Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   We need to find the probability that the service manager cannot meet his commitment, which means the time required to service the transmission is more than 50 minutes (60 - 10).

   Using the z-score formula, we can calculate the z-score as follows:

   z = (x - μ) / σ = (50 - 45) / 8 = 0.625

   Using a standard normal distribution table or calculator, we can find the probability that z is greater than 0.625:

   P(z > 0.625) = 0.2676

   Therefore, the probability that the service manager cannot meet his commitment is **0.2676**.

   So, the answer is **B**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.

```
#q2a
from scipy import stats
p_z1=stats.norm.cdf(44,38,6)
p_z2=stats.norm.cdf(38,38,6)
p_z1,p_z2
p_38_44 = abs(p_z2 - p_z1)
print(f"Therefore, the probability that an employee is between 38 and 44 years old is: {p_38_44}")
p_44 =1- p_z1
print(f"The probability that an employee is older than 44 years old is: {p_44}")
if p_38_44 > p_44:
    print('''Therefore, more employees at the processing center are not older than 44 than between 38 and 44.
          so false''')
else:
    print('''Therefore, more employees at the processing center are older than 44 than between 38 and 44.
          so true''')
```

```
Therefore, the probability that an employee is between 38 and 44 years old is: 0.3413447460685429
The probability that an employee is older than 44 years old is: 0.15865525393145707
Therefore, more employees at the processing center are not older than 44 than between 38 and 44.
          so false
```

   False

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

```
#q2b
from scipy import stats
p_30 =abs(stats.norm.cdf(30,38,6))

print(f'''the probability of selecting 36 or more employees under the age of 30 is approximately {p_30} or {p_30*100}%.
      so false''')
```

```
the probability of selecting 36 or more employees under the age of 30 is approximately 0.09121121972586788 or 9.121121972586
788%.
      so false
```

False

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

   If X1 ~ N(μ, σ^2) and X2 ~ N(μ, σ^2) are independent and identically distributed (iid) normal random variables, then the difference between 2 X1 and X1 + X2 can be calculated as follows:

   2 X1 - X1 + X2 = X1 + X2

   Therefore, the difference between 2 X1 and X1 + X2 is zero.
   The distribution of X1 + X2 is also normal, with a mean of 2μ and a variance of 2σ^2
   The distribution of 2 X1 is also normal, with a mean of 2μ and a variance of 4σ^2

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

```
#q4
import math
z = 2.58
mean = 100
sd = 20
a = mean - z*sd
b= mean + z*sd
a,b
print(f'''Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taki

Therefore, the answer is D. {a}, {b}.''')
```

```
Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taking a valu
e between them is 0.99, are approximately 48.4and 151.6.

Therefore, the answer is D. 48.4, 151.6.
```

Therefore, the two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99, are approximately 48.4and 151.6.

Therefore, the answer is D. 48.4, 151.6.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

```
#q5
import numpy as np
mean = (5 + 7) * 45
variance = 3^2 * 45 + 4^2 * 45
sd= np.sqrt(variance)
z1=stats.norm.ppf(0.025)
a= mean - z1*26
b= mean + z1*26
print(f"A. Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is({
z2=stats.norm.ppf(0.05)
profit = mean+z2*26
print(f"B. The 5th percentile of profit (in Rupees) for the company is {profit}")
z3 =-5/3
profit1=stats.norm.cdf(z3,5,3)
z4=-7/4
profit2=stats.norm.cdf(z4,7,4)
if profit1 < profit2:
    print("C. Profit1 has a smaller probability of making a loss in a given year than Profit2")
else:
    print("C. Profit2 has a smaller probability of making a loss in a given year than Profit1")
```

```
A. Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is(590.9590
635980414,489.04093640195856)
B. The 5th percentile of profit (in Rupees) for the company is 497.2338056992617
C. Profit1 has a smaller probability of making a loss in a given year than Profit2
```

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t of the company is(590.9590635980414,489.04093640195856)
B. The 5th percentile of profit (in Rupees) for the company is 497.2338056992617
C. Profit1 has a smaller probability of making a loss in a given year than Profit2