Question 1 – Exoplanet Characterization
In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

To detect the planet, inclination should be around 90 degrees.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.
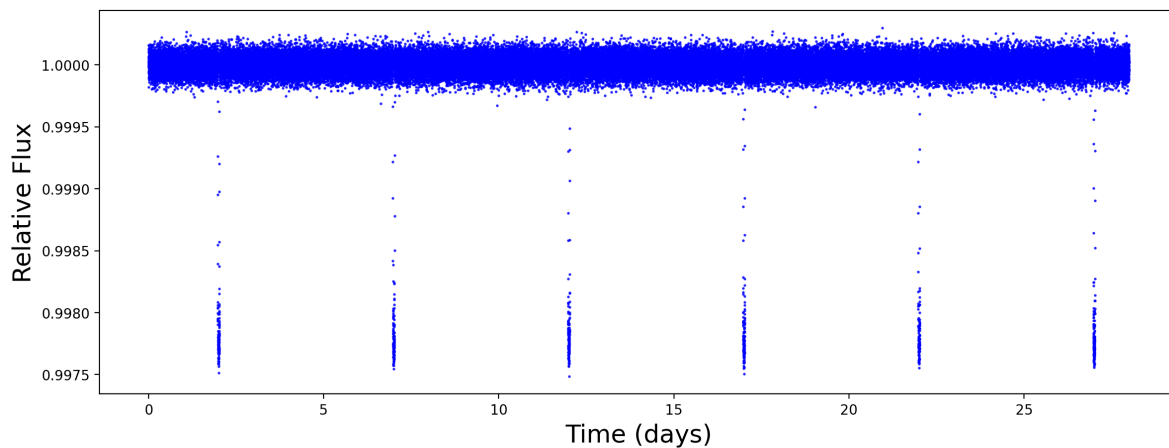

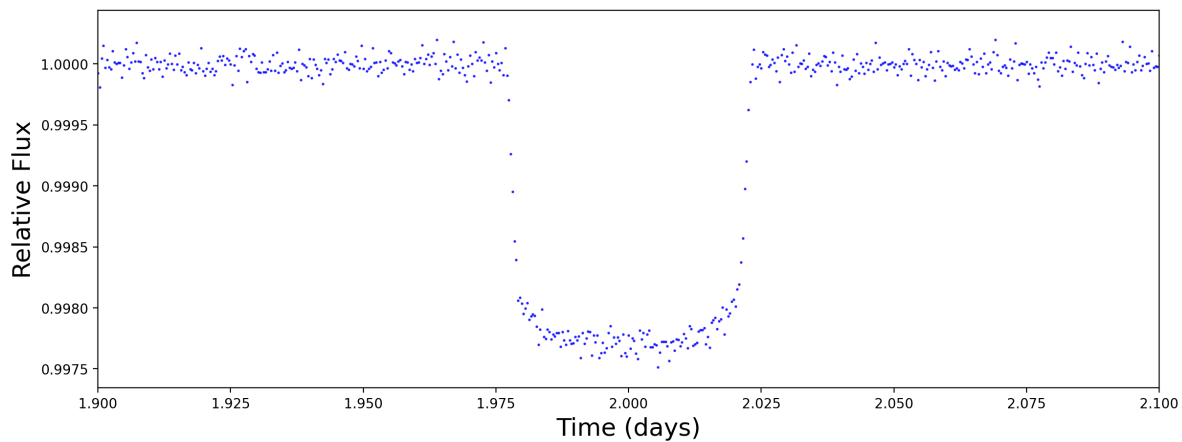Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

As per the Fig. 1,In the period of 28 days, the transit is repeated six times. so,
the period of the given planet is around 4.6 Days. With rounding off, The period is 5 Days

c) What is the radius of this planet?

The radius of the planet depends on the transit of the planet, From Fig 2

stars normal brightness = 1.0000
stars brightness during the transit = 0.9975

transit depth (z) = 1.0000 - 0.9975
                  = 0.0025

Radius of planet(Rp) = Radius of star(Rs) * sqrt(z)

Given :

Rs = 0.2*radius of sun

Rp = 0.2*696,340*0.05
    =6963.4 Km

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.
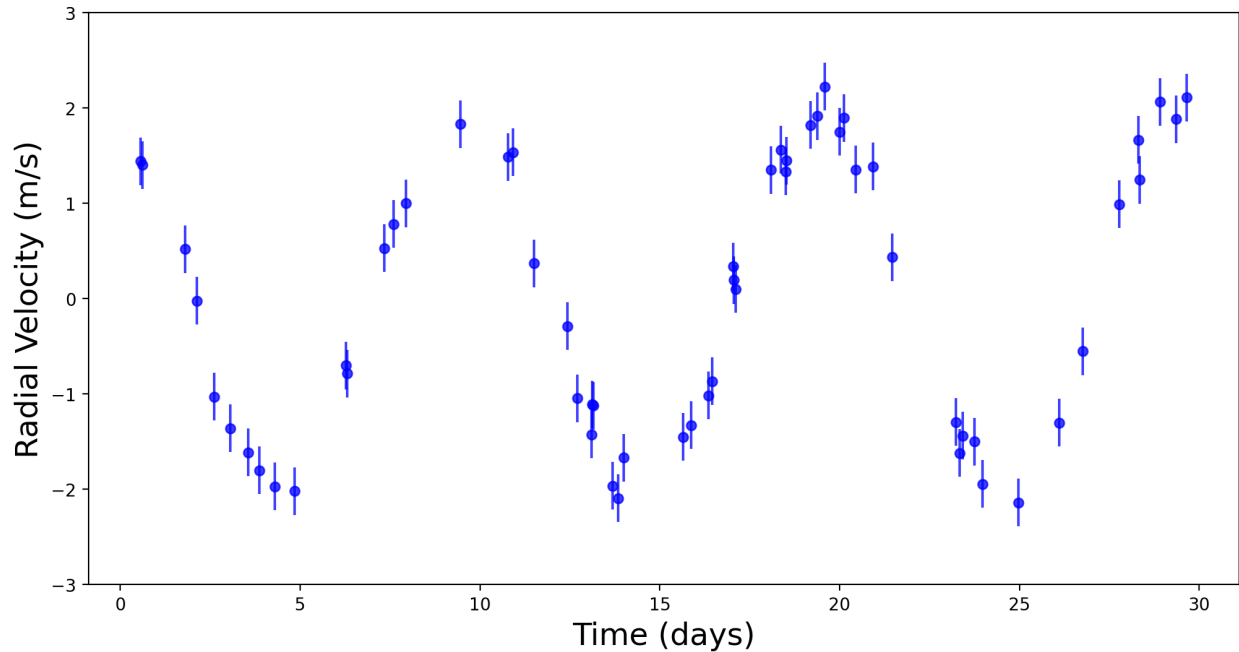

Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

Fig 3 shows, that the radial velocity is periodically shifts between -2 m/s and +2m/s
.
which result in semi amplitude of 2 m/s.

e) What is the mass of this planet?

Mass of the given planet can be determined by the following formula:

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{M_*^{2/3}}$$

So, formula for the mass is:

$$M_p \sin i = K \left(\frac{P}{2\pi G}\right)^{1/3} M_*^{2/3}$$

Where :
sin i = 1 (i = 90 degrees)
K = 2 m/s = 432000 s
P = 5 days
M(mass of star) = 0.2 * mass of sun = 0.2*1.98*10^30 kg
                  = 0.39*10^30 kg
G= 6.674*10^(-11) m^3/kg*s^2

by putting values in above formula, we get

mass of planet = 0.1174* 10^26 Kg

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.
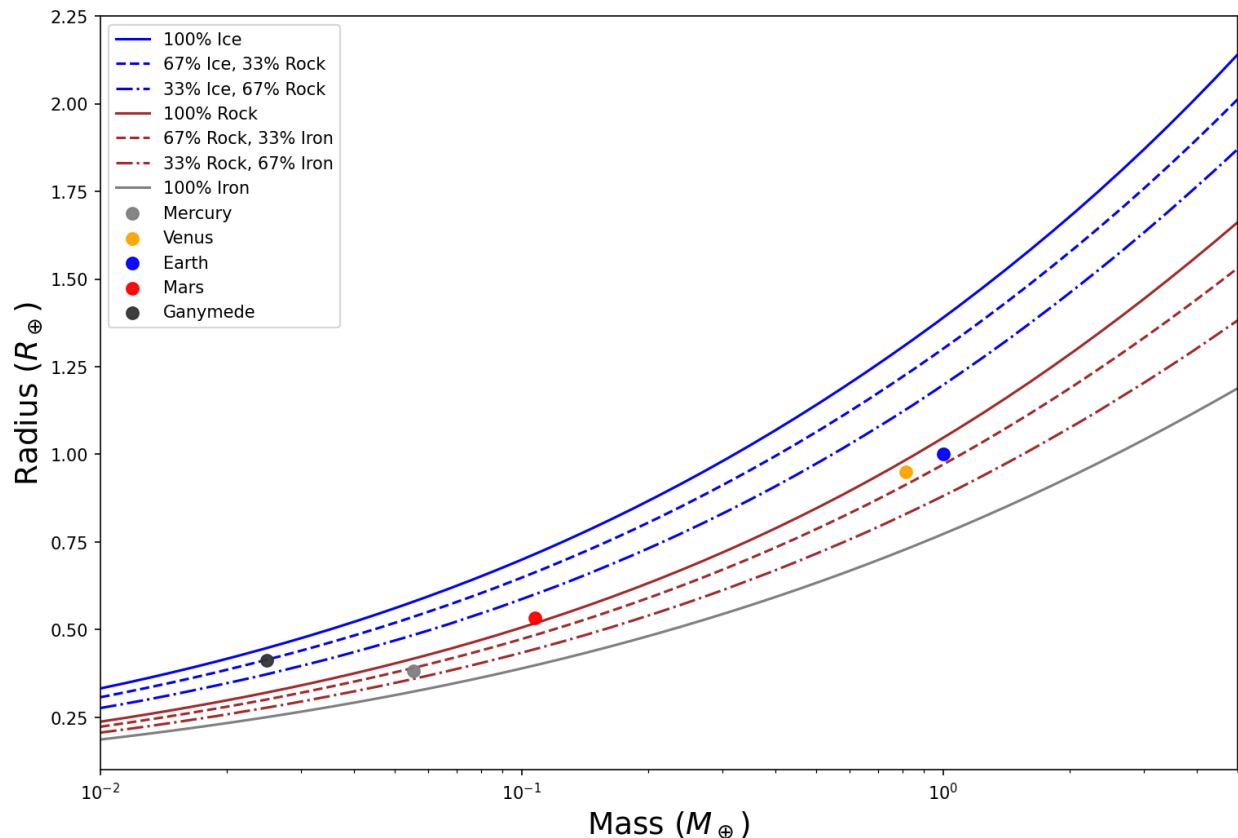

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

radius of planet is 0.01 times of radius of sun
mass of planet is 0.00016 times of mass of sun

the given planet lie in the region of 33% rock and 67% iron