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New Scheme Based On AICTE Flexible Curricula

Civil Engineering, V-Semester

CE501- Fluid Mechanics-I

Unit-I

Review of Fluid Properties: Engineering units of measurement, mass, density, specific weight, specific volume, specific gravity, surface tension, capillarity, viscosity, bulk modulus of elasticity, pressure and vapor pressure. Fluid Static's : Pressure at a point, pressure variation in static fluid, Absolute and gauge pressure, manometers, Forces on plane and curved surfaces (Problems on gravity dams and Tainter gates); buoyant force, Stability of floating and submerged bodies, Relative equilibrium.

Unit-II

Kinematics of Flow : Types of flow-ideal & real , steady & unsteady, uniform & non uniform, one, two and three dimensional flow, path lines, strea klines, streamlines and stream tubes; continuity equation for one and three dimensional flow, rotational & irrotational flow, circulation, stagnation point, separation of flow, sources & sinks, velocity potential, stream function, flow nets- their utility & method of drawing flow nets.

Unit-III

Dynamics of Flow: Euler's equation of motion along a streamline and derivation of Bernoulli's equation, application of Bernoulli's equation, energy correction factor, linear momentum equation for steady flow; momentum correction factor. The moment of momentum equation, forces on fixed and moving vanes and other applications. Fluid Measurements: Velocity measurement (Pitot tube, Prandtl tube, current meters etc.); flow measurement (orifices, nozzles, mouth pieces, orifice meter, nozzle meter, venturimeter, weirs and notches).

Unit-IV

Laminar Flow: Introduction to laminar flow, Reynolds experiment & Reynolds number, relation between shear & pressure gradient, laminar flow through circular pipes, laminar flow between parallel plates, laminar flow through porous media, Stokes law.

Unit-V

Dimensional Analysis and use of Buckingham-pi theorem, Introduction to Turblent flow-Prandtl mixing length hypothesis, Flow over smooth & rough surface. Darcy –weisbach resistance equation , variation of friction factor & Moody's diagram , pipe flow problem.

Reference Books : -

1. Modi & Seth; Fluid Mechanics; Standard Book House, Delhi
2. Som and Biswas; Fluid Mechnics and machinery; TMH
3. Engg fluid mech. – By Grade & Miraj gaonkar , Nem Chand & Bros. Prakashan
4. White ; Fluid Mechanics ; TMH
5. Essential of Engg Hyd. By JNIK DAKE; Afrikan Network & Sc Instt. (ANSTI)
6. A Text Book of fluid Mech. for Engg. Student by Franiss JRD
7. R Mohanty; Fluid Mechanics By; PHI
8. Fluid Mechanics; Gupta Pearson.

List of Experiment (Expandable):

1. To determine the local point pressure with the help of pitot tube.
2. To find out the terminal velocity of a spherical body in water.
3. Calibration of Venturimeter
4. Determination of Cc, Cv, Cd of Orifices
5. Calibration of Orifice Meter
6. Calibration of Nozzle meter and Mouth Piece
7. Reynolds experiment for demonstration of stream lines & turbulent flow
8. Determination of metacentric height
9. Determination of Friction Factor of a pipe
10. To study the characteristics of a centrifugal pump.
11. Verification of Impulse momentum principle

Fundamental Fluid Properties: Engineering units of measurement, mass, density, specific weight, specific volume, specific gravity, surface tension, capillarity, and viscosity, bulk modulus of elasticity, pressure and vapor pressure. Fluid Statics: Pressure at a point, pressure variation in static fluid, Absolute and gauge pressure, manometers, Forces on plane and curved surfaces (Problems on Gravity Dams and Tainter Gates), buoyant force, stability of floating and submerged bodies, relative equilibrium.

Fluid Mechanics is: - Fluid Mechanics is that section of applied mechanics, concerned with the statics and dynamics of liquids and gases.

A knowledge of fluid mechanics is essential for the chemical engineer, because the majority of chemical processing operations are conducted either partially or totally in the fluid phase.

The handling of liquids is much simpler, much cheaper, and much less troublesome than handling solids.

Even in many operations a solid is handled in a finely divided state so that it stays in suspension in a fluid.

Fluid Statics: Which treats fluids in the equilibrium state of no shear stress

Fluid Mechanics: Which treats when portions of fluid are in motion relative to other parts.

Fluids and their Properties

Fluids

In everyday life, we recognize three states of matter: solid, liquid and gas. Although different in many respects, liquids and gases have a common characteristic in which they differ from solids: they are fluids, lacking the ability of solids to offer a permanent resistance to a deforming force.

A fluid is a substance which deforms continuously under the action of shearing forces, however small they may be. Conversely, it follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.

Shear stress in a moving fluid

Although there can be no shear stress in a fluid at rest, shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities, causing the original shape of the fluid to become distorted. If, on the other hand, the velocity of the fluid is same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Differences between solids and fluids

The differences between the behavior of solids and fluids under an applied force are as follows:

i. **For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.**

ii. **The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.**

Differences between liquids and gases

Although liquids and gases both share the common characteristics of fluids, they have many distinctive characteristics of their own. A liquid is difficult to compress and, for many purposes, may be regarded as incompressible. A given mass of liquid occupies a fixed volume, irrespective of the size or shape of its container, and a free surface is formed if the volume of the container is greater than that of the liquid.

A gas is comparatively easy to compress. Changes of volume with pressure are large, cannot normally be neglected and are related to changes of temperature. A given mass of gas has no fixed volume and will expand continuously unless restrained by a containing vessel. It will completely fill any vessel in which it is placed and, therefore, does not form a free surface.

Newtonian and non-Newtonian Fluids

Newtonian fluids:

Fluids which obey the Newton's law of viscosity are called as Newtonian fluids. Newton's law of viscosity is given by

$$t = m\,dv/dy$$

where t = shear stress

m = viscosity of fluid

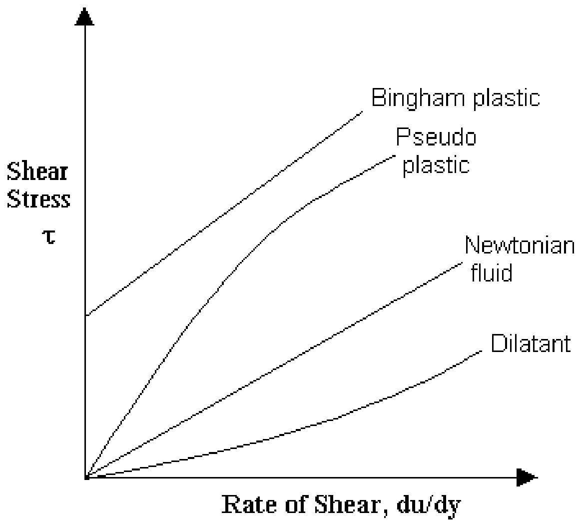
dv/dy = shear rate, rate of strain or velocity gradient

All gases and most liquids which have simpler molecular formula and low molecular weight such as water, benzene, ethyl alcohol, CCl_4, hexane and most solutions of simple molecules are Newtonian fluids.

Non-Newtonian fluids:

Fluids which do not obey the Newton's law of viscosity are called as non-Newtonian fluids.

Generally non-Newtonian fluids are complex mixtures: slurries, pastes, gels, polymer solutions etc.,



Various non-Newtonian Behaviors:

Time-Independent behaviors:

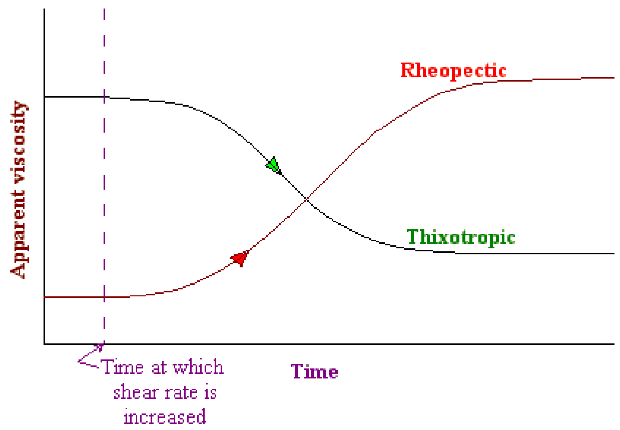
Properties are independent of time under shear.

Bingham-plastic: Resist a small shear stress but flow easily under larger shear stresses. E.g. tooth-paste, jellies, and some slurries.

Pseudo-plastic: Most non-Newtonian fluids fall into this group. Viscosity decreases with increasing velocity gradient. e.g. polymer solutions, blood. Pseudoplastic fluids are also called as Shear thinning fluids. At low shear rates (du/dy) the shear thinning fluid is more viscous than the Newtonian fluid, and at high shear rates it is less viscous.

Dilatant fluids: Viscosity increases with increasing velocity gradient. They are uncommon, but suspensions of starch and sand behave in this way. Dilatant fluids are also called as shear thickening fluids.

Time dependent behaviors:



Effect of sudden change of shear rate on apparent viscosity of time-dependent fluids

Those which are dependent upon duration of shear.

Thixotropic fluids: for which the dynamic viscosity decreases with the time for which shearing forces are applied. e.g. thixotropic jelly paints.

Rheopectic fluids: Dynamic viscosity increases with the time for which shearing forces are applied. E.g. gypsum suspension in water.

Visco-elastic fluids: Some fluids have elastic properties, which allow them to spring back when a shear force is released. e.g. egg white.

Viscosity

The viscosity (m) of a fluid measures its resistance to flow under an applied shear stress. Representative units for viscosity are kg/ (m.sec), g/ (cm.sec) (also known as poise designated by P). The centipoise (cP), one hundredth of a poise, is also a convenient unit, since the viscosity of water at room temperature is approximately 1 centipoise.

The kinematic viscosity (n) is the ratio of the viscosity to the density:

$n = m/r$,

and will be found to be important in cases in which significant viscous and gravitational forces exist.

Viscosity of liquids:

Viscosity of liquids in general, decreases with increasing temperature.

The viscosities (m) of liquids generally vary approximately with absolute temperature T according to:

$\ln m = a - b \ln T$

Viscosity of gases:

Viscosity of gases increases with increase in temperature.

The viscosity (m) of many gases is approximated by the formula:

$m = m_o(T/T_o)^n$

in which T is the absolute temperature, m_o is the viscosity at an absolute reference temperature T_o, and n is an empirical exponent that best fits the experimental data.

The viscosity of an ideal gas is independent of pressure, but the viscosities of real gases and liquids usually increase with pressure.

Viscosity of liquids are generally two orders of magnitude greater than gases at atmospheric pressure. example, at 25°C, m_{water} = 1 centipoise and $m_{air} = 1 \times 10^{-2}$centipoise.

Vapor Pressure

The pressure at which a liquid will boil is called its vapor pressure. This pressure is a function of temperature (vapor pressure increases with temperature). In this context we usually think about the temperature at which boiling occurs. For example, water boils at 100°C at sea-level atmospheric pressure (1 atm abs). However, in terms of vapor pressure, we can say that by increasing the temperature of water at sea level to 100 °C, we increase the vapor pressure to the point at which it is equal to the atmospheric pressure (1 atm abs), so that boiling occurs. It is easy to visualize that boiling can also occur in water at temperatures much below 100°C if the pressure in the water is reduced to its vapor pressure. For example, the vapor pressure of water at 10°C is 0.01 atm. Therefore, if the pressure within water at that temperature is reduced to that value, the water boils. Such boiling often occurs in flowing liquids, such as on the suction side of a pump. When such boiling does occur in the flowing liquids, vapor bubbles start growing in local regions of very low pressre and then collapse in regions of high downstream pressure. This phenomenon is called as cavitation

Compressibility and the Bulk modulus

All materials, whether solids, liquids or gases, are compressible, i.e. the volume V of a given mass will be reduced to V - dV when a force is exerted uniformly all over its surface. If the force per unit area of surface increases from p to p + dp, the relationship between change of pressure and change of volume depends on the bulk modulus of the material.

Bulk modulus (K) = (change in pressure) / (volumetric strain)

Volumetric strain is the change in volume divided by the original volume. Therefore,

(Change in volume) / (original volume) = (change in pressure) / (bulk modulus)

i.e., $-dV/V = dp/K$

Negative sign for dV indicates the volume decreases as pressure increases.

In the limit, as dp tends to 0,

$K = -V\, dp/dV$ **à 1**

Considering unit mass of substance, $V = 1/r$ **à 2**

Differentiating,

$Vdr + rdV = 0$

$dV = -(V/r)dr$ **à3**

putting the value of dV from equn.3 to equn.1,

$K = -V\,dp / (-(V/r)dr)$

i.e. $K = rdp/dr$

The concept of the bulk modulus is mainly applied to liquids, since for gases the compressibility is so great that the value of K is not a constant.

The relationship between pressure and mass density is more conveniently found from the characteristic equation of gas.

For liquids, the changes in pressure occurring in many fluid mechanics problems are not sufficiently great to cause appreciable changes in density. It is therefore usual to ignore such changes and consider liquids as incompressible.

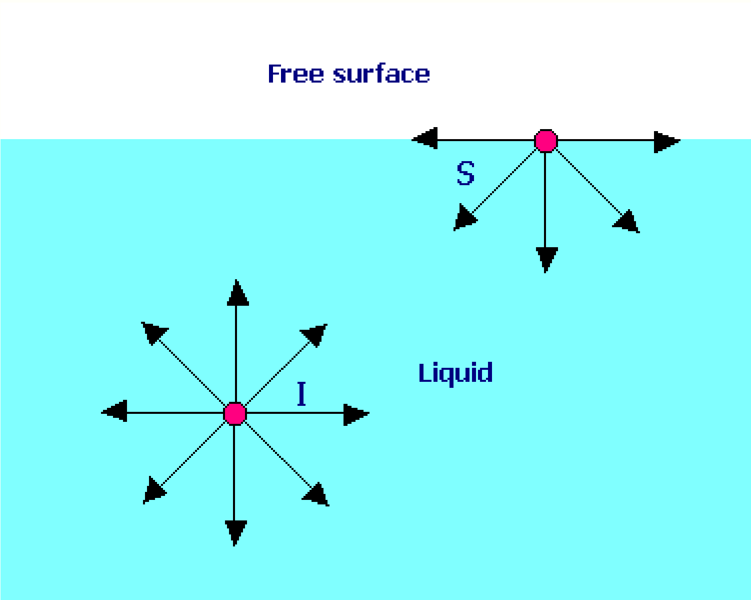
Gases may also be treated as incompressible if the pressure changes are very small, but usually compressibility cannot be ignored. In general, compressibility becomes important when the velocity of the fluid exceeds about one-fifth of the velocity of a pressure wave (velocity of sound) in the fluid.

Typical values of Bulk Modulus:

$K = 2.05 \times 10^9$ N/m^2 for water

$K = 1.62 \times 10^9$ N/m^2 for oil.

+Surface Tension



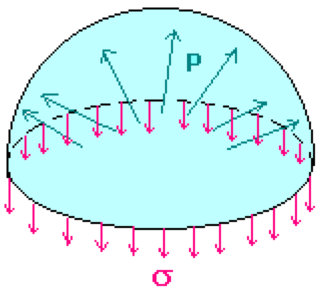
A molecule I in the interior of a liquid is under attractive forces in all directions and the vector sum of these forces is zero. But a molecule S at the surface of a liquid is acted by a net inward cohesive force that is perpendicular to the surface. Hence it requires work to move molecules to the surface against this opposing force, and surface molecules have more energy than interior ones.

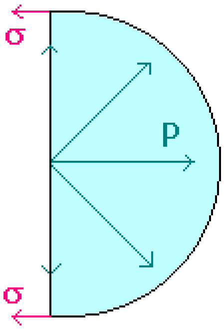
The surface tension (s sigma) of a liquid is the work that must be done to bring enough molecules from inside the liquid to the surface to form one new unit area of that surface (J/m^2 = N/m). Historically surface tensions have been reported in handbooks in dynes per centimeter (1 dyn/cm = 0.001 N/m).

Surface tension is the tendency of the surface of a liquid to behave like a stretched elastic membrane. There is a natural tendency for liquids to minimize their surface area. For this reason, drops of liquid tend to take a spherical shape in order to minimize surface area. For such a small droplet, surface tension will cause an increase of internal pressure p in order to balance the surface force.

We will find the amount D (Dp = p - $p_{outside}$) by which the pressure inside a liquid droplet of radius r, exceeds the pressure of the surrounding vapor/air by making force balances on a hemispherical drop. Observe that the internal pressure p is trying to blow apart the two hemispheres, whereas the surface tension s is trying to pull them together. Therefore, DP pr^2 = 2prs

i.e. Dp = 2s/r



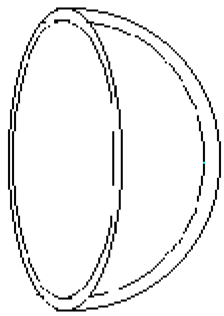


Similar force balances can be made for cylindrical liquid jet. Dp 2r= 2s

i.e. Dp = s/r

Similar treatment can be made for a soap bubble which is having two free surfaces. Dp pr^2 = 2 x 2prs

i.e. Dp = 4s/r



Surface tension generally appears only in situations involving either free surfaces (liquid/gas or liquid/solid boundaries) or interfaces (liquid/liquid boundaries); in the latter case, it is usually called the interfacial tension.

Representative values for the surface tensions of liquids at 20°C, in contact either with air or their vapor (there is usually little difference between the two), are given in Table.

Liquid	Surface Tension s dyne/cm
Benzene	23.70
Benzene	28.85
Ethanol	22.75
Glycerol	63.40
Mercury	435.50
Methanol	22.61

n-Octane	21.78
Water	72.75

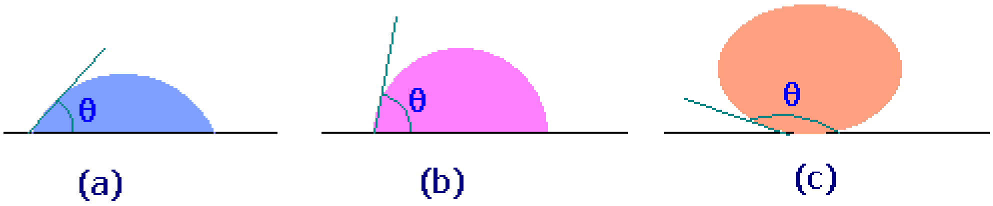
Capillarity

Rise or fall of a liquid in a capillary tube is caused by surface tension and depends on the relative magnitude of cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel.

Liquids rise in tubes if they wet (adhesion > cohesion) and fall in tubes that do not wet (cohesion > adhesion).

Wetting and contact angle

Fluids wet some solids and do not others.



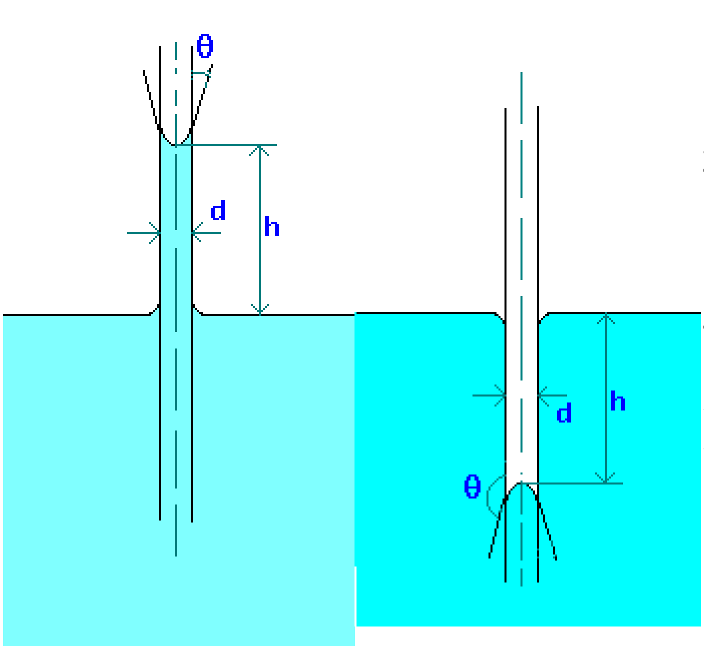
The figure shows some of the possible wetting behaviors of a drop of liquid placed on a horizontal, solid surface (the remainder of the surface is covered with air, so two fluids are present).

Fig.(a) represents the case of a liquid which wets a solid surface well, e.g. water on a very clean copper. The angle q shown is the angle between the edge of the liquid surface and the solid surface, measured inside the liquid. This angle is called the contact angle and is a measure of the quality of wetting. For perfectly wetting, in which the liquid spreads as a thin film over the surface of the solid, q is zero.

Fig.(c) represents the case of no wetting. If there were exactly zero wetting, q would be 180°. However, the gravity force on the drop flattens the drop, so that 180° angle is never observed. This might represent water on Teflon or mercury on clean glass.

We normally say that a liquid wets a surface if q is less than 90° and does not wet if q is more than 90°. Values of q less than 20° are considered strong wetting, and values of q greater than 140° are strong no wetting.

Capillarity is important (in fluid measurements) when using tubes smaller than about 10 mm in diameter.



Capillary rise (or depression) in a tube can be calculated by making force balances. The forces acting are force due to surface tension and gravity.

The force due to surface tesnion, F_s = pdscos(q), where q is the wetting angle or contact angle. If tube (made of glass) is clean q is zero for water and about 140° for Mercury. This is opposed by the gravity force on the column of fluid, which is equal to the height of the liquid which is above (or below) the free surface and which equals

F_g = (p/4) d2hgr,

Where r is the density of liquid.

Equating these forces and solving for Capillary rise (or depression), we find h = 4scos(q)/(rgd)

Surface Tension - Solved Problems

Air is introduced through a nozzle into a tank of water to form a stream of bubbles. If the bubbles are intended to have a diameter of 2 mm, calculate how much the pressure of the air at the tip of the nozzle must exceed that of the surrounding water. Assume that the value of surface tension between air and water as 72.7 x 10^{-3} N/m.

Data:

Surface tension (s) = 72.7 x 10^{-3} N/m

Radius of bubble (r) = 1

Formula:

Dp = 2s/r

Calculations:

Dp = 2 x 72.7 x 10^{-3} / 1 = 145.4 N/m^2

That is, the pressure of the air at the tip of nozzle must exceed the pressure of surrounding water by 145.4 N/m^2

A soap bubble 50 mm in diameter contains a pressure (in excess of atmospheric) of 2 bar. Find the surface tension in the soap film.

Data:

Radius of soap bubble (r) = 25 mm = 0.025 m

Dp = 2 Bar = 2 x 10^5 N/m^2

Formula:

Pressure inside a soap bubble and surface tension (s) are related by, Dp = 4s/r

Calculations:

s = Dpr/4 = 2 x 10^5 x 0.025/4 = 1250 N/m

Water has a surface tension of 0.4 N/m. In a 3 mm diameter vertical tube if the liquid rises 6 mm above the liquid outside the tube, calculate the contact angle.

Data:

Surface tension (s) = 0.4 N/m

Dia of tube (d) = 3 mm = 0.003 m

Capillary rise (h) = 6 mm = 0.006 m

Formula:

Capillary rise due to surface tension is given by

h = 4scos(q)/(rgd), where q is the contact angle.

Calculations:

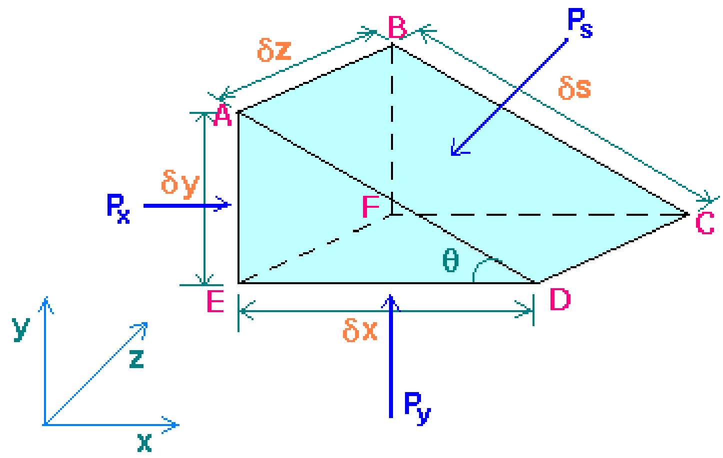
Cos (q) = hrgd/ (4s) = 0.006 x 1000 x 9.812 x 0.003 / (4 x 0.4) = 0.11

Therefore, contact angle q = 83.7^o

Fluid statics

Pascal's law for pressure at a point

The basic property of a static fluid is pressure. Pressure is familiar as a surface force exerted by a fluid against the walls of its container. Pressure also exists at every point within a volume of fluid. For a static fluid, as shown by the following analysis, pressure turns to be independent direction.



By considering the equilibrium of a small fluid element in the form of a triangular prism ABCDEF surrounding a point in the fluid, a relationship can be established between the pressures P_x in the x direction, P_y in the y direction, and Ps normal to any plane inclined at any angle θ to the horizontal at this point.

P_x is acting at right angle to ABEF, and Py at right angle to CDEF, similarly P_s at right angle to ABCD.

Since there can be no shearing forces for a fluid at rest, and there will be no accelerating forces, the sum of the forces in any direction must therefore, be zero. The forces acting are due to the pressures on the surrounding and the gravity force.

Force due to P_x = P_x x Area ABEF = $P_x \delta y \delta z$

Horizontal component of force due to P_s = - (P_s x Area ABCD) sin(θ) = $- P_x \delta s \delta z \delta y / \delta s = -P_x \delta y \delta z$

As P_y has no component in the x direction, the element will be in equilibrium, if

$P_x \delta y \delta z + (-P_x \delta y \delta z) = 0$

I.e. $P_x = P_s$

Similarly in the y direction, force due to $P_y = P_x \delta y \delta z$

Component of force due to P_s = - (P_s x Area ABCD) cos (θ) = $- P_x \delta y \delta z \delta x / \delta s = - P_x \delta y \delta z$

Force due to weight of element = - mg = $- \rho V g = - \rho (\delta Px \delta y \delta z/2) g$

Since δx, δy and δz are very small quantities, $Px \delta y \delta z$ is negligible in comparison with other two vertical force terms, and the equation reduces to,

$P_y = P_s$

Therefore, $P_x = P_y = P_s$

i.e. pressure at a point is same in all directions. This is Pascal's law. This applies to fluid at rest.

Fine powdery solids resemble fluids in many respects but differs considerably in others. For one thing, a static mass of particulate solids, can support shear stresses of considerable magnitude and the pressure is not the same in all directions.

Variation of pressure with elevation

Consider a hypothetical differential cylindrical element of fluid of cross sectional area A and height $(z_2 - z_1)$.

Upward force due to pressure P_1 on the element = P_1A

Downward force due to pressure P_2 on the element = P_2A

Force due to weight of the element = mg = $\rho A (z_2 - z_1) g$

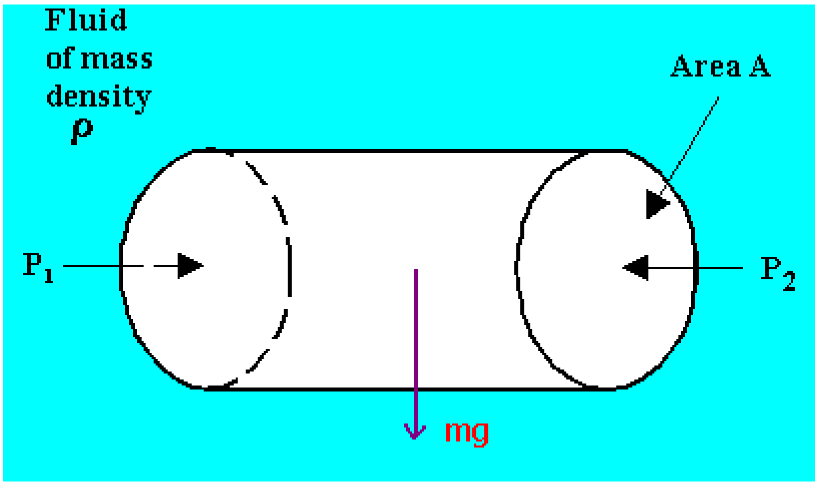
Equating the upward and downward forces,

$P_1A = P_2A + \rho A(z_2 - z_1)g$

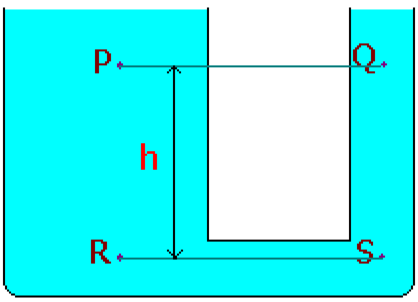
$P_2 - P_1 = - \rho g (z_2 - z_1)$

Thus in any fluid under gravitational acceleration, pressure decreases, with increasing height z in the upward direction.

Equality of pressure at the same level in a static fluid:



Equating the horizontal forces, $P_1A = P_2A$ (i.e. some of the horizontal forces must be zero)



Equality of pressure at the same level in a continuous body of fluid:

Pressures at the same level will be equal in a continuous body of fluid, even though there is no direct horizontal path between P and Q provided that P and Q are in the same continuous body of fluid.

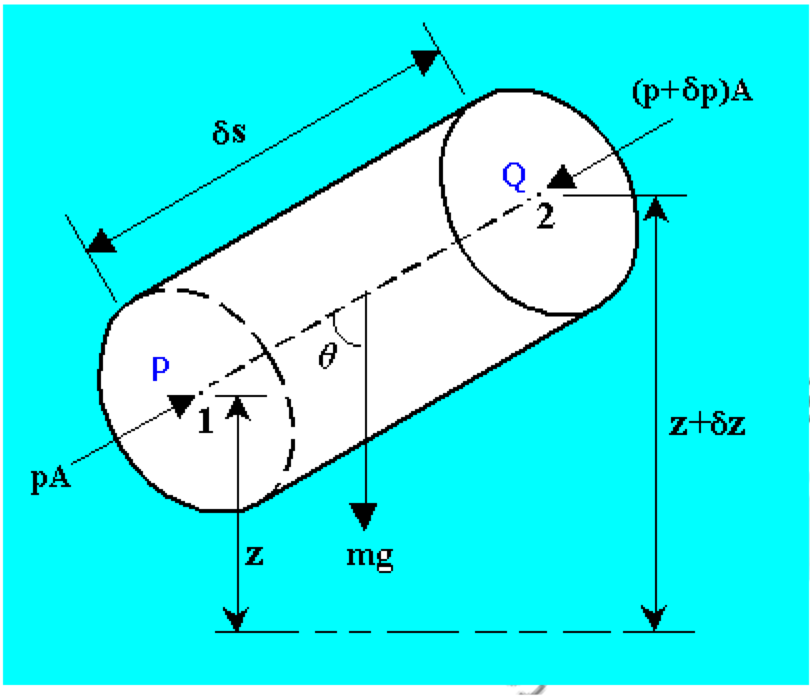
We know that, $P_R = P_S$

$P_R = P_P + \rho gh \rightarrow 1$

$P_S = P_Q + \rho gh \rightarrow 2$

From equn.1 and 2, $P_P = P_Q$

General equation for the variation of pressure due to gravity from point to point in a static fluid:



Resolving the forces along the axis PQ,

$pA - (p + \delta p)A - \rho gA\,\delta s\cos(\theta) = 0$

$\delta p = -\rho g\,\delta s\cos(\theta)$

or in differential form,

$dp/ds = -\rho g\cos(\theta)$

In the vertical z direction, $\theta = 0$.

Therefore,

$dp/dz = -\rho g$

This equation predicts a pressure decrease in the vertically upwards direction at a rate proportional to the local density.

Absolute Pressure, Gage Pressure, and Vacuum

In a region such as outer space, which is virtually void of gases, the pressure is essentially zero. Such a condition can be approached very nearly in a laboratory when a vacuum pump is used to evacuate a bottle. The pressure in a vacuum is called absolute zero, and all pressures refernced with respect to this zero pressure are termed absolute pressures.

Many pressure-measuring devices measure not absolute pressure but only difference in pressure. For example, a Bourdon-tube gage indicates only the difference between the pressure in the fluid to which it is tapped and the pressure in the atmosphere. In this case, then, the reference pressure is actually the atmospheric pressure. This type of pressure reading is called gage pressure. For example, if a pressure of 50 kPa is measured with a gage referenced to the atmosphere and the atmospheric pressure is 100 kPa, then the pressure can be expressed as either

p = 50 kPa gage or p = 150 kPa absolute.

Whenever atmospheric pressure is used as a reference, the possibility exists that the pressure thus measured can be either positive or negative. Negative gage pressure are also termed as vacuum pressures. Hence, if a gage tapped into a tank indicates a vacuum pressure of 31 kPa, this can also be stated as 70 kPa absolute, or -31 kPa gage, assuming that the atmospheric pressure is 101 kPa absolute.

Pressure Measurement

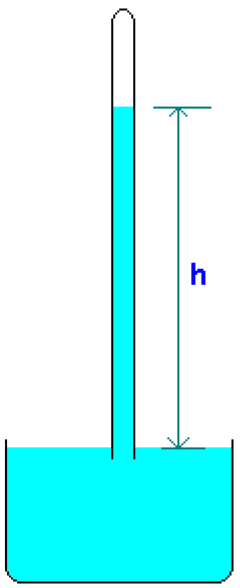
Fluid Pressure

In a stationary fluid the pressure is exerted equally in all directions and is referred to as the static pressure. In a moving fluid, the static pressure is exerted on any plane parallel to the direction of motion. The fluid pressure exerted on a plane right angles to the direction of flow is greater than the static pressure because the surface has, in addition, to exert sufficient force to bring the fluid to rest. The additional pressure is proportional to the kinetic energy of fluid; it cannot be measured independently of the static pressure.

When the static pressure in a moving fluid is to be determined, the measuring surface must be parallel to the direction of flow so that no kinetic energy is converted into pressure energy at the surface. If the fluid is flowing in a circular pipe the measuring surface must be perpendicular to the radial direction at any point. The pressure connection, which is known as a piezometer tube, should flush with the wall of the pipe so that the flow is not disturbed: the pressure is then measured near the walls were the velocity is a minimum and the reading would be subject only to a small error if the surface were not quite parallel to the direction of flow.

The static pressure should always be measured at a distance of not less than 50 diameters from bends or other obstructions, so that the flow lines are almost parallel to the walls of the tube. If there are likely to be large cross-currents or eddies, a piezometer ring should be used. This consists of 4 pressure tappings equally spaced at 90°intervals round the circumference of the tube; they are joined by a circular tube which is connected to the pressure measuring device. By this means, false readings due to irregular flow or avoided. If the pressure on one side of the tube is relatively high, the pressure on the opposite side is generally correspondingly low; with the piezometer ring a mean value is obtained.

Barometers

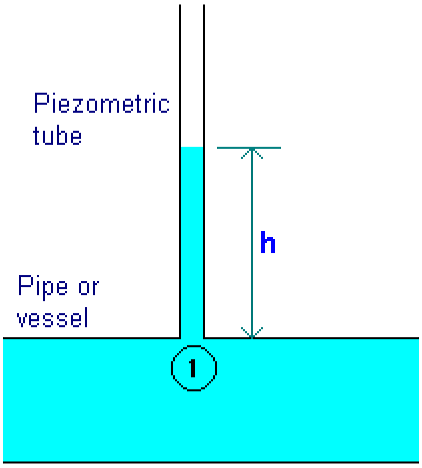


A barometer is a device for measuring atmospheric pressure. A simple barometer consists of a tube more than 30 inch (760 mm) long inserted in an open container of mercury with a closed and evacuated end at the top and open tube end at the bottom and with mercury extending from the container up into the tube. Strictly, the space above the liquid cannot be a true vacuum. It contains mercury vapor at its saturated vapor pressure, but this is extremely small at room temperatures (e.g. 0.173 Pa at 20°C).

The atmospheric pressure is calculated from the relation $P_{atm} = \rho gh$ where ρ is the density of fluid in the barometer.

Piezometer

For measuring pressure inside a vessel or pipe in which liquid is there, a tube may be attached to the walls of the container (or pipe) in which the liquid resides so liquid can rise in the tube. By determining the height to which liquid rises and using the relation $P_1 = \rho gh$, gauge pressure of the liquid can be determined. Such a device is known as piezometer. To avoid capillary effects, a piezometer's tube should be about 1/2 inch or greater.



It is important that the opening of the device to be tangential to any fluid motion, otherwise an erroneous reading will result.

Manometer

A somewhat more complicated device for measuring fluid pressure consists of a bent tube containing one or more liquid of different specific gravities. Such a device is known as manometer.

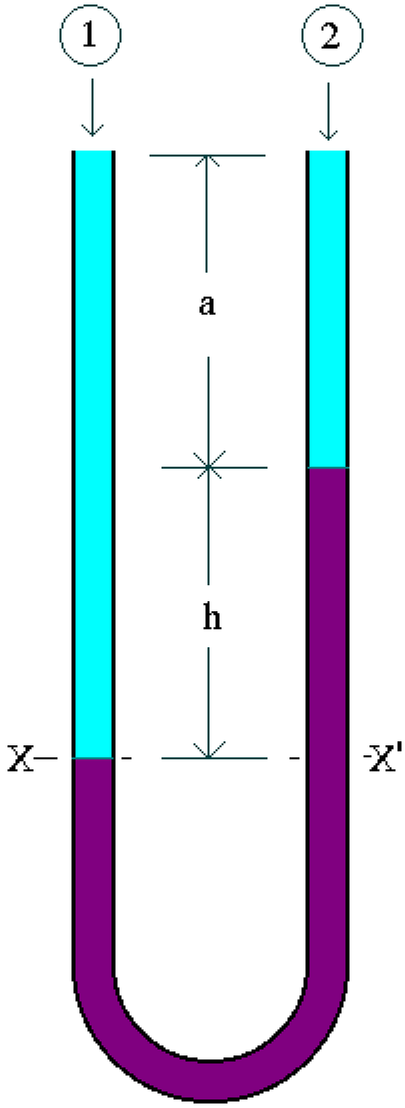
In using a manometer, generally a known pressure (which may be atmospheric) is applied to one end of the manometer tube and the unknown pressure to be determined is applied to the other end.

In some cases, however, the difference between pressure at ends of the manometer tube is desired rather than the actual pressure at the either end. A manometer to determine this differential pressure is known as differential pressure manometer.

Manometers - Various forms

1. Simple U - tube Manometer
2. Inverted U - tube Manometer
3. U - tube with one leg enlarged
4. Two fluid U - tube Manometer
5. Inclined U - tube Manometer

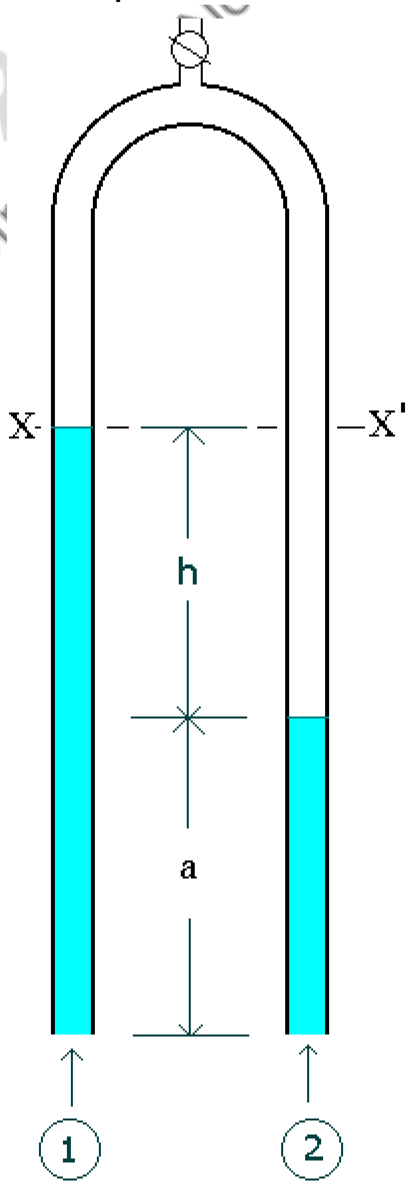
Simple U - tube Manometer



6. Equating the pressure at the level XX'(pressure at the same level in a continuous body of fluid is equal),
7. For the left hand side:
8. $P_x = P_1 + rg(a+h)$
9. For the right hand side:
10. $P_{x'} = P_2 + rga + r_m gh$
11. Since $P_x = P_{x'}$
12. $P_1 + rg(a+h) = P_2 + rga + r_m gh$
13. $P_1 - P_2 = r_m gh - rgh$
14. i.e. $P_1 - P_2 = (r_m - r)gh$.

The maximum value of $P_1 - P_2$ is limited by the height of the manometer. To measure larger pressure differences we can choose a manometer with heigher density, and to measure smaller pressure differences with accuracy we can choose a manometer fluid which is having a density closer to the fluid density.

Inverted U - tube Manometer



Inverted U-tube manometer is used for measuring pressure above the liquid in the be admitted or expelled to adjust the level of the

differences in liquids. The space manometer is filled with air which can through the tap on the top, in order liquid in the manometer.

Equating the pressure at the level level in a continuous body of static

XX'(pressure at the same fluid is equal),

For the left hand side:

$P_x = P_1 - rg\ (h+a)$

For the right hand side:

$P_x' = P_2 - (rga + r_m gh)$

Since $P_x = P_{x'}$

$P_1 - rg(h+a) = P_2 - (rga + r_m gh)$

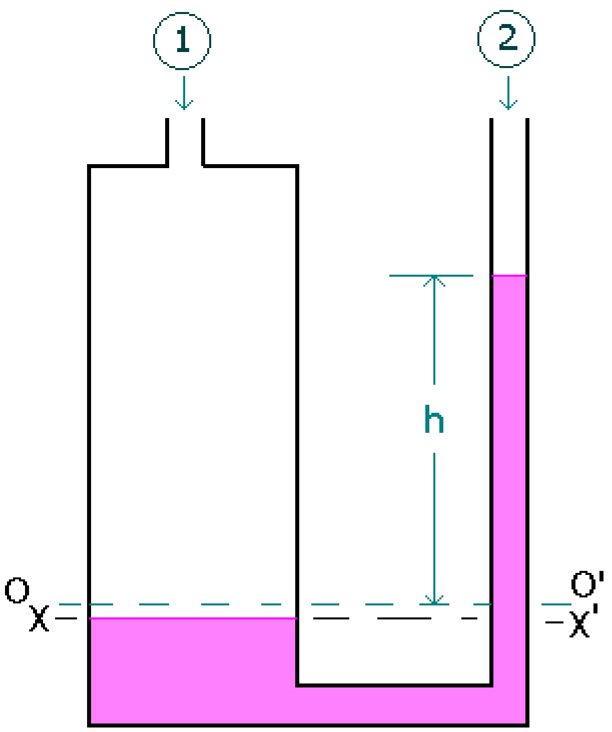
$P_1 - P_2 = (r - r_m)gh$

If the manometric fluid is choosen in such a way that $r_m << r$ then,

$P_1 - P_2 = rgh.$

For inverted U - tube manometer the manometric fluid is usually air

U - tube Manometer with one leg enlarged



Industrially, the simple U - tube manometer has the disadvantage that the movement of the liquid in both the limbs must be read. By making the diameter of one leg large as compared with the other, it is possible to make the movement the large leg very small, so that it is only necessary to read the movement of the liquid in the narrow leg.

In figure, OO' represents the level of liquid surface when the pressure difference $P_1 - P_2$ is zero. Then when pressure is applied, the level in the right hand limb will rise a distance h vertically.

Volume of liquid transferred from left-hand leg to right-hand leg

$= h(??4)d^2$

where d is the diameter of smaller diameter leg. If D is the diameter of larger diameter leg, then, fall in level of left-hand leg

= Volume transferred/Area of left-hand leg

$= (h(??4)d^2) / ((?/4)D^2)$

$= h(d/D)^2$

For the left-hand leg, pressure at X , i.e. $P_x = P_1 + ?g\ (h+a) + ?g\ h(d/D)^2$

For the right-hand leg, pressure at X', i.e. $P_{x'} = P_2 + \rho ga + \rho_m g(h + h(d/D)^2)$

For the equality of pressure at XX',

$P_1 + \rho g(h+a) + \rho g\, h(d/D)^2 = P_2 + \rho ga + \rho_m g(h + h(d/D)^2)$

$P_1 - P_2 = \rho_m g(h + h(d/D)^2) - \rho gh - \rho g\, h(d/D)^2$

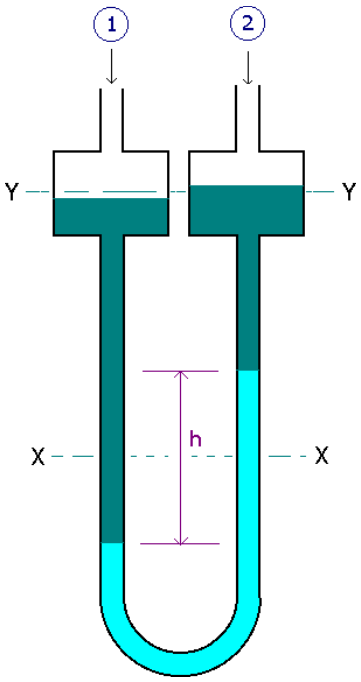
If D>>d then, the term $h(d/D)^2$ will be negligible(i.e approximately about zero)

Then $P_1 - P_2 = (\rho_m - \rho)gh$.

Where h is the manometer liquid rise in the right-hand leg.

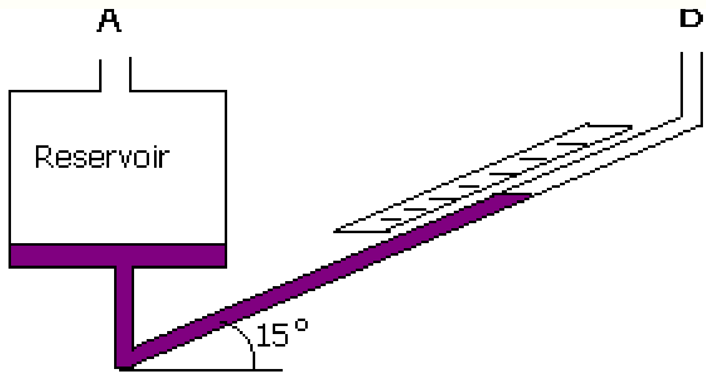
If the fluid density is negligible compared with the manometric fluid density (eg. the case for air as the fluid and water as manometric fluid), then $P_1 - P_2 = \rho_m\, gh$

Two fluid U-tube Manometer



Small differences in pressure in gases are often measured with a manometer of the form shown in the figure.

Inclined U



Manometers - Advantages and Limitations

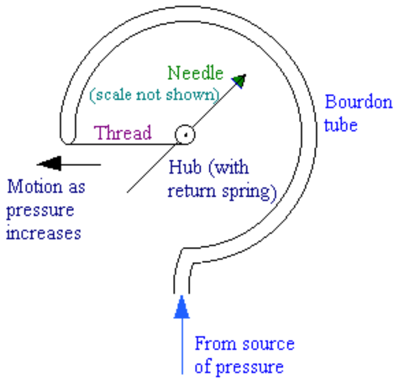
The manometer in its various forms is an extremely useful type of pressure measuring instrument, but suffers from a number of limitations.

- While it can be adapted to measure very small pressure differences, it can not be used conveniently for large pressure differences - although it is possible to connect a number of manometers in series and to use mercury as the manometric fluid to improve the range. (limitation)
- A manometer does not have to be calibrated against any standard; the pressure difference can be calculated from first principles. (Advantage)

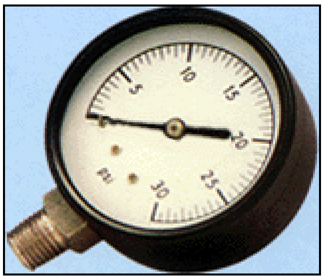
- Some liquids are unsuitable for use because they do not form well-defined menisci. Surface tension can also cause errors due to capillary rise; this can be avoided if the diameters of the tubes are sufficiently large - preferably not less than 15 mm diameter. (limitation)
- A major disadvantage of the manometer is its slow response, which makes it unsuitable for measuring fluctuating pressures.(limitation)
- It is essential that the pipes connecting the manometer to the pipe or vessel containing the liquid under pressure should be filled with this liquid and there should be no air bubbles in the liquid.(important point to be kept in mind)

Pressure Gauges

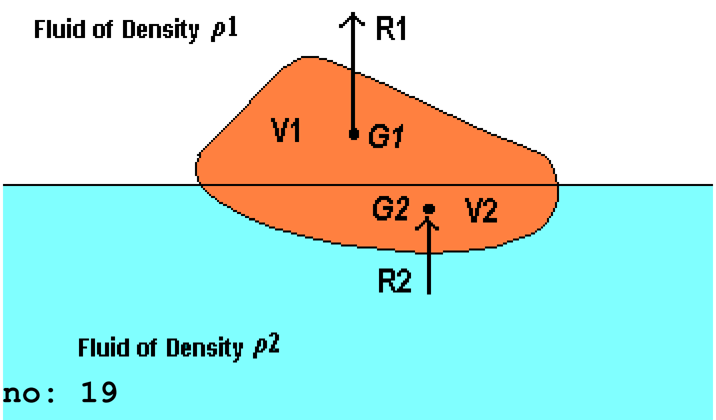
Bourdon Gauge:-



The pressure to be measured is applied to a curved tube, oval in cross section. Pressure applied to the tube tends to cause the tube to straighten out, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for steam and compressed gases. The pressure indicated is the difference between that communicated by the system to the external (ambient) pressure, and is usually referred to as the gauge pressure.



Buoyancy



Upthrust on body = weight of fluid displaced by the body
This is known as Archimedes principle.

If the body is immersed so that part of its volume V1 is immersed in a fluid of

**density ρ_1 and the rest of its volume V2 in another immiscible fluid of mass density ρ_2,
Upthrust on upper part, R1 = $\rho_1 g V1$ acting through G1, the centroid of V1,
Upthrust on lower part,R2 = $\rho_2 g V2$ acting through G2, the centroid of V2,**

Total upthrust = $\rho_1 g V1$ + $\rho_2 g V2$.

The positions of G1 and G2 are not necessarily on the same vertical line, and the centre of buoyancy of the whole body is, therefore, not bound to pass through the centroid of the whole body.

UNIT-2

Kinematics of Flow : Types of flow-ideal & real , steady & unsteady, uniform & non uniform, one, two and three dimensional flow, path lines, streak lines, streamlines and stream tubes; continuity equation for one and three dimensional flow, rotational & irrotational flow, circulation, stagnation point, separation of flow, sources & sinks, velocity potential, stream function, flow nets- their utility & method of drawing flow nets

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1. Classification of Fluid

a) Ideal fluid:

It is hypothetical which represents frictionless flow i.e. fluid without any viscosity. It is also called in viscid fluid. In ideal fluid the internal forces at any internal section are always normal to the section, even during motion. Hence the forces are purely pressure forces.

b) Real fluid:

In a real fluid tangential or shearing force always come into being whenever the motion takes place, thus giving the rise to fluid friction, also known as viscosity.

2.

a) Compressible fluid:

Fluids which will change in volume with the application of force are known as compressible fluid. Gases are compressible.

b) Incompressible fluid:

It implies fluids with constant density i.e. it will not change in volume with the application of force. Though liquids are slightly compressible they are usually assumed to be incompressible.

3.

a) Newtonian fluid: A Newtonian fluid is a fluid whose stress versus strain rate curve is linear and passes through the origin. The constant of proportionality is known as the viscosity.

A simple equation to describe Newtonian fluid behavior is

$$\tau = \mu \frac{du}{dy}$$

Where

t is the shear stress exerted by the fluid [Pa]

μ is the fluid viscosity - a constant of proportionality [Pa·s]

$\frac{du}{dy}$ **is the velocity gradient perpendicular to the direction of shear [s-1]**

For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure (and also the chemical composition of the fluid if the fluid is not a pure substance), not on the forces acting upon it.

b) Non-Newtonian fluid: A non-Newtonian fluid is a fluid whose flow properties are not described by a single constant value of viscosity. Many polymer solutions and molten polymers are non-Newtonian fluids. In a non-Newtonian fluid, the relation between the shear stress and the strain rate is nonlinear, and can even be time-dependent.

Viscosity:

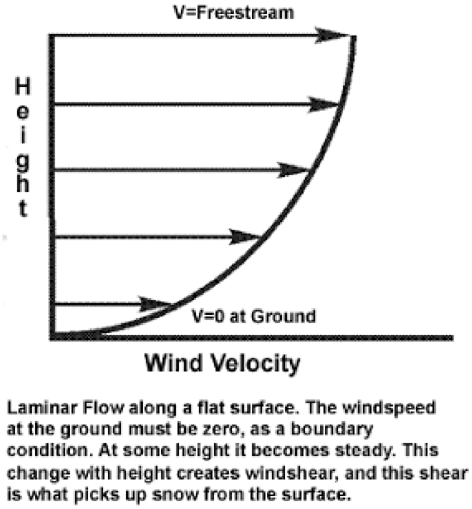
The viscosity of a fluid is a measure of its resistance to shear or angular deformation. The friction forces in fluid flow result from the cohesion &momentum interchange between molecules in fluid.

Viscosity depends on temperature. But this property is different for liquid & gas. As temperature increases, the viscosities of all liquids decrease, while the viscosities of all gases increase.

Classification of flow

Laminar /streamline/ viscous flow:

It occurs when a fluid flows in parallel layers, with no disruption between the layers. The fluid appears to move by the sliding of laminations of infinitesimal thickness relative to the adjacent layer.



b) Turbulent flow:

The main characteristics of turbulent flow is its irregularity, there being no definite frequency, as in wave action, and no observable pattern.

Steady flow:

A steady flow is one in which all conditions at any point in a stream remain constant with respect to time, but the conditions may be different at different points. True steady flow is found only in laminar flow.

Unsteady flow:

An unsteady flow is one in which all conditions at any point in a stream do not remain constant with respect to time.

Uniform flow:

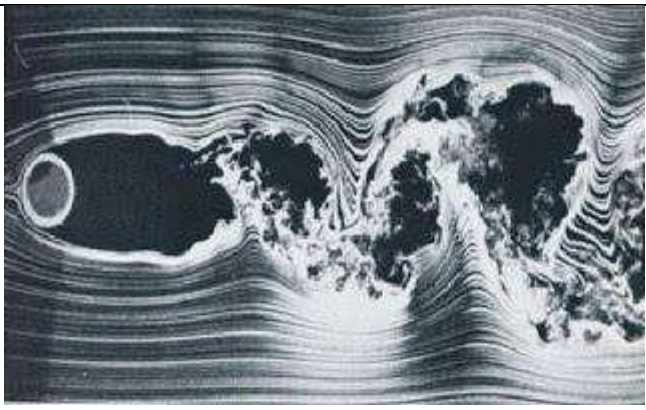
A truly uniform flow is one in which the velocity is the same in both magnitude direction at a given instant at every point in the fluid.

Non uniform flow:

A non-uniform flow is one in which the velocity is not same in magnitude or direction at a given instant at every point in the fluid.

a) Rotational flow:

It implies the flow where the fluid particles rotate about their own axis.



b) Irrigational flow:
It implies the flow where the fluid particles do not rotate about their axis, they only retain their orientation.

OTHER DEFINITIONS

Path lines:
It is the trace made by a single particle over a period of time.

Stream lines:
Streamlines show the mean directions of a number of particles at the same instant of time. Streamlines are a family of curves that are instantaneously tangent to the velocity vector of the flow.

Streak lines:
Streak lines are the locus of points of all the fluid particles that have passed continuously through a particular spatial point in the past. Dye steadily injected into the fluid at a fixed point extends along a streak line. These can be thought of as a "recording" of the path a fluid element in the flow takes over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time.

Elementary flow patterns
Recall the discussion of flow patterns in Chapter 1. The equations for particle paths in a three-dimensional, steady fluid flow are

$$\frac{dx}{dt} = U(\bar{x}) \qquad \frac{dy}{dt} = V(\bar{x}) \qquad \frac{dz}{dt} = W(\bar{x}).$$

Although the position of a particle depends on time as it moves with the flow, the flow pattern itself does not depend on time and the system (4.1) is said to be autonomous. Autonomous systems of deferential equations arise in a vast variety of applications in mechanics, from the motions of the planets to the dynamics of pendulums to velocity vector fields in steady fluid flow. A great deal about the flow can be learned by plotting the velocity vector field Ui (¯x). When the flow pattern is plotted one notices that among the most prominent features are stagnation points also known as critical points that occur where

Ui (¯xc) = 0.

Quite often the qualitative features of the flow can be almost completely described once the critical points of the flow field have been identified and classified.

<u>Stagnation point</u>

In fluid dynamics, a stagnation point is a point in a flow field where the local velocity of the fluid is zero. Stagnation points exist at the surface of objects in the flow field, where the fluid is brought to rest by the object. The Bernoulli equation shows that the static pressure is highest when the velocity is zero and hence static pressure is at its maximum value at stagnation points. This static pressure is called the stagnation pressure.

The Bernoulli equation applicable to incompressible flow shows that the stagnation pressure is equal to the dynamic pressure plus static pressure. Total pressure is also equal to dynamic pressure plus static pressure so, in incompressible flows, stagnation pressure is equal to total pressure. (In compressible flows, stagnation pressure is also equal to total pressure providing the fluid entering the stagnation point is brought to rest isentropic ally.)

<u>Separation of Flow</u>

Pressure gradient is one of the factors that influences a flow immensely. It is easy to see that the shear stress caused by viscosity has a retarding effect upon the flow. This effect can however be overcome if there is a negative pressure gradient offered to the flow. A negative pressure gradient is termed a Favorable pressure gradient. Such a gradient enables the flow. A positive pressure gradient has the opposite effect and is termed the Adverse Pressure Gradient. Fluid might find it difficult to negotiate an adverse pressure gradient. Sometimes, we say the fluid has to climb the pressure hill.

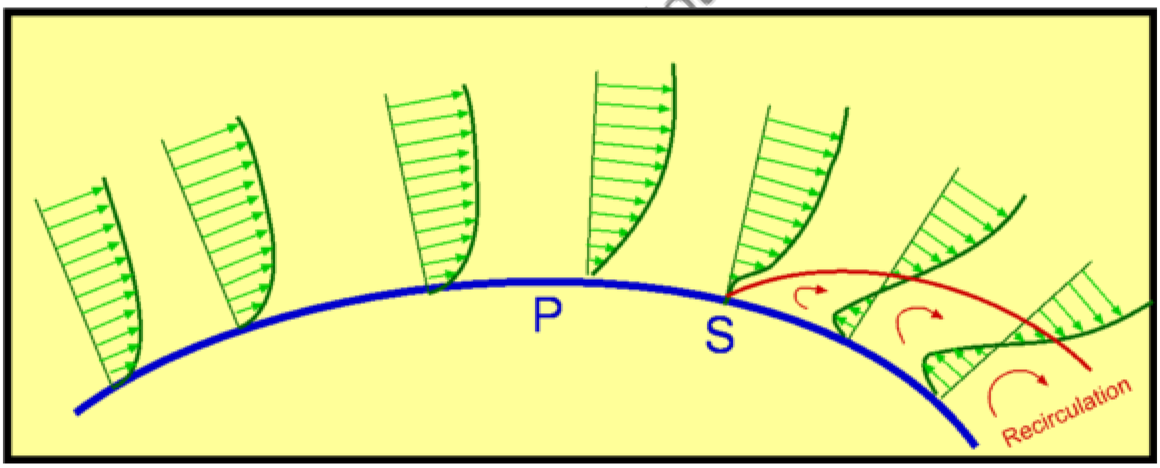


Figure : Separation of flow over a curved surface

One of the severe effects of an adverse pressure gradient is to separate the flow. Consider flow past a curved surface as shown in Fig.6.4. The geometry of the surface is such that we have a favorable gradient in pressure to start with and up to a point P. The negative pressure gradient will counteract the retarding effect of the shear stress (which is due to viscosity) in the boundary layer. For the geometry considered we have an adverse pressure gradient downstream of P.

Now the adverse pressure gradient begins to retard. This effect is felt more strongly in the regions close to the wall where the momentum is lower than in the regions near the free stream. As indicated in the figure, the velocity near the wall reduces and the boundary layer thickens. A continuous retardation of flow brings the wall shear stress at the point S on the wall to zero. From this point Onwards the shear stress becomes negative and the flow reverses and a region of recirculating flow develops. We see that the flow no longer follows the contour of the body. We say that the flow has separated. The point S where the shear stress is zero is called the Point of Separation.

Depending on the flow conditions the recirculating flow terminate and the flow may become reattached to the body. A separation bubble is formed. There are a variety of factors that could influence this reattachment. The pressure gradient may be now favorable due to body geometry and

other reasons. The other factor is that the flow initially laminar may undergo transition within the bubble and may become turbulent. A turbulent flow has more energy and momentum than a laminar flow. This can kill separation and the flow may reattach. A short bubble may not be of much consequence.

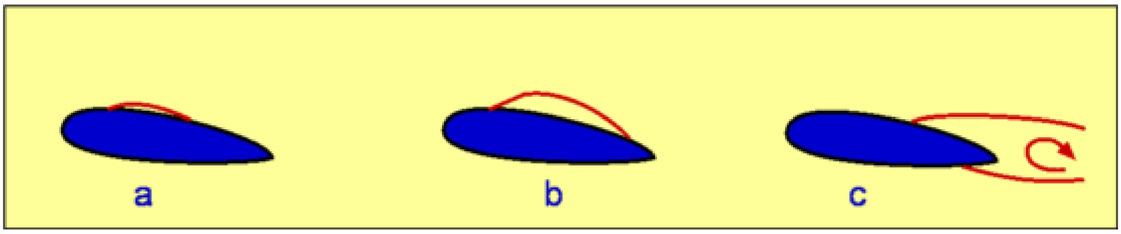


Figure: Separation bubble over an aero foil

On aero foils sometimes the separation occurs near the leading edge and gives rise to a short bubble. What can be dangerous is the separation occurring more towards the trailing edge and the flow not reattaching. In this situation the separated region merges with the wake and may result in stall of the aero foil (loss of lift).

Sources and Sinks

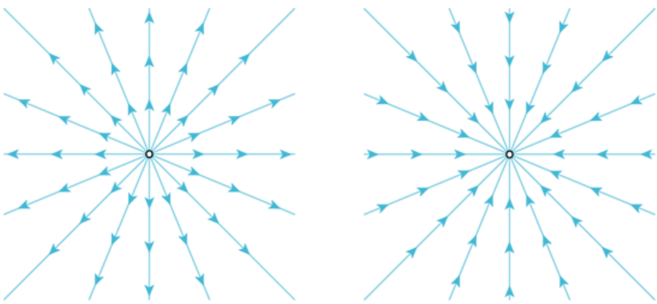
If the two-dimensional motion of an ideal fluid consists of an outward radial flow from a point and is symmetrical in all directions, then the point is called a simple source. A source at the origin can be considered as a line perpendicular to the z plane along which fluid is being emitted. If the rate of emission of volume of fluid per unit length 2π, then the origin is said to be a source of strength m, the complex potential for the flow is

$$F(z) = m \log(z),$$

and the velocity $\mathbf{V}$ at the point (x, y) is given by

$$\mathbf{V}(x, y) = \overline{F'(z)} = \frac{m}{\overline{z}}.$$

For fluid flows, a sink is a negative source and is a point of inward radial flow at which the fluid is considered to be absorbed or annihilated. Sources and sinks for flows are illustrated in Figure.



(a) A source at the origin. (b) A sink at the origin.

Fig. : Sources and sinks for an ideal fluid.

Stream Function

The idea of introducing stream function works only if the continuity equation is reduced to two terms. There are 4-terms in the continuity equation that one can get by expanding the vector equation (3.3.1) i.e.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

For a steady, incompressible, plane, two-dimensional flow, this equation reduces to,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Here, the striking idea of stream function works that will eliminate two velocity components *u and v*into a single variable (Fig. 3.3.1-a). So, the *stream function* $\{\psi(x,y)\}$ relates to the velocity components in such a way that continuity equation is satisfied.

$$u=\frac{\partial \psi}{\partial y};\ v=-\frac{\partial \psi}{\partial x}$$

$$\text{or}, \vec{V}=\frac{\partial \psi}{\partial y}\hat{i}-\frac{\partial \psi}{\partial x}\hat{j}$$

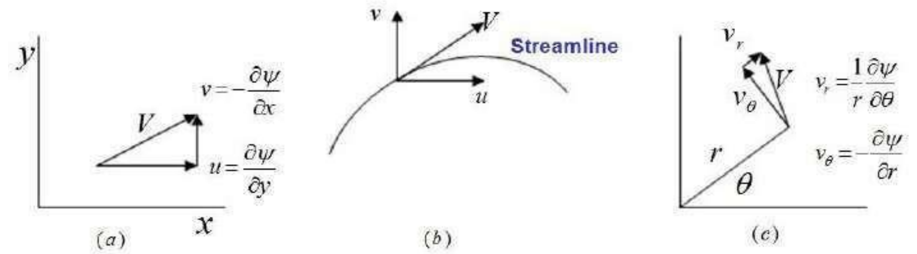


Fig.: Velocity components along a streamline.

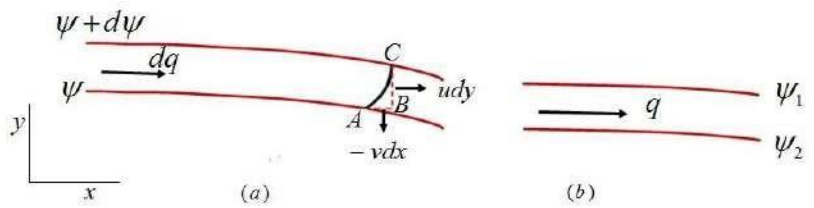
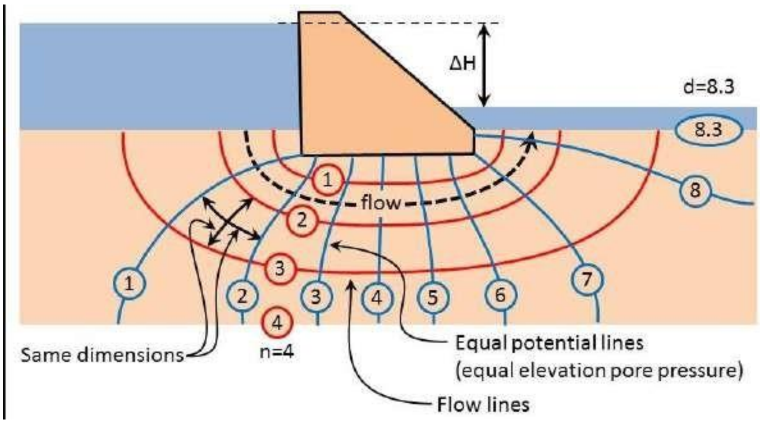


Fig.: Flow between two streamlines.

FLOWNET

A flownet is a grid obtained by drawing a series of streamlines and equipotential lines are known as a flow net.Equi-Potential Line is an imaginary line in a field of flow such that the total head is the same for all points on the line, and therefore the direction of flow is perpendicular to the line at all points.



The graphical properties of a flow net can be used in obtaining solutions for many seepage problems such as:

1. ***Estimation of seepage losses from reservoirs:*** It is possible to use the flow net in the transformed space to calculate the flow underneath the dam.

2. ***Determination of uplift pressures below dams:*** From the flow net, the pressure head at any point at the base of the dam can be determined. The uplift pressure distribution along the base can be drawn and then summed up.

3. ***Checking the possibility of piping beneath dams:*** At the toe of a dam when the upward exit hydraulic gradient approaches unity, boiling condition can occur leading to erosion in soil and consequent piping. Many dams on soil foundations have failed because of a sudden formation of a piped shaped discharge channel. As the stored water rushes out, the channel widens and catastrophic failure results. This is also often referred to as piping failure.

METHOD OF DRAWING FLOW NETS

1. **Hydraulic models:**

a. Streamlines can be traced by injecting a dye in a seepage model or Heleshaw apparatus.
b. They by drawing equipotential lines the flow net is completed.

2. **Analytical Method:**

a. It is only applied to problems with simple and ideal boundaries conditions.
b. The equation corresponding curve ǿ and Ψ are first obtained and the same are plotted to give the flow net pattern for the flow of fluid between the given boundary shape.

3. **Electrical Analogy Method:**

a) This method based on the fact that the flow of fluids and flow of electricity through a conductor are analogues. These two systems are similar in the respect that electric potential is analogues to the velocity potential. The electric current is analogues to the velocity of flow and the homogeneous conductor is analogues to the homogeneous fluid. This method only for a practical method of drawing a flow net for a particular set of boundaries.

4. Graphical Method:

a) This method requires a lot of erasing to get the proper shape of a flow net and also consume a lots of time. A graphical consists of drawing steam lines and equipotential lines such that they cut orthogonally and form curvilinear squares.

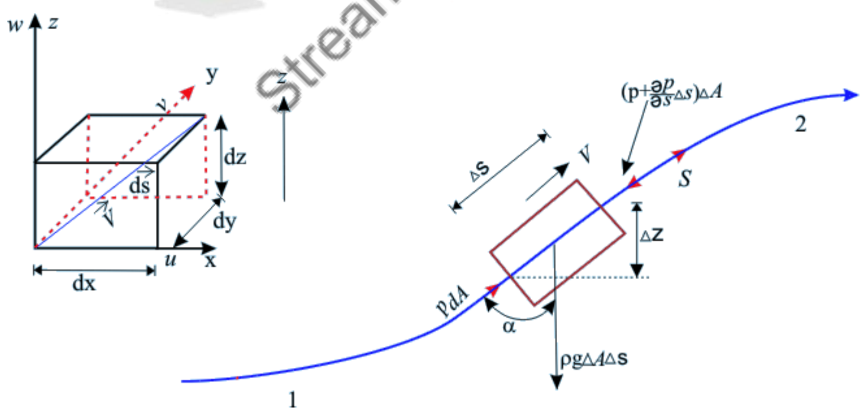
UNIT-3

Dynamics of Flow: Euler's equation of motion along a streamline and derivation of Bernoulli's equation, application of Bernoulli's equation, energy correction factor, linear momentum equation for steady flow; momentum correction factor. The moment of momentum equation, forces on fixed and moving vans and other applications. Fluid Measurements: Velocity Measurement (Pitot tube, prandtl tube, current meters etc.); flow measurement (orifices, nozzles, mouth pieces, orifice meter, nozzle meter, venturimeter, weirs and notches).

- **Euler's equation of motion along a streamline**

The Euler's equation for steady flow of an ideal fluid along a streamline is a relation between the velocity, pressure and density of a moving fluid. It is based on the Newton's Second Law of Motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit weight of the following fluid.

1. **It is based on the following assumptions:**
2. **The fluid is non-viscous (i,e., the frictional losses are zero).**
3. **The fluid is homogeneous and incompressible (i.e., mass density of the fluid is constant).**
4. **The flow is continuous, steady and along the streamline.**
5. **The velocity of the flow is uniform over the section.**
6. **No energy or force (except gravity and pressure forces) is involved in the flow.**



Derivation

Euler's equation along a streamline is derived by applying Newton's second law of motion to a fluid element moving along a streamline. Considering gravity as the only body force component acting vertically downward (Fig.), the net external force acting on the fluid element along the directions can be written as

$$F_s = -\frac{\partial p}{\partial s}\Delta s\Delta A - \rho\Delta s\Delta A g\cos\alpha \quad \text{------------------------3.1}$$

Where ΔA is the cross-sectional area of the fluid element. By the application of Newton's second law of motion in s direction, we get

$$\rho \Delta s \Delta A \frac{DV}{Dt} = -\frac{\partial p}{\partial s}\Delta s \Delta A - \rho \Delta s \Delta A g \cos\alpha$$ **------------------------------3.2**

From geometry we get

$$\cos\alpha = \lim_{\Delta s \to 0} \frac{\Delta z}{\Delta s} = \frac{dz}{ds}$$

Hence, the final form of Eq.

$$\rho \frac{DV}{Dt} = -\frac{\partial p}{\partial s} - \rho g \frac{dz}{ds}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{dz}{ds}$$ **--3.3**

Above Equation is the Euler's equation along a streamline.

Let us consider $d\vec{s}$ **along the streamline so that**

$$d\vec{s} = \vec{i}dx + \vec{j}dy + \vec{k}dz$$

Again, we can write from Fig.

$$\frac{dx}{ds} = \frac{u}{V}, \quad \frac{dy}{ds} = \frac{v}{V} \text{ and } \frac{dz}{ds} = \frac{w}{V}$$

The equation of a streamline is given by

$$\vec{V} \times d\vec{S} = 0$$

$$\begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$

or, which finally leads to

$$udy = vdx; \qquad udz = wdx; \qquad vdz = wdy$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X_x - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = X_y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = X_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

Multiplying Eqs above by dx, dy and dz respectively and then substituting the above mentioned equalities, we get

$$\rho\left(u\frac{\partial u}{\partial t}\cdot\frac{ds}{V}+u\frac{\partial u}{\partial x}dx+u\frac{\partial u}{\partial y}dy+u\frac{\partial u}{\partial z}dz\right)=-\frac{\partial p}{\partial x}dx+X_x dx$$

$$\rho\left(v\frac{\partial v}{\partial t}\cdot\frac{ds}{V}+v\frac{\partial v}{\partial x}dx+v\frac{\partial v}{\partial y}dy+v\frac{\partial v}{\partial z}dz\right)=-\frac{\partial p}{\partial y}dy+X_y dy$$

$$\rho\left(w\frac{\partial w}{\partial t}\cdot\frac{ds}{V}+w\frac{\partial w}{\partial x}dx+w\frac{\partial w}{\partial y}dy+w\frac{\partial w}{\partial z}dz\right)=-\frac{\partial p}{\partial z}dy+X_z dz$$

Adding these three equations, we can write

$$\rho\left(\frac{ds}{V}\cdot\frac{\partial}{\partial t}\left(\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}\right)+\frac{\partial}{\partial x}\left(\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}\right)dx+\frac{\partial}{\partial y}\left(\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}\right)dy+\frac{\partial}{\partial z}\left(\frac{u^2}{2}+\frac{v^2}{2}+\frac{w^2}{2}\right)dz\right)$$

$$=\rho\left(\frac{ds}{V}\cdot\frac{\partial}{\partial t}(\frac{V^2}{2})+\frac{\partial}{\partial x}\left(\frac{V^2}{2}\right)dx+\frac{\partial}{\partial y}\left(\frac{V^2}{2}\right)dy+\frac{\partial}{\partial z}\left(\frac{V^2}{2}\right)dz\right)$$

$$=\rho\left[\frac{\partial V}{\partial t}+V(\frac{\partial V}{\partial x}\cdot\frac{dx}{ds}+\frac{\partial V}{\partial y}\cdot\frac{dy}{ds}+\frac{\partial V}{\partial z}\cdot\frac{dz}{ds})\right]=-\left(\frac{\partial p}{\partial x}\cdot\frac{dx}{ds}+\frac{\partial p}{\partial y}\cdot\frac{dy}{ds}+\frac{\partial p}{\partial z}\cdot\frac{dz}{ds}\right)-\rho g\frac{dz}{ds}$$

Hence, $$\boxed{\rho\left[\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial s}\right]=-\frac{\partial p}{\partial s}-\rho g\frac{dz}{ds}}$$

This is the more popular form of Euler's equation because the velocity vector in a flow field is always directed along the streamline.

Bernoulli's Equation

Energy Equation of an ideal Flow along a Streamline

Euler's equation (the equation of motion of an inviscid fluid) along a stream line for a steady flow with gravity as the only body force can be written as

$$V\frac{dV}{ds}=-\frac{1}{\rho}\frac{dp}{ds}-g\frac{dz}{ds}$$

Application of a force through a distance ds along the streamline would physically imply work interaction. Therefore an equation for conservation of energy along a streamline can be obtained by integrating the Eq. (13.6) with respect to ds as

$$\int V \frac{dV}{ds} ds = -\int \frac{1}{\rho} \frac{dp}{ds} ds - \int g \frac{dz}{ds} ds$$

$$\text{or,} \qquad \frac{V^2}{2} + \int \frac{dp}{\rho} + gz = C$$

Where C is a constant along a streamline. In case of an incompressible flow, Eq. (13.7) can be written as

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = C$$

The Eqs (13.7) and (13.8) are based on the assumption that no work or heat interaction between a fluid element and the surrounding takes place. The first term of the Eq. (13.8) represents the flow work per unit mass, the second term represents the kinetic energy per unit mass and the third term represents the potential energy per unit mass. Therefore the sum of three terms in the left hand side of Eq. (13.8) can be considered as the total mechanical energy per unit mass which remains constant along a streamline for a steady inviscid and incompressible flow of fluid. Hence the Eq. (13.8) is also known as Mechanical energy equation.

This equation was developed first by Daniel Bernoulli in 1738 and is therefore referred to as Bernoulli's equation. Each term in the Eq. (13.8) has the dimension of energy per unit mass. The equation can also be expressed in terms of energy per unit weight as

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = C_1 \text{(constant)}$$

In a fluid flow, the energy per unit weight is termed as head. Accordingly, equation 13.9 can be interpreted as

Pressure head + Velocity head + Potential head =Total head (total energy per unit weight).

Bernoulli's Equation with Head Loss

The derivation of mechanical energy equation for a real fluid depends much on the information about the frictional work done by a moving fluid element and is excluded from the scope of the book. However, in many practical situations, problems related to real fluids can be analysed with the help of a modified form of Bernoulli's equation as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

Where, hf represents the frictional work done (the work done against the fluid friction) per unit weight of a fluid element while moving from a station 1 to 2 along a streamline in the direction of flow. The term hf is usually referred to as head loss between 1 and 2, since it amounts to the loss in total mechanical energy per unit weight between points 1 and 2 on a streamline due to the effect of fluid friction or viscosity. It physically signifies that the difference in the total mechanical energy between stations 1 and 2 is dissipated into intermolecular or thermal energy and is expressed as loss of head hf in

Eq. (13.10). The term head loss, is conventionally symbolized as hL instead of hf in dealing with practical problems. For an in viscid flow hL = 0, and the total mechanical energy is constant along a streamline.

PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation finds wide applications in all types of problems of incompressible flow where there is involvement of energy considerations. The other equation, which is commonly used in the solution of the problems of fluid flow, is the continuity equation. In this section, the applications of Bernoulli's equation and continuity equation will be discussed for the following measuring devices.

1. **Venturi meter**
2. **Nozzle**
3. **Orifice meter**
4. **Pitot tube**

ENERGY CORRECTION FACTOR

The energy equation for ideal flow has a kinetic energy term with a velocity square factor. We consider ideal flows to be in viscid. For in viscid flow shear effects are not present, hence the flow is uniform across the area of cross section of the flow. It means that the velocity is uniform or the same across the cross section of flow. Therefore the average velocity is equal to the velocity at any point on the section. The total kinetic energy at the section can be written for the ideal flow using the average velocity.

We prefer to use average velocity to calculate kinetic energy because it is easy to find average velocity. Average velocity at any cross section of flow is equal to the rate of flow divided by the area of flow. Energy Correction Factor

We have assumed in the derivation of Bernoulli equation that the velocity at the end sections (1) and (2) is uniform. But in a practical situation this may not be the case and the velocity can very across the cross section. A remedy is to use a correction factor for the kinetic energy term in the equation. If $\overline{V}$ is the average velocity at an end section then we can write for energy,

$$\int_A \frac{V^2}{2}\rho V.dA = \alpha \dot{m}\frac{\overline{V}^2}{2}$$

After simplification we find that

$$\alpha = \frac{1}{A} \int_A \left(\frac{u}{\overline{V}}\right)^3 dA$$

Consequently, Eqn.3.70 (Low Speed Application) is written as

$$\left(\frac{p}{\gamma} + z + \alpha_1 \frac{V^2}{2g}\right)_1 = \left(\frac{p}{\gamma} + z + \alpha_2 \frac{V^2}{2g}\right)_2 + h_{friction} - h_{pump} + h_{turbine}$$

Where $\alpha$is the Kinetic Energy Factor? Its value for a fully developed laminar pipe flow is around 2, whereas for a turbulent pipe flow it is between 1.04-1.11. It is usual to take it is 1 for a turbulent flow. It should not be neglected for a laminar flow.

Moment of momentum equation

Moment of momentum equation is derived from moment of momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of chance of moment of momentum.

Let V1V1 = velocity of fluid at section 1

r1r1 = radius of curvature at section 1

Q = rate of flow of fluid

ρρ = density of fluid

and V2V2 and r2r2 = velocity and radius of curvature at section 2

Momentum of fluid at section 1 = mass x velocity = PQ×V1/sPQ×V1/s

Moment of momentum per second at section 1 = PQ×V2×r2PQ×V2×r2

Rate of change of moment of momentum

= PQV2r2−PQV1r1=PQ[v2r2−v1r1]PQV2r2−PQV1r1=PQ[v2r2−v1r1]

According to moment of momentum principle

Resultant torque = rate of change of moment of momentum

T=PQ[V2r2−V1r1]T=PQ[V2r2−V1r1] ----- (1)

Eq (1) is known as moment of momentum equation.

Analysis Of Finite Control Volumes - the application of momentum theorem

We'll see the application of momentum theorem in some practical cases of inertial and non-inertial control volumes.

Inertial Control Volumes

Applications of momentum theorem for an inertial control volume are described with reference to three distinct types of practical problems, namely

Forces acting due to internal flows through expanding or reducing pipe bends.

Forces on stationary and moving vanes due to impingement of fluid jets.

Jet propulsion of ship and aircraft moving with uniform velocity.

Non-inertial Control Volume

A good example of non-inertial control volume is a rocket engine which works on the principle of jet propulsion.

We shall discuss each example separately in the following slides.

Application of Moment of Momentum Theorem

Let us take an example of a sprinkler like turbine as shown in Fig. 12.2. The turbine rotates in a horizontal plane with angular velocity ω. The radius of the turbine is r. Water enters the turbine from a vertical pipe that is coaxial with the axis of rotation and exits through the nozzles of cross sectional area 'a' with a velocity Ve relative to the nozzle.

A control volume with its surface around the turbine is also shown in the fig below.

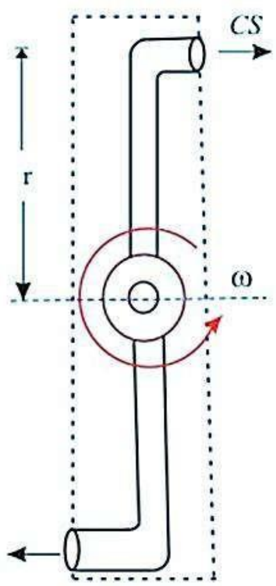


Fig 12.1 A Sprinkler like Turbine

Application of Moment of Momentum Theorem (Eq. 10.20b) gives

$$M_{zc} = \dot{m}\left(\vec{r} \times \vec{V}\right)$$

When Mzc is the moment applied to the control volume. The mass flow rate of water through the turbine is given by

$$\dot{m} = \rho 2 V_e a$$

The velocity must be referenced to an inertial frame so that

$$\vec{r} \times \vec{V} = -r\vec{i}_r \times \left(V_e - \omega r\right)\vec{i}_\theta = -r\left(V_e - \omega r\right)\vec{i}_z$$

$$M_{zc} = -\dot{m}r\left(V_e - \omega r\right)$$

The moment M_z acting on the turbine can be written as

$$M_z = -M_{zc} = \dot{m}r\left(V_e - \omega r\right)$$

The power produced by the turbine is given by

$$P = M_z \omega$$

VELOCITY MEASUREMENT

The methods of measuring the velocity of liquids or gases can be classified into three main groups: kinematic, dynamic and physical.

In kinematic measurements, a specific volume, usually very small, is somehow marked in the fluid stream and the motion of this volume (mark) is registries by appropriate instruments. Dynamic methods make use of the interaction between the flow and a measuring probe or between the flow and electric or magnetic fields. The interaction can be hydrodynamic, thermodynamic or magneto hydrodynamic.

For physical measurements, various natural or artificially organized physical processes in the flow area under study, whose characteristics depend on velocity, are monitored.

Kinematic Methods

The main advantage of kinematic methods of velocity measurements is their perfect character, and also their high space resolution. By these methods, we can find either the time the marked volume covers a given path, or the path length covered by it over a given time interval. The mark can differ from the surrounding fluid flow in temperature, density, charge, degree of ionization, luminous admittance, index of refraction, radioactivity, etc.

The marks can be created by impurities introduced into the fluid flow in small portions at regular intervals. The mark must follow the motion of the surrounding medium accurately. The motion of marks is distinguished by the method of their registration, into non optical and optical kinematic methods. In the probe (no optical) method which traces thermal non uniformities, a probe consisting of three filaments located in parallel plates (see Anemometers, Thermal) is used. The thermal trace is registered by two receiving wires located a distance 1 from the central wire. By registering the time Δt between the

pulse heat emitted from the central wire and the thermal response of the receiving wire, we can determine the velocity $u = l/\Delta t$. Depending on which receiving wire receives a thermal pulse, we can define the direction of flow.

Marks consisting of regions of increased ion content are also widely used. To create ion marks, a spark or a corona discharge or an optical breakdown under the action of high-pulse laser radiation is used. In tracing by radioactive isotopes, the marks are created by injecting radioactive substances into the fluid flow; the times of passing selected locations by the marks are registered with the help of ionizing-radiation detectors.

Optical kinematic methods use cine and still photography to follow the motion of marks. Three main types of photography are used: cine photography, still photography with stroboscopic lighting and photo tracing. In cine photography, to determine the velocity, successive frames are aligned and the distance between the corresponding positions of the mark is measured. In the stroboscopic visualization method, several positions of the mark are registered on a single frame (a discontinuous track), which correspond to its motion between successive light pulses. Two components of the instantaneous velocity vector are determined by the distance between the particle positions. Typical of the marks used are 3-5 mm aluminum powder particles or small bubbles of gas generated electrolytic ally in the circuit of the experimental plant. Of vital importance in this method is the accuracy of measurement of the time intervals between the flashes.

In the photo tracing method, the motion of the mark is recorded by projecting the image of the mark through a diaphragm (in the form of a thin slit oriented along the fluid flow) onto a film located on a drum rotating at a certain speed. The mark image leaves a trace on the film whose trajectory is determined by adding the two vectors: the vector of mark motion and the vector of film motion. The slope angle of a tangent to this trajectory is proportional to the velocity of mark motion. Further information on the photographic technique is given in the article on Tracer Methods.

Laser Doppler anemometers can also be classified as kinematic techniques (see Anemometers, Laser Doppler).

Dynamic Methods

Among the dynamic methods the most generally employed are, because of the simplicity of the corresponding instruments, the methods based on hydrodynamic interaction between the primary converter and the fluid flow. The Pitot tube is used most often (see Pitot tube) whose function is based on the velocity dependence of the stagnation pressure ahead of a blunt body placed in the flow.

The operating principle of fiber-optic velocity converters is based on the deflection of a sensing element, in the simplest case, made in the form of a cantilever beam of diameter D and length L and placed in the fluid flow between the receiving and sending light pipe, depends on the velocity of fluid flowing around it. The change in the amount of light supplied to a receiving light-pipe is measured by a photo detector.

The upper limit of the range of velocities measured u_{max} is limited by the value of $Re = u_{max} D/\nu \lll 50$ and the frequency response is limited by the natural frequency f_0 which depends on the material, diameter and length of the sensing element. However, by varying L and D, we can change the velocities over a wide range. Depending on the fluid in which measurements of umax are made, the dimensions of the sensing elements vary within the limits of 5 D 50 µm, 0.25 L 2.5 mm.

The tachometric methods use the kinetic energy of flow. Typical anemometers using this principle consist of a hydrometric current meter with several semi-spherical cups or an impeller with blades situated at an angle of attack to the direction of flow (see Anemometer, Vane).

What is a pitot tube?

Basically, a pitot tube is used in wind tunnel experiments and on airplanes to measure flow speed. It's a slender tube that has two holes on it. The front hole is placed in the airstream to measure what's called the stagnation pressure. The side hole measures the static pressure. By measuring the difference between these pressures, you get the dynamic pressure, which can be used to calculate airspeed.

On an airplane, the pitot tube can be mounted in a number of ways, including jutting out from the edge of the wing or sticking up from the fuselage.

Flow Through Orifices And Mouthpieces

An orifice is a small aperture through which the fluid passes. The thickness of an orifice in the direction of flow is very small in comparison to its other dimensions.

If a tank containing a liquid has a hole made on the side or base through which liquid flows, then such a hole may be termed as an orifice.The rate of flow of the liquid through such an orifice at a given time will depend partly on the shape, size and form of the orifice.

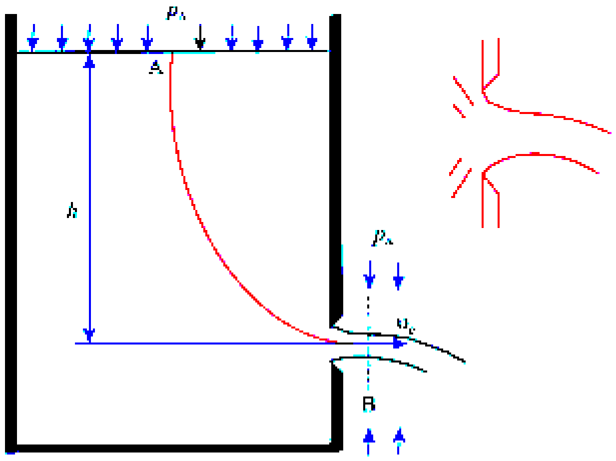
An orifice usually has a sharp edge so that there is minimum contact with the fluid and consequently minimum frictional resistance at the sides of the orifice. If a sharp edge is not provided, the flow depends on the thickness of the orifice and the roughness of its boundary surface too.

Flow from an Orifice at the Side of a Tank under a Constant Head

Let us consider a tank containing a liquid and with an orifice at its side wall as shown in Fig. 16.5. The orifice has a sharp edge with the bevelled side facing downstream. Let the height of the free surface of liquid above the centre line of the orifice be kept fixed by some adjustable arrangements of inflow to the tank.

The liquid issues from the orifice as a free jet under the influence of gravity only. The streamlines approaching the orifice converges towards it. Since an instantaneous change of direction is not possible, the streamlines continue to converge beyond the orifice until they become parallel at the Sec. c-c (Fig. 16.5).

For an ideal fluid, streamlines will strictly be parallel at an infinite distance, but however fluid friction in practice produce parallel flow at only a short distance from the orifice. The area of the jet at the Sec. c-c is lower than the area of the orifice. The Sec. c-c is known as the vena contracta.

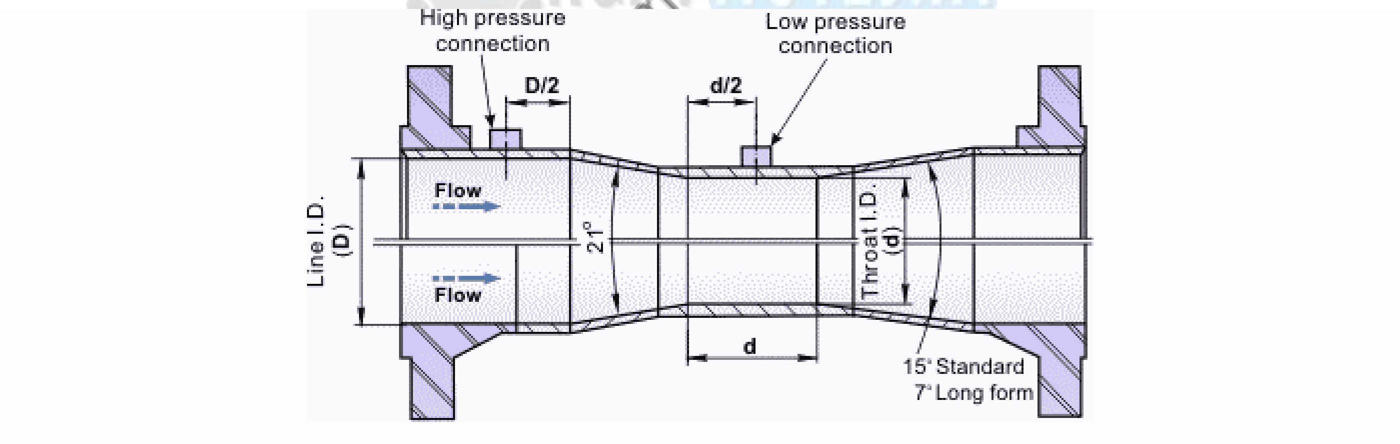


Flow from a Sharp edged Orifice

VENTURI METERS

Venturi meters are flow measurement instruments which use a converging section of pipe to give an increase in the flow velocity and a corresponding pressure drop from which the flow rate can be deduced. They have been in common use for many years, especially in the water supply industry.

The classical Venturi meter, whose use is described in ISO 5167-1: 1991, has the form shown in Figure 1



Classical Vetituri meter design.

For incompressible flow if the Bernoulli Equation is applied between two planes of the tappings,

(1)

$$p_1 + \tfrac{1}{2}\rho \bar{u}_1^2 = p_2 + \tfrac{1}{2}\rho \bar{u}_2^2$$

Where p, ρ and ū are the pressure, density and mean velocity and the subscripts $_1$ and $_2$ refer to the upstream and downstream (throat) tapping planes.

From continuity

(2)

$$\dot{V} = \tfrac{1}{4}\pi D^2 u_1 = \tfrac{1}{4}\pi d^2 u_2,$$

where $\dot{V}$ is the volumetric flow rate and D and d the pipe and throat diameters.

Combining Eqs. (1) and (2)

(3)

$$\dot{V} = \frac{\pi d^2}{4}\frac{1}{\sqrt{(1-\beta^4)}}\sqrt{\left\{\frac{2(p_1-p_2)}{\rho}\right\}},$$

where β is the diameter ratio, d/D. In reality, there is a small loss of total pressure, and the equation is multiplied by the discharge coefficient, C, to take this into account:

(4)

$$\dot{V} = C\frac{\pi d^2}{4}\frac{1}{\sqrt{(1-\beta^4)}}\sqrt{\left\{\frac{2\Delta p}{\rho}\right\}},$$

Where Δp is the differential pressure ($\equiv p_1 - p_2$). The discharge coefficient of a Venturi meter is typically 0.985, but may be even higher if the convergent section is machined. Discharge coefficients for uncelebrated Venturi meters, together with corresponding uncertainties, are given in ISO 5167-1: 1991.

If the fluid being metered is compressible, there will be a change in density when the pressure changes from p_1 to $p_2$on passing through the contraction. As the pressure changes quickly, it is assumed that no heat transfer occurs and because no work is done by or on the fluid, the expansion is isentropic. The expansion is almost entirely longitudinal and an expansibility factor, ε, can be calculated assuming one-dimensional flow of an ideal gas:

(5)

$$\varepsilon = \left(\left(\frac{\kappa\tau^{2/\kappa}}{\kappa-1}\right)\left(\frac{1-\beta^4}{1-\beta^4\tau^{2/\kappa}}\right)\left(\frac{1-\tau^{(\kappa-1)/\kappa}}{1-\tau}\right)\right)^{1/2}.$$

Where τ is the pressure ratio, p_2/p_1, and κ the isentropic exponent. The expansibility factor is applied to the flow equation in the same way as the discharge coefficient.

Various forms of construction of a Venturi meter are employed, depending on size, but all are considerably more expensive than the orifice plate. However, because most of the differential pressure is recovered by means of the divergent outlet section, the Venturi causes less overall pressure loss in a system and thus saves energy: the overall pressure loss is generally between 5 and 20 per cent of the measured differential pressure. The Venturi meter has an advantage over the orifice plate in that it does not have a sharp edge which can become rounded; however, the Venturi meter is more susceptible to errors due to burrs or deposits round the downstream (throat) tapping.

The lengths of straight pipe required upstream and downstream of a Venturi meter for accurate flow measurement are given in ISO 5167-1: 1991. These are shorter than those required for an orifice plate by a factor which can be as large as 9. However, Kochen et al. show that the minimum straight lengths between a single upstream 90° bend and a Venturi meter in the Standard are too short by a factor of about 3.