8.12 RELATION BETWEEN DISPLACEMENT AND PRESSURE AMPLITUDE LO1

Consider a plane monochromatic wave of sound travelling with a phase velocity v_0 along the length of the tube filled with a gas with density ρ_0. We will find the relationship between amplitude of the molecular displacement and that of pressure oscillations.

Proof

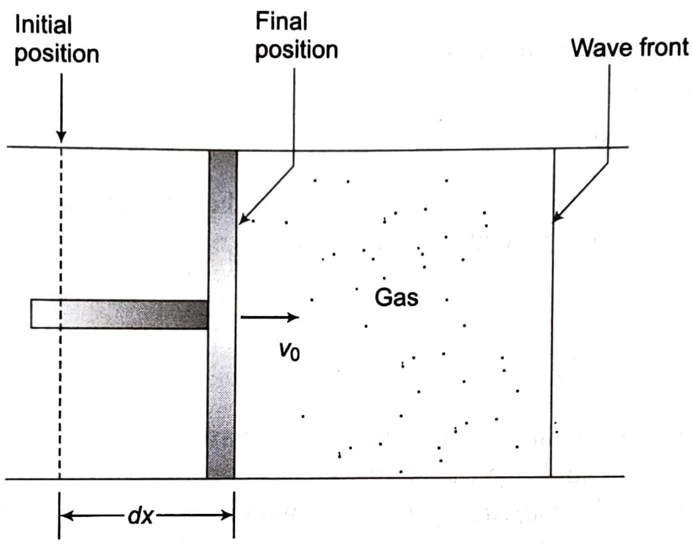

FIGURE 8.7

Consider a long tube of cross-sectional area A, in which a movable piston is fitted easily in the left side. The tube is open at another end with ambient surrounding. Firstly, the piston is at rest and then we apply a force F on the piston so that it moves towards the right side. This way we produce compression so that the wavefront moves dx distance towards right during the small interval dt. If the wavefront moves with velocity v_0, then

$$dx = v_0 dt \tag{i}$$

Due to the moment or displacement of the piston, the gas molecules also move with lower velocity v_m. Since the distance moved by the piston is very small, we can consider that the molecules under that volume have the same speed as that of the wavefront. So the total mass moved by the gas is obtained as

$$dm = \rho_0 A\, dx \tag{ii}$$

Here, ρ_0 is the mass density of the gas.

Linear momentum is given by

$$dP = dm \cdot v_m$$

$$= \rho_0 A\, dx\, v_m \tag{iii}$$

Putting the value of dx from Eq. (i), we get

$$dP = \rho_0 A\, v_m\, v_0\, dt \tag{iv}$$

From Eq. (iv), we can obtain the force as follows:

$$F = \frac{dp}{dt}$$

and the pressure $P = \frac{1}{A}\frac{dp}{dt}$

$$= \rho_0 \, v_0 \, v_m \qquad \text{(v)}$$

If the maximum longitudinal speed is $s\omega$, where s is the displacement of molecules of the gas and ω is the angular frequency of the sound wave, then maximum pressure is given by

$$P_{\max} = \rho_0 \, v_0 \, s\omega \qquad \text{(vi)}$$

This is the required relation between the displacement s and the pressure amplitude P or $P_{\max}$. Clearly, a larger pressure is exerted by a larger velocity of the piston (v_0) or the larger displacement of the molecules (s) of the sound.

EXAMPLE 10 A wave of 1000 Hz frequency travels in air of 1.2 kg m^{-3} density at 340 m/s. If the wave has 10 μ Wm^{-2} intensity, find the displacement and pressure amplitudes.

SOLUTION

$$I = \frac{1}{2}(\rho v)(A\omega)^2 \qquad v \rightarrow \text{wave speed}$$

$$\Rightarrow \quad A = \sqrt{\frac{2I}{\rho v \omega^2}}$$

$$= \sqrt{\frac{2\times 10^{-6}}{1.2\times 340\times(2\pi\times 1000)^2}}$$

$$= 11 \text{ nm}$$

$$p_0 = \rho v A\omega$$

$$= 1.2\times 340\times 11\times 10^{-3}\times 2\pi\times 1000$$

$$= \mathbf{28\ mPa}$$

EXAMPLE 11 Assuming $\rho = 1.29$ kg/m^3 for the density of air and $v = 331$ m/s for the speed of sound, find the pressure amplitude corresponding to the threshold of hearing intensity of 10^{-12} W/m^3.

SOLUTION

$$I = \frac{1}{2} P_{\max}^2 / \rho_0 v$$

$$\Rightarrow \quad P_{\max} = \sqrt{2I\rho_0 v}$$

$$= \sqrt{2 \times 10^{-12} \times 1.29 \times 331}$$

$$= \mathbf{2.92 \times 10^{-5}\ N/m^2}$$

EXAMPLE 12 For ordinary conservation, the intensity level is given as 60 dB. What is the intensity of the wave?

SOLUTION

$$I_L = 10 \log \frac{I}{I_0}$$

$$60 = 10 \log \frac{I}{10^{-12}}$$

$$\log I + \log 10^{12} = 6$$

$$\log I = -6$$

$$\therefore \quad I = 10^{-6}\ \text{W/m}^2 = \mathbf{1\ \mu W/m^2}$$

EXAMPLE 13 A small source of sound radiates energy uniformly at a rate of 4 W. Calculate the intensity level at a point 25 cm from the source if there is no absorption.

SOLUTION

$$I = \frac{\text{Power}}{4\pi r^2} = \frac{4}{4\pi \times 25^2} = 5.093 \times 10^{-4}\ \text{W/m}^2$$

$$I_L = 10 \log \frac{I}{I_0}$$

$$= 10 \log \frac{5.093 \times 10^{-4}}{10^{-12}}$$

$$= 10 \log (5.093 \times 10^8)$$

$$= 10[\log 5.093 + 8]$$

$$= \mathbf{87\ dB}$$

EXAMPLE 14 The maximum pressure variation that the ear can tolerate is about 29 N/m^2. Find the corresponding maximum displacement for a sound wave in air having a frequency of 2000 Hz. Assume the density of air as 1.22 kg/m^3 and the speed of sound as 331 m/s.

SOLUTION

$$A = \frac{P_{\max}}{k \rho_0 v^2} = \frac{P_{\max}}{2\pi \rho_0 f v}$$

$$k = 2\pi/\lambda \quad \text{and} \quad v = f\lambda$$

$$A = \frac{29}{2 \times 3.14 \times 1.22 \times 331 \times 2000} = \mathbf{5.7 \times 10^{-6}\ m}$$

EXAMPLE 15 If two sound waves, one in air and the other in water, have equal pressure amplitude, what is the ratio of intensities of waves? Assume that the density of air is 1.293 kg/m^3, and the speeds of sound in air and water are 330 and 1450 m/s respectively.

SOLUTION

$$I = \frac{P_{max}^2}{2\rho_0 v}$$

$$P_{max}\text{ (air)} = P_{max}\text{ (water)}$$

$$\therefore \quad \frac{I_{Water}}{I_{Air}} = \frac{\rho_A v_A}{\rho_W v_W}$$

$$= \frac{1.293 \times 330}{1000 \times 1450} = \mathbf{2.94 \times 10^{-4}}$$

EXAMPLE 16 The pressure in a progressive sound wave is given by the equation $P = 2.4 \sin \pi(x - 330\,t)$, where x is in metres, t in seconds and P in N/m^2. Find (a) pressure amplitude, (b) frequency, (c) wavelength, and (d) speed of wave.

SOLUTION

$$P = 2.4 \sin \pi(x - 330\,t)$$

$$= 2.4 \sin 2\pi\left(\frac{1}{2}x - 165\,t\right)$$

$$P = P_{max} \sin 2\pi\left(\frac{x}{\lambda} - ft\right)$$

On comparing, we get

Pressure amplitude = 2.4 N/m^2

Frequency = 165 Hz

Wavelength = 2.0 m

Speed of wave $v = f\lambda = 165 \times 2 = 330$ m/s