Leaks, disclosures and internal communication*

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Abstract

We study how increasing whistleblower incentives affects a firm's communication decisions, price informativeness and real efficiency. An informed manager, who can divert cash for private benefit, privately communicates with his employee about project fundamentals and chooses investment. Given her information, the employee maximizes internal alignment and can leak the manager's message with some noise. Stronger whistleblower incentives lead to more informative leaks, less misconduct and higher price informativeness. However, they can decrease firm value and real efficiency by increasing the manager's manipulation of internal communication. More targeted policies (e.g., mandating more public disclosure) improve both price informativeness and real efficiency.

JEL: G10, G12, G14, G32
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1 Introduction

Publicity is justly commended as a remedy for social and industrial diseases. Sunlight is said to be the best of disinfectants; electric light the most efficient policeman.
— Louis Brandeis, Other People's Money – and How Bankers Use It (1914)

Regulators encourage public whistleblowing to deter wrongdoing by firms. For example, the Securities and Exchange Commission (SEC)'s Dodd-Frank Whistleblower Program is designed to improve stakeholder protection and provide incentives to whistleblowers in financial markets.[1] Since inception, the program has awarded more than \$1.9 billion to 397 whistleblowers, and has led to SEC enforcement actions with remedies in excess of \$6 billion. In fiscal year 2023 alone, the SEC's Office of the Whistleblower received over 18,000 tips and awarded nearly \$600 million as part of the program.

While these policies are arguably effective at detecting unlawful behavior by firm insiders, they can also have unintended consequences. By increasing the likelihood that internal communication is leaked publicly, these policies may distort how a firm's employees communicate with each other within the firm and to market participants. As a result, the overall impact of higher whistleblower incentives on real (allocative) efficiency is not clear. We show that increasing whistleblower incentives can reduce the incidence of misconduct within the firm and improve price informativeness. However, it can lead to lower real efficiency, especially when alignment within the firm and, consequently, effective internal communication are sufficiently important for firm value. Our analysis also suggests that other policy interventions, such as mandating more informative public disclosures, may be more effective than increasing whistleblower incentives in such settings.

Model Overview and Intuition. There is a single firm with a manager (he) and an employee (she). First, the manager chooses how much of the firm's internal cash to invest in a new project, but can engage in misconduct by diverting some of this cash for private benefits. The manager's investment increases the mean of the project's interim cash flows, or fundamentals. To incentivize appropriate allocation of corporate funds, the manager's compensation is increasing in the firm's stock price.

Next, there is a mandatory public disclosure (e.g., earnings), which provides all participants a noisy signal about the project's interim cash flows. In addition, the manager

[1] The SEC Whistleblower Program is a recent addition to a number of other, similar financial market regulations (e.g., US Whistleblower Protection Act (1989), Section 806 of Sarbanes-Oxley Act (2002), Whistleblower Protection Enhancement Act (2007)). See the U.S. Securities and Exchange Commission, 2023 Annual Report to Congress on The Dodd-Frank Whistleblower Program (https://www.sec.gov/files/fy23-annual-report.pdf) for more details.

privately communicates with the employee about interim cash flows — importantly, he can bias this report at a private cost. Given the public disclosure and the private communication, the employee's goal is to maximize the alignment between her action and the firm's fundamentals. Moreover, she can leak a noisy signal of the manager's internal communication to the market at a private cost, and her incentives for doing so are increasing in the precision of her leak.[2] The firm's terminal value depends on the project's interim cash flows and the extent of alignment within the firm, and the firm's stock price reflects the market's conditional expectation of this terminal value, given the public disclosure and the employee's leaked signal.

In this setting, we show that increasing public information quality (i.e., increasing the precision of mandatory public disclosures) and increasing whistleblower incentives both lead to (i) higher investment by the manager and (ii) higher price informativeness. However, we show that their impact on real efficiency is different: while an increase in public information quality leads to higher real efficiency, as measured by expected firm value, stronger whistleblower incentives can lead to lower real efficiency.

To see why, first note that the manager's investment decision trades off the benefit from a higher stock price (by investing more) versus the private benefit of diverting more cash (by investing less). Importantly, an increase in the sensitivity of the stock price to either the public disclosure or the employee's leak leads to higher investment by the manager, and consequently, higher firm value. In contrast, the manager's incentive to distort his internal communication increases with the weight that the market puts on the employee's leak, since there is a greater benefit from biasing his message, but this reduces firm value by degrading the alignment of the employee's action.

All else equal, an increase in public information quality increases the weight that the market puts on the public disclosure, while decreasing the weight on the employee's leak. As a result, we show that the manager invests more and distorts his internal communication less, both of which lead to higher firm value. Moreover, the combined informativeness of the public disclosure and the leak is higher, and so the price is more informative about firm fundamentals.

An increase in whistleblower incentives leads the employee to exert more effort in reporting any discrepancy between public disclosure and internal messages, and consequently, leak a more informative signal to the market. This increases the weight that the market puts on the leak. On the one hand, this enhances market discipline on the manager's behavior and increases investment. On the other hand, it also increases the manager's incentives to

[2]Section 7 explores how our results change under alternate assumptions for the manager's and employee's incentives.

distort his internal communication to the employee, which leads to less alignment between her action and fundamentals. When the impact of internal alignment on overall value is relatively small, the first effect dominates, and so firm value increases with whistleblower incentives. However, when internal alignment is sufficiently important for firm value, the latter effect can dominate and real efficiency decreases with stronger whistleblower incentives, when these incentives are sufficiently large. In the latter case, our analysis implies that there is an optimal level of whistleblower incentives beyond which they are counter-productive. This optimal level varies across firms: it is lower for firms with more volatile cash flows and for those in which the relative value of internal alignment is higher. Moreover, we find that the optimal level of whistleblower incentives should decrease when the quality of public disclosures is higher.

We then consider a setting where the manager can distort the public disclosure as well. As in the main analysis, an increase in whistleblower incentives leads the market to put more weight on the employee's leak, which leads the manager to distort his internal communication more. However, we show that the increase in whistleblower incentives leads to more informative public disclosures by the manager — this is because the relative weight that the market puts on such disclosures is lower, which reduces the manager's incentives to bias them. Moreover, as in the main analysis, we show that while the overall impact of increasing whistleblower incentives on investment and price informativeness is always beneficial, the impact on real efficiency can be negative when these incentives are very large.

Our analysis suggests a novel channel through which regulatory policy towards whistleblowers can affect firm behavior. We show that while stronger whistleblower incentives can lead to less misconduct by firm managers, this need not always lead to higher firm value or improve real efficiency. Moreover, our analysis highlights that excessive whistleblower incentives can have a negative impact especially for firms in which internal alignment is an important component of value. This implies that such policies can have qualitatively different effects on firms across different sectors and industries and care must be taken in accounting for these differences when evaluating the overall impact of such regulations. Finally, our analysis also highlights that other regulatory changes (e.g., mandating more informative public disclosures and more scrutiny of such disclosures) may be preferable in that they have a more limited, direct impact on managerial decisions and fewer (negative) unintended consequences.

The rest of the paper is organized as follows. The next section briefly discusses the related literature and the paper's contribution. Section 3 presents the model and discusses the key assumptions, while Section 4 characterizes the equilibrium. Section 5 presents the analysis of how whistleblower incentives and public disclosure quality affect price informativeness and real efficiency in our setting. Section 6 extends the analysis to allow the manager to manipulate the public disclosure. Section 7 explores how our results change under alternate assumptions on manager and employee payoffs. Section 8 concludes. All proofs and additional analysis are in the Appendix.

2 Related Literature

Our model builds on the seminal work by Fischer and Verrecchia (2000), and subsequent work by Frankel and Kartik (2019) and Ball (2019), that studies a sender's incentives to engage in costly manipulation when communicating with a receiver. Unlike this existing work, the manager in our model engages not only in communication, but also chooses investment to affect the distribution of fundamentals. Moreover, the incentives for distorting communication and investment are interdependent and further depend endogenously on the employee's choice of whistleblowing intensity.

Our work contributes to the growing literature on the impact of whistleblower incentives on firm decisions. The most closely related paper is Nan and Zheng (2023) where a manager who probabilistically detects a product defect and chooses whether to share this information with the employee. In turn, an informed employee chooses whether to fix the defect, which improves welfare, or to blow the whistle about the defect, which reduces the stock price. The paper shows that when whistleblower incentives are very strong, the employee is very likely to blow the whistle, which discourages the manager from reporting defects and, therefore, reduces welfare.

We view our analysis as complementary. As in their setting, stronger whistleblowing incentives lead the manager to distort his internal communication, which leads to less informed decisions by the employee, and consequently, can reduce firm value. However, in our setting, higher whistleblowing incentives also reduce the manager's incentives to divert cash flows, which can increase firm value. Our analysis characterizes how the overall impact of whistleblower incentives on firm value and real efficiency depend on the relative importance of these effects. Our analysis also characterizes how such incentives affect price informativeness, which is particularly important, given that investor protection is the main motivation of SEC to enact the whistleblower program.[3]

More generally, our paper adds to the recent literature that points out how stronger whistleblower incentives can have negative effects on social welfare. For instance, Nan, Tang, and Zhang (2024) show that increasing whistleblower incentives leads insiders to leak

[3]See the U.S. Securities and Exchange Commission, 2017 Annual Report to Congress on the Dodd-Frank Whistleblower Program (https://www.sec.gov/files/sec-2017-annual-report-whistleblower-program.pdf).

information more frequently, which can make such reports less informative about the actual incidence of a fraud. Nan, Tang, and Ye (2023) extends this framework to study how an increase in whistleblower incentives can reduce audit quality, misstatement detection, enforcement by regulators and, consequently, social welfare.

While we abstract from analyzing the role of enforcement explicitly, our paper highlights how stronger whistleblower incentives can have opposite effects on measures of price efficiency and real efficiency. As such, our paper relates to the larger literature that distinguishes between forecasting price efficiency and revelatory price efficiency (e.g., Bond, Edmans, and Goldstein (2012)). Our paper highlights a novel channel through which higher incidence of leaks can lead to lower firm value even while making prices more informative about fundamentals.

Finally, the extension we study in Section 6 relates to the literature on communication with multiple receivers that compares effectiveness of private versus public communication.[4] In this setting, the manager communicates differently with the market and the employee, since he wishes to induce different behavior from these parties. The manager's incentives to communicate are influenced by the fact that while his disclosure to the market is public, his private communication with the employee may also be leaked to the market. This is in contrast to much of the existing literature which assume that receivers cannot learn each others' private signals.[5]

3 Model

We consider a setting in which the manager chooses investment in a new project, but can divert internal cash for private benefit, and the employee can blow the whistle on this behavior by providing a (costly) noisy signal to the market. Moreover, the employee can take an action which affects firm value. The overall value of the firm depends on both the manager's investment decision and the extent to which the employee's action is aligned with fundamentals, which depends on the manager's internal communication.

Specifically, there are four dates (i.e., $\{1, 2, 3, 4\}$) and three participants: (i) the firm

[4] See Farrell and Gibbons (1989); Levy and Razin (2004); Johns (2007); Koessler (2008); Goltsman and Pavlov (2011) for general communication games. Newman and Sansing (1993) and Gigler (1994) consider the problem that a firm's disclosure of information about product demand may be simultaneously observed by the capital market, shareholders and competitors. Gertner, Gibbons, and Scharfstein (1988) examine the choice of financial structure when the financing contract is observed by both the capital market and a competing firm in a signaling model. Spiegel and Spulber (1997) consider the audiences as the capital market and a regulator.

[5] An exception is Hagenbach and Koessler (2010), who study formation of a communication network where the agents can exchange private messages.

manager M (he), (ii) the firm employee E (she), and (iii) a representative risk neutral investor. The firm initially begins with \$1 of internal cash. The terminal (date four) value of the firm $V(\omega, a)$ depends on the manager's action through intermediate cash-flows, ω, and the employee's action, a, as described below.

Timing. The timing of events is summarized in Figure 1.

At date one, the manager diverts a fraction $y = 1 - x$ of the internal cash, which generates private utility of $y - (1/2)y^2 = (1/2)(1 - x^2)$. He invests the remaining cash, x, in the new project. The project generates intermediate cash-flows ω, where $\mathbb{E}[\omega|x] = x$, and which is privately observable by the manager.[6]

At date two, a public disclosure $d = \omega + \xi$ about the intermediate payoffs ω is realized. Moreover, the manager engages in internal communication with the employee by sending a signal t about ω that he can manipulate at a cost $\left(\frac{\tau}{2}\right)\frac{(t-\omega)^2}{\varepsilon}$. The manager privately observes his cost parameter ε, but this is unobserved by the employee and the market.

At date three, the employee takes an action a that is aligned to the intermediate cash-flows of the firm, i.e., to minimize $(a - \omega)^2$. Moreover, she can expend effort to collect evidence for whistleblowing. Specifically, she exerts effort z at cost $\left(\frac{c}{2}\right) z^2$ to publicly leak the noisy signal $m = t + \frac{\eta}{z}$ about the manager's internal message t. The employee gets a whistleblower bounty of $w \times z$. The date three price, P, reflects the (representative) investor's expectation of the firm's terminal payoffs $V(\omega, a)$, conditional on the public signals d and m, i.e.,

$$P = \mathbb{E}[V(\omega, a)|d, m]. \tag{1}$$

At date four, the firm's terminal cash flows, given by

$$V(\omega, a) = \omega - \beta(a - \omega)^2, \tag{2}$$

are realized. Here $\beta > 0$ measures the relative importance of alignment between the employee's action and the firm's fundamentals.

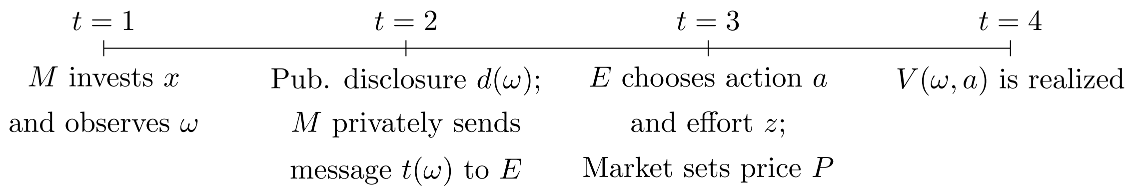
Payoffs. The manager chooses x at date one and t at date two to maximize the conditional expectation of his payoff:

$$u_M = P + \frac{1}{2}\left(1 - x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon}. \tag{3}$$

The employee chooses a and z at date three to maximize the conditional expectation of her

[6] We can alternatively interpret the manager's action x as a choice of effort, where cash flow diversion corresponds to shirking.

Figure 1: Timeline of events



payoff:

$$u_E = V + w \times z - \frac{c}{2} z^2. \tag{4}$$

Distributional Assumptions. We assume that the vector $\theta \equiv (\omega, \varepsilon, \xi, \eta)$ follows a normal distribution where the mean and variance are given by:

$$\mu_\theta = (x, \mu_\varepsilon, \mu_\xi, 0), \quad \text{and} \quad \Sigma_\theta = \begin{pmatrix} \sigma_\omega^2 & 0 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 & 0 \\ 0 & 0 & \sigma_\xi^2 & 0 \\ 0 & 0 & 0 & \sigma_\eta^2 \end{pmatrix}, \tag{5}$$

respectively. Importantly, we assume that the shocks are conditionally uncorrelated.

Equilibrium. We focus on pure strategy, subgame perfect equilibrium in which the price is a linear function of the public signals. In particular, a **linear equilibrium** is characterized by: (i) an optimal choice of investment x and internal message $t(\omega)$ by the manager which maximize (conditional) expectation of u_M in (3); (ii) an optimal choice of action a and effort z by the employee which maximize (conditional) expectation of u_E in (4); (iii) a price of the form $P = b_0 + b_d d + b_m m$ which satisfies (1); and, (iv) participants' beliefs that satisfy Bayes' rule wherever it is well-defined.

3.1 Discussion of Assumptions

In our main analysis, we assume that the public disclosure d cannot be manipulated by the manager. We interpret this as capturing the notion that many public disclosures (e.g., earnings announcements) are mandatory and verifiable, and interpret the variance of the error term ξ, denoted by σ_ξ^2, as an inverse measure of public information quality. Specifically, in Section 5 we compare the impact of increasing whistleblower incentives (w) versus increasing public disclosure quality (decreasing σ_ξ^2) on measures of efficiency in our setting.

Moreover, in the extension in Section 6, we relax the assumption that the public disclosure cannot be manipulated by the manager. Instead, we assume that $\xi > 0$ is a manipulation cost parameter, analogous to ε for internal communication, that affects the manager's ability to distort the public signal d. We study how our results change in this scenario. More generally, we expect the key forces of our analysis to obtain when the manager can engage in other types of communication.[7]

The specifications for the manager and employee payoffs are primarily for tractability. The fact that the manager's payoff depends on the price reflects that most firm executives receive stock based compensation in practice and implies that the manager has an incentive to distort his communication to the employee and the market (in Section 6).[8] We expect our results to be qualitatively similar if, instead, the manager's payoff was driven by a weighted average of P and V, although the analysis would be more cumbersome. Moreover, as we illustrate in Section 7.1, our results are qualitatively similar if the manager's payoff is independent of the price, but he is concerned about a penalty resulting from whistleblowing.

Similarly, the fact that the employee's payoff is sensitive to the terminal value ensures that she has an incentive to align her action to fundamentals, i.e., choose a as close to ω as possible. Making her payoff also depend on the price P would not qualitatively change our results.[9] We assume that the employee's benefit from leaking information depends on the effort she exerts in improving the precision of the signal m. One can think of this as a reduced form specification for a probabilistic reward / bounty that the whistleblower would receive if, conditional on her report, the manager is successfully prosecuted. In such a specification, one expects the likelihood of successful prosecution to be increasing in the precision of the leaked information. In Section 7.2, we study how our results change when, in addition, the whistleblower bounty is larger if the reported misconduct is more extreme. We show that when the employee can manipulate the leak, such incentives can be counter-productive: the employee biases her leak downwards to make the misconduct look more severe, but this makes her report less informative to the market, and leads to lower investment by the manager.

The assumption that the vector of shocks θ follows a normal distribution is made for tractability, since it implies that the conditional distribution of ω, given the signals, is also normal. This ensures that characterizing agents' updating of beliefs is tractable. As in

[7]For instance, an earlier version of the paper considers a setting in which the manager engages in cheap talk for internal communication and verifiable disclosure for public communication. As in the current analysis, we show that real efficiency can decrease with whistleblower incentives when internal alignment is sufficiently important for firm value. Details are available upon request.

[8]If instead, the manager's payoff only depended on the terminal cash-flow V, then the manager would have no incentive to distort his communication (i.e., $t = \omega$) and would invest efficiently a constant amount $x^* = 1$.

[9]Given that the variables ω and η are independent, the linear equilibrium as we characterized continues to exist. Moreover, since she takes the price coefficients as given, the employee's choice of a and z remain unchanged.

Fischer and Verrecchia (2000), this can imply that the cost of manipulating signals can sometimes be negative (since ε in the main model and ξ in the extension in Section 6 can take on negative values). However, by setting μ_ε and μ_ξ appropriately, the likelihood of this can be made arbitrarily small. One could alternatively ensure non-negativity by assuming that the shocks are elliptically distributed (as in Frankel and Kartik (2019) and Ball (2019)), but this makes the characterization of real efficiency in our analysis less tractable.[10]

4 Analysis

We solve the model by working backwards.

Date three. Given her objective in (4), the employee's optimal choice of whistleblowing effort z^* is given by:

$$z^* = \frac{w}{c} \tag{6}$$

and her optimal choice of action a is given by

$$a^* = \arg\max_a \mathbb{E}\left[\omega - \beta\left(a - \omega\right)^2 |d, t\right] = \mathbb{E}\left[\omega | d, t\right]. \tag{7}$$

Next, note that the price P can be expressed as

$$\begin{aligned} P &= \mathbb{E}\left[V(\omega, a^*)|d, m\right] \\ &= \mathbb{E}\left[\omega|\, d, m\right] - \beta\mathbb{E}\left[\mathbb{E}\left[\left(a^* - \omega\right)^2\middle|\, d, t\right]\middle|\, d, m\right] \\ &= \mathbb{E}\left[\omega|\, d, m\right] - \beta\mathbb{E}\left[\mathbb{V}\left[\omega|\, d, t\right]|\, d, m\right], \end{aligned}$$

where the final equality follows from the observation that $a^* = \mathbb{E}[\omega|d, t]$. Intuitively, the more informative the disclosure d and internal message t are about cash flows ω, the better aligned action a^* is. In turn, this translates to a higher valuation P for the firm, since in expectation, $\mathbb{V}[\omega|d, t]$ is smaller.

We shall conjecture, and then verify, that the price is linear in the public signals d and m, i.e.,

$$P = b_0 + b_d d + b_m m.$$

Date two. The manager conditions on the realization of ω and ε when choosing his

[10] Specifically, unless the shocks are normally distributed, the conditional variance of ω depends on the realization of the signals (e.g., see Foster and Viswanathan (1993)), which implies the expected firm value and real efficiency are no longer analytically tractable.

internal communication to the employee. Specifically, he chooses $t(\omega,\varepsilon)$ to maximize:

$$\begin{aligned}\mathbb{E}[u_M|\omega,\varepsilon] &= \mathbb{E}[P|\omega,\varepsilon] + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon} \\ &= b_0 + b_d\omega + b_m t + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon},\end{aligned}$$

since $\mathbb{E}[d|\omega,\varepsilon]=\omega$ and $\mathbb{E}\left[m|\omega,\varepsilon\right]=t(\omega,\varepsilon)$. This implies that the manager's optimal choice is given by

$$t^*(\omega,\varepsilon) = \omega + \frac{b_m}{\tau}\varepsilon. \tag{8}$$

This is intuitive. The higher the weight, b_m, the market puts on the employee's leak, m, the stronger the incentive of the manager to distort the signal t by inflating it (recall that $\varepsilon>0$). The extent of manipulation decreases in the manager's cost of doing so, which is captured by τ.

Let the market's conjecture about x be given by $\hat{x}$. Given the above characterizations of t and the employee's optimal choice of z, we have that the market's beliefs can be expressed as:

$$\mathbb{E}\left[\omega|m,d\right] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d-\mu_\xi}{\sigma_\xi^2} + \frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}$$

and

$$\mathbb{E}\left[\mathbb{V}\left[\omega|\, d,t\right]|\, d,m\right] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}},$$

which implies

$$P = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d-\mu_\xi}{\sigma_\xi^2} + \frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.$$

Note that this verifies the conjecture that the price is linear in the signals, d and m.

Date one. Given these coefficients, the manager's effort choice x maximizes:

$$\begin{aligned}\mathbb{E}[u_M] &= b_0 + b_m\mathbb{E}\left[m\right] + b_d\mathbb{E}\left[d\right] + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\mathbb{E}\left[\frac{(t-\omega)^2}{\varepsilon}\right] \\ &= b_0 + \left(b_m+b_d\right)x + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\mathbb{E}\left[\left(\frac{b_m}{\tau}\right)^2\varepsilon\right],\end{aligned}$$

so the equilibrium investment is

$$x^* = b_m + b_d. \tag{9}$$

Consistent with intuition, the manager's investment is increasing in the market's total weights on the public signal (i.e., b_d) and the employee's leak (i.e., b_m). The following proposition establishes that there exists a unique equilibrium in our setting.

Proposition 1. *There exists a unique linear equilibrium characterized by the optimal choices in (6), (7), (8), and (9), and an equilibrium price $P = b_0 + b_m m + b_d d$, where the price coefficients are pinned down by*

$$b_0 = \frac{\frac{b_m + b_d}{\sigma_\omega^2} - \frac{\mu_\xi}{\sigma_\xi^2} - \frac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}},$$

$$b_d = \frac{\frac{1}{\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}},$$

and $b_m \in (0,1)$ is the unique solution to

$$b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}. \tag{10}$$

The proof establishes that there is a unique solution $b_m \in (0,1)$ to the fixed point characterized by (10) which pins down the equilibrium. This expression reflects the fact that the weight b_m that investors put on the employee's message m depends on the variance of the error in the message, which depends on the extent to which the manager distorts his internal communication (as given by equation (8)), which in turn, increases with the weight b_m that investors put on the employee's message.

5 Measures of Efficiency

In this section, we characterize how changes in whistleblower incentives and public disclosure quality affect two different measures of efficiency: price informativeness and real efficiency.

We define **price informativeness**, denoted by PI, as the conditional precision of interim cash-flows ω, given the price, P i.e., $PI = (\mathbb{V}[\omega|P])^{-1}$. This is analogous to the notion of forecasting price efficiency in Bond et al. (2012) and captures the extent to which the security price is informative about firm fundamentals.[11] The following result characterizes how an increase in whistleblower incentives and public disclosure quality affects the price coefficients b_d and b_m, equilibrium investment, and price informativeness.

Proposition 2. *In equilibrium, an increase in the whistleblower bounty w leads to: (i) an increase in the price coefficient on the employee's signal, b_m; (ii) a decrease in the price coefficient on the public disclosure, b_d; and (iii) an increase in equilibrium investment, $x^* = b_d + b_m$, and price informativeness, PI.*
An increase in the quality of public disclosures (i.e., higher $1/\sigma_\xi^2$) leads to: (i) a decrease in the price coefficient on the employee's signal, b_m; (ii) an increase in the price coefficient on the public disclosure, b_d; and (iii) an increase in equilibrium investment, $x^* = b_d + b_m$, and price informativeness, PI.

Figure 2 provides an illustration of the above result. Intuitively, an increase in whistleblower incentives (w) and an increase in public disclosure quality ($1/\sigma_\xi^2$) improve the overall quality of information available to investors. The former leads to an increase in the precision of the employee's leak, m, which leads to an increase in the weight b_m investors put on this signal. All else equal, this leads to a decrease in the weight b_d investors put on the public disclosure. Similarly, an increase in public disclosure quality leads to a more precise public disclosure d, which leads to an increase in its weight b_d, but a decrease in the weight b_m investors put on the employee's leak.

In both cases, the informativeness of the total information available to investors increases, as suggested by an increase in $b_d + b_m$. This immediately implies that optimal investment x^* increases with either change. Finally, as we show in the proof, one can express price informativeness as:

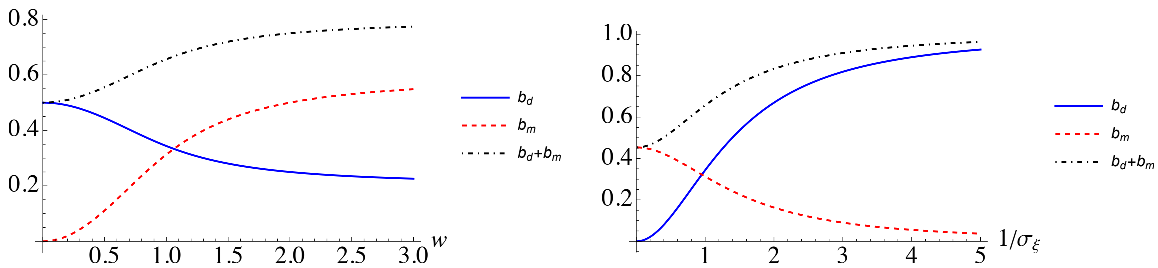
$$PI = (\mathbb{V}[\omega|P])^{-1} = \frac{1}{\sigma_\omega^2(1-(b_d+b_m))},$$

and so price informativeness also increases with whistleblower incentives and public disclosure quality.

While increasing whistleblower incentives and public disclosure quality have qualitatively similar effects on forecasting price efficiency, they do not have similar effects on revelatory

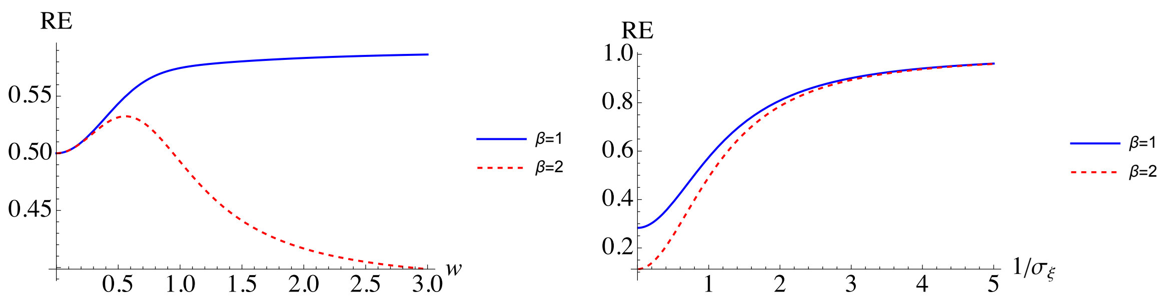
[11] Note that in contrast to settings where investors have heterogeneous information and price informativeness measures the extent to which the price aggregates this information, in our setting investors are symmetrically informed. However, price informativeness captures the extent to which the price reflects information about fundamentals to an econometrician. An alternate measure of price informativeness would be how informative the price is about terminal cash flows i.e., $(\mathbb{V}[V|P])^{-1}$. We prefer our measure of price informativeness because it is more analytically tractable and intuitive, and because it maps more closely to empirical measures of price informativeness proposed in the literature (e.g., Bai, Philippon, and Savov (2016); Dávila and Parlatore (2018)).

Figure 2: Price coefficients as a function of w and σ_ξ
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = c = w = 1$.



(a) b_d, b_m v.s. w (b) b_d, b_m v.s. $1/\sigma_\xi$

Figure 3: Real efficiency as a function of w, σ_ξ, and β.
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = c = w = 1$.



(a) RE v.s. w for differing β (b) RE v.s. $1/\sigma_\xi$ for differing β

price efficiency. Specifically, define **real efficiency**, denoted by RE, as the unconditional expected value of the firm, i.e., $RE = \mathbb{E}[V]$. The following result shows how policy changes affect real efficiency in our setting.

Proposition 3. *An increase in public information quality (i.e., higher $1/\sigma_\xi^2$) always leads to an increase in real efficiency RE. In contrast, there exists $0 < \underline{\beta} \leq \overline{\beta}$, such that:*
(i) when $\beta \leq \underline{\beta}$, real efficiency RE increases in the whistleblower bounty w, and
(ii) when $\beta > \overline{\beta}$, real efficiency RE is hump-shaped in the whistleblower bounty w.

Figure 3 provides an illustration of the above result. To gain some intuition, note that one can express real efficiency as the sum of two components:

$$RE = \mathbb{E}[V] = \mathbb{E}[\omega] - \beta\mathbb{E}[\mathbb{V}[\omega|d,t]].$$

The first component, $\mathbb{E}[\omega]$, is determined by the manager's equilibrium investment i.e., $\mathbb{E}[\omega] = x^* = b_d + b_m$, which is increasing in both whistleblower incentives and public disclosure quality.

The second component, $\beta\mathbb{E}[\mathbb{V}[\omega|d,t]]$, reflects the loss in firm value due to the misalignment between fundamentals and the employee's action. This loss is proportional to the average posterior uncertainty that the employee faces, after observing the public disclosure d and the internal communication t. An increase in public information quality $(1/\sigma_\xi^2)$ leads to a decrease in the employee's uncertainty, and so reduces the loss in value due to misalignment. As a result, firm value and real efficiency always increase in public information quality, as illustrated by panel (b) of Figure 3.

In contrast, an increase in whistleblower incentives (w) increases the manager's incentive to inflate the internal communication, which makes the internal message t a noisier signal of cash flows. As a result, the employee's uncertainty increases, which increases the loss in value due to misalignment. When the relative impact of the loss from misalignment is sufficiently low (i.e., β is sufficiently low), the first component dominates and real efficiency always increases with w. However, when β is sufficiently high, the latter effect can dominate when whistleblower incentives are sufficiently large. In this case, expected firm value and real efficiency can first increase but then eventually decrease in w, as illustrated in panel (a) of Figure 3.

To gain some intuition for this hump-shape, note that an increase in w affects RE through its impact on the market's weight b_m.[12] Moreover, as we show in the proof of Proposition 3, the benefit from increasing w through higher investment (captured by the $\mathbb{E}[\omega]$ term in RE) is linear in b_m. However, the penalty from misalignment (captured by $\beta\mathbb{V}[\omega|d,t]$) is initially convex in b_m.[13]

This implies that when w, and consequently b_m, is very low, the increase in RE due to higher investment dominates the decrease in RE due to more misalignment. In fact, when $w = 0$, internal communication is perfect and so the penalty from misalignment is zero (since $\mathbb{V}[\omega|d,t] = 0$). However, as w (and b_m) increase, the rate of increase in RE due to higher investment is constant, but the rate of decrease due to misalignment increases. Beyond a certain level of w under a sufficiently high β, we show that decrease due to misalignment

[12] Given other parameters, the dependence of b_d on w is also captured by its dependence on b_m, since we can express $b_d = \frac{(1-b_m)\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega^2}$.

[13] As we show in the proof of Proposition 3, the penalty from misalignment can become concave in the right tail of b_m depending on the other parameters. However, even in that case, we always have a hump-shaped real efficiency *locally*.

dominates and so RE decreases with further increases in whistleblower incentives.[14]

This implies that when β is sufficiently high, there exists an interior optimum level of b_m, and correspondingly, whistleblower incentives, w^*, which maximizes real efficiency. One can interpret this as the level of whistleblower incentives a policy maker should target, if their goal is to maximize firm value. The next result characterizes how this optimum level changes with model parameters.

Proposition 4. *Suppose $\beta > \bar{\beta}$, so that real efficiency is hump-shaped in w and maximized at w^*. Then, the optimal level of whistleblower incentives w^* is decreasing in β, σ_ω and $1/\sigma_\xi$, but increasing in σ_η.*

Figure 4 provides an illustration of these results. In each of these cases, a change in the underlying parameter differentially affects the marginal benefit (i.e., $\frac{\partial \mathbb{E}[\omega]}{\partial w}$) and the marginal cost (i.e., $-\frac{\partial \beta \mathbb{E}[\mathbb{V}[\omega|d,t]]}{\partial w}$) of increasing w. For instance, an increase in the sensitivity to misalignment β increases the marginal cost of increasing w, but leaves the marginal benefit unchanged, and as a result, the optimal choice of w^* is lower. Similarly, an increase in public disclosure quality (i.e., higher $1/\sigma_\xi$) reduces the weight market participants put on the leak, and consequently, the marginal benefit increasing whistleblower incentives, leading to a lower choice of w^*.

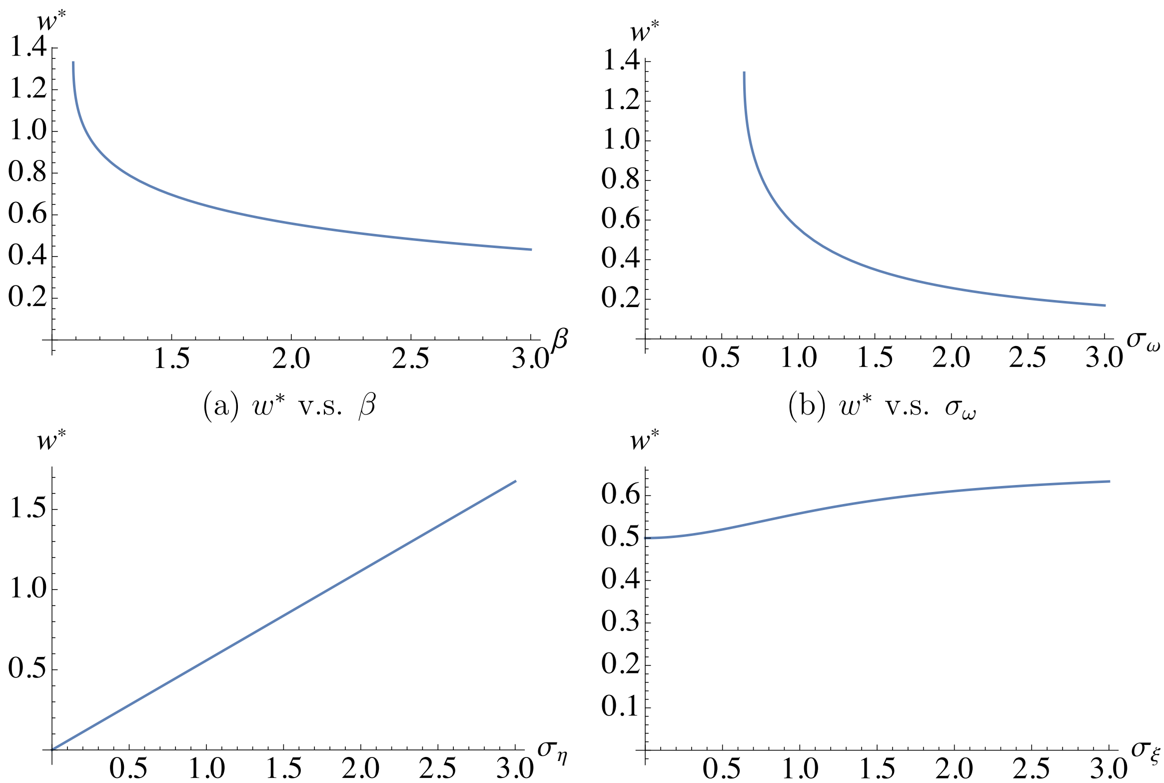
An increase in cash flow volatility, σ_ω, increases both the marginal benefit and the marginal cost. On the one hand, when cash flow volatility is higher, investors put more weight on the employee's leak (b_m is higher) and so the marginal benefit of increasing w is higher. On the other hand, a higher b_m increases the manager's incentive to distort his message, which increases the marginal cost of increasing w. When β is sufficiently large, the increase in the marginal cost due to misalignment is higher than the increase in the marginal benefit due to investment, and so the optimal w^* decreases.

In contrast, an increase in σ_η reduces the quality of the signal that the employee leaks to market, which leads to a decrease in b_m, all else equal. This reduces both the marginal benefit and the marginal cost of increasing w. However, when β is sufficiently large, the reduction in the marginal cost dominates, and the optimal choice of w^* increases.

Importantly, the above results imply that one must be cautious about evaluating the impact of such policies on the cross-section of firms, since they can have qualitatively different effects across different firms. Specifically, whistleblower incentives should be weaker for firms in which internal alignment is more important (i.e., β is higher) and cash flows are more volatile (i.e., σ_ω is higher), but stronger when employee leaks are more likely to be noisy

[14]If β is too low, the supremum of b_m (i.e., $\lim_{w\to\infty} b_m$) is not large enough to reach the domain in which the loss from misalignment dominates.

Figure 4: Optimal level of whistleblower incentives w^* as a function of model parameters. Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = c = w = 1$ and $\beta = 2$.



(a) w^* v.s. β

(b) w^* v.s. σ_ω

(c) w^* v.s. σ_η

(d) w^* v.s. σ_ξ

(i.e., σ_η is higher). Moreover, the optimal level of whistleblower incentives decreases with an increase in the quality of public disclosure (i.e., increases when σ_ξ increases).

The above results also highlight an important difference across the two policy instruments. While an increase in whistleblower incentives and public information quality have similar effects on the equilibrium choice of investment and price informativeness, they can have opposite effects on firm value and real efficiency when the relative value loss from misalignment within the firm is sufficiently high. One might conjecture that this difference arises because the manager can manipulate his internal communication, but cannot distort the public signal. In the next section, we show that our results remain qualitatively similar when the manager can manipulate both the internal communication and the public disclosure.

6 Endogenous public disclosure

In our baseline analysis, we assume that the public disclosure d cannot be manipulated by the manager. In this section, we relax this assumption and instead allow the manager to send a public signal d about ω which can be manipulated at a private cost $\frac{\delta}{2}\frac{(d-\omega)^2}{\xi}$. We maintain all the other assumptions of our benchmark analysis.

One can interpret δ as a measure of the public disclosure scrutiny in this setting — a higher value of δ corresponds to more analysis, monitoring and verification of the public disclosure (e.g., more intensive audits, stricter disclosure requirements), which make it more difficult for the manager to inflate the public disclosure without being detected.[15] In the analysis that follows, we shall compare the effect of increasing whistleblower incentives (w) to that of increasing this measure of public disclosure scrutiny (δ).

It is immediate that the date three choices of the employee are given by (6) and (7), as before. Moreover, given the linear conjecture for the price $P = b_0 + b_d d + b_m m$, the manager's date two objective naturally generalizes to maximize:

$$\begin{aligned}\mathbb{E}[u_M|\omega,\varepsilon,\xi] &= \mathbb{E}[P|\omega,\varepsilon,\xi] + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon} - \frac{\delta}{2}\frac{(d-\omega)^2}{\xi} \\ &= b_0 + b_d d + b_m t + \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon} - \frac{\delta}{2}\frac{(d-\omega)^2}{\xi},\end{aligned}$$

which implies that his optimal choices are given by:

$$t^*(\omega,\varepsilon) = \omega + \frac{b_m}{\tau}\varepsilon, \quad \text{and} \quad d^*(\omega,\xi) = \omega + \frac{b_d}{\delta}\xi. \tag{11}$$

This is also intuitive. The more weight the price puts on the public disclosure (the higher b_d is), the greater the incentive to inflate the signal, and the larger the weight on the "error" $\xi > 0$ in the optimal disclosure.

Given these specifications, we can once again verify the linear conjecture of the price. Specifically, the price can be expressed as:

$$P = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d - \frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{m - \frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.$$

[15] Greater scrutiny increases the probability of detection and the expected cost of misreporting (Ferri, Zheng, and Zou, 2018; Samuels, Taylor, and Verrecchia, 2021).

Finally, the date one investment decision is characterized by

$$\max_x b_0 + b_m \mathbb{E}[m] + b_d \mathbb{E}[d] + \frac{1}{2}\left(1 - x^2\right) - \left(\frac{\delta}{2}\right)\mathbb{E}\left[\frac{(d-\omega)^2}{\xi}\right] - \left(\frac{\tau}{2}\right)\mathbb{E}\left[\frac{(t-\omega)^2}{\varepsilon}\right],$$

which again implies that the optimal choice x^* is given by (9). The following result generalizes Proposition 1 from our benchmark analysis.

Proposition 5. *There exists a unique linear equilibrium characterized by the optimal choices in (6), (7), (9), and (11), and an equilibrium price $P = b_0 + b_m m + b_d d$, where the price coefficient b_0 is pinned down by:*

$$b_0 = \frac{\frac{b_m + b_d}{\sigma_\omega^2} - \frac{\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} - \frac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}} - \beta \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}},$$

and $b_m, b_d \in (0,1)$ is the unique solution to the following system of equations:

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}, \quad b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}. \tag{12}$$

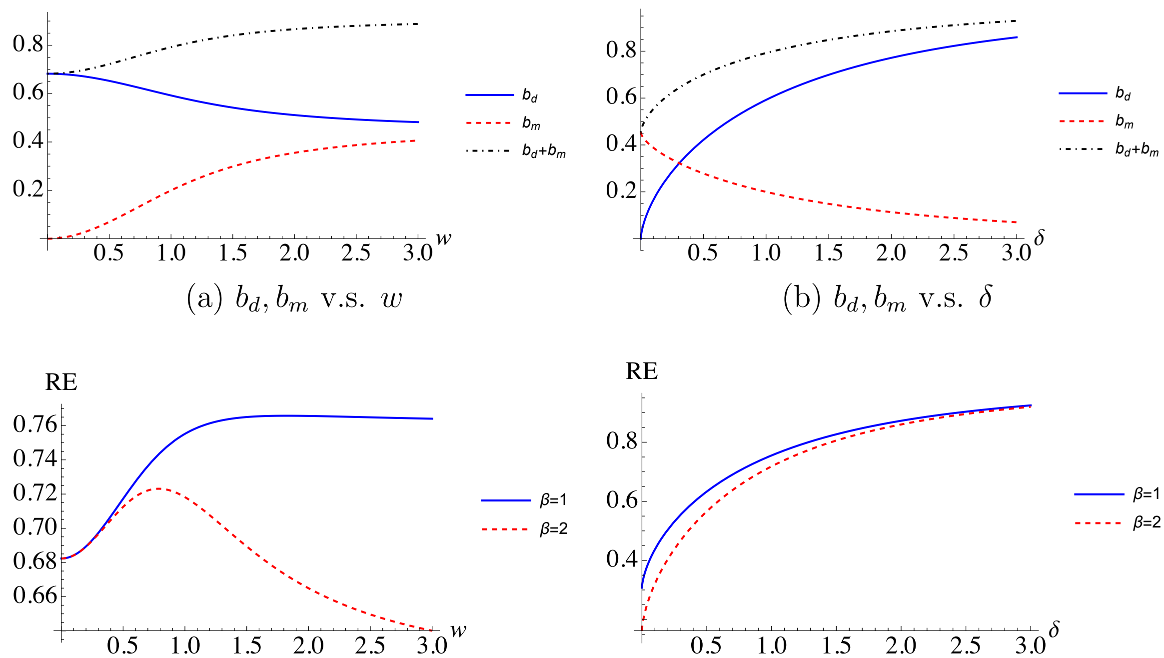
Notably, the equilibrium is now pinned down by the solution to a system of two fixed points: the weights b_d and b_m that investors put on the signals d and m depend on the errors in these signals, which depends on the extent to which the manager distorts his messages d and t, which in turn, increases in the weights b_d and b_m that investors put on these signals. The proof establishes that there exists a unique pair of $b_m, b_d \in (0,1)$, which solves the system of equations in (12).

Having characterized the equilibrium, we now study how changes in whistleblower incentives and public disclosure scrutiny affect measures of efficiency. As the following result illustrates, the key results from our main analysis are qualitatively similar when the manager can manipulate both public and private communications.

Proposition 6. *In equilibrium, an increase in public disclosure scrutiny δ always leads to an increase in investment x^*, price informativeness PI, and real efficiency RE.*
An increase in the whistleblower bounty w always leads to an increase in investment x^* and in price informativeness PI. Moreover, there exists $0 < \underline{\beta} \leq \overline{\beta}$, such that:
(i) when $\beta \leq \underline{\beta}$, real efficiency RE increases in the whistleblower bounty w, and
(ii) when $\beta > \overline{\beta}$, real efficiency RE is hump-shaped in the whistleblower bounty w.

Panels (a) and (b) of Figure 5 illustrate how the market's weights b_d and b_m change as a function of whistleblower incentives w and public disclosure scrutiny δ. As in the main model, an increase in whistleblower incentives leads to more informative leaks by the employee. In turn, this leads to an increase in the market's weight on the leaked information (b_m) and a decrease in the market's weight on the public disclosure (b_d).

Figure 5: Price coefficients and real efficiency when public disclosure is endogenous
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = c = \beta = \delta = w = 1$.



(a) b_d, b_m v.s. w

(b) b_d, b_m v.s. δ

(c) RE v.s. w for differing β

(d) RE v.s. δ for differing β

In contrast to the main analysis, however, note that this latter effect is muted — a lower weight on the public disclosure also leads the manager to distort such reports less i.e., d is a more informative signal about ω as a result. This uncovers a novel implication of increasing whistleblower incentives — by reducing the market's reliance on public disclosures, such policies reduce managerial incentives to manipulate such reports, thereby making them more informative.

Similarly, an increase in public disclosure scrutiny δ makes it costlier for the manager to distort the public report. This leads to an increase in the market weight b_d on the public disclosure,[16] and a corresponding decrease in the weight b_m on the employee's leak. In turn, this reduces the manager's incentive to distort his internal communication.

The proposition establishes that an increase in whistleblower incentives or public disclosure scrutiny leads to higher investment and higher price informativeness, as in the main model. Moreover, as panels (c) and (d) illustrate, while higher scrutiny of public disclosure also unambiguously leads to higher firm value and real efficiency, excessively strong whistleblower incentives can reduce real efficiency when the relative importance of alignment is sufficiently high (i.e., β is sufficiently high).

7 Robustness

In this section, we explore how our results change under alternate specifications of managerial and employee payoffs.

7.1 Alternate managerial payoffs

In our benchmark analysis, the impact of a whistleblower's leak on the manager's payoffs is through its impact on the price. In this subsection, we explore how our results change if instead, the impact of such leaks is through the penalty that the regulator imposes on the manager.

Specifically, consider the benchmark model from Section 3 (with exogenous disclosure), but suppose that the manager's compensation is independent of the price. Instead, the manager incurs a penalty $\Pi(d, m)$ which depends on the public disclosure d and the whistleblower's leak m. Since the goal of encouraging whistleblowing is to deter misconduct by the firm, it is natural to assume that the penalty is increasing in how much firm cash-flows fall below the first-best outcome i.e., increasing in $\mathbb{E}[\omega|x_{FB}] - \mathbb{E}[\omega|d, m] = x_{FB} - \mathbb{E}[\omega|d, m]$, where $x_{FB} = 1$ is the first best choice of investment in the absence of moral hazard. For tractability, suppose the penalty is linear i.e.,

$$\Pi(d, m) \equiv \pi \times (x_{FB} - \mathbb{E}[\omega|d, m]),$$

so that the manager's payoff is given by

$$u_M = \frac{1}{2}\left(1 - x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon} - \pi(x_{FB} - \mathbb{E}[\omega|d, m]).$$

[16] It has been well documented empirically that greater public scrutiny increases the weight that investors place on accounting disclosures in valuing the firm (Collins and Kothari, 1989; Teoh and Wong, 1993).

Here, one can interpret the parameter π as the (expected) intensity of the regulator's penalty. It should increase in the severity of the punishment and in the probability of successful prosecution of the manager.

We can solve for the equilibrium by working backwards as in Section 4. Since the employee's payoffs remain the same, her optimal choice of z^* and a^* are unchanged. Moreover, as in the earlier analysis, one can conjecture and verify that the policymaker's conditional expectation of cash-flows is linear in d and m i.e.,

$$\mathbb{E}[\omega|d,m] = \hat{b}_0 + \hat{b}_d d + \hat{b}_m m.$$

At date two, the manager conditions on the realization of ω and ε when choosing internal communication t^*:

$$t^*(\omega,\varepsilon) = \arg\max_t \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\frac{(t-\omega)^2}{\varepsilon} - \pi\left(x_{FB} - (\hat{b}_0 + \hat{b}_d\omega + \hat{b}_m t)\right) = \omega + \pi\frac{\hat{b}_m}{\tau}\varepsilon. \tag{13}$$

This is intuitive — the higher the intensity of the penalty that the manager faces, the stronger the incentive to manipulate his internal communication.

Finally, at date one, the manager's effort choice maximizes:

$$\mathbb{E}[u_M] = \frac{1}{2}\left(1-x^2\right) - \frac{\tau}{2}\mathbb{E}\left[\frac{(t-\omega)^2}{\varepsilon}\right] - \pi\left(x_{FB} - (\hat{b}_0 + (\hat{b}_d + \hat{b}_m)x)\right)$$

which implies the optimal choice of investment is

$$x^* = \pi(\hat{b}_m + \hat{b}_d). \tag{14}$$

The following result establishes that there exists a unique equilibrium in this case.

Proposition 7. *There exists a unique equilibrium characterized by the optimal choices in (6), (7), (13), and (14), and the regulator's conditional expectation* $\mathbb{E}[\omega|d,m] = \hat{b}_0 + \hat{b}_d d + \hat{b}_m m$, *where the coefficients are pinned down by*

$$\hat{b}_0 = \frac{\frac{\pi(\hat{b}_m+\hat{b}_d)}{\sigma_\omega^2} - \frac{\mu_\xi}{\sigma_\xi^2} - \frac{\frac{\pi\hat{b}_m}{\tau}\mu_\varepsilon}{\left(\frac{\pi\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\pi\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}},$$

$$\hat{b}_d = \frac{\frac{1}{\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\pi\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}},$$

and $\hat{b}_m \in (0,1)$ *is the unique solution to*

$$\hat{b}_m = \frac{\frac{1}{\left(\frac{\pi \hat{b}_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\pi \hat{b}_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z^*}\right)^2 \sigma_\eta^2}}.$$

Moreover, $\hat{b}_m$ *is decreasing in* π*, while* $\hat{b}_d$ *and* x^* *are increasing in* π*.*

The proof follows analogously to that of Proposition 1. Since the manager is penalized more when the policymaker's conditional beliefs about ω are lower, an increase in the intensity π increases his incentives to distort his internal communication. This results in a noisier leak m by the employee, and consequently, a lower weight $\hat{b}_m$ on the leak by the policymaker. Further, the policymaker attaches a higher weight $\hat{b}_d$ to the exogenous disclosure d instead. We show in the proof that the total weight on the signals $\hat{b}_d + \hat{b}_m$ decreases with the intensity π, as a result of the declining quality of the employee's leaked information. Following the arguments in Section 5, this implies that price informativeness PI is *decreasing* in the intensity π, since

$$PI = (\mathbb{V}[\omega|P])^{-1} = \frac{1}{\sigma_\omega^2(1 - (\hat{b}_d + \hat{b}_m))}.$$

Nevertheless, the manager's optimal choice of investment $x^* = \pi(\hat{b}_d + \hat{b}_m)$ increases with the penalty, π, since the cost of the penalty for deviating from the first-best level has a dominant effect.

7.2 Alternate whistleblower payoffs

In our benchmark analysis, the employee's payoffs from leaking information depends only on the precision of the signal she generates. In practice, whistleblower bounties depend not only on the quality of the leaked information, but also on how large the magnitude of the misconduct is. Moreover, given such incentives, the whistleblower may have the ability to distort the information they provide to the regulator.

To capture these effects, suppose the employee chooses z and $\bar{m}$ to publicly leak a noisy signal

$$m = \bar{m} + \frac{\eta}{z}$$

about the internal communication t, where the cost of choosing z is $\frac{c_1}{2}z^2$ and the cost of distorting the mean $\bar{m}$ is given by $\frac{c_2}{2}\frac{(\bar{m}-t)^2}{\nu}$. Moreover, the employee's payoff is given by

$$u_E = V(\omega, a) + w_1 \times z - \frac{c_1}{2}z^2 + w_2 \left(x_{FB} - \mathbb{E}[\omega|d, m]\right) - \frac{c_2}{2}\frac{(\bar{m} - t)^2}{\nu},$$

where ν is privately known to the employee, w_1 parameterizes the whistleblower incentives for higher precision, while w_2 parameterizes the whistleblower incentives for reporting larger misconduct. Specifically, all else equal, the employee gets a larger bounty when the policy maker infers a larger degree of misconduct by the manager (i.e., larger $x_{FB} - \mathbb{E}\left[\omega|d,m\right]$) based on the available information. We assume the rest of the setup is the same as in our benchmark model of Section 3 (with exogenous disclosure), and that the vector $\theta \equiv (\omega, \varepsilon, \xi, \eta, \nu)$ follows an normal distribution where the mean and variance of θ are given by:

$$\mu_\theta = (x, \mu_\varepsilon, \mu_\xi, 0, \mu_\nu)\,, \quad \text{and} \quad \Sigma_\theta = \begin{pmatrix} \sigma_\omega^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\xi^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\nu^2 \end{pmatrix}.$$

As before, we solve the model by working backwards. Note that the employee's optimal choice of a^* remains the same, since it depends only on her conditional expectation of $V(\omega, a)$, given d and t. Moreover, the above payoffs imply that

$$z^* = \frac{w_1}{c_1} \tag{15}$$

as before. Next, suppose that conditional on public information (i.e., d and m), the conditional expectation of ω is given by:

$$\mathbb{E}\left[\omega|d,m\right] = \hat{b}_0 + \hat{b}_d d + \hat{b}_m m.$$

Since the employee can condition on t, d and ν, her optimal choice of $\bar{m}$ is given by:

$$\bar{m} = t - \frac{w_2}{c_2}\hat{b}_m \nu, \tag{16}$$

which in turn implies:

$$m = t - \frac{w_2}{c_2}\hat{b}_m \nu + \frac{\eta}{z}.$$

Note that w_1 and w_2 have qualitatively different effects on the informativeness of the leak: an increase in w_1 leads the employee to increase the precision z of the message, while an increase in w_2 leads the employee to distort the leak downwards by pushing $\bar{m}$ lower.

Since the manager's payoffs are unchanged, his optimal choice of t and x are given by:

$$t^* = \omega + \frac{\hat{b}_m}{\tau}\varepsilon, \quad \text{and} \quad x^* = \hat{b}_m + \hat{b}_d, \tag{17}$$

respectively. This yields the following result.

Proposition 8. *There exists a unique equilibrium characterized by the optimal choices in (7), (15), (16) and (17), and the public conditional expectation* $\mathbb{E}[\omega|d,m] = \hat{b}_0 + \hat{b}_d d + \hat{b}_m m$*, where the coefficients are pinned down by:*

$$\hat{b}_0 = \frac{\frac{\hat{b}_m+\hat{b}_d}{\sigma_\omega^2} - \frac{\mu_\xi}{\sigma_\xi^2} - \frac{\frac{\hat{b}_m}{\tau}\mu_\varepsilon + \frac{w_2}{c_2}\hat{b}_m\mu_\nu}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}$$

$$\hat{b}_d = \frac{\frac{1}{\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}$$

and $\hat{b}_m \in (0,1)$ *is the unique solution to:*

$$\hat{b}_m = \frac{\frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2 + \left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}. \tag{18}$$

Moreover, $\hat{b}_m$ *and* x^* *are increasing in* w_1 *and decreasing in* w_2*, while* $\hat{b}_d$ *is decreasing in* w_1 *but increasing in* w_2*.*

The above result highlights the importance of designing whistleblower incentives appropriately. As our benchmark analysis highlights, an increase in w_1 incentivizes the employee to invest more in providing a more informative leak. This leads the market to put more weight on the leak (higher $\hat{b}_m$) which encourages the manager to (i) make internal communication more noisy, but (ii) increase investment x^*.

However, if the employee can distort her leak, a higher incentive to report greater misconduct (i.e., an increase in w_2) can be counterproductive. This is because an increase in w_2 leads the employee to distort her report downwards by pushing $\bar{m}$ lower. However, from the policymaker's perspective, this makes the leaked information a noisier signal of ω, and as a result, reduces $\hat{b}_m$. While this leads to a reduction in the distortion in internal communication, it leads to lower investment x^* by the manager.

Moreover, as we show in the following proposition, real efficiency depends on the interaction of the two types of incentives. Specifically, the optimal level of one dimension depends on the relative importance of internal alignment (β) and the level of the other dimension.

Proposition 9. *Let* $0 < \underline{\beta} \leq \overline{\beta}$ *be the cutoffs defined in Proposition 3. Price informativeness is always increasing in the whistleblower incentives* w_1 *for higher precision, and decreasing in* *the whistleblower incentives w_2 for reporting larger misconduct. When $\beta \leq \underline{\beta}$, real efficiency RE increases in w_1 but decreases in w_2. When $\beta > \overline{\beta}$, real efficiency RE is hump-shaped in the whistleblower incentives w_i for any given w_j, where $i \neq j$. Furthermore, an increase in w_j leads to an increase in the optimal level of w_i.*

To gain some intuition, note that the incentives w_1 and w_2 affect real efficiency through their impact on the weight $\hat{b}_m$ that the market puts on the employee's leak.[17] This immediately implies the result for price informativeness, which is increasing in $\hat{b}_m + \hat{b}_d$. Moreover, when the relative value of alignment is low (i.e., $\beta < \underline{\beta}$), this implies real efficiency is increasing in $\hat{b}_m$, and consequently, increasing in w_1, but decreasing in w_2.

In contrast, when the relative value of alignment is sufficiently high (i.e., $\beta > \overline{\beta}$), real efficiency attains a maximum for an interior value of $\hat{b}_m$ (as in Proposition 3 of our baseline analysis). In this case, there are multiple pairs of (w_1, w_2) that maximize real efficiency. Moreover, given that these incentives have opposing effects on $\hat{b}_m$, increasing one type of incentive implies that the optimal level of the other type of incentive should also increase.

Our analysis has important implications for the choice of different dimensions of whistleblower policy. In practice, whistleblower bounties are proportional to the size of the sanctions collected by the regulator's enforcement action (see https://www.whistleblower.gov/overview). Such incentives might lead employees to distort their leaked information to make the reported misconduct appear more negative than it is, which can lead to lower real efficiency. Our analysis suggests that the regulator should balance such incentives with stronger, direct incentives for accuracy (i.e., an increase in w_1) to ensure that the negative impact on real efficiency is offset.

8 Conclusion

We develop a model to study how increasing whistleblower incentives affect a manager's incentives for public disclosure, internal communication and engaging in misconduct. We show that while stronger incentives reduce misconduct and can increase the quality of public disclosure, they worsen internal communication. This is because when the market puts more weight on the employee's leak, the manager has a stronger incentive to bias his message to the employee. This deterioration in the quality of internal communication can lead to less informed actions by the employee. When the relative importance of the employee's action is sufficiently high, our analysis implies that stronger whistleblower incentives can reduce firm

[17] As before, one can express $\hat{b}_d = \frac{(1-\hat{b}_m)\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega^2}$, which implies that (w_1, w_2) affect $\hat{b}_d$ through their effect on $\hat{b}_m$, all else equal.

value and real efficiency.

Our model provides a stylized setting to analyze the impact of such policy interventions and provides a number of useful takeaways. First, changes in whistleblower incentives not only have an effect on internal communication, but can have a direct effect on the informativeness of public disclosures themselves. Specifically, our model predicts that public disclosures by firms should become more informative following the implementation of the Whistleblower Program of the Dodd-Frank Act (see http://www.sec.gov/spotlight/dodd-frank/whistleblower.shtml). However, a key challenge in testing such a prediction empirically is to separately identify the impact of such regulation on the incidence of leaks, while controlling for other aspects that might affect firms' disclosure policies directly.

Second, all else equal, stronger whistleblower incentives should be associated with less informative internal communication. Empirical tests of this prediction are confounded by the fact that following (the incidence of) a leak, firms usually make changes to the internal governance and communication policies. As such, a test of this prediction requires identifying firms that are ex-ante more likely to have leaks, and comparing their internal communication to a control group. Bowen, Call, and Rajgopal (2010) provide some preliminary evidence consistent with this prediction: in their sample, targets of employee whistleblowing allegations are more likely to have unclear internal communication channels.

Third, the impact of stronger whistleblower incentives can be qualitatively different across firms: they unambiguously improve real efficiency for firms where internal alignment is not very important, but can reduce firm value when misalignment is very costly. This suggests that targeting stronger whistleblower incentives to certain industries and sectors may be more effective than in others.

Our analysis has implications for policy decisions. In our setting, more direct policy interventions like increasing mandatory disclosure or scrutiny of public reports may be preferable since they do not have the unintended consequence of increasing the manager's incentives to distort internal communication. Our analysis also suggests that when whistleblowers receive larger rewards for reporting larger degrees of misconduct, this should be accompanied by stronger incentives for more accurate reporting.

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A Proofs

Proof of Proposition 1. Given the employee's objective function (4), the solution to the employee's problem satisfies:

$$w - cz = 0 \quad \text{and} \quad \mathbb{E}[-2\beta(a-\omega)|d,t] = 0,$$

implying the optimal choice of z

$$z^* = \frac{w}{c}$$

and the optimal choice of a

$$a^* = \mathbb{E}\left[\omega|d,t\right].$$

In what follows, z corresponds to the optimal choice, z^*. Hence, the price P is equal to

$$\begin{aligned}
P &= \mathbb{E}\left[V|d,m\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[V|\, d,t\right]|\, d,m\right] \\
&= \mathbb{E}\left[\omega|\, d,m\right] - \beta\mathbb{E}\left[\mathbb{E}\left[(a-\omega)^2\middle|\, d,t\right]\middle|\, d,m\right] \\
&= \mathbb{E}\left[\omega|\, d,m\right] - \beta\mathbb{E}\left[\mathbb{E}\left[(\mathbb{E}\left[\omega|d,t\right]-\omega)^2\middle|\, d,t\right]\middle|\, d,m\right] \\
&= \mathbb{E}\left[\omega|\, d,m\right] - \beta\mathbb{E}\left[\mathbb{V}\left[\omega|\, d,t\right]|\, d,m\right],
\end{aligned}$$

where $\mathbb{V}\left[\omega|\, d,t\right]$ is the posterior variance of ω conditional on the realizations of d and t.

We conjecture that there is an equilibrium in which the price takes the form of $P = b_0 + b_d d + b_m m$, where b_0, b_d, b_m are constants such that $b_d, b_m \geq 0$. Since $\mathbb{E}\left[\eta\right] = 0$, we have that $\mathbb{E}\left[m|t\right] = t$. This implies that given the realizations of (ω, ε), the manager chooses t to maximize:

$$\mathbb{E}\left[P|\omega,\varepsilon\right] - \left(\frac{\tau}{2}\right)\frac{(t-\omega)^2}{\varepsilon} = b_0 + b_d d + b_m t - \left(\frac{\tau}{2}\right)\frac{(t-\omega)^2}{\varepsilon}.$$

The FOC imply that

$$b_m - \frac{\tau}{\varepsilon}\left(t-\omega\right) = 0 \Rightarrow t\left(\omega,\varepsilon\right) = \omega + \frac{b_m}{\tau}\varepsilon.$$

Let $\hat{x}$ be the market's conjecture about x. Given the above characterization of t and the employee's optimal choice of z, we have that the market's beliefs can be expressed as:

$$\mathbb{E}\left[\omega|m,d\right] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d-\mu_\xi}{\sigma_\xi^2} + \frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2\sigma_\eta^2}}$$

and

$$\mathbb{E}\left[\mathbb{V}\left[\omega|\,d,t\right]|\,d,m\right]=\frac{1}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}}},$$

which implies

$$P=\frac{\frac{\hat{x}}{\sigma_{\omega}^{2}}+\frac{d-\mu_{\xi}}{\sigma_{\xi}^{2}}+\frac{m-\frac{b_m}{\tau}\mu_{\varepsilon}}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}-\beta\frac{1}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}}}.$$

Note that this verifies the conjecture that the price is linear in the signals d and m. Matching terms, we have:

$$\begin{aligned}
b_0&=\frac{\frac{\hat{x}}{\sigma_{\omega}^{2}}-\frac{\mu_{\xi}}{\sigma_{\xi}^{2}}-\frac{\frac{b_m}{\tau}\mu_{\varepsilon}}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}-\beta\frac{1}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}}},\\
b_d&=\frac{\frac{1}{\sigma_{\xi}^{2}}}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}},\\
b_m&=\frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}.
\end{aligned}\tag{19}$$

It is clear that the equations above are functions of b_m.

Next, we show that there is a unique b_m that solves the equations, which implies existence and uniqueness of the equilibrium market price. Specifically, b_m is given by the solution to $H\left(b_m\right)=0$, where

$$H\left(b_m\right)\equiv b_m-\frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}{\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\left(\frac{b_m}{\tau}\right)^{2}\sigma_{\varepsilon}^{2}+\left(\frac{1}{z}\right)^{2}\sigma_{\eta}^{2}}}.\tag{20}$$

Note that

$$\begin{aligned}
H\left(0\right)&=-\frac{z^{2}}{\sigma_{\eta}^{2}\left(\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\sigma_{\omega}^{2}}+\frac{z^{2}}{\sigma_{\eta}^{2}}\right)}<0\\
H\left(1\right)&=1-\frac{\frac{1}{\frac{\sigma_{\eta}^{2}}{z^{2}}+\frac{\sigma_{\epsilon}^{2}}{\tau^{2}}}}{\left(\frac{1}{\sigma_{\xi}^{2}}+\frac{1}{\sigma_{\omega}^{2}}+\frac{1}{\frac{\sigma_{\eta}^{2}}{z^{2}}+\frac{\sigma_{\epsilon}^{2}}{\tau^{2}}}\right)}>0\\
H'\left(b_m\right)&=1+\frac{2\tau^{2}z^{4}b_m\sigma_{\xi}^{2}\sigma_{\omega}^{2}\sigma_{\epsilon}^{2}\left(\sigma_{\xi}^{2}+\sigma_{\omega}^{2}\right)}{\left(z^{2}b_m^{2}\sigma_{\epsilon}^{2}\left(\sigma_{\xi}^{2}+\sigma_{\omega}^{2}\right)+\tau^{2}\left(\sigma_{\eta}^{2}\left(\sigma_{\xi}^{2}+\sigma_{\omega}^{2}\right)+z^{2}\sigma_{\xi}^{2}\sigma_{\omega}^{2}\right)\right)^{2}}>0.
\end{aligned}$$

Hence there exists a unique solution b_m in $(0, 1)$ to $H(b_m) = 0$ by the intermediate value theorem.

Finally, we solve for the equilibrium investment x^*. Given the coefficient b_m, the manager's investment choice maximizes:

$$b_0 + b_m \mathbb{E}[m] + b_d \mathbb{E}[d] + \frac{1}{2}\left(1 - x^2\right) - \left(\frac{\tau}{2}\right) \mathbb{E}\left[\left(\frac{b_m}{\tau}\right)^2 \varepsilon\right].$$

So the equilibrium investment maximizes $(b_m + b_d)\, x + \frac{1}{2}(1 - x^2)$ and is given by

$$x^* = b_m + b_d.$$

This characterizes the unique linear equilibrium as stated in the proposition. $\square$

Proof of Proposition 2. By (6), a higher whistleblower bounty w induces the employee to exert more effort and to choose a higher z^*. The first part of the proposition shows how the equilibrium quantities change as z^* increases.

The function $H(b_m)$ is defined as in (20). The chain rule implies that

$$\frac{\partial H}{\partial b_m} \times \frac{db_m}{dz} + \frac{\partial H}{\partial z} = 0.$$

It follows that

$$\frac{db_m}{dz} = -\frac{\partial H/\partial z}{\partial H/\partial b_m} = -\frac{-\frac{2\tau^4 z \sigma_\eta^2 \sigma_\xi^2 \sigma_\omega^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)}{\left(z^2 b_m^2 \sigma_\epsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + \tau^2 \left(\sigma_\eta^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + z^2 \sigma_\xi^2 \sigma_\omega^2\right)\right)^2}}{1 + \frac{2\tau^2 z^4 b_m \sigma_\xi^2 \sigma_\omega^2 \sigma_\epsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)}{\left(z^2 b_m^2 \sigma_\epsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + \tau^2 \left(\sigma_\eta^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + z^2 \sigma_\xi^2 \sigma_\omega^2\right)\right)^2}} > 0.$$

We eliminate z^2 from (19) and get

$$b_d = \frac{(1 - b_m)\, \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2},$$

which shows that

$$\frac{db_d}{dz} = -\left(\frac{\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}\right)\left(\frac{\partial b_m}{\partial z}\right) < 0.$$

Moreover, we have

$$x^* = b_d + b_m = \frac{b_m \sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}$$

which is increasing in b_m and z. So if the whistleblower bounty w increases, the employee's effort z^* will increase, the price coefficient b_m on the employee's signal will increase, the price coefficient b_d on the public disclosure will decrease, and the equilibrium investment will increase. Next, we examine how it affects the price informativeness PI.

Recall that the price informativeness is defined as $PI = (\mathbb{V}[\omega|P])^{-1}$. Since $P = b_0 + b_m m + b_d d = b_0 + (b_m + b_d)\left(\frac{b_m}{b_m+b_d} m + \frac{b_d}{b_m+b_d} d\right)$, where $m = \omega + \frac{b_m}{\tau}\varepsilon + \frac{1}{z}\eta$ and $d = \omega + \xi$, we can write

$$\mathbb{V}\left[\omega|P\right] = \mathbb{V}\left[\omega|y_P\right] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_y^2}},$$

where $y_p \equiv \frac{b_m}{b_m+b_d}(\omega + \frac{b_m}{\tau}\varepsilon + \frac{1}{z}\eta) + \frac{b_d}{b_m+b_d}(\omega + \xi) = \omega + \frac{1}{b_m+b_d}\left(b_m\left(\frac{b_m}{\tau}\varepsilon + \frac{1}{z}\eta\right) + b_d\xi\right)$, and

$$\sigma_y^2 \equiv \frac{b_d^2\sigma_\xi^2 + b_m^2\left(\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2\sigma_\eta^2\right)}{\left(b_m + b_d\right)^2}.$$

It hence follows that

$$\begin{aligned}\mathbb{V}\left[\omega|P\right] &= \frac{1}{\frac{\left(b_m\sigma_\xi^2+\sigma_\omega^2\right)^2}{\left(\sigma_\xi^2+\sigma_\omega^2\right)^2\left(\frac{(b_m-1)^2\sigma_\xi^2\sigma_\omega^4}{\left(\sigma_\xi^2+\sigma_\omega^2\right)^2}+\frac{b_m^2\sigma_\eta^2}{z^2}+\frac{b_m^4\sigma_\epsilon^2}{\tau^2}\right)} + \frac{1}{\sigma_\omega^2}} \\ &= \frac{\left(1 - b_m\right)\sigma_\xi^2\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2},\end{aligned}$$

which implies that $\mathbb{V}\left[\omega|P\right]$ decreases in b_m and so decreases in z. As a result, price efficiency increases in z and w.

The second part of the proposition shows how equilibrium quantities change with the quality of public disclosure σ_ξ^2. By the chain rule

$$\frac{\partial H}{\partial b_m} \times \frac{db_m}{d\sigma_\xi^2} + \frac{\partial H}{\partial \sigma_\xi^2} = 0,$$

we have

$$\frac{db_m}{d\sigma_\xi^2} = -\frac{\partial H/\partial\sigma_\xi^2}{\partial H/\partial b_m} = -\frac{-\frac{1}{\left(\frac{b_m^2\sigma_\epsilon^2}{\tau^2}+\frac{\sigma_\eta^2}{z^2}\right)\left(\sigma_\xi^2\left(\frac{1}{\frac{b_m^2\sigma_\epsilon^2}{\tau^2}+\frac{\sigma_\eta^2}{z^2}}+\frac{1}{\sigma_\omega^2}\right)+1\right)^2}}{1 + \frac{2\tau^2z^4b_m\sigma_\xi^2\sigma_\omega^2\sigma_\epsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)}{\left(z^2b_m^2\sigma_\epsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\left(\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+z^2\sigma_\xi^2\sigma_\omega^2\right)\right)^2}} > 0.$$

So the noisier the public disclosure (i.e., higher σ_ξ^2), the more weight investors put on the employee's message. We eliminate σ_ξ^2 from (19) and get:

$$b_d = 1 - b_m\left(1 + \frac{\frac{b_m^2\sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}}{\sigma_\omega^2}\right),$$

which implies that

$$\frac{db_d}{d\sigma_\xi^2} = -\left(1 + \frac{\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}}{\sigma_\omega^2}\right)\frac{db_m}{d\sigma_\xi^2} < 0.$$

So the noisier the public disclosure (i.e., higher σ_ξ^2), the less weight investors put on it. Similarly, we have

$$\frac{dx^*}{d\sigma_\xi^2} = \frac{d}{d\sigma_\xi^2}\left(b_d + b_m\right) = \frac{d}{d\sigma_\xi^2}\left(1 - \frac{b_m\left(\frac{b_m^2\sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}\right)}{\sigma_\omega^2}\right) = -\left(\frac{\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}}{\sigma_\omega^2}\right)\frac{db_m}{d\sigma_\xi^2} < 0.$$

Finally, we examine the impact on price efficiency. Note that we can solve the expression for b_m to get

$$\sigma_\xi^2 = \frac{b_m\sigma_\omega^2\left(z^2b_m^2\sigma_\varepsilon^2 + \tau^2\sigma_\eta^2\right)}{\tau^2\left(z^2\left(1-b_m\right)\sigma_\omega^2 - b_m\sigma_\eta^2\right) - z^2b_m^3\sigma_\varepsilon^2}. \tag{21}$$

By substituting σ_ξ^2 into $\mathbb{V}\left[\omega|P\right]$, it follows that

$$\mathbb{V}\left[\omega|P\right] = \frac{\left(1-b_m\right)\sigma_\xi^2\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2} = \frac{b_m\sigma_\eta^2}{z^2} + \frac{b_m^3\sigma_\varepsilon^2}{\tau^2},$$

which implies that $\mathbb{V}\left[\omega|P\right]$ increases with b_m and σ_ξ^2. Therefore, better quality of public disclosure leads to more price informativeness. □

Proof of Proposition 3. Recall that the real efficiency is given by $RE = \mathbb{E}[V] = b_m + b_d - \beta\mathbb{E}[\mathbb{V}[\omega|d,t]]$. Fix z^*. Substituting σ_ξ^2 by (21), we get

$$\begin{aligned}
RE &= b_m + \frac{1-b_m}{1+\frac{\sigma_\xi^2}{\sigma_\omega^2}} - \beta\frac{b_m^2}{\left(\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}\right)b_m^2 + \frac{\tau^2}{\sigma_\varepsilon^2}} \\
&= b_m + \frac{1-b_m}{1+\frac{b_m\left(z^2b_m^2\sigma_\epsilon^2+\tau^2\sigma_\eta^2\right)}{\tau^2\left(z^2(1-b_m)\sigma_\omega^2-b_m\sigma_\eta^2\right)-z^2b_m^3\sigma_\epsilon^2}} - \beta\frac{b_m^2}{\frac{(1-b_m)b_m\tau^2z^2}{z^2b_m^2\sigma_\varepsilon^2+\tau^2\sigma_\eta^2}+\frac{\tau^2}{\sigma_\varepsilon^2}} \\
&= \frac{\tau^2\left(z^2\sigma_\omega^2 - b_m\sigma_\eta^2\right) - z^2b_m^3\sigma_\varepsilon^2}{\tau^2z^2\sigma_\omega^2} - \beta\frac{b_m^2(z^2b_m^2\sigma_\varepsilon^2+\tau^2\sigma_\eta^2)\sigma_\varepsilon^2}{(\sigma_\varepsilon^2b_mz^2+\tau^2\sigma_\eta^2)\tau^2}.
\end{aligned}$$

It is clear that RE is decreasing with b_m by observing that

$$\frac{dRE}{db_m} = \frac{-\tau^2\sigma_\eta^2 - 3z^2b_m^2\sigma_\varepsilon^2}{\tau^2z^2\sigma_\omega^2} - \beta\left(\frac{\sigma_\varepsilon^2}{\tau^2}\right)\frac{b_m(3z^4\sigma_\varepsilon^4b_m^3 + 4z^2\sigma_\varepsilon^2\tau^2\sigma_\eta^2b_m^2 + z^2\sigma_\varepsilon^2\tau^2\sigma_\eta^2b_m + 2\tau^4\sigma_\eta^4)}{(\sigma_\varepsilon^2b_mz^2+\tau^2\sigma_\eta^2)^2} < 0.$$

As we show in Proposition 2, b_m is strictly increasing with σ_ξ^2. Hence, better public information quality (i.e., higher $1/\sigma_\xi^2$) leads to an increase in real efficiency RE.

Next, we fix σ_ξ^2 and show that there exist $0 < \underline{\beta} \leq \overline{\beta}$ such that if $\beta \leq \underline{\beta}$, RE is increasing with w, while if $\beta > \overline{\beta}$, RE is first increasing and then decreasing with w. First, because $db_m/dz > 0$ as proved in Proposition 2, the value of b_m is bounded above by $\bar{b}_m$, where $\bar{b}_m$ is defined as the equilibrium solution to $H(b_m) = 0$ as $z \to \infty$. We then examine how real efficiency changes with b_m that is increasing in w and z.

Recall that real efficiency is equal to

$$RE = b_d + b_m - \beta \left(\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}} \right) = \frac{b_m \sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2} - \beta \left(\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}} \right).$$

It follows that

$$\frac{dRE}{db_m} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} - \frac{2\beta \frac{\tau^2}{b_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2}.$$

We note that the derivative dRE/db_m is first strictly decreasing and then strictly increasing with b_m by observing that

$$\frac{d^2 RE}{db_m^2} = -\frac{2\beta \tau^2 \sigma_\xi^4 \sigma_\omega^4 \sigma_\varepsilon^2 \left(\tau^2 \sigma_\xi^2 \sigma_\omega^2 - 3 b_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2 \right) \right)}{\left(b_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2 \right) + \tau^2 \sigma_\xi^2 \sigma_\omega^2 \right)^3}.$$

It is clear that dRE/db_m attains the minimum at

$$b_m^\dagger \equiv \sqrt{\frac{\tau^2 \sigma_\xi^2 \sigma_\omega^2}{3 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2 \right)}}.$$

Consider the case in which $\bar{b}_m \leq b_m^\dagger$. Then dRE/db_m is always decreasing with b_m in equilibrium. If $\beta \leq \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \frac{\bar{b}_m^3 \sigma_\varepsilon^2}{2\tau^2} \left(\frac{\tau^2}{\bar{b}_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2$, it must be that

$$\begin{aligned}
\frac{dRE}{db_m} &= \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} - \frac{2\beta \frac{\tau^2}{b_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2} \\
&> \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} - \frac{2\beta \frac{\tau^2}{\bar{b}_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2} \\
&\geq \frac{2\beta \frac{\tau^2}{\bar{b}_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2} - \frac{2\beta \frac{\tau^2}{\bar{b}_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2} \\
&= 0.
\end{aligned}$$

So RE is always increasing with b_m. Otherwise, $\frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{\bar{b}_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2}$ is less than zero.

We also note that

$$\left.\frac{dRE}{db_m}\right|_{b_m\to 0} \to \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} > 0.$$

Then there must exist some b'_m such that $dRE/db_m > 0$ for $b_m < b'_m$ and $dRE/db_m < 0$ for $b_m > b'_m$ by the intermediate value theorem. Hence, RE is hump-shaped in b_m.

Consider the case in which $\bar{b}_m > b_m^\dagger$. Recall that dRE/db_m is strictly decreasing with b_m for $b_m < b_m^\dagger$ and strictly increasing with b_m for $b_m > b_m^\dagger$. If $\beta \leq \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2$, it must be that

$$\begin{aligned}
\frac{dRE}{db_m} &= \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{b_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} \\
&> \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} \\
&\geq \frac{2\beta\frac{\tau^2}{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} - \frac{2\beta\frac{\tau^2}{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} \\
&= 0.
\end{aligned}$$

So RE is always increasing with b_m. Furthermore, because dRE/db_m is strictly increasing (decreasing) in $b_m \geq b_m^\dagger$ ($b_m \leq b_m^\dagger$), we have

$$\frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{b_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} > \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2}$$

for $b_m > b_m^\dagger$ or $b_m < b_m^\dagger$. It follows that

$$\frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\bar{b}_m^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2 > \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2,$$

regardless of whether $\bar{b}_m > b_m^\dagger$ or $\bar{b}_m < b_m^\dagger$. If $\beta > \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\bar{b}_m^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2 >$

$\frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2$, we have

$$\left.\frac{dRE}{db_m}\right|_{\bar{b}_m} = \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \frac{2\beta\frac{\tau^2}{\bar{b}_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} < \frac{2\beta\frac{\tau^2}{\bar{b}_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} - \frac{2\beta\frac{\tau^2}{\bar{b}_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2} = 0.$$

Further because $\frac{dRE}{db_m}|_{b_m\to 0} \to \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} > 0$, there must exist some b_m'' such that $dRE/db_m = 0$ for $b_m = b_m''$ by the intermediate value theorem. Moreover, dRE/db_m is strictly decreasing with $b_m \le b_m^\dagger$ and strictly increasing with $b_m > b_m^\dagger$. So for any $b_m \in [b_m^\dagger, \bar{b}_m)$, $dRE/db_m|_{b_m} < dRE/db_m|_{\bar{b}_m} < 0$. It follows that $b_m'' < b_m^\dagger$, and $dRE/db_m > 0$ for $b_m < b_m''$ while $dRE/db_m < 0$ for $b_m > b_m''$ by the definition of b_m''. Hence, RE is hump-shaped in b_m.

Recall that b_m is strictly increasing with w by Propositions 1 and 2. We summarize our results as follows. Let

$$\underline{\beta} \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\left(b_m^\dagger\right)^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\left(b_m^\dagger\right)^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2 \quad \text{and} \quad \overline{\beta} \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\bar{b}_m^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2,$$

where $\underline{\beta} \le \overline{\beta}$ as shown above. If $\beta \le \underline{\beta}$, RE is always increasing with the whistleblower bounty w; if $\beta > \overline{\beta}$, RE is hump-shaped in the whistleblower bounty w. □

Proof of Proposition 4. Recall that

$$\overline{\beta} = \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2}\frac{\bar{b}_m^3\sigma_\varepsilon^2}{2\tau^2}\left(\frac{\tau^2}{\bar{b}_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2,$$

where $\bar{b}_m$ is the equilibrium solution to $H(b_m) = 0$ as $z \to \infty$. As shown in Proposition 3, when $\beta \le \underline{\beta}$, real efficiency RE increases in the whistleblower bounty w. So the higher the whistleblower bounty, the higher real efficiency. When $\beta > \overline{\beta}$, nevertheless, real efficiency RE is hump-shaped in the whistleblower bounty and maximized at some w^*. We show how w^* varies with the parameters.

Let b_m^* be the value of b_m that maximize

$$RE = \frac{b_m\sigma_\xi^2+\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega^2} - \beta\left(\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}\right).$$

Given that RE is a continuously differentiable function of b_m, we have

$$\left.\frac{dRE}{db_m}\right|_{b_m=b_m^*} = \frac{\sigma_\xi^2}{\sigma_\xi^2+\sigma_\omega^2} - \left.\frac{2\beta\frac{\tau^2}{b_m^3\sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2\sigma_\varepsilon^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)^2}\right|_{b_m=b_m^*} = 0. \tag{22}$$

Let $L \equiv dRE/db_m$. It is clear from the proof of Proposition 3 that $b_m^* < b_m^\dagger = \sqrt{\frac{\tau^2\sigma_\xi^2\sigma_\omega^2}{3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)}}$ which is defined to be the minimizer of L. By the definition of $b_m^\dagger$, $\partial L/\partial b_m < 0$ for $b_m < b_m^\dagger$. It follows that

$$\left.\frac{\partial L}{\partial b_m}\right|_{b_m=b_m^*} = \left.\frac{6\beta\tau^2 b_m^2\sigma_\xi^4\sigma_\omega^4\sigma_\varepsilon^4\left(\sigma_\xi^2+\sigma_\omega^2\right)-2\beta\tau^4\sigma_\xi^6\sigma_\omega^6\sigma_\varepsilon^2}{\left(b_m^2\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3}\right|_{b_m=b_m^*} < 0. \tag{23}$$

Moreover, note that the optimal z^2 is given by

$$z^2 = \frac{\left(\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}\right)\sigma_\eta^2 b_m}{1-b_m-\left(\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}\right)\frac{\sigma_\varepsilon^2}{\tau^2}b_m^3}, \tag{24}$$

where b_m is evaluated at b_m^*. We also have

$$\frac{\partial z^2}{\partial b_m} = \frac{\tau^2\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2+b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2} > 0. \tag{25}$$

Eq (23), (24) and (25) will be used to prove the following comparative statics.

Note that the optimal z^2 satisfies

$$\frac{dz^2}{d\beta} = \frac{\partial z^2}{\partial\beta} + \left(\left.\frac{\partial z^2}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial\beta}\right|_{b_m=b_m^*}\right).$$

Because $\partial z^2/\partial\beta = 0$ by (24) and

$$\left.\frac{\partial b_m}{\partial\beta}\right|_{b_m=b_m^*} = -\left.\frac{\partial L/\partial\beta}{\partial L/\partial b_m}\right|_{b_m=b_m^*} = -\left.\frac{-\frac{2\tau^2 b_m\sigma_\xi^4\sigma_\omega^4\sigma_\varepsilon^2}{\left(b_m^2\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^2}}{\frac{6\beta\tau^2 b_m^2\sigma_\xi^4\sigma_\omega^4\sigma_\varepsilon^4\left(\sigma_\xi^2+\sigma_\omega^2\right)-2\beta\tau^4\sigma_\xi^6\sigma_\omega^6\sigma_\varepsilon^2}{\left(b_m^2\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3}}\right|_{b_m=b_m^*} < 0$$

by the Implicit function theorem, we have

$$\frac{dz^2}{d\beta} = \left(\left.\frac{\partial z^2}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial\beta}\right|_{b_m=b_m^*}\right) < 0$$

by (23) and (25). It follows from $w^* = cz$ that the optimal w^* is decreasing in β.

Note that the optimal z^2 satisfies

$$\frac{dz^2}{d\sigma_\eta} = \frac{\partial z^2}{\partial \sigma_\eta} + \left(\frac{\partial z^2}{\partial b_m}\bigg|_{b_m = b_m^*} \right) \left(\frac{\partial b_m}{\partial \sigma_\eta}\bigg|_{b_m = b_m^*} \right).$$

Because $\partial z^2/\partial\sigma_\eta = 2\sigma_\eta(z^2/\sigma_\eta^2) > 0$ by (24) and

$$\frac{\partial b_m}{\partial \sigma_\eta}\bigg|_{b_m = b_m^*} = -\frac{\partial L/\partial \sigma_\eta}{\partial L/\partial b_m}\bigg|_{b_m = b_m^*} = 0,$$

we have $dz^2/d\sigma_\eta > 0$. It follows from $w^* = cz$ that the optimal w^* is increasing in σ_η.

Note that the optimal z^2 satisfies

$$\frac{dz^2}{d\sigma_\omega} = \frac{\partial z^2}{\partial \sigma_\omega} + \left(\frac{\partial z^2}{\partial b_m}\bigg|_{b_m = b_m^*} \right) \left(\frac{\partial b_m}{\partial \sigma_\omega}\bigg|_{b_m = b_m^*} \right).$$

By the Implicit function theorem, we have

$$\frac{\partial b_m}{\partial \sigma_\omega}\bigg|_{b_m = b_m^*} = -\frac{\partial L/\partial \sigma_\omega}{\partial L/\partial b_m}\bigg|_{b_m = b_m^*} = -\frac{-\frac{8\beta\tau^2 b_m^3 \sigma_\xi^6 \sigma_\omega^3 \sigma_\epsilon^4}{\left(b_m^2\sigma_\epsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3} - \frac{2\sigma_\xi^2\sigma_\omega}{\left(\sigma_\xi^2+\sigma_\omega^2\right)^2}}{\frac{6\beta\tau^2 b_m^2\sigma_\xi^4\sigma_\omega^4\sigma_\epsilon^4\left(\sigma_\xi^2+\sigma_\omega^2\right)-2\beta\tau^4\sigma_\xi^6\sigma_\omega^6\sigma_\epsilon^2}{\left(b_m^2\sigma_\epsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3}}\Bigg|_{b_m = b_m^*} < 0$$

by (23). Moreover, since $\partial z^2/\partial b_m|_{b_m = b_m^*} > 0$ by (25), it follows from (24) that

$$\begin{aligned}
\frac{dz^2}{d\sigma_\omega} &= \frac{\partial z^2}{\partial \sigma_\omega} + \frac{\partial z^2}{\partial b_m^*}\frac{\partial b_m^*}{\partial \sigma_\omega} \\
&= -\frac{2\tau^4(1-b_m)\, b_m \sigma_\eta^2 \sigma_\xi^4 \sigma_\omega}{\left(\tau^2(b_m-1)\,\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\epsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2} + \left(\frac{\partial z^2}{\partial b_m}\bigg|_{b_m = b_m^*} \right) \left(\frac{\partial b_m}{\partial \sigma_\omega}\bigg|_{b_m = b_m^*} \right) \\
&< 0.
\end{aligned}$$

It follows from $w^* = cz$ that the optimal w^* is decreasing in σ_ω.

Note that the optimal z^2 satisfies

$$\frac{dz^2}{d\sigma_\xi} = \frac{\partial z^2}{\partial \sigma_\xi} + \left(\frac{\partial z^2}{\partial b_m}\bigg|_{b_m = b_m^*} \right) \left(\frac{\partial b_m}{\partial \sigma_\xi}\bigg|_{b_m = b_m^*} \right).$$

By the Implicit function theorem, we get

$$\begin{aligned}\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*} &= -\left.\frac{\partial L/\partial \sigma_\xi}{\partial L/\partial b_m}\right|_{b_m=b_m^*} \\ &= \left.\frac{\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^3 - 4(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2 b_m^3\sigma_\varepsilon^4\sigma_\omega^4\sigma_\xi^2}{(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2\sigma_\varepsilon^2\sigma_\xi^3\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3b_m^2\sigma_\varepsilon^2(\sigma_\xi^2+\sigma_\omega^2)\right)}\right|_{b_m=b_m^*}.\end{aligned}$$

It follows from (22) that

$$\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^2 = 2\beta\tau^2 b_m\sigma_\varepsilon^2\sigma_\xi^2\sigma_\omega^4(\sigma_\xi^2+\sigma_\omega^2).$$

By substituting the term $\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^2$ into the equation, we get

$$\begin{aligned}&\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*} \\ =&\left.\frac{\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)2\beta\tau^2 b_m\sigma_\varepsilon^2\sigma_\xi^2\sigma_\omega^4(\sigma_\xi^2+\sigma_\omega^2) - 4(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2 b_m^3\sigma_\varepsilon^4\sigma_\omega^4\sigma_\xi^2}{(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2\sigma_\varepsilon^2\sigma_\xi^3\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3b_m^2\sigma_\varepsilon^2(\sigma_\xi^2+\sigma_\omega^2)\right)}\right|_{b_m=b_m^*} \\ =&\left.\frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2+\sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\right|_{b_m=b_m^*}.\end{aligned}$$

It follows from $b_m^* < b_m^\dagger = \sqrt{\frac{\tau^2\sigma_\xi^2\sigma_\omega^2}{3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)}}$ that

$$[\tau^2\sigma_\xi^2\sigma_\omega^2 - 3\left(\sigma_\xi^2+\sigma_\omega^2\right)b_m^2\sigma_\varepsilon^2]|_{b_m=b_m^*} > 0. \tag{26}$$

So we have

$$\begin{aligned}\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*} &= \left.\frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2+\sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\right|_{b_m=b_m^*} \\ &> \left.\frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2+\sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\right|_{b_m=b_m^*} \\ &= \frac{2b_m^*\sigma_\omega^2}{(\sigma_\omega^2+\sigma_\xi^2)\sigma_\xi}.\end{aligned}$$

It then follows from (24) and (25) that

$$\begin{aligned}
\frac{dz^2}{d\sigma_\xi} =& \frac{\partial z^2}{\partial \sigma_\xi} + \left(\left. \frac{\partial z^2}{\partial b_m} \right|_{b_m=b_m^*} \right) \left(\left. \frac{\partial b_m}{\partial \sigma_\xi} \right|_{b_m=b_m^*} \right) \\
>& \left. \frac{-2\tau^4\sigma_\eta^2 b_m(1-b_m)\sigma_\xi\sigma_\omega^4 + \tau^2\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)\left(\frac{2b_m\sigma_\omega^2}{(\sigma_\omega^2+\sigma_\xi^2)\sigma_\xi}\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2} \right|_{b_m=b_m^*} \\
=& \left. \frac{2\tau^2\sigma_\eta^2\sigma_\omega^2 b_m\left(-\tau^2(1-b_m)\sigma_\xi\sigma_\omega^2 + \left(2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)\left(\frac{1}{\sigma_\xi}\right)\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2} \right|_{b_m=b_m^*} \\
>& \left. \frac{2\tau^2\sigma_\eta^2\sigma_\omega^2 b_m\left(-\tau^2\sigma_\xi\sigma_\omega^2 + 2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(\frac{1}{\sigma_\xi}\right)+\tau^2\sigma_\xi\sigma_\omega^2\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2} \right|_{b_m=b_m^*} \\
>& 0.
\end{aligned}$$

Because $w^* = cz$, the optimal w^* is increasing in σ_ξ.

Finally, note that the optimal z^2 satisfies

$$\frac{dz^2}{d\sigma_\varepsilon} = \frac{\partial z^2}{\partial \sigma_\varepsilon} + \left(\left. \frac{\partial z^2}{\partial b_m} \right|_{b_m=b_m^*} \right) \left(\left. \frac{\partial b_m}{\partial \sigma_\varepsilon} \right|_{b_m=b_m^*} \right).$$

By the Implicit function theorem, we get

$$\left. \frac{\partial b_m}{\partial \sigma_\varepsilon} \right|_{b_m=b_m^*} = - \left. \frac{\partial L/\partial \sigma_\varepsilon}{\partial L/\partial b_m} \right|_{b_m=b_m^*} = - \frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}.$$

It follows from (26) that

$$\left. \frac{\partial b_m}{\partial \sigma_\varepsilon} \right|_{b_m=b_m^*} = - \frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)} < - \frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)} = -\frac{2b_m}{\sigma_\varepsilon}.$$

By (24) and (25), we have

$$\begin{aligned}
\frac{dz^2}{d\sigma_\varepsilon} =& \frac{\partial z^2}{\partial \sigma_\varepsilon} + \left(\left.\frac{\partial z^2}{\partial b_m}\right|_{b_m=b_m^*} \right) \left(\left.\frac{\partial b_m}{\partial \sigma_\varepsilon}\right|_{b_m=b_m^*} \right) \\
<& \left.\frac{2\tau^2\sigma_\eta^2 \left(\sigma_\xi^2+\sigma_\omega^2\right)^2 b_m^4 \sigma_\varepsilon + \tau^2\sigma_\eta^2 \left(\sigma_\xi^2+\sigma_\omega^2\right) \left(2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)\left(-\frac{2b_m}{\sigma_\varepsilon}\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2}\right|_{b_m=b_m^*} \\
=& \left.\frac{2\tau^2\sigma_\eta^2 \left(\sigma_\xi^2+\sigma_\omega^2\right) b_m \left(\left(\sigma_\xi^2+\sigma_\omega^2\right) b_m^3 \sigma_\varepsilon + \left(2b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)\left(-\frac{1}{\sigma_\varepsilon}\right)\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2}\right|_{b_m=b_m^*} \\
=& \left.\frac{2\tau^2\sigma_\eta^2 \left(\sigma_\xi^2+\sigma_\omega^2\right) b_m \left(-\left(\sigma_\xi^2+\sigma_\omega^2\right) b_m^3 \sigma_\varepsilon + \tau^2\sigma_\xi^2\sigma_\omega^2\left(-\frac{1}{\sigma_\varepsilon}\right)\right)}{\left(\tau^2\left(b_m-1\right)\sigma_\xi^2\sigma_\omega^2 + b_m^3\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\right)^2}\right|_{b_m=b_m^*} \\
<& 0.
\end{aligned}$$

Because $w^* = cz$, the optimal w^* is decreasing in σ_ε. $\square$

Proof of Proposition 5. The optimal choice of the employee's effort z^* and the action a^* are the same as derived in Proposition 1. Similarly, the price is equal to

$$P = \mathbb{E}\left[V|d,m\right] = \mathbb{E}\left[\omega|\, d,m\right] - \beta\mathbb{E}\left[\mathbb{V}\left[\omega|\, d,t\right]|\, d,m\right].$$

We conjecture that there is an equilibrium in which the price takes the form of $P = b_0 + b_d d + b_m m$, where b_0, b_d, b_m are constants such that $b_d, b_m \geq 0$. Since $\mathbb{E}\left[\eta\right] = 0$, we have that $\mathbb{E}\left[m|t\right] = t$. This implies that given the realizations of $(\omega, \varepsilon, \xi)$, the manager chooses d and t to maximize:

$$\begin{aligned}
&\mathbb{E}\left[P|\omega,\varepsilon,\xi\right] - \left(\frac{\delta}{2}\right)\frac{\left(d-\omega\right)^2}{\xi} - \left(\frac{\tau}{2}\right)\frac{\left(t-\omega\right)^2}{\varepsilon} \\
=& b_0 + b_d d + b_m \mathbb{E}\left[m|t\right] - \left(\frac{\delta}{2}\right)\frac{\left(d-\omega\right)^2}{\xi} - \left(\frac{\tau}{2}\right)\frac{\left(t-\omega\right)^2}{\varepsilon}.
\end{aligned}$$

The FOC imply that

$$b_d = \frac{\delta}{\xi}\left(d-\omega\right) \Rightarrow d\left(\omega,\xi\right) = \omega + \frac{b_d}{\delta}\xi$$

and

$$b_m = \frac{\tau}{\varepsilon}\left(t-\omega\right) \Rightarrow t\left(\omega,\varepsilon\right) = \omega + \frac{b_m}{\tau}\varepsilon.$$

Let $\hat{x}$ be the market's conjecture about x. Given the above characterizations of d and t, and the employee's optimal choice of z, the market's beliefs can be expressed as:

$$\mathbb{E}\left[\omega|m,d\right]=\frac{\frac{\hat{x}}{\sigma_\omega^2}+\frac{d-\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}$$

and

$$\begin{aligned}\mathbb{E}\left[\mathbb{V}\left[\omega|\,d,t\right]|\,d,m\right]&=\mathbb{E}\left[\left.\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}\right|d,m\right]\\&=\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.\end{aligned}$$

which implies

$$P=\frac{\frac{\hat{x}}{\sigma_\omega^2}+\frac{d-\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}-\beta\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.$$

Note that this verifies the conjecture that the price is linear in the signals d and m. Matching terms, we have:

$$\begin{aligned}b_d&=\frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}\\b_m&=\frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}\\b_0&=\frac{\frac{\hat{x}}{\sigma_\omega^2}-\frac{\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}-\frac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}-\beta\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.\end{aligned}\tag{27}$$

The equilibrium b_d and b_m are given by the solution to the first two equations.

Next, we show that there is a unique solution (b_d, b_m), which implies existence and uniqueness of the equilibrium market price. First, it is clear that the solution, if it exists, must satisfy $b_d, b_m \in [0,1]$. Then we show that there is a unique solution $b_d(b_m) \in (0,1)$ for any

b_m. Define

$$G\left(b_d; b_m\right) \equiv b_d - \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2 \sigma_\eta^2}}.$$

For any given b_m, b_d is the solution to $G\left(b_d; b_m\right) = 0$ by the first equation of (27). Note that

$$\lim_{b_d \downarrow 0} G\left(b_d; b_m\right) = -1 < 0$$

$$G\left(1; b_m\right) = 1 - \frac{\frac{\delta^2}{\sigma_\xi^2}}{\left(\frac{1}{\frac{b_m^2 \sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}} + \frac{\delta^2}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2}\right)} > 0$$

and

$$\frac{\partial G\left(b_d; b_m\right)}{\partial b_d} = 1 + \frac{2\delta^2 \left(\frac{1}{\frac{b_m^2 \sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}} + \frac{1}{\sigma_\omega^2}\right)}{b_d^3 \sigma_\xi^2 \left(\frac{\delta^2}{b_d^2 \sigma_\xi^2} + \frac{1}{\frac{b_m^2 \sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}} + \frac{1}{\sigma_\omega^2}\right)^2} > 0,$$

which implies for every b_m, there exists a unique $b_d\left(b_m\right) \in (0, 1)$ by the intermediate value theorem.

Given $b_d\left(b_m\right)$, we show that there is a unique solution b_m as follows. By (27), we have

$$1 - \left(b_d + b_m\right) = \frac{\frac{1}{\sigma_\omega^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2 \sigma_\eta^2}}.$$

Moreover,

$$\frac{b_d^3}{\delta^2} \sigma_\xi^2 = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2 \sigma_\eta^2}}.$$

It follows that

$$b_m = 1 - \left(b_d + \frac{b_d^3 \sigma_\xi^2}{\delta^2 \sigma_\omega^2}\right). \tag{28}$$

Substituting the solution $b_d(b_m)$ into (28), we solve for the equilibrium b_m as the solution to $F = 0$, where

$$F\left(b_m, b_d(b_m)\right) \equiv b_m + b_d + \frac{b_d^3 \sigma_\xi^2}{\delta^2 \sigma_\omega^2} - 1.$$

Since $b_d\,(b_m) \in (0,1)$, we have

$$
\begin{aligned}
F\left(1, b_d(1)\right) &= 1 + b_d\left(1\right) + \frac{b_d^3\left(1\right)\sigma_\xi^2}{\delta^2\sigma_\omega^2} - 1 > 0 \\
F\left(0, b_d(0)\right) &= -\left(1 - b_d\left(0\right)\right) + \frac{b_d^3\left(0\right)\sigma_\xi^2}{\delta^2\sigma_\omega^2} \\
&= -\left(1 - b_d\left(0\right)\right) + \frac{1}{\sigma_\omega^2}\frac{1 - b_d\left(0\right)}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{1}{z}\right)^2\sigma_\eta^2}} \\
&= \left(1 - b_d\left(0\right)\right)\left(\frac{1}{1 + \frac{\sigma_\omega^2}{\left(\frac{1}{z}\right)^2\sigma_\eta^2}} - 1\right) \\
&< 0,
\end{aligned}
$$

where the second equation of $F(0, b_d(0))$ follows from $G\left(b_d; b_m = 0\right) = 0$ that

$$
\frac{b_d^3\left(0\right)\sigma_\xi^2}{\delta^2} = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d(0)}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{1}{z}\right)^2\sigma_\eta^2}} = \frac{1 - b_d\left(0\right)}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{1}{z}\right)^2\sigma_\eta^2}}.
$$

Further, note that

$$
\frac{dF}{db_m} = 1 + \left(1 + \frac{3\sigma_\xi^2 b_d^2}{\delta^2\sigma_\omega^2}\right)\frac{db_d}{db_m} > 0,
$$

which follows from the implicit function theorem that

$$
\frac{db_d}{db_m} = -\frac{\partial G/\partial b_m}{\partial G/\partial b_d} = \frac{\frac{2\delta^2\tau^2 z^4 b_d^2 b_m \sigma_\xi^2\sigma_\omega^4\sigma_\epsilon^2}{\left(b_d^2\sigma_\xi^2\left(z^2 b_m^2\sigma_\epsilon^2 + \tau^2\left(\sigma_\eta^2 + z^2\sigma_\omega^2\right)\right) + \delta^2\sigma_\omega^2\left(z^2 b_m^2\sigma_\epsilon^2 + \tau^2\sigma_\eta^2\right)\right)^2}}{1 + \frac{2\delta^2\left(\frac{1}{\frac{b_m^2\sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}} + \frac{1}{\sigma_\omega^2}\right)}{b_d^3\sigma_\xi^2\left(\frac{\delta^2}{b_d^2\sigma_\xi^2} + \frac{1}{\frac{b_m^2\sigma_\epsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}} + \frac{1}{\sigma_\omega^2}\right)^2}} > 0.
$$

Hence, there exists a unique solution $b_m, b_d \in (0,1)$ by the intermediate value theorem.

Finally, we solve for the equilibrium investment x^*. Given the coefficients b_d and b_m, the manager's investment choice maximizes:

$$
b_0 + b_m\mathbb{E}\left[m\right] + b_d\mathbb{E}\left[d\right] + \frac{1}{2}\left(1 - x^2\right) - \left(\frac{\delta}{2}\right)\mathbb{E}\left[\left(\frac{b_d}{\delta}\right)^2\xi\right] - \left(\frac{\tau}{2}\right)\mathbb{E}\left[\left(\frac{b_m}{\tau}\right)^2\varepsilon\right].
$$

So the equilibrium investment maximizes $\left(b_m + b_d\right)x + \frac{1}{2}\left(1 - x^2\right)$ and is given by

$$
x^* = b_m + b_d.
$$

This characterizes the unique linear equilibrium as stated in the proposition. $\square$

Proof of Proposition 6. It follows from (27) that

$$\frac{b_d^3 \sigma_\xi^2}{\delta^2} = \frac{b_m^3 \sigma_\varepsilon^2}{\tau^2} + \frac{b_m \sigma_\eta^2}{z^2}. \tag{29}$$

Differentiating (29) with respect to δ, we get

$$\left(\frac{3b_d^2 \sigma_\xi^2}{\delta^2}\right)\left(\frac{db_d}{d\delta}\right) - \frac{2b_d^3 \sigma_\xi^2}{\delta^3} = \left(\frac{3b_m^2 \sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}\right)\left(\frac{db_m}{d\delta}\right). \tag{30}$$

Differentiating (28) with respect to δ, we get

$$\frac{db_m}{d\delta} = -\left(1 + \frac{3b_d^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2}\right)\left(\frac{db_d}{d\delta}\right) + \frac{2b_d^3 \sigma_\xi^2}{\delta^3 \sigma_\omega^2}. \tag{31}$$

Equations (30) and (31) imply that

$$\left(\frac{3b_d^2 \sigma_\xi^2}{\delta^2} + \left(\frac{3b_m^2 \sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}\right)\left(1 + \frac{3b_d^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2}\right)\right)\left(\frac{db_d}{d\delta}\right) = \frac{2b_d^3 \sigma_\xi^2}{\delta^3} + \left(\frac{3b_m^2 \sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}\right)\left(\frac{2b_d^3 \sigma_\xi^2}{\delta^3 \sigma_\omega^2}\right).$$

So $db_d/d\delta > 0$. Eliminating $(b_d^3 \sigma_\xi^2)/\delta^2$ in (28) and (29), we get

$$(1 - (b_d + b_m))\sigma_\omega^2 = \frac{b_m^3 \sigma_\varepsilon^2}{\tau^2} + \frac{b_m \sigma_\eta^2}{z^2}. \tag{32}$$

Differentiating (32) with respect to δ yields

$$-\sigma_\omega^2 \frac{db_d}{d\delta} = \left(\frac{3b_m^2 \sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2} + \sigma_\omega^2\right)\frac{db_m}{d\delta}.$$

So $db_m/d\delta < 0$. We also observe that

$$\frac{dx^*}{d\delta} = \frac{d(b_m + b_d)}{d\delta} = -\frac{1}{\sigma_\omega^2}\left(\frac{3b_m^2 \sigma_\varepsilon^2}{\tau^2} + \frac{\sigma_\eta^2}{z^2}\right)\frac{db_m}{d\delta} > 0.$$

Hence, an increase in public disclosure scrutiny δ leads to an increase in investment x^*.

Next, we examine how public disclosure scrutiny δ affects price informativeness and real efficiency. Since $P = b_0 + b_m m + b_d d$, where $m = \omega + \frac{b_m}{\tau}\varepsilon + \frac{1}{z}\eta$ and $d = \omega + \frac{b_d}{\delta}\xi$, we can write

$$\mathbb{V}[\omega|P] = \mathbb{V}[\omega|y_P] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_y^2}},$$

where $y_P \equiv \omega + \frac{1}{b_m+b_d}\left(b_m\left(\frac{b_m}{\tau}\varepsilon + \frac{1}{z}\eta\right) + b_d\left(\frac{b_d}{\delta}\xi\right)\right)$ and

$$\sigma_y^2 \equiv \frac{\left(b_d^2\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2 + b_m^2\left(\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2\sigma_\eta^2\right)\right)}{\left(b_m+b_d\right)^2}.$$

It follows from (27) that

$$\begin{aligned}
\frac{1}{\sigma_y^2} &= \frac{\left(b_m+b_d\right)^2}{\left(b_d^2\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2 + b_m^2\left(\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2\sigma_\eta^2\right)\right)} \\
&= \frac{\left(b_m+b_d\right)^2}{\frac{b_d}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}} + \frac{b_m}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}} \\
&= \frac{b_m+b_d}{\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{1}{z}\right)^2\sigma_\eta^2}}} \\
&= \frac{b_m+b_d}{\sigma_\omega^2\left(1-\left(b_d+b_m\right)\right)}.
\end{aligned}$$

So we have

$$\begin{aligned}
\mathbb{V}\left[\omega|P\right] &= \frac{1}{\frac{1}{\sigma_y^2}+\frac{1}{\sigma_\omega^2}} \\
&= \frac{1}{\frac{b_m+b_d}{\sigma_\omega^2\left(1-\left(b_d+b_m\right)\right)}+\frac{1}{\sigma_\omega^2}} \\
&= \sigma_\omega^2\left(1-\left(b_d+b_m\right)\right) \\
&= \sigma_\omega^2\left(1-x^*\right),
\end{aligned} \tag{33}$$

implying that $\mathbb{V}\left[\omega|P\right]$ is decreasing with δ. Hence, $PI = \left(\mathbb{V}\left[\omega|P\right]\right)^{-1}$ is increasing with δ. For real efficiency, recall that

$$\begin{aligned}
RE &= \mathbb{E}\left[V^*\right] \\
&= \mathbb{E}\left[\omega - \beta\left(a-\omega\right)^2\right] \\
&= x^* - \beta\mathbb{E}[\mathbb{V}[\omega|d,t]] \\
&= x^* - \beta\left(\frac{1}{\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}\right).
\end{aligned}$$

Taking derivative with respect to δ, we get

$$\frac{dRE}{d\delta} = \frac{dx^*}{d\delta} + 2\beta\left(\frac{-\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(\frac{db_d}{d\delta}\right)+\frac{\delta}{b_d^2\sigma_\xi^2}-\left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right)}{\left(\frac{1}{\sigma_\omega^2}+\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}+\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}\right)^2}\right).$$

By (30), we have

$$\frac{b_d^2\sigma_\xi^2}{\delta^2}\left(3\frac{db_d}{d\delta}-\frac{2b_d}{\delta}\right)=\left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2}+\frac{\sigma_\eta^2}{z^2}\right)\left(\frac{db_m}{d\delta}\right)<0.$$

It follows that

$$\begin{aligned}
-\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(\frac{db_d}{d\delta}\right)+\frac{\delta}{b_d^2\sigma_\xi^2}-\left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right) &= \left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(-\frac{db_d}{d\delta}+\frac{b_d}{\delta}\right)-\left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right)\\
&> -\frac{1}{3}\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(3\frac{db_d}{d\delta}-2\frac{b_d}{\delta}\right)-\left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right)\\
&> 0,
\end{aligned}$$

since $3\frac{db_d}{d\delta}-\frac{2b_d}{\delta}<0$ from the equation above and $db_m/d\delta<0$. Because $dx^*/d\delta>0$, we conclude that $dRE/d\delta>0$, i.e., real efficiency is increasing with public disclosure scrutiny.

We examine how the whistleblower bounty affects investment, price informativeness and real efficiency as follows. Recall that the equilibrium values of b_d and b_m are given by (28) and (29). Differentiating (28) with respect to z, we get

$$\frac{db_m}{dz}=-\frac{db_d}{dz}\left(1+\frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right). \tag{34}$$

Differentiating (29) with respect to z, we get

$$\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2}\right)\left(\frac{db_d}{dz}\right)=\left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2}+\frac{\sigma_\eta^2}{z^2}\right)\left(\frac{db_m}{dz}\right)-\frac{2b_m\sigma_\eta^2}{z^3}. \tag{35}$$

It follows from (34) and (35) that

$$\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2}+\left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2}+\frac{\sigma_\eta^2}{z^2}\right)\left(1+\frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right)\right)\left(\frac{db_d}{dz}\right)=-\frac{2b_m\sigma_\eta^2}{z^3}.$$

So $db_d/dz < 0$. By (34), we also obtain $db_m/dz > 0$ and

$$\frac{dx^*}{dz} = \frac{d(b_m + b_d)}{dz} = -\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right)\left(\frac{db_d}{dz}\right) > 0.$$

Hence, an increase in the whistleblower bounty w leads to an increase in investment x^*. Further, it follows from (33) that

$$\frac{d\mathbb{V}\left[\omega|P\right]}{dz} = -\sigma_\omega^2\frac{dx^*}{dz} < 0.$$

So $PI = (\mathbb{V}\left[\omega|P\right])^{-1}$ is increasing with z and w.

It is useful in the remaining proof to define the limits of b_m and b_d as $w \to 0$ and $w \to \infty$. When $w \to 0$, $z \to 0$ and (27) converges to

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}$$
$$b_m = 0.$$

Thus, $b_{d0} \equiv \lim_{w\to 0} b_d > 0$ and $\lim_{w\to 0} b_m = 0$. Note that b_{d0} is the unique root of the above cubic equation and the supremum of b_d. When $w \to \infty$, $z \to \infty$ and (27) converges to

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}} \tag{36}$$

$$b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}} \tag{37}$$

implying

$$\frac{\sigma_\xi^2}{\delta^2}b_d^3 = \frac{\sigma_\varepsilon^2}{\tau^2}b_m^3.$$

With this relation, (36) and (37) imply a cubic equation for each of b_m and b_d with a unique positive root. The roots are denoted by $b_{m\infty} > 0$ and $b_{d\infty} > 0$. To summarize, as w increases, b_m increases starting from 0 to $b_{m\infty}$, while b_d decreases from b_{d0} to $b_{d\infty}$.

Turning to real efficiency, (28) implies

$$t \equiv \frac{db_d}{db_m} = -\frac{\delta^2\sigma_\omega^2}{\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2}$$

as we change z. Here, we view b_d as a function of a single variable b_m, which is uniquely determined by z implicitly. It follows that

$$\begin{aligned}\frac{dt}{db_m} &= \frac{6b_d\delta^2\sigma_\omega^2\sigma_\xi^2}{\left(\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2\right)^2}t \\ &= -\frac{6b_d\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2}t^2 \\ &= -\frac{2}{b_d}\left(1+t\right)t^2\end{aligned}$$

Note that

$$t \in \left(-\frac{\delta^2\sigma_\omega^2}{\delta^2\sigma_\omega^2 + 3b_{d\infty}^2\sigma_\xi^2}, -\frac{\delta^2\sigma_\omega^2}{\delta^2\sigma_\omega^2 + 3b_{d0}^2\sigma_\xi^2}\right),$$

and $1+t$ has a strictly positive infimum. Define

$$A\left(b_m\right) \equiv \frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}.$$

Then,

$$\frac{dA}{db_m} = -\frac{2\delta^2}{b_d^3\sigma_\xi^2}t - \frac{2\tau^2}{b_m^3\sigma_\varepsilon^2}$$

and

$$\begin{aligned}\frac{d^2A}{db_m^2} &= \frac{6\delta^2}{b_d^4\sigma_\xi^2}t^2 - \frac{2\delta^2}{b_d^3\sigma_\xi^2}\frac{dt}{db_m} + \frac{6\tau^2}{b_m^4\sigma_\varepsilon^2} \\ &= \frac{6\delta^2}{b_d^4\sigma_\xi^2}t^2 + \frac{2\delta^2}{b_d^3\sigma_\xi^2}\frac{6b_d\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2}t^2 + \frac{6\tau^2}{b_m^4\sigma_\varepsilon^2} \\ &= \frac{6\delta^2}{b_d^3\sigma_\xi^2}\left[\frac{1}{b_d} + \frac{2b_d\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2}\right]t^2 + \frac{6\tau^2}{b_m^4\sigma_\varepsilon^2}\end{aligned}$$

Note that

$$\begin{aligned}\frac{dA}{db_m} &= -\frac{2\delta^2}{b_d^3\sigma_\xi^2}t - \frac{2\tau^2}{b_m^3\sigma_\varepsilon^2} \\ &\leq -\frac{2\delta^2}{b_d^3\sigma_\xi^2}t - \frac{2\delta^2}{b_d^3\sigma_\xi^2} \\ &= -\frac{2\delta^2}{b_d^3\sigma_\xi^2}\left(1+t\right) \\ &< 0,\end{aligned}$$

because (29) implies that

$$\frac{b_d^3 \sigma_\xi^2}{\delta^2} \geq \frac{b_m^3 \sigma_\varepsilon^2}{\tau^2}.$$

As

$$RE = b_m + b_d - \frac{\beta}{A},$$

we have

$$\frac{dRE}{db_m} = 1 + t + \frac{\beta}{A^2} \frac{dA}{db_m}.$$

Therefore,

$$\frac{dRE}{db_m} \gtreqless 0 \Longleftrightarrow \frac{A^2 (1+t)}{(-dA/db_m)} \gtreqless \beta.$$

Define from above that

$$h(b_m) = \frac{A(b_m)^2 (1 + t(b_m))}{(-dA/db_m)}.$$

First, we consider the behavior of $h(b_m)$ when w is close to 0. As noted above, $1+t$ is bounded in $\left(\frac{3b_{d\infty}^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_{d\infty}^2 \sigma_\xi^2}, \frac{3b_{d0}^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_{d0}^2 \sigma_\xi^2}\right)$. Since $b_m \to 0$ and $b_d \to b_{d0} > 0$ as $w \to 0$,

$$\begin{aligned}
\lim_{w \to 0} h(b_m) &= \lim_{w \to 0} \left[\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}\right]^2 \frac{3b_d^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2} \frac{1}{\frac{2\delta^2}{b_d^3 \sigma_\xi^2} t + \frac{2\tau^2}{b_m^3 \sigma_\varepsilon^2}}, \\
&= \infty.
\end{aligned}$$

Thus, when w is close to zero, $h(b_m)$ becomes arbitrarily large. It implies that, for any value of β, $h(b_m) > \beta$ for sufficiently small w and RE is increasing in w. Next, we consider $\frac{dh}{db_m}$. Denote $\frac{dA}{db_m}$ and $\frac{dt}{db_m}$ by A' and t', respectively. Then,

$$\frac{dh(b_m)}{db_m} = \frac{[2AA'(1+t) + A^2 t'](-A') + A^2 A''(1+t)}{A'^2}, \tag{38}$$

the sign of which is driven by $\frac{1}{b_m}$ terms because $\lim_{w \to 0} b_m = 0$ and $\lim_{w \to 0} b_d = b_{d0} > 0$. If we collect the highest-order terms of $(1/b_m)$ in the numerator of (38), which is a polynomial of $(1/b_m)$, and drop all other terms, we are left with

$$\begin{aligned}
A(1+t)\left(AA'' - 2A'^2\right) &\approx A(1+t)\left(\frac{\tau^2}{b_m^2 \sigma_\varepsilon^2} \frac{6\tau^2}{b_m^4 \sigma_\varepsilon^2} - 2\left(\frac{2\tau^2}{b_m^3 \sigma_\varepsilon^2}\right)^2\right) \\
&= A(1+t)\left(\frac{-2\tau^4}{b_m^6 \sigma_\varepsilon^4}\right)
\end{aligned}$$

$$< 0,$$

implying that $\frac{dh(b_m)}{db_m} < 0$ for small w. In other words, there exists a $\hat{w}$ such that, for all $w \in [0, \hat{w}]$, $\frac{dh(b_m(w))}{dw} < 0$. As $h(b_m)$ is continuous and $\lim_{w\to\infty} h(b_m)$ is finite, $h(b_m(w))$ attains a maximum on $[\hat{w}, \infty]$ on the extended real number line by the extreme value theorem. Call the maximum of $h(b_m)$ M. Since $\lim_{w\to 0} h(b_m) = \infty$, there exists a $\overline{w} \in (0, \hat{w}]$ such that $h(b_m(\overline{w})) > M$. If we choose β to be

$$\overline{\beta} \equiv h(b_m(\overline{w})),$$

- For all $w \in [0, \overline{w})$, $h(b_m(w)) > \overline{\beta}$, because $h(b_m)$ is decreasing in the range.
- For all $w \in (\overline{w}, \infty)$, $h(b_m(w)) < \overline{\beta}$, because $h(b_m(w))$ is decreasing on $(\overline{w}, \hat{w})$ and cannot be larger than M on $[\overline{w}, \infty)$, which is smaller than $\overline{\beta}$.

All combined, RE is hump-shaped when $\beta = \overline{\beta}$. It can be easily seen that RE is hump-shaped for all $\beta > \overline{\beta}$.

As noted above, $1 + t$ and $-\frac{dA}{db_m}$ are bounded above and below away from 0, while A is bounded below away from 0 as it is always larger than $\frac{1}{\sigma_\omega^2}$. It follows that $\frac{A^2(1+t)}{(-dA/db_m)}$ is bounded below away from 0. Thus, if β is sufficiently small, $\frac{A^2(1+t)}{(-dA/db_m)} > \beta$ for all w and RE is increasing everywhere. □

Proof of Proposition 7. The proof is analogous to the proof of Proposition 1. Note that $\hat{b}_m$ is given by the unique solution to $\hat{H}(\hat{b}_m) = 0$, where

$$\hat{H}\left(\hat{b}_m\right) \equiv \hat{b}_m - \frac{\frac{1}{\left(\frac{\pi \hat{b}_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2 \sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{\pi \hat{b}_m}{\tau}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{z}\right)^2 \sigma_\eta^2}}. \tag{39}$$

This immediately follows from the arguments in Proposition 1 once we observe that $\hat{H}(\cdot)$ is identical to $H(\cdot)$ if we replace τ by τ/π. Given the unique solution to $\hat{b}_m$, the remaining coefficients are characterized by the stated expressions and the rest of the proof follows.

Applying the Implicit function theorem to (39) yields

$$\frac{d\hat{b}_m}{d\pi} = -\frac{\partial \hat{H}/\partial \pi}{\partial \hat{H}/\partial \hat{b}_m} < 0,$$

because

$$\frac{\partial \hat{H}}{\partial \pi} = \frac{2\pi\tau^2 z^4 \hat{b}_m^2 \sigma_\xi^2 \sigma_\omega^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)}{\left(\pi^2 z^2 \hat{b}_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + \tau^2 \left(\sigma_\eta^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + z^2 \sigma_\xi^2 \sigma_\omega^2\right)\right)^2} > 0$$

and

$$\frac{\partial \hat{H}}{\partial \hat{b}_m} = \frac{2\pi^2\tau^2 z^4 \hat{b}_m \sigma_\xi^2 \sigma_\omega^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)}{\left(\pi^2 z^2 \hat{b}_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + \tau^2 \left(\sigma_\eta^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right) + z^2 \sigma_\xi^2 \sigma_\omega^2\right)\right)^2} + 1 > 0.$$

Moreover,

$$\begin{aligned}
\frac{d\hat{b}_d}{d\pi} &= \left(\frac{\partial \hat{b}_d}{\partial \hat{b}_m}\right)\left(\frac{d\hat{b}_m}{d\pi}\right) + \frac{\partial \hat{b}_d}{\partial \pi} \\
&= \frac{2\pi\tau^2 z^4 \hat{b}_m^2 \sigma_\xi^2 \sigma_\omega^4 \sigma_\varepsilon^2}{\pi^4 z^4 \hat{b}_m^4 \sigma_\varepsilon^4 \left(\sigma_\xi^2+\sigma_\omega^2\right)^2 + 2\pi^2\tau^2 z^2 \hat{b}_m \sigma_\varepsilon^2 \left(\sigma_\xi^2+\sigma_\omega^2\right)\left(\hat{b}_m \sigma_\eta^2 \sigma_\xi^2 + \sigma_\omega^2 \left(\hat{b}_m \sigma_\eta^2 + z^2 \left(\hat{b}_m+1\right)\sigma_\xi^2\right)\right) + \tau^4 \left(\sigma_\eta^2 \left(\sigma_\xi^2+\sigma_\omega^2\right) + z^2 \sigma_\xi^2 \sigma_\omega^2\right)^2} \\
&> 0.
\end{aligned}$$

Further, as in the baseline model, we have

$$\hat{b}_d = \frac{\left(1 - \hat{b}_m\right)\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2},$$

implying

$$x^* = \pi \frac{\hat{b}_m \sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}.$$

Note that, because

$$\frac{\partial \hat{H}}{\partial \hat{b}_m} = \left(\frac{\partial \hat{H}}{\partial \pi}\right)\left(\frac{\pi}{\hat{b}_m}\right) + 1,$$

it follows that

$$\frac{d\hat{b}_m}{d\pi} = -\frac{\partial \hat{H}/\partial \pi}{\partial \hat{H}/\partial \hat{b}_m} > -\frac{\hat{b}_m}{\pi}.$$

Therefore, we have

$$\frac{dx^*}{d\pi} = \frac{\hat{b}_m \sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2} + \pi \left(\frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2}\right)\left(\frac{d\hat{b}_m}{d\pi}\right) > \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2}\left[\hat{b}_m + \pi \frac{d\hat{b}_m}{d\pi}\right] > 0,$$

which completes the proof. □

Proof of Proposition 8. Let

$$H\left(\hat{b}_m\right)=\hat{b}_m-\frac{\frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2+\left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\left(\frac{\hat{b}_m}{\tau}\right)^2\sigma_\varepsilon^2+\left(\frac{w_2}{c_2}\hat{b}_m\right)^2\sigma_\nu^2+\left(\frac{1}{z^*}\right)^2\sigma_\eta^2}}.$$

Note that

$$H\left(0\right)=-\frac{w_1^2}{c_1^2\sigma_\eta^2\left(\frac{w_1^2}{c_1^2\sigma_\eta^2}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)}<0$$

$$H\left(1\right)=1-\frac{\frac{1}{\left(\frac{c_1^2\sigma_\eta^2}{w_1^2}+\frac{w_2^2\sigma_\nu^2}{c_2^2}+\frac{\sigma_\epsilon^2}{\tau^2}\right)}}{\left(\frac{1}{\frac{c_1^2\sigma_\eta^2}{w_1^2}+\frac{w_2^2\sigma_\nu^2}{c_2^2}+\frac{\sigma_\epsilon^2}{\tau^2}}+\frac{1}{\sigma_\xi^2}+\frac{1}{\sigma_\omega^2}\right)}>0$$

$$\frac{\partial H}{\partial\hat{b}_m}=1+\frac{2c_2^2\tau^2w_1^4\hat{b}_m\sigma_\xi^2\sigma_\omega^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(c_2^2\sigma_\epsilon^2+\tau^2w_2^2\sigma_\nu^2\right)}{\left(w_1^2\hat{b}_m^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(c_2^2\sigma_\epsilon^2+\tau^2w_2^2\sigma_\nu^2\right)+c_2^2\tau^2\left(c_1^2\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+w_1^2\sigma_\xi^2\sigma_\omega^2\right)\right)^2}>0$$

which implies there exists a unique solution $\hat{b}_m\in(0,1)$ to $H\left(\hat{b}_m\right)=0$.

Next, note that

$$\frac{\partial H}{\partial w_1}=-\frac{2c_1^2c_2^4\tau^4w_1\sigma_\eta^2\sigma_\xi^2\sigma_\omega^2\left(\sigma_\xi^2+\sigma_\omega^2\right)}{\left(w_1^2\hat{b}_m^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(c_2^2\sigma_\epsilon^2+\tau^2w_2^2\sigma_\nu^2\right)+c_2^2\tau^2\left(c_1^2\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+w_1^2\sigma_\xi^2\sigma_\omega^2\right)\right)^2}<0$$

$$\frac{\partial H}{\partial w_2}=\frac{2c_2^2\tau^4w_1^4w_2\hat{b}_m^2\sigma_\nu^2\sigma_\xi^2\sigma_\omega^2\left(\sigma_\xi^2+\sigma_\omega^2\right)}{\left(w_1^2\hat{b}_m^2\left(\sigma_\xi^2+\sigma_\omega^2\right)\left(c_2^2\sigma_\epsilon^2+\tau^2w_2^2\sigma_\nu^2\right)+c_2^2\tau^2\left(c_1^2\sigma_\eta^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+w_1^2\sigma_\xi^2\sigma_\omega^2\right)\right)^2}>0$$

which implies $\frac{d\hat{b}_m}{dw_1}=-\frac{\partial H/\partial w_1}{\partial H/\partial\hat{b}_m}>0$ and $\frac{d\hat{b}_m}{dw_2}=-\frac{\partial H/\partial w_2}{\partial H/\partial\hat{b}_m}<0$.

Moreover, note that

$$\hat{b}_d=\frac{\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega}\left(1-\hat{b}_m\right)\text{ and }x^*=\hat{b}_d+\hat{b}_m=\frac{\hat{b}_m\sigma_\xi^2+\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega}$$

which implies $\hat{b}_d$ is decreasing in w_1 and increasing in w_2, while x^* is increasing in w_1 and decreasing in w_2. □

Proof of Proposition 9. We define $b_m^\dagger$, $\underline{\beta}$, and $\overline{\beta}$ in the same way as in Proposition 3. $\bar{b}_m$ is defined similarly as the largest possible equilibrium solution to $H(\hat{b}_m)=0$ that is obtained by letting $w_1\to\infty$ and $w_2\to 0$.

By following the same steps in the proof of Proposition 2, we can easily see that

$$\mathbb{V}[\omega|P] = \frac{\left(1-\hat{b}_m\right)\sigma_\xi^2\sigma_\omega^2}{\sigma_\xi^2+\sigma_\omega^2},$$

and hence $PI = (\mathbb{V}[\omega|P])^{-1}$ is increasing in w_1 and decreasing in w_2 by Proposition 8.

The proof of Proposition 3 shows that when $\beta \leq \underline{\beta}$ (that is independent of w_1 and w_2), RE is increasing in $\hat{b}_m$. Because $\hat{b}_m$ is increasing in w_1 and decreasing in w_2 as shown in Proposition 8, RE is increasing in w_1 and decreasing in w_2 for $\beta \leq \underline{\beta}$.

When $\beta > \overline{\beta}$, RE is hump-shaped in $\hat{b}_m$. Further, $\hat{b}_m$ changes monotonically with the incentive w_i given w_j. Since w_i affects RE only through $\hat{b}_m$, RE is hump-shaped in w_i. By (18), the optimal (w_1, w_2) satisfies

$$Q \equiv \left(\frac{\hat{b}_m^*}{\tau}\right)^2\sigma_\varepsilon^2 + \left(\frac{w_2}{c_2}\hat{b}_m^*\right)^2\sigma_\nu^2 + \left(\frac{c_1}{w_1}\right)^2\sigma_\eta^2 - \frac{1-\hat{b}_m^*}{\left(\frac{1}{\sigma_\omega^2}+\frac{1}{\sigma_\xi^2}\right)\hat{b}_m^*} = 0,$$

where $\hat{b}_m$ is evaluated at $\hat{b}_m^*$ that maximizes RE. Differentiating with respect to w_1, we have

$$\left(\frac{\partial Q}{\partial \hat{b}_m}\right)\left(\frac{d\hat{b}_m}{dw_1}\right) + \left(\frac{2w_2\hat{b}_m^2\sigma_\nu^2}{c_2^2}\right)\left(\frac{dw_2}{dw_1}\right) - 2\frac{c_1^2\sigma_\eta^2}{w_1^3}\bigg|_{\hat{b}_m=\hat{b}_m^*} = 0.$$

Note that

$$\frac{d\hat{b}_m}{dw_1}\bigg|_{\hat{b}_m=\hat{b}_m^*} = -\frac{\partial L/\partial w_1}{\partial L/\partial \hat{b}_m}\bigg|_{\hat{b}_m=\hat{b}_m^*} = 0.$$

It follows that

$$\frac{dw_2}{dw_1} = \frac{\frac{2c_1^2\sigma_\eta^2}{w_1^3}}{\frac{2w_2\hat{b}_m^2\sigma_\nu^2}{c_2^2}} > 0.$$

Similarly,

$$\frac{dw_1}{dw_2} = \frac{\frac{2w_2\hat{b}_m^2\sigma_\nu^2}{c_2^2}}{\frac{2c_1^2\sigma_\eta^2}{w_1^3}} > 0,$$

which completes the proof. □