

Quick Sort Average case Runtime

Assume a bad split $\Rightarrow T(\frac{9}{10}n) + T(\frac{1}{10}n)$

Solve with Substitution method : Can assume best & worst
 $n \lg n \leq \text{avg} \leq n^2$.

Guess: $T(n) \leq cn \lg n$

$$T(n) = T(\frac{9n}{10}) + T(\frac{n}{10}) + n$$

Assume : $T(n) \leq cn \lg n$. Have to prove this

$$T(n) = T(\frac{9n}{10}) + T(\frac{n}{10}) + n \leq cn \lg n$$

$$T(\frac{9n}{10}) \leq c \frac{9n}{10} \lg \frac{9n}{10}$$

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$$\therefore T(n) = T(\frac{9n}{10}) + T(\frac{n}{10}) + n \leq c \frac{9n}{10} \lg(\frac{9}{10}n) + \frac{c}{10} \lg(\frac{n}{10}) + n.$$

$$= \frac{c}{10} \left[9n \lg(\frac{n}{10/9}) + \lg(n) - \lg(10) \right]$$

$$= \frac{c}{10} \left[9n (\lg(n) + \lg(9) - 1) + \lg(n) - 1 \right]$$

$$= \frac{c}{10} \left[9n \lg n + \underbrace{n(9 \lg(9) - 9)}_{\text{negative}} + \lg(n) - 1 \right]$$

$$\leq \frac{c}{10} \left[9n \lg n + \lg(n) \right]$$

$$\leq \frac{c}{10} \left[9n \lg n + n \lg n \right]$$

$$\leq \frac{c}{10} \left[10 n \lg n \right] \Rightarrow c n \lg n$$

$$\therefore T(n) = O(n \lg n)$$