No. of Printed Pages : 5 **MCS-031**

MCA (Revised)

Term-End Examination 12240

December, 2011

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHM

Time : 3 hours *Maximum Marks : 100*

Note : *Question No. 1 is compulsory. Attempt any three from the rest of the questions.*

1. (a) Explain the relation/difference between a problem and its instance through an example of each. 2

   (b) State and describe any one of the following two problems : 2

   (i) The Four - Colour Problem

   (ii) The Fermat's Last Theorem

   (c) State any four characteristics of an algorithm, with an appropriate examples. 4

   (d) Let $\lfloor x \rfloor$ denote floor function of x and $\lceil x \rceil$ denote ceiling function of x. Find values of : 4

   (i) $\lfloor 3.4 \rfloor$ (ii) $\lfloor -4.6 \rfloor$

   (iii) $\lceil 2.7 \rceil$ (iv) $\lceil -9.8 \rceil$

(e) Using Insertion Sort or Bubble Sort (state before starting the solution, which algorithm for sorting, you are using), sort the following sequence of integers in decreasing order : 4

85 36 34 109 49 36

(f) Arrange the following growth rates in increasing order : O (4^n), O(n^4), O(1), O$(n^3 \log n)$, where 'O' denotes 'big oh'. 4

(g) Using Principle of Mathematical Induction, prove that $3^0 + 3^1 + .. + 3^n$ is equal to 4

$$\left(\frac{1}{2}\right).\left(3^{n+1} - 1\right) \text{ for all } n \geq 1.$$

(h) Explain how Binary Search Method finds or fails to find the given value 43 in the sorted array : 4

9, 13, 76, 27, 36, 49, 58, 79, 86.

(i) Write important properties of Depth - first search strategy for traversing a tree and cite an example of its use. 4

(j) Explain the essential idea of Dynamic Programming. How does Dynamic Programming differ from Divide and conquer approach for solving problems ? 4

(k) Available currency notes in India are : Rupee 1, 2, 5, 10, 20, 50, 100, 500 and 1000. Explain how to make Rupees 289 by using minimum number of currency notes. 4

2. (a) Discuss the three control mechanisms in an algorithm : 10

(i) Direct sequencing

(ii) Selection

(iii) Repetition

(b) Define the function $f(n) = a^n$ recursively, where a is a constant real number and $n \geq 0$ is an integer. 5

(c) Compare the following sorting algorithms on the basis of comparasion of keys and number of assignments : 5

(i) Selection sort

(ii) Insertion sort

3. (a) Multiply the following two matrices A and B using Strassen's algorithm : 8

$$A = \begin{bmatrix} 5 & 6 \\ -4 & 3 \end{bmatrix} ; B = \begin{bmatrix} -7 & 6 \\ 5 & 9 \end{bmatrix}$$

(b) By taking a suitable example of a tree with at least ten nodes, explain the three rules of traversal : 12

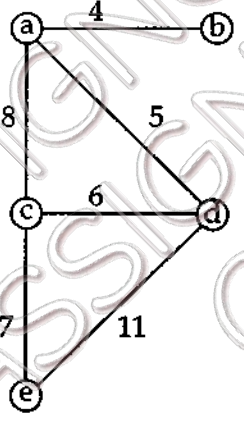
(i) Pre - order traversal

(ii) In - order traversal

(iii) Post - order traversal

4. (a) Let $C(n, k) = \binom{n}{k}$ denote the number of combinations of k things out of n given things. Let $C(i, o) = 1$ for all $i = 0, 1, 2, ..., n$ and $C(o, j) = 0$ for all $j = 1, 2, ..., k$. Explain, using Dynamic Programming, how to compute $C(n, k)$ for positive integers n and k. 12

(b) Using Prim's algorithm, find a minimal spanning tree for the graph, given as follows : 8



5. (a) Define the following, with at least one appropriate example : 6

(i) Directed Graph

(ii) Single - Source Shortest Path Problem

(b) Find a regular expression for each of the following languages : 6

(i) {a, b, ab, ba, abb, baa, ...}

(ii) {^, a, abb, abbbb, ...}

(c) Define the following : 8

(i) Halting Problem

(ii) Undecidable Problem

No. of Printed Pages : **4**

MCS-031

13465

MCA (Revised)

Term-End Examination

December, 2012

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : **100***

Note : *Question **No. 1** is **compulsory**. Attempt **any three** from the rest of questions.*

1. (a) (i) Prove that function $f(x) = 5x^4 + 7x + 3$ is $0(x^4)$. Is $f(x)$ also $0(x^4)$? Explain. **5+2=7**

(ii) Arrange the following growth rates in increasing order of time : **3**

$0(x^3)$, $0(2^x)$, $0(x^2)$, $0(\sqrt{x} \log x)$, $0(x \log x)$, $0(x^2 \log x)$

(b) (i) Differentiate between dynamic programming and greedy approach to solve different problems. **5**

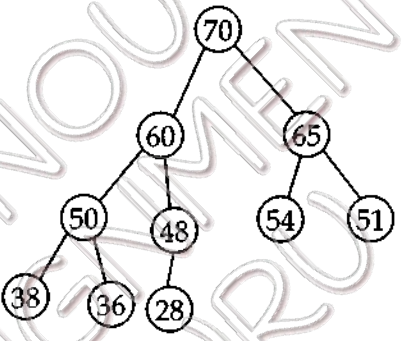
(ii) Write a recursive function to multiply two natural numbers. **5**

(c) (i) Define a Kleene Star (*) of a language L. Prove that if L is context free then L* is also context free. **7**

(ii) Explain the difference between Push Down Automata (PDA) and Finite Automata (FA). 3

(d) (i) Define 'Halting Problem" of Turing Machines. 5

(ii) Show stepwise sorting of elements using Heapsort algorithm to the following max heap. 5

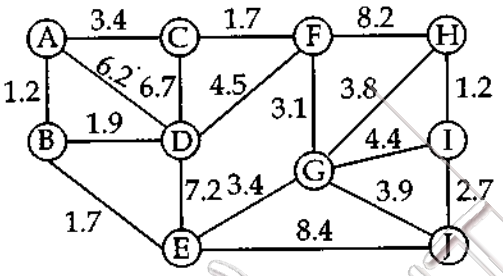


2. (a) (i) Define ambiguity in Context-Free Grammar (CFG). Show that the grammar : 4

$E \rightarrow E+E/E*E/a$ is ambiguous.

(ii) If L_1 and L_2 are two Context-Free languages, then show that $L_1.L_2$ is also Context - Free language. 3

(b) Use Prim's algorithm to construct a minimum spanning tree from the following graph (by using starting node A). 8



(c) Write a short note on NP-hard problems. 5

3. (a) Explain the complete steps of Strassen's algorithm for multiplying two $(n \times n)$ matrices. 10

Show that the Running time of Strassen's Algorithm is $0\ (n^{2.81})$.

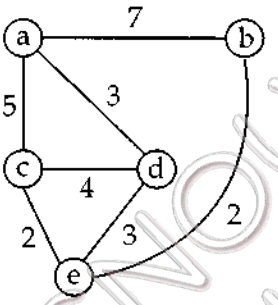
(b) Build regular grammar and corresponding finite automata (FA) for the following languages over the alphabet $\Sigma = \{a, b\}$. 10

(i) Language in which words do not end with ab.

(ii) Language having even number of a's.

4. (a) Describe 0-1 Knapsack problem. Which approach among greedy algorithm or dynamic programming is applicable for this problem ? 7

(b) Write a Dijkstra's algorithm for single source shortest path problem. Apply Dikstra's Algorithm for the following graph : 10



(c) What is the best case, average case and worst case running time of merge sort ? 3

5. (a) (i) Explain the purpose of randomization of quicksort. 4

(ii) What is the best case running time of quicksort ? In which situation does it occur ? 4

(b) Why do we perform amortized analysis of a problem ? 5

(c) Define θ notation and 0 notation. Explain, how these two notations are different. 7

No. of Printed Pages : 4 **MCS-031**

MCA (Revised)

Term-End Examination

December, 2013

12105

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question No. 1 is compulsory. Attempt any three from the remaining questions.*

1. (a) What is an algorithm ? Explain characteristics of an algorithm with the help of an example. **6**

(b) What is big O notation ? Find O ($f(x)$) for $f(x) = 3x^3 + 2x^2 + 4x$. **4**

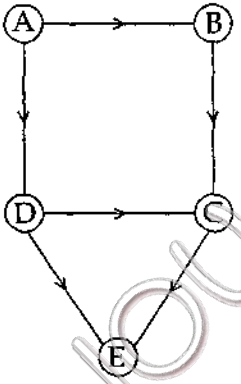
(c) What is dynamic programming ? How it is different from greedy technique of solving problems ? Also give a greedy solution for the change making problem, considering the denominations : **8**

{500, 200, 100, 50, 10, 5, 2, 1}

(d) Sort the following list using insertion sort. Show all intermediate stages while sorting : **6**

70, 40, 60, 80, 20, 6

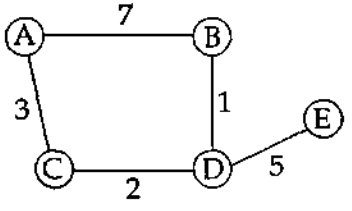
(e) Write algorithm of Depth-First Search and trace how Depth First-Search traverses the graph given below, when starting node is A : 6



(f) What is a heap ? Explain how a heap is built, using a simple example. 5

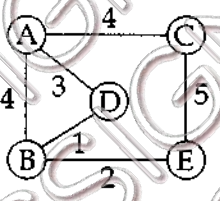
(g) List and explain any five properties of regular expressions. 5

2. (a) What is minimum spanning tree ? Write Prim's algorithm for finding minimum spanning tree and evaluate its time complexity. Also find minimum spanning tree of the following graph, using Prim's algorithm : 10



(b) What is binary search ? Explain its time complexity. Consider an array A = [2, 5, 9, 20, 25, 30, 60]. Find the average number of comparisons made by binary search for successful search in array A's and 'unsuccessful search in array A'. **10**

3. (a) What is Single - Source Shortest Path Problem (SSSPP) ? Explain Dijkstra's algorithm for SSSPP. Also find the minimum distances of all the nodes from node A, which is taken as the source node for the following graph : **10**



(b) Explain the meaning and the language described by each of the following expression : **6**

(i) $(a + b)^*$

(ii) $ab^*a^*(a + b)$

(iii) $ab\ (a + b)^*$

Where '*' is Kleene closure.

(c) Explain NP-hard problem with an example. **4**

4. (a) What is a Turing Machine ? Design a Turing Machine that recognizes language L of all strings over $\Sigma = \{a, b\}$ such that : $a^n b^n$, $n \geq 1$. 8

(b) What is topological sort ? Explain its application with an example. 6

(c) What is Quick Sort ? Explain/analyse the average case time complexity of Quick Sort. 6

5. (a) Draw the recursion tree for the following, also write the following in θ notations 4

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

(b) Prove that the Halting Problem is undecidable. 6

(c) Explain the following problems, together with their respective significance. 10

(i) Undecidable problem

(ii) NP-complete problem

No. of Printed Pages : 4

MCS-031

MCA (Revised)

11324 **Term-End Examination**

December, 2014

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is **compulsory**. Answer any **three** questions from the rest.*

1. (a) (i) Write an algorithm to build a heap from a given sequence.

(ii) Illustrate the heap sort algorithm on the sequence <10, 5, 12, 6, 9, 2, 8, 16>.

6+6

(b) (i) Solve the recurrence equation

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + O(n^2), & n > 1 \\ 1 & n \leq 1 \end{cases}$$

(ii) Prove that $f(n) = 2n^3 + 3n + 5$ is $O(n^3)$.

(iii) Solve the recurrence

$$T(n) = T\left(\frac{n}{2}\right) + 1 \quad \text{for } (n \geq 2)$$

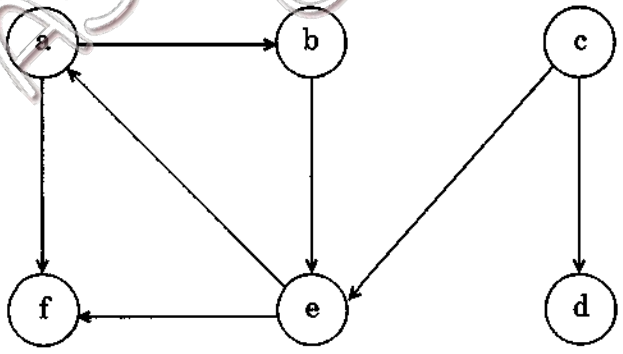
$$= 1 \qquad n < 2. \qquad 4+4+4$$

(c) (i) List the major differences between Divide and Conquer and dynamic programming design techniques for solving problems.

(ii) Define fractional Knap-Sack problem, and give a greedy algorithm to solve this problem efficiently. 5+5

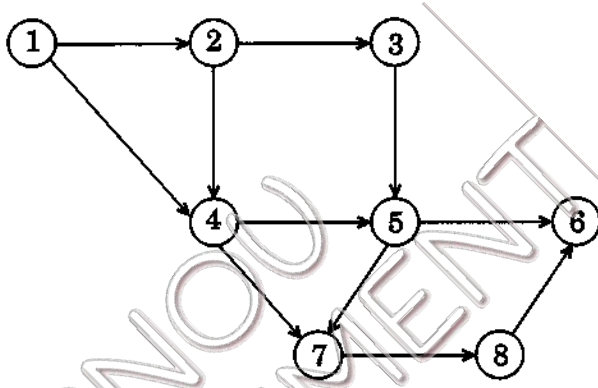
(d) Give a recursive function to find the height of a binary tree. What is the running time of this algorithm ? 6

2. (a) What is depth first search ? Give the DFS traversal for the following graph, starting with node 'a'. 6



(b) Identify the tree edges, back edges and forward edges. *6*

(c) Give an algorithm for topological sort. Obtain a topological ordering for the following graph : *8*



3. (a) Explain the Kruskal-algorithm for Minimum Spanning Tree construction. Derive the running time of the algorithm. *10*

(b) Show the MST corresponding to the following adjacency matrix representation of a graph : *10*

	a	b	c	d	e
a	–	1	15	–	5
b	1	–	2	–	10
c	15	2	–	8	6
d	–	–	8	–	3
e	5	10	6	3	–

4. (a) Define Regular Languages. Write regular expression corresponding to the following languages over alphabet {a, b}.

(i) Strings with even length.

(ii) Strings with odd number of a's and even number of b's. *8*

(b) Write context free grammar for the following languages.

(i) Even palindromes over {a, b}.

(ii) Odd palindromes over {a, b}. *6*

(c) If L_1 and L_2 are context free languages, prove that $L_1 \cup L_2$ is also context free. *6*

5. (a) Define the Class P, NP and NP-complete problems. *6*

(b) Show a polynomial time reduction from the Clique problem to the Vertex Cover problem by giving an example. *9*

(c) Give the formal definition of a Turing machine. *5*

09822

MCA (Revised)

Term-End Examination

June, 2011

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHM

Time : 3 hours *Maximum Marks : 100*

Note : ***Question No. 1*** *is* ***compulsory.*** *Attempt* ***any three*** *from the rest of the questions.*

1. (a) Arrange the following growth rates in increasing order : $O(3^n)$, $O(n^2)$, $O(1)$, $O(n \log n)$ **4**

   (b) Briefly discuss three basic actions and instructions that build a program in Von Newmann architecture machine. **4**

   (c) Write a recursive algorithm that finds the sum of first n natural numbers. **4**

   (d) Explain briefly The Fermat's Last Theorem. **4**

   (e) Using Principle of Mathematical Induction, Prove that the sum $2^0 + 2^1 + \ldots + 2^n$ is $2^{n+1} - 1$ for all $n \geq 1$. **4**

   (f) Using Insertion Sort or Bubble Sort, sort the following sequence in increasing order : 35, 37, 18, 15, 40, 12 **4**

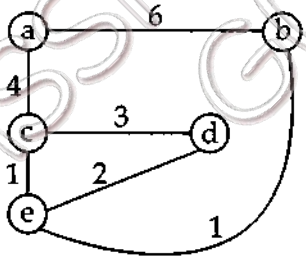
(g) Define Knapsack Problem and cite one instance of the problem. 4

(h) Consider a (hypothetical) country in which only notes available are of denominations 10, 40 and 60. Using Greedy algorithm, how do we collect an amount of 80. 4

(i) Briefly explain Kruskal's OR Prim's algorithm for finding minimal spanning tree of a graph. 4

(j) Name four undecidable problems, each with brief description. 4

2. (a) Using Dijkstra's algorithm, find the minimum distances of all the nodes from node 'b' which is taken as the source node, for the following graph. 10

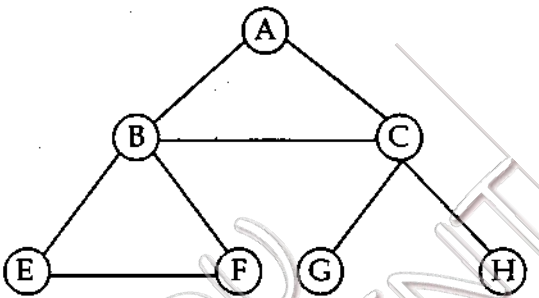


(b) Find a regular expression for the language 5

{∧, a, a b b, a b b b, a b b b b b b, ...}

(c) Briefly discuss Chomsky classification for Grammars. 5

3. (a) Trace how BFS (Breadth - First Search) traverses, i.e, discovers and visits the graph given below when starting at node A. 8



(b) Write pseudo-code for Depth-First search. 5

(c) Find the value of $(12)^{31}$ using not more than SIX multiplications and/or divisions. 7

4. (a) Write a program that computes the length of the diagonal of a right - angled triangle, the length of the two sides of which are given. 6

(b) For the function $f(x) = 4x^3 + 6x + 1$ show that (i) $f(x) = O(x^4)$ but (ii) $x^4 \neq O(f(x))$ 6

(c) Sort the following sequence of numbers using Quick Sort : 8, 6, 4, 12, 11, 5, 7 and 9. 8

5. (a) Design a Turing Machine that recognises the languages of all strings of even lengths over the alphabet {c, d}. **10**

(b) For each of the following pairs of lists, discuss whether PCP (Post Correspondence Problem) has a solution : **10**

(i) List A = (b, b a b b b, b a)

and List B = (b b b, b a, a)

(ii) List C = (a b, b, b) and

D = (a b b, b a, b b)

No. of Printed Pages : 4

MCS-031

08277

MCA (Revised)

Term-End Examination

June, 2012

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHM

Time : 3 hours *Maximum Marks : 100*

Note : *Question No. 1 is compulsory. Attempt any three from the remaining questions.*

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1. (a) Write Euclid's algorithm for finding Greatest Common Divisor (G.C.D) of two natural numbers m and n. **4**

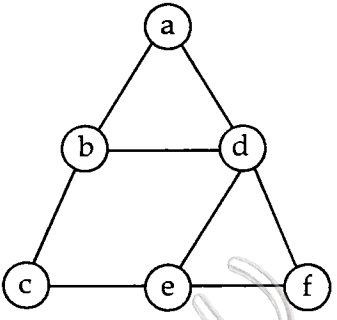
(b) Let fact (n) = 1 * 2 * 3.....* n where '*' denotes product of two integers and n is a natural number. Give a recursive definition of fact (n). **4**

(c) Name at least four well-known techniques for solving problems algorithmically. **2**

(d) For the function $f(x) = 3x^3 + 2x^2 + 1$, show that $f(x) = O(x^3)$ where 'O' denotes 'big oh'. **2**

(e) Explain how binary search method finds or fails to find the number 25 in the sorted list : 8, 12, 26, 35, 48, 57. **4**

(f) Explain how the nodes of the following the graph will be traced using breadth first search, starting at node 'a': 4



(g) Explain the method of pre-order traversal of a tree, using a suitable example of a tree, with at least eight nodes. 4

(h) Using Principle of Mathematical Induction, show that sum of first n natural numbers is $n(n+1)/2$. 4

(i) Explain the essential idea of Dynamic Programming. How does Dynamic Programming differ from Divide and Conquer approach for solving problems ? 4

(j) Using selection sort, sort the following sequence of numbers, in the increasing order : 4

15, 12, 18, 24, 13, 27,

(k) Name four undecidable problems, along with their special significance. 4

2. (a) Write an algorithm that finds the real roots, if any, of a quadratic equation : 5

$$ax^2 + bx + c = 0,$$

Where $a \neq 0$, b and c are real numbers.

(b) Explain Chomsky classification for grammers, with an appropriate example. 5

(c) Write a recursive algorithm to find the sum of first n natural numbers. 5

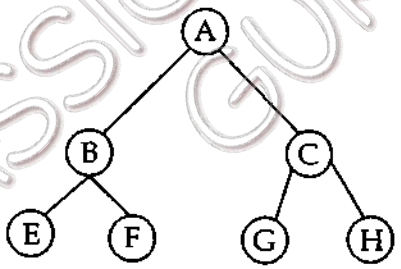
(d) Sort the following list using Merge Sort : 5

7, 9, 10, 8, 4, 6, 5.

3. (a) Find solution of the recurrence equation, given as follows : 5

$$F(n) - 4F(n-1) + 4F(n-2) = 0$$

(b) Trace how Depth First Search Traverses the following tree, when starting at node B : 7



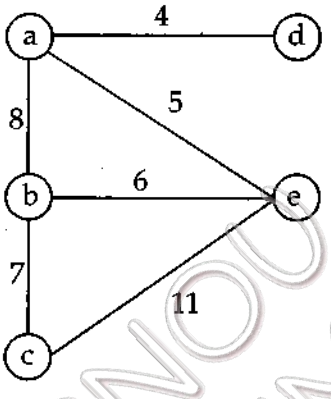
(c) Explain each of the following, with an appropriate example : 8

(i) Minimax Problem

(ii) Topological Sort

4. (a) Find the value of $(12)^{31}$, using not more than SIX (6) multiplications and/or divisions. **6**

(b) Using either Prim's algorithm or Kruskal's algorithm, find a minimal spanning tree for the graph given as follows : **8**



(c) Explain the meaning of each of the following expression : **6**

(i) $(a+b)^*$

(ii) $a^*\ b\ a^*\ b\ a^*$

(iii) $(a+b)^*\ a\ b$

Where '*' denotes Kleene Closure.

5. (a) Find a grammer for the following language : **8**

$\{ a^m b^n;\ m, n \in N,\ n > m \}$

(b) Explain each of the following problems, together with their respective significance. **12**

(i) Halting Problem

(ii) Post Correspondence Problem

(iii) Undecidable Problem

No. of Printed Pages : 3

MCS-031

10829

MCA (Revised)

Term-End Examination

June, 2014

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question **No. 1** is **compulsory**. Attempt **any three** from the remaining questions.*

1. (a) What is big O notation ? How is it different from Ω notation ? 4

   (b) Give an analysis of merge-sort. For simplicity, assume that the number of elements i.e. n is an exact power of two. 6

   (c) Explain limitations of Strassen's Algorithm for matrix multiplication. 5

   (d) Use Master's method to find tight asymptotic bounds for the following recurrence : 5

   $$T(n) = T\left(\frac{n}{2}\right) + n$$

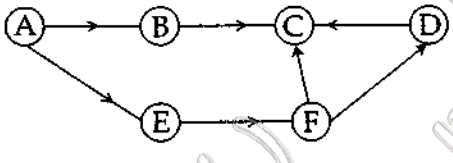
   (e) Give a divide and conquer based algorithm to find i^{th} largest element in an array of size n. 4

   (f) What is regular expression ? Write a regular expression over $\sum = \{a, b\}$ to generate all string that start with a and end with two b's. 6

(g) Write binary search algorithm and evaluate its time complexity in the best, average and worst cases. 6

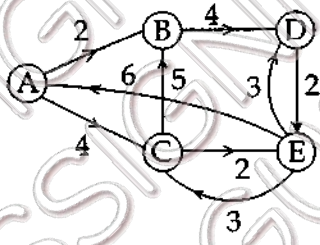
(h) Explain NP- complete problem with the help of an example. 4

2. (a) Find the topological ordering of the following graph : 6



(b) Write Kruskal's algorithm and determine its time complexity. 8

(c) Represent the following graph using 6
(i) Array ; and (ii) Adjancy List



3. (a) Sort the given list using bubble sort and show the steps involved in the process : 6
2, 7, 5, 10, 21, 3

(b) Write Euclid's algorithm for finding Greatest Common Divisor (G.C.D) of two natural numbers m and n. 4

(c) What is the benefit of preconditioning a problem space ? Explain using an example. 5

(d) Consider the CFG : 5

$S \rightarrow SS \mid X\ a\ X\ a\ X \mid \wedge$

$X \rightarrow bX \mid \wedge$

Explain the language generated by this CFG

4. (a) What is Push Down Automata ? How is it different from Finite Automata. 5

(b) What is MinMax Algorithm ? Explain how Alpha-Beta pruning helps in improving MinMax algorithm. 10

(c) What is best case analysis ? Perform the best case analysis for Quick Sort. 5

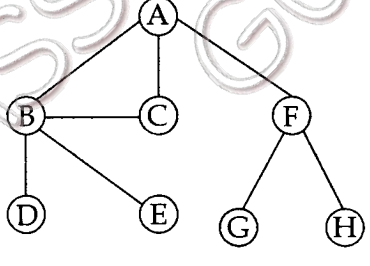
5. (a) Explain each of the following, with an appropriate example : 12

(i) NIM/Marienbad Game

(ii) Principle of Mathematical Induction

(iii) Halting Problem

(b) Trace how Depth First Search Traverses the following tree, when starting at node A. 8



No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

December, 2018

04363

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from the rest.*

1. (a) Differentiate between P and NP class of problems with example of each. *5*

(b) Write an algorithm that finds the real roots, if any, of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, b and c are real numbers. *5*

(c) By using Principle of Mathematical Induction, show that $n^3 - n$, is divisible by 6, where n is a non-negative integer. *5*

(d) Sort the following sequence of numbers using Bubble sort :

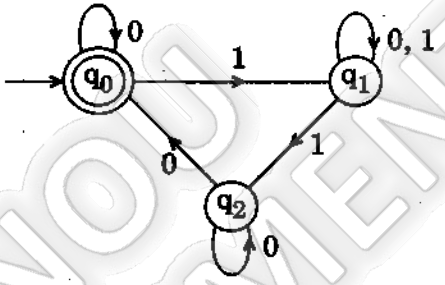
15, 10, 13, 9, 12, 17.

Find the total number of comparisons required by the algorithm in sorting the list. *6*

(e) Explain the "Principle of Optimality" in dynamic programming with suitable example. 6

(f) Compute x^{29} by using divide and conquer technique. 5

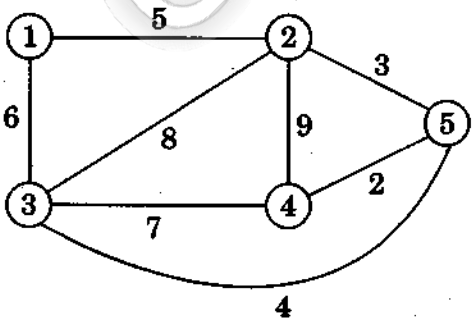
(g) Define Regular Expression. Find the Regular Expression for the following Finite Automata : 8



2. (a) Apply Dynamic programming to multiply the following chain of matrices :

M_1, M_2, M_3 and M_4 with respective dimensions (5×10), (10×3), (3×7), (7×15). 10

(b) Differentiate between Kruskal's and Prim's algorithms. Apply Prim's algorithm to find the minimum spanning tree for the following graph : 10



3. (a) Write Euclid's algorithm to find the GCD of two natural numbers m and n. 5

(b) Write Merge Sort Algorithm. Apply the same to sort the array of elements

15, 10, 5, 9, 7, 20, 25, 18, 16. 10

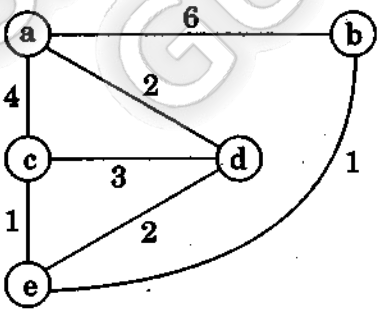
(c) Show that the context-free grammar $S \rightarrow S + S \mid S * S \mid a$ is ambiguous. 5

4. (a) Define Θ-Notation. Show that

(i) $3x^2 + 2x + 1 = \Theta(x^2)$

(ii) $2x^3 + x + 5 \neq \Theta(x^4)$ 10

(b) Write Dijkstra's Algorithm. Using Dijkstra's Algorithm, find the minimum distances of all the nodes from starting node a. 10



5. (a) Write short notes on any ***three*** of the following : *3×5=15*

(i) Kleene Closure

(ii) Push-down Automata (PDA)

(iii) Chomsky's Classification of Grammar

(iv) Amortize Analysis

(b) Find context-free grammar for the following : *5*

(i) $L = \{a^m b^n \mid m, n \in N,\ n > m\}$

(ii) $L = \{a^m b\, c^n \mid n \in N\}$

No. of Printed Pages : **4**

MCS-031

11604

MCA (Revised)

Term-End Examination

June, 2013

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question **No. 1** is **compulsory**. Attempt **any three** from the rest of questions.*

1. (a) (i) Write an algorithm to sort the given array of numbers using Insertion Sort and explain the necessary steps. Also write the time complexity in the worst case. 6

(ii) Prove that function : 6

$f(x) = 2x^3 + x^2/4 + 100$ is $\theta(x^3)$.

(b) (i) Solve the following recurrence relation : 5

$$f_n - f_{n-1} - f_{n-2} = 0 \text{ s.t. } f_0 = 0, \; f_1 = 1$$

(ii) Prove that the running time of binary search algorithm in worst case is $0(\log_2 n)$. 5

(c) (i) Define undecidability. List any three undecidable problem. 5

(ii) Discuss, why do we use dynamic programming approach to solve matrix chain multiplication problem. 5

(d) (i) There are five bottles of medicine, namely A, B, C, D and E. The capacity of each bottle in term of number of tablets it can hold is given below : 5

Bottle A : 10 tablets, Bottle B : 60 tables,

Bottle C : 130 tablets,

Bottle D : 240 tablets, Bottle E : 100 tablets.

Give a greedy approach to store 560 tablets by using minimum number of bottles.

(ii) Analyse the time complexity for the following : 3

```
for (i=1 ; i≤n ; i=i*2)
for (j=1 ; j≤n ; j++)
Count=count+2 ;
```

2. (a) If L_1 and L_2 are regular languages. Prove that $L_1 \cup L_2$ is also regular. 5

(b) Write a recursive function to calculate the sum of all elements in an integer array. 5

(c) Define a Context-Free Grammar (CFG). Write a CFG for the following language. 10

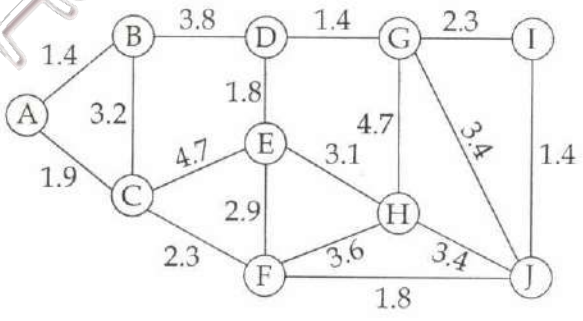
(i) $L = \{a^m b^n : m > n, m, n \geq 0\}$

(ii) $L = \{\omega\omega^R : \omega \in \{a, b\}^*\}$ where ω^R is a reverse of ω.

3. (a) What is "principle of optimality" in Dynamic Programming ? Explain how dynamic programming can be used to solve a chain of matrix multiplication ? Apply Dynamic Programming to multiply the following (4) matrices. $<M_1, M_2, M_3, M_4>$ with dimensions $<(15, 3), (3, 8), (8, 9), (9, 7)>$. 10

(b) Write an algorithm for Quick Sort. Analyse the Running time of Quick Sort in Best and worst case. 10

4. (a) Use Kruskal's algorithm to construct a minimum spanning tree from the following graph. 5



(b) Define O/I Knapsack Problem. Using Dynamic programming. Solve the following O/I Knapsack problem. 10

No. of objects n = 5,

Capacity of Knapsack, M = 16

Weight	2	3	6	5	9
Profit	2	7	15	20	12

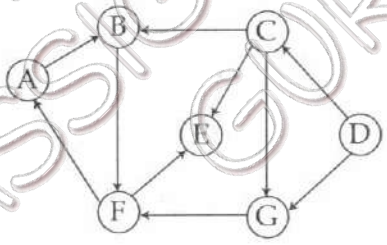
(c) Write an algorithm for Topological Sort. 5

5. (a) Sort the following elements using Heap Sort : 7

47, 29, 82, 11, 48, 32, 28, 17, 65, 36

Show each step, while creating a heap and processing a heap.

(b) Use breadth first search to traverse the following graph by using C as the starting node. 5



(c) Write an algorithm to multiply given two n-bit decimal number $X = x_{n-1}\, x_{n-2} \ldots x_1\, x_0$ and $Y = y_{n-1}\, y_{n-2} \ldots y_0$, using Divide and Conguer method. Also analyse the time complexity of the algorithm used ? 8

No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

00995

June, 2018

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

1. (a) Multiply the following n-digit decimal numbers, x and y, using the Karatsuba technique, where

$x = x_{n-1}, x_{n-2}, \ldots, x_0$

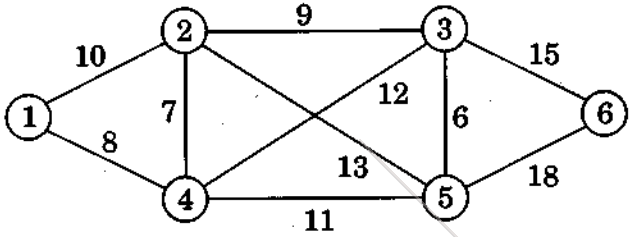
$y = y_{n-1}, y_{n-2}, \ldots, y_0$. 7

(b) Sort the following sequence in ascending order using Insertion sort : 5

{28, 13, 12, 25, 38, 11, 15, 9, 36}

(c) Differentiate between Asymptotic notations, O (Big "oh"), Ω (Big "omega") and Θ (Theta) notations. 6

(d) Define Minimum Cost Spanning Tree (MCST). Find the MCST using the Prim's algorithm for the following graph : 6



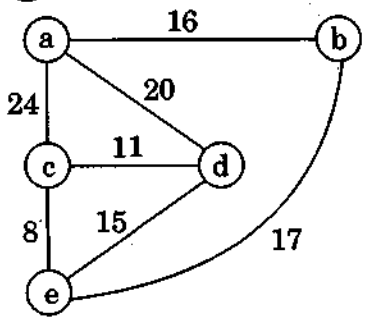
(e) Construct a Deterministic Finite Automata (DFA) for the following Regular Expression : 5

$$(0 + 1)^* (00 + 11) (0 + 1)^*$$

(f) Write the Recursive and Iterative algorithm to compute the Greatest Common Divisor (GCD) of two numbers X and Y. 5

(g) Explain the V. Strassen's matrix multiplication method of multiplying two matrices of size $(n \times n)$. Show that its running time is $O(n \log_2 7)$. 6

2. (a) Write the Dijkstra's algorithm for shortest path. Apply the same for the following graph, to find the shortest path from node (a) : 8



(b) Explain the Chomsky's classification of grammars. 6

(c) Differentiate between the Greedy and Dynamic programming approaches to solve a problem. List 3 problems which use these approaches, respectively. 6

3. (a) Write the Context-Free Grammar (CFG) for the following language : 6

(i) $L = \{a^n b^m c^m d^n \mid m, n \geq 1\}$

(ii) $L = \{\omega d \omega^R \mid \omega \in \{a, b\}^*$ and ω^R is the reverse of $\omega\}$.

(b) Explain the 0/1 Knapsack problem. Solve the following 0/1 Knapsack problem :

Given number of objects n = 6

Capacity of Knapsack (M) = 12

$(p_1, p_2, ..., p_6) = (1, 6, 18, 22, 28, 43)$

and $(w_1, w_2, ..., w_6) = (1, 2, 5, 6, 7, 10)$.

Where p_i's and w_i's are the profits and weights of the corresponding objects. 7

(c) Apply the dynamic programming method to solve the following chain-matrix-multiplication : 7

(M_1, M_2, M_3, M_4) with dimensions $(15 \times 6, 6 \times 50, 50 \times 9, 9 \times 12)$

4. (a) Solve the following recurrence using the Master method : 6

(i) $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

(ii) $T(n) = 2T\left(\frac{n}{2}\right) + n$

(b) Find the best case and worst case time complexity of Quick Sort by writing their recurrence relation. 7

(c) Define Turing Machine (TM). Design TM for the language

$L = \{a^n b^n \mid n \geq 1\}$. 7

5. (a) Differentiate between P, NP, NP-Complete and NP-Hard problem. 8

(b) Define Push-Down Automata (PDA). Design a PDA that accepts the language EVEN PALINDROME over $\Sigma = \{a, b\}$. 6

(c) Write short notes on the following : 6

(i) Halting Problem of TM

(ii) Post Correspondence Problem (PCP)

08754

No. of Printed Pages : 3

MCS-031

MCA (Revised)

Term-End Examination

December, 2015

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

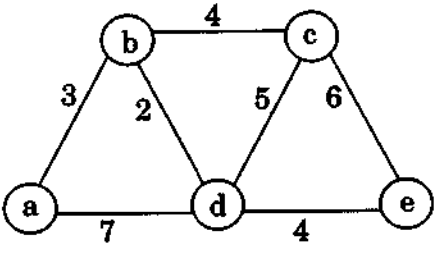
Note : *Question no. **1** is **compulsory**. Attempt any **three** questions from the rest.*

1. (a) Write recursive binary search algorithm and analyse its run time complexity. 7

(b) Solve the recurrence :

$$T(n) = 2T(n/2) + n; \quad n \geq 2$$
$$= 1 \; ; \; n < 2.$$ 4

(c) Using Dijkstra's algorithm, find the minimum distances of all the nodes from source node 'a' for the following graph : 7

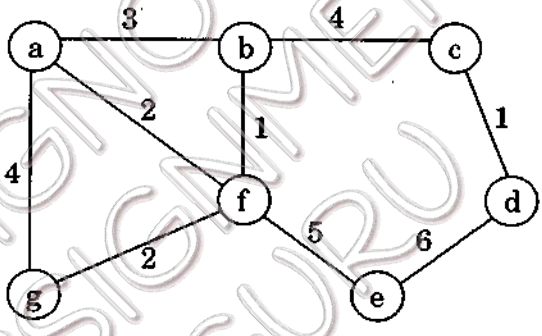


(d) Construct a Turing Machine (TM) to accept all languages of palindromes on alphabet $\Sigma = (a, b)$. 6

(e) Explain matrix multiplication using dynamic programming. 10

(f) What is minimax principle ? Explain with the help of an example. 6

2. (a) Obtain the minimum cost spanning tree for the following graph using Prim's algorithm. 5



(b) Obtain the DFS tree for the graph given in Q.no. 2(a); considering node (a) as root node. 5

(c) Explain the Chomsky's classification of grammars. 10

3. (a) Enumerate any five well-known techniques for designing algorithms for solving problems. 5

(b) Sort the following elements using Heap Sort :

10, 28, 46, 39, 15, 12, 18, 9, 56, 2.

Show each step, while creating a heap and processing a heap. *8*

(c) For any set S of strings prove that $S^* = (S^*)^* = S^{**}$. *7*

4. (a) Arrange the following growth rates in increasing order : *5*

$O(n^3)$, $O(3^n)$, $O(n \log n)$, $O(1)$, $O(\log n)$.

(b) For the function

$f(x) = 4x^3 + 6x + 5$,

show that (i) $f(x) = O(x^4)$

but (ii) $x^4 \neq O(f(x))$. *5*

(c) What is Pushdown Automata (PDA) ? Build a PDA that accepts the language even palindrome. *10*

5. (a) What is Satisfiability problem ? Explain briefly. *5*

(b) Prove that the running time of binary search algorithm in worst case is $O(\log_2 n)$. *5*

(c) Using Bubble Sort, sort the following sequence in increasing order : *5*

11, 21, 6, 14, 8, 12, 28, 32.

(d) Write a note on regular languages. *5*

No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

03935

December, 2016

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is **compulsory**. Attempt any **three** from the remaining questions.*

1. (a) Use Mathematical Induction to prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. 5

(b) For a problem P, two algorithms A_1 and A_2 have time complexities $T_1(n) = 5n^2$ and $T_2(n) = 100\ n \log n$. Find the range for n, the size of instance of the given problem P, for which A_1 is more efficient than A_2. 5

(c) Define the big theta notation. Show that $n^2 + 3 \log n = \theta(n^2)$. 5

(d) (i) Explain the bottom-up build heap procedure.

(ii) Illustrate heapsort algorithm on the sequence <10, 5, 12, 25, 2, 8, 13, 7>. *10*

(e) Solve the following recurrence equations : *10*

(i) $T(n) = 2T(n/2) + O(n)$

(ii) $T(n) = T(n - 1) + O(n)$

(f) Write a Regular expression to generate strings of even length over the alphabet $\Sigma = \{a, b\}$. *5*

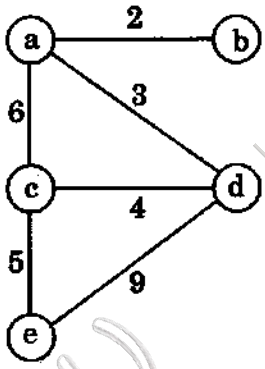
2. (a) Give a divide and conquer algorithm to find the i^{th} smallest in an unsorted list of n integers. Show that the algorithm works in O(n) time. *10*

(b) Write a recursive function to calculate the sum of all elements in an integer array. *5*

(c) Explain any two applications of DFS traversal algorithm. *5*

3. (a) Given the currency coins of denomination 1, 4 and 6. Design a dynamic programming algorithm to obtain minimum number of coins for a given amount. *10*

(b) Using Prim's algorithm, find a Minimal Spanning tree for the graph given below : 10

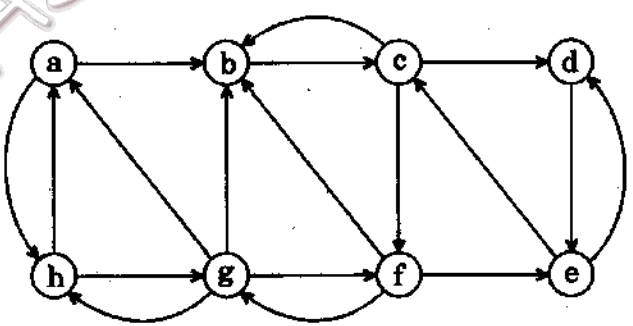


4. (a) (i) Write a context-free grammar to generate all palindromes of even length over the alphabet $\Sigma = \{a, b\}$. 5

(ii) Derive the parse tree and derivation for the string aabbaa. 5

(b) (i) Explain the algorithm to find the Strongly Connected Component in an undirected graph. 5

(ii) Find the Strongly Connected Components in the following graph : 5



5. (a) Explain the following : *10*

(i) Undecidable problems

(ii) Turing machines

(b) Define the Class P, NP and NP-complete problems. *5*

(c) Write a Turing machine to recognize the language of all strings of even length over the alphabet {a, b}. *5*

No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

June, 2016

08022

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest. Parts of the same question may be attempted together.*

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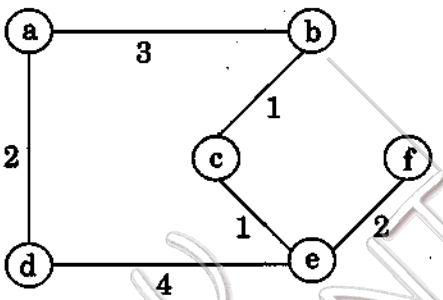
1. (a) Explain five characteristics of an algorithm briefly. *5*

(b) Write and explain recursive algorithm to find the factorial of any given number $n \geq 0$. *5*

(c) Explain the importance of asymptotic analysis for running time of an algorithm with the help of an example. *5*

(d) Briefly describe Chomsky classification for Grammars. *5*

(e) Using Dijkstra's algorithm, find the minimum distances of all the nodes from node 'a' which is taken as the source node, for the following graph : 10



(f) "The best-case analysis is not as important as the worst-case analysis of an algorithm." Yes or No ? Justify your answer with the help of an example. 10

2. (a) Explain how greedy approach is useful to find the solution to fractional knapsack problem. 7

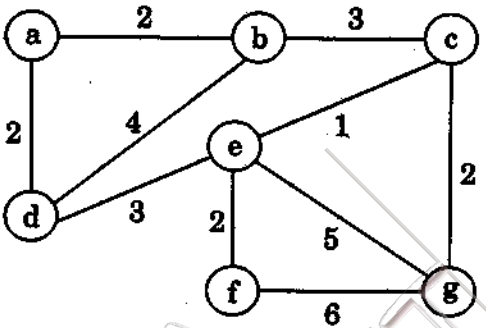
(b) Solve the following recurrence relation : 5

$$f_n - f_{n-1} - f_{n-2} = 0$$

such that $f_0 = 0$ and $f_1 = 1$.

(c) Explain Turing Machine (TM) as a computer of functions, with the help of an example. 8

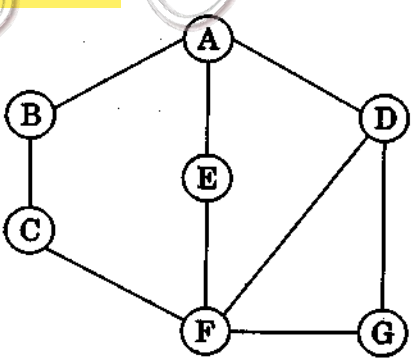
3. (a) Using Prim's algorithm, find a minimal spanning tree for the graph given below : 10



(b) Sort the following sequence of numbers, using Selection Sort. Also find the number of comparisons and copy operations required by the algorithm in sorting this list : 10

20 5 15 8 6 28

4. (a) Using Depth First Search (DFS) traverse the following graph by using A as the starting node : 5



(b) Define Ω notation used for comparing two functions.

For $f(x) = 2x^3 + 3x^2 + 1$

$h(x) = 2x^3 - 3x^2 + 2$

show that

(i) $f(x) = \Omega(x^3)$

(ii) $x^2 \neq \Omega(h(x))$ 5

(c) What is dynamic programming ? Explain briefly the optimal substructure property of a dynamic programming problem. 5

(d) What is NP-complete problem ? Is it necessary that every NP-complete problem must also be a NP-hard problem ? Justify. 5

5. (a) Write an algorithm for Heap Sort and analyse its Best and Worst run time complexity. 10

(b) Define a Turing Machine. 5

(c) Consider the CFG :

$S \rightarrow SS/XaXaX/\wedge$

$X \rightarrow bX/\wedge$

Find the language generated by this CFG. 5

No. of Printed Pages : 3

MCS-031

MCA (Revised)

Term-End Examination

June, 2017

03512

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is **compulsory**. Attempt any **three** from the remaining questions.*

1. (a) Use mathematical induction to prove the following expression : 5

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 1$$

(b) Define Big-O and Big Omega notation, and prove that

$$f(n) = 3 \log n + \log \log n = O(\log n).$$ 6

(c) Write a regular expression to generate strings of odd lengths over the alphabet $\Sigma = \{a, b\}$. 5

(d) Solve the following recurrence equations : 9

(i) $T(n) = 2T(n/2) + n$

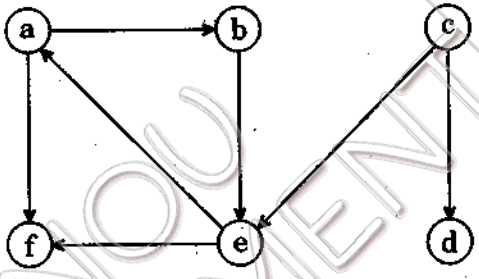
(ii) $T(n) = T(n/2) + 1$

(iii) $T(n) = T(n/2) + n$

(e) Write an algorithm for Merge Sort. Analyze its time complexity. 10

(f) What is the essence of Greedy technique ? Give an example. 5

2. (a) Obtain the DFS traversal for the following graph :



Identify the tree edges, back edges and cross edges. 8

(b) Explain any three applications of DFS traversal. 6

(c) Explain Kruskal's algorithm to compute the minimum cost spanning tree. 6

3. (a) Explain how dynamic programming can be used to solve matrix chain multiplication. Apply the algorithm to multiply the following : 10

3 matrices, $< M_1, M_2, M_3 >$ with

dimensions $< (15, 3), (3, 10), (10, 2) >$

(b) Give a divide and conquer based algorithm to find the i^{th} smallest element in an array of size n. Trace your algorithm to find 3^{rd} smallest in the array

A = {10, 2, 5, 15, 50, 6, 20}. 10

4. (a) Define Regular Languages. Write regular expressions for the following : 9

(i) Strings of even length over the alphabet $\Sigma = \{a, b\}$.

(ii) Strings with odd number of a's and even number of b's over the alphabet $\Sigma = \{a, b\}$.

(b) Explain Chomsky's classification for grammars. 6

(c) Show that the following CFG is ambiguous : 5

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow a/b$

5. (a) Define a Turing machine. 5

(b) If L_1 and L_2 are context-free languages, then prove that $L_1 \cup L_2$ is also context-free. 5

(c) Explain the term 'Polynomial time reduction'. Explain how the clique problem can be transformed to the vertex cover problem. 10

No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

December, 2017

04290

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from the rest.*

---

1. (a) Show that the partition problem is NP. *5*

(b) For the functions $f(x) = 2x^3 + 3x^2 + 1$ and $h(x) = 2x^3 - 3x^2 + 2$, prove that : *2×5=10*

(i) $f(x) = \theta(h(x))$

(ii) $f(x) \neq O(x^2)$

(c) Show that the state entry problem is undecidable. *5*

(d) If S = {a, aa, aaa}, T = {bb, bbb}, then prove that ST = {abb, abbb, aabb, aabbb, aaabb, aaabbb}. *5*

(e) Differentiate between NP-Complete and NP-Hard problems. Give one example for each. *5*

(f) Analyze the time complexity of binary search in worst case. 5

(g) Construct a Deterministic Finite Automata (DFA) over $\Sigma = \{a, b\}$, which accepts all strings over Σ that start and end with the same letter. 5

2. (a) Explain Strassen's Matrix Multiplication Algorithm and apply the same to multiply the following two matrices : 10

$$A = \begin{bmatrix} 5 & 6 \\ -4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -7 & 6 \\ 5 & 9 \end{bmatrix}$$

(b) Solve the following 0/1 knapsack problem using dynamic programming : 10

Number of objects n = 6

Weights of objects $w_i = (1,2,5,6,8,10)$

Profit of objects $p_i = (1,6,18,22,30,43)$

Capacity of knapsack, M = 12

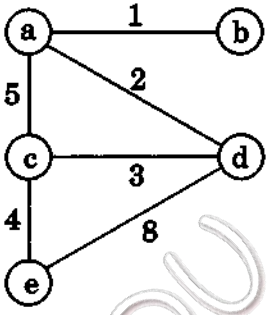
3. (a) Multiply the following two numbers using Karatsuba's method : 5

1026732 and 732912

(b) Define Context-Free Grammar (CFG). Find CFG for the language

$L = \{a^n b^m c^n \mid m, n \geq 1\}$. 5

(c) Write Kruskal's Algorithm and apply the same to find a minimum spanning tree for the following graph : 5



(d) Solve the following recurrence relation : 5

$t_n = 2t_{n-1} + 1$, such that $t_0 = 1$.

4. (a) Write short notes on any *three* of the following : 3×5=15

(i) Chomsky's Classification of Grammar

(ii) Push-Down Automata (PDA)

(iii) Depth-First Search (DFS)

(iv) Asymptotic Notation (O, Ω and θ)

(b) Design a Push Down Automata that accepts the language of odd palindromes. 5

5. (a) Design a Turing Machine that accepts the following language : 8

$L = \{a^n b^n \mid n \geq 1\}$

(b) **Write an algorithm for Quick Sort. Sort the following sequence of numbers using Quick Sort :**

15, 10, 13, 9, 12, 7

Analyze the time complexity of Quick Sort in best and worst cases. *6+6=12*

No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

10423 **June, 2015**

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number **1** is **compulsory**. Attempt any **three** questions from the rest.*

1. (a) (i) Solve the recurrence equation

$T(n) = 2.T(n/4) + n^3$ for $n > 1$ and $T(1) = 1$.

(ii) Obtain the asymptotic upper bound for $f(n) = (6n^2 - 5n + 2)^2$. *4+4*

(b) A binomial coefficient is defined by the following recurrence relation :

$C(n, 0) = 1$ and $C(n, n) = 1$ for $n > 0$.

$C(n, k) = C(n - 1, k) + C(n - 1, k - 1)$ for $n > k > 0$.

(i) Draw the recursion tree for C(6, 4). *4*

(ii) Write a recursive function to generate C(n, k). *4*

(iii) Give an algorithm based on Dynamic Programming to solve C(n, k). *4*

(iv) Compare the time and space requirements of the algorithm in part (iii). *4*

(c) (i) You are given currency of denominations {500, 100, 50, 20, 10, 5}. Give a greedy algorithm to obtain the minimum number of denomination for any amount which is a multiple of 5. *6*

(ii) Write a procedure to merge two sorted arrays. Analyze the running time of your algorithm. *6*

(d) Is the following sequence a heap ? If not, convert it into a heap. *4*

< 10, 5, 3, 8, 6, 1, 7 >

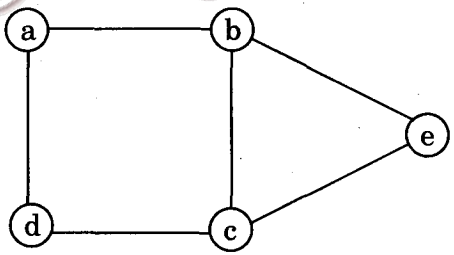
2. (a) (i) Write an algorithm to find the i^{th} smallest element in O(n) time. *6*

(ii) Illustrate the working of your algorithm on the input

< 1, 5, 8, 6, 13, 4, 3 >

to find the 4^{th} smallest element. *4*

(b) (i) Define a BFS tree. Give the breadth first traversal for the undirected graph given below, starting from vertex 'a'. *6*



(ii) Give any two applications of Depth first search. *4*

3. (a) (i) Explain Dijkstra's shortest path algorithm. 6

(ii) Find the shortest path in the following graph represented by adjacency matrix, from vertex 'a'. 4

	a	b	c	d	e
a	–	1	15	–	5
b	1	–	2	–	10
c	15	2	–	8	6
d	–	–	8	–	3
e	5	10	6	3	–

(b) (i) Explain the principle of greedy algorithm. 4

(ii) Explain Prim's algorithm for Minimum Spanning Tree, and obtain the MST for the graph in question 3 (a) (ii). 6

4. (a) (i) Define Finite Automata and Regular Expression. 4

(ii) Write Regular Expression for the following : 6

(1) $L = (01)^n$, $n \geq 1$.

(2) Strings that start with '1' and end with '0'.

(b) Obtain the CFG for the following : *10*

(i) Strings of matching parenthesis.

(ii) Expression of the form E = (E + E) * E. The expression contains : parenthesis, operators : +, –, * and /.

5. (a) Explain the class-P, NP and NP-complete problems. *6*

(b) (i) What is undecidability ? Give an example for an undecidable problem. *5*

(ii) Design a polynomial time reduction from the vertex cover problem to the clique problem. *9*