Assignment 2

8.3

(a) $x_{n+1} = (a x_n) \bmod 2^4$

Maximum period $= 2^4 - 2$

$= 4$

$5 x_n \bmod 2^4$, 5

(b) $a = 5, 11$

(c) Seed must be odd.

8.4

$x_{n+1} = (6 x_n) \bmod 13$

Sequence : 1, 6, 10, 8, 9, 2, 12, 7, 3, 5, 4, 11, 1....

$x_{n+1} = (7 x_n) \bmod 13$

Seq : 1, 7, 10, 5, 9, 11, 12, 6, 3, 8, 4, 2, 1, ...

Because of the patterns seen in 2nd half of the latter sequence, most people would consider it to be less random than the first sequence.

(8.6) (a) Use a key of length 255 bytes
The first two bytes are zero,
$K[0] = K[1] = 0$

Thereafter we have
$K[2] = 255, \quad K[3] = 254, \quad \ldots,$
$K[255] = 2.$

This key will leave S unchanged.

(8.7) (a) Let us consider i, j, S which requires

$$8 + 8 + (256 * 8) = 2064 \text{ bits}.$$

(b) The number of states is
$(2561 \times 2562) \times 21700$

Therefore 1700 bits are required.

8.8 (a) By taking the first 80 bits of $V \| C$ we obtain the intialization vector V.
Since V, C, K are known,
the message can be recovered
by computing $RC4(V \| K) * C$

(b) If adversary observes the $v_i = v_j$ for distinct i, j than he/ she

knows that the same key stream was used to encrypt both m_i and m_j.
In this case, the messages m_i and m_j may be vulnerable to the type of cryptanalysis carried out in above part (a).

(c) Since the key is fixed, the key stream varies with the choice of the 80 bit V, which is selected randomly. Thus after approximately messages are sent, we expect the same V and hence the same key stream, to be used more than once.

(d) The key K should be changed sometime before 240 messages are sent.

8.5 Algorithm

(1) Generate 'n' number of any random numbers

(2) Finding GCD for pairs in random numbers using Euclids algorithm. (for 'n' numbers, we will see n/2 pairs)

(3) Find probability of GCD = 1 (wherever this occurs)

(4) Estimate π = sqrt(6/(probability gcd as 1)) ← calculated in (3)

Program:

```
#include <random>
#include <iostream>
#include <chrono>
int number_generator( )
{
    unsigned int num;
    unsigned int seed = chrono::system_clock::now().
                        time_since_epoch().count();

    minstd_rand0 generator (seed);
    // standard linear_congruential engine.

    num = generator();

    return num;
}

int Euclids_algo( int n, int m)
{
    ~~return (n!=0) ? gcd(m, n%m);~~

    if (n == 0)
        return m.
    return euclids_algo(n%m, n);
}
```

```
int squareroot( int n)
{
   int sq = sqrt(n)
   return sq;
}

int main( )
{
      float phi;    int val;..
int x= number_generator();
      int probability;
      int n;
      int random[100]; // array of random
      int GCD[50];                numbers.
         cin >> n; // number of nos.
for (int i=0; i<n; i++)
      random[i] = number_generator( );

for(int i=0; i<n; i+=2)
{
      GCD[i] = euclids_algo(random[i],
                            random[i+1]);
}
  for (int i=0; i< n/2; i+1)
{
      if (GCD[i] == 1)
         num_1 ++;
}
```

probability = $\frac{num-1}{(n/2)}$;

val = $\frac{6}{probability}$;

phi = squareroot(val);

~~if(phi <= 3)~~

cout << " π value is " << phi

return 0;