

# Expected Data Rate: An Accurate High-Throughput Path Metric For Multi-Hop Wireless Routing

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**Abstract**—We present a new metric, Expected Data Rate (EDR), for accurately finding high-throughput paths in multi-hop ad hoc wireless networks. Our metric is based upon a new model for transmission interference which is a critical factor in determining path throughput. We construct a realistic and practical transmission interference model by (1) determining transmission contention degree of each link as a function of the wireless link loss, (2) quantifying the impact of the wireless link loss on medium access backoff, and (3) considering possible concurrent transmissions when two links do not interfere with each other. Our transmission interference model also takes the non-optimality of IEEE 802.11 medium access scheduling into account. Using extensive *ns-2* simulations of IEEE 802.11 ad hoc networks, we find that EDR can accurately determine the achievable data rates of ad hoc paths, thereby significantly outperforming the other existing metrics.

## I. INTRODUCTION

Multi-hop ad hoc wireless networks provide a flexible solution to applications where wireless users, mobile or not, wish to communicate with each other without a fixed wired infrastructure. Efficient ad hoc routing protocols are necessary to extract optimal performance from these networks. There has been a lot of research on designing and developing efficient ad hoc routing protocols. Some of these efforts have extended existing wired network routing protocols to the wireless scenario. More recently, research on ad hoc routing has been focusing on understanding the characteristics of the shared wireless medium, and on incorporating these characteristics in determining ad hoc path performance metrics for finding best ad hoc paths. Although interesting, the existing research on finding suitable path metrics only provides a good beginning. It fails to fully address some of the fundamental properties of multi-hop ad hoc wireless networks and hence the existing path metrics are not very accurate.

A simple metric for finding ad hoc paths in multi-hop wireless networks is minimum hop count. There are many existing routing protocols based on this metric including Dynamic Source Routing (DSR) [1], and Ad Hoc On-Demand Distance Vector Routing (AODV) [2]. These approaches use shortest-path routing, implicitly assuming that links either work perfectly or do not work at all. They do not consider the wireless link loss. The shortcomings of shortest-path routing using only the minimum hop count metric have been investigated recently [3]–[5]. The results of [3]–[5] consistently show that the wireless link loss must also be considered in the path metric for finding high performance ad hoc paths.

In this paper, we develop a new path metric that we call *Expected Data Rate* or EDR in short. Our metric accurately determines the data rates of ad hoc paths in multi-hop ad hoc wireless networks. In order to develop EDR, we use an accurate understanding for transmission interference in the shared wireless medium. Transmission interference is a fundamental property of shared wireless networks. It is a critical factor in determining ad hoc path throughput. We find that transmission interference behavior is highly dependent upon the wireless link loss rates. Existing path metrics, including even those that consider wireless link loss, do not address this dependence. Interestingly, we find that the transmission interference does not only depend upon the wireless link loss rates, but also on the ordering of link loss rates along the ad hoc path. We construct a realistic and practical transmission interference model in the context of the IEEE 802.11 medium access control protocol [6] by (1) determining transmission contention degree of each link as a function of the wireless link loss, (2) quantifying the impact of the wireless link loss on medium access backoff, and (3) considering possible concurrent transmissions when two links do not interfere with each other. Our transmission interference model also takes the non-optimality of IEEE 802.11 medium access scheduling into account.

Using extensive *ns-2* simulations, we find that our new path metric EDR, can fairly accurately determine the achievable data rates of ad hoc paths. Unlike existing path metrics, EDR can find best ad hoc paths in almost all cases that we study.

The remainder of this paper is organized as follows. In the next section, we present the existing work. In Section III, we motivate our ideas to demonstrate the limitation of the existing metric. Section IV describes our problem setting. In Section V, we develop the transmission interference model in the presence of lossy links. Our new metric EDR is also presented in the section. Issues about incorporating EDR into existing ad hoc routing algorithms are discussed in Section VI. In Section VII we evaluate the performance of EDR and demonstrate its superior performance over the existing path metrics. We conclude our work in Section VIII.

## II. RELATED WORK

A vast amount of research has been done on ad hoc routing [1], [2], [7]–[11]. Instead of presenting a survey of the existing literature on the ad hoc routing, we mainly focus on one research effort: ETX [3] that is closest to our work

on EDR. Although [3] considers wireless link loss in its metric for determining ad hoc path performance, it is deficient in modeling the transmission interference. [12]–[18] model the transmission interference to determine the capacity of ad hoc paths based on graph theoretic approaches or geometric analysis, but they do not consider wireless link loss. In fact, none of the existing work has *comprehensively* addressed these two important issues, wireless link loss and transmission interference, together in determining ad hoc path performance. We now summarize the existing work on ETX and on transmission interference.

#### A. ETX

[3] proposed the metric Expected Transmission Count (or ETX) for finding high-throughput paths in multi-hop wireless networks. In contrast to the minimum hop count metric [1], [2], ETX incorporated the effects of wireless link loss, and transmission interference among successive links of an ad hoc path. For a link with loss probability,  $p$ , ETX was defined to be the number of transmissions (including retransmissions) required to successfully transmit a packet on that link, i.e.,  $ETX = 1/(1 - p)$ . The transmission interference along an ad hoc path was taken into account by adding the ETX of all the links along the path. ETX has several drawbacks. First, a total sum of ETXes of the links as a path metric is too simplistic and as we will show in Section III, two ad hoc paths with the same ETX sum could achieve very different data rates. Further, ETX does not consider the effects of medium access control backoff. Moreover, for longer ad hoc paths, the ETX sum underestimates the achievable data rates. This is because it ignores the possibility of concurrent transmissions on links that are far apart and that do not interfere with each other. Another drawback of ETX is the lack of multi-radio support. ETX cannot find ad hoc paths that have the maximal achievable data rates for multi-radio wireless networks. The actual performance gain of ETX in their DSR experiments are small, as observed in [3].

To overcome the single-radio limitation of ETX, [4] proposed a new metric called Weighted Cumulative Expected Transmission Time (WCETT) that is based on Expected Transmission Time (ETT) of the wireless links. However, for a single-radio, WCETT essentially reduces to the total ETX sum. Consequently, it has the same drawbacks as ETX for a single radio. We believe our single-radio EDR could be extended to the multi-radio scenario. A preliminary version of this extension is presented in [19].

#### B. Transmission Interference

In wired networks, any transmissions on a link do not interfere with transmissions on the other links because they are independent. However, in ad hoc wireless networks, transmissions on one wireless link can interfere with those on another if they both use the same radio and are within interference range of each other. Therefore, transmission interference is a critical factor in determining the achievable data rates of ad hoc paths.

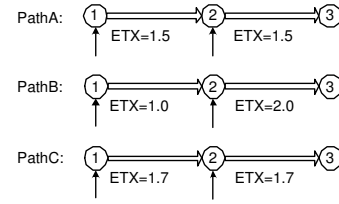


Fig. 1. Three 2-hop ad hoc paths

Models of transmission interference in ad hoc wireless networks have been researched to determine the capacity of ad hoc paths [12]–[16] and to study the correlation between transmission interference and medium access scheduling [20]–[22]. These existing efforts model the ad hoc network as a graph where vertices in the graph represents ad hoc nodes and edges are constructed between those vertices (nodes) that are within the interference range. However, such graph theoretic transmission interference models do not consider *the presence of lossy links and retransmissions of packets due to packet loss*. The geometric approaches [17], [18] to analyze the capacity of ad hoc networks also do not consider link loss. This simplification results in unrealistic transmission interference models which cannot be practically applied for ad hoc wireless networks in the face of lossy links.

To comprehensively address wireless link loss and transmission interference together, our work on EDR extends the work on ETX in the following significant ways. First, we consider the transmission interference in the shared wireless medium by determining the transmission contention degree of each node as a function of wireless link loss. Second, we include the impact of wireless link loss on medium access backoff. Third, we consider the possibility of concurrent transmissions on links that are “far” apart. We also take the non-optimality of IEEE 802.11 medium access scheduling into account.

### III. LIMITATIONS OF ETX SUM

In this section, for motivating our work, we demonstrate the limitations of ETX sum using *ns-2* simulations of three ad hoc paths.

TABLE I  
RESULTS OF *ns-2* SIMULATION

Path	ETX Sum	Throughput in <i>ns-2</i>
A	3.0	1.98 Mbps
B	3.0	0.84 Mbps
C	3.4	1.60 Mbps

Consider the three ad hoc paths shown in Fig. 1 where the send rate of each source node is the maximal one-hop data rate of IEEE 802.11b. UDP packets of 1500 bytes are used to generate the traffic at the first hops in the three paths. Table I shows the throughput of the three ad hoc paths that is obtained using the *ns-2* simulations.

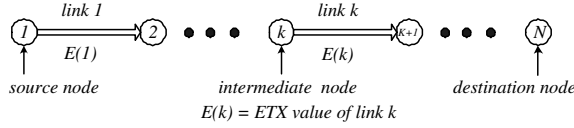


Fig. 2. ETX for each link

First, consider PathA and PathB that have the same ETX sum, equal to 3.0. An ETX sum equal to three implies that whenever a source node sends a packet, three transmissions of the packet are required on average before the packet is successfully received at the last hop. Interestingly, the throughput of PathA is approximately 230% better than that of PathB.

Second, consider PathB and PathC. The ETX sum on PathC is larger than that of PathB. According to the ETX path-metric, PathB will be selected as a better ad hoc path. However, the throughput of PathC is 190% higher than that of PathB.

These simple examples show that an ETX sum cannot accurately differentiate ad hoc paths with different achievable data rates. The main reason is that, when successive hops have different loss rates, the transmission interference of the two nodes is different. The changed transmission interference considerably affects the achievable data rates. ETX sum is unable to capture this phenomenon.

#### IV. PROBLEM SETTING

Our goal, similar to [3], is to develop a load insensitive path metric and load insensitive routing. Load sensitive routing has been shown to be unstable in the Internet [23]. Today's Internet routing protocols including RIP (routing information protocol), and OSPF (open shortest path first), are load insensitive.

In contrast to wired links, ad hoc paths in the shared wireless medium have the interesting property that links within the interference range of each other could experience transmission interference due to packets belonging to the same flow that are being sent/forwarded at the same time on different links. In our transmission interference model and our metric, EDR, for determining the data rate along an ad hoc path, we consider only the "unavoidable" interference along the links of the ad hoc path due to a single flow. This interference arises due to transmission of packets belonging to the same flow on different links along the ad hoc path. We do not consider the *dynamic interference* on wireless links that changes with time, potentially due to changes in traffic conditions in the close vicinity. Nor do we consider, the characteristics of the ad hoc path that could change due to other flows that can come and go along the links on the ad hoc path.

Our metric requires measurement of wireless link losses. We use the approach proposed in [3] for periodically measuring these losses. We note that, although the short term dynamic changes are avoided in our transmission interference model and our path metric, the measured losses do reflect some changes in the link characteristics.

In the rest of the paper we will focus on a single ad hoc path with one data flow from the source node to the destination as shown in Fig. 2. We will assume that the loss rate or ETX value of each link along the ad hoc path is known. The ETX value of link  $k$  is denoted by  $E(k)$ . We also assume that the ideal maximal data rate of each link is same and constant.

#### V. EXPECTED DATA RATE METRIC

In order to construct a realistic and practical transmission interference model for IEEE 802.11, we need to clearly understand the IEEE 802.11 distributed coordination function (DCF) standard. In this section, we first explain the medium access control mechanism of IEEE 802.11 DCF. Next, we present three important observations on transmission interference relevant to the IEEE 802.11 DCF when links are lossy: transmission contention as a function of packet loss, exponentially increased contention window size due to packet loss, and possible concurrent transmissions over non-interfering links. We quantify these observations and use them in developing our EDR metric.

##### A. IEEE 802.11 DCF

We first describe the medium access mechanism of IEEE 802.11 distributed coordination function (DCF) standard.

When a node wants to transmit a packet and senses the medium to be free for a DIFS (DCF Interframe Space) time, it immediately occupies the medium by transmitting the packet. If the medium is sensed busy, this node selects a backoff counter which is randomly chosen within its current contention window size, and starts the countdown of the backoff counter whenever the medium is free. The node suspends the countdown of the counter whenever it senses the medium busy. This mechanism implies that the node should wait for the completion of other nodes' transmissions *without counting down its backoff counter*. When the medium becomes free again, it resumes the remainder of the counter. When the counter eventually becomes zero, the node sends a packet after a DIFS. After sending the packet, the node waits to be acknowledged by the receiver node. On failure to receive an acknowledgment, the transmitting node restarts the backoff procedure for retransmitting the unacknowledged packet. However, the contention window size is doubled and the backoff counter value is chosen from the increased contention window size. Note that there is no mechanism in DCF to differentiate between loss due to *collisions* and that due to channel noise of wireless links, as observed in [24], [25]. This implies that a transmitting node always doubles its contention window size upon transmission failure regardless of whether the failure is caused by collision or packet loss. Since no mechanisms for differentiating between different types of transmission failures have been implemented yet, we use the original DCF in this work.

When an ad hoc node transmits a packet, it needs to keep the sent packet in its outgoing queue (or some system buffer) for possible retransmissions. The sent packet can be removed from the outgoing queue only when it is acknowledged. As a result,

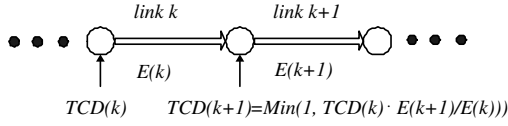


Fig. 3. Transmission contention degree

whenever there are any transmission failures, the time a node's outgoing queue is not empty increases due to preserving the lost packet for retransmissions. This time contributes to the node's transmission contention against the other nodes that are within its interference range. This is because the node tries to occupy the medium as long as its outgoing queue has at least one packet. Therefore, transmission contention of a node is associated with the status of its outgoing queue. Intuitively, the transmission contention represents how busy a link  $k$  is transmitting or retransmitting packet. The transmission contention of ad hoc nodes along an ad hoc path affects the achievable data rate along the path. In the next subsection we define the transmission contention *degree* of a link, provide an approximate method to determine it, and show how the transmission contention degree affects ad hoc path throughput.

#### B. Transmission Contention Degree

We define Transmission Contention Degree (TCD) of a link  $k$ , denoted by  $TCD(k)$ , as the average time outgoing queue of node  $k$  is not empty over a given time period. TCD captures not only the original transmission load on a link, but also the increased load due to the retransmissions of lost packets.

For example, suppose that the traffic rate generated by a node  $k$  is half of the maximal achievable data rate of its outgoing wireless link. If the loss rate of this link is 0,  $TCD(k)$  is 0.5 because in a given time period, the node is busy transmitting half the time and thereby having a packet in its outgoing queue for half the time in that period. However, if the loss rate of the link is 0.5, the outgoing queue of node  $k$  is never empty owing to the retransmission of every other transmission. Then,  $TCD(k)$  becomes 1.0.

The TCD of a link in an ad hoc path can be approximated by the rate at which data arrives at the link and the rate at which it is transmitted on the link. The arrival and departure rates depend upon its own loss rate, and the loss rate and TCD of the previous adjacent link in the ad hoc path. Figure 3 shows two links  $k$  and  $k+1$  adjacent to node  $k+1$  in an ad hoc path.  $TCD(k+1)$  can be approximated in terms of  $TCD(k)$ , and the ratio  $E(k+1)/E(k)$  as follows:

When  $E(k+1)/E(k)$  is less than 1.0, link  $k+1$  will have more idle time than link  $k$ . Then,  $TCD(k+1)$  becomes  $TCD(k) \times E(k+1)/E(k)$ . When  $E(k+1)/E(k)$  is greater than 1.0, link  $k+1$  transmits more packets than link  $k$  due to more retransmissions on link  $k+1$ . Therefore,  $TCD(k+1)$  becomes  $TCD(k) \times E(k+1)/E(k)$ . Now,  $TCD(k) \times E(k+1)/E(k)$  may exceed 1.0. This means that node  $k+1$  may drop some packets because its outgoing queue is full. However,  $TCD(k+1)$  can never exceed 1.0. Thus,  $TCD(k+1)$  can

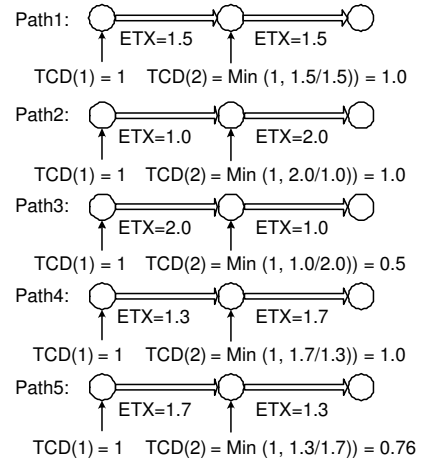


Fig. 4. TCD Examples

be determined using the following equation.

$$TCD(k+1) = \text{Min}(1, TCD(k) \times \frac{E(k+1)}{E(k)}) \quad (1)$$

We are interested in finding the maximum achievable data rate along an ad hoc path. So, we assume that the source node of the ad hoc path is always backlogged. This assumption results in  $TCD(1) = 1$ . In order to quantify the affect of transmission contention degree on ad hoc path throughput, we define another quantity called *total transmission contention degree*. The total transmission contention degree (total TCD) of a link  $k$  is defined as a total sum of TCDs of all links that are within the interference range of link  $k$ . This sum also includes the TCD of link  $k$ . Let  $I(k)$  denote total TCD of a link  $k$ .

$$I(k) = \sum_{i=n_s}^{n_e} TCD(i) \quad (2)$$

where  $n_s, \dots, k, \dots, n_e$  are the links (including link  $k$ ) along the ad hoc path within the interference range of link  $k$ .

Fig. 4 shows TCD examples for five two-hop ad hoc paths with different loss rates. The ETX sum of all the five paths is equal to 3.0. Table II shows the values  $TCD(k)$  and  $I(k)$  for these five paths. Since the links of the two-hop ad hoc paths are within the interference range with each other,  $I(1)$  and  $I(2)$  are same and both equal  $TCD(1) + TCD(2)$ .

TABLE II  
TOTAL TRANSMISSION CONTENTION DEGREE

Path	E(1)	E(2)	TCD(1)	TCD(2)	I(1) and I(2)
1	1.5	1.5	1.0	1.0	2.0
2	1.0	2.0	1.0	1.0	2.0
3	2.0	1.0	1.0	0.5	1.5
4	1.3	1.7	1.0	1.0	2.0
5	1.7	1.3	1.0	0.76	1.76

### C. Initial Expected Data Rate

We use  $I(k)$  to determine an initial *Expected Data Rate*,  $EDR_{init}$ . We will refine this initial value in the subsequent sections. As before, let  $E(k)$  denote the ETX of link  $k$  and  $I(k)$  denote a total transmission contention degree of link  $k$ . Due to interference from other neighboring links, the data rate on link  $k$  reduces by  $E(k) \times I(k)$ . This is because, for every transmission attempt (including retransmissions) on link  $k$ , the sender node of the link should contend against other nodes as many times as  $I(k)$ . Therefore, the achievable data rate of a link  $k$  can be represented by  $\Gamma$  divided by  $E(k) \times I(k)$  where  $\Gamma$  is the ideal maximal data rate of a one-hop link. Thus, our initial *Expected Data Rate* (EDR) for a link  $k$  is defined, as follows:

$$EDR_{init}(k) = \frac{\Gamma}{E(k) \times I(k)} \quad (3)$$

However, our main goal is to find the achievable data rate of the entire ad hoc path (not of a link  $k$ ). The achievable data rate of the ad hoc path will be determined by the send rate of the bottleneck link  $k'$  which has the largest data rate reduction. The send rate of the link  $k'$  will depend upon  $E(k') \times I(k')$ . We argue that the bottleneck link of the ad hoc path would be one that has the highest loss rate among all the links of the ad hoc path.

This argument is supported by the following two points. First, the highest loss rate link has the largest number of retransmissions among all the links. Second, due to its highest loss rate, and thereby largest contention window size, it has the lowest probability of occupying the shared medium. Although in this paper we do not consider auto-rate capability of nodes, even when auto-rate capability is enabled, the highest loss rate link could be the bottleneck link because the highest loss rate link is likely to lower data rate to increase its own loss rate. The lowered data rate can eventually exacerbate the achievable data rate of multi-hop ad hoc paths. We will quantify the impact of contention window size on the data rate in Section V-D. Consequently, we use the highest loss rate link as the bottleneck link of an ad hoc path.

Note that we are not looking at the highest loss rate link ( $k'$ ) in isolation. As noted in [3], the bottleneck loss rate by itself cannot be used as a metric for ad hoc path performance. In our metric, we also take the total transmission interference around the bottleneck link ( $I(k')$ ) into account.

Let  $E_{max}$  and  $I$  denote the ETX and the total transmission contention degree of the highest loss rate link respectively. We define the initial *Expected Data Rate* (EDR) of an ad hoc path as shown in Equation (4).

$$EDR_{init} = \frac{\Gamma}{E_{max} \times I} \quad (4)$$

The actual achievable data rate of a one-hop link is reduced due to several other overheads including packet overhead for control fields, backoff procedure overhead, and the use of RTS/CTS/DATA/ACK or DATA/ACK mechanism. Earlier work has quantified the reduction in achievable data rate of a one-hop link by accounting for these overheads [26]–[28]. Let

$r$  be the reduction in one-hop link data rate. By taking this  $r$  into account, we redefine  $EDR_{init}$  as  $EDR_r$  in the following equation.

$$EDR_r = \frac{r\Gamma}{E_{max} \times I} \quad (5)$$

TABLE III  
COMPARISON OF ETX SUM,  $EDR_r$  AND  $ns-2$

Path	E(1)	E(2)	ETX sum	$EDR_r$	$ns-2$
1	1.5	1.5	3.0	2.02 Mbps	1.98 Mbps
2	1.0	2.0	3.0	1.51 Mbps	0.84 Mbps
3	2.0	1.0	3.0	2.02 Mbps	1.58 Mbps
4	1.3	1.7	3.0	1.78 Mbps	1.56 Mbps
5	1.7	1.3	3.0	2.02 Mbps	1.82 Mbps

Table III shows the comparison of throughput of ad hoc paths of Fig. 4 using ETX sum,  $EDR_r$ , and  $ns-2$  simulated throughput. In these experiments,  $\Gamma$  is 11 Mbps, and for packet sizes of 1500 bytes with a DATA/ACK mechanism,  $r$  can be approximated fairly accurately by 0.55 [27], [28]. As shown in Table III, although  $EDR_r$  is able to differentiate the worst path (Path2) and the second worst path (Path4) in contrast to ETX sum, there are still three paths (Path1, 3, and 5) that have the same  $EDR_r$  but different  $ns-2$  data rates. The main reason for the discrepancy between  $EDR_r$  and  $ns-2$  results is that  $EDR_r$  does not consider different contention window sizes. We refine  $EDR_r$  by considering the effect of contention window size in the next section.

### D. Adjusted TCD With Contention Window Size

In the last section, we assumed that the contention window size does not change with transmission failures. This meant that every node in the ad hoc path could occupy the shared wireless medium with equal probability regardless of the wireless link loss. However, according to IEEE 802.11 DCF, the contention window size of each node increases exponentially with transmission failures. Although  $TCD(k)$  in Equation (1) can represent how busy a link  $k$  is in transmitting data including retransmissions due to loss rates, the actual transmission contention degree of a link  $k$  or  $k+1$  can be relatively higher when adjacent nodes have different loss rates. This is because different loss rates amount to different contention window sizes. This relative difference in contention window sizes causes additional reduction in data rates on the link ( $k$  or  $k+1$ ) with a higher loss rate. In this section, we incorporate *relatively increased contention degree (RTCD)* into  $EDR_r$  by quantifying the impact of relative contention window size on data rates of ad hoc paths.

Note that when a node senses that the wireless medium is busy, it does not change its current contention window size. Rather, only when it fails to receive an acknowledgment due to a loss, it doubles the contention window size<sup>1</sup>. This char-

<sup>1</sup>Contention window sizes can also be doubled due to collisions. However, in our experiments we find that the probability of collision is typically much smaller than the probability of packet loss.

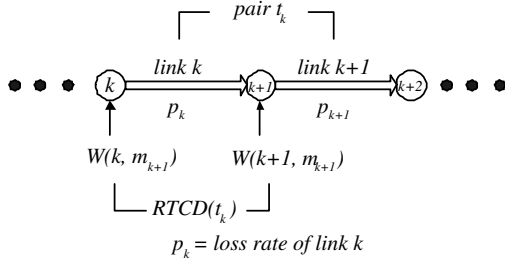


Fig. 5. RTCD between two adjacent links

acteristic is different from some CSMA/CA medium access protocol implementations such as one in the Lucent Wavelan cards, where a node doubles its current contention window size whenever it senses the medium busy [29]. According to IEEE 802.11 DCF, the contention window size of a node depends only upon its link loss rate (or collisions). We will stick to this interpretation in this paper.

Consider a scenario where  $p_k$  and  $p_{k+1}$  are the loss rates of adjacent links  $k$  and  $k+1$ , respectively, as shown in Fig. 5. When  $p_k$  and  $p_{k+1}$  are different, nodes  $k$  and  $k+1$  have different contention window sizes.

First, suppose that  $p_k \geq p_{k+1}$ . When nodes  $k$  and  $k+1$  contend for the shared wireless medium, node  $k$  loses out to node  $k+1$ . This is because whenever node  $k$  gets a chance to transmit it is more likely to fail and thereby increase its contention window. Whereas, whenever node  $k+1$  gets a chance to transmit, it is likely to be successful and thus able to keep its contention window small. Interestingly, this relative difference in contention window could result in node  $k+1$  enjoying some idle time when it does not receive any packets from the previous node that it must forward towards the ad hoc path destination. Since the contention window size of the node  $k+1$  increases until a successful transmission, we focus on determining the number of transmission attempts (including the first transmission) with which it is highly likely to be successful in transmitting a packet for the loss rate  $p_{k+1}$ . This number of attempts, denoted by  $m_{k+1}$ , from node  $k+1$  depends upon the loss probability  $p_{k+1}$ . We would like to find the average contention window size, at node  $k+1$ , at which node  $k+1$  stops contending with node  $k$  as a function of  $m_{k+1}$ . On average, it is at this window size that node  $k$  gets a chance to transmit without contention from node  $k+1$ . This window size of node  $k+1$  is then compared with the average window size of node  $k$  to determine the RTCD.

Let  $s(k+1, j)$  denote the probability that a node  $k+1$  can successfully transmit a packet on the  $j^{th}$  transmission attempt, assuming that each retransmission is independent. Then,  $s(k+1, j)$  can be represented by Equation (6).

$$s(k+1, j) = (1 - p_{k+1})p_{k+1}^{j-1} \quad (6)$$

Then, we can find  $m_{k+1}$ , the number of transmission attempts on links, which is a minimal number required to satisfy the

following inequality.

$$\sum_{j=1}^{m_{k+1}} s(k+1, j) > \alpha \quad (7)$$

where  $\alpha$  is a number close to  $1^2$ .

Let  $W(k, m_{k+1})$  be the average contention window size of node  $k$  until  $m_{k+1}$  transmission attempts. Likewise, let  $W(k+1, m_{k+1})$  denote the average contention window size of node  $k+1$  until  $m_{k+1}$  transmission attempts. Then, the value of  $W(k, m_{k+1})/W(k+1, m_{k+1}) - 1$  can be viewed as the RTCD on link  $k$ . If  $W(k, m_{k+1})$  and  $W(k+1, m_{k+1})$  are equal, there is no relative increase in transmission contention. That is why the ratio of two average contention window sizes needs to be subtracted by 1.

Similarly, when  $p_k < p_{k+1}$ , the value of  $W(k+1, m_{k+1})/W(k, m_{k+1}) - 1$  can be viewed as the RTCD on link  $k+1$ . The formula for determining  $W(k+1, m_{k+1})$  and  $W(k, m_{k+1})$  is presented in Appendix A.

More formally, let  $RTCD(t_k)$  denote the RTCD of a pair  $t_k$  of adjacent links  $k$  and  $k+1$ . Then,

$$RTCD(t_k) = \begin{cases} (\frac{W(k, m)}{W(k+1, m)} - 1) \times TCD(k) & \text{if } p_k \geq p_{k+1} \\ (\frac{W(k+1, m)}{W(k, m)} - 1) \times TCD(k+1) & \text{if } p_k < p_{k+1} \end{cases} \quad (8)$$

The transmission interference is refined by adding RTCD to  $I$  as shown in Equation (9). Here,  $I_b$  denotes the total transmission interference around the highest loss rate link  $k'$ , considering the effects of medium access control backoff.

$$I_b = I + \sum_{i=t_s}^{t_e} RTCD(i) \quad (9)$$

where  $t_s$  is the first pair of links,  $n_s$  and  $n_s+1$ , and  $t_e$  is the last pair of links,  $n_e-1$  and  $n_e$ . Recall that  $n_s$  and  $n_e$  are the links within the interference range of the highest loss rate link  $k'$ .

We refine  $EDR_r$  by incorporating the effects of relatively increased transmission contention degree as follows.

$$EDR_b = \frac{r\Gamma}{E_{max} \times I_b} \quad (10)$$

Here,  $EDR_b$  denotes the refined  $EDR_r$ .

Table IV shows all the values of  $EDR_b$  for the five two-hop ad hoc paths in Fig. 4 when  $\alpha = 0.9$ . We find that the throughput obtained using  $EDR_b$  very accurately match the throughput obtained by  $ns-2$  simulations.

### E. Interference Range

To determine  $EDR_b$  for an ad hoc path we must determine the range  $[n_s, n_e]$  of neighbor nodes that interfere with the highest loss rate link. Recall that  $n_s$  and  $n_e$  are respectively the start and end nodes of the range. Whether or not a

<sup>2</sup>Note that, in IEEE 802.11 DCF, the maximal value of  $m_{k+1}$  is 7, because a packet is discarded after its  $6^{th}$  retransmission.

TABLE IV  
RESULTS OF  $EDR_b$

Path	E(1)	E(2)	$I_b \times E_{max}$	$EDR_b$	$ns-2$
1	1.5	1.5	3.00	2.02 Mbps	1.98 Mbps
2	1.0	2.0	7.00	0.87 Mbps	0.84 Mbps
3	2.0	1.0	4.00	1.52 Mbps	1.58 Mbps
4	1.3	1.7	3.93	1.55 Mbps	1.56 Mbps
5	1.7	1.3	3.25	1.87 Mbps	1.82 Mbps

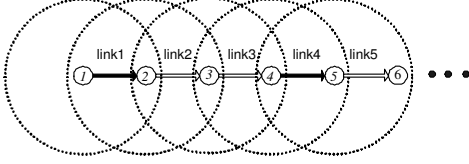


Fig. 6.  $IR=TR$

neighbor node interferes with a given node  $k$  depends upon the interference range of wireless networks. In general, it is known that interference range,  $IR$ , of a node is larger than or equal to transmission range,  $TR$ . We define this relationship as  $IR = cTR$  ( $c \geq 1$ ).

In order to look into the effect of interference range in our path metric, we first conduct basic experiments with zero loss ad hoc paths, shown in Fig. 6. Since the loss in every link is zero, the TCD of each link is one. Thus, the transmission interference effects described in the last two sections are excluded in these zero loss scenarios.

In Fig. 6, where  $IR = TR$ , nodes 1 and 4, can potentially transmit on links *link1* and *link4* (shown in bold black) *simultaneously*. Likewise, in case of  $IR = 2TR$ , as shown in Fig. 7, node 5 can transmit a packet on *link5* at the same time as node 1 transmits a packet on *link1*. However, each node executes its media access algorithm *independently*. This independence results in non-optimal use of the shared wireless medium. For example, if *link4* in Fig. 6 does not transmit packets at the same time as *link1* then the achievable data rate can be reduced.

In order to analyze the non-optimal scheduling of the shared wireless media as a function of the hop length of ad hoc paths, we simulate IEEE 802.11 DCF ad hoc paths with

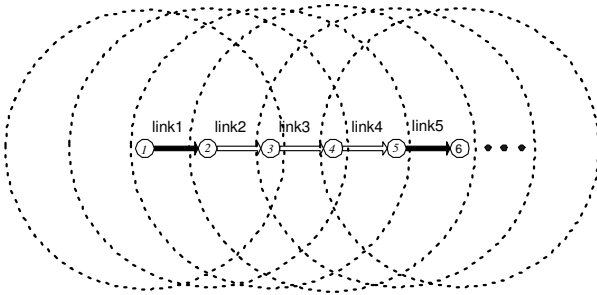


Fig. 7.  $IR=2TR$

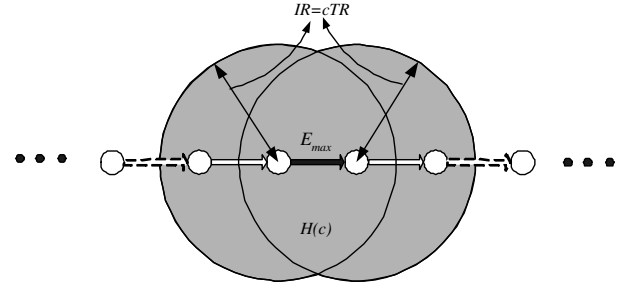


Fig. 8. Range of neighbor nodes around the highest loss link

increasing hop lengths in  $ns-2$ . We summarize the results of our simulations in Table V. In this table, “RF” stands for the Reduction Factor of each ad hoc path. RF is the ratio of the one-hop achievable data rate ( $r\Gamma = 6.07$  Mbps, where  $r$  from Section V-C is  $\approx 0.55$  and  $\Gamma$  is 11 Mbps) to the throughput of each path.

TABLE V  
RESULTS OF  $ns-2$  SIMULATION

Len	IR=TR		IR=2TR		ETX Sum
	Throughput	RF	Throughput	RF	
1	6.07 Mbps	1.00	6.07 Mbps	1.00	1.0
2	3.01 Mbps	2.01	3.01 Mbps	2.01	2.0
3	2.11 Mbps	2.87	2.04 Mbps	2.98	3.0
4	1.68 Mbps	3.61	1.54 Mbps	3.95	4.0
5	1.32 Mbps	4.59	1.24 Mbps	4.90	5.0
6	1.24 Mbps	4.89	1.06 Mbps	5.72	6.0
7	1.18 Mbps	5.14	0.93 Mbps	6.54	7.0

First, consider the case of  $IR = TR$ . When the hop length of an ad hoc path is greater than 4, theoretically, the reduction factor should be 3. However, due to the non-optimal use of the shared medium this is not the case as shown in Table V. We see that the reduction factor of a four-hop path is 3.61. The reduction factor of a five-hop path is 4.59. Now if we use the ETX sum as the path metric, the reduction factor is simply given by the ETX sum. We observe that there is a gap between the simulated reduction factors and the ETX sum. This gap suggests that *some* concurrent transmissions do take place. We observe a similar behavior for  $IR = 2TR$ .

Using the above observations for no loss ad hoc paths, we now describe a heuristic to determine the range  $[n_s, n_e]$ . Consider the ad hoc path shown in Fig. 8. Let the link shown in dark black be this ad hoc path’s highest loss link with ETX value  $E_{max}$ . Let  $H(c)$  denote the union of the interference ranges of two nodes on the  $E_{max}$  link (link  $k$ ) where  $c$  is the ratio of the interference range to the transmission range (i.e.,  $IR = cTR$ ). Now, any node along the ad hoc path inside the area of  $H(c)$  could cause interference on the the  $E_{max}$  link. Ideally, with optimal media scheduling, the nodes in the area  $H(c)$  should construe the range  $[n_s, n_e]$ . However, as observed earlier, the media scheduling has somewhat more indirect

transmission interference from the nodes that are outside  $H(c)$ . In order to account for such non-optimal media scheduling we conservatively extend the *effective* interference range by another  $cTR (=IR)$ . We do not have a concrete reasoning for this range extension. However, intuitively, we believe that some links at the periphery of  $H(c)$  can *indirectly* interfere with the  $E_{max}$  link. Thus,

$$[n_s, n_e] = \text{All nodes in } (H(c) + cTR) \quad (11)$$

We use this range in Equation (9) to finally obtain our EDR path metric.

## VI. AD HOC ROUTING USING EDR

Our EDR metric could be easily incorporated in ad hoc routing protocols such as Dynamic Source Routing (DSR), and Ad hoc On-demand Distance Vector Routing (AODV). As an example, we describe how EDR can be incorporated in DSR. DSR is a reactive ad hoc routing protocol where a node initiates a *route request* to neighboring nodes when it needs to find a path to transmit data packets to a destination node. The route request is flooded on the network and forwarded by the intermediate ad hoc nodes towards the destination node. The intermediate nodes attach their addresses to the route requests and update their hop counts before forwarding them. Once the route requests reach the destination node it uses the hop counts to determine the shortest path and returns the route request to the source. EDR could be easily incorporated in the DSR mechanism. For using EDR to find highest throughput paths we just need to append ETX values (or link loss rates) of each link to the route request (in addition to the intermediate hop address). Note that appending ETX of each link in an ad hoc path to a route request message has more overhead than computing a running ETX sum of the links as proposed in [3]. EDR needs more space in the route request packet. This additional space is proportional to the ad hoc path hop length. However, since the route requests already require  $O(n)$  space to store the addresses of the intermediate nodes of an  $n$ -hop ad hoc path, this space requirement of EDR only slightly increases the overall space requirements as  $n$  increases. WCETT [4] also uses a 32-bit link-quality metric for each hop in route-related packets. In comparison to ETX sum [3], EDR also requires more processing at the destination nodes. This additional processing is in  $O(n)$  where  $n$  is the ad hoc path length. We believe that the relative increase in the processing cost is not too high.

In [3], the authors have proposed an optimization of their routing algorithm where an intermediate node restricts the forwarding of duplicate route request messages that arrive from different neighboring nodes but are meant for the same destination node. An intermediate node forwards a duplicate route request only if its ETX sum is lower than the route request it forwarded earlier. We could use the same optimization in EDR as well but incur a slightly higher processing costs at the intermediate nodes.

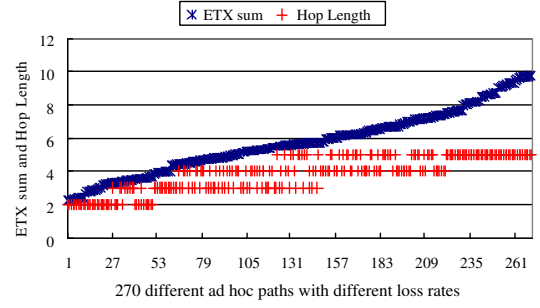


Fig. 9. 270 randomly generated ad hoc paths

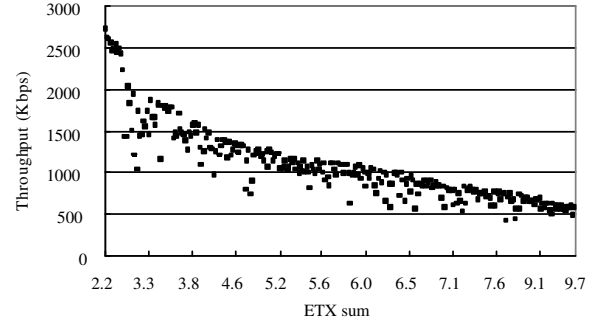


Fig. 10. Throughput in  $ns-2$  simulation

## VII. EVALUATION

In this section, we evaluate the performance of *EDR* using extensive  $ns-2$  simulations [30]. We slightly modify the current implementation of DCF [31] in  $ns-2$  so that an ad hoc node *does not double* its current contention window size when it senses the medium busy at the first transmission. This modified mechanism complies with IEEE 802.11 DCF, as opposed to one in the Lucent Wavelan cards.

To study the performance of *EDR*, we implement two loss models - one where loss events are independent, another where loss is temporally correlated. A two-state Markov chain is used to model the temporally correlated loss.

### A. Test Cases

For a thorough evaluation of EDR, we construct a variety of ad hoc paths with different hop lengths and link loss rates. We randomly generate 270 ad hoc paths whose hop lengths range from 2 to 5 and link loss rates range from 0% to 50% (correspondingly the link ETX range from 1 to 2). We summarize the properties of these paths in Fig. 9 and Fig. 10. Fig. 9 shows the ETX sum and hop length of each path. For an  $n$ -hop ad hoc path, the ETX sum of the ad hoc path varies between  $n$  and  $2n$ . Fig. 10 shows the  $ns-2$  path throughput for each ETX sum. As expected, we find that smaller ETX sum does not always mean higher throughput.

We now create subsets of the set of all ad hoc paths. Our goal is to determine the  $ns$  throughput of the best path in



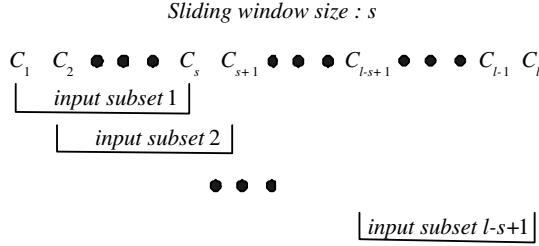


Fig. 11. Input cases

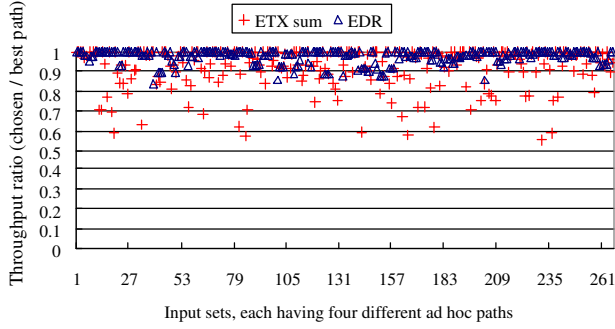


Fig. 12. Performance comparison between ETX sum and EDR

each subset and compare it with the throughput of the paths selected by ETX sum and by EDR.

Assume that there are totally  $l$  different ad hoc paths denoted by  $C_1, \dots, C_l$ , as shown in Fig. 11. An input subset with  $s$  ad hoc paths can be constructed by sliding a window of size  $s$  over the  $l$  ad hoc paths. The first  $s$  ad hoc paths in Fig. 11 compose the first input subset. The total number of input sets is  $l - s + 1$ . In our experiments,  $l = 270$  and we choose  $s = 4$ . Therefore, the total number of input subsets is equal to 267. The window size of 4 implies that there are 4 ad hoc paths to choose from in making a routing decision.

### B. Independent Loss

We first evaluate the performance of EDR over ETX sum under an independent link loss model. In all experiments, unless specified, the send rate of a source node of an ad hoc path is 11 Mbps, the maximal one-hop data rate of IEEE 802.11b. UDP packets of 1500 bytes are used to generate the traffic at each source node. We show the results of EDR performance when  $IR = 2TR$ . We use 250  $m$  for the interference range and 125  $m$  for the transmission range. The Cartesian distance between any two nodes is 100  $m$ .

Fig. 12 and 13 show the performance comparison between EDR and ETX sum. In Fig. 12, the x-axis represents the input subset number. The y-axis represents the ratio of the throughput of the chosen ad hoc path by EDR or ETX sum to the throughput of the best ad hoc path for each subset containing four ad hoc paths. We observe that EDR finds the best paths for most of the input sets. In contrast, there are many cases where ETX sum finds paths whose throughput is much less than the best path throughput.

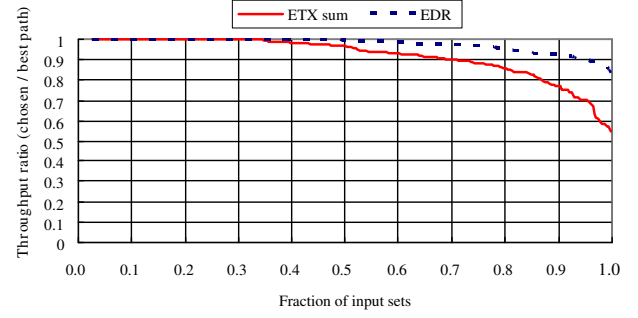


Fig. 13. Performance comparison between ETX sum and EDR based on the fraction of input sets

Fig. 13 shows how the throughput ratio, throughput of the chosen ad hoc path by EDR or ETX sum over the throughput of the best ad hoc path, varies with the fraction of the input subsets. For each curve (ETX sum or EDR), the x-axis represents the fraction of the input cases sorted by decreasing value of the throughput ratio. For example,  $x = 0.2$  represents 20% of the 267 input cases when the ETX sum (or EDR) throughput ratios are sorted in decreasing order.

We observe that when ETX sum is used for finding the best ad hoc paths, 30% of the input cases (corresponding to  $x \geq 0.7$ ) have a throughput less than 90% of the throughput of the best ad hoc paths. The relative throughput of the paths obtained from ETX sum reduces with increasing  $x$ . In some cases ETX sum even selects ad hoc paths that have less than 60% of the throughput of the best paths. In contrast, EDR is able to find ad hoc paths that have more than 80% of the throughput of the best ad hoc paths in all the input cases. Moreover, for 90% of the input cases, it can select ad hoc paths that have more than 90% of the throughput of the best ad hoc paths. When the input sets are constructed with a larger sliding window size, we find (although not shown here) that EDR performance is even better.

### C. Temporally Correlated Loss

To study the effect of a temporally correlated loss, we model packet burst loss in wireless links using a two-state continuous-time Markov chain  $\{X_t\}$  where  $X_t \in \{0, 1\}$  [32]. If  $X_t = 1$ , a packet is lost at time  $t$  and not lost if  $X_t = 0$ . In [33], the authors have observed significant temporal correlation but little spatial correlation in packet loss in their experimental wireless LAN setup. In our work, we use the experimental results of [33] for choosing the burst and arrival rate parameters described below.

The Markov chain  $\{X_t\}$  can be described by the following generator matrix.

$$Q = \begin{pmatrix} -\mu_0 & \mu_0 \\ \mu_1 & -\mu_1 \end{pmatrix}$$

The stationary distribution is determined by  $\pi = (\pi_0, \pi_1)$ , where  $\pi_0 = \mu_1 / (\mu_0 + \mu_1)$  and  $\pi_1 = \mu_0 / (\mu_0 + \mu_1)$ . The parameters of  $\mu_0$  and  $\mu_1$  for this model are determined in

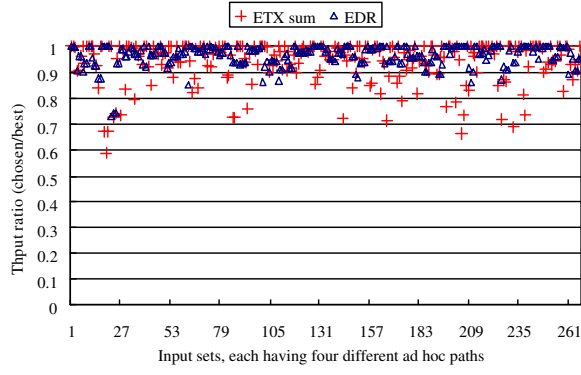


Fig. 14. Performance comparison of EDR and ETX sum in the presence of temporally correlated loss

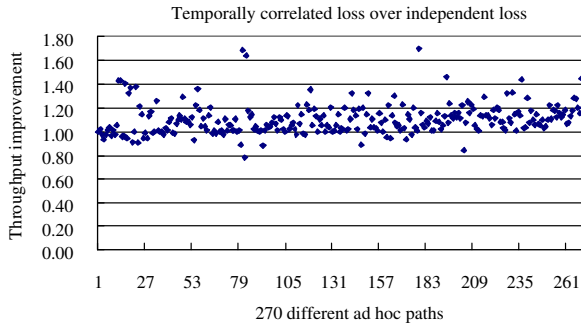


Fig. 15. Performance improvement in a temporally correlated loss

Equation (12). Let  $\lambda$  be packet transmission time rate,  $\bar{b}$  be the expected burst loss length in packets, and  $p$  be packet loss rate.

$$\begin{aligned}\mu_0 &= -p\lambda \log(1 - 1/\bar{b}) \\ \mu_1 &= -(1 - p)\lambda \log(1 - 1/\bar{b})\end{aligned}\quad (12)$$

Based on the results of [33], we choose  $\bar{b} = 1.6$  (Figure 2 from [33]) and  $\lambda = 1/(2.1ms)$ .

Fig. 14 shows how well EDR finds the best paths for each input subset of four ad hoc paths in the presence of temporally correlated packet loss. EDR is still able to find the best ad hoc paths in most input sets, however it occasionally underestimates the path throughput. In order to understand why EDR underestimates path throughput, we plot in Fig. 15 the relative change in the throughput of each ad hoc path (270 in all) when a temporally correlated loss model is used instead of an independent loss model, while keeping the average loss same. Interestingly, for most ad hoc paths, the throughput is improved. Investigation of this behavior will be an important future work.

## VIII. CONCLUSIONS

In this paper, we proposed a new metric, EDR, for accurately finding high-throughput paths in multi-hop ad hoc

wireless networks. This metric is based upon our new transmission interference model that considered the transmission interference degree of each link, the impact of medium access backoff procedure, and possible concurrent transmissions. We used an independent loss model and a temporally correlated loss model for simulating wireless link loss. Using  $ns-2$  simulation, we showed that EDR performs very well and finds best ad hoc paths in most of the cases.

Although EDR found the best paths in the presence of temporally correlated loss, it underestimated the path throughput in some cases. We would like to further investigate ad hoc network performance under temporally correlated loss and refine EDR to remove this inaccuracy. We also plan to implement EDR in the extended Emulab testbed [34].

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attempt,

$$\begin{aligned}
 W(k, m_{k+1}) &= \sum_{j=1}^{m_{k+1}} (s(k, j) \times \frac{w(j)}{2}) \\
 &+ \sum_{j=m_{k+1}+1}^{\infty} (s(k, j) \times \frac{w(m_{k+1})}{2}) \\
 &= \sum_{j=1}^{m_{k+1}} (s(k, j) \times \frac{w(j)}{2}) \\
 &+ p_k^{m_{k+1}} \times \frac{w(m_{k+1})}{2} \tag{14}
 \end{aligned}$$

## APPENDIX

### A. Average Contention Window Size

In this appendix, we derive the expression for  $W(k, m_{k+1})$ . Recall from Section V-D that  $W(k, m_{k+1})$  is the average contention window size of node  $k$  until  $m_{k+1}$  transmission attempts.

Let  $w(j)$  denote the contention window size of a node on the  $j^{th}$  transmission attempt including the first transmission. Then, the average selected backoff counter value on the  $j^{th}$  transmission attempt is  $w(j)/2$ . Let  $C_{min}$  ( $=w(1)$ ) be the initial minimum contention window size. When the node  $k$  fails to transmit a packet in the first attempt, it doubles its contention window size, and selects a random number as a backoff counter within  $[0, 2 \times C_{min}]$ . Equation (13) represents the expression for the contention window size on the  $j^{th}$  transmission attempt.

$$w(j) = C_{min} \times 2^{j-1} \tag{13}$$

Assuming that the contention window on links  $k$  or  $k + 1$  does not increase any more after the  $m_{k+1}^{th}$  transmission