Pinball Simulation Project

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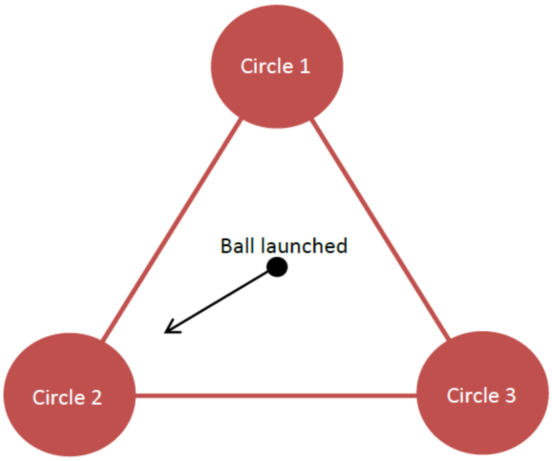
Math 2605 – Professor Louinici

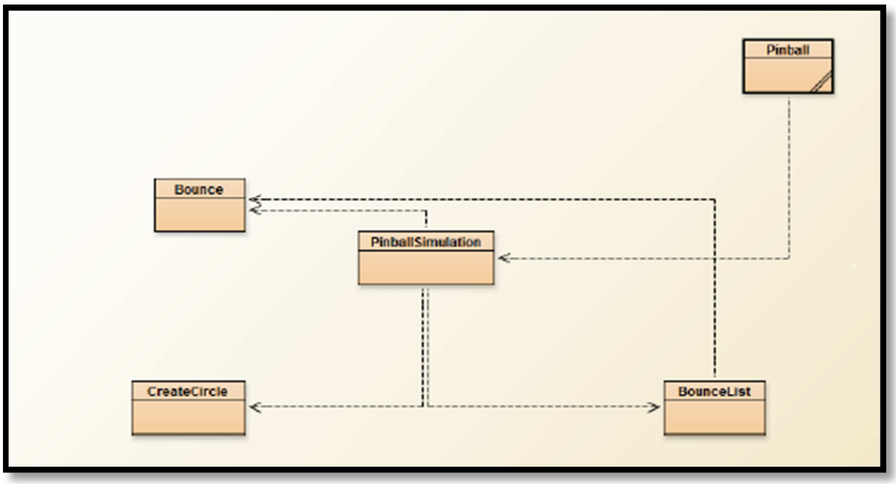
Defining the Simulation

This project requires us to experiment the launching of a 'ball' from the center of an equilateral triangle with circles at the corners of the triangle which reflect the ball at the appropriate angle. The object was to run this simulation an adequate amount of times to try and determine the initial launch angle which would create the most number of hits of a circle with a specified radius and triangle with specified lengths. A visual representation of the project is shown below:



Starting the Project

When I started this project, I planned out all the classes that I needed for this project to simulate the pinball structure properly. I decided the following classes would be the most effective in helping me solve the problem at hand[1].The image to the right shows a screen shot of all the classes from the BlueJ interface.



1. Bounce class
2. BounceList class
3. CreateCircle class
4. PinballSimulation class
5. Pinball class

[1] Read Javadocs for individual Class Descriptions

Calculating relative frequency for Targeted Run – Case 1

The first table represents the data when the sides of the triangle are 6.0, the radius is 1.0 and the number of runs is 100,000,000.

Number of Bounces	Frequency of Bounce	Number of Trials	Relative Frequency
0	72,035,575	100,000,000	**72.04%**
1	22,381,245	100,000,000	**22.38%**
2	4,556,654	100,000,000	**4.56%**
3	838,600	100,000,000	**0.84%**
4	153,532	100,000,000	**0.15%**
5	28,098	100,000,000	**0.03%**
6	5,148	100,000,000	**0.01%**
7	940	100,000,000	**0.00094%**
8	166	100,000,000	**0.00017%**
9	34	100,000,000	**0.000030%**
10	6	100,000,000	**0.000010%**
11	0	100,000,000	**0.000000%**
12	2	100,000,000	**0.0000020%**

From this table we can make the resulting conclusion. Firstly, there is a geometric decrease between the relative frequency and the number of bounces. Secondly, from the program simulation that was run to generate the required data, the maximum number of bounces that was achieved was **12**. This was accomplished at two separate angles (measured in radians) which were:

(1) 1.4757566773429387°

(2) 1.6658359762468540°

The following output is taken directly from the program to show the angle generated for the top 10 bounces.

```
Top 10 Bounces
|*| 1.4757566773429387 radians, 12 bounces |*| Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: Circle 2
|*| 1.6658359762468540 radians, 12 bounces |*| Circle 3: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 1
|*| 1.4741731889818237 radians, 10 bounces |*| Circle 3: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1
|*| 1.4757688038905816 radians, 10 bounces |*| Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1
|*| 1.6658238496992115 radians, 10 bounces |*| Circle 3: Circle 2: Circle 3: Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 2
|*| 1.6674194646079694 radians, 10 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2
|*| 3.7618309451708660 radians, 10 bounces |*| Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2
|*| 5.6629470155985140 radians, 10 bounces |*| Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1
|*| 1.4743180164031542 radians, 09 bounces |*| Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1
|*| 1.4757678614127856 radians, 09 bounces |*| Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 2
..Simulation Over
```

Calculating relative frequency for targeted run – case 2

The table below represents the data when the sides of the triangle are 6.0, the radius is 2.0 and the number of runs is 100,000,000. It is clear that the number of bounces increases since the ball has a larger surface to hit:

Number of Bounces	Frequency of Bounce	Number of Trials	Relative Frequency
0	41,226,017	100,000,000	41.23%
1	28,726,885	100,000,000	28.73%
2	15,980,926	100,000,000	15.98%
3	7,609,726	100,000,000	7.61%
4	3,504,626	100,000,000	3.50%
5	1,603,296	100,000,000	1.60%
6	732,540	100,000,000	0.73%
7	334,630	100,000,000	0.33%
8	152,832	100,000,000	0.15%
9	69,822	100,000,000	0.070%
10	31,904	100,000,000	0.032%
11	14,534	100,000,000	0.015%
12	6,692	100,000,000	0.0067%
13	3,050	100,000,000	0.0031%
14	1,330	100,000,000	0.0013%
15	688	100,000,000	0.00069%
16	288	100,000,000	0.00029%
17	116	100,000,000	0.00012%
18	54	100,000,000	0.000054%
19	26	100,000,000	0.000026%
20	10	100,000,000	0.000010%
21	6	100,000,000	0.0000060%
22	2	100,000,000	0.0000020%

From this table our findings in the first trial were emphasized. We can firstly see a geometric decrease between the number of bounces and the relative frequency. Secondly, our program simulation clearly accounts for a change in radius as the maximum number of bounces almost doubled to **22**. This occurred at two separate angles (measured in radians) which are:

(1) 3.8278013121354510°

(2) 5.5969766486339285°

```
Top 10 Bounces
|*| 3.8278013121354510 radians, 22 bounces |*| Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: C
|*| 5.5969766486339285 radians, 22 bounces |*| Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: C
|*| 3.4870932640750740 radians, 21 bounces |*| Circle 2: Circle 3: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: C
|*| 3.9238015977890230 radians, 21 bounces |*| Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: C
|*| 3.9414315246105858 radians, 21 bounces |*| Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: C
|*| 5.4833464361587945 radians, 21 bounces |*| Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: C
|*| 5.5009763629803565 radians, 21 bounces |*| Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 1: Circle 2: Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: C
|*| 5.9376846966943050 radians, 21 bounces |*| Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: C
|*| 1.2931292990451917 radians, 20 bounces |*| Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: C
|*| 1.4084132458931760 radians, 20 bounces |*| Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: C
..Simulation Over
```

Calculating Relative frequency for Random Run – Case 1

We will now run the simulation when the sides of the triangle are 6.0, the radius is 1.0 and the number of runs is 100,000,000. However, the only difference is that the angle will be randomly chosen. The table below compiles the data received:

Number of Bounces	Frequency of Bounce	Number of Trials	Relative Frequency
0	72040213	100,000,000	**72.04%**
1	22377740	100,000,000	**22.38%**
2	4556315	100,000,000	**4.56%**
3	838377	100,000,000	**0.84%**
4	153137	100,000,000	**0.15%**
5	27936	100,000,000	**0.028%**
6	5104	100,000,000	**0.0051%**
7	967	100,000,000	**0.00097%**
8	174	100,000,000	**0.00017%**
9	29	100,000,000	**0.000029%**
10	7	100,000,000	**0.000070%**
11	0	100,000,000	**0.000000%**
12	1	100,000,000	**0.000010%**

From this data, we can see that the randomly compiled data is almost exactly the same as the data generated from the targeted data approach with a couple of differences in the total number of hits and other areas. However, the geometric decrease is still the present and the maximum number of hits that occurs is **12** however it occurs only on one occasion when the angle (measured in radians) was:

(1) 1.6499864195275822°

```
Top 10 Bounces
|*| 1.6499864195275822 radians, 12 bounces |*| Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3
|*| 5.6805250585137745 radians, 10 bounces |*| Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3
|*| 5.6645467744456140 radians, 10 bounces |*| Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3
|*| 1.4903993113480478 radians, 10 bounces |*| Circle 3: Circle 1: Circle 2: Circle 3: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1
|*| 3.7455981954166186 radians, 10 bounces |*| Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1
|*| 3.5701517805469830 radians, 10 bounces |*| Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2
|*| 3.7601004902981434 radians, 10 bounces |*| Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 1
|*| 5.6791885607756900 radians, 10 bounces |*| Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3
|*| 3.5687262426817900 radians, 09 bounces |*| Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3
|*| 3.7442397587780890 radians, 09 bounces |*| Circle 2: Circle 1: Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 3: Circle 1
..Simulation Over
```

Calculating Relative frequency for Random Run – Case 2

The final run I'm going to perform will be for the following set of data with random angles. The sides of the triangle are 6.0, the radius is 2.0 and the number of runs is 100,000,000. The results are shown in the table below:

Number of Bounces	Frequency of Bounce	Number of Trials	Relative Frequency
0	41227167	100,000,000	**41.23%**
1	28721523	100,000,000	**28.72%**
2	15980736	100,000,000	**15.98%**
3	7611248	100,000,000	**7.61%**
4	3506192	100,000,000	**3.51%**
5	1603930	100,000,000	**1.60%**
6	732506	100,000,000	**0.73%**
7	335119	100,000,000	**0.34%**
8	152892	100,000,000	**0.15%**
9	70132	100,000,000	**0.070%**
10	31638	100,000,000	**0.032%**
11	14694	100,000,000	**0.015%**
12	6559	100,000,000	**0.0066%**
13	3145	100,000,000	**0.0031%**
14	1387	100,000,000	**0.0014%**
15	649	100,000,000	**0.00065%**
16	256	100,000,000	**0.00026%**
17	123	100,000,000	**0.00012%**
18	59	100,000,000	**0.000059%**
19	28	100,000,000	**0.000028%**
20	9	100,000,000	**0.0000090%**
21	5	100,000,000	**0.0000050%**
22	3	100,000,000	**0.0000030%**
23	3	100,000,000	**0.0000030%**

This table is the final consolidation that the data decreases geometrically as well as proves the program simulation works. The only difference is that the maximum number of hits was **23** at an angle (measured in radians) of:

(1) 1.8283904967630935°

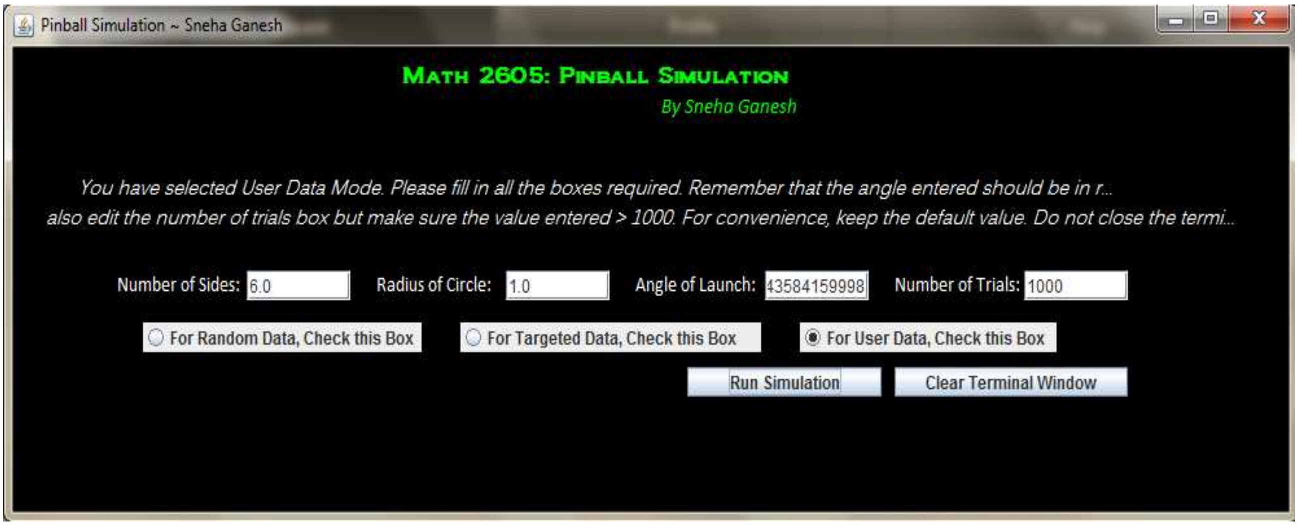
(2) 6.0134474346232270°

(3) 3.8278754363785197°

```
Top 10 Bounces
|*| 1.8283904967630935 radians, 23 bounces |*| Circle 3: Circle 2: Circle 3: Circle 1: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Cir
|*| 6.0134474346232270 radians, 23 bounces |*| Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Cir
|*| 3.8278754363785197 radians, 23 bounces |*| Circle 2: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Cir
|*| 1.8531211833308925 radians, 22 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Cir
|*| 3.9429395175953030 radians, 22 bounces |*| Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 1: Circle 3: Cir
|*| 3.4113199066282440 radians, 22 bounces |*| Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Circle 2: Circle 1: Circle 2: Circle 3: Circle 2: Cir
|*| 6.0434544276168220 radians, 21 bounces |*| Circle 1: Circle 3: Circle 1: Circle 3: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 3: Cir
|*| 5.9412942290696940 radians, 21 bounces |*| Circle 1: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 3: Circle 1: Cir
|*| 3.4835282423112230 radians, 21 bounces |*| Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 1: Circle 3: Cir
|*| 1.3883192491839310 radians, 21 bounces |*| Circle 3: Circle 1: Circle 2: Circle 3: Circle 2: Circle 3: Circle 1: Circle 3: Circle 1: Circle 2: Circle 1: Circle 3: Circle 2: Circle 3: Cir
...Simulation Over
```

Conclusion

From this data, we can see that the random method generally came up with the longest orbit, although the results were very similar. However this can vary, as since it is random sometimes it may end up smaller than the systematic (targeted) run.
By testing and doing research, I found the longest orbit case when the radius is 1.0 and the sides are 6.0 to be **16** bounces at an angle of 1.66743584159998°. The screen shot below show how this run was completed:




```
Top 10 Bounces
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
|*| 1.6674358415999800 radians, 16 bounces |*| Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 3: Circle 2: Circle 1: Circle 2: Circle 3: Circle 1: Circle 2: Circle 1: Cir
..Simulation Over
```

BlueJ: Terminal Window - Pinball

Options

```
++++++++++++++++++++++++++++++++++++++
***                                  *
***    PINBALL SIMULATION PROJECT    *
***        ---   RUN   ---           *
***                                  *
***           CREATED BY             *
***          SNEHA GANESH            *
***                                  *
***     MATH 2605: PROF LOUNICI      *
++++++++++++++++++++++++++++++++++++++

Please wait...large amounts of data be

Number of Bounces          Frequency of
        0                       0
        1                       0
        2                       0
        3                       0
        4                       0
        5                       0
        6                       0
        7                       0
        8                       0
        9                       0
        10                      0
        11                      0
        12                      0
        13                      0
        14                      0
        15                      0
        16                      1000
        17                      0
        18                      0
        19                      0
        20                      0
        21                      0
        22                      0
        23                      0
        24                      0
```