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A fuzzy and bipolar approach to preference modeling with application to need and desire

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Abstract

Fuzziness and bipolarity allow representing human knowledge, taking into account the gradual and the dialectic properties of language, focusing on the meaning of concepts. Under this cognitive and linguistic approach, we explore preference relations, examining their semantic decomposition through fuzzy preference structures and the specification of meaningful opposites. In particular, we introduce the Preference–Aversion (P–A) model, which allows analyzing, under an independent aggregation methodology, the possible gains and losses, like *pros* and *cons*, towards a given set of alternatives. As an attractive feature of this proposal, we show that the P–A model allows distinguishing between need and desire, contrary to common preference models where both notions are indistinguishable.

Keywords: Preference representation; Decision making

1. Introduction

Standard preference models usually assume a decomposition of individual preferences, each one being basically conceived as a one-dimensional concept. For example, the weak preference binary relation can be decomposed in terms of basic relations, such as strict preference, indifference, incomparability or ignorance. While incomparability suggests the existence of underlying and conflicting criteria, ignorance suggests the existence of undiscovered information, but the only immediate consequence of incomparability and ignorance is a certain doubt about the reliability of the subsequent decision.

From the classical perspective of economic utility theory [6,39,47], the individual's decision rests on maximizing the satisfaction of desire, according to the reasons existing for preferring one alternative over another, where no satiation is possible. Therefore, such wishful thinking regarding desire finds its limits on an aggregated level, as an economic equilibrium, only because of the conflict between individuals and their blocking strategies (see e.g., [28,32]). But a socioeconomic equilibrium should also be possible, based on the particular condition of self-control of the individuals who have the capacity of finding their own limitations, guided only by their *desires* and *needs* (as argued in [16,20]).

Motivated by this intuition, we consider an alternative approach, examining the *meaning* of the preference predicate by means of a *bipolar* approximation (see, e.g., [5,22,31] but also [8,18]), allowing the introduction of at least two families

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of arguments (like *pros* and *cons*). Notice that such a bipolar approach can be understood as a particular decomposition analysis, where information is received and then organized by means of a particular preference structure (see e.g., [12]), which defines a semantic frame according to two different reference poles, one of them with a positive charge and the other with a negative one. In this way, it is possible to frame the alternatives in terms of *gains* and *losses*, following the terminology of Cumulative Prospect Theory [21,45], although not necessarily in a monetary sense.

Here, we represent an intelligent individual by means of the *gains–losses rationality*, which allows distinguishing between arguments in favor and against possible decisions. This basic postulate is supported by relevant results in distinct scientific fields such as psychology [5,22,31], neuroscience [30,48] economics [21,45] and philosophy [19,41]. But let alone, such a bipolar approach is not always sufficient for identifying one optimal solution. Therefore, we propose a methodology for obtaining such a solution, by focusing on what is needed apart from what it is desired.

Based on the *negative quality of justice* [41], the negative or damaging aspects of things have a general and heavier connotation than the positive ones, due to the fact that the detrimental attributes can be universally stated as anything that stands against life or any of its vital qualities. In this sense, need refers to goods or conditions that are imperative because the lack of them would be harmful. Here, it is proposed that from this basic distinction, an intelligent individual is able to find a personal state of equilibrium given by the *necessary course of action* (one, out of other possible courses of action).

To do so, this paper is organized as follows. In Section 2, we review classical preference modeling [37,39,47], where preference, desire and need are taken as equivalent. In Section 3, we examine fuzzy preference structures [10,27,46], where more than one basic relation can be used for describing the cognitive state of the individual facing a decision problem. Then, in Section 4, we introduce the Preference–Aversion (P–A) model [13,14], where it is possible to treat preference and aversion intensities in an independent manner, such that the epistemic state of *ambivalence* [17,22] can be represented in a non-symmetric way. Finally, in Section 5, we use the P–A model to distinguish between desire and need, following our proposal for need-preference structures.

2. Basic concepts on preference modeling

This section introduces the preference predicate as it is represented by binary preference relations. We review some basic features of preference modeling, pointing out the role of bipolarity and desire in the specification of preference structures.

2.1. Binary preference relations

The preference predicate usually refers to the priority of one object of interest over another, based on a general perception of value. In this sense, the perception of preference allows ordering some given set of alternatives according to their degree of satisfaction. Hence, standard preference models build a certain order over a given set of alternatives by means of pair-wise comparisons between its elements, by means of binary preference relations (see e.g., [37]).

Recall that a binary relation R is a subset of $A \times A$, such that for every pair $(a, b) \in A \times A$, $R(a, b)$ basically represents the particular relation that exists between a and b. In this way, for any given pair $(a, b) \in A \times A$, the preference relation $R(a, b)$ represents the preference predicate "a is at least as *preferred* as b". Here we assume that the individual evaluates the intensity in which the predicate R holds for every $a, b \in A$.

Then, the order that R induces over A is its *primary meaning*. That is, the primary use that is given to the predicate R refers to the ordering that it assigns on the given set of alternatives A. From this first and basic observation, it is natural to take the intensity in which such predicate can be verified as its *secondary meaning* (following the general approach of [42]). Hence, under classical logic, such intensity is absolute, equal to 1, or inexistent, equal to 0, such that no gradualism is allowed.

In classical decision theory, take e.g., economic utility theory [6,39,47], the individual's decision rests on maximizing the satisfaction of preference or desire. In this way, preference and desire are taken as synonyms. Hence, the negative aspects towards the alternatives are not considered, and the component of need cannot be included in the individual's decision process. Such process by which the preference order is obtained results in the identification of an optimal choice, ignoring the harmful aspects of the alternatives and focusing on the insatiable quality of desire, which is at the same time the foundation for the development of empirical demand analysis [1,6].

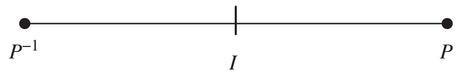

Fig. 1. The preference structure $R = \langle P, I\rangle$.

Now, we examine standard preference modeling and the decomposition of the preference predicate into particular structures. Here we explore two commonly used preference structures [37], where, as we have mentioned, the need component of preference is excluded and only the desire component of preference is considered.

2.2. Preference structures

From a historical perspective, preference orders are in the very epistemological foundation of the economic theory of utility under uncertainty (see, e.g., [39,47]), the core of classical decision theory. Such a theory generally assumes the existence of a *total* or *weak* order over the set of alternatives (also referred to as *lotteries* in [47], *acts* in [39] or *prospects* in [21]), where the preference predicate "at least as *preferred* as" is equivalent to "at least as *desired* as" (see e.g., [6]).

For example, under a *total order*, it is assumed that for every $a, b \in A$, one and only one relation from *strict preference* (P), its inverse (P^{-1}) or *indifference* (I) occurs, being P transitive and I an equality. The same happens under a *weak order*, only indifference is defined as an equivalence relation, i.e., reflexive, symmetric and transitive.

Hence, under the *classical preference structure* $R = P \cup I = \langle P, I\rangle$, the meaning of R is built regarding the opposite poles P and P^{-1}. In this sense, P stands for the positive pole and P^{-1} stands for the negative pole of R. Then, the decision situation can be represented by either P, P^{-1}, or in case the alternatives verify both R and R^{-1}, by I (see Fig. 1). Notice that this model makes reference only to the desire component of preference, even if the negative aspects regarding one alternative are assumed to be equivalent to the positive aspects regarding the other one. In this way, need is equivalent to that which is desired or preferred ($P \cup I$), provided that P^{-1} does not hold.

But such classical preference structure seems incomplete, given that no situation exists for identifying when neither R nor R^{-1} can be verified. So, if we define a *partial order*, such that at most one situation from P, I or P^{-1} can be satisfied, then *incomparability* (J) can be defined by exclusion, such that J occurs if none of the other situations occur. In this way, we are able to distinguish between a situation where both alternatives are equally desired (I) and a situation where they cannot be compared (J). This situation of incomparability emerges every time that the decision situation does not exactly agrees with one and only one of the situations represented by P, I or P^{-1}.

Taking into account the basic relation of incomparability (J) allows the specification of the *complete preference structure* $R = \langle P, I, J\rangle$, which allows modeling the subjective decision process in a less restrictive way than the classical approach of the first utility models. Notice that economic utility and general equilibrium theory has continued developing in this direction (see e.g., [2,23]), where the basic economical concepts survive given a partially ordered set of alternatives.

Hence, the complete preference structure $R = \langle P, I, J\rangle$ allows building the meaning of R regarding the reference poles P and P^{-1}, only now two semantic situations of indecision or hesitancy are identified. In this sense, as it has been mentioned before, I holds when both R and R^{-1} are verified, as in the classical case, and J holds when neither R nor R^{-1} can be verified (see Fig. 2).

Under this approach, it is assumed that the comparison between every pair of alternatives generates four basic relations that compose the complete preference structure, given by indifference I (a symmetric and reflexive relation), strict preference P (an asymmetric and irreflexive relation), its inverse, and incomparability J (a symmetric and irreflexive relation). In this context, the relation R is decomposed into three basic relations as follows [37]:

$$P = R \cap R^d, \tag{1}$$

$$I = R \cap R^{-1}, \tag{2}$$

$$J = \neg R \cap R^d, \tag{3}$$

where $\neg R$ is the complementary of R, such that $\neg R = 1 - R$, and R^d is the dual of R, such that $R^d = \neg R^{-1}$.

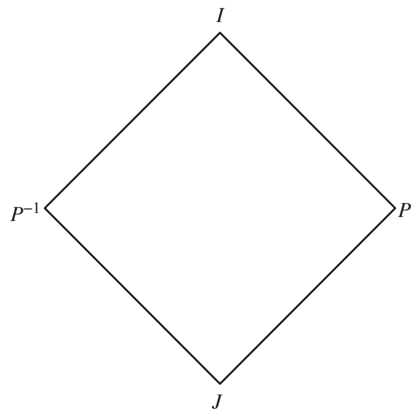

Fig. 2. The preference structure $R = \langle P, I, J \rangle$.

Hence, under this Boolean setting, the complete preference structure satisfies the three following properties:

$$P \cup I = R, \tag{4}$$

$$P \cup I \cup P^{-1} = R \cup R^{-1}, \tag{5}$$

$$P \cup J = R^d. \tag{6}$$

Given this basic framework, we emphasize that the basic relational situations defined in (1)–(3) represent possible epistemic states in the following sense. If P or P^{-1} are obtained, then a clear decision can be observed in favor of one of the alternatives. Instead, if I or J are obtained, a certain kind of *decision neutrality* is identified, where I reveals a certain *compatibility* between the desirable attributes of the alternatives, while J expresses a certain *incompatibility* between them.

Notice that as in the classical case, P stands for the positive pole and P^{-1} stands for the negative pole of R. Again, we point out that such opposite poles are subject of a particular interpretation, given that both P and P^{-1} refer to equally positive attributes when comparing any pair of alternatives in A. But it is observed that when thinking of *opposites*, the distinction and confrontation of positive and negative poles suggests considering distinct negative attributes separately from positive ones (as pertaining to different sources).

In this sense, the complete preference structure continues ignoring the need component of preference, i.e., that which is desired but is not harmful, like in the classical case. This is, because the only possibility where *pseudo-negative* aspects can be measured is the inverse valuation, something that certainly refers to the same kind of desirable attributes regarding the alternatives.

It seems that for a better understanding of the individual's cognitive space facing a decision problem, we have to focus on more meaningful preference structures. Hence, developing different semantics over the meaning of the preference predicate, nearer to the known (bipolar) attributes of the brain (see e.g., [30,48]), it can be possible to differentiate between the preferred and rejected choices of action, such that the needed ones can be properly identified.

If we focus on the negative as well as the positive aspects towards a decision, we may refer to their simultaneous existence as a situation of ambivalence. Such situation of ambivalence occurs any time that opposites coexist. In this way, we have to follow a methodology that allows representing such natural ambiguity/ambivalence of common life problems under the same relational approach (e.g., where P and P^{-1} can be simultaneously verified).

So, in order to examine such methodology, where more than one basic situation can be verified, we review fuzzy preference modeling. In this way, under the gradual frame of fuzzy set theory [49], the alternatives satisfy the properties of the preference predicate according to a continuous valuation scale L, which will be taken as the interval [0,1]. Hence, basic situations receive different intensities from the unitary interval, allowing the verification of more than one basic relation from the complete preference structure.

3. Fuzzy preference relations

In this section we examine the preference predicate and its representation under a fuzzy logic approach. We explore fuzzy preference relations, such that a gradual and combined verification of their properties can be developed, maintaining a relational treatment of the information. Here we review the *standard fuzzy preference model* (see e.g., [10]) and explore its particular preference semantics.

3.1. The preference predicate under a fuzzy environment

The genealogy of fuzzy sets follows a basic argument on what is referred to as *humanistic* systems (see [49,50]). Such systems are strongly influenced by human judgements, perceptions or emotions, as for example the role of the individual's decision process in the political, social or economical system. Therefore, in order to understand the complexity of human thought processes and decision-making, the standards of rigor and precision of the mathematical analysis, based on mechanistic systems, can be extended by more tolerant approaches which are gradual in nature [26].

Then, a fuzzy preference binary relation for a set of alternatives A is characterized by the fuzzy set

$$R(a,b)=\{\langle a,b,\mu_R(a,b)\rangle | a,b\in A\},$$

where $\mu_R(a,b):A\times A\to[0,1]$ is the membership function and $\mu_R(a,b)\in[0,1]$ is the membership intensity for every $a,b\in A$, according to the verification of the property of being "at least as *desired* as".

In this way, the natural inexactness of language and concepts can be examined in detail. Such inexactness or fuzziness may refer to different phenomena (see e.g., [17]), such as *vagueness*, where there are no precise boundaries between concepts, *ambiguity,* where more than one distinguishable concept can be described, or *ambivalence*, where conflicting valuations coexist.

Now, under this fuzzy environment we explore the preference predicate, paying special attention to its classical properties and particular logical operators.

3.2. The standard fuzzy preference model

In the context of fuzzy preference modeling, diverse difficulties arise when (1)–(6) are replicated using fuzzy logical connectives (see for instance [10]). As a result, the fuzzy relations P, I, J have to be defined in terms of the fuzzy preference relation R using fuzzy logical operations that preserve definitions (1)–(3) and the essence of properties (4)–(6) as much as possible.

The following standard fuzzy preference model (see e.g., [10,27,46]) assumes the three general axioms of Independence of Irrelevant Alternatives (*IA*), Positive Association (*PA*) and Symmetry (*SM*).

Axiom *IA*. There exist three functions $p,i,j:[0,1]^2\to[0,1]$, such that

$$P(a,b)=p(x,y),$$

$$I(a,b)=i(x,y),$$

$$J(a,b)=j(x,y),$$

where $x=\mu_R(a,b)$ and $y=\mu_R(b,a)$.

Axiom *PA*. The functions $p(x,n(y))$, $i(x,y)$ and $j(n(x),n(y))$ are non-decreasing regarding both arguments, where n is a strong negation.

Axiom *SM*. The functions i and j are symmetric.

Hence, using traditional fuzzy logic operators [40], where the degree of union and intersection can be represented by some continuous t-conorm S and t-norm T, respectively, the three classical properties (4)–(6) can be translated as follows:

$$S(p(x, y), i(x, y)) = x, \tag{7}$$

$$S(p(x, y), i(x, y), p(y, x)) = S(x, y), \tag{8}$$

$$S(p(y, x), j(x, y)) = n(x). \tag{9}$$

One recognized solution for this axiomatic model takes the standard Lukasiewicz structure $\langle T^L, S^L, n\rangle$, the only one that satisfies both the ‘principles of the excluded middle and non-contradiction [10], along with its corresponding residual *max–min* $\langle T^m, S^m, n\rangle$, for the characterization of the continuous preference structure $\langle p, i, j\rangle_\varphi$, defined up to an automorphism φ of the unit interval [10]. Such characterization finds the following bounds:

$$T^L(x, n(y)) \le p(x, y) \le T^m(x, n(y)),$$

$$T^L(x, y) \le i(x, y) \le T^m(x, y),$$

$$T^L(n(x), n(y)) \le j(x, y) \le T^m(n(x), n(y)),$$

where the only solution for $\langle p, i, j\rangle_\varphi$ that fulfills all the properties (7)–(9) is given by [10]

$$p(x, y) = T^m(x, n(y)), \tag{10}$$

$$i(x, y) = T^L(x, y), \tag{11}$$

$$j(x, y) = T^L(n(x), n(y)), \tag{12}$$

being i and j are mutually exclusive (notice that other solutions exist, but satisfying less of the basic properties, as it can be seen in [10,27,46]).

Hence, the solution (10)–(12) allows the verification of various basic situations, which coexist by means of distinct intensities. As a result, it is possible to construct a gradual description of the different epistemic states of the individual facing decision. Now we explore the particular semantics of this model, in particular, the way in which the information is interpreted and subsequently used.

3.3. On the semantics of the standard fuzzy preference model

The general meaning of preference refers to a common perception of value, which allows deciding in favor of a certain alternative. Hence, this meaning has to be measured, such that the available data can be organized or absorbed into a structured frame of information (see e.g., [12,15]). In this sense, preference structures organize data and, under a fuzzy approach, measure the degree or intensity in which the different properties of preference are verified.

From this perspective, the measurement of meaning is a central point in the development of scientific knowledge. In particular, examining the alternate meanings of attitude, psychology has explored the existence of *meaningful opposites*, which define reference poles according to which any concept of interest can be semantically valued (see e.g., [5,22,31]). In this way, the complete fuzzy preference structure can be interpreted in order to evaluate the attitude that the individual has towards decision, with the purpose of understanding, describing and predicting the possible courses of action.

Hence, the different basic situations of the complete preference structure can be combined under the respective fuzzy values, taking a simultaneous valuation of one positive pole P and the respective *pseudo-negative* pole P^{-1} (regarding the standard model *IA–SM* and (7)–(9)), along with the specification of an intermediate state of decision neutrality given by I or J. Here the meaning of the preference predicate is measured by the respective intensities x and y, which are considered as opposites because of an implicit interpretation of taking the inverse as the meaningful opposite, which in fact have the same source of information, i.e., the rational evaluation of the positive attributes for a given pair of alternatives a and b.

As a result, the opposite preference predicate is not verified over the negative attributes of the alternatives, but rather, by checking the violation of the same positive aspects. But focusing on the preference concept, we find that its linguistic antonym refers to rejection or aversion, which effectively can be verified in the same way as preference, i.e., measuring the degree in which negative attributes exist for rejecting any pair of alternatives.

For example, if a person thinks of the existing reasons to vote in favor of a candidate a instead of b, then the voter pays attention to the proposals that each candidate presents. Hence, it is different to think on the proposals of b as negative aspects against a, than to think of reasons why not to vote for a. In the former case, the voter thinks of everything that is good about b, but in the latter one, the voter is able to explicitly value everything that is bad about a, like e.g., that a is connected with illegal activities.

Then, preference structures have to be as meaningful as possible in order to express the conditions of subjectivity and inexactness natural to human intelligence or rationality (see, e.g., [24]). Hence, a more flexible preference structure can be explored, taking into account the way in which an intelligent individual builds the meaning of concepts and predicates, referring explicitly to a positive pole of preference and a negative pole of aversion.

4. The Preference–Aversion (P–A) model

Different studies in social sciences show that people weight gains differently from losses (see, e.g., [21,41,45]), and more generally stated, value arguments in favor and arguments against alternatives distinctly (see, e.g., [5,22,31]). In this way, the gains–losses rationality, where the *pros* and *cons* for a decision are considered, suggests a balanced evaluation for organizing the available alternatives under a given viewpoint or (not necessarily linear) collection of criteria.

Recently the term *bipolarity* [8,35] has been used to examine different types of mathematical models for the representation of *ambivalence* (see e.g., [17,22]), the situation that emerges naturally when positive and negative attributes are considered regarding the verification of some concept or predicate. The basic notion behind the term bipolarity has a long trace back to the nature of *knowledge* and *meaning*, from philosophy (see, e.g., [19,41]) and psychology (see, e.g., [5,22,31]) to social sciences (see, e.g., [21,38,45]) and neurology (see, e.g., [30,48]), under the view that the explicit identification of opposites allows for the dialectic construction of knowledge from reality.

In this section, following the initial proposals of [13,14], we introduce the Preference–Aversion model (P–A), where it is possible to represent the independence of positive and negative arguments when facing a decision. As a result, positive and negative aspects are measured over different sources of information, and this distinction is maintained through the entire inference process of the individual.

4.1. Axiomatics for the fuzzy P–A model

Notice that under a cognitive and linguistic approach, the predicate that functions as the contrary to “at least as *desired* as” has to make reference to its antonym (see e.g., [42], but also [25]) or in case such antonym does not exist, we should try to find its antagonistic (see e.g., [36]) counterpart, an equally verifiable or positive concept. In this way, we have that the linguistic antonym of desire is rejection or aversion.

The P–A model takes as input data the verification of two opposite predicates, one expressing positive preference, given by R^+, which represents the property of being “at least as *desired* as”, and another one expressing negative preference, given by R^-, which represents the property of being “at least as *rejected* as”. Then, we assume the existence of the intensities

$$\mu_{R^+}, \mu_{R^-} : A \times A \to [0,1],$$

representing the strength with which any pair of elements $a, b \in A$ verify the respective predicate.

The inclusion of these two independent evaluations allows the characterization of two basic structures, on the one hand, the complete preference structure, composed by strict preference P, indifference I and incomparability J, and on the other hand, the complete aversion structure, composed by strict aversion Z, indifference on aversion G, and incomparability on aversion H. In this way, the P–A basic structure is represented by

$$R = \langle R^+, R^- \rangle = \langle (P, I, J), (Z, G, H) \rangle,$$

such that the basic properties of the preference structure are extended to the aversion one, in the following way.

Axiom *IA2*. There exist six functions $p, i, j, z, g, h : [0,1]^2 \to [0,1]$ such that

$$P(a,b) = p(x^+, y^+),$$

$$I(a,b) = i(x^+, y^+),$$

$$J(a,b) = j(x^+, y^+),$$

$$Z(a,b) = z(x^-, y^-),$$

$$G(a,b) = g(x^-, y^-),$$

$$H(a,b) = h(x^-, y^-),$$

where $x^+ = \mu_{R^+}(a,b)$, $y^+ = \mu_{R^+}(b,a)$, $x^- = \mu_{R^-}(a,b)$ and $y^- = \mu_{R^-}(b,a)$.

Axiom *PA2*. The functions $p(x^+, n(y^+))$, $i(x^+, y^+)$, $j(n(x^+), n(y^+))$, $z(x^-, n(y^-))$, $g(x^-, y^-)$, $h(n(x^-), n(y^-))$, are non-decreasing regarding both arguments.

Axiom *SM2*. The functions i, j, g, and h are symmetric.

Hence, the basic properties (7)–(9) are extended for the basic aversion structure as follows:

$$S(z(x^-, y^-), g(x^-, y^-)) = x^-, \tag{13}$$

$$S(z(x^-, y^-), g(x^-, y^-), z(y^-, x^-)) = S(x^-, y^-), \tag{14}$$

$$S(z(y^-, x^-), h(y^-, x^-)) = n(x^-), \tag{15}$$

such that the only solution for $\langle z, g, h\rangle_\varphi$ that fulfills all the properties (13)–(15) is given by the Lukasiewicz structure and its respective residual, where g and h are mutually exclusive, i.e.,

$$z(x^-, y^-) = T^m(x^-, n(y^-)), \tag{16}$$

$$g(x^-, y^-) = T^L(x^-, y^-), \tag{17}$$

$$h(x^-, y^-) = T^L(n(x^-), n(y^-)). \tag{18}$$

Therefore, the solutions (10)–(12) and (16)–(18), which take into account a type of rationality that frames alternatives in terms of gains and losses, allows to construct one positive order, based on the preference basic structure $\langle p, i, j\rangle_\varphi$, and a negative order over A, based on the aversion basic structure $\langle z, g, h\rangle_\varphi$. The first one makes reference to the positive attributes, while the second one to the negative attributes of x relative to y. In this way, the opposition over preference R^+ is measured upon the aversion predicate R^-.

Now, the basic P–A structure can be explored in a conjunctive way, representing the ambivalence of the individual and examining the different epistemic states that result from the independent aggregation of gains and losses.

4.2. The complete P–A structure

The combination of the preference and aversion basic structures can now be examined by means of a continuous t-norm T, such that

$$R = \langle R^+, R^-\rangle = T(\langle p, p^{-1}, i, j\rangle_\varphi, \langle z, z^{-1}, g, h\rangle_\varphi).$$

As a result, there are 16 different situations representing the cognitive space of the individual, characterized by 10 distinct relations. These relations are specified in Table 1, building up the complete P–A structure

$$\langle pz, pa, pg, ph, iz, ig, ih, jz, jg, jh\rangle,$$

defined by

- *Ambivalence*: $pz = T(p, z)$.
- *Strong preference*: $pa = T(p, z^{-1})$.
- *Pseudo-preference*: $pg = T(p, g)$.

Table 1
The complete P–A cognitive space.

$R = \langle R^+, R^- \rangle$	z	z^{-1}	g	h
p	pz	pa	pg	ph
p^{-1}	pa^{-1}	pz^{-1}	pg^{-1}	ph^{-1}
i	iz	iz^{-1}	ig	ih
j	jz	jz^{-1}	jg	jh

- *Semi-strong preference*: $ph = T(p, h)$.
- *Pseudo-aversion*: $iz = T(i, z)$.
- *Strong indifference*: $ig = T(i, g)$.
- *Positive indifference*: $ih = T(i, h)$.
- *Semi-strong aversion*: $jz = T(j, z)$.
- *Negative indifference*: $jg = T(j, g)$.
- *Strong incomparability*: $jh = T(j, h)$.

This characterization of the complete P–A structure represents, in a relational and gradual way, the different epistemic states of the individual whose evaluation of the alternatives develops under an independent methodology. In this way, this structure labels the subjective attitude existing over the decision problem. Hence, the complete P–A structure defines a flexible framework for mapping the cognitive space of a gains–losses rational individual.

Examining such cognitive space, we start by pz, pa, pg and ph. The first situation, called *ambivalence* (pz), expresses maximum conflict, due to the coexistence of the two completely opposite valuations of strict preference (p) and strict aversion (z). In this case there is no easy decision because a is at the same time better (more desired) and worse (more rejected) than b. Hence, the P–A representation of the relation of ambivalence is non-symmetrical, as it is the conjunction of two non-symmetrical situations, p and z.

The second situation is *strong preference* (pa), which represents the most favorable situation towards decision, such that strict preference for a becomes even stronger given that b is worse. Then we have *pseudo-preference* (pg), which reflects a situation in which strict preference is verified although the two alternatives are bad or unsatisfactory, revealing a certain degree of conflict and unconformity. Lastly, we have *semi-strong preference* (ph), representing the situation where there is strict preference and incomparability on aversion.

Now, regarding positive indifference, its combination with strict aversion generates *pseudo-aversion* (iz), where the two alternatives are equally desired but one is worse than the other. Then, from the combination of positive and negative indifference, *strong indifference* (ig) is obtained, where both alternatives are equally desired and rejected, i.e., they are very similar. And by the combination of indifference and incomparability on aversion, *positive indifference* (ih) is obtained, where a and b are desired and their negative attributes cannot be compared.

Finally, we have the different combinations between incomparability and the basic relations of the aversion dimension. On the one hand, *semi-strong aversion* (jz) represents the case where there is strict aversion and the positive attributes of both options cannot be compared, and on the other hand, *negative indifference* (jg) represents the situation where there is rejection over both alternatives and the positive attributes of both options cannot be compared, reflecting total unconformity on the given alternatives. Lastly, the individual's cognitive space expresses a situation where absolutely no comparability can be identified, under the name of *strong incomparability* (jh).

In the preference literature we find some similar approaches to the P–A approach, but that should not be taken as equivalent (see e.g., [13]). One of them is based on the Concordance–Discordance (C/D) principle of Partial Comparability Theory (PCT) [38,44], where it is assumed that a decision always relies on arguments in favor and against the given alternatives. This proposal builds the PCT preference structure from the joint evaluation of independent preference and non-preference predicates [43,44], such that, under a continuous setting, a four-valued extension is obtained for each predicate, by means of the same functional characterization of (10)–(12) (for more details see [33,34], but also [13]).

It is worth noting that a different characterization for the continuous solutions of the PCT structure is given in [11], but over an ordinal scale (for other approaches examining the aggregation of *pros* and *cons* in a qualitative setting, see e.g., [4,7]). A detailed investigation of the PCT proposal and the ordinal one can be found in [33].

Table 2
The P–A structure and the verification of the epistemic states of the voter.

$R = \langle R^+, R^- \rangle$	z	z^{-1}	g	h
p	0.5	0	0.3	0
p^{-1}	0.2	0	0.2	0
i	0.3	0	0.3	0
j	0	0	0	0.1

Either way, such a bipolar approach allows verifying the preference and non-preference or aversion predicates independently, taking as the source for preference and aversion the respective positive and negative aspects over the given set of alternatives. In this way, each family of criteria (*pros* and *cons*) is valued by different neural areas of the brain, as suggested by recent results in neuroscience (see e.g., [30,48]).

Now we present a simple example on the construction of the complete P–A structure over a voting decision problem, analyzing the epistemic states of the voter and the predicted course of action. Then, in Section 5, we apply the P–A model to the identification of need apart from desire.

Example. Consider a voter who has to decide for one out of two candidates. When this voter looks for positive arguments in favor of a, he finds out that a has better proposals for managing social problems than b, but when the voter thinks of negative arguments, he finds out that a is formally accused of corruption, something that does not happens with b, although the voter suspects that b is also corrupt. So, thinking of positive arguments, the voter verifies the preference predicate by the following intensities, $x^+ = \mu_{R^+}(a,b) = 0.9$ and $y^+ = \mu_{R^+}(b,a) = 0.1$, while thinking of negative ones, the voter verifies the aversion predicate by $x^- = \mu_{R^-}(a,b) = 0.9$ and $y^- = \mu_{R^-}(b,a) = 0.4$.

Then, applying the P–A model for obtaining the complete description of the epistemic states of the voter, we take the solutions (10)–(12) and (16)–(18) for the specification of the basic P–A intensities, and then construct the complete P–A structure by means of the t-norm $T^m = \min$. In this way, we identify a greater degree of verification on strict preference and strict aversion of a over b, i.e., on the relational situation of ambivalence (pz). This situation expresses a complex cognitive state, where the most opposite forces come into action with greatest intensity (the complete result is given in Table 2).

Then, it is possible to describe the other situations that go with this ambivalence, in order to understand in a better way the course of action for the voter. Besides ambivalence, the other relations that are verified with less intensity correspond with pseudo-preference (pg) and pseudo-aversion (iz) of a over b, strong preference (pa) and pseudo-preference (pg) of b over a, and finally, strong indifference (ig).

Now, as it can be seen in Table 2, the situation of ambivalence can be explained with the help of the other states and their corresponding intensities. In this way, we know that the voter recognizes pseudo-preference and pseudo-aversion of a over b, but also, strong preference and pseudo-aversion of b over a. It is evident that both candidates are very similar, but one of them receives strong preference and the other one does not. Then, according to the P–A representation of the cognitive attitude of the voter, it can be predicted that the vote will go for b instead of a.

5. Application of the P–A model to need and desire

In classical and general equilibrium economic theory (see e.g., [1,6,39,47]), a *rational* decision maximizes the satisfaction of desire according to the preferences of the individual. But a different approach recognizes, as a basic characteristic of the individual's social and economical rationality, that besides the maximization of desire, a different course of action can be identified based on what the individual needs.

Here we propose a methodology for identifying such a necessary course of action, focusing on what is needed apart from what it is desired. It is proposed that from this basic distinction, an intelligent individual is able to find a personal state of equilibrium given by the satiation level of necessity. This satiation level is the prominent attribute of necessity, on the contrary to desire, which, from the point of view of the general equilibrium economic theory [6], has no possible satiation.

Hence, we explore the particular semantics of the preference–aversion approach, with the purpose of identifying need apart from desire, by means of the *need-preference structure*. Finally, we explore some examples on this proposal.

5.1. The principle of unsatisfied needs

Classical economic theory [1,6] denies that need can be distinguished from desire. In this sense, it is assumed that the preferences of a *rational* individual can be represented by a utility function, which expresses the magnitude of desire for every given alternative, such that the best option is the result of a maximization procedure over the whole set of utilities (see e.g., [39,47]). This type of individual is referred to as the *homo-economicus*, whose desire is insatiable (referred to as the *no satiation* assumption [6]), and finds its limits only at an aggregated level, by the conflict with other individuals and their blocking strategies (see e.g., [23,28,32]).

As a result, the classical paradigm of rationality considers that a *rational* individual will always want more instead of less of a good. Maximizing utility, the *homo-economicus* will never stop in this desire, guided by a rationality principle where no ethic considerations are involved. But the idea of need is an inescapable dimension of economic life, suggesting a reconsideration of the foundations of such economic individual. Essential to this re-examination of economic thought is a reconsideration of human beings in their two economic roles, both as a worker and as a consumer [29]. This is the cornerstone for a social approach on economics.

Hence, the concept of need is a basic notion. This statement can be cleared up by focusing on the distinction between desire and need. On the one hand, while desire is commonly associated with preference, we recognize that need expresses a priority for maintaining or protecting life. In this sense, following the *negative quality of justice* [41], need refers in the same way to desire, but only when such desire is not harmful or damaging. On the other hand, need has a characteristic satiation level, such that if the desire intensity continuously increases, then it is possible for the need intensity either to increase, until it arrives to a certain threshold, revealing some synergy in their behavior, or to decrease, revealing an opposite or antagonistic (in the sense of [36]) behavior.

Notice that under our approach, the distinction between desire and need does not require that the individual directly reveals his needs and desires, but instead, it is deduced from the explicit valuation of the positive and negative attributes over the given set of alternatives. As a result, this proposal on the concept of need shall not be confused with other proposals found in the literature, in particular in the fields of Artificial Intelligence and the Theory of Possibility (see e.g., [3,9,11,33]), where the individual is supposed to directly reveal his needs or requirements.

In this way, we propose to consider an individual with a gains–losses rationality that is capable of expressing self-control, such that the state of equilibrium can exist on an individual level, and not only on an aggregated one. This type of individual, capable of telling apart desire from necessity and behaving according to his/her desires and needs, will be here referred to as the *homo-socioeconomicus*.

Hence, we present the *principle of unsatisfied needs*, as an evident postulate stating that it is more important to satisfy a need than to satisfy a desire. So, for every $a, b \in A$, if an individual needs a over b with greater intensity than needing b over a, and at the same time, desires b over a with equal or greater intensity than desiring a over b, then, by the principle of unsatisfied needs, the necessary course of action is to choose a over b.

Therefore, an unsatisfied need is certainly different from an unsatisfied desire, in the sense that the former receives priority over the latter. In this way, need has to be properly identified apart from desire in order to give it priority. For this purpose, we formalize our proposal for distinguishing between the need and desire components of preference, by means of the need-preference structure.

5.2. Need-preference structure

As argued in [16,20], the individual's behavior is not always determined by utility, or any other single element, but by the individual desires and needs. In this way, it is stated that in the most basic level, the desires or wishes arise from physiological and psychological needs. Hence, it is very important to state that the concept of need, as human material need, is an inescapable dimension of every day economic life, although it escapes any formal recognition in mainstream economics and its theory of rational choice (for a discussion on this matter, see e.g., [29]).

This statement may be better understood if we examine the relation of need and desire in common language, where need implies desire but desire does not imply need. In other words, something may be desired without it being needed, but on the other hand, if something is needed, and just because it is needed, then it is also desired. Notice that this basic observation refers to the characteristic duality between need and desire, given that they can have similar or antagonistic meanings.

For instance, a person might affirm that he needs to take some time for rest, but it will be the case that the person certainly *needs* to rest if he has worked the whole day, and more likely *desires* to rest if he feels like not doing anything. Therefore, an unsatisfied desire is something essentially different from an unsatisfied need. The latter is a requirement for human existence, coming from the materiality of human nature, while the former may be just a wish, whether it is materially needed or not.

In this sense, here we refer explicitly to this kind of material need, accepting that there may be other types of need. In the same way that desire has been examined in decision theory as that which is preferred because it represents general satisfaction, which can be measured by numerical functions (such as utility), need represents here that which is preferred because of its basic and urgent condition for preserving life and any of its vital qualities (based on the negative quality of justice [41] and the principle of unsatisfied needs).

Hence, we decompose the preference perception into need and desire, two basic components which are not disjoint, but as noted before, relate between themselves in a gradual manner. Such gradualism follows our basic observation, which points out that something may be desired without it being needed, but if something is needed, then it is forcefully desired. Therefore, desire can exist without need, but on the other hand, need cannot exist without desire, because need is a desire that cannot be denied.

Once we acknowledge that need is the basic component from where preference develops, then a minimal implicative condition between the need and desire components can be formalized. This condition can be modeled using an implication operator satisfying at least the order property [10], such that for any two elements x and y, $x \to y$ holds if and only if $x \leq y$ holds.

Then, under a fuzzy environment, the implication operator $I_i^{\to}$ is such that for any pair of elements $x, y \in [0, 1]$, $I_i^{\to}(x, y) = 1$ if and only if $x \leq y$. In this way, let $\mu_N(a, b) \in [0, 1]$ be the intensity in which the predicate "a is at least as *needed* as b" is verified, and let $\mu_D(a, b) \in [0, 1]$ be the respective intensity for the predicate "a is at least as desired as b". Hence, for the components of need and desire, the implicative condition $I_i^{\to}(\mu_N, \mu_D) = 1$ holds. Following this line of thought we define a need-preference structure.

Definition 1. A need-preference structure is given by the pair $\langle \mu_N, \mu_D \rangle$, such that for every $a, b \in A$, it holds that

$$I_i^{\to}(\mu_N(a, b), \mu_D(a, b)) = 1.$$

In this way, if $\mu_D(a, b) = 0$, then $\mu_N(a, b)$ cannot be greater than 0 and if $\mu_N(a, b) = 1$, then $\mu_D(a, b) = 1$. Hence, if $\mu_D(a, b) = \mu_N(a, b)$, then the intensity of preference expresses the will, grounded on the material identification of need, for satisfying such need. Otherwise, if $\mu_D(a, b) > \mu_N(a, b)$, then the intensity of preference expresses desire, confronted with a diminishing intensity of need.

Therefore, the individual can prefer different alternatives with given intensities, but this does not necessarily mean that all the preferred alternatives are equally needed. Then, although the individual reveals that he prefers one alternative over the other, he also has the capacity of distinguishing if such preference implies the satisfaction of desire or, on the contrary, if such preference implies the satisfaction of need, taking into account the negative aspects of desire in order to avoid them.

As a result, following Definition 1, an example of such a need-preference structure is given by

$$\mu_D(a, b) = \mu_{R^+}(a, b), \tag{19}$$

$$\mu_N(a, b) = T(\mu_{R^+}(a, b), n(\mu_{R^-}(a, b))), \tag{20}$$

where $\mu_{R^+}(a, b)$ represents the intensity in which the predicate "a is at least as desired as b" is verified, $\mu_{R^-}(a, b)$ represents the respective intensity for the predicate "a is at least as rejected as b", n is a strong negation and T is a continuous t-norm. In this way we show an efficient way on how to distinguish desire from need, referring only to the preference and aversion intensities over the set of given alternatives.

This example verifies Definition 1, given that it will always be the case that the need intensity (20) implies the desire intensity (19), where it is always true that

$$\mu_N = T(\mu_{R^+}, n(\mu_{R^-})) \leq \mu_D = \mu_{R^+}.$$

Besides, it follows the intuition given by the negative quality of justice, where need refers to desire provided that such desire is not harmful or negative in any way. Notice that, under this example, we can formally see that the complete preference structure $R = \langle P, I, J\rangle$ is insufficient for the separate identification of desire and need. This is, if we take the inverse intensity as the pseudo-negative verification of preference, such that $\mu_{R^-}(a,b) = \mu_{R^+}(b,a)$, then we have that the intensity for need agrees with the one for strict preference P, where

$$P(a,b) = p(\mu_{R^+}(a,b), \mu_{R^+}(b,a)) = T(\mu_{R^+}(a,b), n(\mu_{R^+}(b,a))) = \mu_N(a,b).$$

Notice that our approach on need should not be confused with other proposals such as the bipolar-possibilistic one presented in [3,9], where positive preferences represent desire and negative preferences represent constraints. Then, from such possibilistic perspective, *need* is everything that is tolerated or desired, while in our subjective approach, as we have mentioned before, need refers explicitly to the perception of desire whose intensity diminishes with the perception of aversion.

Now, we present an example of a need-preference structure which is implicit in our P–A proposal.

5.3. The P–A need-preference structure

Following our proposal on a need-preference structure, we have a formal procedure for distinguishing between need and desire. Such procedure is supported by three cornerstones. First, need refers to desire provided that such desire is not harmful or negative in any way (negative quality of justice), second, an unsatisfied need is more important than an unsatisfied desire (principle of an unsatisfied need), and third, need implies desire (definition of a need-preference structure).

In this way, we have presented an example of a need-preference structure, given by (19) and (20), which we will now examine over the P–A model. Hence, we have that the intensity for the P–A component of desire is given by (19), such that

$$\mu_D^{P\text{–}A} = \mu_{R^+} = S(p,i),$$

and that the respective intensity for need is given by (20), such that

$$\mu_N^{P\text{–}A} = T(S(p,i), S(z^{-1},h)),$$

where, taking Eqs. (7) and (15) of the P–A model, we have that $\mu_{R^+} = S(p,i)$ and $n(\mu_{R^-}) = S(z^{-1},h)$. Besides, notice that for the case of the need component, $\mu_N^{P\text{–}A} = S(T(p,z^{-1}), T(p,h), T(i,z^{-1}), T(i,h)) = S(pa, ph, iz^{-1}, ih)$ holds for any t-norm T if and only if $S = S^m = \max$, given that S^m is the only continuous t-conorm that satisfies the distributive law (such that for every $a, b, c \in [0,1]$ and for any t-norm T, it is true that $T(a, S(b,c)) = S(T(a,b), T(a,c))$ if and only if $S = S^m$ [10]). In this way, we can define a particular P–A need intensity by

$$\mu_N^{P\text{–}A} = S^m(pa, ph, iz^{-1}, ih).$$

Given this proposal for the P–A need-preference structure $\langle \mu_N, \mu_D\rangle^{P\text{–}A}$, we can re-examine the example of Section 5.3, in order to identify the necessary alternative. Then, according to the principle of unsatisfied needs, the prioritary course of action is the one that agrees with the alternative with the highest need intensity. In this way, the individual is capable of specifying the needed alternatives, and because they are necessary, they receive priority over the other preferred alternatives (which lack of the attribute of being effectively necessary).

Hence, recall the example of Section 5.3 (see Table 2), where the epistemic state of a voter, trying to decide between two candidates, was described by ambivalence, followed up by pseudo-preference and pseudo-aversion of a over b, strong preference and pseudo-aversion of b over a, and strong indifference. Then, it was predicted that the individual would vote for b instead of a, due to the fact that there was a positive degree of strong preference of b over a, but not the inverse.

Now, we can formulate the necessary course of action, following our proposal of the P–A need-preference structure. Here, we obtain that the desire intensities between a and b are $\mu_D^{P\text{–}A}(a,b) = 0.5$ and $\mu_D^{P\text{–}A}(b,a) = 0.3$, while the respective need intensities are $\mu_N^{P\text{–}A}(a,b) = 0$ and $\mu_N^{P\text{–}A}(b,a) = 0.3$. Then, although a is the most desired option, it results that it lacks of the attribute of being necessary. On the contrary, although b is not the most desired option, it is the most necessary. Hence, b is the necessary course of action, and on behalf of the principle of the unsatisfied need, the individual should be expected to vote for it.

6. Concluding remarks

Our proposal of the complete P–A structure and the need-preference structure supports decision in two different levels. The first level consists in describing the epistemic states of the individual facing a decision problem, which allows understanding the kind of conflicts existing over the available information. And on a second level, it points out a definite course of action based on the needs of the individual, and not just his/her desires.

This approach recognizes the complexity of the decision problem, but at the same time, offers a possible solution based on the preferences of the individual and basic ethical (socioeconomic) principles. It has been shown that the distinction between positive and negative attributes is essential for properly identifying desire from need. In this way, it is not enough to interpret the inverse preference as the opposite negative preference predicate, because negative aspects come from different sources of information than positive ones.

Now, referring to need and desire, both concepts are closely related. Sometimes they are even confused in common language, given their duality/antagonism. This condition is nothing but the expression of the duality of the individual, conceived here in particular as the *homo-socioeconomicus*, whose personal dimension exists in relation to his/her social dimension. In this sense, the desire intensity expresses the part of the human nature that responds to egoism, while the need intensity expresses the other part of the human nature which responds to solidarity and vitality.

This work pretends to be a first step for the development of models where need and desire are the basis for preference representation, i.e., for examining the subjective perception towards socioeconomic objectives. In this way, an inner limitation on the search for individual desire is introduced, allowing the possibility for social behavior by considering the negative aspects of desire, along with satiation levels over the individual's preferences. Such behavior opens the possibility for examining the existence and stability of a socioeconomic equilibrium. This situation deserves further examination and stands as an open line for future research.

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