→ S.K. Jha
→ A.K. Jha
→ D.S. Mathur
→ A.K. Bajaj
→ Energy-mass equivalence, $E=mc^2$
→ Lorentz Transformation Application Qs
→ Engineering Physics 1 – P.K. Palanisamy

First End Semester Syllabus

1. Interference of Light
2. Diffraction of Light
3. Polarization of Light
4. Laser
5. Mechanics from 8.13
6. Wave Motion
7. Relativity – GTR, STR ....
8. Fibre Optics

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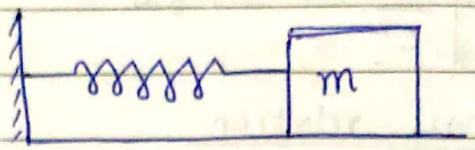
OSCILLATIONS

Damped Oscillations

$F = -kx$

$F = -bv$ (frictional force)



$$F = -kx - bv$$

$$ma = -kx - b\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

① — $$\boxed{\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega_0^2 x = 0}$$ Second order homogeneous differential equation

where $\frac{b}{m} = 2r$ = damping constant (always +ve)

$\frac{k}{m} = \omega_0^2$ = natural frequency of oscillator

→ Assumed Solution :

$$\boxed{x = Ae^{\alpha t}}$$ — ④ To find : A, α

$$\frac{dx}{dt} = \alpha A e^{\alpha t} \quad \text{— ②}$$

$$\frac{d^2x}{dt^2} = \alpha^2 A e^{\alpha t} \quad \text{— ③}$$

Put ②, ③, ④ in ①

$$\alpha^2 Ae^{\alpha t} + 2rA\alpha e^{\alpha t} + \omega_0^2 Ae^{\alpha t} = 0$$

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$\Rightarrow \alpha^2 + 2\gamma\alpha + \omega_0^2 = 0$

$$\alpha = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

$$\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$\Rightarrow \alpha_1 = -\gamma + \sqrt{\gamma^2 - \omega_0^2}$$

$$\alpha_2 = -\gamma - \sqrt{\gamma^2 - \omega_0^2}$$

$\rightarrow$ Final solution is

$$x = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

$$x = A_1 \exp\left[\left(-\underbrace{\gamma}_{\text{Damping constant}} + \underbrace{\sqrt{\gamma^2 - \omega_0^2}}_{\text{Frequency}}\right)t\right] + A_2 \exp\left[\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t\right]$$

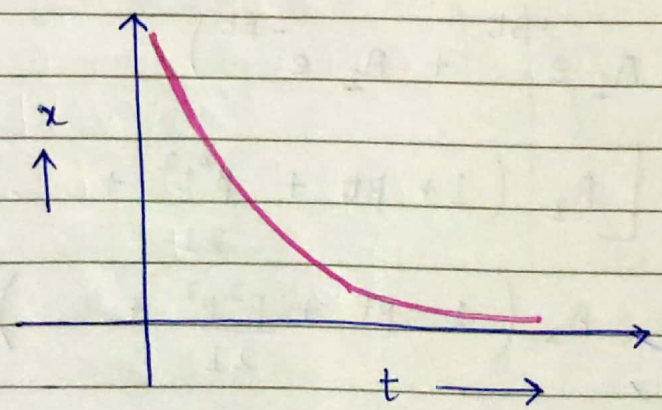
<u>CASES</u> : $\gamma^2 > \omega_0^2$

$\gamma^2 < \omega_0^2$

$\gamma^2 = \omega_0^2$

<u>Case I</u> $\gamma^2 > \omega_0^2$

Both terms of x become negative. $\Rightarrow$ Exponential decrease



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Case II : $\gamma^2 = \omega_0^2$

$$\alpha_1 = -\gamma + \sqrt{\gamma^2 - \omega_0^2} = \gamma$$
$$\alpha_2 = -\gamma - \sqrt{\gamma^2 - \omega_0^2} = \gamma$$

Solution :

Since it satisfies only the displacement condition

$x = (A_1 + A_2)\, e^{\alpha t}$ → But this is a partial solution not the complete solution.

$$x = (A_1 + A_2)\, e^{\alpha t}$$

Assume $A_1 + A_2 = B$

$$\Rightarrow \quad x = B e^{\alpha t} \quad \text{—} ①$$
$$v = \frac{dx}{dt} = B\alpha e^{\alpha t} \quad \text{—} ②$$

Assume the equilibrium position :

$\boxed{t = 0, \quad v = v_0, \quad x = 0}$

From ①, we get

$B = 0$

From ②, we get

$\boxed{v_c = 0}$!! Should not happen !!
!! Solution incomplete !!

$$x = A_1 e^{(-\gamma + \beta)t} + A_2 e^{(-\gamma - \beta)t}$$

$$= e^{-\gamma t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$$

$$= e^{-\gamma t}\left[A_1\left(1 + \beta t + \frac{\beta^2 t^2}{2!} + \dots\right) + A_2\left(1 - \beta t + \frac{\beta^2 t^2}{2!} + \dots\right)\right]$$

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$$x = e^{-\gamma t}\left[(A_1+A_2)t + \beta + (A_1-A_2)\right]$$

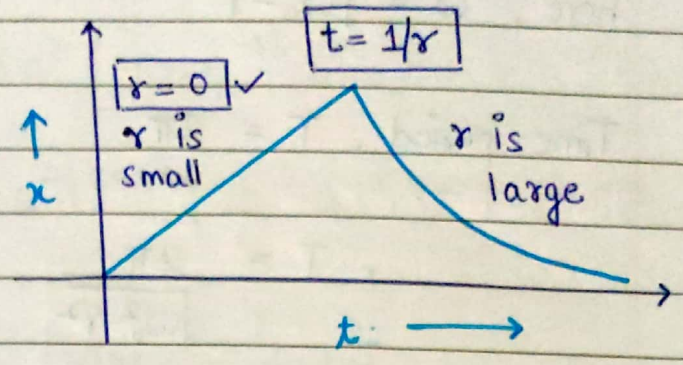
Neglecting higher order terms :

$$\boxed{x = e^{-\gamma t}(A+\beta t)}$$

where

$A = A_1 + A_2$

$B = A_1 - A_2$



Case III : $\omega_0^2 > \gamma^2$ ($\sqrt{\gamma^2-\omega_0^2}$ becomes imaginary)

$$\Rightarrow \sqrt{\gamma^2-\omega_0^2} = j\sqrt{\omega_0^2-\gamma^2}$$
$$= j\omega$$

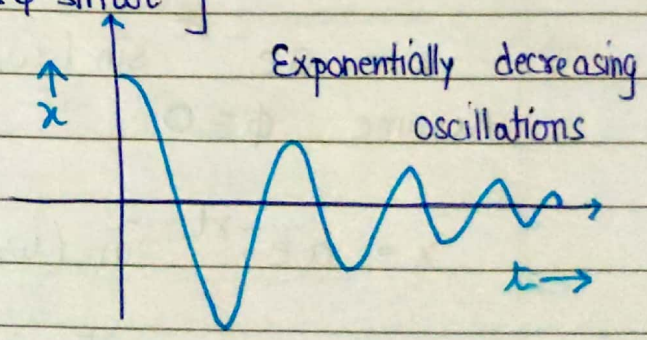
$\hookrightarrow$ Imaginary term ($j = \sqrt{-1}$)

$$x = A_1 e^{(-\gamma+j\omega)t} + A_2 e^{(-\gamma-j\omega)t}$$
$$x = e^{-\gamma t}\left(A_1 e^{i\omega t} + A_2 e^{-j\omega t}\right)$$
$$= e^{-\gamma t}\left[A_1(\cos\omega t + j\sin\omega t) + A_2(\cos\omega t - j\sin\omega t)\right]$$
$$= e^{-\gamma t}\left[(A_1+A_2)\cos\omega t + j(A_1-A_2)\sin\omega t\right]$$

Let $A_1 + A_2 = a\sin\phi$; $j(A_1-A_2) = a\cos\phi$

$$x = e^{-\gamma t}\left[a\sin\phi\cos\omega t + a\cos\phi\sin\omega t\right]$$
$$x = e^{-\gamma t}\, a\sin(\omega t+\phi)$$

$$\boxed{x = a e^{-\gamma t}\sin(\omega t+\phi)}$$



where a is amplitude

ϕ is phase difference between natural frequency and damping

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Time Period

$$x = a e^{-\gamma t} \sin(\omega t + \phi)$$

here, $\omega = \sqrt{\omega_0^2 - \gamma^2}$

Time period, $T = \frac{2\pi}{\omega}$.

$$T = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$$

$$\boxed{T = \frac{2\pi}{\sqrt{k/m - b^2/4m^2}}}$$

Power Dissipation

$$x = a e^{-\gamma t} \sin(\omega t - \phi)$$

For weak damping,

$$\gamma^2 << \omega_0^2$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega = \omega_0$$

$$\Rightarrow \quad x = a e^{-\gamma t} \sin(\omega_0 t + \phi)$$

Assume $\phi = 0$

$$x = a e^{-\gamma t} \sin(\omega_0 t)$$

$$\frac{dx}{dt} = -a\gamma e^{-\gamma t} \sin\omega_0 t + a e^{-\gamma t} \omega_0 \cos\omega_0 t$$

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Kinetic energy, K.E. $= \frac{1}{2} m v^2$

$$= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2} m a^2 \left[\omega_0 e^{-\gamma t} \cos\omega_0 t - \gamma e^{-\gamma t} \sin\omega_0 t\right]^2$$

$$= \frac{1}{2} m a^2 e^{-2\gamma t} \left[\omega_0^2 \cos^2\omega_0 t + \gamma^2 \sin^2\omega_0 t - 2\gamma\omega_0 \sin\omega_0 t \cos\omega_0 t\right]$$

Average KE $= K_{avg} = \frac{1}{T}\int_0^T \frac{1}{2} m a^2 e^{-2\gamma t} \left[\omega_0^2 \cos^2\omega_0 t + \gamma^2 \sin^2\omega_0 t - 2\gamma\omega_0 \sin\omega_0 t \cos\omega_0 t\right]$

$$= \frac{1}{2} m a^2 e^{-2\gamma t} \left[\frac{1}{2}\omega_0^2 + \frac{\gamma^2}{2}\right]$$

$$K_{avg} = \frac{1}{4} m a^2 e^{-2\gamma t} (\omega_0^2 + \gamma^2)$$

$$= \frac{1}{4} m a^2 e^{-2\gamma t} \omega_0^2$$ (as this is of weak damping $\omega_0^2 \gg \gamma^2$)

Potential Energy, $U = \frac{1}{2} k x^2$

$$= \frac{1}{2} k \left(a e^{-\gamma t} \sin\omega_0 t\right)^2$$

$$= \frac{1}{2} k a^2 e^{-2\gamma t} \sin^2\omega_0 t$$

$$= \frac{1}{2} m \omega_0^2 a^2 e^{-2\gamma t} \sin^2\omega_0 t$$

Average PE, $U_{avg} = \frac{1}{2} m \omega_0^2 a^2 \int_0^T \frac{e^{-2\gamma t} \sin^2\omega_0 t}{T}$

$$= \frac{1}{4} m \omega_0^2 a^2 e^{-2\gamma t}$$

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Total average energy, $E_{avg} = U_{avg} + K_{avg}$

$$= \frac{1}{4} m\omega_0^2 a^2 e^{-2\gamma t} + \frac{1}{4} m\omega_0^2 a^2 e^{-2\gamma t}$$

$$= \frac{1}{2} m\omega_0^2 a^2 e^{-2\gamma t}$$

At $t=0$, $E_0 = \frac{1}{2} m\omega_0^2 a^2$

$$\boxed{E = E_0 e^{-2\gamma t}}$$

Relaxation Time

Time at which total mechanical energy decay 1/e times of its initial value:

$E = E_0 e^{-2\gamma t}$

at $t = \tau$ (relaxation time)

$$\frac{1}{e} E_0 = E_0 e^{-2\gamma \tau}$$

$$e^{-1} = e^{-2\gamma \tau}$$

$$1 = 2\gamma \tau$$

$$\boxed{\tau = \frac{1}{2\gamma}}$$

Quality Factor

which tells about sharpness of tuning

OR

rate of change of energy decay of the oscillation

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy lost per second}}$$

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Energy lost $= \frac{dE}{dt} =$ Power $= E_0 \, 2\gamma \, e^{-2\gamma t}$

$= 2\gamma E$

$$Q = \frac{2\pi E}{PT}$$

↳ (Time period)

$$= \frac{2\pi E \omega}{P \, 2\pi}$$

$$Q = \frac{\omega E}{P}$$

$$\Rightarrow \quad Q = \frac{\omega E}{2\gamma E}$$

$$\boxed{Q = \omega \tau}$$

— Yahan tak test —

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Quality factor Q of driven oscillator

We know,

$$Q = 2\pi \times \frac{\text{average energy stored}}{\text{energy lost per cycle}}$$

$$= 2\pi \frac{E}{P \cdot T}$$

Power ← P. T → Time period

$$\text{Energy} = K.E + P.E$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kv^2$$

$$= \frac{1}{2}mp^2A^2\cos^2(pt-\phi) + \frac{1}{2}m\omega_0^2A^2\sin^2(pt-\phi)$$

$(k = m\omega_0^2)$

$$\text{Average energy} = \text{Average KE} + \text{Average PE}$$

$$\text{Average energy} = \frac{1}{T}\int_0^T \frac{1}{2}mp^2A^2\cos(pt-\phi) + \frac{1}{T}\int_0^T \frac{1}{2}\omega_0^2mA^2\sin^2(pt-\phi)$$

$$= \frac{1}{4}mp^2A^2 + \frac{1}{4}mA^2\omega_0^2$$

$$\text{Average power} = \frac{mv_0^2}{2\tau} = \frac{mA^2p^2}{2\tau}$$

τ → relaxation time

$$\Rightarrow Q = \frac{2\pi\left(\frac{1}{4}mp^2A^2 + \frac{1}{4}m\omega_0^2A^2\right)}{\frac{mA^2p^2}{2\tau} \times T}$$

$$Q = \frac{2\pi\left[\frac{1}{4}mA^2(p^2+\omega_0^2)\right]}{\frac{mA^2p^2}{2\tau} \times \frac{2\pi}{p}}$$

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$$Q = \frac{\frac{1}{2}(p^2 + \omega_0^2)\tau}{p}$$

$$\left(\omega = \frac{2\pi}{T}\right)$$

$$= \frac{1}{2}\left(p + \frac{\omega_0^2}{p}\right)\tau$$

In case of forced oscillation driven force has freq p

$p = 2\pi / T$

At resonance,

$\omega_0 = p = p_r$

$$Q = \frac{1}{2} \times 2p\tau$$

$$= p\tau$$

$$= \omega_0 \tau$$

$$\boxed{Q = \frac{\omega_0}{2\,\Delta p \longrightarrow \text{Band}}}$$

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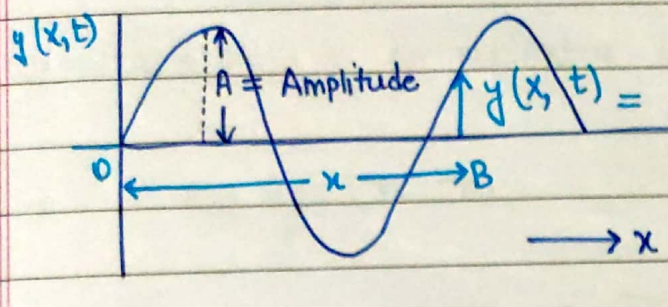
WAVES

Plane Progressive Harmonic Wave



→ travelling with a velocity, v

y(x,t) = displacement of particle from its mean position

At point O

$$y(0,t) = A \sin \frac{2\pi t}{T}$$

↳ Time period

At point B $= t - \frac{x}{v} = t$

= displacement of particle at $t = 0$

$$\Rightarrow y(x,t) = A \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \quad \text{— ①}$$

Eq ① is valid when the motion is along +x- direction. If the motion is along x- direction.

Then

$$y(x,t) = A \sin \frac{2\pi}{T}\left(t + \frac{x}{v}\right) \quad \text{— ②}$$

We can also rewrite eq ① & ② in terms of wavelength λ

$$y(x+\lambda,t) = A \sin\left\{\frac{2\pi}{T}\left(t - \frac{x+\lambda}{t}\right)\right\}$$

$$= A \sin\left\{\frac{2\pi}{T}\left(t - \frac{x}{v}\right) - 2\pi\right\} \quad (\because \lambda = vT)$$

Eq ① can also be rewritten as

$$y(x,t) = A \sin\left\{\frac{2\pi}{\lambda} v \left(t - \frac{x}{v}\right)\right\}$$

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$$= A \sin\left\{ \frac{2\pi}{\lambda} (vt - x) \right\}$$

$k = \frac{2\pi}{\lambda}$ $\qquad$ $\omega = \frac{2\pi}{T}$

↳ wave number

$$\boxed{y(x,t) = A \sin(\omega t - kx)}$$

Classical Wave Equation

Differentiate eq. ① wrt 't'

$$\frac{d^2y}{dt^2} = -A\left(\frac{2\pi v}{\lambda}\right)^2 \sin\left\{ \frac{2\pi}{\lambda} (vt - x) \right\}$$

$$= -\left(\frac{2\pi v}{\lambda}\right)^2 y \qquad \text{— ③}$$

Diff ① wrt x

$$\frac{d^2y}{dx^2} = -A\left(\frac{2\pi}{\lambda}\right)^2 \sin\left\{ \frac{2\pi}{\lambda} (vt - x) \right\}$$

$$= -\left(\frac{2\pi}{\lambda}\right)^2 y \qquad \text{— ④}$$

Combine ③ & ④

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

$$= A \sin\left\{ \frac{2\pi}{\lambda} (vt - x) \right\}$$

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Energy of a plane progressive wave

Energy = Potential energy + Kinetic energy

We know, motion of an oscillator is represented as

$$y = A \sin \frac{2\pi}{\lambda} (vt - x)$$

particle velocity, $u = \frac{dy}{dt} = \frac{2\pi v}{\lambda} A \cos \frac{2\pi}{\lambda} (vt - x)$

$$\frac{du}{dt} = \frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} A \sin \frac{2\pi}{\lambda} (vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2} y$$

Consider a unit volume of the medium in the form of extremely thin element

$\Rightarrow$ mass of element = density x volume

$= \rho \times V$

Since the element is extremely thin

KE per unit volume $= \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$

$$KE = \frac{1}{2} \rho u^2$$

$$= \frac{1}{2} \rho \frac{4\pi^2}{\lambda^2} v^2 A^2 \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

$$= \frac{2\pi v^2 \rho}{\lambda^2} A^2 \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

Force acting on a unit volume, $F = ma$

$$= \rho \frac{d^2y}{dt^2}$$

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$$F = -\frac{4\pi^2 v^2 y \rho}{\lambda^2}$$

$$= \frac{4\pi^2 v^2 \rho y}{\lambda^2}$$

-ve sign indicates acceleration & displacement are oppositely directed.

Work done = Force × displacement

$$dW = \frac{4\pi^2 v^2 \rho}{\lambda^2} y\, dy$$

↳ represents that work is done for the small dy of the layer

Total work done, $W = \int_0^y \frac{4\pi^2 v^2 \rho}{\lambda^2} y\, dy$

$$W = \frac{2\pi^2 v^2 \rho y^2}{\lambda^2}$$

$$= \frac{2\pi^2 v^2 \rho}{\lambda^2} A^2 \sin^2 \frac{2\pi}{\lambda}(vt - x)$$

Also,

Total work done = PE per unit volume

$$PE = \frac{2\pi^2 v^2}{\lambda^2} \rho A^2 \sin^2(vt - x)$$

Total energy = PE + KE

$$= \frac{2\pi^2 v^2 \rho}{\lambda^2} A^2 \cos^2 \frac{2\pi}{\lambda}(vt - x) + \frac{2\pi^2 v^2 \rho}{\lambda^2} A^2 \sin^2 \frac{2\pi}{\lambda}(vt - x)$$

$$= \frac{2\pi^2 v^2 \rho A^2}{\lambda^2}$$

$$= 2\pi^2 \nu^2 \rho A^2 \qquad (v = \nu\lambda)$$

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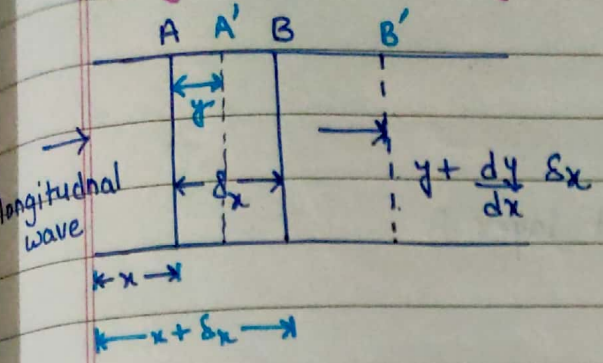
Longitudinal Waves (Sound waves) in a Gaseous (or fluid) medium



Initial A was at distance x

B was at distance $x + \delta_x$

When wave passes through the medium A displace to A' & B " " B' s.t.

$$AA' = y$$

$$BB' = y + \frac{dy}{dx}\delta_x$$

Distance between A & B $= \delta_x$

" A' & B' $= BB' + A'B$

$$= BB' + (AB - AA')$$

$$= y + \frac{dy}{dx}\delta_x + \delta_x - y$$

$$= \frac{dy}{dx}\delta_x + \delta_x \quad \text{—— (1)}$$

where in eq (1)

δ_x is the original distance between A & B

$\frac{dy}{dx}\delta_x \longrightarrow$ is the increase in distance b/w A & B.

Increase in volume of cylindrical slab $= \frac{dy}{dx}\delta_x \alpha$

$\alpha \rightarrow$ cross section of cylindrical slab

$$\text{Volume strain} = \frac{\text{increase in vol.}}{\text{original vol.}}$$

$$= \frac{\frac{dy}{dx}\delta_x \alpha}{\delta_x \alpha}$$

$$\text{Volume strain} = \frac{dy}{dx} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{dy}{dx}$$

If K is the elasticity of a medium p is the applied pressure.

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$$k = \frac{-p}{dy/dx}$$

$$p = -k\,(dy/dx)$$

↓

This is the pressure which applied at layer A

Now, pressure applied at

$$B' = p + \left(\frac{dp}{dx}\right)\delta_x$$

$$= p + \frac{d}{dx}\left(-k\frac{dy}{dx}\right)\delta_x$$

$$= p - k\frac{d^2y}{dx}\,\delta_x$$

Now pressure b/w A' & B' $= BB' + AA'$

$$= BB + AB - A'B$$

$$= p - k\left(\frac{d^2y}{dx^2}\right)\delta_x + 0 - p$$

$$= -k\left(\frac{d^2y}{dx^2}\right)\delta_x$$

Force = p x area

$$= -k\left(\frac{d^2y}{dx^2}\right)\delta_x \times \alpha \quad \text{— (1)}$$

Also, $F = ma$

$$= -\delta_x \alpha \rho \frac{d^2y}{dt^2} \quad \text{— (2)}$$

$$-k^2 \frac{d^2y}{dx^2}\,\delta_x \alpha = -\delta_x \alpha \rho \frac{d^2y}{dt^2}$$

$$\boxed{\frac{d^2y}{dt^2} = \frac{k}{\rho}\frac{d^2y}{dx^2}} \quad \text{— (3)}$$

If compare eq (3) with classical wave eq^n, we get

$$\boxed{v^2 = k/\rho}$$

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Acoustic Impedance or characteristic impedance

$$Z = P/V$$

$$= \frac{P}{(dy/dt)}$$

We know,

$$K = \frac{-P}{dV/V} = \frac{-P}{dy/dx}$$

$$P = -K\frac{dV}{V} = -K\frac{dy}{dx} \quad \text{—— (4)}$$

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dt} = \frac{2\pi v A}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dx} = -\frac{2\pi A}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dt} = -v\frac{dy}{dx} \quad \text{—— (5)}$$

Put eq (5) in eq (4)

$$P = \frac{K}{v}\frac{dy}{dt}$$

Also, $v = \sqrt{\frac{K}{\rho}}$

$$\Rightarrow P = \frac{K}{\sqrt{K/\rho}}\frac{dy}{dt} = \sqrt{\rho K}\frac{dy}{dt}$$

$$\frac{P}{dy/dt} = \sqrt{\rho K} \quad \text{—— (6)}$$

Compare eq (3) & (6)

$$Z = \sqrt{\rho K} \quad \text{or} \quad Z = v\rho$$

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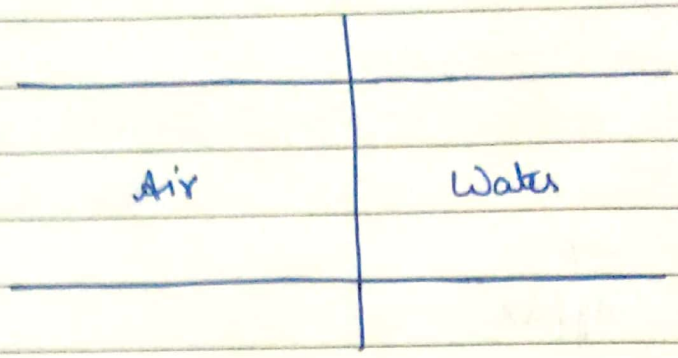
Reflection and transmission of longitudinal waves at discontinuity or boundary



$y_1 = A_1 \sin(\omega t - k_1 x) \longrightarrow$ incident wave

$y_2 = B_1 \sin(\omega t - k_1 x) \longrightarrow$ reflected wave

$y_3 = A_2 \sin(\omega t - k_2 x) \longrightarrow$ transmitted wave

The I^{st} boundary conditions

$y_1 + y_2 = y_3$ —— (1)

$p_1 + p_2 = p_3$ —— (2)

From (1)

$A_1 \sin(k_1 x - \omega t) + B_1 \sin(k_1 x - \omega t) = A_2 \sin(k_2 x - \omega t)$

Put $x = 0 \longrightarrow$ at boundary

$A_1 \sin\omega t + B_1 \sin\omega t = A_2 \sin\omega t$

$A_1 + B_1 = A_2$ —— (3)

Using eq (2)

$$\rho_1 v_1^2 \frac{dy_1}{dx} + \rho_1 v_1^2 \frac{dy_2}{dx} = \rho_2 v_2 \frac{dy_3}{dx}$$

$$p_1 = \rho_1 v^2 \frac{dy_1}{dx} = -\rho_1 v_1^2 A_1 k_1 \cos(\omega t - kx)$$

$$= -\rho_1 v_1 A_1 \omega \cos(\omega t - kx) \qquad (\omega = vk)$$

$p_2 = \rho_1 v_1 \omega B_1 \cos(\omega t + k_1 x)$

$p_3 = -\rho_2 v_2 \omega A_2 \cos(\omega t - k_2 x)$

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At $x=0$

$P_1 = -\rho_1 V_1 A_1 \omega \cos\omega t$

$P_2 = \rho_1 V_1 B_1 \omega \cos\omega t$

$P_3 = -\rho_2 V_2 A_2 \omega \cos\omega t$

$P_1 + P_2 = P_3$

$-\rho_1 V_1 A_1 + \rho_1 V_1 B_1 = -\rho_2 V_2 A_2$

As $Z = \rho V$

$-Z_1 A_1 + Z_1 B_1 = -Z_2 A_2$ —— (4)

Add eq (3) & (4)

$Z_2 (A_1 + B_1 = A_2)$

$-Z_1 A_1 + Z_1 B_1 = -Z_2 A_2$

$(Z_2 - Z_1) A_1 + (Z_1 + Z_2) B_1 = 0$

$\rightarrow \dfrac{B_1}{A_1} = -\dfrac{(Z_2 - Z_1)}{Z_1 + Z_2}$

$\downarrow$

Reflection coefficient

$R = \dfrac{B_1}{A_1} = \dfrac{Z_1 - Z_2}{Z_1 + Z_2}$

$T = A_2 / A_1$

↳ transmission coefficient

For that,

subtract (3) & (4)

$(A_1 + B_1 = A_2) Z_1$

$$\begin{array}{rl} & -Z_1 A_1 + Z_1 B_1 = -Z_2 A_2 \\ (+) & \\ \hline & 2Z_1 A_1 = (Z_1 + Z_2) A_2 \end{array}$$

(−) (+)

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$$T = \frac{A_2}{A_1} = \frac{2Z_1}{Z_1+Z_2}$$

From R & T, we get

$$T = R +$$

So we obtained

$$R = \frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1+Z_2} = \frac{V_2}{V_1}$$

$$T = \frac{A_2}{A_1} = \frac{2Z_1}{Z_1+Z_2} = \frac{V_3}{V_1}$$

In terms of pressure,

$$R = \frac{P_2}{P_1} = \frac{\rho_1 V_1 B_1}{\rho_1 V_1 A_1}$$

$$R = \frac{P_2}{P_1} = -\frac{B_1}{A_1} = \frac{Z_2 - Z_1}{Z_1+Z_2}$$

$$T = \frac{P_3}{P_1} = \frac{\rho_2 V_2 A_2}{\rho_1 V_1 A_1}$$

$$= \frac{Z_2 A_2}{Z_1 A_1}$$

$$= \frac{Z_2}{Z_1} \times \frac{2Z_1}{Z_1 \pm Z_2}$$

$$= \frac{2Z_2}{Z_1+Z_2}$$

Special cases

Case I $\quad Z_2 > Z_1$

Z_1	Z_2
less dense	more dense

velocity of incident & reflected wave have a phase difference of π. no change in pressure phase.

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Red Shift

Doppler Effect

Impedance in string

Case II $Z_1 > Z_2$

$$\boxed{h\nu' = h\nu + \frac{h\nu}{c^2} gH}$$ → Planck's constant

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