8 3 REFLECTION AND TRANSMISSION OF LONGITUDINAL (SOUND) WAVES AT A BOUNDARY BETWEEN TWO MEDIA

When a sound wave meets a boundary separating two media of different acoustic impedances, it is partly reflected and partly transmitted at the boundary. Consider a plane sound wave travelling in a medium *1* of density ρ_1 and incident normally on a plane boundary at $x = 0$ separating medium *1* from another medium *2* of density ρ_2. The acoustic impedances of the two media respectively are

$$Z_1 = \rho_1 v_1$$

$$Z_2 = \rho_2 v_2$$

where v_1 and v_2 are the sound speeds in medium *1* and medium *2* respectively (see Fig. 8.5). The incident, reflected and transmitted waves are respectively given by

$$\xi_i(x, t) = A_i \sin(\omega t - k_1 x)$$

$$\xi_r(x, t) = A_r \sin(\omega t + k_1 x)$$

$$\xi_t(x, t) = A_t \sin(\omega t - k_2 x)$$

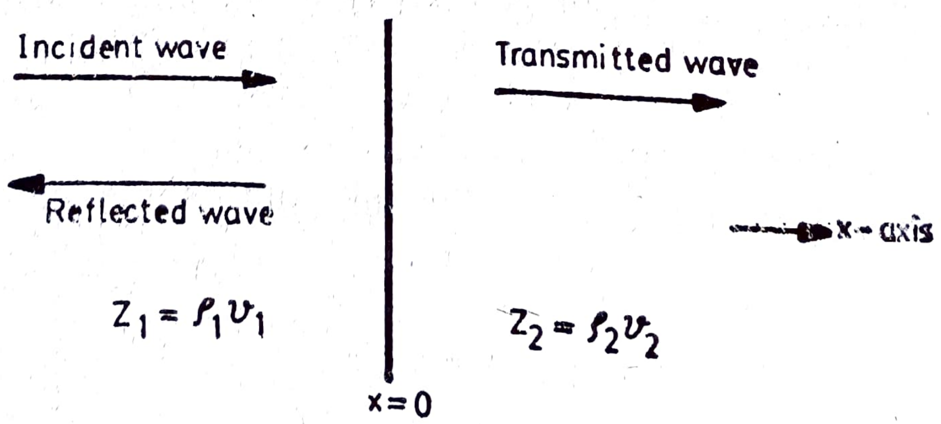

Fig 8.5 Reflection and transmission of plane sound waves at a plane boundary separating two media of acoustic impedances Z_1 and Z_2.

The boundary conditions are :—

(1) The particle displacement $\xi(x, t)$ is continuous across the boundary. Hence the particle velocity $\frac{\partial \xi}{\partial t}$ is also continuous.

$E = \text{volume elasticity} = -\frac{\Delta P}{\Delta V} V$

(2) The excess pressure $\left(p = \Delta P = -E\dfrac{\partial \xi}{\partial x} = -\gamma P_0 \dfrac{\partial \xi}{\partial x}\right)$ is continuos across the boundary [(see Eq. 6.34 of Chap. 6)]. Here $\gamma = C_P/C_V$ and P_0 is the equilibrium pressure. Thus the two boundary conditions to be satisfied are (at $x = 0$)

$$\xi_i(x, t) + \xi_r(x, t) = \xi_t(x, t)$$

$\Delta V = \text{area} \times \text{inc. in length}$
$= \text{area}\{\xi(x_2) - \xi(x_1)\}$

and
$$p_i + p_r = p_t$$

$= \text{area}\ \frac{\partial \xi}{\partial x} \cdot \Delta x$

or
$$+\frac{\partial \xi_i}{\partial x} - \frac{\partial \xi_r}{\partial x} = -\frac{\partial \xi_t}{\partial x}$$

$\frac{\Delta V}{V} = \frac{\partial \xi}{\partial x} = \text{volume strain}$

The two boundary conditions give

$$A_i + A_r = A_t \tag{8.16}$$

and
$$k_1(A_r - A_i) = k_2 A_t$$

But
$$k_1 = \frac{\omega}{v_1} = \frac{\omega}{\rho_1 v_1^2}\rho_1 v_1 = \frac{\omega Z_1}{\gamma P_0} \qquad \left(\because Z_1 = \rho_1 v_1\ ;\ v_1 = \sqrt{\frac{\gamma P_0}{\rho_1}}\right)$$

and
$$k_2 = \frac{\omega}{\gamma P_0} Z_2$$

$\text{Excess Pressure} = P(x_2) - P(x_1)$
$= \frac{\partial}{\partial x}P(x) \cdot \Delta x = \frac{\partial}{\partial x}(P_0 - \Delta P) \cdot \Delta x$
$= -\frac{\partial}{\partial x}\Delta P \cdot \Delta x$

Therefore, we have

$$Z_1(A_r - A_i) = Z_2 A_t \tag{8.17}$$

Equations (8.16) and (8.17) give

$$r_{12} = \frac{A_r}{A_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{8.18}$$

$$t_{12} = \frac{A_t}{A_i} = \frac{2Z_1}{Z_1 + Z_2} \tag{8.19}$$

These equations give the amplitude reflection and transmission coefficients. It is clear that if $Z_1 > Z_2$; A_r/A_i is positive indicating that the incident and reflected displacements are in phase. But if $Z_1 < Z_2$; A_r/A_i is negative showing that the reflected wave undergoes a phase change of π with respect to the incident wave. Furthermore, it is clear from Eq. (8.19) that A_t/A_i remains positive independent of whether Z_1 is less or more than Z_2 which means that the transmitted wave does not undergo any phase change.

At a rigid wall where Z_2 is infinity, $A_r = -A_i$ showing that the wave is completely reflected.

Reflection and Transmission of Sound Energy

The intensity of a sound wave of amplitude A and frequency ν travelling with a speed v in a medium of density ρ is given by [see Eq. (7.45) of Chap. 7].

$$I = 2\pi^2\nu^2 A^2 \rho v = 2\pi^2 \nu^2 A^2 Z$$

where Z is the characteristic acoustic impedance offered by the medium. The intensity coefficients of reflection and transmission are, therefore, given by

$$\text{Reflection coefficient} = \frac{I_r}{I_i} = \frac{2\pi^2\nu^2 A_r^2 Z_1}{2\pi^2\nu^2 A_i^2 Z_1} = \frac{A_r^2}{A_i^2} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \quad (8.20)$$

$$\text{Transmission coefficient} = \frac{I_t}{I_i} = \frac{2\pi^2\nu^2 A_t^2 Z_2}{2\pi^2\nu^2 A_i^2 Z_1} = \frac{Z_2 A_t^2}{Z_1 A_i^2} = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2} \quad (8.21)$$

Notice that $\frac{I_r}{I_i} + \frac{I_t}{I_i} = 1$ or $I_i = I_r + I_t$

which means that the energy is conserved.

Experiments show that there is almost total reflection of sound energy at the air-water interface, whereas, in the case of steel-water Interface, the reflection coefficient is about 0.85 or 85 per cent. Thus only about 15 per cent of sound energy is transmitted at a steel-water interface. This severely limits the transmission and detection devices used in submarines using ultrasonic waves.