THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Venue ____________________

Seat Number __________

Student Number |__|__|__|__|__|__|__|__|

Family Name ____________________

First Name ____________________

This exam paper must not be removed from the venue

School of Electrical Engineering & Computer Science

Semester One Examinations, 2024

ELEC3004 Signals, Systems and Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Planning Time: 10 minutes

For Examiner Use Only

Question	Mark

Exam Conditions:

•Specified written or printed material permitted
•Casio FX82 series or UQ approved and labelled calculator only •During Planning Time - Students are encouraged to review and plan responses to the exam questions

Materials Permitted in the Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)

One A4 sheet of handwritten notes double sided is permitted

Materials to be supplied to Students:
Additional exam materials (e.g. answer booklets, rough paper) will be provided upon request.

1 x Gradescope Bubble Sheet

Instructions to Students:
If you believe there is missing or incorrect information impacting your ability to answer any question, please state this when writing your answer.

Total _________

Section 1: Multiple Choice (30 marks)

Select only ONE option for each question. If multiple options are selected, a mark of ZERO will be awarded for that question.

1. Which of the following is a property of an odd signal $f(t)$?

(a) $f(t) = f(-t)$

(b) t only extends from 0 to $+\infty$

(c) t only extends from $-\infty$ to 0

(d) $f(t) = -f(-t)$

2. Given the complex number $x = 9 - 2j$, what is $\text{Im}\{x\}$?

(a) -2

(b) 9

(c) $9 - 2 = 7$

(d) $\sqrt{9^2 + (-2)^2} = 85$

3. Which of the following statements is TRUE?

(a) A continuous-time analogue signal can only take on finite values and exists at fixed time steps

(b) A continuous-time analogue signal can take on any value and exists at fixed time steps

(c) A discrete-time digital signal can only take on finite values and exists at fixed time steps

(d) A discrete-time analogue signal can only take on finite values and exists at fixed timesteps

4. Given the signal $f(t) = \sin\left(3\pi t + \frac{\pi}{6}\right)$, what is the frequency of $f(t)$ in Hz?

(a) 1.5

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{6}$

(d) 3π

5. Given input $x(t)$ and output $f(t)$, which of the following is a linear time invariant signal?

(a) $f(t) = \frac{d^2x}{dt^2} - 6\frac{dx}{dt}$ **+4**

(b) $f(t) = 2\frac{dx}{dt} - 9x(t) + 1$

(c) $f(t) = t^2x(t) + \cos(x(t))$

(d) $f(t) = 3x^2(t) + 7x(t)$

6. When two linear time-invariant (LTI) systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in series then the equivalent impulse response is given by:

(a) $h(t) = h_1(t) * h_2(t)$

(b) $h(t) = h_1(t) + h_2(t)$

(c) $h(t) = h_1(t) - h_2(t)$

(d) $h(t) = h_1(t) \cdot h_2(t)$

7. Which of the following describes the properties of the Discrete Time Fourier Transform?

(a) Discrete in the frequency domain and is a function of time and frequency

(b) Continuous in the frequency domain and is a function of time and frequency

(c) Discrete in the frequency domain and is a function of frequency only

(d) Continuous in the frequency domain and is a function of frequency only

8. Given that the Fourier transform of $x(t) = e^{-at}u(t)$, $a > 0$ gives $X(\omega) = \frac{1}{a+j\omega}$, apply the duality property to determine the Fourier Transform of $y(t) = \frac{1}{a+jt}$.

(a) $Y(\omega) = 2\pi e^{-a\omega}u(\omega)$, $a > 0$

(b) $Y(\omega) = 2\pi e^{a\omega}u(-\omega)$, $a > 0$

(c) $Y(\omega) = e^{-a\omega}u(\omega)$, $a > 0$

(d) $Y(\omega) = \frac{1}{2\pi}e^{-a\omega}u(\omega)$, $a > 0$

9. What is the relationship between the impulse response and the step response?

(a) The impulse response is equal to the step response convolved with $e^{j\omega t}$.

(b) The impulse response is the derivative of the step response.

(c) The impulse response is the integral of the step response.

(d) The relationship between the two is undefined because the step response has a discontinuity in it (i.e., a step).

10. Which of the following signals does NOT have a Fourier transform?

(a) $e^{t}u(-t)$

(b) $e^{-3t}u(t)$

(c) $\delta(t)$

(d) $|t|u(t)$

11. Given the Fourier transform of $x(t)$ is $X(\omega)$, what is the Fourier transform of $x(\frac{1}{5}t)$?

(a) $5X(5\omega)$

(b) $\frac{1}{5}X(\frac{\omega}{5})$

(c) $5X(\frac{\omega}{5})$

(d) $\frac{1}{5}X(5\omega)$

12. Find the Nyquist rate of $x(t) = 3\pi + cos(2\pi t)$.

(a) 3 Hz

(b) 2 Hz

(c) 5 Hz

(d) 0.5 Hz

13. When a complex exponential function, $e^{j\omega t}$, is input into a linear time invariant system, which property is NOT modified at the output?

(a) The magnitude of each harmonic component

(b) The phase of each harmonic component

(c) The frequency of each harmonic component

(d) The real component of the signal

14. Which operation(s) in an analogue to digital converter (ADC) *always* results in information loss?

(a) Sampling

(b) Quantization

(c) Reconstruction filter

(d) Both sampling and quantization

Would this not be both? Sampling does not always result in information loss (if you sample at a rate higher than nyquist frequency then you can represent all frequencies of the signal with no information loss). Hence the answer is just quantisation. +1

15. When aliasing has occurred in a reconstructed audio signal, what happens to the aliased frequencies relative to their original values before sampling?

(a) They can appear higher or lower depending on the Nyquist rate

(b) They appear higher pitched (higher frequency)

(c) The signals are lost in reconstruction so they are not heard

(d) They appear deeper pitched (lower frequency) +6

16. Find the Laplace transform of $(e^{4t} + 5)u(t)$ and its region of convergence (ROC).

(a) $\frac{1}{s-4} + \frac{5}{s}, Re(s) > 4$

(b) $\frac{1}{s-4} - \frac{5}{s}, Re(s) > 0$

(c) $\frac{1}{4-s} - \frac{5}{s}, Re(s) > 4$

(d) $\frac{1}{4-s} + \frac{5}{s}, Re(s) > 0$

17. Find the poles of a system with the transfer function $H(s) = \frac{s^2+2s+2}{s^2+10s+34}$.

(a) $-1 \pm j$

(b) $1 \pm j$

(c) $-5 \pm 3j$

(d) $5 \pm 3j$

18. For an LTI system that has a single pole at -2.6, which of the following statements about the impulse response of the system is TRUE?

(a) It is a growing exponential function

(b) It is a decaying exponential functionS

(c) It is a sine wave with a growing exponential envelope

(d) It is a sine wave with a decaying exponential envelope

19. A necessary and sufficient condition for the stability of a system is that:

(a) All poles of the transfer function, $H(z)$, lie outside the unit circle of the z-plane

(b) All poles of the transfer function, $H(z)$, lie inside the unit circle of the z-plane

(c) The transfer function, $H(z)$, does not present poles

(d) The zero of the transfer function $H(z)$ is the origin of the z-plane

20. What is the Z-transform of the signal $x[n] = u[-n]$?

(a) $\frac{1}{1-z}$

(b) $\frac{1}{z-1}$

(c) $\frac{1}{1-z^{-1}}$

(d) $\frac{z-1}{2z}$

21. For a system with transfer function $H(s) = \frac{2s-7}{(s+0.2)(s+0.9)}$, which of the following statements is TRUE?

(a) The system is unstable

(b) The system is underdamped

(c) The system is critically damped

(d) The system is overdamped

22. A critically damped second order system has which of the following properties?

(a) Two distinct and real poles in the left-hand plane

(b) Two complex conjugate poles in the left-hand plane

(c) Two distinct and real poles in the right-hand plane

(d) A double pole in the left-hand plane

23. A signal is corrupted with high-frequency noise. What basic filter type is best suited to removing the noise?

(a) Low-pass filter

(b) High-pass filter

(c) Band-stop filter

(d) Band-pass filter

24. The gain response of an LTI system is:

(a) The change in an input signal's phase over varying input signal frequencies.

(b) The change in an input signal's magnitude over varying input signal frequencies.

(c) The change in an input signal's stability over varying input signal frequencies.

(d) The change in an input signal's order over varying input signal frequencies.

25. Which of the following is a potential effect of quantisation?

(a) Decreased system order

(b) Conversion from an IIR filter to an FIR filter

(c) Increased sampling period

(d) Limit cycles

26. The absolute stability of a feedback control system is defined by its:

(a) Closed-loop transfer function zeros

(b) Closed-loop transfer function poles

(c) Open-loop transfer function poles

(d) Order

27. Apart from the closed-loop transfer function poles, the transient response of a feedback control system is also influenced by its:

(a) Open-loop transfer function zeros

(b) Open-loop transfer function poles

(c) Closed-loop transfer function zeros

(d) Nothing else influences a system's transient response

28. For an underdamped system, the time it takes for the system to reach its maximum value is known as the:

(a) Delay time T_d

(b) Rise time T_r

(c) Peak time T_p

(d) Settling time T_s

29. Which of the following is TRUE regarding disturbances to a feedback control system:

(a) A fixed disturbance will have a different impact on the system depending on where it enters the system

(b) A disturbance only impacts the system response when it is injected in the feedforward path

(c) It doesn't matter where it enters the system, a fixed disturbance will always have the same impact on the system response

(d) A disturbance only impacts the system response when it is injected in the feedback path

30. A control system is said to be controllable if

(a) The controller gains are very high

(b) The system response decays to zero

(c) Any state variable is independent of the control signal

(d) It is possible to transfer between any two arbitrary states in a finite time period

Section 2: Short Answer (20 marks)

List and explain any assumptions.

Question 31. **(3 marks)**

Name two advantages and one disadvantage of digital signals versus analogue signals.

Digital:
- Infinite resolution, good for HD signals such as audio
- Directly interfaces with real world signals without need for ADC
- Can be susceptible to noise
- Requires bigger hardware to store

Analogue:
- Less affected by noise
- Easy to store
- Quantisation error
- Requires ADC for real world interfacing

Question 32. **(2 marks)**

An audio CD uses a sampling frequency f_s of 44.1 kHz. Human hearing has a cut-off frequency f_c of approximately 20 kHz. Why is the sampling frequency significantly larger than $2f_c$?

To avoid undesirable behaviour, there should always be some headroom. This then poses less demand on the anti-aliasing filter. +4

Sample greater than the nyquist rate, 2*f_max to avoid aliasing

Question 33. **(2 marks)**

Describe how the Laplace Transform is related to the Fourier Transform.

Fourier Transform is a case of Laplace Transform where s = j\omega and the ROC includes the j\omega complex axis

Question 34. **(2 marks)**

What are the window effects when a signal is truncated in the time domain? How to reduce the window effects?

Window effects is where a signal is truncated in time, introducing spectral leakage, i.e. signal smearing or distortion.
Reduce by:
- Choosing suitable window function
- Increase number of samples for a larger window and increased resolution

Good job little bro stay in school kid +69

Question 35. **(1 mark)**

For a fast Fourier transform (FFT) operation, how do you define the frequency resolution Δf using the number of points of a discrete-time signal N and the time resolution Δt?

Time resolution Δt = time per each sample.
Fs = 1/Δt.
Frequency resolution Δf is the width of each frequency bin. If there are N samples this becomes Fs/N = Δf = 1/NΔt

Question 36. **(1 mark)**

Give one reason why you might want to use a Chebyshev or Elliptical filter instead of a Butterworth filter.

Cheby and Elliptic both have faster transition bands which better replicate the shape of an ideal brick filter +1

Question 37. **(2 marks)**

Identify one benefit and one drawback of increasing the order of an FIR filter.

1) Increasing order sharpens the transition band, better modelling the behaviour of an ideal brick filter
2) Increasing order requires more complex computation, higher cost to cascade multiple filters together,
3) increased potential for instability
4) limit cycles

FIR filters can't be unstable, the last two points are for IIR filters.
Increases delay? (Slower response)

Question 38. **(3 marks)**

Draw the Direct Form I for the filter shown in Figure 1:

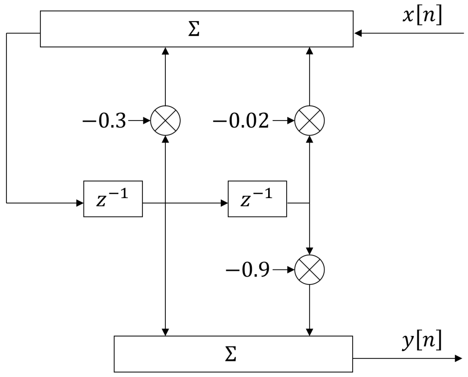
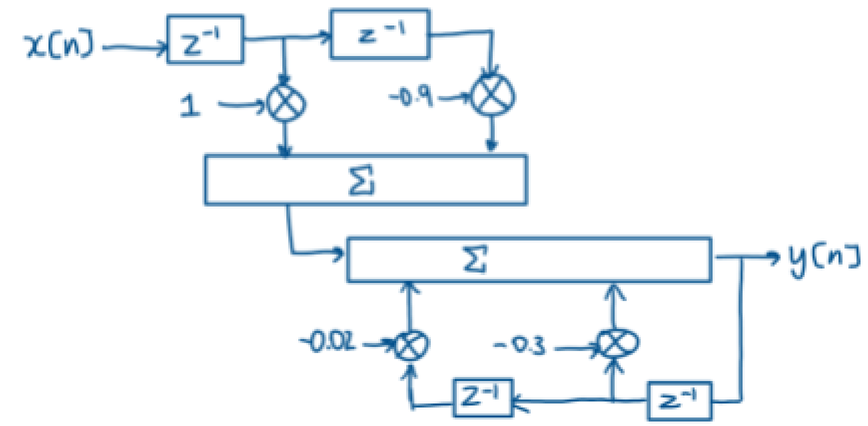


Figure 1: Canonical form of the filter in Question 38.

$y[n] = x[n-1] - 0.9x[n-2] - 0.3y[n-1] - 0.02y[n-2]$



Question 39. **(2 marks)**

Explain the role of the lead component and the lag component in a lead-lag compensator. Give your answer in terms of the system's transient and steady-state response.

Lead:

Improves transient response by increasing phase margin, hence improving system stability and efficiency

Lag:

Improves steady state response by increasing low frequency gain

Question 40. **(2 marks)**

Name one advantage and one disadvantage of the state-space method over conventional (root locus or frequency response) methods for control system design.

State space response allows analysis of MIMO systems, that is complex to do with root locus or state space. However state space requires all states to be observable, and implementing state observers can be complex or impossible sometimes

You also can't control easily for zeroes in the state-space representation

Section 3: Long Answer (60 marks)

Show all working out in your answer. List and explain any assumptions.

Question 41. **(10 marks)**

Consider a discrete-time linear time-invariant (LTI) system, with example input, $x[n]$, and corresponding output, $y[n]$, as below.

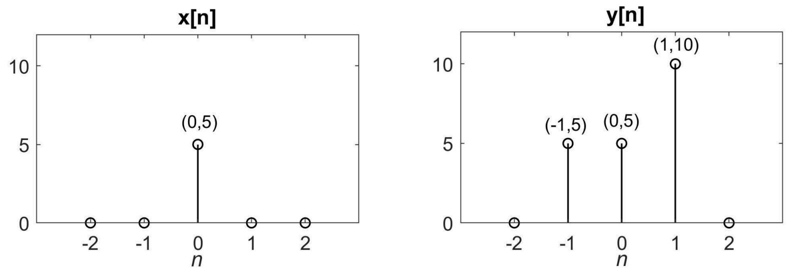


Figure 2: Input and output signals in Question 41.

(a) Determine the impulse response, $h[n]$, of the system. (4 marks)

$x[n] = 5\delta[n]$, $y[n] = 5\delta[n+1] + 5\delta[n] + 10\delta[n-1]$

Impulse response of LTI system: $y[n] = x[n] * h[n]$

$y[n] = 5\delta[n] * h[n] = 5h[n]$

$h[n] = y[n]/5 = \delta[n+1] + \delta[n] + 2\delta[n-1]$

+2

for y[n], why is the 10δ delayed if it's appearing after n = 0? Why isn't it 10[n+1]?

The inside of the brackets need to equal to 0.

it isn't delayed, the delta form of it has the points before zero + and the points after zero -

(b) For the input sequence,

$$x[n] = [1, \quad 3, \quad 1] \qquad n = 0...2,$$

determine the output, $y[n]$. (6 marks)

y[n] = [1, 4, 6, 7, 2], n= -1…3

Use this table ↓↓↓ fastest method no cap (it even works when you switch the order)

		x[n]				
	*	1z^0	3z^-1	1z^-2		
	1z^1	1z^1	3z^0	1z^-1		
h[n]	1z^0	1z^0	3z^-1	1z^-2		
	2z^-1	2z^-1	6z^-2	2z^-3		
	y[n]	1z^1	4z^0	6z^-1	7z^-2	2z^-3

		h[n]				
	*	1z^1	1z^0	2z^-1		
	1z^0	1z^1	1z^0	2z^-1		
x[n]	3z^-1	3z^0	3z^-1	6z^-2		
	1z^-2	1z^-1	1z^-2	2z^-3		
	y[n]	1z^1	4z^0	6z^-1	7z^-2	2z^-3

Convolution in time → multiplication in Z

$Y(Z) = H(Z)X(Z) = Z\{\delta[n+1] + \delta[n] + 2\delta[n-1]\} \cdot Z\{\delta[n] + 3\delta[n-1] + \delta[n-2]$

$z + 3 + z^{-1} + 1 + 3z^{-1} + z^{-2} + 2z^{-1} + 6z^{-2} + 2z^{-3} = 2z^{-3} + 7z^{-2} + 6z^{-1} + z + 4$

$Z^{-1}\{2z^{-3} + 7z^{-2} + 6z^{-1} + z + 4\} = 2\delta[n-3] + 7\delta[n-2] + 6\delta[n-1] + \delta[n+1] + 4\delta[n]$

$y[n] = 2\delta[n-3] + 7\delta[n-2] + 6\delta[n-1] + 4\delta[n] + \delta[n+1]$ (Why is this reversed lol? - it's not I don't think, the [n+1] corresponds to the n = -1 term. I had it written as y[n] = [1,4,6,7,2] for n = [-1,0,1,2,3] which I think is the same someone correct me if I'm wrong [+1])

n	x[n]	-1	0	1	2	3
0	1	1	1	2		
1	3		3	3	6	
2	1			1	1	2
sum:		1	4	6	7	2

therefore, y[n] = {1, 4, 6, 7, 2} for n = {-1, 0, 1, 2, 3}

Question 42. **(15 marks)**

Miggzy was performing a song at his latest Epochs concert, consisting of frequencies in the range of 20 Hz to 8 kHz, and lasting for a total of 2 minutes. The audio was recorded so that it could later be listened to on the Tisney streaming service.

(a) What is the minimum sampling frequency that will allow resolution of all the frequencies of the song and their perfect reconstruction? (2 marks)

f_s must be $> 2 \times f_{max}$, minimum sampling frequency must be greater than 16kHz

+2

(b) The song was recorded using a sampling rate of 20 kHz. If the samples are quantized into 1024 amplitude levels, determine the number of binary digits (bits) required to encode the 2 minute long song. (5 marks)

Recorded at $f_s = 20kHz$, quantised into 1024 levels, 2 minute $log_2 1024 = log_{10}1024/log_{10}2 = 10$

$2\, minutes = 120\, seconds \rightarrow 20000 \times 120 = 240000$

$240000 \times 10 = 24000000\, bits = 24Mb$

+2

(c) If the recorded song was reconstructed with an ideal lowpass filter, what cut-off frequency should it have? (2 marks)

LPF: Gain = $\{1,\ \omega < \omega_c;\ 0,\ \omega > \omega_c\}$

Hence $f_c = 8$ kHz

(d) If instead a practically realisable reconstruction filter was used, how large a transition band does a sampling rate of 20 kHz allow for the signal? (2 marks)

Transition bandwidth = 16kHz - 8kHz = 8kHz ?

Since the Q say's "sampling rate of 20 kHz" isn't it transition bandwidth = 20kHz - 8kHz = 12kHz?

It's from like half the sampling rate to the maximum frequency, so 10Kz - 8kHz = 2kHz. (+5)

(e) The vocals of the song contained a high number of "S's" and "T's", resulting in *sibilance* or a hissing sound, which is most prominent in the frequency range of 5 kHz to 8 kHz. It was proposed to eliminate the sibilance and all frequencies greater than 5 kHz by sampling at twice this maximum desired frequency, sampling at f_s = 10 kHz.

What would happen? Will there be aliasing? If so, what frequencies will alias? (4 marks)

If sampling occurred at 10kHz, aliasing would occur due to the Nyquist rate of the original signal being 16kHz. Aliasing would occur between f_s /2 = 5 kHz and f_max = 8 kHz. This would cause a fold from 5kHz to 2kHz, thus the 8kHz component will alias to 2kHz
+3

Question 43. **(10 marks)**

Consider the sequences $x_1[n] = 6^n u[n]$ and $x_2[n] = 8^n u[-n+1]$.

(a) Find the z-transform and the region of convergence for $x_1[n]$. (3 marks)n

$x_1[n] = 6^n u[n]$

$Z\{x_1[n]\} = 1/(1 - 6z^{-1})$, ROC: $|z| > 6$

+1

(b) Find the z-transform and the region of convergence for $x_2[n]$. (4 marks)

$x_2[n] = 8^n u[-n+1]$

$X(Z) = \sum_{-\infty}^{\infty} x[n] z^{-n}$

$X(Z) = \sum_{-\infty}^{-1} 8^n\ z^{-n}$ (Should it not be from -inf to 1)? [+1]

$X(Z) = \sum_{k=1}^{\infty} (z/8)^k$

$X(Z) = [\sum_{k=0}^{\infty} (z/8)^k] - 1$

$X(Z) = (z/8)/(1 - z/8)$, given that |z/8|<1

$X(Z) = z/(8 - z)$, therefore ROC |z| < 8

$X(Z) = \sum_{-\infty}^{1} 8^n z^{-n}$

$X(Z) = \sum_{k=-1}^{\infty} (z/8)^k$

r^-1 / (1-r) = 8/(z-(z^2/8)) = 64/(8z-z^2), ROC |z| < 8 [+1] (Miguel Confirmed lol GET OWNED RED)

(c) What is the region of convergence of $x_1[n] + x_2[n]$? Would the answer change for $x_1[n] - x_2[n]$? (3 marks)

The ROC for both operations is the intersection of the two ROCs, hence
ROC: 6 < |z| < 8

Question 44. **(10 marks)**

Consider the discrete-time unity-feedback control system (with sampling period $T = 1$ s) shown below:

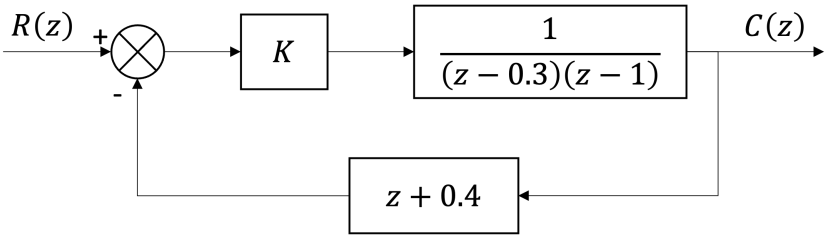


Figure 3: Discrete-time feedback control system in Question 44.

(a) Compute the closed-loop pulse transfer function and identify the characteristic equation of the system. (5 marks)

$$A = \frac{K}{(z-0.3)(z-1)}, \quad B = (z+0.4)$$

$$\frac{A}{AB+1} = \frac{K}{(z-0.3)(z-1)} \frac{1}{\frac{K(z+0.4)}{(z-0.3)(z-1)}+1}$$

$$= \frac{K}{K(z+0.4)+(z-0.3)(z-1)}$$

After expanding denominator:

$$= \frac{K}{z^2+(K-1.3)z+(0.4K+0.3)}$$

So the characteristic equation is:

$$z^2+(K-1.3)z+(0.4K+0.3) = 0$$

(b) Determine the range of K for stability by use of the Jury stability test. (5 marks)

2nd order system (even) (ie $n = 2$ therefore the Jury Table only has 1 row, and only the first three conditions need to be checked. Therefore the criteria is:

$$|a_2| < a_0 = (0.4K - 0.3) < 1 = K < 3.25$$

$$a_2 > -1 = 0.4K - 0.3 > -1 = K > \frac{0.7}{0.4}$$

$$P(1) = 1 + K - 1.3 + 0.4K - 0.3 > 0 \Rightarrow K > \frac{0.6}{1.4}$$

$$P(-1) = 1 - K + 1.3 + 0.4K - 0.3 > 0 \Rightarrow K < \frac{-2}{-0.6}$$

Therefore the criteria are:

1. $\frac{0.7}{0.4} < K < 3.25$
2. $K > \frac{0.6}{1.4}$
3. $K < \frac{2}{0.6}$

So therefore to be stable, $\frac{0.6}{1.4} < K < 3.25$

$P(z) = \underbrace{z^2}_{a_0} + \underbrace{(k-1.3)z}_{a_1} + \underbrace{(0.4K+0.3)}_{a_2}$

1. $|a_2| < a_0$

$|0.4k + 0.3| < 1$

$-1 < 0.4K + 0.3 < 1$

$-1.3 < 0.4K < 0.7$

$\frac{-1.3}{0.4} < K < \frac{0.7}{0.4}$

$-3.25 < K < 1.75$

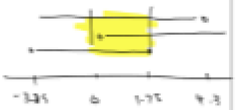
2. $P(z)|_{z=1} > 0$

$(1)^2 + K - 1.3 + 0.4K + 0.3 > 0$

$1 + 1.4K - 1 > 0$

$1.4K > 0$

$K > 0$



3. $P(z)|_{z=-1} > 0$

$(-1)^2 - K + 1.3 + 0.4K + 0.3 > 0$

$1 + 1.6 - 0.6K > 0$

$-0.6K > -2.6$

$K < \frac{2.6}{0.6}$

$K < 4.3$

$0 < K < 1.75$

I think there's a mistake above, where they suggest a0 = 0.4K - 0.3, when I think it should be 0.4K+0.3. [+2] I got K stability range 0 -> 1.75 [+8]

Answer should be 0 < K < 7/4 (1.75)a

can someone explain why is it 0 in 0 < K <1.75?

Question 45. **(15 marks)**

Consider the following state-space representation of a feedback control system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.9 & -2.6 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Where $y(k)$ is the output, $u(k)$ is the input and $\mathrm{x}(k) = [x_1(k)\ x_2(k)]^\top$ is the state vector.

(a) Derive the corresponding pulse transfer function.(7 marks) *Hint: Recall that the inverse of a* 2×2 *matrix* $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *is given by* $M^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, *where* $\det(M)$ *is the determinant of matrix* M.

$$G(z) = C(zI - A)^{-1}B$$

$$A = \begin{bmatrix} 0 & 1 \\ -0.9 & -2.6 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0.5 & 1 \end{bmatrix}$$

find $zI - A$

$$\begin{bmatrix} z & 1 \\ -0.9 & z + 2.6 \end{bmatrix}$$

take inverse

$$(zI - A)^{-1} = \frac{1}{\det zI - A} \begin{bmatrix} z + 2.6 & -1 \\ 0.9 & z \end{bmatrix}$$
$$= \frac{1}{z^2 + 2.6z + 0.9} \begin{bmatrix} z + 2.6 & -1 \\ 0.9 & z \end{bmatrix}$$

Now do full

$$\frac{1}{z^2 + 2.6z + 0.9} \begin{bmatrix} 0.9 \\ z \end{bmatrix}$$

Now with C,

$$\frac{1}{z^2 + 2.6z + 0.9} \begin{bmatrix} 0.9 \\ z \end{bmatrix} \times \begin{bmatrix} 0.5 & 1 \end{bmatrix} = 0.45 + z$$

Thus

$$G(z) = \frac{z + 0.45}{z^2 + 2.6z + 0.9}$$

I think inverse * B is wrong. should be (1, z) i think?
which means the final numerator is z + 0.5 [+10]

(b) Is the system observable? (2 marks)

Observable defined as

$$\mathcal{O} = \begin{bmatrix} C \\ AC \\ A^2C \end{bmatrix}$$

only have 2 x 2 so up-to AC

$$\mathcal{O} = \begin{bmatrix} \begin{bmatrix} 0.5 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 \\ -0.9 & -2.6 \end{bmatrix} \times \begin{bmatrix} 0.5 & 1 \end{bmatrix} \end{bmatrix}$$
$$= \begin{bmatrix} 0.5 & 1 \\ -0.9 & -2.6 \end{bmatrix}$$

As this system has full rank it is obeservable.

CA = [-0.9 -2.1] -> matrix: [0.5 1
-0.9 -2.1] (+3)

but still det(OB) != 0 so full rank and observable

(c) Determine a suitable state feedback gain matrix K = $[k_1\ k_2]$ such that the system will have closed-loop poles at $z = -0.7 \pm j0.8$. (6 marks)

$Poles\ at\ z = -\ 0.7 \pm j0.8$D
$(z + 0.7 + j0.8)(z + 0.7 - j0.8) = z^2 + 1.13 + 1.4z$
$Characteristic\ Equation\colon\ |zI - A + Bk| = [z,\ -1;\ 0.9 + k,\ z + 2.6 + k]$
$det(zI - A + Bk) = z^2 + 2.6z + kz + 0.9 + k = z^2 + (2.6 + k)z + (0.9 + k)$
$Matching\ coefficients\ of\ z^2 + 1.13 + 1.4z\ and\ z^2 + (2.6 + k)z + (0.9 + k),$
$k = -\ 1.2,\ k = 0.23$

```
A = [0 1; -0.9 -2.6];
B = [0; 1];
placed_poles = [-0.7 + 0.8j, -0.7 - 0.8j];
K = acker(A, B, placed_poles)
```

Command Window

```
>> a3q3

K =

    0.2300   -1.2000
```

END OF EXAMINATION