RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA, BHOPAL

New Scheme Based On AICTE Flexible Curricula

Electrical & Electronics Engineering, IV-Semester

EX402 Electrical Machine-I

Transformer-I: Working principle, e.mf. equation, construction, phasor diagrams, equivalent circuit, voltage regulation, losses, separation of hysteresis and eddy current losses, efficiency, tests: open circuit and short circuit, load, Sumpner's test, Condition for maximum efficiency and regulation, Power and distribution transformer, allday efficiency, Excitation phenomenon. Autotransformer: working, advantages, its equivalent circuit and phasor diagram.

Transformer-II: Three phase transformer: its construction, groups and connections, their working and applications; Scottconnection; Parallel operation of Transformers: application, advantages, requirement and load sharing; Tap changers, cooling, conservator and breather. Pulse and high frequency transformers.

Three phase Induction Motor- I: Working principle, construction, comparison of slip ring and squirrel cage motors, steady state analysis, phasor diagram and equivalent circuit, power flow diagram, torque-speed and power-speed characteristics, Losses and efficiency, No load and block rotor test, circle diagram

Three phase Induction Motor-II: Starting of squirrel cage and slip ring motors, power factor control, Cogging & Crawling, Double cage &Deep bar Indication Motor, impact of unbalanced supply and harmonics on performance, speed control, braking, Induction Generator. Applications

Single Phase Motors: Single Phase Induction motor; double revolving field theory, equivalent circuit and its determination, performance calculation, starting methods and types of single phase Induction motors: their working principle and applications, comparison with three phases Induction Motor. Single phase A.C. series motor, Servo motors, Linear Induction Motor

List of Experiments (expandable)

Experiments can cover any of the above topics, following is a suggestive list:

1. Perform turn ratio and polarity test on 1-phasetransformer
2. Perform load test on a 1-phase transformer and plot its loadcharacteristic
3. Perform OC and SC tests on a 1-phase transformer and determine its equivalent circuit. Also find its efficiency and regulation at different load and powerfactor.
4. Perform OC and SC tests on a 3-phase transformer and determine its equivalent circuit. Also find its efficiency and regulation at different load and powerfactor.
5. Perform Sumpner's test on two 1-phase transformer and determine its efficiency at variousload.
6. Perform No-load and block rotor test on a 3- phase IM and determine its equivalentcircuit.

7. Perform load test on a 3- phase IM and plot its performancecharacteristics.
8. Study various types of starters used for 3- IMs.
9. Perform No-load and block rotor test on a 1- phase IM and determine its equivalentcircuit.

TEXT BOOKS

1. Electrical Machines by Nagrath and Kothari, McGraw-Hill
2. P.S.Bimbhra, Electrical Machines,Khanna Publishers

REFERENCES

1.V.Del Toro, "Electrical Machines & Power Systems", 1985, Prentice-Hall, Inc., EnglewoodCliffs
2.S K Bhattacharya, Electrical Machines, McGraw-Hill
3. Ashfaq Hussain, Electrical Machines, Dhanpat Rai & Co
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5. Samarajit Ghosh, Electrical Machines, Pearson

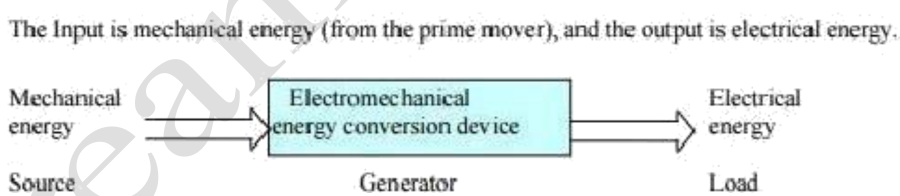
UNIT I
DC GENERATORS

Generators

There are two types of generators, one is ac generator and other is dc generator. Whatever may be the types of generators, it always converts mechanical power to electrical power. An ac generator produces alternating power.

A DC generator produces direct power. Both of these generators produce electrical power, based on same fundamental principle of Faraday's law of electromagnetic induction. According to these law, when an conductor moves in a magnetic field it cuts magnetic lines force, due to which an emf is induced in the conductor. The magnitude of this induced emf depends upon the rate of change of flux (magnetic line force) linkage with the conductor. This emf will cause an current to flow if the conductor circuit is closed. Hence the most basic two essential parts of a generator are

1. a magnetic field
2. Conductors which move inside that magnetic field.



Constructional Features

A DC generator has the following parts

1. Yoke
2. Pole of generator
3. Field winding
4. Armature of DC generator
5. Brushes of generator
6. Bearing

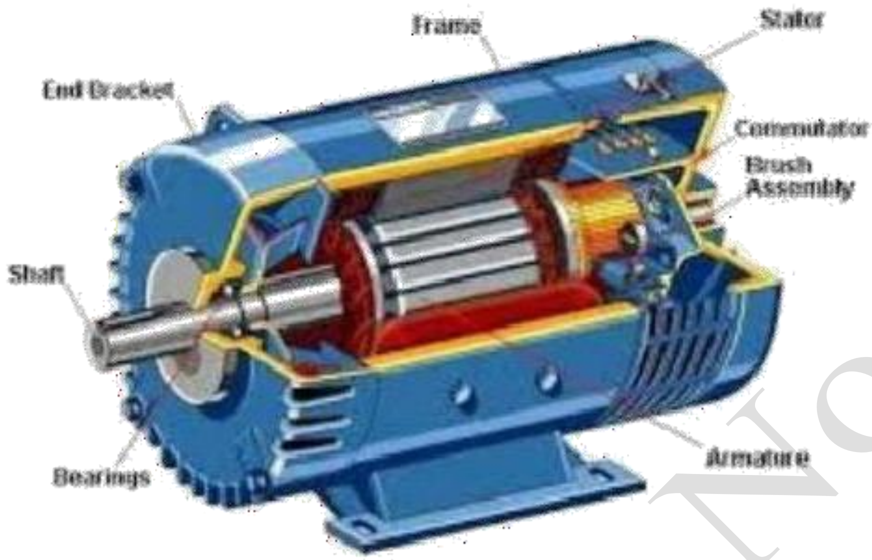


Figure: A Cut Away View of Practical DC Generator

Yoke of DC Generator

Yoke of DC generator serves two purposes,

1. It holds the magnetic pole cores of the generator and acts as cover of the generator.
2. It carries the magnetic field flux.

In small generator, yoke are made of cast iron. Cast iron is cheaper in cost but heavier than steel. But for large construction of DC generator, where weight of the machine is concerned, lighter cast steel or rolled steel is preferable for constructing yoke of DC generator. Normally larger yokes are formed by rounding a rectangular steel slab and the edges are welded together at the bottom. Then feet, terminal box and hangers are welded to the outer periphery of the yoke frame.

Armature Core of DC Generator

The purpose of armature core is to hold the armature winding and provide low reluctance path for the flux through the armature from N pole to S pole. Although a DC generator provides direct current but induced current in the armature is alternating in nature. That is why, cylindrical or drum shaped armature core is build up of circular laminated sheet. In every circular lamination, slots are either die - cut or punched on the outer periphery and the key way is located on the inner periphery as shown. Air ducts are also punched of cut on each lamination for circulation of air through the core for providing better cooling.

Armature Winding of DC Generator

Armature winding are generally formed wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots, which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in it and secured in place by special hard wooden or fiber wedges.

Commutator of DC Generator

The commutator plays a vital role in dc generator. It collects current from armature and sends it to the load as direct current. It actually takes alternating current from armature and converts it to direct current and then send it to external load. It is cylindrical structured and is build up of wedge - shaped segments of high conductivity, hard drawn or drop forged copper. Each segment is insulated from the shaft by means of insulated commutator segment shown below. Each commutator segment is connected with corresponding armature conductor through segment riser or lug.

Brushes of DC Generator

The brushes are made of carbon. These are rectangular block shaped. The only function of these carbon brushes of DC generator is to collect current from commutator segments. The brushes are housed in the rectangular box shaped brush holder. As shown in figure, the brush face is placed on the commutator segment with attached to the brush holder.

Bearing of DC Generator

For small machine, ball bearing is used and for heavy duty dc generator, roller bearing is used.

The bearing must always be lubricated properly for smooth operation and long life of generator.

Emf equation for dc generator

The derivation of EMF equation for DC generator has two parts:

1. Induced EMF of one conductor
2. Induced EMF of the generator

Derivation for Induced EMF of One Armature Conductor

For one revolution of the conductor,

Let,

Φ = Flux produced by each pole in weber (Wb) and P

= number of poles in the DC generator. therefore,

Total flux produced by all the poles = ø*p

And,

Time taken to complete one revolution = 60/N

Where,

N = speed of the armature conductor in rpm.

Now,according to Faraday's law of induction, the induced emf of the armature conductor is denoted by "e" which is equal to rate of cutting the flux.

Therefore,

$$e = \frac{d\phi}{dt} \ \ and\ e = \frac{total\ flux}{time\ take}$$

Induced emf of one conductor is

$$e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

Derivation for Induced EMF for DC Generator

Let us suppose there are Z total numbers of conductor in a generator, and arranged in such a manner that all parallel paths are always in series. Here,

Z = total numbers of conductor

A = number of parallel paths

Then,

Z/A = number of conductors connected in series

We know that induced emf in each path is same across the line Therefore,

Induced emf of DC generator

E = emf of one conductor × number of conductor connected in series.

Induced emf of DC generator is

$$e = \phi P \frac{N}{60} X \frac{Z}{A} \, volts$$

Simple wave wound generator

Numbers of parallel paths are only 2 = A

Therefore,

Induced emf for wave type of winding generator is

$$\frac{\phi PN}{60} X \frac{Z}{2} = \frac{\phi ZPN}{120} \, volts$$

Simple lap-wound generator

Here, number of parallel paths is equal to number of conductors in one path i.e. P = A

Therefore,

$$E = \frac{\phi ZN}{} X \frac{P}{}$$

Induced emf for lap-wound generator is

Methods Of Excitation

An electric generator or electric motor consists of a rotor spinning in a magnetic field. The magnetic field may be produced by permanent magnets or by field coils. In the case of a machine with field coils, a current must flow in the coils to generate the field, otherwise no power is transferred to or from the rotor. The process of generating a magnetic field by means of an electric current is called ***excitation.***

For a machine using field coils, which is most large generators, the field current must be supplied, otherwise the generator will be useless. Thus it is important to have a reliable supply. Although the output of a generator can be used once it starts up, it is also critical to be able to start the generators reliably. In any case, it is important to be able to control the field since this will maintain the system voltage.

Types of excitation

(1)seperately excited generator.

(2)self excited generator.

self generator is classified into 3 types.

1.shunt generator.

2.series generator.

3.compound generator.

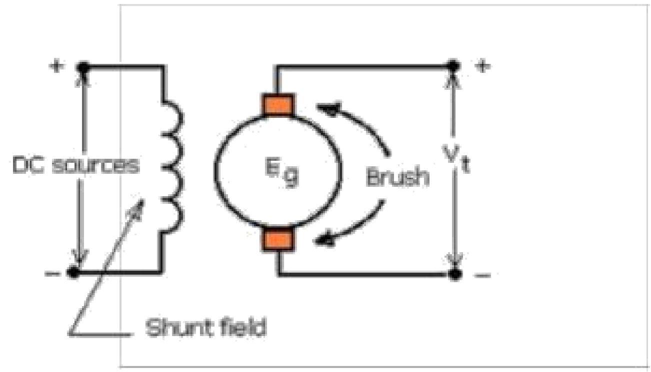
compoud generator is again classified into 2 types.

1.short shunt generator.

2.long shunt generator.

Separately excited generators.

These kind of generators has provided field exciter terminals which are external DC voltage source is supplies to produce separately magnetic field winding (shunt field) for magnetize of the generator as illustrated in figure as below.



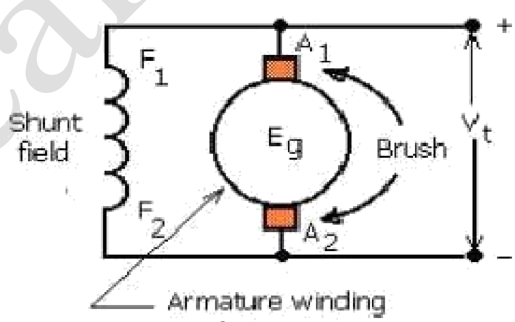
Separately excited generators.

Self excited field generators.

This type of generator has produced a magnetic field by itself without DC sources from an external. The electromotive force that produced by generator at armature winding is supply to a field winding (shunt field) instead of DC source from outside of the generator. Therefore, field winding is necessary connected to the armature winding. They may be further classified as

a) Shunt generator.

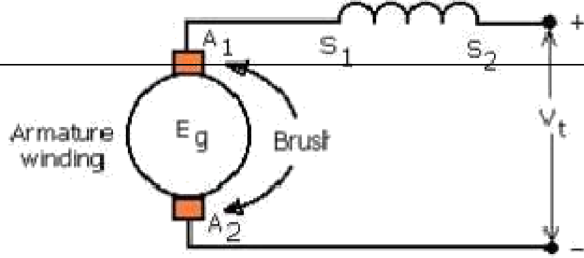
This generator, shunt field winding and armature winding are connected in parallel through commutator and carbon brush as illustrated in the figure below.



Shunt generator

b) Series generator

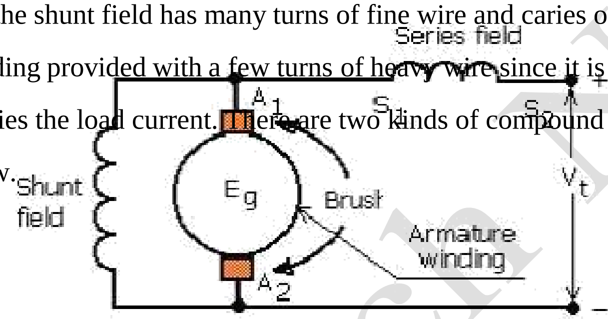
The field winding and armature winding is connected in series. There is different from shunt motor due to field winding is directly connected to the electric applications (load). Therefore, field winding conductor must be sized enough to carry the load current consumption and the basic circuit as illustrated below



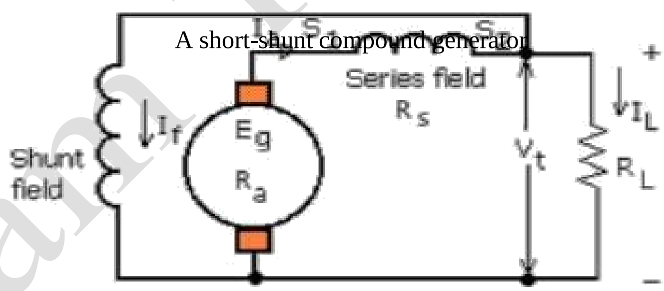
Series generator

c) Compound generator

The compound generator has provided with magnetic field in combine with excitation of shunt and series field winding, the shunt field has many turns of fine wire and caries of a small current, while the series field winding provided with a few turns of heavy wire since it is in series with an armature winding and caries the load current. There are two kinds of compound generator as illustrated in figures below.



A short-shunt compound generator



A long-shunt compound generator

Characteristic of separately excited generator

The generated electromotive force (EMF) is proportional to both of a magnetic density of flux per pole and the speed of the armature rotated as expression by the relation as following.

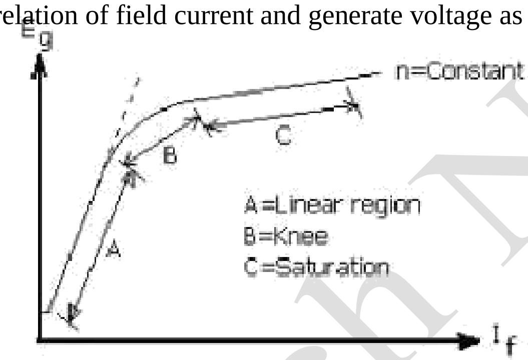
$$E_g = \kappa \varphi n$$

Where

K = Constant for a specific machine

φ = The density of flux per pole n
= Speed of the armature rotation
Eg = Generator voltage

By holding the armature speed (n) at a constant value it can show that generator voltage (Eg) is directly proportional to the magnetic flux density. Which, flux density is proportionately to the amount of field current (If). The relation of field current and generate voltage as impressed by figure .



The magnetization curve on open circuit of the separately excited dc machine.

From the figure when the field current (If) is become zero a small generate voltage is produce due to a residual magnetism.

As the field current increases cause to increase generated voltage linearly up to the knee of the magnetization curve. Beyond this point by increasing the field current still further causes saturation of the magnetic structure.

Generator voltage (Eg) is also directly to the armature speed. The formula and a magnetization curve can be both impressed about this relation.

$$E_g' = E_g \times \frac{n'}{n}$$

Where

Eg = Generator voltage or the value of EMF at speed n

Eg' = Generator voltage or the value of EMF at speed

n’ n = Speed of the generator armature (n’ ≠ n)

Voltage Regulation

When we add load on the generator, the terminal voltage will decrease due to

(a) The armature winding resistance is mainly of armature resistance. It is cause directly decrease in terminal voltage as following relation.

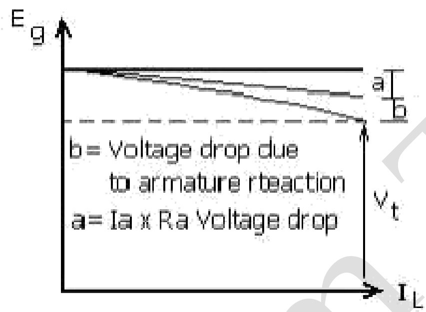
$$Vt = Eg - Ia\ Ra$$

Where,

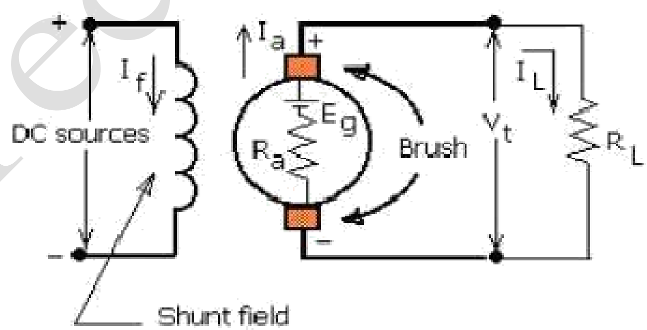
Vt = Terminal or output voltage

Ia = Armature current or load current

Ra = Armature resistance



(a) Load characteristic o generator



(b) Circuit diagram a separately excited DC

The decrease in magnetic flux due to armature reaction. The armature current establishes a magneto motive force (MMF), which it distorts to main flux, and makes result in weakened flux. We can put inter-pole between main field poles to reduce the armature reaction.

To have some measure by how much the terminal voltage change from no-load condition and on load condition, which is called "voltage regulation".

$$\text{Voltage regulation} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100 = \%$$

Where Vnl = No-load terminal voltage Vfl = Full-load terminal voltage

Remark:

A separately excited generator has disadvantage of requiring an external DC source. It is therefore used only where a wide range of terminal voltage required.

Example 2

The separately excited generator of example 1 is driven at revolving speed 1000 rpm and the field current is adjusted to 0.6 Amp. If the armature circuit resistance is 0.28 ohm, plot the output voltage as the load current is varied from 0 to 60 Amp. Neglect armature reaction effects. If the full-load current is 60 Amp, what is the voltage regulation?

Solution

From example 1, Eg = 153 volts when the field current is 0.6 Amp, which is the open circuit terminal voltage. When the generator is loaded, the terminal voltage is decreased by internal voltage drop,

namely.

$$V_t = E_g - I_a R_a$$

For a load current of, say 40 Amp.

$$V_t = 153 - (40 \times 0.28) = 141.80 \text{ Volts.}$$

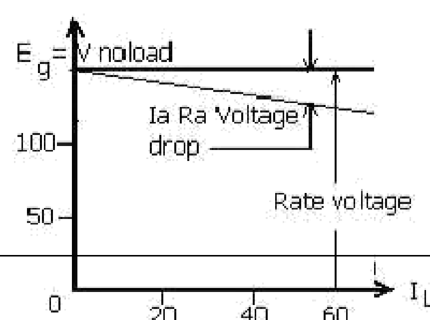
This calculation is for a number of load currents and the external characteristic can be plotted as show in fig. 10 at full load the terminal voltage.

$$V_t = 153 - (60 \times 0.28) = 136.20 \text{ Volts.}$$

Therefore the voltage regulation is

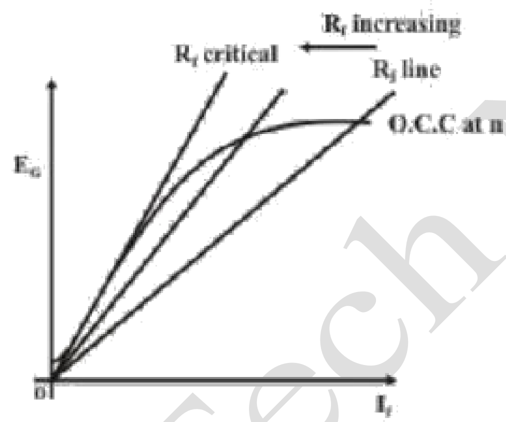
$$\text{Voltage regulation} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100 = \%$$

$$= \frac{153 - 136.2}{136.2} \times 100 = 12.3\,\%$$

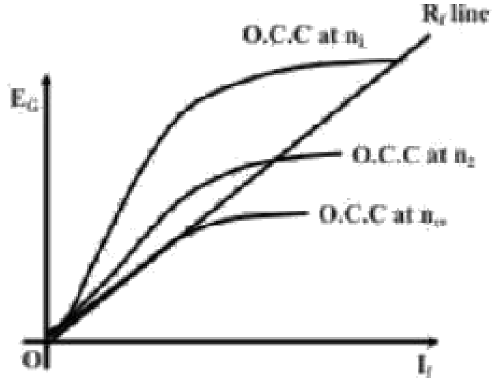


Critical Field Resistance And Critical Speed

The **critical field resistance** is the maximum field circuit resistance for a given speed with which the shunt generator would excite. The shunt generator will build up voltage only if field circuit resistance is less than critical field resistance. It is a tangent to the open circuit characteristics of the generator at a given speed.



Suppose a shunt generator has built up voltage at a certain speed. Now if the speed of the prime mover is reduced without changing Rf, the developed voltage will be less as because the O.C.C at lower speed will come down (refer to figure). If speed is further reduced to a certain critical speed (ncr), the present field resistance line will become tangential to the O.C.C at ncr. For any speed below ncr, no voltage built up is possible in a shunt generator.

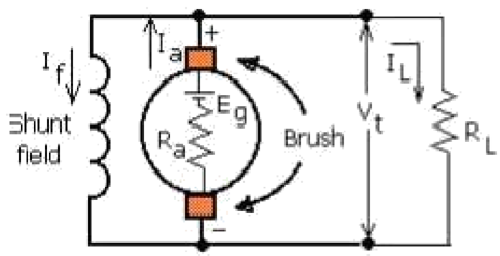


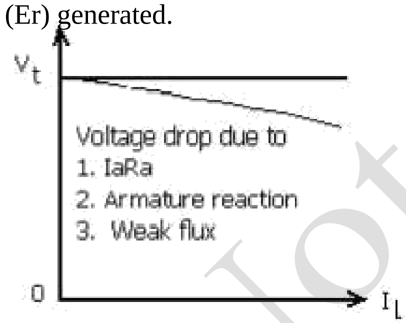
Critical Speed

Load characteristics

Self excited DC shunt generator

A shunt generator has its shunt field winding connected in parallel with the armature so that the machine provides it own excitation. For voltage to build up, there must be some residual magnetism in the field poles. There will be a small voltage (Er) generated.





(a) Shunt generator circuit (b) load characteristic of shunt generator

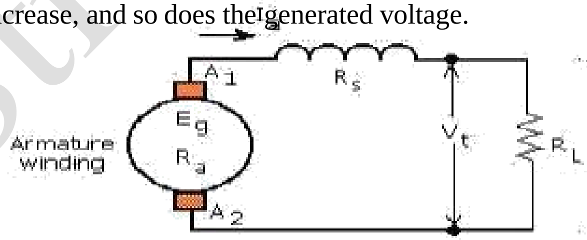
If the connection of the field and armature winding are such that the weak main pole flux aids to the residual flux, the induced voltage will become larger. Thus more voltage applied to the main field pole and cause to the terminal voltage increase rapidly to a large value. When we add load on the generator, the terminal voltage will decrease due to. The armature winding resistance

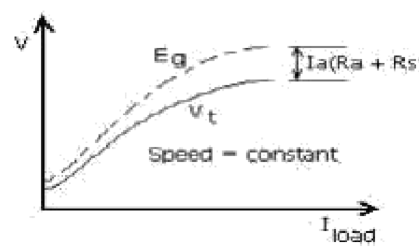
a) The armature reaction

c) The weakened flux due to the connection of the generator to aids or oppose to the residual

Series Generator

The field winding of a series generator is connect in series with the armature winding. Since it carries the load current, the series field winding consists of only a few turns of thick wire. At no-load, the generator voltage is small due to residual field flux only. When a load is added, the flux increase, and so does the generated voltage.





a) Circuit diagram of series generator (b) load characteristics

Figure shows the load characteristic of a series generator driven at a certain speed. The dash line indicated the generated EMF of the same machine with the armature opencircuited and the field separated excited. The different between the two curves is simply the voltage drop (IR) in the series field and armature winding.

$$V_t = Eg - Ia\ Ra + Rf$$

Where

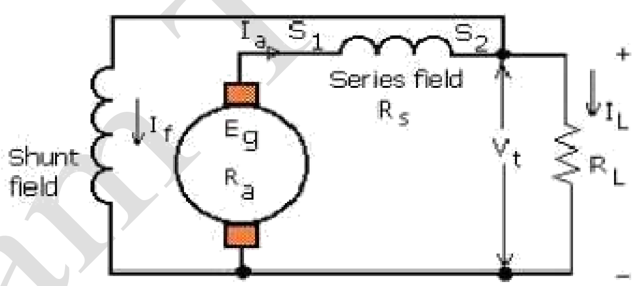
Rf = The series field winding resistance

Ra = The armature winding resistance

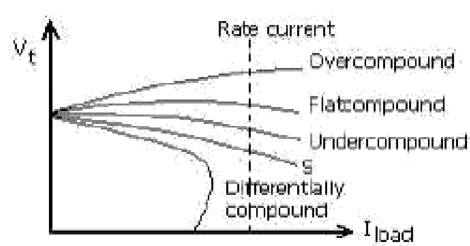
The series generators are obviously not suited for applications requiring good voltage regulation. Therefore, they have been used very little and only in special applications for example, as voltage booster. The generator is placed in series with a supply line. When the current consumption is increase, the generated voltage of the series machine goes up because the magnetic field current is increases.

Compound generator

The compound generator has both a shunt and a series winding. The series field winding usually wound on the top of a shunt field. The two winding are usually connected such that their ampere-turns act in the same direction. As such the generator is said to be cumulatively compound.



Simple circuit for compound generator



Terminal voltage characteristic of compound generator

(a) Curve s is represent the terminal voltage characteristic of shunt field winding alone. Under-compound, this condition the addition of series field winding too short it is cause the terminal voltage no rise to certain value and reduce while increasing in load current.

(b) Flat compound by increasing the number of a series field turns. It is cause to rise up in terminal voltage and when no-load and full load condition a terminal voltage is made nearly same value or equal.

(c) Over-compound, if the number of series field turns is more than necessary to compensated of the reduce voltage. In this case while a full load condition a terminal voltage is higher than a no-load voltage. Therefore over-compound generator may use where load is at some distance from generator. Voltage drop in the line has compensated by used of an over-compound generator.

(d) If a reversing the polarity of the series field occur this cause to the relation between series field and shunt field, the field will oppose to each other more and more as the load current increase. Therefore terminal voltage will drop, such generator is said to be a differentially compound.

The compound generator are used more extensively than the other type of dc generator because its design to have a wide variety of terminal voltage characteristics.

Machine Efficiency

The efficiency of any machine is the ratio of the ratio of the output power to the input power. The input power is provided by the prime mover to drive the generator. Because part of the energy delivered to the generator is converted into heat, it represents wasted energy. These losses are generally minimized in the design stage; however, some of these losses are unavoidable.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\% \quad \text{or}$$

$$\text{Efficiency} = \frac{\text{Output power} \times 100\%}{\text{Input power} + \text{losses}}$$

Losses of generator

The losses of generators may be classified as

1) Copper losses

The copper losses are present because of the resistance of the windings. Currents flowing through these windings create ohmic losses. The windings that may be present in addition to the (I2 R) armature winding are the field windings, inter-pole and compensate windings.

2) Iron losses

As the armature rotates in the magnetic field, the iron parts of the armature as well as the conductors cut the magnetic flux. Since iron is a good conductor of electricity, the EMF s induced in the iron parts courses to flow through these parts. These are the eddy currents. Another loss occurring in the iron is due to the Hysteresis loss is present in the armature core.

3) Other rotational losses consist of

3.1 bearing friction loss

3.2 friction of the rushes riding on the commutator

3.3 windage losses

Windage losses are those associated with overcoming air friction in setting up circulation currents of air inside the machine for cooling purposes. These losses are usually very small.

Applications Of Dc Generators

Applications of Separately Excited DC Generators

These types of DC generators are generally more expensive than self-excited DC generators because of their requirement of separate excitation source. Because of that their applications are restricted. They are generally used where the use of self-excited generators are unsatisfactory.

1. Because of their ability of giving wide range of voltage output, they are generally used for testing purpose in the laboratories.
2. Separately excited generators operate in a stable condition with any variation in field excitation. Because of this property they are used as supply source of DC motors, whose speeds are to be controlled for various applications. Example- Ward Leonard Systems of speed control.

Applications of Shunt Wound DC Generators

The application of shunt generators are very much restricted for its dropping voltage characteristic. They are used to supply power to the apparatus situated very close to its position. These type of DC generators generally give constant terminal voltage for small distance operation with the help of field regulators from no load to full load.

1. They are used for general lighting.
2. They are used to charge battery because they can be made to give constant output voltage.
3. They are used for giving the excitation to the alternators.
4. They are also used for small power supply.

Applications of Series Wound DC Generators

These types of generators are restricted for the use of power supply because of their increasing terminal voltage characteristic with the increase in load current from no load to full load. We can clearly see this characteristic from the characteristic curve of series wound generator. They give constant current in the dropping portion of the characteristic curve. For this property they can be used as constant current source and employed for various applications.

1. They are used for supplying field excitation current in DC locomotives for regenerative breaking.

2. This types of generators are used as boosters to compensate the voltage drop in the feeder in various types of distribution systems such as railway service.

3. In series arc lightening this type of generators are mainly used.

Applications of Compound Wound DC Generators

Among various types of DC generators, the compound wound DC generators are most widely used because of its compensating property. We can get desired terminal voltage by compensating the drop due to armature reaction and ohmic drop in the in the line. Such generators have various applications.

1. Cumulative compound wound generators are generally used lighting, power supply purpose and for heavy power services because of their constant voltage property. They are mainly made over compounded.
2. Cumulative compound wound generators are also used for driving a motor.
3. For small distance operation, such as power supply for hotels, offices, homes and lodges, the flat compounded generators are generally used.
4. The differential compound wound generators, because of their large demagnetization armature reaction, are used for arc welding where huge voltage drop and constant current is required.

At present time the **applications of DC generators** become very limited because of technical and economic reasons. Now days the electric power is mainly generated in the form of alternating current with the help of various power electronics devices.

UNIT – II

DC MOTORS

Direct Current Motor (DC motor)

DC motor is similar to dc generator; in fact the same machine can act as motor or generator. The only difference is that in a generator the EMF is greater than terminal voltage, whereas in motor the generated voltage EMF is less than terminal voltage. Thus the power flow is reversed, that is the motor converts electrical energy into mechanical energy. That is the reverse process of generator.

DC motors are highly versatile machines. For example, dc motors are better suited fore many processes that demand a high degree of flexibility in the control of speed and torque. The dc motor can provided high starting torque as well as high decelerating torque for application requiring quick stop or reversals.

DC motors are suited in speed control with over wide range is easily to achieve compare with others electromechanical.

The input is electrical energy (from the supply source), and the output is mechanical energy (to the load).

DC Motor Basic Principles

(a) Energy Conversion

If electrical energy is supplied to a conductor lying perpendicular to a magnetic field, the interaction of current flowing in the conductor and the magnetic field will produce mechanical force (and therefore, mechanical energy).

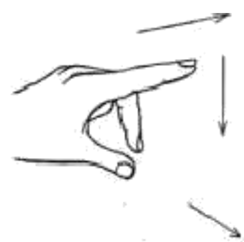
(b) Value of Mechanical Force

There are two conditions which are necessary to produce a force on the conductor. The conductor must be carrying current, and must be within a magnetic field. When these two conditions exist, a force will be applied to the conductor, which will attempt to move the conductor in a direction perpendicular to the magnetic field. This is the basic theory by which all DC motors operate.

The force exerted upon the conductor can be expressed as follows.

$$F = B\, i\, l \text{ Newton} \qquad (1)$$

where B is the density of the magnetic field, l is the length of conductor, and i the value of current flowing in the conductor. The direction of motion can be found using Fleming's Left Hand Rule.



Fleming's Left Hand Rule

The first finger points in the direction of the magnetic field (first - field), which goes from the North pole to the South pole. The second finger points in the direction of the current in the wire (second - current). The thumb then points in the direction the wire is thrust or pushed while in the magnetic field (thumb - torque or thrust).

Principle of operation

Consider a coil in a magnetic field of flux density B (figure). When the two ends of the coil are connected across a DC voltage source, current I flows through it. A force is exerted on the coil as a result of the interaction of magnetic field and electric current. The force on the two sides of the coil is such that the coil starts to move in the direction of force.

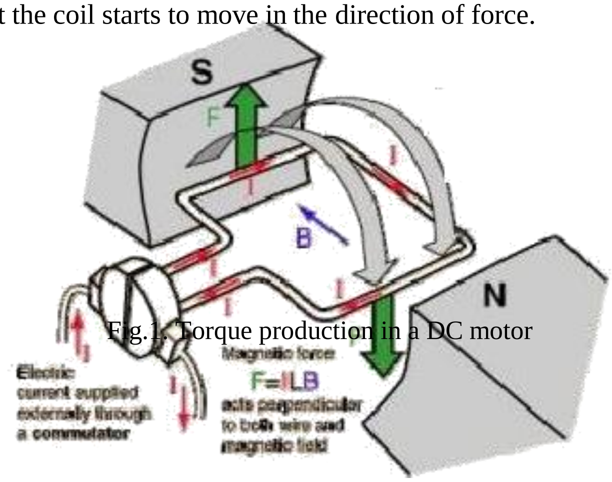


Fig.1. Torque production in a DC motor

In an actual DC motor, several such coils are wound on the rotor, all of which experience force, resulting in rotation. The greater the current in the wire, or the greater the magnetic field, the faster the wire moves because of the greater force created.

At the same time this torque is being produced, the conductors are moving in a magnetic field. At /dt) as shown in different positions, the flux linked with it changes, which causes an emf to be induced (e = d figure 5. This voltage is in opposition to the voltage that causes current flow through the conductor and is referred to as a counter-voltage or back emf.

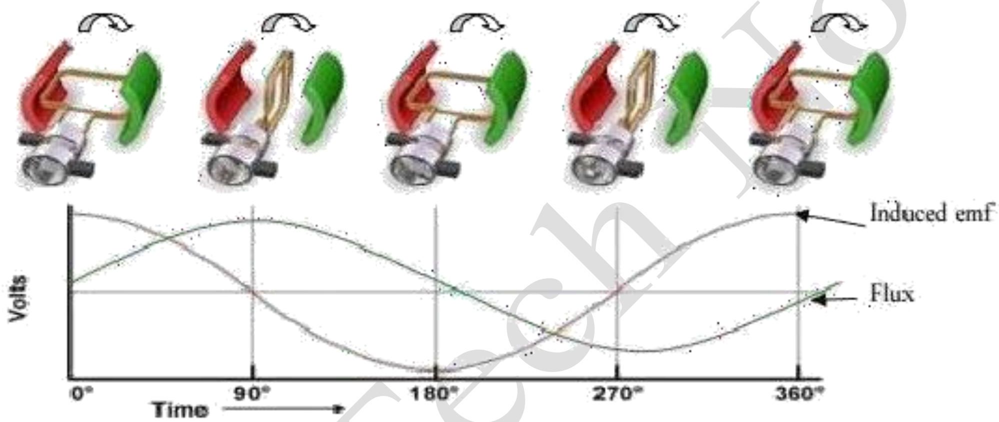


Fig.2. Induced voltage in the armature winding of DC motor

The value of current flowing through the armature is dependent upon the difference between the applied voltage and this counter-voltage. The current due to this counter-voltage tends to oppose the very cause for its production according to Lenz's law. It results in the rotor slowing down.

Eventually, the rotor slows just enough so that the force created by the magnetic field (F = Bil) equals the load force applied on the shaft. Then the system moves at constant velocity.

Construction

DC motors consist of one set of coils, called armature winding, inside another set of coils or a set of permanent magnets, called the stator. Applying a voltage to the coils produces a torque in the armature, resulting in motion.

Stator

The stator is the stationary outside part of a motor.

- The stator of a permanent magnet dc motor is composed of two or more permanent magnet pole pieces.
- The magnetic field can alternatively be created by an electromagnet. In this case, a DC coil (field winding) is wound around a magnetic material that forms part of the stator.

Rotor

The rotor is the inner part which rotates.

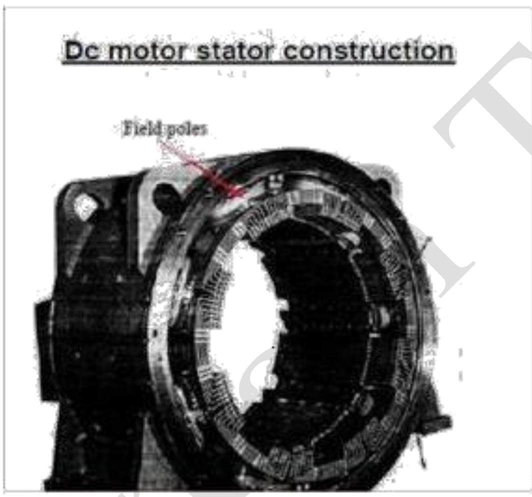
- The rotor is composed of windings (called armature windings) which are connected to the external circuit through a mechanical commutator. Both stator and rotor are made of ferromagnetic materials. The two are separated by air-gap.

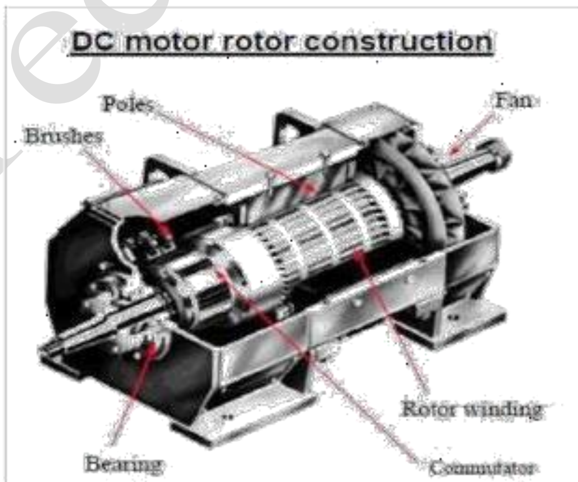
Winding

A winding is made up of series or parallel connection of coils.

- Armature winding - The winding through which the voltage is applied or induced.
- Field winding - The winding through which a current is passed to produce flux

  (for the electromagnet)
- Windings are usually made of copper.





<u>**Torque Developed**</u>

The turning or twisting moment of a force about an axis is called torque. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius meter acted upon by a circumferential force of newton which causes it to rotate at rpm.

Then torque $T = F \times r$ newton-metre(N-m

Work done by this force in one revolution

=Force $\times$ distance

$= F \times 2\pi r$ joule

Power developed $= F \times 2\pi r \times N$ joule/second or watt $= (F \times r) \times 2\pi N$ watt

Now, $2\pi N$ = angular velocity ω in radian per second and $F \times r$ =torque T

Hence, power developed $= T \times \omega$ watt or $P = T\omega$ watt

Moreover, if N is in rpm, then

$\omega = 2\pi N / 60$ rad/s

Hence, $P = \frac{2\pi N}{60} \times T$ or $P = \frac{2\pi}{60} NT = \frac{NT}{9.55}$

Armature torque of a motor

Let T_a be the torque developed by the armature of a motor running at N rps. If T_a is in N-m, then

power developed $= T_a \times 2\pi N$ watt

We also know that electrical power converted into mechanical power in the armature $= E_b I_a$ watt.

Comparing above equations, we get $T_a \times 2\pi N = E_b I_a$

After simplification, if N in rps, $T_a = \frac{E_b I_a}{2\pi N}$

If N is in rpm, then $T_a = 9.55 \frac{E_b I_a}{N}$ N-m

Shaft torque

The whole of the armature torque, as calculated above, is not available for doing useful work, because of iron and friction losses in the motor. The torque which is available for doing useful work is known as shaft torque T_{sh}. The motor output is given by

Output $= T_{sh} \times 2\pi N$ watt provided T_{sh} is in N-m and N in rps.

Hence, $T_{sh} = \frac{Output\_in\_watts}{2\pi N}$; if N is in rps

Induced Counter-voltage (Back emf):

Due to the rotation of this coil in the magnetic field, the flux linked with it changes at different positions, which causes an emf to be induced (refer to figure 2).

The induced emf in a single coil, $e = d\o_c/dt$

Since the flux linking the coil, □c= □Sin □

Induced voltage , e = □□□cos t□ (4)

Note that equation (4) gives the emf induced in one coil. As there are several coils wound all around the rotor, each with a different emf depending on the amount of flux change through it, the total emf can be obtained by summing up the individual emfs.

The total emf induced in the motor by several such coils wound on the rotor can be obtained by integrating equation (4), and expressed as:

Eb = K □□m (5)

where K is an armature constant, and is related to the geometry and magnetic properties of the motor, and □m is the speed of rotation.

The electrical power generated by the machine is given by:

Pdev = Eb Ia = K □□m Ia (6)

DC Motor Equivalent circuit

The schematic diagram for a DC motor is shown below. A DC motor has two distinct circuits: Field circuit and armature circuit. The input is electrical power and the output is mechanical power. In this equivalent circuit, the field winding is supplied from a separate DC voltage source of voltage Vf. Rf and Lf represent the resistance and inductance of the field winding. The current If produced in the winding establishes the magnetic field necessary for motor operation. In the armature (rotor) circuit, VT is the voltage applied across the motor terminals, Ia is the current flowing in the armature circuit, Ra is the resistance of the armature winding, and Eb is the total voltage induced in the armature.

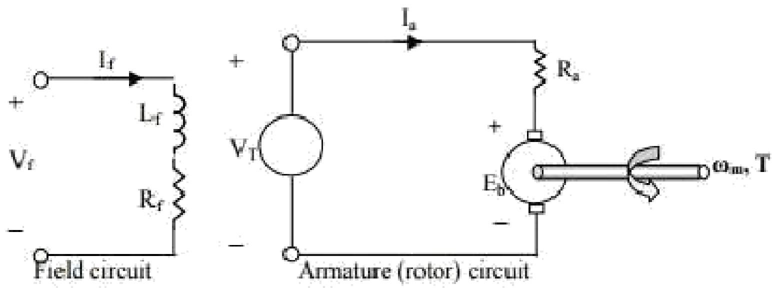


Fig.4. DC motor representation

~~Counter EMF in DC motor~~

When voltage is applied to dc motor, current will flow into the positive brush through the commutator into the armature winding. The motor armature winding is identical to the generator armature winding. Thus the conductors on the north field poles are carry current in one direction, while all conductors on the south field poles carry the current in opposite direction. When the armature carry current it will produce a magnetic field around the conductor of it own which interact with the main field. It is cause to the force developed on all conductors and tending to turn the armature.

The armature conductors continually cut through this resultant field. So that voltages are generated in the same conductors that experience force action. When operating the motor is simultaneously acting as generator. Naturally motor action is stronger than generator action.

Although the counter EMF is opposite with the supplied voltage, but it cannot exceed to applied voltage. The counter EMF is serves to limit the current in an armature winding. The armature current will be limited to the value just sufficient to take care of the developed power needed to drive the load.

In the case of no load is connected to the shaft. The counter EMF will almost equal to the applied voltage. The power develops by the armature in this case is just the power needed to overcome the rotational losses. It's mean that the armature current IA is controlled and limited by counter EMF therefore

$$Ia = \frac{VL - Ea}{Ra}$$

Where:

V_L = Line voltage across the armature winding

R_a = Resistance of the armature winding

E_a = Induced EMF or generated voltage

I_a = Armature current

Since, EA is induced or generated voltage it is depend on the flux per pole and the speed of the armature rotate (n) in rpm.

Therefore

$$E_a = K\varphi n$$

Where:

K = the constant value depending on armature winding and number of pole of machine.

φ = Rotation of the armature

And,

$$K = \frac{Z \times P}{a}$$

Where:

Z = Total number of conductor in the armature winding

a = Number of parallel circuit in the armature winding between positive and negative brushes. For wave wound armature “a” = 2

Lap wound armature “a” = P

Mechanical power develop in dc motor (Pd)

Let,

Pd = Mechanical power develop

T = Torque exerted on the armature

$$P_d = \omega T$$

$$= \left(\frac{2\pi n}{60}\right) T$$

$$\text{Where: } T = P_d / \omega$$

$$= \frac{Ea \times Ia}{2\pi n / 60} = \frac{K\varphi n \times Ia}{(2\pi n)/60}$$

DC Machine Classification

DC Machines can be classified according to the electrical connections of the armature winding and the field windings. The different ways in which these windings are connected lead to machines operating with different characteristics. The field winding can be either self-excited or separately-excited, that is, the terminals of the winding can be connected across the input voltage terminals or fed from a separate voltage source (as in the previous section). Further, in self-excited motors, the field winding can be connected either in series or in parallel with the armature winding. These different types of connections give rise to very different types of machines, as we will study in this section.

(a) Separately excited machines

- The armature and field winding are electrically separate from each other.
- The field winding is excited by a separate DC source.

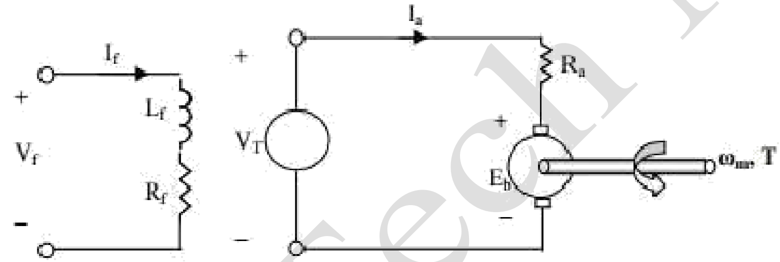


Fig.5. Separately excited dc motor

The voltage and power equations for this machine are same as those derived in the previous section. Note that the total input power = $V_f I_f + V_T I_a$

(b)Self excited machines

In these machines, instead of a separate voltage source, the field winding is connected across the main voltage terminals.

1.Shunt machine

- The armature and field winding are connected in parallel.
- The armature voltage and field voltage are the same.

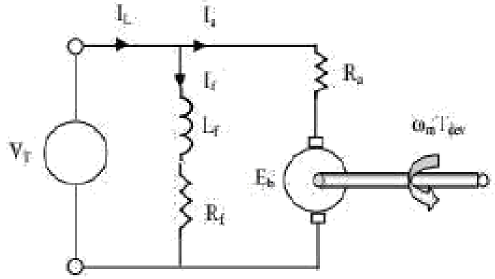


Fig.6. shunt motor

Total current drawn from the supply, IL = If + Ia

Total input power = VT IL

Voltage, current and power equations are given in equations (7), (8) and (9).

2. Series DC machine

- The field winding and armature winding are connected in series.
- The field winding carries the same current as the armature winding.

A series wound motor is also called a universal motor. It is universal in the sense that it will run equally well using either an ac or a dc voltage source.

Reversing the polarity of both the stator and the rotor cancel out. Thus the motor will always rotate the same direction regardless of the voltage polarity.

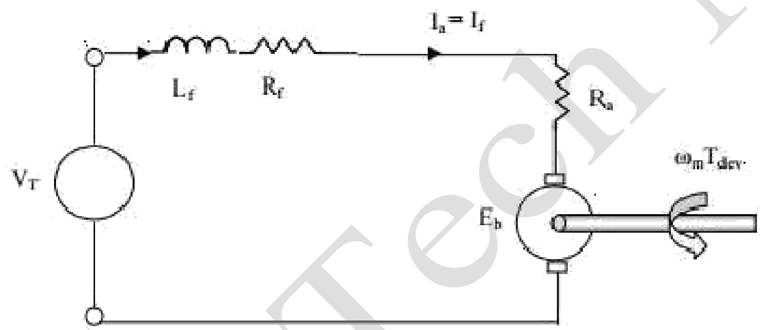


Fig.7.Series Motor

Compound DC machine

If both series and shunt field windings are used, the motor is said to be compounded. In a compound machine, the series field winding is connected in series with the armature, and the shunt field winding is connected in parallel. Two types of arrangements are possible in compound motors:

Cumulative compounding **-** If the magnetic fluxes produced by both series and shunt field windings are in the same direction (i.e., additive), the machine is called cumulative compound.

Differential compounding **-** If the two fluxes are in opposition, the machine is differential compound.

In both these types, the connection can be either short shunt or long shunt.

Speed control of DC motor

Many applications require the speed of a motor to be varied over a wide range. One of the most attractive features of DC motors in comparison with AC motors is the ease with which their speed can be varied.

We know that the back emf for a separately excited DC motor:.

$$E_b = K\phi\omega_m = V_T - I_a R_a$$

Rearranging the terms,

$$\text{Speed } \omega_m = (V_T - I_a R_a)/K\phi$$

From the above equation, it is evident that the speed can be varied by using any of the following methods:

- Armature voltage control (By varying VT)
- Field Control (By Varying)
- Armature resistance control (By varying Ra)

Armature voltage control

This method is usually applicable to separately excited DC motors. In this method of speed control, Ra and are kept constant.

In normal operation, the drop across the armature resistance is small compared to Eb and therefore: EbV_T

Since, Eb = Kø ω_m

Angular speed can be expressed as:

$$\omega_m = V_T / K\text{ø} \qquad (8)$$

From this equation, If flux is kept constant, the speed changes linearly with VT.

- As the terminal voltage is increased, the speed increases and vice versa.

The relationship between speed and applied voltage is shown in figure 8. This method provides smooth variation of speed control.

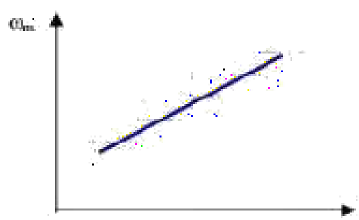


Fig.8.Variation of speed with applied voltage

Field Control , (□)

In this method of speed control, Ra and VT remain fixed.

Therefore, from equation (7):

□ □ □ $_m$ I/

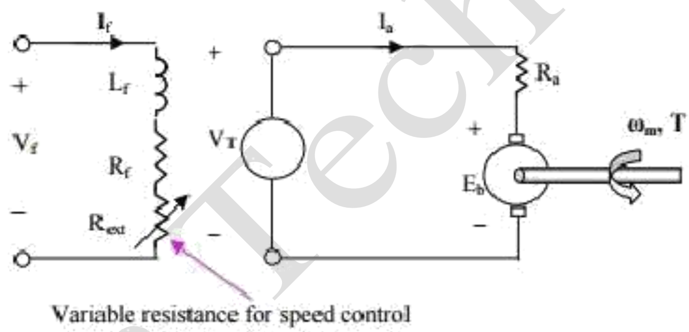
Assuming magnetic linearity, □□ If

(OR) □$_m$ □ I/I$_F$ (9)

i.e., Speed can be controlled by varying field current If.

□ The field current can be changed by varying an adjustable rheostat in the field circuit (as shown in figure 9).

□ By increasing the value of total field resistance, field current can be reduced, and therefore speed can be increased.



The relationship between the field winding current and angular speed is shown in figure 10

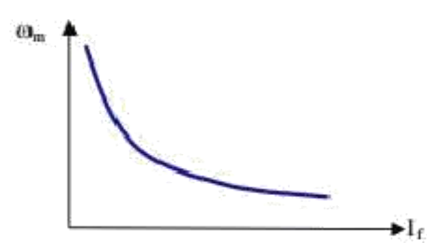


Fig.10: Variation of speed with field current

Armature Resistance Control

The voltage across the armature can be varied by inserting a variable resistance in series with the armature circuit.

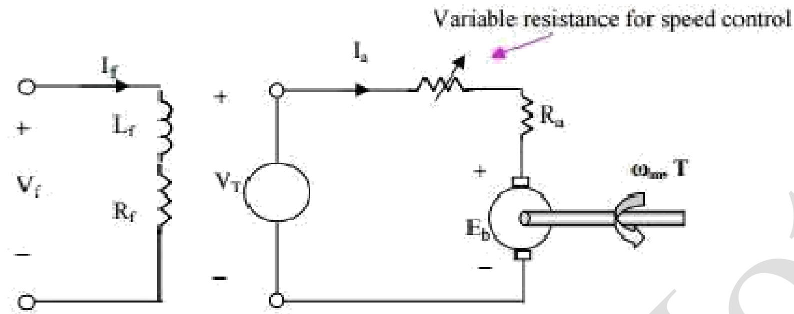


Fig.11. Armature resistance method for speed control

From speed-torque characteristics , we know that:

$$T_{dev} = \frac{K\phi}{R_a}(V_T - K\phi\omega_m)$$

For a load of constant torque V_T and □ are kept constant, as the armature resistance Ra is increased, speed decreases. As the actual resistance of the armature winding is fixed for a given motor, the overall resistance in the armature circuit can be increased by inserting an additional variable resistance in series with the armature. The variation if speed with respect to change in this external resistance is shown in figure 12. This method provides smooth control of speed..

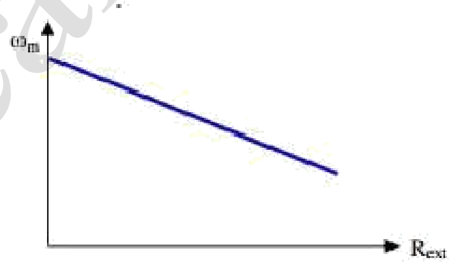


Fig. 12: Variation of speed with external armature resistance

DC Shunt Motor speed control

All three methods described above can be used for controlling the speed of DC Shunt Motors.

Series Motor speed control

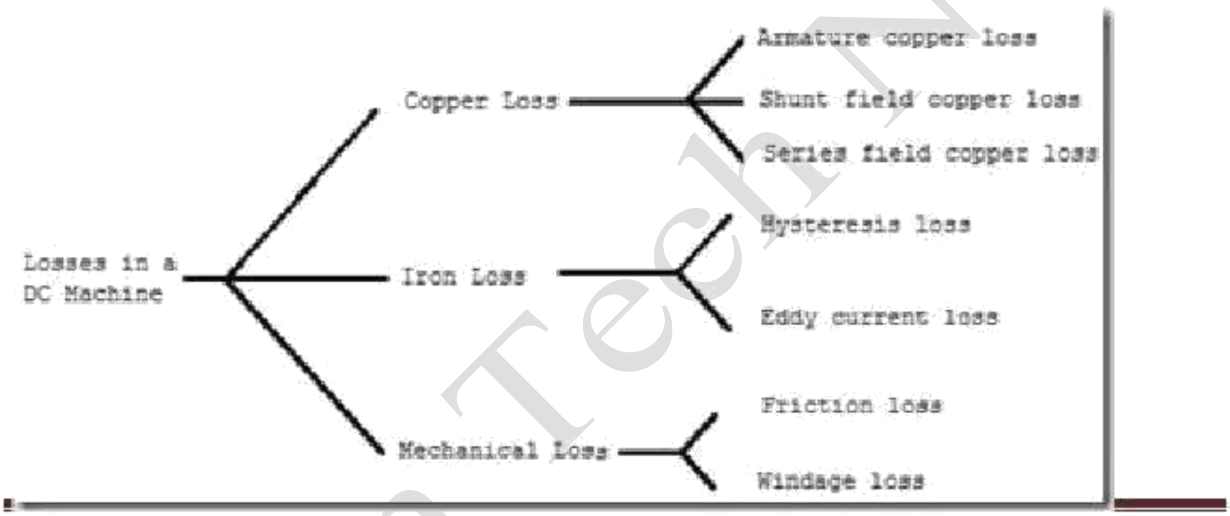
The speed is usually controlled by changing an external resistance in series with the armature.

The other two methods described above are not applicable to DC series motor speed control.

Applications of dc motors

d.c. shunt motor	lathes,fans,pumps disc and band saw drive requiring moderate torques.
d.c. series motor	Electric traction, high speed tools
d.c. compound motor	Rolling mills and other loads requiring large momentary toques.

Types of Losses in a DC Machines



The losses can be divided into three types in a dc machine (Generator or Motor). They are

1. Copper losses
2. Iron or core losses and
3. Mechanical losses.

All these losses seem as heat and therefore increase the temperature of the machine. Further the efficiency of the machine will reduce.

1. Copper Losses:

This loss generally occurs due to current in the various windings on of the machine. The different winding losses are;

Armature copper loss = $I_a^2 R_a$

Shunt field copper loss = $I^2_{sh}R_{sh}$

Series field copper loss = $I^2_{se}\,R_{se}$

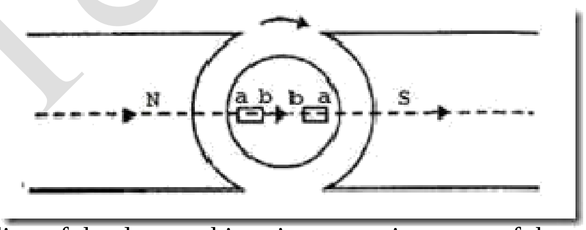
Note: There's additionally brush contact loss attributable to brush contact resistance (i.e., resistance in the middle of the surface of brush and commutator). This loss is mostly enclosed in armature copper loss.

2. Iron Losses

This loss occurs within the armature of a d.c. machine and are attributable to the rotation of armature within the magnetic field of the poles. They're of 2 sorts viz.,

(i) Hysteresis loss
(ii)eddy current loss.

Hysteresis loss:



Hysteresis loss happens in the armature winding of the d.c. machine since any given part of the armature is exposed to magnetic field of reverses as it passes underneath sequence poles. The above fig shows the 2 pole DC machine of rotating armature. Consider a tiny low piece ab of the armature winding. Once the piece ab is underneath N-pole, the magnetic lines pass from a to b. Half a revolution well along, identical piece of iron is underneath S-pole and magnetic lines pass from b to a in order that magnetism within the iron is overturned. So as to reverse constantly the molecular magnets within the armature core, particular quantity of power must be spent that is named hysteresis loss. It's given by Steinmetz formula.

The steinmetz formula is

$$\text{Hysteresis loss } P_h = \eta B_{16max} fV \text{ watts}$$

Where,

η = Steinmetz hysteresis co-efficient

B_{max} = Maximum flux Density in armature winding

F= Frequency of magnetic reversals

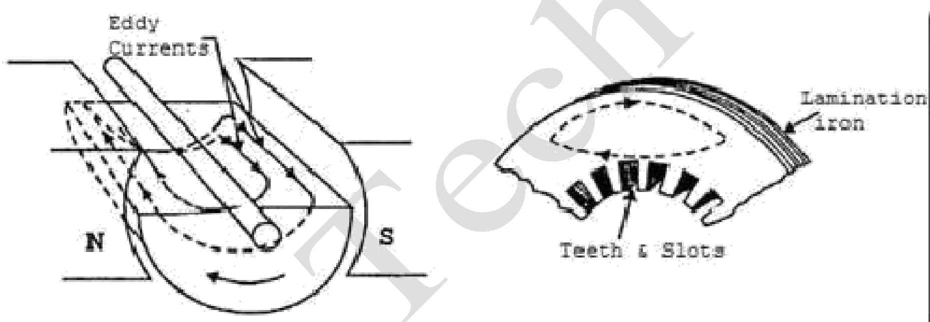
= NP/120 (N is in RPM)

V= Volume of armature in m^3

If you want to cut back this loss in a d.c. machine, armature core is created of such materials that have an lesser value of Steinmetz hysteresis co-efficient e.g., silicon steel.

Eddy current loss:

In addition to the voltages evoked within the armature conductors, some of other voltages evoked within the armature core. These voltages turn out current currents within the coil core as shown in Fig. These are referred to as eddy currents and power loss attributable to their flow is named eddy current loss. This loss seems as heat that increases the temperature of the machine and efficiency will decrease.



If never-ending cast-iron core is employed, the resistance to eddy current path is tiny attributable to massive cross-sectional space of the core. Consequently, the magnitude of eddy current and therefore eddy current loss are massive. The magnitudes of eddy current are often decreased by creating core resistance as high as sensible. The core resistances are often greatly exaggerated by making the core of skinny, spherical iron sheets referred to as lamination's shown in the fig. The lamination's are insulated from one another with a layer of varnish. The insulating layer features a high resistance, thus only small amount of current flows from one lamination to the opposite. Also, as a result of every lamination is extremely skinny, the resistance to current passing over the breadth of a lamination is additionally quite massive. Therefore laminating a core will increase the core resistance that drops the eddy current and therefore the eddy current loss.

$$\text{Eddy Current loss } P_e = K_e B^2_{max} f^2 t^2 V \text{ Watts}$$

Where, k_e = constant

B_{max} = Maximum flux density in wb/m

T = Thickness of lamination in m

V = Volume of core in m^3

Note: Constant (K_e) depend upon the resistance of core and system of unit used.

It may well be noted that eddy current loss be subject to upon the sq. of lamination thickness. For this reason, lamination thickness ought to be unbroken as tiny as potential.

***3.** Mechanical Loss*

These losses are attributable to friction and windage.

- Friction loss occurs due to the friction in bearing, brushes etc.
- windage loss occurs due to the air friction of rotating coil.

These losses rely on the speed of the machine. Except for a given speed, they're much constant.

Constant and Variable Losses

The losses in a d.c. machine is also further classified into (i) constant losses (ii) variable losses.

Constant losses

Those losses in a d.c. generator that stay constant at all loads are referred to as constant losses.

The constant losses in a very d.c. generator are:

(a)iron losses

(b)mechanical losses

(c)shunt field losses

Variable losses

Those losses in a d.c. generator that differ with load are referred to as variable losses. The variable losses in a very d.c. generator are:

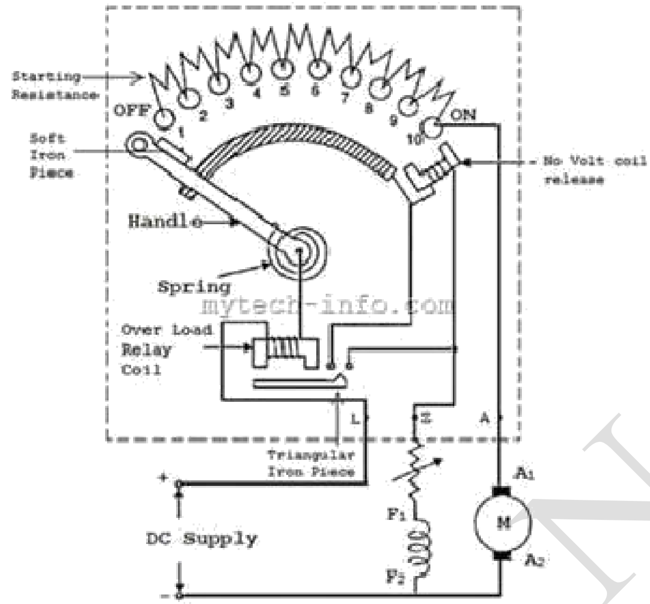
Copper loss in armature winding ($I^2 R_a$)

Copper loss in series field winding ($I^2_{se} R_{se}$)

Total losses = Constant losses + Variable losses.

Generally this copper loss is constant for shunt and compound generators.

Three point starter



The figure above shows that typical representation diagram of a 3 point starter for DC shunt motors with its protective devices. It contains 3 terminals namely L, Z, & A; hence named 3 point starter. The starter is made up of of starting resistances divided into many section and which are connected in series within the armature. The each tapping point on the starting resistances is carried out to a no. of studs. The starter 3 terminals L,Z & A are connected to the positive terminal of line, shunt field and armature terminal of motor respectively. The remaining terminal of the shunt and armature are connected to the negative line terminal. The No volt coil release is connected in series with field winding. The handle one end is connected to the L terminal by means of over load release coil. Then another end of handle travels against the twisting spring & make touching base with every single stud in the course of starting operation, tripping out the starting resistance as it moves above every stud in clockwise.

Armature Reaction in DC Motor

Working:

- Initially the DC supply is turned on with the handle is in OFF position.
- Now the handle is moved towards clockwise direction to the 1st stud. Once it contacts with 1st

the 1 stud, immediately the shunt field coil is connected to the supply, however the entire starting resistances is injected with armature circuit in series.

- As the handle moved gradually towards the final stud, so that the starting resistance is cut out step by step in armature circuit. And finally the handle is detained magnetically by the No volt coil release since it is energized by the filed winding.
- In case if the shunt field winding excitation is cut out by accident or else the supply is interrupted then the no volt coil release gets demagnetized and handle returned back to the original position under the influence of spring.

Note: If we were not used No volt coil release; then if the supply is cut off the handle would remain in the same position, causing an extreme current in armature.

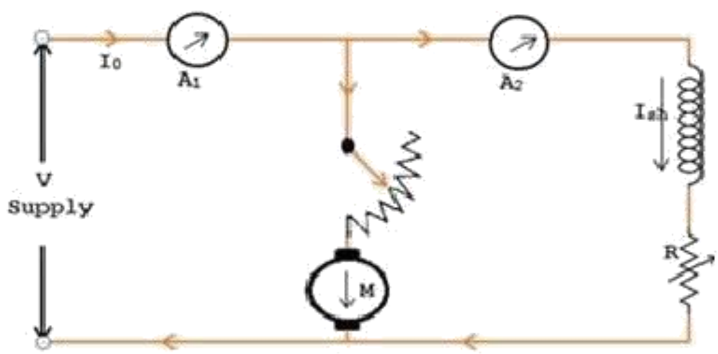
- If any fault occurs on motor or overload, it will draw extreme current from the source. This current raise the ampere turns of OLR coil (over load relay) and pull the armature Coil, in consequence short circuiting the NVR coil (No volt relay coil). The NVR coil gets demagnetized and handle comes to the rest position under the influence of spring. Therefore the motor disconnected from the supply automatically.

Characteristic of DC Shunt Motor

Disadvantage:

In point starter, no volt relay coil is connected in series with field circuit; hence it carries shunt current in the field. When the speed control of DC motor through field regulator, it may be weakened the shunt field current to such extent the no volt coil release might not in a position to hold the starter handle in ON position. This might the motor disconnected from the source when it is not anticipated. This can be overcome by using the point starter.

Swinburne's Test for DC Machines



In this technique, the DC Generator or DC Motor is run as a motor at no load; with that losses of the DC machines are determined. When the losses of DC machine well-known, then we can find the efficiency of a DC machine in advance at any desired load. In DC machines this test is applicable only throughout the flux is constant at all load (DC Shunt machine and DC Compound Machine). This test maintains of two steps;

Determination of Hot Resistance of Windings:

The resistance of armature windings and shunt field windings are measured with the help of a battery, ammeter and voltmeter. Since these armature and shunt filed resistances are measured while the DC machine is cold, it should be transformed to values equivalent to the temperature at which the DC machine would work at full load. These values are measured generally when the room temperature increases above 40 ^{o}C. Take on the hot resistance of armature winding and shunt field winding be R_a and R_{sh} correspondingly.

Determination of Constant Losses:

On no load the DC machine run as a motor with the supply voltage is varied to the normal rated voltage. With the use of the field regulator R the motor speed is varied to run the rated speed which is shown in the figure.

Let

V = Supply Voltage

I_0 = No load current read by A_1

I_{sh} = Shunt Field current ready by A_2

No load armature current $I_{ao} = I_0 - I_{sh}$

No load Input power to motor = VI_0

No load Input power to motor = VI_{a0}

$$= V (I_0 - I_{sh})$$

As the output power is nil, the no loads input power to the armature provides Iron loss, armature copper loss, friction loss and windage loss.

Constant loss W_c = Input power to Motor – Armature copper loss

$$W_c = VI_0 - (I_0 - I_{sh} R_a)^2$$

As the constant losses are identified, the efficiency of the DC machine at any loads can be determined. Suppose it is desired to determine the DC machine efficiency at no load current. Then,

Armature current $I_a = I - I_{sh}$ (For Motoring)

$I_a = I + I_{sh}$ (For Generating)

To find the Efficiency when running as a motor:

Input power to motor = VI

Armature copper loss $= I_a R_a^2 = (I - I_{sh} R_a)^2$

Constant Loss $= W_c$

Total Loss $= (I - I_{sh} 2 R_a) + W_c$

Motor Efficiency η = (Input power – Losses)/ Input

$\boldsymbol{\eta = \{VI - (I - I_{sh} R_a^2)\} / VI}$

To find the Efficiency when running as a Generator:

Output Power of Generator = VI

Armature copper loss $= I_a R_a^2 = (I + I_{sh} R_a)^2$

Constant Loss $= W_c$

Total Loss $= (I + I_{sh}^2 R_a) + W_c$

Motor Efficiency η= Output power/ (Output power + Losses)

$\boldsymbol{\eta = VI / \{VI + (I + I_{sh} R_a)^2 + W_c\}}$

Merits:

- Since this test is no load test, power required is less. Hence the cost is economic.
- The efficiency of the machine can be found very easily, because the constant losses are well known.
- This test is appropriate.

Demerits:

- When the DC machine is loaded, this test does not deliberate the stray load loss that occurs.
- Using this method we cannot check the DC machine performances at full load.

UNIT – IV

SINGLE PHASE TRANSFORMERS

TRANSFORMERS

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems.

SINGLE PHASE TRANSFORMERS

INTRODUCTION

In its simplest form a single-phase transformer consists of two windings, wound on an iron core one of the windings is connected to an ac source of supply f. The source supplies a current to this winding (called primary winding) which in turn produces a flux in the iron core. This flux is alternating in nature (Refer Figure 4.1). If the supplied voltage has a frequency f, the flux in the core also alternates at a frequency f. the alternating flux linking with the second winding, induces a voltage E2 in the second winding (called secondary winding). [Note that this alternating flux linking with primary winding will also induce a voltage in the primary winding, denoted as E1. Applied voltage V1 is very nearly equal to E1]. If the number of turns in the primary and secondary windings is N1 and N2 respectively, we shall see later in this unit that

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}.$$

The load is connected across the secondary winding, between the terminals a1, a2. Thus, the load can be supplied at a voltage higher or lower than the supply voltage, depending upon the ratio N1/N2

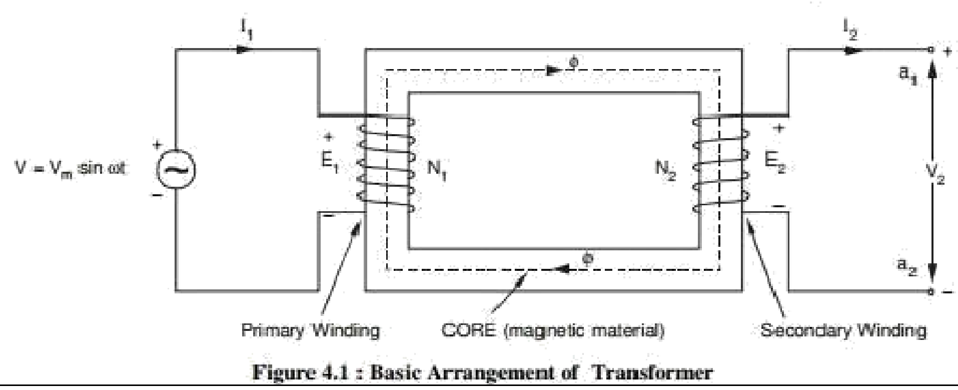


Figure 4.1 : Basic Arrangement of Transformer

When a load is connected across the secondary winding it carries a current I2, called load current. The primary current correspondingly increases to provide for the load current, in addition to the small no load current. The transfer of power from the primary side (or source) to the secondary side (or load) is through the mutual flux and core. There is no direct electrical connection between the primary and secondary sides.

In an actual transformer, when the iron core carries alternating flux, there is a power loss in the core called core loss, iron loss or no load loss. Further, the primary and secondary windings have a resistance, and the currents in primary and secondary windings give rise to I 2 R losses in transformer windings, also called copper losses. The losses lead to production of heat in the transformers, and a consequent temperature rise. Therefore, in transformer, cooling methods are adopted to ensure that the temperature remains within limit so that no damage is done to windings' insulation and material.

In the Figure 4.1 of a single-phase transformer, the primary winding has been shown connected to a source of constant sinusoidal voltage of frequency f Hz and the secondary terminals are kept open. The primary winding of N1 turns draws a small amount of alternating current of instantaneous value i0, called the exciting current. This current establishes flux φ in the core (+ve direction marked on diagram). The strong coupling enables all of the flux φ to be confined to the core (i.e. there is no leakage of flux).

CONSTRUCTION OF A TRANSFORMER

There are two basic parts of a transformer:

1. Magnetic core
2. Winding or coils

MAGNETIC CORE:

The core of a transformer is either square or rectangular in size. It is further divided in two parts. The vertical portion on which the coils are bound is called limb, while the top and bottom horizontal portion is called yoke of the core as shown in fig. 2.

Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimized. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the airgap at the joints. For this generally _L' shaped or _I' shaped laminations are used which are shown in the fig. 3 below.

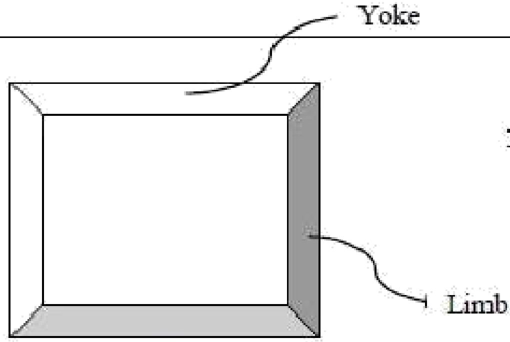


Fig. 2

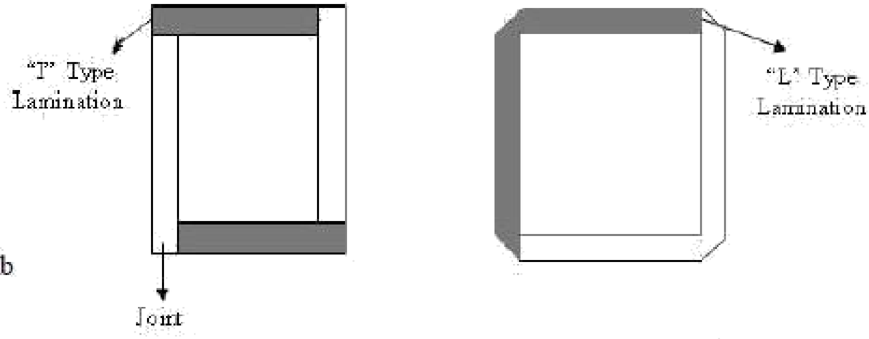


Fig. 3

WINDING:

There are two windings, which are wound on the two limbs of the core, which are insulated from each other and from the limbs as shown in fig. 4. The windings are made up of copper, so that, they possess a very small resistance. The winding which is connected to the load is called secondary winding and the winding which is connected to the supply is called primary winding. The primary winding has N_1 number of turns and the secondary windings have N_2 number of turns.

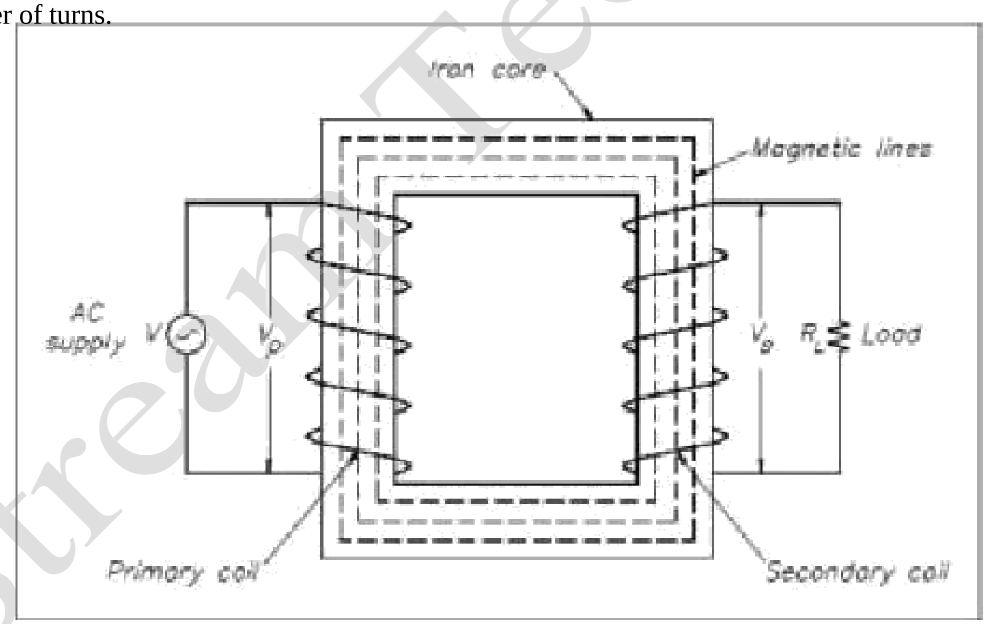
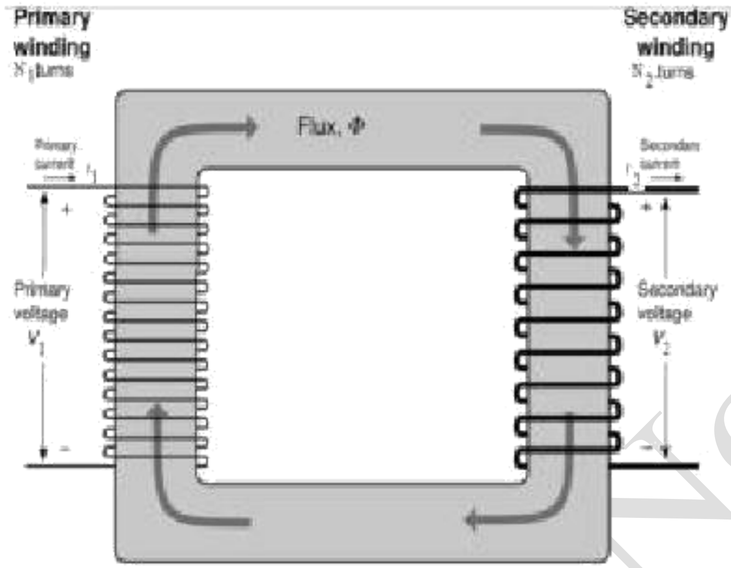


Fig. 4 Single Phase Transformer

PRINCIPLE OF OPERATION OF A SINGLE PHASE TRANSFORMER



A single phase transformer works on the principle of mutual induction between two magnetically coupled coils. When the primary winding is connected to an alternating voltage of r.m.s value, V1 volts, an alternating current flows through the primary winding and setup an alternating flux in the material of the core. This alternating flux ϕ, links not only the primary windings but also the secondary windings. Therefore, an e.m.f e1 is induced in the primary winding and an e.m.f e2 is induced in the secondary winding, e1 and e2 are given:

$$e1 = -N1\frac{d\varphi}{dt} \text{ --------- (a)}$$

$$e2 = -N2\frac{d\varphi}{dt} \text{ ----------(b)}$$

If the induced e.m.f is e1 and e2 are represented by their rms values E1 and E2 respectively, then

$$E1 = -N1\frac{d\varphi}{dt} \text{ --------- (1)}$$

$$E2 = -N2\frac{d\varphi}{dt} \text{ ---------- (2)}$$

$$\text{Therefore, } \frac{E2}{E1} = \frac{N2}{N1} = k \text{ --------------- (3)}$$

K is known as the transformation ratio of the transformer. When a load is connected to the secondary winding, a current I_2 flows through the load, V_2 is the terminal voltage across the load. As the power transfered from the primary winding to the secondary winding is same, Power input to the primary winding = Power output from the secondary winding.

The directions of emf‘s E1 and E2 induced in the primary and secondary windings are such that, they always oppose the primary applied voltage V1.

EMF Equation of a transformer:

Consider a transformer having,

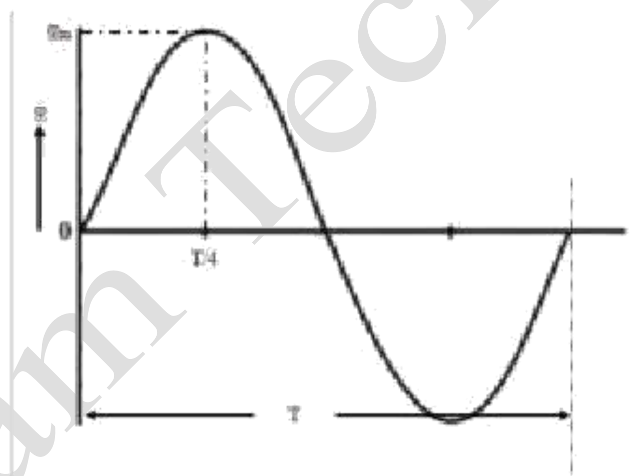
N_1 =Primary turns

N_2 = Secondary turns

Φ_m = Maximum flux in the core

$\Phi_m = B_m \times A$ webers

f= frequency of ac input in hertz (Hz)



The flux in the core will vary sinusoidal as shown in figure, so that it increases from zero to maximum “ϕ_m” in one quarter of the cycle i.e, 1/4f second.

Therefore, average rate of change of flux = $\phi m / _{1/4f}$

$$= 4f\varphi_m$$

We know that, the rate of change of flux per turn means that the induced emf in volts.

Therefore, average emf induced per turn $= 4f\varphi_m$ volts.

Since the flux is varying sinusoidally, the rms value of induced emf is obtained by multiplying the average value by the form factor .

Therefore, rms value of emf induced per turns $= 1.11 \times 4f \times \varphi_m$

i.e, $E_1 = 4.44 f \varphi_m \times N_1 = 4.44 f B_m \times A \times N_1$

Similarly;

$E_2 = 4.44\ f\ \varphi_m \times N_2 = 4.44\ f\ B_m \times A \times N_2$

Transformation Ratio:

(1) Voltage Transformation Ratio
(2) Current Transformation Ratio

Voltage Transformation Ratio:

Voltage transformation ratio can be defined as the ratio of the secondary voltage to the primary voltage denoted by K.

Mathematically given as $K = \frac{Secondary\ Voltage}{PrimarPrimar\ Voltage} = \frac{V_2}{V_1}$

$$K = \frac{E_2}{E_1} = \frac{4.44\ f\ \varphi_m N_2}{4.44\ f\ \varphi_m N_1} = \frac{N_2}{N_1}$$

$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Current Transformation Ratio:

Consider an ideal transformer and we have the input voltampere is equal to output voltampere. Mathematically, Input Voltampere = Output Voltampere

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

$$\therefore,\ K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

TRANSFORMER ON NO-LOAD

Theory of Transformer On No-load, and Having No Winding Resistance and No Leakage Reactance of Transformer

Let us consider one electrical transformer with only core losses, which means, it has only core losses but no copper loss and no leakage reactance of transformer. When an alternating source is applied in the primary, the source will supply the current for magnetizing the core of transformer.

But this current is not the actual magnetizing current, it is little bit greater than actual magnetizing current. Actually, total current supplied from the source has two components, one is magnetizing current which is merely utilized for magnetizing the core and other component of the source current is consumed for compensating the core losses in transformer. Because of this core loss component, the source current in **transformer on no-load** condition supplied from the source as source current is not exactly at 90° lags of supply voltage, but it lags behind an angle θ is less than 90°. If total current supplied from source is I_o, it will have one component in phase with supply voltage V_1 and this component of the current I_w is core loss component. This component is taken in phase with source voltage, because it is associated with active or working losses in transformer. Other component of the source current is denoted as I_μ. This component produces the alternating magnetic flux in the core, so it is watt-less; means it is reactive part of the transformer source current. Hence I_μ will be in quadrature with V_1 and in phase with alternating flux Φ.

Hence, total primary current in **transformer on no-load** condition can be represented as

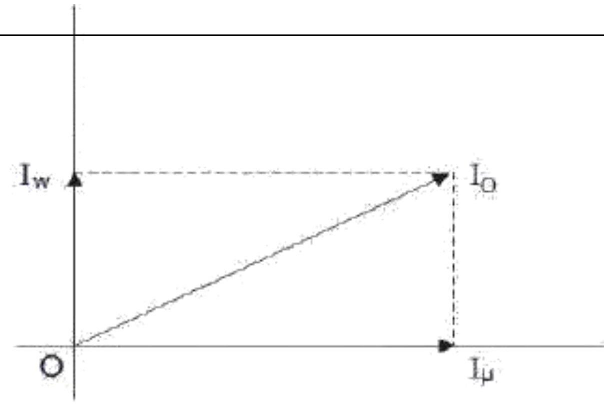
$$I_0 = I_\mu + I_w$$

$$|I_\mu| = |I_0| \cos\theta$$

$$|I_w| = |I_0| \sin\theta$$

$$|I_w| = \sqrt{|I_\mu|^2 + |I_w|^2}$$

Now you have seen how simple is to explain the **theory of transformer** in no-load.

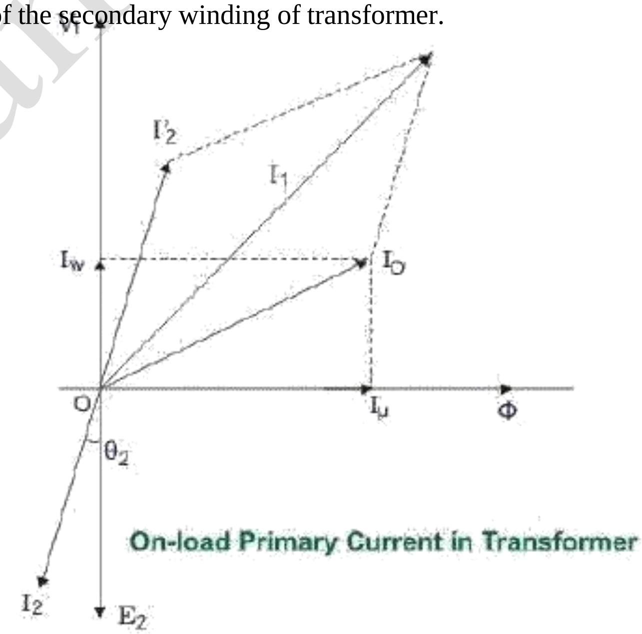


Excitation Current of Transformer

TRANSFORMER ON LOAD

Theory of Transformer On Load But Having No Winding Resistance and Leakage Reactance

Now we will examine the behavior of above said **transformer on load,** that means load is connected to the secondary terminals. Consider, transformer having core loss but no copper loss and leakage reactance. Whenever load is connected to the secondary winding, load current will start to flow through the load as well as secondary winding. This load current solely depends upon the characteristics of the load and also upon secondary voltage of the transformer. This current is called secondary current or load current, here it is denoted as I_2. As I_2 is flowing through the secondary, a self mmf in secondary winding will be produced. Here it is N_2I_2, where, N_2 is the number of turns of the secondary winding of transformer.



On-load Primary Current in Transformer

This mmf or magneto motive force in the secondary winding produces flux φ_2. This φ_2 will oppose the main magnetizing flux and momentarily weakens the main flux and tries to reduce primary self induced emf E_1. If E_1 falls down below the primary source voltage V_1, there will be an extra current flowing from source to primary winding. This extra primary current I_2' produces extra flux φ' in the core which will neutralize the secondary counter flux φ_2. Hence the main magnetizing flux of core, Φ remains unchanged irrespective of load.

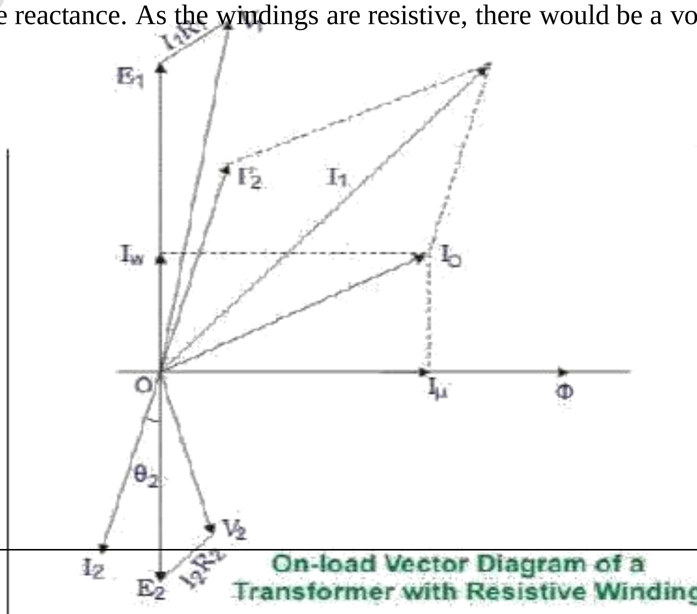
So total current, this transformer draws from source can be divided into two components, first one is utilized for magnetizing the core and compensating the core loss i.e. I_o. It is no-load component of the primary current. Second one is utilized for compensating the counter flux of the secondary winding. It is known as load component of the primary current. Hence total no load primary current I_1 of a electrical power transformer having no winding resistance and leakage reactance can be represented as follows

$$I_1 = I_0 + I_2'$$

Where θ_2 is the angle between Secondary Voltage and Secondary Current of transformer.
Now we will proceed one further step toward more practical aspect of a transformer.

Transformer On Load, With Resistive Winding, But No Leakage Reactance

Now, consider the winding resistance of transformer but no leakage reactance. So far we have discussed about the transformer which has ideal windings, means winding with no resistance and leakage reactance, but now we will consider one transformer which has internal resistance in the winding but no leakage reactance. As the windings are resistive, there would be a voltage drop in the windings.



On-load Vector Diagram of a Transformer with Resistive Winding

We have proved earlier that, total primary current from the source on load is I_1. The voltage drop in the primary winding with resistance, R_1 is R_1I_1. Obviously, induced emf across primary winding E_1, is not exactly equal to source voltage V_1. E_1 is less than V_1 by voltage drop I_1R_1.

$$V_1 = E_1 + I_1R_1$$

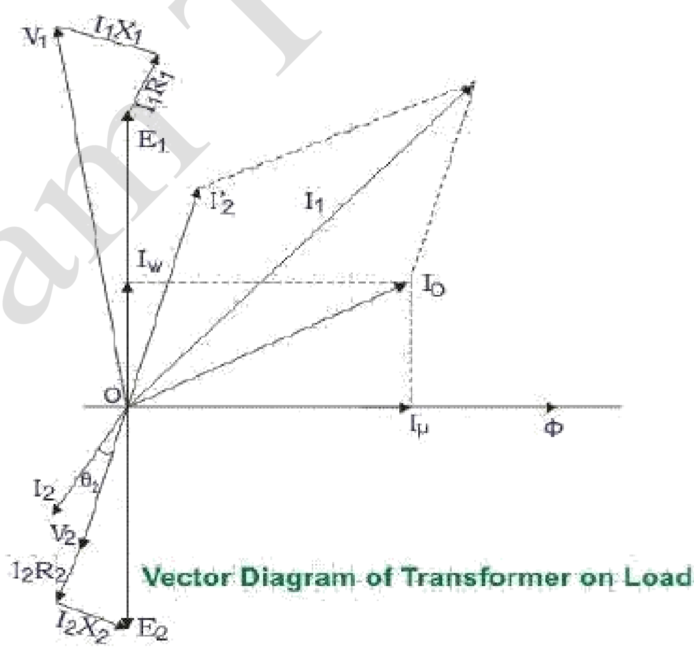
Again in the case of secondary, the voltage induced across the secondary winding, E_2 does not totally appear across the load since it also drops by an amount I_2R_2, where R_2 is the secondary winding resistance and I_2 is secondary current or load current.

Similarly, voltage equation of the secondary side of the transformer will be

$$V_2 = E_2 - I_2R_2$$

Theory of Transformer On Load, With Resistance As Well As Leakage Reactance in Transformer Windings

Now we will consider the condition, when there is leakage reactance of transformer as well as winding resistance of transformer.



Let leakage reactances of primary and secondary windings of the transformer are X_1 and X_2 respectively.

Hence total impedance of primary and secondary winding of transformer with resistance R_1 and R_2 respectively, can be represented as,

$$Z_1 = R_1 + jX_1 \; (impedance\ of\ primary\ winding)$$

$$Z_2 = R_2 + jX_2 \; (impedance\ of\ secondary\ winding)$$

We have already established the voltage equation of a **transformer on load,** with only resistances in the windings, where voltage drops in the windings occur only due to resistive voltage drop. But when we consider leakage reactances of transformer windings, voltage drop occurs in the winding not only because of resistance, it is because of impedance of transformer windings. Hence, actual voltage equation of a transformer can easily be determined by just replacing resistances R_1 & R_2 in the previously established voltage equations by Z_1 and Z_2.

Therefore, the voltage equations are,

$$V_1 = E_1 + I_1 Z_1 \ \& \ V_2 = E_2 - I_2 Z_2$$

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

$$\Rightarrow V_1 = E_1 + I_1 R_1 + jI_1 X_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$$\Rightarrow V_2 = E_2 - I_2 R_2 - j\,I_2 X_2$$

Resistance drops are in the direction of current vector but, reactive drop will be perpendicular to the current vector as shown in the above **vector diagram** of **transformer.**

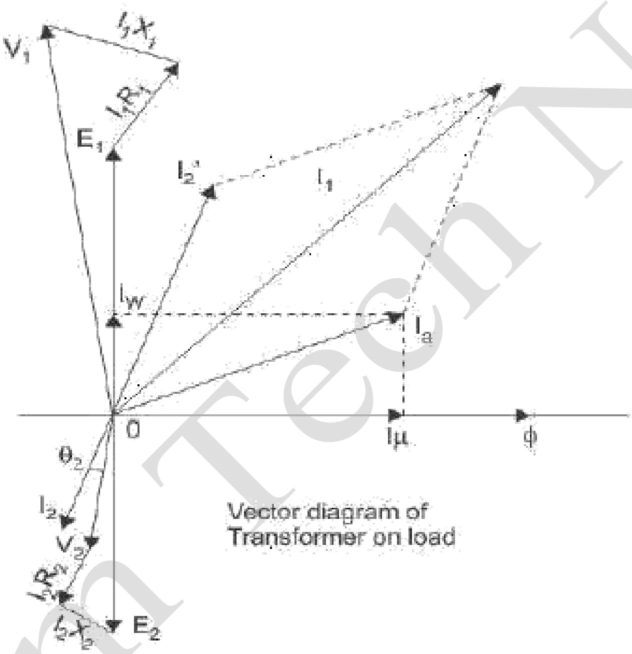
Equivalent Circuit of Transformer

Equivalent impedance of transformer is essential to be calculated because the electrical power transformer is an electrical power system equipment for estimating different parameters of electrical power system which may be required to calculate total internal impedance of an electrical power transformer, viewing from primary side or secondary side as per requirement. This calculation requires equivalent circuit of transformer referred to primary **or** equivalent circuit of transformer referred to secondary sides respectively. Percentage impedance is also very essential parameter of transformer. Special attention is to be given to this parameter during installing a transformer in an existing electrical power system. Percentage impedance of different power transformers should be properly matched during parallel operation of power transformers. The percentage impedance can be derived from equivalent impedance of transformer so, it can be said that equivalent circuit of transformer is also required during calculation of % impedance.

Equivalent Circuit of Transformer Referred to Primary

For drawing equivalent circuit of transformer referred to primary, first we have to establish general equivalent circuit of transformer then, we will modify it for referring from primary side. For doing this, first we need to recall the complete vector diagram of a transformer which is shown in the figure below

.



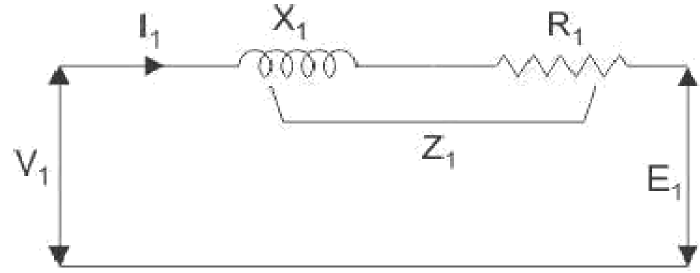
Let us consider the transformation ratio be,

$$K = \frac{N_1}{N_2} = \frac{E_1}{E_2}$$

In the figure above, the applied voltage to the primary is V_1 and voltage across the primary winding is E_1. Total current supplied to primary is I_1. So the voltage V_1 applied to the primary is partly dropped by I_1Z_1 or $I_1R_1 + j.I_1X_1$ before it appears across primary winding. The voltage appeared across winding is countered by primary induced emf E_1. So voltage equation of this

$$\dot{V_1} - (\dot{I_1}R_1 + jI_1X_1) = \dot{E_1}$$

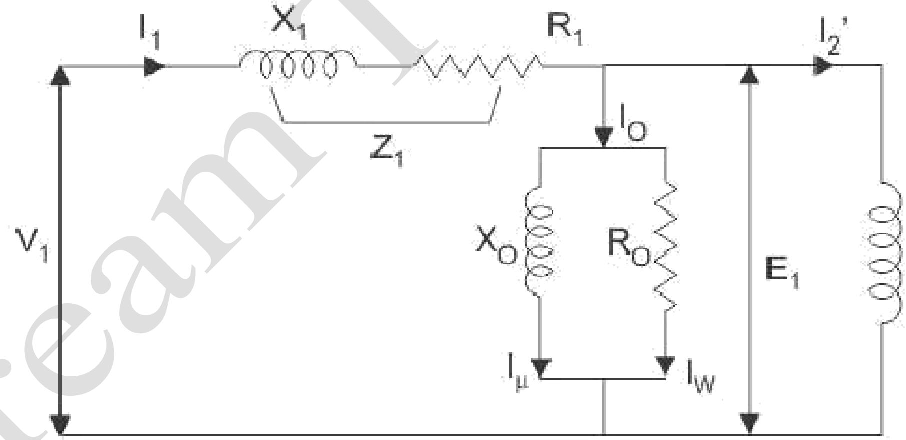
portion of the transformer can be written as, The equivalent circuit for that equation can be drawn as below,



Equivalent Circuit

From the vector diagram above, it is found that the total primary current I_1 has two components, one is no - load component I_o and the other is load component I_2'. As this primary current have two components or branches, so there must be a parallel path with primary winding of transformer. This parallel path of current is known as excitation branch of equivalent circuit of transformer. The resistive and reactive branches of the excitation circuit can be represented as

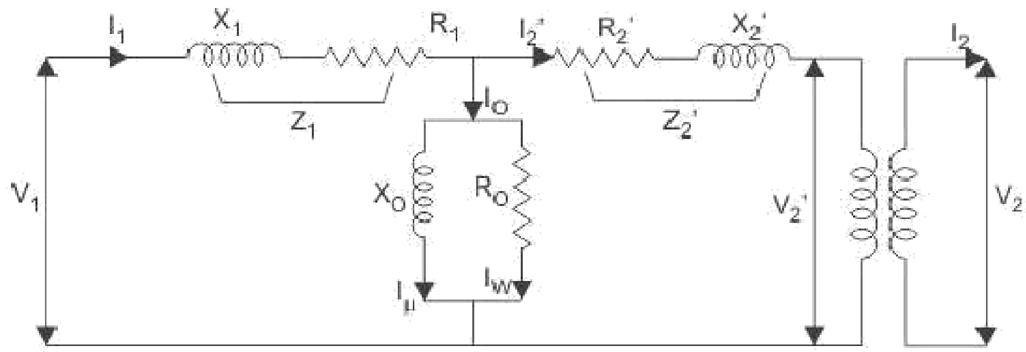
$$R_0 = \frac{E_1}{I_w} \; and \; X_0 = \frac{E_1}{I_\mu}$$



Equivalent Circuit of Primary Side of Transformer

The load component I_2' flows through the primary winding of transformer and induced voltage across the winding is E_1 as shown in the figure right. This induced voltage E_1 transforms to secondary and it is E_2 and load component of primary current I_2' is transformed to secondary as secondary current I_2. Current of secondary is I_2. So the voltage E_2 across secondary winding is partly dropped by I_2Z_2 or $I_2R_2 + j.I_2X_2$ before it appears across load. The load voltage is V_2.

The complete equivalent circuit of transformer is shown below.



Equivalent Circuit of Transformer referred to Primary

Now if we see the voltage drop in secondary from primary side, then it would be ′K′ times greater and would be written as $K.Z_2.I_2$. Again $I_2'.N_1 = I_2.N_2$

$$\Rightarrow I_2 = I_2'\frac{N_1}{N_2}$$

$$\Rightarrow I_2 = KI_2'$$

Therefore,

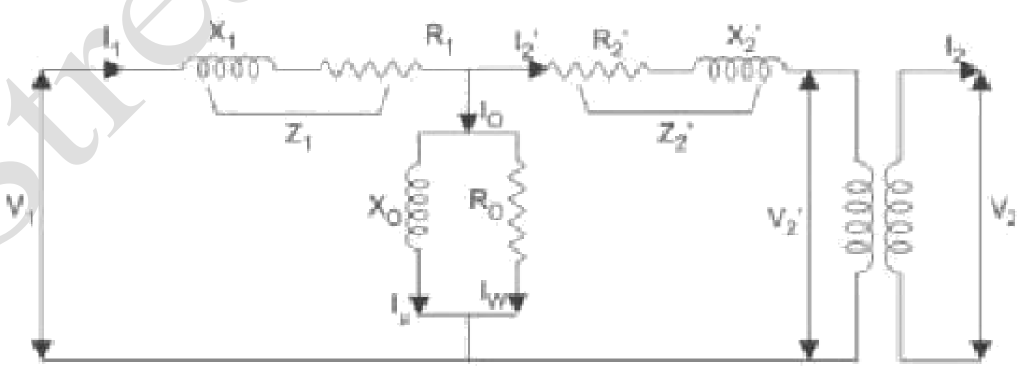
$$KZ_2I_2 = KZ_2KI_2' = K^2Z_2I_2'$$

From above equation, secondary impedance of transformer referred to primary is,

$$Z_2' = K^2Z_2$$

$$Hence,\ R_2' = K^2R_2\ and\ X_2 = K^2X_2$$

So, the complete equivalent circuit of transformer referred to primary is shown in the figure below,

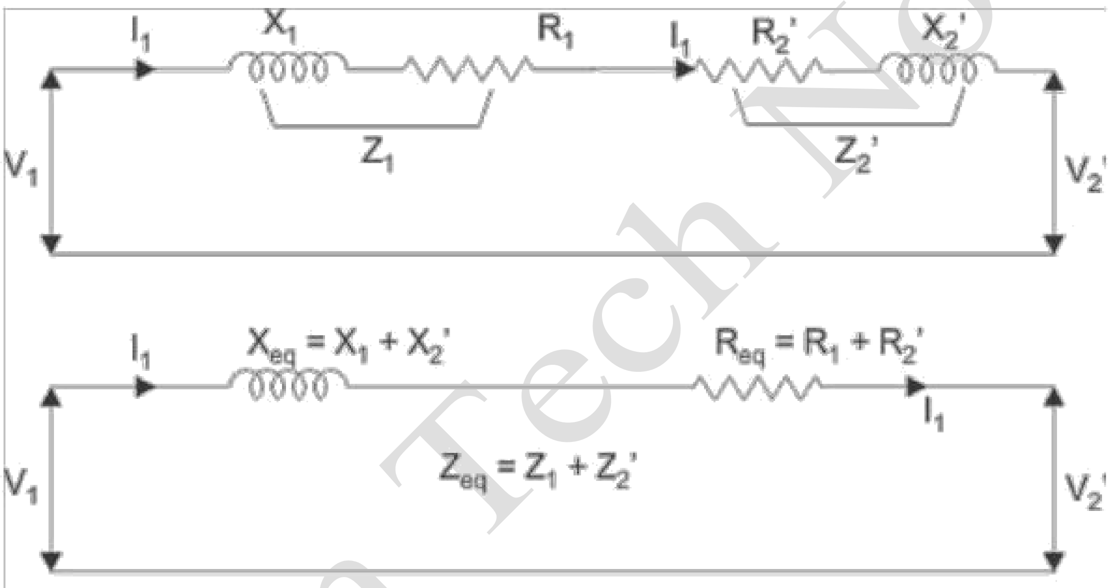


Equivalent Circuit of Transformer referred to Primary

Approximate Equivalent Circuit of Transformer

Since I_0 is very small compared to I_1, it is less than 5% of full load primary current, I_0 changes the voltage drop insignificantly. Hence, it is good approximation to ignore the excitation circuit in approximate equivalent circuit of transformer. The winding resistance and reactance being in series can now be combined into equivalent resistance and reactance of transformer, referred to any particular side. In this case it is side 1 or primary side.

$$Here,\ V_2' = KV_2$$



Approximate Equivalent Circuit of Transformer referred to Primary

Equivalent Circuit of Transformer Referred to Secondary

In similar way, approximate equivalent circuit of transformer referred to secondary can be drawn.

Where equivalent impedance of transformer referred to secondary, can be derived as

$$Z_1 = \frac{Z_1}{K^2}$$

$$Therefore,\ R_1' = \frac{R_1}{K^2}$$

$$X_1' = \frac{X_1}{K^2}$$

$$Here,\ V_1' = \frac{V_1}{K}$$

Losses in Transformer:

Losses of transformer are divided mainly into two types:

1. Iron Loss
2. Copper Losses

IRON LOSS:

This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

a) Eddy current loss

b) Hysterisis loss

a) Eddy Current Loss:

This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

b) Hysterisis Loss:

This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents. Hysterisis loss can be minimized by using the core material having high permeability.

COPPER LOSS:

This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltage and frequency are constants

EFFICIENCY:

It is the ratio of the output power to the input power of a

transformer Input = Output + Total losses

= Output + Iron loss + Copper loss

Efficiency =

$$\eta = \frac{outputpower}{outputpower + Ironloss + copperloss}$$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_{eron} + W_{copper}}$$

Where, V_2 is the secondary (output) voltage, I_2 is the secondary (output) current and $\cos\Phi$ is the power factor of the load.

The transformers are normally specified with their ratings as KVA,

Therefore,

$$Efficiency; \eta = \frac{(KVA) \times 10^3 \times cos\varphi}{(KVA) \times 10^3 \times cos\varphi \times W_{iron} + W_{copper}}$$

Since the copper loss varies as the square of the load the efficiency of the transformer at any desired load n is given by

$$Efficiency; \eta = \frac{n \times (KVA) \times 10^3 \times cos\varphi}{n \times (KVA) \times 10^3 \times cos\varphi \times W_{iron} + n^2 \times W_{copper}}$$

where,

Wcopper is the copper loss at full load

Wcopper = $I^2 R$ watts

CONDITION FOR MAXIMUM EFFICIENCY:

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.

Therefore the condition for maximum efficiency in a transformer is Copper loss = Iron loss

VOLTAGE REGULATION:

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage between no load and full load at a specified power factor expressed as a percentage of the full load terminal voltage.

$$\%Voltage\ Regulation = \frac{(no\ load\ Sec.Voltage) - (full\ load\ Sec.Voltage)}{full\ load\ Sec.Voltage} \times 100$$

Voltage regulation is a measure of the change in the terminal voltage of a transformer between No load and Full load. A good transformer has least value of the regulation of the order of ±5%

O.C. and S.C. Tests on Single Phase Transformer

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

1. Open circuit test (O.C Test)
2. Short circuit test (S.C.Test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters.

Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C test is shown in the Fig. 1.

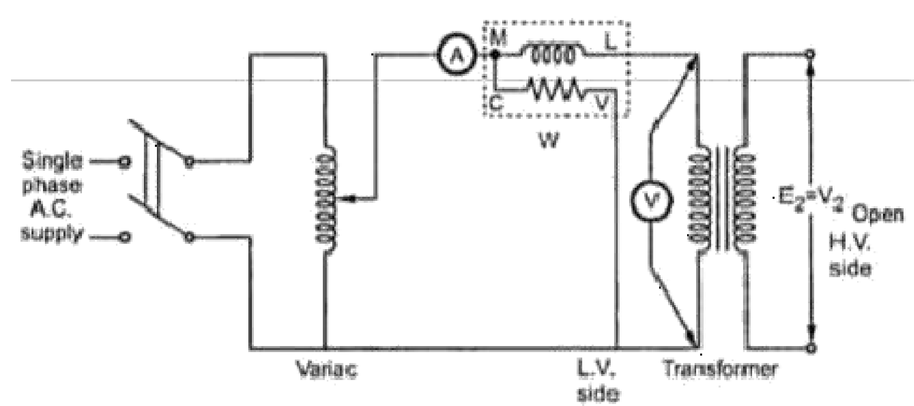


Fig 1. Experimental circuit for O.C. test

The transformer primary is connected to a.c. supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C test.

The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltemeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.

Let,

V_o = Rated voltage

W_o = Input power

I_o = Input current = no load current

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$$I_m = I_o \sin \Phi_o$$

$$I_c = I_o \cos \Phi_o$$

where $\cos \Phi_o$ = No load power factor

And hence power input can be written as,

$$W_o = V_o I_o \cos \Phi_o$$

The phasor diagram is shown in the Fig.

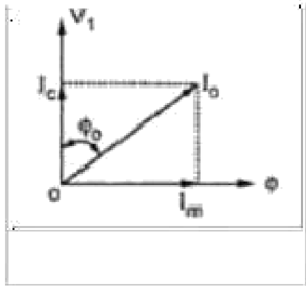


Fig.

As secondary is open, $I_2 = 0$. Thus its reflected current on primary is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses. This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses

which remain constant for all the loads.

$$\therefore \quad W_o = P_i = \text{Iron losses}$$

Calculations : We know that,

$$W_o = V_o \, I_o \cos \Phi$$

$$\cos \Phi_o = W_o / (V_o \, I_o) = \text{no load power factor}$$

Once $\cos \Phi_o$ is known we can obtain,

$$I_c = I_o \cos \Phi_o$$

and $\quad I_m = I_o \sin \Phi_o$

Once I_c and I_m are known we can determine exciting circuit parameters as,

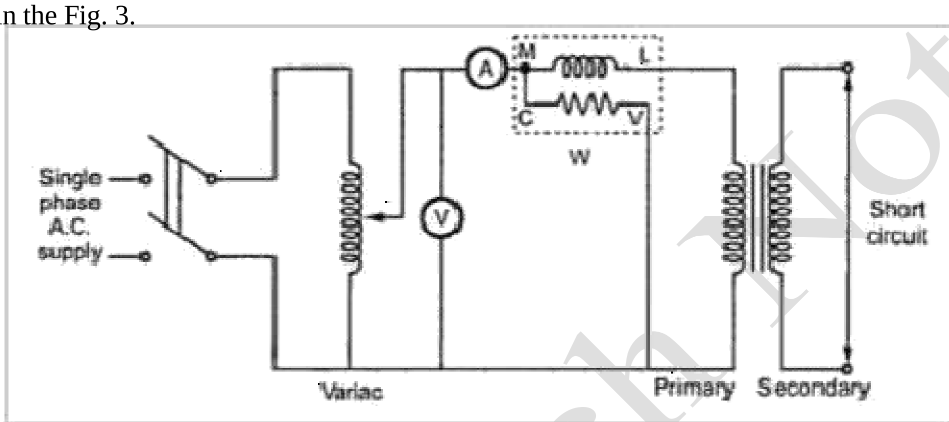
$$R_o = V_o / I_c \ \Omega$$

and $X_o = V_o / I_m\ \Omega$

Key Point : The no load power factor cos Φ_o is very low hence wattmeter used must be low power factor type otherwise there might be error in the results. If the meters are connected on secondary and primary is kept open then from O.C. test we get R_o'**and** X_o' with which we can obtain R_o and X_o knowing the transformation ratio K.

Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 3.



Experimental circuit for O.C. test

The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.

As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage test. The wattmeter reading as well as voltmeter, ammeter readings are recorded.

Now the current flowing through the windings are rated current hence the total copper loss is full load copper loss. Now the voltage supplied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as iron losses are very low.

$\therefore$ $W_{sc} = (P_{cu})$ F.L. = Full load copper loss Calculations :

From S.C. test readings we can write,

$$W_{sc} = V_{sc} I_{sc} \cos \Phi_{sc}$$

$$\therefore \quad \cos \Phi_{sc} = V_{sc} I_{sc} / W_{sc} = \text{short circuit power factor}$$

$$W_{sc} = I_{sc}^2 R_{1e} = \text{copper loss}$$

$$\therefore \quad R_{1e} = W_{sc} / I_{sc}^2$$

while $$Z_{1e} = V_{sc} / I_{sc} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$$

$$\therefore \quad X_{1e} = \sqrt{(Z_{1e}^2 - R_{1e}^2)}$$

Thus we get the equivalent circuit parameters R_{1e}, X_{1e} and Z_{1e}. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

Important Note : If the transformer is step up transformer, its primary is L.V. while secondary is H.V. winding. In S.C. test, supply is given to H.V. winding and L.V is shorted. In such case we connect meters on H.V. side which is transformer secondary through for S.C. test purpose H.V side acts as primary. In such case the parameters calculated from S.C. test readings are referred to secondary which are R_{2e}, Z_{2e} and X_{2e}. So before doing calculations it is necessary to find out where the readings are recorded on transformer primary or secondary and accordingly the parameters are to be determined. In step down transformer, primary is high voltage itself to which supply is given in S.C. test. So in such case test results give us parameters referred to primary i.e. R_{1e}, Z_{1e} and X_{1e}.

Key point : In short, if meters are connected to primary of transformer in S.C. test, calculations give us R_{1e} and Z_{1e} if meters are connected to secondary of transformer in S.C. test calculations give us R_{2e} and Z_{2e}.

Calculation of Efficiency from O.C. and S.C. Tests

We know that,

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu})$ F.L.

$$\therefore \quad \% \eta \text{ on full load} = \frac{V_2 (I_2) \text{ F.L.} \cos \phi}{V_2 (I_2) \text{ F.L.} \cos + W_o + W_{sc}} \times 100$$

Thus for any p.f. cos Φ_2 the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\boxed{\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_0 + n^2 W_{sc}} \times 100}$$

where n = fraction of full load

$$\text{or} \qquad \% \eta = \frac{n V_2 I_2 \cos\phi}{n V_2 I_2 \cos\phi + W_0 + n^2 W_{sc}} \times 100$$

where $I_2 = n (I_2)$ F.L.

Calculation of Regulation

From S.C. test we get the equivalent circuit parameters referred to primary or secondary.

The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\boxed{\begin{aligned} \% R &= \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100 \\ &= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100 \end{aligned}}$$

where I_1, I_2 are rated currents for full load regulation.

For any other load the currents I_1, I_2 must be changed by fraction n.

$\therefore$ I_1, I_2 at any other load = n (I_1) F.L., n (I_2) F.L.

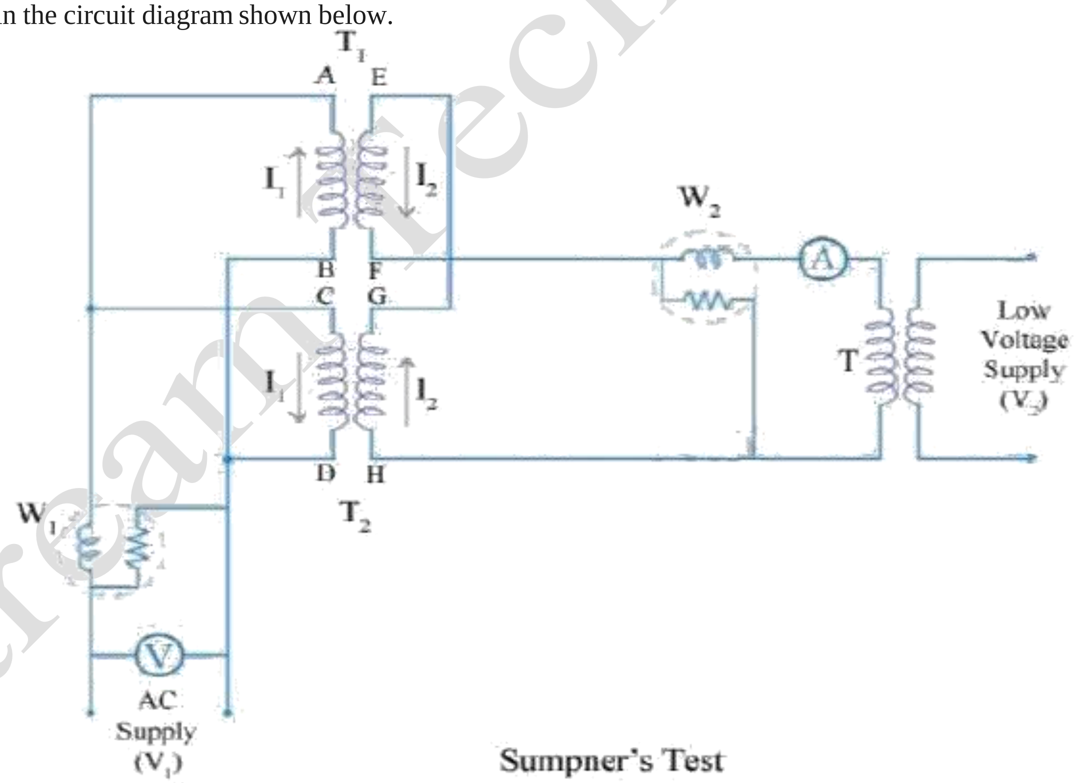
Key Point : Thus regulation at any load and any power factor can be predetermined, without actually loading the transformer.

Sumpner's Test Or Back-To-Back Test On Transformer

Sumpner's test or back to back test on transformer is another method for determining transformer efficiency, voltage regulation and heating under loaded conditions. Short circuit and open circuit tests on transformer can give us parameters of equivalent circuit of transformer, but they cannot help us in finding the heating information. Unlike O.C. and S.C. tests, actual loading is simulated in Sumpner's test. Thus the Sumpner's test give more accurate results of regulation and efficiency than O.C. and S.C. tests.

Sumpner's Test

Sumpner's test or back to back test can be employed only when two identical transformers are available. Both transformers are connected to supply such that one transformer is loaded on another. Primaries of the two identical transformers are connected in parallel across a supply. Secondaries are connected in series such that emf's of them are opposite to each other. Another low voltage supply is connected in series with secondaries to get the readings, as shown in the circuit diagram shown below.



Sumpner's Test

In above diagram, T_1 and T_2 are identical transformers. Secondaries of them are connected in voltage opposition, i.e. E_{EF} and E_{GH}. Both the emf's cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated. The current drawn from V_1 is $2I_0$, where I_0 is equal to no load current of each transformer. Thus input power measured by wattmeter W_1 is equal to iron losses of both transformers.

i.e. iron loss per transformer $P_i = W_1/2$.

Now, a small voltage V_2 is injected into secondary with the help of a low voltage transformer. The voltage V_2 is adjusted so that, the rated current I_2 flows through the secondary. In this case, both primaries and secondaries carry rated current. Thus short circuit test is simulated and wattmeter W_2 shows total full load copper losses of both transformers.

i.e. copper loss per transformer $P_{Cu} = W_2/2$.

From above test results, the **full load efficiency of each transformer** can be given as –

$$\text{\% full load efficiency of each transformer} = \frac{\text{output}}{\text{output} + \frac{W_1}{2} + \frac{W_2}{2}} \times 100$$

Predetermination of Voltage Regulation

Modern power systems operate at some standard voltages. The equipments working on these systems are therefore given input voltages at these standard values, within certain agreed tolerance limits. In many applications this voltage itself may not be good enough for obtaining the best operating condition for the loads. A transformer is interposed in between the load and the supply terminals in such cases. There are additional drops inside the transformer due to the load currents. While input voltage is the responsibility of the supply provider, the voltage at the load is the one which the user has to worry about.

If undue voltage drop is permitted to occur inside the transformer the load voltage becomes too low and affects its performance. It is therefore necessary to quantify the drop that takes place inside a transformer when certain load current, at any power factor, is drawn from its output leads. This drop is termed as the voltage regulation and is expressed as a ratio of the terminal voltage (the absolute value per se is not too important).

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage as can be seen below. Regulation down: This is defined as ‖ the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage.

Expressed in symbolic form we have,

$$Regulation = \frac{|V_{nl}| - |V_l|}{|V_l|}$$

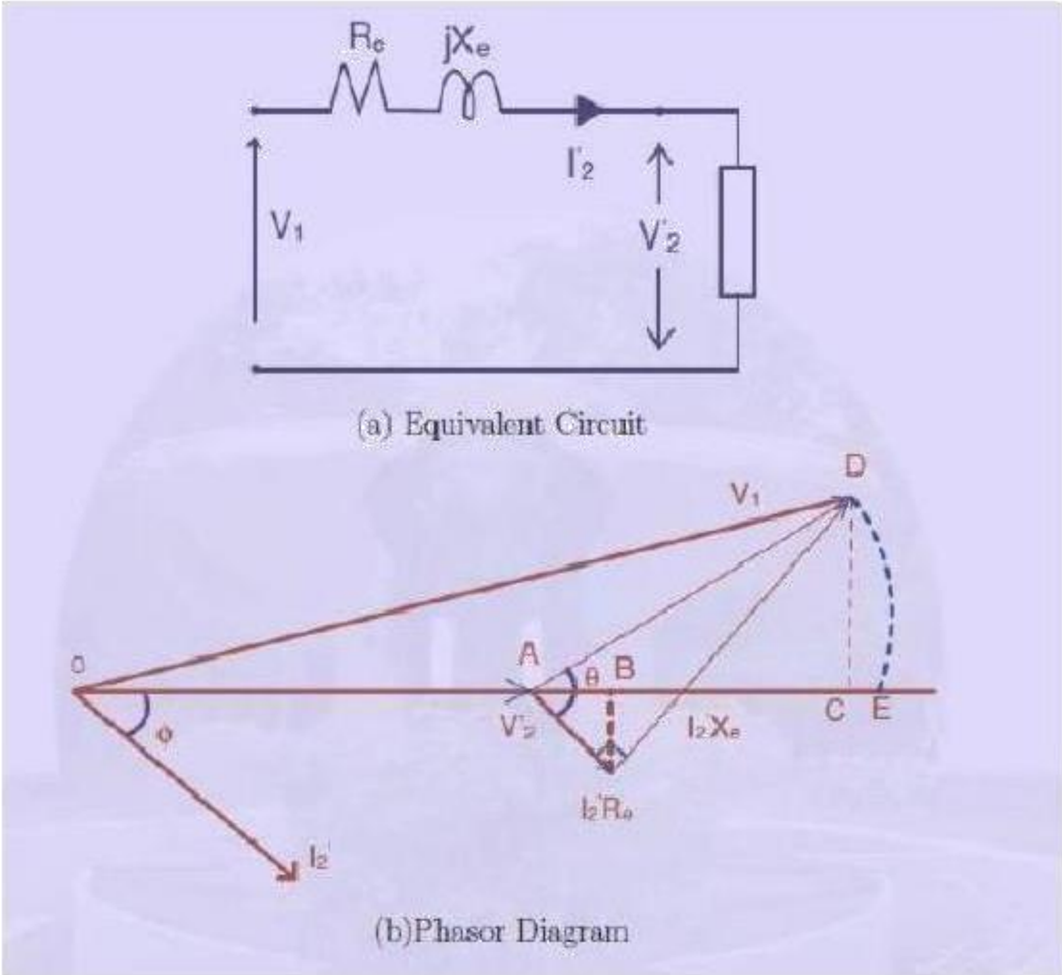
Where,

Vnl is the no-load terminal voltage.

Vl is load voltage.

Normally full load regulation is of interest as the part load regulation is going to be lower. This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application. Fig. 23 shows the phasor diagram of operation of the transformer under loaded condition. The no-load current I0 is neglected in view of the large magnitude of I‘2.



(a) Equivalent Circuit

(b)Phasor Diagram

Fig. Regulation of Transformer

$I_1 = I_2'$.

$$
\begin{aligned}
V_1 &= I_2'(R_e + jX_e) + V_2' \\
OD &= V_1 = \sqrt{[OA + AB + BC]^2 + [CD]^2} \\
&= \sqrt{[V_2' + I_2' R_e \cos\phi + I_2' X_e \sin\phi]^2 + [I_2' X_e \cos\phi - I_2' R_e \sin\phi]^2}
\end{aligned}
$$

ϕ - power factor angle,

θ- internal impedance angle$=\tan^{-1}\frac{X_e}{R_e}$

Also,

$$
\begin{aligned}
V_1 &= V_2' + I_2'.(R_e + jX_e) \\
&= V_2' + I_2'(\cos\phi - j\sin\phi)(R_e + jX_e) \\
\therefore \text{Regulation} R &= \frac{|V_1| - |V_2'|}{|V_2'|} = \sqrt{(1+v_1)^2 + v_2^2} - 1
\end{aligned}
$$

$$
(1+v_1)^2 + v_2^2 \simeq (1+v_1)^2 + v_2^2.\frac{2(1+v_1)}{2(1+v_1)} + [\frac{v_2^2}{2(1+v_1)}]^2 = (1 + v_1 + \frac{v_2^2}{2(1+v_1)})^2
$$

Taking the square root

$$
\sqrt{(1+v_1)^2 + v_2^2} = 1 + v_1 + \frac{v_2^2}{2(1+v_1)}
$$

where $v_1 = e_r\cos\phi + e_x\sin\phi$ and $v_2 = e_x\cos\phi - e_r\sin\phi$

$e_r = \frac{I_2' R_e}{V_2'}$=per unit resistance drop

$e_x = \frac{I_2' X_e}{V_2'}$=per unit reactance drop

as v_1 and v_2 are small.

$$
\therefore R \simeq 1 + v_1 + \frac{v_2^2}{2(1+e_1)} - 1 \simeq v_1 + \frac{v_2^2}{2}
$$

$$
\therefore \text{regulation } R = e_r\cos\phi \pm e_x\sin\phi + \frac{(e_x\sin\phi - e_r\cos\phi)^2}{2}
$$

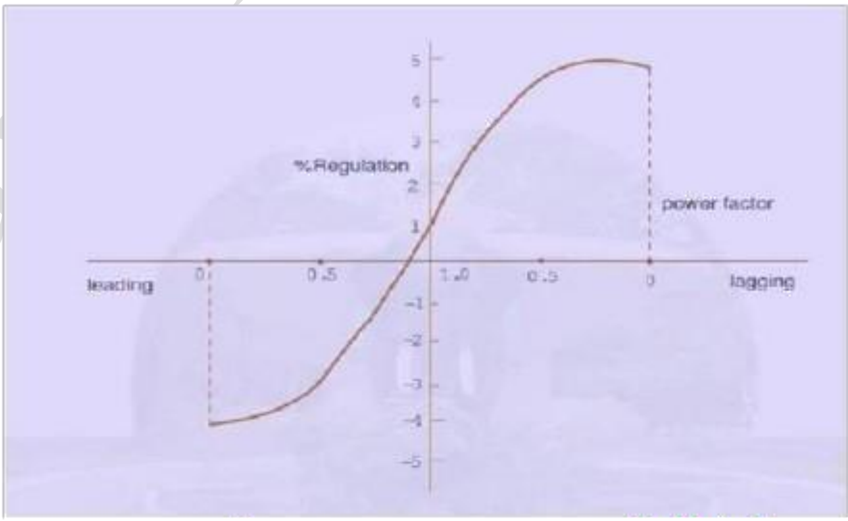


Fig. Variation of full load regulation with power factor

Predetermination of Efficiency

Transformers which are connected to the power supplies and loads and are in operation are required to handle load current and power as per the requirements of the load. An unloaded transformer draws only the magnetization current on the primary side, the secondary current being zero. As the load is increased the primary and secondary currents increase as per the load requirements. The volt amperes and wattage handled by the transformer also increases. Due to the presence of no load losses and I2R losses in the windings certain amount of electrical energy gets dissipated as heat inside the transformer. This gives rise to the concept of efficiency.

Efficiency of power equipment is defined at any load as the ratio of the power output to the power input. Putting in the form of an expression,

$$Efficiency\ \eta = \frac{output\ power}{input\ power} = \frac{Input\ power - losses\ inside\ the\ machine}{Input\ power}$$

$$= 1 - \frac{losses\ inside\ the\ machine}{input\ power} = 1 - defficiency$$

$$= \frac{output\ power}{output + losses\ inside\ the\ machine}$$

More conveniently the efficiency is expressed in percentage. $\%\eta = \frac{output\ power}{input\ power} * 100$

While the efficiency tells us the fraction of the input power delivered to the load, the deficiency focuses our attention on losses taking place inside transformer. As a matter of fact the losses heat up machine. The temperature rise decides the rating of the equipment.

The temperature rise of the machine is a function of heat generated the structural configuration, method of cooling and type of loading (or duty cycle of load). The peak temperature attained directly affects the life of the insulations of the machine for any class of insulation.

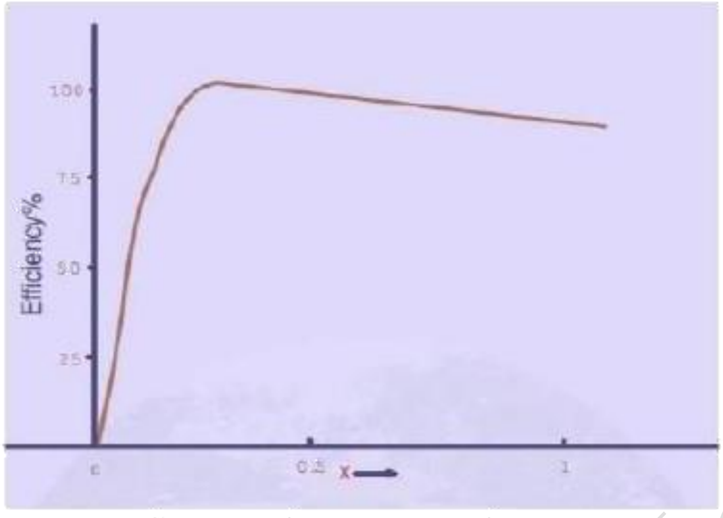


Fig. Efficiency

A typical curve for the variation of efficiency as a function of output is given in Fig. The losses that take place inside the machine expressed as a fraction of the input is sometimes termed as deficiency. Except in the case of an ideal machine, a certain fraction of the input power gets lost inside the machine while handling the power. Thus the value for the efficiency is always less than one. In the case of a.c. machines the rating is expressed in terms of apparent power. It is nothing but the product of the applied voltage and the current drawn. The actual power delivered is a function of the power factor at which this current is drawn. As the reactive power shuttles between the source and the load and has a zero average value over a cycle of the supply wave it does not have any direct effect on the efficiency. The reactive power however increases the current handled by the machine and the losses resulting from it. Therefore the losses that take place inside a transformer at any given load play a vital role in determining the efficiency. The losses taking place inside a transformer can be enumerated as below:

1. Primary copper loss
2. Secondary copper loss
3. Iron loss
4. Dielectric loss
5. Stray load loss

These are explained in sequence below.

Primary and secondary copper losses take place in the respective winding resistances due to the flow of the current in them

$$P_c = I_1^2 r_1 + I_2^2 r_2 = I_2'^2 R_e$$

The primary and secondary resistances differ from their d.c. values due to skin effect and the temperature rise of the windings. While the average temperature rise can be approximately used, the skin effect is harder to get analytically. The short circuit test gives the value of Re taking into account the skin effect.

The iron losses contain two components - Hysteresis loss and Eddy current loss. The Hysteresis loss is a function of the material used for the core.

$$P_h = K_h B^{1.6} f$$

For constant voltage and constant frequency operation this can be taken to be constant. The eddy current loss in the core arises because of the induced emf in the steel lamination sheets and the eddies of current formed due to it. This again produces a power loss Pe in the lamination.

$$P_e = K_e B^2 f^2 t^2$$

where t is the thickness of the steel lamination used. As the lamination thickness is much smaller than the depth of penetration of the field, the eddy current loss can be reduced by reducing the thickness of the lamination. Present day laminations are of 0.25 mm thickness and are capable of operation at 2 Tesla. These reduce the eddy current losses in the core. This loss also remains constant due to constant voltage and frequency of operation. The sum of hysteresis and eddy current losses can be obtained by the open circuit test.

The dielectric losses take place in the insulation of the transformer due to the large electric stress. In the case of low voltage transformers this can be neglected. For constant voltage operation this can be assumed to be a constant.

The stray load losses arise out of the leakage fluxes of the transformer. These leakage fluxes link the metallic structural parts, tank etc. and produce eddy current losses in them. Thus they take place 'all round' the transformer instead of a definite place , hence the name 'stray'. Also the leakage flux is directly proportional to the load current unlike the mutual flux which is proportional to the applied voltage. Hence this loss is called 'stray load' loss. This can also be estimated experimentally. It can be modeled by another resistance in the series branch in the equivalent circuit. The stray load losses are very low in air-cored transformers due to the absence of the metallic tank

Thus, the different losses fall in to two categories Constant losses (mainly voltage dependant) and Variable losses (current dependant). The expression for the efficiency of the transformer operating at a fractional load x of its rating, at a load power factor of Θ2, can be written as

$$\eta = \frac{xS\cos\theta_2}{xS\cos\theta_2 + P_{const} + x^2 P_{var}}$$

Here S in the volt ampere rating of the transformer (V'2 I'2 at full load), Pconst being constant losses and Pvar the variable losses at full load.

UNIT - V

THREE PHASE TRANSFORMERS

5.1 Introduction

Electric power is generated in generating stations, using three phase alternators at 11 KV. This voltage is further stepped up to 66 KV, 110 KV, 230 KV or 400 KV using 3 phase power transformers and power is transmitted at this high voltage through transmission lines. At the receiving substations, these high voltages are stepped down by 3 phase transformers to 11 KV. This is further stepped down to 400 volts at load centers by means of distribution transformers. For generation, transmission and distribution, 3 phase system is economical. Therefore 3 phase transformers are very essential for the above purpose. The sectional view of a 3 phase power transformer is shown in Fig.4.1.

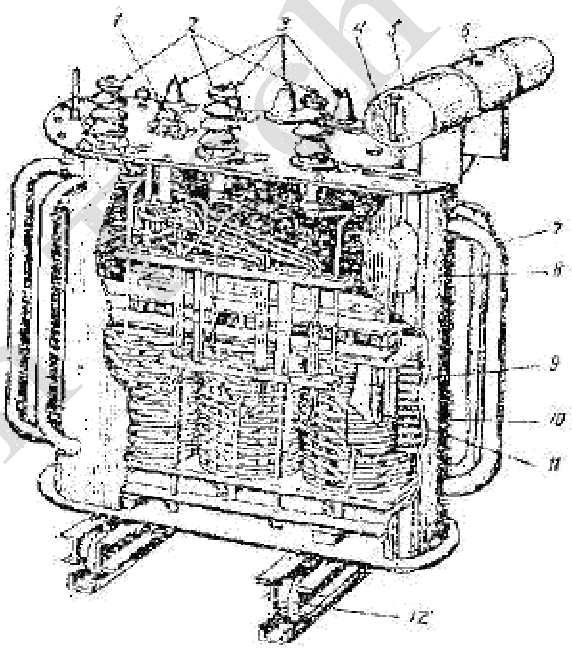


Fig. 4.1 100 KVA oil immersed power transformer

1. Tap-changer switch handle
2. Porcelain-bushing insulator (For high voltage)
3. Bushing insulators (For low voltages)
4. Oil gauge
5. Oil tank
6. Breather plug
7. Cooling pipes
8. Tank front wall

9. Core,
10. High voltage winding
11. Low voltage winding
12. Wheels or rollers.

4.2 Construction of Three phase Transformer

Three phase transformers comprise of three primary and three secondary windings. They are wound over the laminated core as we have seen in single phase transformers. Three phase transformers are also of core type or shell type as in single phase transformers. The basic principle of a three phase transformer is illustrated in fig 4.2 in which the primary windings and secondary windings of three phases are shown. The primary windings can be inter connected in star or delta and put across three phase supply.

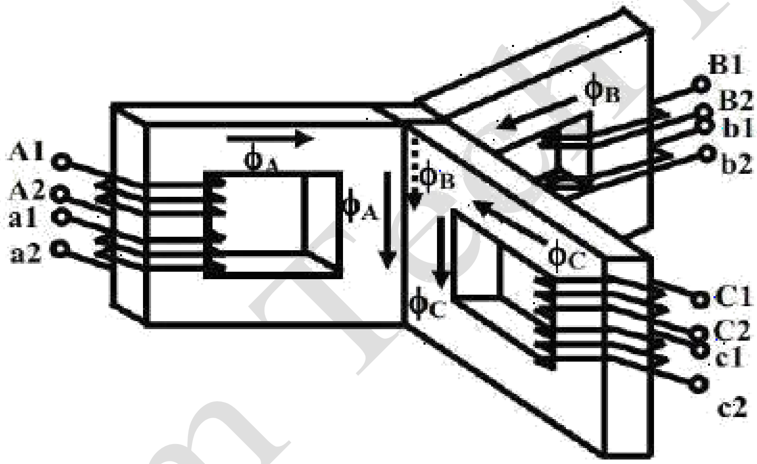


Fig. 4.2 3-phase core-type Transformer

The three cores are 120° apart and their unwound limbs are shown in contact with each other. The center core formed by these three limbs, carries the flux produced by the three phase currents I_R, I_Y and I_B. As at any instant $I_R+I_y+I_B=0$, the sum of three fluxes (flux in the center limb) is also zero.

Therefore it will make no difference if the common limb is removed. All the three limbs are placed in one plane in case of a practical transformer as shown in fig 4.3.

The core type transformers are usually wound with circular cylindrical coils. The construction and assembly of laminations and yoke of a three phase core type transformer is shown in fig 4.4 one method of arrangement of windings in a three phase transformer is shown.

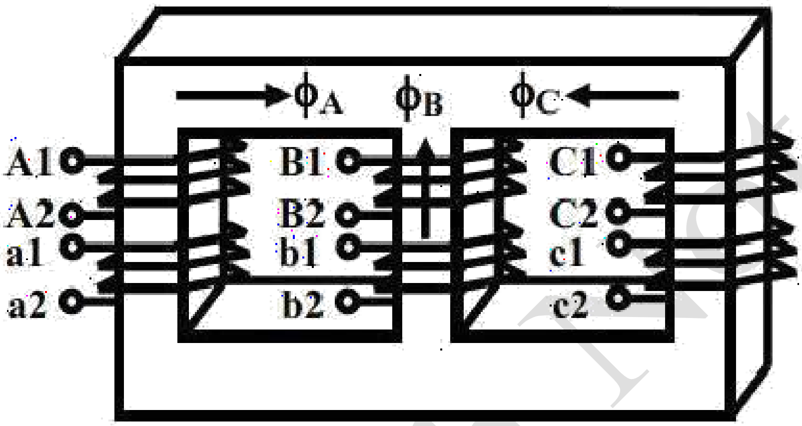


Fig. 4.3 A practical core type three phase transformer

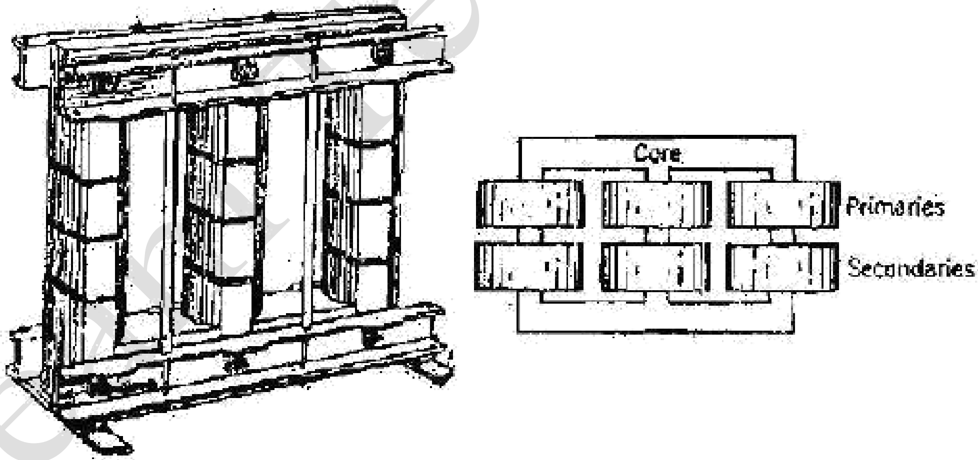


Fig. 4.4 Core type transformer windings and construction

In the other method the primary and secondary windings are wound one over the other in each limb. The low-tension windings are wound directly over the core but are, of course, insulated for it. The high tension windings are wound over the low— tension windings and adequate insulation is provided between the two windings.

The primary and secondary windings of the three phase transformer can also
be interconnected as star or delta.

4.3 Three Phase Transformer connections:-

The identical single phase transformers can be suitably inter-connected and used instead of a single unit 3—phase transformer. The single unit 3 phase transformer is housed in a single tank. But the transformer bank is made up of three separate single phase transformers each with its own, tanks and bushings. This method is preferred in mines and high altitude power stations because transportation becomes easier. Bank method is adopted also when the voltage involved is high because it is easier to provide proper insulation in each single phase transformer.

As compared to a bank of single phase transformers, the main advantages of a single unit 3-phase transformer are that it occupies less floor space for equal rating, less weight costs about 20% less and further that only one unit is to be handled and connected.

There are various methods available for transforming 3 phase voltages to higher or lower 3 phase voltages. The most common connections are (i) star — star (ii) Delta—Delta (iii) Star —Delta (iv) Delta — Star.

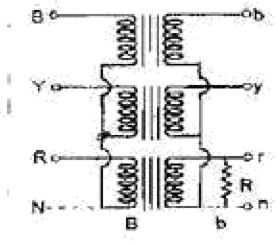


Fig 4.5 Star-star connection

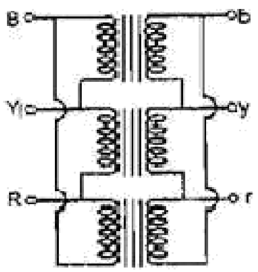


Fig. 4.6 Delta-delta connection

The star-star connection is most economical for small, high voltage transformers because the number of turns per phase and the amount of insulation required is minimum (as phase voltage is only 1/3 of line voltage. In fig. 4.5 a bank of three transformers connected in star on both the primary and the secondary sides is shown. The ratio of line voltages on the primary to the secondary sides is the same as a transformation ratio of single phase transformer.

The delta— delta connection is economical for large capacity, low voltage transformers in which insulation problem is not a serious one. The transformer connection are as shown in fig. 4.6.

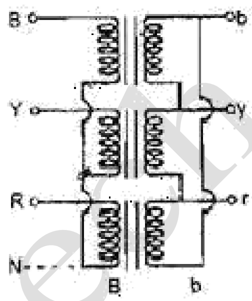


Fig. 4.7 Star-delta connection

The main use of star-delta connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is star connected with grounded neutral as shown in Fig. 4.7. The ratio between the secondary and primary line voltage is 1/3 times the transformation ratio of each single phase transformer. There is a 30° shift between the primary and secondary line voltages which means that a star-delta transformer bank cannot be paralleled with either a star-star or a delta-delta bank.

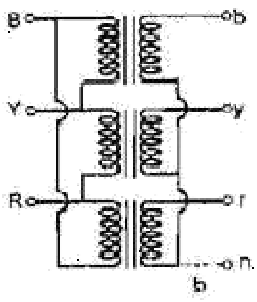


Fig. 4.8 Delta-star connection

Delta-Star connection is generally employed where it is necessary to step up the voltage. The connection is shown in fig. 4.8. The neutral of the secondary is grounded for providing 3-phase, 4-wire service. The connection is very popular because it can be used to serve both the 3-phase power equipment and single phase lighting circuits.

4.4Vector Group of 3-phase transformer

The secondary voltages of a 3-phase transformer may undergo a *phase shift* of either +30° leading or - 30° lagging or 0° i.e, no phase shift or 180° reversal with respective line or phase to neutral voltages. On the name plate of a three phase transformer, the vector group is mentioned. Typical representation of the vector group could be Ydl or Dy 11 etc. The first capital latter Y indicates that the primary is connected in star and the second lower case latter d indicates delta connection of the secondary side. The third numerical figure conveys the angle of phase shift based on *clock convention.* The minute hand is used to represent the primary phase to neutral voltage and always shown to occupy the position 12. The hour hand represents the secondary phase to neutral voltage and may, depending upon phase shift, occupy position other than 12 as shown in the figure 4.9. The angle between two consecutive numbers on the clock is 30°.

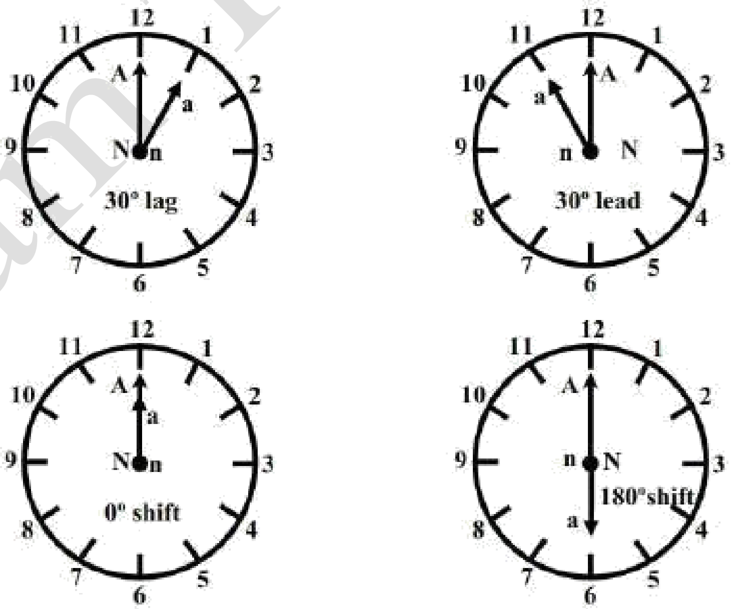


Fig. 4.9 Clock convention representing vector groups

4.4.1 Delta/delta (Dd0, Dd6) connection

The connection of Dd0 is shown in fig. 4.10 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is zero degree (0°).

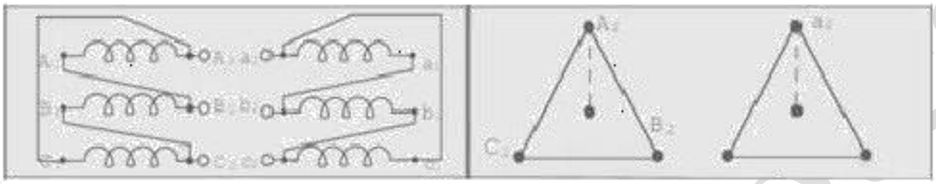


Fig 4.10 Dd0 connection and phasor diagram

The connection of Dd6 is shown in fig. 4.11 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

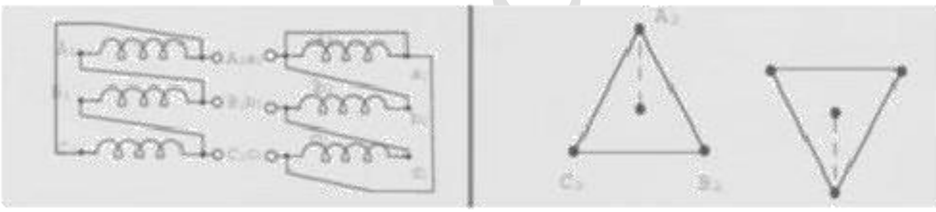


Fig 4.11 Dd6 connection and phasor diagram

This connection proves to be economical for large low voltage transformers as it increases number of turns per phase. Primary side line voltage is equal to secondary side line voltage. Primary side phase voltage is equal to secondary side phase voltage. There is no phase shift between primary and secondary voltages for Dd0 connection. There is 180° phase shift between primary and secondary voltages for Dd6 connection.

Advantages

- **Sinusoidal Voltage at Secondary:**

  In order to get secondary voltage as sinusoidal, the magnetizing current of transformer must contain a third harmonic component. The delta connection provides a closed path for circulation of third harmonic component of current. The flux remains sinusoidal which results in sinusoidal voltages.

- **Suitable for Unbalanced Load:** Even if the load is unbalanced the three phase voltages remains constant. Thus it suitable for unbalanced loading also.
- **Carry 58% Load if One Transfer is Faulty in Transformer Bank** : If there is bank of single phase transformers connected in delta-delta fashion and if one of the transformers is disabled then the supply can be continued with remaining tow transformers of course with reduced efficiency.

No Distortion in Secondary Voltage: there is no any phase displacement between primary and secondary voltages. There is no distortion of flux as the third harmonic component of magnetizing current can flow in the delta connected primary windings without flowing in the line wires .there is no distortion in the secondary voltages.

- **Economical for Low Voltage:** Due to delta connection, phase voltage is same as line voltage hence winding have more number of turns. But phase current is $(1/\sqrt{3})$ times the line current. Hence the cross-section of the windings is very less. This makes the connection economical for low voltages transformers.
- **Reduce Cross section of Conductor:** The conductor is required of smaller Cross section as the phase current is $1/\sqrt{3}$ times of the line current. It increases number of turns per phase and reduces the necessary cross sectional area of conductors thus insulation problem is not present.
- **Absent of Third Harmonic Voltage:** Due to closed delta, third harmonic voltages are absent.
- The absence of star or neutral point proves to be advantageous in some cases.

Disadvantages

- Due to the absence of neutral point it is not suitable for three phase four wire system.
- More insulation is required and the voltage appearing between windings and core will be equal to full line voltage in case of earth fault on one phase.

Application

- Suitable for large, low voltage transformers.
- This Type of Connection is normally uncommon but used in some industrial facilities to reduce impact of SLG faults on the primary system
- It is generally used in systems where it need to be carry large currents on low voltages and especially when continuity of service is to be maintained even though one of the phases develops fault.

4.4.2 Star/star (YyO, Yy6) connection

This is the most economical one for small high voltage transformers. Insulation cost is highly reduced. Neutral wire can permit mixed loading. Triplen harmonics are absent in the lines. These triplen harmonic currents cannot flow, unless there is a neutral wire. This connection produces oscillating neutral. Three phase shell type units have large triplen harmonic phase voltage. However three phase core type transformers work satisfactorily. A tertiary mesh connected winding may be required to stabilize the oscillating neutral due to third harmonics in three phase banks.

The connection of Yy0 is shown in fig. 4.12 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is zero degree (0°).

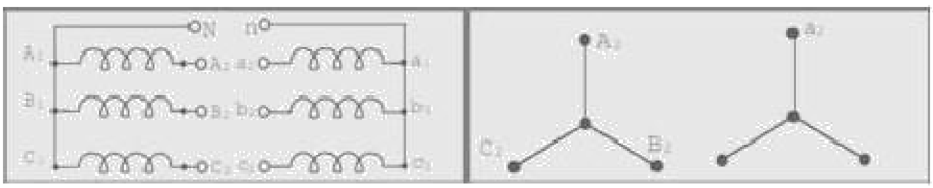


Fig .4.12 Yy0 connection and phasor diagram

The connection of Yy6 is shown in fig. 4.13 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

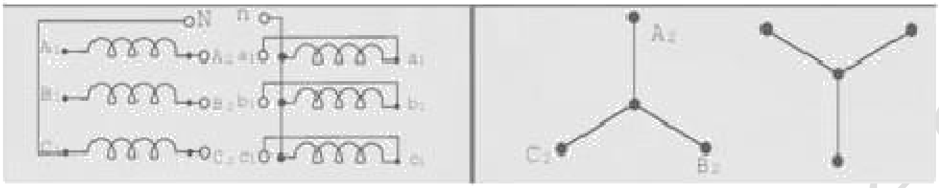


Fig 4.13. Yy6 connection and phasor diagram

- In Primary Winding Each Phase is120°electrical degrees out of phase with the other two phases.

- In Secondary Winding Each Phase is120°electrical degrees out of phase with the other two phases.
- Each primary winding is magnetically linked to one secondary winding through a common core leg. Sets of windings that are magnetically linked are drawn parallel to each other in the vector diagram. In the Y-Y connection, each primary and secondary winding is connected to a neutral point.

- The neutral point may or may not be brought out to an external physical connection and the neutral may or may not be grounded.

Advantages of Y-y connection

- **No Phase Displacement:** The primary and secondary circuits are in phase; i.e., there are no phase angle displacements introduced by the Y-Y connection. This is an important advantage when transformers are used to interconnect systems of different voltages in a cascading manner. For example, suppose there are four systems operating at 800, 440, 220, and 66 kV that need to be interconnected. Substations can be constructed using Y-Y transformer connections to interconnect any two of these voltages. The 800 kV systems can be tied with the 66 kV systems through a single 800 to 66 kV transformation or through a series of cascading transformations at 440,220 and 66 kV.

- **Required Few Turns for winding:** Due to star connection, phase voltages is (1/√3) times the line voltage. Hence less number of turns is required. Also the stress on insulation is less. This makes the connection economical for small high voltage purposes.

- **Required Less Insulation Level**: If the neutral end of a Y-connected winding is grounded, then there is an opportunity to use reduced levels of insulation at the neutral end of the winding. A winding that is connected across the phases requires full insulation throughout the winding.

- **Handle Heavy Load:** Due to star connection, phase current is same as line current. Hence windings have to carry high currents. This makes cross section of the windings high. Thus the windings are mechanically strong and windings can bear heavy loads and short circuit current.

- **Use for Three phases Four Wires System:** As neutral is available, suitable for three phases four wiresystem.

- **Eliminate Distortion in Secondary Phase Voltage:** The connection of primary neutral to the neutral of generator eliminates distortion in the secondary phase voltages by giving path to triple frequency currents toward to generator.

- **Sinusoidal voltage on secondary side:** Neutral give path to flow Triple frequency current to flow Generator side thus sinusoidal voltage on primary will give sinusoidal voltage on secondary side.

- **Used as Auto Transformer:** A Y-Y transformer may be constructed as an autotransformer, with the possibility of great cost savings compared to the two-winding transformer construction.

- **Better Protective Relaying:** The protective relay settings will be protecting better on the line to ground faults when the Y-Y transformer connections with solidly grounded neutrals are applied.

Disadvantages

- **The Third harmonic issue:**

  The voltages in any phase of a Y-Y transformer are 1200 apart from the voltages in any other phase. However, the third-harmonic components of each phase will be in phase with each other. Nonlinearities in the transformer core always lead to generation of third harmonic. These components will add up resulting in large (can be even larger than the fundamental component) third harmonic component.

- **Overvoltage at Lighting Load:** The presence of third (and other zero-sequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load. When constructing a Y-Y transformer using single-phase transformers connected in a bank, the measured line-to-neutral voltages are not 57.7% of the system phase-to-phase voltage at no load but are about 68% and diminish very rapidly as the bank is loaded. The effective values of voltages at different frequencies combine by taking the square root of the sum of the voltages squared. With sinusoidal phase-to-phase voltage, the third-harmonic component of the phase-to-neutral voltage is about 60%.

- **Voltage drop at Unbalance Load:** There can be a large voltage drop for unbalanced phase-to-neutral loads. This is caused by the fact that phase-to-phase loads cause a voltage drop through the leakage reactance of the transformer whereas phase-to-neutral loads cause a voltage drop through the magnetizing reactance, which is 100 to 1000 times larger than the leakage reactance.

- **Overheated Transformer Tank:** Under certain circumstances, a Y-Y connected three-phase trans-can produce severe tank overheating that can quickly destroy the transformer. This usually occurs with an open phase on the primary circuit and load on the secondary.

- **Over Excitation of Core in Fault Condition:** If a phase-to-ground fault occurs on the primary circuit with the primary neutral grounded, then the phase-to-neutral voltage on the un faulted phases increases to 173% of the normal voltage. This would almost certainly result in over excitation of the core, with greatly increased magnetizing currents and core losses

- If the neutrals of the primary and secondary are both brought out, then a phase-to-ground fault on the secondary circuit causes neutral fault current to flow in the primary circuit. Ground protection re- laying in the neutral of the primary circuit may then operate for faults on the secondary circuit

- **Neutral Shifting:** If the load on the secondary side unbalanced then the performance of this connection is not satisfactory then the shifting of neutral point is possible. To prevent this, star point of the primary is required to be connected to the star point of the generator.

- **Distortion of Secondary voltage:** Even though the star or neutral point of the primary is earthed, the third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages.

- **Over Voltage at Light Load:** The presence of third (and other zero-sequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load.

Difficulty in coordination of Ground Protection: In Y-Y Transformer, a low-side ground fault causes primary ground fault current, making coordination more difficult.

- **Increase Healthy Phase Voltage under Phase to ground Fault:** If a phase-to-ground fault occurs on the

  primary circuit with the primary neutral grounded, then the phase-to-neutral voltage on the UN faulted phase's increases to 173% of the normal voltage. If the neutrals of the primary and secondary are both brought out, then a phase-to-ground fault on the secondary circuit causes neutral fault current to flow in the primary circuit.

- **Trip the T/C in Line-Ground Fault:** All harmonics will propagate through the transformer, zero-sequence current path is continuous through the transformer, one line-to-ground fault will trip the transformer.

- **Suitable for Core Type Transformer:** The third harmonic voltage and current is absent in such type of connection with three phase wire system or shell type of three phase units, the third harmonic phase voltage may be high. This type of connection is more suitable for core type transformers.

Application

- This Type of Transformer is rarely used due to problems with unbalanced loads.

- It is economical for small high voltage transformers as the number of turns per phase and the amount of insulation required is less.

4.4.3 Star/Delta connection(Yd1/Yd11)

There is a +30 Degree or -30 Degree Phase Shift between Secondary Phase Voltage to Primary Phase Voltage. The connection of Yd1 is shown in fig. 4.14 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

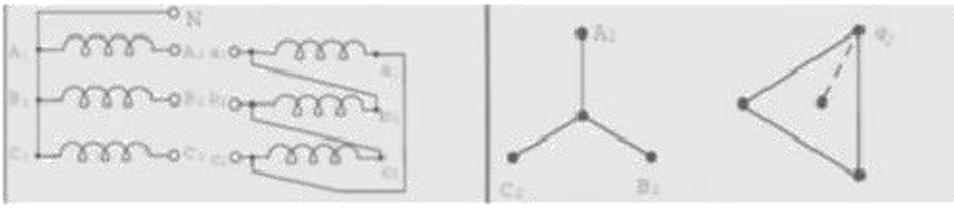


Fig 4.14. Yd1 connection and phasor diagram

The connection of Yd11 is shown in fig. 4.15 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

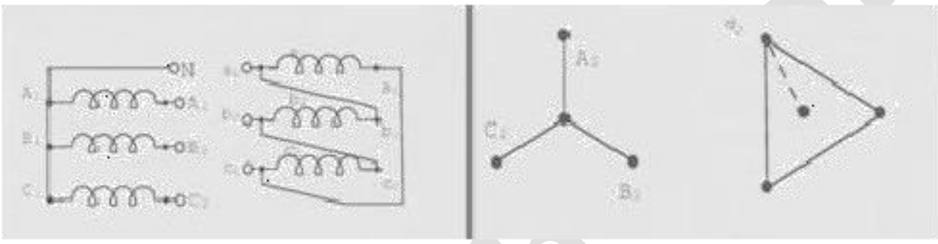


Fig 4.15. Yd11 connection and phasor diagram

Advantages

- The primary side is star connected. Hence fewer numbers of turns are required. This makes the connection economical for large high voltage step down power transformers.
- The neutral available on the primary can be earthed to avoid distortion.
- The neutral point allows both types of loads (single phase or three phases) to be met.
- Large unbalanced loads can be handled satisfactory.
- The Y-D connection has no problem with third harmonic components due to circulating currents inD. It is also more stable to unbalanced loads since the D partially redistributes any imbalance that occurs.

- The delta connected winding carries third harmonic current due to which potential of neutral point is stabilized. Some saving in cost of insulation is achieved if HV side is star connected. But in practice the HV side is normally connected in delta so that the three phase loads like motors and single phase loads like lighting loads can be supplied by LV side using three phase four wire system.

- **As Grounding Transformer:** In Power System Mostly grounded Y- Δ transformer is used for no other purpose than to provide a good ground source in ungrounded Delta system.

Disadvantages

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.

- One problem associated with this connection is that the secondary voltage is shifted by 30^0 with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.

- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem

Application

- It is commonly employed for power supply transformers.

- This type of connection is commonly employed at the substation end of the transmission line. The main use with this connection is to step down the voltage. The neutral available on the primary side is grounded. It can be seen that there is phase difference of 30° between primary and secondary line voltages.

- Commonly used in a step-down transformer, Y connection on the HV side reduces insulation costs the neutral point on the HV side can be grounded, stable with respect to unbalanced loads. As for example, at the end of a transmission line. The neutral of the primary winding is earthed. In this system, line voltage ratio is 1/√3 Times of transformer turn-ratio and secondary voltage lags behind primary voltage by 30°. Also third harmonic currents flows in

4.4.4 Delta-star connection (Dy1/Dy11)

In this type of connection, the primary connected in delta fashion while the secondary current is connected in star. There is s +30 Degree or -30 Degree Phase Shift between Secondary Phase Voltage to Primary Phase Voltage.

The connection of Dy1 is shown in fig. 4.16 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

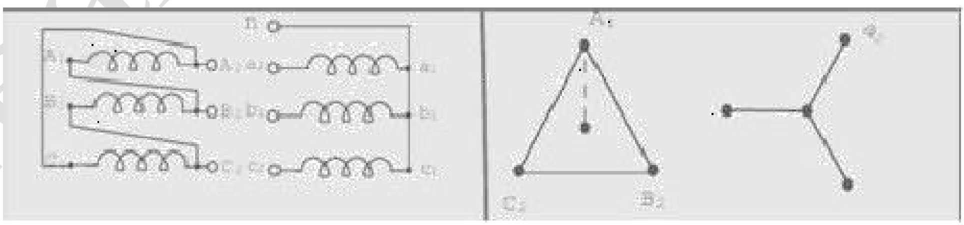


Fig 4.16. Dy1 connection and phasor diagram

The connection of Dy11 is shown in fig. 4.17 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

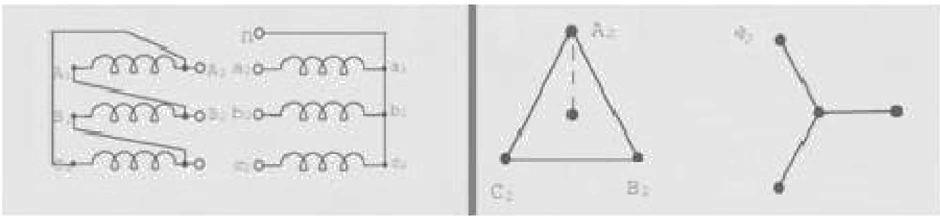


Fig 4.17. Dy11 connection and phasor diagram

Advantages

- **Cross section area of winding is less at Primary side:** On primary side due to delta connection winding cross-section required is less.
- **Used at Three phase four wire System:** On secondary side, neutral is available, due to which it can be used for 3-phase, 4 wire supply system.
- **No distortion of Secondary Voltage:** No distortion due to third harmonic components.
- **Handled large unbalanced Load:** Large unbalanced loads can be handled without any difficulty.
- **Grounding Isolation between Primary and Secondary:** Assuming that the neutral of the Y-connected secondary circuit is grounded, a load connected phase-to-neutral or a phase-to-ground fault produces two equal and opposite currents in two phases in the primary circuit without any neutral ground current in the primary circuit. Therefore, in contrast with the Y-Y connection, phase-to-ground faults or current unbalance in the secondary circuit will not affect ground protective relaying applied to the primary circuit. This feature enables proper coordination of protective devices and is a very important design consideration.
- The neutral of the Y grounded is sometimes referred to as a grounding bank, because it provides a local source of ground current at the secondary that is isolated from the primary circuit.

- **Harmonic Suppression:** The magnetizing current must contain odd harmonics for the induced voltages to be sinusoidal and the third harmonic is the dominant harmonic component. In a three-phase system the third harmonic currents of all three phases are in phase with each other because they are zero-sequence currents. In the Y-Y connection, the only path for third harmonic current is through the neutral. In the Δ -Y connection, however, the third harmonic currents, being equal in amplitude and in phase with each other, are able to circulate around the path formed by the Δ connected winding. The same thing is true for the other zero-sequence harmonics.

- **Grounding Bank:** It provides a local source of ground current at the secondary that is isolated from the primary circuit. For suppose an ungrounded generator supplies a simple radial system through Δ-Y transformer with grounded Neutral at secondary as shown Figure. The generator can supply a single-phase-to-neutral load through the -grounded Y transformer.

Disadvantages

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.

- One problem associated with this connection is that the secondary voltage is shifted by 30^0 with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled.

Therefore, we must pay attention to these shifts.

- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem.

Application

- **Commonly used in a step-up transformer:** As for example, at the beginning of a HT transmission

  line. In this case neutral point is stable and will not float in case of unbalanced loading. There is no distortion of flux because existence of a -connection allows a path for the third-harmonic components. The line voltage ratio is √3 times of transformer turn-ratio and the secondary voltage leads the primary one by 30°. In recent years, this arrangement has become very popular for distribution system as it provides 3- Ø, 4-wire system.

- **Commonly used in commercial, industrial, and high-density residential locations**: To supply three-

  phase distribution systems. An example would be a distribution transformer with a delta primary, running on three 11kV phases with no neutral or earth required, and a star (or wye) secondary providing a 3-phase supply at 400 V, with the domestic voltage of 230 available between each phase and an earthed neutral point.

- **Used as Generator Transformer:** The Δ-Y transformer connection is used universally for connecting generators to transmission systems.

Delta-zigzag and Star zigzag connections (Dz0/Dz6 & Yz1/Yz6) –

The connection of Dz0 is shown in fig. 4.18 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 0°.

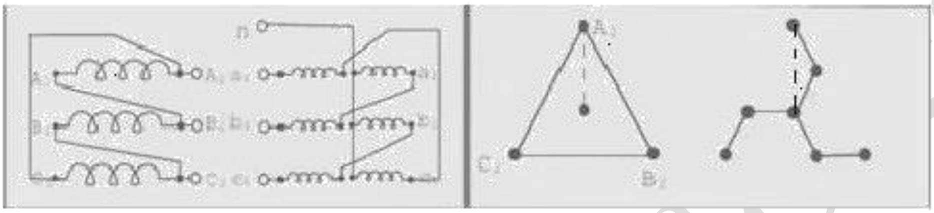


Fig 4.18. Dz0 connection and phasor diagram

The connection of Dz6 is shown in fig. 4.19 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

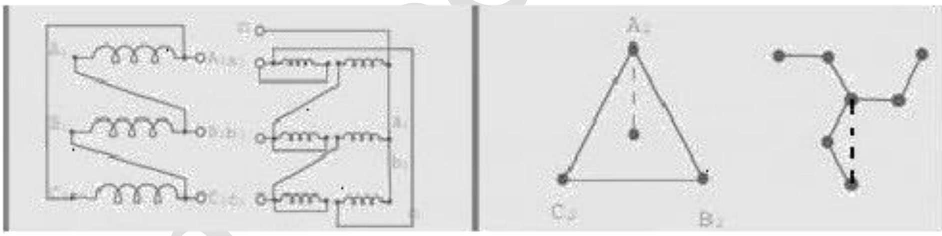


Fig 4.19. Dz6 connection and phasor diagram

The connection of Yz1 is shown in fig. 4.20 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

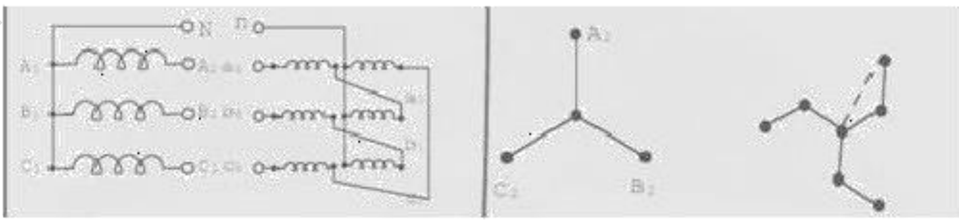


Fig 4.20. Yz1 connection and phasor diagram

The connection of Yz11 is shown in fig. 4.21 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

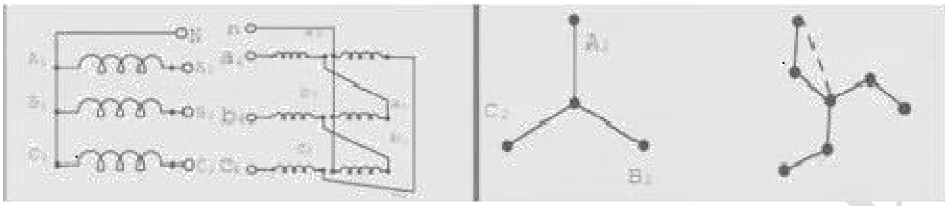


Fig 4.22 Yz11 connection and phasor diagram

- These connections are employed where delta connections are weak. Interconnection of phases in zigzag winding effects a reduction of third harmonic voltages and at the same time permits unbalanced loading.

- This connection may be used with either delta connected or star connected winding either for step-up or step-down transformers. In either case, the zigzag winding produces the same angular displacement as a delta winding, and at the same time provides a neutral for earthing purposes.

- The amount of copper required from a zigzag winding in 15% more than a corresponding star or delta winding. This is extensively used for earthing transformer.

- Due to **zigzag** connection (interconnection between phases), third harmonic voltages are reduced. It also allows unbalanced loading. The zigzag connection is employed for LV winding. For a given total voltage per phase, the zigzag side requires 15% more turns as compared to normal phase connection. In cases where delta connections are weak due to large number of turns and small cross sections, then zigzag star connection is preferred. It is also used in rectifiers.

4.5 Scott connection

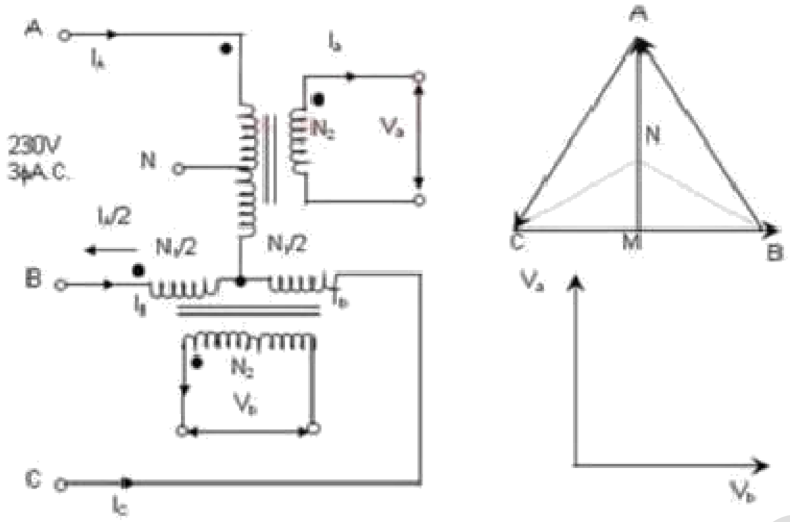
There are two main reasons for the need to transform from three phases to two phases,

1. To give a supply to an existing two phase system from a three phase supply.
2. To supply two phase furnace transformers from a three phase source.

Two-phase systems can have 3-wire, 4-wire, or 5-wire circuits. It is needed to be considering that a two-phase system is not 2/3 of a three-phase system. Balanced three-wire, two-phase circuits have two phase wires, both carrying approximately the same amount of current, with a neutral wire carrying 1.414 times the currents in the phase wires. The phase-to-neutral voltages are 90° out of phase with each other.

Two phase 4-wire circuits are essentially just two ungrounded single-phase circuits that are electrically 90° out of phase with each other. Two phase 5-wire circuits have four phase wires plus a neutral; the four phase wires are 90° out of phase with each other.

A Scott-T transformer (also called a Scott connection) is a type of circuit used to derive two-phase power from a three-phase source or vice-versa. The Scott connection evenly distributes a balanced load between the phases of the source. Scott T Transformers require a three phase power input and provide two equal single phase outputs called Main and Teaser. The MAIN and Teaser outputs are 90 degrees out of phase. The MAIN and the Teaser outputs must not be connected in parallel or in series as it creates a vector current imbalance on the primary side. MAIN and Teaser outputs are on separate cores. An external jumper is also required to connect the primary side of the MAIN and Teaser sections. The schematic of a typical Scott T Transformer is shown below:



4.23 Connection diagram of Scott-connected transformer and vector relation of input and output

From the phasor diagram it is clear that the secondary voltages are of two phases with equal magnitude and 90° phase displacement.

Scott T Transformer is built with two single phase transformers of equal power rating. Assuming the desired voltage is the same on the two and three phase sides, the Scott-T transformer connection consists of a center-tapped 1:1 ratio main transformer, T1, and an 86.6% (0.5√3) ratio teaser transformer, T2. The center-tapped side of T1 is connected between two of the phases on the three-phase side. Its center tap then connects to one end of the lower turn count side of T2, the other end connects to the remaining phase. The other side of the transformers then connects directly to the two pairs of a two-phase four-wire system.

If the main transformer has a turn’s ratio of 1: 1, then the teaser transformer requires a turn’s ratio of

0.866: 1 for balanced operation. The principle of operation of the Scott connection can be most easily seen by first applying a current to the teaser secondary windings, and then applying a current to the main secondary winding, calculating the primary currents separately and superimposing the results.

The primary three-phase currents are balanced; i.e., the phase currents have the same magnitude and their phase angles are 120° apart. The apparent power supplied by the main transformer is greater than the apparent power supplied by the teaser transformer. This is easily verified by observing that the primary currents in both transformers have the same magnitude; however, the primary voltage of the teaser transformer is only 86.6% as great as the primary voltage of the main transformer. Therefore, the teaser transforms only 86.6% of the apparent power transformed by the main.

- The total real power delivered to the two phase load is equal to the total real power supplied from the three-phase system, the total apparent power transformed by both transformers is greater than the total apparent power delivered to the two-phase load.

- The apparent power transformed by the teaser is 0.866 X IH1= 1.0 and the apparent power transformed by the main is 1.0X IH2 =1.1547 for a total of 2.1547 of apparent power transformed.

- The additional 0.1547 per unit of apparent power is due to parasitic reactive power owing between the two halves of the primary winding in the main transformer.

- Single-phase transformers used in the Scott connection are specialty items that are virtually impossible to buy ‘‘off the shelf ’’ nowadays. In an emergency, standard distribution transformers can be used.

If desired, a three phase, two phase, or single phase load may be supplied simultaneously using scott-connection. The neutral points can be available for grounding or loading purposes. The Scott T connection in theory would be suitable for supplying a three, two and single phase load simultaneously, but such loads are not found together in modern practice.

The Scott T would not be recommended as a connection for 3 phase to 3 phase applications for the following reasons:

The loads of modern buildings and office buildings are inherently unbalanced and contain equipment that can be sensitive to potential voltage fluctuations that may be caused by the Scott T design.

A properly sized Scott T transformer will have to be a minimum of 7.75% larger than the equivalent Delta-Wye transformer. Properly sized, it would be a bulkier and heavier option and should not be considered a less expensive solution.

4.6 Open Delta or V-Connection

As seen previously in connection of three single phase transformers that if one of the transformers is unable to operate then the supply to the load can be continued with the remaining two transformers at the cost of reduced efficiency. The connection that obtained is called V-V connection or open delta connection.

Consider the Fig. 4.24 in which 3 phase supply is connected to the primaries. At the secondary side three equal three phase voltages will be available on no load.

The voltages are shown on phasor diagram. The connection is used when the three phase load is very very small to warrant the installation of full three phase transformer.

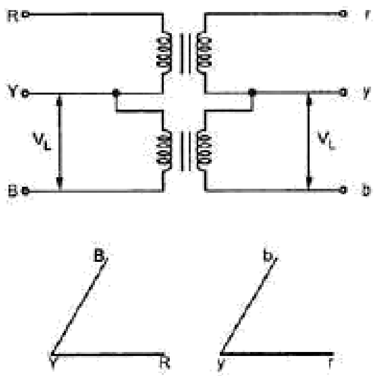


Fig. 4.24 Open delta connection of transformer at noload

If one of the transformers fails in Δ - Δ bank and if it is required to continue the supply even though at reduced capacity until the transformer which is removed from the bank is repaired or a new one is installed then this type of connection is most suitable.

When it is anticipated that in future the load increase, then it requires closing of open delta. In such cases open delta connection is preferred. It can be noted here that the removal of one of the transformers will not give the total load carried by V - V bank as two third of the capacity of Δ - Δ bank.

The load that can be carried by V - V bank is only 57.7% of it.

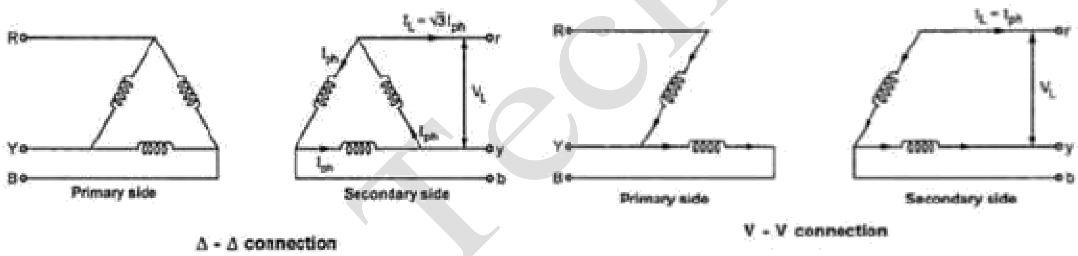


Fig. 4.25 Delta-delta and V-V connection

It can be seen from the Fig. 4.25 of delta delta connection that

$\Delta - \Delta$ capacity $= \sqrt{3}\, V_L I_L = \sqrt{3}\, V_L (\sqrt{3}\, I_{ph})$

$\Delta - \Delta$ capacity $= 3\, V_L I_{ph}$

It can also be noted from the Fig. 4.25 V-V connection that the secondary line current I_L is equal to the phase current I_{ph}.

V- V capacity $= \sqrt{3}\, V_L I_L = \sqrt{3}\, V_L I_{ph}$

$$\text{So, } \frac{\text{V-V capacity}}{\Delta\text{-}\Delta \text{ capacity}} = \frac{\sqrt{3} V_L I_{ph}}{3 V_L I_{ph}} = \frac{1}{\sqrt{3}} = 0.577 \simeq 58\%$$

Thus the three phase load that can be carried without exceeding the ratings of the transformers is 57.5 percent of the original load. Hence it is not 66.7 % which was expected otherwise.

The reduction in the rating can be calculated as {(66.67 - 57.735)/(57.735)}x 100 = 15.476

Suppose that we consider three transformers connected in Δ - Δ fashion and supplying their rated load. Now one transformer is removed then each of the remaining two transformers will be overloaded. The overload on each transformer will be given as,

$$\frac{\text{Total load in V-V}}{\text{VA rating of each transformer}} = \frac{\sqrt{3} V_L I_{ph}}{V_L I_{ph}} = \sqrt{3} = 1.73$$

This overload can be carried temporarily if provision is made to reduce the load otherwise overheating and breakdown of the remaining two transformers would take place.

- <u>The limitation with V -V connection are given below :</u>

  The average p.f. at which V- V bank is operating is less than that with the load .

  This power p.f is 86.6 % of the balanced load p.f.

- The two transformers in V -V bank operate at different power factor except for balanced unity p.f .load.

- The terminals voltages available on the secondary side become unbalanced. This may happen even though load is perfectly balanced.

- Thus in summary we can say that if tow transformers are connected in V - V fashion and are loaded to rated capacity and one transformer is added to increase the total capacity by √3 or

  173.2 %. Thus the increase in capacity is 73.2 % when converting from a V - V system to a Δ-Δ system.

- With a bank of tow single phase transformers connected in V-V fashion supplying a balanced

3 phase load with cosΦ asp.f., one of the transformer operate at a p.f. of cos (30-Φ) and other at cos (30+Φ). The powers of tow transformers are given by,

$$P_1 = KVA \cos (30-\Phi)$$

$$P_2 = KVA \cos (30+\Phi)$$

4.7Oscillating Neutral

In addition to the operation of transformers on the sinusoidal supplies, the harmonic behavior becomes important as the size and rating of the transformer increases. The effects of the harmonic currents are

1. Additional copper losses due to harmonic currents
2. Increased core losses
3. Increased electro-magnetic interference with communication circuits. On the other hand the harmonic voltages of the transformer cause

1. Increased dielectric stress on insulation
2. Electro static interference with communication circuits.
3. Resonance between winding reactance and feeder capacitance.

In the present times a greater awareness is generated by the problems of harmonic voltages and currents produced by non-linear loads like the power electronic converters. These combine with non-linear nature of transformer core and produce severe distortions in voltages and currents and increase the power loss. Thus the study of harmonics is of great practical significance in the operation of transformers.

In the case of single phase transformers connected to form three phase bank, each transformer is magnetically decoupled from the other. The flow of harmonic currents are decided by the type of the electrical connection used on the primary and secondary sides. Also, there are three fundamental voltages in the present case each displaced from the other by 120 electrical degrees. Because of the symmetry of the a.c. wave about the time axis only odd harmonics need to be considered. The harmonics which are triplen (multiples of three) behave in a similar manner as they are co-phasal or in phase in the three phases. The non-triplen harmonics behave in a similar manner to the fundamental and have ±120° phase displacement between them.

When the connection of the transformer is Yy without neutral wires both primary and secondary connected in star no closed path exists. As the triplen harmonics are always in phase, by virtue of the Y connection they get canceled in the line voltages. Non-triplen harmonics like fundamental, become 0 times phase value and appear in the line voltages. Line currents remain sinusoidal except for non-triplen harmonic currents. Flux wave in each transformer will be flat topped and the phase voltages remain peaked. The potential of the neutral is no longer steady. The star point oscillates due to the third harmonic voltages. This is termed as "oscillating neutral".

4.8 Tertiary winding

Apart from the Primary & Secondary windings, there sometimes placed a third winding in power transformers called "Tertiary Winding". Its purpose is to provide a circulating path for the harmonics (especially third harmonics) produced in the transformers along with power frequency (50Hz. third harmonic means 150 Hz oscillations). In delta-delta, delta-star and star-delta transformers all voltages are balanced and there is no floating of neutral or oscillating neutral. The floating of neutral is developed in the case star-star connection only. The transformers are sometimes constructed with three windings. The main windings are connected to form star-star connection and the third winding known as tertiary winding is used to make a closed delta connection to stabilize the neutrals of both primary and secondary circuits. The tertiary winding carries the third-harmonic currents.

4.9 Three Winding Transformers

Thus far we have looked at transformers which have one single primary winding and one single secondary winding. But the beauty of transformers is that they allow us to have more than just one winding in either the primary or secondary side. Transformers which have three winding are known commonly as **Three Winding Transformers**.

The principal of operation of a *three winding transformer* is no different from that of an ordinary transformer. Primary and secondary voltages, currents and turns ratios are all calculated the same, the difference this time is that we need to pay special attention to the voltage polarities of each coil winding, the dot convention marking the positive (or negative) polarity of the winding, when we connect them together.

Three winding transformers, also known as a three-coil, or three-winding transformer, contain one primary and two secondary coils on a common laminated core. They can be either a single-phase transformer or a three-phase transformer, (three-winding, three-phase transformer) the operation is the same.

Three Winding Transformers can also be used to provide either a step-up, a step-down, or a combination of both between the various windings. In fact a three winding transformers have two secondary windings on the same core with each one providing a different voltage or current level output.

As transformers operate on the principal of mutual induction, each individual winding of a three winding transformer supports the same number of volts per turn, therefore the volt-ampere product in each winding is the same, that is $N_P/N_S = V_P/V_S$ with any turns ratio between the individual coil windings being relative to the primary supply.

In electronic circuits, one transformer is often used to supply a variety of lower voltage levels for different components in the electronic circuitry. A typical application of three winding transformers is in power supplies and Triac Switching Converters. So a transformer have two secondary windings, each of which is electrically isolated from the others, just as it is electrically isolated from the primary. Then each of the secondary coils will produce a voltage that is proportional to its number of coil turns.

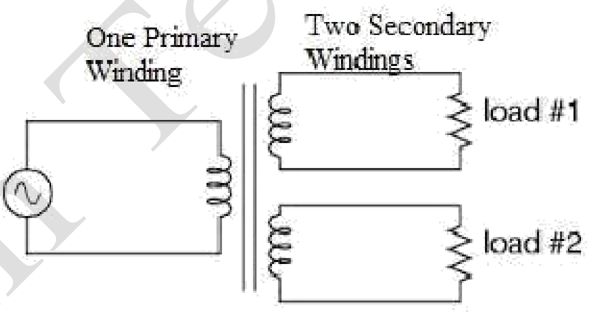


Fig. 4.27 A three winding transformer

The secondary windings can be connected together in various configurations producing a higher voltage or current supply. It must be noted that connecting together transformer windings is only possible if the two windings are electrically identical. That is their current and voltage ratings are the same.

4.10 Parallel operation of three phase transformer

4.10.1Advantages of using transformers in parallel

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfill the total demand. In this way we can run the system with maximum efficiency.

2. To maximize electrical power system availability: If numbers of transformers run in parallel, we can shut down any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

3. To maximize power system reliability: If any one of the transformers run in parallel, is tripped due to fault of other parallel transformers is the system will share the load, hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

4. To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfill the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

4.10.2 Conditions for parallel operation

Certain conditions have to be met before two or more transformers are connected in parallel and share a common load satisfactorily. They are,

1. The voltage ratio must be the same.

2. The per unit impedance of each machine on its own base must be the same.

3. The polarity must be the same, so that there is no circulating current between the transformers.

4. The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.

Same voltage ratio : Generally the turns ratio and voltage ratio are taken to be the same. If the ratio is large there can be considerable error in the voltages even if the turns ratios are the same. When the primaries are connected to same bus bars, if the secondaries do not show the same voltage, paralleling them would result in a circulating current between the secondaries. Reflected circulating current will be there on the primary side also. Thus even without connecting a load considerable current can be drawn by the transformers and they produce copper losses. In two identical transformers with percentage impedance of 5 percent, a no-load voltage difference of one percent will result in a circulating current of 10 percent of full load current. This circulating current gets added to the load current when the load is connected resulting in unequal sharing of the load. In such cases the combined full load of the two transformers can never be met without one transformer getting overloaded.

- **Per unit impedance:** Transformers of different ratings may be required to operate in parallel. If they have to share the total load in proportion to their ratings the larger machine has to draw more current. The voltage drop across each machine has to be the same by virtue of their connection at the input and the output ends. Thus the larger machines have smaller impedance and smaller machines must have larger ohmic impedance. Thus the impedances must be in the inverse ratios of the ratings. As the voltage drops must be the same the per unit impedance of each transformer on its own base, must be equal. In addition if active and reactive power are required to be shared in proportion to the ratings the impedance angles also must be the same. Thus we have the requirement that per unit resistance and per unit reactance of both the transformers must be the same for proper load sharing.

Polarity of connection: The polarity of connection in the case of single phase transformers can be either same or opposite. Inside the loop formed by the two secondaries the resulting voltage must be zero. If wrong polarity is chosen the two voltages get added and short circuit results. In the case of polyphase banks it is possible to have permanent phase error between the phases with substantial circulating current. Such transformer banks must not be connected in parallel. The turns ratios in such groups can be adjusted to give very close voltage ratios but phase errors cannot be compensated. Phase error of 0.6 degree gives rise to one percent difference in voltage. Hence poly phase transformers belonging to the same vector group alone must be taken for paralleling.

Transformers having —30° angle can be paralleled to that having +30° angle by reversing the phase sequence of both primary and secondary terminals of one of the transformers. This way one can overcome the problem of the phase angle error.

- **Phase sequence-** The phase sequence of operation becomes relevant only in the case of poly phase systems. The poly phase banks belonging to same vector group can be connected in parallel. A transformer with +30° phase angle however can be paralleled with the one with — 30° phase angle, the phase sequence is reversed for one of them both at primary and secondary terminals. If the phase sequences are not the same then the two transformers cannot be connected in parallel even if they belong to same vector group. The phase sequence can be found out by the use of a phase sequence indicator.

4.11 Load Sharing

When the transformers have equal voltage ratios, the magnitudes of secondary no-load voltages are equal. Further if the primary leakage impedance drops due to exciting currents are also equal, then

$\bar{E}_a = \bar{E}_b$ and the circulating current at no load is zero.

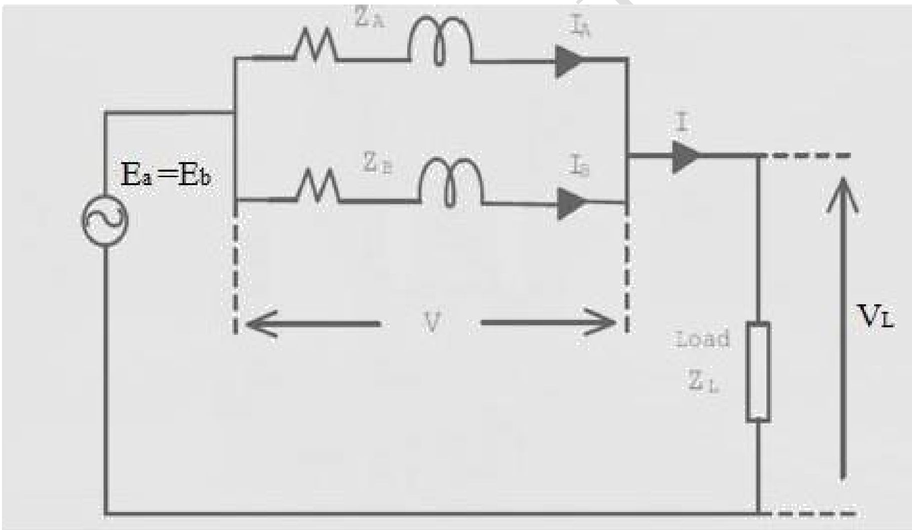


Fig. 4.28 Circuit modelling of two transformer in parallel

The equivalent circuit of two three phase transformer connected in parallel connected with a load of

Z_L impedance on per phase basis is drawn in fig 4.28. In this figure transformer A and

B are operating in parallel. I_A and I_B are the load current of the two transformer.

The voltage equation of transformer A is

$$\bar{E}_a = \bar{I}_a\bar{Z}_a + \bar{V}_L = \bar{I}_a\bar{Z}_a + \bar{I}\bar{Z}_L$$

Since $\bar{E}_a = \bar{E}_b$; $\bar{E}_b = \bar{I}_a\bar{Z}_a + \bar{V}_L = \bar{I}_a\bar{Z}_a + \bar{I}\bar{Z}_L$

The voltage equation of transformer B is

$$\bar{E}_b = \bar{I}_b \bar{Z}_b + \bar{V}_L = \bar{I}_b \bar{Z}_b + \bar{I}\bar{Z}_L$$

$$\bar{E}_b - \bar{I}_a \bar{Z}_a = \bar{E}_b - \bar{I}_b \bar{Z}_b$$

$$\bar{I}_a \bar{Z}_a = \bar{I}_b \bar{Z}_b$$

According to the voltage drops across the two equivalent leakage impedance Za and Zb are equal.

According to KCL we can write

$$\bar{I} = \bar{I}_a + \bar{I}_b$$

$$\bar{I} = \bar{I}_a + \frac{\bar{I}_a \bar{Z}_a}{\bar{Z}_b}$$

$$\bar{I}_a = \bar{I}\frac{\bar{Z}_b}{\bar{Z}_a + \bar{Z}_b}$$

similarly, $\bar{I}_b = \bar{I}\dfrac{\bar{Z}_a}{\bar{Z}_a + \bar{Z}_b}$

Multiplying both the current equations by terminal voltage we get,

$$\bar{S}_a = \bar{S}\frac{\bar{Z}_b}{\bar{Z}_a + \bar{Z}_b}$$

similarly, $\bar{S}_b = \bar{S}\dfrac{\bar{Z}_a}{\bar{Z}_a + \bar{Z}_b}$

Thus the power sharing in between two transformer is given in above equation in VA rating.