Relativity

→ Special theory of relativity (Inertial frame)

a) all laws of physics are applicable in inertial frame

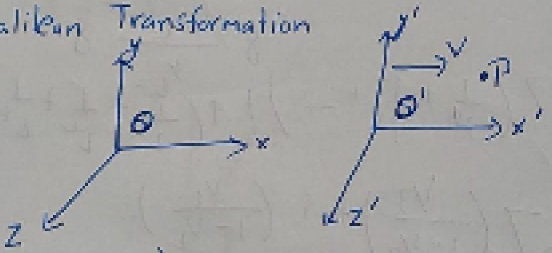
b) c is same in all inertial frame.

→ General theory of Relativity (accelerated frame)

→ Photon, gravity, Black hole

Transformation eq – if frame of reference is changed the eq of coordinate is also changed. The changed eq is called transformed eq.

→ Galilean Transformation



$O \Rightarrow P(x, y, z, t)$

$O' \Rightarrow P(x', y', z', t')$

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

⇒ Galilean violate both Postulate of Special Relativity

1) 1st Postulate – ~~electricity & magnetism~~

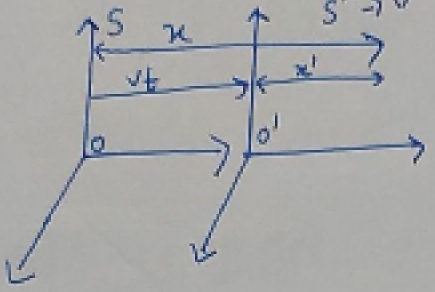
Same eq of Phy in both frame S & S', but eq of electricity & magnetism become very different when Galilean Transformation convert quantities in 1 frame to another frame

2) 2^{nd} Postulate

→ Same value of speed of light in S & S'.

In S frame ~~Ix~~ Speed of light in x-direction be c, but in S' it will be $c' = c - v$

→ Lorentz Transformation



$$x' = k(x - vt) \quad -(1)$$

$$x = k(x' + vt') \quad -(2)$$

Using Einstein Postulate 1 [All Phy law valid & same for all inertial Frame]

$$x = k[k(x - vt) + vt']$$

$$\frac{x}{k} = kx - kvt + vt'$$

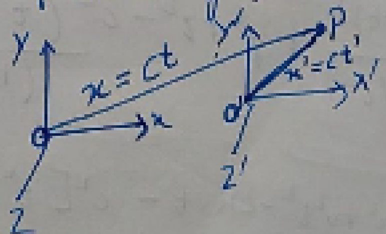
$$vt' = \frac{x}{k} - kx + kvt$$

$$t' = \frac{x}{kv} - \frac{kx}{v} + kt$$

$$t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right) \quad -(3)$$

Using IInd Postulate (Constancy of speed of light)

$x = ct \qquad x' = ct'$



eq1 $\quad ct' = k(ct - vt) \quad -(4)$

eq2 $\quad ct = k(ct' + vt') \quad -(5)$

⇒ Multiply eq 4 & 5

$$c^2\cancel{t}\cancel{t'} = k^2\cancel{t}(c-v)\cancel{t}(c+v) \rightarrow k^2 \Rightarrow \frac{c^2}{c^2 - v^2} \Rightarrow \frac{c^2}{c^2\left(1 - \frac{v^2}{c}\right)}$$

$$1/k^2 = 1 - v^2/c^2 \rightarrow \boxed{k = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}}$$

$$t' = kt - \frac{kx}{v}\left(1 - 1/k^2\right) \Big/ \sqrt{1 - v^2/c^2}$$

$$t' = k\left[t - \frac{x}{v}\left(v - v + v^2/c^2\right)\right] \Big/ \sqrt{1 - v^2/c^2}$$

$$\boxed{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}}$$

$$y' = y$$
$$z' = z$$

<u>Note</u> : At low velocity lorentz transformation convert to galilean transformation.

→ Space Time Interval

Q) Show that space time interval is invariant under lorentz transformation

Q) show that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under L.T

If space time interval is invarient then

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

<u>RHS</u>

$$x'^2 + y'^2 + z'^2 - c^2t'^2$$

$$\frac{(x - vt)^2}{(1 - v^2/c^2)} + y^2 + z^2 - c^2\frac{\left(t - \frac{xv}{c^2}\right)^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= \frac{x^2 + v^2t^2 - 2xvt - c^2\left[t^2 + \frac{x^2v^2}{c^4} - \frac{2txv}{c^2}\right]}{1 - v^2/c^2} + y^2 + z^2$$

$$\frac{x^2+v^2t^2-\cancel{2xvt}-c^2t^2-\frac{x^2v^2}{c^2}+\cancel{2txv}}{(1-v^2/c^2)}+y^2+z^2$$

$$\frac{x^2+v^2t^2-c^2t^2-\frac{x^2v^2}{c^2}}{(1-v^2/c^2)}+y^2+z^2$$

$$\frac{x^2(1-v^2/c^2)-c^2t^2(1-v^2/c^2)}{(1-v^2/c^2)}+y^2+z^2$$

$$\frac{[x^2-c^2t^2][1-v^2/c^2]}{1-v^2/c^2}+y^2+z^2$$

$$x^2+y^2+z^2-c^2t^2 = LHS$$

→ Inverse L.T.E

1) $x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}}$ 2) $y=y'$ 3) $z=z'$

4) $t = \frac{t'+\frac{x'v}{c^2}}{\sqrt{1-v^2/c^2}}$

→ Space time interval are invarient of Lorentz Transformation

→ space & time interval taken alone then they are varient

⇒ Consequence of Lorentz Transformation

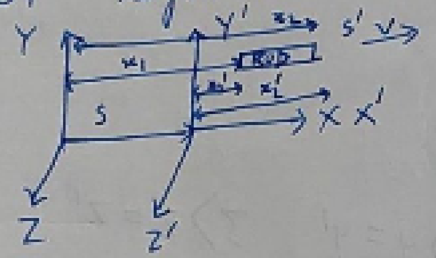
1) length Contraction

The length of any object moving with high velocity (approchable to c) relative to observer is measured contracted in the direction of motion while no change in other direction perpendicular to direction of motion. Phenomena is called length Contraction.

2) Proper length

The length of any object measured by observer at rest w.r.t obj is called proper length.

→ Derivation of length Contraction (STR)



l_0 → Proper length

l – Contracted length

S – rest ref frame

S' = moving ref frame velocity v along direction

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \qquad x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = x_2' - x_1' \quad -(1)$$

$$l = x_2 - x_1 \quad -(2)$$

$$l_0 = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}} - \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} = \frac{l}{\sqrt{1 - v^2/c^2}} \Rightarrow \boxed{l = l_0\sqrt{1 - v^2/c^2}}$$

Condition 1 when $v << c$

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

$l = l_0$ [No contraction]

Condition 2 when $v = c$

$l = 0$

Not valid

Condition 3 when $v \approx c$

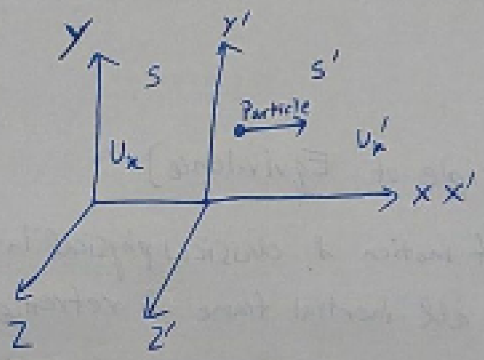
$l = l_0\sqrt{1-0}$.

$l_0 > 0$

* length contraction independent on direction of velocity

* L.C is always in direction of Relative motion. (along x-axis)

→ velocity addition



S frame

$U_x = \frac{dx}{dt}$ - ① $\quad U_y = \frac{dy}{dt} \quad U_z = \frac{dz}{dt}$

S' frame

$U_x' = \frac{dx'}{dt'}$ - ②

From LT

$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}}$ - ③ $\quad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-v^2/c^2}}$ - ④

To find eq2 values dx' & dt' differentiate ③ & ④

$\boxed{dx' = \frac{dx - vdt}{\sqrt{1-v^2/c^2}}}$ - ⑤ $\quad \boxed{dt' = \frac{dt - \frac{vdx}{c^2}}{\sqrt{1-v^2/c^2}}}$ - 6

Put ⑤ & ⑥ in eq ②

$$U_x' = \frac{dx - vdt}{dt - \frac{v}{c^2}dx} \quad \frac{dt\left(\frac{dx}{dt} - v\right)}{dt\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

$$\boxed{U_x' = \frac{U_x - v}{1 - \frac{vU_x}{c^2}}}$$ → velocity seen by O' observer

$$\boxed{U = \frac{U_x' + v}{1 + \frac{vU_x'}{c^2}}}$$ → velocity seen by O observer

→ Einstein Postulates

a) I^{st} Postulate [Principle of Equivalence]

All newton law of motion & classical physical laws are valid & remain constant in all inertial frame of refrence.

b) II^{nd} Postulate

speed of light remain constant in all inertial form.