Risk Sharing, Investment Efficiency, and Welfare with Feedback Effects*

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Abstract

Financial markets enable risk sharing and efficient allocation of capital. We characterize how these roles interact in a "feedback effects" model with diversely informed, risk-averse investors and a manager who learns from prices when making an investment decision. While learning from prices always improves investment efficiency, we identify a novel channel by which it can reduce welfare. Namely, investment decisions change the stock's exposure to underlying shocks and, consequently, investors' ability to hedge risk. We show that this is a robust feature of general investment decisions, and outline implications for firm governance and policy.

JEL: D82, D84, G12, G14

Keywords: feedback effects, welfare, investment efficiency, hedging, market completeness, risk sharing

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1 Introduction

Financial markets serve two central roles in the economy. First, they enable investors to hedge and share risks. Existing work has explored how the payoffs and risk exposures of a security affects its use for hedging and speculation, and its impact on investor welfare.[1] Second, markets aggregate information across investors, which can directly impact firms' real decisions. The literature on "feedback effects" underscores the importance of this role, demonstrating how managers incorporate the information in prices when investing, and studying whether this improves real efficiency.[2] However, the analysis so far largely ignores the interaction between these two roles. Importantly, a firm's investment choices endogenously change its exposure to underlying economic shocks, which affects investors' ability to use its stock for hedging and, consequently, its market price.

For example, consider a traditional automaker deciding whether to adopt electric vehicle technology. In addition to other information sources (e.g., research, peer comparisons), the firm may rely on the market response to the announcement of such a strategy before deciding whether to pursue it. At the same time, the firm's investment decision determines how its stock covaries with climate-change news, which affects investors' ability to hedge their diverse exposures to correlated risks (e.g., rising gasoline prices or sea levels). This affects the information aggregated by the stock price, which feeds back into the firm's decisions. Similarly, mergers, acquisitions, and spin-offs often lead to discrete changes in firms' risk exposures, which directly impact the ability of investors to hedge and trade the underlying risks. This influences the market reaction to the announcement of these events, which often impacts firms' decisions of whether to complete them.[3]

These examples highlight that firms' decisions affect investor welfare not only through their effect on firm value but also by changing the firm's risk exposures, which affects investors' ability to hedge. Understanding this interaction is especially important for policy analysis, and naturally leads to a number of questions. When do better-informed real investment decisions improve welfare in the presence of risk-sharing motives for trade? Does a manager who bases real decisions on the information in prices improve investor welfare relative to one who ignores such information? When do changes in the information environment

[1]See, for example, Allen and Gale (1988), Boot and Thakor (1993), and Marín and Rahi (2000), and the reviews by Allen, Gale, et al. (1994) and Duffie and Rahi (1995).

[2]See Bond, Edmans, and Goldstein (2012) for a survey on feedback effects, and Goldstein and Sapra (2014) and Goldstein and Yang (2017) for policy related discussions.

[3]A large literature (e.g., Asquith (1983) has explored the relation between announcement returns and eventual likelihood of completions for mergers and acquisitions. More directly, Dasgupta and Gao (2004) suggest that the decision to complete or cancel an announced acquisition is sensitive to the market response to the announcement and Luo (2005) show that this sensitivity depends upon the quality of investors' private information.

that improve investment efficiency also improve welfare?

To better understand the consequences of endogenous risk exposures, we develop a benchmark model with both feedback effects and risk-sharing motives for trade. The firm's manager is considering whether to invest in a new project, and can condition on the stock price when doing so. The firm's stock is traded by a continuum of investors, who are risk averse, own shares of the firm, and observe dispersed information about the profitability of the new project. They are also exposed to endowment shocks that are correlated with the project payoff and which they desire to hedge.[4] Key for our analysis, the firm's investment decision changes the risk exposures of the firm's cash flows which, in turn, affects the ability of investors to use the stock to hedge their endowment shocks.

We find that endogeneity of risk exposures has important implications, especially for investor well-being. We focus on two relevant measures in closed form: (i) *investment efficiency*, which reflects the (ex-ante) expected firm value, and (ii) *welfare*, which reflects the (ex-ante) expected utility for investors. Not surprisingly, these two measures differ: welfare depends not only on the expected firm value, but also on the ability of investors to hedge their endowment shocks. Clarifying this distinction in the context of feedback effects is important because existing analysis has largely studied welfare only in the absence of feedback, or has focused on proxies involving price and investment efficiency in the presence of feedback.[5] Moreover, investment efficiency is often used as a proxy for empirical and policy analysis because it is easier to measure (e.g., using empirical estimates of firm profitability) than welfare. As such, it is crucial to understand precisely how these measures are related and identify situations in which they are in tension.

First, we show that, state-by-state, the welfare-maximizing level of investment is (weakly) greater than the value-maximizing, equilibrium level of investment. Next, we apply this result to characterize how feedback affects investment efficiency and welfare. Feedback unambiguously improves investment efficiency because better informed investment decisions always increase the expected value of the firm. However, its effect on welfare is more nuanced. We show that feedback tends to reduce welfare when (i) the ex-ante NPV of the project is positive, (ii) investors' initial share endowments are sufficiently small, and (iii) the new project is correlated with risks that investors would like to hedge. In this case, welfare and

[4]We restrict attention to a setting with binary investment decisions and a risk-neutral market maker in the benchmark analysis in order to clarify the key forces, but relax these assumptions in Section 6. One contribution of our paper is that we are able to incorporate all these features in an analytically tractable, yet general, model and explicitly characterize welfare by extending the approach of Breon-Drish (2015).

[5]In general, this is because of analytical tractability: feedback models often rely on exogenous noise trading or risk-neutrality of investors, which make welfare difficult to characterize. Important exceptions to this are Dow and Rahi (2003), Bond and Garcia (2021), and Gervais and Strobl (2021), which we discuss in Section 2.

investment efficiency can move in opposite directions.[6]

To see the intuition for our results, note that the investment decision affects welfare via two channels. First, the investment affects the expected value of the firm, which influences welfare because investors own shares in the firm. We refer to this as the **valuation channel**. However, the endogeneity of risk exposures leads to a second channel. More investment in the new project makes the stock more sensitive to the risk factor governing project cash flows and consequently, more useful for hedging exposures that are correlated with this risk factor. Holding the expected firm value constant, this implies that more investment improves welfare – we refer to this effect as the **hedging channel**.

Because the firm's manager does not account for the hedging channel when making investment choices, she generically under-invests relative to the welfare-maximizing rule – this immediately implies our first result. Moreover, since feedback improves investment efficiency, its impact on welfare via the valuation channel is always positive. In contrast, feedback's impact on welfare via the hedging channel hinges on the project's ex-ante NPV. If the project's NPV is negative, then the manager does not invest in the project in the absence of feedback and the stock is not useful for hedging. In this case, feedback increases investment incrementally when the information in prices is sufficiently positive, which improves investors' ability to use the stock for hedging, and so increases welfare via the hedging channel.

On the other hand, when the project's NPV is positive, the manager always invests in the absence of feedback. In this case, feedback *reduces* investment incrementally when the information in prices is sufficiently negative. This leads to a decrease in welfare via the hedging channel because the stock becomes less useful for hedging purposes in those states. We show that the negative impact of the hedging channel dominates the positive effect of the valuation channel when the firm is relatively small, i.e., when investors' endowments of the firm are small relative to their hedging motives.[7] Intuitively, the impact of the firm's investment efficiency on investors' expected payoffs is proportional to the firm's size, while the efficacy of the firm's stock as a hedge is independent of its size.

The hedging channel we uncover is novel and distinct from the Hirshleifer (1971) effect. The latter refers to the mechanism whereby, *holding fixed a security's exposure to underlying*

[6]As we discuss in Section 2, a number of other recent papers have highlighted the role of the project's ex-ante NPV in determining information choices and market outcomes (e.g., Dow, Goldstein, and Guembel (2017), Davis and Gondhi (2019), and Goldstein, Schneemeier, and Yang (2020)) in settings where the firm's investment decision is discrete. However, the economic mechanism underlying our analysis is distinct and also applies in settings where investment choice is continuous (e.g., see Section 6.1).

[7]Formally, our model has a single risky asset. In the context of a richer model with multiple assets, the size of the share endowment proxies for the weight of the firm in the market portfolio. When a particular firm is a small, the efficiency of its investment decision has little effect on aggregate investment efficiency in the economy.

shocks, more information about those shocks makes the security less useful as a hedging tool. In contrast, our hedging channel is a consequence of the inherent incompleteness of markets and the endogeneity of risk exposures.[8] In principle, this implies that if there were a separate tradeable security that was perfectly correlated with endowment shocks, then the value-maximizing investment rule would coincide with the welfare-maximizing rule. In practice, however, it is unlikely that such perfect hedges are available for a comprehensive set of risk exposures. In particular, markets appear to be incomplete with respect to hedging climate risk or shocks to human capital.[9]

Our results are also distinct from, but complement, those in the existing literature that show why higher price informativeness need not improve investment efficiency if the information is not useful for real decisions (e.g., Dow and Gorton (1997), Bond, Edmans, and Goldstein (2012), Goldstein and Yang (2014)).[10] In contrast to this work, we show that higher price informativeness can decrease welfare by affecting the endogenous risk exposures of a tradeable security, even if it always improves investment efficiency.

While our benchmark model considers a stylized setting for tractability and expositional clarity, our key results are robust to alternate assumptions and extensions. Importantly, we show that our results remain qualitatively unchanged if we replace the firm's binary investment decision (invest or not) with a general investment technology. Similarly, we can generalize the specification of investors' endowment shocks. The key feature for our result is that the investment decision changes the exposure of the firm's cash flows to underlying shocks, and consequently, affects how useful the stock is for hedging. The underlying tension between valuation and hedging also obtains in a setting in which the price is determined by market clearing, as opposed to by a risk-neutral competitive market maker. However, the overall impact of feedback on welfare is more nuanced in this case.[11] Finally, we also discuss how our results change if investors' information and endowment shocks are correlated with

[8]In contrast to the Hirshleifer effect, the hedging channel leads to a disparity between investment efficiency and welfare even if investors were able to trade prior to the receipt of information.

[9]For instance, Engle, Giglio, Kelly, Lee, and Stroebel (2020) find that a dynamic equity portfolio optimized to hedge climate risk is at most 30% correlated with news on such risk. Similarly, Lustig and Van Nieuwerburgh (2006) document a low correlation between market returns and human capital, and Attanasio and Davis (1996) and Eiling (2013) provide direct evidence of the failure of perfect risk sharing in the context of labor markets).

[10]In particular, Bond et al. (2012) distinguish between the notions of forecasting price efficiency (FPE), which measures the extent to which prices convey information about fundamentals, and revelatory price efficiency (RPE), which measures the extent to which prices reveal information relevant for real decision making.

[11]As we discuss in Section 6.2, this is because the market clearing price reflects not just the aggregate private information across investors, but also a risk premium term (which depends on the per-capita endowment of shares). This risk premium term affects welfare through two additional channels – an endowment channel which lowers investor wealth (since shares are worth less) and a cost of hedging channel (which affects the cost of using the stock as a hedge).

assets in place.

Our analysis speaks to the recent debate about a firm's objectives that distinguishes between shareholder value maximization and welfare maximization (e.g., Hart and Zingales (2017)). Specifically, we show that dampening the feedback effect may improve shareholder welfare for firms with (i) small size and dispersed ownership, and (ii) positive NPV projects that are in new lines of business, but reduce welfare in other firms. Directly contracting on whether or not the firm engages in feedback may not be practical because the manager is usually unable to commit to learn from (or ignore) price information. However, the existing literature on variation in feedback effects across firms suggests possible approaches. For example, since overconfident managers are more likely to ignore price information (e.g., Bond and Goldstein (2015)), hiring such managers may improve shareholder welfare for small firms with risky, positive NPV projects. On the other hand, pre-announcement of investment strategies enables feedback from prices, and so is likely to improve shareholder welfare and investment efficiency for larger firms.[12] Finally, our results suggest that encouraging the manager to focus on the (short-term) price instead of (long-term) firm value may lead to higher welfare, because this leads the manager to partially account for the hedging role of the firm's stock.

The key feature we model – the impact of investment decisions on risk exposures – is particularly salient in mergers, acquisitions, and spin-offs.[13] That is, one may interpret the investment decision as the decision to pursue an acquisition, or conversely, to spin off an existing division. Naturally, a merger (spin-off) destroys (creates) the opportunity for investors to separately hedge exposures to the two entities, such as employees in the firms' industries who desire to hedge their human-capital exposures (e.g., see Eiling (2013)). Thus, our results suggest that mergers, even when value-enhancing in expectation, can reduce investor welfare.

Our model also provides a set of policy implications. Since equilibrium under-investment (relative to the welfare-maximizing level) is most severe for small firms with (ex-ante) negative NPV projects and limited feedback from prices, our analysis suggests that direct investment or product-market subsidies might have the largest positive impact on welfare in these cases. Our results also recommend care when evaluating policies that encourage information production and disclosure about a firm's cash-flows. Specifically, we show that policies to encourage information production about a firm's assets in place always improve welfare in our setting. This is consistent with standard accounting principles, which focus on provid-

[12]See Luo (2005), Edmans, Goldstein, and Jiang (2012), and Jayaraman and Wu (2020) for evidence of how firm disclosures may lead to more feedback.

[13]SPACs are another an example where the risk exposure of a publicly traded firm's cash flows change discretely.

ing information regarding assets in place. In contrast, encouraging information production and disclosure for new projects may reduce welfare, especially for small firms with risky investment opportunities.

The rest of the paper is organized as follows. The next section discusses the related literature and clarifies our contribution. Section 3 presents the benchmark model and discusses key assumptions. Section 4 characterizes the equilibrium in our setting and Section 5 presents our main results regarding investment efficiency and welfare. Section 6 explores the robustness of our analysis and discusses extensions. Section 7 highlights some implications of our model, and Section 8 concludes. A general characterization of welfare in our setting is presented in Appendix A and proofs of our results are in Appendix B.

2 Related Literature

Our paper adds to the literature on feedback effects (see Bond et al. (2012) and Goldstein and Yang (2017) for recent surveys). Our primary contribution is to develop a tractable and general framework that allows for a unified analysis of investor welfare in the presence of feedback effects and risk-sharing. Most existing models of feedback effects assume either risk-neutral investors or the presence of noise traders with unmodeled utility functions, which makes welfare difficult to characterize. Instead, we assume that investors are risk-averse and incorporate noise in prices via their endowment shocks (e.g., Diamond and Verrecchia (1981), Wang (1994), Ganguli and Yang (2009), Manzano and Vives (2011) and Bond and Garcia (2021)).

The most closely related papers are Dow and Rahi (2003), Gervais and Strobl (2021), and Hapnes (2020). Dow and Rahi (2003) explore how increases in informed trading affect investment efficiency and risk-sharing in a setting with feedback from market prices to investment decisions. They argue that investment efficiency always improves with more informed trading, but risk-sharing may worsen due to the Hirshleifer (1971) effect. Our analysis, which we view as complementary, is distinct in its focus and results. Our primary goal is to understand when managerial learning from prices, and better informed investment decisions more generally, improves investor welfare. This distinct focus has different implications for regulatory policy, as we discuss in Section 7.

Furthermore, in contrast to our setting, the attractiveness of the investment opportunity faced by the firm does not affect investor welfare in Dow and Rahi (2003). This reflects two crucial differences between our model and theirs. First, the investors in their model are not endowed with shares of the firm itself, and consequently, the impact of the investment decision on expected cash flows has no impact on welfare (i.e., there is no analogous valuation channel in investor welfare in their model).[14] Second, they do not analyze the effect of the firm's investment decision on the stock's usefulness as a hedge. In contrast, we study, via the hedging channel, how the firm's investment decision *endogenously* affects market completeness in our setting, which also distinguishes our analysis from Marín and Rahi (1999), Marín and Rahi (2000), and Eckwert and Zilcha (2003), who consider how *exogenous* differences in market completeness influence investor welfare.

Gervais and Strobl (2021) consider a related setting with feedback from prices where investors are endowed with shares of firms and can either directly trade these stocks or allocate their money to an informed money manager. They study how the gross and net performance of the actively managed fund compares with the market portfolio and study how the presence of an informed money manager affects welfare. We view their analysis as complementary to ours. As in our model, they show that improvements in firm information quality generally improve investment efficiency and can improve welfare through a valuation channel (i.e., by making investment decisions more informationally efficient). However, more informative prices in their model can reduce welfare via a classic Hirshleifer (1971) effect. Our analysis highlights a distinct effect (the hedging channel) that can arise when investment decisions affect the exposure of the firm's stock to underlying risks.[15]

Hapnes (2020) characterizes the financial equilibrium in a two-types CARA-Normal model à la Grossman and Stiglitz (1980), with feedback and risk-averse investors. However, his focus is on managerial investment behavior and investor information acquisition; the analysis does not study the effect of feedback on welfare. Furthermore, because there are noise traders in the model and two asymmetric classes of rational investors, it is difficult to define an unambiguous welfare measure. This contrasts with our model, in which all agents in the economy maximize well-defined utility functions and are ex-ante symmetric.

Our focus on analytically characterizing and studying welfare is also complementary to recent work by Bond and Garcia (2021), who study the consequences of index investing. By making use of the market clearing condition, they are able to characterize welfare in closed form, which they use to study the effects of decreasing the cost of indexing. They point out that while indexing may reduce aggregate price efficiency, in so doing it improves

[14] While they specify that the asset is in unit supply, this does not play a direct role in the welfare analysis since investors have endowments that are correlated with the shocks driving investment profitability, not shares of the firm itself. This is apparent from the observation that their expressions for investor utility do not depend on the cost of investment.

[15] In Gervais and Strobl (2021), the production function has a linear-quadratic form, which implies the level of investment is linear in the conditional expectation of fundamentals. Given normally distributed payoffs, this ensures that investment is non-zero (although potentially negative) almost everywhere, which implies that the stock is always useful for hedging. Furthermore, the investment opportunity comprises the entire firm value, which implies that changes in the investment level, on the intensive margin, do not affect the effectiveness of the stock as a hedge.

retail investor welfare due to improvements in risk sharing. Tension between notions of firm profitability and welfare also appears in Goldstein and Yang (2019a), who propose a model of commodity financialization. They show that improvements in price informativeness always increase producer profits due to better-informed real investment, but may ultimately harm producer welfare by destroying risk-sharing opportunities, similar to the Hirshleifer (1971) effect.

A key feature of our model is that, because the firm learns from price and adjusts investment accordingly, its expected cash flow, and thus its price, is a non-linear function of investors' private information. This implies the payoff distribution is non-normal and generally truncated below, which breaks the linearity important for solving standard CARA-normal models. Prior work that studies feedback in alternative settings has analyzed related non-linearities. Albagli, Hellwig, and Tsyvinski (2011) generate an analogous non-linearity and study its implications for how managerial incentives based on prices versus cash flows impact investment efficiency. Davis and Gondhi (2019) find that debt leads to a non-linear relationship between information and prices, and explore its impact on agency problems between equity and debt holders. Dow et al. (2017) show that non-linearities in a feedback setting can lead to multiplicity in investors' equilibrium information acquisition decisions.

Similar to our findings, other papers studying discrete investment choice emphasize the importance of the firm's "default" investment decision (i.e., the decision the firm would make in the absence of feedback) in determining information choices and market outcomes. Dow, Goldstein, and Guembel (2017) show that the nature of investors' equilibrium information acquisition decisions in a feedback setting hinges on whether the firm defaults to a risky or a riskless project. Davis and Gondhi (2019) show that complementarity in learning by investors depends not only on the default investment decision, but also on the correlation of the investment return and cash flows from assets in place. Goldstein et al. (2020) study the interaction between investor and managerial information acquisition in a feedback model with multiple sources of uncertainty. They show that investors seek to acquire the same information as management for positive NPV projects, but different information for NPV negative projects. This tension leads to a disconnect between real efficiency and price efficiency, and can generate a negative relationship between managerial signal precision and real efficiency. Our analysis complements these works by identifying a novel tension between real efficiency and welfare, and suggests that while better-informed managerial investment is socially desirable for (ex-ante) negative NPV projects, it may not be for positive NPV projects.

3 The Benchmark Model

Payoffs. There are three dates $t \in \{1, 2, 3\}$ and two securities. The risk-free security is normalized to the numeraire. The risky security is a claim to the terminal cash flows V generated by the firm at date $t = 3$, and trades at a price P.

The firm. The firm generates cash flows $A \sim N(\mu_A, \tau_A)$ from assets in place.[16] In addition, it is deciding whether to invest in a new project. The investment decision is binary, and denoted by $y \in Y \equiv \{0, 1\}$. The gross payoff to the investment is given by $\theta \sim N(\mu_\theta, \tau_\theta)$, where θ is independent of A, and the investment cost is $c(y)$, where $c(1) = c > 0 = c(0)$. Hence, the total cash flow given an investment level y is

$$V(y) = A + y\theta - c(y). \tag{1}$$

Investors. There is a continuum of investors, indexed by $i \in [0, 1]$, with CARA utility over terminal wealth with risk aversion γ. Investor i has initial endowment of n shares of the risky asset and $z_i = Z + \zeta_i$, units of exposure to a non-tradeable asset that is perfectly correlated with θ, and where $Z \sim N(\mu_Z, \tau_Z)$ and $\zeta_i \sim N(0, \tau_\zeta)$ are independent across each other and all other random variables.

Investor i also observes a signal $s_i = \theta + \varepsilon_i$, where the errors $\varepsilon_i \sim N(0, \tau_\varepsilon)$ are independent of all other random variables and each other. Let $\mathcal{F}_i = \sigma(s_i, z_i, P)$ denote investor i's information set at the trading stage, with associated expectation, covariance, and variance operators, $\mathbb{E}_i[\cdot]$, $\mathbb{C}_i[\cdot]$, and $\mathbb{V}_i[\cdot]$, respectively. Then, investor i chooses *trade* X_i to maximize her expected utility i.e.,

$$W_i \equiv \sup_{x \in \mathbb{R}} \mathbb{E}_i \left[-e^{-\gamma(x(V-P)+z_i\theta+nV)} \right]. \tag{2}$$

Timing of events. Figure 1 summarizes the timing of events. At date $t = 1$, investors observe their signals and endowments and submit trades $X_i(s_i, z_i, P)$. At date $t = 2$, the firm manager chooses the optimal level of investment $y(P)$, given the information conveyed by the equilibrium price P, to maximize the expected value of the firm subject to investment costs i.e.,

$$y(P) \equiv \arg\max_{y \in Y} \mathbb{E}[V(y)|P]. \tag{3}$$

At date 3, the firm's cash flows are realized and the risky security pays off the terminal dividend V.

[16] We let $\tau_{(\cdot)}$ denote the unconditional precision and $\sigma^2_{(\cdot)}$ the unconditional variance of all random variables.

Figure 1: Timeline of events

$t = 1$	$t = 2$	$t = 3$
Investor i observes s_i, z_i submits trades X_i Asset price is P	Firm chooses $y(P)$	Firm pays off $V(y) = A + y\theta - c(y)$

Equilibrium. An equilibrium consists of a price P, trades $\{X_i\}$, and an investment rule $y(P)$ such that (i) trade X_i maximizes investor i's expected utility, given the price and investment rule, (ii) the investment rule $y(P)$ maximizes firm value, and (iii) the equilibrium price P reflects the conditional expectation of the firm's cash flows, i.e.,

$$P = \mathbb{E}[V(y(P))|P]. \tag{4}$$

3.1 Discussion of Assumptions

The assumption that the non-tradeable asset is perfectly correlated with the payoff of the new project θ can be relaxed without changing our results. Similarly, the investment decision can be generalized without qualitatively changing our results. In Section 6.1, we show that the equilibrium characterization in Proposition 1 obtains with the appropriate modifications when we generalize (i) investment opportunities and (ii) endowment payoffs. Moreover, the key tension between investment efficiency and hedging obtains naturally in this setting.

The positivity of the investment level (i.e., we do not consider dis-investment) can be relaxed by re-interpreting y. Suppose that the firm begins with y_0 invested in the project and must make an *incremental* investment/dis-investment decision $\hat{y}$ with $\hat{y} \geq -y_0$. Then, defining $y = y_0 + \hat{y} \geq 0$ as the total amount invested in the project and re-normalizing the cost function so that $c(y_0) = 0$, with the understanding that negative costs correspond to the proceeds from disinvestment, our analysis goes through essentially unchanged but with y interpreted at the total amount invested in the project, including any pre-existing investment.[17] The assumption that the firm's assets in place and the project are uncorrelated can be relaxed in a similar manner. If they are instead positively correlated, by projecting the firm's assets in place onto θ, we can write $A = \lambda\theta + \varepsilon_A$ for $\lambda > 0$ and $\mathbb{C}(\varepsilon_A, \theta) = 0$. Thus, re-defining the firm's assets in place to equal ε_A, this case is equivalent to the firm

[17]Technically, this requires generalizing the firm's investment choice set $Y = \{0, 1\}$ to $Y = \{y_0, y_0 + \hat{y}\}$. However, the proofs of our results in Sections 4 and 5 readily apply to the more general case in which Y consists of any two distinct non-negative constants.

having a pre-existing investment of λ.[18]

The assumption that the equilibrium price reflects the conditional expectation of the firm's cash flow (as in (4)) makes transparent the key tension between investment efficiency and risk-sharing. One can interpret the price as being determined by a competitive, risk-neutral market maker (e.g., Kyle (1985), Hirshleifer, Subrahmanyam, and Titman (1994), Vives (1995)) who conditions on all observable public information (including the submitted demand schedules). In Section 6.2, we characterize the equilibrium in a fully general version of our model with a general investment opportunity set, and in which prices are set by market clearing, and discuss how our welfare results change due to the presence of a risk premium in the equilibrium asset price.

The mere presence of assets in place is not qualitatively important for our results. Indeed, in the baseline model, the hedging channel would become more stark if there were no assets in place ($\tau_A \to \infty$). In such a case, in the event of investment ($y > 0$) the stock would be a perfect hedge for θ risk, while in the event of no investment it would be useless for hedging. This contrasts with the $\tau_A < \infty$ case, in which the stock is an imperfect hedge in the event of investment and useless otherwise.[19] Finally, in our benchmark model, investors have private information about the new project being considered, and their endowment shocks are correlated with the cash flows of this project. In Section 6.3, we discuss how our results would change if investors' private information and/or hedging needs were instead correlated with the firm's assets in place.

4 Equilibrium

Conjecture a threshold $\bar{s}_p$ such that the date 1 price P reveals a linear signal about θ of the form:

$$s_p = \theta - \frac{1}{\beta} Z, \tag{5}$$

when $s_p \geq \bar{s}_p$ and reveals noting otherwise. Here β is a constant that is determined in equilibrium. Denote the precision of the signal s_p by $\tau_p = \mathrm{var}(s_p|\theta)$, which will be determined

[18] Gao and Liang (2013) study a feedback-effects model with an asset in place and growth opportunity that are subject to the same underlying shock. This can be accommodated in our model by letting $\tau_A \to \infty$ and setting the pre-existing investment level to equal the size of the firm's assets in place. As in their model, in this case, an increase in the firm's growth opportunity relative to its asset in place raises the importance of feedback in driving the firm's expected cash flows.

[19] It is worth noting that if there are no assets in place, $\tau_A \to \infty$, then in states in which $y = 0$ investors face no risk and so effectively trade as if they are risk-neutral i.e., their demand for the risky asset is unbounded for any 'off equilibrium' price $P \neq \overline{A}$. This poses no difficulties for the equilibrium construction or economic implications.

in equilibrium.

In order for our conjecture to be confirmed, $\bar{s}_p$ must be such that the manager finds it optimal to invest when she infers s_p from the price and to not invest when she infers $s_p < \bar{s}_p$. That is, $\bar{s}_p$ must be such that $\mathbb{E}[\theta|s_p < \bar{s}_p] < c$ and $\mathbb{E}[\theta|s_p] \geq c$ for $s_p \geq \bar{s}_p$. The unique continuous equilibrium of this class is characterized by the threshold at which the investment has exactly zero NPV:

$$\mathbb{E}[\theta|s_p = \bar{s}_p] = \frac{\tau_\theta \mu_\theta + \tau_p \bar{s}_p}{\tau_\theta + \tau_p} = c. \tag{6}$$

We focus on this equilibrium threshold in the following analysis.[20] Denote the optimal investment choice by $y^*(s_p) = \mathbf{1}_{\{s_p \geq \bar{s}_p\}}$ and the maximized firm value by $V^* = V(y^*(s_p))$.

Given the joint normality of all random variables, the conditional distribution of θ given (s_i, z_i, n_i, s_p) is itself normal with mean and variance

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{7}$$

where the coefficients b_s, b_p and b_z, and the precision τ, are determined in equilibrium. Note that since all investors are symmetrically informed, $\mathbb{V}_i[\theta]$ is identical across all investors. Given the (conjectured) investment rule $y = y^*(s_p)$, investor i's optimal trade X_i maximizes (2) and is given by:

$$X_i = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau} z_i\right) - c - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau} y^2\right)} - n. \tag{8}$$

Finally, the equilibrium price P is determined by

$$P = \mathbb{E}[V(y^*(s_p))|s_p] = \begin{cases} \mu_A + \mathbb{E}[\theta|s_p] - c & \text{if } s_p \geq \bar{s}_p \\ \mu_A & \text{if } s_p < \bar{s}_p. \end{cases} \tag{9}$$

Given the feedback effect from the equilibrium price to the investment rule, the price is no longer a linear signal about fundamentals. However, as in Breon-Drish (2015), we can verify that it is an invertible function of s_p, which is a linear signal about fundamentals.

[20] There also exists a continuum of equilibria for which price exhibits a discontinuity in s_p. Let $\hat{s}_p$ denote the unique solution to $\mathbb{E}[\theta|s_p < \hat{s}_p] = c$. Then, there is an equilibrium in which the manager invests if and only if $s_p > T$ for any threshold $T \in [\bar{s}_p, \hat{s}_p]$, where $\bar{s}_p$ is defined in equation (6). As such equilibria lead to strictly less investment relative to the continuous equilibrium we study, feedback's effect on both investment efficiency and welfare is attenuated relative to the equilibrium we study. Details are available upon request.

Proposition 1. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let*

$$\beta^{SUB} \equiv \frac{1}{2\tau_\zeta}\left(\gamma - \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right), \quad \textit{and} \quad \beta^{COM} \equiv \frac{1}{2\tau_\zeta}\left(\gamma + \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right). \tag{10}$$

There exists a equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that
(i) the investment rule is $y^*(s_p) = \mathbf{1}_{\{s_p \geq \bar{s}_p\}}$ where the threshold $\bar{s}_p$ is given by (6),
(ii) the optimal demand is characterized by (7) and (8), and
(iii) the equilibrium price is given by

$$P(s_p) = \mu_A + y^*(s_p)\left(\mathbb{E}[\theta|s_p] - c\right), \tag{11}$$

where $s_p = \theta - \frac{1}{\beta}Z$, and $\mathbb{E}[\theta|s_p]$ follows from Bayesian updating for normal distributions, as characterized in the appendix.

As in Ganguli and Yang (2009) and Manzano and Vives (2011), the financial market features two possible equilibria corresponding to the two solutions of β. This multiplicity arises because investors' beliefs about fundamentals depend on their endowment shocks z_i. We focus on the equilibrium in which information acquisition is a substitute (i.e., $\beta = \beta^{SUB}$) for our comparative statics results because it has (i) more intuitive properties, and (ii) is more stable.[21] In particular, an increase in the precision of private information τ_ε decreases β and so increases the informativeness of the price signal s_p in this equilibrium. However, it is important to note that our main results (i.e., Propositions 1 and Proposition 4) hold for both equilibria.

Note also that the investor demand function can be decomposed explicitly into three trading motives, which helps clarify the role of the real investment decision in determining the hedging properties of the stock. Specifically, each investor desires to speculate on her private information, to hedge her endowment of the tradeable stock, and to hedge her endowment of the non-tradeable payoff:

$$X_i = \underbrace{\frac{\mathbb{E}_i[V] - P}{\gamma \mathbb{V}_i(V)}}_{\text{Speculative trading}} - \underbrace{n}_{\text{Hedging stock endowment}} - \underbrace{\frac{\frac{\gamma}{\tau} z_i}{\gamma \mathbb{V}_i(V)}}_{\text{Hedging non-tradeable endowment}} \tag{12}$$

The speculative component takes the standard mean-variance form, and the trader optimally sells her entire share endowment to hedge her initial exposure to the stock itself. The final component, which captures the investor's desire to hedge the nontradeable payoff, illustrates the importance of the firm's investment decision for hedging. The non-tradeable hedging

[21]See Manzano and Vives (2011) for the notion of stability and corresponding arguments.

term can be written as

$$\frac{y\frac{\gamma}{\tau}z_i}{\gamma\mathbb{V}_i(V)} = \frac{\mathbb{C}_i(\theta,V)}{\mathbb{V}_i(V)}z_i = Corr_i(\theta,V)\sqrt{\frac{\mathbb{V}_i(\theta)}{\mathbb{V}_i(V)}}z_i. \tag{13}$$

Hence, holding fixed the riskiness of θ and V, the investor naturally uses the asset more intensively for hedging when it is more highly correlated with the payoff, θ, on her exposure. Since

$$Corr_i(\theta,V) = \sqrt{\frac{y^2\frac{1}{\tau}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}}, \tag{14}$$

it is clear that the asset is more effective for hedging θ risk, and the trader hedges more aggressively when investment y is higher or the assets in place are less risky ($\tau_A \uparrow$).

5 Investment Efficiency and Welfare

In this section, we characterize how feedback from market prices to investment decisions affects investment efficiency and welfare. First, note that without feedback, the manager would invest in the project if and only if the ex-ante net present value of the project is positive i.e., if $\mu_\theta \geq c$. Let us denote the no-feedback investment rule by $y_{NF} = \mathbf{1}_{\{\mu_\theta \geq c\}}$.

A natural measure of investment efficiency is the expected value of the firm under the chosen investment rule. Feedback from market prices to investment decisions always improves investment efficiency in our setting, as summarized by the following result.

Proposition 2. *The expected value of the firm is higher with feedback than without, i.e.,*

$$\mathbb{E}[V(y_{NF})] \leq \mathbb{E}[V(y(s_p))].$$

Moreover, in the β^{SUB} equilibrium, the increase in expected value with feedback increases with the precision of private signals and the precision of endowment shocks τ_ζ, but decreases with γ.

The first part of the result follows from the observation that managers can always choose to ignore the information in prices even if they can observe it.[22] The second part follows from the fact that the increase in expected value increases with the posterior precision of the managers' beliefs about θ.

[22]It is important to note that the comparison is across equilibria, and consequently, accounts for the fact that the equilibrium prices are different with and without feedback.

In our setting, welfare can be measured by the average ex-ante expected utility of investors in the economy i.e.,

$$W = \int_i \mathbb{E}\left[W_i\right] di.$$

Since all investors are ex-ante symmetric, this is equivalent to the unconditional expected utility of investors i.e.,

$$W(y) = \mathbb{E}[W_i] = \mathbb{E}\left[-e^{-\gamma(W_0+(X_i+n)(A+y(\theta-c))-X_iP+z_i\theta)}\right], \tag{15}$$

where the argument y emphasizes the dependence of welfare on the investment rule. We will also write $W(y|s_p)$ to denote the conditional expected utility $\mathbb{E}\left[U_i|s_p\right]$.

The following result characterizes the conditional expected utility of an arbitrary trader i, given s_p, and is a special case of Corollary 1 in the Appendix in which the nontradeable is perfectly correlated with θ.

Lemma 1. *Consider an arbitrary investment rule $y(s_p)$, with associated asset value $V(y(s_p))$. Suppose that the asset price is characterized as the conditional expected cash flow $P = \mathbb{E}[V|P]$. Then the conditional expected utility given $\mathcal{F}_R \equiv \sigma(s_p)$ is*

$$\begin{aligned} W(y|s_p) = -&\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2}\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((\theta,z_i)|V-P)\right|^{-1/2} \\ &\times \exp\left\{-\gamma\mathbb{E}_R[V]n - \frac{1}{2}\begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix}'\begin{pmatrix} \mathbb{V}_R(\theta|V-P) & \mathbb{C}_R(\theta,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,\theta|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1}\begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix}\right\}, \end{aligned} \tag{16}$$

where $\mathbb{E}_R$ and $\mathbb{V}_R$ denote the conditional expectation and variance given $\mathcal{F}_R$.

This expression in Lemma 1 presents welfare in an intuitive form that is amenable to analytical calculations. There are four components to expected utility. The first, $\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|$, is standard and captures investor i's utility gain from speculative trading on private information. The second, $\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((\theta,z_i)|V-P)\right|^{-1/2}$ captures the 'residual' endowment risk to which the investor is exposed after hedging. That is, endowment risk that is uncorrelated with the return $V-P$ on the tradeable asset. The third term, $\gamma\mathbb{E}_R[V]n$ represents the expected value of the investor's share endowment and captures her concern for value maximization. The fourth and final term $\begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix}'\begin{pmatrix} \mathbb{V}_R(\theta|V-P) & \mathbb{C}_R(\theta,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,\theta|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1}\begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix}$ captures the expected value of the 'residual' endowment shock to which the investor is exposed after hedging.

It is illustrative to compare the above expression for expected utility to the analogous $\mathcal{F}_R$ conditional expected utility in the event that the manager never invests $y \equiv 0$. In this case, the asset payoff does not depend on θ and therefore the trader cannot not engage in speculative trading, nor can she hedge her nontradeable exposure. It is easy to confirm that the expected utility in this case is

$$\mathbb{E}_R[-e^{-\gamma(nP+z_i\theta)}] = -\left| I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix} \mathbb{V}_R((\theta, z_i)) \right|^{-1/2} \tag{17}$$

$$\times \exp\left\{ -\gamma \mathbb{E}_R[V]n - \frac{1}{2} \begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(\theta) & \mathbb{C}_R(\theta,z_i)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,\theta)+\frac{1}{\gamma} & \mathbb{V}_R(z_i) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[\theta] \\ \mathbb{E}_R[z_i] \end{pmatrix} \right\}. \tag{18}$$

This expression takes the same form as that in Lemma 1, but reflects that the trader does not enjoy any speculative trading profit and remains fully exposed to the risk of her nontradeable endowment.

Using the characterization in Lemma 1, we establish our first main result, which shows that the equilibrium investment rule $y^*(s_p)$ invests (weakly) less state-by-state than the welfare maximizing s_p-dependent investment rule.

Proposition 3. *The investment rule $y(s_p)$ that maximizes expected investor utility $W(y)$ is (weakly) higher state-by-state than the equilibrium investment rule $y^*(s_p)$.*

The proposition implies that the welfare-maximizing investment rule has a lower threshold for investment than the firm-value maximizing rule. This highlights the key tension between investment efficiency and welfare in our setting. Intuitively, the manager does not invest for sufficiently low realizations of the signal s_p (i.e., when $s_p < \bar{s}_p$). While this is efficient from the perspective of maximizing firm value (i.e., rejecting projects with negative NPV, conditional on s_p), it limits the ability of investors to hedge their exposure to θ by trading the stock. Hence, the welfare-maximizing rule accepts some projects with slightly negative NPV because they nevertheless allow investors to hedge θ risk.

In our benchmark model, this tension between investment efficiency and welfare is particularly stark because when the manager does not invest, the firm's cash flows do not depend on θ and so the stock is useless for hedging. However, as we illustrate in Section 6.1, this effect arises more generally in settings where the investment decision is not binary. In general, lower investment y reduces the sensitivity of the firm's cash flows, $V = A + y\theta - c(y)$, to θ, which makes the stock a less effective hedge for investors' endowment risk.

Using Propositions 2 and 3, one can show the second main result of our analysis.

Proposition 4. *(i) Suppose the ex-ante NPV of the project is negative i.e., $\mu_\theta < c$. Then, welfare is higher with feedback than with no feedback i.e., $W(y^*(s_p)) \geq W(y_{NF})$.*
(ii) Suppose the ex-ante NPV of the project is positive i.e., $\mu_\theta \geq c$, and the per-capita endowment of shares n is sufficiently small. Then, welfare is higher without feedback than with feedback i.e., $W(y^*(s_p)) \leq W(y_{NF})$.

To gain some intuition for this result, note that the firm's investment rule affects welfare through two channels. First, when the firm decides to invest in the project, it affects the cash flows that investors receive through their endowment n of shares. We refer to this as the **valuation channel**. Importantly, feedback from market prices to the firm's investment decisions increases the expected value of the firm and so increases welfare through this channel.

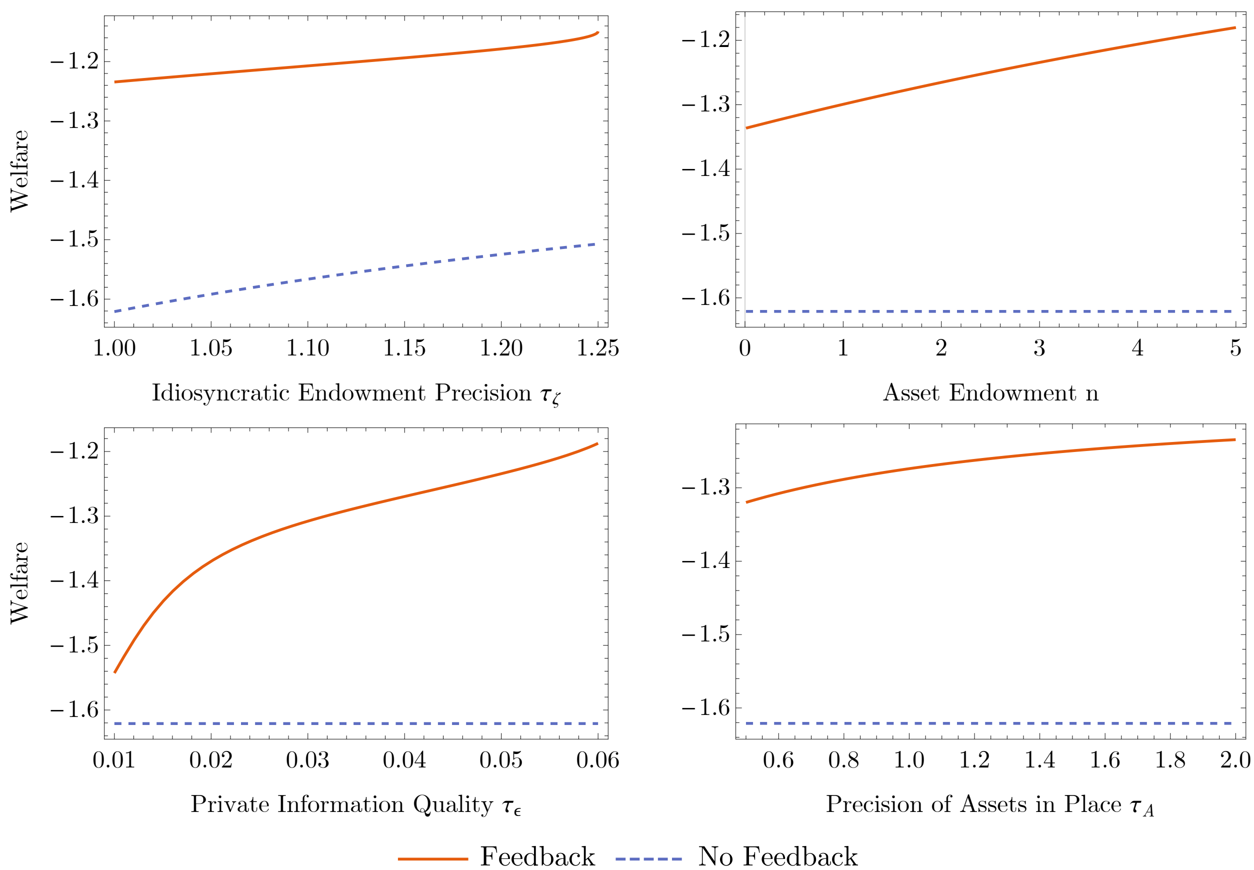
Second, the firm's choice of whether or not to invest in the project impacts investors' ability to use the risky security to hedge their exposure to θ risk. We refer to this as the **hedging channel**. This channel implies that all else equal, investors are better off when the firm invests in the project than when it does not.

When the ex-ante NPV of the project is negative, the no-feedback investment rule leads to no investment in the project. In this case, feedback from market prices to the investment rule improves welfare through both the valuation and hedging channels. Investors are better off with feedback because (i) the expected value of their firm is higher, and (ii) they are able to hedge their θ risk by trading the risky security.

However, when the ex-ante NPV of the project is positive, the no-feedback investment rule leads the firm to always invest. In this case, feedback implies the firm does not invest for sufficiently low realizations of s_p, and so can have an ambiguous impact on welfare. On the one hand, feedback improves the expected value of the firm and so increases welfare via the valuation channel. On the other hand, because it leads to no-investment in some states, feedback reduces the ability of investors to use the risky security as a hedge, and so reduces welfare via the hedging channel. When the per-capita endowment of shares n is large relative to hedging needs $(1/\tau_\zeta)$, the investment channel dominates and feedback improves welfare. However, when n is sufficiently small, the hedging channel dominates and feedback can reduce welfare.

Figures 2 and 3 provide an illustration of the above result, and depict how the difference in welfare with and without feedback depends on the underlying parameters for the β^{SUB} equilibrium. When the project is negative NPV, Figure 2 verifies that feedback always improves welfare. Moreover, as the plots illustrate, the improvement in welfare increases in the precision of private information τ_ε and the per-capita asset endowment n, but decreases in the precision of endowment shocks τ_ζ and the prior precision about fundamentals τ_θ.

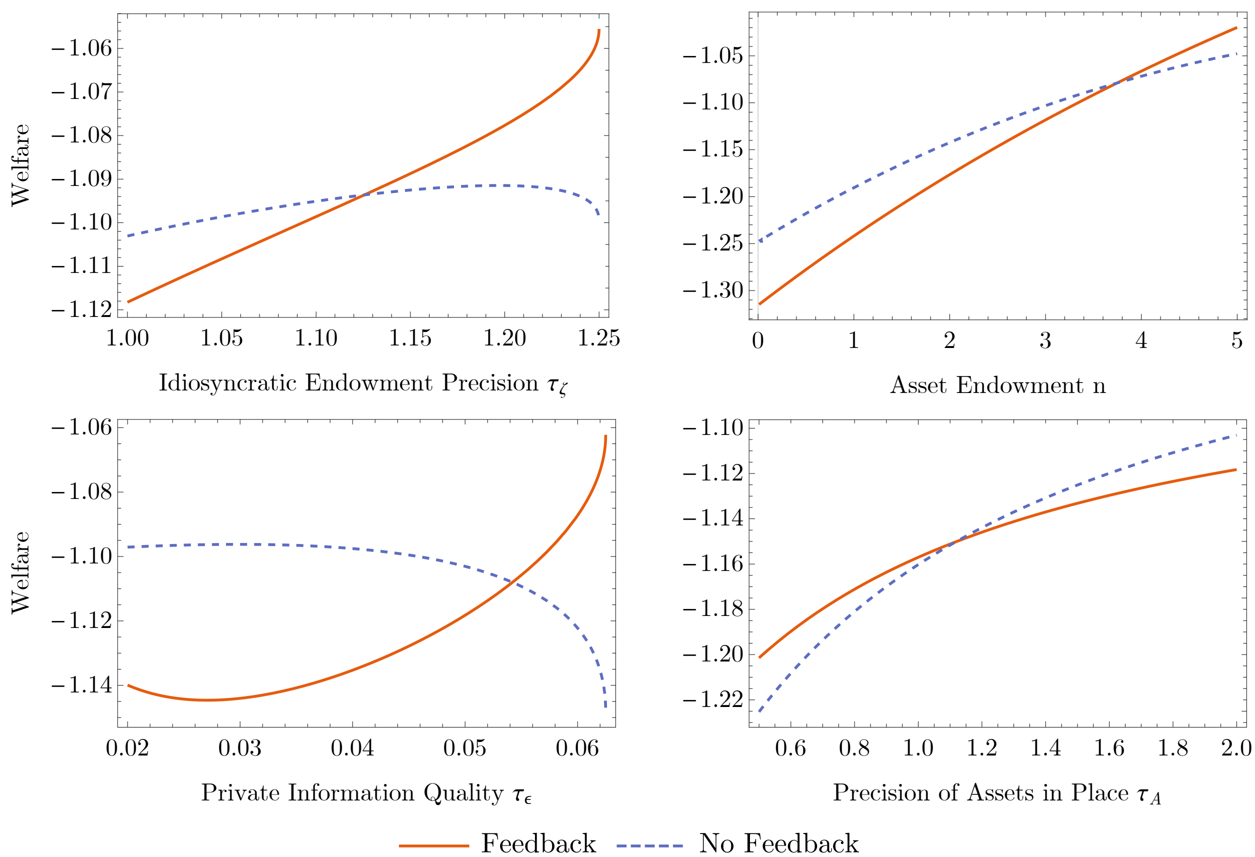
Figure 2: This figure plots investor welfare as a function of the underlying parameters when the project has negative NPV ex ante (i.e., $\mu_\theta < c$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.5; c = 0.55$; $\gamma = 0.5$; $\tau_\theta = 1$; $\tau_\varepsilon = 0.05$; $\tau_\zeta = 1$; $\tau_z = 1$; $n = 3$; $\mu_A = 0$; $\tau_A = 2$; $\mu_Z = 0$.



These results are intuitive and follow from the fact that feedback improves welfare through both the valuation and hedging channels.

When the project has positive NPV, however, the relative strength of the valuation and hedging channels determines the direction of these comparative statics. As Figure 3 illustrates, feedback increases welfare only when either endowment precision or the asset endowment is high. This follows because an increase in endowment precision attenuates the hedging channel, while an increase in the asset endowment raises the valuation effect. Moreover, feedback increases welfare only when private information quality τ_ε is high. This is a consequence of the Hirshleifer (1971) effect: when τ_ε is high, prices accurately reflect the firm's true value, such that investors cannot use the stock to share risk. Finally, the impact of feedback on welfare is positive when assets in place are imprecise. Intuitively, when the firm's assets in place drive most of the firm's value, even if the firm invests in the

Figure 3: This figure plots investor welfare as a function of the underlying parameters when the project has positive NPV ex ante (i.e., $\mu_\theta > c$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.5; c = 0.45;\ \gamma = 0.5;\ \tau_\theta = 1;\ \tau_\varepsilon = 0.05;\ \tau_\zeta = 1;\ \tau_z = 1;\ n = 3;$ $\mu_A = 0;\ \tau_A = 2;\ \mu_Z = 0$.



new project, its usefulness as a hedge is limited.

6 Robustness and extensions

6.1 General investment opportunities and endowment shocks

Our baseline setting considers only the situation in which the manager must make a discrete choice of whether to pursue the project, and in which investors' non-tradeable endowment payoff is perfectly correlated with the project payoff. In this section, we generalize our results to allow arbitrary investment opportunities $Y \subseteq [0, \infty)$, with increasing investment cost function $c(y)$, and imperfectly correlated endowment payoffs $U = \theta + \xi$, with $\xi \sim N(0, \tau_\xi)$.

The following Proposition formally characterizes optimal investment and the financial market equilibrium in the general case.[23]

Proposition 5. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let β^{SUB} and β^{COM} be given by (10). Then, there exists a equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that*
(i) the investment rule is

$$y^*(s_p) = \begin{cases} 0 & s_p < \bar{s}_p \\ \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) & s_p \geq \bar{s}_p \end{cases} \tag{19}$$

where the threshold $\bar{s}_p = \inf\{s_p : \max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$,
(ii) the optimal demand is

$$X_i = \frac{\mathbb{E}_i[V(y)] - \mathbb{C}_i(V(y),\theta)z_i - P}{\gamma\mathbb{V}_i(V(y))} - n = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau}z_i\right) - c(y) - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau}y^2\right)} - n, \tag{20}$$

where

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{21}$$

and
(iii) the equilibrium price is given by

$$P(s_p) = \begin{cases} \mu_A & s_p < \bar{s}_p \\ \mu_A + y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p)) & s_p \geq \bar{s}_p, \end{cases} \tag{22}$$

where $s_p = \theta - \frac{1}{\beta}Z$ and $\mathbb{E}[\theta|s_p]$ follows from Bayesian updating for normal distributions, as characterized in the appendix.

Importantly, the key tension between the valuation and hedging channels remains in this setting, and a general analogue of Proposition 3 holds in this setting, which we record here.

Proposition 6. *Consider the setting with general investment opportunities and endowment payoffs introduced above. The investment rule $y(s_p)$ that maximizes expected investor utility $W(y)$ is (weakly) higher state-by-state than the equilibrium investment rule $y^*(s_p)$.*

[23] To avoid unnecessary technical detail, we assume that the investment opportunity set Y and cost function $c(y)$ are such that for each s_p there exists a unique, finite y that solves $\max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y)$. That is, the optimal investment correspondence is single-valued and finite. Sufficient conditions to ensure this are straightforward (e.g., $c(y)$ strictly convex, and $Y = [0,\infty)$ or Y compact). A complete enumeration of necessary and sufficient conditions would take us too far afield and add little economic insight, so we do not pursue it here.

The above result implies that by reducing investment in some states, feedback from prices can lead to lower welfare even though it improves investment efficiency. This will be the case when n is sufficiently small, and the production technology is such that feedback reduces investment.

6.2 Market clearing price

Our benchmark analysis, and its generalization in Section 6.1, assumes that the price reflects the conditional expected value of cash flows, given publicly available information. We can establish existence of equilibrium (i.e., establish the analog to Proposition 1) in a setting where prices are determined by market clearing as in classic noisy rational expectations models. However, due to the presence of the investment level in the risk premium, we must generally impose additional parameter restrictions to ensure that the price function is monotone, as required in any equilibrium. Alternatively, if one assumes that investors are able to condition on both the equilibrium price and the aggregate demand schedule (analogous to conditioning order flow in a market-order model), no such additional restrictions are needed.[24]

The following Proposition formally characterizes optimal investment and the financial market equilibrium in the general case.

Proposition 7. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let β^{SUB} and β^{COM} be given by (10). An equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that*
(i) the investment rule is

$$y^*(s_p) = \begin{cases} 0 & s_p < \bar{s}_p \\ \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) & s_p \geq \bar{s}_p \end{cases} \tag{23}$$

where the threshold $\bar{s}_p = \inf\{s_p : \max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$,
(ii) the optimal demand is

$$X_i = \frac{\mathbb{E}_i[V(y)] - \mathbb{C}_i(V(y),\theta)z_i - P}{\gamma\mathbb{V}_i(V(y))} - n = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau}z_i\right) - c(y) - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau}y^2\right)} - n, \tag{24}$$

[24] Practically speaking, conditioning on the aggregate demand schedule is equivalent to conditioning on the limit order book for the stock, which reflects aggregate investor demand at each possible price level, as there are no market order traders in the model. Similar assumptions are common in the literature. For instance, the equilibrium definitions in Dow and Rahi (2003) (p.442), Goldstein and Guembel (2008) (p.152), and Edmans, Goldstein, and Jiang (2015) (p.3775) allow the manager's investment strategy to depend directly on order flow.

where

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{25}$$

and
(iii) the equilibrium price is given by

$$P(s_p) = \begin{cases} \mu_A - \gamma \frac{1}{\tau_A} n & s_p < \overline{s}_p \\ \mu_A + y^*(s_p)\left(b_0 + (b_s + b_p)s_p\right) - c(y^*(s_p)) - \gamma\left(\frac{1}{\tau_A} + (y^*(s_p))^2 \frac{1}{\tau}\right) n & s_p \geq \overline{s}_p, \end{cases} \tag{26}$$

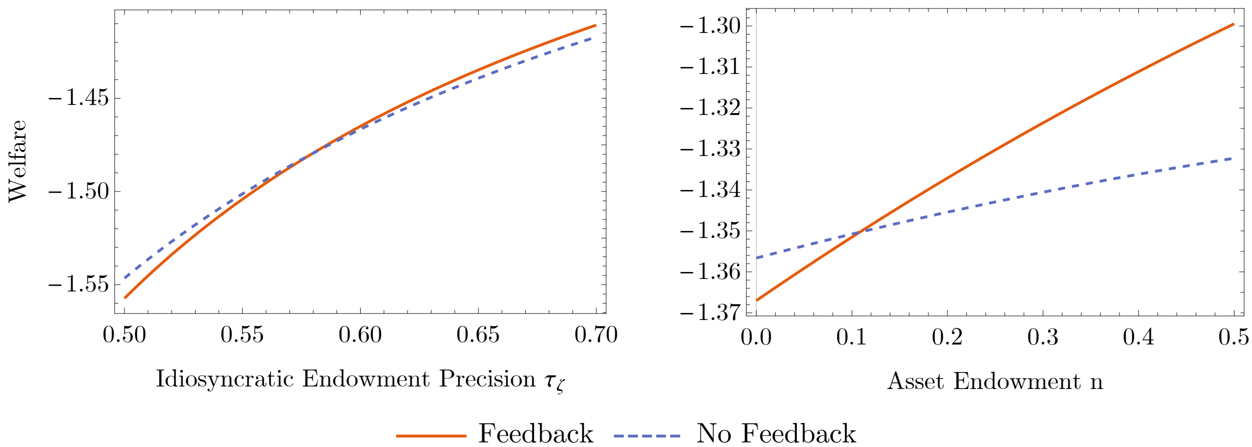
where $s_p = \theta - \frac{1}{\beta}Z$, and b_0, b_s, b_p, b_z, and τ are characterized in the appendix, exists if the function P so defined is strictly increasing in s_p in the region $s_p \geq \overline{s}_p$.

When prices are set by market clearing, the ranking of welfare, analogous to Proposition 4, becomes more challenging. This is because in addition to the price aggregating the private information across investors, it also reflects a risk premium (which depends on the per-capita supply of the asset n). The risk premium affects welfare through two additional channels. First, because the price is discounted (relative to expected cash flows) and investors have a per-capita endowment of the stock, an increase in the risk premium lowers wealth and consequently, expected utility. This implies that the investment level that maximizes investor utility need not always be higher than the equilibrium investment level (i.e., Proposition 3 need not hold).

Second, the risk premium affects the cost that investors' incur to use the risky asset as a hedge. Specifically, an investor who would like to sell (buy) the risky asset to hedge their z_i shock has to pay more (less) when the price is discounted by the risk premium. This can lead to greater disparity in wealth across investors, reducing average investor utility. As such, while we expect the valuation and hedging channels to continue to play a role in this environment, the overall welfare implications are more nuanced due to the risk premium component.

Figure 4 provides an illustration of the ranking between feedback and no-feedback when investment is subject to a quadratic cost and the price is determined by market clearing. For the parameters considered, we note that the results of our benchmark analysis continue to hold in this setting — welfare is lower with feedback when hedging channel is important (e.g., τ_ζ is low), and the valuation channel is not (n is small). This suggests that our benchmark results are robust to the extensions considered in Sections 6.1 and 6.2, even though an analytic characterization of this ranking is not tractable in the latter case.

Figure 4: This figure plots investor welfare as a function of the underlying parameters when the manager selects non-negative investment ($Y = [0, \infty)$) under the market-clearing price. The manager is subject to a quadratic cost ($c(y) = c * y^2$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.25; c = 0.5$; $\gamma = 0.5$; $\tau_\theta = 1$; $\tau_\varepsilon = 0.05$; $\tau_\zeta = 1$; $\tau_z = 1$; $n = 0.25$; $\mu_A = 0$; $\mu_Z = 0$; and τ_A is arbitrarily large.

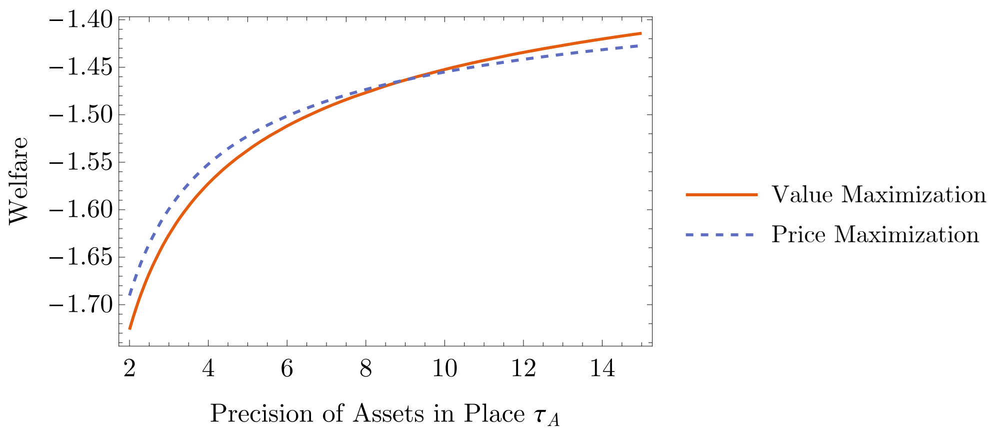


More broadly, the disconnect between the manager's optimal investment decision and the welfare-maximizing decision stems from the fact that the manager is incentivized to maximize the firm's fundamental value, which ignores both investor risk aversion and the risk-sharing role of the firm's stock. These two forces trade-off: the fact that the manager ignores the risk premium (risk-sharing) tends to push them towards investing more (less) than the welfare-maximizing investment level, all else equal. We have also analyzed a version of the model in which the manager maximizes price rather than expected value. In this case, the equilibrium takes the same form as in Proposition 7, where the investment function $y^*(s_p)$ is simply replaced by an analogous price-maximizing investment function $y_P^*(s_p)$, which also accounts for the risk premium.[25] In this case, since the manager maximizes the stock price, their investment is disciplined by its effect on the risk premium and can be lower than under "value maximization."

Under the same specification and parameter choices as Figure 4, Figure 5 compares welfare under value maximization and price maximization. The plots suggest that investor welfare is greater with value maximization only when τ_A is sufficiently large. Conditional on investment, the firm's cash flows are more sensitive to θ when τ_A is high, which makes the stock more useful for hedging. This implies that the relative "over-investment" that results from value maximization increases welfare in this region. On the other hand, when τ_A is

[25] It is worth noting that in the price maximization case the equilibrium price function is guaranteed to be monotone in s_p and so it is not necessary to assume that investors and the manager can condition on the aggregate demand schedule.

Figure 5: This figure plots investor welfare as a function of τ_A in both the cases in which the manager maximizes price and expected firm value. The manager selects non-negative investment ($Y = [0, \infty)$) under the market-clearing price. Moreover, the manager is subject to a quadratic cost ($c(y) = c * y^2$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.25; c = 0.5;\ \gamma = 0.5;\ \tau_\theta = 1;\ \tau_\varepsilon = 0.05;\ \tau_\zeta = 0.6;\ \tau_z = 1;\ n = 1.5;\ \mu_A = 0;\ \mu_Z = 0$.



low, welfare can be higher under price maximization.

An implication of these results is that optimal managerial incentives should vary across firms, and depend on the relative volatility of their assets in place to their growth options (new projects). For firms with relatively more volatile assets in place, welfare may be higher when the manager is incentivized to maximize the expected stock price rather than expected value. While a complete analysis of the optimal compensation contract for the manager is beyond the scope of the current paper, our analysis uncovers a novel tradeoff between (short-term) price-based and (long-term) value-based compensation.

6.3 Hedging and information about assets in place

Our benchmark analysis focuses on the natural case in which investors have private information about the new project, and have risk exposures that are correlated with the cash flows of the new project. The potential tension between the valuation and hedging channels is clearest in this case. While we expect these underlying mechanisms to be in effect for other specifications, their overall implications for how feedback affects welfare may be different.

For instance, suppose investors have private information about the cash flows of the new project but their endowment shocks are correlated with assets in place (i.e., z_i units of exposure to a shock that is correlated with A and not θ). In this case, investment in the new project makes the stock less useful for risk sharing, as the resulting exposure to θ makes the security a worse hedge for A risk. As a result, in contrast to the benchmark model, one would expect that feedback would always improve welfare for positive NPV projects (by reducing investment in the new project for some states), while it may reduce welfare for negative NPV projects.

If both investors' private information and endowment shocks relate to assets in place as opposed to the new project, then there is no direct role for market feedback since we have assumed the manager's investment decision does not affect cash flows from the assets in place. Finally, while we expect similar forces to be at play in a richer model that allows for arbitrary correlations between assets in place and new project cash flows, and for investor information and risk exposures to both components, this is beyond the scope of the current paper.

7 Model Implications

Our model highlights that higher investment efficiency does not always translate to higher welfare. This complements the insights of Bond et al. (2012) who make the point that more informative prices (i.e., higher forecasting price efficiency) do not always lead to more efficient investment decisions (i.e., higher revelatory price efficiency). Moreover, while feedback from prices always improves investment efficiency in our setting, it need not increase welfare. As such, our model cautions against exclusively focusing on measures of investment efficiency when evaluating policy changes, especially when such changes affect the ability of investors to hedge their exposures to shocks. Specifically, the analysis in Section 5 suggests that investment efficiency measures are most likely to give a misleading picture of welfare for small firms pursuing risky, yet (ex-ante) positive NPV projects.

Our results have clear predictions on when feedback improves welfare and when it harms it. In our model, feedback is always socially beneficial for risky projects with low ex-ante probabilities of success (i.e., negative ex-ante NPV with low τ_θ). Feedback is also beneficial for ex-ante profitable projects with relatively low uncertainty, especially when such projects are not sensitive to systematic or macroeconomic risks (high τ_ζ). Moreover, large firms (high n) with well-informed investors (high τ_ε) benefit from feedback. In contrast, feedback tends to reduce welfare for projects that are ex-ante attractive but risky, and that are correlated with non-tradeable risks (e.g., human capital exposures or climate-change risk). These effects are amplified when investors are not well-informed, and per-capita ownership is low (i.e., the firm is small).

These results speak to the recent debate that distinguishes between shareholder value and shareholder welfare (e.g., Hart and Zingales (2017)). The above results suggest that investors would prefer that managers allow price information to feed back into investment decisions under some circumstances, but not others, even though feedback always increases shareholder value (in expectation). In practice, implementing such contingent feedback behavior is not likely to be contractually feasible, especially since it is difficult for the manager to commit to ignore (or use) information. However, the existing literature on cross-firm variation in feedback effects provides some guidance on how shareholders can encourage welfare improving investment decisions. For instance, Bond and Goldstein (2015) suggest managers who are overconfident in the precision of their own private information are more likely to ignore the information in price. Our results suggest that, all else equal, hiring overconfident managers may improve shareholder welfare for small firms facing risky, ex-ante attractive projects that are correlated with non-tradeable risks, especially when investors are not well informed.

On the other hand, certain corporate decisions can strengthen feedback effects. Foucault and Gehrig (2008) and Foucault and Frésard (2012) show that cross-listing improves firms' price informativeness and consequently increases their investment sensitivity to price. Jayaraman and Wu (2020) show that voluntary capital expenditure disclosures induce information acquisition and stimulate feedback. Similarly, Dye and Sridhar (2002) suggest that pre-announcing potential investment decisions may encourage feedback, consistent with Luo (2005) and Edmans et al. (2012), who show that, following publicly announced M&A deals, managers place heightened weight on prices. Our analysis implies that such activities are likely to be welfare improving when investors are well-informed, the firm is large, and the projects are less correlated with investors' hedging needs.

The case of M&A deals is a particularly salient application of our setting, given the effect of such decisions on both the firms' cash flows and the set of tradeable risks. One may interpret the investment in our model as the decision of the firm to pursue an acquisition, or conversely, to spin off a component of its business. Naturally, a spin-off (merger) creates (destroys) the opportunity for investors to separately hedge exposures to the two entities, such as employees in the firms' industries who desire to hedge their human-capital exposures (e.g., see Eiling (2013)). One testable implication that this mechanism is at work is that risk-factor loadings should change and trading volume should increase around a merger announcement. Moreover, our results suggest that mergers, even when value-enhancing in expectation, can reduce investor welfare when at least one of the entities involved is small and used as a hedge (and vice versa for spin-offs). Thus, while timely pre-announcements of merger discussions that are likely to go through (i.e., have ex ante positive NPV) may prevent value-decreasing mergers, they need not improve investor welfare.

Our analysis can also shed light on the welfare consequences of common managerial compensation schemes. The discussion in Section 6.2 suggests that incentivizing the manager to focus on (short-term) stock price instead of (long-run) firm value can improve welfare by causing her to partially internalize the hedging role of the firm's stock (which enters the price through the aggregate exposure Z). This represents a new channel by which shareholders may benefit from tying compensation to short-term stock prices (e.g., Bolton, Scheinkman, and Xiong (2006)) and is broadly consistent with evidence of a positive relationship between price informativeness and sensitivity of executive compensation to stock prices (Kang and Liu, 2008). Moreover, Dessaint, Foucault, Frésard, and Matray (2019) and Foucault and Frésard (2019) suggest that firms with closer peers may place greater weight on prices (in particular, peers' prices). These observations suggest that features of managerial compensation that appear to be sub-optimal from a value maximization perspective (e.g., sensitivity to short-term price performance, or peer / industry benchmarking) may be useful in improving shareholder welfare.

Our model also generates a number of policy implications. A robust conclusion of our analysis is that the firm's optimal strategy leads to under-investment relative to the socially optimal benchmark because the manager fails to account for the hedging channel. This under-investment problem is particularly severe in small firms that are considering (ex-ante) positive NPV projects which are correlated with risks that investors would like to hedge, and when there is limited feedback from prices. The model recommends that interventions that improve the 'conditional' NPV of the project and therefore stimulate investment (e.g., product-market subsidies, such as tax deductions for electric vehicles or residential solar panels) may improve welfare in such settings.

Finally, our model highlights the importance of regulations that encourage disclosure and information production about a firm's assets in place, but not necessarily about future projects and growth options.[26] This is consistent with standard accounting principles, which focus on providing information regarding assets in place, but discourage recognition of future growth options.[27] Specifically, improving the precision of private information about the new project can decrease welfare, even though it improves investment efficiency, by changing the set of traded risks.[28] On the other hand, improving information about assets in place tends to improve welfare. This is because, all else equal, reducing uncertainty about cash flows from existing assets increases the correlation with the new project, which makes the stock a more effective hedge.

[26] This is reminiscent of the observations in Goldstein and Yang (2019b) who suggest that disclosures about some components of fundamentals enhance investment efficiency, while disclosures about others reduce it.

[27] As detailed in FASB's Concept Statement 6, firms are only allowed to record assets on their balance sheets when they are the result of a past transaction.

[28] As previously discussed, this is distinct from the standard Hirshleifer (1971) effect by which more private information about θ directly reduces investors ability to share risks, even in the absence of feedback.

8 Conclusions

Our key insight is that investment decisions change the risk exposures of the firm's cash flows, and so affect investors' ability to use the stock to hedge. We explore the implications of this observation in a setting with feedback effects, where the manager conditions on the information aggregated by prices before choosing how to invest in a new project. The linkage between real investment decisions and the stock's risk exposures has important consequences: improvements in investment efficiency need not be associated with improvements in welfare. Specifically, while feedback always improves investment efficiency, it can reduce welfare, especially when small firms are choosing investment in (ex-ante) positive NPV projects which are correlated with risks that investors would like to hedge.

A further contribution of our analysis is to provide a benchmark feedback effects framework that can be extended naturally along various dimensions for future work. Immediate extensions would include generalizations to the structure of cash flows and information. For instance, allowing for correlation between assets in place and future projects would make the analysis richer, although we expect the underlying economic forces we highlight to still be in play. Similarly, it would be interesting to study how introducing multiple dimensions of fundamentals (e.g., Goldstein and Yang (2019b), Goldstein et al. (2020)) would interact with the forces we highlight. Moreover, it may be interesting to consider how information acquisition by investors and managers interact in our setting.

An extension of our model that incorporates direct preferences over the firm's investment choices (e.g., as in Pástor, Stambaugh, and Taylor (2020)) would be a natural setting to study how the environmental, social and governance (ESG) factors affect financial markets and investment choices. In such settings, the firm's investment decisions not only affect welfare via the valuation and hedging channels that we focus on, but also via a direct "externality" channel. We leave this analysis for future work.

References

Albagli, E., C. Hellwig, and A. Tsyvinski (2011). Information aggregation, investment, and managerial incentives. Technical report, National Bureau of Economic Research.

Allen, F. and D. Gale (1988). Optimal security design. *The Review of Financial Studies 1*(3), 229–263.

Allen, F., D. Gale, et al. (1994). *Financial innovation and risk sharing*. MIT press.

Asquith, P. (1983). Merger bids, uncertainty, and stockholder returns. *Journal of Financial Economics 11*(1-4), 51–83.

Attanasio, O. and S. J. Davis (1996). Relative wage movements and the distribution of consumption. *Journal of Political Economy 104*(6), 1227–1262.

Bolton, P., J. Scheinkman, and W. Xiong (2006). Executive compensation and short-termist behaviour in speculative markets. *Review of Economic Studies 73*(3), 577–610.

Bond, P., A. Edmans, and I. Goldstein (2012). The real effects of financial markets. *The Annual Review of Financial Economics is 4*, 339–60.

Bond, P. and D. Garcia (2021). The equilibrium consequences of indexing. *Forthcoming in Review of Financial Studies*.

Bond, P. and I. Goldstein (2015). Government intervention and information aggregation by prices. *Journal of Finance 70*(6), 2777–2812.

Boot, A. W. and A. V. Thakor (1993). Security design. *The Journal of Finance 48*(4), 1349–1378.

Breon-Drish, B. (2015). On existence and uniqueness of equilibrium in a class of noisy rational expectations models. *Review of Economic Studies 82*(3), 868–921.

Dasgupta, S. and N. Gao (2004). Cancelled acquisitions, market information, and corporate governance. *Market Information, and Corporate Governance (March 2004)*.

Davis, J. and N. Gondhi (2019). Learning in financial markets: Implications for debt-equity conflicts.

Dessaint, O., T. Foucault, L. Frésard, and A. Matray (2019). Noisy stock prices and corporate investment. *Review of Financial Studies 32*(7), 2625–2672.

Diamond, D. W. and R. E. Verrecchia (1981). Information aggregation in a noisy rational expectations economy. *Journal of Financial Economics 9*(3), 221–235.

Dow, J., I. Goldstein, and A. Guembel (2017). Incentives for information production in markets where prices affect real investment. *Journal of the European Economic Association 15*(4), 877–909.

Dow, J. and G. Gorton (1997). Stock market efficiency and economic efficiency: Is there a connection? *Journal of Finance 52*(3), 1087–1129.

Dow, J. and R. Rahi (2003). Informed trading, investment, and welfare. *Journal of Business 76*(3), 439–454.

Duffie, D. and R. Rahi (1995). Financial market innovation and security design: An introduction. *Journal of Economic Theory 65*(1), 1–42.

Dye, R. A. and S. Sridhar (2002). Resource allocation effects of price reactions to disclosures. *Contemporary Accounting Research 19*(3), 385–410.

Eckwert, B. and I. Zilcha (2003). Incomplete risk sharing arrangements and the value of information. *Economic Theory 21*(1), 43–58.

Edmans, A., I. Goldstein, and W. Jiang (2012). The real effects of financial markets: The impact of prices on takeovers. *Journal of Finance 67*(3), 933–971.

Edmans, A., I. Goldstein, and W. Jiang (2015). Feedback effects, asymmetric trading, and the limits to arbitrage. *American Economic Review 105*(12), 3766–97.

Eiling, E. (2013). Industry-specific human capital, idiosyncratic risk, and the cross-section of expected stock returns. *Journal of Finance 68*(1), 43–84.

Engle, R. F., S. Giglio, B. Kelly, H. Lee, and J. Stroebel (2020). Hedging climate change news. *Review of Financial Studies 33*(3), 1184–1216.

Foucault, T. and L. Frésard (2012). Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *Review of Financial Studies 25*(11), 3305–3350.

Foucault, T. and L. Frésard (2019). Corporate strategy, conformism, and the stock market. *The Review of Financial Studies 32*(3), 905–950.

Foucault, T. and T. Gehrig (2008). Stock price informativeness, cross-listings, and investment decisions. *Journal of Financial Economics 88*(1), 146–168.

Ganguli, J. V. and L. Yang (2009). Complementarities, multiplicity, and supply information. *Journal of the European Economic Association 7*(1), 90–115.

Gao, P. and P. J. Liang (2013). Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research 51*(5), 1133–1158.

Gervais, S. and G. Strobl (2021). Money management and real investment. *Working Paper*.

Goldstein, I. and A. Guembel (2008). Manipulation and the allocational role of prices. *The Review of Economic Studies 75*(1), 133–164.

Goldstein, I. and H. Sapra (2014). Should banks' stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends in Finance 8*(1), 1–54.

Goldstein, I., J. Schneemeier, and L. Yang (2020). Market feedback: Who learns what? *Unpublished working paper*.

Goldstein, I. and L. Yang (2014). Market efficiency and real efficiency: The connect and disconnect via feedback effects. *Available at SSRN 2378120*.

Goldstein, I. and L. Yang (2017). Information disclosure in financial markets. *Annual Review of Financial Economics 9*, 101–125.

Goldstein, I. and L. Yang (2019a). Commodity financialization and information transmission. In *Working Paper*.

Goldstein, I. and L. Yang (2019b). Good disclosure, bad disclosure. *Journal of Financial Economics 131*(1), 118–138.

Grossman, S. and J. Stiglitz (1980). On the impossibility of informationally efficient markets. *American Economic Review 70*(3), 393–408.

Hapnes, E. (2020). Endogenous information acquisition with feedback effects and risk-aversion. Unpublished working paper, EPFL and Swiss Finance Institute.

Hart, O. and L. Zingales (2017). Companies should maximize shareholder welfare not market value. *ECGI-Finance Working Paper* (521).

Henderson, H. and S. Searle (1981). On deriving the inverse of a sum of matrices. *SIAM Review 23*(1), 53–60.

Hirshleifer, D., A. Subrahmanyam, and S. Titman (1994). Security analysis and trading patterns when some investors receive information before others. *Journal of Finance 49*(5), 1665–1698.

Hirshleifer, J. (1971). The private and social value of information and the reward to inventive activity. *American Economic Review*, 561–574.

Jayaraman, S. and J. Wu (2020). Should I stay or should I grow? Using voluntary disclosure to elicit market feedback. *Review of Financial Studies 33*(8), 3854–3888.

Kang, Q. and Q. Liu (2008). Stock trading, information production, and executive incentives. *Journal of Corporate Finance 14*(4), 484–498.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica 53*(6), 1315–1335.

Luo, Y. (2005). Do insiders learn from outsiders? evidence from mergers and acquisitions. *Journal of Finance 60*(4), 1951–1982.

Lustig, H. and S. Van Nieuwerburgh (2006, 09). The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street. *The Review of Financial Studies 21*(5), 2097–2137.

Manzano, C. and X. Vives (2011). Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity. *Journal of Mathematical Economics 47*(3), 346–369.

Marín, J. M. and R. Rahi (1999). Speculative securities. *Economic Theory 14*(3), 653–668.

Marín, J. M. and R. Rahi (2000). Information revelation and market incompleteness. *The Review of Economic Studies 67*(3), 563–579.

Milgrom, P. and C. Shannon (1994, January). Monotone comparative statics. *Econometrica 62*(1), 157–180.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2020). Sustainable investing in equilibrium. *Journal of Financial Economics*.

Vives, X. (1995). Short-term investment and the informational efficiency of the market. *Review of Financial Studies 8*(1), 125–160.

Wang, J. (1994). A model of competitive stock trading volume. *Journal of Political Economy 102*(1), 127–168.

A Explicit expressions for investor welfare

In this section, we characterize investors' expected utilities, which is a key step in establishing the welfare results in the text. Because we will apply this result in a number of different contexts, we characterize the expected utility for an arbitrary investment rule and asset price.

First, we require an assumption to guarantee that expected utility exists and is finite.

Assumption 1. Given the investment rule y, the unconditional expected utility for a investor who refrains from trading (i.e., her autarky expected utility) is finite

$$\mathbb{E}[-e^{-\gamma(nV(y)+z_i U)}] > -\infty. \tag{27}$$

This assumption implies that the conditional expected utility given any conditioning information that we consider, is finite. Because CARA utility is bounded above, expected utility always exists but is possibly $-\infty$. To ensure that it is finite, it suffices to ensure that the expected utility, in the event that the investor does not trade is finite, since her equilibrium utility will always be weakly greater than this.

Proposition 8. *Consider an arbitrary investment rule $y(s_p)$, with associated asset value $V(y(s_p))$ and pricing rule $P(s_p)$. Consider an arbitrary investor i and define the 5×1 random vector*

$$\eta = \begin{pmatrix} V - P \\ \vec{V} \\ \vec{Z}_i \end{pmatrix}, \tag{28}$$

where $\vec{V} = \begin{pmatrix} V \\ U \end{pmatrix}$ and $\vec{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ are the vectors of the payoffs and endowments of the tradeable and non-tradeable asset. Let $\mathcal{F}_R$ be any information set that is (i) weakly coarser than $\sigma(s_p)$ and (ii) under which η is conditionally normally distributed (e.g., $\mathcal{F}_R = \sigma(s_p)$ satisfies these conditions). Finally, let $\mu_{\eta|R}$ and $\Sigma_{\eta|R}$ denote the conditional mean and variance of η given $\mathcal{F}_R$ and define the 5×5 matrix $\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}$. The conditional expected utility of the investor i given $\mathcal{F}_R$ is given by

$$-\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2} \left|I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix} \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P)\right|^{-1/2} \exp\left\{-\tfrac{1}{2}\mu'_{\eta|R}(\Sigma_{\eta|R}+\mathcal{I})^{-1}\mu_{\eta|R}\right\} \tag{29}$$

where

$$(\Sigma_{\eta|R}+\mathcal{I})^{-1} = \begin{pmatrix} \mathbb{V}_R(V-P)^{-1} + \vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1}\vec{h}_R & -\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1}\vec{h}_R & \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1} \end{pmatrix} \tag{30}$$

and $\vec{h}_R = \mathbb{C}_R((\vec{V}, \vec{Z}_i), V-P)\mathbb{V}_R^{-1}(V-P)$ *is the* 4×1 *vector of* $\mathcal{F}_R$*-conditional regression coefficients of* $\vec{V}$ *and* $\vec{Z}$ *on* $V - P$.

Proof. Consider an arbitrary investment rule $y(s_p) \in \{0, 1\}$ with associated asset value $V(y) = A + y\theta - c(y)$. We would like to compute the conditional expected utility, given s_p, for an arbitrary investor. Given the optimal demand $x^* = \frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n$, the *realized* utility is

$$-e^{-\gamma(x^*(V-P)+z_iU+nV)} = -e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)}-n\right)(V-P)+z_i\theta+nV\right)}. \tag{31}$$

To compute the expected utility, we will use the law of iterated expectations, first computing the expectation conditional on $\mathcal{F}_{i^+} = \sigma(\{s_i, z_i, s_p\})$, which is the investor information set augmented with s_p, and then conditional on $\mathcal{F}_R$. We emphasize that the first step is not identical to computing the expectation given the investor's information set $\mathcal{F}_i$ itself since s_p is only observed by the investor in states in which investment is positive.

Note that because $V = A$ in any state with zero investment (i.e., in any state in which the investor does not infer s_p from the price) and because the $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ information sets coincide in any states with positive investment, we have

$$\mathbb{E}_{i^+}[V] = \mathbb{E}_i[V] = \begin{cases} \mathbb{E}[A] & y = 0 \\ \mathbb{E}[A + y\theta - c(y)|s_i, z_i, s_p] & y > 0 \end{cases} \tag{32}$$

$$\mathbb{V}_{i^+}[V] = \mathbb{V}_i[V] = \begin{cases} \mathbb{V}[A] & y = 0 \\ \mathbb{V}[A + y\theta - c(y)|s_i, z_i, s_p] & y > 0 \end{cases}. \tag{33}$$

Similarly,

$$\mathbb{C}_{i^+}(V, U) = \mathbb{C}_i(V, U) = \begin{cases} 0 & y = 0 \\ y\mathbb{V}[\theta|s_i, z_i, s_p] & y > 0 \end{cases}. \tag{34}$$

However, the conditional variance of the endowment payoff itself U is not always identical under the two information sets since it differs in states in which there is no investment and the investor does not infer s_p.

Given $\{s_i, z_i, s_p\}$ the investor's terminal wealth is conditionally normally distributed with mean

$$\mathbb{E}_{i^+}\left[\left(\frac{\mathbb{E}_i[V-P] - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right)(V-P) + z_iU + nV\right] \tag{35}$$

$$= \left(\frac{\mathbb{E}_i[V-P] - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right)(\mathbb{E}_{i^+}[V] - P) + z_i\mathbb{E}_{i^+}[U] + n\mathbb{E}_{i^+}[V] \tag{36}$$

$$= \left(\frac{\mathbb{E}_i^2[V-P]}{\gamma\mathbb{V}_i(V)}\right) - \left(n + \frac{\mathbb{C}_i(V,U)}{\mathbb{V}_i(V)}z_i\right)\mathbb{E}_i[V-P] + z_i\mathbb{E}_i[U] + n\mathbb{E}_i[V] \tag{37}$$

$$= \left(\frac{\mathbb{E}_i^2[V-P]}{\gamma \mathbb{V}_i(V-P)} \right) - \left(n + \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)} z_i \right) \mathbb{E}_i[V-P] + z_i \mathbb{E}_i[U] + n\mathbb{E}_i[V], \qquad (38)$$

where the next to last line uses the equality of the conditional means and conditional variances of V under $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ and the final line uses the fact that P is $\mathcal{F}_i$ measurable. Similarly, the conditional variance of wealth is

$$\mathbb{V}_{i^+} \left(\left(\frac{\mathbb{E}_i[V-P] - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n \right) (V-P) + z_i U + nV \right) \qquad (39)$$

$$= \mathbb{V}_{i^+} \left(\left(\frac{\mathbb{E}_{i^+}[V-P] - \gamma \mathbb{C}_{i^+}(V,U) z_i}{\gamma \mathbb{V}_{i^+}(V)} \right) V + z_i U \right) \qquad (40)$$

$$= \frac{\mathbb{E}_{i^+}^2[V-P]}{\gamma^2 \mathbb{V}_{i^+}(V)} + \left(\mathbb{V}_{i^+}(U) - \frac{\mathbb{C}_{i^+}^2(V,U)}{\mathbb{V}_{i^+}(V)} \right) z_i^2 \qquad (41)$$

$$= \frac{\mathbb{E}_i^2[V-P]}{\gamma^2 \mathbb{V}_i(V)} + \mathbb{V}_{i^+}(U|V) z_i^2 \qquad (42)$$

$$= \frac{\mathbb{E}_i^2[V-P]}{\gamma^2 \mathbb{V}_i(V-P)} + \mathbb{V}_{i^+}(U|V-P) z_i^2. \qquad (43)$$

Hence, computing the conditional expected utility given $\mathcal{F}_{i^+}$ yields

$$\mathbb{E}_{i^+} \left[-e^{-\gamma \left(\left(\frac{\mathbb{E}_i[V-P] - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n \right) (V-P) + z_i U + nV \right)} \right] \qquad (44)$$

$$= -e^{-\gamma n_i \mathbb{E}_i[V] - \gamma z_i \mathbb{E}_i[U] + \gamma (n_i + z_i h_i) \mathbb{E}_i[V-P] - \frac{1}{2} \frac{\mathbb{E}_i^2[V-P]}{\mathbb{V}_i(V-P)} + \frac{1}{2} \gamma^2 \mathbb{V}_{i^+}(U|V-P) z_i^2} \qquad (45)$$

where $h_i = \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}$ is the conditional regression coefficient of the endowment payoff U on the asset return $V-P$.

To complete the proof, we need to compute the conditional expectation of this quantity given $\mathcal{F}_R$. Define the 5×1 random vector

$$\eta = \begin{pmatrix} V-P \\ \begin{pmatrix} V \\ U \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix}, \qquad (46)$$

partitioned as indicated by parentheses. Let

$$\eta_i \equiv \mathbb{E}_{i^+}[\eta] = \begin{pmatrix} \mathbb{E}_{i^+}[V-P] \\ \begin{pmatrix} \mathbb{E}_{i^+}[V] \\ \mathbb{E}_{i^+}[U] \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \mathbb{E}_i[V-P] \\ \begin{pmatrix} \mathbb{E}_i[V] \\ \mathbb{E}_i[U] \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix} \qquad (47)$$

be the $\mathcal{F}_{i^+}$ conditional expectation of η. Finally, let $\vec{h}_i = (1, h_i)$ be the 2×1 vector of conditional regression coefficients of (V,U) on V and define the conformably partitioned 5×5 matrix

$$\alpha_i = \begin{pmatrix} \mathbb{V}_i^{-1}(V-P) & \vec{0}' & -\gamma \vec{h}_i' \\ \vec{0} & \mathbf{0} & \gamma I_2 \\ -\gamma \vec{h}_i & \gamma I_2 & -\gamma^2 \mathbb{V}_i((V,U)|V-P) \end{pmatrix}, \tag{48}$$

where I_k denotes an identity matrix of dimension k, and $\vec{0}$ and $\mathbf{0}$ denote a conformable vector and matrix of all zeros, respectively. Below, we will typically just use 0, with no vector notation or bolding, for conformable vectors or matrices of zeros, except where confusion would result.

With this notation, we can concisely write the $\mathcal{F}_{i+}$ expected utility above as

$$\mathbb{E}_{i+}\left[-e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P]-\gamma \mathbb{C}_i(V,U)z_i}{\gamma \mathbb{V}_i(V)}-n\right)(V-P)+z_iU+nV\right)}\right] = -e^{-\frac{1}{2}\eta_i' \alpha_i \eta_i}. \tag{49}$$

The random vector η_i is conditionally jointly normally distributed given $\mathcal{F}_R$.[29]. Let $\mu_{\eta_i|R} = \mathbb{E}_R[\eta_i]$ and $\Sigma_{\eta_i|R} = \mathbb{V}_R(\eta_i)$ denote the $\mathcal{F}_R$ conditional mean and variance matrix of η_i, and let $\mu_{\eta|R} = \mathbb{E}_R[\eta]$ and $\Sigma_{\eta|R} = \mathbb{V}_R(\eta)$ denote the $\mathcal{F}_R$ conditional mean and variance of η itself. We can use standard formulas for expected exponential-quadratic forms of normal random vectors to compute

$$\mathbb{E}_R\left[-e^{-\frac{1}{2}\eta_i'\alpha_i\eta_i}\right] = -\left|\alpha_i\right|^{-1/2}\left|\Sigma_{\eta_i|R}+\alpha_i^{-1}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mu_{\eta_i|R}'\left(\Sigma_{\eta_i|R}+\alpha_i^{-1}\right)^{-1}\mu_{\eta_i|R}\right\}. \tag{50}$$

This expression requires that the matrix α_i is invertible. However, using standard formulas for determinants of partitioned matrices (e.g., eq. (5) in Henderson and Searle (1981)) and inverses of partitioned matrices (e.g., eq. (8) in Henderson and Searle (1981)) we can compute

$$|\alpha_i| = \left|\mathbb{V}_i^{-1}(V-P)\right|\left|-\gamma^2 I_2\right| = \gamma^4\left|\mathbb{V}_i^{-1}(V-P)\right| > 0, \tag{51}$$

so that α_i is invertible and we have

$$\alpha_i^{-1} = \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,(V,U)) & 0 \\ \mathbb{C}_i\left((V,U),V-P\right) & \mathbb{V}_i\left(V,U\right) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}. \tag{52}$$

With this expression for α_i^{-1} we can compute

$$\Sigma_{\eta_i} + \alpha_i^{-1} = \mathbb{V}_R(\mathbb{E}_i[\eta]) + \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,(V,\theta)) & 0 \\ \mathbb{C}_i((V,\theta),V-P) & \mathbb{V}_i(V,U) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{53}$$

[29]Note that η_i follows a singular normal distribution since n is a constant. That is, the conditional variance matrix of η_i is only positive semidefinite. However, defining the random vector in this way causes no difficulties in the derivation below, and both simplifies the algebra and provides guidance for how to handle more general situations in which the share endowment is random.

$$= \Sigma_\eta + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{54}$$

$$\equiv \Sigma_\eta + \mathcal{I}, \tag{55}$$

where the second equality follows from the law of total variance, and the final equality defines the 5×5 matrix $\mathcal{I} = \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{smallmatrix}\right)$.

Putting together everything above, the conditional expected utility can be written

$$-\left|\alpha_i\right|^{-1/2} \left|\Sigma_{\eta_i|R} + \alpha_i^{-1}\right|^{-1/2} \exp\left\{-\frac{1}{2}\mu'_{\eta_i|R}\left(\Sigma_{\eta_i|R} + \alpha_i^{-1}\right)^{-1}\mu_{\eta_i|R}\right\} \tag{56}$$

$$= -\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\Sigma_{\eta|R} + \mathcal{I}\right|^{-1/2} \exp\left\{-\frac{1}{2}\mu'_{\eta|R}\left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1}\mu_{\eta|R}\right\}. \tag{57}$$

The key step to evaluating this expression is to characterize the inverse $\left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1}$ and the determinant $\left|\Sigma_{\eta|R} + \mathcal{I}\right|$. Partition $\Sigma_{\eta|R} + \mathcal{I}$ as

$$\begin{pmatrix} A & U \\ V & D \end{pmatrix}$$

$$\equiv \left(\begin{matrix} \mathbb{V}_R(V-P) & \left(\mathbb{C}_R(V-P,(V,U)) \quad \mathbb{C}_R(V-P,(n,z_i))\right) \\ \begin{pmatrix} \mathbb{C}_R((V,U),V-P) \\ \mathbb{C}_R((n,z_i),V-P) \end{pmatrix} & \left(\begin{pmatrix} \mathbb{V}_R((V,U)) & \mathbb{C}_R((V,U),(n,z_i)) \\ \mathbb{C}_R((n,z_i),(V,U)) & \mathbb{V}_R((n,z_i)) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right) \end{matrix}\right), \tag{58}$$

where A is 1×1, $U = V'$ is 1×4 and D is 4×4.

Using standard methods for inverting a partitioned matrix (e.g., eq. (8) in Henderson and Searle (1981)) we therefore have

$$(\Sigma^{-1}_{\eta|R} + \mathcal{I})^{-1} = \begin{pmatrix} A^{-1} + A^{-1}U(D - VA^{-1}U)^{-1}VA^{-1} & -A^{-1}U(D-VA^{-1}U)^{-1} \\ -(D-VA^{-1}U)^{-1}VA^{-1} & (D-VA^{-1}U)^{-1} \end{pmatrix}. \tag{59}$$

Note that

$$D - VA^{-1}U \tag{60}$$

$$= \left(\begin{pmatrix} \mathbb{V}_R((V,U)) & \mathbb{C}_R((V,U),(n,z_i)) \\ \mathbb{C}_R((n,z_i),(V,U)) & \mathbb{V}_R((n,z_i)) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right) \tag{61}$$

$$- \begin{pmatrix} \mathbb{C}_R((V,U),V-P) \\ \mathbb{C}_R((n,z_i),V-P) \end{pmatrix} \mathbb{V}_R^{-1}(V-P)\left(\mathbb{C}_R(V-P,(V,U)) \quad \mathbb{C}_R(V-P,(n,z_i))\right) \tag{62}$$

$$= \begin{pmatrix} \mathbb{V}_R((V,U)|V-P) & \mathbb{C}_R((V,U),(n,z_i)|V-P) \\ \mathbb{C}_R((n,z_i),(V,U)|V-P) & \mathbb{V}_R((n,z_i)|V-P) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{63}$$

or if we concisely let $\vec{V} = (V, U)$ and $\vec{Z}_i = (n, z_i)$,

$$D - VA^{-1}U = \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V - P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}. \tag{64}$$

Similarly, we have

$$\begin{aligned} VA^{-1} &= \begin{pmatrix} \mathbb{C}_R(\vec{V}, V - P) \\ \mathbb{C}_R(\vec{Z}_i, V - P) \end{pmatrix} \mathbb{V}_R^{-1}(V - P) & (65) \\ &\equiv \vec{h}_R, & (66) \end{aligned}$$

where the last line defines $\vec{h}_R$ as the 4×1 vector of $\mathcal{F}_R$-conditional regression coefficients of $\vec{V}$ and $\vec{Z}$ on $V - P$. Hence, we can concisely write our desired matrix inverse as

$$\begin{aligned} &(\Sigma_{\eta|R}^{-1} + \mathcal{I})^{-1} & (67) \\ &= \begin{pmatrix} A^{-1} + A^{-1}U(D - VA^{-1}U)^{-1}VA^{-1} & -A^{-1}U(D - VA^{-1}U)^{-1} \\ -(D - VA^{-1}U)^{-1}VA^{-1} & (D - VA^{-1}U)^{-1} \end{pmatrix} & (68) \\ &= \begin{pmatrix} \mathbb{V}_R(V-P)^{-1} + \vec{h}'_R \left(\mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \vec{h}_R & -\vec{h}'_R \left(\mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \vec{h}_R & \left(\mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \end{pmatrix}. & (69) \end{aligned}$$

Using the same partitioning and applying eq. (5) of Henderson and Searle (1981) its determinant is

$$\begin{aligned} \left|\Sigma_{\eta|R} + \mathcal{I}\right| &= |A| \left|D - VA^{-1}U\right| & (70) \\ &= |\mathbb{V}_R(V - P)| \left| \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V - P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right|. & (71) \end{aligned}$$

Putting everything together, the $\mathcal{F}_R$ expected utility is

$$\begin{aligned} &-\frac{1}{\gamma^2} \left|\mathbb{V}_i^{-1}(V - P)\right|^{-1/2} \left|\Sigma_{\eta|R} + \mathcal{I}\right|^{-1/2} \exp\left\{ -\frac{1}{2}\mu'_{\eta|R} \left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1} \mu_{\eta|R} \right\} & (72) \\ &= -\frac{1}{\gamma^2} \left| \frac{\mathbb{V}_i(V - P)}{\mathbb{V}_R(V - P)} \right|^{1/2} \left| \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V - P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right|^{-1/2} \exp\left\{ -\frac{1}{2}\mu'_{\eta|R}(\Sigma_{\eta|R} + \mathcal{I})^{-1}\mu_{\eta|R} \right\} & (73) \\ &= -\left| \frac{\mathbb{V}_i(V - P)}{\mathbb{V}_R(V - P)} \right|^{1/2} \left| I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix} \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V - P) \right|^{-1/2} \exp\left\{ -\frac{1}{2}\mu'_{\eta|R}(\Sigma_{\eta|R} + \mathcal{I})^{-1}\mu_{\eta|R} \right\} & (74) \end{aligned}$$

where

$$(\Sigma_{\eta|R}+\mathcal{I})^{-1}=\begin{pmatrix} \mathbb{V}_R(V-P)^{-1}+\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix}0 & \frac{1}{\gamma}I_2\\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\vec{h}_R & -\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix}0 & \frac{1}{\gamma}I_2\\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix}0 & \frac{1}{\gamma}I_2\\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\vec{h}_R & \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix}0 & \frac{1}{\gamma}I_2\\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\end{pmatrix}. \tag{75}$$

□

The expression for the expected utility in Proposition 8 simplifies further in the case where prices are characterized as the public expectation of future cash flows.

Corollary 1. *Suppose that the asset price is characterized as the conditional expected cash flow $P=\mathbb{E}[V|P]$. Then the conditional expected utility given $\mathcal{F}_R$ is given by*

$$-\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2}\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right|^{-1/2} \tag{76}$$

$$\times\exp\left\{-\gamma\mathbb{E}_R[V]n-\frac{1}{2}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}'\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}\right\} \tag{77}$$

Proof. Set $\mathbb{E}_R[V-P]=0$ in Proposition 8, write out the matrix products explicitly, and collect terms. □

B Proofs of primary results

Proof of Proposition 1

This is a special case of Proposition 5 with $Y=\{0,1\}$, $c(0)=0, c(1)=c$, and $1/\tau_\xi=0$.

Proof of Proposition 2

The first part follows from the observations that the firm manager chooses investment to maximize V and can always choose to ignore the information in s_p when making her decision.

For the second part, let $\omega=\mathbb{E}\left[\theta|s_p\right]-c\sim N\left(\mu_\theta-c,\mathrm{var}\left(\mathbb{E}\left[\theta|s_p\right]\right)\right)\equiv N\left(\mu_\omega,\sigma_\omega^2\right)$. Then,

$$\mathbb{E}\left[V\left(y\left(s_p\right)\right)\right]=\mathbb{E}[A]+\mathbb{E}\left[\left(\mathbb{E}\left[\theta|s_p\right]-c\right)\mathbf{1}_{\{s_p>\bar{s}_p\}}\right] \tag{78}$$

$$=\mu_A+\mathbb{E}\left[\omega\mathbf{1}_{\{\omega>0\}}\right] \tag{79}$$

$$=\mu_A+\mu_\omega+\sigma_\omega\frac{\phi\left(\frac{\mu_\omega}{\sigma_\omega}\right)}{\Phi\left(\frac{\mu_\omega}{\sigma_\omega}\right)}. \tag{80}$$

Holding fixed μ_ω, the above expectation is increasing in σ_ω. This implies the expected value with feedback is increasing in $\mathrm{var}\left(\mathbb{E}\left[\theta|s_p\right]\right)=\mathrm{var}\left(\theta\right)-\mathrm{var}\left(\theta|s_p\right)$. Moreover, in the β^{SUB}

equilibrium, $\text{var}(\theta|s_p)$ decreases in τ_ε and τ_ζ, but increases in γ. The result follows from noting that without feedback, the expected value is unaffected by τ_ε, τ_ζ and γ.

Proof of Proposition 3

This is a special case of Proposition 6 with $Y = \{0, 1\}$, $c(0) = 0, c(1) = c$, and $1/\tau_\xi = 0$.

Proof of Proposition 4

The conditional expected utility of an arbitrary investor given $\mathcal{F}_R = \sigma(s_p)$, as represented in the proof of Proposition 6 below, is

$$\mathbb{E}_R[-e^{-\gamma W_i^*}] = -e^{-n\gamma \mathbb{E}_R[V] - F(y|s_p)} \tag{81}$$

where F is an increasing function of y. If we can show that the claimed welfare ranking holds state-by-state in s_p then it is immediate that it holds unconditionally.

First consider the case in which the project has negative ex-ante NPV (i.e., the no-feedback default investment is $y = 0$). Applying the transformation $-\log(-x)$, the conditional expected utility satisfies

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=0} + F(y|s_p)\Big|_{y=0} \tag{82}$$

$$\leq n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_P)} + F(y|s_p)\Big|_{y=0} \tag{83}$$

$$\leq n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_P)} + F(y|s_p)\Big|_{y=y^*(s_P)} \tag{84}$$

where the first inequality follows from the fact that the expected asset value (i.e., the NPV) is weakly higher with feedback, and the second inequality follows from the fact that Proposition 6 establishes that F is increasing in y, and $y^* \geq 0$. Because this inequality holds state-by-state in s_P with strict inequality on a set of positive probability, it follows that the investor is strictly better off with feedback in this case $U(0) < U(y^*(s_p))$.

Next, consider the case in which the project has positive ex-ante NPV (i.e., the no-feedback default investment is $y = 1$) and $n = 0$. Following similar steps to the negative NPV case, we have

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=1,n=0} + F(y|s_p)\Big|_{y=1} \tag{85}$$

$$= F(y|s_p)\Big|_{y=1} \tag{86}$$

$$\geq F(y|s_p)\Big|_{y=y^*(s_p)} \tag{87}$$

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_p),n=0} + F(y|s_p)\Big|_{y=y^*(s_p)}. \tag{88}$$

Because this inequality holds state-by-state in s_P with strict inequality on a set of positive probability, it follows that the investor is strictly worse off with feedback in this case $U(0) > U(y^*(s_p))$. Now, because the unconditional expected utility is continuous in n and because the investor is strictly worse off with feedback when $n = 0$, it follows that for n sufficiently small but positive, the investor remains strictly worse off with feedback.

Proof of Proposition 5

Conjecture that there exists an equilibrium in which the asset price reveals $s_p = \theta - \frac{1}{\beta}Z$ when $s_p \geq \overline{s}_p$, for constants β and $\overline{s}_p$ to be determined.

We proceed by working backwards, starting with the manager's problem at $t = 2$. In any equilibrium of the conjectured form, the manager's can infer s_p if and only if $s_p \geq \overline{s}_p$ and otherwise can infer only that $s_p < \overline{s}_p$. Her optimal investment is therefore

$$y^*(\cdot) = \begin{cases} \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p] - cy & s_p \geq \overline{s}_p \\ \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p < \overline{s}_p] - cy & \text{otherwise} \end{cases}. \tag{89}$$

The manager can always guarantee a project payoff of zero by not investing. Hence, in order for her to find it optimal to invest when she observes any value $s_p \geq \overline{s}_p$ and not invest otherwise, the threshold $\overline{s}_p = T$ must be such that

$$\begin{cases} \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) \geq 0 & \forall s_p \geq T \\ \max_{y \in Y} y\mathbb{E}[\theta|s_p < \overline{s}_p] - c(y) = 0 & \text{otherwise} \end{cases}. \tag{90}$$

Clearly any threshold T that lies (weakly) between $\inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$ (i.e., T such that the optimal investment conditional on observing any realization of s_p below the threshold is zero) and $\sup\{T : \max_{y \in Y} y\mathbb{E}[\theta|s_p < T] - c(y) = 0\}$ (i.e., T such that the optimal investment given knowledge only that s_p is less that then threshold is zero) satisfies these conditions.

We select the lowest admissible threshold, defined by $\overline{s}_p \equiv \inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$. In the baseline model with $Y = \{0, 1\}$, this is the unique equilibrium within this class in which the price function is continuous in s_p. This equilibrium is also robust to the manager directly observing s_p is all states (regardless of the investment opportunities Y). That is, if the manager were to observe a realization $s_p < \overline{s}_p$, so defined, she would still find it optimal to not invest.

Now, step back to $t = 1$ and consider the problem of an arbitrary investor i:

$$\sup_{x \in \mathbb{R}} \mathbb{E}_i \left[-e^{-\gamma(x(V-P)+z_i U+nV)} \right]. \tag{91}$$

Under the conjecture that the investment occurs and the price reveals s_p if and only if $s_p \geq \overline{s}_p$, the investor's conditional beliefs about payoffs are conditionally normal with

$$\mathbb{E}_i[\theta] = \begin{cases} \mu_\theta & s_p < \overline{s}_p \\ \mathbb{E}[\theta|s_i, z_i, s_p] & s_p \geq \overline{s}_p \end{cases} \tag{92}$$

$$\mathbb{V}_i(\theta) = \begin{cases} \frac{1}{\tau_\theta} & s_p < \overline{s}_p \\ \mathbb{V}(\theta|s_i, z_i, s_p) & s_p \geq \overline{s}_p \end{cases} \tag{93}$$

and

$$\mathbb{E}_i[V] = \mu_A + y\mathbb{E}_i[\theta] - c(y) \tag{94}$$
$$\mathbb{E}_i[U] = \mathbb{E}_i[\theta] \tag{95}$$
$$\mathbb{V}_i(V) = \frac{1}{\tau_A} + y^2\mathbb{V}_i(\theta) \tag{96}$$
$$\mathbb{C}_i(V, U) = \mathbb{C}_i(V, \theta) = y\mathbb{V}_i(\theta) \tag{97}$$
$$\mathbb{V}_i(U) = \mathbb{V}_i(\theta) + \frac{1}{\tau_\xi}. \tag{98}$$

Furthermore, the conditional moments of θ in the $s_p \geq \overline{s}_p$ region can be written explicitly as

$$\mathbb{E}[\theta|s_i, z_i, s_p] = \mu_\theta + \underbrace{\tfrac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}}_{=b_s}(s_i - \mu_\theta) + \underbrace{\tfrac{\beta^2(\tau_Z+\tau_\zeta)}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}}_{=b_p}\left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z\right)\right) \tag{99}$$

$$+ \underbrace{\tfrac{\beta^2(\tau_Z+\tau_\zeta)}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}\tfrac{\tau_\zeta}{\tau_Z+\tau_\zeta}\frac{1}{\beta}}_{=b_z}(z_i - \mu_Z), \tag{100}$$

and

$$\frac{1}{\tau} \equiv \mathbb{V}(\theta|s_i, z_i, s_p) = \frac{1}{\tau_\theta + \tau_\varepsilon + \beta^2(\tau_Z + \tau_\zeta)}. \tag{101}$$

Hence, computing the expectation, the investor's objective is

$$\sup_{x\in\mathbb{R}} -e^{-\gamma(x\mathbb{E}_i[V-P]+z_i\mathbb{E}_i[U]+n\mathbb{E}_i[V])+\frac{1}{2}\gamma^2\left((x+n)^2\mathbb{V}_i(V)+2z_i(x+n)\mathbb{C}_i(V,U)+z_i^2\mathbb{V}_i(U)\right)}. \tag{102}$$

It is immediate that this problem is strictly concave on all of $\mathbb{R}$, so the first-order condition (FOC) characterizes the maximum

$$x^* = \frac{\mathbb{E}_i[V] - P - \gamma\mathbb{C}_i(V, U)z_i}{\gamma\mathbb{V}_i(V)} - n. \tag{103}$$

Now aggregate the demand of the investors. For states $s_p < \overline{s}_p$, plugging in for the conditional moments and computing the aggregate demand

$$\int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma\mathbb{C}_i(V, U)z_i}{\gamma\mathbb{V}_i(V)} - n\right) di = \frac{\mu_A - P}{\gamma\frac{1}{\tau_A}} - n \tag{104}$$

so that the aggregate order does not reveal any information to the market maker, as conjectured. For states $s_p \geq \overline{s}_p$, we have

$$\int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma \mathbb{C}_i(V,\theta) z_i}{\gamma \mathbb{V}_i(V)} - n \right) di = \int_i \left(\frac{\mu_A + y\mathbb{E}[\theta | s_i, z_i, s_p] - c(y) - P - \gamma \frac{1}{\tau} z_i}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n \right) di \tag{105}$$

$$= \frac{\mu_A + y \int_i \left(\mathbb{E}[\theta | s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - P}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n. \tag{106}$$

Hence, the aggregate order reveals

$$\int_i \left(\mathbb{E}[\theta | s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di \tag{107}$$

$$= \int_i \left(\mu_\theta + b_s (s_i - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta} \mu_Z \right) \right) + b_z (z_i - \mu_Z) - \gamma \frac{1}{\tau} z_i \right) di \tag{108}$$

$$= \mu_\theta + b_s (\theta - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta} \mu_Z \right) \right) + \left(b_z - \gamma \frac{1}{\tau} \right) Z - b_z \mu_Z. \tag{109}$$

It follows that the market maker can infer

$$b_s \theta - \left(\gamma \frac{1}{\tau} - b_z \right) Z = b_s \left(\theta - \frac{\gamma \frac{1}{\tau} - b_z}{b_s} Z \right), \tag{110}$$

which satisfies our conjecture $s_p = \theta - \frac{1}{\beta} Z$ if and only if

$$\frac{1}{\beta} = \frac{\gamma \frac{1}{\tau} - b_z}{b_s}. \tag{111}$$

Plugging in using the explicit expressions for b_s, b_z, and $1/\tau$ from above yields:

$$\frac{1}{\beta} = \frac{\gamma - \beta \tau_\zeta}{\tau_\varepsilon}, \tag{112}$$

which has two solutions

$$\beta = \frac{1}{2\tau_\zeta} \left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon \tau_\zeta} \right), \tag{113}$$

or, equivalently,

$$\frac{1}{\beta} = \frac{1}{2\tau_\varepsilon} \left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon \tau_\zeta} \right), \tag{114}$$

where the + solution for β corresponds to the − solution for $1/\beta$ and vice versa. This is easily confirmed by multiplying the expressions, which yields 1 if and only if we choose opposing signs for each.

With the precise expression for s_p pinned down, our candidate price function is

$$P = \begin{cases} \mu_A & s_p < \overline{s}_p \\ \mu_A + y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p)) & s_p \geq \overline{s}_p \end{cases}. \tag{115}$$

To confirm that this represents an equilibrium price function, it remains to confirm that investors can infer s_p if and only if $s_p \geq \overline{s}_p$. It suffices to establish that the candidate price function is constant for $s_p < \overline{s}_p$, is strictly increasing in s_p for $s_p \geq \overline{s}_p$ and does not jump downward at the threshold.

Clearly P is constant for $s_p < \overline{s}_p$. Since the threshold is defined as $\overline{s}_p = \inf\{s_p : \max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$, we have

$$\mu_A + y(\overline{s}_p)\mathbb{E}[\theta|s_p = \overline{s}_p] - c(y(\overline{s}_p)) \geq \mu_A, \tag{116}$$

which establishes that the price is increasing at the threshold.

Finally, consider the region $s_p > \overline{s}_p$. We need to show that the optimized project payoff $y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p))$ is strictly increasing in s_p. Note first that by Theorem 4 in Milgrom and Shannon (1994), the optimal investment $y(s_p)$ is increasing in s_p since the manager's objective function $y\mathbb{E}[\theta|s_p]-c(y)$ satisfies the single-crossing property in $(y; s_p)$. Furthermore, for any $s_{p2} > s_{p1} > \overline{s}_p$ we have

$$y(s_{p2})\mathbb{E}[\theta|s_{p2}] - c(y(s_{p2})) \geq y(s_{p1})\mathbb{E}[\theta|s_{p2}] - c(y(s_{p1})) > y(s_{p1})\mathbb{E}[\theta|s_{p1}] - c(y(s_{p1})), \tag{117}$$

where the first inequality follows from the optimality of $y(s_{p2})$ at s_{p2} and the second inequality follows from the fact that $\mathbb{E}[\theta|s_p]$ is strictly increasing in s_p and $y > 0$ for $s_p > \overline{s}_p$. This establishes that $P(s_p)$ is strictly increasing above the threshold and completes the proof.

Proof of Proposition 6

We begin by observing that the s_p-dependent investment rule that maximizes the unconditional expected utility is equivalent to the s_p-dependent rule that maximizes the conditional expected utility $\mathbb{E}[U_i|s_p]$ state-by-state in s_p.

Fix any s_p. Using the expression for expected utility from Corollary 1 with $\mathcal{F}_R = \sigma(s_p)$ and taking the monotonic transformation $-\log(-x)$, the welfare-maximizing investment y maximizes

$$\gamma n\mathbb{E}_R[V] - \frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} + \frac{1}{2}\log\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{118}$$

$$+\frac{1}{2}\begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}. \tag{119}$$

Letting

$$F(y|s_p) = -\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} + \frac{1}{2}\log\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{120}$$

$$+\frac{1}{2}\begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}'\begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1}\begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}, \tag{121}$$

we can write the welfare maximization problem concisely as

$$\max_{y\in Y}\left(\gamma n\mathbb{E}_R[V] + F(y|s_p)\right), \tag{122}$$

which is the sum of the manager's objective function (scaled by a constant) and the function F. Let $\nu \in \{0,1\}$ parameterize the manager's objective function and the objective function for welfare maximization, respectively. We would like to show that the maximizer of the parameterized problem

$$y(s_p;\nu) = \arg\max_{y\in Y}\left(\gamma n\mathbb{E}_R[V] + \nu F(y|s_p)\right) \tag{123}$$

is an increasing function of ν (holding the state s_p fixed). Theorem 4 in Milgrom and Shannon (1994) implies that if we can show that $\gamma n\mathbb{E}_R[V] + F(y|s_p)$ satisfies the single-crossing property in $(y;\nu)$ then we are done. That is, we want to show that the incremental payoff to higher investment crosses zero at most once, and from below, as ν increases from 0 to 1. It suffices to show that $F(y|s_p)$ is an increasing function of y.

Define $\tau_R \equiv \frac{1}{\mathbb{V}_R(\theta)}$. Recall that the $\mathcal{F}_R$ conditional variance matrix is

$$\Sigma_{\eta|R} = \begin{pmatrix} \mathbb{V}(V-P|s_p) & \mathbb{C}(V-P,V|s_p) & \mathbb{C}(V-P,U|s_p) & 0 & \mathbb{C}(V-P,Z_i|s_p) \\ \mathbb{C}(V-P,V|s_p) & \mathbb{V}(V|s_p) & \mathbb{C}(V,U|s_p) & 0 & \mathbb{C}(V,Z_i|s_p) \\ \mathbb{C}(V-P,U|s_p) & \mathbb{C}(V,U|s_p) & \mathbb{V}(U|s_p) & 0 & \mathbb{C}(U,Z_i|s_p) \\ 0 & 0 & 0 & 0 & 0 \\ \mathbb{C}(V-P,Z_i|s_p) & \mathbb{C}(V,Z_i|s_p) & \mathbb{C}(U,Z_i|s_p) & 0 & \mathbb{V}(Z_i|s_p) \end{pmatrix} \tag{124}$$

$$= \begin{pmatrix} \frac{1}{\tau_A}+y^2\frac{1}{\tau_R} & \frac{1}{\tau_A}+y^2\frac{1}{\tau_R} & y\frac{1}{\tau_R} & 0 & \beta y\frac{1}{\tau_R} \\ \frac{1}{\tau_A}+y^2\frac{1}{\tau_R} & \frac{1}{\tau_A}+y^2\frac{1}{\tau_R} & y\frac{1}{\tau_R} & 0 & \beta y\frac{1}{\tau_R} \\ y\frac{1}{\tau_R} & y\frac{1}{\tau_R} & \frac{1}{\tau_R}+\frac{1}{\tau_\xi} & 0 & \beta\frac{1}{\tau_R} \\ 0 & 0 & 0 & 0 & 0 \\ \beta y\frac{1}{\tau_R} & \beta y\frac{1}{\tau_R} & \beta\frac{1}{\tau_R} & 0 & \beta^2\frac{1}{\tau_R}+\frac{1}{\tau_\zeta} \end{pmatrix}, \tag{125}$$

where we use the fact that since $s_p = \theta - \frac{1}{\beta}Z$ we have $\mathbb{C}(\theta, z_i|s_p) = \beta\mathbb{V}(\theta|s_p)$ and $\mathbb{V}(Z|s_p) = \beta^2\mathbb{V}(\theta|s_p)$. We can thus directly compute

$$\mathbb{V}_R((U,z_i)|V-P) = \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P) \\ \mathbb{C}_R(z_i,U|V-P) & \mathbb{V}_R(z_i|V-P) \end{pmatrix} \tag{126}$$

$$= \begin{pmatrix} \frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\xi} & \beta\frac{1}{\tau_R+y^2\tau_A} \\ \beta\frac{1}{\tau_R+y^2\tau_A} & \beta^2\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\zeta} \end{pmatrix}. \tag{127}$$

Now, moving to the quadratic form,

$$\frac{1}{2}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}'\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}, \tag{128}$$

note than in equilibrium, both $\mathbb{E}_R[U]=\mathbb{E}_R[\theta]$ and $\mathbb{E}_R[z_i]=\mathbb{E}_R[Z]$ do not depend on y. Hence, differentiating with respect to y, using standard results for differentiating a matrix inverse, yields

$$\frac{\partial}{\partial y}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}'\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix} \tag{129}$$

$$=\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix}'\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\left[-\frac{\partial}{\partial y}\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}\right] \tag{130}$$

$$\times\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\begin{pmatrix}\mathbb{E}_R[U]\\ \mathbb{E}_R[z_i]\end{pmatrix} \tag{131}$$

If we establish that the matrix $-\frac{\partial}{\partial y}\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}$ is positive semidefinite then it will follow that the above expression is (weakly) positive and hence the quadratic form is increasing in y. But this is immediate since, applying expression (126), we have:

$$-\frac{\partial}{\partial y}\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma}\\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}=\begin{pmatrix}\frac{2y\tau_A}{(\tau_R+y^2\tau_A)^2} & \beta\frac{2y\tau_A}{(\tau_R+y^2\tau_A)^2}\\ \beta\frac{2y\tau_A}{(\tau_R+y^2\tau_A)^2} & \beta^2\frac{2y\tau_A}{(\tau_R+y^2\tau_A)^2}\end{pmatrix}, \tag{132}$$

for which all principal minors are non-negative.

Next consider the determinant term:

$$-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}+\frac{1}{2}\log\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right|. \tag{133}$$

Using the conditional variances computed above, we immediately have:

$$-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}=-\frac{1}{2}\log\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}. \tag{134}$$

Furthermore,

$$\frac{1}{2}\log\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{135}$$

$$=\frac{1}{2}\log\left|\begin{pmatrix}1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A} & \gamma\left(\beta^2\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\zeta}\right)\\ \gamma\left(\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\xi}\right) & 1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\end{pmatrix}\right| \tag{136}$$

$$= \frac{1}{2}\log\left(\left(1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\right)^2 - \gamma^2\left(\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\xi}\right)\left(\beta^2\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\zeta}\right)\right) \tag{137}$$

$$= \frac{1}{2}\log\left(\left(1+2\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\right) - \gamma^2\left(\frac{1}{\tau_R+y^2\tau_A}\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)+\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\right) \tag{138}$$

$$= \frac{1}{2}\log\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\tau_R+y^2\tau_A}\right). \tag{139}$$

So, combining terms yields:

$$-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}+\frac{1}{2}\log\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{140}$$

$$= \frac{1}{2}\log\left[\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\tau_R+y^2\tau_A}\right)\right] \tag{141}$$

$$= \frac{1}{2}\log\left[\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{1}{\tau_R\tau_A}\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}\right)\right]. \tag{142}$$

Differentiating the argument of the log with respect to y implies that signing the dependence of this expression on y is equivalent to signing

$$\frac{\partial}{\partial y}(\cdot) \propto 2y\frac{1}{\tau_R}\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau}\right) \tag{143}$$

$$-2y\frac{1}{\tau}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}\right)+\frac{1}{\tau_R\tau_A}\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right) \tag{144}$$

$$= 2y\left[\frac{1}{\tau_A}\left(\frac{1}{\tau_R}-\frac{1}{\tau}\right)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\frac{1}{\tau}\frac{1}{\tau_R\tau_A}\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right] \tag{145}$$

$$= 2y\frac{1}{\tau_A\tau_R\tau}\left[(\tau-\tau_R)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right] \tag{146}$$

$$= 2y\frac{1}{\tau_A\tau_R\tau}\left[\left(\tau_\varepsilon+\beta^2\tau_\zeta\right)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+k^2\frac{1}{\tau_\xi}\right)\right)\right]. \tag{147}$$

We claim that this expression is positive. To establish this, we need to sign the term inside the brackets, which can be further simplified as

$$\left(\tau_\varepsilon+\beta^2\tau_\zeta\right)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right) \tag{148}$$

$$= \left(\tau_\varepsilon+\beta^2\tau_\zeta\right)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-2\gamma\beta+\frac{\gamma^2}{\tau_\xi\tau_\zeta}\left(\tau_\xi+\beta^2\tau_\zeta\right) \tag{149}$$

$$= \gamma\beta\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-2\gamma\beta+\frac{\gamma^2}{\tau_\xi\tau_\zeta}\left(\tau_\xi+\gamma\beta-\tau_\varepsilon\right) \tag{150}$$

$$= \frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta + \left(1 - \frac{\gamma^2}{\tau_\xi \tau_\zeta}\right)\tau_\varepsilon, \tag{151}$$

where the second line rearranges terms, the third line comes from substituting in $\tau_\varepsilon + \beta^2\tau_\zeta = \gamma\beta$ from the equation defining the price informativeness parameter β, and the remaining lines rearrange and collect terms.

We know that $\left(1 - \frac{\gamma^2}{\tau_\xi\tau_\zeta}\right)\tau_\varepsilon \geq 0$ due to the parameter restriction $1 - \frac{\gamma^2}{\tau_\xi\tau_\zeta} > 0$ required for the existence of expected utility. It remains to show that $\frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta > 0$. We have

$$\frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta = \gamma\beta\left(\left(\frac{\gamma^2}{\tau_\zeta\tau_\varepsilon} - 1\right)\frac{\tau_\varepsilon}{\gamma\beta} - 1\right) \tag{152}$$

$$= \gamma\beta\left(\frac{1}{2}\left(\frac{\gamma^2}{\tau_\zeta\tau_\varepsilon} - 1\right)\left(1 \pm \sqrt{1 - 4\frac{\tau_\varepsilon\tau_\zeta}{\gamma^2}}\right) - 1\right) \tag{153}$$

$$\equiv \gamma\beta\left(\frac{1}{2}(a-1)\left(1 \pm \sqrt{1 - \frac{4}{a}}\right) - 1\right), \tag{154}$$

where the first line groups terms, the second line substitutes in the equilibrium $\frac{\tau_\varepsilon}{\gamma}\frac{1}{\beta} = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\frac{\tau_\varepsilon\tau_\zeta}{\gamma^2}}\right)$ and cancels terms, and the third line simplifies notation by defining $a = \frac{\gamma^2}{\tau_\zeta\tau_\varepsilon}$. The admissible parameters are such that $a \geq 4$ (i.e, the parameters such that equilibrium in the financial market exists). Hence, we need to show that this expression is positive for any $a \in [4, \infty)$.

Note that the expression is clearly positive if we select the β^{SUB} equilibrium, since that corresponds to selecting the $+$ sign for the solution for $1/\beta$. With the selection of the $+$ in the above, the expression is trivially increasing in a and therefore

$$\frac{1}{2}(a-1)\left(1 + \sqrt{1 - \frac{4}{a}}\right) - 1 \geq \frac{1}{2}(4-1) - 1 = \frac{1}{2} \tag{155}$$

as desired.

Consider next the β^{COM} equilibrium, which corresponds to the negative square root above. We claim that $\frac{1}{2}(a-1)\left(1 - \sqrt{1 - \frac{4}{a}}\right) - 1$ is strictly decreasing in a, takes value $1/2$ at $a = 4$ and tends to 0 as $a \to \infty$. Once shown this establishes that it is strictly positive for any finite a. Computing the derivative, we have:

$$\frac{\partial}{\partial a}\left(\frac{1}{2}(a-1)\left(1 - \sqrt{1 - \frac{4}{a}}\right) - 1\right) \tag{156}$$

$$\propto 1 - \frac{\partial}{\partial a}\left((a-1)\sqrt{1 - \frac{4}{a}}\right) \tag{157}$$

$$= 1 - \frac{a^2 - 2a - 2}{a^2\sqrt{1 - \frac{4}{a}}}. \tag{158}$$

Note that

$$1-\frac{a^2-2a-2}{a^2\sqrt{1-\frac{4}{a}}}=\frac{a^2\sqrt{1-\frac{4}{a}}-a^2-2a-2}{a^2\sqrt{1-\frac{4}{a}}}<\frac{-2a-2}{a^2\sqrt{1-\frac{4}{a}}}<0, \tag{159}$$

which establishes that the derivative is negative and therefore $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ is strictly decreasing. Next, it is immediate that the expression takes value $1/2$ at $a=4$:

$$\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1=\frac{1}{2}3-1=\frac{1}{2}. \tag{160}$$

Finally, to confirm that $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ limits to 0 as $a\to\infty$, we have

$$\begin{aligned}\lim_{a\to\infty}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)&=\lim_{a\to\infty}a\left(1-\sqrt{1-\frac{4}{a}}\right) &(161)\\ &=\lim_{a\to\infty}\frac{1-\sqrt{1-\frac{4}{a}}}{a^{-1}} &(162)\\ &=\lim_{a\to\infty}\frac{-\frac{2}{a^2\sqrt{1-\frac{4}{a}}}}{-\frac{1}{a^2}} &(163)\\ &=\lim_{a\to\infty}\frac{2}{\sqrt{1-\frac{4}{a}}} &(164)\\ &=2, &(165)\end{aligned}$$

where the first equality uses the fact that the limit does not change if we replace $a-1$ with a, the second equality moves the a to the denominator, the third equality uses L'Hospital's rule, the next-to-last equality cancels and collects terms, and the final equality takes the limit directly.

Putting things together, the expression $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ is strictly decreasing from $1/2$ to 0 as a ranges from 4 to ∞. This establishes that the original expression we desired to sign, $\left(\frac{1}{2}(a-1)\left(1\pm\sqrt{1-\frac{4}{a}}\right)-1\right)$, is strictly positive for all $a\in[4,\infty)$. Owing to eq. (154) this implies that $\frac{\gamma^2}{\tau_\zeta}-\tau_\varepsilon-\gamma k>0$ for all admissible values of γ,τ_ε, and τ_ζ. It follows that the derivative in eq. (147) is strictly positive and therefore $-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}+\frac{1}{2}\log\left|I_2+\begin{pmatrix}0&\gamma\\ \gamma&0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right|$ is strictly increasing in y as claimed.

Combined with the above result that the quadratic form that appears in the function F is increasing in y, it follows that $F(y|s_p)$ is increasing in y. This establishes that the welfare-maximizing investment rule invests more than the manager state-by-state in s_p.

Proof of Proposition 7

Much of this proof is analogous to that of Proposition 5, so we highlight mostly the essential differences.

Conjecture that there exists an equilibrium in which the asset price reveals $s_p = \theta - \frac{1}{\beta}Z$ when $s_p \geq \overline{s}_p$ for constants β and $\overline{s}_p$ to be determined. In the conjectured equilibrium, the manager's optimal investment is:

$$y^*(\cdot) = \begin{cases} \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p] - cy & s_p \geq \overline{s}_p \\ \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p < \overline{s}_p] - cy & \text{otherwise} \end{cases}, \tag{166}$$

where we select the threshold $\overline{s}_p \equiv \inf\{s_p : \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$.

Stepping back to $t = 1$, the problem of an arbitrary investor i is

$$\sup_{x\in\mathbb{R}} \mathbb{E}_i\left[-e^{-\gamma(x(V-P)+z_iU+nV)}\right]. \tag{167}$$

Under the conjecture that the investment occurs and the price reveals s_p if and only if $s_p \geq \overline{s}_p$, the investor's conditional beliefs about payoffs are conditionally normal with moments specified in the proof of Proposition 5.

The investor's optimal demand is

$$x^* = \frac{\mathbb{E}_i[V] - P - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n. \tag{168}$$

Now turn to the market-clearing condition and solve for the candidate asset price. For states $s_p < \overline{s}_p$, we have

$$0 = \int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right) di \Rightarrow P = \mu_A - \gamma\frac{1}{\tau_A}n, \tag{169}$$

so that the aggregate order does not reveal any information to the market maker, as conjectured. For states $s_p \geq \overline{s}_p$, we have

$$\int_i \left(\frac{\mathbb{E}_i[V]-P-\gamma\mathbb{C}_i(V,\theta)z_i}{\gamma\mathbb{V}_i(V)} - n\right) di = \int_i \left(\frac{\mu_A+y\mathbb{E}[\theta|s_i,z_i,s_p]-c(y)-P-\gamma\frac{1}{\tau}z_i}{\gamma\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau}\right)} - n\right) di \tag{170}$$

$$= \frac{\mu_A+y\int_i\left(\mathbb{E}[\theta|s_i,z_i,s_p]-\gamma\frac{1}{\tau}z_i\right)di-c(y)-P}{\gamma\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau}\right)} - n, \tag{171}$$

so that the asset price is

$$P = \mu_A + y\int_i \left(\mathbb{E}[\theta|s_i,z_i,s_p] - \gamma\frac{1}{\tau}z_i\right) di - c(y) - \gamma\left(\frac{1}{\tau_A} + y^2\frac{1}{\tau}\right)n. \tag{172}$$

Integrating over i, we have

$$\int_i \left(\mathbb{E}[\theta|s_i,z_i,s_p] - \gamma\frac{1}{\tau}z_i\right) di \tag{173}$$

$$= \int_i \left(\mu_\theta + b_s(s_i - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z \right) \right) + b_z(z_i - \mu_Z) - \gamma \frac{1}{\tau} z_i \right) di \tag{174}$$

$$= \mu_\theta + b_s(\theta - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z \right) \right) + \left(b_z - \gamma \frac{1}{\tau} \right) Z - b_z \mu_Z \tag{175}$$

$$= b_0 + b_s \left(\theta - \frac{\gamma \frac{1}{\tau} - b_z}{b_s} Z \right) + b_p s_p, \tag{176}$$

where the last line groups terms and defines the constant

$$b_0 = \mu_\theta - b_s \mu_\theta - b_p \left(\mu_\theta - \frac{1}{\beta}\mu_Z \right) - b_z \mu_Z. \tag{177}$$

Thus, the conjecture that the equilibrium price reveals a statistic of the form $s_p = \theta - \frac{1}{\beta} Z$ holds if and only if

$$\frac{1}{\beta} = \frac{\gamma \frac{1}{\tau} - b_z}{b_s}, \tag{178}$$

which is identical to the case of Proposition 5 and has solutions

$$\beta = \frac{1}{2\tau_\zeta} \left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon \tau_\zeta} \right). \tag{179}$$

With the precise expression for s_p and our candidate price function is

$$P = \begin{cases} \mu_A - \gamma \frac{1}{\tau_A} n & s_p < \overline{s}_p \\ \mu_A + y \int_i \left(\mathbb{E}[\theta | s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - \gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right) n & s_p \geq \overline{s}_p \end{cases} \tag{180}$$

$$= \begin{cases} \mu_A - \gamma \frac{1}{\tau_A} n & s_p < \overline{s}_p \\ \mu_A + y \left(b_0 + (b_s + b_p) s_p \right) - c(y) - \gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right) n & s_p \geq \overline{s}_p \end{cases}. \tag{181}$$

As long as this function is increasing in s_p the region $s_p \geq \overline{s}_p$ then investors can infer s_p when $s_p \geq \overline{s}_p$, as conjectured.