A Differential GPS Carrier Phase Technique for Precision Outdoor AR Tracking

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ABSTRACT

This paper presents a differential GPS carrier phase technique for 3D outdoor position tracking in mobile augmented reality (AR) applications. It has good positioning accuracy, low drift and jitter, and low computation requirement. It eliminates the resolution of integer ambiguities. The position from an initial point is tracked by accumulating the displacement in each time step, which is determined using Differential Single Difference. Preliminary results using low cost GPS receivers show that the position error is 10cm, and the drift is 0.001ms^{-1}, which can be compensated using linear models. Stable and accurate augmentations in outdoor scenes are demonstrated.

KEYWORDS: Position tracking, Global Positioning System, differential carrier phase, outdoor augmented reality.

INDEX TERMS: H.5.1 Multimedia Information Systems: Artificial, augmented, and virtual realities H.5.2 User Interfaces: Input devices

1 INTRODUCTION

The Global Positioning System (GPS) is a satellite tracking system that enables users to determine their absolute position on the Earth's surface. This service is freely available to users equipped with GPS receivers operating on a radio frequency of 1.57542 GHz. This GPS measurement has high jitter and a typical positioning accuracy of 10m, rendering it unsuitable for outdoor Augmented Reality (AR) applications, except those which do not require high positioning accuracy. The paper presents the preliminary results of a novel technique using GPS carrier phase measurements from two low cost GPS receivers to achieve an accuracy of 10cm, with low drift and jitter. The use of carrier phase measurement is common for GPS surveying systems, which can often achieve 20cm accuracy in real-time and 1mm accuracy with post-processing. The main errors in the GPS measurements are removed by differencing techniques. The proposed method differs from previous methods with the use of a new quantity, namely, the Differential Single Difference, to compute the relative position of the mobile receiver from the starting position without having to determine the baseline vector relative to the stationary receiver. This method achieves the accuracy of current real-time GPS trackers without the need for heavy computing resources, which are required for resolving the integer ambiguities. There is a resultant linear drift due to the accumulation of minute errors of the actual GPS modules. However, the drift rate is less than 0.001ms^{-1}, varies slowly and is highly linear within a period of several minutes. Therefore, the drift can be easily corrected using linear regression.

The rest of the paper is organized as follows. A brief overview of the use of GPS in AR and surveying is presented in Section 2. This is followed by a description of the proposed method in Section 3, the experimental setup and results are presented in Sections 4 and 5 respectively. A discussion on the use of GPS for AR and future research directions are included in Section 6.

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2 RELATED WORKS

One of the earliest applications of GPS [1][2] as a positional tracker in an AR system is the Touring Machine [3]. This prototype is used for augmentation when navigating in a city, and in this application, the accuracy of the GPS receiver suffices. A recent example of a lightweight wearable system reported by Peternier et al [4] illustrates the rapid minimization of both the weight and power consumption of GPS receivers, which allows the GPS to be used as a robust and lightweight absolute position tracker. However, the accuracy level and jitter are not sufficient for applications that require higher level of precision. Therefore, the use of Differential GPS is often considered for accuracy of 1cm. Many work have been reported on the use of differential GPS carrier phase for determining the relative positions between two or more GPS receivers [5][6][7]. As surveying deals with distances over several kilometers or more, these techniques are not often directly applicable. Outdoor AR applications are expected to work using shorter baselines, as raw measurements are most likely transmitted in real-time using wireless links with a limited range. Techniques for short baselines are often more applicable [5][8][9]. When the baseline is less than a kilometer, the differential GPS measurements can be approximated using interferometry [5][9]. Such approximation is widely used in GPS attitude determination [9]. This approximation is applied in the method proposed in this paper. The main research issue in differential GPS is the resolution of the integer ambiguity in the presence of measurement noise and in real-time [5][6][7][10]. The proposed method avoids solving the integer ambiguities. A recent work [11] also tracks the relative position from an initial point, rather than from the stationary GPS receiver. This method differences the GPS Doppler measurements to obtain accurate velocities, which are in turn integrated to give the position. The method proposed here uses carrier phase measurements instead.

3 DESCRIPTION OF PROPOSED METHOD

Models for the carrier phase measurement and the associated errors can be found in numerous references [5][6][7][8]. Equation (1) is the model for the phase measurement from satellite i to receiver s at time t_k, which accounts for most of the significant errors. The measurement unit here is in carrier cycles.

$$\Phi_s^i(t_k) = \rho_s^i(t_k)\lambda^{-1} + N_s^i + f\tau_s(t_k) + f\tau^i(t_k) - \beta_{iono}(t_k) + \delta_{tropo}(t_k) + \mu_s^i \quad (1)$$

where $\Phi_s^i(t_k)$ is the carrier phase. $\rho_s^i(t_k)$ is the range between the satellite and the receiver. N_s^i is the integer ambiguity that is constant with respect to time except during cycle slips. $\tau_s(t_k), \tau^i(t_k)$ are the receiver and satellite clock errors respectively. $\beta_{iono}(t_k)$ and $\delta_{tropo}(t_k)$ are errors due to transmission through the Earth's ionosphere and troposphere respectively. μ_s^i includes both random noise and un-modeled errors. λ and f are the wavelength and frequency respectively.

The Single Difference (SD) is the difference between two simultaneous measurements of the carrier phase by the receivers s and r, for satellite i. Due to the short baseline between the receivers in this configuration, the ionospheric and tropospheric errors are common to both and removed by differencing [5]. The satellite clock error is also removed, while the noise level doubles but remains uncorrelated between the satellites [6]. The model for SD is shown in Equation (2).

$$
\begin{aligned}
SD_{sr}^i(t_k) &= \Phi_r^i(t_k) - \Phi_s^i(t_k) \\
&= \lambda^{-1}[\rho_r^i(t_k) - \rho_s^i(t_k)] + [N_r^i - N_s^i] + [f\tau_r(t_k) - f\tau_s(t_k)] + \mu_{sr}^i \\
&= \lambda^{-1}\rho_{sr}^i(t_k) + N_{sr}^i + f\tau_{sr}(t_k) + \mu_{sr}^i
\end{aligned} \tag{2}
$$

In GPS surveying, the SD for satellites i and j are differenced to form the Double Difference (DD), which removes the common inter-receiver clock errors. The DD at well separated time epochs, t_k and t_{k+n}, can be further differenced to form the Triple Difference (TD), which removes the time invariant integer ambiguities, assuming no cycle slips. The TD is a robust method to determine the static baseline vector to an accuracy of 1m. However, as the noise levels increase greatly due to differencing and become highly correlated, a minimum of one hour of data is often recommended in practice [6], rendering this an ineffectual method for dynamic real-time AR tracking. Although DD enables real-time tracking with at least four visible satellites, it is also not utilized in the proposed method. This is because the noise becomes correlated across satellites. As the DDs in each time epoch are computed against one common reference satellite, this causes the measurement to be overly dependent on the noise level of the reference satellite measurement [8]. Furthermore, these techniques are often complicated by the signal outage or setting of the reference satellite, especially for techniques where some forms of data from previous epochs are kept. The removal of the receiver clock errors in DD enables "float solution", where the integer ambiguities can be computed as real values and not integers. The accuracy is often quoted as within the decimeter level. Resolving the integer ambiguities to true integer values using techniques, such as LAMBDA [6][10], gives the "fixed solution" with millimeter accuracy level. This is possible due to the very low noise level of the carrier phase measurements, which is less than 1mm in today's receivers. The resolution of the integer ambiguities in the presence of noise is non-trivial. The short wavelength of the GPS carrier implies a large search space, and therefore a high computational load. Several minutes of data is typically required to increase the confidence in the accuracy.

The proposed method avoids the resolution of the integer ambiguity through differencing the SD between two consecutive time epochs t_k and t_{k+1}, for the same satellite i. This is shown in Equation (3)

$$
\begin{aligned}
SD_{sr}^i(t_k, t_{k+1}) = {}& \lambda^{-1}[\rho_{sr}^i(t_{k+1}) - \rho_{sr}^i(t_k)] + f[\tau_{sr}(t_{k+1}) - \tau_{sr}(t_k)] \\
&+ \mu_{sr}^i(t_k, t_{k+1})
\end{aligned} \tag{3}
$$

$SD_{sr}^i(t_k, t_{k+1})$ is a derived GPS quantity that can be described as the Differential Single Difference (DSD). DSD has been applied for the study of receiver hardware delay [12]. To convert the obtained DSDs to position measurements, the following steps are needed. First, $\rho_{sr}^i(t_k)$ is approximated using interferometry principles [5][9]. Consider receivers s and r that are less than a kilometer apart. As the GPS satellites are typically 23x10^{6}m away, the unit vectors $\mathbf{e}_s^i$ and $\mathbf{e}_r^i$, of the lines of sight from the two receivers to the same satellite i can be assumed to be parallel, i.e., $\mathbf{e}_s^i \cong \mathbf{e}_r^i$. Let $\mathbf{b}$ be the baseline vector between the receivers, in meters, and receiver s is stationary. The approximation is given by vector dot product, as shown in Equation (4).

$$
\rho_{sr}^i(t_k) = (\mathbf{e}_s^i(t_k) \bullet \mathbf{b}(t_k)) \tag{4}
$$

As the satellite moves several kilometers per second, $\mathbf{e}_s^i(t_k) \cong \mathbf{e}_s^i(t_{k+1})$ due to the large receiver to satellite range. Substituting Equation (4) into Equation (3) and omitting $\mu_{sr}^i(t_k, t_{k+1})$ for greater clarity, gives Equation (5).

$$
\begin{aligned}
&SD_{sr}^i(t_k, t_{k+1}) \\
&= \lambda^{-1}[\mathbf{e}_s^i(t_{k+1}) \bullet \mathbf{b}(t_{k+1}) - \mathbf{e}_s^i(t_k) \bullet \mathbf{b}(t_k)] + f[\tau_{sr}(t_{k+1}) - \tau_{sr}(t_k)] \\
&= \lambda^{-1}\mathbf{e}_s^i(t_{k+1}) \bullet [\mathbf{b}(t_{k+1}) - \mathbf{b}(t_k)] + f[\tau_{sr}(t_{k+1}) - \tau_{sr}(t_k)] \\
&= \lambda^{-1}\mathbf{e}_s^i(t_{k+1}) \bullet \Delta\mathbf{b}(t_{k+1}) + f\Delta\tau_{sr}(t_{k+1})
\end{aligned} \tag{5}
$$

There are four unknowns in Equation (5), namely, the three components in the position change vector $\Delta\mathbf{b}(t_{k+1})$ and inter-receiver time drift $\Delta\tau_{sr}(t_{k+1})$. $SD_{sr}^i(t_k, t_{k+1})$ is derived from raw GPS phase measurements, while $\mathbf{e}_s^i(t_{k+1})$ is obtained using satellite ephemeris, the receiver position using the standalone GPS measurement and the GPS time measured by the receiver. Although the standalone GPS position has an error of 10m, applying the same reasoning for the approximation, $\mathbf{e}_s^i(t_k) \cong \mathbf{e}_s^i(t_{k+1})$, the large range between the satellite to the receiver causes the resultant error in $\mathbf{e}_s^i(t_{k+1})$ to be insignificant. With at least four satellites, the unknowns can be solved using the following linear system.

$$
\begin{pmatrix}
SD_{sr}^1(t_k, t_{k+1}) \\
SD_{sr}^2(t_k, t_{k+1}) \\
SD_{sr}^3(t_k, t_{k+1}) \\
SD_{sr}^4(t_k, t_{k+1}) \\
\vdots
\end{pmatrix}
=
\begin{pmatrix}
\mathbf{e}_s^1(t_{k+1})^T, & 1 \\
\mathbf{e}_s^2(t_{k+1})^T, & 1 \\
\mathbf{e}_s^3(t_{k+1})^T, & 1 \\
\mathbf{e}_s^4(t_{k+1})^T, & 1 \\
\vdots & \vdots
\end{pmatrix}
\begin{bmatrix}
\lambda^{-1}\Delta\mathbf{b}(t_{k+1}) \\
f\Delta\tau_{sr}(t_{k+1})
\end{bmatrix} \tag{6}
$$

The position change from one epoch to the next can be accumulated to give the position vector of receiver r from the starting position. This is in contrast to measuring the baseline vector from receiver s to r, and avoids the resolution of integer ambiguities. The main issue with accumulative approaches is that minute errors and biases are also accumulated, resulting in positional drift. Furthermore, the noise also becomes time correlated. However, the experimental results in the section 5 show that with phase measurements from low cost GPS receivers, the drift is low, less than 0.001ms^{-1} and highly linear with time. The position derived is in the Earth-Centered Earth-Fixed (ECEF) frame [1]. The position vector can be rotated from ECEF to the local level, North-East-Down (NED) frame, at Longitude Λ, and Latitude Φ, using the following rotation matrix $\mathbf{R}$ [1]:

$$
\mathbf{R} = \begin{pmatrix}
-\sin(\Phi)\cos(\Lambda) & -\sin(\Phi)\sin(\Lambda) & \cos(\Phi) \\
-\sin(\Lambda) & \cos(\Lambda) & 0 \\
-\cos(\Phi)\cos(\Lambda) & -\cos(\Phi)\sin(\Lambda) & -\sin(\Phi)
\end{pmatrix} \tag{7}
$$

4 EXPERIMENTAL SETUP

To determine the effectiveness of the proposed method, two LEA-4T GPS modules from U-Blox are used to collect raw carrier phase measurements. The data is recorded using serial links and the vendor supplied software. The maximum measurement rate of the LEA-4T is 10Hz.

Two experiments, E1 and E2, were conducted. For E1, the receivers are placed three metres apart on a level ground. The direction from the static receiver *s* to the mobile receiver *r* with respect to the North is measured using an InertiaCube from InterSense. Two data sets, D1 and D2, are collected in E1. D1 consists of 1Hz GPS raw measurements collected over a period of one hour with both receivers static, so as to determine the drift characteristics. D2 consists of 10Hz GPS raw measurements, where the receiver *r* is first left static for approximately 180 seconds, moved 20cm along the baseline vector towards the static receiver *s*, after which the receiver *r* is then moved back to the starting position. The same motion profile is repeated, but at a distance of one metre instead of 20cm. The controls in E1 enable the accuracy of the proposed method to be illustrated. E2 is conducted to determine the suitability of the proposed method for outdoor AR applications. In E2, the receiver *r* is mounted rigidly with an InertiaCube and a Firewire video camera. The InertiaCube measures the orientation in the NED frame, which when combined with the proposed GPS setup becomes a hybrid six degrees of freedom tracker. The resultant position and orientation tracking data is used to augment virtual objects onto the video recorded. In this case, the assembly of the receiver *r*, the InertiaCube and the camera are handheld and moved over a distance of 1.2m. Both E1 and E2 are conducted in open areas, where there are minimal obstructions from both buildings and trees, which can cause signal outages and multipath errors.

5 Experimental Results

The position vector from the initial position of receiver *r*, **r1**, in the ECEF frame is computed using the proposed method based on the data set D1, and the values of *x*, *y* and *z* axes of the vector shown as a plot against time in Figure 1. As both receivers are stationary, Figure 1 shows the drift characteristics and the maximum position vector drift is 2.5m over a period of 3,000 seconds. This translates to a drift of less than $0.001 ms^{-1}$,, which is sufficient for maintaining the stability of virtual objects augmented onto a real environment. Figure 1 also shows that the drift varies slowly with time and is highly linear within a period of several hundred seconds. This is similar to the clock drift, and indicates that this drift is due to the residual clock errors in the GPS receivers and satellites. Finally, the drift has low jitter.

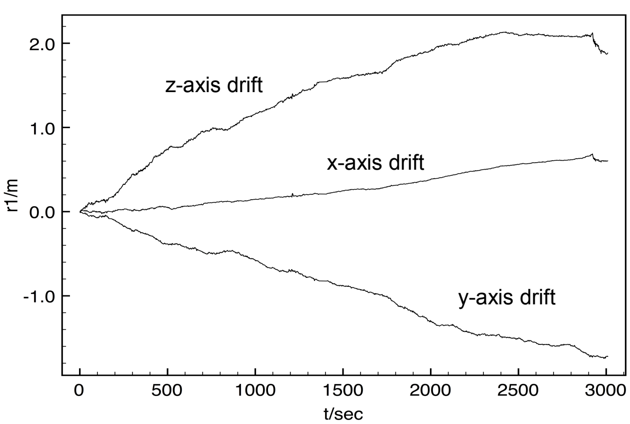

Figure 1. Plot of the drift of position vector from the initial position of a stationary receiver, **r1**, against time, *t*.

The position vector from the initial position of the receiver r, **r2,** in the NED frame derived using D2 is shown in Figure 2. For data set D2, the receiver *r* is moved 20cm to and fro, followed by one meter to and fro. The initial static period of 180 seconds allows for the determination of the linear drift, which can be effectively removed using linear regression analysis.

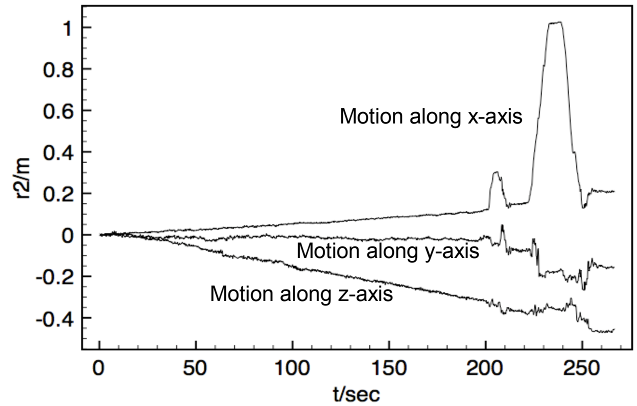

Figure 2. Plots of position vector from the initial position of mobile receiver *r*, **r2**, derived using D2 against time *t*.

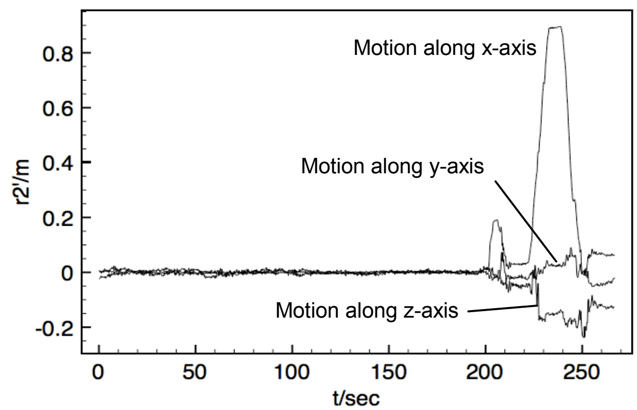

Figure 3. Drift corrected position vector from the initial position of mobile receiver *r*, **r2'**, derived using D2 against time *t*.

Figure 3 shows the plot for **r2'** derived using D2 with the error corrected. The result shows that the linear drift is effectively removed and the prescribed motions are measured with a good level of accuracy. From Figure 3, the magnitude of the distance moved is accurate for the first prescribed motion profile of 20cm and within 10cm for the second prescribed motion profile of 1m. The error after receiver *r* has been returned to the starting position is 15cm. These errors are mainly due to the noise introduced by the effects of the motion on the reception of the radio signal by the antenna on the receiver *r*. Further work will be required to determine the antenna designs where the reception is minimally affected by motion. The low jitter and high precision indicate that the proposed method is suitable for outdoor AR applications.

Figure 4 shows the plot of the linear drift corrected position vector **r3**, in the NED frame derived using the data collected in experiment E2. For this data set, the linear drift is low and effectively removed using linear regression analysis on the initial 60 seconds of the static data. Here, the main motion is the picking up of the camera and panning to record the scene. There is also a certain amount of lateral motion and tilting of the camera. The plot shows that the motion of the camera is tracked with a high level of accuracy and with low jitter. As the motion profile is not exactly known, as in the case for D2, the effectiveness of the method is demonstrated by augmenting virtual objects onto a video, thereby directly checking the effectiveness of the proposed method for outdoor AR. Qualitatively, the video shows that the motion of the camera is well-tracked, allowing for fairly realistic augmentation.

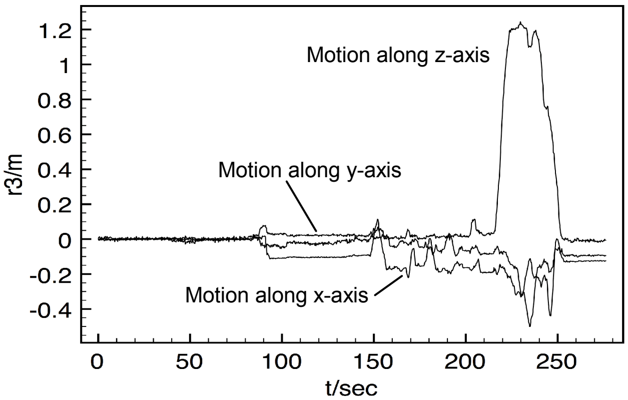

Figure 4. Drift corrected position vector from the initial position of mobile receiver *r*, **r3**, against time *t*.

6 Conclusion and Further Work

This paper presents the preliminary results of a novel use of GPS carrier phase measurements from two GPS receivers, one stationary and the other mobile, for high precision position tracking in outdoor environments with a focus toward AR applications. A quantity, Differential Single Difference (DSD), is derived from raw phase measurements. A novel method is proposed to use the DSD to compute the relative position of the mobile receiver from its initial starting position. This method works by accumulating the positional change in each time epoch computed using DSD. The current work shows that the quality of the phase measurements from low cost GPS modules is sufficient to achieve an accuracy of 10cm in precision tracking. The results show that the error in using the proposed method drifts slowly with time, is highly linear within a period of several minutes and has low jitter. The experimental results also show that the proposed method has an accuracy of 10cm. This result is obtained without sophisticated signal processing or filtering. As the carrier phase can be measured with an accuracy of 1mm, a tracker accuracy of 1cm is possible with further improvements in the design of the antenna and the receiver, as well as signal processing techniques. Such a level of precision is comparable to indoor tracking systems, allowing for accurate tracking for new large scale, outdoor AR applications.

In the context of traditional GPS surveying techniques, the results show that DSD is useful as a derived quantity with good noise characteristics. In GPS surveying, the main goal is to derive accurate measurements of the relative vector between two points that are several kilometers apart, thus limiting the usefulness of DSD. In contrast, the relative position from the initial position is a useful quantity for AR applications. In a fully developed setup, the static receivers may be parts of an existing infrastructure. In locations without a static receiver, a receiver can be carried by the user and left at a static position for the duration of use. In both cases, the raw measurements can be transmitted wirelessly to the mobile unit. On initialization, the starting point may be determined automatically or set by the user, after which tracking continues using the proposed method. Other than having high precision, low drift and jitter, which have been shown experimentally, the proposed method has low computational load and is robust as compared to traditional GPS surveying techniques, allowing it to be used in real-time. If the low drift is assumed to be insignificant, the tracking system is immediately usable, after the GPS signals are locked by the receivers. Furthermore, traditional GPS relative positing techniques can be used to periodically correct the drift by measuring the actual baseline between the two receivers. This allows for highly accurate real-time tracking while avoiding the high computational load associated with integer ambiguity resolution.

The main issues with the proposed method are common to all GPS based trackers, namely the need for a clear line of sight to the satellites and frequent signal outages. This is particularly acute for carrier-based techniques as a good signal to noise ratio is required for the phase lock. In GPS, a channel noise measure, C/N_0, is used and a value greater than 35 is generally needed in experiments, otherwise this can cause the GPS receiver to lose the signal phase lock due to obstructions and antenna motions. When the number of phase measurements drops below four, the proposed method is no longer effective. Subsequent phase lock will render the DSD to be inaccurate as cycle slips would have occurred. Therefore, the preliminary results here can be used for creating hybrid trackers through combining GPS with inertial and computer vision based trackers. Under good operating conditions, the proposed method can also be used as a valuable tool for the development and validation of other outdoor trackers. Finally, with the modernization of GPS with increased signal power levels, the addition of the European Galileo system and the increasing performance of GPS receivers in recent years, GPS is expected to serve as a valuable and easily accessible tool for high precision, real-time and wide area outdoor tracking.

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