Choosing to Disagree in Financial Markets

Snehal Banerjee
UC San Diego

Jesse Davis
UNC Chapel Hill

Naveen Gondhi
INSEAD

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Rational expectations implies learning is efficient

- **Assumes** subjective beliefs agree with objective distribution
- Why? Objective beliefs are *accurate*, forward looking

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e.g., excess predictability, volatility and volume

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Behavioral literature explores role of cognitive frictions and biases (e.g., over-confidence, dismissiveness), but is silent on **when / why** such distortions arise

Given a choice, how do investors interpret information?

We allow investors to choose how to interpret information in a standard, Hellwig (1980) setting

- Observe conditionally i.i.d. private signals and (noisy) price
- Well-being *also* depends on anticipation of future outcomes
- Investors choose precision of private / price signals ex-ante

Subjective beliefs trade off:

Desirability higher anticipatory utility

versus

Accuracy higher experienced utility

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Return predictability, volume, volatility and welfare can be higher under chosen beliefs than under rational expectations

Related Literature

Belief Choice See survey by Benabou and Tirole (2016)

- Caplin and Leahy (2019)'s model of "wishful thinking"
- Brunnermeier and Parker (2005)'s model of "optimal expectations"

Deviations from Rational Expectations

- Overconfidence: Odean (1998); Daniel et al. (1998); Daniel, Hirshleifer, and Subrahmanyam (2001); Gervais and Odean (2001)
- Under-weighting price information:
  Difference of opinions (e.g., Banerjee, Kaniel and Kremer, 2009)
  Rational inattention (e.g., Kacperczyk et. al. 2016)
  Cursedness (e.g., Eyster, Vayanos and Rabin, 2018)
  Costly learning from prices (e.g., Vives and Yang, 2018)

What drives choice of beliefs?
(a.k.a. motivating motivated beliefs)

Choice of subjective beliefs depends on overall goal

Discounted expected utility: Goal is to maximize future, experienced (ex-post) utility

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- Accurate beliefs $\Rightarrow$ accurate decisions

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Robust control: Goal is to optimize against bad (worse-case) scenarios

- Prefer to choose *pessimistic* subjective beliefs
- But, distortion in beliefs $\Rightarrow$ distorted actions, lower ex-post utility

Trade-off: accuracy vs. robustness (down-side protection)

$$\max_a \min_\mu \mathbb{E}_\mu[u(a)] + C(\mu, \mu_0)$$

where $C(\mu, \mu_0)$ is **cost** of choosing beliefs $\mu \neq$ objective beliefs μ_0

Anticipatory Utility and Wishful thinking

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Figure 1: Balcetis & Dunning (2006)

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When desirable, 72% saw “B” and 61% saw “13”

Wishful thinking and motivated reasoning affects the acquisition and interpretation of information in many settings

- Oster, Shoulson & Dorsey (2013): Don't want to learn if at risk for Huntington's even if test is cheap and perfectly predictive
- Ganguly and Tasoff (2016): Pay to avoid getting tested for HSV-1 / HSV-2
- Eli & Rao (2011): People under-react to negative feedback on intelligence / beauty, but respond to good news
- Karlsson, Loewenstein & Seppi (2009): Investors monitor their portfolios more in rising markets
- Babcock and Loewenstein (1997): Randomly assigned "prosecutors" interpret the same evidence to be more consistent with defendant's guilt than assigned "defense attorneys"
- Exley and Kessler (2019): Interpret uninformative signals about ability as favorable

Moreover, expertise / cognitive ability can exacerbate the biases e.g., political bias in Kahan (2013), Kahan, Peters, Dawson & Slovic (2014)

Model Setup

Payoffs, Signals and Preferences

There are three dates $t = 0, 1, 2$ and two assets:

- Risk-free asset is normalized to numeraire
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Continuum of investors with CARA (γ) utility over terminal ($t = 2$) wealth
Normalize initial wealth to $W_0 = 0$ for presentation.

At date $t = 1$, investor i

(i) observes private signal $s_i = F + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, 1/\tau_e)$ is i.i.d.

(ii) infers a private signal $s_p = F + \beta z$ from the equilibrium price P

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Aggregate supply of the asset is $z \sim \mathcal{N}(0, 1/\tau_z)$, so market clearing:

$$\int_i x_i(s_i, P) di = z$$

Subjective Beliefs

Investor i's subjective beliefs about:

- error in private signal: $\varepsilon_i \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{e,i}\tau_e}\right)$
- aggregate supply shock: $z \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{z,i}\tau_z}\right)$

where $\delta_{e,i}, \delta_{z,i} \in [0, \infty]$ parameterize the degree to which the investor over- or under-estimates the precision of her information.

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Aside: Beliefs about supply noise $\Leftrightarrow$ Beliefs about others

For example, suppose that investor i's subjective beliefs are given by

$$\varepsilon_j = \alpha_i F + \sqrt{1 - \alpha_i} u_j \qquad \text{where } \alpha_i \in (-1, 1) \text{ and } u_j \sim \mathcal{N}\left(0, \frac{1}{\tau_e}\right)$$

Then, we can show that $\delta_{z,i} = (1 + \alpha_i)^2$

Anticipated Utility

Each investor adopts her chosen beliefs as her “true” model.

- At date $t = 1$, optimal demand is

$$x_i(s_i, P; \delta_{e,i}, \delta_{z.i}) = \frac{\mathbb{E}_i[F] - P}{\gamma \mathrm{var}_i[F]}$$

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- At date $t = 0$, anticipated utility is

$$AU(\delta_{e,i}, \delta_{z,i}) = \mathbb{E}_i \left[-\gamma \mathbb{E}_i \left[-e^{\gamma x_i(s_i, P) \times (F-P)} \middle| s_i, P \right] \right]$$

Anticipated utility is *current utility* derived from expectation of the future.

Expected utility is current expectation of future, ex-post utility.

Cost of Belief Distortion

Deviations from objective distribution impose a cost $C(\delta_{e,i}, \delta_{z,i})$, so investor i chooses $\delta_{e,i}$ and $\delta_{z,i}$ to maximize:

$$AU(\delta_{e,i}, \delta_{z,i}) - C(\delta_{e,i}, \delta_{z,i})$$

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Experienced utility penalty: The cost reflects utility loss from distorted actions i.e.,

$$C(\delta_{e,i}, \delta_{z,i}) \equiv \mathbb{E}\left[-\gamma e^{-\gamma x_i(\delta_{e,i}, \delta_{z,i})(F-P)}\right] - \mathbb{E}\left[-\gamma e^{-\gamma x_i(1,1)(F-P)}\right]$$

- Similar to Brunnermeier and Parker (2005)'s optimal expectations

Well-behaved cost function: $C(\cdot)$ is increasing, strictly convex, and

$$C(1,1) = \frac{\partial C(1,1)}{\partial \delta_{e,i}} = \frac{\partial C(1,1)}{\partial \delta_{z,i}} = 0$$

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Note: Investor need not "know" objective distribution, convenience / discipline for economist

Solving the Model

"Standard" Financial Market Equilibrium

Lemma: Given investors' subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$ $\forall i \in [0,1]$, there always exists a unique linear equilibrium with

$$P = \Lambda s_p, \text{ where } \Lambda = \frac{\int_i \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p di}, \quad s_p = F + \beta z$$

and with $\tau_p \equiv \tau_z/\beta^2$, and $\beta \equiv -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}$.

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Subjective beliefs affect prices through two channels:

- Higher private signal precision $\delta_{e,i}$ increases signal to noise ratio of s_p increases $|\beta|$
- Higher precision of either signal increases price sensitivity to fundamentals i.e., Λ

Return and Volume Characteristics

We will compare predictions on the following observables:

(i) Return volatility $\sigma_R = \sqrt{\mathrm{var}(F - P)}$

(ii) Return predictability $\theta = \frac{\mathrm{cov}(F-P,P)}{\mathrm{var}(P)}$

regression coefficient of return on lagged return

(iii) Price informativeness $\tau_p = \tau_z / \beta^2$

(iv) Expected trading volume $\mathbb{E}[\mathcal{V}] = \mathbb{E}\left[\int_i |x_i| di\right]$

Subjective Beliefs and Anticipated Utility

Anticipated utility increases in the volatility of conditional Sharpe Ratio:

$$AU(\delta_{e,i}, \delta_{z,i}) = -\sqrt{\frac{\text{var}_i[F|s_i, P]}{\text{var}_i[F - P]}} = -\sqrt{\frac{1}{\text{var}_i(SR_i)}},$$

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Lemma: Anticipated utility increases in perceived private precision ($\delta_{e,i}$), but is U-shaped in perceived price precision ($\delta_{z,i}$)

Results

Benchmark: Overconfidence in private information

Suppose investors have objective beliefs about price information $\delta_{z,i} = 1$.

Theorem: There exists a unique symmetric equilibrium in which the investors are overconfident about private information i.e., $\delta_{e,i} > 1$.

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Relative to RE equilibrium, we have
(i) lower return volatility,
(ii) higher predictability in returns,
(iii) higher price informativeness, and
(iv) higher expected volume.

Subjective beliefs about price information

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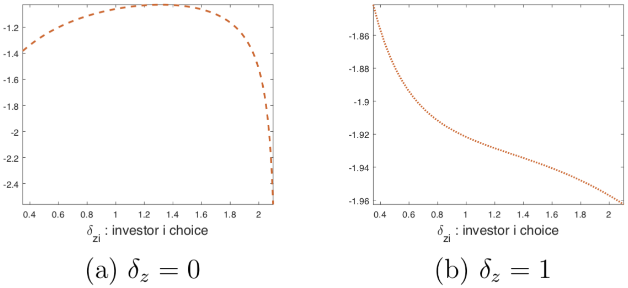

Figure 2: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

Dismissiveness in symmetric equilibria

Theorem: In **any** symmetric equilibrium, all investors are:
(i) (weakly) over-confident about their private info i.e., $\delta_{e,i} \geq 1$
(ii) dismissive of price info i.e., $\delta_{z,i} < 1$

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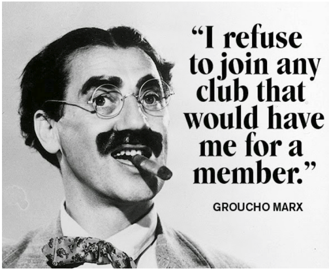
Intuition:



"I refuse to learn from prices when others are doing so."

Risk aversion and symmetric equilibria

Note that in a symmetric equilibrium, the price is

$$P = \bar{\mathbb{E}}_i[F \big| s_i, P] - \gamma \mathrm{var}_i[F \big| s_i, P] z$$

$\Rightarrow$ All else equal, price is less informative as risk aversion γ increases

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Theorem: With exp. utility penalty, there exist cutoffs $\underline{\gamma} < \bar{\gamma}$ such that

(i) For $\gamma \geq \bar{\gamma}$, there exists a unique, symmetric equilibria in which all investors **ignore** price information and correctly interpret private information (i.e., $\delta_{z,i} = 0$ and $\delta_{e,i} = 1$).

(ii) For $\gamma \leq \underline{\gamma}$, there does not exist a symmetric equilibrium.

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(ii) For $\gamma \leq \underline{\gamma}$, there does not exist a symmetric equilibrium.

Intuition: When prices are sufficiently uninformative ($\gamma \geq \bar{\gamma}$), ignoring prices is not *too* costly, so symmetric equilibrium can be sustained

Risk tolerance and asymmetric equilibria

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There exist **asymmetric** equilibria characterized by $(\lambda, \delta_e, \delta_z)$ where

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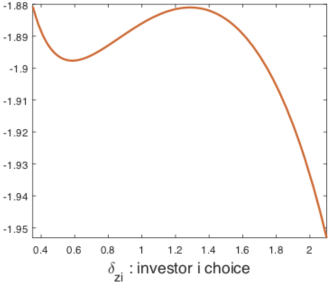

Figure 3: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

Implications of Asymmetric Equilibria

Observed heterogeneity in investment styles arise endogenously:

- **Value investors** who find *mispriced securities* using their private info, but *dismiss* the information in prices
- **Technical traders** use price trends, reminiscent of overweighting of price information

This is not a difference in degrees, but **in kind**: bias in opposite directions

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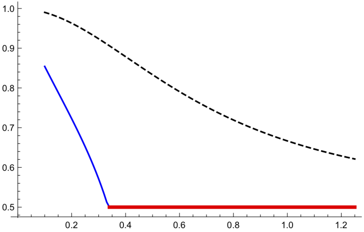
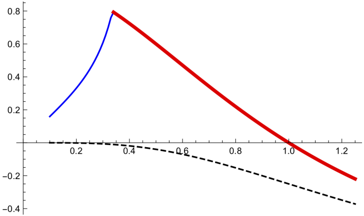
Model predicts such heterogeneity arises when risk tolerance and price informativeness are high:

- in more developed financial markets
- in larger (more widely held) assets

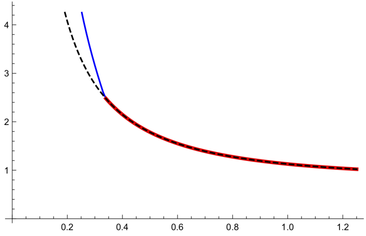
Return Volume Implications



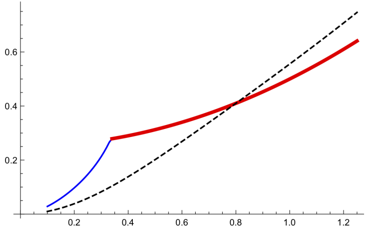
(a) Price sensitivity Λ vs. γ



(b) Predictability θ vs. γ



(c) Volume vs. γ



(d) Volatility vs. γ

Rational expectations - dashed; Symmetric - red; Asymmetric - blue

Welfare

How do subjective beliefs affect investor utility?

- Under the subjective measure, investors are better off
- Under the objective measure, investors are worse off

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- Under the subjective measure, investors are better off
- Under the objective measure, investors are worse off

How do investor's subjective beliefs affect other participants?

- Price may be more informative about fundamentals
- Utility cost to liquidity traders can be lower, because prices track fundamentals more closely

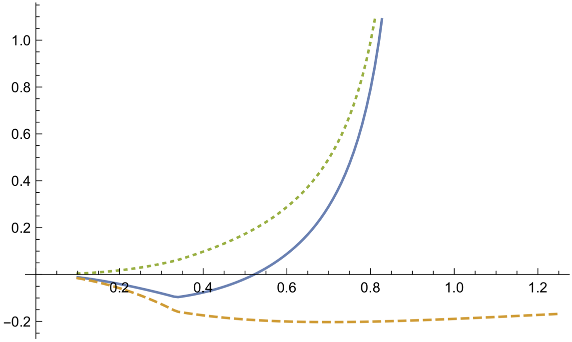
$$U_z(z) \equiv \mathbb{E}\left[-\gamma_z e^{-\gamma_z(W_0 - z(F-P))}\right]$$

Welfare is higher subjective beliefs when γ is high

Theorem: Suppose $\gamma_z \leq \gamma$. Then,

(i) liquidity traders **always** have higher expected utility under symmetric equilibrium that in RE

(ii) when $\Lambda_{AE} < \Lambda_{RE}$, liquidity traders have higher expected utility under the asymmetric equilibrium than in RE



ΔU for investors (dashed), noise traders (dotted), both (solid)

Extensions: Public signals and Ex-post belief choice

Public Signals: Consider signals of the type $s = F + \eta$

- Tradeoff between information effect vs. speculative effect
- Anticipated utility is U-shaped in perceived precision
- But, in any symmetric equilibrium, investors **overweight** public signal

Ex-post belief choice: Choose perceived precision *after* observing signal

- Not tractable to solve for general equilibrium prices are not linear
- Taking others actions as fixed, partial equilibrium analysis suggest results are robust:

  When private signal realizations are sufficiently far from priors, investors are over-confident in their private info, but dismissive of prices

Conclusions and Future Work

Subjective belief choice tells us **when** investors exhibit biases:

- Naturally gives rise to over-confidence and dismissiveness
- Can generate **endogenous** differences in behavior

Fruitful approach to explore how different biases arise in different settings

- Financial markets are characterized by strategic substitutability
- How do results change in settings with strategic complementarity (e.g., coordination games)

Land or Sea?

Land or Sea?

Land or Sea?

66.7% horse vs. 72.7% seal