Summary of quantum Simulator

Introduction

The paper ‘Universal Statistical Simulator’ by Mark Carney and Ben Varcoe demonstrated the quantum circuit for galton boards.

The Galton Board, invented by Sir Francis Galton tells about the probability distribution of random walks. Classical Galton boards have pegs, each peg has the probability of 50% to direct the ball to right and to the left. Balls after passing through the pegs get collected in a bin. Strangely as it seems, the ball in the bin gives the nice and clean bell curve also known as normal or binomial distribution.

This property of galton boards makes it useful for many natural phenomena and computation complexities. The random walk, a path sequencing by any random steps, is a widely known application of galton boards. It can be used in solving phenomena like brownian motion, entropy, thermodynamic randomness, modelling of stock prices fluctuations, machine learning etc.

Mark and Ben in the given paper demonstrate the circuit for quantum random walk. They build a circuit for a quantum galton board that mimics the physical action of a classical galton board.

The quantum galton board

To mimic the action of classical galton board, they created the circuit in which a set of controlled swap and controlled not, working as peg.
Peg in a layer is arranged in the manner of triangle number [1,3,6,10, 15 ,- - -]. That means in the first layer we have 1 peg and then in the second layer we have 3 peg and in the third layer we have 6 peg etc.

For one peg circuit, they need four qubits, where the 0th qubit acts as a controlled qubit(all swap gates are controlled by this) and the middle qubit acts as a ball. The hadamard gate is applied on the 0th qubit which creates an equal superposition, and the not gate gate is applied at the middle qubit, followed by the series of controlled swaps and controlled not. Cnot is applied to rebalance the circuit. Apply measurement and 1st and 3rd qubit.

They also give the circuit for 3 peg (2 level) quantum galton board. For level 2, They use 6 qubit, the one peg circuit remains as it is and they reset the controlled qubit and apply hadamard again to create a superposition. And then repeat the peg module 3 times. Take measurement from 1st, 3rd and 5th qubit.

They claim that quantum gates and measurement bits needed by this method are very less as compared to the previous research's. The upper limit of the gate used in this method is given by: $2n^2 + 5n + 2$
The Author also mentions that though gate counts are reduced, however this circuit needs post - processing.

Experimental results
They run the 4 level quantum galton board on an IBM-QX simulator which produces a nice bell curve.
But for full QGB, the results were not that promising as the author mentioned the action of the transpiler increases the gates and eventually the noise in the circuit is increased. To solve this problem, they run the circuit on local simulator and stored all 20000 shots generated and they reassigned them with an integer 0 to 4 such that their mean average, standard deviation and variance are 1.9977, 1.001521 and 1.002995 respectively. After this reassignment, they took blocks of 8 output and summed each block, which scaled up the circuit from 0 to 32. When the sum of each block is plotted against this array of 32 we get the nice bell curve, a normal distribution.

Biased QGB

Authors also discussed the biased QGB, to get the desired distribution plots.
To achieve this, they replace the hadamard gate applied on the 0th qubit with the rotation gate(about x-axis).
This will create an unequal superposition depending on the degree of rotation, and hence we get a different distribution plot.
The number of gates is still limited. The total number of gates used in this are:
$3n^2 + 4n + 2$

They then run this biased circuit on IBm manilla which like the previous one offers some noise and disturbed the plot. To solve the problem, they follow the same procedure. Firstly they run the circuit on a local simulator and then assign values to output and then add each block with the same values. And he noticed a skewed distribution, which aligns the experiment's goals.

Fine grained biased QGB circuit

The authors also tried to implement the new circuit in which they introduced 3 different rotation gates(about x-axis), each at the start of a layer, in a 3 layer circuit. And apply reset at the end of 4th qubit and cnot before that to balance the output.

This circuit possesses $3n^2 + 3n + 2$ gates, which is very limited compared to previous research as per the authors.