Chapter - 1

Force and Moment (Resultant and Equillibrium)

- Engineering mechanics :→ It is the branch of physical science which deals with the state of a body under the action of external agency.

State → state of rest
State → state of motion

External agency → Force
External agency → Moment

- Force :→ Force is an action or an external agency which changes or tries to change the state of a body. (unit = Newton) (Push / Pull)
- Moment :→ It is a measure of turning effect.

unit = Nm

Moment = Force × ⊥r distance

Moment about A = F × d

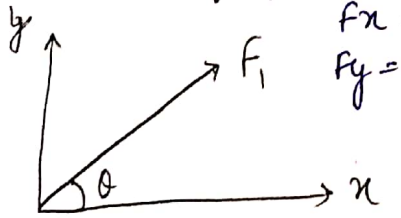
- Quantity :→ Anything which can be measured directly or indirectly.

(i) Scalar (mag) (ii) vector (mag - direcⁿ)

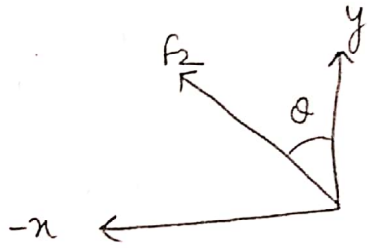
- Rigid body :→ When there is no deformaⁿ in the body after the application of external force, the body is said to rigid.

⇒ Deformaⁿ is very negligible, so neglected.
⇒ No body is perfectly Rigid.

- Resolution of forces :→



$F_x = F_1 \cos\theta$
$F_y = F_1 \sin\theta$

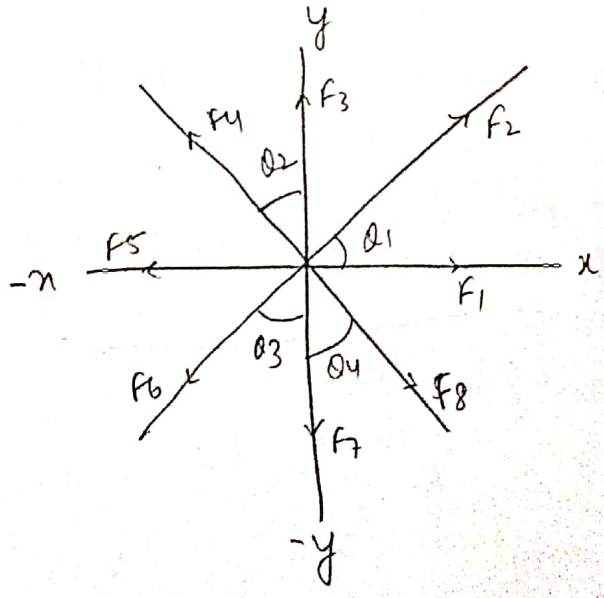


$F_x = -F_2 \sin$
$F_y = F_2 \cos\theta$

- Composition of forces :→

ΣF_x = Horizontal $= F_1 + F_2 \cos\theta_1 - F_4 \sin\theta_2 - F_5 - F_6 \sin\theta_3 + F_8 \sin\theta_4$

ΣF_y = Vertical $= F_2 \sin\theta_1 + F_3 + F_4 \cos\theta_2 - F_6 \cos\theta_3 - F_7 - F_8 \cos\theta_4$



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- **Equilibrium of a body :→** A body is in equilibrium if resultant force and resultant moment acting on the body are separately equals to 0.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$$

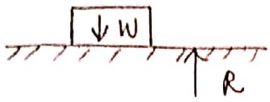
If a body is not under equilibrium, then we find :

(i) Resultant, R
(ii) angle of direction, θ

$$\left[R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \quad \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) \right]$$

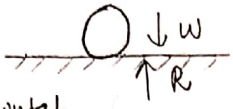
- **Types of support :→**

(i) Frictionless support :→



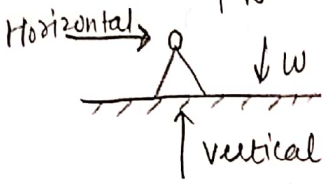
[only 1 Rxn which is ⊥r to surface]

(ii) Roller support :→



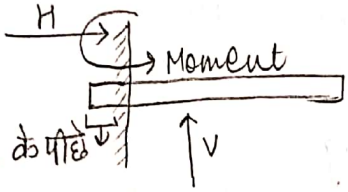
[1 Rxn ⊥r surface]

(iii) Hinged support :→



[2 Rxn ⊥r and ∥al surface]

(iv) Fixed support :→



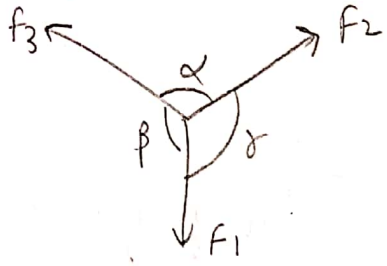
[2 Rxn + 1 Moment]

- **FBD**

It is the diagram of a body or part of a body or system of bodies on which all forces acting are shown.
• weight of a body • External applied forces • all reactions.

- **Lami's theorem :→**

If a body is in equilibrium under the action of <u>three forces</u> then each force is proportional to the sine of angle b/w the other two forces.

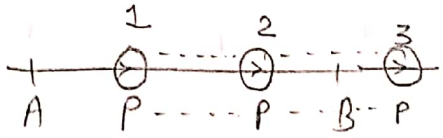


∴ according to Lami's theorem

$$\frac{f_1}{\sin\alpha} = \frac{f_2}{\sin\beta} = \frac{f_3}{\sin\gamma}$$

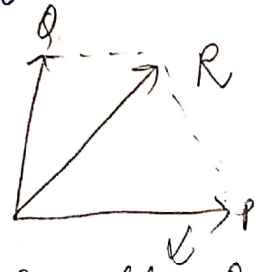
- **Transmibility of force :→**

If we change the position of force (P) on the line of action without changing its direction, then there is no disturbance to equilibrium. This is called principle of transimibility.



- **∥gm law**

$$\vec{R} = \vec{P} + \vec{Q}$$



[उसे Numerical solve करना है] generally P का Mag. ↑
Q का कम

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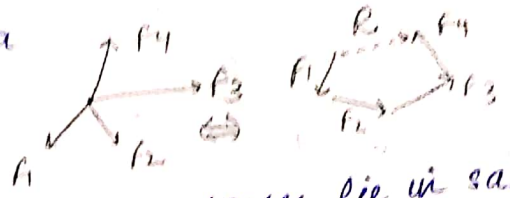
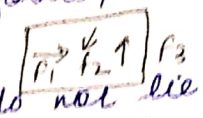
- Polygon law

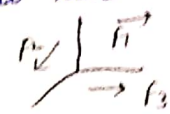
In this, no of forces passes through a single point and we have to make a polygon by assuming forces as sides the side which complete the polygon is Resultant i.e. R.



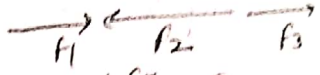
- Coplanar forces :→ the line of action of all forces, lie in same plane.



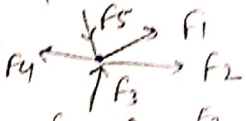
- Non-coplanar ⇒ line of action of all forces, do not lie in same plane



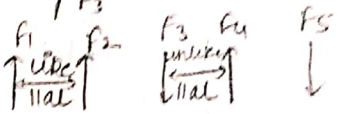
- collinear ⇒ line of action of all forces, passing through single line.



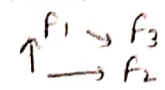
- concurrent :→ line of action of all forces, passing through a single point.



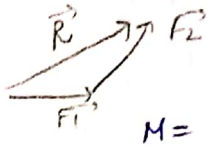
- ||al forces ⇒ line of action of all forces are ||al to each other.



- Non ||al forces ⇒ are not ||al to each other



- Triangle law :→ $\vec{R} = \vec{F_1} + \vec{F_2}$



- Moment of force :→

Moment :→ the rotational effect of a force

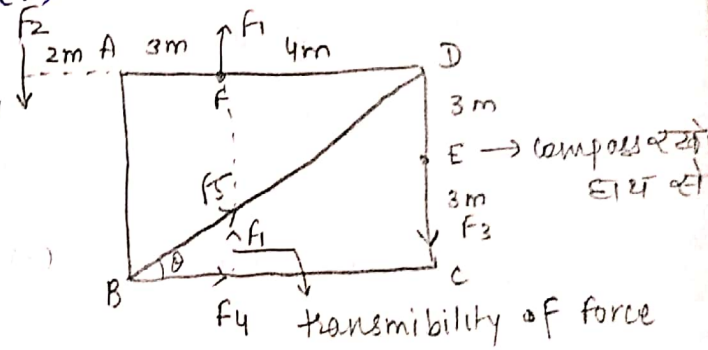
Example ⇒ गाड़ियों में बच्चे tyre घुमाते हैं,

$M = F \times d$ (⊥r dist.)

(F) → Tangential (2) घुमेगा

Sense :→ clock (−)) (anticlock. (+)

- Problem 1 #



$F_1 = 10N$ $F_2 = 20N$ $F_3 = 30N$ $F_4 = 40N$ $F_5 = 50N$

(i) Moment of F_1 about B & E

Sol^n ⇒ at E

$M = F_1 \times d$

$= 10 \times 4 = 40 Nm$ (clockwise)

at B

$M = F_1 \times d$

$= 10 \times 3 = 30 Nm$ (anticlockwise)

[Force is a sliding vector]

(ii) F_2 about A and C

at A ⇒ $M = F_2 \times d$

$= 20 \times 2 = 40 Nm$ (anticlock)

at C ⇒ $M = F_2 \times d$

$= 20 \times 9 = 180 Nm$ (anticlock)

(iii) F_3 about F

$M = F_3 \times d$

$= 30 \times 4 = 120 Nm$ (clockwise)

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(iv) F_y about B + D

at B ⇒ $F_y \times d = 40 \times 0 = 0$ Nm [Bcoz force is passing through pt of rotation → no ⊥r distance]

Lets take instance → अगर दरवाजे के Hinge पर force लगाए (Handle के बराबर). Then there is no moment

at D = $F_y \times d = 40 \times 6 = 240$ Nm (anticlock)

(v) F_5 about A + F + D

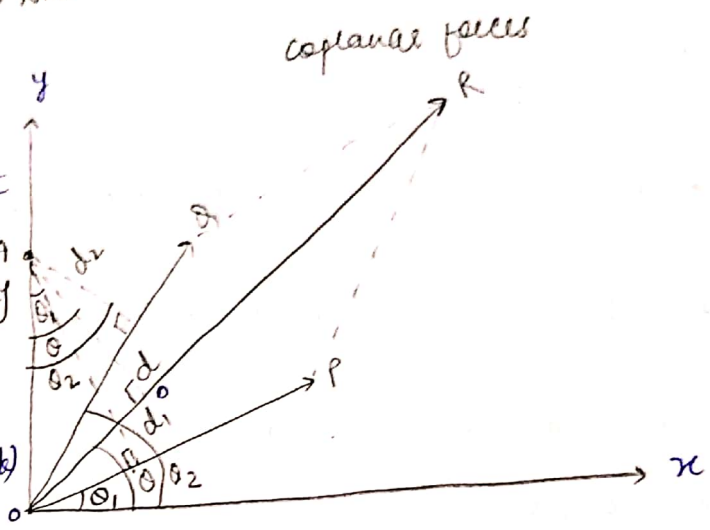
at A ⇒ $M = F_5 \times d = 50 \times \cos\theta \times 6$ (anticlock)

at F ⇒ $M = F_5 \times d = 50 \times \cos\theta \times 6$ (anticlock) $- 50 \times \sin\alpha \times 3$ [Total +ve ∴ anticlockwise]

at D ⇒ $M = F_5 \times d = 50 \times 0 = 0$ Nm

- Varignon's Theorem :→

The sum of moments of a coplanar force system at any point (say A) is equal to the moment of the resultant of the two arbitrary coplanar forces about the same pt.



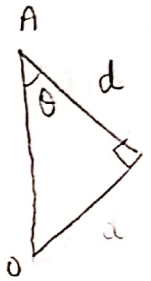
Proof :→

① Moment of P at A = $P \times d_1$ (anticlock)

② Moment of Q at A = $Q \times d_2$ (")

Moment of R at A = $R \times d$ (anti)

In △ OAD ⇒



$\cos\theta = \frac{d}{OA}$

$OA \cos\theta = d$

$$M_A^R = R(OA)\cos\theta$$
$$= OA\ R\cos\theta = OA\ R_x \rightarrow ③$$

adding eqⁿ ① + ②

$$M_A^P + M_A^Q = \Sigma M_A^F = P d_1 + Q d_2$$

Similarly $d_1 = OA \cos\theta_1$ $\quad d_2 = OA \cos\theta_2$

$$\therefore \Sigma M_A^F = P\,OA \cos\theta_1 + Q\,OA \cos\theta_2$$
$$= OA\,(P\cos\theta_1 + Q\cos\theta_2)$$
$$= OA\,(P_x + Q_x)$$
$$= OA \cdot R_x \rightarrow ④$$

∴ equating ③ and ④

$$\Sigma M_A^F = M_A^R$$

Hence Proved.

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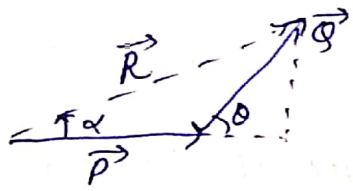
* Triangle law of vector addition :→ If two sides of a triangle taken in order represent two vectors in magnitude + direction then the third side taken in opposite order will represent the resultant of these 2 vectors in magnitude and direction

$$\vec{R} = \vec{P} + \vec{Q}$$

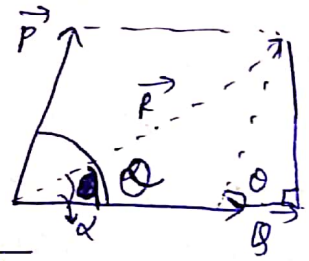


magnitude $|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

direction $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$\therefore \alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

* ||gm law :→ If two vectors are represented in magnitude and direction by the two adjacent sides of a ||gm, then their diagonal will be the resultant drawn from that point.



$$\vec{R} = \vec{P} + \vec{Q}$$

Magnitude ⇒ $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

direction ⇒ $\tan\alpha = \dfrac{P\sin\theta}{Q + P\cos\theta}$

* Kinematics :→ Part of dynamics. It deals with the study of motion of bodies without considering the force acting on them. For example :→ velocity, acceleration, etc.

* Kinetics ⇒ It considers the force acting upon them. Also the part of dynamics.

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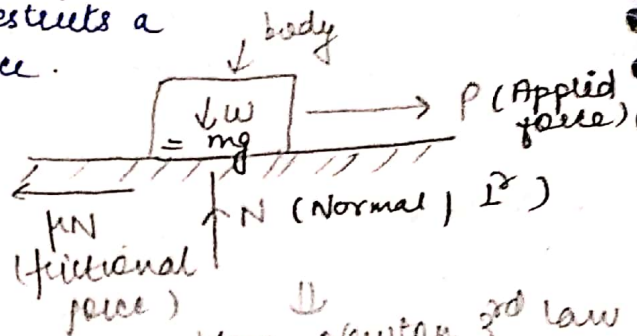
Chapter - 1 (Friction)

* The opposing force that opposes or restricts a body to move is called frictional force.

Friction force = μN
$\downarrow$
Normal Rxn

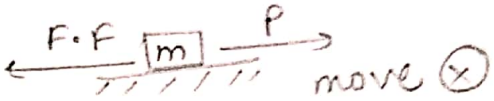


$\Downarrow$
Here Newton 3rd law is also verified.

* Types of friction

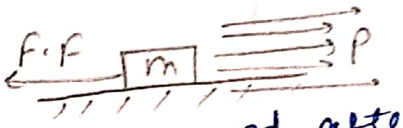
(i) Static :→ स्थिरता, in this type of friction, the body does not moves.

Here $F.F \geq P$, so that body is not moving



(ii) Limiting :→ सीमांत

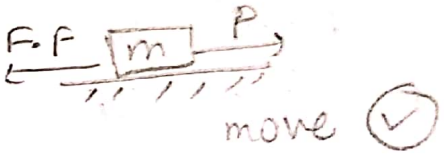
the maximum value of frictional force upto which body does not moves, but if limit of F.F has been crossed after ↑ the value of P (applied force), then the body will move.

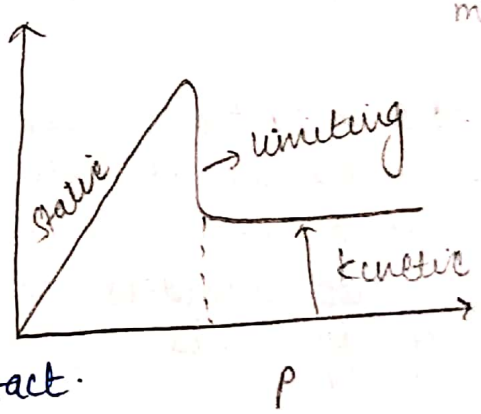


(iii) Kinetic :→ गति

the body now starts moving as limit has been crossed ~~and~~

$F.F < < < < < P$



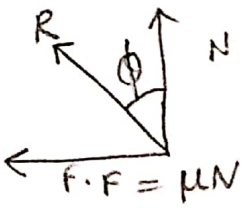


* Laws of friction

(i) direction of $F.F \propto$ direction of P
(ii) $F.F \propto$ Normal rxn b/w the body
(iii) It depends upon the surface, on which the body is moving
(iv) independent of area of body/contact.

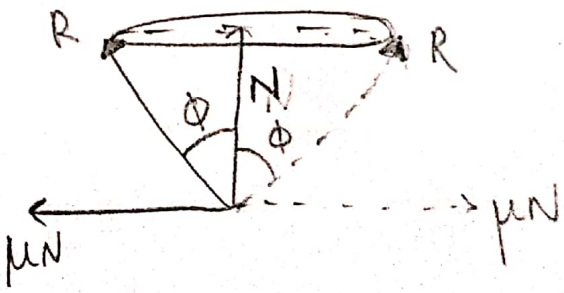
* Angle of friction

It is an angle b/w Normal Rxn force & resultant of N & F.F



* Cone of friction

If the resultant rxn is rotated about Normal rxn force, it will form a cone.



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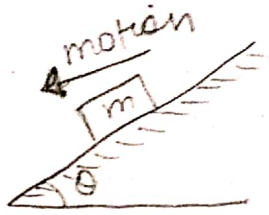
* Angle of Repose.

When the body is about to slide down on an inclined plane due to its own weight, then the angle b/w the plane with the horizontal is known as angle of repose



For example:- $\theta = 10 \cdots 15 \cdots 30 \cdots 90$ ↑

then at some particular angle, the body m will slide down

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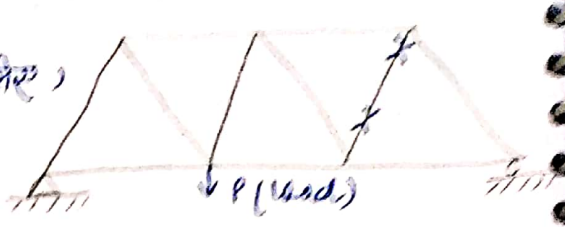
TRUSS

A structure made up of several bars joined together by welding or pin joints.

- The members of a truss are straight members and the loads are applied only at the joints. (loads को संतुलन के लिए)
- Every member of a truss is a 2-force member
  → Tensile force (जो point से दूर) → compressive force (जो point के पास)

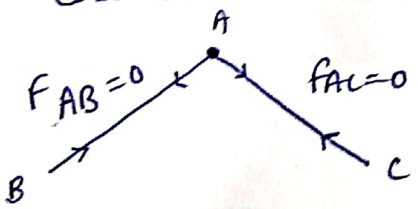


- m = no. of member in a truss
  j = no. of joints in a truss.
  $m + 3 = 2j$ → Perfect truss
  $m + 3 > 2j$ → Over Rigid truss.
  $m + 3 < 2j$ → Collapsible truss
- the joints of a simple truss are pin jointed and frictionless
- weight of members are negligible small.
- truss is statically determinate
  ↳ जितनी unknown value, उतनी eqn (atleast 2)
- How to use/solve any Truss?

(i) Method of joints ($\Sigma F_x = 0$, $\Sigma F_y = 0$) [जब सभी Members का निकाल (calculation करते समय सब tensile force लेना है)]

(ii) Method of sections ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_A = 0$) [जब कुछ particular Members का निकाल]

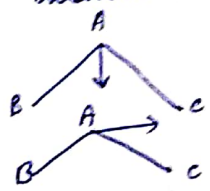
- Case 1:→



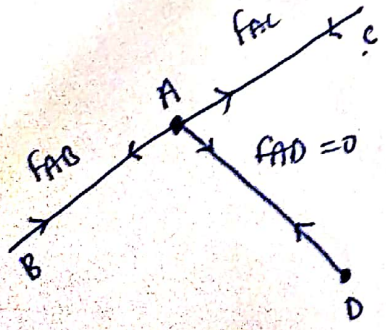
BCOZ AB and AC are non-collinear and does not balance each other

$F_{AB} = 0$ $F_{AC} = 0$

If there is any load at A then $F_{AB} \neq 0$ $F_{AC} \neq 0$

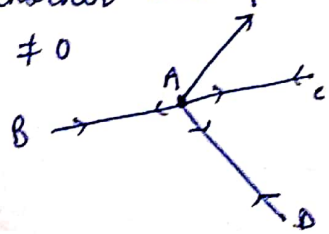


- Case 2:→



BCOZ AB and AC are collinear then $F_{AD} = 0$

If there is another load at A, then $F_{AD} \neq 0$



then $F_{AD} \neq 0$

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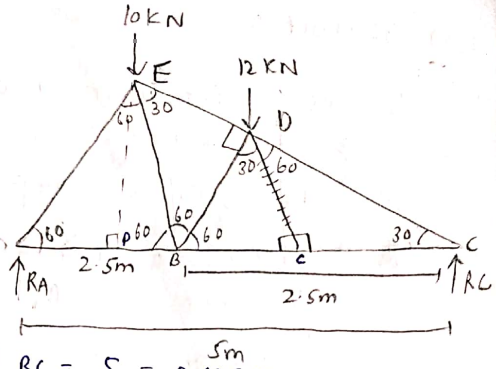
Numerical on method of joints



① In ΔAEC

$\sin 30 = \frac{AE}{AC}$

$\frac{1}{2} = \frac{AE}{5}$

$2.5\,m = AE = AB = BE$

In ΔBED

$\sin 30 = \frac{BD}{BE} \Rightarrow \frac{1}{2} = \frac{BD}{5} \times 2$

$\Rightarrow \frac{5}{4} m = BD = 1.25\,m$

In ΔBDC

$\sin 30 = \frac{BDC}{BD} \Rightarrow \frac{1}{2} = \frac{BC \times 4}{5} \Rightarrow BC = \frac{5}{8} = 0.625\,m$

Now taking moment across A = 0

[anticlock = +ve
clock = −ve]

$\Sigma M_A = 0$

$-10 \times 1.25 - 12 \times 3.125 + R_C \times 5 = 0$

$R_C = 10\ KN$

$R_A + R_C = 22$

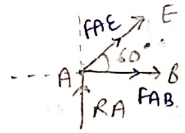
$R_A = 12\ KN$

In ΔAEP

$\sin 30 = \frac{AP}{AE}$

$\frac{1}{2} = \frac{AP}{5} \times 2$

$AP = \frac{5}{4} = 1.25\,m$



(i) at point A

$F_{AE} \sin 60 + R_A = 0$

$F_{AE} \frac{\sqrt{3}}{2} + 12 = 0$

$F_{AE} = \frac{-24}{\sqrt{3}} = -8\sqrt{3} = 8\sqrt{3}\ (C)$

$F_{AE} \cos 60 = -F_{AB}$

$-8\sqrt{3}\,\frac{1}{2} = -F_{AB}$

$4\sqrt{3} = F_{AB}\ (T)$

[जब हम overall truss का Moment निकालते हैं, तो anti clock का चक्कर होता है (− +) में]

[But जब हम individual का force निकालते हैं F_x, F_y, then only direc^n का चक्कर होता है]

(ii) at point E

$F_x = 0$

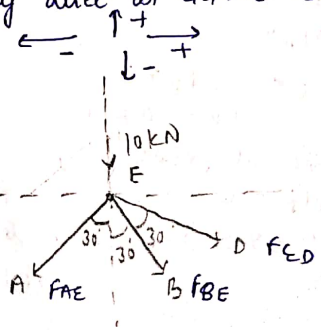
$-F_{AE} \sin 30 + F_{BE} \sin 30 + F_{ED} \sin 60 = 0$

$\frac{8\sqrt{3}}{2} + \frac{F_{BE}}{2} + \frac{F_{ED}\sqrt{3}}{2} = 0 \Rightarrow F_{BE} + F_{ED}\sqrt{3} = -8\sqrt{3}$

$F_y = 0$

$F_{AE} \cos 30 + F_{BE} \cos 30 + F_{ED} \cos 60 = -10$

$-8\sqrt{3} \cdot \frac{\sqrt{3}}{2} + F_{BE}\frac{\sqrt{3}}{2} + \frac{F_{ED}}{2} = -10 \Rightarrow F_{BE}\sqrt{3} + F_{ED} = 4$



$F_{ED} = -14\,KN = 14\,KN\ (C)$

$F_{BE} = 6\sqrt{3}\ KN\ (T)$

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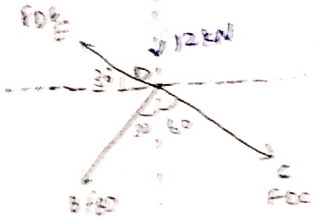
(iii) at point D



$\Sigma F_x = 0$

$-F_{DE}\cos 30 - F_{BD}\sin 30 + F_{DC}\sin 60 = 0$

$14\frac{\sqrt{3}}{2} - \frac{F_{BD}}{2} + \frac{F_{DC}\sqrt{3}}{2} = 0$

$F_{DC}\sqrt{3} - F_{BD} = -14\sqrt{3}$

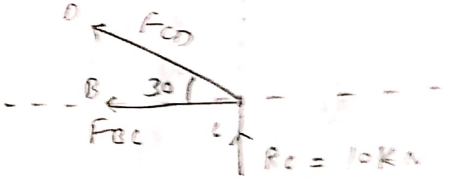
$\Sigma F_y = 0$ $\quad F_{DE}\sin 30 - F_{BD}\cos 30 - F_{CD}\cos 60 = 0$

$-\frac{14}{2} - F_{BD}\frac{\sqrt{3}}{2} - \frac{F_{CD}}{2} = 12$

$\frac{F_{CD}}{2} + F_{BD}\frac{\sqrt{3}}{2} = -14 + 12 = -2$

$F_{BD} = -6\sqrt{3}$ (C) $\qquad F_{DC} = \cancel{8\sqrt{3}}$ -20 (C)

(iv) at Point C



$\Sigma F_x = 0$

$-F_{CD}\cos 30 - F_{BC} = 0$

$\Sigma F_y = 0$

$F_{CD}\sin 30 + \overset{(R_C)}{10} = 0$

$20\frac{\sqrt{3}}{2} = F_{BC} = 10\sqrt{3}$ KN (T)

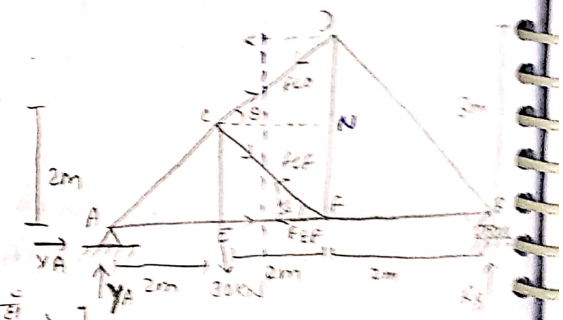
[जब हम value निकालते हैं force की, then उत्तर -ve आती है तो वाकि eqn में put करें (-) के साथ- वाली value ही रखनी है]

Numerical based on methods of sections (CD, EF, CF)

① Taking value of θ

In Δ CDN, $\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$



Taking moment about A

$\Sigma M_A = 0$

$-30\times 2 + R_B\times 6 = 0$

$R_B = 10$ KN

Now, at point F [जो point होते हैं उनको पर हो force converge कर वाकि उनका Moment = 0]

$\Sigma M_F = 0$

$10\times 2 + 2(F_{CD}\cos 26.56) = 0$

$F_{CD} = -7.45$ KN (C)

Now,

$\Sigma F_x = 0$ $\quad -F_{CD}\cos 26.56 - F_{CF}\cos 45 - F_{EF} = 0$

$\Sigma F_y = 0$ $\quad -F_{CD}\sin 26.56 + F_{CF}\sin 45 + R_B = 0$

[Now we can find F_{CF} and F_{EF}]

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Chapter - 2 (Strength of a material)

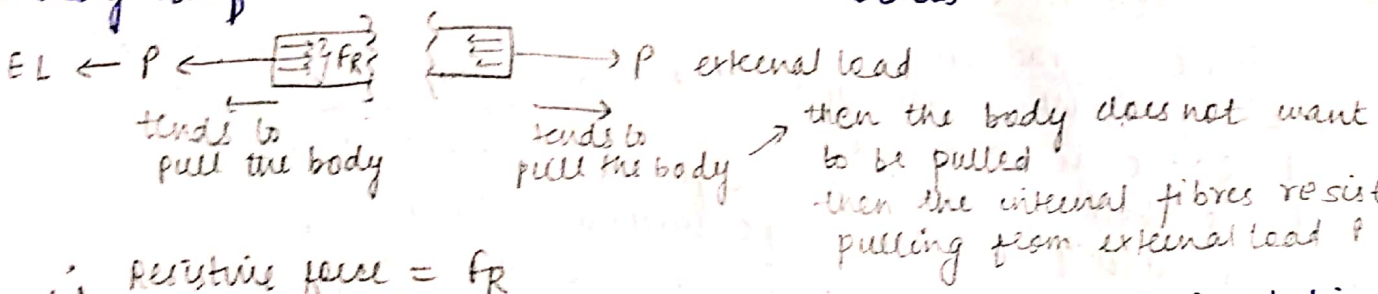
* stress :→

external → load → deformaⁿ → Elastically / Plastically

when we apply some external load to the body, then the body deforms

Every engineer does not want their material bodies to be get deformed. So to overcome deformaⁿ, a resistive force is applied i.e. stress

[internal resistance is provided by body itself]



∴ Resistive force = F_R

stress $(\sigma) = \frac{F_R}{A}$

$\sigma \Rightarrow \frac{F_R}{A} = \frac{P}{A}$

And the whole body is in static equilibrium

∴ $F_R = P$ (it is not going anywhere

unit N/m^2 or Pascal (Pa)

stress is defined as internal resistive force per unit area

* Types of stress and loads.

(i) Axial load ⇒ along the axis

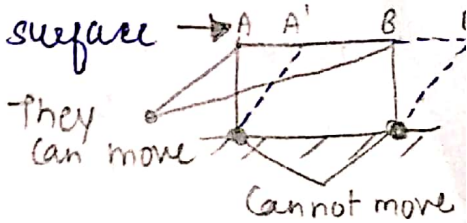
- Tensile ⇒ elongates a body, Pulling force, Tensile stress σ_T
- compressive ⇒ shortens a body, Pushing force, compressive stress σ_C

(ii) Transverse load ⇒ ⊥ to the axis

It will create bending stresses

(iii) surface | shear loads ⇒ tangential to the surface or (along the surface)

(T) brings deformation

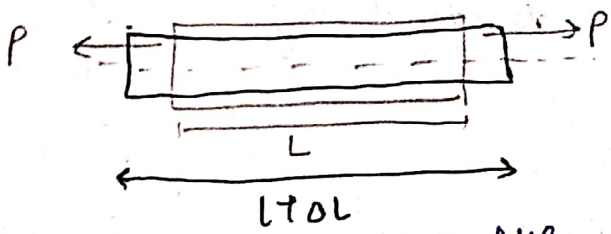


* strain ⇒ strain meaning deformation

$$\text{strain} = \frac{\text{deformation}}{\text{original size}}$$

(i) tensile strain

- thickness reduces
- $S_T = \frac{\text{deformⁿ}}{\text{O.S}}$



deformⁿ = New length − original length = $L + \Delta L - L = \Delta L$ = +ve

$S_T = \frac{\Delta L}{L}$ = +ve strain

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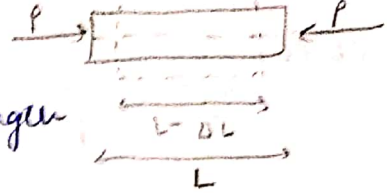
(ii) compressive strain :→



- thickness increases
- $S_c = \frac{\text{deformaⁿ}}{\text{original}}$ ⇒ deformaⁿ :→ New length − original length

$L - \Delta L - L = -\Delta L$

$S_c = \frac{-\Delta L}{L}$ → -ve strain.

(iii) shear strain :→

ϕ = angular strain | shear strain

Unit = radian

* elasticity (property) :→ when we put load, it brings out deformaⁿ. and if we remove the load, then the deformaⁿ vanishes and the body regain its original position and shape. This is called elastic deformation (Temporary)
* Plasticity :→ it is the property when the body does not regain its original position after load removal. and the deformaⁿ is called Plastic deformaⁿ (Permanent)
* Elastic limit :→ when we keep applying load and removing it, then a particular stage comes when ~~the get~~ the body gets its original shape back. and after that point the body will behave as plastic body.

↳ load corresponding across Elastic limit → Elastic limit load
↳ stress " " " " → " " stress
↳ strain " " " " → " " strain

* Hooke's law :→ If our body is loaded under Elastic limit only, not in Permanent deformaⁿ

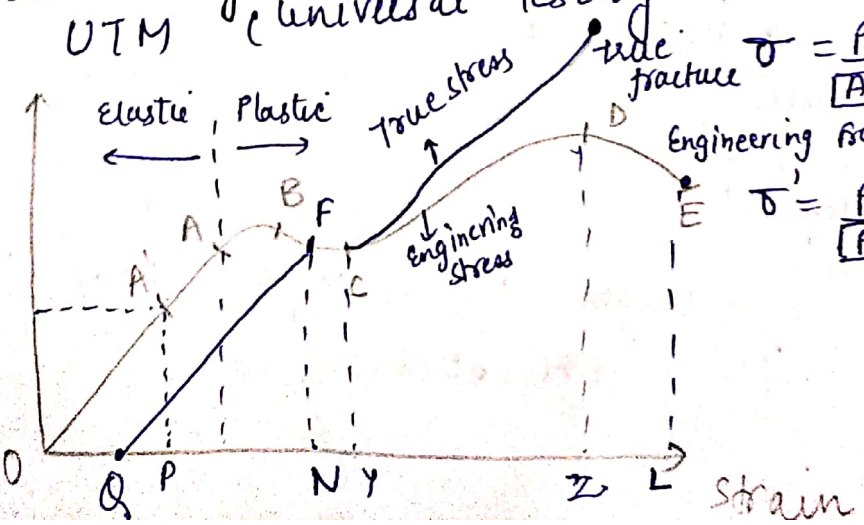
then stress ∝ strain

$\sigma \propto S$

$\sigma = Y S$ → Youngs modulus of elasticity.

unit of Y = unit of stress (Pascal)

* stress − strain diagram designed by UTM (Universal Testing Machine)



$\sigma = \frac{P}{A}$ → Engine. stress
A → constant

$\sigma' = \frac{P}{A'}$ → True stress
A' → instantaneous

[~~area of~~ area of AOP = Youngs modulus]

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(i) from pt O to A' ⇒ A' point is proportionality limit.
∴ In this region, stress ∝ strain. This is not the elastic limit

(ii) from pt A' to A ⇒ A point is elastic limit. At point A, elasticity will be finished. stress is not very ~~less~~ clearly proportional to strain due to unconsiderable distance b/w A & A'
∴ This region does not follows Hooke's law.

(iii) from pt A to B ⇒ Point B is upper yield point. Initially when we apply load, the strain was FN. But when we remove load, then the strain decreases to FQ. Therefore QN (strain) vanishes and OQ strain is still left which tells us that the body is in permanent deformation. (Yielding ⇒ deformation is producing) ⇒ flow of material started

(iv) from point B to C ⇒ Point C is lower yield point

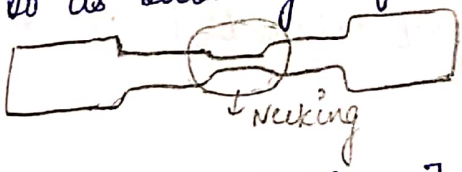
① Is σ a function of strain (S) ?
∴ load to body → deformaⁿ (S) → will be resisted by stress
Hence $\sigma = f(S)$ (dependent, independent variable)

② why there is dip b/w B & C ?
Because there is strain / deformaⁿ / flow (NY) has been started which is not properly resisted by stress. Therefore stress (↓).

(v) from point C to D ⇒ Point D is ultimate stress. Because there is too much strain YZ, which has to resisted by very high stress. The body is becoming hard, hence hard to deform, hence more stress. This YZ region is called strain hardening point.

(vi) from point D to E ⇒ Point E is Rupture Point

Q ⇒ why the true fracture point is higher than engineering fracture pt ?
The maximum stress can be applied till D (ultimate stress). Now the material cannot be resisted by stress. So its becoming soft. Therefore at point D, Necking process begins



the region ZL is geometrical softening

⇒ क्यों true stress ऊपर ?

Engineering stress $\frac{P}{A}$ < True stress $\frac{P}{A'}$ [loads are same but A' is too low as area is decreasing on stretching]

⇒ क्यों Engine stress dip ? ⇒ main answer! the dip occurs because we tried to delay the stretching in necking. We are actually reducing loads to stretch more.

$\sigma = \frac{P}{A}$ P↓ σ↓ for engineering stress (for dip)

$\sigma' = \frac{P}{A'}$ P↓ A' still ↓ due to stretching, then also true stress is always higher than engineering stress.

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* shear strain (ϕ) and shear stress (τ)

$\therefore \quad \tau \propto \phi$

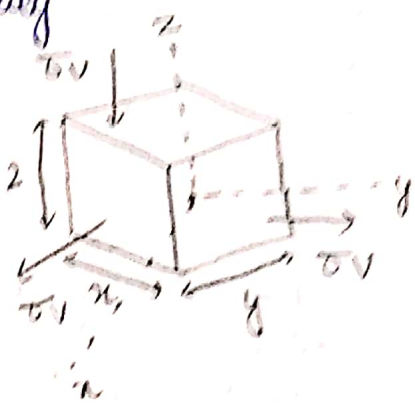
$\tau = C\phi$ → Modulus of Rigidity.

* volumetric strain (V_s) and volumetric stress (σ_s)

$\therefore \quad \sigma_s \propto V_s$

$\sigma_s = K V_s$ → Bulk modulus of elasticity

* volumetric (σ_v) stress →



x, y, z are the dimensions

∴ initial volume before applying stress $= xyz$

Now, let the dimensions are changed due to stress,

∴ dimensions would be

$x+dx \quad y+dy \quad z+dz$

volume after stress $= (x+dx)(y+dy)(z+dz)$

$= (xy + xdy + ydx + dydx)(z+dz)$

$= xyz + xydz + xzdy + xdydz + yzdx + ydxdz + zdydx + dydxdz$

[neglecting multiples of dx, dy, dz] $= xyz + xydz + xzdy + yzdx$

Now change in volume $= xyz + xydz + xzdy + yzdx - xyz$

$= xydz + xzdy + yzdx$

Therefore volumetric strain $= \dfrac{\text{change in volume}}{\text{Original volume}}$

$= \dfrac{xydz}{xyz} + \dfrac{xzdy}{xyz} + \dfrac{yzdx}{xyz} = \dfrac{dx}{x} + \dfrac{dy}{y} + \dfrac{dz}{z}$

volumetric strain at particular axis

* Working stress :→ stress which is considered as the allowable stress for design purpose.

working stress < ultimate stress

* factor of safety :→ $\dfrac{\text{Ultimate stress}}{\text{working stress}}$

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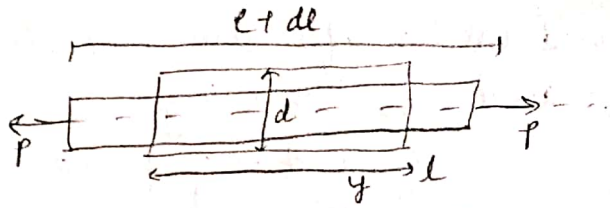
* Poisson's Ratio

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

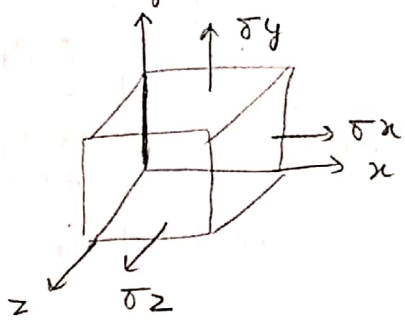
$$= \frac{-\Delta d/d}{\Delta l/l} = -ve \text{ [in tensile]}$$



similarly, in compressive

$$\mu = \frac{\Delta d/d}{-\Delta l/l} = -ve \text{ too.}$$

lateral strain = $-\mu$ · longitudinal strain



Now in 3D system,

$$(S)_x = \frac{\sigma_x}{Y} - \mu\frac{\sigma_y}{Y} - \mu\frac{\sigma_z}{Y}$$

$$(S)_y = \frac{\sigma_y}{Y} - \mu\frac{\sigma_x}{Y} - \mu\frac{\sigma_z}{Y}$$

$$(S)_z = \frac{\sigma_z}{Y} - \mu\frac{\sigma_x}{Y} - \mu\frac{\sigma_y}{Y}$$

for total strain $= S_x + S_y + S_z$

$$= \frac{1}{Y}\left[\sigma_x + \sigma_y + \sigma_z - 2\mu(\sigma_x + \sigma_y + \sigma_z)\right]$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{Y}\left[1 - 2\mu\right]$$

* Stress in the Bars of uniformly tapering circular section or stress / Extension of tapered bar.

Given :→

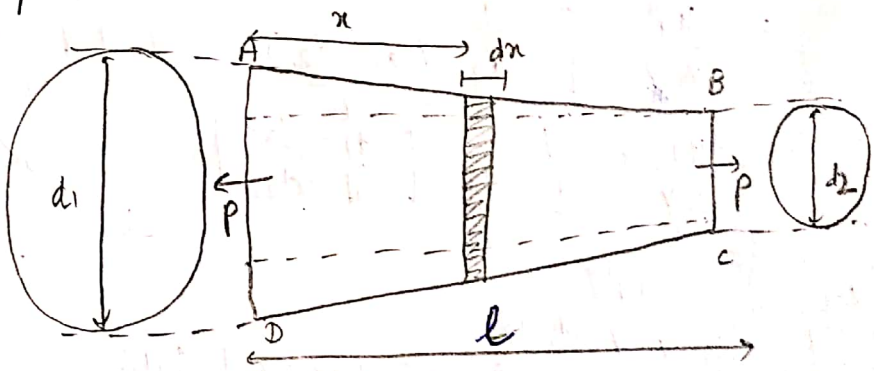
P = Pull (Tensile load)

l = length of bar

Sol^n :→

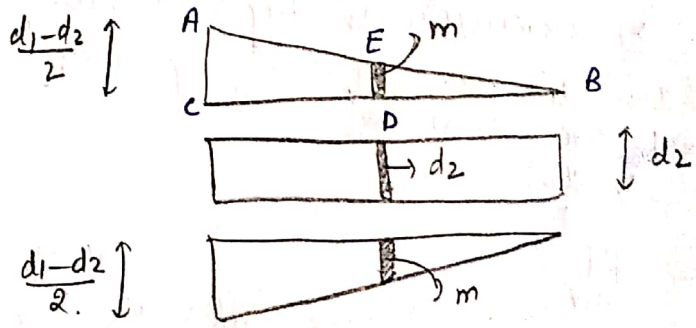
In similar triangle,

ABC and EDB



$$\frac{AC}{BC} = \frac{ED}{BD}$$

$$\frac{d_1 - d_2}{2(l)} = \frac{m}{(l-x)}$$



$$m = \left(\frac{d_1 - d_2}{2}\right)\left(\frac{l-x}{l}\right)$$

$$= \left(1 - \frac{x}{l}\right)\left(\frac{d_1}{2} - \frac{d_2}{2}\right)$$

$$= \frac{d_1}{2} - \frac{d_2}{2} - \frac{x d_1}{2l} + \frac{x d_2}{2l}$$

$$\therefore 2m + d_2 = d_1 - \cancel{d_2} - \frac{x d_1}{L} + \frac{x d_2}{L} + \cancel{d_2} = d_1 - \frac{x}{L}\left[d_1 - d_2\right]$$

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diameter of elementary strip $= d_1 - \frac{x}{L}(d_1 - d_2)$

area of elementary strip (circular) $= \pi r^2 = \frac{\pi d^2}{4}$

$$\frac{\pi}{4}\left[d_1 - \frac{x}{L}(d_1 - d_2)\right]^2 \qquad \text{let } \frac{d_1 - d_2}{L} = K$$

$$= \frac{\pi}{4}\left[d_1 - Kx\right]^2$$

now $\sigma_x = \frac{\text{force}}{\text{area}} = \frac{4P}{\pi[d_1 - Kx]^2}$

now strain $= \frac{\sigma_x}{Y} = \frac{4P}{\pi Y [d_1 - Kx]^2}$

Now, we know for total extension in tapered bar, we use integration

$$\Delta L = \int_0^L \frac{4P}{\pi Y}[d_1 - Kx]^2$$

$$= \frac{4P}{\pi Y}\left[\frac{[d_1 - Kx]^{-1}}{-1 \times -K}\right]_0^L$$

$$= \frac{4P}{\pi Y}\left[\frac{1}{(d_1 - Kx)K}\right]_0^L$$

$$= \frac{4P}{\pi Y}\left[\frac{1}{\left(d_1 - \left(\frac{d_1 - d_2}{L}\right) \cdot L\right)} \frac{d_1 - d_2}{L} - \frac{1}{d_1}\left(\frac{d_1 - d_2}{L}\right)\right]$$

$$= \frac{4P}{\pi Y}\left[\frac{1}{d_2}\left(\frac{d_1 - d_2}{L}\right) - \frac{1}{d_1}\left(\frac{d_1 - d_2}{L}\right)\right]$$

$$= \frac{4P}{\pi Y} \times \frac{L}{\cancel{d_1 - d_2}}\left[\frac{\cancel{d_1 - d_2}}{d_1 \cdot d_2}\right]$$

$$\boxed{\Delta L = \frac{4PL}{\pi Y d_1 d_2}}$$

Y = young's modulus.

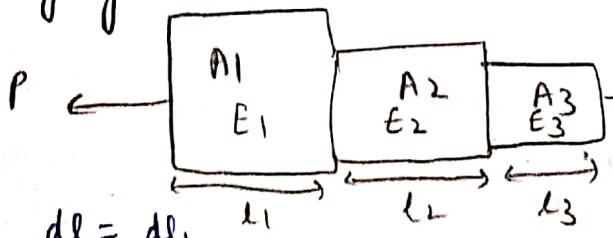
→ extension in tapered bar

* strain in bars of varying cross-section

from hookes law

$$\frac{P}{A} = Y\frac{dl}{l}$$

$$dl = \frac{Pl}{AY}$$



(for diff. material)

$E_1 \neq E_2 \neq E_3$

or may be

$E_1 = E_2 = E_3 = E$

(for same material)

∴ total strain in bars $dl = dl_1 + dl_2 + dl_3$

$$dl = \frac{Pl_1}{A_1 Y_1} + \frac{Pl_2}{A_2 Y_2} + \frac{Pl_3}{A_3 Y_3}$$

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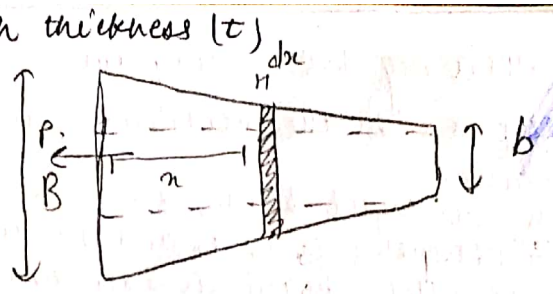
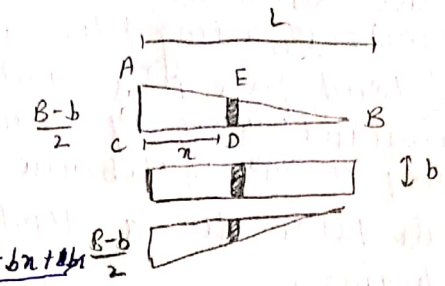
* Extension of tapered strip with thickness (t)



∴ ~~from the previous case,~~

$$\cancel{\text{length} = d_1 - \frac{x}{L}(d_1 - d_2)}$$
$$\cancel{= d_1 + \frac{x}{L}(d_2 - d_1)}$$
$$\cancel{= B + \frac{x}{L}(b - B)}$$



In △ ABC & BDE

$$\frac{AC}{BC} = \frac{ED}{BD} \qquad \frac{B-b}{2L} = \frac{ED}{(L-x)}$$

$$\frac{(B-b)(L-x)}{2L} = ED$$

$$2ED + b = \frac{(B-b)(L-x) + 2b}{2} = \frac{BL - Bx - bL + bx + 2bL}{L}$$

$$= B + \frac{x}{L}(b - B)$$

area = thickness × length
(rectangular strip)

$$= t\left(B + \frac{x}{L}(b - B)\right) \qquad \text{let } \frac{b-B}{L} = K$$

$$= \cancel{Bt + \frac{xt}{L}(b - B)} \quad t(B + Kx)$$

$$\sigma = \frac{\text{force}}{\text{area}} = \frac{P}{t(B + Kx)}$$

$$\text{strain} = \frac{\sigma}{Y} = \frac{P}{Yt(B + Kx)}$$

Now, we use integral, for total extension or strain in rectangular tapered bar.

$$\Delta L = \int_0^L \frac{P}{Yt(B + Kx)} = \frac{P}{Yt}\int_0^L \frac{1}{Kx + B} = \frac{P}{YtK}\left[\log Kx + B\right]_0^L$$

$$= \frac{P}{YKt}\left[\log(KL + B) - \log B\right]$$

$$= \frac{PL}{Y(b-B)t}\left[\log(b) - \log(B)\right]$$

Y = Young's modulus.

---

① $-\frac{dS}{dx} = w$ — -ve slope of shear force = loading rate (w)

② $\frac{dM}{dx} = S$ M = moment S = shear force

③ If S.F changes sign, B.M is either min or max

④ the pt. of change of sign of B.M is called point of contraflexture or point of inflexion.

⑤ B.M sign changes, curvature will also change sign

⑥ B.M is constant & max, SF = 0. This is called Pure Bending.

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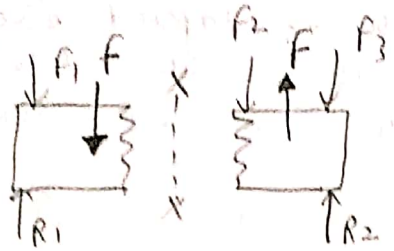
* Shearing force diagram

Let XX be the section of the beam.

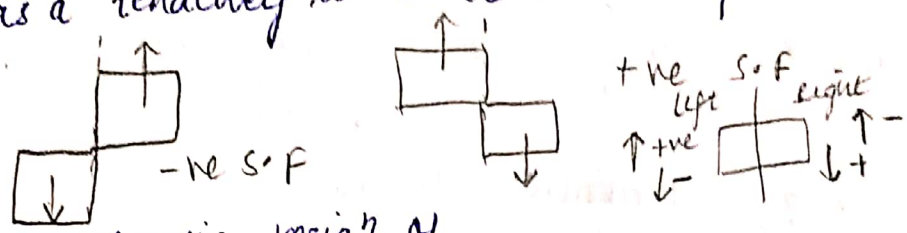


On the right beam, let F be the resultant force of F_2 and F_3 and to keep the beam in equilibrium the F force is acting downwards in left secⁿ.

∴ This load force 'F' is called shear force which ~~acts along~~ is resultant force of loads acting on any section of beam to keep it in equilibrium.

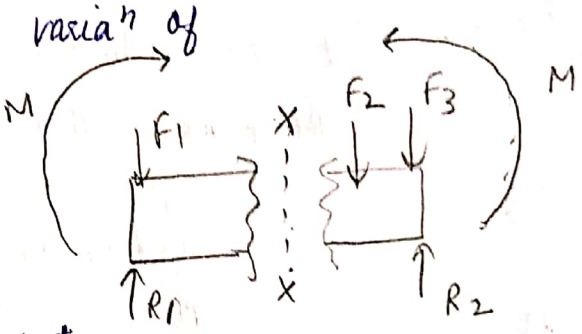
⇒ shear force has a tendency to slide these respective secⁿ of beams.

sign convenⁿ :→



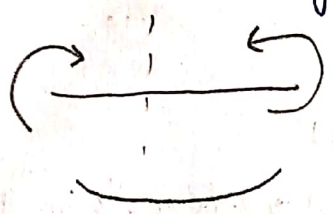
SFD :→ diagram which is showing variaⁿ of S.F vs length of beam

* Bending moment diagram

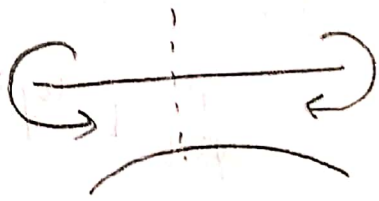


Similarly, on the right beam let M be the resultant moment of F_2 and F_3 and to keep the beam in equilibrium, the ~~bending~~ moment M is acting clockwise in left section.

∴ This moment which is resultant moment of all the loads acting on any secⁿ of beam to keep it in equilibrium is called Bending moment.



Sagging (+ve B.M)

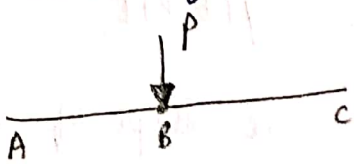


Hogging (-ve B.M)

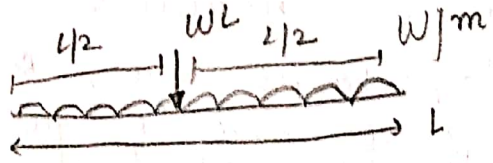
B.M.D :→ diagram which is showing variaⁿ of B.M vs length of beam

* Types of loads

(i) concentrated / Point load :→ acts at a point



(ii) Uniformly distributed load (UDL) :→ acts at the centroid of the beam means at the centre of beam.



Total load = WL

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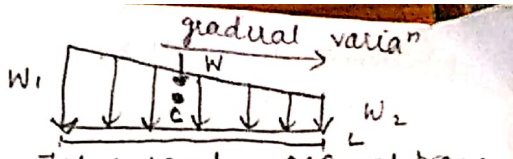
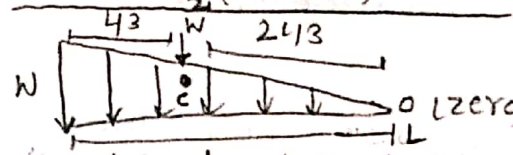
(iii) Uniformly varying load (UVL) :→
here load varies from a particular value to 0 or some another value (gradual varia^n)
It acts at a centroid (C)



Total load = area of trape.
$W = \frac{1}{2}(W_1 + W_2)L$



Total load = area of Δ
$W = \frac{1}{2}(WL)$

~~long~~ beam

sec^n

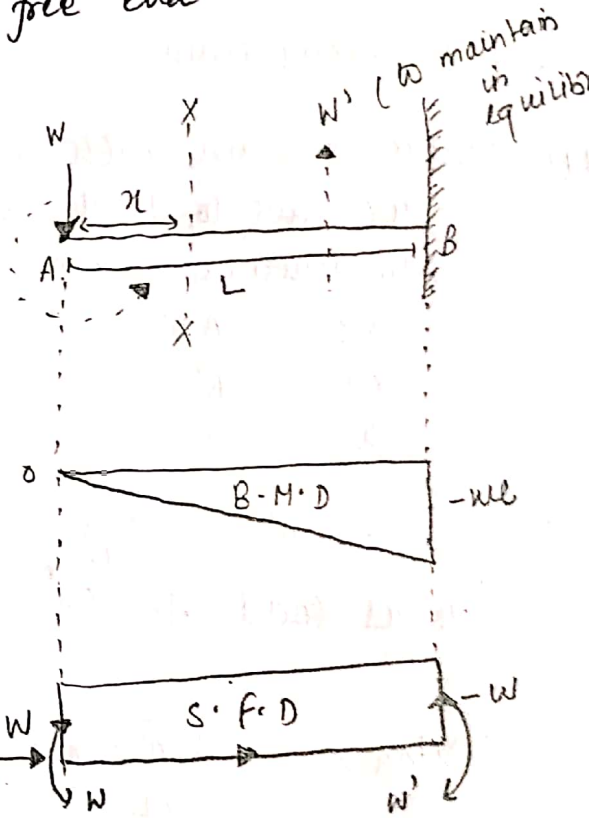
* Types of beams and their SFD + BMD

(i) Cantilever with point load at free end.

$B.M_{X-X} = -Wx$ (Hogging)

$M_A\ (x=0) = 0$

$M_B\ (x=L) = -WL$



$S.F_{X-X} = -W$ (see sign conve^n)

Similarly $S.F_A = -W$
$S.F_B = -W$

Always note, that S.F.D is always closed

(ii) Cantilever with ~~Point~~ UDL

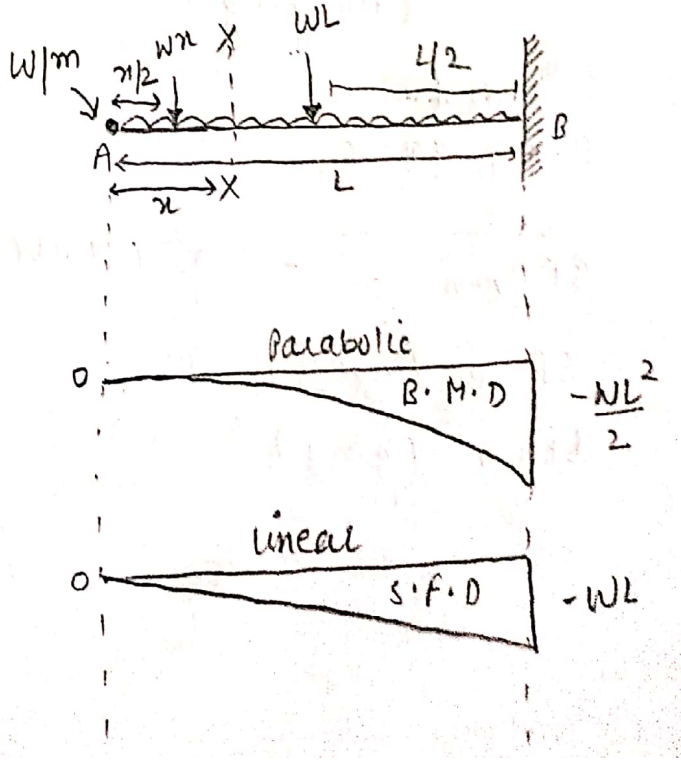
$B.M_{X-X} = -Wx \cdot \frac{x}{2}$ (Hogging)
$= -\frac{Wx^2}{2}$ (Parabolic)

$B.M_A\ (x=0) = 0$
$B.M_B\ (x=L) = -\frac{WL^2}{2}$

$S.F.D_{X-X} = -Wx$ (see sign conven^n)
(linear)

$S.F_A\ (x=0) = 0$

$S.F_B\ (x=L) = -WL$



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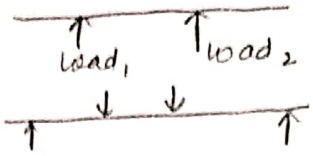
* Types of Beams.

(i) Cantilever

(ii) fixed beam

(iii) overhang beam



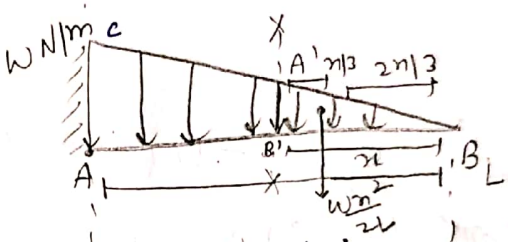
(iv) simple supported

(v) continuous beam

(iii) cantilever beam with gradually varying load :→

we have to find load A'B'

∴ In similar Δ ABC and B'B A'



$$\frac{AC}{AB} = \frac{A'B'}{B'B}$$

$$\frac{W}{L} = \frac{A'B'}{x}$$

$$\therefore A'B' = \frac{Wx}{L}$$

Total load b/w X-X section = area under A'B'B

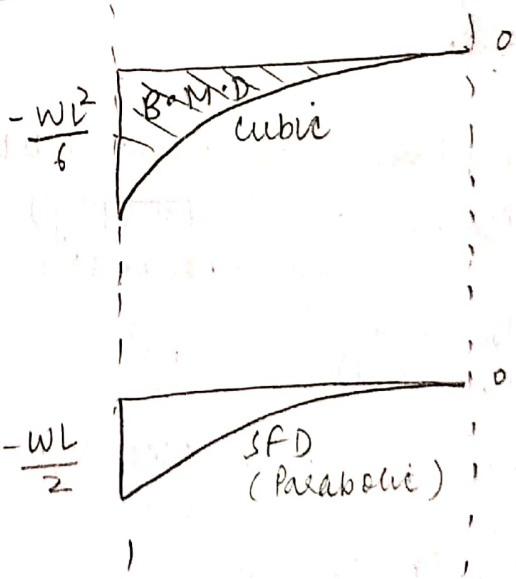
$$= \frac{1}{2} \times x \times \frac{Wx}{L} = \frac{Wx^2}{2L}$$

$$BMD_{x-x} = -\frac{Wx^2}{2L} \times \frac{x}{3} = \frac{-Wx^3}{6L}$$

(Hogging)

$$BMD_A (x=L) = \frac{-WL^2}{6}$$

$$BMD_B (x=0) = 0$$



$$SFD_{x-x} = -\frac{Wx^2}{2L} \text{ (Parabolic)}$$

$$SFD_B (x=0) = 0$$

$$SFD_A (x=L) = \frac{-WL}{2}$$

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(iv) Shear force diagram and bending moment diagram for simply supported load with point load.

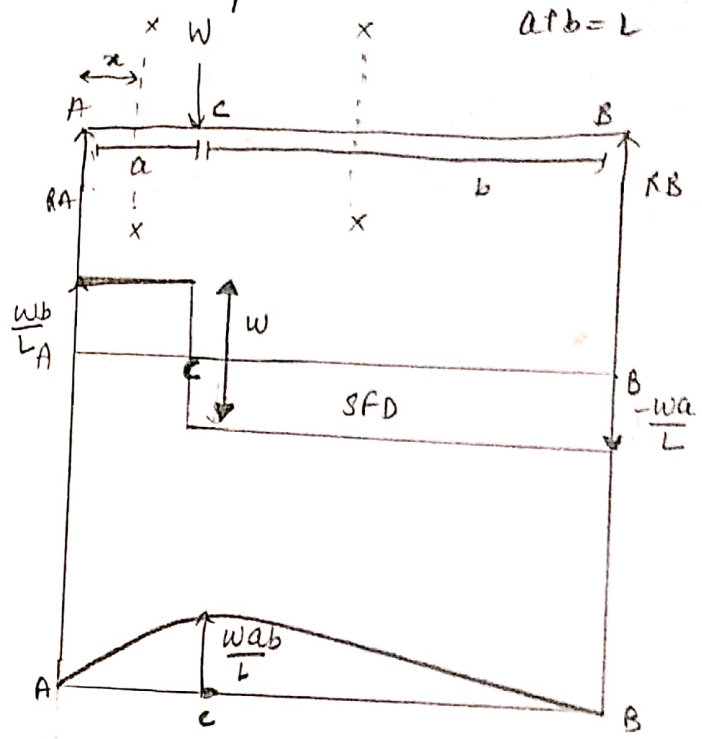
W = Point load

$M_A = 0$

$-Wa + R_B L = 0$

$$\boxed{R_B = \frac{Wa}{L}}$$

$$\therefore R_A = W - \frac{Wa}{L} = \boxed{\frac{Wb}{L} = R_A}$$



Now shear force, in the length AC

at A = $R_A = \frac{Wb}{L}$

Now, S.F in the length AB

at B = $R_A - W = \frac{Wb}{L} - W = -\frac{Wa}{L}$

New S.F on the point C

at C = $-W$

Bending moment,

- in the line AC,
- $B.M = R_A \cdot x$

$M_A (x=0) = 0$ ✓

$M_C (x=a) = R_A \cdot a = \frac{Wa}{L} b$ ✓

- B.M in the line BC

$B.M = R_A \cdot x - W(x-a)$

$M_C (x=a) = R_A \cdot a = \frac{Wa}{L} b$ ✓

$M_B (x=L) = R_A \cdot L - W(L-a)$

$= Wb - WL + Wa$

$= 0$ ✓

(v) Simply supported load with UDL

Shear force in the

- line AL

$SF = R_A = \frac{WL}{2} - Wx$

at A $(x=0) = \frac{WL}{2}$

at C $(x=\frac{L}{2}) = 0$

at B $(x=L) = -\frac{WL}{2}$

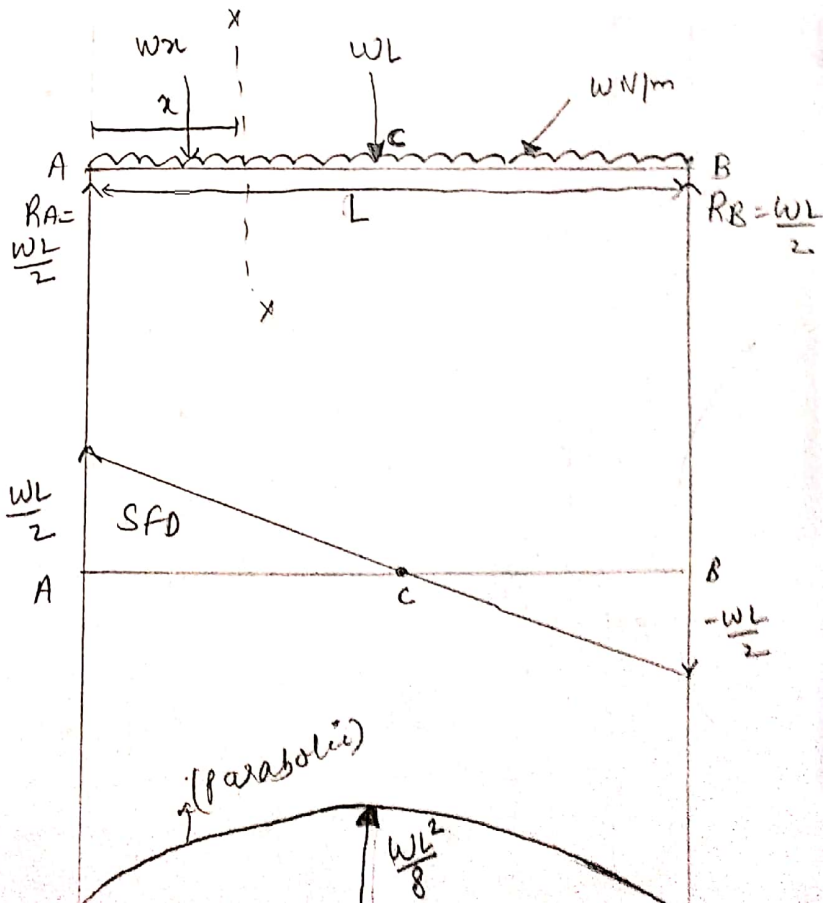
- B.M.D

$B.M = R_A \cdot x - \frac{Wx^2}{2} = \frac{WLx}{2} - \frac{Wx^2}{2}$

at A $(x=0) = 0$

at C $(x=\frac{L}{2}) = \frac{WL^2}{4} - \frac{WL^2}{8} = \frac{WL^2}{8}$

at B $(x=L) = 0$



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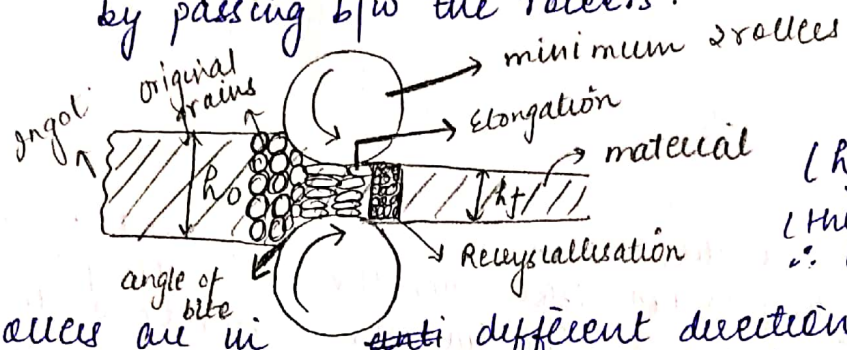
Chapter (Unit-3) Manufacturing Process.

Metal forming process :→ shaping processes in which mass of metal or alloy is subjected to mechanical forces. By these processes, the given shape and size of a machine part can be manufactured.

(i) Rolling :→ the process of reducing thickness of a metal by passing b/w the rollers.



→ final → original
$(h_f < h_o)$
(thickness is reduced)
∴ length ↑

Both the rollers are in ~~anti~~ different directions so that the material may go further. If both will be in same diren then material wont go further.

⇒ used for materials like steel, copper, Mg, Aluminium

⇒ Instruments used are; working & supporting roller, drive motor, fly wheel, reducing gear, coupling gear.

⇒ Rollers apply compressive force to material so that it can be reduced

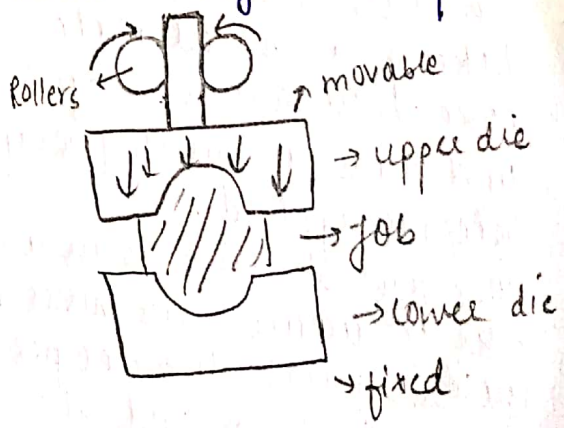
⇒ here Ingot means raw material which ~~is~~ is reshaped into bars, plates, L-shape, pipes, etc.

⇒ Roller 1 → fixed Roller 2 → movable. (or vice versa)

⇒ This is almost hot rolling process and sometimes cold rolling process also.

⇒ Mechanical property is improved

(ii) Forging :→ In this process material is squeezed b/w 2 or more dies to deform the shape and size in such a way that required final product is obtained.



⇒ raw material is heated first.

⇒ Its of 5 types
hand forging
Machine forging
Drop forging
Press forging
Upset forging.

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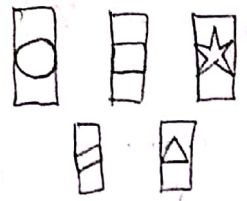
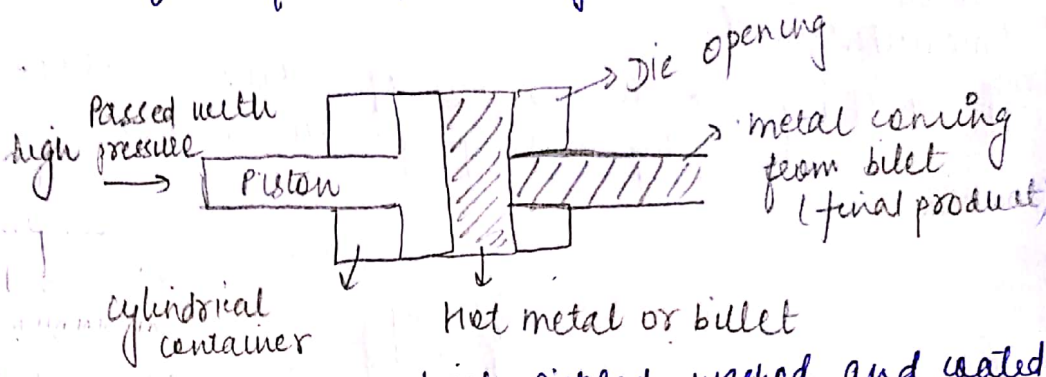
(iii) Extrusion :→ This is a process of forcing a heated metal from a die opening by applying too much compressive force.

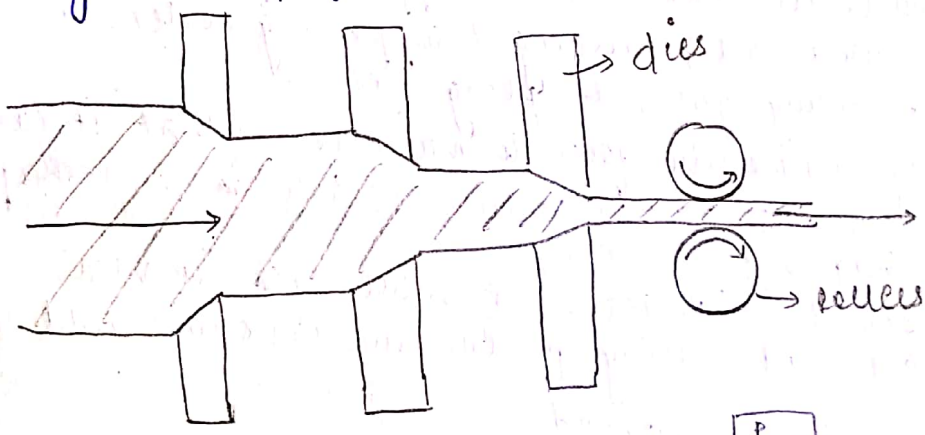
Example :→ Extraction of toothpaste from colgate tube



(These all are die openings)

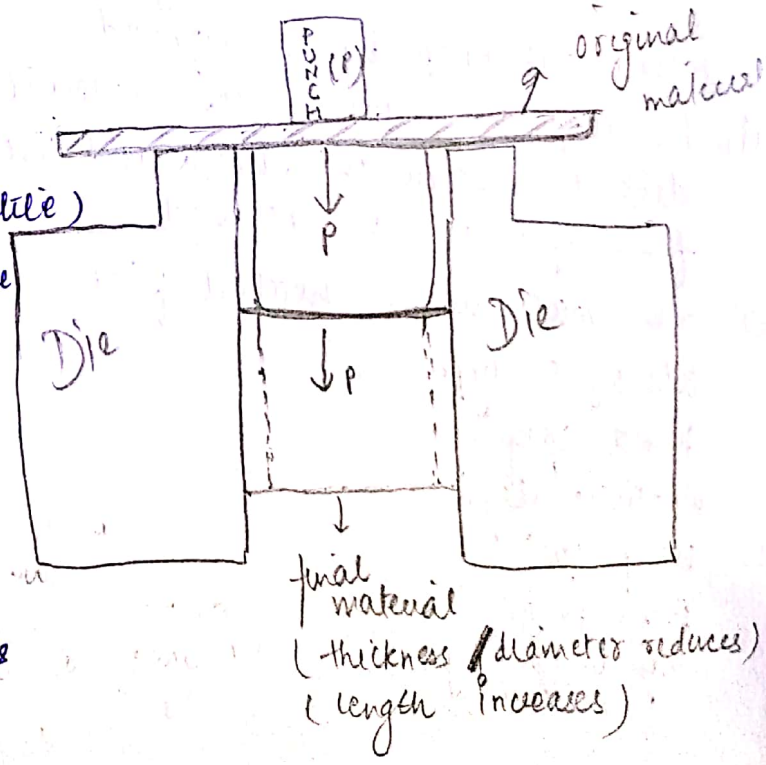


(iv) Drawing :→ In this, the bars are first pickled, washed and coated to prevent any oxidation. After this the rods are passed through several dies of decreasing diameter to provide a desired reduction in the size (diameter). The dies used for the reduction process is generally made up of carbide materials.



(v) deep drawing :→

In this, initially we are taking a metal plate (like aluminium, copper → ductile) and then a punch (compressive force) is applied. Therefore, it will form a cup like structure. The more we increase punch, the deeper cup will form and its thickness decreases & length increases. That's why it is called deep drawing.



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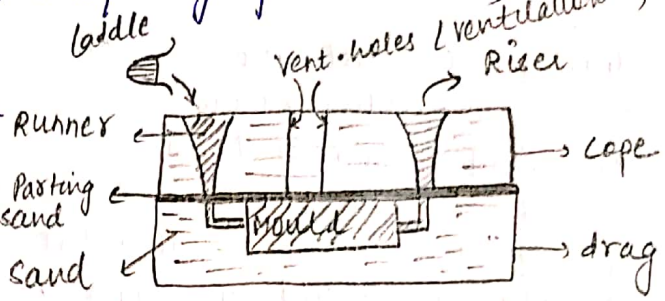
Casting Process :→ It is the process in which molten metal is drop into mould from runner and inspected by the riser and then cut off to get the desirable shape & structure.

⇒ Instruments
Drag (lower box) cope (upper box) [Laddle (जिसमें से Molten Metal डालते हैं)
Runner (Passing way to molten metal)
Riser (Inspects whether mould is completely filled or not)
Pattern (replica of mould)



⇒ when we pour molten metal from laddle to runner than it will pass into mould. when mould is completely filled than, the metal goes out from riser.

Hence, when the molten metal comes out from Riser than we stop pouring. Let it rest and cool down. Cut the runner & riser from mould we get a desirable shape as final product.

⇒ Funⁿ of riser :→ (i) tells whether mould is filled or not.
(ii) completes the vacant space (if any) in mould so that we may get our desirable shape.
(riser flow back)

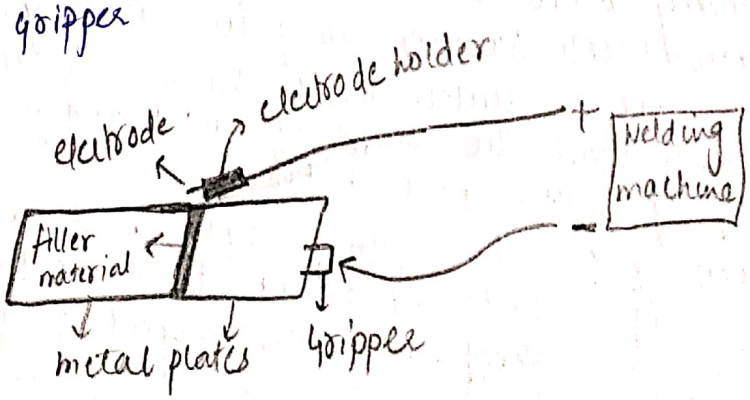
⇒ Joining Process :→ Most of the products cannot be manufactured as single unit. They are manufactured as different compounds using one or more of the manufacturing processes. These compounds are assembled to get the desirable shape. Some of the common joing process are as follows :→

(i) welding :→ It is the process of joining similar/different metals by heating them to a suitable temperature with filler material.

⇒ filler material is used to joing metals by melting itself. Its melting point is almost same as that of metal but above 430°c.

⇒ Weldability :→ joinability or ability of be welded

जैसे ही electrode में +ve and gripper में -ve, than at electrode, the circuit completes. Therefore produce a spark and causes heating effect of electric current which melts the metal + filler electrode and finally they diseolve into each other + get joined by welding process.



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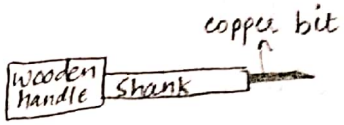
(ii) Soldering :→ It is the process of joining similar / dissimilar metals by means of a filler metal (lead + tin alloy solder)

⇒ Melting point of solder is less than 450°c but less than parent material

⇒ Flux is also required while soldering. It is made up of $ZnCl_2$, $AlCl_3$)
↓
prevents oxidations

Initially, we ~~heat~~ give electricity to soldering iron. Then shank is heated + heat is transfered from shank to copper (good conductor of heat + electricity) which heats immediately. After that solder is applied on copper bit. and it gets melted + applied at the joints.



Soldering iron

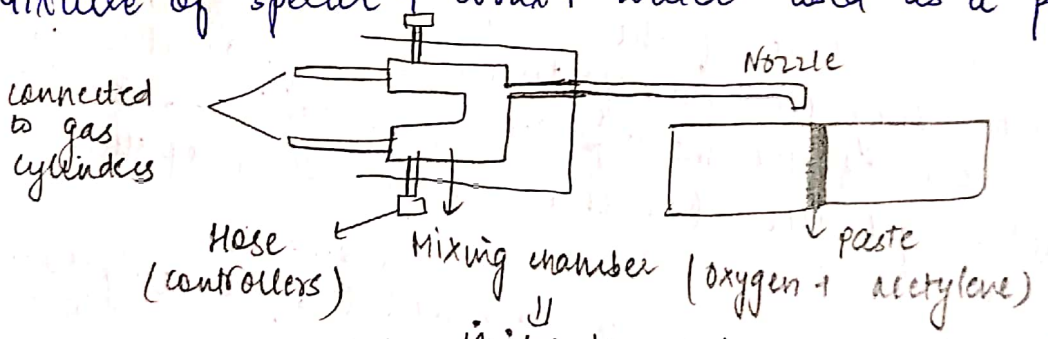
Solder (lead + tin alloy)

⇒ Before soldering, we apply flux to prevent oxidan.

(iii) Brazing :→ It is the process of joing metals by means of filler metal (copper alloy spelter)

⇒ Melting point of spelter is above than 450°c but less than parent material (Parent material → जिसके ऊपर joint करना है)

⇒ Mixture of spelter + Borax + water used as a paste on metal parts

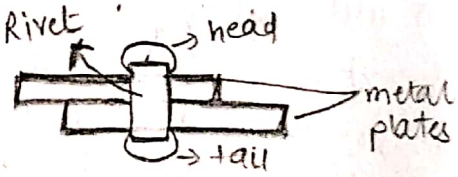


↓
ignites temperature + produces heat + then paste melted + metal joined.

Brazing joints > soldering joints.

(iv) Riveting :→ It is the process of joining the metal plates by making a cavity inside them and place a rivet b/w them, and beaten with hammer to join properly.

⇒ Here tail is beaten with hammer so that it may look the shape of head and joined properly from top + bottom.



(v) Screwing :→ It is the process used to f/to a small or thin piece of metal to something bulky. Screwing doesn't compromise the properties of metals screwed together

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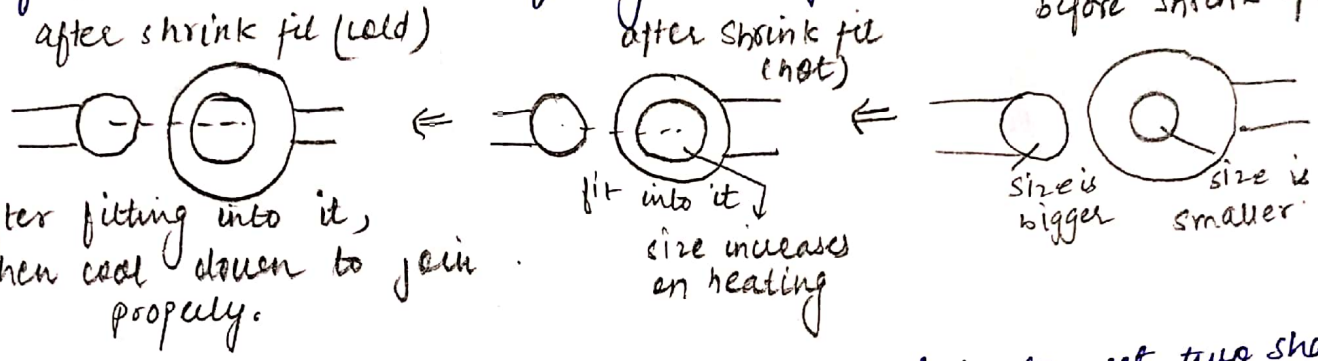
(vi) Sintering :→ Also known as pittage. It is the process of compacting and forming a solid mass of material from powdered form by pressure / heat without melting it to the point of liquefaction [Powdered goli → Tablets]

(vii) Adhesive bonding :→ Also known as gluing. It is a process in which the different materials are joined by water bonding technique [means applying a liquid layer like glues, epoxies b/w different types of materials]

(viii) Shrink fitting :→ It is the process in which we change the size (diameter) of materials to fix the objects. ~~For example :→ If smaller one is not fitting in larger one, then~~



once. after fitting into it, then cool down to join properly.

(ix) Coupling :→ A coupling is a device used to connect two shafts together at their ends for the purpose of transmitting power. It prevents misalignment and vibrations (cancels out) in rotating equipment. It can be a mechanical device that connects the ends of adjacent parts or objects.

(X) Nut and Bolt joints :→ These are the fasteners which are working together with each other for any permanent or temporary connection or joint.

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