

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
 - A. 0.3875
 - B. 0.2676
 - C. 0.5
 - D. 0.6987

We need to find the probability that the service manager cannot meet his commitment, which means the time required to service the transmission is more than 50 minutes (60 - 10).

Using the z-score formula, we can calculate the z-score as follows:

$$z = (x - \mu) / \sigma = (50 - 45) / 8 = 0.625$$

Using a standard normal distribution table or calculator, we can find the probability that z is greater than 0.625:

$$P(z > 0.625) = 0.2676$$

Therefore, the probability that the service manager cannot meet his commitment is **0.2676**.

So, the answer is **B**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.
 - A. More employees at the processing center are older than 44 than between 38 and 44.

```
#q2a
from scipy import stats
p_z1=stats.norm.cdf(44,38,6)
p_z2=stats.norm.cdf(38,38,6)
p_z1,p_z2
p_38_44 = abs(p_z2 - p_z1)
print(f"Therefore, the probability that an employee is between 38 and 44 years old is: {p_38_44}")
p_44 =1- p_z1
print(f"The probability that an employee is older than 44 years old is: {p_44}")
if p_38_44 > p_44:
    print('Therefore, more employees at the processing center are not older than 44 than between 38 and 44.
          so false')
else:
    print('Therefore, more employees at the processing center are older than 44 than between 38 and 44.
          so true')
```

Therefore, the probability that an employee is between 38 and 44 years old is: 0.3413447460685429
The probability that an employee is older than 44 years old is: 0.15865525393145707
Therefore, more employees at the processing center are not older than 44 than between 38 and 44.
so false

False

- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

```
#q2b
from scipy import stats
p_30 = abs(stats.norm.cdf(30,38,6))

print(f'''the probability of selecting 36 or more employees under the age of 30 is approximately {p_30} or {p_30*100}%.
so false''')
```

the probability of selecting 36 or more employees under the age of 30 is approximately 0.09121121972586788 or 9.121121972586788%.
so false

False

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are iid normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are independent and identically distributed (iid) normal random variables, then the difference between $2X_1$ and $X_1 + X_2$ can be calculated as follows:

$$2X_1 - X_1 + X_2 = X_1 + X_2$$

Therefore, the difference between $2X_1$ and $X_1 + X_2$ is zero.

The distribution of $X_1 + X_2$ is also normal, with a mean of 2μ and a variance of $2\sigma^2$

The distribution of $2X_1$ is also normal, with a mean of 2μ and a variance of $4\sigma^2$

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

```
#q4
import math
z = 2.58
mean = 100
sd = 20
a = mean - z*sd
b = mean + z*sd
a,b
print(f'''Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taki

Therefore, the answer is D. {a}, {b}.''' )
```

Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99, are approximately 48.4 and 151.6.

Therefore, the answer is D. 48.4, 151.6.

Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99, are approximately 48.4 and 151.6.

Therefore, the answer is D. 48.4, 151.6.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
 - Specify the 5th percentile of profit (in Rupees) for the company
 - Which of the two divisions has a larger probability of making a loss in a given year?

```
#q5
import numpy as np
mean = (5 + 7) * 45
variance = 3^2 * 45 + 4^2 * 45
sd = np.sqrt(variance)
z1 = stats.norm.ppf(0.025)
a = mean - z1*26
b = mean + z1*26
print(f"A. Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is({a}, {b})")
z2 = stats.norm.ppf(0.05)
profit = mean + z2*26
print(f"B. The 5th percentile of profit (in Rupees) for the company is {profit}")
z3 = -5/3
profit1 = stats.norm.cdf(z3, 5, 3)
z4 = -7/4
profit2 = stats.norm.cdf(z4, 7, 4)
if profit1 < profit2:
    print("C. Profit1 has a smaller probability of making a loss in a given year than Profit2")
else:
    print("C. Profit2 has a smaller probability of making a loss in a given year than Profit1")
```

A. Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is(590.9590635980414, 489.04093640195856)

B. The 5th percentile of profit (in Rupees) for the company is 497.2338056992617

C. Profit1 has a smaller probability of making a loss in a given year than Profit2

A. Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is(590.9590635980414, 489.04093640195856)

B. The 5th percentile of profit (in Rupees) for the company is 497.2338056992617

C. Profit1 has a smaller probability of making a loss in a given year than Profit2