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New Scheme Based On AICTE Flexible Curricula

Mechanical Engineering, IV-Semester

ME403 -THEORY OF MACHINES

[1] Introduction, kinematics and kinetics, mechanisms and machines, degree of freedom, types of motions, kinematic concept of links, basic terminology and definitions, joints and kinematic chains, inversions, absolute and relative motions, displacement, velocity and acceleration diagrams, different mechanisms and applications,

[2] kinematic synthesis of linkages, dynamic motion analysis of mechanisms and machines, D'Alembert's principle, number synthesis, free body diagrams, kinematic and dynamic quantities and their relationships, analytical method and graphical method

[3] Cams, introduction, classifications of cams and followers, nomenclature, analysis of cam and follower motion, analytical cam design with specific contours, pressure angle, radius and undercutting, motion constrains and program, critical path motion, torque on cam shaft

[4] Power transmission, kinematics of belt- pulley, flat and v –belt, rope, condition of maximum power transmission, efficiency, friction, friction devices, pivot and collars, power screw, plate and cone clutch, brakes, classifications, bock, band, internal and external, friction circle, friction axis,

[5] Gears, laws of gearing, classification and basic terminology, tooth profiles, kinematic considerations, types of gears, spur, bevel, worm, helical, hypoid etc, gear trains, epicyclic, compound,, balancing- static and dynamic, in same/ different planes, Introduction to vibration, single degree of freedom.

BOOKS:

[1] R.L.Norton,kinematics& dynamics of machinery,Tata McGraw Hill, ISBN13 978 0 07 014480 4
[2] A.Ghosh & A.Malik, Theory of Mechanisms and Machines,EWP Pvt Ltd,ISB 81 85095 72 8

Tutorials:

1. Displacement diagrams of slider crank and other linkages, analytical and graphical
2. Velocity diagrams and acceleration diagrams
3. Diagrams of cam and followers for different applications
4. Gears and gear trains transmission diagrams, analytical and graphical applications
5. Solutions to problems of industrial application using software

Unit-IV

Cams: Classification of Cams and Followers, Radial Cam Terminology, Analysis of Follower motion (uniform, modified uniform, simple harmonic, parabolic, cycloidal), Pressure Angle, Radius of Curvature, Cam Profile for radial and offset followers Synthesis of Cam Profile by Graphical Approach, Cams with Specified Contours.

Introduction

A **cam** is a rotating machine element which gives reciprocating or oscillating motion to another is known as a **follower.** The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at a uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are used to operate the inlet and exhaust valves of internal combustion engines, automatic attachment of machinery, paper cutting machines, spinning and weaving textile machinery, feed mechanism of automatic lathes etc.

Classification of Followers

The followers may be classified as discussed below:

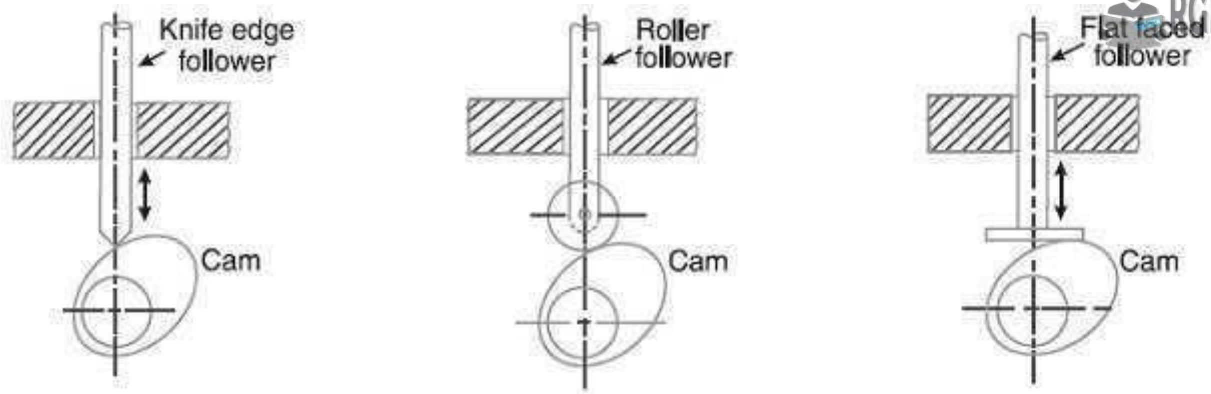
1. According to the surface in contact. The followers, according to the surface in contact, are as follows:

(a) Knife edge follower. Knife edge follower has sharp knife edge at the contact end, as shown in Fig. 4.1 (a). The sliding motion takes place between the contacting surfaces of cam and follower. It is rarely used in practice because the small area of contacting surface results in excessive wear. A considerable side thrust exists between the follower and the guide in case of these followers.

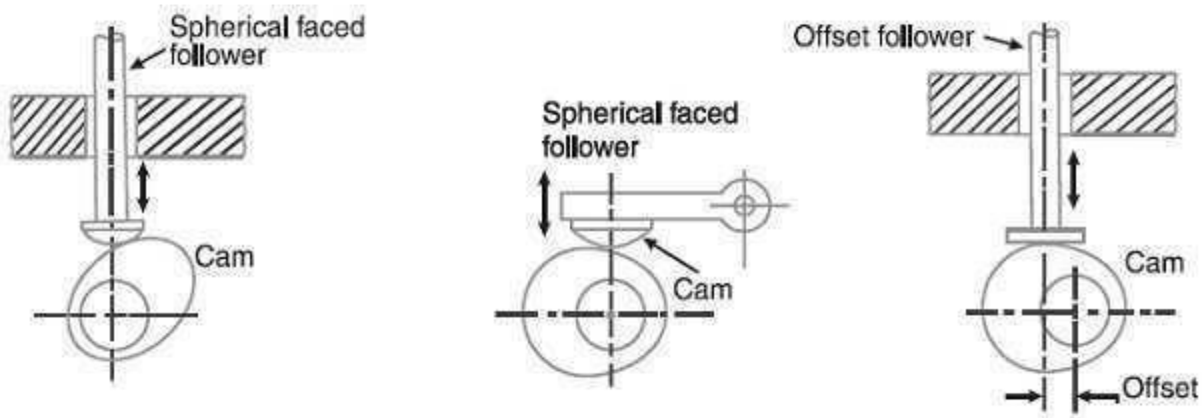
(b) Roller follower. In a roller follower, the contacting end of the follower is a roller, as shown in Fig. 4.1 (b). In case of roller followers, rolling motion takes place between the contacting surfaces (i.e. the roller and the cam). Therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are used only where more space is available such as in stationary gas and oil engines and aircraft engines.

(c) Flat faced or mushroom follower. In a Flat Face follower contacting end of the follower is a perfectly flat face, as shown in Fig. 4.1 (c). In case of flat-faced followers, the side thrust between the guide and the follower is reduced. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between the surfaces of Cam and follower sliding nature but wear may be reduced by offsetting the axis of the follower, as shown in Fig. 4.1 (f) so that when the cam rotates, the follower also rotates about its own axis. These are generally used where space is limited such as in cams which operate the valves of automobile engines.

(d) Spherical faced follower. In Spherical faced follower, the contacting end of the follower is in the form of spherical shape, as shown in Fig. 4.1 (d). In automobile engines, high surface stresses are produced. In order to minimize these stresses when a flat-faced follower is used, the flat end of the follower is machined to a spherical shape.



(a) Cam with knife edge follower (b) Cam with roller follower (c) Cam with flat faced follower



(d and e) Cam with spherical faced follower (f) Cam with offset follower

Fig. 4.1 Classification of followers

2. According to the motion of the follower. According to the motion follower there are following two types:
(a) Reciprocating or translating follower. When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. All reciprocating or translating followers are shown in Fig. 4.1 (a) to 4.1 (d).

(b) Oscillating or rotating follower. In case of an oscillating follower, the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower. The follower, as shown in Fig. 4.1 (e), is an oscillating or rotating follower.

3. According to the path of motion of the follower. The followers, according to its path of motion, are of the following two types:
(a) Radial follower. When the motion of the follower is along an axis passing through the centre of the cam, it is known as a radial follower. The followers, as shown in Fig. 4.1 (a) to 4.1 (e), are all radial followers.

(b) Off-set follower. In off-set followers, the motion of the follower is along an axis away from the axis of the cam centre. The follower, as shown in Fig. 4.1 (f), is an off-set follower.

Classification of Cams

Following cams are important in day to day life:

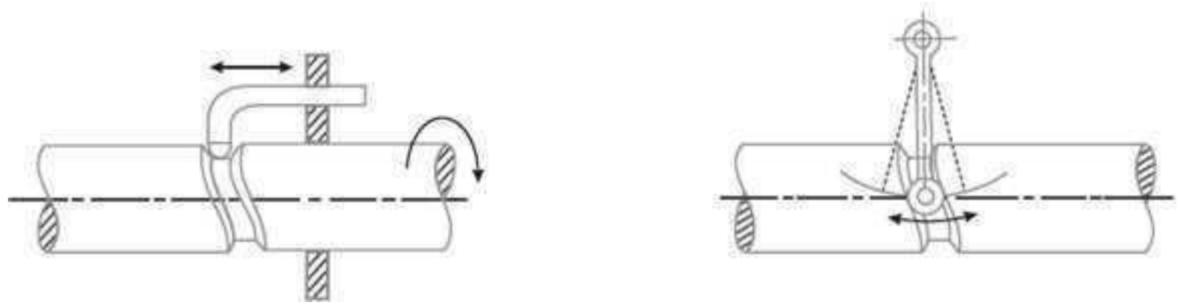


Fig. 4.2 Cylindrical cam

1. Radial or disc cam. The follower reciprocates or oscillates in a direction perpendicular to the cam axis in case of radial cams. The cams as shown in Fig. 4.2 are all radial cams.

2. Cylindrical cam. In cylindrical cams, the follower reciprocates or oscillates along the parallel direction to the cam axis. The follower rides in a groove on its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 4.2 (a) and 4.2 (b) respectively.

Terms Used in Radial Cams

Fig. 4.3 shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. Base circle: It is the smallest circle that can be drawn to the cam profile.

2. Tracepoint: Tracepoint is used to generate the pitch curve and also it is a reference point on the follower. The knife edge represents the tracepoint and the pitch curve corresponds to the cam profile, in case of knife edge follower. In a roller follower, the centre of the roller represents the tracepoint.

3. Pressure Angle: It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. Pitch point: It is a point on the pitch curve having the maximum pressure angle.

5. Pitch circle: The circle drawn from the centre of the cam through the pitch points.

6. Pitch curve: The curve generated by the tracepoint as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas, for a roller follower, they are separated by the radius of the roller.

7. Prime circle: The smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve is known as a prime circle. The prime circle and the base circle are identical in case of the knife edge and flat faced follower. The prime circle is larger than the base circle of the radius of the roller in case of roller follower.

8. Lift or stroke: It is the maximum travel of the follower from its lowest position to the topmost position.

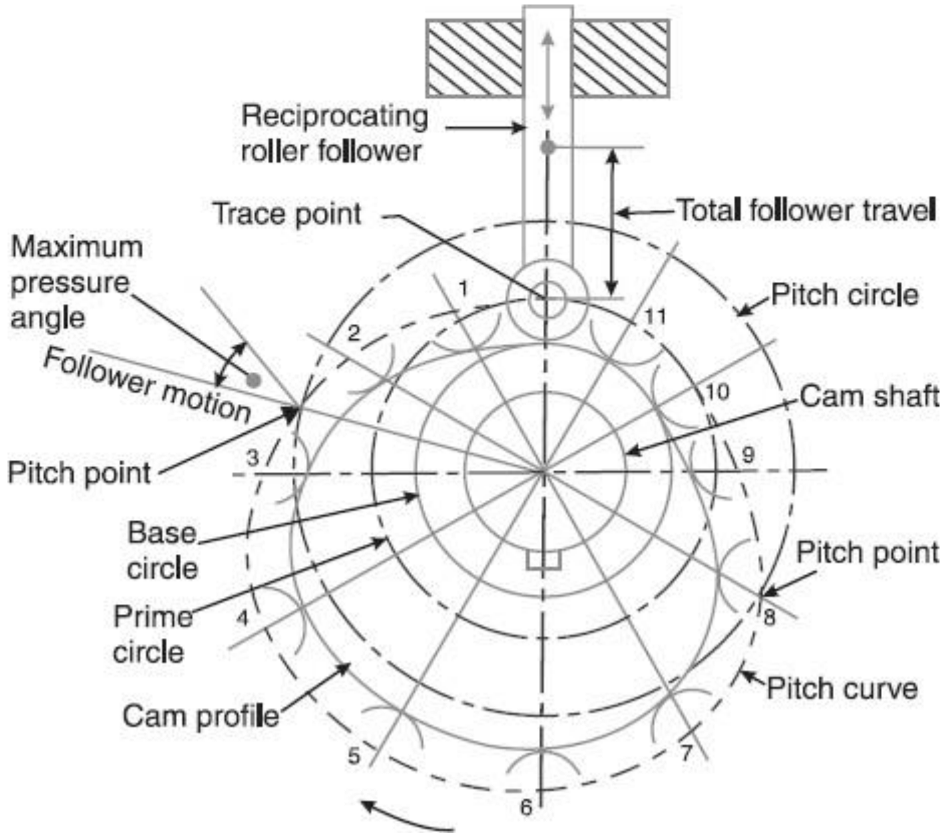


Fig. 4.3 Terms used in radial cams

Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity,

2. Simple harmonic motion,

3. Uniform acceleration and retardation, and

4. Cycloidal motion

1. Displacement, Velocity and Acceleration Diagrams when the Follower is moving with Uniform Velocity:

The displacement, velocity, and acceleration diagrams when a knife-edged follower moving with uniform velocity are shown in Fig. 4.4 (a), (b) and (c) respectively. The time or the angular displacement of the cam is represented on the abscissa (base) in degrees. The ordinate represents the displacement or velocity or acceleration of the follower.

As the follower is moving with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1D must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. Dwell period is the period of time for which the follower remains at rest, as shown by lines B_1C_1 and DE in Fig. 4.4 (a). From Fig. 4.4 (c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is occurring due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are, however, impracticable.

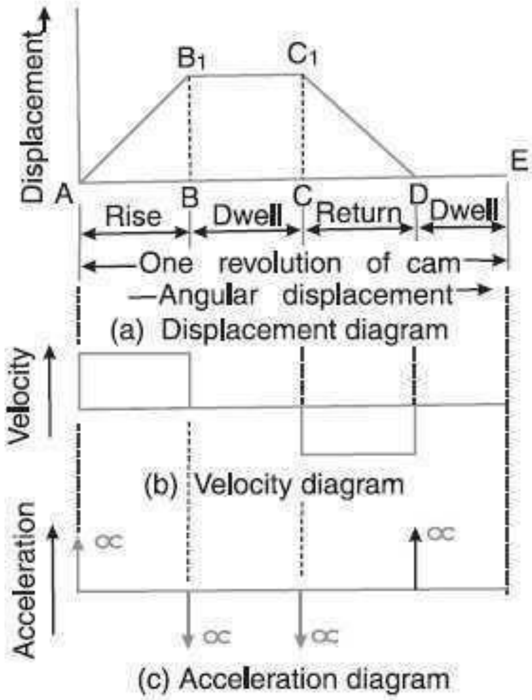


Fig. 4.4 Displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

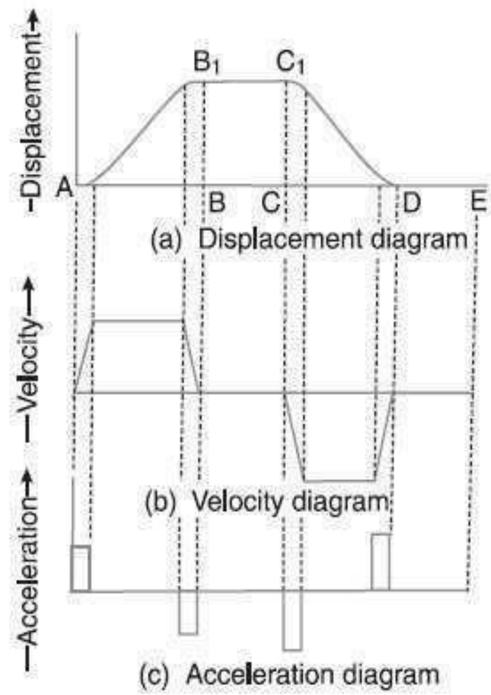


Fig. 4.5 Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

It is necessary to modify the conditions which govern the motion of the follower for the acceleration and retardation to be in within the limits. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 4.5 (a). By doing this, the velocity of the follower is increasing gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 4.5 (b). The modified displacement, velocity and acceleration diagrams are shown in Fig. 4.5 (c). The round corners in the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

2. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity, and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 4.6 (a),(b) and (c) respectively. The displacement diagram is drawn as follows:

1. Draw a semi-circle on the follower stroke as diameter.

2. Divide the semi-circle into any number of even equal parts (say eight).

3. Divide the angular displacement of the cam during out stroke and return stroke into the same number of equal parts.

4. The displacement diagram is obtained by projecting the points as shown in Fig. 4.6 (a). The velocity and acceleration diagrams are shown in Fig. 4.6 (b) and 4.6 (c) respectively. The velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve as the follower is moving with simple harmonic motion. We see from Fig. 4.6 (b) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the end of the stroke and diminishes to zero at mid-stroke.

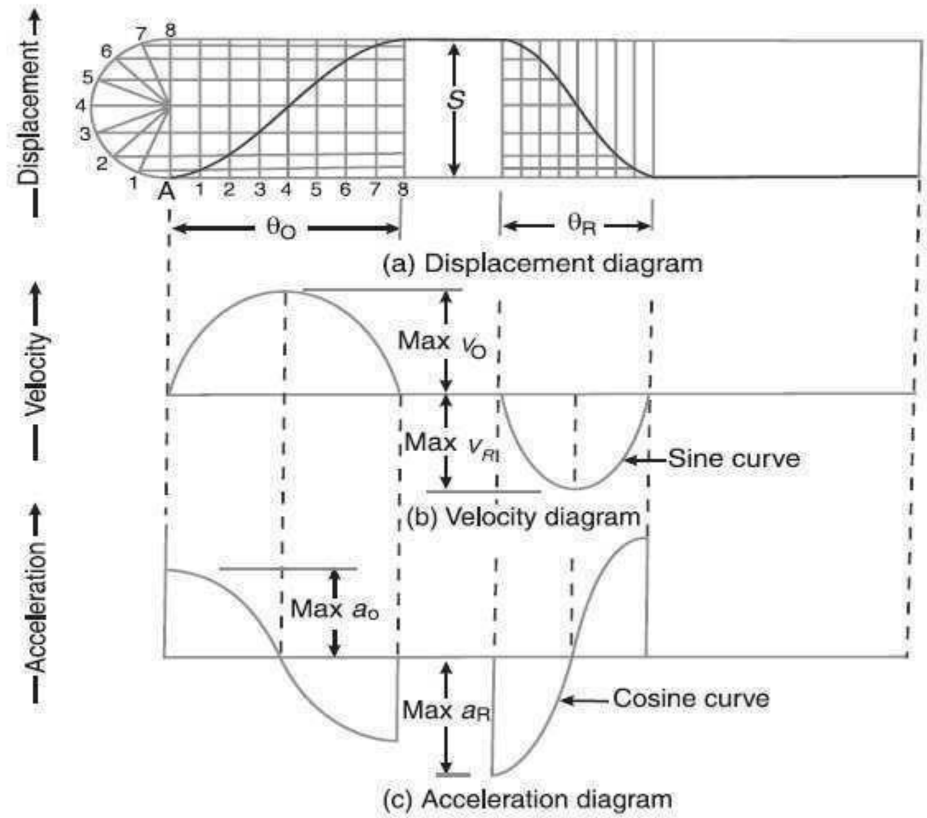


Fig. 4.6 Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴Time required for the out stroke of the follower in seconds,

$t_O = \theta_O / \omega$

Consider a point P moving at a uniform speed ω_P radians per sec around the circumference of a circle with the stroke S as diameter, as shown in Fig. 4.6. The point P′ (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P′.

∴ Peripheral speed of the point P'

$v_P = uS/2 \text{ X } 1/t_0 = uS/2 \text{ X } \omega/\theta_0$

and maximum velocity of the follower on the outstroke,

$v0 = v_P = uS/2 \text{ X } \omega/\theta_0 = u^2\omega^2. S / 2 \theta_0^2$

We know that the centripetal acceleration of the point P,

$a_P = vP2/OP = (u\omega. S / 2 \theta_0)^2 \text{ X } 2/S = u^2\omega^2. S / 2 \theta_0^2$

∴ Maximum acceleration of the follower on the outstroke,

$a_0 = a_P = u^2\omega^2. S / 2 \theta_0^2$

Similarly, maximum velocity of the follower on the return stroke,

$v_R = u\omega. S / 2 \theta_R$

and maximum acceleration of the follower on the return stroke,

$a_R = u^2\omega^2. S / 2 \theta_R^2$

3. Displacement, Velocity and Acceleration Diagrams when the Follower is moving with Uniform Acceleration and Retardation

The displacement, velocity, and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 4.7 (a), (b) and (c) respectively. From the displacement diagram we see that it consists of a parabolic curve and may be drawn as discussed below :

1. Divide the angular displacement of the cam during outstroke (θ_O) into an even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig. 4.7 (a).

2. The stroke of the follower(S) is divided into the equal same number of even parts.

3. Now join Aa which intersects the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 4.7 (a). Now join these points to obtain the parabolic curve for the out stroke of the follower.

4. In a similar way, as discussed above, the displacement diagram for the follower during return stroke may be drawn.

Since the acceleration and retardation are uniforms, therefore the velocity varies directly with the time. The velocity diagram is shown in Fig. 4.7 (b).

Let S = Stroke of the follower,

θ_O and θ_R are the angular displacement of the cam during out stroke and return stroke of the follower respectively, and

ω = Angular velocity of the cam.

We know that time required for the follower during outstroke, $T_O = \theta_O / \omega$

and time required for the follower during the return stroke, $t_R = \theta_R / \omega$

Mean velocity of the follower during outstroke is equal to S/t_O and mean velocity of the follower during return stroke will be equal to S/t_R

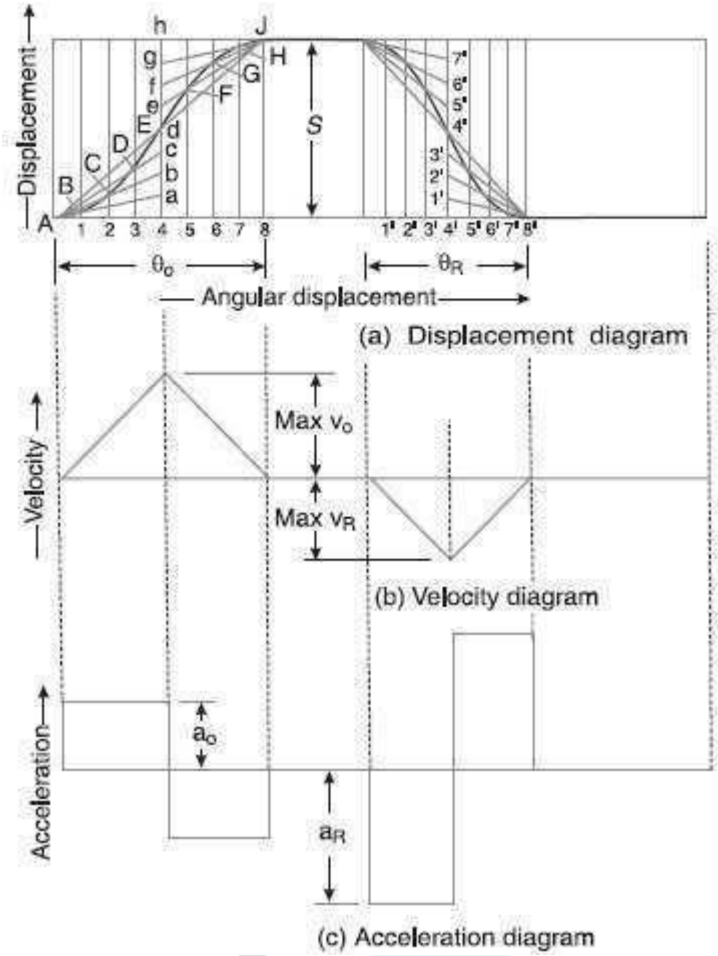


Fig. 4.7 Displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation

Since the maximum velocity of the follower is equal to twice the mean velocity, therefore the maximum velocity of the follower during outstroke

$v_O = 2S/t_o = 2\ \omega.\ S / \theta_O$

Similarly, the maximum velocity of the follower during return stroke will be,

$v_R = 2\ \omega.\ S / \theta_R$

We see from the acceleration diagram, as shown in Fig. 4.7 (c), that during the first half of the outstroke there is uniform acceleration and during the second half of the out stroke, there is uniform retardation. Therefore, the maximum velocity of the follower is reached at time $t_O/2$ (during out stroke) and $t_R/2$ (during return stroke).

∴ Maximum acceleration of the follower during outstroke,

$a_O = v_O /(t_O/2) = 2 \times 2\ \omega.\ S / t_O.\ \theta_O = 4\ \omega^2.\ S / \theta_O^2$

Similarly, maximum acceleration of the follower during return stroke,

$a_R = 4\ \omega^2.\ S / \theta_O^2$

4. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Cycloidal Motion

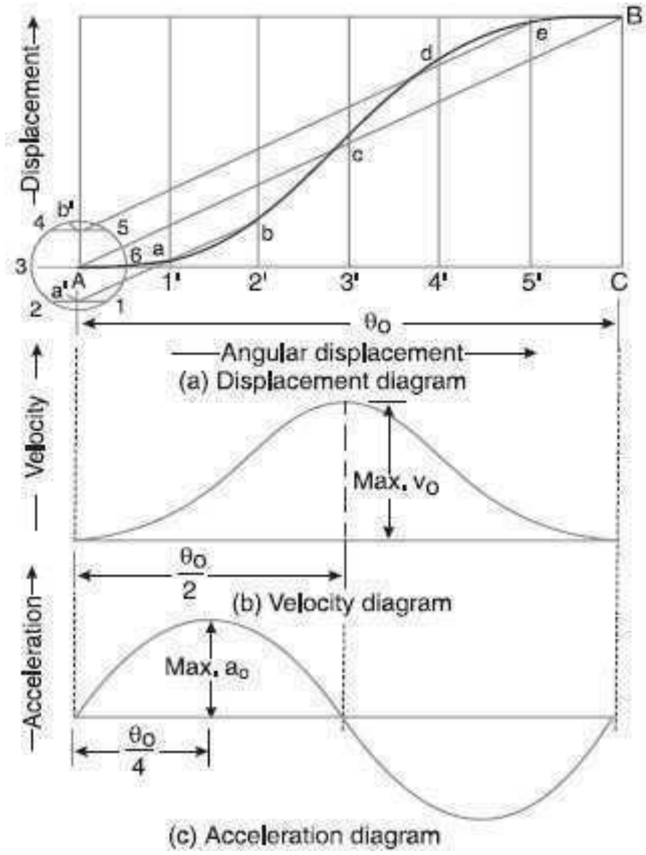


Fig. 4.8 Displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion

When the follower is moving with cycloidal motion the displacement, velocity, and acceleration diagrams are shown in Fig. 4.8 (a), (b) and (c) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.

In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (S) of the follower. Therefore the radius of the rolling circle is S/2u. The displacement diagram is drawn as discussed below:

1. Draw a circle of radius S/2u with A as a centre.

2. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical centre line of the circle. These points are shown by a‘ and b‘ in Fig. 4.8 (a).

3. Divide the angular displacement of the cam during outstroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.

4. Join AB intersecting the vertical line through 3‘ at c. From a‘ draw a line parallel to AB intersecting the vertical lines through 1‘ and 2‘ at a and b respectively.

5. Similarly, from b‘ draw a line parallel to AB intersecting the vertical lines through 4‘ and 5‘ at d and e respectively.

6. Join the points A, a, b, c, d, e, and B by a smooth curve. This is the required cycloidal curve for the follower during outstroke.

Let θ = Angle through which the cam rotates in time t seconds, and

ω = Angular velocity of the cam.

We know that displacement of the follower after time t seconds,

$x = S\,[\theta/\theta_O - 1/2u \sin(2u\theta / \theta_O)]$

∴ Velocity of the follower after time t seconds,

$dx/dt = S[1/\theta_O \times d\theta/dt - 2u/2u\,\theta_O \cos(2u\,\theta/\theta_O)\,d\theta/dt]$

$dx/dt = S/\theta_O \times d\theta/dt\,[1 - \cos(2u\,\theta/\theta_O)] = \omega S/\theta_O\,[1 - \cos(2u\,\theta/\theta_O)]$

The velocity is maximum, when

$\cos(2u\,\theta/\theta_O) = -1$ Or $2u\,\theta/\theta_O = u$ Or $\theta = \theta_O/2$

Substituting $\theta = \theta_O / 2$ in equation, we have maximum velocity of the follower during outstroke,

$v_o = \omega.\,S/\theta_O\,(1 + 1) = 2\omega.\,S/\theta_O$

Similarly, maximum velocity of the follower during return stroke,

$v_R = 2\omega.\,S/\theta_R$

Now, acceleration of the follower after time t sec,

$d^2x/dt^2 = \omega.\,S/\theta_O\,[2u/\theta_O \sin(2u\,\theta/\theta_O)\,d\theta/dt]$

$d^2x/dt^2 = 2\,u\,\omega^2.\,S/\theta_O^2 \sin(2u\,\theta/\theta_O)$

The acceleration is maximum, when

$\sin(2u\,\theta/\theta_O) = 1$ Or $2u\,\theta/\theta_O = u/2$ Or $\theta = \theta_O/4$

Substituting $\theta = \theta_O / 4$ in equation (iii), we have maximum acceleration of the follower during outstroke,

$a_O = 2\,u\,\omega^2.\,S/\theta_O^2$

Similarly, maximum acceleration of the follower during return stroke,

$a_R = 2\,u\,\omega^2.\,S/\theta_R^2$

The velocity and acceleration diagrams are shown in Fig. (b) and (c) respectively.

Construction of Cam Profile for a Radial Cam

To draw the cam profile for a radial cam, the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

The principle of kinematic inversion is used in the construction of cam profile, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the **opposite direction** to the **cam rotation.**

Unit-VI

Belt Rope & Chain Drive : Types of Belts, Velocity ratio of a belt drive, Slip in belts, Length of open belt and crossed belt, Limiting ratio of belt-Tensions, Power transmitted by a belt, Centrifugal tension, Maximum tension in a belt, Condition for maximum power transmitted, Initial tension in a belt, Creep in belt, Applications of V-Belt, Rope and Chain drives.

Introduction

To transmit power from one shaft to another the belts or ropes are used by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The arc of contact between the belt and the pulley.

4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to ensure uniform tension across the belt section.

(b) The arc of contact on the smaller pulley may be as large as possible for that the pulleys should not be too close together

(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings

(d) The belt to run out of the pulleys caused due to the long belt tends to swing from side to side, which in turn develops crooked spots in the belt.

(e) The tight side of the belt should be at the bottom so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(f) The maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley in order to obtain good results with flat belts.

Selection of a Belt Drive

The selection of belt drives depends on the following important factors:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available, and

8. Service conditions.

Types of Belt Drives

The belt drives are classified into the following three groups:

1. Light drives: To transmit small powers (at belt speeds up to about 10 m/s) light drives are used, as in agricultural machines and small machine tools.

2. Medium drives. To transmit medium power (at belt speeds over 10 m/s but up to 22 m/s) medium drives are used, as in machine tools.

3. Heavy drives. To transmit large powers (at belt speeds above 22 m/s) heavy drives are used, as in compressors and generators.

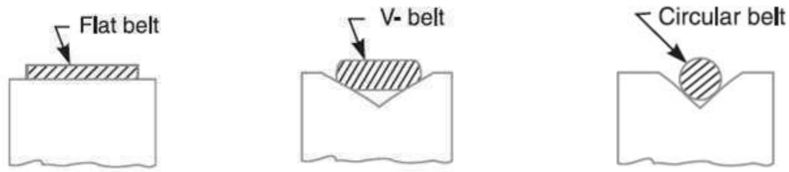
Types of Belts

There are many types of belts used these days, the following are important:

1. Flat belt. The flat belt, as shown in Fig. 6.1 (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 meters apart.

2. V-belt. The V-belt, as shown in Fig. 6.1 (b), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope. The rope or circular belt, as shown in Fig. 6.1 (c), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.



(a) Flat Belt (b) V- Belt (c) Circular Belt

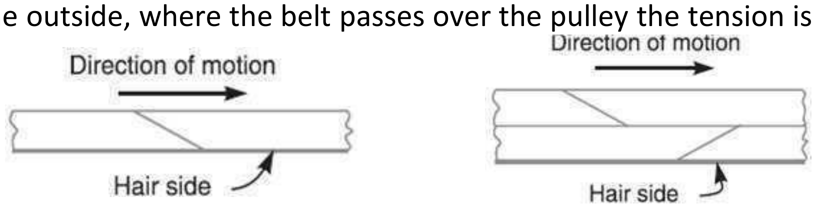
Fig. 6.1 Types of belts

For transmitting a huge amount of power a single belt may not be sufficient. Wide pulleys (for V-belts or circular belts) with a number of grooves are used to transmit a huge amount of power.

Material used for Belts

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows:

1. Leather belts. The most important material for the belt is leather. The best leather belts are made from 1.2 meters to 1.5 meters long strips cut from either side of the backbone of the top grade steer hides. In leather, the hair side is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. 6.2. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the belt passes over the pulley the tension is maximum.



(a) Single Layer Belt (b) Double Layer Belt

Fig. 6.2 Leather belts

The leather may be either oak-tanned or mineral salt tanned e.g. chrome tanned. In order to increase the thickness of the belt, the strips are cemented together. The belts are specified according to the number of layers used e.g. single, double or triple ply and according to the thickness of hides used e.g. light, medium or heavy. These belts must be cleaned periodically and treated with a compound or dressing containing neat foot or other suitable oils so that the belt will remain soft and flexible.

2. Cotton or fabric belts. By folding canvass or cotton duck to three or more layers (depending on the thickness desired) and stitching together fabric belts are made. Cotton or fabric belts are woven also into a strip of the desired width and thickness. To make the belts waterproof and to prevent injury to the fibres, they are impregnated with some filler like linseed oil. These belts are cheaper and much more suitable in warm climates, in damp atmospheres and in exposed positions. Cotton belts are mostly used in farm machinery, belt conveyor etc as they require little attention.

3. Rubber belt. Rubber belts are made up of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. The principal advantages of these belts are that they may be easily made endless. These belts are found suitable for sawmills, paper mills where they are exposed to moisture.
4. Balata belts. Balata gum is used in these belts otherwise these belts are similar to rubber belts. Balata belts are acid proof as well as waterproof and it is not affected by animal oils or alkalis. These belts should not be used at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The balata belts are having 25 percent higher strength than the rubber belts.

Types of Flat Belt Drives
The power from one pulley to another may be transmitted by any of the following types of belt drives:
1. Open belt drive. The open belt drive, as shown in Fig. 6.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ) and delivers it to the other side (i.e. upper side LM). Thus the tension in the lower side belt will be more than that on the upper side belt. The lower side belt (because of more tension) is known as **tight side** whereas the upper side belt (because of less tension) is known as a **slack side**, as shown in Fig. 6.3.

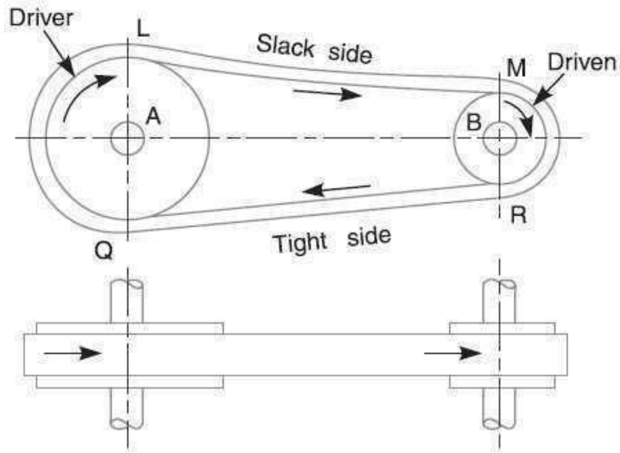


Fig. 6.3 Open belt drive

2. Crossed or twist belt drive. The crossed or twist belt drive, as shown in Fig. 6.4, is used with shafts arranged parallel and rotating in the opposite directions.

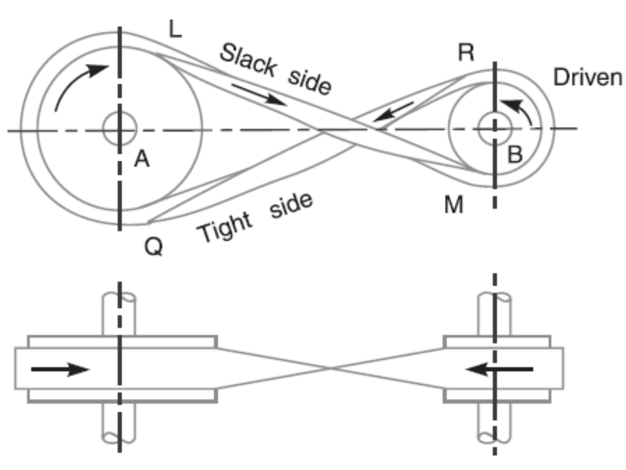


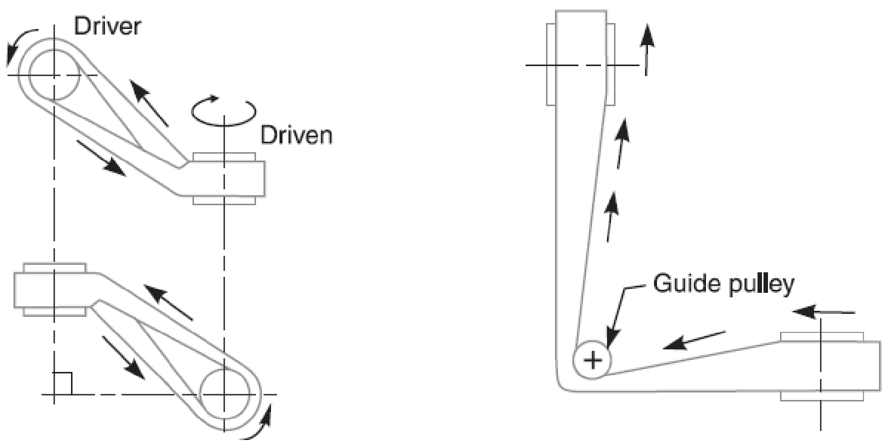
Fig. 6.4 Crossed or twist belt drive

In crossed belt drive the driver pulls the belt from RQ side and delivers it to the LM side. Thus the tension in the side RQ will be more than that in the side LM. The belt RQ (because of more tension) is known as a **tight side,** whereas the belt LM (because of less tension) is known as a **slack side**, as shown in Fig. 6.4.

Here let us consider that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of the belt and the speed of the belt should be less than 15 m/s.

3. Quarter turn belt drive. The quarter turn belt drives also known as right angle belt drive, as shown in Fig. 6.5 (a), is used when the shafts are arranged at right angles and rotating in one definite direction. The width of the face of the pulley should be greater or equal to 1.4 b to prevent the belt from leaving the pulley, where b is the width of the belt.

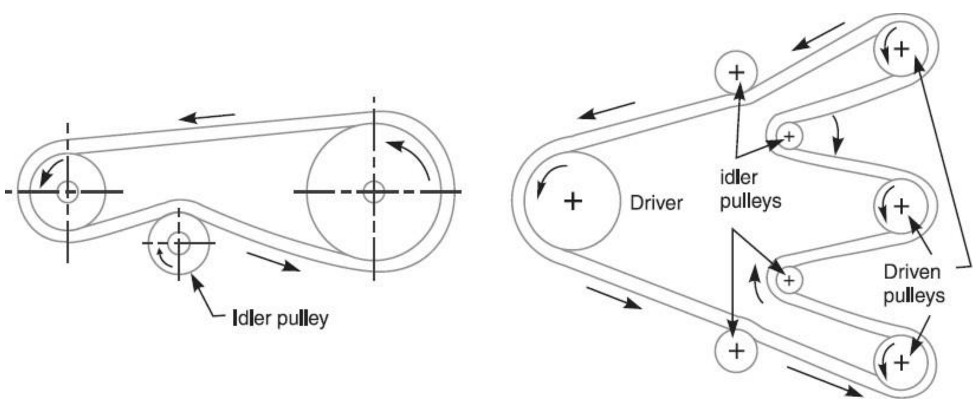
When the pulleys are not arranged, as shown in Fig. 6.5 (a), or when the reversible motion is desired, then a **quarter turn belt drive with guide pulley**, as shown in Fig. 6.5 (b), may be used.



(a) Quarter turn belt drive (b) Quarter turn belt drive with guide pulley

Fig. 6.5 Quarter Turn Belt Drive

4. Belt drive with idler pulleys. A belt drives with an idler pulley, as shown in Fig. 6.6 (a), is used with shafts arranged parallel and when an open belt drive cannot be used due to the small angle of contact on the smaller pulley. To obtain high-velocity ratio and when the required belt tension cannot be obtained by other means belt drives with idler pulleys are used.



(a) Belt drive with single idler pulley (b) Belt drive with many idler pulleys

Fig. 6.6 Belt Drive with Idler Pulleys

When one has to transmit motion from one shaft to several numbers of shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 6.6 (b), may be employed.

5. Compound belt drive. A compound belt drive, as shown in Fig. 6.7, is used when power is transmitted from one shaft to another through a number of pulleys.

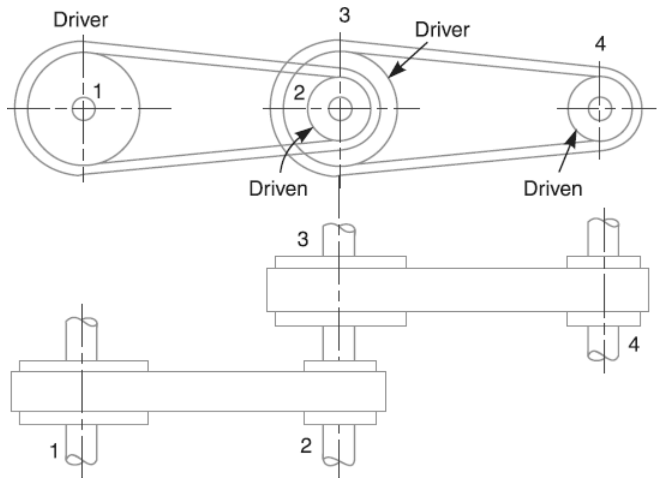


Fig. 6.7 Compound belt drive

6. Stepped or cone pulley drive. A stepped or cone pulley drive (Fig. 6.8) is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

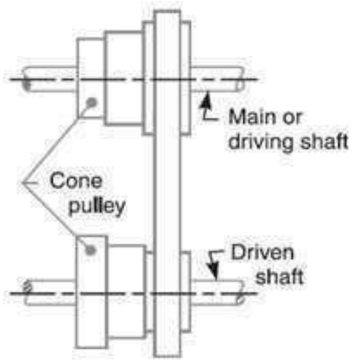


Fig. 6.8 Stepped or cone pulley drive

7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig. 6.9, is used when the driven or machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. The fast pulley is keyed to the machine shaft and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed onto the loose pulley by means of a sliding bar having belt forks.

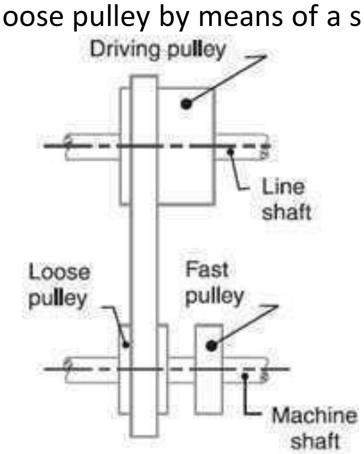


Fig. 6.9 Fast and loose pulley drive

Velocity Ratio of Belt Drive

The ratio between the velocities of the driver and the follower or driven is known as the velocity ratio of belt drives**.** It may be expressed, mathematically, as discussed below:

Let d1 = Diameter of the driver,

d2 = Diameter of the follower,

N1 = Speed of the driver in r.p.m., and

N2 = Speed of the follower in r.p.m.

∴ Length of the belt that passes per minute over the driver = $\pi\ d_1.N_1$

Similarly, length of the belt that passes over the follower, in one minute = $\pi\ d_2.\ N_2$

Since the length of belt that passes over the driver in one minute = the length of belt that passes over the follower in one minute, therefore

$\pi\ d_1.N_1 = \pi\ d_2.\ N_2$

∴ Velocity ratio,

$N_2 / N_1 = d_1 / d_2$

When the thickness of the belt (t) is considered, then velocity ratio,

$N_2 / N_1 = d_1 + t / d_2 + t$

Slip of Belt

We know that the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward movement of the belt without carrying the driven pulley with it. This is called **slip of the belt** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As we know that the most common phenomena are the slipping of the belt, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of the hour, minute and second arms in a watch).

Let s1 % = Slip between the driver and the belt, and

s2 % = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$v = \pi.\ d_1.\ N_1 / 60 - \pi.\ d_1.\ N_1 / 60 \times s_1/100 = \pi.\ d_1.\ N_1 / 60\ (1 - s_1/100)$

and velocity of the belt passing over the follower per second,

$\pi\ d_2.\ N_2/60 = v - v \times s_2/100 = v\ (1 - s_2/100)$

Substituting the value of v from equation,

$\pi\ d_2.\ N_2/60 = \pi.\ d_1.\ N_1 / 60\ (1 - s_1/100)\ (1 - s_2/100)$

$N_2/N_1 = d_1/d_2\ (1 - s_1/100 - s_2/100)$

$N_2/N_1 = d_1/d_2\ [1 - (s_1 + s_2)/100] = = d_1/d_2\ (1 - s/100)$

If thickness of the belt (t) is considered, then

$N_2/N_1 = d_1 + t/d_2 + t\ (1 - s/100)$

Length of Open Belt Drive

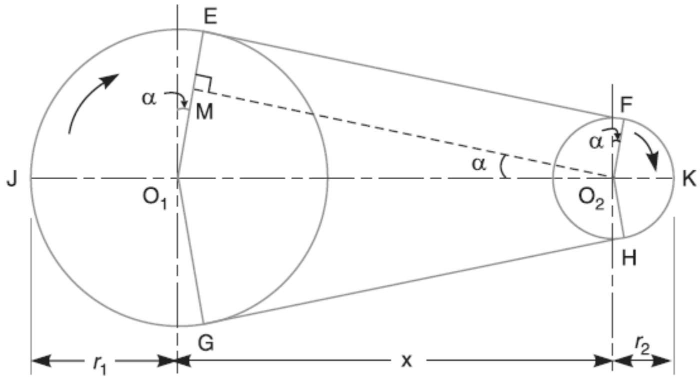


Fig. 6.10 Length of an open belt drive

In an open belt drive, both the pulleys rotate in the **same** direction as shown in Fig. 6.10.
Let r_1 and r_2 are the radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (i.e. $O_1 O_2$), and
L = Total length of the belt.
Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 6.10. Through O_2, draw O_2 M parallel to FE.
From the geometry of the fig. 6.10, we find that O_2 M will be perpendicular to O_1 E.
Let the angle $MO_2 O_1$ will be equal to α radians
We know that the length of the belt,
L = Arc GJE + EF + Arc FKH + HG = 2 (Arc JE + EF + Arc FK) ... (i)
From the geometry of the figure, we find that
$\sin \alpha = O_1M/O_1O_2 = (O_1E - EM) / O_1O_2 = (r_1 - r_2)/x$

Since α is very small, therefore putting
$\sin \alpha = \alpha$ (in radians) = $r_1 - r_2 / x$ ... (ii)
∴ Arc JE = $r_1 (u/2 + \alpha)$ ... (iii)
Similarly Arc FK = $r_2 (u/2 - \alpha)$ ... (iv)
$EF = MO_2 = [(O_1O_2)^2 - (O_1M)^2]^{1/2} = [x^2 - (r_1 - r_2)^2]^{1/2}$
$EF = x [1 - \{(r_1 - r_2)/x\}^2]^{1/2}$
Expanding this equation by binomial theorem,
$EF = x [1 - ½ \{(r1 - r_2)/x\}^2 + ...] = x - (r_1 - r_2)^2/2x$ ... (v)
Substituting the values of arc JE from equation (ii), arc FK from equation (iii), and EF from equation (v), in equation (i), we get
$L = 2[r_1 (u/2 + \alpha) + x - (r_1 - r_2)^2/2x + r_2 (u/2 - \alpha)]$
On solving above expression we get,
$L = u (r_1 + r_2) + 2\alpha (r_1 - r_2) + 2x - (r_1 - r_2)^2/x$
Substituting the value of $\alpha = r_1 - r_2 / x$ from above said equation,
$L = u (r_1 + r_2) + 2(r_1 - r_2) (r_1 - r_2)/x + 2x - (r_1 - r_2)^2/x$
On solving above expression we get,
$L = u (r_1 + r_2) + 2x + (r_1 - r_2)^2/x$

Length of a Cross Belt Drive

In a cross belt drive, both the pulleys rotate in **opposite** directions as shown in Fig. 6.11.

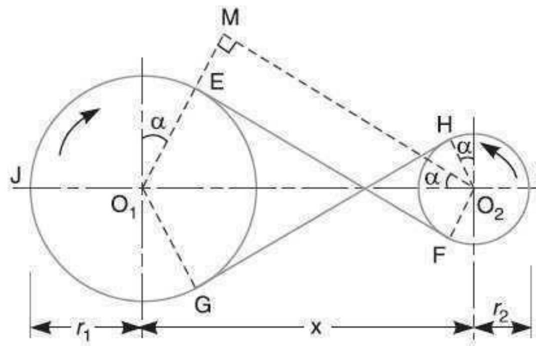


Fig. 6.11 Length of a cross belt drive

Let r_1 and r_2 are the radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (i.e. $O_1 O_2$), and
L = Total length of the belt.
Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. 6.11. Through O_2, draw an O_2M parallel to FE.
From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.
Let the angle $MO_2\ O_1 = \alpha$ radians.
We know that the length of the belt,

$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG = 2\,(\text{Arc } JE + EF + \text{Arc } FK)$ ... (i)

From the geometry of the figure, we find that

$\sin\alpha = O_1M/O_1O_2 = (O_1E + EM) / O_1O_2 = (r_1 + r_2)/x$

$\sin\alpha = \alpha$ (in radians) $= r_1 + r_2 / x$ ... (ii)

$\therefore$ Arc $JE = r_1\,(u/2 + \alpha)$ ... (iii)

Similarly Arc $FK = r_2\,(u/2 + \alpha)$ ... (iv)

$EF = MO_2 = [(O_1O_2)^2 - (O_1M)^2]^{1/2} = [x^2 - (r_1 - r_2)^2]^{1/2}$

$EF = x\,[1 - \{(r_1 - r_2)/x\}^2]^{1/2}$

Expanding this equation by binomial theorem,

$EF = x\,[1 - ½\,\{(r_1 + r_2)/x\}^2 + \ldots\ldots] = x - (r_1 + r_2)^2/2x$ ... (v)

Substituting the values of arc JE from equation (iii), arc FK from equation (iv) and EF from equation (v) in equation (i), we get

$L = 2[r_1\,(u/2 + \alpha) + x - (r_1 + r_2)^2/2x + r_2\,(u/2 + \alpha)]$

On solving above expression we get,

$L = u\,(r_1 + r_2) + 2\alpha\,(r_1 + r_2) + 2x - (r_1 + r_2)^2/x$

Substituting the value of $\alpha = r_1 + r_2 / x$ from equation,

$L = u\,(r_1 + r_2) + 2(r_1 + r_2)\,(r_1 + r_2)/x + 2x - (r_1 + r_2)^2/x$

On solving above expression we get,

$L = u\,(r_1 + r_2) + 2x + (r_1 + r_2)^2/x$

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus obvious that if the sum of the radii of the two pulleys is constant, the length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

Power Transmitted by a Belt

Fig. 6.12 shows the driving pulley (or driver) A and the driven pulley (or follower) B. We know that the driving pulley pulls the belt from one side and delivers the same to the other side. Thus it is obvious that the tension on the former side (i.e. tight side) will be greater than the latter side (i.e. slack side) as shown in Fig. 6.12.
Let T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in Newton,

r_1 and r_2 are the radii of the driver and follower respectively, and
v = Velocity of the belt in m/s.

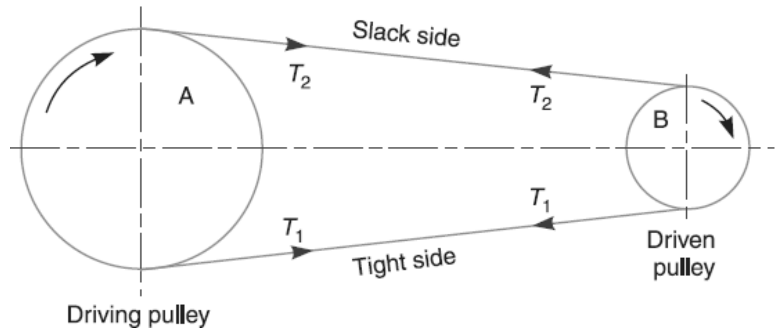


Fig. 6.12 Power transmitted by a belt

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e. $T_1 - T_2$).

∴ Work done per second = $(T_1 - T_2)$ v N-m/s

And the power transmitted will be, $P = (T_1 - T_2)$ v W ... (∴ 1 N-m/s = 1 W)

Let us consider that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$.

Similarly, the torque exerted on the driven pulley i.e. follower is $(T_1 - T_2)\, r_2$.

Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. 6.13.

Let T_1 will be the Tension in the belt on the tight side and T_2 will be the Tension in the belt on the slack side,

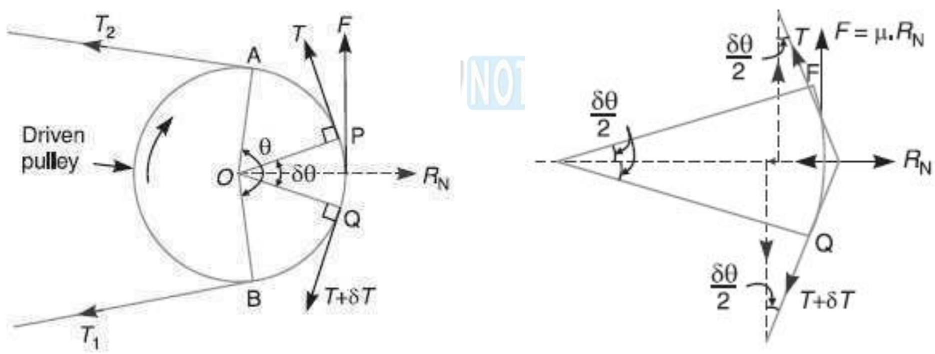


Fig. 6.13 Ratio of driving tensions for flat belt

θ = Angle of contact in radians

Now consider a small portion of the belt PQ, subtending an angle δθ at the centre of the pulley as shown in Fig. 6.13. Under the following forces the belt PQ is in equilibrium:

1. Tension T in the belt at P,

2. Tension (T + δ T) in the belt at Q,

3. Normal reaction RN, and

4. Frictional force, F = µ × RN, where µ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$R_N = (T + \delta T) \sin \delta\theta/2 + T \sin \delta\theta/2$ ... (i)

As we know that the angle δθ is too small, therefore putting sin δθ / 2 = δθ / 2 in equation (i),

$R_N = = (T + \delta T)\, \delta\theta/2 + T\, \delta\theta/2 = T\, \delta\theta/2 + \delta T\, \delta\theta/2 + T\, \delta\theta/2 = T\, \delta\theta/2$ ... (ii)

... (Neglecting δT δθ/2)

Now resolving the forces vertically, we have

$\mu \times R_N = (T + \delta T) \cos \delta\theta/2 - T \cos \delta\theta/2$ ... (iii)

As we know that the angle δθ is too small, therefore putting cos δθ / 2 = 1 in equation (iii),

$\mu \times R_N = T + \delta T - T$ or $RN = \delta T/\mu$ ... (iv)

Equating the values of RN from equations (ii) and (iv),

$T\, \delta\theta/2 = \delta T/\mu$ or $\delta T/T = \mu.\, \delta\theta$

Integrating both sides between the limits T2 and T1 and from 0 to θ respectively,

$\int_{T2}^{T1} \delta T/T = \mu. \int_{\theta}^{\theta} \delta\theta$ or $\log_e (T_1/T_2) = \mu.\, \theta$ or $T_1/T_2 = e^{\mu\theta}$ ...(v)

Equation (v) can be expressed in terms of corresponding logarithm to the base 10, i.e.

$2.3 \log (T_1/T_2) = \mu.\, \theta$

The above expression gives the relation between the tensions of tight side and slack side, in terms of coefficient of friction and the angle of contact.

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. Centrifugal tension is the tension caused by centrifugal force. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion PQ of the belt subtending an angle dθ the centre of the pulley as shown in Fig. 6.13.

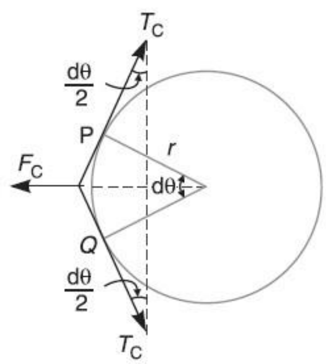


Fig. 6.14 Centrifugal Tension

Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in meters, and

TC = Centrifugal tension acting tangentially at P and Q in Newton.

We know that length of the belt PQ = r. dθ

And the mass of the belt PQ = m. r. dθ

∴ Centrifugal force acting on the belt PQ,

$F_C = (m.\, r.\, d\theta)\, v^2/r = m.\, d\theta.\, v^2$

The centrifugal tension TC acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (i.e. centrifugal force and centrifugal tension) horizontally and equating the same, we have

$T_C \sin (d\theta/2) + T_C \sin (d\theta/2) = F_C = m.\, d\theta.\, v^2$

Since the angle dθ is very small, therefore, putting sin (dθ/2) = dθ / 2 in the above expression,

$2T_C \sin (d\theta/2) = m.\, d\theta.\, v^2$ Or $TC = m.\, v^2$

Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension on the tight side of the belt (T_{t1}).

Let σ will be the maximum safe stress in N/mm^2,

b will be the width of the belt in mm, and

t will be the thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = σ. b. t
When centrifugal tension is neglected, then
T (or T_{t1}) = T_1, i.e. Tension on the tight side of the belt
And when centrifugal tension is considered, then
T (or T_{t1}) = $T_1 + T_C$

Condition for the Transmission of Maximum Power
The power transmitted by a belt,
$P = (T_1 – T_2)\ v$ ... (i)
Where T_1 = Tension on the tight side of the belt in Newton,
T_2 = Tension on the slack side of the belt in Newton, and
v = Velocity of the belt in m/s.
The ratio of driving tensions is
$T_1 / T_2 = e^{\mu\theta}$ or $T_2 = T_1 / e^{\mu\theta}$ ... (ii)
Substituting the value of T_2 in equation (i),
$P = [T_1 – (T_1/e^{u\theta})]\ v = T_1\ [1 – (1/e^{u\theta})]\ v = T_1.\ v.\ C$ ... (iii)
Where $C = 1 – 1 / e^{\mu\theta}$
We know that $T_1 = T – T_C$
Where T = Maximum tension to which the belt can be subjected in Newton, and
T_C = Centrifugal tension in Newton.
Substituting the value of T_1 in equation (iii),
$P = (T – T_C)\ v.\ C$
$= (T – m.v^2)\ v.\ C = (T.\ v – m\ v^3)\ C$ ... (Substituting $TC = m.\ v^2$)
For maximum power, differentiate the above expression with respect to v and equate to zero, i.e.
$dP / dv = 0$ or $d/dv\ (T.\ v – m\ v^3)\ C = 0$
$\therefore T – 3\ m\ .\ v^2 = 0$
Or $T – 3\ T_C = 0$ or $T = 3\ T_C$ ... (iv)
It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Initial Tension in the Belt
When a belt is wound around the two pulleys (i.e. driver and follower), its two ends are joined together ; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase the grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called **initial tension**.
When the driver pulls the belt from one side (increasing tension in the belt on this side) and delivers it to the other side (decreasing the tension in the belt on that side). The increased tension on one side of the belt is called tension on the tight side and the decreased tension on the other side of the belt is called tension on the slack side.
Let T_0 = Initial tension in the belt,
T_1 = Tension on the tight side of the belt,
T_2 = Tension on the slack side of the belt, and
α = Coefficient of increase of the belt length per unit force.
A little consideration will show that the increase of tension on the tight side = $T_1 – T_0$
and increase in the length of the belt on the tight side = $\alpha\ (T_1 – T_0)$ ... (i)
Similarly, the decrease in tension of the slack side = $T_0 – T_2$
and decrease in the length of the belt on the slack side = $\alpha\ (T_0 – T_2)$ ... (ii)
Now let us assume that the belt is made up of perfectly elastic material such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to

decrease in the length on the slack side. Thus, equating equations (i) and (ii),
$\alpha (T_1 - T_0) = \alpha (T_0 - T_2)$ or $T_1 - T_0 = T_0 - T_2$
$T_0 = (T_1 + T_2) / 2$ ... (Neglecting centrifugal tension)
$T_0 = (T_1 + T_2 + 2T_C) / 2$ ... (Considering centrifugal tension)

Creep of Belt
A certain portion of the belt extends when the belt passes from the slack side to the tight side and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as a **creep.** The speed of the driven pulley or follower is slightly reduced due to the total effect of creep. Considering creep, the velocity ratio is given by
$N_2/N_1 = d_1/d_2 \times [E + (\sigma_2)^{1/2}] / [E + (\sigma_1)^{1/2}$
Where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and
E = Young's modulus of the material of the belt.

Advantages and Disadvantages of V-belt Drive over Flat Belt Drive
Following are the advantages and disadvantages of the V-belt drive over a flat belt drive.

Advantages
1. The V-belt drive gives compactness due to the small distance between the centres of pulleys.
2. The slip between the belt and the pulley groove is negligible thus the drive is positive.
3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.
4. It provides longer life, 3 to 5 years.
5. These are easy to install and remove.
6. The operations of the belt and pulley are quiet.
7. The belts have the ability to cushion the shock when machines are started.
8. The high-velocity ratio (maximum 10) may be obtained.
9. The wedging action of the belt in the groove gives a high value of limiting ratio of tensions. Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.
10. The V-belt may be operated in either direction with a tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages
1. The V-belt drive cannot be used with large centre distances.
2. The V-belts are not so durable as flat belts.
3. The construction of V-grooved pulleys for V-belts is more complicated than flat belt pulleys.
4. Since the V-belts are subjected to a certain amount of creep, therefore these are not suitable for constant speed application such as synchronous machines, and timing devices.
5. The belt life is greatly influenced by temperature changes, improper belt tension and mismatching of belt lengths.
6. The centrifugal tension prevents the use of V-belts at speeds below 5 m/s and above 50m/s.

Advantages of Rope Drives
The fibre rope drives have the following advantages:
1. They offer smooth, steady and quiet service.
2. They are not much affected by outdoor conditions.
3. The shafts may be out of strict alignment.
4. The power may be taken off in any direction and in fractional parts of the whole amount.
5. They give high mechanical efficiency.

Advantages and Disadvantages of Chain Drives over Belt or Rope Drive

Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. As the chains are made of metal, thus they occupy less space in width than a belt or rope drive.
3. The chain drives may be used when the distance between the shafts is less.
4. The chain drive gives high transmission efficiency (up to 98 percent).
5. The chain drive gives fewer loads on the shafts.
6. The chain drive has the ability to transmit motion to several shafts by one chain only.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance.
3. The velocity fluctuations are offered by chain drives especially when unduly stretched.

FRICTION

INTRODUCTION

When a body moves or tends to move on another body, a force appears between the surfaces. This force is called force of friction and it acts opposite to the direction of motion. Its line of action is tangential to the contacting surfaces. The magnitude of this force depends on the roughness of surfaces.

In engineering applications friction is desirable and undesirable. We can walk on the ground because of friction. Friction is useful in power transmission by belts. It is useful in appliances like brakes, bolts, screw jack, etc. It is undesirable in bearing and moving machine parts where it results in loss of energy and, thereby, reduces efficiency of the machine.

In this unit, you will study screw jack, clutches and different type of bearings.

Objectives

After studying this unit, you should be able to

- know application of friction force,
- know theory background of screw jack,
- know different type of bearings,
- analyze bearings, and
- know function of cluthes

PIVOT AND COLLAR FRICTION

The shafts of ships, steam and water turbines are subjected to axial thrust. In order to take up the axial thrust, they are provided with one or more bearing surfaces at right angle to the axis of the shaft. A bearing surface provided at the end of a shaft is known as a pivot and that provided at any place along with the length of the shaft with bearing surface of revolution is known as a callar. Pivots are of two forms : flat and conical. The bearing surface provided at the foot of a vertical shaft is called foot step bearing.

Due to the axial thrust which is conveyed to the bearings by the rotating shaft, rubbing takes place between the contacting surfaces. This produces friction as well as wearing of the bearing. Thus, power is lost in over-coming the friction, which is ultimately to be determined in this unit. Obviously, the rate of wearing depends upon the intensity of thrust (pressure) and relative velocity of rotation. Since velocity is proportional to the radius, therefore, □ Rate of wear □ pr.

Assumptions Taken

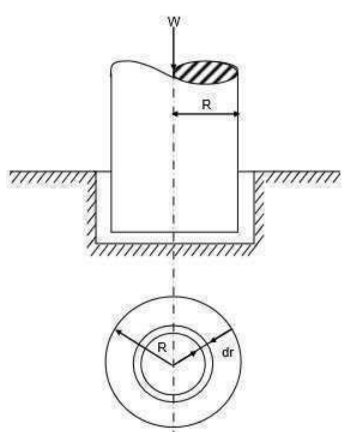
(a) Firstly, the intensity of pressure is uniform over the bearing surface. This assumption only holds good with newly fitted bearings where fit between the two contacting surfaces is assumed to be perfect. After the shaft has run for quite sometime the pressure distribution will not remain uniform due to varying wear at different radii.

(b) Secondly, the rate of wear is uniform. As given previously, the rate of wear is proportional to pr which means that the pressure will go on increasing radially inward and at the centre where r = 0, the pressure will be infinite which is not true in practical sense. However, this assumption of uniform wear gives better practical results when bearing has become older.

The various types of bearings mentioned above will be dealt with separately for each assumption.

Flat Pivot

A flat pivot is shown in Figure



Let

W = Axial thrust or load on the bearing,

R = External radius of the pivot,

p = Intensity of pressure, and

μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr as shown in Figure.

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total load $W = \int_0^R p \times 2\pi r \times dr$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of rotation

$$dM = dF \times r = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_0^R dM = \int_0^R \mu \times p \times 2\pi r^2 \times dr$$

Uniform Pressure

If the intensity of pressure p is assumed to be uniform and hence constant. From Eq.

$$W = p \times 2\pi \int_0^R r \times dr = p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \times \frac{R^2}{2}$$

$$\Rightarrow \qquad \boldsymbol{W = p \times \pi R^2}$$

$$W = p \times 2\pi \int_0^R r \times dr = p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \times \frac{R^2}{2}$$

$$\Rightarrow \qquad \boldsymbol{W = p \times \pi R^2}$$

And from Eq.

$$M = \mu \times p \times 2\pi \int_0^R r^2\, dr$$

$$M = \mu p \times 2\pi \times \left[\frac{r^3}{3}\right]_0^R = \frac{2}{3}\mu \times p \times \pi \times R^3$$

But $\quad p\,\pi\,R^2 = W$

$$\Rightarrow \quad \boldsymbol{M = \frac{2}{3}\mu WR = \mu W \times \frac{2}{3}R}$$

The friction force μW can be considered to be acting at a radius of $\frac{2}{3}R$.

Uniform Rate of Wear

By Eq.,

$$W = \int_0^R p \times 2\pi r \times dr$$

As the rate of wear is taken as constant and proportional to pr = a constant say c. Substituting for pr = c in the above equation.

$$W = \int_0^R 2\pi \times c \times dr = 2\pi R \times c$$

$$c = \frac{W}{2\pi R}$$

As the rate of wear is taken as constant and proportional to pr = a constant say c. Substituting for pr = c in the above equation.

$$W = \int_0^R 2\pi \times c \times dr = 2\pi R \times c$$

$$\Rightarrow \quad c = \frac{W}{2\pi R}$$

By Eq. (2.9), total frictional moment

$$M = \int_0^R \mu \times p \times 2\pi r^2 \times dr = \int_0^R \mu \times 2\pi \times c \times r \times dr$$

$$= \mu \times 2\pi \times c \times \frac{R^2}{2} = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2}$$

$$\therefore \quad \boldsymbol{M = \mu W \times \frac{R}{2}}$$

Thus, the frictional force : μW acts at a distance $\frac{R}{2}$ from the axis.

Conical Pivot

A truncated conical pivot is shown in Figure

Let 2α = The cone angle,

W = The axial load/thrust,

R_1 = The outer radius of the cone,

R_2 = The inner radius of the cone, and

p = Intensity of pressure which will act normal to the inclined surface of the cone as shown in Figure

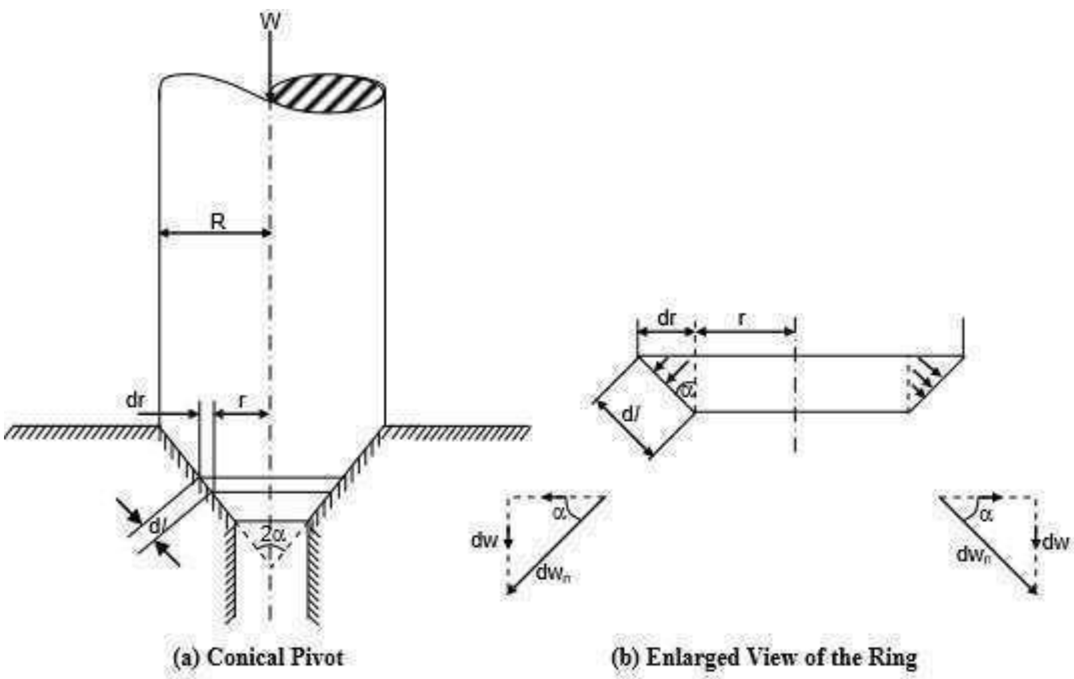


Figure: Truncated Conical Pivot

Consider an elementary ring of the cone, of radius r thickness dr and of sloping length dl as shown. Enlarged view of the ring is shown in Figure

Normal load on the ring

$$dW_n = p \times 2\pi r \times dl = p \times 2\pi r \times \frac{dr}{\sin \alpha}$$

Axial load on the ring

$$dW = dW_n \times \sin \alpha$$

$$= p \times 2\pi r \times \frac{dr}{\sin \alpha} \times \sin \alpha = p \times 2\pi r \times dr$$

Total axial load on the bearing

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr$$

Frictional force on the elementary ring

$$dF = \mu\, dW_n = \mu \times p \times 2\pi r \times dl = \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

Moment of the frictional force about the axis of rotation

$$dM = \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha}$$

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha}$$

Uniform Pressure

Uniform pressure, p is constant

$$W = p \times 2\pi \int_{R_2}^{R_1} r\, dr = p \times 2\pi \left[\frac{r^2}{2}\right]_{R_2}^{R_1}$$

or $$W = p \times \frac{2\pi}{2}(R_1^2 - R_2^2) = p \times \pi\,(R_1^2 - R_2^2)$$

$\therefore$ $$M = \frac{\mu \times p \times 2\pi}{\sin\alpha}\int_{R_2}^{R_1} r^2\, dr = \frac{\mu \times p \times 2\pi}{\sin\alpha} \times \frac{1}{3}(R_1^3 - R_2^3)$$

$$\boldsymbol{W = p \times \pi\,(R_1^2 - R_2^2)}$$

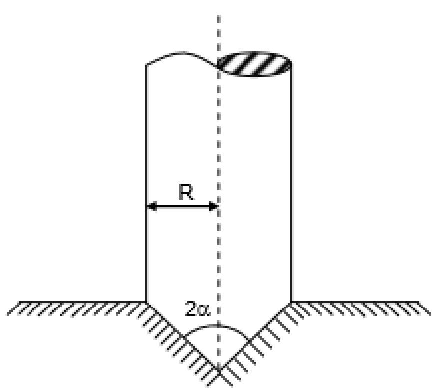
$\therefore$ $$p = \frac{W}{(\pi R_1^2 - R_2^2)}$$

Substituting for p above,

$\Rightarrow$ $$\boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{2}{3}\frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}}$$

Thus, frictional force $\frac{\mu W}{\sin\alpha}$ acts at a radius of $\frac{2}{3} \times \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$

For a full conical pivot R_2=0



$$r = 0$$

$\Rightarrow$ $$\boldsymbol{W = p\,\pi\,R_1^2}$$

and $$\boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{2}{3}\,R_1}$$

In this case the frictional force $\left(\dfrac{\mu W}{\sin\alpha}\right)$ acts at a radius $\dfrac{2}{3}$ from the axis.

Uniform Wear

From Eq.,

$$W = \int_r^R p \times 2\pi r \times dr$$

Since the rate of wear is uniform.

Therefore, pr = constant c

Substituting for pr

$$W = \int_{R_2}^{R_1} 2\pi c \times dr = 2\pi c \int_{R_2}^{R_1} dr = 2\pi c \times (R_1 - R_2)$$

$\Rightarrow$ $$c = \frac{W}{2\pi\,(R_1 - R_2)}$$

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin\alpha}$$

$$= \int_{R_2}^{R_1} \mu \times 2\pi r \times c \times \frac{dr}{\sin\alpha} \qquad [\text{since } pr = c]$$

$$= \frac{\mu \times 2\pi c}{\sin\alpha} \int_{R_2}^{R_1} r\,dr$$

$$= \frac{\mu \times 2\pi c}{\sin\alpha}\,\frac{(R_1^2 - R_2^2)}{2} = \frac{\mu \times \pi c}{\sin\alpha}\,(R_1^2 - R_2^2)$$

But $$c = \frac{W}{2\pi (R_1 - R_2)}$$

$$\Rightarrow \quad \boldsymbol{M = \frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}}$$

[Thus, the frictional force $\frac{\mu W}{\sin \alpha}$ acts at a radius $= \frac{(R_1 + R_2)}{2}$]

For the full conical pivot (Figure 2.10), $R_2 = 0$

$$\boldsymbol{M = \frac{\mu W}{\sin \alpha} \times \frac{R_1}{2}}$$

Thus, the friction force $\frac{\mu W}{\sin \alpha}$ acts at a radius $\frac{R_1}{2}$.

Collar Bearing

A collar bearing which is provided on to a shaft, is shown in Figure

Let

W = The axial load/thrust,

R1 = External radius of the collar, and

R2 = Internal radius of the collar.

Consider an elementary ring of the collar surface, of radius r and of thickness dr as shown

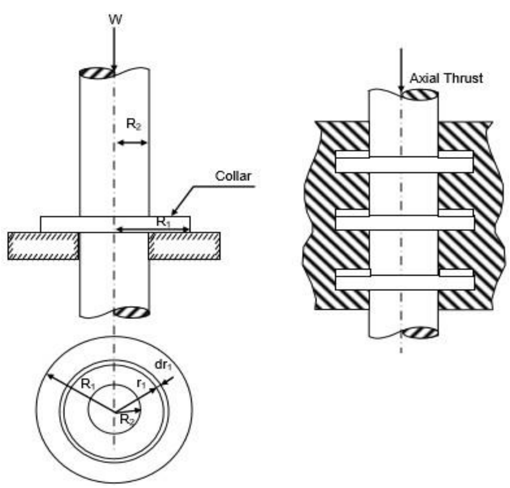


Figure: Collar Bearing

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total axial load

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr$$

Frictional force on the ring

$$dF = \mu \times p \times 2\pi r \times dr$$

Frictional moment of the ring

$$dM = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr$$

Uniform Pressure

Uniform pressure, p is constant

From Eq. (2.17),

$$W = p \times 2\pi \int_{R_2}^{R_1} r \times dr = p \times 2\pi \frac{(R_1^2 - R_2^2)}{3}$$

$$\Rightarrow \quad \boldsymbol{W = p \times \pi (R_1^2 - R_2^2)}$$

$$M = \mu \times p \times 2\pi \int_{R_2}^{R_1} r^2 \times dr = \mu \times p \times 2\pi \frac{(R_1^3 - R_2^3)}{3}$$

Substituting for p from above

$$\Rightarrow \quad \boldsymbol{M = \mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}}$$

Uniform Wear

$\therefore \quad pr$ = constant c

Substituting for pr above,

$$W = \int_r^R c \times 2\pi dr = c \times 2\pi \int_{R_2}^{R_1} dr = c \times 2\pi \times (R_1 - R_2)$$

$$c = \frac{W}{2\pi (R_1 - R_2)}$$

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr = \mu \times 2\pi \times c \int_{R_2}^{R_1} r \times dr$$

$$= \mu \times 2\pi \times c \frac{(R_1 - R_2)}{2}$$

Substituting for c from Eq. (2.21),

$$\Rightarrow \quad \boldsymbol{M = \mu W \times \frac{(R_1 + R_2)}{2}}$$

There is a limit to the bearing pressure on a single collar and it is about 40 N/cm2. Where the axial load is more and pressure on each collar is not to be allowed to exceed beyond the designed limit, then more collars are provided as shown in Figure.

Number of collars =

N= Total axial load / Permissible axial load on each collar

It may be pointed out there is no change in the magnitude of frictional moments with more number of collars. The number of collars, as given above, only limits the maximum intensity of pressure in each collar.

Table: Pivot and Collars Summary of Formulae

Sl. No.	Particular	Frictional Moments : M	
		Uniform Pressure	**Uniform Wear**
1.	Flat pivot	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
2.	Conical pivot (a) Truncated	$\frac{\mu W}{\sin \alpha} \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$
	(b) Full conical	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
3.	Collar	$\mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\mu W \times \frac{(R_1 + R_2)}{2}$

CLUTCH

It is a mechanical device which is widely used in automobiles for the purpose of engaging driving and the driven shaft, at the will of the driver or the operator. The driving shaft is the engine crankshaft and the driven shaft is the gear-box driving shaft. This means that the clutch is situated between the engine flywheel mounted on the crankshaft and the gear box.

In automobile, gears are required to be changed for obtaining different speeds. It is possible only if the driving shaft of the gear box is stopped for a while without stopping the engine. These two objects are achieved with the help of a clutch.

Broadly speaking, a clutch consists of two members; one fixed to the crankshaft or the flywheel of the engine and the other mounted on a splined shaft, of the gear box so that this could be engaged or disengaged as the case may be with the member fixed to the engine crankshaft.

TYPES OF CLUTCHES

Clutches can be classified into two types as follows :

(a) conical clutch, and

(b) the place or disc clutches can be of single plate or of multiple plates

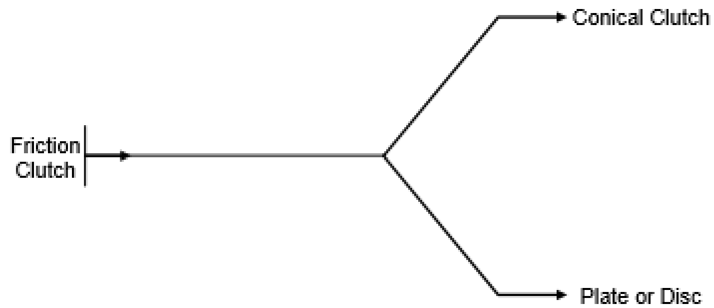


Figure: The Single Cone Clutch

Conical Clutch

A conical clutch is shown in Figure, It consists of a cone A mounted on engine crankshaft. Cone B has internal splines in its boss which fit into the corresponding splines provided in the gear box shaft. Cone B could rotate the gear box shaft as well as may slide along with it. The outer surface of cone B is lined with friction material. In the normal or released position of the clutch pedal P, cone B fits into the inner conical surface of cone A and by means of the friction between the contacting surfaces, power is transmitted from crankshaft to the gear box shaft. When the clutch pedal is pressed, pivot D being the fulcrum provided in it, the collar E is pressed towards the right side, thus disengaging cone B from cone A and keeping the compression spring S compressed. On releasing the pedal, by the force of the spring, the cone B is thrusted back to engage cone A for power transmission.

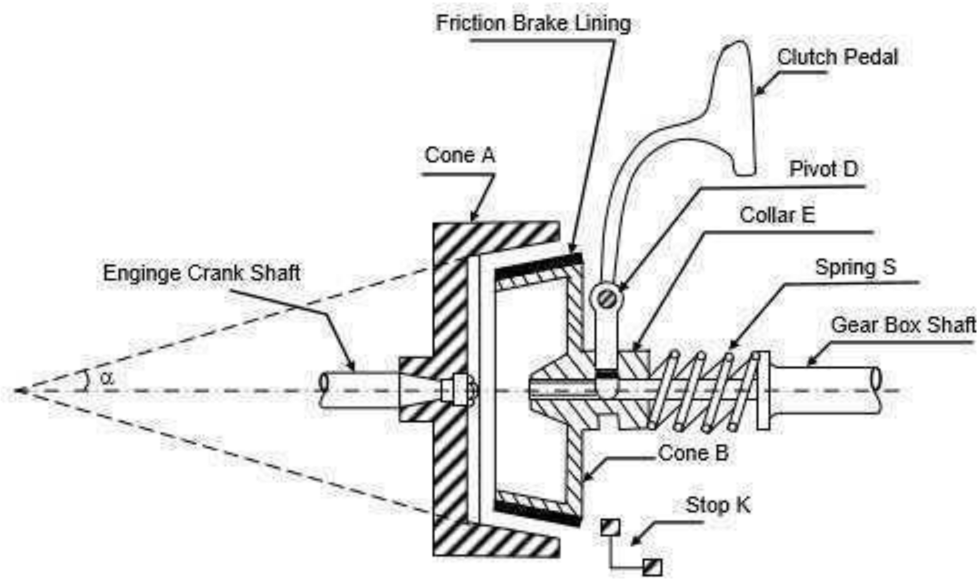


Figure: Conical Clutch

For calculating frictional moments or torque transmitted on account of friction in clutches, unless otherwise specifically stated uniform rate of wear is assumed. For torque transmitted formulae of conical pivot can be used.

Single Plate Clutch

A single plate clutch is known as single disc clutch. It is shown in Figure, It has two sides which are driving and the driven side. The driving side comprises of the driving shaft or engine crankshaft A. A boss B is keyed to it to which flywheel C is bolted as shown. On the driven side, there is a driven shaft D. It carries a boss E which can freely slide axially along with the driven shaft through splines F. The clutch plate is mounted on the boss E. It is provided with rings of friction material – known as friction linings, on the both sides indicated H. One friction lining is pressed on the flywheel face and the other on the pressure plate I. A small spigot, bearing J, is provided in the end of the driving shaft for proper alignment.

The pressure plate provides axial thrust or pressure between clutch plate G and the flywheel C and the pressure plate I through the linings on its either side, by means of the springs, S.

The pressure plate remains engaged and as such clutch remains in operational position. Power from the driving shaft is transferred to the driven shaft from flywheel to the clutch plate through the friction lining between them.

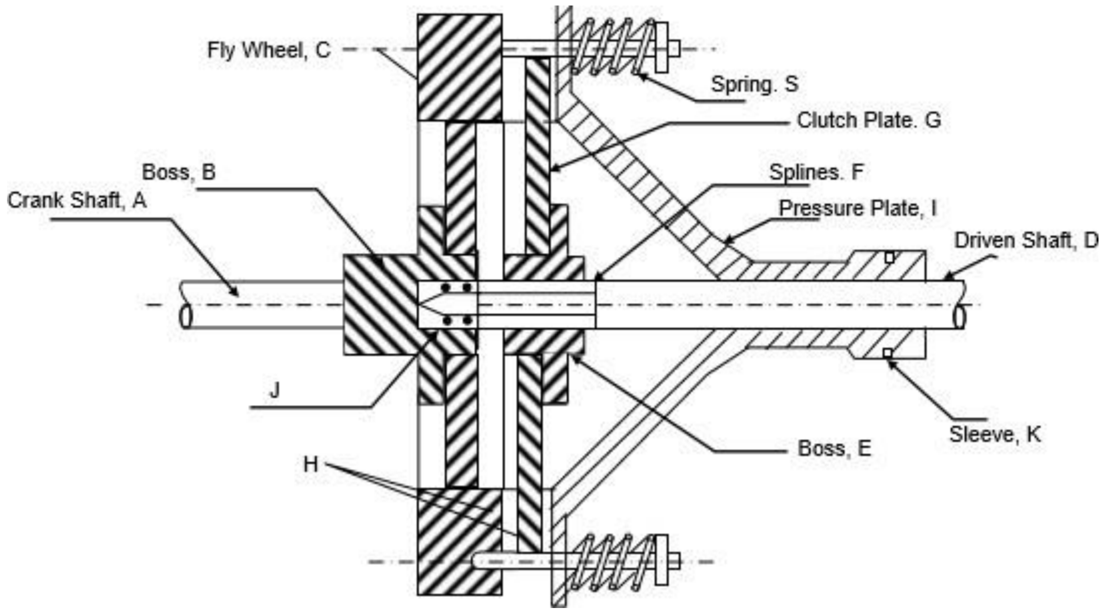


Figure: Single Plate Clutch

From pressure plate the power is transmitted to clutch plate through friction linings. Both sides of the clutch plate are effective. When the clutch is to be disengaged the sleeve K is moved towards right hand side by means of clutch pedal mechanism (it is not shown in the figure). By doing this, there is no pressure between the pressure plate, flywheel and the clutch plate and no power is transmitted. In medium size and heavy vehicles, like truck, single plate clutch is used.

Multi Plate Clutch

As already explained in a plate clutch, the torque is transmitted by friction between one or more pairs of co-axial annular faces kept in contact by an axial thrust provided by springs. In a single plate clutch, both sides of the plate are effective so that it has two pairs of surfaces in contact or $n = 2$. Obviously, in a single plate clutch limited amount of torque can be transmitted. When large amount of torque is to be transmitted, more pair of contact surfaces are needed and it is precisely what is obtained by a multi-plate clutch.

KEY WORDS

Dry Friction : It is the friction between two dry surfaces.

Lubricated Surfaces : The lubricant which is put between surfaces forms a film between surfaces to reduce friction.

Static Friction : It is the friction between the two static surfaces.

Kinetic Friction : It is the friction between the surfaces having motion.

Coefficient of Friction : It is the ratio of force of limiting friction to the normal reaction.

Limiting Friction : It is the maximum force of friction which is developed when slide is impending.

Angle of Repose : It is the angle of the inclined plane to the horizontal at which body slides on its own.

Screw Jack : It is machine to lift or lower the load.

Pivots : It is a surface which bears axial thrust of shaft.

Collars : Collars are like plates which support axial thrust.

Clutch : It is a friction coupling which is used between engine and gear box in an automobile.

Journal Bearing : In these bearings, lubricant layer separate metal surfaces to lower the friction.

Ball Bearing : In this bearing, balls are used to separate rotating surface from stationary surface.

Roller Bearing : In this bearing rollers are used to separate rotating surface from stationary surface.