

1a)

**Part a)** A pure Nash equilibrium is a situation **where no player has an incentive to unilaterally change their strategy.**

The game has **two** pure strategy Nash equilibria: (S, S) and (H, H).

A Pareto optimal outcome is a situation where it is not possible to make one player better off without making the other player worse off.

**(S, S) with payoffs [5, 5] is the Pareto optimal outcome.**

**Part b)**

This is precisely the strength of the Nash equilibrium concept. If each player anticipates that the other will not join forces, then he knows that going out to hunt the stag alone is not likely to be a successful enterprise and that going after the hare will be better. This belief would result in a society of individualists who do not cooperate to achieve a better outcome. In contrast, if the players expect each other to be cooperative in going after the stag, then this anticipation is self-fulfilling and results in what can be considered a cooperative society.

1b)

Imagine that a mafia member who finks on another member is very seriously reprimanded, which will change the payoff structure of the Prisoner's Dilemma if he is caught. If the pain from mafia punishment is equivalent to  $z$ , then we have to subtract  $z$  units of payoff for each player who finks. The "mafia-modified" Prisoner's Dilemma is represented by the following matrix:

		Player 2	
		<i>M</i>	<i>F</i>
Player 1	<i>M</i>	$-2, -2$	$-5, -1 - z$
	<i>F</i>	$-1 - z, -5$	$-4 - z, -4 - z$

If  $z$  is strictly greater than 1 then this punishment will be enough to flip our predicted equilibrium outcome of the game because then *M* becomes the strict dominant strategy (and (M,M) is Pareto optimal).

$$z=2$$

2a)

- (a) Is there an alternative that a rational player will never take regardless of  $p$ ? (i.e., it is dominated for any  $p \in [0, 1]$ .)

**Answer:** For this decision maker choosing the hike is always worse (dominated) by going to the football game, and he should never go on a hike. ■

- (b) What is the optimal decision, or best response, as a function of  $p$ .

**Answer:** The expected payoffs from each of the remaining two choices are given by,

$$v(\text{Football}) = p \times 1 + (1 - p) \times 2 = 2 - p,$$

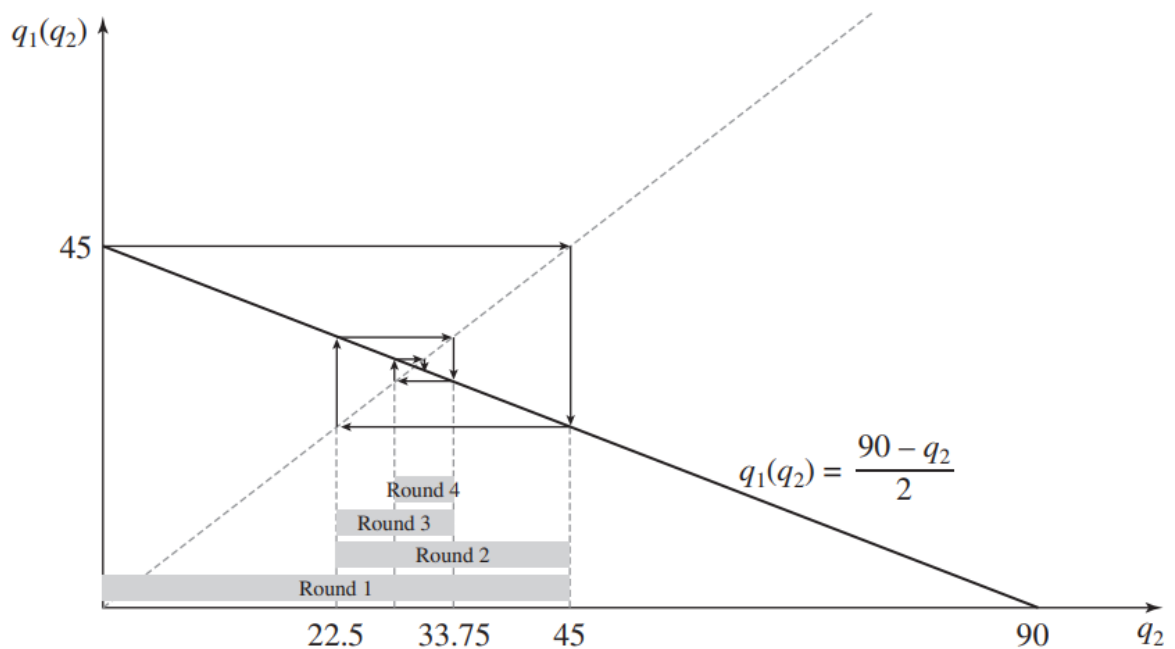
$$v(\text{Boxing}) = p \times 3 + (1 - p) \times 0 = 3p,$$

which implies that football is a better choice if and only if

$$2 - p \geq 3p,$$

or,  $p \leq \frac{1}{2}$ , and boxing is better otherwise. ■

2b) (30,30)



3(a)

**Answer:** If player  $i$  believes that player  $j$  chooses  $e_j$  then  $i$ 's first order optimality condition for maximizing his payoff is,

$$a + e_j - 2e_i = 0 ,$$

yielding the best response function,

$$BR_i(e_j) = \frac{a + e_j}{2} \text{ for all } e_j \geq 0.$$

3b)

**Answer:** Here the best response function of player  $i$  is *increasing* in the choice of player  $j$  whereas in the Cournot model it is *decreasing* in the choice of player  $j$ . This is because in this game the choices of the two players are strategic complements while in the Cournot game they are strategic substitutes. ■

3c)

**Answer:** We solve two equations with two unknowns,

$$e_1 = \frac{a + e_2}{2} \text{ and } e_2 = \frac{a + e_1}{2},$$

which yield the solution  $e_1 = e_2 = a$ . It is easy to see that it is unique because it is the only point at which these two best response functions cross. ■

4a)

The inverse demand function, also known as **the market demand function**, describes the relationship between the **total quantity** of the good produced in the market and the **market price** at which the good is sold.

The inverse demand function is typically represented as  $P(Q)$ , where:

- $P(Q)$  is the **market price** of the good.
- $Q$  is the **total quantity** of the good produced in the market.

**The inverse demand function indicates how the market price (P) changes as the total quantity produced (Q) in the market changes.**

The linear form of the inverse demand function is:

$$P(Q) = a - bQ$$

In this linear form:

- "a" represents the **intercept** of the demand curve on the price axis. It is the price at which no units are produced ( $Q = 0$ ), often referred to as the **monopoly price**.

- "b" is the **slope** of the demand curve, representing how much the price decreases as one additional unit of the good is produced and added to the market.

The firms' profit-maximizing decisions are made based on this inverse demand function. They determine how much to produce in order to maximize their profits, taking into account the prices determined by the market demand function and the quantities produced by other firms in the industry.

#### 4b)

If one country decides to invest in Healthcare over Defence, then the neighbouring country can attack and get a higher payoff while decreasing the payoff of the first country and vice versa. Whereas if both decide to invest in Defence, they will both get a lower payoff as compared to the scenario where both choose to invest in Healthcare but the risk of getting dominated is averted by picking defence over healthcare.

You may also provide the game table as discussed in class.

#### 5a)

The Game table is as follows:

	Rock	Paper	Scissor
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissor	(-1,1)	(1,-1)	(0,0)

#### 5b)

- i) **Yes, C**
- ii) **L** (The payoff is less than the payoff for Player 1 in all 3 choices for L)
- iii) **L** (worse payoff for all 3 options as compared to Player 1)