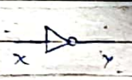
UNIT - 5

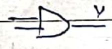
Boolean Algebra

NOT $(A', \bar{A})$



x	y
0	1
1	0

AND $(A \cdot B)$



A	B	y
0	0	0
0	1	0
1	0	0
1	1	1

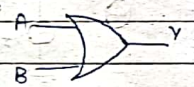
$A \cdot A = A$

$A \cdot 1 = 0$

$A \cdot 0 = 0$

$A \cdot \bar{A} = 0$

OR $(Y = A + B)$



A	B	y
0	0	0
0	1	1
1	0	1
1	1	1

$A + A = A$

$A + 1 = 1$

$A + 0 = A$

$A + \bar{A} = 1$

Q. $AB + A\bar{B}$

$= A(\underbrace{B + \bar{B}}_{1})$

$= A(1)$

Ans $= A$

Q. $AB + A\bar{B}C + A\bar{B}\bar{C}$

$= AB + A\bar{B}(\underbrace{C + \bar{C}}_{1})$

$= AB + AB'$

$= A(B + \bar{B})$

Ans $= A$

Q. $A\bar{B} + AB\bar{C} + A\bar{B}\bar{C}D$

Q. $(A+B)(A+C)$ minimize the expression.

Q. $AB + \bar{A}\bar{B} + A\bar{B}$

$\Rightarrow A(B + \bar{B}) + \bar{A}\bar{B}$

$\Rightarrow A + \bar{A}\bar{B}$

$= (A + \bar{A})(A + \bar{B})$

$= (A + \bar{B})$

$= AA + 0 + \bar{B}A + \bar{A}\bar{B}$

Property

$A + \bar{A}B$

$= (A + \bar{A})(A + B)$

$= 1 \, (A + B)$

$= (A + B)$

Q. $AB + \bar{A}B + A\bar{B}$

$= B(A + \bar{A}) + A\bar{B}$

$= B + A\bar{B}$

$= (B + \bar{B})(B + A)$

Ans. $\Rightarrow B + A$

Eg. $AB + \bar{A}B + BC$

$= AB + \bar{A}C + BC(A + \bar{A})$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB(1 + C) + \bar{A}C(B + 1)$

$= AB + \bar{A}C$

comparing we get that BC is of no use in question

Hence $BC \Rightarrow$ redundant term

Demorgan's Theorem

$$\overline{A.B.C} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{A+B+C} = \bar{A}.\bar{B}.\bar{C}$$

Minimization

SOP (Sum of Product) — generally used

eg. $ABC + ABD + AC$

minimal, canonical

POS (Product of Sum)

Eg. $(A+B+C)(A+C)(C+D)$

minimal, canonical

Q. $A\bar{B} + AB\bar{C} + A\bar{B}\bar{C}D$

$= AB\bar{C} + A\bar{B}\underbrace{(1+\bar{C}D)}_{1}$

$= AB\bar{C} + A\bar{B}$

$= A(B\bar{C} + \bar{B})$

$= A(\bar{B}+B)(\bar{B}+\bar{C}) = A(\bar{B}+\bar{C}) = A\bar{B} + A\bar{C}$

Q. $(A+B)(A+C)$

$= A.A + A.C + B.A + B.C$

$= A + A(C+B) + BC$

$= A\underbrace{[1 + (B+C)]}_{1} + BC$

Ans $= A + BC$

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A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

$\bar{A}\bar{B}$ $A\bar{B}$

Q. Minimize the expression using SOP.

Sol SOP form is used when output of logical expression is 1.

$$Y = \bar{A}\bar{B} + A\bar{B}$$
$$= \bar{B}(\bar{A} + A)$$
$$= \bar{B}$$

(4) $Y = \Sigma M(0,2)$ ⇒

Y is 1 at 0 and 2

0 0 → 0 → $\bar{A}\bar{B}$

0 1 → 1

1 0 → 2 → $A\bar{B}$

1 1 → 3

⇒ $\bar{A}\bar{B} + A\bar{B}$

$Y = \Sigma M(0, 2, 3)$

A B	Y
0 0	1
0 1	
1 0	1
1 1	1

⇒ $Y = \bar{A}\bar{B} + A\bar{B} + AB$

Q. Expression in SOP form.

Sol
$$Y = \bar{A}\bar{B} + A\bar{B} + AB$$
$$= \bar{B}(\bar{A} + A) + AB$$
$$= \bar{B} + AB$$
$$= (\bar{B} + A)(\bar{B} + B)$$
$$= \bar{B} + A$$
$$= A + \bar{B} \quad \text{Ans}$$

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SOP

Minimal form | Cannonical form

Eg. $A + AB + BC + CA$ — Eg. $A\bar{B}C + B\bar{C}A + CAB$

Eg. $AB + BCA + CD$ — Eg. ~~AB + AC + DB + AD~~

Eg. $ABC + BCA$

no definite no. of variables in an expression — each term has same no. of variable and all variable included.

Q. Covert into cannonical $A + \bar{B}C$

Sol

$$= A(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$$

$$= (AB + A\bar{B})(C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

POS form (Product of Sum)

Look for zero (0).

Q.

A	B	Y	
0	0	1	
0	1	0	$\Rightarrow A+\bar{B}$
1	0	1	
1	1	0	$\Rightarrow \bar{A}+\bar{B}$

$$Y = (A+\bar{B})(\bar{A}+\bar{B}) = A\bar{A} + A\bar{B} + \bar{B}.\bar{B} + \bar{A}\bar{B}$$

$$= 0 + A\bar{B} + \bar{B} + \bar{A}\bar{B}$$

$$= \bar{B}(\underbrace{A + 1 + \bar{A}}_{1 \text{ as there is } 1}) = \bar{B}$$ Same as in canonical form.

SOP ⇒ consider 0

POS ⇒ consider 1

Kmap

Q. $\underbrace{\Sigma m(0,2)}_{\text{for SOP}} = \underbrace{\Pi M(1,3)}_{\text{for POS}}$

Q. $F(A,B,C) = \Sigma m(0,1,4,7) = \Pi M(2,3,5,6)$

	A	B	C	
$\bar{A}\bar{B}\bar{C}$ ← 0	0	0	0	1
$\bar{A}\bar{B}C$ ← 1	0	0	1	1
2	0	1	0	
3	0	1	1	
$A\bar{B}\bar{C}$ ← 4	1	0	0	1
5	1	0	1	
6	1	1	0	
ABC ← 7	1	1	1	1

By SOP,

$\Rightarrow Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

By POS,

Y =

2 ⇒ 0 1 0 → 0 ⇒ $A+\bar{B}+C$

3 ⇒ 0 1 1 → 0 ⇒ $A+\bar{B}+\bar{C}$

5 ⇒ 1 0 1 → 0 ⇒ $\bar{A}+B+\bar{C}$

6 ⇒ 1 1 0 → 0 ⇒ $\bar{A}+\bar{B}+C$

Product of all this.

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

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Dual Expression :-

(1) AND $(\cdot)$ $\xleftrightarrow{\text{replace}}$ OR $(+)$

(2) $1 \longleftrightarrow 0$

(3) Keep the variable as it is.

Q. Find Dual.

$$A B \bar{C} + \bar{A} B C + A B C$$

Sol

$$(A + B + \bar{C}) \cdot (\bar{A} + B + C) \cdot (A + B + C)$$

Complement Expression :-

(A) AND $\longleftrightarrow$ OR

$\cdot \longleftrightarrow +$

(B) $1 \longleftrightarrow 0$

(C) Complement of each variable

Eg. Find the complement

$$Y = ABC + \bar{A} B \bar{C} + A \bar{B} C.$$

Sol

$$\bar{Y} = (\bar{A} + \bar{B} + \bar{C}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + \bar{C})$$

Basic Building Blocks :-

NOT
AND } → Basic gate
OR

NAND
NOR } → Universal gates

EXOR
EXNOR } → Arithematic gate

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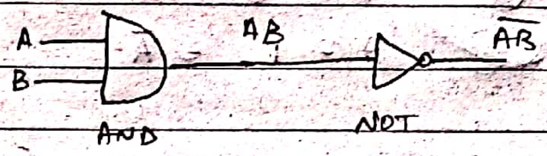
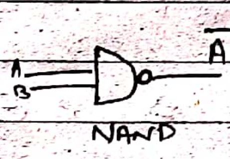
NAND Gate :-

AND + NOT

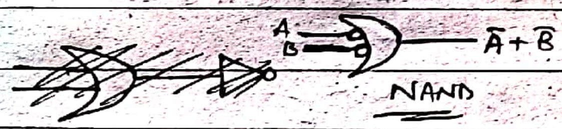


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



$\overline{AB} = (\bar{A} + \bar{B})$

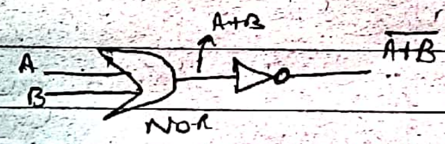
using this



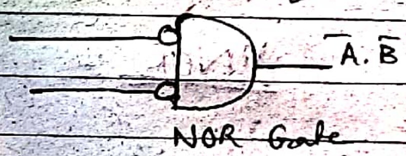
NOR Gate :-

OR + NOT

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



$\overline{A+B} = \bar{A}.\bar{B}$



EXOR OR XOR :-

(Exclusive OR)

$Y = A \oplus B$
$= A\bar{B} + \bar{A}B$

$$Y = A \oplus B$$
$$= A\bar{B} + \bar{A}B$$
$$= \bar{A}B + A\bar{B}$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

When $A = B$, Output is 0. When $A \neq B$, ⇒ output 1.

(a) $A \oplus A = 0$

(b) $A \oplus \bar{A} = 1$

(c) $A \oplus 0 = A$

(d) $A \oplus 1 = \bar{A}$

Q. $\underbrace{A \oplus A}_{0} \oplus A = A$?

Sol $\Rightarrow 0 \oplus A = A$

Q. $A \oplus A \oplus A \oplus A$

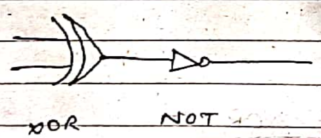
Sol

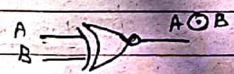
$= 0$

$A \oplus A \oplus A \oplus \ldots n = A$, if n = odd

$= 0$, if n = even

XNOR :-





$Y = A \odot B = AB + \bar{A}\bar{B}$

$\overline{A\bar{B} + \bar{A}B}$

$(\bar{A}+B)(A+\bar{B})$

$= 0 + \bar{A}\bar{B} + BA + 0$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$A \odot A =$

$A = B \Rightarrow$ output $= 1$

$A \neq B \Rightarrow$ output $= 0$

Q. $A \odot A = 1$

$A \odot A \odot A = ?$

$A \odot A \odot A \odot A = ?$

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