Unit - 3
Wave Optics

Wave Front

* It is locus of all that points whic are vibrating in same Phase

* Point Source (•) → Spherical Wavefront
  Line Source (|) → Cylindrical Wavefront
  Source at ∞ → Plane Wavefront

Coherent Sources

* Sources are said to be coherent if they have Same Wavelength, Same Amplitude and Same phase or Const. phase differance. Laser is a fully coherent source & Sun is a partial coherent source.

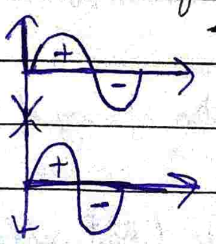
Interference

* When two sources (lights / Waves) propagating in same direc- super imposes each other then redistribution of light intensity takes place this phenomenon is called interference.

* There are 2 types of Interference, Constructive (Maxima) & Distructive (minima).

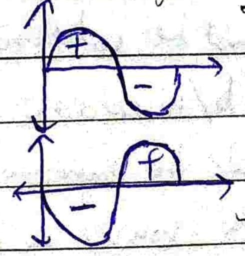
(i) Constructive :- When we get Max. intensity output after interference.
(ii) Distructive :- When we get Min. intensity output after interference.

* Condition of Constructive Interference :-



(i) Crest + Crest
or
(ii) Trough + Trough

* Condi. for Distructive Interference :-



(i) Crest + Trough
or
(ii) Trough + Crest

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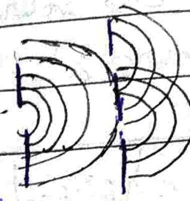
☆ Note:- If we choose 2 source of a monochromatic light with same power, they may emit wave of same wavelength & same Amplitude but since waves are emitted spontaneously, they will have random phase difference.

Production of Interference or Coherent source



Method 1 → By Division of Wave Front:
(i) Y.D.S.E
(ii) Lloyd's Single Mirror
(iii) Fresne's bi-Prism

☆ In this method Wave Fronts are divided into 2 parts either by Reflection or Refraction.
These 2 ~~parts~~ of Same Wave Front travell unequal dist. & Reunite at small angle to Produce interference Band.

Method 2 → Division of Amplitude:
☆ In this the Amplitude of the incoming beam is divided into 2 Parts, by partial reflection or refraction. Then these Parts reunite ~~& travell~~ after travelling different paths & interfere constructively & distructively.
☆ For eg:- (i) Thin Film. (ii) Newton's Ring (iii) Wedge Shaped Film

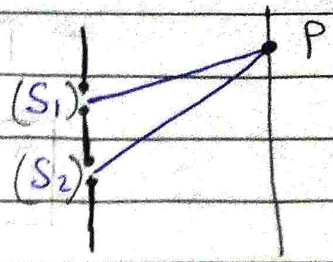
(2 Marks)

Conditions For Interference (Sustained/Continous)

☆ Conditis for Sustained or Continous Interference are:
(i) Amplitude of Both the Waves should be Same
(ii) Frequency or Wavelength of both waves should be Same
(iii) Both wave should have same pase or a Const. Phase difference. i.e Sources must be coherent.
(iv) Seperation b/w coherent sources must be Small. Also opening of both the sources should be narrow.
(v) The dist. b/w source & Screen should be reasonable (Not too Close also not too Far).

Path Difference

☆ It is the diff. of Path travelled by 2 Waves.

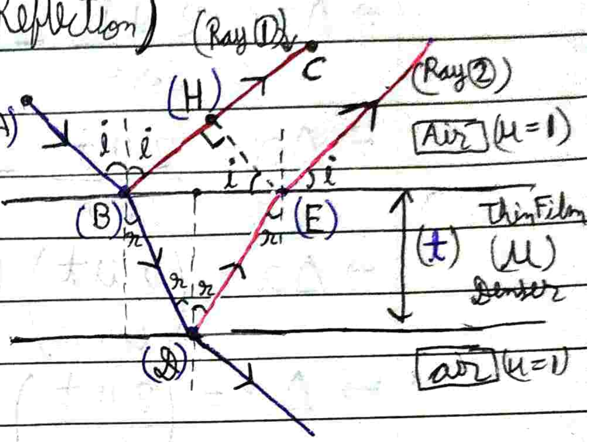


☆ $\therefore$ $\boxed{\text{Path difference} = S_2P - S_1P}$

☆ $\boxed{\underset{(\Delta\phi)}{\text{Phase difference}} = \underset{K}{\left(\frac{2\pi}{\lambda}\right)} \underset{(\Delta x)}{\text{Path difference}}}$ or $\boxed{\Delta\phi = K(\Delta x)}$

K → Angular Wave No. or Propagation Const.

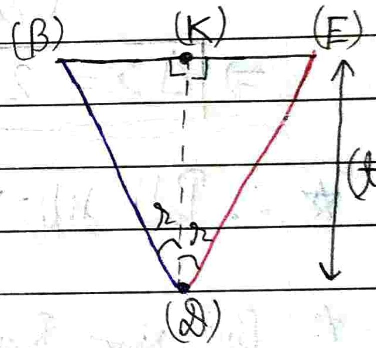
Interference Due to Thin Films (Reflection)



☆ Path diff., $\boxed{\Delta x = (BD + DE)_{\text{in Film}} - (BH)_{\text{in Air}}}$

$\Rightarrow \Delta x = (BD + DE)\mu - BH(1)$

$\because$ $\boxed{BD = DE}$

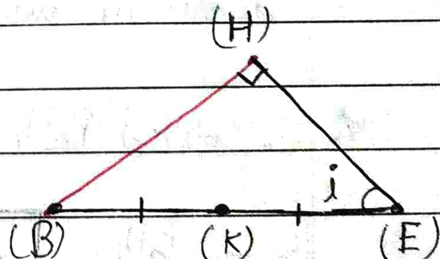


☆ $\therefore$ Path diff. $\rightarrow$ $\boxed{\Delta x = 2\mu(BD) - (BH)}$

☆ Now in ΔBDK, $\rightarrow$ $\cos r = \frac{DK}{BD} = \frac{t}{BD}$

[To Find (BD)]

$\Rightarrow$ $\boxed{BD = \frac{t}{\cos r}}$



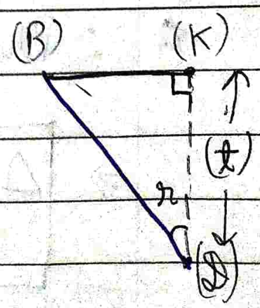
☆ Now in ΔBEH, $\rightarrow$ $\sin i = \frac{BH}{BE}$

[To Find (BH)]

$\Rightarrow BH = (BE)(\sin i)$

$\Rightarrow BH = (BK + KE)(\sin i)$

$\Rightarrow$ $\boxed{BH = 2(BK)(\sin i)}$



☆ Now in ΔBDK $\rightarrow$ $\because \tan r = \frac{BK}{D}$

[To Find (BK)]

$\Rightarrow BK = (KD) \tan r$

$$\Rightarrow BK = (t)(\tan r)$$

$$\therefore \boxed{BH = 2t(\tan r)(\sin i)}$$

★ $\because \Delta x = 2\mu(BD) - (BH)$

$$\Rightarrow \Delta x = \left[2\mu\left(\frac{t}{\cos r}\right)\right] - \left[2t(\tan r)(\sin i)\right]$$

$$\Rightarrow \Delta x = \frac{2\mu t}{\cos r} - 2t\left(\frac{\sin r}{\cos r}\right)\sin i \qquad [\sin i = \mu \sin r]$$

$$\Rightarrow \Delta x = \frac{2\mu t}{\cos r} - 2t\left(\frac{\sin r}{\cos r}\right)(\mu \sin r)$$

$$\Rightarrow \Delta x = \left(\frac{2\mu t}{\cos r}\right)(1 - \sin^2 r)$$

$$\Rightarrow \Delta x = \left(\frac{2\mu t}{\cos r}\right)(\cos^2 r)$$

$$\Rightarrow \boxed{\Delta x = 2\mu t(\cos r)}$$

★ $\therefore$ Path diff. $\rightarrow \boxed{\Delta x = 2\mu t(\cos r)}$

★ But since Ray BC suffers reflection at A on the surface of a denser medium it undergoes path diff. of $\left(\frac{\lambda}{2}\right)$ on reflection

★ $\therefore$ Correct Path diff. b/w (BC) & (EH) is,

(Correct Path diff.) $\rightarrow \boxed{\Delta x = 2\mu t(\cos r) \pm \left(\frac{\lambda}{2}\right)}$

★ $\because$ Phase diff. $= \left(\frac{2\pi}{\lambda}\right)$ Path diff.

$$\Rightarrow \boxed{\Delta \phi = \left(\frac{2\pi}{\lambda}\right)\left[2\mu t(\cos r) \pm \left(\frac{\lambda}{2}\right)\right]}$$

☆ For Maxima or Constructive Interference :

☆ $\Delta x =$ Even Multiples of $\left(\frac{\lambda}{2}\right)$ $\Rightarrow \Delta x = 2n\left(\frac{\lambda}{2}\right) = n\lambda$

$$\Rightarrow 2\mu t(\cos r) + \frac{\lambda}{2} = 2n\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow 2\mu t(\cos r) = 2n\left(\frac{\lambda}{2}\right) - \frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t(\cos r) = (2n-1)\left(\frac{\lambda}{2}\right)}$$

→ Condition of Maxima
→ t :- thickness of Film
→ μ :- Refractive index of film

☆ For Minima or Distructive Interference :

$\Delta x =$ Odd Multiples of $\left(\frac{\lambda}{2}\right)$

$$\Rightarrow 2\mu t(\cos r) + \frac{\lambda}{2} = (2n+1)\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow 2\mu t(\cos r) = \cancel{2}n\left(\frac{\lambda}{\cancel{2}}\right) + \cancel{\frac{\lambda}{2}} - \cancel{\frac{\lambda}{2}}$$

$$\Rightarrow \boxed{2\mu t(\cos r) = n\lambda}$$ → Condition of Minima

Interference Due To Transmitted Light in Thin Film

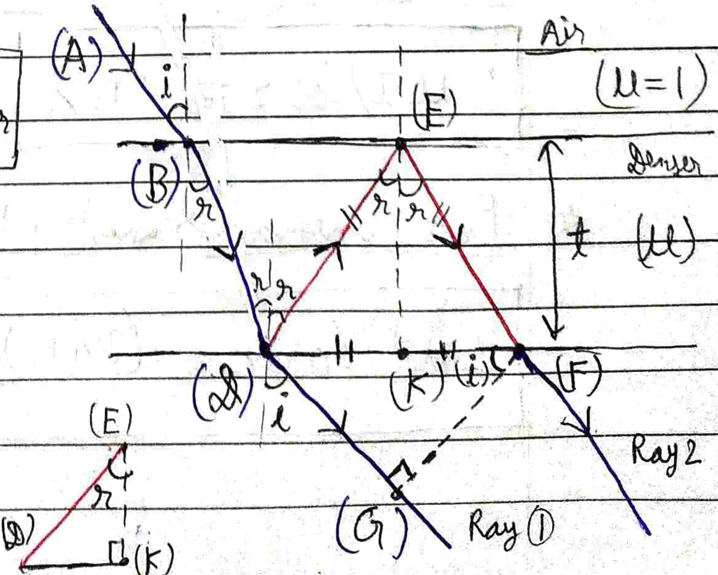
☆ $\because \boxed{\Delta x = \underbrace{(DE + EF)}_{\text{in Film}} - (DG)_{\text{in Air}}}$

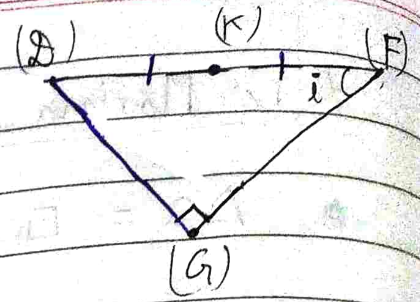
$$\Rightarrow \Delta x = (2DE)\mu - (DG)(1)$$

↳ $\because \boxed{DE = EF}$

☆ In Δ DEK → $\cos r = \frac{EK}{DE}$
[To Find DE]

$$\Rightarrow \boxed{DE = \frac{t}{\cos r}}$$



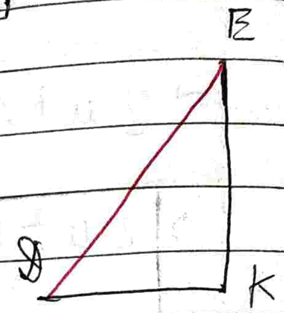


☆ Now, In $\Delta DGF \longrightarrow \sin i = \frac{DG}{DF}$
[To Find (DG)]

$\because DK = KF$

$\Rightarrow DF = 2(DK)$

$\Rightarrow \boxed{DG = 2(DK)(\sin i)}$



☆ Now, In $\Delta DKE \longrightarrow \tan r = \frac{DK}{EK}$
[To Find DK]

$\Rightarrow DK = (EK)\tan r$

$\Rightarrow DK = (t)(\tan r)$

$\therefore DG = 2(t)(\tan r)(\sin i)$

$\therefore \Delta x = \left(\frac{2t\mu}{\cos r}\right) - [2(t)(\tan r)(\sin i)]$ $[\because \sin i = \mu \sin r]$
[Snell's Law]

$\Rightarrow \Delta x = \frac{2t\mu}{\cos r} - (2t)\left(\frac{\sin r}{\cos r}\right)(\mu \sin r)$

$\Rightarrow \Delta x = \frac{2t\mu(1-\sin^2 r)}{\cos r}$

☆ $\therefore$ Path diff. $\rightarrow \boxed{\Delta x = (2t\mu)(\cos r)}$ (There will be No. additional Phase Change due to refraction unlike reflection)

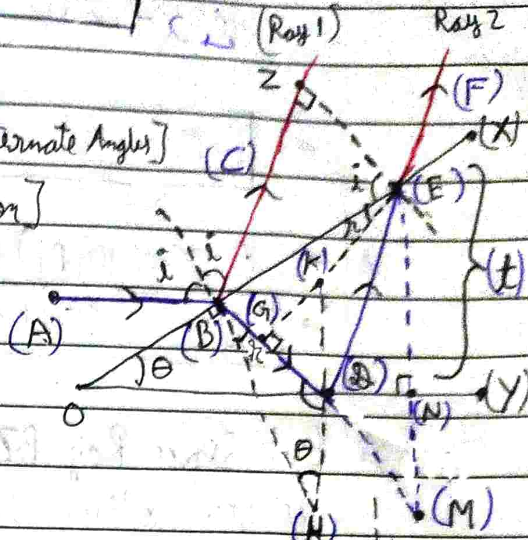
☆ For Maxima or Constructive: (Bright Band)

$$\boxed{(2\mu t)\cos r = n\lambda}$$

☆ For Minima or Distructive: (Dark Band)

$$\boxed{(2\mu t)\cos r = (2n+1)\frac{\lambda}{2}}$$

Wedge Shape Thin Film Interference



☆ $\because \angle XOY = \angle BHK = \theta$

$\angle DMN = \angle DEN = \angle HDM = (r+\theta)$ [Alternate Angles]

$\angle GDK = \angle KDE = (r+\theta)$ [Law of Reflection]

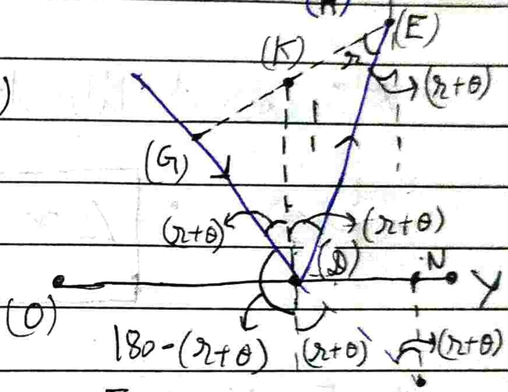
$\angle GEB = \angle r$

$\Delta DNE = \Delta DNM$ (Congruent)

☆ Path Difference :-

$\Delta x = (BD + DE)\mu - BZ$

$$\boxed{\Delta x = (BG + GD + DE)\mu - BZ} \quad -(i)$$

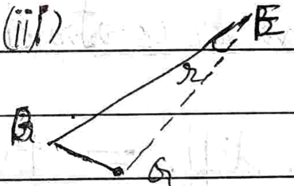


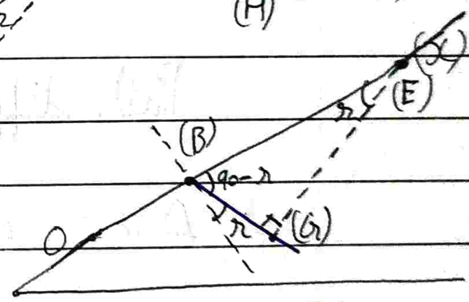
☆ Now in $\Delta BZE \longrightarrow$ $\because \sin i = \dfrac{BZ}{BE}$ $-(ii)$

[To Find BZ]

☆ Now in $\Delta BGE \rightarrow$ $\sin r = \dfrac{BG}{BE}$ $-(iii)$

[To Find BE]





Now By Snells Law, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{BZ/BE}{BG/BE}$

$\Rightarrow \boxed{\mu = \dfrac{BZ}{BG}} \Rightarrow \boxed{BZ = \mu(BG)}$

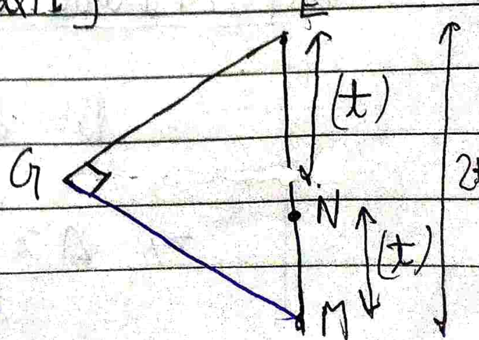
☆ On substituting Value of BZ in Δx we get,

$\Delta x = (BG)\mu + (GD)\mu + (DE)\mu - (BG)\mu$ $\quad \Delta DEN = \Delta DMN$

$\Rightarrow \boxed{\Delta x = (GD + DE)\mu}$ $\quad [\because DE = DM]$

$\Rightarrow \boxed{\Delta x = (GD + DM)\mu = (GM)\mu}$

In ΔGME



$\cos(r+\theta) = \dfrac{GM}{EM} = \dfrac{GM}{EN + NM} = \dfrac{GM}{(t+t)}$

$$\therefore \cos(r+\theta) = \frac{GM}{2t}$$

$$\Rightarrow \boxed{GM = 2t\cos(r+\theta)}$$

★ $\because \Delta x = (GM)\mu$

$$\Rightarrow \boxed{\Delta x = 2\mu t\cos(r+\theta)}$$

★ But since Ray BZ suffers Reflection at B on the surface of denser medium so it undergoes a addition path diff. of $\frac{\lambda}{2}$.

★ $\therefore$ Correct Path diff. b/w (BZ) & (EF),

$$\therefore \boxed{\Delta x = (2\mu t)\cos(r+\theta) + \frac{\lambda}{2}}$$

★ For Maxima or Constructive : (Bright Band)

Path diff. = Even multiple of $\frac{\lambda}{2}$

$$\Rightarrow \Delta x = 2n\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow (2\mu t)\cos(r+\theta) + \frac{\lambda}{2} = 2n\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow \boxed{2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}} \rightarrow \text{Condition of Maxima}$$

★ For Minima or Destructive : (Dark Band)

Path diff. = Odd Multiples of $\frac{\lambda}{2}$

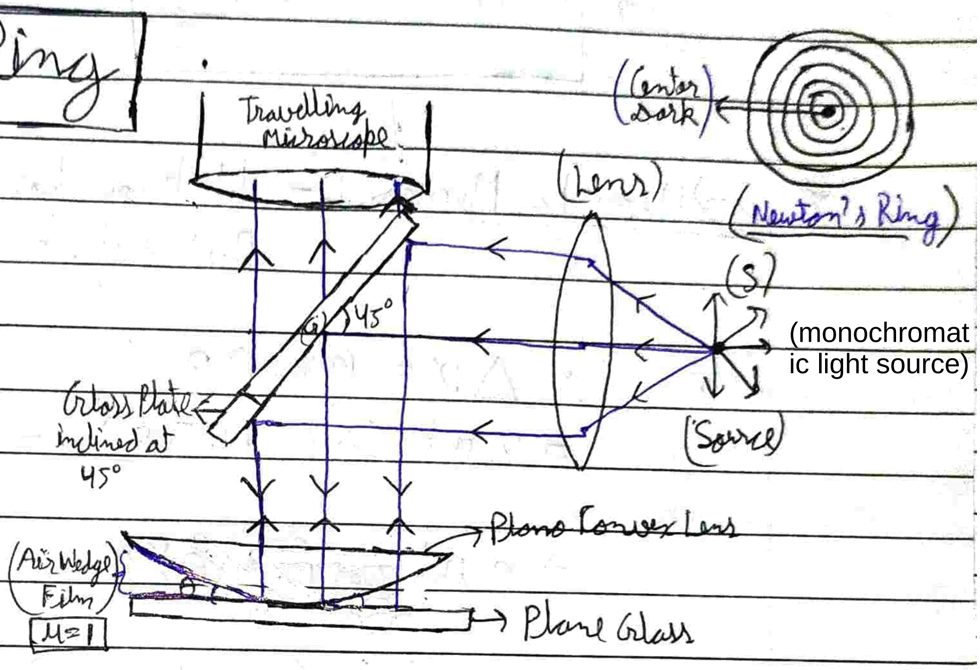
$$\Rightarrow \Delta x = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \Delta x = n\lambda + \frac{\lambda}{2}$$

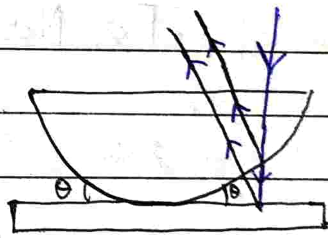
$$\Rightarrow 2\mu t \cos(r+\theta) + \frac{\lambda}{2} = n\lambda + \frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t \cos(r+\theta) = n\lambda} \rightarrow \text{Condition of Minima}$$

Newton's Ring



Interference in Newton's Ring Exp.



☆ We know that from wedge shape film interference path diff. is:-

$$\boxed{\Delta x = 2\mu t \cos(r+\theta) \pm \frac{\lambda}{2}}$$

☆ Condition for Maxima :- We know for constructive interference,
Path diff. = Even multiples of $\frac{\lambda}{2}$

$$\left[2\mu t \cos(r+\theta)\right] + \frac{\lambda}{2} = 2n\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow \boxed{2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2}}$$

$\because$ For Normal incident case i.e. ($\angle r = 0^\circ$)
& For very Small Wedge Angle ($\theta \simeq 0^\circ$)

then, $2\mu t \cos(0+\theta) = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow \boxed{2\mu t = (2n-1)\frac{\lambda}{2}}$$

★ <u>Condition for Minima</u> :- We know for destructive Interference :
Path diff. = Odd multiples of $\frac{\lambda}{2}$

$\therefore \Delta x = (2n+1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos(r+\theta) = 2n\left(\frac{\lambda}{2}\right) + \frac{\lambda}{2} - \frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos(r+\theta) = n\lambda$

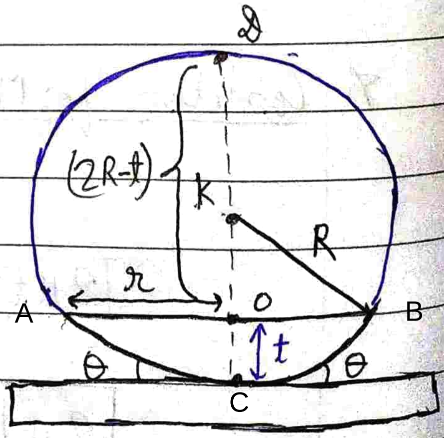
$\because$ For Normal incidence & very Small wedge angle ($\angle r = 0$ & $\theta \simeq 0$)

then, $2\mu t \cos(0+0) = n\lambda$

$$\Rightarrow \boxed{2\mu t = n\lambda}$$

Diameter Of Newtons Ring

★ From the property of Circle,



$[OD \times OC] = [AO \times OB]$

$\Rightarrow [(2R-t) \times t] = [(r) \times (r)]$

$\Rightarrow t = \frac{r^2}{2R-t}$

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$\because t = \frac{r^2}{2R-t}$ $\because [t <<< 2R]$

$\Rightarrow \boxed{t = \frac{r^2}{2R}}$ — (i)

☆ <u>diameter of Bright Rings :-</u>

$\because$ For Maxima :- $\boxed{2\mu t = (2n-1)\frac{\lambda}{2}}$

Now on Putting Value of 't' in above eq.^{n} we get,

$$2\mu\left(\frac{r^2}{2R}\right) = (2n-1)\frac{\lambda}{2}$$ $\because [\mu = 1] \to$ For Air

$$\Rightarrow r^2 = \frac{(2n-1)\lambda R}{2\mu}$$

[Let $r = \frac{D_n}{2}$ where D_n is the diameter of n^{th} Ring]

$$\Rightarrow \left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$

$$\Rightarrow (D_n)^2 = 2(2n-1)\lambda R$$

$$\Rightarrow \boxed{D_n = \sqrt{(2n-1)2\lambda R} \quad \& \quad D_n \propto \sqrt{(2n-1)}}$$

D_n (Bright) $\propto \sqrt{(2n-1)}$ (Odd)

→ From Here We can conclude that Diameter of bright Rings are in $(\sqrt{odd})$ multiples of $\sqrt{2\lambda R}$.

→ ~~For~~ $n = 1, 2, 3, \ldots$

☆ <u>Diameter of Dark Ring :—</u>

$\because$ For Minima :- $\boxed{2\mu t = n\lambda}$

Now on putting Value of 't' in above eq.^{n} we get,

$$2\mu\left(\frac{r^2}{2R}\right) = n\lambda$$

$$\Rightarrow r^2 = \frac{n\lambda R}{\mu} \quad [\because \mu = 1]$$

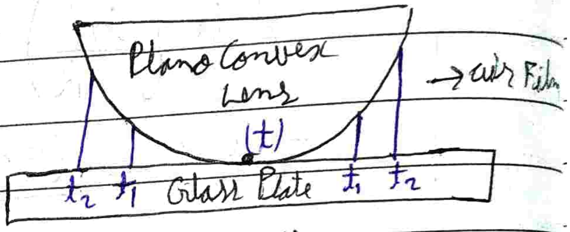
$$\Rightarrow \left(\frac{D_n}{2}\right)^2 = n\lambda R$$

$$\Rightarrow (D_n)^2 = 4n\lambda R$$

$$\boxed{\begin{aligned} \therefore\ D_n &= \sqrt{4n\lambda R} \\ \Rightarrow D_n &\propto \sqrt{2n} \end{aligned}}$$

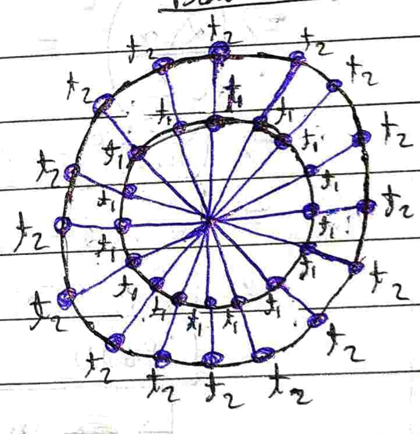
Why Newton's Rings Are Circular

At, $t_1 \rightarrow$ Bright Ring
At, $t_2 \rightarrow$ Dark Ring



☆ Locus of Constant thickness of air film lie on a circle.

Front View



☆ Here At (t_1) thickness we are getting maxima so the Locus of all Points at (t_1) will form a Circle of Bright Ring.

☆ At (t_2) thickness we are getting minima so the Locus of all Points at (t_2) will form a Circle of Dark Spots. So we will get a Dark Ring at t_2.