Investment Efficiency and Welfare with Feedback Effects

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Abstract

Financial markets enable risk sharing and efficient allocation of capital, but these roles can be at odds. We illustrate this tension in a "feedback effects" model with diversely informed, risk-averse investors and a manager who learns from prices before deciding whether to adopt a new project. While managerial learning from prices always improves investment efficiency, it can reduce welfare when the ex-ante net present value of the project is positive. This is because investment decisions change the stock's exposure to underlying shocks and consequently, investors' ability to hedge risk. We show that this tension applies broadly to investment decisions beyond the simple project adoption setting, and outline implications for regulatory policy and incentive provision for managers.

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1 Introduction

Financial markets serve two important roles in the real economy. First, they aggregate information across investors, which affects production decisions and investment efficiency. Second, they enable investors to hedge and share risks. In general, information aggregation, production decisions, and risk-sharing are determined jointly in equilibrium and affect one another. For instance, the adoption of a new project or technology (e.g., electric vehicles or green energy) by a firm affects investors' ability to hedge their exposures to correlated risks (e.g., rising gasoline prices or climate change). At the same time, demand shocks driven by hedging needs can distort the information that prices convey about the profitability of the project, and so influence investment decisions.

Understanding the interaction of these roles is crucial for regulatory policy. Given the difficulty in directly assessing investor welfare, academics and regulators often evaluate policy using proxies that measure price and investment efficiency instead. However, the link between such measures and welfare is less clear when risk-sharing is also important. This naturally leads to a number of questions. Do more informed investment decisions necessarily improve welfare in the presence of risk-sharing motives for trade? Does a manager who bases real decisions on the information in prices improve investor welfare relative to one who ignores such information? When do changes in the information environment that improve investment efficiency also improve welfare?

To answer these questions, we develop a feedback effects model in which a manager, who seeks to maximize firm value, can condition on prices when choosing whether to invest in a new project. Investors are risk averse, own shares of the firm, and observe dispersed information about the profitability of the new project. They are also exposed to endowment shocks that are correlated with the project payoff. The stock price is set by a competitive, risk-neutral market maker.[1] We distinguish between *investment efficiency*, which reflects the (ex ante) expected firm value, and *welfare*, which reflects the (ex-ante) expected utility for investors. Notably, welfare increases in both the expected value of the firm and the ability of investors to hedge their endowment shocks.

We compare the optimal investment rule with feedback from market prices, to the investment rule without feedback. Without feedback, the manager invests in the new project if and only if its ex-ante net present value (NPV) is positive. We show that feedback unambiguously

[1] We restrict attention to a setting with binary investment decisions and a risk-neutral market maker in the benchmark analysis in order to clarify the key forces, but relax these assumptions in Section 6. One contribution of our paper is that we are able to incorporate all these features in an analytically tractable, yet general, model and explicitly characterize welfare by extending the generalized linear equilibrium approach of Breon-Drish (2015).

improves investment efficiency, because more informed investment decisions always increase the expected value of the firm. Feedback also increases investor welfare when the ex-ante NPV of the project is negative. However, we show that feedback reduces welfare when (i) the ex-ante NPV of the project is positive and (ii) investors' initial share endowments are sufficiently small.

To see the intuition for this result, note that the investment decision affects welfare via two channels. First, the investment affects the expected value of the firm, which in turn, influences welfare because investors own shares in the firm. We refer to this as the **valuation channel**. This channel is standard in the existing feedback literature. The key insight of our analysis is that there is a second channel. Investment decisions affect the firm's exposures to underlying shocks, which affect investors' ability to use the stock as a hedge. Specifically, more investment in the new project makes the stock more sensitive to these cash flows and consequently, more useful for hedging endowment shocks that are correlated with these cash flows.[2] We refer to this as the **hedging channel**.

Since feedback improves investment efficiency, its impact on welfare via the valuation channel is always positive. In contrast, feedback's impact on welfare via the hedging channel hinges on the project's ex-ante NPV. If the project's NPV is negative, the manager does not invest in the project without feedback and the stock is not useful for hedging. In this case, feedback increases investment by the manager when the information in prices is sufficiently positive, which improves investors' ability to use the stock for hedging, and so increases welfare via the hedging channel.

On the other hand, when the project's NPV is positive, the manager always invests in the absence of feedback. In this case, feedback reduces investment by the manager when the information revealed by prices is sufficiently negative. This leads to a decrease in welfare via the hedging channel because the stock becomes less useful for hedging purposes. We show that the negative impact of the hedging channel dominates the positive effect of the valuation channel when the firm is relatively small, i.e., when investors' endowments of the firm are small relative to their hedging motives.[3] Intuitively, the impact of the firm's investment efficiency on investors' expected payoffs is proportional to the firm's size, while the efficacy of the firm's stock as a hedge is independent of its size.

[2]As we discuss further in the next section, this effect is distinct from, but related to, a channel often termed the Hirshleifer (1971) effect, where more information about the payoff of a stock makes it less useful as a hedging tool. Note that the latter channel is also present in our model, but also arises in settings where the firm's exposure to shocks is held fixed.

[3]Formally, our model has a single risky asset. In the context of a richer model with multiple assets, the size of the share endowment proxies for the weight of the firm in the market portfolio. When a particular firm is a small, the efficiency of its investment decision has little effect on aggregate investment efficiency in the economy.

Our analysis illustrates that improvements in price informativeness or investment efficiency need not be associated with improvements in welfare. As such, it complements the existing literature that has pointed out how more informative prices can (i) reduce welfare (e.g., Hirshleifer (1971), Dow and Rahi (2003)), and (ii) lower investment efficiency (e.g., Dow and Gorton (1997), Bond, Edmans, and Goldstein (2012)).[4] In our setting, even though higher price informativeness leads to higher investment efficiency and higher firm value, it can still reduce welfare.

As such, our analysis also speaks to the recent debate about a firm's objectives that distinguishes between shareholder value maximization and welfare maximization (e.g., Hart and Zingales (2017)). Specifically, we show that dampening the feedback effect may improve shareholder welfare for firms with (i) small size and dispersed ownership, and (ii) positive NPV projects that are in new lines of business. In these cases, value maximization does not necessarily correspond to shareholder welfare maximization. On the other hand, encouraging managers to pay attention to prices when making investment decisions is particularly important for firms that (i) are large and have concentrated ownership, (ii) have speculative projects with negative (ex ante) NPV, or (iii) have positive NPV projects that are in the same line of business as existing assets or other firms.

As in other rational expectations models with noisy endowments (e.g., Ganguli and Yang (2009), Manzano and Vives (2011)), our model gives rise to two financial market equilibria. Our main results (i.e., feedback improves investment efficiency but can reduce welfare) obtain in both equilibria. However, we follow the literature and focus on the more stable equilibrium to develop additional implications of the model.[5] For instance, with feedback, we show that an increase in the precision of private information can reduce welfare when this information is noisy, but increases welfare when it is sufficiently precise. In contrast, we find that better information about assets in place tends to uniformly increase welfare. Together, these results highlight the importance of nuanced disclosure policy that distinguishes between different types of information.

While our benchmark model considers a stylized setting for tractability and expositional clarity, our key takeaways are robust to alternate assumptions and extensions. For instance, we show that our results remain qualitatively unchanged if we replace the firm's binary investment decision (invest vs. not) with a general investment technology. Similarly, we can

[4]Bond et al. (2012) distinguish between the notions of forecasting price efficiency (FPE), which measures the extent to which prices convey information about fundamentals, and revelatory price efficiency (RPE), which measures the extent to which prices reveal information relevant for real decision making.

[5]Using the terminology of Ganguli and Yang (2009), we focus on the *SUB* equilibrium in which information acquisition is a strategic substitute. As Manzano and Vives (2011) argue, this equilibrium has desirable stability properties. Importantly, in this equilibrium, more precise private information leads to more informative prices.

generalize the specification of investors' endowment shocks. The key feature for our result is that the investment decision changes the exposure of the firm's cash flows to underlying shocks, and consequently, affects how useful the stock is for hedging. The underlying tension between valuation and hedging also obtains in a setting in which the price is determined by market clearing, as opposed to by a risk-neutral competitive market maker. However, the overall impact of feedback on welfare is more nuanced in this case.[6] Finally, we also discuss how our results change if investors information and endowment shocks are correlated with assets in place.

The rest of the paper is organized as follows. The next section discusses the related literature and clarifies our contribution. Section 3 presents the benchmark model and discusses key assumptions. Section 4 characterizes the equilibrium in our setting and Section 5 presents our main results regarding investment efficiency and welfare. Section 6 explores the robustness of our analysis and discusses some extensions and Section 7 concludes. A general characterization of welfare in our setting is presented in Appendix A and proofs of our results are in Appendix B.

2 Related Literature

Our paper adds to the literature on feedback effects (see Bond et al. (2012) and Goldstein and Yang (2017) for recent surveys). Our primary contribution is to develop a tractable and general framework that allows for a unified analysis of investor welfare in the presence of feedback effects and risk-sharing. Most existing models of feedback effects assume either risk-neutral investors or the presence of noise traders with unmodeled utility functions, which makes welfare difficult to characterize. Instead, we assume that investors are risk-averse and incorporate noise in prices via their endowment shocks (e.g., Diamond and Verrecchia (1981), Wang (1994), Ganguli and Yang (2009), Manzano and Vives (2011) and Bond and Garcia (2018)).

The most closely related papers are Dow and Rahi (2003), Gervais and Strobl (2021), and Hapnes (2020). Dow and Rahi (2003) explore how increases in informed trading affect investment efficiency and risk-sharing in a setting with feedback from market prices to investment decisions. They argue that investment efficiency always improves with more informed trading, but risk-sharing may worsen due to the Hirshleifer (1971) effect. Our analysis,

[6]As we discuss in Section 6.2, this is because the market clearing price reflects not just the aggregate private information across investors, but also a risk premium term (which depends on the per-capita endowment of shares). This risk premium term affects welfare through two additional channels - an endowment channel which lowers investor wealth (since shares are worth less) and a cost of hedging channel (which affects the cost of using the stock as a hedge).

which we view as complementary, is distinct in its focus and results. Our primary goal is to understand when managerial learning from prices, and better informed investment decisions more generally, improves welfare of investors. This distinct focus has different implications for regulatory policy, as we discuss in Section 5.

Furthermore, in contrast to our setting, the attractiveness of the investment opportunity faced by the firm does not affect investor welfare in Dow and Rahi (2003). This reflects two crucial differences between our model and theirs. First, the investors in their model are not endowed with shares of the firm itself, and consequently, the impact of the investment decision on expected cash flows has no impact on welfare (i.e., there is no analogous valuation channel in investor welfare in their model).[7] Second, they do not analyze the effect of the firm's investment decision on the stock's usefulness as a hedge. In contrast, we study, via the hedging channel, how the firm's investment decision *endogenously* affects market completeness in our setting, which also distinguishes our analysis from Marín and Rahi (1999), Marín and Rahi (2000), and Eckwert and Zilcha (2003), who consider how *exogenous* differences in market completeness influence investor welfare.

Gervais and Strobl (2021) consider a related setting with feedback from prices where investors are endowed with shares of firms and can either directly trade these stocks or allocate their money to an informed money manager. They study how the gross and net performance of the actively managed fund compares with the market portfolio and study how the presence of an informed money manager affects welfare. We view their analysis as complementary to ours. As in our model, they show that improvements in firm information quality generally improve investment efficiency and can improve welfare through a valuation channel (i.e., by making investment decisions more informationally efficient). However, more informative prices in their model can reduce welfare via a classic Hirshleifer (1971) effect. Our analysis highlights a distinct effect (the hedging channel) that can arise when investment decisions affect the exposure of the firm's stock to underlying risks.[8]

Hapnes (2020) characterizes the financial equilibrium in a specific two-types CARA-Normal model à la Grossman and Stiglitz (1980), with feedback and risk-averse traders. However, his focus is on managerial investment behavior and investor information acquisition;

[7]While they specify that the asset is in unit supply, this does not play a direct role in the welfare analysis since traders have endowments that are correlated with the shocks driving investment profitability, not shares of the firm itself. This is apparent from the observation that their expressions for investor utility do not depend on the cost of investment.

[8]In Gervais and Strobl (2021), the production function has a linear-quadratic form, which implies the level of investment is linear in the conditional expectation of fundamentals. Given normally distributed payoffs, this ensures that investment is non-zero (although potentially negative) almost everywhere, which implies that the stock is always useful for hedging. Furthermore, the investment opportunity comprises the entire firm value, which implies that changes in the investment level, on the intensive margin, do not affect the effectiveness of the stock as a hedge.

the analysis does not consider the effect of feedback on welfare. Furthermore, because there are noise traders in the model and two asymmetric classes of rational traders, it is difficult to define an unambiguous welfare measure. This contrasts with our model, in which all agents in the economy maximize well-defined utility functions and are ex-ante symmetric.

A key feature of our model is that, because the firm learns from price and adjusts investment accordingly, its expected cash flow, and thus its price, is a non-linear function of investors' private information. This implies the payoff distribution is non-normal and generally truncated below, which breaks the linearity important for solving standard CARA-normal models. Prior work that studies feedback in alternative settings has analyzed related non-linearities. Albagli, Hellwig, and Tsyvinski (2011) generate an analogous non-linearity and study its implications for how managerial incentives based on prices versus cash flows impact investment efficiency. Davis and Gondhi (2019) find that debt leads to a non-linear relationship between information and prices, and explore its impact on agency problems between equity and debt holders. Dow, Goldstein, and Guembel (2017) show that non-linearities in a feedback setting can lead to multiplicity in investors' equilibrium information acquisition decisions.

Similar to our findings, other papers studying discrete investment choice emphasize the importance of the firm's "default" investment decision (i.e., the decision the firm would make in the absence of feedback) in determining market outcomes. Dow et al. (2017) show that the nature of investors' equilibrium information acquisition decisions in a feedback setting hinges on whether the firm defaults to a risky or a riskless project. Davis and Gondhi (2019) show that complementarity in learning by investors depends not only on the default investment decision, but also on the correlation of the investment return and cash flows from assets in place. Goldstein, Schneemeier, and Yang (2020) show that investors seek to acquire the same information possessed by management for NPV positive projects, but different information for NPV negative projects. Our analysis suggests that feedback from prices is always socially desirable for (ex-ante) negative NPV projects, but may not be for positive NPV projects.

3 The Benchmark Model

Payoffs. There are three dates $t \in \{1, 2, 3\}$ and two securities. The risk-free security is normalized to the numeraire. The risky security is a claim to the terminal cash flows V generated by the firm at date $t = 3$, and trades at a price P.

The firm. The firm generates cash flows $A \sim N(\mu_A, \tau_A)$ from assets in place.[9] In addition, it is deciding whether to invest in a new project. The investment decision is binary,

[9] We let $\tau_{(\cdot)}$ denote the unconditional precision and $\sigma^2_{(\cdot)}$ the unconditional variance of all random variables.

and denoted by $y \in Y \equiv \{0, 1\}$. The gross payoff to the investment is given by $\theta \sim N(\mu_\theta, \tau_\theta)$, where θ is independent of A, and the investment cost is $c(y)$, where $c(1) = c > 0 = c(0)$. Hence, the total cash flow given an investment level y is

$$V(y) = A + y\theta - c(y). \tag{1}$$

Investors. There is a continuum of investors, indexed by $i \in [0, 1]$, with CARA utility over terminal wealth with risk aversion γ. Investor i has initial endowment of n shares of the risky asset and $z_i = Z + \zeta_i$, units of exposure to a non-tradeable asset that is perfectly correlated with θ, and where $Z \sim N(\mu_Z, \tau_Z)$ and $\zeta_i \sim N(0, \tau_\zeta)$ are independent across each other and all other random variables.

Investor i also observes a signal $s_i = \theta + \varepsilon_i$, where the errors $\varepsilon_i \sim N(0, \tau_\varepsilon)$ are independent of all other random variables and each other. Let $\mathcal{F}_i = \sigma(s_i, z_i, P)$ denote investor i's information set at the trading stage, with associated expectation, covariance, and variance operators, $\mathbb{E}_i[\cdot]$, $\mathbb{C}_i[\cdot]$, and $\mathbb{V}_i[\cdot]$, respectively. Then, investor i chooses *trade* X_i to maximize her expected utility i.e.,

$$U_i \equiv \sup_{x \in \mathbb{R}} \mathbb{E}_i \left[-e^{-\gamma(x(V-P)+z_i\theta+nV)} \right]. \tag{2}$$

Timing of events. Figure 1 summarizes the timing of events. At date $t = 1$, investors observe their signals and endowments and submit trades $X_i(s_i, z_i, P)$. At date $t = 2$, the firm manager chooses the optimal level of investment $y(P)$, given the information conveyed by the equilibrium price P, to maximize the expected value of the firm subject to investment costs i.e.,

$$y(P) \equiv \underset{y \in Y}{\arg\max}\ \mathbb{E}[V(y)|P]. \tag{3}$$

At date 3, the firm's cash flows are realized and the risky security pays off the terminal dividend V.

Figure 1: Timeline of events

$t = 1$	$t = 2$	$t = 3$
Investor i observes s_i, z_i submits trades X_i Asset price is P	Firm chooses $y(P)$	Firm pays off $V(y) = A + y\theta - c(y)$

Equilibrium. An equilibrium consists of a price P, trades $\{X_i\}$, and an investment rule $y(P)$ such that (i) trade X_i maximizes investor i's expected utility, given the price and investment rule, (ii) the investment rule $y(P)$ maximizes firm value, and (iii) the equilibrium price P reflects the conditional expectation of the firm's cash flows, i.e.,

$$P = \mathbb{E}[V(y(P))|P]. \tag{4}$$

3.1 Discussion of Assumptions

The assumption that the non-tradeable asset is perfectly correlated with the payoff of the new project θ can be relaxed without changing our results. Similarly, the investment decision can be generalized without qualitatively changing our results. In Section 6.1, we show that the equilibrium characterization in Proposition 1 obtains with the appropriate modifications when we generalize (i) investment opportunities and (ii) endowment payoffs. Moreover, the key tension between investment efficiency and hedging obtains naturally in this setting.

The assumption that the investment must be positive (i.e., we do not consider dis-investment) can be relaxed by re-interpreting y. Suppose that the firm begins with y_0 invested in the project and must make an incremental investment/dis-investment decision $\hat{y} \geq -y_0$. Then defining $y = y_0 + \hat{y} \geq 0$ as the total amount invested in the project and re-normalizing the cost function so that $c(y_0) = 0$, with the understanding that negative costs correspond to the proceeds from disinvestment, our analysis goes through essentially unchanged but with y interpreted at the total amount invested in the project, including any pre-existing investment.[10] The assumption that the firm's assets in place and the project are uncorrelated can be relaxed in a similar manner. If they are instead positively correlated, by projecting the firm's assets in place onto θ, we can write $A = \lambda\theta + \varepsilon_A$ for $\lambda > 0$ and $\mathbb{C}(\varepsilon_A, \theta) = 0$. Thus, re-defining the firm's assets in place to equal ε_A, this case is equivalent to the firm having a pre-existing investment of λ.[11]

The assumption that the equilibrium price reflects the conditional expectation of the firm's cash flow (as in (4)) makes transparent the key tension between investment efficiency and risk-sharing. One can interpret the price as being determined by a competitive, risk-neutral market maker (e.g., Kyle (1985), Hirshleifer, Subrahmanyam, and Titman (1994))

[10]Technically, this requires generalizing the firm's investment choice set $Y = \{0, 1\}$ to $Y = \{y_0, y_0 + \hat{y}\}$. However, the proofs of our results in Sections 4 and 5 readily apply to the more general case in which Y consists of any two distinct non-negative constants.

[11]Gao and Liang (2013) study a feedback-effects model with an asset in place and growth opportunity that are subject to the same underlying shock. This can be accommodated in our model by letting $\tau_A \to \infty$ and setting the pre-existing investment level to equal the size of the firm's assets in place. As in their model, in this case, an increase in the firm's growth opportunity relative to its asset in place raises the importance of feedback in driving the firm's expected cash flows.

who conditions on all observable public information (including the submitted demand schedules). In Section 6.2, we characterize the equilibrium in a fully general version of our model with a general investment opportunity set, and in which prices are set by market clearing, and discuss how our welfare results change due to the presence of a risk-premium in the equilibrium asset price.

Finally, in our benchmark model, investors have private information about the new project being considered, and their endowment shocks are correlated with the cash flows of this project. In Section 6.3, we discuss how our results would change if investors' private information and/or hedging needs were instead correlated with the firm's assets in place.

4 Equilibrium

Conjecture a threshold $\bar{s}_p$ such that the date 1 price P reveals a linear signal about θ of the form:

$$s_p = \theta - \frac{1}{\beta} Z, \tag{5}$$

when $s_p \geq \bar{s}_p$ and reveals noting otherwise. Here β is a constant that is determined in equilibrium. Denote the precision of the signal s_p by $\tau_p = \text{var}(s_p|\theta)$, which will be determined in equilibrium.

In order for our conjecture to be confirmed, $\bar{s}_p$ must be such that the manager finds it optimal to invest when she infers s_p from the price and to not invest when she infers $s_p < \bar{s}_p$. That is, $\bar{s}_p$ must be such that $\mathbb{E}[\theta|s_p < \bar{s}_p] < c$ and $\mathbb{E}[\theta|s_p] \geq c$ for $s_p \geq \bar{s}_p$. The unique continuous equilibrium of this class is characterized by the threshold at which the investment has exactly zero NPV:

$$\mathbb{E}[\theta|s_p = \bar{s}_p] = \frac{\tau_\theta \mu_\theta + \tau_p \bar{s}_p}{\tau_\theta + \tau_p} = c. \tag{6}$$

We focus on this equilibrium threshold in the following analysis.[12] Denote the optimal investment choice by $y^*(s_p) = \mathbf{1}_{\{s_p \geq \bar{s}_p\}}$ and the maximized firm value by $V^* = V(y^*(s_p))$.

Given the joint normality of all random variables, the conditional distribution of θ given

[12] There also exists a continuum of equilibria for which price exhibits a discontinuity in s_p. Let $\hat{s}_p$ denote the unique solution to $\mathbb{E}[\theta|s_p < \hat{s}_p] = c$. Then, there is an equilibrium in which the manager invests if and only if $s_p > T$ for any threshold $T \in [\bar{s}_p, \hat{s}_p]$, where $\bar{s}_p$ is defined in equation (6). As such equilibria lead to strictly less investment relative to the continuous equilibrium we study, feedback's effect on both investment efficiency and welfare is attenuated relative to the equilibrium we study. Details are available upon request.

(s_i, z_i, n_i, s_p) is itself normal with mean and variance

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{7}$$

where the coefficients b_s, b_p and b_z, and the precision τ, are determined in equilibrium. Note that since all investors are symmetrically informed, $\mathbb{V}_i[\theta]$ is identical across all investors. Given the (conjectured) investment rule $y = y^*(s_p)$, investor i's optimal trade X_i maximizes (2) and is given by:

$$X_i = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau} z_i\right) - c - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau} y^2\right)} - n. \tag{8}$$

Finally, the equilibrium price P is determined by

$$P = \mathbb{E}[V(y^*(s_p))|s_p] = \begin{cases} \mu_A + \mathbb{E}[\theta|s_p] - c & \text{if } s_p \geq \bar{s}_p \\ \mu_A & \text{if } s_p < \bar{s}_p. \end{cases} \tag{9}$$

Given the feedback effect from the equilibrium price to the investment rule, the price is no longer a linear signal about fundamentals. However, as in Breon-Drish (2015), we can verify that it is an invertible function of s_p, which is a linear signal about fundamentals.

Proposition 1. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let*

$$\beta^{SUB} \equiv \frac{1}{2\tau_\zeta}\left(\gamma - \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right), \quad \textit{and} \quad \beta^{COM} \equiv \frac{1}{2\tau_\zeta}\left(\gamma + \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right). \tag{10}$$

There exists a equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that
(i) the investment rule is $y^*(s_p) = \mathbf{1}_{\{s_p \geq \bar{s}_p\}}$ where the threshold $\bar{s}_p$ is given by (6),
(ii) the optimal demand is characterized by (7) and (8), and
(iii) the equilibrium price is given by

$$P(s_p) = \mu_A + y^*(s_p)\left(\mathbb{E}[\theta|s_p] - c\right), \tag{11}$$

where $s_p = \theta - \frac{1}{\beta}Z$, and $\mathbb{E}[\theta|s_p]$ follows from Bayesian updating for normal distributions, as characterized in the appendix.

As in Ganguli and Yang (2009) and Manzano and Vives (2011), the financial market features two possible equilibria corresponding to the two solutions of β. This multiplicity arises because investors' beliefs about fundamentals depend on their endowment shocks z_i. We focus on the equilibrium in which information acquisition is a substitute (i.e., $\beta = \beta^{SUB}$)

for our comparative statics results because it has (i) more intuitive properties, and (ii) is more stable.[13] In particular, an increase in the precision of private information τ_ε decreases β and so increases the informativeness of the price signal s_p in this equilibrium. However, it is important to note that our main results (i.e., Propositions 1 and Proposition 4) hold for both equilibria.

5 Investment Efficiency and Welfare

In this section, we characterize how feedback from market prices to investment decisions affects investment efficiency and welfare. First, note that without feedback, the manager would invest in the project if and only if the ex-ante net present value of the project is positive i.e., if $\mu_\theta \geq c$. Let us denote the no-feedback investment rule by $y_{NF} = \mathbf{1}_{\{\mu_\theta \geq c\}}$.

A natural measure of investment efficiency is the expected value of the firm under the chosen investment rule. Feedback from market prices to investment decisions always improves investment efficiency in our setting, as summarized by the following result.

Proposition 2. *The expected value of the firm is higher with feedback than without, i.e.,*

$$\mathbb{E}[V(y_{NF})] \leq \mathbb{E}[V(y(s_p))].$$

Moreover, in the β^{SUB} equilibrium, the increase in expected value with feedback increases with the precision of private signals and the precision of endowment shocks τ_ζ, but decreases with γ.

The first part of the result follows from the observation that managers can always choose to ignore the information in prices even if they can observe it. The second part follows from the fact that the increase in expected value increases with the posterior precision of the managers' beliefs about θ.

In our setting, welfare can be measured by the average ex-ante expected utility of investors in the economy i.e.,

$$W = \int_i \mathbb{E}\left[U_i\right] di.$$

Since all investors are ex-ante symmetric, this is equivalent to the unconditional expected utility of investors i.e.,

$$W(y) = \mathbb{E}[U_i] = \mathbb{E}\left[-e^{-\gamma(W_0+(X+n)(A+y(\theta-c))-XP+z_i\theta)}\right], \tag{12}$$

[13]See Manzano and Vives (2011) for the notion of stability and corresponding arguments.

where the argument y emphasizes the dependence of welfare on the investment rule. We will also write $W(y|s_p)$ to denote the conditional expected utility $\mathbb{E}[U_i|s_p]$.

We begin with an intermediate result which establishes that the equilibrium investment rule $y^*(s_p)$ invests (weakly) less state-by-state than the welfare maximizing s_p-dependent investment rule.

Proposition 3. *The investment rule $y(s_p)$ that maximizes expected trader utility $W(y)$ is (weakly) higher state-by-state than the equilibrium investment rule $y^*(s_p)$.*

The proposition implies that the welfare-maximizing investment rule has a lower threshold for investment than the firm-value maximizing rule. This highlights the key tension between investment efficiency and welfare in our setting. Intuitively, the manager does not invest for sufficiently low realizations of the signal s_p (i.e., when $s_p < \bar{s}_p$). While this is efficient from the perspective of maximizing firm value (i.e., rejecting projects with negative NPV, conditional on s_p), it limits the ability of investors to hedge their exposure to θ by trading the stock. Hence, the welfare-maximizing rule accepts some projects with slightly negative NPV because they nevertheless allow investors to hedge θ risk.

In our benchmark model, this tension between investment efficiency and welfare is particularly stark because when the manager does not invest, the firm's cash flows do not depend on θ and so the stock is useless for hedging. However, we illustrate in Section 6.1, this effect arises more generally in settings where the investment decision is not binary. In general, lower investment y reduces the sensitivity of the firm's cash flows, $V = A + y\theta - c(y)$, to θ, which makes the stock a less effective hedge for investors' endowment risk.

Given this observation, we present the main result of our analysis.

Proposition 4. *(i) Suppose the ex-ante NPV of the project is negative i.e., $\mu_\theta < c$. Then, welfare is higher with feedback than with no feedback i.e., $W(y^*(s_p)) \geq W(y_{NF})$.*
(ii) Suppose the ex-ante NPV of the project is positive i.e., $\mu_\theta \geq c$, and the per-capita endowment of shares n is sufficiently small. Then, welfare is higher without feedback than with feedback i.e., $W(y^*(s_p)) \leq W(y_{NF})$.

To gain some intuition for this result, note that the firm's investment rule affects welfare through two channels. First, when the firm decides to invest in the project, it affects the cash flows that investors receive through their endowment n of shares. We refer to this as the **valuation channel**. Importantly, feedback from market prices to the firm's investment decisions increases the expected value of the firm and so increases welfare through this channel.

Second, the firm's choice of whether or not to invest in the project impacts investors' ability to use the risky security to hedge their exposure to θ risk. We refer to this as the

hedging channel. This channel implies that all else equal, investors are better off when the firm invests in the project than when it does not.

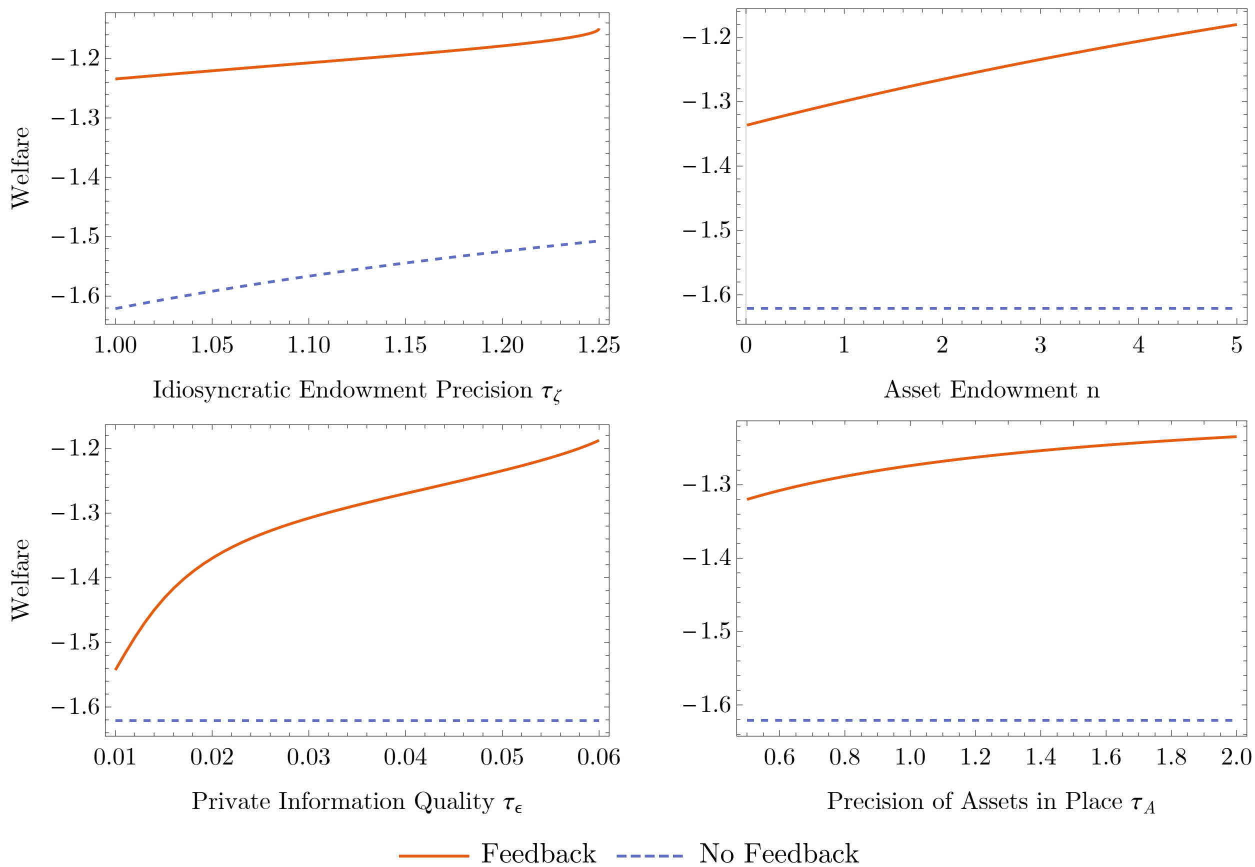
When the ex-ante NPV of the project is negative, the no-feedback investment rule leads to no investment in the project. In this case, feedback from market prices to the investment rule improves welfare through both the valuation and hedging channels. Investors are better off with feedback because (i) the expected value of their firm is higher, and (ii) they are able to hedge their θ risk by trading the risky security.

However, when the ex-ante NPV of the project is positive, the no-feedback investment rule leads the firm to always invest. In this case, feedback implies the firm does not invest for sufficiently low realizations of s_p, and so can have an ambiguous impact on welfare. On the one hand, feedback improves the expected value of the firm and so increases welfare via the valuation channel. On the other hand, because it leads to no-investment in some states, feedback reduces the ability of investors to use the risky security as a hedge, and so reduces welfare via the hedging channel. When the per-capita endowment of shares n is relatively large to hedging needs $(1/\tau_\zeta)$, the investment channel dominates and feedback improves welfare. However, when n is sufficiently small, the hedging channel dominates and feedback can reduce welfare.

Figures 2 and 3 provide an illustration of the above result, and depict how the difference in welfare with and without feedback depends on the underlying parameters for the β^{SUB} equilibrium. When the project is negative NPV, Figure 2 verifies that feedback always improves welfare. Moreover, as the plots illustrate, the improvement in welfare increases in the precision of private information τ_ε and the per-capita asset endowment n, but decreases in the precision of endowment shocks τ_ζ and the prior precision about fundamentals τ_θ. These results are intuitive and follow from the fact that feedback improves welfare through both the valuation and hedging channels.

When the project has positive NPV, however, the relative strength of the valuation and hedging channels determines the direction of these comparative statics. As Figure 3 illustrates, feedback increases welfare only when either endowment precision or the asset endowment is high. This follows because an increase in endowment precision attenuates the hedging channel, while an increase in the asset endowment raises the valuation effect. Moreover, feedback increases welfare only when private information quality τ_ε is high. This is a consequence of the Hirshleifer (1971) effect: when τ_ε is high, prices accurately reflect the firm's true value, such that investors cannot use the stock to share risk. Finally, the impact of feedback on welfare is positive when assets in place are imprecise. Intuitively, when the firm's assets in place drive most of the firm's value, even if the firm invests in the new project, its usefulness as a hedge is limited.

Figure 2: This figure plots investor welfare as a function of the underlying parameters when the project has negative NPV ex ante (i.e., $\mu_\theta < c$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.5; c = 0.55;\ \gamma = 0.5;\ \tau_\theta = 1;\ \tau_\varepsilon = 0.05;\ \tau_\zeta = 1;\ \tau_z = 1;\ n = 3;$ $\mu_A = 0;\ \tau_A = 2;\ \mu_Z = 0$.

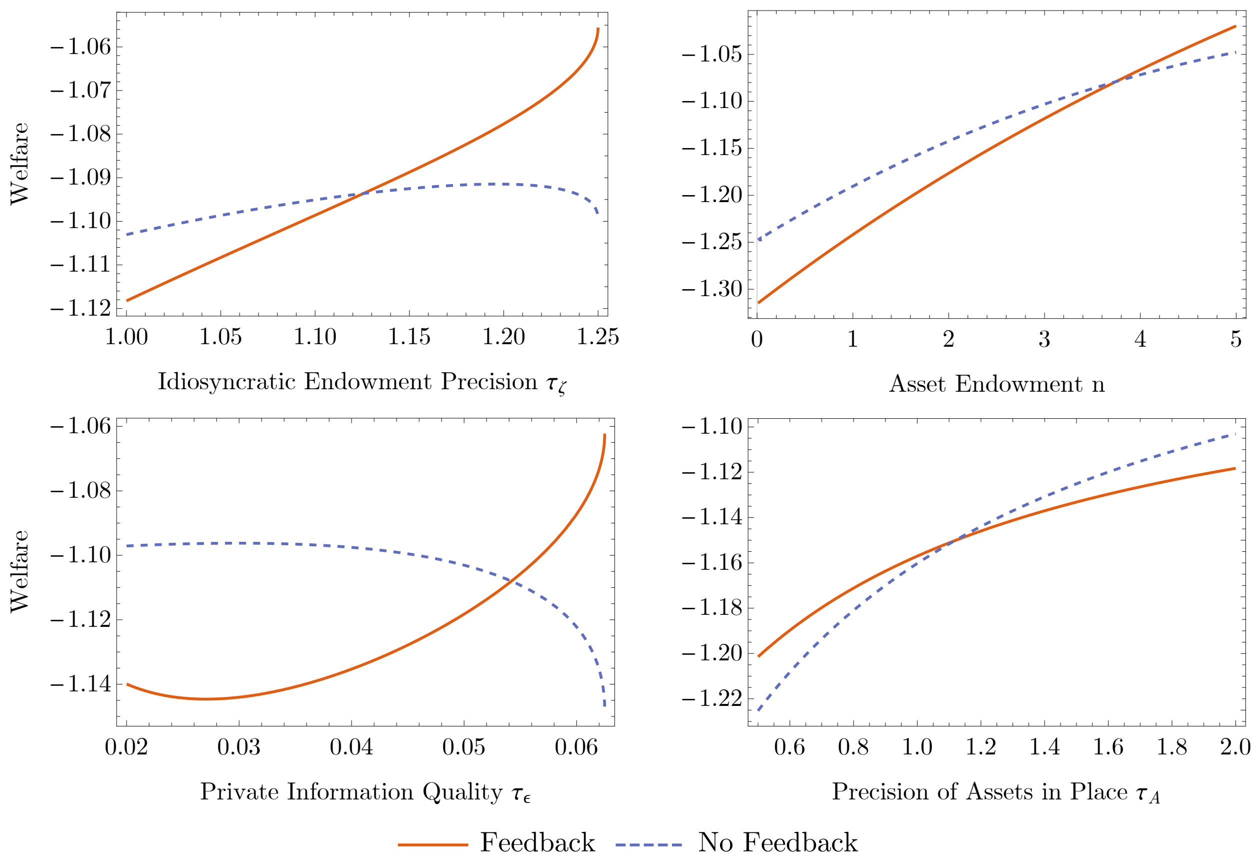


5.1 Implications and policy interventions

Our results highlight that higher investment efficiency does not always translate to higher welfare. This complements the insights of Bond et al. (2012) who make the point that more informative prices (i.e., higher forecasting price efficiency) do not always lead to more efficient investment decisions (i.e., higher revelatory price efficiency). Moreover, while feedback from prices always improves investment efficiency in our setting, it need not increase welfare. As such, our model cautions against exclusively focusing on measures of price informativeness and investment efficiency when evaluating policy changes, especially when such changes have an impact on the ability of investors to hedge their exposures to shocks.

Our results have clear predictions on when managers should be encouraged to update their beliefs about future investments from market prices, and when such feedback should be discouraged. Feedback is always socially beneficial for risky projects with low ex-ante

Figure 3: This figure plots investor welfare as a function of the underlying parameters when the project has positive NPV ex ante (i.e., $\mu_\theta > c$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.5; c = 0.45$; $\gamma = 0.5$; $\tau_\theta = 1$; $\tau_\varepsilon = 0.05$; $\tau_\zeta = 1$; $\tau_z = 1$; $n = 3$; $\mu_A = 0$; $\tau_A = 2$; $\mu_Z = 0$.



probability of success (i.e., negative ex-ante NPV with low τ_θ). Feedback is also beneficial for ex-ante profitable projects with relatively low uncertainty, especially when such projects are not sensitive to systematic or macroeconomic risks (high τ_ζ). Moreover, large firms (high n) with well-informed investors (high τ_ε) benefit from feedback.

In contrast, feedback tends to reduce welfare when projects that are ex-ante profitable but risky, and correlated with macroeconomic risks. Moreover, these effects are amplified when investors are not well-informed, and per-capita ownership is low (i.e., the firm is small). As such, investors in small startups with risky bets may be better off when their managers dismiss the firm's stock price as a source of information.[14]

In the equilibrium we focus on (i.e., the β^{SUB} equilibrium), improving the precision of private information can decrease welfare, even though it improves investment efficiency. In

[14]This is consistent with the rationale of Elon Musk in August 2018, when he announced he would consider taking Tesla private. See https://www.tesla.com/blog/taking-tesla-private.

the absence of feedback, more private information about θ reduces investors ability to share risks (via the standard Hirshleifer (1971) effect). In the presence of feedback, this channel still operates when the precision of private signals is sufficiently low — in such cases, the benefit from more efficient decisions is dominated by the loss in welfare from reduced risk sharing.

On the other hand, improving information about assets in place tends to improve welfare in our setting. This is because less uncertainty about cash flows generated by existing assets implies that the stock is less noisy hedge to θ shocks. As such, our model highlights the importance of nuanced regulations that encourage disclosure and information production about a firm's assets in place, but not necessarily about future projects and growth options.[15]

6 Robustness and extensions

6.1 General investment opportunities and endowment shocks

Our baseline setting considers only the situation in which the manager must make a discrete choice of whether to pursue the project, and in which traders' non-tradeable endowment payoff is perfectly correlated with the project payoff. In this section, we generalize our results to allow arbitrary investment opportunities $Y \subseteq [0, \infty)$, with increasing investment cost function $c(y)$, and imperfectly correlated endowment payoffs $U = \theta + \xi$, with $\xi \sim N(0, \tau_\xi)$.

The following Proposition formally characterizes optimal investment and the financial market equilibrium in the general case.[16]

Proposition 5. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let*

$$\beta^{SUB} \equiv \frac{1}{2\tau_\zeta}\left(\gamma - \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right), \quad \text{and} \quad \beta^{COM} \equiv \frac{1}{2\tau_\zeta}\left(\gamma + \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right). \tag{13}$$

There exists a equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that

[15] This is reminiscent of the observations in Goldstein and Yang (2019) who suggest that disclosures about some components of fundamentals enhance investment efficiency, while disclosures reduce it.

[16] To avoid unnecessary technical detail, we assume that the investment opportunity set Y and cost function $c(y)$ are such that for each s_p there exists a unique, finite y that solves $\max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y)$. That is, the optimal investment correspondence is single-valued and finite. Sufficient conditions to ensure this are straightforward (e.g., $c(y)$ strictly convex, and $Y = [0, \infty)$ or Y compact). A complete enumeration of necessary and sufficient conditions would take us too far afield and add little economic insight, so we do not pursue it here.

(i) the investment rule is

$$y^*(s_p) = \begin{cases} 0 & s_p < \bar{s}_p \\ \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) & s_p \geq \bar{s}_p \end{cases} \tag{14}$$

where the threshold $\bar{s}_p = \inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$*,*
(ii) the optimal demand is

$$X_i = \frac{\mathbb{E}_i[V(y)] - \mathbb{C}_i(V(y),\theta)z_i - P}{\gamma \mathbb{V}_i(V(y))} - n = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau}z_i\right) - c(y) - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau}y^2\right)} - n. \tag{15}$$

where

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{16}$$

and
(iii) the equilibrium price is given by

$$P(s_p) = \begin{cases} \mu_A & s_p < \bar{s}_p \\ \mu_A + y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p)) & s_p \geq \bar{s}_p \end{cases} \tag{17}$$

where $s_p = \theta - \frac{1}{\beta}Z$ *and* $\mathbb{E}[\theta|s_p]$ *follows from normal-normal updating as characterized in the appendix.*

Importantly, the key tension between the valuation and hedging channels remains in this setting, and a general analogue of Proposition 3 holds in this setting, which we record here.

Proposition 6. *Consider the setting with general investment opportunities and endowment payoffs introduced above. The investment rule* $y(s_p)$ *that maximizes expected trader utility* $W(y)$ *is (weakly) higher state-by-state than the equilibrium investment rule* $y^*(s_p)$*.*

The above result implies that by reducing investment in some states, feedback from prices can lead to lower welfare even though it improves investment efficiency. This will be the case when n is sufficiently small, and the production technology is such that feedback reduces investment.

6.2 Market clearing price

Our benchmark analysis, and its generalization in Section 6.1, assumes that the price reflects the conditional expected value of cash flows, given publicly available information. We can establish existence of equilibrium (i.e., establish the analog to Proposition 1) in a setting where prices are determined by market clearing as in classic noisy rational expectations models. However, due to the presence of the investment level in the risk premium, we must generally impose additional parameter restrictions to ensure that the price function is monotone, as required in any equilibrium. Alternatively, if one assumes that traders are able to condition on both the equilibrium price and order flow, no such additional restrictions are needed.[17]

The following Proposition formally characterizes optimal investment and the financial market equilibrium in the general case.

Proposition 7. *Suppose $\gamma^2 > 4\tau_\varepsilon\tau_\zeta$, and let*

$$\beta^{SUB} \equiv \frac{1}{2\tau_\zeta}\left(\gamma - \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right), \quad \textit{and} \quad \beta^{COM} \equiv \frac{1}{2\tau_\zeta}\left(\gamma + \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right). \tag{18}$$

An equilibrium corresponding to each value of $\beta \in \{\beta^{SUB}, \beta^{COM}\}$, such that
(i) the investment rule is

$$y^*(s_p) = \begin{cases} 0 & s_p < \bar{s}_p \\ \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) & s_p \geq \bar{s}_p \end{cases} \tag{19}$$

where the threshold $\bar{s}_p = \inf\{s_p : \max_{y\in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$,
(ii) the optimal demand is

$$X_i = \frac{\mathbb{E}_i[V(y)] - \mathbb{C}_i(V(y),\theta)z_i - P}{\gamma\mathbb{V}_i(V(y))} - n = \frac{\mu_A + y\left(\mathbb{E}_i[\theta] - \frac{\gamma}{\tau}z_i\right) - c(y) - P}{\gamma\left(\frac{1}{\tau_A} + \frac{1}{\tau}y^2\right)} - n. \tag{20}$$

where

$$\mathbb{E}_i[\theta] = \mu_\theta + b_s(s_i - \mu_\theta) + b_p(s_p - \mathbb{E}[s_p]) + b_z(z_i - \mu_Z), \quad \mathbb{V}_i[\theta] \equiv \frac{1}{\tau}, \tag{21}$$

and
(iii) the equilibrium price is given by

$$P(s_p) = \begin{cases} \mu_A - \gamma\frac{1}{\tau_A}n & s_p < \bar{s}_p \\ \mu_A + y^*(s_p)\left(b_0 + (b_s + b_p)s_p\right) - c(y^*(s_p)) - \gamma\left(\frac{1}{\tau_A} + (y^*(s_p))^2\frac{1}{\tau}\right)n & s_p \geq \bar{s}_p \end{cases} \tag{22}$$

[17] This assumption is common in the literature. For instance, the equilibrium definitions in Dow and Rahi (2003) (p.442), Goldstein and Guembel (2008) (p.152), and Edmans, Goldstein, and Jiang (2015) (p.3775) allow the manager's investment strategy to depend directly on order flow.

where $s_p = \theta - \frac{1}{\beta}Z$, and b_0, b_s, b_p, b_z, and τ are characterized in the appendix, exists if the function P so defined is strictly increasing in s_p in the region $s_p \geq \overline{s}_p$.

The ranking of welfare in Proposition 4 becomes more challenging. This is because in addition to the price aggregating the private information across investors, it also reflects a risk premium (which depends on the per-capita supply of the asset n). The risk premium affects welfare through two additional channels. First, because the price is discounted (relative to expected cash flows) and investors have a per-capita endowment of the stock, an increase in the risk premium lowers wealth and consequently, expected utility. This implies that the investment level that maximizes trader utility need not always be higher than the equilibrium investment level (i.e., Proposition 3 need not hold).

Second, the risk premium affects the cost that investors' incur to use the risky asset as a hedge. Specifically, an investor who would like to sell (buy) the risky asset to hedge their z_i shock has to pay more (less) when the price is discounted by the risk premium. This can lead to greater disparity in wealth across investors, reducing average investor utility. As such, while we expect the valuation and hedging channels to continue to play a role in this environment, the overall welfare implications are more nuanced due to the risk premium component.

Figure 4: This figure plots investor welfare as a function of the underlying parameters when the manager selects non-negative investment ($Y = [0, \infty)$) under the market-clearing price. The manager is subject to a quadratic cost ($c(y) = c * y^2$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.25; c = 0.5$; $\gamma = 0.5$; $\tau_\theta = 1$; $\tau_\varepsilon = 0.05$; $\tau_\zeta = 1$; $\tau_z = 1$; $n = 0.25$; $\mu_A = 0$; $\mu_Z = 0$; and τ_A is arbitrarily large.

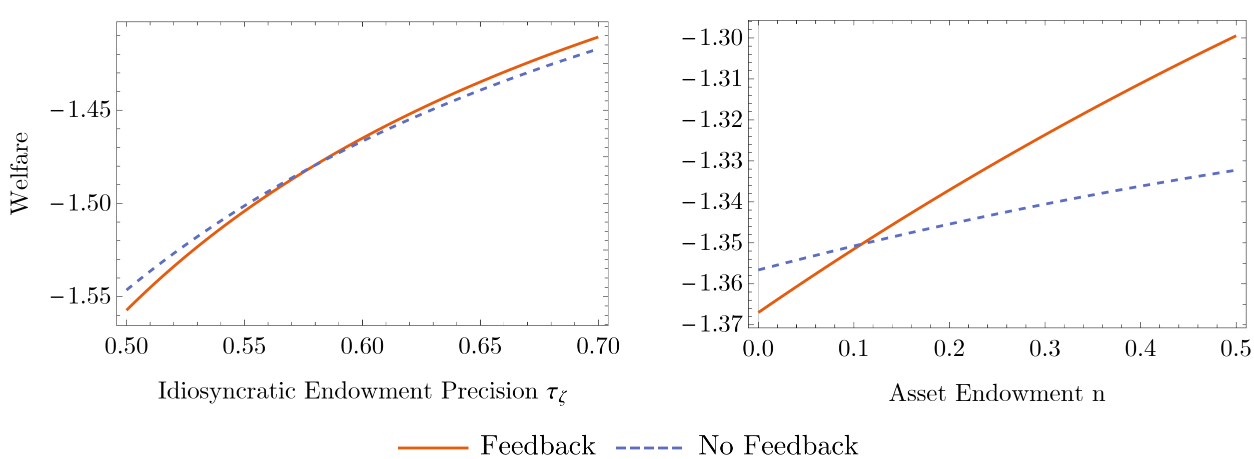
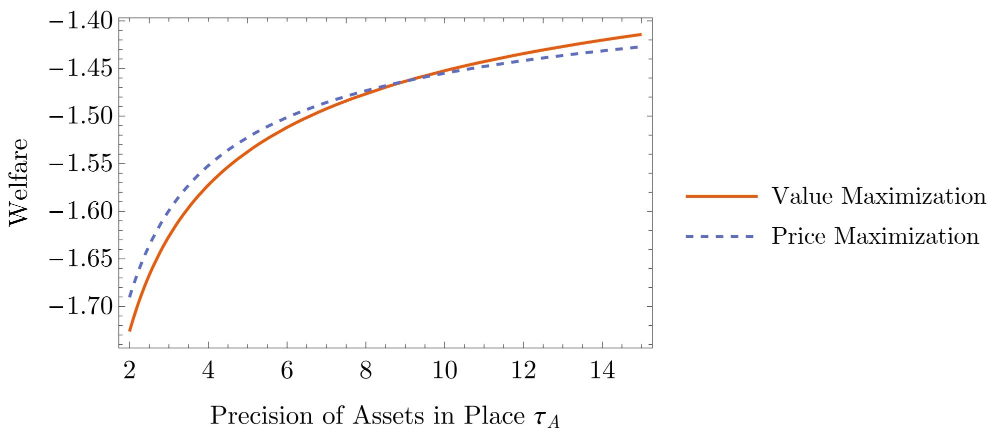


Figure 4 provides an illustration of the ranking between feedback and no-feedback when investment is subject to a quadratic cost and the price is determined by market clearing. For the parameters considered, we note that the results of our benchmark analysis continue to hold in this setting — welfare is lower with feedback when hedging channel is important (e.g., τ_ζ is low), and the valuation channel is not (n is small). This suggests that our benchmark results are robust to the extensions considered in Sections 6.1 and 6.2, even though an analytic characterization of this ranking is not tractable in the latter case.

More broadly, the disconnect between the manager's optimal investment decision and the welfare-maximizing decision stems from the fact that the manager is incentivized to maximize the firm's expected value, which ignores both investor risk aversion and the risk-sharing role of the firm's stock. The fact that the manager ignores the risk premium tends to, on average, lead them to over-invest from the perspective of an investor who does not use the stock to hedge. We have also analyzed a version of the model in which the manager maximizes price rather than expected value. In this case, the equilibrium takes the same form as in Proposition 7, where the investment function $y^*(s_p)$ is simply replaced by the price-maximizing investment, which also accounts for the risk premium.[18] In this case, since the manager maximizes the stock price, their investment is disciplined by its effect on the risk premium and can be lower than under "value maximization."

Figure 5: This figure plots investor welfare as a function of τ_A in both the cases in which the manager maximizes price and expected firm value. The manager selects non-negative investment ($Y = [0, \infty)$) under the market-clearing price. Moreover, the manager is subject to a quadratic cost ($c(y) = c * y^2$) and $\beta = \beta^{SUB}$. Unless otherwise stated, the parameters are $\mu_\theta = 0.25; c = 0.5; \gamma = 0.5; \tau_\theta = 1; \tau_\varepsilon = 0.05; \tau_\zeta = 0.6; \tau_z = 1; n = 1.5; \mu_A = 0; \mu_Z = 0$.



Under the same specification and parameter choices as Figure 4, Figure 5 compares welfare under value maximization and price maximization. The plots suggest that investor welfare is greater with value maximization only when τ_A is sufficiently large. Conditional on investment, the firm's cash flows are more sensitive to θ when τ_A is high, which makes the

[18] In fact, the equilibrium continues to hold even when investors observe price only, as opposed to the order flow.

stock more useful for hedging. This implies that the relative "over-investment" that results from value maximization increases welfare in this region. On the other hand, when τ_A is low, welfare can be higher under price maximization.

An implication of these results is that optimal managerial incentives should vary across firms, and depend on the relative volatility of their assets in place to their growth options (new projects). For firms with relatively more volatile assets in place, welfare may be higher when the manager is incentivized to maximize the expected stock price rather than expected value. While a complete analysis of the optimal compensation contract for the manager is beyond the scope of the current paper, our analysis uncovers a novel tradeoff between price-based and (long-term) value-based compensation.

6.3 Hedging and information about assets in place

Our benchmark analysis focuses on the natural case in which investors have private information about the new project, and have risk exposures that are correlated with the cash flows of the new project. The potential tension between the valuation and hedging channels is clearest in this case. While we expect these underlying mechanisms to be in effect for other specifications, their overall implications for how feedback affects welfare may be different.

For instance, suppose investors have private information about the cash flows of the new project but their endowment shocks are correlated with assets in place (i.e., z_i units of exposure to a shock that is correlated with A and not θ). In this case, investment in the new project makes the stock less useful for risk sharing, as the resulting exposure to θ makes the security a worse hedge for A risk. As a result, in contrast to the benchmark model, one would expect that feedback would always improve welfare for positive NPV projects (by reducing investment in the new project for some states), while it may reduce welfare for negative NPV projects.

If both investors' private information and endowment shocks relate to assets in place as opposed to the new project, then there is no direct role for market feedback since we have assumed the manager's investment decision does not affect cash flows from the assets in place. Finally, while we expect similar forces to be at play in a richer model that allows for arbitrary correlations between assets in place and new project cash flows, and for investor information and risk exposures to both components, this is beyond the scope of the current paper.

7 Conclusions

Higher investment efficiency does not necessarily improve welfare. We illustrate this in a setting with feedback effects where the manager conditions on the information aggregated by prices before choosing how to invest in a new project. In our setting, market feedback always improves investment efficiency, but it can reduce welfare for (ex-ante) positive NPV projects. Our key insight is that investment decisions change the exposure of the firm's cash flows, and so affect investors' ability to use the stock to hedge endowment shocks. As such, feedback can lead the manager to reduce investment in some states of the world, and consequently lead to less efficient risk sharing and lower welfare.

A contribution of our analysis is to provide a benchmark framework that can be naturally extended along various dimensions for future work. Immediate extensions would include generalizations to the structure of cash flows and information. For instance, allowing for correlation between assets in place and future projects would make the analysis richer, although we expect the underlying economic forces we highlight to still be in play. Similarly, it would be interesting to study how introducing multiple dimensions of fundamentals (e.g., Goldstein and Yang (2019), Goldstein et al. (2020)) would interact with the forces we highlight. Moreover, it may be interesting to consider how information acquisition by investors and managers interact in our setting.

An extension of our model that incorporates direct preferences over the firm's investment choices (e.g., as in Pástor, Stambaugh, and Taylor (2020)) would be a natural setting to study how the environmental, social and governance (ESG) factors affect financial markets and investment choices. In such settings, the firm's investment decisions not only affect welfare via the valuation and hedging channels that we focus on, but also via a direct "externality" channel. We leave this analysis for future work.

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A Explicit expressions for investor welfare

In this section, we characterize traders' expected utilities, which is a key step in establishing the welfare results in the text. Because we will apply this result in a number of different contexts, we characterize the expected utility for an arbitrary investment rule and asset price.

First, we require an assumption to guarantee that expected utility exists and is finite.

Assumption 1. Given the investment rule y, the unconditional expected utility for a trader who refrains from trading (i.e., her autarky expected utility) is finite

$$\mathbb{E}[-e^{-\gamma(nV(y)+z_i U)}] > -\infty. \tag{23}$$

This assumption implies that the conditional expected utility given any conditioning information that we consider, is finite. Because CARA utility is bounded above, expected utility always exists but is possibly $-\infty$. To ensure that it is finite, it suffices to ensure that the expected utility, in the event that the trader does not trade is finite, since her equilibrium utility will always be weakly greater than this.

Proposition 8. *Consider an arbitrary investment rule $y(s_p)$, with associated asset value $V(y(s_p))$ and pricing rule $P(s_p)$. Consider an arbitrary trader i and define the 5×1 random vector*

$$\eta = \begin{pmatrix} V - P \\ \vec{V} \\ \vec{Z}_i \end{pmatrix}, \tag{24}$$

where $\vec{V} = \begin{pmatrix} V \\ U \end{pmatrix}$ and $\vec{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ are the vectors of the payoffs and endowments of the tradeable and non-tradeable asset. Let $\mathcal{F}_R$ be any information set that is (i) weakly coarser than $\sigma(s_p)$ and (ii) under which η is conditionally normally distributed (e.g., $\mathcal{F}_R = \sigma(s_p)$ satisfies these conditions). Finally, let $\mu_{\eta|R}$ and $\Sigma_{\eta|R}$ denote the conditional mean and variance of η given $\mathcal{F}_R$ and define the 5×5 matrix $\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}$. The conditional expected utility of the trader i given $\mathcal{F}_R$ is given by

$$-\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2} \left| I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix} \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) \right|^{-1/2} \exp\left\{-\tfrac{1}{2}\mu'_{\eta|R}(\Sigma_{\eta|R} + \mathcal{I})^{-1}\mu_{\eta|R}\right\} \tag{25}$$

where

$$(\Sigma_{\eta|R} + \mathcal{I})^{-1} = \begin{pmatrix} \mathbb{V}_R(V-P)^{-1} + \vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1}\vec{h}_R & -\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1}\vec{h}_R & \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right)^{-1} \end{pmatrix} \tag{26}$$

and $\vec{h}_R = \mathbb{C}_R((\vec{V}, \vec{Z}_i), V - P)\mathbb{V}_R^{-1}(V - P)$ *is the* 4×1 *vector of* $\mathcal{F}_R$*-conditional regression coefficients of* $\vec{V}$ *and* $\vec{Z}$ *on* $V - P$.

Proof. Consider an arbitrary investment rule $y(s_p) \in \{0, 1\}$ with associated asset value $V(y) = A + y\theta - c(y)$. We would like to compute the conditional expected utility, given s_p, for an arbitrary trader. Given the optimal demand $x^* = \frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n$, the *realized* utility is

$$-e^{-\gamma(x^*(V-P)+z_iU+nV)} = -e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)}-n\right)(V-P)+z_i\theta+nV\right)}. \tag{27}$$

To compute the expected utility, we will use the law of iterated expectations, first computing the expectation conditional on $\mathcal{F}_{i^+} = \sigma(\{s_i, z_i, s_p\})$, which is the trader information set augmented with s_p, and then conditional on $\mathcal{F}_R$. We emphasize that the first step is not identical to computing the expectation given the trader's information set $\mathcal{F}_i$ itself since s_p is only observed by the trader in states in which investment is positive.

Note that because $V = A$ in any state with zero investment (i.e., in any state in which the trader does not infer s_p from the price) and because the $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ information sets coincide in any states with positive investment, we have

$$\mathbb{E}_{i^+}[V] = \mathbb{E}_i[V] = \begin{cases} \mathbb{E}[A] & y = 0 \\ \mathbb{E}[A + y\theta - c(y)|s_i, z_i, s_p] & y > 0 \end{cases} \tag{28}$$

$$\mathbb{V}_{i^+}[V] = \mathbb{V}_i[V] = \begin{cases} \mathbb{V}[A] & y = 0 \\ \mathbb{V}[A + y\theta - c(y)|s_i, z_i, s_p] & y > 0 \end{cases}. \tag{29}$$

Similarly

$$\mathbb{C}_{i^+}(V, U) = \mathbb{C}_i(V, U) = \begin{cases} 0 & y = 0 \\ y\mathbb{V}[\theta|s_i, z_i, s_p] & y > 0 \end{cases} \tag{30}$$

However, the conditional variance of the endowment payoff itself U is not always identical under the two information sets since it differs in states in which there is no investment and the trader does not infer s_p.

Given $\{s_i, z_i, s_p\}$ the trader's terminal wealth is conditionally normally distributed with mean

$$\mathbb{E}_{i^+}\left[\left(\frac{\mathbb{E}_i[V-P] - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right)(V - P) + z_iU + nV\right] \tag{31}$$

$$= \left(\frac{\mathbb{E}_i[V-P] - \gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right)(\mathbb{E}_{i^+}[V] - P) + z_i\mathbb{E}_{i^+}[U] + n\mathbb{E}_{i^+}[V] \tag{32}$$

$$= \left(\frac{\mathbb{E}_i^2[V-P]}{\gamma\mathbb{V}_i(V)}\right) - \left(n + \frac{\mathbb{C}_i(V,U)}{\mathbb{V}_i(V)}z_i\right)\mathbb{E}_i[V - P] + z_i\mathbb{E}_i[U] + n\mathbb{E}_i[V] \tag{33}$$

$$= \left(\frac{\mathbb{E}_i^2[V-P]}{\gamma \mathbb{V}_i(V-P)}\right) - \left(n + \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)} z_i\right) \mathbb{E}_i[V-P] + z_i \mathbb{E}_i[U] + n\mathbb{E}_i[V] \tag{34}$$

where the next to last line uses the equality of the conditional means and conditional variances of V under $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ and the final line uses the fact that P is $\mathcal{F}_i$ measurable. Similarly, the conditional variance of wealth is

$$\mathbb{V}_{i^+}\left(\left(\frac{\mathbb{E}_i[V-P] - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n\right)(V-P) + z_i U + nV\right) \tag{35}$$

$$= \mathbb{V}_{i^+}\left(\left(\frac{\mathbb{E}_{i^+}[V-P] - \gamma \mathbb{C}_{i^+}(V,U) z_i}{\gamma \mathbb{V}_{i^+}(V)}\right) V + z_i U\right) \tag{36}$$

$$= \frac{\mathbb{E}_{i^+}^2[V-P]}{\gamma^2 \mathbb{V}_{i^+}(V)} + \left(\mathbb{V}_{i^+}(U) - \frac{\mathbb{C}_{i^+}^2(V,U)}{\mathbb{V}_{i^+}(V)}\right) z_i^2 \tag{37}$$

$$= \frac{\mathbb{E}_i^2[V-P]}{\gamma^2 \mathbb{V}_i(V)} + \mathbb{V}_{i^+}(U|V) z_i^2 \tag{38}$$

$$= \frac{\mathbb{E}_i^2[V-P]}{\gamma^2 \mathbb{V}_i(V-P)} + \mathbb{V}_{i^+}(U|V-P) z_i^2 \tag{39}$$

Hence, computing the conditional expected utility given $\mathcal{F}_{i^+}$ yields

$$\mathbb{E}_{i^+}\left[-e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)} - n\right)(V-P) + z_i U + nV\right)}\right] \tag{40}$$

$$= -e^{-\gamma n_i \mathbb{E}_i[V] - \gamma z_i \mathbb{E}_i[U] + \gamma(n_i + z_i h_i)\mathbb{E}_i[V-P] - \frac{1}{2}\frac{\mathbb{E}_i^2[V-P]}{\mathbb{V}_i(V-P)} + \frac{1}{2}\gamma^2 \mathbb{V}_{i^+}(U|V-P) z_i^2} \tag{41}$$

where $h_i = \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}$ is the conditional regression coefficient of the endowment payoff U on the asset return $V-P$.

To complete the proof, we need to compute the conditional expectation of this quantity given $\mathcal{F}_R$. Define the 5×1 random vector

$$\eta = \begin{pmatrix} V-P \\ \begin{pmatrix} V \\ U \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix}, \tag{42}$$

partitioned as indicated by parentheses. Let

$$\eta_i \equiv \mathbb{E}_{i^+}[\eta] = \begin{pmatrix} \mathbb{E}_{i^+}[V-P] \\ \begin{pmatrix} \mathbb{E}_{i^+}[V] \\ \mathbb{E}_{i^+}[U] \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \mathbb{E}_i[V-P] \\ \begin{pmatrix} \mathbb{E}_i[V] \\ \mathbb{E}_i[U] \end{pmatrix} \\ \begin{pmatrix} n \\ z_i \end{pmatrix} \end{pmatrix} \tag{43}$$

be the $\mathcal{F}_{i^+}$ conditional expectation of η. Finally, let $\vec{h}_i = (1, h_i)$ be the 2×1 vector of conditional regression coefficients of (V,U) on V and define the conformably partitioned 5×5 matrix

$$\alpha_i = \begin{pmatrix} \mathbb{V}_i^{-1}(V-P) & \vec{0}' & -\gamma \vec{h}_i' \\ \vec{0} & \mathbf{0} & \gamma I_2 \\ -\gamma \vec{h}_i & \gamma I_2 & -\gamma^2 \mathbb{V}_i((V,U)|V-P) \end{pmatrix} \tag{44}$$

Where I_k denotes an identity matrix of dimension k, and $\vec{0}$ and $\mathbf{0}$ denote a conformable vector and matrix of all zeros, respectively. Below, we will typically just use 0, with no vector notation or bolding, for conformable vectors or matrices of zeros, except where confusion would result.

With this notation, we can concisely write the $\mathcal{F}_{i^+}$ expected utility above as

$$\mathbb{E}_{i^+}\left[-e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P]-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)}-n\right)(V-P)+z_iU+nV\right)}\right] = -e^{-\frac{1}{2}\eta_i'\alpha_i\eta_i} \tag{45}$$

The random vector η_i is conditionally jointly normally distributed given $\mathcal{F}_R$.[19]. Let $\mu_{\eta_i|R} = \mathbb{E}_R[\eta_i]$ and $\Sigma_{\eta_i|R} = \mathbb{V}_R(\eta_i)$ denote the $\mathcal{F}_R$ conditional mean and variance matrix of η_i, and let $\mu_{\eta|R} = \mathbb{E}_R[\eta]$ and $\Sigma_{\eta|R} = \mathbb{V}_R(\eta)$ denote the $\mathcal{F}_R$ conditional mean and variance of η itself. We can use standard formulas for expected exponential-quadratic forms of normal random vectors to compute

$$\mathbb{E}_R\left[-e^{-\frac{1}{2}\eta_i'\alpha_i\eta_i}\right] = -\left|\alpha_i\right|^{-1/2}\left|\Sigma_{\eta_i|R}+\alpha_i^{-1}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mu_{\eta_i|R}'\left(\Sigma_{\eta_i|R}+\alpha_i^{-1}\right)^{-1}\mu_{\eta_i|R}\right\}. \tag{46}$$

This expression requires that the matrix α_i is invertible. However, using standard formulas for determinants of partitioned matrices (e.g., eq. (5) in Henderson and Searle (1981)) and inverses of partitioned matrices (e.g., eq. (8) in Henderson and Searle (1981)) we can compute

$$|\alpha_i| = \left|\mathbb{V}_i^{-1}(V-P)\right|\left|-\gamma^2 I_2\right| = \gamma^4\left|\mathbb{V}_i^{-1}(V-P)\right| > 0 \tag{47}$$

so that α_i is invertible and we have

$$\alpha_i^{-1} = \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,(V,U)) & 0 \\ \mathbb{C}_i\left((V,U),V-P\right) & \mathbb{V}_i\left(V,U\right) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{48}$$

[19]Note that η_i follows a singular normal distribution since n is a constant. That is, the conditional variance matrix of η_i is only positive semidefinite. However, defining the random vector in this way causes no difficulties in the derivation below, and both simplifies the algebra and provides guidance for how to handle more general situations in which the share endowment is random.

With this expression for α_i^{-1} we can compute

$$\Sigma_{\eta_i} + \alpha_i^{-1} = \mathbb{V}_R(\mathbb{E}_i[\eta]) + \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,(V,\theta)) & 0 \\ \mathbb{C}_i((V,\theta),V-P) & \mathbb{V}_i(V,U) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{49}$$

$$= \Sigma_\eta + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{50}$$

$$\equiv \Sigma_\eta + \mathcal{I} \tag{51}$$

where the second equality follows from the law of total variance, and the final equality defines the 5×5 matrix $\mathcal{I} = \left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{smallmatrix}\right)$.

Putting together everything above, the conditional expected utility can be written

$$-\left|\alpha_i\right|^{-1/2}\left|\Sigma_{\eta_i|R} + \alpha_i^{-1}\right|^{-1/2} \exp\left\{-\frac{1}{2}\mu'_{\eta_i|R}\left(\Sigma_{\eta_i|R} + \alpha_i^{-1}\right)^{-1}\mu_{\eta_i|R}\right\} \tag{52}$$

$$= -\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\Sigma_{\eta|R} + \mathcal{I}\right|^{-1/2} \exp\left\{-\frac{1}{2}\mu'_{\eta|R}\left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1}\mu_{\eta|R}\right\}. \tag{53}$$

The key step to evaluating this expression is to characterize the inverse $\left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1}$ and the determinant $\left|\Sigma_{\eta|R} + \mathcal{I}\right|$. Partition $\Sigma_{\eta|R} + \mathcal{I}$ as

$$\begin{pmatrix} A & U \\ V & D \end{pmatrix}$$

$$\equiv \begin{pmatrix} \mathbb{V}_R(V-P) & \begin{pmatrix} \mathbb{C}_R(V-P,(V,U)) & \mathbb{C}_R(V-P,(n,z_i)) \end{pmatrix} \\ \begin{pmatrix} \mathbb{C}_R((V,U),V-P) \\ \mathbb{C}_R((n,z_i),V-P) \end{pmatrix} & \left(\begin{pmatrix} \mathbb{V}_R((V,U)) & \mathbb{C}_R((V,U),(n,z_i)) \\ \mathbb{C}_R((n,z_i),(V,U)) & \mathbb{V}_R((n,z_i)) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right) \end{pmatrix} \tag{54}$$

where A is 1×1, $U = V'$ is 1×4 and D is 4×4.

Using standard methods for inverting a partitioned matrix (e.g., eq. (8) in Henderson and Searle (1981)) we therefore have

$$(\Sigma_{\eta|R}^{-1} + \mathcal{I})^{-1} = \begin{pmatrix} A^{-1} + A^{-1}U(D - VA^{-1}U)^{-1}VA^{-1} & -A^{-1}U(D - VA^{-1}U)^{-1} \\ -(D - VA^{-1}U)^{-1}VA^{-1} & (D - VA^{-1}U)^{-1} \end{pmatrix} \tag{55}$$

Note that

$$D - VA^{-1}U \tag{56}$$

$$= \left(\begin{pmatrix} \mathbb{V}_R((V,U)) & \mathbb{C}_R((V,U),(n,z_i)) \\ \mathbb{C}_R((n,z_i),(V,U)) & \mathbb{V}_R((n,z_i)) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right) \tag{57}$$

$$- \begin{pmatrix} \mathbb{C}_R((V,U), V-P) \\ \mathbb{C}_R((n,z_i), V-P) \end{pmatrix} \mathbb{V}_R^{-1}(V-P) \begin{pmatrix} \mathbb{C}_R(V-P,(V,U)) & \mathbb{C}_R(V-P,(n,z_i)) \end{pmatrix} \tag{58}$$

$$= \begin{pmatrix} \mathbb{V}_R((V,U)|V-P) & \mathbb{C}_R((V,U),(n,z_i)|V-P) \\ \mathbb{C}_R((n,z_i),(V,U)|V-P) & \mathbb{V}_R((n,z_i)|V-P) \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{59}$$

or if we concisely let $\vec{V} = (V,U)$ and $\vec{Z}_i = (n, z_i)$

$$D - VA^{-1}U = \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{60}$$

Similarly, we have

$$VA^{-1} = \begin{pmatrix} \mathbb{C}_R(\vec{V}, V-P) \\ \mathbb{C}_R(\vec{Z}_i, V-P) \end{pmatrix} \mathbb{V}_R^{-1}(V-P) \tag{61}$$

$$\equiv \vec{h}_R \tag{62}$$

where the last line defines $\vec{h}_R$ as the 4×1 vector of $\mathcal{F}_R$-conditional regression coefficients of $\vec{V}$ and $\vec{Z}$ on $V-P$. Hence, we can concisely write our desired matrix inverse as

$$(\Sigma_{\eta|R}^{-1} + \mathcal{I})^{-1} \tag{63}$$

$$= \begin{pmatrix} A^{-1} + A^{-1}U(D-VA^{-1}U)^{-1}VA^{-1} & -A^{-1}U(D-VA^{-1}U)^{-1} \\ -(D-VA^{-1}U)^{-1}VA^{-1} & (D-VA^{-1}U)^{-1} \end{pmatrix} \tag{64}$$

$$= \begin{pmatrix} \mathbb{V}_R(V-P)^{-1} + \vec{h}'_R \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \vec{h}_R & -\vec{h}'_R \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \vec{h}_R & \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right)^{-1} \end{pmatrix} \tag{65}$$

And using the same partitioning and applying eq. (5) of Henderson and Searle (1981) its determinant is

$$\left|\Sigma_{\eta|R} + \mathcal{I}\right| = |A| \left|D - VA^{-1}U\right| \tag{66}$$

$$= |\mathbb{V}_R(V-P)| \left| \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right| \tag{67}$$

Putting everything together, the $\mathcal{F}_R$ expected utility is

$$-\frac{1}{\gamma^2} \left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2} \left|\Sigma_{\eta|R} + \mathcal{I}\right|^{-1/2} \exp\left\{ -\frac{1}{2} \mu'_{\eta|R} \left(\Sigma_{\eta|R} + \mathcal{I}\right)^{-1} \mu_{\eta|R} \right\} \tag{68}$$

$$= -\frac{1}{\gamma^2} \left| \frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} \right|^{1/2} \left| \mathbb{V}_R((\vec{V}, \vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \right|^{-1/2} \exp\left\{ -\frac{1}{2} \mu'_{\eta|R} (\Sigma_{\eta|R} + \mathcal{I})^{-1} \mu_{\eta|R} \right\} \tag{69}$$

$$= -\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2}\left|I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix}\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)\right|^{-1/2}\exp\left\{-\frac{1}{2}\mu'_{\eta|R}(\Sigma_{\eta|R}+\mathcal{I})^{-1}\mu_{\eta|R}\right\} \tag{70}$$

where

$$(\Sigma_{\eta|R}+\mathcal{I})^{-1} = \begin{pmatrix} \mathbb{V}_R(V-P)^{-1}+\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\vec{h}_R & -\vec{h}'_R\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1} \\ -\left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\vec{h}_R & \left(\mathbb{V}_R((\vec{V},\vec{Z}_i)|V-P)+\begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0\end{pmatrix}\right)^{-1}\end{pmatrix}. \tag{71}$$

□

The expression for the expected utility in Proposition 8 simplifies further in the case where prices are characterized as the public expectation of future cash flows.

Corollary 1. *Suppose that the asset price is characterized as the conditional expected cash flow $P = \mathbb{E}[V|P]$. Then the conditional expected utility given $\mathcal{F}_R$ is given by*

$$-\left|\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}\right|^{1/2}\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right|^{-1/2} \tag{72}$$

$$\times \exp\left\{-\gamma\mathbb{E}_R[V]n - \frac{1}{2}\begin{pmatrix}\mathbb{E}_R[U] \\ \mathbb{E}_R[z_i]\end{pmatrix}'\begin{pmatrix}\mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P)\end{pmatrix}^{-1}\begin{pmatrix}\mathbb{E}_R[U] \\ \mathbb{E}_R[z_i]\end{pmatrix}\right\} \tag{73}$$

Proof. Take $\mathbb{E}_R[V-P] = 0$ in Proposition 8, write out the matrix products explicitly, and collect terms. □

B Proofs of primary results

Proof of Proposition 1

This is a special case of Proposition 5 with $Y = \{0,1\}$, $c(0) = 0, c(1) = c$, and $1/\tau_\xi = 0$.

Proof of Proposition 2

The first part follows from the observations that the firm manager chooses investment to maximize V and can always choose to ignore the information in s_p when making her decision.

For the second part, let $\omega = \mathbb{E}[\theta|s_p] - c \sim N(\mu_\theta - c, \mathrm{var}(\mathbb{E}[\theta|s_p])) \equiv N(\mu_\omega, \sigma^2_\omega)$. Then,

$$\mathbb{E}[V(y(s_p))] = \mathbb{E}[A] + \mathbb{E}\left[(\mathbb{E}[\theta|s_p] - c)\mathbf{1}_{\{s_p > \bar{s}_p\}}\right] \tag{74}$$

$$= \mu_A + \mathbb{E}\left[\omega\mathbf{1}_{\{\omega>0\}}\right] \tag{75}$$

$$= \mu_A + \mu_\omega + \sigma_\omega \frac{\phi\left(\frac{\mu_\omega}{\sigma_\omega}\right)}{\Phi\left(\frac{\mu_\omega}{\sigma_\omega}\right)}. \tag{76}$$

Holding fixed μ_ω, the above expectation is increasing in σ_ω. This implies the expected value with feedback is increasing in $\text{var}\left(\mathbb{E}\left[\theta|s_p\right]\right) = \text{var}\left(\theta\right) - \text{var}\left(\theta|s_p\right)$. Moreover, in the β^{SUB} equilibrium, $\text{var}(\theta|s_p)$ decreases in τ_ε and τ_ζ, but increases in γ. The result follows from noting that without feedback, the expected value is unaffected by τ_ε, τ_ζ and γ.

Proof of Proposition 3

This is a special case of Proposition 6 with $Y = \{0, 1\}$, $c(0) = 0, c(1) = c$, and $1/\tau_\xi = 0$.

Proof of Proposition 4

The conditional expected utility of an arbitrary trader given $\mathcal{F}_R = \sigma(s_p)$, as represented in the proof of Proposition 6 below, is

$$\mathbb{E}_R[-e^{-\gamma W_i^*}] = -e^{-n\gamma \mathbb{E}_R[V] - F(y|s_p)} \tag{77}$$

where F is an increasing function of y. If we can show that the claimed welfare ranking holds state-by-state in s_p then it is immediate that it holds unconditionally.

First consider the case in which the project has negative ex-ante NPV (i.e., the no-feedback default investment is $y = 0$). Applying the transformation $-\log(-x)$, the conditional expected utility satisfies

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=0} + F(y|s_p)\Big|_{y=0} \tag{78}$$

$$\leq n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_P)} + F(y|s_p)\Big|_{y=0} \tag{79}$$

$$\leq n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_P)} + F(y|s_p)\Big|_{y=y^*(s_P)} \tag{80}$$

where the first inequality follows from the fact that the expected asset value (i.e., the NPV) is weakly higher with feedback, and the second inequality follows from the fact that Proposition 6 establishes that F is increasing in y, and $y^* \geq 0$. Because this inequality holds state-by-state in s_P with strict inequality on a set of positive probability, it follows that the trader is strictly better off with feedback in this case $U(0) < U(y^*(s_p))$.

Next, consider the case in which the project has positive ex-ante NPV (i.e., the no-feedback default investment is $y = 1$) and $n = 0$. Following similar steps to the negative NPV case, we have

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=1,n=0} + F(y|s_p)\Big|_{y=1} \tag{81}$$

$$= F(y|s_p)\Big|_{y=1} \tag{82}$$

$$\geq F(y|s_p)\Big|_{y=y^*(s_p)} \tag{83}$$

$$= n\gamma \mathbb{E}_R[V]\Big|_{y=y^*(s_p),n=0} + F(y|s_p)\Big|_{y=y^*(s_p)}. \tag{84}$$

Because this inequality holds state-by-state in s_P with strict inequality on a set of positive probability, it follows that the trader is strictly worse off with feedback in this case $U(0) > U(y^*(s_p))$. Now, because the unconditional expected utility is continuous in n and because the trader is strictly worse off with feedback when $n = 0$, it follows that for n sufficiently small but positive, the trader remains strictly worse off with feedback.

Proof of Proposition 5

Conjecture that there exists an equilibrium in which the asset price reveals $s_p = \theta - \frac{1}{\beta}Z$ when $s_p \geq \overline{s}_p$, for constants β and $\overline{s}_p$ to be determined.

We proceed by working backwards, starting with the manager's problem at $t = 2$. In any equilibrium of the conjectured form, the manager's can infer s_p if and only if $s_p \geq \overline{s}_p$ and otherwise can infer only that $s_p < \overline{s}_p$. Her optimal investment is therefore

$$y^*(\cdot) = \begin{cases} \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p] - cy & s_p \geq \overline{s}_p \\ \arg\max_{y \in Y} y\mathbb{E}[\theta|s_p < \overline{s}_p] - cy & \text{otherwise} \end{cases}. \tag{85}$$

The manager can always guarantee a project payoff of zero by not investing. Hence, in order for her to find it optimal to invest when she observes any value $s_p \geq \overline{s}_p$ and not invest otherwise, the threshold $\overline{s}_p = T$ must be such that

$$\begin{cases} \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) \geq 0 & \forall s_p \geq T \\ \max_{y \in Y} y\mathbb{E}[\theta|s_p < \overline{s}_p] - c(y) = 0 & \text{otherwise} \end{cases} \tag{86}$$

Clearly any threshold T that lies (weakly) between $\inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$ (i.e., T such that the optimal investment conditional on observing any realization of s_p below the threshold is zero) and $\sup\{T : \max_{y \in Y} y\mathbb{E}[\theta|s_p < T] - c(y) = 0\}$ (i.e., T such that the optimal investment given knowledge only that s_p is less that then threshold is zero) satisfies these conditions.

We select the lowest admissible threshold, defined by $\overline{s}_p \equiv \inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$. In the baseline model with $Y = \{0, 1\}$, this is the unique equilibrium within this class in which the price function is continuous in s_p. This equilibrium is also robust to the manager directly observing s_p is all states (regardless of the investment opportunities Y). That is, if the manager were to observe a realization $s_p < \overline{s}_p$, so defined, she would still find it optimal to not invest.

Now, step back to $t = 1$ and consider the problem of an arbitrary trader i

$$\sup_{x\in\mathbb{R}} \mathbb{E}_i \left[-e^{-\gamma(x(V-P)+z_iU+nV)}\right] \tag{87}$$

Under the conjecture that the investment occurs and the price reveals s_p if and only if $s_p \geq \overline{s}_p$, the trader's conditional beliefs about payoffs are conditionally normal with

$$\mathbb{E}_i[\theta] = \begin{cases} \mu_\theta & s_p < \overline{s}_p \\ \mathbb{E}[\theta|s_i, z_i, s_p] & s_p \geq \overline{s}_p \end{cases} \tag{88}$$

$$\mathbb{V}_i(\theta) = \begin{cases} \frac{1}{\tau_\theta} & s_p < \overline{s}_p \\ \mathbb{V}(\theta|s_i, z_i, s_p) & s_p \geq \overline{s}_p \end{cases} \tag{89}$$

and

$$\mathbb{E}_i[V] = \mu_A + y\mathbb{E}_i[\theta] - c(y) \tag{90}$$

$$\mathbb{E}_i[U] = \mathbb{E}_i[\theta] \tag{91}$$

$$\mathbb{V}_i(V) = \frac{1}{\tau_A} + y^2\mathbb{V}_i(\theta) \tag{92}$$

$$\mathbb{C}_i(V, U) = \mathbb{C}_i(V, \theta) = y\mathbb{V}_i(\theta) \tag{93}$$

$$\mathbb{V}_i(U) = \mathbb{V}_i(\theta) + \frac{1}{\tau_\xi}. \tag{94}$$

Further, the conditional moments of θ in the $s_p \geq \overline{s}_p$ region can be written explicitly as

$$\mathbb{E}[\theta|s_i, z_i, s_p] = \mu_\theta + \underbrace{\tfrac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}}_{=b_s}(s_i - \mu_\theta) + \underbrace{\tfrac{\beta^2(\tau_Z+\tau_\zeta)}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}}_{=b_p}\left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z\right)\right) \tag{95}$$

$$+ \underbrace{\tfrac{\beta^2(\tau_Z+\tau_\zeta)}{\tau_\theta+\tau_\varepsilon+\beta^2(\tau_Z+\tau_\zeta)}\tfrac{\tau_\zeta}{\tau_Z+\tau_\zeta}\frac{1}{\beta}}_{=b_z}(z_i - \mu_Z), \tag{96}$$

and

$$\frac{1}{\tau} \equiv \mathbb{V}(\theta|s_i, z_i, s_p) = \frac{1}{\tau_\theta + \tau_\varepsilon + \beta^2(\tau_Z + \tau_\zeta)}. \tag{97}$$

Hence, computing the expectation, the trader's objective is

$$\sup_{x\in\mathbb{R}} -e^{-\gamma(x\mathbb{E}_i[V-P]+z_i\mathbb{E}_i[U]+n\mathbb{E}_i[V])+\frac{1}{2}\gamma^2\left((x+n)^2\mathbb{V}_i(V)+2z_i(x+n)\mathbb{C}_i(V,U)+z_i^2\mathbb{V}_i(U)\right)} \tag{98}$$

It is immediate that this problem is strictly concave on all of $\mathbb{R}$, so the first-order condition

(FOC) characterizes the maximum

$$x^* = \frac{\mathbb{E}_i[V] - P - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n. \tag{99}$$

Now aggregate the demand of the traders. For states $s_p < \overline{s}_p$, plugging in for the conditional moments and computing the aggregate demand

$$\int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n \right) di = \frac{\mu_A - P}{\gamma \frac{1}{\tau_A}} - n \tag{100}$$

so that the aggregate order does not reveal any information to the market maker, as conjectured. For states $s_p \geq \overline{s}_p$, we have

$$\int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma \mathbb{C}_i(V,\theta) z_i}{\gamma \mathbb{V}_i(V)} - n \right) di = \int_i \left(\frac{\mu_A + y\mathbb{E}[\theta|s_i, z_i, s_p] - c(y) - P - \gamma \frac{1}{\tau} z_i}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n \right) di \tag{101}$$

$$= \frac{\mu_A + y \int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - P}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n. \tag{102}$$

Hence, the aggregate order reveals

$$\int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di \tag{103}$$

$$= \int_i \left(\mu_\theta + b_s(s_i - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z \right) \right) + b_z(z_i - \mu_Z) - \gamma \frac{1}{\tau} z_i \right) di \tag{104}$$

$$= \mu_\theta + b_s(\theta - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta}\mu_Z \right) \right) + \left(b_z - \gamma \frac{1}{\tau} \right) Z - b_z \mu_Z. \tag{105}$$

It follows that the market maker can infer

$$b_s \theta - \left(\gamma \frac{1}{\tau} - b_z \right) Z = b_s \left(\theta - \frac{\gamma \frac{1}{\tau} - b_z}{b_s} Z \right), \tag{106}$$

which satisfies our conjecture $s_p = \theta - \frac{1}{\beta} Z$ if and only if

$$\frac{1}{\beta} = \frac{\gamma \frac{1}{\tau} - b_z}{b_s}. \tag{107}$$

Plugging in using the explicit expressions for b_s, b_z, and $1/\tau$ from above yields

$$\frac{1}{\beta} = \frac{\gamma - \beta \tau_\zeta}{\tau_\varepsilon}, \tag{108}$$

which has two solutions

$$\beta = \frac{1}{2\tau_\zeta}\left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right) \tag{109}$$

or, equivalently,

$$\frac{1}{\beta} = \frac{1}{2\tau_\varepsilon}\left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon\tau_\zeta}\right), \tag{110}$$

where the $+$ solution for β corresponds to the $-$ solution for $1/\beta$ and vice versa, which is easily confirmed by multiplying the expressions, which yields 1 if and only if we choose different signs for each.

With the precise expression for s_p pinned down, our candidate price function is

$$P = \begin{cases} \mu_A & s_p < \overline{s}_p \\ \mu_A + y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p)) & s_p \geq \overline{s}_p \end{cases}. \tag{111}$$

To confirm that this represents an equilibrium price function, it remains to confirm that traders can infer s_p if and only if $s_p \geq \overline{s}_p$. It suffices to establish that the candidate price function is constant for $s_p < \overline{s}_p$, is strictly increasing in s_p for $s_p \geq \overline{s}_p$ and does not jump downward at the threshold.

Clearly P is constant for $s_p < \overline{s}_p$. Since the threshold is defined as $\overline{s}_p = \inf\{s_p : \max_{y \in Y} y\mathbb{E}[\theta|s_p] - c(y) > 0\}$, we have

$$\mu_A + y(\overline{s}_p)\mathbb{E}[\theta|s_p = \overline{s}_p] - c(y(\overline{s}_p)) \geq \mu_A \tag{112}$$

which establishes that the price is increasing at the threshold.

Finally, consider the region $s_p > \overline{s}_p$. We need to show that the optimized project payoff $y(s_p)\mathbb{E}[\theta|s_p] - c(y(s_p))$ is strictly increasing in s_p. Note first that by Theorem 4 in Milgrom and Shannon (1994), the optimal investment $y(s_p)$ is increasing in s_p since the manager's objective function $y\mathbb{E}[\theta|s_p] - c(y)$ satisfies the single-crossing property in $(y; s_p)$. Furthermore, for any $s_{p2} > s_{p1} > \overline{s}_p$ we have

$$y(s_{p2})\mathbb{E}[\theta|s_{p2}] - c(y(s_{p2})) \geq y(s_{p1})\mathbb{E}[\theta|s_{p2}] - c(y(s_{p1})) > y(s_{p1})\mathbb{E}[\theta|s_{p1}] - c(y(s_{p1})) \tag{113}$$

where the first inequality follows from the optimality of $y(s_{p2})$ at s_{p2} and the second inequality follows from the fact that $\mathbb{E}[\theta|s_p]$ is strictly increasing in s_p and $y > 0$ for $s_p > \overline{s}_p$. This establishes that $P(s_p)$ is strictly increasing above the threshold and completes the proof.

Proof of Proposition 6

We begin by observing that the s_p-dependent investment rule that maximizes the unconditional expected utility is equivalent to the s_p-dependent rule that maximizes the conditional expected utility $\mathbb{E}[U_i|s_p]$ state-by-state in s_p.

Fix any s_p. Using the expression for expected utility from Corollary 1 with $\mathcal{F}_R = \sigma(s_p)$ and taking the monotonic transformation $-\log(-x)$, the welfare maximizing investment y

maximizes

$$\gamma n\mathbb{E}_R[V] - \frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} + \frac{1}{2}\log\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{114}$$

$$+\frac{1}{2}\begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}. \tag{115}$$

Letting

$$F(y|s_p) = -\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} + \frac{1}{2}\log\left|I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{116}$$

$$+\frac{1}{2}\begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U,z_i|V-P)+\frac{1}{\gamma} \\ \mathbb{C}_R(z_i,U|V-P)+\frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}. \tag{117}$$

we can write the welfare maximization problem concisely as

$$\max_{y\in Y}\left(\gamma n\mathbb{E}_R[V] + F(y|s_p)\right). \tag{118}$$

which is the sum of the manager's objective function (scaled by a constant) and the function F. Let $\nu \in \{0,1\}$ parameterize the manager's objective function and the objective function for welfare maximization, respectively. We would like to show that the maximizer of the parameterized problem

$$y(s_p;\nu) = \arg\max_{y\in Y}\left(\gamma n\mathbb{E}_R[V] + \nu F(y|s_p)\right) \tag{119}$$

is an increasing function of ν (holding the state s_p fixed). Theorem 4 in Milgrom and Shannon (1994) implies that if we can show that $\gamma n\mathbb{E}_R[V] + F(y|s_p)$ satisfies the single-crossing property in $(y;\nu)$ then we are done. That is, we want to show that the incremental payoff to higher investment crosses zero at most once, and from below, as ν increases from 0 to 1. It suffices to show that $F(y|s_p)$ is an increasing function of y.

Define $\tau_R \equiv \frac{1}{\mathbb{V}_R(\theta)}$. Recall that the $\mathcal{F}_R$ conditional variance matrix is

$$\Sigma_{\eta|R} = \begin{pmatrix} \mathbb{V}(V-P|s_p) & \mathbb{C}(V-P,V|s_p) & \mathbb{C}(V-P,U|s_p) & 0 & \mathbb{C}(V-P,Z_i|s_p) \\ \mathbb{C}(V-P,V|s_p) & \mathbb{V}(V|s_p) & \mathbb{C}(V,U|s_p) & 0 & \mathbb{C}(V,Z_i|s_p) \\ \mathbb{C}(V-P,U|s_p) & \mathbb{C}(V,U|s_p) & \mathbb{V}(U|s_p) & 0 & \mathbb{C}(U,Z_i|s_p) \\ 0 & 0 & 0 & 0 & 0 \\ \mathbb{C}(V-P,Z_i|s_p) & \mathbb{C}(V,Z_i|s_p) & \mathbb{C}(U,Z_i|s_p) & 0 & \mathbb{V}(Z_i|s_p) \end{pmatrix} \tag{120}$$

$$= \begin{pmatrix} \frac{1}{\tau_A} + y^2\frac{1}{\tau_R} & \frac{1}{\tau_A} + y^2\frac{1}{\tau_R} & y\frac{1}{\tau_R} & 0 & \beta y\frac{1}{\tau_R} \\ \frac{1}{\tau_A} + y^2\frac{1}{\tau_R} & \frac{1}{\tau_A} + y^2\frac{1}{\tau_R} & y\frac{1}{\tau_R} & 0 & \beta y\frac{1}{\tau_R} \\ y\frac{1}{\tau_R} & y\frac{1}{\tau_R} & \frac{1}{\tau_R} + \frac{1}{\tau_\xi} & 0 & \beta\frac{1}{\tau_R} \\ 0 & 0 & 0 & 0 & 0 \\ \beta y\frac{1}{\tau_R} & \beta y\frac{1}{\tau_R} & \beta\frac{1}{\tau_R} & 0 & \beta^2\frac{1}{\tau_R} + \frac{1}{\tau_\zeta} \end{pmatrix} \tag{121}$$

where we use the fact that since $s_p = \theta - \frac{1}{\beta} Z$ we have $\mathbb{C}(\theta, z_i | s_p) = \beta \mathbb{V}(\theta | s_p)$ and $\mathbb{V}(Z | s_p) = \beta^2 \mathbb{V}(\theta | s_p)$.

We can then directly compute

$$\mathbb{V}_R((U, z_i)|V - P) = \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) \\ \mathbb{C}_R(z_i, U|V-P) & \mathbb{V}_R(z_i|V-P) \end{pmatrix} \tag{122}$$

$$= \begin{pmatrix} \frac{1}{\tau_R + y^2\tau_A} + \frac{1}{\tau_\xi} & \beta \frac{1}{\tau_R + y^2\tau_A} \\ \beta \frac{1}{\tau_R + y^2\tau_A} & \beta^2 \frac{1}{\tau_R + y^2\tau_A} + \frac{1}{\tau_\zeta} \end{pmatrix}. \tag{123}$$

Let's first deal with the quadratic form.

$$\frac{1}{2} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix} \tag{124}$$

Note than in equilibrium, both $\mathbb{E}_R[U] = \mathbb{E}_R[\theta]$ and $\mathbb{E}_R[z_i] = \mathbb{E}_R[Z]$ do not depend on y. Hence, differentiating with respect to y, using standard results for differentiating a matrix inverse, yields

$$\frac{\partial}{\partial y} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix} \tag{125}$$

$$= \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix}' \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \left[-\frac{\partial}{\partial y} \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix} \right] \tag{126}$$

$$\times \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}^{-1} \begin{pmatrix} \mathbb{E}_R[U] \\ \mathbb{E}_R[z_i] \end{pmatrix} \tag{127}$$

If we establish that the matrix $-\frac{\partial}{\partial y} \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix}$ is positive semidefinite then it will follow that the above expression is (weakly) positive and hence the quadratic form is increasing in y. But this is immediate since

$$-\frac{\partial}{\partial y} \begin{pmatrix} \mathbb{V}_R(U|V-P) & \mathbb{C}_R(U, z_i|V-P) + \frac{1}{\gamma} \\ \mathbb{C}_R(z_i, U|V-P) + \frac{1}{\gamma} & \mathbb{V}_R(z_i|V-P) \end{pmatrix} = \begin{pmatrix} \frac{2y\tau_A}{(\tau_R + y^2\tau_A)^2} & \beta \frac{2y\tau_A}{(\tau_R + y^2\tau_A)^2} \\ \beta \frac{2y\tau_A}{(\tau_R + y^2\tau_A)^2} & \beta^2 \frac{2y\tau_A}{(\tau_R + y^2\tau_A)^2} \end{pmatrix} \tag{128}$$

for which all principal minors are non-negative.

Next consider the determinant term:

$$-\frac{1}{2} \log \frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} + \frac{1}{2} \log \left| I_2 + \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix} \mathbb{V}_R((U, z_i)|V-P) \right|. \tag{129}$$

Using the conditional variances computed above, we immediately have:

$$-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)} = -\frac{1}{2}\log\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}. \tag{130}$$

Furthermore,

$$\frac{1}{2}\log\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{131}$$

$$= \frac{1}{2}\log\left|\begin{pmatrix}1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A} & \gamma\left(\beta^2\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\zeta}\right)\\ \gamma\left(\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\xi}\right) & 1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\end{pmatrix}\right| \tag{132}$$

$$= \frac{1}{2}\log\left(\left(1+\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\right)^2-\gamma^2\left(\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\xi}\right)\left(\beta^2\frac{1}{\tau_R+y^2\tau_A}+\frac{1}{\tau_\zeta}\right)\right) \tag{133}$$

$$= \frac{1}{2}\log\left(\left(1+2\gamma\beta\frac{1}{\tau_R+y^2\tau_A}\right)-\gamma^2\left(\frac{1}{\tau_R+y^2\tau_A}\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)+\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\right) \tag{134}$$

$$= \frac{1}{2}\log\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\tau_R+y^2\tau_A}\right) \tag{135}$$

So, combining terms yields

$$-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}+\frac{1}{2}\log\left|I_2+\begin{pmatrix}0 & \gamma\\ \gamma & 0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right| \tag{136}$$

$$= \frac{1}{2}\log\left[\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\tau_R+y^2\tau_A}\right)\right] \tag{137}$$

$$= \frac{1}{2}\log\left[\frac{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}{\frac{1}{\tau_A}+y^2\frac{1}{\tau}}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)+\frac{1}{\tau_R\tau_A}\frac{2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)}{\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}}\right)\right] \tag{138}$$

Differentiating the argument of the log with respect to y implies that signing the dependence of on y is equivalent to signing

$$\frac{\partial}{\partial y}(\cdot) \propto 2y\frac{1}{\tau_R}\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau}\right) \tag{139}$$

$$-2y\frac{1}{\tau}\left(\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)\left(\frac{1}{\tau_A}+y^2\frac{1}{\tau_R}\right)+\frac{1}{\tau_R\tau_A}\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right) \tag{140}$$

$$= 2y\left[\frac{1}{\tau_A}\left(\frac{1}{\tau_R}-\frac{1}{\tau}\right)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\frac{1}{\tau}\frac{1}{\tau_R\tau_A}\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right] \tag{141}$$

$$= 2y\frac{1}{\tau_A\tau_R\tau}\left[(\tau-\tau_R)\left(1-\gamma^2\frac{1}{\tau_\xi}\frac{1}{\tau_\zeta}\right)-\left(2\gamma\beta-\gamma^2\left(\frac{1}{\tau_\zeta}+\beta^2\frac{1}{\tau_\xi}\right)\right)\right] \tag{142}$$

$$= 2y \frac{1}{\tau_A \tau_R \tau} \left[\left(\tau_\varepsilon + \beta^2 \tau_\zeta\right) \left(1 - \gamma^2 \frac{1}{\tau_\xi} \frac{1}{\tau_\zeta}\right) - \left(2\gamma\beta - \gamma^2 \left(\frac{1}{\tau_\zeta} + k^2 \frac{1}{\tau_\xi}\right)\right)\right] \tag{143}$$

We claim that this expression is positive. To establish this, we need to sign the term inside the brackets, which can be further simplified as

$$\left(\tau_\varepsilon + \beta^2 \tau_\zeta\right) \left(1 - \gamma^2 \frac{1}{\tau_\xi} \frac{1}{\tau_\zeta}\right) - \left(2\gamma\beta - \gamma^2 \left(\frac{1}{\tau_\zeta} + \beta^2 \frac{1}{\tau_\xi}\right)\right) \tag{144}$$

$$= \left(\tau_\varepsilon + \beta^2 \tau_\zeta\right) \left(1 - \gamma^2 \frac{1}{\tau_\xi} \frac{1}{\tau_\zeta}\right) - 2\gamma\beta + \frac{\gamma^2}{\tau_\xi \tau_\zeta} \left(\tau_\xi + \beta^2 \tau_\zeta\right) \tag{145}$$

$$= \gamma\beta \left(1 - \gamma^2 \frac{1}{\tau_\xi} \frac{1}{\tau_\zeta}\right) - 2\gamma\beta + \frac{\gamma^2}{\tau_\xi \tau_\zeta} \left(\tau_\xi + \gamma\beta - \tau_\varepsilon\right) \tag{146}$$

$$= \frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta + \left(1 - \frac{\gamma^2}{\tau_\xi \tau_\zeta}\right) \tau_\varepsilon \tag{147}$$

where the second line rearranges terms, the third line comes from substituting in $\tau_\varepsilon + \beta^2 \tau_\zeta = \gamma\beta$ from the equation defining the price informativeness parameter β, and the remaining lines rearrange and collect terms.

We know that $\left(1 - \frac{\gamma^2}{\tau_\xi \tau_\zeta}\right) \tau_\varepsilon \geq 0$ due to the parameter restriction $1 - \frac{\gamma^2}{\tau_\xi \tau_\zeta} > 0$ required for the existence of expected utility. It remains to show that $\frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta > 0$. We have

$$\frac{\gamma^2}{\tau_\zeta} - \tau_\varepsilon - \gamma\beta = \gamma\beta \left(\left(\frac{\gamma^2}{\tau_\zeta \tau_\varepsilon} - 1\right) \frac{\tau_\varepsilon}{\gamma\beta} - 1\right) \tag{148}$$

$$= \gamma\beta \left(\frac{1}{2} \left(\frac{\gamma^2}{\tau_\zeta \tau_\varepsilon} - 1\right) \left(1 \pm \sqrt{1 - 4\frac{\tau_\varepsilon \tau_\zeta}{\gamma^2}}\right) - 1\right) \tag{149}$$

$$\equiv \gamma\beta \left(\frac{1}{2} (a - 1) \left(1 \pm \sqrt{1 - \frac{4}{a}}\right) - 1\right) \tag{150}$$

where the first line groups terms, the second line substitutes in the equilibrium $\frac{\tau_\varepsilon}{\gamma} \frac{1}{\beta} = \frac{1}{2} \left(1 \pm \sqrt{1 - 4\frac{\tau_\varepsilon \tau_\zeta}{\gamma^2}}\right)$ and cancels terms, and the third line simplifies notation by defining $a = \frac{\gamma^2}{\tau_\zeta \tau_\varepsilon}$. The admissible parameters are such that $a \geq 4$ (i.e, the parameters such that equilibrium in the financial market exists). Hence, we need to show that this expression is positive for any $a \in [4, \infty)$.

Note that the expression is clearly positive if we select the β^{SUB} equilibrium, since that corresponds to selecting the + sign for the solution for $1/\beta$. With the selection of the + in the above, the expression is trivially increasing in a and therefore

$$\frac{1}{2} (a - 1) \left(1 + \sqrt{1 - \frac{4}{a}}\right) - 1 \geq \frac{1}{2} (4 - 1) - 1 = \frac{1}{2} \tag{151}$$

as desired.

Consider next the β^{COM} equilibrium, which corresponds to the negative square root above. We claim that $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ is strictly decreasing in a, takes value $1/2$ at $a=4$ and tends to 0 as $a\to\infty$. Once shown this establishes that it is strictly positive for any finite a. Compute the derivative

$$\frac{\partial}{\partial a}\left(\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1\right) \tag{152}$$

$$\propto 1-\frac{\partial}{\partial a}\left((a-1)\sqrt{1-\frac{4}{a}}\right) \tag{153}$$

$$=1-\frac{a^2-2a-2}{a^2\sqrt{1-\frac{4}{a}}} \tag{154}$$

Note that

$$1-\frac{a^2-2a-2}{a^2\sqrt{1-\frac{4}{a}}}=\frac{a^2\sqrt{1-\frac{4}{a}}-a^2-2a-2}{a^2\sqrt{1-\frac{4}{a}}}<\frac{-2a-2}{a^2\sqrt{1-\frac{4}{a}}}<0 \tag{155}$$

which establishes that the derivative is negative and therefore $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ is strictly decreasing.

Next, it is immediate that the expression takes value $1/2$ at $a=4$:

$$\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1=\frac{1}{2}3-1=\frac{1}{2} \tag{156}$$

Finally, to confirm that $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ limits to 0 as $a\to\infty$, we have

$$\lim_{a\to\infty}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)=\lim_{a\to\infty}a\left(1-\sqrt{1-\frac{4}{a}}\right) \tag{157}$$

$$=\lim_{a\to\infty}\frac{1-\sqrt{1-\frac{4}{a}}}{a^{-1}} \tag{158}$$

$$=\lim_{a\to\infty}\frac{-\frac{2}{a^2\sqrt{1-\frac{4}{a}}}}{-\frac{1}{a^2}} \tag{159}$$

$$=\lim_{a\to\infty}\frac{2}{\sqrt{1-\frac{4}{a}}} \tag{160}$$

$$=2, \tag{161}$$

where the first equality uses the fact that the limit does not change if we replace $a-1$ with a, the second equality moves the a to the denominator, the third equality uses L'Hospital's rule, the next-to-last equality cancels and collects terms, and the final equality takes the limit directly.

Putting things together, the expression $\frac{1}{2}(a-1)\left(1-\sqrt{1-\frac{4}{a}}\right)-1$ is strictly decreasing from $1/2$ to 0 as a ranges from 4 to ∞. This establishes that the original expression we desired to sign, $\left(\frac{1}{2}(a-1)\left(1\pm\sqrt{1-\frac{4}{a}}\right)-1\right)$, is strictly positive for all $a\in[4,\infty)$. Owing to eq. (150) this implies that $\frac{\gamma^2}{\tau_\zeta}-\tau_\varepsilon-\gamma k>0$ for all admissible values of γ,τ_ε, and τ_ζ. It follows that the derivative in eq. (143) is strictly positive and therefore $-\frac{1}{2}\log\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_R(V-P)}+\frac{1}{2}\log\left|I_2+\begin{pmatrix}0&\gamma\\\gamma&0\end{pmatrix}\mathbb{V}_R((U,z_i)|V-P)\right|$ is strictly increasing in y as claimed.

Combined with the above result that the quadratic form that appears in the function F is increasing in y, it follows that $F(y|s_p)$ is increasing in y. This establishes that the welfare maximizing investment rule invests more than the manager state-by-state in s_p.

Proof of Proposition 7

Much of this proof is analogous to that of Proposition 5, so we highlight mostly the essential differences.

Conjecture that there exists an equilibrium in which the asset price reveals $s_p=\theta-\frac{1}{\beta}Z$ when $s_p\geq\overline{s}_p$ for constants β and $\overline{s}_p$ to be determined.

In the conjectured equilibrium, the manager's optimal investment is

$$y^*(\cdot)=\begin{cases}\arg\max_{y\in Y} y\mathbb{E}[\theta|s_p]-cy & s_p\geq\overline{s}_p\\ \arg\max_{y\in Y} y\mathbb{E}[\theta|s_p<\overline{s}_p]-cy & \text{otherwise}\end{cases}. \tag{162}$$

where we select the threshold $\overline{s}_p\equiv\inf\{s_p:\arg\max_{y\in Y} y\mathbb{E}[\theta|s_p]-c(y)>0\}$.

Stepping back to $t=1$, the problem of an arbitrary trader i is

$$\sup_{x\in\mathbb{R}}\mathbb{E}_i\left[-e^{-\gamma(x(V-P)+z_iU+nV)}\right] \tag{163}$$

Under the conjecture that the investment occurs and the price reveals s_p if and only if $s_p\geq\overline{s}_p$, the trader's conditional beliefs about payoffs are conditionally normal with moments specified in the proof of Proposition 5.

The trader's optimal demand is

$$x^*=\frac{\mathbb{E}_i[V]-P-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)}-n. \tag{164}$$

Now turn to the market-clearing condition and solve for the candidate asset price. For states $s_p<\overline{s}_p$, we have

$$\int_i\left(\frac{\mathbb{E}_i[V]-P-\gamma\mathbb{C}_i(V,U)z_i}{\gamma\mathbb{V}_i(V)}-n\right)di\Rightarrow P=\mu_A-\gamma\frac{1}{\tau_A}n \tag{165}$$

so that the aggregate order does not reveal any information to the market maker, as conjectured. For states $s_p \geq \overline{s}_p$, we have

$$\int_i \left(\frac{\mathbb{E}_i[V] - P - \gamma \mathbb{C}_i(V,\theta) z_i}{\gamma \mathbb{V}_i(V)} - n \right) di = \int_i \left(\frac{\mu_A + y\mathbb{E}[\theta|s_i, z_i, s_p] - c(y) - P - \gamma \frac{1}{\tau} z_i}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n \right) di \tag{166}$$

$$= \frac{\mu_A + y \int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - P}{\gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right)} - n. \tag{167}$$

so that the asset price is

$$P = \mu_A + y \int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - \gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right) n. \tag{168}$$

Performing the integration over i

$$\int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di \tag{169}$$

$$= \int_i \left(\mu_\theta + b_s(s_i - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta} \mu_Z \right) \right) + b_z(z_i - \mu_Z) - \gamma \frac{1}{\tau} z_i \right) di \tag{170}$$

$$= \mu_\theta + b_s(\theta - \mu_\theta) + b_p \left(s_p - \left(\mu_\theta - \frac{1}{\beta} \mu_Z \right) \right) + \left(b_z - \gamma \frac{1}{\tau} \right) Z - b_z \mu_Z \tag{171}$$

$$= b_0 + b_s \left(\theta - \frac{\gamma \frac{1}{\tau} - b_z}{b_s} Z \right) + b_p s_p \tag{172}$$

where the last line groups terms and defines the constant

$$b_0 = \mu_\theta - b_s \mu_\theta - b_p \left(\mu_\theta - \frac{1}{\beta} \mu_Z \right) - b_z \mu_Z. \tag{173}$$

It follows that the conjecture that the equilibrium price reveals a statistic of the form $s_p = \theta - \frac{1}{\beta} Z$ can hold if and only if

$$\frac{1}{\beta} = \frac{\gamma \frac{1}{\tau} - b_z}{b_s} \tag{174}$$

which is identical to the case of Proposition 5 and has solutions

$$\beta = \frac{1}{2\tau_\zeta} \left(\gamma \pm \sqrt{\gamma^2 - 4\tau_\varepsilon \tau_\zeta} \right). \tag{175}$$

With the precise expression for s_p and our candidate price function is

$$P = \begin{cases} \mu_A - \gamma \frac{1}{\tau_A} n & s_p < \overline{s}_p \\ \mu_A + y \int_i \left(\mathbb{E}[\theta|s_i, z_i, s_p] - \gamma \frac{1}{\tau} z_i \right) di - c(y) - \gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right) n & s_p \geq \overline{s}_p \end{cases} \tag{176}$$

$$= \begin{cases} \mu_A - \gamma \frac{1}{\tau_A} n & s_p < \overline{s}_p \\ \mu_A + y \left(b_0 + (b_s + b_p) s_p \right) - c(y) - \gamma \left(\frac{1}{\tau_A} + y^2 \frac{1}{\tau} \right) n & s_p \geq \overline{s}_p \end{cases}. \tag{177}$$

As long as this function is increasing in s_p the region $s_p \geq \overline{s}_p$ then traders can infer s_p when $s_p \geq \overline{s}_p$, as conjectured.