$T = 1.26$ s

EXAMPLE 13 The relaxation time for damped harmonic oscillator is 50 s. Determine the time in which the amplitude and energy of oscillator falls to $1/e$ times of its initial value.

SOLUTION The amplitude of dampled harmonic oscillator at time t is given by

$$A(t) = A_0 e^{-st}$$

Relaxation time $\tau = \dfrac{1}{2s}$

given $\tau = 50$ s

Now $\tau = \dfrac{1}{2s} = \dfrac{1}{2 \times 50} = \dfrac{1}{100}$ per s

A_0 is the amplitude at $t = 0$ and at time t the amplitude will be A_0/e. Hence

$$\frac{A_0}{e} = A_0 e^{-st} \Rightarrow \frac{1}{e} = e^{-st} \Rightarrow -1 = -st$$

or $$t = \frac{1}{s} = \mathbf{100\ s}$$

EXAMPLE 14 Considering quality factor of sonometer wire of frequency 260 Hz as 2000, calculate the time in which the amplitude decreases to $1/e^2$ of its initial value.

SOLUTION The quality factor is given by

$$Q = \omega\tau$$

Here $Q = 2000$ and $\omega = 2\pi n = 2 \times 3.14 \times 260$ rad/s

Relaxation time $\tau = \dfrac{Q}{\omega} = \dfrac{2000}{2 \times 260 \times 3.14}$

$$= 1.225 \text{ s}$$

The formula for amplitude of damped oscillator at time t is

$$A(t) = A_0 e^{-st}$$

Given $$A(t) = \frac{A_0}{e^2}$$

$\therefore$ $$\frac{A_0}{e^2} = \frac{A_0}{e^{st}}$$

or $$t = \frac{2}{s} = 2\tau$$

$$= 2 \times 1.225 = \mathbf{2.450\ s}$$

8.3 SOUND DISPLACEMENT

LO1

So far, we have understood that the particles of the medium execute SHM in order to generate the sound wave, which means they vibrate about their equilibrium position. *Sound displacement* is defined as the displacement of the vibrating particles of the medium from their rest positions. This can be represented by

$$y(x, t) = y_0 \sin(\omega t - kx) \tag{i}$$

where ω is the angular frequency of oscillations ($\omega = 2\pi f$) and k is the wave number ($k = 2\pi/\lambda$, λ is the wavelength). If we differentiate y w.r.t. time t, we get the velocity of vibrating particles, which is also known as *sound-particle velocity*.

This is obtained as

$$v = \frac{dy}{dt} = y_0 \omega \cos(\omega t - kx)$$

or $$v(x, t) = y_0\, \omega \cos(\omega t - kx) \tag{ii}$$

This equation shows that there is a phase difference of $\pi/2$ between y and v. Moreover, the velocity v is different for different sound particles.

8.9 INTENSITY OF SOUND AND ITS DERIVATION

LO1

Intensity of sound at a point in a progressive wave is defined as the sound energy per unit area per unit time perpendicular to the direction of propagation of the wave. It is measured in W/m^2 in the SI system of units.

We consider a plane progressive simple harmonic wave travelling along the positive x-direction with velocity $v(=\omega/k)$. The displacement y at a time t can be represented as

$$y = a \sin(\omega t - kx) \tag{i}$$

From this, we find the velocity of particle by differentiating it w.r.t. time

$$\frac{dy}{dt} = \omega a \cos(\omega t - kx) \tag{ii}$$

In order to calculate the energy or the intensity of sound, we consider a medium of density ρ. Taking unit area of medium having thickness dx perpendicular to the direction of propagation of wave, we find the kinetic energy as

$$dK = \frac{1}{2}\rho\, dx\left(\frac{dy}{dt}\right)^2 \tag{iii}$$

Putting the value of $\frac{dy}{dt}$ from Eq. (ii) and using $\omega = 2\pi f$, where f is the linear frequency of the wave, Eq. (iii) reads

$$dK = \frac{1}{2}\rho a^2 (2\pi f)^2 \cos^2 (\omega t - kx)\, dx$$

$$= 2\pi^2 a^2 \rho f^2 \cos^2 (\omega t - kx)\, dx \tag{iv}$$

This shall give the total energy of the wave as

$$dE = dK_{max} \text{ (when potential energy is zero)}$$

$$= 2\pi^2 a^2 \rho f^2\, dx \tag{v}$$

dx can be written in terms of the velocity v as $dx = vdt$.

Hence,

$$dE = 2\pi^2 a^2 f^2 \rho\, v\, dt \tag{vi}$$

The integration gives

$$E = 2\pi^2 a^2 f^2 \rho\, vt \tag{vii}$$

The energy flow per unit time is obtained from Eq. (vii) as $2\pi^2 a^2 f^2 \rho v$, which is nothing but the intensity of the sound wave. Hence,

$$I = 2\pi^2 a^2 f^2 \rho\, v \tag{viii}$$

The other forms of the formula of sound intensity I are

$$I = 2\pi^2 \rho f^2\, v\, A^2_{max} \text{ (in terms of displacement)}$$

$$I = \frac{\rho v \Delta v^2_{max}}{2} \left(\text{in terms of velocity, where } \Delta v_{max} = 2\pi f\, A_{max}\right)$$

$$I = \frac{\Delta p^2_{max}}{2\rho v} \left(\text{in terms of pressure, where } \Delta p_{max} = 2\pi \rho f v\, A_{max}\right)$$

8.10 SOUND-INTENSITY LEVEL LO1

The level of sound intensity (say I_L) can be defined in terms of decibels (dB) and neper (Np), as follows:

$$I_L = 10 \log_{10}\left(\frac{I}{I_0}\right) \quad \text{in dB} \tag{i}$$

or $$I_L = \log_{10}\left(\frac{I}{I_0}\right) \quad \text{in B (Bel)}$$

$$I_L = \frac{1}{2}\log_e\left(\frac{I}{I_0}\right) \quad \text{in Np} \tag{ii}$$

Here, I is the sound intensity, I_0 is the reference intensity and B is the unit bel (1 B = 10 dB).

Since the sound intensity I is directly proportional to square of the pressure p, we have

$$\frac{I}{I_0} = \frac{p^2}{p_0^2} \qquad (p_0 \text{ is the reference pressure})$$

With the help of this, I_L can be defined in terms of p as follows:

$$I_L = 10 \log_{10}\left(\frac{p^2}{p_0^2}\right)$$

$$= 20 \log_{10}\left(\frac{p}{p_0}\right)$$