6}C_{1} \left(\frac{1}{5}\right)^{1} \left(\frac{4}{5}\right)^{5} \right\}$

$= 1 - \left\{ \left(\frac{4}{5}\right)^{6} + \frac{6}{5} \times \left(\frac{4}{5}\right)^{5} \right]$

$= 1 - \left[\frac{4096}{15625} + \frac{6144}{15625} \right]$

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$$= 1 - \frac{10240}{15625} = \frac{5385}{15625}$$

$$= 0.34464$$

Que. 13 The following data are the number of seeds germinating out of 10 on dam filter for 80 sets of seeds. Fit a Binomial distribution to these data.

x:	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

Sol. Total $\Sigma f = 80$

$$\text{Mean} = \frac{\Sigma f(x)}{\Sigma f}$$

$$= \frac{20 + 56 + 36 + 32 + 30}{80}$$

$$= \frac{174}{80} = \frac{87}{40}$$

$$m = np = \frac{174}{80} \qquad [n = 10]$$

$$10\,p = \frac{174}{80} \qquad p = \frac{174}{800} = 0.2175$$

$q = 1 - p = 0.7825$

$P(x) = (0.2175 + 0.7825)^{10}$

Also: Expected frequency $= N \cdot P(x)$

$= 80(0.2175 + 0.7825)^{10}$ //

Que: 14 In litters of 4 mice. The no. of ~~eat~~ litters, which contains 0, 1, 2, 3, 4 female were noted. The data are given below.

No of female mice :	0	1	2	3	4	Total
No. of litters :	8	32	34	24	5	103

If the chance of obtaining a female in a single trial is assumed constant, Estimate this const of unknown probability find also the expected frequency.

$$\text{mean} = \frac{\sum fx}{\sum f}$$

$$= \frac{32 + 68 + 72 + 20}{103} = \frac{192}{103}$$

$$= \underline{\underline{1.86}}$$

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$$\text{mean} = np = \frac{192}{103}$$

$$p = \frac{192}{103 \times 4} = \frac{48}{103} = 0.466$$

$$q = 1 - p = \frac{55}{103} = 0.533$$

$$P(r) = (0.466 + 0.533)^4$$

Expected frequency $= N \cdot P(r)$

$$= 103 \cdot (0.466 + 0.533)^4$$

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Poisson - distribution :

$n \longrightarrow$ infinite

$$P(r) = \sum_{r=0}^{\infty} \frac{e^{-m} m^{r}}{\lfloor r}$$

where $m \rightarrow$ mean

Que: Find the mean & Variance of Poisson distribution
For mo

Sol: For mean, we will find first moment about origin.

$$\mu_1' = r . P(r)$$

$$= \sum_{r=0}^{\infty} r . \frac{e^{-m} m^{r}}{\lfloor r}$$

$$= \sum_{r=0}^{\infty} \cancel{r} . \frac{e^{-m} m^{r-1} m}{\cancel{r} \lfloor r-1}$$

$$= m \sum_{r=\infty} e^{-m} \frac{m^{r-1}}{\lfloor r-1}$$

$$= m \sum_{r=1}^{\infty} e^{m} \frac{m^{r-1}}{\lfloor r-1}$$

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$$= m \cdot e^{-m}\left[1 + m + \frac{m^2}{\lfloor 2} + \cdots\cdots\right\}$$

$$= m \cdot e^{-m} \cdot e^{m}$$

$$= \underline{\underline{m}}$$

$$\mu_1' = m = \text{mean} = \underline{\underline{m}}$$

Similarly $\mu_2' = \sum x^2 \cdot P(x)$

Second moment about origin

$$\mu_2 = m^2 + m$$

$$\text{Variance} = \mu_2' - (\mu_1')^2$$

$$= m^2 + m - m^2$$

$$= \underline{m}$$

In poisson distribution mean & variance are equal.

$$S.D = \sqrt{m}$$

Que: 2 Find the probability that atmost 5 defective fuses will be found in a ~~box~~ box of 200 fuses. If experience shows that 2 % of such fuses are defective.

Sol: $P = \frac{2}{100} = 0.02$

$m = np$
$= 200 \times 0.02$
$= 4.$

$\boxed{mean = 4}$

$P(r) = \sum \frac{e^{-m} \cdot m^{r}}{\lfloor r}$

$= \sum \frac{e^{-4} \cdot 4^{r}}{\lfloor r}$

$\therefore$ at most 5 deffective fuse

$P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$.

$e^{-4} + \left[4e^{-4}\right] + \left[\frac{e^{-4}\,16}{2}\right] + \left[\frac{64e^{-4}}{6}\right] + \left[\frac{e^{-4}\,256}{24}\right]$

$+ \left[\frac{e^{-4}\;1024}{120}\right]$

$5e^{-4} + 8e^{-4} + \frac{32e^{-4}}{3} + \frac{32e^{-4}}{3} + \frac{256e^{-4}}{30}$

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$$13e^{-4} + \frac{64e^{-4}}{3} + \frac{256}{30}e^{-4}$$

$$\frac{390e^{-4} + 640e^{-4} + 256e^{-4}}{30}$$

$$= \frac{1286e^{-4}}{30}$$

$$= \frac{643}{15}e^{-4} = \frac{643 \times 0.018}{15}$$

$$= 0.7849$$

Que.2 In a certain factory turning razor blades. There is a small chance of 0.002 for ~~f~~ any blade to be defective. The blades are in packets of 10. Use ~~Pis~~ Poisson distribution to calculate the appropriate number of packets.

Containing, (i) No defective
(ii) 1 defective
(iii) Two defective blades

respectively. In a consignment of 50000 packets.

Sol:

$$p = 0.002$$

$$m = np = m = 10 \times 0.002 = 0.02$$

$$\text{Mean} = 0.02$$

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$$P(r) = \sum \frac{e^{-m} m^r}{\lfloor r}$$

$$= \sum \frac{e^{-0.02} (0.02)^r}{\lfloor r}$$

(i) No defective

$$P(0) = \frac{e^{-0.02} \times (0.02)^0}{\lfloor 0} = e^{-0.02}$$

$$= 0.9802$$

$$P(1) = \frac{e^{-0.02} (0.02)^1}{1} = 0.01960$$

$$P(2) = \frac{e^{-0.02} (0.02)^2}{\lfloor 2} = 0.00019604$$

Now In Consinment = Expected frequency

$$\therefore N\,P(0) = 50{,}000 \times 0.9802$$
$$= 49010$$

$$N(P(1)) = 50000 \times 0.01960$$
$$= 980$$

$$N\,P(2) = 50000 \times 0.00019604$$
$$= 9.802$$

Ques:4 If 3% of the electric bulbs manufactured by a company are defective. Find the probability that in a Sample of 100 bulbs exactly 5 bulbs are defective.

$$p = \frac{3}{100} = 0.03$$

$$m = n \times p = 3$$

$$n = 100.$$

$$P(r) = \sum \frac{e^{-m} m^r}{\lfloor r}$$

$$P(5) = \frac{e^{-3} 3^5}{120} = \underline{\underline{0.10081}}$$

Ques:5 A Car hire firm two Cars which it hires out day by day. The number of demands for a car on each day is distributed as a poission distribution. If both mean = 1.5, Calculate the proportion of days on which neither car is used and the proportion of days on which Some demand is refused.

Sol:
$$P(r) = \sum \frac{e^{-m} m^r}{\lfloor r}$$

$$P(r) = \sum \frac{e^{-1.5} (1.5)^0}{\lfloor 0}$$

Neither car is used

$$P(0) = \frac{e^{-1.5}(1.5)^0}{L0} = 0.2231$$

(ii) Some demand is refused :

$x > 2$

$P(3) + P(4) + \cdots$

$= 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \{0.223 + 0.33465 + 0.2509875\}$

$= 1 - \underline{0.8086375}$ 0.1385

$= \underline{0.1913625}$ 0.8615

Que: 6 Fit a Poission distribution to the following Calculate theoritical frequencies

x :	0	1	2	3	4
f :	122	60	15	2	1

$$m = \frac{\Sigma f(x)}{\Sigma f} = 60 + 30 + 6 + 4$$

$$= \frac{100}{200} = \frac{1}{2}$$

$$= \underline{0.5}$$

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$$P(r) = \sum \frac{e^{-m} \cdot m^r}{\lfloor r}$$

$$P(0r) = \sum \frac{e^{-0.5} \cdot (0.5)^r}{\lfloor r}$$

$$P(0) = \sum \frac{e^{-0.5} \cdot (0.5)^0}{\lfloor 0} = 0.6065$$

$$P(1) = \sum \frac{e^{-0.5} (0.5)^1}{\lfloor 1} = 0.3032$$

$$P(2) = \sum \frac{e^{-0.5} (0.5)^2}{\lfloor 2} = 0.07581$$

$$P(3) = \sum \frac{e^{-0.5} (0.5)^3}{\lfloor 3} = 0.01263$$

$$P(4) = \sum \frac{e^{-0.5} (0.5)^4}{\lfloor 4} = 0.0015794$$

$N \cdot P(0) = 121.3$

$N \cdot P(1) = 60.64$

$N\,P(2) = 15.162$

$N\,P(3) = 2.526$

$N\,P(4) = 0.31588$

derivation

x:	0	1	2	3	4	5	6
f:	128	110	49	14	3	1	0

Que: 7 A Skilled typist on routine work kept a record of mistakes made per day during 300 working days

Mistakes per day :	0	1	2	3	4	5	6
No. of days :	143	90	42	12	9	3	1

Fit a poission distribution to the above data & calculate expected frequencies

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{90+84+36+36+15+6}{143+90+42+12+9+3+1}$$

$$= \frac{267}{300} = \underline{\underline{0.89}}$$

$$P(r) = \Sigma \frac{e^{-m}\, m^r}{\lfloor r}$$

$$P(r) = \Sigma \frac{e^{-(0.89)}\,(0.89)^r}{\lfloor r}$$

$$P(0) = \frac{e^{-(0.89)}\,(0.89)^0}{\lfloor 0} = 0.41065$$

$$P(1) = \frac{e^{-(0.89)}\,(0.89)^1}{\lfloor 1} = 0.36548$$

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$$P(2) = \frac{e^{-(0.89)}(0.89)^2}{\lfloor 2} = 0.16264$$

$$P(3) = \frac{e^{-(0.84)}(0.89)^3}{\lfloor 3} = 0.04824$$

$$P(4) = \frac{e^{-(0.89)}(0.89)^4}{\lfloor 4} = 0.010735$$

$$P(5) = \frac{e^{-(0.84)}(0.84)^5}{\lfloor 5} = 0.001910$$

$$P(6) = \frac{e^{-(0.84)}(0.89)^6}{\lfloor 6} = 0.000283$$

$N(P(0)) = 123.195$

$NP(1) = 109.643$

$NP(2) = 48.792$

$NP(3) = 14.4947$

$NP(4) = 3.2206$

$NP(5) = 0.5732$

$NP(6) = 0.08503557$

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Que: 8 Prove that Poisson distribution has a limiting case of Binomial distribution when

1. n is large i.e $n \to \infty$
2. p is small.
3. $np = m$ is finite.

Sol: By defination of Binomial distribution

$$P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$$

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} \cdot p^r \cdot (1-p)^{n-r} \quad \because p+q=1$$

$$= \frac{\{n(n-1)(n-2)\cdots\cdots\{n-(r-1)\}\} \cdot \cancel{\lfloor n-r}}{\lfloor r \cdot \cancel{\lfloor n-r}} \cdot \left(\frac{m}{n}\right)^r \left\{1-\frac{m}{n}\right\}^{n-r}$$

$$= \frac{n^r\left\{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left[1-\frac{r-1}{n}\right]\right.}{\lfloor r \cdot n^r} \cdot m^r \cdot \frac{(1-m/n)^n}{(1-m/n)^r}$$

$$= \lim_{n\to\infty} \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left\{1-\frac{r-1}{n}\right\}}{\lfloor r} \cdot n^r \cdot \frac{\left\{(1-m/n)^{-n/m}\right\}^{-m}}{(1-m/n)^r}$$

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Normal distribution

$$y = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-m)^2/2\sigma^2}$$

where $m \rightarrow$ mean
$\sigma \rightarrow$ S.D

★ Area under the normal curve :- $Z = \frac{x-m}{\sigma}$ → (i)

Que: 1 If X is a normal variate with mean = 30 & std deviation = 5
Find the probability that

(i) $26 \le x \le 40$

(ii) $X \ge 45$

(iii) $|X-30| > 5$

Sol: W.K.T (i) $Z = \frac{x-m}{\sigma}$

$$Z = \frac{x-30}{5} = \frac{26-30}{5} = \frac{-4}{5}$$

$$= -0.8$$

also $Z = \frac{40-30}{5} = \frac{10}{5} = 2$

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$P(26 \le X \le 40) = P(-0.8 \le z \le 2)$

$= P(-0.8 \le z \le 0) + P(0 \le z \le 2)$.

$= 0.2881 + 0.4772$

$= 0.7653$

(ii) $Z = \frac{x - m}{\sigma} = \frac{x - 30}{6}$

$Z = \frac{45 - 30}{5} = 3$

$P(x \geq 45) = P(z \geq 3)$

$= 0.5 - P(0 \le z \le 3)$

$= 0.5 - 0.4986$

$= \underline{\underline{0.0014}}$

(iii) $|x - 30| > 5$

$P(|x - 30| \cancel{>} 5) = P(25 \le x \le 35)$

$= P(-1 \le x \le 1)$

$= 2P(0 \le x \le 1)$

$= 2 \times 0.3413$

$= \underline{\underline{0.6826}}$

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$= 1 - 0.6826$

$= 0.3174$

Que. 2 A Certain type of wooden beam as a mean breaking Strength 1500 kg and Standard deviation of $\sigma = 100$ kg Find the relative frequencies of all Such beam where breaking Strength are between 1450 and 1600 kg

$$Z = \frac{x - m}{\sigma} \qquad (1450 \le x \le 1600)$$

$$Z = \frac{1450 - 1500}{100} = \frac{-50}{100} = -0.5$$

$$Z = \frac{1600 - 1500}{100} = \frac{100}{100} = 1$$

$$P(1450 \le x \le 1600) = P(-0.5 \le Z \le 1)$$

$$= P(-0.5 \le Z \le 0) + P(0 \le x \le 1)$$

$$= 0.1915 + 0.3413$$

$$= 0.5328$$

Que: 3 The mean height of 500 Students is 151 cm and the Standard deviation is 15 cm assuming that the heights are normally distributed, Find how many Student have height between $P(122 < x < 155)$ cm

$$z = \frac{x - m}{\sigma}$$

$$(122 \leq x \leq 155)$$

$$z = \frac{122 - 151}{15} = \frac{-29}{15} = -1.93$$

$$z = \frac{155 - 151}{15} = 0.26$$

$$P(122 < x < 155) = P(-1.93 < z < 0.26)$$

$$= P(-1.93 < z < 0) + P(0 \leq z < 0.26)$$

$$= 0.4713 + 0.0793$$

$$= 0.5506$$

Expected freq $N = 500 \times 0.5506$
$= 275.3$

Que: 4 The distribution of weekly wages of 500 workers in a factory is approximately Normal with mean and S.D of

Rupees 75 and 15 respectively. Find the No. of workers who receive weekly wages

1. more than 90 (>90)
2. Less than 45 (<45)

Sol:

$$z = \frac{x - m}{\sigma}$$

$$z = \frac{90 - 75}{15} = \frac{15}{15} = 1$$

$$z = \frac{45 - 75}{15} = -2$$

$$P(x > 90) = P(z > 1)$$

$$= 0.5 - P(0 < x < 1)$$

$$= 0.5 - 0.3413$$

$$= \underline{\underline{0.1587}}$$

$$P(x < 45) = P(z < -2)$$

$$= 0.5 - P(-2 > x > 0)$$

$$= 0.5 - 0.4772$$

$$= \underline{\underline{0.0228}}$$

Expected freq $(x > 90) = 0.1587 \times 500$
$= \underline{\underline{79.35}}$

Expected freq $(x < 45) = 0.228 \times 500$
$= \underline{\underline{11.4}}$

Que:5 For the normal (curve) Equation

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-m)^2/2\sigma^2}$$

Find the mean and Standard deviation.

Sol: $\because$ $f(x)$ is P.d.f

Then mean $= \int_{-\infty}^{\infty} x.f(x)$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-m)^2/2\sigma^2} \, dx$$

put $\frac{(x-m)}{\sqrt{2}\sigma} = t$

$$\frac{dx}{\sqrt{2}\sigma} = dt$$

$\therefore$ $x - m = \sqrt{2}\sigma t$ $\quad \therefore \boxed{x = m + \sqrt{2}\sigma t}$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (m + \sqrt{2}\sigma t) \cdot e^{-t^2} \sqrt{2}\, \sigma \, dt.$$

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$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} m \cdot e^{-t^2} dt + \int_{-\infty}^{\infty} \sqrt{2}\, \sigma\, t\, e^{-t^2} dt$$

$$= \frac{2m}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt + 0$$

$$= \frac{2m}{\sqrt{\pi}} \cdot \frac{1}{2} \int_{0}^{\infty} e^{-x} \cdot x^{-1/2} dx$$

putting $t^2 = x$

$2t\, dt = dx$

$dt = \frac{dx}{2\sqrt{x}}$

$$= \frac{m}{\sqrt{\pi}} \int_{0}^{\infty} e^{x} \cdot x^{1/2 - 1} dx$$

$$= \frac{m}{\sqrt{\pi}} \Gamma\left(\tfrac{1}{2}\right)$$

$$\Gamma(n) = \int_{0}^{\infty} e^{-x} x^{n-1} dx$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$= \frac{m}{\cancel{\sqrt{\pi}}} \cancel{\sqrt{\pi}}$$

$$= m$$

Similarly

$$\text{Variance} = \int_{-\infty}^{\infty} (x-m)^2 \cdot f(x)\, dx = \sigma^2$$

$$S.D = \sigma$$

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Que:6 The life time of a certain kind of battery has a mean of 300 hours and Standard deviation 35 hours. Assuming that the distribution of life time which are measured to the nearest hour is normal. Find the percentage of battery of more than 370 hour

$$m = 300$$
$$\sigma = 35$$

$$Z = \frac{x - m}{\sigma} = \frac{370 - 300}{35}$$

$$Z = 2$$

$$(P > 370) = (z > 2)$$
$$= 0.5 - (0 < z < 2)$$
$$= 0.5 - 0.4772$$
$$= \underline{\underline{0.0228}}$$

$$\text{Percentage} = \underline{\underline{2.28\ \%}}$$

Que: Find the expected Value of discrete random Variable x ppm is

$$f(x) = P(x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x}, \quad x = 0, 1, 2, \ldots$$

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Que: Calculate mean & variance of the throwing of an unbiased dice.

$$E(x) = \Sigma x . P(x)$$

$$= \{1 . P(1) + 2 . P(2) + 3P(3) + 4(P(4)) + 5P(5) + 6P(6))\}$$

$$= \left\{1 . \frac{1}{6} + 2 . \frac{1}{6} + 3 . \frac{1}{6} + 4 . \frac{1}{6} + 5 . \frac{1}{6} + 6 . \frac{1}{6}\right\}$$

$$= \frac{1+2+3+4+5+6}{6}$$

$$= \frac{6(6+1)}{2 \times 6} = \frac{7}{2} = 3.5$$

$$\Sigma(x^2) = \Sigma x^2 P(x)$$

$$= \{1^2 . P(1) + 2^2 P(2) + 3^2 P(3) + 4^2 P(4) + 5^2 P(5) + 6^2 P(6)\}$$

$$= \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2}{6}$$

$$= \frac{6(6+1)(2 \times 6 + 1)}{6 \times 6}$$

$$= \frac{7 \times 13}{6}$$

$$= 15.6$$

$$Var(x) = E(x^2) - \{E(x)\}^2$$

$$= 15.16 - (3.5)^2$$

$$= \underline{\underline{2.91}}$$

The bernoulli's distribution or trials is introduced by Jacob Bernoulli, is one of the simplest yet most important random process in probability.

There are 3 assumptions in bernoulli's trial distribution :-

1) Each trial has 2 possible outcomes called success & failure.

2) The trials are independent, the outcome of 1 trial has no influence over the outcome of other trial.

3) On each trial, the probability of success is small P, the probability of failure is (1 - P).

A discrete random variable X is said to bernoulli distribution with parameter P with its mass function, is given by :

$$P(X=x) = \begin{cases} p^x (1-p)^{1-x} & ; x = 0, 1 \\ 0 & ; \text{otherwise.} \end{cases}$$

$P(X=0) = 1-p.$
$P(X=1) = p.$

Chebyshev inequality :-

$\mu \pm 2\sigma \longrightarrow 75\%.$
$\mu \pm 3\sigma \longrightarrow 88.9\%$

The variance of random variable X gives us an idea about the variability of the observations about mean.

Chebyshev inequality gives us bound on probability that how a random variable is deviated when both mean & variance are known / given.

<u>Statement</u> → If x is random variable with mean μ and variance σ^2 then for any value of $k > 0$.

$$\boxed{P\{|x-\mu| \geq k\} \leq \frac{\sigma^2}{k^2}}$$

Proof :- We prove the above inequality for continuous random variable.

$$E(x) = \int_0^\infty x \cdot P(x)\, dx \quad \text{--- (1)}$$

$$= \int_0^a x \cdot P(x)\, dx + \int_a^\infty x \cdot P(x)\, dx.$$

$$E(x) \geq a \int_a^\infty P(x)\, dx$$

$$\frac{E(x)}{a} \geq \int_a^\infty P(x)\, dx$$

$$\frac{E(x)}{a} \geq P(x \geq a) \quad \text{--- (2)}$$

Eq (2) is called Markov inequality.

In eq (2) we put $x = (x-\mu)^2$
$a = k^2$

$$P((x-\mu)^2 \geq k^2) \leq \frac{E(x-\mu)^2}{k^2}$$

$$P(|x-\mu| \geq k) \leq \frac{\text{Var}\, x}{k^2} = \frac{\sigma^2}{k^2}$$

$$\boxed{P(|x-\mu| \geq k) \leq \frac{\text{Var}\, x}{k^2} = \frac{\sigma^2}{k^2}}$$

Hence proved.

Que: A random variable X with unknown distribution has mean $\mu = 8$ and Varian[ce] $\sigma^2 = 9$

(i) $P(-4 < x < 20)$
(ii) $P(|x-8| \geq 6)$

Sol: W.K.T $P(|x-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$

(ii) $P(|x-8| \geq k) \leq \frac{9}{k^2}$

$\because k = 6$

$P(|x-8| \geq 6) \leq \frac{9}{36} = \frac{1}{4}$.

eg: $\because k = 3\lambda$

$P(|x-8| \geq 3\lambda) \leq \frac{1}{\lambda^2}$

$P(|x-8| < 3\lambda) \leq 1 - \frac{1}{\lambda^2}$

$P(8-3\lambda < x < 8+3\lambda) \leq 1 - \frac{1}{\lambda^2}$

$P(-4 < x < 20) \leq \frac{15}{16}$

[Q]ue: $f(x) = k(1-x^2)$ $\quad 0 < x < 1$

1. Find k
2. Mean
3. Variance.

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Sol: As we know that

Total probability = 1

$$\int_0^1 f(x)\, dx = 1$$

$$\int_0^1 k(1-x^2)\, dx = 1$$

$$k\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[1 - \frac{1}{3}\right] = 1$$

$$\frac{2k}{3} = 1$$

$$\boxed{k = \frac{3}{2}}$$

$$\text{Mean} = \int_0^1 x \cdot f(x)\, dx$$

$$= \int_0^1 x \cdot k(1-x^2)\, dx$$

$$= \frac{3}{2}\int_0^1 x - x^3\, dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1}{2} - \frac{1}{4}\right] \Rightarrow \frac{3}{8}$$

Variance $= E(x^2) - [E(x)]^2$

Now. $E(x^2) = \frac{3}{2}\int_0^1 x^2 - x^4 \, dx.$

$$= \frac{3}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{15}} = \frac{1}{5}$$

$\therefore$ Variance $= \frac{1}{5} - \left(\frac{3}{8}\right)^2$

$$= \frac{1}{5} - \frac{9}{64} = \frac{64-45}{320} = \frac{19}{320}$$

Median of Normal Distribution

Median (Md)

Let Median be Md.

$$\int_0^{md} f(x)\,dx = \int_{Md}^1 f(x)\,dx = \frac{1}{2}$$

$$\int_0^{md} \frac{3}{\cancel{2}}(1-x^2)\,dx = \frac{1}{\cancel{2}}$$

$$\left[x - \frac{x^3}{3}\right]_0^{md} = \frac{1}{3}$$

$$Md - \frac{md^3}{3} = \frac{1}{3}$$

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$md^3 - 3md + 1 = 0$

Que: Fit a Normal Curve to the following data

Length of Line Lin(cm): ~~8.60 8.59 8.58~~

Length of line (in cm)	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.53	8.
Frequency	2	3	4	9	10	8	4	1	1

Sol: Let the normal curve by $y = \frac{N}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$

$$N = \Sigma f = 42$$

$$\text{mean } (m) = A + \frac{\Sigma fu}{\Sigma f}$$

$$S.D\ (\sigma) = \sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{0.0133}{42} - \left(\frac{0.11}{42}\right)^2}$$

$$= \sqrt{\frac{0.133}{42} - \frac{0.0121}{1764}}$$

$$= 0.0175$$

$$\sigma = \sqrt{\frac{234.1038}{74088}} = \cancel{0.0}0562$$

$$\sigma = \sqrt{0.0081598}$$

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x	f	$u = x - A$ $A = 8.56$	u^2	fu	fu^2
8.60	2	0.04	0.0016	0.08	0.0032
8.59	3	0.03	0.0009	0.09	0.0027
8.58	4	0.02	0.0004	0.08	0.0016
8.57	9	0.01	0.0001	0.09	0.0009
8.56	10	0	0	0	0
8.55	8	-0.01	0.0001	-0.08	0.0008
8.54	4	-0.02	0.0004	-0.08	0.0016
8.53	1	-0.03	0.0009	-0.03	0.0009
8.52	1	-0.04	0.0016	-0.04	0.0016
				$\Sigma fu = 0.11$	$\Sigma fu^2 = 0.0133$

$$\text{Mean} = 8.56 + \frac{0.11}{42}$$

$$= \underline{\underline{8.5626}}$$

$$y = \frac{42}{0.0562 \times \sqrt{2\pi}} e^{-(x-m)^2/2(0.0175)^2}$$

Gamma distribution

A continuous random variable X is said to be gamma distribution with parameter λ if its p.d.f is given by

$$\Gamma\lambda = \int_0^\infty e^{-x} \cdot x^{\lambda-1}\, dx$$

$$f(x) = \begin{cases} \dfrac{e^{-x} \cdot x^{\lambda-1}}{\Gamma\lambda}, & 0 < x < \infty, \text{ where } \lambda > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Que:1 Find the mean and variance of the gamma distribution.

(1) mean $\mu_1' = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx.$

$$\mu_1' = \int_0^\infty \frac{x \cdot e^{-x} \cdot x^{\lambda-1}}{\Gamma\lambda}\, dx.$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x} \cdot x^{\lambda}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x}\, x^{(\lambda+1)-1}\, dx$$

By defination of gamma

$$= \frac{1}{\Gamma\lambda} \cdot \Gamma\lambda+1 =$$

$$\Gamma\lambda+1 = \lambda\Gamma\lambda$$

$$= \frac{1}{\cancel{\Gamma\lambda}} \cdot \lambda \cancel{\Gamma\lambda}$$

$\boxed{\text{Mean} = \lambda}$

(2) Variance $= \int_{-\infty}^{\infty} x^2 . f(x)\, dx$

$$u_2^2 = \int_0^{\infty} \frac{x^2 . e^{-x} . x^{\lambda-1}}{\Gamma\lambda}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^{\infty} e^{-x}\, x^{\lambda+1}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^{\infty} e^{-x} . x^{(\lambda+2)-1}\, dx$$

$$= \frac{1}{\Gamma\lambda}\, \Gamma\lambda+2 = \frac{1}{\Gamma\lambda} \cdot \Gamma(\lambda+1)+1$$

$$= \frac{1}{\Gamma\lambda} (\lambda+1)\, \Gamma\lambda+1$$

$$= \frac{1}{\cancel{\Gamma\lambda}} (\lambda+1)\, \lambda \cancel{\Gamma\lambda}$$

$$= \lambda(\lambda+1)$$

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Exponential Distribution:

1/ A random variable 'x' is said to be exponential distribution with parameter $\lambda > 0$ if x its p.d.f is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; \text{Otherwise.} \end{cases}$$

Que: 1 Show that the total p.d.f of exponential distribution is unity.

Proof : Total area $= \int_{-\infty}^{\infty} f(x)\, dx$

$$= \int_{0}^{\infty} f(x) \cdot dx$$

$$= \int_{0}^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= \left| \frac{\cancel{\lambda} \cdot -e^{\lambda x}}{\cancel{\lambda}} \right|_{0}^{\infty}$$

$$= -e^{-\infty} + e^{0}$$

$$= 1$$

Hence proved

Que 2 Find the mean and variance of exponential distribution.

Mean → first moment about origin.

$$\mu_1' = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{\infty} x \cdot f(x)\, dx$$

$\overset{\downarrow}{0}$

$$= \int_{0}^{\infty} \lambda \cdot e^{-\lambda x} \cdot x\, dx$$

[U.V.

$$= \lambda \int_{0}^{\infty} e^{-\lambda x} \cdot x\, dx$$

$$= \lambda \left[\frac{\cancel{x}\,x \cdot e^{-\lambda x}}{\lambda} + \frac{\cancel{x^2}\, e^{-\lambda x}}{\lambda^2} \right]_0^{\infty}$$

$$= \frac{1}{\lambda} \,//$$

$$\text{Mean} = \frac{1}{\lambda}$$

$$\mu_2' = \int_{0}^{\infty} x^2 \cdot f(x)\, dx$$

$$= \int_{0}^{\infty} x^2 \cdot \lambda e^{-\lambda x}\, dx$$

$$= \lambda \left[x^2 \cdot e^{-\lambda x}\, dx \right]_0^{\infty}$$

$$= \lambda \left[x^2 \left\{ \frac{e^{-\lambda x}}{-\lambda} \right\} - 2x \left\{ \frac{e^{-\lambda x}}{\lambda^2} \right\} + 2 \left\{ \frac{e^{-\lambda x}}{-\lambda^3} \right\} \right]_0^{\infty}$$

$$= 0 + \lambda \left[2 \cdot \frac{1}{\lambda^3} \right]$$

$$\mu_2' = \frac{2}{\lambda^2}$$

$$\text{Variance} = \mu_2' - (\mu_1')^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$= \frac{1}{\lambda^2}$$

$$S.D = \sqrt{\text{Variance}}$$

$$= \frac{1}{\lambda}$$

SHEPPARD'S CORRECTION OF MOMENTS:

~~Dee~~ When moments are calculated in case of class interval, It is supposed that the frequency of each class interval is centered at the midpoint of the respective class. There is probability of some error in the values of the moment. To remove these error is called the Sheppard's Correction of moments.

$$\mu_1 \text{ (Corrected)} = \mu_1 = 0$$

$$\mu_2 \text{ (—"—)} = \mu_2 - \frac{1}{12}h^2$$

$$\mu_3 \text{ (—"—)} = \mu_3$$

$$\mu_4 \text{ (—"—)} = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4$$

Note

σ_1^2 is the value of Variance which is obtained from the grouped data and σ^2 be the Corrected Value then

$$\sigma^2 = \sigma_1^2 - \frac{1}{12}h^2$$

Que: The Value of 2nd, 3rd & 4th moment are $\mu_2 = 88.75$

$\mu_3 = -131.25$

$\mu_4 = 25445.3125$

Calculate the Corrected moment when the class interval is 10

$$\mu_2 \text{ (Corrected)} = 88.75 - \frac{1}{12} \times 100$$

$$= 80.417$$

μ_3 (corrected) $= -181.25$

μ_4 (corrected) $= 25445.3125 - \frac{100}{2} \times 80.41$

$+ \frac{7}{240} \times 10000$

$= 25445.3125 - 4020.85 + 291.6667$

$= \underline{\underline{21716.1295}}$

KARL PEARSON'S , α, β, γ Coefficients.

ALPHA coeff	Beta coeff	gamma coeff
$\alpha_1 = \frac{\mu_1}{\sigma} = 0$	$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$	$\gamma_1 = \pm\sqrt{\beta_1}$
$\alpha_2 = \frac{\mu_2}{\sigma^2} = 0$	$\beta_2 = \frac{\mu_4}{\mu_2^2}$	$\gamma_2 = \beta_2 - 3 =$ $\frac{\mu_4 - 3\mu_2^2}{\mu_2^3}$
$\alpha_3 = \frac{\mu_3}{\sigma^3} = \frac{\mu_3}{(\mu_2)^{3/2}}$		
$\alpha_4 = \frac{\mu_4}{\sigma^4} = \mu_4/(\mu_2)^2$		

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Que: The sign of gamma 1 depends on μ_3.
If μ_3 is positive then gamma 1 is +ve
If μ_3 is -ve then gamma 1 is -ve.

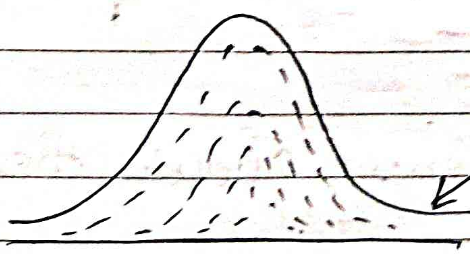
* Coefficient of Skewness based on moments

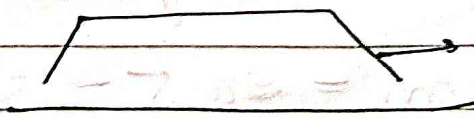
When there is Symmetrical distribution, all the moment of odd order about the arithmatic mean i.e. ($\mu_1, \mu_3, \mu_5 \ldots$) Vanish.
If the value of those Coefficient do not Vanish then there is Skewness in the frequency distribution.

- First Coefficient of Skewness = μ

$$= \frac{\mu_3}{\sqrt{\mu_2^3}} = \sqrt{B_1} = \alpha_3 = \gamma_1$$

- Second $= \dfrac{\sqrt{B_1}\,(B_2+3)}{2\{5B_2 - 6B_1 - 9\}}$


Leptokurtosis


Platukurtosis.


Mesokurtists

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Kurtosis measures the degree of peakedness of a distribution and is given by

$$B_2 = \frac{\mu_4}{\mu_2^2}$$

$$\mu_2 = B_2 - 3$$

gives the access of Kurtosis

if $B_2 > 3$ is Leptokurtosis

if $B_2 = 3$ is Mesokurtosis.

if $B_2 < 3$ is Platykurtosis.

Que:1 The first 4 moment about the working mean i.e 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98 Calculate the moment about the mean. also Evaluate B_1, B_2 & Comments upon the Skewness and Kurtosis of the distribution.

Sol: The first 4 moments about the arbitrary

$m = 28.5$ $\qquad \mu_4' = 454.98$

$\mu_1' = 0.294$.

$\mu_2' = 7.144$.

$\mu_3' = 42.409$

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Now $\mu_1' = \frac{1}{N}\sum f_i x_i - 28.5$

$0.294 = \bar{x} - 28.5$

$\bar{x} = 28.794$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 7.144 - 0.0864$

$= \underline{\underline{7.0576}}$

$\mu_3 = \mu_3' - 3\mu_2' . \mu_1' + 2\mu_1'^3$

$= 42.409 - 6.301008 + 0.05082$

$= \underline{\underline{36.158}}$

$\mu_4 = \mu_4' - 4\mu_3' . \mu_1' + 6\mu_2' . \mu_1'^2 - 3(\mu_1')^4$

$= 454.98 - 49.872984 + 3.70499 - 0.0224$

$= \underline{\underline{408.789}}$

$B_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{1307.40}{49.81} = 26$

Que: Calculate the median Quartiles and the Quartile Coefficient of Skewness from the following data

Weight (Lbs):	70-80	80-90	90-100	100-110
No of persons:	12	18	35	42
	110-120	120-130	130-140	140-150
	50	45	20	8

$\Sigma f = 230$:

CF : = 12, 30, 65, 107, 157, 202, 222, 230

Sol: Now $\frac{N}{2} = \frac{230}{2} = 115^{th}$ Lies between 110-120 group

$\therefore$ Median or $Q_2 = L + \frac{\frac{N}{2} - C^f}{f} \times h$

$= 110 + \frac{115 - 107}{5\cancel{0}} \times 1\cancel{0}$

$= 110 + \frac{8}{5} = 111.6$

also $\frac{N}{4} = \frac{230}{4} = 57.5$ i.e $Q_1 = 58^{th}$

which lies between 90-100

$$Q_1 = L + \frac{N/4 - C}{f} \times h = 9$$

$$= 90 + \frac{57.5 - 30}{35} \times 10 = 97.85$$

Similarly $\frac{3N}{4} = 172.5$ lies Q_3 in 120 – 130

$$Q_3 = L + \frac{\frac{3N}{4} - C}{f} \times h$$

$$= 120 + \frac{175.5 - 157}{45} \times 10$$

$$= 123.44$$

Hence quartile coefficient of skewness

$$= \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1} = \frac{-1.91}{25.59}$$

$$= \underline{\underline{-0.0746}}$$

PROBLEMS RELATED TO EXPONENTIAL DISTRIBUTION

Que: 1 A random variable x has an expo. distribution with p.d.f is given by.

$$f(x) = \begin{cases} 2e^{-x} ; & \text{if } x > 0 \\ 0 ; & \text{if } x \leq 0. \end{cases}$$

Sol: Compute the probability that $x \nless 3$. also find the mean and std dev. and PT Coefficient of variation is unity.

Sol:- By defination of exponential distribution

$$E(\text{dis}) = \int_0^{\infty} f(x)\, dx$$

$$= 2\int_3^{\infty} e^{-x}\, dx$$

$$= 2 - [e^{-x}|_3^{\infty}$$

$$= 2[-e^{-\infty} + e^{-3}]$$

$$= e^{-6}$$

(ii) To find mean $\Rightarrow \mu_1' = E(x)$

$$= \int_{-\infty}^{\infty} x.f(x)\, dx = 2\int_0^{\infty} x.e^{-x}\, dx$$

$$= 2\{-x.e^{-x} - 1.e^{-x}\}_0$$

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$$= \left[2e^{-x} [+x+1] \right]_0^{\infty}$$

$$= +2e^{0}$$

$$= \underline{2}$$

$$\mu_2' = \int_0^{\infty} x^2 \cdot (2e^{-x})\, dx.$$

$$= 2 \left[-x^2 e^{-x} - 2xe^{-x} + 2e^{-x} \right]_0^{\infty}$$

$$= +2 \left[2 \right]$$

$$= \underline{\underline{+4}}$$

$$\text{Variance} = \mu_2' - (\mu_1')^2$$
$$= 4 - 4$$
$$= 0$$

Que:2 The Income tax X, of a man has an exponential distribution with P.d.f is given by

$$f(x) = \begin{cases} \frac{1}{4} e^{-x/4} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

If income tax is levied at the rate of 5%. What is the probability that his income exceed Rs 10,000.

Soln: If Income exceed Rs 10000, then income tax will be income

$$\text{income tax} = 10000 \times \frac{5}{100} = 500$$

∴ required probability :-

$$P(x > 500) = \int_{500}^{\infty} f(x)\, dx$$

$$= \frac{1}{4} \int (e^{-x/4})\, dx$$

$$= \frac{1}{4} \left(\frac{e^{-x/4}}{-1/4} \right)_{500}^{\infty}$$

$$= \cancel{\frac{1}{4}} - (0 - e^{-125})$$

$$= e^{-125}$$

Que: The lifetime of a certain kind of battery is a random variable, which is as an exponential distribution with a mean of 200 hrs.

Find the probability that such a battery will last.

1) at most 100 hrs and

2) last any where from 400 to 600 hrs.

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$$\text{exp. dis} = \int_0^\infty \lambda e^{-\lambda x} \, dx$$

$$m = \frac{1}{\lambda}$$

$$\frac{1}{\lambda} = 200$$

$$\lambda = \frac{1}{200}$$

① atmost 100 $= \frac{1}{200} \int_0^{100} e^{-\frac{1}{200}x} \, dx.$

$$= \frac{1}{\cancel{200}} \left[\frac{-\cancel{200}}{\cancel{200}} e^{-1/200 x} \right]_0^{100}$$

$$= -e^{-1/2} + 1$$

② 400 - 600 $= \int_{400}^{600} \frac{1}{200} \times e^{-\frac{1}{200}x} \, dx$

CORRELATION:

If an increase (or decrease) in the values of one variable corresponds to an increase (or decrease) in the other. The Correlation is said to be positive.

If an increase (or decrease) in the values of one variable corresponds to or decrease (or increase) in the other. The Correlation is said to be negative.

If Variables are independent then they are independent of Correlation.

Coefficient of Co-relation:

$$r = \frac{\Sigma XY}{n \sigma x \cdot \sigma y}$$

where $X = x - \bar{x}$

$Y = y - \bar{y}$

σx = S.D of x series

σy = S.D of y series

n = No of values of the two variable

Direct method.

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

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another form

$$r = \frac{n\sum xy - \sum x \cdot \sum y}{\sqrt{\{n\sum x^2 - (\sum x)^2\} \times \{n\sum y^2 - (\sum y)^2\}}}$$

Que: Psychological tests of intelligence and of engineering ability were applied to 10 Students. Here is a record of ungrouped data showing intelligence ratio (I.R) and engineering ratio (E.R). Calculate the Co-efficient of Correlation.

Student :	A	B	C	D	E	F	G	H	I	J
IR :	105	104	102	101	100	99	98	96	93	92
ER :	101	103	100	98	95	96	104	92	97	94

$\bar{Y} = 98$

Student's	Intelligence x; $x - \bar{x} = x$	x^2	y^2	Engineering ratio Y; $Y - \bar{Y} = Y$	xy
A	6	36	9	3	18
B	5	25	25	5	25
C	3	9	4	2	6
D	2	4	0	0	0
E	1	1	9	-3	-3
F	0	0	4	-2	0
G	-1	1	36	6	-6
H	-3	9	36	-6	18
I	-6	36	1	-1	6
J	-7	49	16	-4	28
		$\sum x^2 = 170$	$\sum y^2 = 140$		$\sum xy = 92$

$\Sigma x^2 = 170$ $\Sigma y^2 = 140$ $\Sigma xy = 92$

$$\therefore \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

$$\frac{92}{\sqrt{170 \cdot 140}} = \frac{92}{154.27} = \underline{\underline{0.596}}$$

Q- A computer operator while calculating the coefficient between 2 variates x & y for 25 pairs of observations obtained the following constants :-

$n = 25$; $\Sigma x = 125$; $\Sigma x^2 = 650$;
$\Sigma xy = 508$; $\Sigma y = 100$; $\Sigma y^2 = 460$

It was however latter discovered at the time of checking that he had copied down 2 pairs as (6, 14) & (8, 6) while the correct pairs were (8, 12) & (6, 8) obtain the correct value of the correlation coefficient.

Sol: Given:

	Incorrect data	Correct data
x_1	6	8
x_2	8	6
y_1	14	12
y_2	6	8

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Incorrect data are $\Sigma x = 125$; $\Sigma y = 100$; $\Sigma x^2 = 650$; $\Sigma y^2 = 460$; $\Sigma xy = 508$.

Then Corrected Σx = Incorrected $\Sigma x - (6+8)+(8+6)$

$= 125 - 14 + 14 = \underline{\underline{125}}$

Corrected Σy = Incorrected $\Sigma y - (14+6)+(12+8)$

$= 100 - 20 + 20 = \underline{\underline{100}}$

Corrected x^2 = Incorrected $\Sigma x^2 - (6^2+8^2) + (8^2+6^2)$

$= 650$

Corrected y^2 = Incorrected $\Sigma y^2 - (14^2+6^2)+(12^2+8^2)$

$= \underline{\underline{436}}$

Corrected Σxy = Incorrected $\Sigma xy - (6 \times 14 + 8 \times 6) + (8 \times 12 + 6 \times 8)$

$= \underline{\underline{520}}$.

$$r = \frac{n\Sigma xy - (\Sigma x)\cdot(\Sigma y)}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \times \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{25(520) - 125 \times 100}{\sqrt{16250 - 15625} \times \sqrt{10900 - 10000}}$$

$$\underbrace{\qquad}_{25} \times 30$$

$= \underline{\underline{0.6667}}$

Rank Correlation : Imp

$$r = 1 - \frac{6 \Sigma d_i^2}{n^3 - n}$$

1 Ten participant in a Contest are ranked by two Judges as follows

x : 1 6 5 10 3 2 4 9 7 8

y : 6 4 9 8 1 2 3 10 5 7

Calculate the rank Correlation Coefficient

Sol: $d_i = \Sigma x_i - y_i$

$d_i = -5, 2, -4, 2, 2, 0, 1, -1, 2, 1$

$\Sigma d_i^2 = 25 + 4 + 16 + 4 + 4 + 1 + 1 + 4 + 1$
$= 60$

$$r = 1 - \frac{6 \times 60}{10^3 - 10}$$

$$r = \frac{630}{990} = 0.636$$

① Three Judges A, B, C give the following rank, find which pair of Judges has common approach.

A : 1 6 5 10 3 2 4 9 7 8

B : 3 5 8 4 7 10 2 1 6 9

C : 6 4 9 8 1 2 3 10 5 7

$A = x$	Rank by $B = y$	Rank by $C = z$	d_1 $x - y$	d_2 $y - z$	d_3 $z - x$	d_1^2	d_2^2	d_3^2
1	3	6	−2	−3	5	4	9	25
6	5	4	1	1	−2	1	1	4
5	8	9	−3	−1	4	9	1	16
10	4	8	6	−4	−2	36	16	4
3	7	1	−4	6	−2	16	36	4
2	10	2	−8	8	0	64	64	0
4	2	3	2	−1	−1	4	1	1
9	1	10	8	−9	1	64	81	1
7	6	5	1	1	−2	1	1	4
8	9	7	−1	2	−1	1	04	1
						200	214	60

$$r = 1 - \frac{6\,[200 + 214 + 60]}{990}$$

$$= 1 - 2.8727$$

$$= -1.8727$$

For Comparision

$$\rho(x,y) = 1 - \frac{6\sum d_1^2}{n^3-n}$$

$$= 1 - \frac{6 \times 200}{990}$$

$$= -0.21$$

$$\rho(y,z) = 1 - \frac{6\sum d_2^2}{n^3-n}$$

$$= 1 - \frac{6 \times 214}{990}$$

$$= -0.29$$

$$\rho(z,x) = 1 - \frac{6\sum d_3^2}{n^3-n}$$

$$= 1 - \frac{6 \times 60}{990}$$

$$= 0.6$$

Que:2 The marks obtain by 9 Students in Chemistry and maths are given below:

Marks in Chemistry : 35 23 47 17 10 43 9 6 28

Marks in Maths : 30 33 45 23 8 49 12 4 31

Sol:

Chem. (X)	Maths (Y)	rank R_x	rank R_y	d $R_x - R_y$	d^2
35	30	3	5	-2	4
23	33	5	3	2	4
47	45	1	2	-1	1
17	23	6	6	0	0
10	8	7	8	-1	1
43	49	2	1	1	1
9	12	8	7	1	1
6	4	9	9	0	0
28	31	4	4	0	0
					$\Sigma d^2 = 12$

Now $\Sigma d^2 = 12$

$$r = 1 - \frac{6 \times 12}{729 - 9} =$$

$$= \underline{\underline{0.9}}$$

$r =$

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Que: 3 10 Student get the following percentage of marks in MATHS and PHYSICS.

Maths : 8 36 98 25 75 82 92 62 69 35

Physics : 84 51 91 60 68 62 86 58 35 49

Find rank Corelation.

Maths	Physics	R_x	R_y	d $R_x - R_y$	d^2
8	84	10	3	7	49
36	51	7	8	-1	1
98	91	1	1	0	0
25	60	9	6	3	9
75	68	4	4	0	0
82	62	3	5	-2	4
92	86	2	2	0	0
62	58	6	7	-1	1
69	35	5	10	-5	25
35	49	8	9	-1	1
					$\Sigma d^2 = 90$

$$r = 1 - \frac{6 \times 90}{10^3 - 10}$$

$$= 1 - \frac{540}{990}$$

$$= \underline{\underline{0.454}}$$

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Que: From the following table find the rank Carelation Coefficient

X : 48 33 40 9 16 16 65 24 16 57

Y : 13 13 24 6 15 4 20 9 6 19

Here in the Series X the value of

We can say $m_1 = 3$

in Series Y the value of 6 and 13 are repeated to say $m_2 = 2$ and $m_3 = 2$

$\therefore$ the rank Correlation Coefficient is given by

$$r = 1 - \frac{6\left\{\sum d^2 + \frac{1}{12} m_1(m_1^2 - 1) + \frac{1}{12} m_2(m_2^2 - 1) + \dots\right\}}{n(n^2 - 1)}$$

Here $n = 10$, $m_1 = 3$, $m_2 = 2$, $m_3 = 2$

X	Y	Rank X in (RX)	Rank in Y (RY)	d = Rx-Ry	d2
48	13	8	5.5	2.5	6.25
33	13	6	5.5	0.5	0.25
40	24	7	10	-3.0	9.0
9	6	1	2.5	-1.5	2.25

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			1	2.0	4.0
16	4	3	9	1.0	1.0
65	20	10	4	1.0	1.0
24	9	5	2.5	0.5	0.25
16	6	3	8	1.0	1.0
57	19	9			
Total				$\Sigma d = 0$	$\Sigma d^2 = 41$

$$r = 1 - \frac{6\left\{41 + \frac{1}{12}3(9-1) + \frac{1}{12}2(4-1)\right\}}{10(100-1)}$$

$$= \underline{\underline{0.733}}$$

Que: Find the rank-correlation coefficient for the following data.

X: 68 64 75 50 64 80 75 40 55 64

Y: 62 58 68 45 81 60 68 48 50 70

X. $m_1 = 2$, $m_2 = 3$, $m_3 = 2$

$$r = 1 - \frac{6\left\{\Sigma d^2 + \frac{1}{12}m_1(m_1^2-1) + \frac{1}{12}m_2(m_2^2-1) + \dots\right\}}{n(n^2-1)}$$

X	Y	Rx	Ry	$d = Rx - Ry$	d^2
68	62	7	6	1	1
64	58	5	4	1	1
75	68	8.5	7.5	1	1
50	45	2	1	1	1
64	81	5	10	-5	25
80	60	10	5	5	25
75	68	8.5	7.5	1	1
40	48	1	2	-1	1
55	50	3	3	0	0
64	70	5	9	-4	16

$\Sigma d^2 = 72$

$$r = 1 - \frac{6\left\{72 + \frac{6}{12} + \frac{24}{12} + \frac{6}{12}\right\}}{10(99)}$$

$$= 1 - 0.4545$$

$$= 0.5455$$

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Assignment - 2

Que: 1 Find first four moments of Gamma distribution.

Sol: First moment about origin $= \int_0^\infty x \cdot f(x)\, dx$

$$= \int_0^\infty x \cdot \frac{e^{-x}}{\Gamma\lambda} x^{\lambda-1}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x} x^{\lambda}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x} x^{(\lambda+1)-1}\, dx$$

By defination of gamma function

$$= \frac{1}{\Gamma\lambda} \Gamma\lambda+1$$

$$= \frac{1}{\cancel{\Gamma\lambda}} \lambda\cancel{\Gamma\lambda} = \lambda$$

$$\boxed{\mu_1' = \lambda}$$

Second moment about origin $\mu_2' = \int_0^\infty x^2 \cdot f(x)\, dx$

$$= \int_0^\infty x^2 \cdot \frac{e^{-x}}{\Gamma\lambda} x^{\lambda-1}\, dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x} x^{(\lambda+2)-1}\, dx$$

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By defination of gamma function.

$$= \frac{1}{\Gamma\lambda} \Gamma\lambda+2$$

$$= \frac{1}{\cancel{\Gamma\lambda}} (\lambda+1)\lambda \cancel{\Gamma\lambda}$$

$$\boxed{\mu_2' = \lambda(\lambda+1)}$$

Third moment about origin. $\mu_3' = \int_0^\infty x^3 . f(x) dx$

$$\mu_3' = \int_0^\infty x^3 . f(x) \, dx$$

$$= \int_0^\infty x^3 . \frac{e^{-x} x^{\lambda-1}}{\Gamma\lambda} dx$$

$$= \frac{1}{\Gamma\lambda} \int \cancel{x^3} . e^{-x} \, x^{(\lambda+3)-1} \, dx .$$

By defination of gamma function

$$= \frac{1}{\Gamma\lambda} \Gamma\lambda+3$$

$$= \frac{1}{\cancel{\Gamma\lambda}} (\lambda+2)(\lambda+1)\lambda \cancel{\Gamma\lambda}$$

$$\boxed{\mu_3' = \lambda(\lambda+1)(\lambda+2)}$$

Fourth moment about origin $\mu_4' = \int_0^\infty x^4 \cdot f(x)$

$$\mu_4' = \int_0^\infty x^4 \cdot \frac{e^{-x} x^{\lambda-1}}{\Gamma\lambda} dx$$

$$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-x} x^{(\lambda+4)-1} dx$$

By defination of gomma function

$$= \frac{1}{\Gamma\lambda} \Gamma\lambda+4$$

$$= \frac{1}{\cancel{\Gamma\lambda}} (\lambda+3)(\lambda+2)(\lambda+1)(\lambda) \cancel{\Gamma\lambda}$$

$$\boxed{\mu_4' = \lambda(\lambda+1)(\lambda+2)(\lambda+3)}$$

Que:3 In a Normal Distribution 7% of items are under 35 and 89% are under 63. determine mean & variance of Distribution

∵ 7% items are below 35, i.e. 50 − 7 = 43% items are between 35 & m &
∵ 89 items are below 63, 89 − 50 = 39 items are between m & 63.

for area 0.43; Z = 1.48.
∵ it is less than m
Z = −1.48 and for area 0.39 Z = 1.23

$\therefore \dfrac{(35 - m)}{\sigma} = -1.48$ —— (1)

$\dfrac{63 - m}{\sigma} = 1.23$ —— (2)

from (1) & (2)

$m = 50.3$ $\sigma = 10.33$

$\boxed{\text{Mean} = 50.3}$

Lines of Regression

(I) Y on x : $Y - \bar{Y} = r . \frac{\sigma y}{\sigma x}(x - \bar{x})$

(II) x on Y : $x - \bar{x} = r . \frac{\sigma x}{\sigma y}(y - \bar{y})$.

Que: If there is the angle between the two regression line. then Show that

$$\tan\theta = \frac{1 - r^2}{r} \cdot \frac{\sigma x . \sigma y}{\sigma x^2 + \sigma y^2}$$

Sol: W.K.T eq of the line of regression

(1) Y on x : $Y - \bar{Y} = r . \frac{\sigma y}{\sigma x}(x - \bar{x})$

(2) x on y : $x - \bar{x} = r . \frac{\sigma x}{\sigma y}(y - \bar{y})$

Slopes are :

$$m_1 = r . \frac{\sigma y}{\sigma x} \quad ; \quad m_2 = \cancel{r} . \frac{\cancel{\sigma x}}{\cancel{\sigma y}} \quad \frac{\sigma y}{r . \sigma x}$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{r\left[\frac{\sigma x}{\sigma y} - \frac{\sigma y}{\sigma x}\right]}{1 + r^2}$$

$$= \frac{r\left[\frac{\sigma x^2 - \sigma y^2}{\sigma x \sigma y}\right]}{1 + r^2}$$

$$= \frac{\frac{\sigma y}{r \cdot \sigma x} - \frac{r \sigma y}{\sigma x}}{1 + \frac{\sigma y^2}{\sigma x^2}}$$

$$= \frac{\frac{\sigma y \sigma x - r^2 \sigma x \sigma y}{\cancel{\sigma x}(r \cdot \cancel{\sigma x})}}{\frac{\sigma x^2 + \sigma y^2}{\cancel{\sigma x^2}}}$$

$$= \frac{\sigma x \sigma y (1 - r^2)}{r(\sigma x^2 + \sigma y^2)}.$$

→ When $r = 0$; $\tan\theta = \infty$ or $\theta = \frac{\pi}{2}$.

i.e when the variables are independent the two line of regression are perpendicular to each other.

→ When $r = \pm 1$ $\tan\theta = 0$ i.e $\theta = 0$ or π.

Thus the two lines of regression coincide i.e there is perfect co-relation between the two variables.

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Que: 1 In a partially destroyed record, only the line of regression of Y on x and x on y are available as $4x-5y+33=0$, $20x-9y=107$ respectively; calculate $\bar{x}, \bar{y}$ and the coefficient of co-relation between x and y.

Sol: Since the regression line passes through $(\bar{x}, \bar{y})$

then,
$$4\bar{x} - 5\bar{y} + 33 = 0 \quad \text{--- (1)}$$
$$20\bar{x} - 9\bar{y} - 107 = 0 \quad \text{--- (2)}$$

$$\bar{x} = \frac{107 + 9\bar{y}}{20}$$

$$\frac{107 + 9\bar{y}}{5} - 5y + 33 = 0$$

$$107 + 9\bar{y} - 25\bar{y} + 165 = 0$$

$$272 - 16\bar{y} = 0$$

$$\boxed{\bar{y} = 17}$$

$$\boxed{\bar{x} = 13}$$

Rewriting the line of regression of y on x

$$Y = \frac{4}{5}x + \frac{33}{5} \quad \text{we get}$$

$$b(y\,x) = r\,\frac{\sigma y}{\sigma x} = \frac{4}{5} \quad \text{--- (3)}$$

Rewriting the line of regression of x on y ;

$$x = \frac{9}{20} y + \frac{107}{9}$$

$$b_{xy} = r \cdot \frac{\sigma x}{\sigma y} = \frac{9}{20} \quad \text{--- } (9_1)$$

$$r^2 = \frac{36}{100} = 0.36$$

$$r = 0.6$$

Coefficient of co-relation b/w x and y $= 0.6$

Que 2 Find the regression equation of y and x and the coefficient of co-relation from the following data $\Sigma x = 60$; $\Sigma y = 40$; $\Sigma xy = 1150$; $\Sigma x^2 = 4160$; $\Sigma y^2 = 1720$ and $n = 10$.

Sol: we have $\bar{x} = \frac{\Sigma x}{n} = \frac{60}{10} = 6$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{10} = 4$$

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x \cdot \Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

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$$= \frac{1150 - \frac{60 \times 4\cancel{0}}{1\cancel{0}}}{1720 - \frac{160\cancel{0}}{1\cancel{0}}}$$

$$= \frac{910}{1560}$$

$$bxy = \underline{\underline{0.5833}}$$

$$r = \sqrt{bxy \times byx}$$

$$byx = \frac{\Sigma xy - \frac{\Sigma x . \Sigma y}{n}}{\Sigma x^2 - \frac{\Sigma (x)^2}{n}}$$

$$= \frac{1150 - \frac{60 \times 4\cancel{0}}{1\cancel{0}}}{4160 - \frac{360\cancel{0}}{1\cancel{0}}}$$

$$= \frac{910}{3800}$$

$$byx = \underline{\underline{0.239}}$$

$$r = \sqrt{bxy \times byx}$$

$$= \sqrt{0.5833 \times 0.239}$$

$$= \underline{\underline{0.3733}}$$

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Also regression eqn of Y on x :-

$$Y - \overline{Y} = byx\,(x - \overline{x})$$

$$Y - 4 = 0.239(x - 6)$$

regression eqn of x on y.

$$x - \overline{x} = bxy\,(y - \overline{y})$$

$$x - 6 = 0.5833\,(Y - 4)$$

Que: If regression eqn of x on y : $5x - y = 22$ and Y on x : $64x - 45y = 24$ are two lines of regression Find:

① The mean value of x and y.
② The regression coefficient
③ The coefficient of correlation b/w x and y.
④ The S.D of y; if the variance of x is 25.

Sol: Since the regression line passes through $(\overline{x}, \overline{y})$.

$$5\overline{x} - \overline{y} = 22 \quad \text{— (1)}$$

$$64\overline{x} - 45\overline{y} = 24 \quad \text{— (2)}$$

from (1) & (2)

$$64\overline{x} - 45(5\overline{x} - 22) = 24$$

$$64\overline{x} - 225\overline{x} + 990 = 24$$

$$+161\overline{x} = \cancel{1014} + 966$$

$$\overline{x} = -\cancel{6.29} \quad 6$$

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$$\bar{y} = \cancel{5x = 6.24} - 22$$

$$\bar{y} = 5\bar{x} - 22$$

$$\bar{y} = 30 - 22 = \underline{\underline{8}}$$

Rewriting the equation y on x:

$$y = \frac{64\bar{x}}{45} - \frac{24}{45}$$

$$byx = r \cdot \frac{\sigma y}{\sigma x} = \frac{64}{45}$$

Rewriting the equation x on y

$$x = \frac{y}{5} + \frac{22}{5}$$

$$bxy = r \cdot \frac{\sigma x}{\sigma y} = \frac{1}{5}$$

$$r = \sqrt{\frac{64}{45} \times \frac{1}{5}} = \frac{8}{15}$$

(ii) Since Variance ~~bxy~~ $\boxed{\sigma x = 25}$

$$byx = r \cdot \frac{\sigma y}{\sigma x}$$

$$\frac{64}{45} = \frac{8}{15} \cdot \frac{\sigma y}{25}$$

$$\sigma y = \underline{\underline{66.667}}$$

Que: 4 If $2x + 3y = 7$ and $5x + 4y = 9$ are 2 lines of regression, Find.

① The mean value of x and y
② The regression coefficients
③ The coefficient of correlation b/w x and y.

Sol: Since the lines of regression passes through $(\bar{x}, \bar{y})$

$$\cancel{10x} + 15y = 35$$
$$\cancel{10x} + 8y = 18$$
$$- \quad - \quad -$$

$$7y = 17$$

$$\bar{y} = \underline{\underline{2.42}}$$

$$x = \frac{7 - 3(2.42)}{2}$$

$$\bar{x} = -\cancel{0.565} = \underline{\underline{-0.13}}$$

Rewriting the equation y on x

$$y = \frac{7}{3} - \frac{2x}{3}$$

$$b_{yx} = r \cdot \frac{\sigma y}{\sigma x} = -\frac{2}{3}$$

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Rewriting the equation x on y:

$$x = \frac{9}{5} - \frac{4y}{5}$$

$$b_{xy} = r \cdot \frac{\sigma x}{\sigma y} = -\frac{4}{5}$$

$$r = \sqrt{\frac{8}{15}}$$

$$r = -\sqrt{\frac{8}{15}}$$

Que:5 Find the two lines of regression from the following data.

Age of husband:	25	22	28	26	35	20	22	40	20	18
Age of wife:	18	15	20	17	22	14	16	21	15	14

Hence estimate: (1) The age of husband; when the age of wife 19

(2) The age of wife; when the age of husband. 30

(3) The Correlation Coefficient between them.

Sol: Let x = Age of husband

Y = Age of wife