THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Venue	____________________
Seat Number	__________
Student Number	|__|__|__|__|__|__|__|__|
Family Name	____________________
First Name	____________________

This exam paper must not be removed from the venue

School of Information Technology and Electrical Engineering

Semester One Examinations, 2023

ELEC3004 Signals, Systems and Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Planning Time: 10 minutes

For Examiner Use Only

Question	Mark

Exam Conditions:

•This is a Closed Book examination - specified written materials permitted
•Casio FX82 series or UQ approved and labelled calculator only •During Planning Time - Students are encouraged to review and plan responses to the exam questions
•This examination paper will be released to the Library

Materials Permitted in the Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)
One A4 sheet of handwritten notes double sided is permitted

Materials to be supplied to Students:
Additional exam materials (e.g. answer booklets, rough paper) will be provided upon request.

1 x Multiple Choice Answer Sheet

Instructions to Students:
If you believe there is missing or incorrect information impacting your ability to answer any question, please state this when writing your answer.

Total available marks: 110

Total **0 you failed smh**

Section 1: Multiple Choice (30 marks)

Select only ONE option for each question. If multiple options are selected, a mark of ZERO will be awarded for that question.

1. Which of the following is a property of a periodic signal $f(t)$?

(a) t extends from $-\infty$ to $+\infty$

(b) t only extends from 0 to $+\infty$

(c) t only extends from $-\infty$ to 0

(d) t extends from finite time t_0 to t_f

2. Which of the following statements is true?

(a) A signal with infinite energy is an energy signal

(b) A signal can be both an energy and a power signal

(c) A signal can be neither an energy nor a power signal

(d) A signal with finite energy is a power signal

3. Given the complex number $x = -3 + 4j$, what is $\mathrm{Re}\{x\}$?

(a) -3

(b) 4

(c) $-3 + 4 = -1$

(d) $\sqrt{(-3)^2 + 4^2} = 5$

4. Given the signal $f(t) = \cos\left(9\pi t + \frac{\pi}{3}\right)$, what is the frequency of $f(t)$ in Hz?

(a) 2π

(b) 9

(c) $\frac{\pi}{3}$

(d) 4.5

5. Given input $x(t)$ and output $y(t)$, which of the following is a linear time-invariant signal?

(a) $y(t) = 3\frac{dx}{dt} - 5x(t)$

(b) $y(t) = 2t^2x(t)$

(c) $y(t) = \sin(2x(t))$

(d) $y(t) = -7x^2(t)$

6. Which of the following is correct regarding the impulse signal?

(a) $x[n]\delta[n] = x[n]$

(b) $x[n]\delta[n] = \delta[n]$

(c) $x[n]\delta[n] = x[0]\delta[n]$

(d) $x[n]\delta[n] = x[0]$

7. When two linear time-invariant (LTI) systems with impulse responses $h_a(t)$ and $h_b(t)$ are cascaded then the equivalent impulse response is given by

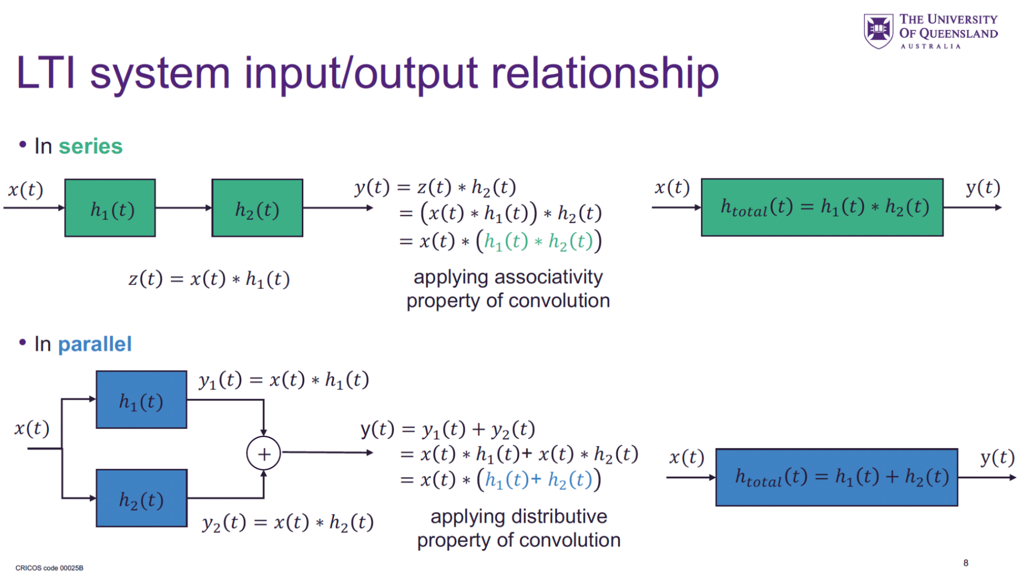
(a) $h(t) = h_a(t) + h_b(t)$

(b) $h(t) = h_a(t) - h_b(t)$

(c) $h(t) = h_a(t)h_b(t)$

(d) $h(t) = h_a(t) * h_b(t)$

'cascaded' = 'series' = convolution as per screenshot in the lecture notes



8. **The impulse response of an LTI system $h(t)$ is given by $e^{-2t}u(t)$. What is the step response of the system?**
   (a) $y(t) = \frac{1}{2}(1 - e^{-2t})$

   (b) $y(t) = \frac{1}{2}(1 - e^{-2t})u(t)$ **+3**

   (c) $y(t) = (1 - e^{-2t})u(t)$

   (d) $y(t) = \frac{1}{2}(e^{-2t})u(t)$

   is it b because it's the integral from t to infty?

   Yes it is B but you integrate between 0 to t. And do convolution where x(t) = u(t)

9. **Which of the following describes the properties of the Fourier Series?**

   (a) Discrete in the frequency domain and is a function of the time and frequency

   Since the fourier series has both time and frequency in its formula

   (b) Continuous in the frequency domain and is a function of the time and frequency

   (c) Discrete in the frequency domain and is a function of the frequency only

   (d) Continuous in the frequency domain and is a function of the frequency only

10. **Which of the following signals do NOT have the Fourier transform?**

    (a) Periodic signals

    (b) Decay exponential functions

    (c) Growing exponential functions

    (d) Step functions

(since Fourier transform requires convergence/periodicity)

11. **Given the Fourier transform of $f(t)$ is $F(\omega)$, what is the Fourier transform of $f(5t)$?**
    (a) $5F(\omega)$

    (b) $\frac{1}{5}F(\frac{\omega}{5})$

    (c) $5F(\frac{\omega}{5})$

    (d) $\frac{1}{5}F(5\omega)$

12. Which of the following is NOT a property of the spectrum of a time domain sampled signal?

(a) Periodic in the frequency domain

(b) Discrete in the frequency domain

Miguel confirmed its b on ed #406 **+3**

(c) Continuous in the frequency domain

would it not be this as frequencies only occur at discrete time periods? +4

[from chatgpt] When a continuous signal is sampled in the time domain, its spectrum in the frequency domain becomes discrete and periodic, with repetitions of the original spectrum. Therefore, the frequency domain representation is not continuous.

Is a time domain sampled signal not DTFT? Which is cts & Periodic in frequency domain but discrete in time

Wouldn't it be the act of applying the DTFT that makes the signal continuous in the frequency domain, not an inherent property of the spectrum of the signal??

(d) Repetitions of the original spectrum in the frequency domain

13. Which of the following does NOT describe practical difficulties for signal sampling and reconstruction?

(a) Practical signals are bandlimited **+6**

(b) Practical signals are not bandlimited

(I believe the answer is B according to #348 on ed) +2

But practical signals aren't bandlimited, which is a practical difficulty - and it's asking for things that are NOT practical difficulties, so it should be (a) **+4**

(c) Instead of an impulse train, the practical sampling pulses have a finite width

(d) Real low-pass filters have unlimited bandwidth spectrum

14. Find the Nyquist rate and Nyquist interval of $sin(2\pi t)$.

(a) 2 Hz, 2 seconds

(b) 0.5 Hz, 0.5 seconds

(c) 2 Hz, 0.5 seconds

(d) 0.5 Hz, 2 seconds

15. The highest frequency component of a speech signal needed for telephonic communications is about 3.1 kHz. What is a suitable value for the sampling rate?

(a) 1 kHz

(b) 2 kHz

(c) 4 kHz

(d) 8 kHz

16. Find the Laplace transform of e^{-at}$u(t)$ and its region of convergence (ROC).

(a) $\frac{1}{s-a}, Re(s) > -a$

(b) $\frac{1}{s}, Re(s) > a$

(c) $\frac{1}{s \times a}, Re(s) > a$

(d) $\frac{1}{s+a}, Re(s) > -a$

17. What is the necessary and sufficient condition to have a stable system?

(a) All poles of the transfer function lie in the left half of the s-plane

(b) All poles of the transfer function lie in the right half of the s-plane

(c) All zeros of the transfer function lie in the left half of the s-plane

(d) All zeros of the transfer function lie in the right half of the s-plane

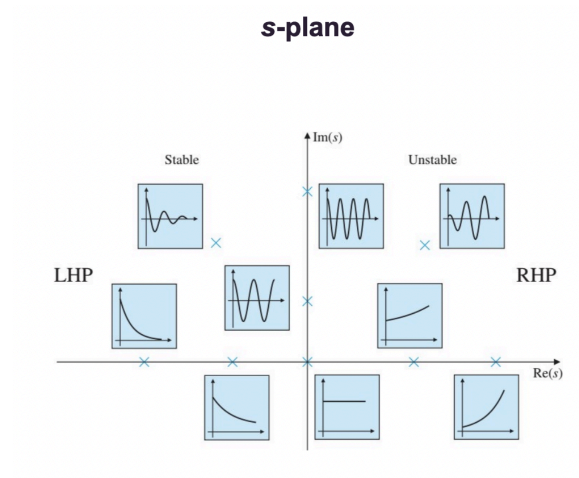
18. **For an LTI system that has a single pole at $-2+j3$, which of the following statement about the impulse response of the system is true?**

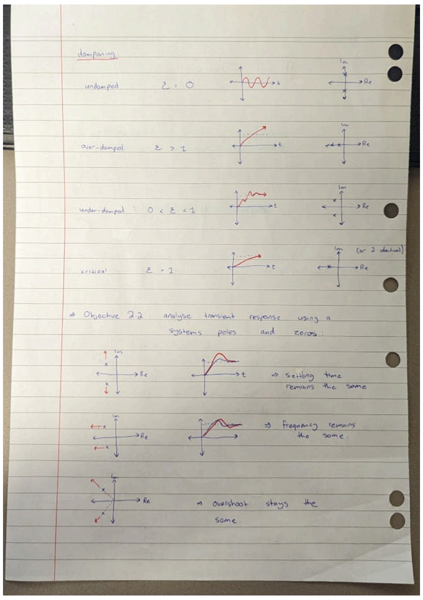
- **(a)** It is a decaying exponential function
- **(b)** It is a growing exponential function
- **(c)** It is a sine wave with a decaying exponential envelope
- **(d)** It is a sine wave with a growing exponential envelope

Can anyone explain this? +1

Poles which have a negative real component will decay exponentially over time. For imaginary (complex conjugate) poles z = a ± jb, we have an underdamped response (i.e. sinusoid which decays according to e^-at). For identical negative-real poles, we have critical damping, etc. (see below). Adding imaginary components to the poles introduces the frequency which is why the options are between sine and exponential functions. This is from my metr4201 notes which might be helpful

Lec 18 slide 12:





(frequency should increase a little when increasing the imaginary components of the poles in the third last pair of plots)

Also see this (L13 2025-Sem1):

RECAP LECTURE 10

Transfer Function and Impulse Response

- Consider a second order system with (complex conjugate) poles $\{p_1, p_2\}$ at $\sigma_a \pm j\omega_a$
- Assume it has an all-pole transfer function

$$H(s) = \frac{1}{\underbrace{(s-\sigma_a-j\omega_a)}_{p_1}\underbrace{(s-\sigma_a+j\omega_a)}_{p_2}}$$

- To find the impulse response, solve inverse Laplace transform
  - First: partial fraction expansion (PFE)

$$H(s) = \frac{\frac{1}{2j\omega_a}}{(s-\sigma_a-j\omega_a)} + \frac{\frac{-1}{2j\omega_a}}{(s-\sigma_a+j\omega_a)}$$

  - Second: use lookup tables! (Table 6.1 in Lathi, p.320)

RECAP LECTURE 10

Impulse Response

- Find $h(t) = \mathcal{L}^{-1}\{H(s)\}$ from Laplace transform tables

$$h(t) = \underbrace{\frac{1}{2j\omega_a}\exp((\sigma_a+j\omega_a)t)}_{p_1} - \underbrace{\frac{1}{2j\omega_a}\exp((\sigma_a-j\omega_a)t)}_{p_2}$$
$$= \frac{1}{2j\omega_a}\exp(\sigma_a t)[\exp(j\omega_a t) - \exp(-j\omega_a t)]$$
$$= \frac{1}{\omega_a}\exp(\sigma_a t)\sin(\omega_a t)$$

- Impulse response is exp × sine, where
  - σ_a (real part of pole) controls rate of decay
  - ω_a (imaginary part of pole) controls oscillation frequency

19. The delay theorem allows us to:

(a) Transform a delayed term in the z-domain into a multiplication in the discrete domain

(b) Transform a delayed term in the discrete domain into a multiplication in the z-domain

(c) Calculate the delay of a signal

(d) Calculate the sum of the geometric series

20. What is the Z-transform of the signal $x[n] = a^n u[n]$**?**

(a) z

(b) $\frac{z}{z+a}$

(c) $\frac{z}{z-a}$ **+3**

(d) $\frac{1}{z}$

21. A second order system with poles at $s = -0.5 \pm j$ **is**

(a) overdamped

(b) underdamped **+4**

(c) critically damped

(d) unstable

22. For a system with transfer function $H(s) = \frac{s-0.5}{(s+0.8)(s+0.3)}$**, which of the following statements is true?**

(a) The system is unstable

(b) The system is critically stable

(c) The system is underdamped

(d) The system is overdamped

23. A signal exhibits frequencies within the range of 100 MHz to 1 GHz. Which basic filter type is best suited to isolating the signal?

(a) Low-pass filter

(b) High-pass filter

(c) Band-stop filter

(d) Band-pass filter **+53**

24. Which of the following is used to transform between a prototype analogue low-pass filter with unity cut-off, to a band-stop filter with cut-out frequency ω_l and cut-in frequency ω_u ?

(a) $s' = \frac{\omega_l}{s}$

(b) $s' = \frac{\omega_u}{s}$

(c) $s' = \frac{s^2+\omega_l\omega_u}{s(\omega_u-\omega_l)}$

(d) $s' = \frac{s(\omega_u-\omega_l)}{s^2+\omega_l\omega_u}$ **+2** (lecture 13 slide 70)

25. Which of the following is NOT true of limit cycles?

(a) They are a type of quantisation error

(b) They are undesirable oscillations in the output signal

(c) They impact IIR filters

(d) They impact FIR filters

26. The absolute stability of a feedback control system is defined by its:

(a) Order

(b) Open-loop transfer function poles

(c) Closed-loop transfer function zeros

(d) Closed-loop transfer function poles

27. The transient response of a feedback control system is defined by its:

(a) Order

(b) Open-loop transfer function poles

(c) Closed-loop transfer function zeros

(d) Closed-loop transfer function poles +5 (Lec 18 slide 7)

Transient Response of feedback control system -> Closed loop system -> Closed-loop poles are the dominant affector of system response

Feedback response = closed-loop poles, feedforward = open-loop poles

Should be (c). 2025 Sem 1 Lec 18 slide 13 shows two systems with different transient responses where the only difference is the closed loop zeros. +2

28. The time it takes for a system to reach and remain within an allowable tolerance of its final value is known as the:

(a) Delay time T_d

(b) Rise time T_r

(c) Peak time T_p

(d) Settling time T_s

29. For a digital control system to track a reference ramp input with zero steady-state error, at least how many poles must it have at the origin?

(a) 0

(b) 1

(c) 2

(d) It is impossible for any system to track a ramp input with zero steady-state error

Summary: Steady-state Error

- e_{ss} is error between reference input $r(t)$ and feedback signal $b(t)$
  - Important: $e_{ss} \neq r(t) - c(t)$

Type	# Poles at origin	Step input response	Ramp input response	Acceleration input response
0	0	$\frac{1}{1+K_p}$	∞	∞
1	1	0	$\frac{1}{K_v}$	∞
2	2	0	0	$\frac{1}{K_a}$

2025 L18 slide 24

-> 1 is for a step-input so a ramp input needs 2 f Ramp input = 1/s^2 - 2nd order - Double pole +1

30. A control system is said to be uncontrollable if

(a) The controller gains are too high

(b) The system response decays to zero

(c) Any state variable is independent of the control signal

(d) It is possible to transfer between any two arbitrary states in a finite time period

Section 2: Short Answer (20 marks)

List and explain any assumptions.

Question 31. **(3 marks)**

Name three advantages of digital signals versus analogue signals.

Easier storage & Processing
Better Transmission Performance - less vulnerable to noise
Lower cost +1

Question 32. **(2 marks)**

What are the two operations to convert a signal from analogue to digital?

First sample the signal then quantify (quantisation) +2

Question 33. **(2 marks)**

Describe how the Discrete Fourier Transform (DFT) is related to the Discrete-Time Fourier Transform (DTFT).

DFT is a frequency sampled version of DTFT (?)
DFT is DTFT truncated in time and sampled in frequency domain +2

Question 34. **(2 marks)**

What is zero padding for a sampled signal? Does it improve the accuracy of the samples?

Zero padding is adding zeroes to a sequence used to improve display resolution and for faster computation. Does not improve accuracy +1
Ed post #361 (2025)

Question 35. **(1 marks)**

For a fast Fourier transform (FFT) operation, how do you define the frequency resolution Δf using the number of points of a discrete-time signal N and the time resolution Δt?

f=1/(N \delta t) $$\Delta f = \frac{1}{N\Delta t}$$

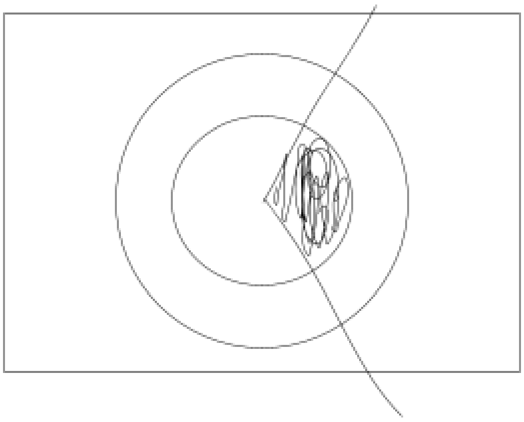
Question 36. **(2 marks)**

Describe the negative side effects of windowing on FIR filter performance.

Does not result in minimum possible order for filter +1
What about the ringing at edges and the gradual gain dropoff due to windowing? Also can cause spectral leakage?
What about smearing and distortions

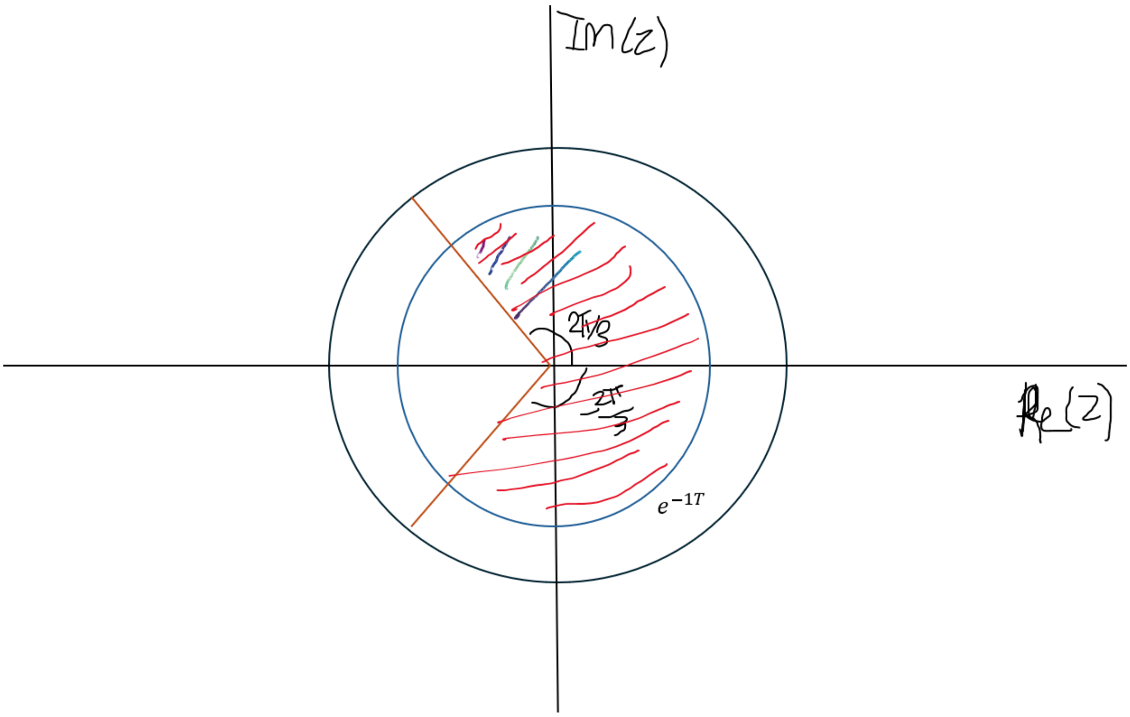
Question 37. **(4 marks)**

Sketch (with labels) the region in the z-plane that corresponds to a desirable region in the s-plane bounded by lines $\sigma = -1$ and.



Are you sure this is correct, this has constant attenuation σ = -1 so there isn't a region bounded-1

The drawing should be like this. The value is w = ±2pi/3 in s-plane which corresponds to a radial line at the angle 2pi/3 and -2pi/3 radians in the z-plane +1
I believe the region you've shaded is the part that shouldn't be included.
(+1 - the desirable region is from -2pi/3 to 2pi/3 so includes 0 hence includes the positive real axis)
Also i think the radius should be exp(-1T) Isn't radius exp(sigma*t) therefore is this case it will be exp(-1t)? +2 Yes, can confirm it's exp(-1T)



This is what I got - thoughts?
yep this is what I got too & matches revision lec answers slide 22 where it says "Desirable region is towards the origin and the positive real axis"
yeah this is it other blokes trippin **+9**

Question 38. **(2 marks)**

Explain the role of the derivative component and the integral component in a PID controller. Give your answer in terms of the system's transient and steady-state response.

Integral -> Improve steady state
Derivative -> Improve oscillations, improves transient response
Both improve stability
What about P?-question doesn't ask for it - it moves poles fyi

Question 39. **(1 mark)**

What is the relationship between the sampling period of a discrete-time control system and the stability of the system?

Sampling period influences poles -> - poles influence stability. (it's a one mark question) +5

↓Overkill alert +3

For a discrete-time system to be stable, all poles of its transfer function G(z)) must lie inside the unit circle in the complex z-plane. The location of these poles depends on the sampling period Ts

A very small Ts (high sampling frequency) typically results in a discrete-time system that closely approximates the behavior of the continuous-time system, maintaining its stability characteristics.

If Ts is too large, the discretization can cause the poles of the discrete-time system to move outside the unit circle, leading to instability. This is because the transformation from the s-plane to the z-plane can distort the pole-zero locations significantly if the sampling period is not chosen appropriately.

Question 40. **(1 mark)**

Name one advantage of the state-space method over conventional (root locus or frequency response) methods for control system design.

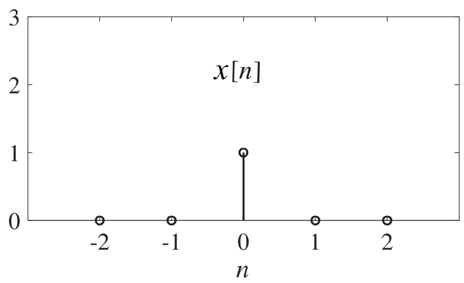
The ability to work with MIMO (multi-input multi-output) easier.

(not 100% but heard Alina mention this) uses linear algebra / matrix operations so can be very efficiently implemented in most software suites (MATLAB, python, etc)

Section 3: Long Answer (60 marks)

Show all working out in your answer. List and explain any assumptions.

Consider a discrete-time linear time-invariant (LTI) system, with example input, $x[n]$, and corresponding output, $y[n]$, as below.



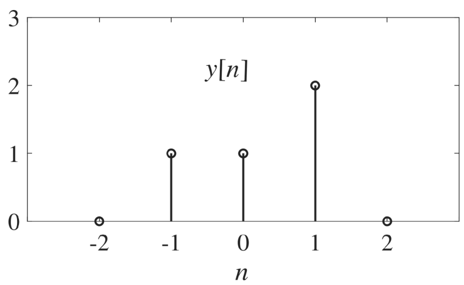


Figure 1: Input and output signals in Question 41

Question 41. **(10 marks)**

(a) Determine the impulse response, $h[n]$, of the system. (2 marks)

Because x[n]=delta[n], h[n] must be equal to y[n] so h[n] = [1, 1, 2] +1

h[n]=[1,1,2] at n={−1,0,1} +2

Shouldn't it be:
h[n] = delta[n+1] + delta[n] + 2delta[n-1] ? +2

They both mean the same thing, since delta[n] has a magnitude of 1

(b) If the input is scaled by a factor of 2 (i.e. $x[n] = 2\delta[n]$), determine the output $y[n]$. (3 marks)

y[n]=[2,2,4] at n={−1,0,1}
y[n] = [2, 2, 4], n=-1…1 (same thing)

be careful about using curly vs. square brackets for y[n] vs. n (Miguel mentioned as much in a revision session)

(c) For the input sequence:

$$x[n] = [1, \quad 3, \quad 1] \qquad n = 0 \ldots 2,$$

determine the output, $y[n]$. (5 marks)

y[n]=[1,4,6,7,2] at n={−1,0,1,2,3} +11

This is the correct answer, bounds for the result is equal to the sum of the bounds of the inputs which were convolved. +2

Btw Miguel (GOAT) showed this table method, it's for a different Q but you can see how it works:

x		h[n]			
		$1z^{0}$	$1z^{-1}$	$1z^{-2}$	$1z^{-3}$
x[n]	$0.25z^{0}$	$0.25z^{0}$	0.25^{-1}	$0.25z^{-2}$	$0.25z^{-3}$
	$0.5z^{-1}$	$0.5z^{-1}$	$0.5z^{-2}$	$0.5z^{-3}$	$0.5z^{-4}$
	$0.75z^{-2}$	$0.75z^{-2}$	$0.75z^{-3}$	$0.75z^{-4}$	$0.75z^{-5}$
	$1z^{-3}$	$1z^{-3}$	$1z^{-4}$	$1z^{-5}$	$1z^{-6}$

y[n]	$0.25z^{0}$	$0.75z^{-1}$	$1.5z^{-2}$	$2.5z^{-3}$	$2.25z^{-4}$	$1.75z^{-5}$	$1z^{-6}$

First response (RED) is correct; refer to this video:

[Discrete Time Convolution (Tabular Method) (youtube.com)]

Question 42. **(10 marks)**

Dr Bee plans to play a lullaby on her cello, and record this digitally to compact disc (CD). The lullaby utilises the full range of the cello, with a lowest note of C2 (65 Hz) and harmonics up to a maximum frequency of 8 kHz.

(a) In this case, what is the Nyquist rate? (2 marks)

>16kHz sample/sec 16kHz is sufficient, nyquist rate defined as 2*fmax

didnt Alina say in revision lecture if u sample at exactly double you sample 0??

They're asking for nyquist rate, not an appropriate sampling frequency +1

(b) A CD uses a sampling rate of 44.1 kHz. If the samples are quantized into 65,536 amplitude levels, determine the number of binary digits (bits) required to encode a 1:20 (1 minute and 20 seconds) long lullaby. (4 marks)

Total samples = (60+20)*44100 = 3 528 000
L = 65 536 -> log2(L) = 16 bits
Total bits = 16 * 3 528 000 = 56 448 000 bits

(c) Dr Bee wants to record the lullaby using 15,870 samples/s instead of the regular CD sampling rate.
What will happen? Will there be aliasing? If so, what frequencies will alias?
(4 marks)

Aliasing will occur as fs=15870<Nyquist rate=16000. Frequencies within range (fs/2,fmax)=(7935,8000)Hz will fold to frequencies within range (fs-fmax,fs/2)=(7870,7935)Hz// – maybe write fold range backwards

(7935 Hz -> 8000 Hz) -> (7935 Hz -> 7870 Hz) +5** is this not more representative due to the folding. Holl up ill get the lecture slide. (Lecture 24a page 22). In the lecture slide they go backwards. The notation above shows that as 7935 Hz increases it folds from 7935 Hz to 7870 Hz aliasing backwards. On the triangle this is section B aliasing to section A.

oh so the lecturer does it backwards. i think it doesnt matter too much but probably safer to do it same order as they do

Question 43. **(15 marks)**

Consider the following digital filter:

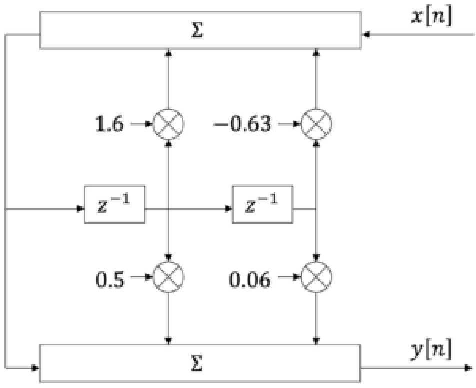


Figure 2: Canonical form of the filter in Question 43.

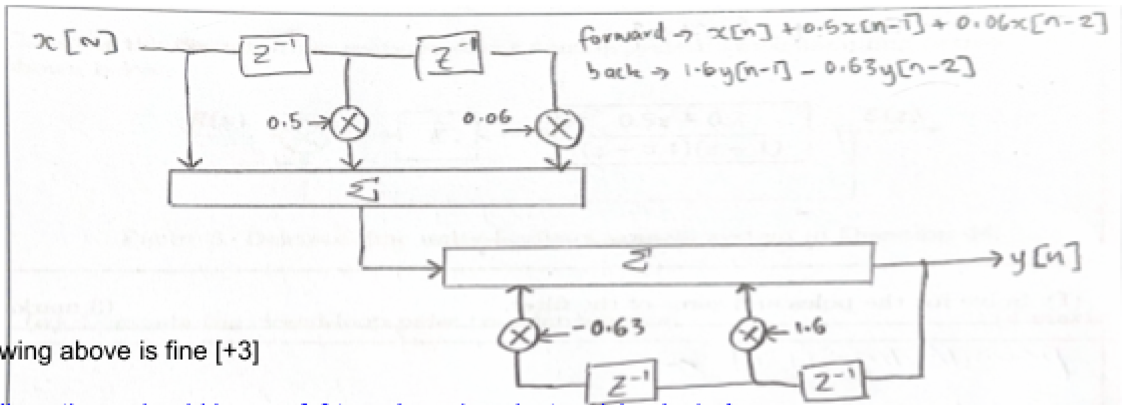
(a) Is this an IIR filter or FIR filter? (1 mark)

IIR has feed back

(b) Name one advantage and one disadvantage of using an IIR filter vs. an FIR filter. (2 mark)

Advantage: IIR is less complex for a given transition band width?
Disadvantage: FIR provides linear phase whereas IIR cannot easily do so?

(c) Draw the Direct Form I for this filter.z (3 marks)



Drawing above is fine [+3]

I believe there should be no x[n] term here (see last revision lec) **-1**

In this case there is a x[n] term as seen in the canonical form where there is a term into the bottom summation which has not yet been delayed hence is still just x[n] **+1**

(d) Give one reason why the canonical form is preferred for filter implementation over Direct Form I. (1 mark)

Has Minimum memory
Has less components, and therefore cheaper

What is the transfer function for this filter? (4 marks)

y[n] = x[n] + 0.5x[n-1] + 0.06x[n-2] +1.6y[n-1] -0.63y[n-2] [+4]

Y(z)/X(z)=(1 + 0.5z^-1 + 0.06z^-2) / (1 - 1.6z^-1 + 0.63z^-2) [+2]

Can someone tell me why it's not y[n] = 0.5x[n-1] + 0.06x[n-2] +1.6y[n-1] -0.63y[n-2]?
Look at far left line, x[n] goes direct to y[n] without any delay block hence y[n] = x[n] + next bits **+1**

Isn't this the difference equation, does it need to be converted to H(z) form? +1
This is the difference equation it should be converted to z domain +2

Do you have to make it causal so Y(z)/X(z) = (z^2 + 0.5z + 0.06) / (z^2 - 1.6z + 0.63) +1
Yes you do (that's standard form) +1

(e) Solve for the poles and zeros of the filter. (3 marks)

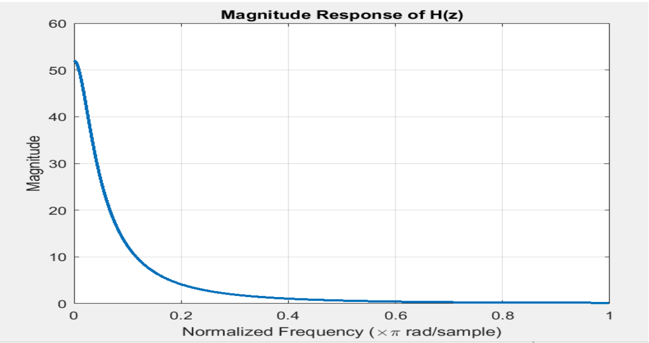
y[n] = x[n] + 0.5x[n-1] + 0.06x[n-2] +1.6y[n-1] -0.63y[n-2]
y[n] -1.6y[n-1] +0.63y[n-2] = x[n] + 0.5x[n-1] + 0.06x[n-2]

Zero = -0.2 and -0.3
Pole = 0.7 and 0.9 [+8]

For the Poles, Since 0 = z^2 + 1.6z − 0.63, shouldn't:
Z1 ≈ 0.327
Z2 ≈ −1.927
Using the Quadratic Formula? [-4, should be z^2 - 1.6z + 0.63, +1]

(f) Which of the four basic filter types (low-pass, high-pass, band-pass, band-stop) does this represent? (1 mark)

Low pass.



Question 44. (10 marks)
Consider the discrete-time unity-feedback control system (with sampling period $T = 1$ s) shown below:

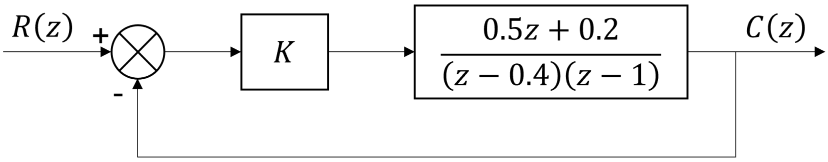


Figure 3: Discrete-time unity-feedback control system in Question 44.

(a) Compute the closed loop pulse transfer function (4 marks)

$$\frac{K\,(0.5z + 0.2)}{z^2 + (0.5K - 1.4)\,z + (0.2K + 0.4)}$$

K(0.5z+0.2) / z^2 +(0.5K−1.4)z+(0.2K+0.4) +1

(b) What is the characteristic equation for the system? (1 mark)

$$z^2 + (0.5K - 1.4)\,z + (0.2K + 0.4) = 0$$

z^2 +(0.5K−1.4)z+(0.2K+0.4)=0 +1

(c) Determine the range of K for stability by use of the Jury stability test. (5 marks)

0<K<3 +5

This is from the first two criteria but doesn't doing P(z) -> z = -1 result in 0 < K < 8/3 -3
No P(z) -> z = -1 should get k<9.33
So 0<K<3 is correct +7

i) |a_n| < a_0
-1 < 0.4 + 0.2K < 1
-7 < K < 3

ii) P(1) > 0
1 +0.5K - 1.4 + 0.4 +0.2K > 0
K > 0

iii) P(-1) > 0 (because even)
1 -0.5K +1.4 + 0.4 + 0.2K > 0
2.8>0.3K
9.33 > K

Therefore 0<K<3 +1

Question 45. **(15 marks)**

Consider the following pulse transfer function of a feedback control system:

$$\frac{Y(z)}{U(z)} = \frac{z+1}{z^2+1.3z+0.4}$$

Where y(k) is the corresponding output vector, $u(k)$ is the input vector and x(k) is the state vector.

(a) Derive the controllable canonical form of the state-space representation using the direct programming method and write the A, B, C and D matrices. (5 marks)

a) $b_0 = 0 \quad b_1 = 1 \quad b_2 = 1$

$a_1 = 1.3 \quad a_2 = 0.4$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$D = 0$$

(b) Is the system controllable? (2 marks) k

Yes, Since we put in controllabel canonical form. BUt since its 2 marks maybe also so the tests C_m [B AB] LOL

CM = [0 1
1 -1.3]

$$C_M = \begin{bmatrix} 0 & 1 \\ 1 & -1.3 \end{bmatrix}, \quad \det(C_M) = \begin{vmatrix} 0 & 1 \\ 1 & -1.3 \end{vmatrix} = -1 \neq 0$$

det(CM) = -1 which is not equal to 0, therefore full rank and controllable

(c) Is the system observable? (2 marks)

Observable yes

OM=[1 1
-0.4 -0.3]

$$O_M = \begin{bmatrix} 1 & 1 \\ -0.4 & -0.3 \end{bmatrix}, \quad \det(O_M) = \begin{vmatrix} 1 & 1 \\ -0.4 & -0.3 \end{vmatrix} = 0.1 \neq 0$$

det(OM) = 0.1, not equal to zero, full rank,

(d) Determine a suitable state feedback gain matrix K = $[k_1 \ k_2]$ such that the system will have the closed-loop poles at $z = 0.5 \pm j0.5$. (6 marks)

K=[0.1 −2.3] correct [+69*420] +100 karma (respect +) +5 +1 + 26 gamer cred
K=[1.9 0.3]-100 karma (L bozo)

K = [0.1 0.77] -100 karma (L bozo)

The following is entirely incorrect… (wrong A matrix whoops!) +1

$$|zI - A + BK| = [z - (0.5 + j0.5)][z - (0.5 - j0.5)]$$

$$BK = \overset{2\times1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \overset{1\times2}{[k_1 \quad k_2]} = \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$$

$$\left| \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -0.4 & -0.3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} \right| = (z - 0.5 - 0.5j)(z - 0.5 + 0.5j)$$

$$\left| \begin{bmatrix} z & -1 \\ 0.4 + k_1 & z + 0.3 + k_2 \end{bmatrix} \right| = z^2 - 0.5z + \cancel{0.5j}z - 0.5z + 0.25 \cancel{-0.25j} \cancel{-0.5j}z + \cancel{0.25j} + 0.25$$

$$z^2 + 0.3z + k_2 z + 0.4 + k_1 = z^2 - z + 0.5$$

$$z^2 + (0.3 + k_2)z + 0.4 + k_1 = z^2 - z + 0.5$$

equate coefficients

$$0.3 + k_2 = -1$$
$$k_2 = -1.3$$

$$0.4 + k_1 = 0.5$$
$$k_1 = 0.1$$

$$\therefore K = [0.1 \;,\; -1.3]$$

I got k1=0.1 and k2=2.3 i think your A matrix is wrong.

(I got k1=0.1 and k2= - 2.3)

A matrix should have -1.3 not -0.3, assuming you've taken CA instead of A carry through error.

END OF EXAMINATION