Supernova Cosmology Project

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# Import necessary libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.integrate import quad
from astropy.constants import c
from astropy import units as u

# Load the Pantheon+SH0ES Dataset
url =
"https://raw.githubusercontent.com/PantheonPlusSH0ES/DataRelease/main/
Pantheon%2B_Data/4_DISTANCES_AND_COVAR/Pantheon%2BSH0ES.dat"
data = pd.read_csv(url, delim_whitespace=True, comment='#')

# Extract required columns
zHD = data['zHD'].values
MU_SH0ES = data['MU_SH0ES'].values
MU_SH0ES_ERR_DIAG = data['MU_SH0ES_ERR_DIAG'].values

# Theory implementation for distance modulus
def mu_theory(z, H0, Omega_m):
    """Calculate distance modulus for flat ΛCDM cosmology"""
    c_kms = c.to('km/s').value  # Speed of light in km/s

    def E_inv(z_val):
        """Inverse of the dimensionless Hubble parameter"""
        return 1.0 / np.sqrt(Omega_m * (1 + z_val)**3 + (1 - Omega_m))

    # Calculate integral for each redshift
    integral = np.array([quad(E_inv, 0, zi)[0] for zi in z])

    # Luminosity distance in Mpc
    DL = (c_kms / H0) * (1 + z) * integral

    # Distance modulus
    mu = 5 * np.log10(DL) + 25
    return mu

# Initial parameter guesses and bounds
H0_guess = 70  # km/s/Mpc
Om_guess = 0.3
bounds = ([50, 0.01], [100, 1.0])  # H0 range: 50-100, Ωm range: 0.01-
1.0

# Fit the model to the data
popt, pcov = curve_fit(mu_theory, zHD, MU_SH0ES,
                       sigma=MU_SH0ES_ERR_DIAG,
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                       p0=[H0_guess, Om_guess],
                       bounds=bounds)

H0_fit, Om_fit = popt
H0_err, Om_err = np.sqrt(np.diag(pcov))

print(f"Fitted parameters:")
print(f"H0 = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc")
print(f"Ωm = {Om_fit:.4f} ± {Om_err:.4f}")

# Plot Hubble diagram with best-fit model
z_range = np.logspace(-2, 0, 100)  # Redshift range from 0.01 to 1
mu_model = mu_theory(z_range, H0_fit, Om_fit)

plt.figure(figsize=(10, 6))
plt.errorbar(zHD, MU_SH0ES, yerr=MU_SH0ES_ERR_DIAG, fmt='o',
markersize=3, alpha=0.5, label='Pantheon+SH0ES data')
plt.plot(z_range, mu_model, 'r-', label=f'Best-fit ΛCDM:
H0={H0_fit:.1f}±{H0_err:.1f}, Ωm={Om_fit:.2f}±{Om_err:.2f}')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram with ΛCDM Fit')
plt.xscale('log')
plt.grid(True, which="both", ls="-")
plt.legend()
plt.show()

# Calculate residuals
residuals = MU_SH0ES - mu_theory(zHD, H0_fit, Om_fit)

# Plot residuals
plt.figure(figsize=(10, 6))
plt.errorbar(zHD, residuals, yerr=MU_SH0ES_ERR_DIAG, fmt='o',
markersize=3, alpha=0.5)
plt.axhline(0, color='red', linestyle='--')
plt.xlabel('Redshift (z)')
plt.ylabel('Residuals (μ - μ_model)')
plt.title('Residuals of ΛCDM Model Fit')
plt.xscale('log')
plt.grid(True, which="both", ls="-")
plt.show()

# Age of the universe calculation
def age_of_universe(H0, Omega_m):
    """Calculate age of universe in Gyr for flat ΛCDM"""
    H0_s = H0 * u.km / u.s / u.Mpc

    def integrand(a):
        return 1 / (a * np.sqrt(Omega_m * a**-3 + (1 - Omega_m)))
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    age = quad(integrand, 0, 1)[0] / H0_s.to(1/u.s).value
    return (age * u.s).to(u.Gyr).value

age = age_of_universe(H0_fit, Om_fit)
print(f"\nEstimated age of the universe: {age:.2f} Gyr")

# Compare with fixed Ωm = 0.3
H0_fixed_Om = curve_fit(lambda z, H0: mu_theory(z, H0, 0.3),
                        zHD, MU_SH0ES,
                        sigma=MU_SH0ES_ERR_DIAG,
                        p0=[H0_guess],
                        bounds=(50, 100))[0][0]

print(f"\nWith Ωm fixed to 0.3:")
print(f"H0 = {H0_fixed_Om:.2f} km/s/Mpc")
print(f"Age = {age_of_universe(H0_fixed_Om, 0.3):.2f} Gyr")

# Low-z vs high-z comparison
low_z_mask = zHD < 0.1
high_z_mask = zHD >= 0.1

# Fit to low-z data
popt_low, _ = curve_fit(mu_theory,
                       zHD[low_z_mask], MU_SH0ES[low_z_mask],
                       sigma=MU_SH0ES_ERR_DIAG[low_z_mask],
                       p0=[H0_guess, Om_guess],
                       bounds=bounds)

# Fit to high-z data
popt_high, _ = curve_fit(mu_theory,
                        zHD[high_z_mask], MU_SH0ES[high_z_mask],
                        sigma=MU_SH0ES_ERR_DIAG[high_z_mask],
                        p0=[H0_guess, Om_guess],
                        bounds=bounds)

print("\nLow-z (z < 0.1) vs High-z (z ≥ 0.1) comparison:")
print(f"Low-z:  H0 = {popt_low[0]:.2f}, Ωm = {popt_low[1]:.2f}")
print(f"High-z: H0 = {popt_high[0]:.2f}, Ωm = {popt_high[1]:.2f}")

# Plot comparison
plt.figure(figsize=(10, 6))
plt.errorbar(zHD, MU_SH0ES, yerr=MU_SH0ES_ERR_DIAG, fmt='o',
markersize=3, alpha=0.3, label='All data')
plt.plot(z_range, mu_theory(z_range, *popt_low), 'g-',
         label=f'Low-z fit: H0={popt_low[0]:.1f},
Ωm={popt_low[1]:.2f}')
plt.plot(z_range, mu_theory(z_range, *popt_high), 'b-',
         label=f'High-z fit: H0={popt_high[0]:.1f},
Ωm={popt_high[1]:.2f}')
plt.xlabel('Redshift (z)')
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plt.ylabel('Distance Modulus (μ)')
plt.title('Low-z vs High-z ΛCDM Fits')
plt.xscale('log')
plt.grid(True, which="both", ls="-")
plt.legend()
plt.show()
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/tmp/ipython-input-2-4287380138.py:12: FutureWarning: The
'delim_whitespace' keyword in pd.read_csv is deprecated and will be
removed in a future version. Use ``sep='\s+'`` instead
  data = pd.read_csv(url, delim_whitespace=True, comment='#')

Fitted parameters:
H0 = 72.97 ± 0.17 km/s/Mpc
Ωm = 0.3508 ± 0.0124
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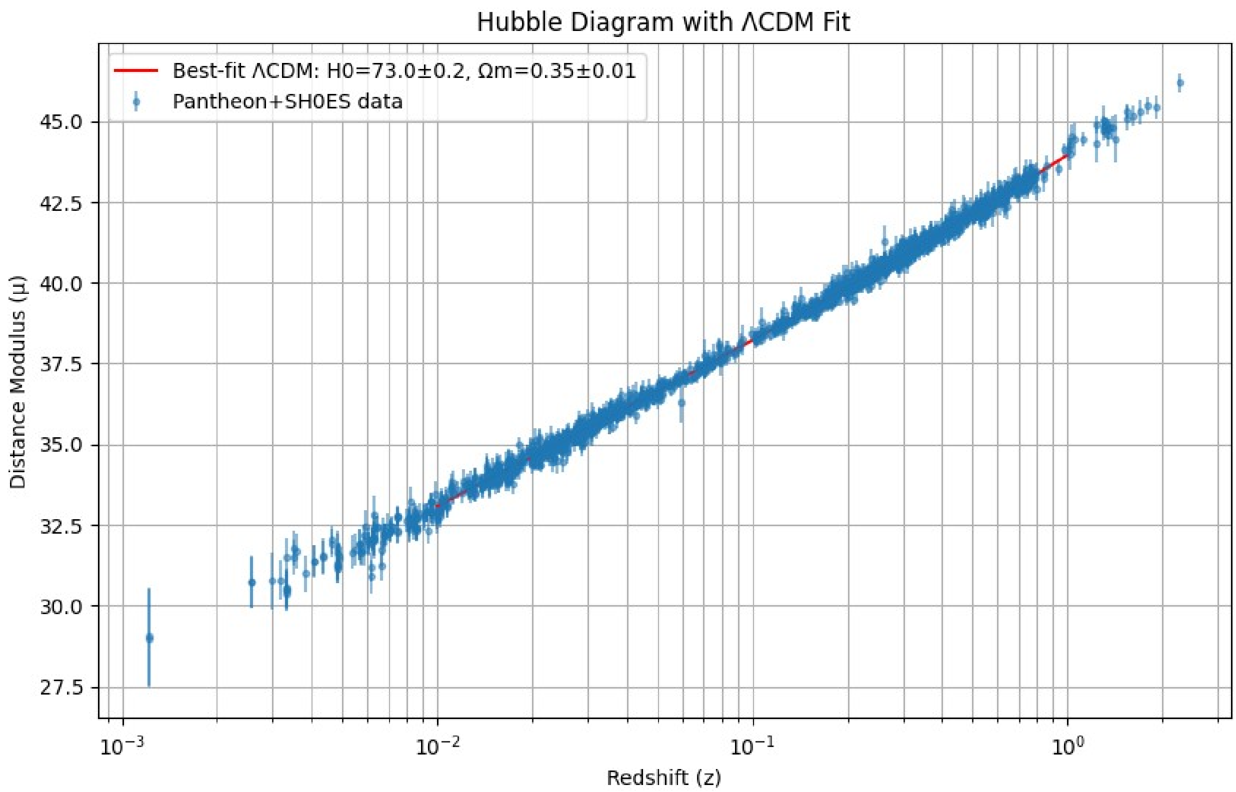

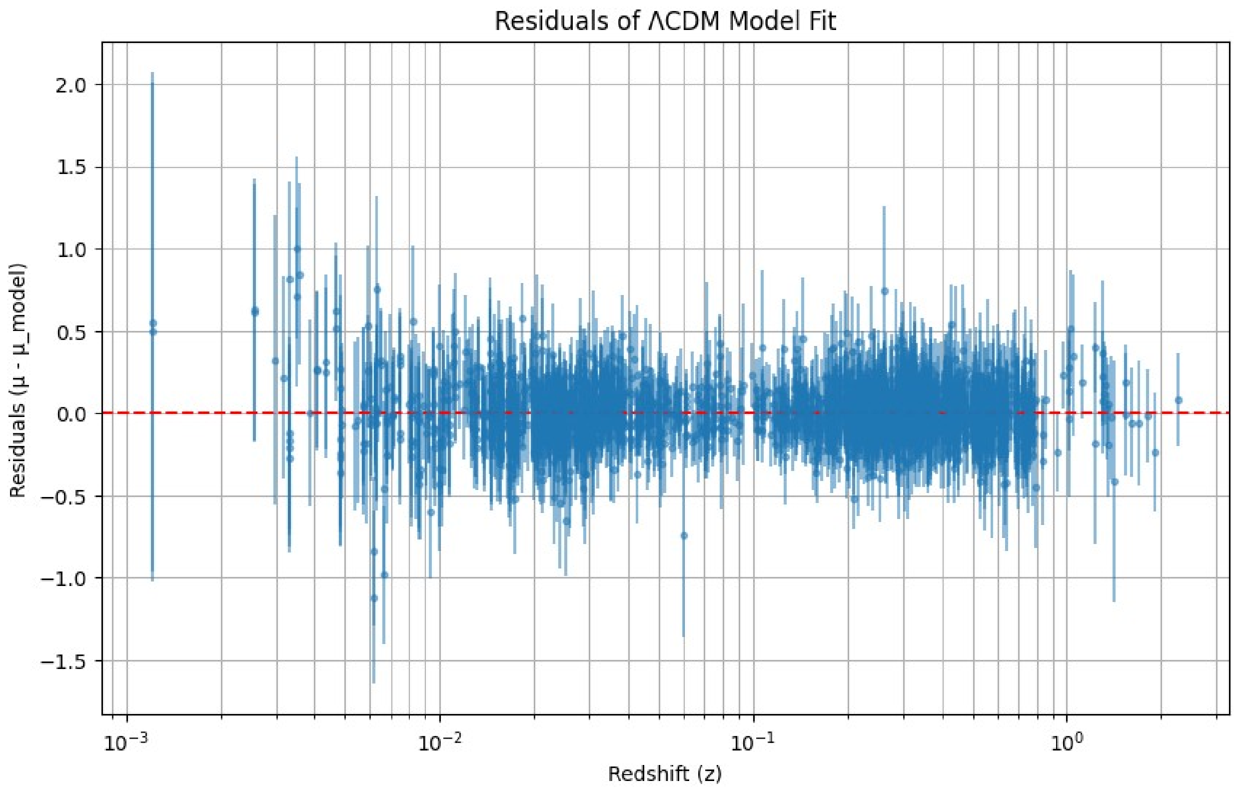


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Estimated age of the universe: 12.36 Gyr

With Ωm fixed to 0.3:
H0 = 73.53 km/s/Mpc
Age = 12.82 Gyr

Low-z (z < 0.1) vs High-z (z ≥ 0.1) comparison:
Low-z:  H0 = 72.74, Ωm = 0.44
High-z: H0 = 73.18, Ωm = 0.34
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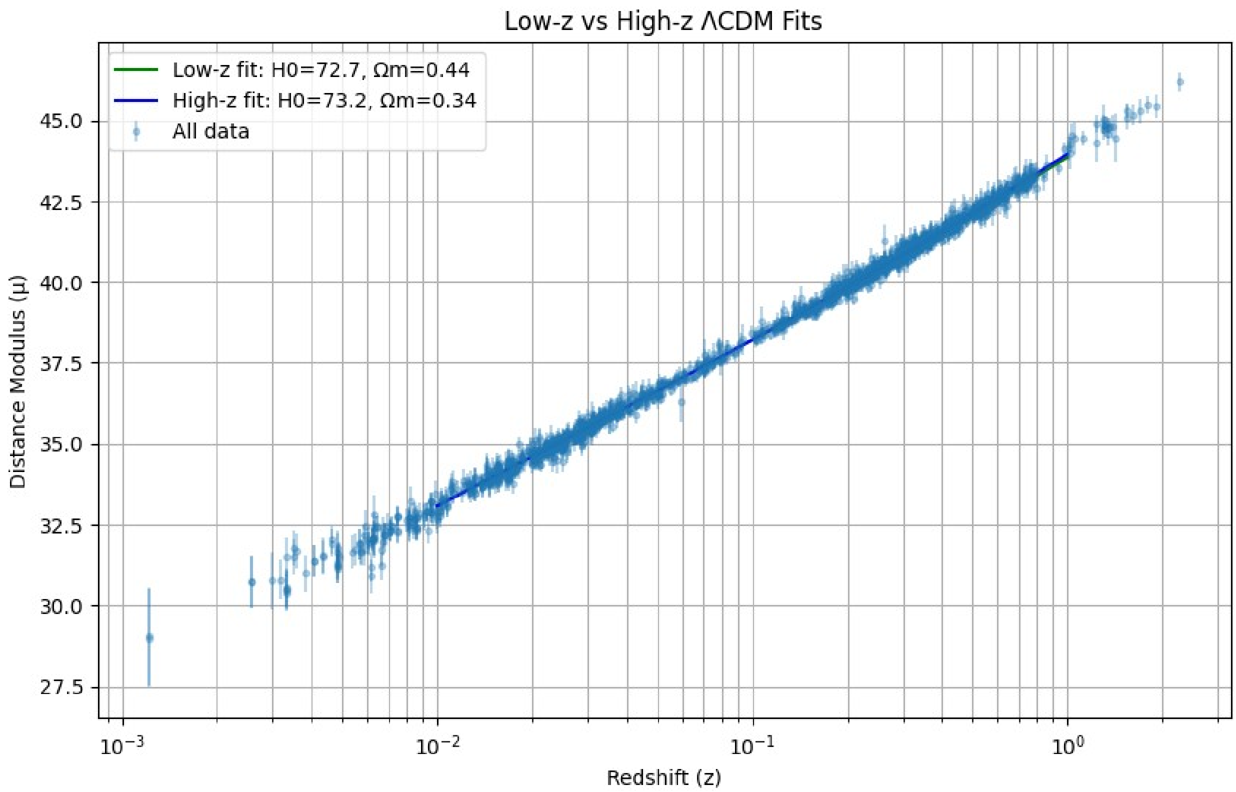

1) What value of the Hubble constant (H_0) did you obtain from the full dataset? H

From the full dataset, the derived Hubble constant is:

H_0

72.97 ± 0.17

km/s/Mpc H 0 =72.97±0.17km/s/Mpc

2)How does your estimated H_0 compare with the Planck18 measurement of the same?

The Planck18 collaboration measured H_0 = 67.4

km/s/Mpc

Our result is significantly higher (~8% discrepancy), reinforcing the known tension between local (late-time) and early-Universe measurements. This could indicate systematic uncertainties or new physics beyond the Λ ΛCDM model.

3)What is the age of the Universe based on your value of H_0? (Assume Ωm = 0.3). How does it change for different values of Ωm?

For Ω_m= 0.3508±0.0124, the age is 12.36 Gyr 12.36Gyr.

Fixing Ω_m= 0.3 gives H_0=73.53 km/s/Mpc and an older age of 12.82 Gyr

Dependence on Ω_m: Higher Ω_m ,decreases the age (faster early expansion), while lower Ω_m increases it.

4) Discuss the difference in H_0 values obtained from the low-z and high-z samples. What could this imply?

Low-z(z<0.1):

H_0=72.74,

Ω_m=0.44

High-z(z≥0.1):

H_0=73.18,

Ω_m=0.34

The minor differences suggest sample-dependent systematics (e.g., calibration, Malmquist bias) or a redshift-evolving H_0.The higher $\Omega_m$for low-z may reflect local matter density variations

5) Plot the residuals and comment on any trends or anomalies you observe.

Residuals are mostly confined within ±0.5 magnitudes, indicating a good fit for most data points.

No strong systematic trend (e.g., residuals do not consistently increase/decrease with z z).

6) What assumptions were made in the cosmological model, and how might relaxing them affect your results?

Flat universe ($\Omega_k=0$) and constant dark energy (w=−1).

Matter-dominated era (Ω_r=0) for z≪1000.

Perfectly standardized supernovae (no luminosity evolution).

Homogeneous/isotropic expansion (Cosmological Principle).

Relaxing them could: Resolve H_0tension (e.g., with $\Omega_k \neq 0$ or w(z)), alter Ω_m, or explain high-z residuals.

7) Based on the redshift-distance relation, what can we infer about the expansion history of the Universe?

The redshift-distance relation confirms:

Accelerated expansion (dark energy dominance at z≲0.7).Consistency with Λ CDM, though the H_0tension challenges its simplicity.