Topics: Confidence Intervals

1. For each of the following statements, indicate whether it is True/False. If false, explain why.

   I. The sample size of the survey should at least be a fixed percentage of the population size in order to produce representative results.
   False. The sample size of a survey should be determined based on the desired level of precision and confidence, not as a fixed percentage of the population size. A larger population size does not necessarily require a larger sample size, but it can increase the margin of error

   II. The sampling frame is a list of every item that appears in a survey sample, including those that did not respond to questions.
   False. The sampling frame is a list of all the items in the population from which the sample is drawn, not the sample itself.

   III. Larger surveys convey a more accurate impression of the population than smaller surveys.
   False. The accuracy of a survey depends on the sample size, not the survey size. A larger sample size can provide a more accurate representation of the population, but it also requires more resources and can increase the margin of error.

2. *PC Magazine* asked all of its readers to participate in a survey of their satisfaction with different brands of electronics. In the 2004 survey, which was included in an issue of the magazine that year, more than 9000 readers rated the products on a scale from 1 to 10. The magazine reported that the average rating assigned by 225 readers to a Kodak compact digital camera was 7.5. For this product, identify the following:

   A. The population: **all users of different brands of electronics**
   B. The parameter of interest: **average rating assigned to a Kodak compact digital camera**
   C. The sampling frame: **all the items in the population from which the sample is drawn**
   D. The sample size: **225 readers**
   E. The sampling design: voluntary response
   F. Any potential sources of bias or other problems with the survey or sample
      - The survey was conducted by PC Magazine, which may have a biased readership.
      - The sample size is relatively small, which can increase the margin of error.
      - The sampling design is voluntary response, which can lead to self-selection bias.
      - The survey only includes readers of PC Magazine, which may not be representative of the entire population of users of different brands of electronics

- The survey only includes ratings of products on a scale from 1 to 10, which may not provide enough granularity to accurately measure satisfaction with different brands of electronics.

3. For each of the following statements, indicate whether it is True/False. If false, explain why.

   I. If the 95% confidence interval for the average purchase of customers at a department store is \$50 to \$110, then \$100 is a plausible value for the population mean at this level of confidence.
   True. If the 95% confidence interval for the average purchase of customers at a department store is \$50 to \$110, then \$100 is a plausible value for the population mean at this level of confidence. The confidence interval indicates that we are 95% confident that the true population mean falls between \$50 and \$110. Since \$100 is within this range, it is a plausible value for the population mean.

   II. If the 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that fewer than half of all moviegoers purchase concessions.
   False. If the 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that we are 95% confident that the true proportion of moviegoers who purchase concessions falls between 30% and 45%. It does not mean that fewer than half of all moviegoers purchase concessions. In fact, we cannot make any conclusions about the proportion of moviegoers who purchase concessions outside of this range.

   III. The 95% Confidence-Interval for μ only applies if the sample data are nearly normally distributed.
   False. The 95% confidence interval for μ applies to any sample size and any distribution of the sample data, as long as the sample is random and independent. However, if the sample size is small (less than 30) and the population standard deviation is unknown, then the sample data should be approximately normally distributed in order to use the t-distribution to calculate the confidence interval.

4. What are the chances that $\overline{X} > \mu$?

   A. ¼
   B. ½
   C. ¾
   D. 1

Without more information about the population and sample, I cannot provide a more specific answer to this question. Therefore, I cannot determine which of the options A, B, C, or D is correct.

5. In January 2005, a company that monitors Internet traffic (WebSideStory) reported that its sampling revealed that the Mozilla Firefox browser launched in 2004 had grabbed a 4.6% share of the market.

   I. If the sample were based on 2,000 users, could Microsoft conclude that Mozilla has a less than 5% share of the market?

   II. WebSideStory claims that its sample includes all the daily Internet users. If that's the case, then can Microsoft conclude that Mozilla has a less than 5% share of the market?

If the sample were based on 2,000 users, Microsoft could not conclude that Mozilla has a less than 5% share of the market. To determine whether Mozilla has a less than 5% share of the market, we need to calculate a confidence interval for the population proportion. The formula for calculating a confidence interval for a population proportion is:
Confidence Interval = p +/- z*sqrt(p(1-p)/n)

where:

- p is the sample proportion
- z is the z-score corresponding to the desired level of confidence
- n is the sample size

If we assume that the sample proportion of users who use Mozilla Firefox is 4.6%, then we can calculate the confidence interval for the population proportion using a z-score of 1.96 (corresponding to a 95% confidence level) and a sample size of 2,000. The confidence interval is:
Confidence Interval = 0.046 +/- 1.96*sqrt(0.046(1-0.046)/2000)

Confidence Interval = 0.046 +/- 0.016

Confidence Interval = (0.03, 0.062)

Since the confidence interval includes values greater than 5%, we cannot conclude that Mozilla has a less than 5% share of the market.

6. A book publisher monitors the size of shipments of its textbooks to university bookstores. For a sample of texts used at various schools, the 95% confidence interval for the size of the shipment was 250 ± 45 books. Which, if any, of the following interpretations of this interval are correct?

   A. All shipments are between 205 and 295 books.

   B. 95% of shipments are between 205 and 295 books.

   C. The procedure that produced this interval generates ranges that hold the population mean for 95% of samples.

D. If we get another sample, then we can be 95% sure that the mean of this second sample is between 205 and 295.

E. We can be 95% confident that the range 160 to 340 holds the population mean.

The 95% confidence interval for the size of the shipment is **250 ± 45 books**. This means that we are 95% confident that the true mean size of the shipment is between **205 and 295 books** Therefore, the correct interpretation is:

C. **The procedure that produced this interval generates ranges that hold the population mean for 95% of samples.**

Option A is incorrect because the confidence interval only provides a range of plausible values for the population mean, not for individual shipments.

Option B is incorrect because the confidence interval only provides a range of plausible values for the population mean, not for individual shipments. We cannot make any conclusions about the proportion of shipments that fall within this range.

Option D is incorrect because the confidence interval only provides a range of plausible values for the population mean, not for individual samples. We cannot make any conclusions about the mean of a second sample based on the confidence interval of the first sample.

Option E is incorrect because the confidence interval only provides a range of plausible values for the population mean, not for individual shipments.

7. Which is shorter: a 95% z-interval or a 95% t-interval for μ if we know that $\sigma = s$?

   A. The z-interval is shorter
   B. The t-interval is shorter
   C. Both are equal
   D. We cannot say

   For a 95% confidence level, the z-critical value is approximately 1.96. The t-critical value depends on the degrees of freedom (n-1), but for any positive degrees of freedom, the t-critical value will be greater than the z-critical value. Therefore, the t-interval will be longer than the z-interval. Hence, the answer is **B. The t-interval is shorter**.

Questions 8 and 9 are based on the following: To prepare a report on the economy, analysts need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

8. How many randomly selected employers (minimum number) must we contact in order to guarantee a margin of error of no more than 4% (at 95% confidence)?

A. 600
B. 400
C. 550
D. 1000

Sample size = p * (1 - p) * (z / E)^2

p=0.5 (the most conservative estimate)assume
z-score corresponding to a 95% confidence level is 1.96.
E= 0.04

Sample size = 0.5 * (1 - 0.5) * (1.96 / 0.04)^2 Sample size ≈ 600

Therefore, the minimum sample size required to achieve a margin of error of 4% at a 95% confidence level is **600**.

9. Suppose we want the above margin of error to be based on a 98% confidence level. What sample size (minimum) must we now use?

A. 1000
B. 757
C. 848
D. 543

Sample size = p * (1 - p) * (z / E)^2

p=0.5 (the most conservative estimate)assume
z-score corresponding to a 98% confidence level is 2.33.
E= 0.04

Sample size = 0.5 * (1 - 0.5) * (2.33 / 0.04)^2 Sample size ≈ 848

Therefore, the minimum sample size required to achieve a margin of error of 4% at a 98% confidence level is **848**.