Incentivizing Effort and Informing Investment: The Dual Role of Stock Prices

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Abstract

Stock prices reflect firm performance and aggregate investor information about investment opportunities. We show that these dual roles are in tension: when prices are more informative about future investment, they are less effective at incentivizing managerial effort. Overall firm value need not increase with price informativeness; both decreasing transparency and allowing for ex-post inefficient investment can increase firm value. We show that standard empirical measures of price efficiency are incomplete and derive testable predictions for the price-sensitivity and composition of managerial compensation.

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1 Introduction

Stock prices reflect information about firm performance and managerial decisions, and as such, play an important role in *incentive provision* via compensation contracts. Standard arguments suggest that contracting efficiency and hence, firm value, increase when prices more precisely capture managerial actions, i.e., if the "noise" in the contracting signal can be reduced. In practice, however, stock prices are inherently multi-dimensional: what may be noise from a contracting perspective may be valuable information from the perspective of the firm. For example, when stock prices affect firm investment decisions, commonly referred to as the *feedback effect* (e.g., Bond, Edmans, and Goldstein (2012)), this noise captures investor information about a potential investment opportunity.

Given the importance of both the incentive provision and feedback effects channels in practice, our paper's primary contribution is to study how these dual roles interact in equilibrium. In our model, the principal of a firm must compensate her risk-averse manager for costly effort while also choosing whether or not to pursue a new investment opportunity. In equilibrium, the price provides a contractable signal of managerial effort *and* aggregates investors' information about future investment opportunities. Our key observation is that the incentive provision and feedback roles are often in tension: as the price becomes more informative about future opportunities, it becomes less useful for incentive provision. Intuitively, when the price is more informative about new investment opportunities, it is also more volatile, making it more difficult for the principal to incentivize the manager to exert effort. As a result, more informative prices need not lead to higher firm value, even when they lead to more informationally efficient investment decisions.

Our analysis implies that understanding the interaction between these dual roles is particularly important for policy. First, we show that existing measures of real price efficiency provide an incomplete measure of firm value, and so should be interpreted carefully when evaluating policy changes. Second, we show that an increase in transparency (i.e., a decrease in information acquisition costs for investors) can decrease firm value and social surplus, even when it increases investment efficiency. We characterize conditions under which increasing transparency is desirable from the firm's perspective, and when it is desirable from a social perspective.

Our model also generates a number of testable predictions for managerial compensation contracts and firm behavior. The key tradeoff between incentive provision and the feedback effect implies that the price-sensitivity of managerial compensation should decrease as the price becomes more informative about future investment opportunities. This is consistent with empirical evidence in Lin, Liu, and Sun (2019). We also show that price-sensitivity can decrease even when the price becomes more informative about managerial effort, in contrast to standard wisdom from existing analysis (e.g., Holmström and Tirole (1993)). Since price informativeness is negatively related to liquidity, our model predicts that there is a positive relation between total pay-for-performance and liquidity, which is consistent with the evidence in Fang, Noe, and Tice (2009) and Jayaraman and Milbourn (2012). Moroever, we show that the firm may prefer to adopt an inefficient investment rule (by delegating investment to the manager) in order to improve the stock price's ability to provide incentives. Finally, our model generates novel predictions about how the relative weights on short-term versus long-term performance vary with the ex-ante profitability of future investment opportunities.

Section 3 presents the model. There is a single firm with a risk-neutral principal (she) and a risk-averse manager (he). The manager can exert costly effort to increase the terminal cash flows of the firm. This effort is incentivized by a contract set by the principal to maximize the value of firm, net of this compensation. The principal chooses whether or not to invest in a new project, given the information available to her. The firm's stock is a claim to the terminal cash flows net of payments, and is traded by a group of investors, some of who are privately informed about the profitability of the new project.

In Section 4, we solve for the financial market equilibrium and the optimal contract offered by the principal, which depend on each other in equilibrium. The model captures the two roles of the stock price. First, the principal uses a stock price-based contract to compensate the manager for exerting costly effort — this is the **incentive provision** role.[1] Second, because the price aggregates investors' information about the future investment opportunity, the principal conditions on the price when making her investment decision — this captures the **feedback effect** from prices to real investment. The key insight of our analysis is that these two roles often operate in opposite directions. On the one hand, when the price is more informative about the new project, the principal makes a more informed investment decision and this increases firm value. On the other hand, more informative prices are also more volatile, *conditional* on a given level of effort. Since the manager is risk averse, this makes compensating him for effort more costly and so the principal optimally reduces the price-sensitivity of the payment to the manager. This, however, reduces his effort and, consequently, firm value.

Section 5 characterizes the implications of this tradeoff. A key parameter of interest is the contribution of the investment opportunity *relative* to the contribution of managerial effort

[1] As is common in the literature, we focus on linear contracts. In Section 6.3, we allow the contract to depend on both the short-term price and the (long-term) terminal value. More generally, our analysis is based on the premise that, while there may be other signals of managerial effort, the stock price provides an unbiased and natural instrument for compensation contracts.

to terminal cash flows — we denote this by δ. When δ is high (low), the relative impact of learning about future investment opportunities is higher than the impact of incentivizing managerial effort. In the absence of any feedback, more informative prices always lower firm value: the optimal contract is lower-powered which discourages managerial effort.[2] When the principal's investment decision depends upon the stock price, expected firm value unambiguously increases (decreases) with price informativeness when δ is sufficiently high (sufficiently low) but is U-shaped for intermediate levels of δ. Together these results imply that the impact of price informativeness on firm value can vary across firms (with different δ) and also within firms, for different levels of price informativeness. Moreover, these results imply that controlling for the relative importance of future opportunities versus assets in place is crucial in understanding both the empirical relationship between price informativeness and firm value, as well as evaluating the impact of policy changes.

Our analysis implies that standard measures of price efficiency provide an incomplete picture of firm value. Following Bond et al. (2012), we distinguish between the notions of *forecasting price efficiency* (FPE), which captures the extent to which information in prices forecast future firm cashflows, and *revelatory price efficiency* (RPE), which measure the extent to which prices reveal information relevant for future investment decisions.[3] The literature on feedback effects has clarified the difference across these measures, and argues that when learning from prices plays an important role for investment decisions, RPE is the relevant measure of real efficiency. However, our results imply that this may no longer the case when the price is also used to incentivize managerial effort. We introduce the notion of *contracting price efficiency* (CPE), which measures the extent to which the price reflects the manager's actions and, consequently, is useful for contracting purposes. We show that when investors are better informed about future investment opportunities, this increases RPE but decreases CPE, and only increases FPE when the relative impact of the investment opportunity, δ, is sufficiently high. Importantly, none of the three measures always agree with firm value (and social surplus), which suggests one must be cautious in using these measures in isolation for empirical and policy analysis.

The implications of our main insight are wide-ranging. In Section 6, we highlight some of these in a series of extensions to the benchmark model. In Section 6.1, we endogenize the information content of prices by allowing investors to choose whether or not to acquire (costly) information about the new project (e.g., as in Grossman and Stiglitz (1980)). This allows us to characterize the impact of an increase in transparency (decrease in the cost

[2]In our model, when the ex-ante profitability of the project is sufficiently high (sufficiently low), the principal will always (never) invest in the project, and so there is no feedback from market prices.

[3]It is worth noting that we follow the feedback literature and use RPE to denote "revelatory price efficiency" and not "relative performance evaluation."

of information) on firm value, net of managerial compensation, as well as a measure of social surplus. Notably, an increase in transparency always (weakly) increases investment efficiency and RPE. However, an increase in transparency may decrease net firm value when δ is sufficiently low or if there is no feedback from prices to investment. If firms can affect transparency by changing the clarity of their disclosures, this implies that firms should be more opaque when the incentive provision role of prices is more important, even if the information in the price is utilized for investment decisions.[4] We find similar results when analyzing the impact of transparency on social surplus, suggesting that it is important to account for firm heterogeneity when considering the consequences of regulations that affect transparency in financial markets. More broadly, our analysis also suggests that technological advances that have led to greater transparency and price informativeness over time (e.g., see Bai, Philippon, and Savov (2016)) need not be associated with increases in firm value or social surplus.

In our benchmark model, we assume that investors can perfectly observe the manager's effort when submitting their demand for the stock. In Section 6.2, we relax this assumption and instead assume that investors observe a noisy, public, non-contractable signal of managerial effort (as in Holmström and Tirole (1993)). We find that the key tradeoff in our benchmark analysis still obtains in this setting: as the price becomes more informative about future investment opportunities, price-sensitivity of the equilibrium contract decreases. However, we show that the price-sensitivity of the equilibrium contract can decrease as the public signal about effort becomes more informative. An increase in the precision of the public signal has two offsetting effects: (i) it increases how informative the price is about effort, which leads to higher effort (holding fixed the contract), and (ii) for a fixed level of effort, increases the volatility of price, which discourages effort. This latter effect can dominate when the signal precision is sufficiently high which leads the principal to offer a less price-sensitive contract.

In section 6.3, the principal is allowed to contract on both the short-term price and the long-term terminal value of the firm. In our setting, the principal generically chooses to put positive weight on both short-term and long-term performance measures. This is because the price and terminal value are imperfectly correlated: by using both measures, the equilibrium contract reduces the risk exposure of the manager through diversification. As in our benchmark analysis, as the price becomes more informative about future investment opportunities, the *overall* pay-performance sensitivity decreases. Interestingly, however, we show that the impact on the relative weight placed on long-term and short-term components

[4] This is consistent with the evidence in Bebchuk and Fried (2004), who find that hard-to-value and opaque firms tend to use more aggressive stock-based pay for managers.

can be non-monotonic. While the intuition from the benchmark model suggests that the relative weight on the short-term price should decrease with price informativeness, we find that the opposite can arise when the ex-ante profitability of the new project is low. More generally, this analysis provides novel predictions about how the composition of compensation contracts should vary with measures of price informativeness, the relative importance of feedback effects, and the ex-ante profitability of investment opportunities.

In the benchmark analysis, the principal chooses investment to maximize long-term value. In Section 6.4, we characterize how our results change if, instead the principal *delegates* the decision to the manager. The manager chooses whether or not to invest in order to maximize his expected payoff, and as a result, his investment rule maximizes the short-term price instead of long-term value. Relative to the principal's investment policy, the manager over-invests for low (ex-ante) profitability projects, but under-invests for high (ex-ante) profitability projects. While this decreases investment efficiency, we show that delegation can increase overall firm value. In particular, because the manager under-invests in high profitability projects, the price is less volatile and, consequently, less costly to use for incentive provision. As a result, the principal increases the performance sensitivity of the contract she offers to the manager, which induces more effort in equilibrium. When the relative importance of the feedback effect (i.e., δ) is low, this latter effect can dominate and expected firm value increases. These results suggests that the principal may prefer to the delegate investment decision to the manager when ex-ante profitability is high, even though the manager invests more "conservatively" because it can encourage effort provision and generate higher firm value.

The rest of the paper is as follows. The next section briefly discusses the related literature and the paper's contribution. Section 3 presents the model and discusses the key assumptions. Section 4 characterizes the equilibrium and Section 5 characterizes the implications for the relation between price informativeness, expected value and the various measures of efficiency. Section 6 presents the extensions and Section 7 concludes. All proofs and robustness exercises are in the Appendix.

2 Related Literature

Our paper contributes to the literature on how asset prices affect real decisions (see Bond et al. (2012) for a survey). The literature broadly identifies two roles for prices through which secondary markets may affect real decisions. The early part of this literature largely focused on how prices provide information about managerial effort and, consequently, how stock based compensation affects the value of the firm through incentive provision (e.g., see

Scholes (1991), Paul (1992), Holmström and Tirole (1993), Calcagno and Heider (2021)). More recently, the literature has focused on "feedback effects," whereby stock prices aggregate dispersed information of investors and managers can learn new information from stock prices and use this information to guide their real decisions (see Goldstein and Yang (2017) for a recent survey). However, the literature has largely focused on each role in isolation, while abstracting from the other.[5] Our analysis implies that the interaction between the "incentives" role and the "feedback" role of prices is important in understanding how measures of price efficiency and investors incentives to acquire information are related to firm value.

The work of Lin, Liu, and Sun (2019) and Dow and Gorton (1997) are perhaps the closest to our own analysis. In both, the manager can utilize either their own private information to make an investment decision or, alternatively, rely on the price. Price information can substitute for managerial information and so when investors are better-informed, the manager's optimal contract is lower-powered. Effectively, high price informativeness makes contracting with the manager easier. In contrast, we focus on settings in which the manager's effort does not affect the investment decision and so increasing investors' information about the investment project makes the price less informative about the manager's effort. More generally, our results highlight that investors' information can affect managerial actions even when the two dimensions independently affect firm value. In this sense, our paper is related to the theoretical literature which considers the implications when firm value is multi-dimensional. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein, Kopytov, Shen, and Xiang (2021) consider settings where investors are heterogeneously informed about two components of fundamentals. Similarly, in our model, we can view managerial effort and information about future investment as different components of fundamentals. The key difference is that the first fundamental is ain unobservable choice made by the manager in our setting.

Gjesdal (1981) shows that, in a generalized information environment, the information system which is preferable for compensating an agent may not be preferable for decision-making. Our paper demonstrates that this tension naturally arises in settings where the signal of interest is the traded stock price. This result is related but distinct from the findings of Paul (1992), Bushman and Indjejikian (1993), and Kim and Suh (1993) who emphasize that the weight investors place on their information reflects the contribution of the project to firm value instead of the contribution of the manager to the project (which would be preferable for contracting purposes). In our setting, investors efficiently weight the manager's contribution to the firm's value but their information about other projects make the price a

[5] For instance, while Bresnahan, Milgrom, Paul, et al. (1992) analyzes both roles for prices, they shut down one dimension when analyzing the impact of change in investors' information on the other dimension.

noisier signal of his effort. Perhaps more importantly, our analysis highlights how investor information also affects investment efficiency and its interaction with contracting efficiency. This interaction also distinguishes our paper from Chaigneau, Edmans, and Gottlieb (2018) who also find that increases in investor information can reduce managerial effort.

Our paper is related to the broader literature on the impact of transparency on firm outcomes and choices. The recent literature has highlighted a number of channels through which changes in transparency (e.g., due to public disclosures or a reduction in the cost of information acquisition) can have unintended consequences for price informativeness and real efficiency (see Bond et al. (2012), Goldstein and Sapra (2014), and Goldstein and Yang (2017) for recent surveys). Our model sheds light on this debate from a new angle: is transparency desirable when the principal also uses the price to incentivize managers? In our framework, the answer to this question depends on the relative impact of managerial effort versus new investment opportunities on firm value.

In our setting, an increase in transparency (i.e., reduction in investors' cost of information acquisition) leads to more informative prices (higher revelatory price efficiency), but can reduce both firm value and social value due to its impact on contracting efficiency. This is in contrast to, but complements, existing analyses that have focused on how increases in disclosure or transparency can lead to "crowding out" of information that is relevant for investment decisions.[6] Moreover, our analysis implies that the impact of increased transparency can have qualitatively different implications for (i) investment efficiency, (ii) overall firm value, and (iii) social surplus. Our results caution against the use of measures of price informativeness as proxies for firm value or social welfare when evaluating the impact of changes in transparency. These results are also related to the recent literature analyzing analyzing the effects of new technologies which make it easier for investors to acquire financial information (e.g., Dugast and Foucault (2018), Banerjee et al. (2018), Farboodi and Veldkamp (2017)). Our analysis shows how advances in information technologies can be costly for society and suggests that the use of such technology may warrant further scrutiny by regulators.

[6] For instance, Diamond (1985), Gao and Liang (2013), and Colombo, Femminis, and Pavan (2014) discuss how public disclosure can crowd out private learning by investors. Goldstein and Yang (2019) show how disclosure about one component of fundamentals affects price informativeness about other components. Banerjee, Davis, and Gondhi (2018) focus on a related effect, where an increase in transparency can encourage learning about fundamentals, but also the behavior of other investors, and thereby make prices less informationally efficient.

3 Model

Payoffs. There are three dates $t \in \{1, 2, 3\}$, and two equally-likely states of the world, denoted by $\omega \in \{H, L\}$. The state is realized at date one, but is unobservable. The firm pays terminal cash flows V at date three, which consist of two components: (i) assets-in-place that generate x_ω and (ii) a zero-cost investment opportunity that generates δy_ω.[7] We assume that $x_H > x_L > 0$, $y_H > 0 > y_L$. The parameter $\delta \geq 0$ captures the importance of the new investment opportunity relative to the firm's existing assets. There are two traded securities. The risk-free security is normalized to the numeraire. The risky security is a claim to the net cash flows of the firm, and trades at a price P at date two.

The firm. The firm consists of a principal (she) and a manager (he). The principal, indexed by p, is risk neutral and offers a take it or leave it contract to the manager at date one to maximize expected firm value, net of payments. We assume that the principal offers the manager a linear compensation contract of the form:

$$W(P) = \alpha + \beta P, \tag{1}$$

where α denotes the manager's fixed wage and β denotes the sensitivity of her compensation to the market price at date two.[8]

The manager, indexed by m, has mean-variance preferences over his payoff W, with risk aversion coefficient γ, and an outside option u_0, which we normalize to zero. At date one, the manager can exert costly effort e to increase the cash flows from assets-in-place from x_ω to $x_\omega + e$. We assume that the effort is observable but not verifiable, and requires the manager to incur a private cost, $c(e)$, where $c' \geq 0, c'' > 0, c(0) = c'(0) = 0$. The manager chooses his effort level to maximize his utility from compensation net of effort costs i.e.,

$$u_m(e; \alpha, \beta) \equiv \mathbb{E}[W(P)] - \frac{\gamma}{2}\mathbb{V}[W(P)] - c(e), \tag{2}$$

subject to $u_m(e; \alpha, \beta) \geq u_0 = 0$.

At date two, the principal chooses whether or not to invest in the new opportunity in order to maximize the expected value of the terminal cash flows, given the security price. We denote this investment decision by $I(P) \in \{0, 1\}$, where $I(P) = 1$ indicates that the

[7] The assumption of zero cost is isomorphic to a setting in which the required investment is non-zero but can be made using the firm's existing cash (contained in the assets-in-place), i.e., it does not require equity holders to contribute additional capital.

[8] In line with the literature (e.g., Holmström and Tirole (1993), Prendergast (1999)), we consider incentive contracts that are linear in performance measures. In a large class of models, Banker and Datar (1989) show that it is optimal to contract linearly.

investment is made. Together, this implies that the terminal cash flows of the firm are given by

$$V(\omega, e, I) \equiv x_\omega + e + \delta y_\omega \times I. \tag{3}$$

The principal's optimal contract (α^*, β^*) maximizes the terminal value of the firm, net of payments, u_P, subject to the manager's incentive compatibility, investment rule, and participation constraints i.e.,

$$u_p \equiv \max_{\alpha,\beta} \mathbb{E}\left[V(\omega, e^*, I_m^*) - W(P)\right], \quad \text{where} \tag{4}$$

$$e^* = \arg\max u_m(e; \alpha, \beta), \tag{5}$$

$$I^* = \arg\max_I \mathbb{E}\left[V(\omega, e^*, I) | P\right], \text{ and} \tag{6}$$

$$u_m(e^*; \alpha, \beta) \geq u_0. \tag{7}$$

Investors. At date two, a unit-measure continuum of risk-neutral investors indexed by $i \in [0,1]$ trade the claim to the firm's net cash flows, i.e., $V - W$. While all investors condition on the information in the equilibrium price, P, a fraction $\lambda < 1$ of these investors also observe a private, conditionally-independent signal, $s_i \in \{s_H, s_L\}$, where

$$\mathbb{P}\left[s_i = s_H | \omega = H\right] = \mathbb{P}\left[s_i = s_L | \omega \in L\right] = \rho > \tfrac{1}{2}. \tag{8}$$

The remaining fraction, $1-\lambda$, are uninformed, which we denote by $s_i = \emptyset$ with some slight abuse of notation.[9] Investors can take both long and short positions in the risky security, but they are subject to position limits – specifically, we normalize the positions so that $d_i \in [-1, 1]$. This implies that the optimal demand is given by

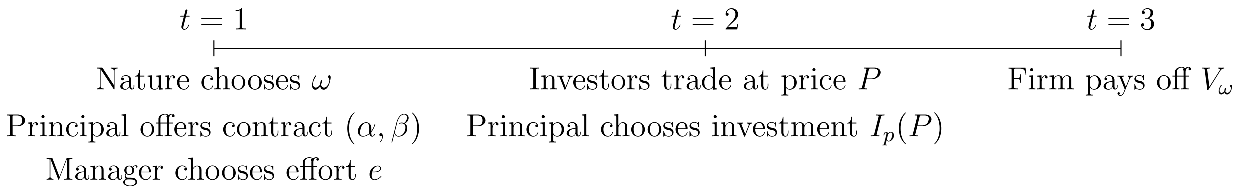
$$d(s_i, P) \equiv \arg\max_{d_i \in [-1,1]} \mathbb{E}\left[d_i \times (V - W - P) | s_i, P\right]. \tag{9}$$

To ensure that the equilibrium price is not fully revealing, the aggregate supply of the risky security available for trade is stochastic and given by $u \sim U[-1, 1]$, where u is independent of all other random variables in the economy.

Timing of events. The timeline of events is summarized in Figure 1. At date one, nature chooses $\omega \in \{H, L\}$. The principal offers the manager a contract (α, β), and the manager

[9] We endogenize the investors' decision to acquire information in Section 6.1.

Figure 1: The timeline of the economy



chooses costly effort e to maximize his expected compensation. At date two, investors observe their signals (if informed) and the price, and trade the risky security. The principal observes the equilibrium price, P, and chooses whether or not to invest in the new opportunity. At date three, the firm's assets pay off cashflows V_ω given by (3).

Equilibrium. An equilibrium consists of a contract $(\alpha.\beta)$, effort level e, investment rule $I(P)$, equilibrium demands $\{d(s_i, P)\}_i$ and equilibrium price P, such that:

(i) $d(s_i, P)$ maximizes the investor's objective in (9),

(ii) the equilibrium price P clears the risky security market,

(iii) the investment rule maximizes the expected firm cash flows (i.e., solves (6)),

(iv) the optimal effort level e maximizes the manager's expected utility (i.e., solves (5)),

(v) the optimal contract (α, β) solves the principal's problem, characterized by (4)-(7), and

(vi) posterior beliefs satisfy Bayes' rule whenever applicable.

3.1 Discussion of Assumptions

Our benchmark analysis makes a number of simplifying assumptions for tractability and expositional clarity. We refer to the positive impact of the manager's decision as arising through "effort" but this need not be interpreted literally. For instance, the private cost borne by the manager can reflect his disutility from completing value-enhancing but monotonous tasks or making difficult but valuable decisions.

Our benchmark analysis assumes that some investors are exogenously endowed with private information about the state of the world, ω. In Section 6.1, we extend the analysis to allow for costly information acquisition. Specifically, we assume that before date one, investors choose whether or not to pay a cost, c_0, to become informed. We then derive the equilibrium fraction of informed investors.This allows us to characterize how changes in transparency (driven by changes in c_0) affect the optimal contract, the expected value of the firm, as well as a measure of social surplus.

We assume that the principal and investors can observe the effort level chosen by the manager but cannot directly contract on this information. This allows us to introduce the tradeoff between the allocative and contracting roles of prices in a transparent manner. In Section 6.2, we consider a setting in which investors have access to a noisy signal of the manager's effort. We show that our key tradeoff obtains in this setting: when the price is more informative about future investment opportunities, it is less effective from a contracting perspective.

For our benchmark model, we assume that the principal cannot contract on the firm's realized value. This is a standard assumption in the literature, commonly motivated by the observation that the manager's tenure at the firm may be shorter than the life of the project. However, in Section 6.3, we relax this assumption and allow the equilibrium contract to depend on both the date two price as well as the terminal value, $V(\omega, e, I)$. We show that the principal always chooses to include the price in the equilibrium contract and, moreover, the interaction between investors' information and equilibrium managerial effort is preserved. Similarly, we show that in Appendix B.2 that our main tradeoff obtains when the manager is risk-neutral, but the optimal contract must satisfy a limited liability constraint.

In our baseline model, the principal's investment policy maximizes the firm's terminal cash flows given the information in the price in our benchmark model. We think of this principal as representing long-term shareholders or the board of directors. In Section 6.4, we relax this constraint and assume that the *manager* chooses whether or to invest to maximize his payoff and, consequently, the price. While this can lead to inefficient investment decisions, the main results are qualitatively similar. Moreover, we show that delegation to manager can be optimal, even though manager's investment policy is sub-optimal, due to its positive impact on managerial effort. This result suggests that it could be optimal for the principal to commit to an ex-post inefficient investment rule. We rule such commitment out to focus on the economic mechanism we wish to highlight; moreover, such a commitment would attenuate but not eliminate the impact of this mechanism.

4 Equilibrium

We solve the model by working backwards.

4.1 Principal's investment decision

The principal can condition on the equilibrium price when choosing whether or not to invest in the new project. Let $q(P) \equiv \mathbb{P}[\omega = H|P]$ denote the her posterior beliefs given his information set. The principal optimally chooses to invest if and only if doing so is ex-post efficient and so in equilibrium she follows a threshold strategy:

$$I(P) = \begin{cases} 1 & \text{if } q(P) \geq \frac{-y_L}{y_H - y_L} \equiv K \\ 0 & \text{if } q(P) < K. \end{cases} \tag{10}$$

The principal invests only if she is sufficiently optimistic that the investment will pay off positively, i.e., if $q(P)$ is sufficiently large. Intuitively, the investment threshold, K, increases with the size of the potential loss ($-y_L$) but decreases with the potential gain (y_H).

4.2 Financial market equilibrium

Given investor i's objective function 9, he optimally adopts a threshold strategy given his risk-neutrality:

$$d(s_i, P) = \begin{cases} 1 & \text{if } P < \mathbb{E}[V - W|s_i, P] \\ [-1, 1] & \text{if } P = \mathbb{E}[V - W|s_i, P] \\ -1 & \text{if } P > \mathbb{E}[V - W|s_i, P]. \end{cases} \tag{11}$$

Investors can observe the manager's effort choice e and infer the principal's investment choice, $I(P)$, since neither the manager nor the principal possess any private information and condition only on public information, i.e., P. As such, conditional on (ω, P), all investors agree on the *state-dependent* value of the firm, $V(\omega, e^*, I(P))$, defined in (3).

However, investors generally differ in their belief about the likelihood of each state. Let $q(s_i, P) \equiv \mathbb{P}[\omega = H|s_i, P]$ denote investor i's beliefs conditional on observing the price P and the private signal $s_i \in \{s_H, s_L, \emptyset\}$. To emphasize the crucial role of the traded price and to simplify our notation, let $V_\omega(P) \equiv V(\omega, e^*, I(P))$. Then investor i's conditional expectations about V are given by

$$\mathbb{E}[V|s_i, P] = V_L(P) + q(s_i, P) \times \left[\underbrace{V_H(P) - V_L(P)}_{\equiv \Delta V(P)} \right], \tag{12}$$

where $\Delta V(P)$ reflects the information sensitivity of cash flows. Note that since $x_H > x_L$ and $y_H > y_L$, we have $\Delta V(P) > 0$ for any P, and so investor i's valuation $\mathbb{E}[V|s_i, P]$ is increasing in $q(s_i, P)$.[10]

To establish the existence of an equilibrium, we conjecture and verify that there are three, distinct price levels $p_H > p_U > p_L$, each of which correspond to the market-clearing price when the marginal investor's valuation of the firm's cash flows is given by

$$\mathbb{E}[V|s_H, p_H] > \mathbb{E}[V|\emptyset, p_U] > \mathbb{E}[V|s_L, p_L], \tag{13}$$

respectively.

For instance, suppose that the supply is relatively high. Then, in order for the market to clear, it must be the case that the marginal investor is a "pessimistic" informed trader (i.e., someone who observed $s_i = s_L$). His valuation of the claim determines the market-clearing price ($P = p_L$) and all such "pessimistic" investors are indifferent between buying and selling. However, both the uninformed and "optimistic" investors (i.e., those who observed $s_i = s_H$) optimally take long positions since

$$\mathbb{E}[V|s_H, p_L] > \mathbb{E}[V|\emptyset, p_L] > \mathbb{E}[V|s_L, p_L]. \tag{14}$$

Whether this price can be supported at a given supply level depends on ω. For instance, if $\omega = H$, the market clearing implies that $P = p_L$ as long as

$$u > \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 1 - 2\lambda(1-\rho). \tag{15}$$

If the supply of the risky asset, u, is any lower, then the market can only clear if one of the uninformed investors is indifferent between buying and selling. But for this to be the case, the price must reflect the valuation of the uninformed, i.e., $P = p_U \neq p_L$. Suppose instead that $\omega = L$. In this case, the measure of informed investors who observed $s_i = s_L$ is higher while the measure that observed $s_i = s_H$ is lower. Thus, the threshold supply such that $P = p_L$ falls to

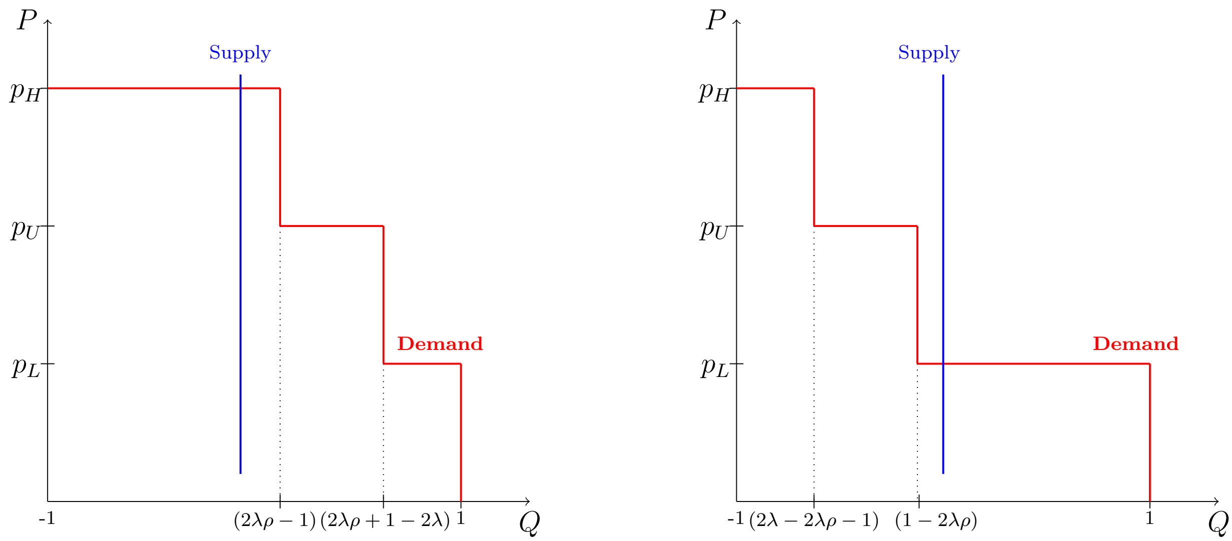
$$u > \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda\rho}_{\text{observed } s_i = s_L} = 1 - 2\lambda\rho. \tag{16}$$

Following similar steps, we can determine the supply thresholds for both p_U and p_H.

Figure 2 illustrates this market-clearing process. While the supply of the traded claim is inelastic, aggregate demand depends upon the realized state, ω. This is clear when comparing panel (a), where $\omega = H$ to panel (b), where $\omega = L$. While the supply is the same in both

[10] This is because $\Delta V(P) = \delta(x_H - x_L) + (1-\delta)(y_H - y_L) \times I_m(P) > 0$.

Figure 2: Market clearing price



(a) Equilibrium price when $\omega = H$

(b) Equilibrium price when $\omega = L$

panels, there are only enough optimistic investors to support the high price when $\omega = H$, while the price falls to p_L when the true state is low.

The information contained in the price is also determined by these thresholds. For instance, given the distribution of u and the thresholds found in (15) and (16), we know that

$$\mathbb{P}\left[P = p_L | \omega = L\right] = \lambda\rho \quad \mathbb{P}\left[P = p_L | \omega = H\right] = \lambda\left(1 - \rho\right). \tag{17}$$

By Bayes' rule, this implies that $\mathbb{P}\left[\omega = H | P = p_L\right] = 1 - \rho$. Following similar steps yields the corresponding belief when the the marginal investor is "optimistic": $\mathbb{P}\left[\omega = H | P = p_H\right] = \rho$. However, the price is not always informative since, independent of the state, the marginal investor is uninformed in equilibrium (i.e., $P = p_U$) with probability $1 - \lambda$. In this latter case, conditioning on the price provides no additional information to the agents in the model.

From this, we can derive the beliefs of both the manager and the marginal investor. Note that, for the manager, who observes only the price, we have:

$$q_m\left(p_L\right) = 1 - \rho, \quad q_m\left(p_U\right) = \frac{1}{2}, \text{ and } \quad q_m\left(p_H\right) = \rho. \tag{18}$$

Plugging these beliefs into (10) gives the equilibrium investment rule. In addition to accounting for the information in the price, the beliefs of the marginal investor must also account for any private information they possess. For example, when the marginal investor is optimistic (so that $P = p_H$), his valuation must also account for his private signal, $s_i = s_H$ and so his expectation of cash flows is given by

$$\tilde{\mu}(p_H) \equiv \mathbb{E}[V|s_H, p_H] = V_L(p_H) + \tilde{\rho}\Delta V(p_H), \tag{19}$$

where

$$\tilde{\rho} \equiv \mathbb{P}[\omega = H|s_i = s_H, P = p_H] = \tfrac{\rho^2}{\rho^2+(1-\rho)^2}. \tag{20}$$

Similar logic implies that the marginal investor is more pessimistic after conditioning on p_L, since

$$\tilde{\mu}(p_L) \equiv \mathbb{E}[V|s_L, p_L] = V_L(p_L) + (1-\tilde{\rho})\Delta V(p_L). \tag{21}$$

In contrast, when the marginal investor is uninformed, the price provides no additional information and so his valuation is given by

$$\tilde{\mu}(p_U) \equiv \mathbb{E}[V|\emptyset, p_U] = V_L(p_U) + \frac{1}{2}\Delta V(p_U). \tag{22}$$

From these conditional expectations of the firm's cash flows, we can derive the market-clearing price since, in equilibrium, the marginal investor is indifferent between buying and selling i.e.,

$$P = \tilde{\mu}(P) - W \tag{23}$$

$$= \frac{1}{1+\beta}(\tilde{\mu}(P) - \alpha). \tag{24}$$

With this, we can formally establish the existence of a financial market equilibrium with feedback to firm investment.

Proposition 1. *There exists a unique financial market equilibrium with price P and investment rule $I_p(P)$, where:*
(i) the equilibrium price $P(\omega, u)$ is given by

$$P(\omega, u) = \begin{cases} \frac{1}{1+\beta}(\tilde{\mu}(p_L) - \alpha) \equiv p_L & \text{if } u < u_\omega - (1-\lambda) \\ \frac{1}{1+\beta}(\tilde{\mu}(p_U) - \alpha) \equiv p_U & \text{if } u_\omega - (1-\lambda) \leq u \leq u_\omega + (1-\lambda) \\ \frac{1}{1+\beta}(\tilde{\mu}(p_H) - \alpha) \equiv p_H & \text{if } u > u_\omega + (1-\lambda) \end{cases}, \tag{25}$$

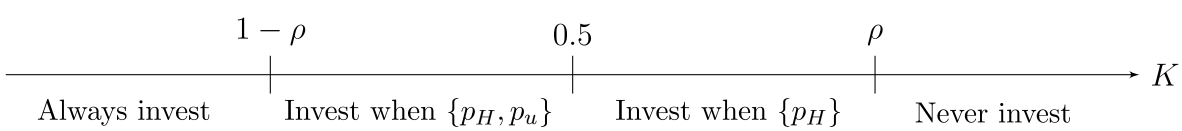
where $u_H \equiv \lambda(2\rho - 1)$, $u_L = \lambda(1-2\rho)$, $p_H > p_U > p_L$, and where the marginal investor's conditional expectations $\tilde{\mu}(P)$ are given by (19)–(22).; and *(ii) the equilibrium investment rule is given by*

$$I_p(P) = \begin{cases} 1 & \textit{if } 1-\rho > K \\ \begin{cases} 1 & P \in \{p_U, p_H\} \\ 0 & P = p_L \end{cases} & \textit{if } \frac{1}{2} > K > 1-\rho \\ \begin{cases} 1 & P = p_H \\ 0 & P \in \{p_U, p_L\} \end{cases} & \textit{if } \rho > K > \frac{1}{2} \\ 0 & \textit{if } \rho < K \end{cases}. \tag{26}$$

In equilibrium, there exists a feedback loop between firm investment and the financial market. The principal's decision to invest depends upon the price through the beliefs of the marginal investor while the realized price level is determined by the investment policy through the state-dependent firm value, $V_\omega(P)$. The first part of the above proposition characterizes the financial market equilibrium, given the investment rule.

The second part of the result characterizes the investment rule, given the equilibrium price - Figure 3 provides an illustration of this. Recall that the principal only invests if she is sufficiently optimistic that the true state is high. Given the ordering of her beliefs (see equation (18)) and the investment threshold K, this gives rise to the four distinct investment policies described above. These results are intuitive. When the investment threshold K is sufficiently high (low), the principal never invests (always invests, respectively). When the threshold is in the intermediate region, the principal conditions on the information in prices: she never invests when the price is low (i.e., $P = p_L$) but always invests when the price is high (i.e., $P = p_H$). In these regions, she only invests when the price is uninformative if the threshold K is below $1/2$ which corresponds to an investment opportunity with an ex-ante positive return (i.e., $\frac{y_H + y_L}{2} > 0$).

Figure 3: Different investment policies



Finally, we note that investors' information impacts the investment decision through both ρ and λ. While the precision of their information, ρ, determines the price level(s) at which she invests, as discussed above, it is the measure of informed investors, λ, which determines the likelihood of observing each price since

$$\mathbb{P}\left[P=p_H\right]=\mathbb{P}\left[P=p_L\right]=\tfrac{\lambda}{2},\mathbb{P}\left[P=p_U\right]=1-\lambda. \tag{27}$$

In Section 6.1, we allow λ to be endogenous and analyze its determinants.

4.3 Managerial effort

Given the terms of the principal's offered contract (α, β), we can characterize the manager's optimal effort choice at date one. Specifically, the manager maximizes his expected utility over compensation $W=\alpha+\beta P$, net of effort costs i.e.,

$$\max_e \mathbb{E}\left[\alpha+\beta P\right]-\frac{\gamma}{2}\mathbb{V}\left[\alpha+\beta P\right]-c\left(e\right), \tag{28}$$

where P is the equilibrium price of the traded claim, characterized in Proposition 1.

Note that the cash-flows $V_\omega\left(P\right)$ shift upward uniformly in each state with an increase in the manager's effort, e, and consequently, so does the price P. Importantly, this implies that the variance of the price, and hence, the variance of the manager's payoff does not depend upon his effort level, given β. As a result, and by utilizing (24), the first order condition for the manager's problem in (28) is given by

$$\frac{\beta}{1+\beta}=c'\left(e\right). \tag{29}$$

Given the convexity of the cost function, the second order condition always holds, and so the optimal choice of effort is completely characterized by (29).

As expected, the manager's optimal level of effort increases with the sensitivity of his compensation to the market price. In contrast to the canonical contracting models with exogenous signals, the increase in effort e (as β increases) is attenuated due to the nature of the endogenous signal on which the contract is written. In particular, the risky security price accounts for the manager's payoff and so is scaled by $\frac{1}{1+\beta}$. While this suggests that incentivizing the manager will be more costly, we discuss in the next section that this leads the principal to optimally provide *higher*-powered incentives (i.e., higher β) relative to a setting in which the price reflects only the value of the firm's expected cash flows.

4.4 Optimal Contract

Given the date two equilibrium characterized in Proposition 1, and the manager's optimal effort choice given by (29), the principal chooses the contract (α, β) that maximizes the (unconditional) expected firm value, net of the manager's payoff. Specifically, the principal's problem can be re-written as

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_p - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{30}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{31}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c(e) \geq 0. \tag{32}$$

Since α does not effect the manager's effort choice, the principal can adjust it, given β, so that the manager's participation constraint (32) binds. Moreover, note that the variance of the price can be expressed in terms of the variance of the marginal investor's valuation, $\tilde{\mu}(P)$:

$$\mathbb{V}\left[P\right] = \frac{1}{(1+\beta)^2}\mathbb{V}\left[\tilde{\mu}(P)\right]. \tag{33}$$

With these observations, the principal's objective simplifies to

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{34}$$

The following proposition provides a characterization of the optimal contract.

Proposition 2. *Suppose the financial equilibrium, (P, I_m), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (29). Then the principal's optimal contract is given by (α, β), where*

$$\alpha = c(e) + \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - \beta\mathbb{E}\left[P\right], \textit{ and} \tag{35}$$

$$\beta = \frac{1}{c''(e)\gamma\mathbb{V}\left[\tilde{\mu}(P)\right]}. \tag{36}$$

The principal's payoff in equation (34) has three components. The first component, $\mathbb{E}\left[x_\omega + e + \delta y_\omega I_m\right]$, is the expected value of the firm, which increases with the effort choice of the manager. The second term, $\frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right]$, captures the reduction in the manager's expected utility because his payoff is tied to the price of the risky security. The third term, $c(e)$, is the manager's private cost of providing effort.

The principal must compensate the manager for both of these costs in order for the contract to be accepted. As a result, she faces a trade-off: while an increase in β increases the effort exerted by the manager and, hence, firm value, it also increases the compensation he demands. Intuitively, when the manager is more risk-averse (higher γ), the price is more variable (higher $\mathbb{V}[\tilde{\mu}(P)]$), or the cost of effort increases quickly ($c''(e)$), the optimal β is lower. This is seen clearly in the expression for the optimal β i.e., equation (36).

As we noted at the end of Section 4.3, increasing β makes the traded price less sensitive to the manager's effort; however, there is also a reduction in the variance of the price (33). This countervailing effect makes it cheaper for the manager to increase β. In Appendix B.1, we consider a setting in which investors trade a claim to the firm's terminal cash flow *without* netting out the manager's payoff.[11] We show that, relative to our benchmark (36), the manager optimally provides lower-powered incentives but the equilibrium level of effort, firm value, and manager's payoff are unchanged. Intuitively, this is because the *signal* on which the principal contracts is $\tilde{\mu}(P)$ in both settings.

5 Implications

In this section, we characterize the implications of our benchmark model. The first subsection highlights the key tradeoff in our setting: more informative prices lead to more efficient investment decisions, but also increase the cost of incentivizing managerial effort. This implies that expected firm value is non-monotonic in the informativeness of prices. The second subsection explores the implications of this observation for various measures of efficiency that have been proposed in the literature.

5.1 Price informativeness and expected value

We begin by characterizing the impact of investment and managerial effort on the expected value, EV, where

$$\begin{aligned} EV &= \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I_m\right] \\ &= \frac{x_H + x_L}{2} + e + \delta \times \begin{cases} 0 & \text{if } \rho < K, \\ \frac{\lambda}{2}\left(y_L + \rho\left(y_H - y_L\right)\right) & \text{if } \rho > K > \frac{1}{2}, \\ \frac{y_H + y_L}{2} - \frac{\lambda}{2}\left(y_L + (1-\rho)\left(y_H - y_L\right)\right) & \text{if } \frac{1}{2} > K > 1 - \rho \\ \frac{y_H + y_L}{2} & \text{if } 1 - \rho > K \end{cases} \end{aligned}$$

[11] This could correspond to a setting in which the principal, who owns the firm, pays the manager directly instead of utilizing the firm's cash flows. In practice, this arrangement is akin to a private equity or venture capital firm paying one of its employee to improve and oversee a firm in its portfolio.

The above expression implies that, holding fixed managerial effort, the expected value increases in λ and ρ as long as the principal's investment decision depends on the price, that is, when $\rho > K > 1-\rho$.[12] Similarly, holding fixed investors' information, the expected value increases in managerial effort, e.

However, both the optimal contract and, hence, equilibrium managerial effort, depend upon the informativeness of the price in equilibrium. Intuitively, since ρ measures the precision of informed investors' information and λ measures the fraction of informed traders, an increase in either makes the price more informative about the new investment opportunity, y_ω. This improves the efficiency of the investment decision since she is more likely to invest when the payoff is positive. However, it also makes the price more volatile, which increases the principal's cost of incentivizing effort. In equilibrium, the principal responds to an increase in either ρ or λ by reducing the the sensitivity of the manager's compensation to the price (i.e., she reduces β), which in turn reduces the effort, e, exerted by the manager. We show this formally in the proof of the following proposition.

Proposition 3. *The optimal choice of β and e decrease with both λ and ρ.*

The proposition highlights the key tradeoff in our model. While the expected value increases in both the manager's effort choice and the efficiency of the investment decision, an increase in λ or ρ always leads to a decrease in managerial effort in equilibrium. Thus, the impact of more informed investors depends upon the relative strength of these two countervailing effects. In particular, in the following proposition, we characterize how the impact of changes in λ depend on both (i) the quality of investors' information and (ii) the relative importance of the investment project, δ.

Proposition 4. *If $\rho < K$ or $1-\rho > K$, then firm value is decreasing in ρ and λ. If $\rho > K > 1-\rho$ and if the cost of effort satisfies the conditions given in (77), then*

(i) EV is increasing in λ if $\delta > \bar{\delta}$,

(ii) EV is U-shaped in λ if $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$,

(iii) EV is decreasing in λ if $\delta \leq \underline{\delta}$.

Specifically, condition (77) is satisfied if the cost of effort is quadratic.

Note that the impact of increase in the fraction of informed investors on expected value

[12] While this is straightforward to see when $\rho > K > \frac{1}{2}$, it is also the case when $\frac{1}{2} > K > 1-\rho$, since

$$y_L + (1-\rho)(y_H - y_L) < 0 \iff \tag{37}$$

$$1-\rho < \frac{-y_L}{y_H - y_L} = K. \tag{38}$$

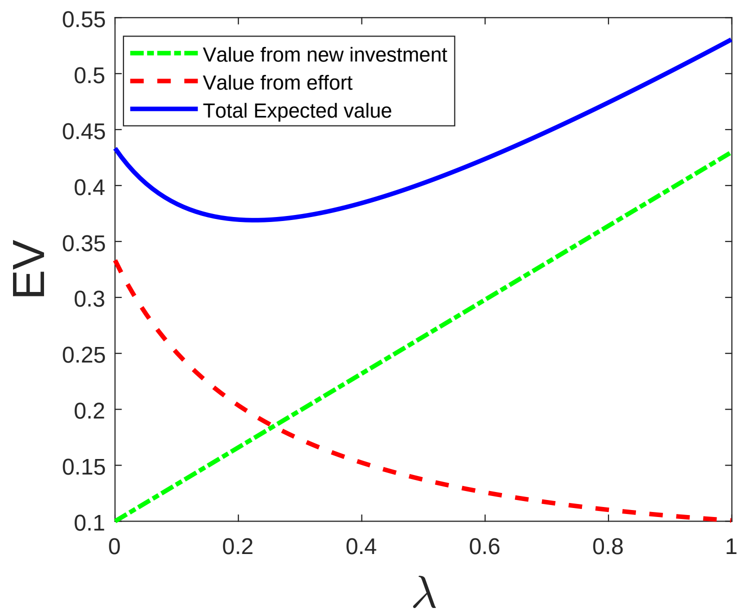
can be expressed as:

$$\frac{\partial EV}{\partial \lambda} = \underbrace{\frac{\partial e}{\partial \lambda}}_{<0} + \delta \times \begin{cases} 0 & \text{if } \rho < K, \\ \frac{y_H - y_L}{2}(\rho - K) & \text{if } \rho > K > \frac{1}{2}, \\ \frac{y_H - y_L}{2}(K - (1-\rho)) & \text{if } \frac{1}{2} > K > 1-\rho \\ 0 & \text{if } 1-\rho > K. \end{cases} \tag{39}$$

When the principal never invests (i.e., if $\rho < K$) or always invests (i.e., if $1-\rho > K$), increasing λ has no impact on investment efficiency. As a result, more informative prices necessarily decrease the expected value because they lead to a decrease in managerial effort.

Figure 4: Expected value versus fraction of informed investors

The figure plots the expected value of the firm as a function of λ. The cost of effort is $\frac{c_e}{2}e^2$. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -0.9$.



When $\rho > K > 1-\rho$, the impact depends upon δ, which parameterizes the relative importance of the investment project. Intuitively, when the investment project is sufficiently important (i.e., δ is sufficiently high), expected value increases with λ. On the other hand, if the investment project is sufficiently small relative to assets-in-place, the effect of λ on effort choice dominates and expected value decreases with λ. Condition (77) provides a sufficient condition on the cost function to ensures this is the case. Specifically, the condition implies that the manager's effort choice is convex and decreasing in λ. Note that the impact of λ

on investment efficiency is constant. When δ is sufficiently high, the decrease in managerial effort which follows from the increase in λ is always smaller than the increase in investment efficiency. As a result, firm value strictly increases. When δ is sufficiently low, this is reversed: the increased investment efficiency is always outweighed by the decline in effort, even as λ approaches one, and so the expected value strictly declines.

For $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, however, the effect is no longer monotonic. The decline in effort for "low" λ causes the expected value to decrease initially, then increase. The plots in Figure 4 provide a numerical illustration for this case. The figure plots expected value (solid blue line) and its components: the manager's equilibrium effort level (red dashed line) and the expected value of the investment opportunity (green dotted line). Since the manager's effort is decreasing and convex in λ, while the value from the new investment opportunity increases linearly, the expected value is U-shaped in λ when $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$.

5.2 Efficiency measures

Next, we characterize how alternate measures of price efficiency proposed in the literature depend on the fraction of informed investors and the precision of their signals in our setting.

1. **Revelatory price efficiency (*RPE*):** In our model, the price guides the principal's real investment decision. Revelatory price efficiency measures the extent to which prices reveal information which is relevant for investment decisions. In our model, this can be measured as
$$RPE = \frac{var\left[y_\omega\right] - var\left[y_\omega|p\right]}{var\left[y_\omega\right]}$$

2. **Contracting price efficiency (*CPE*):** Importantly, the price not only conveys information about the investment opportunity, but also affects the manager's incentive to exert effort. Intuitively, given the manager's risk-aversion, the price is more useful for contracting if it tracks the manager's effort more precisely. This is captured by the signal-to-noise ratio of the price about the manager's action. Since the signal which the principal can extract is $\tilde{\mu}\left(P\right)$, which moves one-for-one with managerial effort, this measure is simply the inverse of the noise in price signal:
$$CPE = \left[\mathbb{V}\left(\tilde{\mu}\left(P\right)|e\right)\right]^{-1}.$$

3. **Forecasting price efficiency (*FPE*):** Forecasting price efficiency captures the extent to which the information in the price correctly forecast future firm cash flows. A common measure of forecasting price efficiency is the inverse of the variance of cash

flows conditional on observing the price. In our model, this measure is given by

$$FPE = \left[var\left(x_\omega + e + \delta y_\omega \mathbb{I}_{d_m=I} | p\right)\right]^{-1}.$$

With these definitions in mind, we formally characterize how each type of efficiency varies when investors, in the aggregate, have more information.

Proposition 5. *Consider the unique financial market equilibrium described in proposition 1 and the optimal contract described in proposition 2. Then,*

(i) RPE increases in λ and ρ,

(ii) CPE decreases in λ and ρ,

(iii) FPE decreases in λ iff $\rho > K > \frac{1}{2}$ and δ is sufficiently high i.e., $\left(1 + \delta \frac{y_H - y_L}{x_H - x_L}\right)^2 > \left(\frac{\rho^2 + (1-\rho)^2}{2\rho(1-\rho)}\right)$. FPE decreases in ρ iff $K > 0.5$ at $\rho = K$.

As ρ increases, the signal which the principal can extract from the price is more precise. As λ increases, it is more likely that the price provides an informative signal to the principal. This implies that revelatory price efficiency increases in both λ and ρ. Panel (a) of Figure 5 illustrates this. On the other hand, as the price becomes more informative about investors' information, it becomes relatively less informative about the manager's action, that is, the signal to noise ratio of the price signal about e decreases. This leads to lower contracting price efficiency, as illustrated by Panel (b) of Figure 5.

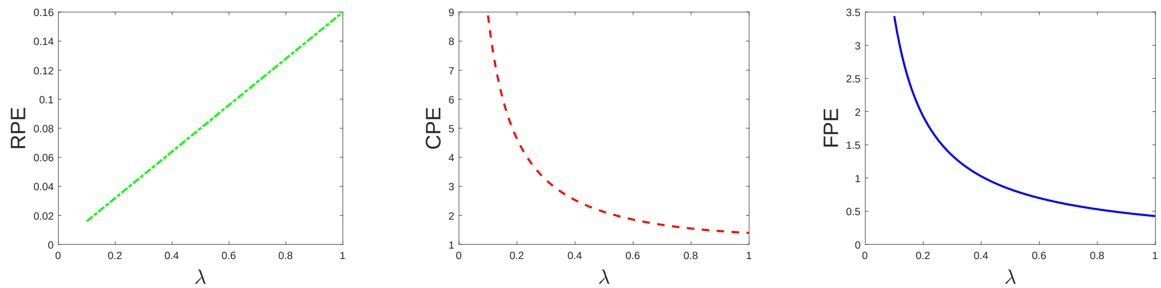
The link between investors' information and forecasting price efficiency is more nuanced because the variance of the firm's cash flows is endogenous. Holding fixed the likelihood of investment, an increase in either λ or ρ increases the precision of the marginal investor's forecast, on average, which increases *FPE*. However, λ and ρ also affect the likelihood of investment, which can increase the variance in the firm's cash flows and push *FPE* downward. For example, when $\rho < K$, the manager never invests; once ρ crosses the threshold K, there is a discontinuous drop in *FPE* due to the likelihood of investment. Once in this region (i.e., when $\rho > K > \frac{1}{2}$), an increase in λ makes investment more likely. When the relative importance of the investment is sufficiently large (i.e., δ is sufficiently high) relative to the precision of investors' information, ρ, such increases in λ can lead to a decrease in *FPE*.[13] As we see in Figure 5 panel (c), forecasting price efficiency can decrease with λ.

The recent literature on feedback effects has emphasized the tension between forecasting price efficiency (FPE) and revelatory price efficiency (RPE) (e.g., see Bond et al. (2012)). In these models, what matters for real efficiency is whether the price reveals information necessary for decision makers to take value-maximizing actions; as a result, this literature

[13]Note that the right-hand side of the inequality found in the proposition is increasing in ρ.

Figure 5: Efficiency metrics versus fraction of informed

The figure plots the RPE, CPE and FPE as a function of λ. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -1.1$.



has focused on RPE as the relevant measure of real efficiency. Our results highlight that RPE is an incomplete measure of real efficiency when stock prices are used to provide managerial incentives. In fact, Proposition 5 implies that our measure of contracting price efficiency (CPE), which reflects the extent to which prices reflect managerial effort, often moves in the opposite direction as RPE. Since both RPE and CPE are relevant for firm value, our analysis suggests that using any of these measures in isolation as a proxy for real efficiency can be misleading.

6 Extensions

6.1 Costly Information Acquisition and Transparency

Our benchmark analysis shows that while an increase in λ (i) increases the extent to which prices reveal information about y_ω (i.e., increases RPE), it (ii) can decrease firm value by making it costlier for the principal to incentivize the manager to exert effort. Given this tradeoff, we explore the incentives of investors to acquire costly information by endogenizing λ.

Specifically, suppose that before date one, each investor chooses whether or not to become informed by paying a cost c_0 to obtain an informative private signal $s_i \in \{s_H, s_L\}$. Each investor takes as given the manager's optimal effort level (i.e., the solution to equation (29)) as well as the information acquisition decision of all other investors. Acquiring a private signal is only valuable if the incremental increase in his expected profits exceed the fixed cost, c_0, given the nature of the financial market equilibrium. This implies that, in any interior equilibrium (i.e., when $\lambda \in (0, 1)$) where investors are indifferent between being informed and uninformed, it must be the case that

$$\mathbb{E}\left[d\left(s_i, P\right)\left(V-P\right)\right]-\mathbb{E}\left[d\left(\emptyset, P\right)\left(V-P\right)\right]=c_0. \tag{40}$$

This yields the following equilibrium characterization.

Proposition 6. *There exists a $\hat{c}$, defined in the appendix, such that:*

(i) If $c_0 \geq \hat{c}$, then $\lambda = 0$.

(ii) If $c_0 < \hat{c}$, the measure of informed investors in equilibrium is given by

$$\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right) \Delta V\left(p_U\right)} < 1,$$

and is decreasing in the cost of information c_0, increasing in the precision of the signal ρ, and the information sensitivity of the risky security when the price is uninformative ($\Delta V\left(p_U\right)$).

When information acquisition costs are sufficiently high (i.e., $c_0 > \hat{c}$), the net benefit from acquiring information is too low, even when all other investors are uninformed. As a result, all investors optimally choose not to acquire any information. On the other hand, when costs are sufficiently low, a fraction of investors choose to acquire information. This fraction of informed investors increases with the precision of the signal ρ and decreases with the fixed cost c_0.

Intuitively, the measure of informed investors depends upon the information-sensitivity of the risky security. However, in our setting, it only depends on the sensitivity when the price is uninformative. As we show in the proof, the expected trading gains when the price is high or low (i.e., $p = p_H$ and $p = p_L$,respectively) are the same for both informed and uninformed investors. This is a consequence of our assumption that investors are risk-neutral and face position limits. As such, the incremental benefit from acquiring private information arises when the price is uninformative (i.e., $p = p_U$). Since investors differ only in their beliefs about the payoff across the two states, the benefit of trading at the uninformed price is $\Delta V\left(p_U\right) = V_H\left(p_U\right) - V_L\left(p_U\right)$ and is scaled by their information advantage relative to the uninformed, $\rho - \frac{1}{2}$. This is also the reason why there are always uninformed investors in equilibrium — if $\lambda = 1$, the price is never uninformative and thus, no benefit to being informed.

Given the impact of λ on the expected value, this result suggests a natural role for policy changes that increase transparency by reducing the cost of information acquisition, c_0. We begin by characterizing the impact of transparency on firm value FV, which captures the expected value of the firm net of compensation to the manager, i.e.,

$$FV = \text{Expected Value } - \text{Managerial Compensation.} \tag{41}$$

Since the optimal contract sets the manager's compensation to ensure that he is at his reservation utility, this implies that firm value can be expressed as

$$FV = \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I_m\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right). \tag{42}$$

The following result characterizes how an increase in transparency, or equivalently a decrease in c_0, affects firm value.

Proposition 7. *Firm Value increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large. Firm Value decreases with transparency (increases with c_0) if either (i) $1-\rho > K$, or (ii) $\rho < K$, or (iii) $1-\rho < K < \rho$ and δ is sufficiently small.*

The proposition provides sufficient conditions when an increase in transparency increases firm value, and when it decreases firm value. To gain some intuition, note that a change in information acquisition costs affects firm value via two channels:

$$\frac{d}{dc_0}FV = \frac{\partial}{\partial\lambda}FV \times \frac{d\lambda}{dc_0} \tag{43}$$

$$= \delta \times \underbrace{\frac{\partial}{\partial\lambda}\mathbb{E}\left[y_\omega \times I_m\right]\frac{d\lambda}{dc_0}}_{\text{investment channel}} + \underbrace{\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right)\frac{d\lambda}{dc_0}}_{\text{incentive channel}} \tag{44}$$

The first channel, which we call the **investment channel**, captures the impact of transparency on the efficiency of the investment decision. Since an increase in λ increases the expected value of the investment, an increase in c_0 reduces firm value (FV) through this term. This channel only arises when there is feedback from prices (i.e., when $1-\rho < K < \rho$) and is increasing in the scale of the investment, δ.

The second channel, which we refer to as the **incentive channel**, captures the impact of transparency on equilibrium effort provision net of costs. As the proof of Proposition 7 establishes,

$$\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right) = -\frac{\gamma}{2}\left(\frac{1}{1 + c''\left(e\right)\gamma\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}\right)^2\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) < 0, \tag{45}$$

which reflects the fact that when the price becomes more informative about the investment project, it increases the variance of the price. This increases the potential risk borne by the manager due to the compensation contract and, in equilibrium, decreases the effort the manager exerts. As such, an increase in c_0, which reduces λ, decreases firm value (FV) through this channel.

In general, the impact of transparency on firm value depends on the relative magnitude of these two channels, and can be ambiguous. Proposition 7 provides sufficient conditions for when the direction of the effect is unambiguous. Specifically, the investment channel dominates when the feedback effect is operational (i.e., $1-\rho < K < \rho$) and the relative importance of the new investment opportunity is sufficiently large (i.e., δ is sufficiently large). In this case, an increase in transparency unambiguously leads to an increase in firm value. On the other hand, when the feedback effect is not operational (i.e., if $1-\rho > K$ or $\rho < K$) or the relative importance of the new investment is sufficiently low, the incentive channel dominates and firm value decreases with transparency.

Our measure of firm value does not account for the impact of transparency on either the manager or investors. To characterize the aggregate impact of transparency in our setting, we characterize a measure of social surplus (or social value, SV). This measure accounts for not only the expected value of the firm net of compensation (i.e., FV), but also the manager's expected utility net of effort costs, and the aggregate trading gains and losses net of information acquisition costs, i.e.,

$$SV = \begin{array}{c} \text{Expected Value } - \text{Managerial Compensation} \\ +\text{Managerial Utility} - \text{Manager's Cost of Effort} \\ +\text{Net Trading Gains / Losses} \\ -\text{Investor Cost of Information} \end{array} \tag{46}$$

Note that since (i) aggregate trading gains / losses sum to zero and (ii) the manager is exactly indifferent given his compensation (i.e., Manager utility - Manager's Cost of Effort matches his reservation utility, $u_0 = 0$), this simplifies to

$$SV = \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I_m\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right) - \lambda c_0 \tag{47}$$

$$= FV - \lambda c_0. \tag{48}$$

The following result characterizes how an increase in transparency, or equivalently a decrease in c_0, affects social value.

Proposition 8. *Social Value increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large. Social Value decreases with transparency (increases with c_0) if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)\left(x_H - x_L\right)$ and if either (i) $1-\rho > K$, or (ii) $\rho < K$, or (iii) $1-\rho < K < \rho$ and δ is sufficiently small.*

The above result parallels Proposition 7 in that it provides sufficient conditions when an increase in transparency has an unambiguous effect on social value. Given the expression for social value in (47), one can express the impact of a change in c_0 on social value as:

$$\frac{d}{dc_0}SV = \frac{d}{dc_0}FV - \left(\lambda + c_0\frac{d\lambda}{dc_0}\right). \tag{49}$$

In addition to the investment and incentive channels, transparency has a direct effect on the SV through the aggregate cost of information acquisition. Whether this increases or decreases the social value depends on the equilibrium level of λ, since

$$\lambda + c_0\frac{d\lambda}{dc_0} = 2\lambda - 1. \tag{50}$$

When information acquisition costs c_0 are sufficiently high (i.e., $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$), the equilibrium λ is lower than $\frac{1}{2}$. As a result, any further increase in the cost of learning improves social value by decreasing the fraction of investors who become informed, thereby reducing aggregate expenditure on private information acquisition. On the other hand, when information acquisition costs are low (so that $\lambda > \frac{1}{2}$), the resultant decrease in λ is not sufficiently large to offset the increase in c_0 paid by those investors who still choose to become informed. In this case, overall expenditure on information acquisition is higher and, consequently, social value is lower. We note that this is more likely to be the case when investors have access to more precise information (i.e., if ρ increases) and if the information sensitivity of assets in place is high (i.e., if $x_H - x_L$ increases). The overall impact of transparency then follows from how this channel interacts with the impact on firm value.

Our analysis highlights that an increase in transparency can have nuanced effects on both firm value and social surplus. Importantly, an increase in transparency leads to more information acquisition by investors and greater RPE, but can decrease firm value. To the extent that firms influence transparency e.g., by changing the clarity of their financial reporting, our results predict that the level of transparency should predictably vary with both firm and investor characteristics (see Li (2008); You and Zhang (2009); Miller (2010)). In particular, firms should be more opaque when price based managerial compensation plays an important role, even when feedback from market prices can improve the firm's investment decisions. This is consistent with empirical evidence that documents hard-to-value and opaque firms tend to use more aggressive stock-based pay for their managers (Bebchuk and Fried (2004)).

Similarly, our results caution against an "one-size-fits-all" approach to regulatory policy that affects transparency in financial markets. Notably our results are not driven by the Hirshleifer (1971) effect, whereby more public information reduces opportunities for risk sharing, and therefore reduce welfare. Our results are also distinct from the "crowding out" channel highlighted by the feedback effects literature (e.g, Goldstein and Yang (2019)), whereby greater transparency about some component of fundamentals can crowd out learning about other, investment-relevant dimensions. Our analysis implies that higher transparency may not be desirable from a social perspective, even when it improves revelatory price efficiency and improves investment outcomes, because of its impact on contracting efficiency and overall firm value.

6.2 Noisy signal about effort

In the baseline model, we assumed that investors can perfectly observe the manager's equilibrium effort. In this section, we relax this restriction and assume that investors can only condition on a noisy signal of his chosen effort level.

Specifically, suppose that the terminal value of the firm's cash flows, V, is given by

$$V(\omega, e, I_m, \theta) = x_\omega + (e + \theta) + \delta y_\omega \times I_m, \tag{51}$$

where $\theta \sim N(0, \sigma_\theta^2)$ is independent of ω. Further suppose that in addition to observing private signals s_i about ω, all investors observe a *public, non-contractable* signal $s_\theta = e + \theta + \eta$, where $\eta \sim N\left(0, \sigma_\eta^2\right)$ is independent of all other random variables.

Let $V_\omega(P; s_\theta)$ denote the investors' expected cash-flow in state ω given the price P and signal s_θ. Then, we can express

$$V_\omega(P, s_\theta) = \mathbb{E}\left[V(\omega, e, I_m, \theta)\,|\omega, P, s_\theta\right] = x_\omega + \hat{e} + \kappa(s_\theta - \hat{e}) + \delta y_\omega \times I_m(P), \tag{52}$$

where $\hat{e}$ denotes the investors' inference of the manager's choice of effort, and $\kappa \equiv \frac{\mathbb{C}(s_\theta, \theta)}{\mathbb{V}(s_\theta)} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2} \in [0, 1]$ is a measure of the informativeness of s_θ. As in Holmström and Tirole (1993), investors correctly infer the manager's effort level in equilibrium. The public signal, however, provides information about θ and so investors utilize it when forming beliefs about the firm's value. Since the public signal is observed by all investors, its inclusion does not change the nature of the financial market equilibrium but it does alter the equilibrium price. Specifically, using steps analogous to those in the benchmark analysis, one can show that the equilibrium price in this setting is given by

$$P(\omega, u, s_\theta) = \begin{cases} \frac{1}{1+\beta}\left(\tilde{\mu}(p_L; s_\theta) - \alpha\right) \equiv p_L(s_\theta) & \text{if } u < u_\omega - (1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}(p_U; s_\theta) - \alpha\right) \equiv p_U(s_\theta) & \text{if } u_\omega - (1-\lambda) \le u \le u_\omega + (1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}(p_H; s_\theta) - \alpha\right) \equiv p_H(s_\theta) & \text{if } u > u_\omega + (1-\lambda) \end{cases}, \tag{53}$$

where the marginal investor's beliefs are given by

$$\tilde{\mu}(p_H; s_\theta) \equiv \mathbb{E}[V|s_H, p_H, s_\theta] = V_L(p_H, s_\theta) + \tilde{\rho}\Delta V(p_H, s_\theta), \tag{54}$$

$$\tilde{\mu}(p_L; s_\theta) \equiv \mathbb{E}[V|s_H, p_H, s_\theta] = V_L(p_L, s_\theta) + (1-\tilde{\rho})\Delta V(p_L, s_\theta), \text{ and} \tag{55}$$

$$\tilde{\mu}(p_U; s_\theta) \equiv \mathbb{E}[V|s_U, p_U, s_\theta] = V_L(p_U, s_\theta) + \frac{1}{2}\Delta V(p_U, s_\theta) \tag{56}$$

and the equilibrium investment rule is equation (26).

Given this characterization of the financial market equilibrium, the manager's optimal choice of effort is given by the first order condition

$$\frac{\beta}{1+\beta}\kappa = c'(e). \tag{57}$$

While investors' inference of the manager's effort level, $\hat{e}$, is correct in equilibrium, changes in the manager's effort affect the price only through the public signal. Since this public signal of managerial effort is noisy (i.e., $\kappa \leq 1$), the equilibrium level of effort is lower (holding fixed β) than in the benchmark (see equation (29)). As the investors' signal becomes more precise, κ increases which leads to an increase in the manager's effort as well.

The following proposition characterizes the optimal contract in this setting.

Proposition 9. *Suppose the financial market equilibrium is characterized by the price in (53) and beliefs (54)-(56), and the manager's optimal choice of effort, e, is given by (57). Then, the principal's optimal contract is given by (α, β) where*

$$\alpha = c(e) + \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \left(\mathbb{V}(\tilde{\mu}(P; s_\theta)|s_\theta) + \kappa\sigma_\theta^2\right) - \beta\mathbb{E}[P], \text{ and} \tag{58}$$

$$\beta = \frac{\kappa}{c''(e)\gamma(\mathbb{V}(\tilde{\mu}(P; s_\theta)|s_\theta) + \kappa\sigma_\theta^2) - \kappa(1-\kappa)}. \tag{59}$$

Moreover, the optimal choice of β:

(i) decreases with both λ and ρ, and

(ii) increases with κ if and only if $\gamma c''(e)\mathbb{V}(\tilde{\mu}(P; s_\theta)|s_\theta) > \kappa^2$.

The optimal contract now depends on the two sources of investor information reflected in the price. To gain some intuition, note that the variance in the manager's compensation can be expressed as

$$\mathbb{V}(\alpha + \beta P) = \beta^2\mathbb{V}(P) = \left(\frac{\beta}{1+\beta}\right)^2 \left(\mathbb{V}(\tilde{\mu}(P; s_\theta)|s_\theta) + \kappa\sigma_\theta^2\right). \tag{60}$$

As in the benchmark model, for a fixed β, when the price is more informative about the true state of the world, ω, (i.e., if either λ or ρ are higher), the volatility of the price too, (captured by the $\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)$ term). This makes the manager's compensation more risky, and hence it becomes costlier for the principal to induce the manager to exert effort. As in our benchmark analysis, the principal optimally responds to such an increase by reducing the optimal β.

When the price is more informative about effort (i.e., as κ increases), there are two opposing effects on the optimal contract. First, as (57) highlights, when the signal is more informative increasing β induces more effort, on the margin. However, this increase in κ also increases the volatility of the price (as captured by the $\kappa\sigma_\theta^2$) term, which in turn, makes it more costly to incentivize effort provision. When κ is sufficiently low, Proposition 9 shows that β is increasing in κ — in this case, the positive impact on the equilibrium effort choice dominates the negative impact of more costly risk compensation. As a result, the principal optimally chooses to increase the price-sensitivity of the optimal contract. On the other hand, when κ is sufficiently high, the former effect dominates, and the principal optimally chooses to decrease β when κ increases.

In short, our key tradeoff obtains even when investors cannot observe the manager's effort perfectly: all else equal, when the price becomes more informative about investment opportunities, the price-sensitivity of the optimal contract and managerial effort decrease. In contrast, when the price becomes more informative about managerial effort, the price-sensitivity of the optimal contract can increase or decrease, depending upon on the relative informativeness of the two signals.[14]

6.3 Contracting on price and value

In the baseline model, we assume that the principal cannot contract on the firm's realized cash flow. In this section, we relax this assumption, that is, the manager's compensation is

$$W\left(P,V\right)=\alpha+\beta\left(\pi P+\left(1-\pi\right)Z\right) \tag{61}$$

where $Z\equiv V-W$ denotes the firm's net cash flows and $\alpha,\ \beta,$ and π are chosen optimally by the principal.

Given the terms of the principal's offered contract (α,β,π), the manager maximizes his expected utility over his compensation, $W\left(P,V\right)$, net of the cost of effort costs, that is, he

[14] This result differs superficially from the findings of Holmström and Tirole (1993) due to the signals they utilize in their optimal contract. Specifically, the manager's compensation depends upon a transformation of the price so that the only impact of an increase in κ is on the informativeness of this signal. As a result, more information about managerial effort always leads to a more price-sensitive contract in equilibrium.

solves

$$\max_{e} \mathbb{E}\left[\alpha+\beta\left(\pi P+(1-\pi) Z\right)\right]-\frac{\gamma}{2} \mathbb{V}\left[\alpha+\beta\left(\pi P+(1-\pi) Z\right)\right]-c(e), \tag{62}$$

As before, the manager optimally chooses his effort so that

$$\frac{\beta}{1+\beta}=c'(e),$$

since both the price and the firm's terminal value increase one-for-one with effort. The principal chooses the contract (α, β, π) that maximizes the (unconditional) expected firm value, net of the manager's compensation. Specifically, she now solves

$$\max_{\alpha, \beta, \pi} \mathbb{E}\left[x_{\omega}+e+\delta y_{\omega} I_{m}-\left(\alpha+\beta\left(\pi P+(1-\pi) Z\right)\right)\right], \qquad \text{subject to :} \tag{63}$$

$$\frac{\beta}{1+\beta}=c'(e), \tag{64}$$

$$\mathbb{E}\left[\alpha+\beta\left(\pi P+(1-\pi) Z\right)\right]-\frac{\gamma}{2} \mathbb{V}\left[\alpha+\beta\left(\pi P+(1-\pi) Z\right)\right]-c(e) \geq 0. \tag{65}$$

The following proposition provides a characterization of the optimal contract.

Proposition 10. *Suppose the financial equilibrium, (P, I_m), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (64). Then the principal's optimal contract is given by (α, β, π), where*

$$\pi^{*}=1+\frac{cov\left(V-\tilde{\mu}(P), \tilde{\mu}(P)\right)}{\left(1+\beta^{*}\right) var\left(V-\tilde{\mu}\right)+\beta^{*} cov\left(V-\tilde{\mu}, \tilde{\mu}\right)}<1, \text{ and} \tag{66}$$

$$\beta=\frac{1}{c''(e)\,\gamma\left(\mathbb{V}\left[\tilde{\mu}(P)\right]-\frac{cov\left(V-\tilde{\mu}(P), \tilde{\mu}(P)\right)^{2}}{var\left(V-\tilde{\mu}(P)\right)}\right)}. \tag{67}$$

Since the manager's participation constraint is always binding, the principal's objective simplifies to

$$\max_{\beta, \pi} U_{P}=\mathbb{E}\left[x_{\omega}+e+\delta y_{\omega} I_{m}\right]-\left(\frac{\beta^{2} \gamma}{2} \mathbb{V}\left[\pi P+(1-\pi) Z\right]+c(e)\right). \tag{68}$$

Moreover, note the manager's choice of effort depends upon β, only. As a result, the optimal π is chosen to minimize

$$\mathbb{V}\left[\pi P+(1-\pi) Z\right] \tag{69}$$

which, it is straightforward to show, yields equation (66). Given this choice of π, the optimal level of β is then given by equation (67). In this setting, the manager's contract is more sensitive to her effort relative to the baseline model (i.e, the optimal β is always higher, see equation (36)). This increase in β, and the resultant increase in managerial effort and firm value, arises because $\mathbb{V}[P] \geq \mathbb{V}[\pi P + (1-\pi) Z]$ at the optimal π. Intuitively, this is because $\text{cov}(V - \tilde{\mu}(P), \tilde{\mu}(P)) < 0$, and this implies $\pi < 1$: the manager always receives a positive loading on both price and value. This arises in our model because prices exhibit reversal, a feature which arises across a wide range of rational expectations equilibria models (e.g., Hellwig (1980)). As a result, putting positive weights on the price and the terminal cash flow provides diversification, lowering the total risk borne by the manager. In contrast, if the price were set by a risk-neutral market maker (as in Vives (1995)), π would be one in our setting. That is, the principal would only contract on the price since $\text{cov}(V - \tilde{\mu}(P), \tilde{\mu}(P)) = 0$. This is because the price is a less noisy signal of the manager's effort relative to the terminal value.[15]

In Figure 6, we plot the optimal π and a scaled measure of performance-sensitivity, $\frac{\beta}{1+\beta}$ as a function of the fraction of informed investors, λ. As the measure of informed investors increases, the volatility of both the price and the realized cash flow increases. In both cases, this leads to lower-powered incentives, i.e., β falls, as in our benchmark model. The impact on the weight the principal places on the price, however, may be non-monotonic. Recall that when the investment threshold, K, is lower than $1/2$ (panel (a)), the principal invests in the project as long as the price is not $P = p_L$. In this case, the relative weight on the price, π, decreases with λ. On the other hand, when the investment threshold is higher than $1/2$ (panel (b)), the relative weight is U-shaped in λ: the principal optimally puts relatively more weight on short-term signals (the price) relative to long term signals (V) when the mass of informed investors is sufficiently low or sufficiently high. These results suggest a novel channel through which the manager's optimal short-term compensation varies as a function of investors' information.[16]

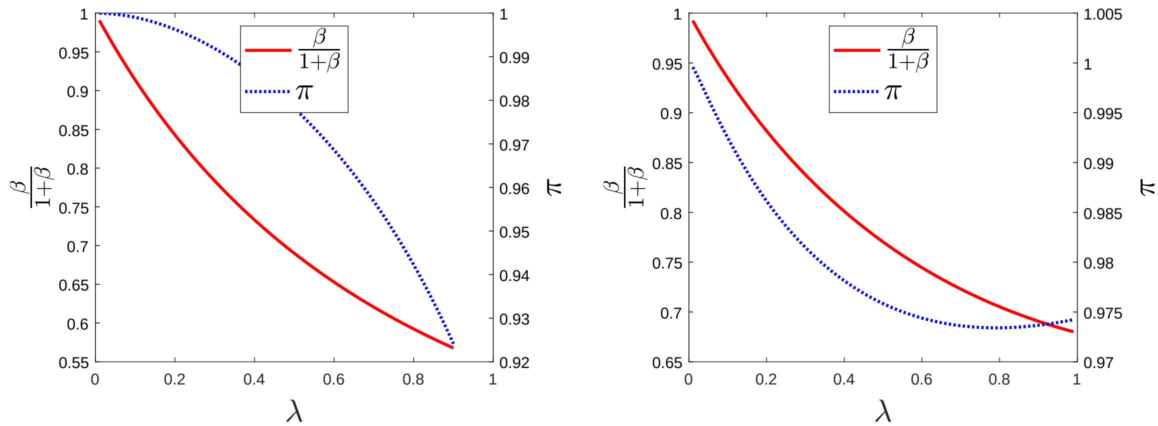
These results also speak to the literature which analyzes the impact of liquidity on stock-based compensation. In our model, the fraction of informed investors, λ, measures the extent to which the average price changes in response to an increase in (noise) trading and, as such, is a proxy for market liquidity. As such, we find an unambiguously positive relation between performance-based compensation and market liquidity, consistent with the empirical evidence in the literature (e.g., Fang et al. (2009), Jayaraman and Milbourn (2012)). Moreover, while our results imply that the relative weight on short-term price compensation (i.e., π) increases with market liquidity (decreases with λ) when investment opportunities are ex-ante

[15] If $\text{cov}(V - P, P) \geq 0$, conditioning on the terminal value adds volatility (since investors only observe a noisy signal of the true state, ω) and provides no additional incentive to exert effort.

[16] For instance, in Peng and Röell (2014), changes in short-term compensation reflect changes in the manager's ability to manipulate the firm's value.

Figure 6: Optimal pay-for-performance versus λ

The figure plots the optimal contract chosen (β, π) by the principal as a function of λ. The cost of effort is $\frac{e^2}{2}$. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$, $\rho = 0.7$.



(a) $K = 0.33$ (b) $K = 0.67$

positive NPV (i.e., K is low), such compensation may decrease with market liquidity when investment opportunities are ex-ante negative NPV. This suggests that the the impact of market liquidity on stock-based compensation and market liquidity depends in part on the nature of investment opportunities available to firms and, as a result, should vary across industries and over market conditions.

6.4 Price maximization versus value maximization

In the benchmark model, we assumed that the principal's investment rule, $I(P)$, maximized her expectation of the firm's terminal cash flows, given the price. In this section, we examine the implications of relaxing this assumption and allow the *manager* to choose the investment policy. Specifically, suppose the manager invests (i.e., chooses $I_m(P) \in \{0, 1\}$) in order to maximize his payoff $(\alpha + \beta P)$ which, given his contract, is equivalent to maximizing the price.

The date-one price is maximized when the manager invests (i.e., $I_m(P) = 1$), as long as

$$q_m(P) > \frac{-y_L}{y_H - y_L} \equiv K,$$

where $q_m(P)$ denotes the beliefs of the marginal investor. This change in the investment policy does not alter the financial market equilibrium given in proposition 1 except for the new investment rules:

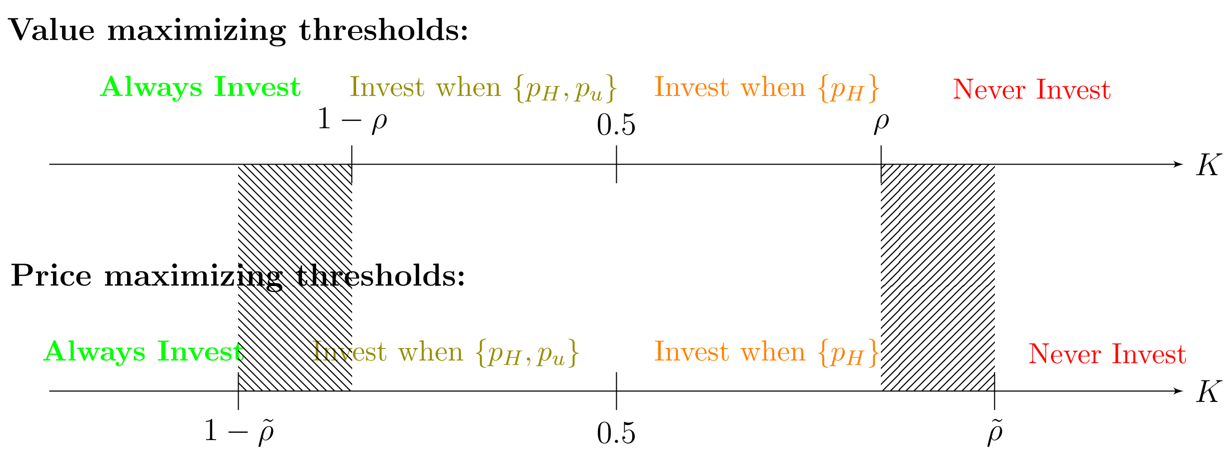
- If $1-\tilde{\rho} > K$, the manager always invests.
- If $\frac{1}{2} > K > 1-\tilde{\rho}$, the manager invests if $P \in \{p_u, p_H\}$.
- If $\tilde{\rho} > K > \frac{1}{2}$, the manager invests if $P = p_H$.
- If $\tilde{\rho} < K$, the manager never invests.

In Figure 7, we compare the new investment thresholds to those found in our baseline specification. There are two regions, shaded in the figure, where there the investment decision flips:

1. If $K \in (\rho, \tilde{\rho})$ and $P = p_H$, the manager doesn't invest if he maximizes his expectation of the firm's terminal value, but does invest if he wants to maximizes the date-one price.
2. If $K \in (1-\tilde{\rho}, 1-\rho)$, the manager always invests if he maximizes his expectation of the firm's terminal value, but refrains from investing when $P = p_L$ when he maximizes the date-one price.

Figure 7: Thresholds

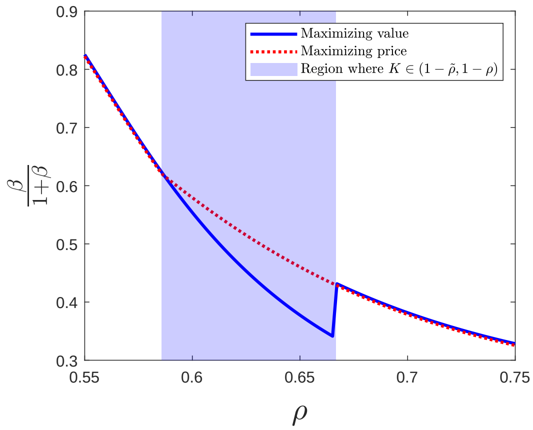
The figure characterizes the investment rule when the manager maximizes value (top) to the one when the manager maximizes price (bottom), as a function of $K = \frac{-y_L}{y_H - y_L}$,



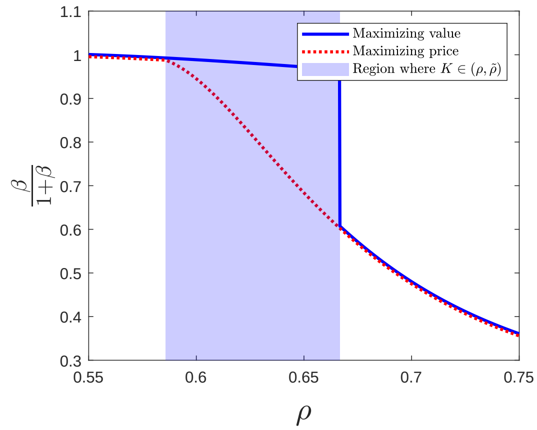
These thresholds suggests that contracting on date-one price has a downside when the manager also chooses the firm's investment policy: it can encourage the manager to focus too much on maximizing the firm's short-term value at the expense of its long-term cash flows. As a result, an increase in revelatory price efficiency (i.e., an increase in either λ or ρ) can lead to a decrease in firm value, even when managerial effort is held constant.

Figure 8: Optimal pay-for-performance versus ρ

The figure plots the optimal contract chosen by the principal as a function of ρ. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$.



(a) $K = 0.33$



(b) $K = 0.67$

Proposition 11. *Holding e fixed, when the manager invests to maximize price, firm value decreases in λ if and only if $\rho < K < \tilde{\rho}$ or $1 - \tilde{\rho} < K < 1 - \rho$.*

In the baseline model, an increase in either the quality of investors' information (ρ) or the fraction of informed investors (λ) increases investment efficiency. More information increases investment efficiency because the principal's investment rule is also efficient. However, Proposition 11 highlights this is no longer the case when the manager maximizes expected price. This reflects the fact that, as in other rational expectations models with a continuum of investors, the marginal investors' equilibrium expectation of cashflows (reflected in the price) need not coincide with expected cashflows conditional on the information in the price. As a result, an investment policy that maximizes the expected price need not maximize expected value. This wedge between price maximization and value maximization arises more generally (e.g., Albagli, Hellwig, and Tsyvinski (2011), Banerjee, Breon-Drish, and Smith (2021)).

Despite this source of inefficiency, however, the price maximization investment rule may be desirable in our setting because of its impact on managerial effort. Note that the manager's effort and the optimal contract are the same as in the baseline model. However, the variance in prices $\mathbb{V}[\tilde{\mu}(\mathbb{P})]$, which affects β and e in equilibrium, depends upon the investment policy. Figure 8 plots the optimal pay-for-performance component when the investment policy maximizes the terminal value (blue, solid line) and when it maximizes the price (red, dashed line). In the shaded region of panel (a), $K \in (1 - \tilde{\rho}, 1 - \rho)$, which implies that the manager always invests if he maximizes the firm's terminal value, but refrains from investing when $P = p_L$ when he maximizes the price. This reduction in investment lowers the unconditional variance of the price which allows the principal to choose a higher β in equilibrium. As a result, the effort chosen by the manager will also be higher in this region when he maximizes the price. In the shaded region of panel (b), $K \in (\rho, \tilde{\rho})$, which implies that the manager never invests if he maximizes the terminal value, but does invest when $P = p_H$ when he maximizes the price. This decision to invest increases the unconditional variance of the price which leads the principal to choose a lower β, which lowers managerial effort.

These results suggest that allowing the manager to determine the firm's investment policy when the project's unconditional net present value is high (i.e., when K is low) can be optimal even if the manager's preferred policy is ex-ante inefficient. For instance, suppose that the conditional value of the investment when the price is low ($P = p_L$) is close to zero, i.e., the difference between $1 - \rho$ and K is arbitrarily small. If the manager maximizes the price, he will not invest in this project (since $1 - \tilde{\rho} < K$). This has an arbitrarily small effect on the value of the investment project but, as panel (a) makes clear, can lead to a non-trivial increase in β and, hence, managerial effort. When the latter outweighs the former, it is optimal for the principal to delegate the investment decision to the manager.

7 Conclusions

We develop a model in which the price has a dual role. First, it provides the principal a contractable signal about managerial effort, and so is used for managerial compensation. Second, it aggregates investor information about a new project and so affects real investment via feedback effects. Our key insight is that these roles are often at odds: when the price is more informative about future investment opportunities, it is more volatile and, therefore, less effective for incentive provision. We show that, as a result, firm value and social surplus can decrease with price informativeness and existing measures of price efficiency (including forecasting price efficiency and revelatory price efficiency) are incomplete. Our model also provides novel implications for how the composition and price-sensitivity of managerial compensation depends on price informativeness.

Our model is stylized for tractability and clarity of exposition, but lends itself to a number of natural extensions. For instance, we assume that the impact of managerial effort and the new project on firm cash flows are additively separable. This is a natural theoretical benchmark to consider, and empirically relevant in many settings. However, it would be interesting to consider the case where the manager's effort affects the payoffs of the new project, if undertaken by the principal. In this case, the manager's effort choice would depend not only on his compensation, but also his beliefs about the likelihood of the new project being adopted. We leave this analysis for future work. Similarly, we restrict compensation contracts to be linear in the price (and value, in Section 6.3). It would be interesting to study how our results extend if the principal is allowed to use a larger set of instruments (e.g., options) to incentivize the manager, and what the impact of the feedback effect on such compensation schemes is. Finally, our benchmark analysis assumes that the principal's investment decision maximizes expected value, while the extension in Section 6.4 considers the case when the decision is delegated to the manager, and thus, maximizes expected price. While these seem to be the most natural benchmarks, given the evidence on firm behavior, it would be interesting to consider a richer set of investment rules to which the principal could commit.

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A Proofs

A.1 Proof of Proposition 1

First, we establish the market-clearing price as a function of u and ω. In equations (15) and (16), we establish the thresholds for $P = p_L$. We now establish analogous thresholds for when the price is p_u and p_H.

If $\omega = H$, then $P = p_H$ as long as

$$u < \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 2\lambda\rho - 1, \tag{70}$$

while if $\omega = L$, then $P = p_H$ as long as

$$u < \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(\rho)}_{\text{observed } s_i = s_L} = 2\lambda(1-\rho) - 1. \tag{71}$$

As a result, if $\omega = H$, then $P = p_u$ as long as

$$2\lambda\rho - 1 \leq u \leq 1 - 2\lambda(1-\rho), \tag{72}$$

while if $\omega = L$, then $P = p_u$ as long as

$$2\lambda(1-\rho) - 1 \leq u \leq 1 - 2\lambda\rho. \tag{73}$$

Note, then, that if we define $u_H \equiv \lambda(2\rho - 1)$, $u_L = \lambda(1-2\rho)$, the thresholds above correspond to those found in the proposition.

With these market-clearing conditions, we confirm the signal that both investors and the principal can extract from the price and, in so doing, confirm the beliefs of the marginal investor at each price level. Specifically,

$$\begin{aligned}
\mathbb{P}[\omega = H|p_h] &= \frac{\mathbb{P}[\omega = H \cap P = p_H]}{\mathbb{P}[P = p_h]} = \frac{\lambda\rho}{\lambda\rho + \lambda(1-\rho)} = \rho \\
\mathbb{P}[\omega = H|p_l] &= \frac{\mathbb{P}[\omega = H \cap P = p_l]}{\mathbb{P}[P = p_l]} = \frac{\lambda(1-\rho)}{\lambda\rho + \lambda(1-\rho)} = 1 - \rho \\
\mathbb{P}[\omega = H|p_u] &= \frac{\mathbb{P}[\omega = H \cap P = p_u]}{\mathbb{P}[P = p_u]} = \frac{1-\lambda}{1-\lambda+1-\lambda} = \tfrac{1}{2}.
\end{aligned}$$

Note that, given these beliefs, the optimal investment rule, (6), yields the investment rule specified in the proposition.

Finally, in order for the conjectured equilibrium to exist, the equilibrium price levels must be distinct. To do so, we show that it is always the case that $p_H > p_u > p_L$. If the principal's investment decision is the same across any two prices, then this ordering holds trivially: V_H and V_L are the same across the two prices (given the investment policy) while $\tilde{\rho} > \frac{1}{2} > 1 - \tilde{\rho}$. Suppose instead that the principal's investment decision differs across two adjacent prices. There are two cases to consider.

(1) Suppose that the principal only invests if she observes $P = p_H$. Then, given the logic above, $p_u > p_L$ and it remains to be shown that $p_H > p_u$. Note that

$$\begin{aligned}
p_H &= \frac{1}{1+\beta}\left[(1-\tilde{\rho})\, V_L\left(p_H\right) + \tilde{\rho} V_H\left(p_H\right) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[(1-\rho)\, V_L\left(p_H\right) + \rho V_H\left(p_H\right) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[(1-\rho)\, V_L\left(p_u\right) + \rho V_H\left(p_u\right) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L\left(p_u\right) + \tfrac{1}{2} V_H\left(p_u\right) - \alpha\right] \\
&= p_u,
\end{aligned}$$

where the first and third inequalities follows from $\tilde{\rho} > \rho > \frac{1}{2}$, while the second inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows.

(2) Suppose that the principal only invests if she observes $P \in \{p_H, p_u\}$. Then, given the logic above, $p_H > p_u$ and it remains to be shown that $p_u > p_L$. Note that

$$\begin{aligned}
p_u &= \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L\left(p_u\right) + \tfrac{1}{2} V_H\left(p_u\right) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tfrac{1}{2} V_L\left(p_L\right) + \tfrac{1}{2} V_H\left(p_L\right) - \alpha\right] \\
&> \frac{1}{1+\beta}\left[\tilde{\rho} V_L\left(p_L\right) + (1-\tilde{\rho})\, V_H\left(p_L\right) - \alpha\right] \\
&= p_L,
\end{aligned}$$

where the second inequality follows from $\frac{1}{2} > 1 - \tilde{\rho}$, while the first inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows. This completes the proof.

A.2 Proof of Proposition 2

Principal's objective is

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + y_\omega I_m\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{74}$$

The FOC is

$$\frac{\partial e}{\partial \beta}\left(1 - c'(e)\right) - \gamma \frac{\beta}{(1+\beta)^3} \mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which implies

$$\frac{1}{(1+\beta)^2}\left(\frac{1 - c'(e)}{c''(e)}\right) - \gamma \frac{\beta}{(1+\beta)^3} \mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which in turn implies

$$\beta = \frac{1}{\gamma c''(e) \mathbb{V}\left[\tilde{\mu}(P)\right]}.$$

Let $B = \frac{\beta}{1+\beta}$. The second order condition is

$$\frac{\partial^2 e}{\partial B^2}\left(1 - c'(e)\right) - \frac{\partial e}{\partial B}\frac{\partial e}{\partial B} c''(e) - \gamma \mathbb{V}\left[\tilde{\mu}(P)\right]$$

Note that

$$B = c'(e) \implies 1 = c''(e)\frac{\partial e}{\partial B} \implies 0 = c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B} + c''(e)\frac{\partial^2 e}{\partial B^2} = 0.$$

Substituting these into SOC, we get

$$-\frac{c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B}}{c''(e)}\left(1 - c'(e)\right) - \frac{1}{c''(e)} - \gamma \mathbb{V}\left[\tilde{\mu}(P)\right]$$

which simplifies to

$$-\frac{c'''(e)}{\left[c''(e)\right]^2} c'(e)\left(1 - c'(e)\right) - 1.$$

For the SOC to hold, we need

$$\frac{c'''(e)}{\left[c''(e)\right]^2} c'(e)\left(1 - c'(e)\right) + 1 > 0$$

A.3 Proof of Proposition 3

Note that

$$\beta = \frac{1}{\gamma c''(e) \mathbb{V}\left[\tilde{\mu}(P)\right]}$$

which implies that

$$\frac{\partial \beta}{\partial \lambda} = -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial \mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial \lambda}$$

$$\frac{\partial \beta}{\partial \rho} = -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial \mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial \rho}.$$

$$\mathbb{V}\left[\tilde{\mu}(P)\right] = \frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\tilde{\mu}(p_u)^2 - \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)\right) + (1-\lambda)\tilde{\mu}(p_u)\right]^2 \tag{75}$$

which implies

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial \lambda} = \left(\tilde{\mu}(p_H) - \tilde{\mu}(p_u)\right)\left(\tilde{\mu}(p_u) - \tilde{\mu}(p_L)\right) + \frac{1-\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L) - 2\tilde{\mu}(p_u)\right)^2 > 0$$

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial \rho} > 0$$

This implies that

$$\frac{\partial \beta}{\partial \lambda} < 0 \qquad \frac{\partial \beta}{\partial \rho} < 0$$

Note that effort solves

$$\frac{\beta}{1+\beta} = c'(e)$$

which implies

$$\frac{1}{(1+\beta)^2}\frac{\partial \beta}{\partial \lambda} = c''(e)\frac{\partial e}{\partial \lambda}$$

$$\frac{1}{(1+\beta)^2}\frac{\partial \beta}{\partial \rho} = c''(e)\frac{\partial e}{\partial \rho}$$

which implies that effort decreases with λ and ρ.

A.4 Proof of Proposition 4

Let $B = \frac{\beta}{1+\beta}$. Note that B and e solves

$$B = c'(e) \qquad B = \frac{1}{1 + \gamma c''(e)\,\mathbb{V}\left[\tilde{\mu}(P)\right]}$$

Differentiating with respect to λ, we get

$$c''(e)\frac{\partial e}{\partial \lambda} = \frac{\partial B}{\partial \lambda} \qquad \frac{\partial B}{\partial \lambda} = -\gamma B^2\left[c''(e)\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda} + \mathbb{V}\left[\tilde{\mu}\right]c'''(e)\frac{\partial e}{\partial \lambda}\right]$$

This implies that

$$\frac{\partial e}{\partial \lambda} = -\gamma B^2 \left[\frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} + \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)} \frac{\partial e}{\partial \lambda} \right]$$

$$\frac{\partial e}{\partial \lambda} \left(\underbrace{1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}}_{>0} \right) = -\gamma B^2 \frac{\partial \mathbb{V}[P]}{\partial \lambda}$$

The term in the underlying brace in the above equation is

$$1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)} = 1 + c'(e)(1 - c'(e)) \frac{c'''(e)}{[c''(e)]^2} > 0$$

because of the second order condition. This implies

$$\frac{\partial e}{\partial \lambda} = -\frac{\gamma B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}}{1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}} < 0. \tag{76}$$

Note that firm value is given by

$$\frac{\partial FV}{\partial \lambda} = \frac{\partial e}{\partial \lambda} + \delta \frac{\partial E[\mathbb{I}_{d_m = I} y_\omega]}{\partial \lambda}$$

If $\rho < K$ or $1 - \rho > K$, then firm value is decreasing in λ. For the rest of the proof, assume that $\rho > K > 1 - \rho$. Differentiating equation 76 , we get

$$\frac{\partial^2 e}{\partial \lambda^2} = -\frac{\left(1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}\right)\left(\gamma B^2 \frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} + 2\gamma B \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{\partial B}{\partial \lambda}\right) - \gamma B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \gamma \left(B^2 \mathbb{V}[\tilde{\mu}] \frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + 2B \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)} \frac{\partial B}{\partial \lambda} + B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{c'''(e)}{c''(e)}\right)}{\left(1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}\right)^2}$$

$$= -\frac{\gamma B^2 \left(1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}\right) \frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} + 2\gamma B \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{\partial B}{\partial \lambda} - \gamma^2 B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \left(B^2 \mathbb{V}[\tilde{\mu}] \frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{c'''(e)}{c''(e)}\right)}{\left(1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c'''(e)}{c''(e)}\right)^2}$$

Note that $\frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} = -\frac{1}{2}(p_H + p_L - 2p_u)^2 \leq 0$. This implies

$$\frac{\partial^2 e}{\partial \lambda^2} = \gamma B^2 \left(1 + \gamma B^2 \mathbb{V}[\tilde{\mu}] \frac{c''(e)}{c'(e)}\right) \frac{-\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} + 2\gamma B c''(e) \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{-\partial e}{\partial \lambda} + \gamma^2 B^4 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \left(\mathbb{V}[\tilde{\mu}] \frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{c'''(e)}{c''(e)}\right)$$

The first two terms are positive. So, the function is convex iff the third term is also positive .i.e.,

$$\mathbb{V}[\tilde{\mu}] \frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda} \frac{c'''(e)}{c''(e)} > 0. \tag{77}$$

If the above condition is true, effort is convex in λ, which implies that firm value is also convex in λ. This implies that there are only three possible shapes for FV: increasing, decreasing, U shaped. There exists $\bar{\delta} > \underline{\delta} > 0$ such that, for $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, FV is U shaped in λ. If $\delta \leq \underline{\delta}$, FV is decreasing in λ. If $\delta > \bar{\delta}$, FV is increasing in λ.

A.5 Proof of Proposition 5

We begin by characterizing how the efficiency measures depend on the underlying parameters, given the financial market equilibrium and the optimal contract.

Lemma 1. *Consider the unique financial market equilibrium described in proposition 1 and the optimal contract described in proposition 2. Then,*

(i) Revelatory price efficiency is

$$RPE = \lambda \left(2\rho - 1\right)^2$$

(ii) Forecasting price efficieny is

$$FPE^{-1} = \begin{cases} \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \textit{if } 1-\rho > K \\ \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) + \left(x_H - x_L\right)^2 \frac{\rho(1-\rho)\lambda}{2} & \textit{if } \frac{1}{2} > K > 1-\rho \\ \left(x_H - x_L + \delta y_H - \delta y_L\right)^2 \frac{\rho(1-\rho)\lambda}{2} + \left(x_H - x_L\right)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) & \textit{if } \rho > K > \frac{1}{2}, \\ \left(x_H - x_L\right)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \textit{if } \rho < K \end{cases} \tag{78}$$

(iii) Contracting price efficiency is

$$CPE = \left[\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right)^2 + \tilde{\mu}\left(p_L\right)^2\right) + \left(1-\lambda\right)\tilde{\mu}\left(p_U\right)^2 - \left(\frac{\lambda}{2}\left(\tilde{\mu}\left(p_H\right) + \tilde{\mu}\left(p_L\right)\right) + \left(1-\lambda\right)\tilde{\mu}\left(p_U\right)\right)^2\right]^{-1}$$

(iv) Firm value is

$$FV = \frac{x_H + x_L}{2} + e + \begin{cases} \delta \frac{y_H + y_L}{2} & \textit{if } 1-\rho > K \\ \left(1-\lambda\right)\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta\left(\rho y_H + \left(1-\rho\right) y_L\right) & \textit{if } \frac{1}{2} > K > 1-\rho \\ \frac{\lambda}{2}\delta\left(\rho y_H + \left(1-\rho\right) y_L\right) & \textit{if } \rho > K > \frac{1}{2}, \\ 0 & \textit{if } \rho < K \end{cases}$$

where $\tilde{\mu}\left(p_H\right)$,$\tilde{\mu}\left(p_U\right)$ and $\tilde{\mu}\left(p_L\right)$ are defined in proposition (1).

Proof. Note that

$$RPE = \frac{V\left[y_\omega\right] - V\left[y_\omega | p\right]}{V\left[y_\omega\right]}$$

$$= \frac{\frac{(y_H - y_L)^2}{4} - \left[\frac{(y_H - y_L)^2}{4}(1-\lambda) + \lambda\rho(1-\rho)(y_H - y_L)^2\right]}{\frac{(y_H - y_L)^2}{4}}$$

$$= \lambda (2\rho - 1)^2 \tag{79}$$

Moreover, by definition, $FPE^{-1} = \mathbb{V}(x_\omega + y_\omega \mathbb{I}_{d_m = I}|p)$ and this simplifies to 78. Contracting price efficiency is captured by

$$CPE = [\mathbb{V}[\tilde{\mu}(p)|e]]^{-1}$$
$$= \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\tilde{\mu}(p_U)^2 - \left(\frac{\lambda}{2}(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)) + (1-\lambda)\tilde{\mu}(p_U)\right)^2\right]^{-1}$$

□

A.6 Proof of Proposition 6

Recall that investors differ only in their belief about the relative likelihood of each state. This implies that

$$\mathbb{E}[d(\{s_i, P\})(x_\omega + e + y_\omega \mathbb{I}_{d_m=I} - P)] = \left[\frac{\rho\lambda(1-\rho)}{2} + \frac{(1-\rho)\lambda\rho}{2}\right](q(s_i = s_H, p_H) - q(s_i = s_L, p_H))\Delta V(p_H) \tag{80}$$

$$+ \left[\frac{\rho\lambda(1-\rho)}{2} + \frac{(1-\rho)\lambda\rho}{2}\right](q(s_i = s_H, p_L) - q(s_i = s_L, p_L))\Delta V(p_L) \tag{81}$$

$$+ \left[\frac{(1-\lambda)\rho}{2} + \frac{(1-\lambda)(1-\rho)}{2}\right](q(s_i = s_H, p_U) - q(p_U))\Delta V(p_U) \tag{82}$$

$$+ \left[\frac{(1-\lambda)\rho}{2} + \frac{(1-\lambda)(1-\rho)}{2}\right](q(p_U) - q(s_i = s_L, p_U))\Delta V(p_U) \tag{83}$$

while

$$\mathbb{E}[d(\{P\})(x_\omega + e + y_\omega \mathbb{I}_{d_m=I} - P)] = \left[\frac{\rho\lambda}{2} + \frac{(1-\rho)\lambda}{2}\right](q(s_i = s_H, p_H) - q(p_H))\Delta V(p_H) \tag{84}$$

$$+ \left[\frac{\rho\lambda}{2} + \frac{(1-\rho)\lambda}{2}\right](q(p_L) - q(s_i = s_L, p_L))\Delta V(p_L) \tag{85}$$

We can simplify by noting that

$$\left[\frac{\rho\lambda(1-\rho)}{2} + \frac{(1-\rho)\lambda\rho}{2}\right](q(s_i = s_H, p_H) - q(s_i = s_L, p_H)) = \rho\lambda(1-\rho)\left(\tilde{\rho} - \tfrac{1}{2}\right) \tag{86}$$

$$\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(s_i=s_H,p_H\right)-q\left(p_H\right)\right)=\tfrac{\lambda}{2}\left(\tilde{\rho}-\rho\right) \tag{87}$$

$$\left[\tfrac{\rho\lambda(1-\rho)}{2}+\tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i=s_H,p_L\right)-q\left(s_i=s_L,p_L\right)\right)=\rho\lambda\left(1-\rho\right)\left(\tfrac{1}{2}-\left(1-\tilde{\rho}\right)\right) \tag{88}$$

$$\left[\tfrac{\rho\lambda}{2}+\tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(p_L\right)-q\left(s_i=s_L,p_L\right)\right)=\tfrac{\lambda}{2}\left(\left(1-\rho\right)-\left(1-\tilde{\rho}\right)\right) \tag{89}$$

Substituting in the expression for $\tilde{\rho}$ reveals that all four expressions are equal to $=\frac{\rho\lambda(1-\rho)}{2}\left(\frac{2\rho-1}{\rho^2+(1-\rho)^2}\right)$.

As a result, the indifference condition reduces to

$$\tfrac{1-\lambda}{2}\left(\rho-\tfrac{1}{2}\right)\Delta V\left(p_U\right)+\tfrac{1-\lambda}{2}\left(\tfrac{1}{2}-\left(1-\rho\right)\right)\Delta V\left(p_U\right)=c \tag{90}$$

and so in an interior equilibrium, the measure of informed investors is

$$\lambda=1-\frac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}. \tag{91}$$

$$=1-\frac{c}{\left(\rho-\frac{1}{2}\right)(x_H-x_L+\mathbb{P}[d_m=I|p_U](y_H-y_L))} \tag{92}$$

More generally,

$$\lambda=\max\left\{0,1-\frac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}\right\}. \tag{93}$$

Thus, in any setting, the measure of informed investors is increasing in ρ but does *not* depend upon e.

A.7 Proof of Proposition 7

Let $B=\frac{\beta}{1+\beta}$ and note that $c'\left(e\right)=B$ and $\beta=\frac{1}{c''(e)\gamma\mathbb{V}(\tilde{\mu}(P))}$, which implies $c''\left(e\right)\frac{\partial e}{\partial\lambda}=\frac{\partial B}{\partial\lambda}$ and

$$B=\frac{\beta}{1+\beta}=\frac{1}{1+c''\left(e\right)\gamma\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)} \tag{94}$$

$$\Rightarrow\frac{\partial B}{\partial\lambda}=-B^2\gamma\left(c''\left(e\right)\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)+\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)c'''\left(e\right)\frac{\partial e}{\partial\lambda}\right). \tag{95}$$

This implies

$$\frac{\partial}{\partial\lambda}\left(e-c\left(e\right)-\frac{\gamma}{2}B^2\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)\right)=\left(1-c'\left(e\right)\right)\frac{\partial e}{\partial\lambda}-\gamma\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)B\frac{\partial B}{\partial\lambda}-\frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) \tag{96}$$

$$=\left(1-B-\gamma B\mathbb{V}\left(\tilde{\mu}\left(P\right)\right)c''\left(e\right)\right)\frac{\partial e}{\partial\lambda}-\frac{\gamma}{2}B^2\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) \tag{97}$$

$$= \left(1 - B - \gamma \mathbb{B}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) c''\left(e\right)\right) \frac{\partial e}{\partial \lambda} + \frac{1}{2}\left(1 + \gamma B^2 \mathbb{V}\left(\tilde{\mu}\left(P\right)\right) \frac{c'''\left(e\right)}{c''\left(e\right)}\right) \frac{\partial e}{\partial \lambda} \tag{98}$$

$$= \left(\frac{1}{2} + \frac{\gamma}{2} B^2 \mathbb{V}\left(\tilde{\mu}\left(P\right)\right) \frac{c'''\left(e\right)}{c''\left(e\right)}\right) \frac{\partial e}{\partial \lambda} \tag{99}$$

$$= -\frac{\gamma}{2} B^2 \frac{\partial}{\partial \lambda} \mathbb{V}\left(\tilde{\mu}\left(P\right)\right) < 0 \tag{100}$$

and so we can express the change in firm value as

$$\frac{\partial}{\partial c_0} FV = \frac{\partial}{\partial \lambda} \mathbb{E}\left[\delta y_\omega \times I_m\right] \frac{d\lambda}{dc_0} + \frac{\partial}{\partial \lambda}\left(e - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left(\tilde{\mu}\left(P\right)\right) - c\left(e\right)\right) \frac{d\lambda}{dc_0} \tag{101}$$

$$= \left(\delta \frac{\partial}{\partial \lambda} \mathbb{E}\left[y_\omega \times I_m\right] - \frac{\gamma}{2} B^2 \frac{\partial}{\partial \lambda} \mathbb{V}\left(\tilde{\mu}\left(P\right)\right)\right) \frac{d\lambda}{dc_0}, \tag{102}$$

Next, note that

$$\frac{\partial \mathbb{E}\left[\delta y_\omega \times I_m\right]}{\partial \lambda} = \begin{cases} 0 & \text{if } 1 - \rho > K \\ \frac{\delta(y_H - y_L)}{2}\left(\rho - 1 + K\right) & \text{if } \frac{1}{2} > K > 1 - \rho \\ \frac{\delta(y_H - y_L)}{2}\left(\rho - K\right) & \text{if } \rho > K > \frac{1}{2}, \\ 0 & \text{if } \rho < K \end{cases}$$

and

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \lambda} = \frac{1}{2}\left(\mu_H^2 + \mu_L^2 - 2\mu_U^2\right) - \mu_U\left(\mu_H + \mu_L\right) - \frac{1}{2}\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \tag{103}$$

$$= \frac{1}{2}\left(\left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2\right) - \frac{1}{2}\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \tag{104}$$

$$\geq \left(\mu_H - \mu_U\right)\left(\mu_U - \mu_L\right) > 0 \tag{105}$$

So that

$$\frac{\partial}{\partial c_0} FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda} \mathbb{E}\left[\delta y_\omega \times I_m\right] \\ -\frac{\gamma}{4} B^2 \left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{106}$$

Next, note that if $1 - \rho > K$ or $\frac{1}{2} > K > 1 - \rho$:

$$\lim_{\delta \to 0} \lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\left(x_H - x_L\right)}, \tag{107}$$

$$\lim_{\delta\to\infty} \lambda = 1 \tag{108}$$

$$\lim_{\delta\to\infty} (1-\lambda)\,\delta = \lim_{\delta\to\infty} \frac{c_0\delta}{\left(\rho-\frac{1}{2}\right)\left(x_H - x_L + \delta\left(y_H - y_L\right)\right)} \tag{109}$$

$$= \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H - y_L\right)} \tag{110}$$

and if $\rho > K > \frac{1}{2}$ or $\rho < K$, then

$$\lambda = 1 - \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(x_H - x_L\right)}. \tag{111}$$

We have to consider four cases:

(1) If $1-\rho > K$, then

$$\frac{\partial}{\partial c_0} FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c}\left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2\\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2\end{array}\right)\right), \tag{112}$$

which is positive.

(2) If $\frac{1}{2} > K > 1-\rho$, then

$$\frac{\partial}{\partial c_0} FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c}\frac{\partial}{\partial\lambda}\mathbb{E}\left[\delta y_\omega \times I_m\right]\\ -\frac{\gamma}{4}B^2\left(\begin{array}{c}\left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2\\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2\end{array}\right)\end{array}\right) \tag{113}$$

$$\Rightarrow \lim_{\delta\to\infty}\frac{\partial}{\partial c_0} FV = \lim_{\delta\to\infty} -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c}\frac{\partial}{\partial\lambda}\mathbb{E}\left[\delta y_\omega \times I_m\right]\\ -\frac{\gamma}{4}B^2\left(\begin{array}{c}\left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2\\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2\end{array}\right)\end{array}\right) \tag{114}$$

$$= -\frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H-y_L\right)}\mathbb{E}\left[y_\omega \times I_m\right] - 1 < 0 \tag{115}$$

which implies, SV decreases with c_0 when when δ is sufficiently large. On the other hand,

$$\lim_{\delta\to 0}\frac{\partial}{\partial c_0} FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c}\left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2\\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2\end{array}\right)\right), \tag{116}$$

which is positive.

(3) If $\rho > K > \frac{1}{2}$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$, and

$$\frac{\partial}{\partial c_0} FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I_m\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right)\end{array}\right) \tag{117}$$

$$\Rightarrow \lim_{\delta \to 0} \frac{\partial}{\partial c_0} FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right)\right), \tag{118}$$

which is positive. However, as

$$\Rightarrow \lim_{\delta \to \infty} \frac{\partial}{\partial c_0} SV = \lim_{\delta \to \infty} -\delta\left(\frac{1-\lambda}{c_0}\right)\left(\frac{\partial}{\partial \lambda}\mathbb{E}\left[y_\omega \times I_m\right]\right) \tag{119}$$

which is negative for δ large enough.

(4) If $\rho < K$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$, and

$$\frac{\partial}{\partial c_0} SV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right)\right) \tag{120}$$

which is positive.

A.8 Proof of Proposition 8

The proof of this follows from the observation that

$$\frac{\partial}{\partial c_0} SV = \frac{\partial}{\partial c_0} FV - \left(\lambda + c_0 \frac{d\lambda}{dc_0}\right) \tag{121}$$

$$= \frac{\partial}{\partial c_0} FV - (2\lambda - 1) \tag{122}$$

since

$$\lambda = \begin{cases} 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L + \delta(y_H - y_L))} & \text{if } 1 - \rho > K \text{ or } \frac{1}{2} > K > 1 - \rho \\ 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)} & \text{if } \rho > K > \frac{1}{2} \text{ or } \rho < K \end{cases}, \tag{123}$$

and so

$$\lim_{\delta \to 0} \lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}. \tag{124}$$

This implies that $2\lambda - 1 < 0$ if and only if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$.

A.9 Proof of Proposition 9

Let $B \equiv \frac{\beta}{1+\beta}$, and note that choosing β optimally is equivalent to choosing B optimally. First, note that α can be chosen to ensure that the manager's participation constraint binds. Then, one can rewrite the principal's objective as

$$\max_{\beta} \mathbb{E}\left[x_\omega + (e+\theta) + \delta y_\omega \times I_m\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c\left(e\right) \tag{125}$$

subject to $B\kappa = c'\left(e\right)$, which implies:

$$\kappa = c''\left(e\right) e_B, \quad \text{and } 0 = c'''\left(e\right) e_B^2 + c''\left(e\right) e_{BB}. \tag{126}$$

The FOC w.r.t. B for the principal is given by

$$e_B\left(1 - c'\left(e\right)\right) - \gamma B\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) = 0 \tag{127}$$

$$\Leftrightarrow \frac{\kappa}{c''\left(e\right)}\left(1 - \kappa B\right) - \gamma B\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) = 0 \tag{128}$$

which follows from plugging in $e_B = \frac{\kappa}{c''(e)}$. This implies that the optimal choice of B is given by:

$$B = \frac{\kappa}{\gamma c''\left(e\right)\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) + \kappa^2} \tag{129}$$

or equivalently, the optimal choice of β is given by:

$$\beta = \frac{\kappa}{\gamma c''\left(e\right)\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) - \kappa\left(1-\kappa\right)}. \tag{130}$$

The SOC is given by $\mathcal{S} < 0$, where

$$\mathcal{S} = e_{BB}\left(1 - c'\left(e\right)\right) - e_B^2 c''\left(e\right) - \gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) \tag{131}$$

Since $e_{BB} = -\frac{c'''(e)}{c''(e)} e_B^2$ and

$$\gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P; s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) = \frac{e_B\left(1 - c'\left(e\right)\right)}{B} \tag{132}$$

$$= \frac{\kappa e_B\left(1 - c'\left(e\right)\right)}{c'\left(e\right)} \tag{133}$$

$$= \frac{e_B^2 c''\left(e\right)\left(1 - c'\left(e\right)\right)}{c'\left(e\right)} \tag{134}$$

we have

$$\mathcal{S} = -\frac{c'''(e)}{c''(e)} e_B^2 (1 - c'(e)) - e_B^2 c''(e) - \gamma \left(\mathbb{V}(\tilde{\mu}(P; s_\theta) | s_\theta) + \kappa \sigma_\theta^2 \right) \tag{135}$$

$$= -e_B^2 \left(\frac{c'''(e)}{c''(e)} (1 - c'(e)) + c''(e) + \frac{c''(e)(1 - c'(e))}{c'(e)} \right) \tag{136}$$

$$= -e_B^2 \left(\frac{c''(e)}{c'(e)} + \frac{c'''(e)(1 - c'(e))}{c''(e)} \right) \tag{137}$$

So we need

$$\frac{c''(e)}{c'(e)} + \frac{c'''(e)(1 - c'(e))}{c''(e)} > 0. \tag{138}$$

A.10 Proof of Proposition 10

Note that

$$W = \alpha + \beta (\pi P + (1 - \pi)(V - W)) \implies W = \frac{\alpha}{(1 + \beta(1 - \pi))} + \frac{\beta}{(1 + \beta(1 - \pi))} (\pi P + (1 - \pi) V)$$

Moreover, the market clearing condition implies

$$\begin{aligned} P &= E[V - W] \\ &= E\left[V - \frac{\alpha}{(1 + \beta(1 - \pi))} - \frac{\beta}{(1 + \beta(1 - \pi))} (\pi P + (1 - \pi) V) \right] \end{aligned}$$

which implies that

$$P \left(1 + \frac{\beta \pi}{(1 + \beta(1 - \pi))} \right) = E[V] \left(1 - \frac{\beta(1 - \pi)}{(1 + \beta(1 - \pi))} \right) - \frac{\alpha}{(1 + \beta(1 - \pi))}$$

$$P = \frac{E[V] - \alpha}{1 + \beta} \equiv \frac{\tilde{\mu}(P) - \alpha}{1 + \beta}$$

Manager's effort problem is

$$\max_e \mathbb{E}\left[\frac{\alpha}{(1 + \beta(1 - \pi))} + \frac{\beta(\pi P + (1 - \pi) V)}{(1 + \beta(1 - \pi))} \right] - \frac{\gamma}{2} \mathbb{V}[\alpha + \beta(\pi P + (1 - \pi) Z)] - c(e),$$

The FOC is

$$\frac{\beta}{1 + \beta} = c'(e).$$

The optimal contract solves

$$\max_{\alpha,\beta,\pi} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m - \left(\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right)\right], \qquad \text{subject to :} \tag{139}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{140}$$

$$\mathbb{E}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta\left(\pi P + (1-\pi) Z\right)\right] - c(e) \geq 0. \tag{141}$$

This simplifies to

$$\max_{\beta,\pi} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m\right] - \frac{\gamma}{2}\beta^2\mathbb{V}\left[\left(\pi P + (1-\pi) Z\right)\right] - c(e) \quad \text{subject to } \frac{\beta}{1+\beta} = c'(e).$$

Moreover

$$\begin{aligned}
\beta^2\mathbb{V}\left[\left(\pi P + (1-\pi) Z\right)\right] &= \frac{\beta^2\mathbb{V}\left[\left(\pi P + (1-\pi) V\right)\right]}{\left(1+\beta(1-\pi)\right)^2} \\
&= \frac{\beta^2\mathbb{V}\left[\left(\pi \frac{\tilde{\mu}}{1+\beta} + (1-\pi) V\right)\right]}{\left(1+\beta(1-\pi)\right)^2} \\
&= \frac{\beta^2}{(1+\beta)^2}\mathbb{V}\left[\left(\tilde{\mu} + \frac{(1-\pi)(1+\beta)}{\left(1+\beta(1-\pi)\right)}\left(V-\tilde{\mu}\right)\right)\right]
\end{aligned}$$

Note that $\tilde{\mu}$ is independent of π and the optimal π is characterized by minimizing

$$\min_{e} \ \mathbb{V}\left[\left(\tilde{\mu} + \frac{(1-\pi)(1+\beta)}{\left(1+\beta(1-\pi)\right)}\left(V-\tilde{\mu}\right)\right)\right]$$

The FOC is

$$\pi = 1 + \frac{\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}{(1+\beta)\operatorname{var}\left(V-\tilde{\mu}\right) + \beta\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}$$

Substituting this into the principal's objective, we get

$$\begin{aligned}
\mathbb{V}[W] &= \frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] + \left(\frac{(1-\pi)(1+\beta)}{\left(1+\beta(1-\pi)\right)}\right)^2\mathbb{V}\left(V-\tilde{\mu}\right) + 2\frac{(1-\pi)(1+\beta)}{\left(1+\beta(1-\pi)\right)}\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right)\right] \\
&= \frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] - \frac{\left[\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\operatorname{var}\left(V-\tilde{\mu}\right)}\right)\right]
\end{aligned}$$

The objective is

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m\right] - \frac{\gamma}{2}\frac{\beta^2}{(1+\beta)^2}\left[\left(\mathbb{V}[\tilde{\mu}] - \frac{\left[\operatorname{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\operatorname{var}\left(V-\tilde{\mu}\right)}\right)\right] - c(e) \quad \text{subject to } \frac{\beta}{1+\beta} = c'(e).$$

The FOC is

$$(1 - c'(e)) \frac{\partial e}{\partial \beta} = \gamma \frac{\beta}{(1+\beta)^3} \left(\mathbb{V}[\tilde{\mu}] - \frac{[\mathrm{cov}(V - \tilde{\mu}, \tilde{\mu})]^2}{\mathrm{var}(V - \tilde{\mu})} \right)$$

$$\beta = \frac{1}{\gamma c''(e) \left[\left(\mathbb{V}[\tilde{\mu}] - \frac{[\mathrm{cov}(V - \tilde{\mu}, \tilde{\mu})]^2}{\mathrm{var}(V - \tilde{\mu})} \right) \right]}.$$

A.11 Proof of Proposition 11

Note that firm value (ignoring assets in place) is given by

$$\begin{aligned} FV &= E[e + \delta y_\omega \times I] \\ &= \begin{cases} e + \delta \frac{y_H + y_L}{2} & \text{if } 1 - \tilde{\rho} > K \\ e + (1-\lambda)\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta(\rho y_H + (1-\rho) y_L) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ e + \frac{\lambda}{2}\delta(\rho y_H + (1-\rho) y_L) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ e & \text{if } \tilde{\rho} < K \end{cases} \end{aligned}$$

Note that,

$$\frac{\partial FV}{\partial \lambda} = \begin{cases} \frac{\partial e}{\partial \lambda} & \text{if } 1 - \tilde{\rho} > K \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}(\rho - 1 + K) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}(\rho - K) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ \frac{\partial e}{\partial \lambda} & \text{if } \tilde{\rho} < K \end{cases}$$

- Let's focus on the third case above. As λ increases, the second term is positive only when $\rho > K$. In the region where $\rho < K < \hat{\rho}$, the second term is negative. High λ can be bad for firm value in this region.

- Let's focus on the second case above. As λ increases, the second term is positive only when $K > 1 - \rho$. In the region where $1 - \hat{\rho} < K < 1 - \rho$, the second term is negative. Again, high λ can be bad for firm value in this region.

B Robustness

B.1 Traded Cash Flows

In what follows, we assume that investors trade a claim to the firm's terminal cash flows *without* netting out the compensation to the manager. We denote the price of this claim as

$\hat{P}$. Note that

$$\hat{P} = \tilde{\mu}(P) \tag{142}$$

$$= (1+\beta) P + \alpha \tag{143}$$

The proof of the existence of the financial market equilibrium is unchanged since α and β are scalars and do not play a role in the proof of Proposition 1.

As a result of this change, the manager's optimal level of effort now solves

$$\beta = c'(e), \tag{144}$$

which becomes the principal's new IC constraint. Specifically, we can rewrite the principal's objective as

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + y_\omega I_m\right] - \frac{\gamma}{2}\beta^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \beta = c'(e). \tag{145}$$

Then the principal's optimal contract is given by $\left(\hat{\alpha}, \hat{\beta}\right)$, where

$$\hat{\alpha} = c(e) + \frac{\gamma}{2}\hat{\beta}^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - \hat{\beta}\mathbb{E}[P], \text{ and} \tag{146}$$

$$\hat{\beta} = \frac{1}{1 + c''(e)\gamma \mathbb{V}\left[\tilde{\mu}(P)\right]}. \tag{147}$$

Thus, the contract is "lower-powered", i.e., less sensitive to the stock price. Note that, relative to the baseline model, this has no impact on the manager's equilibrium level of effort since $\hat{\beta} = \frac{\beta}{1+\beta}$, i.e., in equilibrium the manager's first-order condition is unchanged.

B.2 Risk neutral managers

The key tradeoff in our analysis is that when the price is more informative about future investment opportunities, it becomes more volatile signal about effort and, consequently, it is more costly for the principal to incentivize managerial effort. While the assumption of managerial risk aversion makes this tradeoff particularly transparent in our benchmark analysis, the analysis in this section establishes that it is not necessary. Specifically, we show that the same tradeoff can obtain when the manager is risk neutral, but the optimal contract satisfies a limited liability constraint, defined as follows.

Definition 1. *The optimal contract satisfies limited liability if for all $P \in \{p_H, p_U, p_L\}$, the contract satisfies $\alpha + \beta P \geq 0$.*

Note that the manager's effort choice is still given by (29). However, the objective of the principal is now given by:

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{148}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{149}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - c(e) \geq 0, \tag{150}$$

$$\alpha + \beta p_L \geq 0. \tag{151}$$

Given that $p_H > p_u > p_L$, condition (151) ensures that the limited liability constraint holds for all price levels. The above problem implies that the principal can always decrease α to improve her payoff till the manager's participation constraint (150) or (151) binds. This implies that the principal's objective can be rewritten as

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I_m\right] - \max\left\{\beta E\left[P\right] - \beta p_L, c(e)\right\} \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{152}$$

The following lemma provides a characterization of how more information in prices affects the contract offered in equilibrium.

Lemma 2. *$E[P]$ increases in λ. Moreover, suppose at the optimal contract, the limited liability constraint (151) binds. Then, an increase in λ leads to a decrease in β, and consequently, the equilibrium level of effort e.*

Intuitively, the above result implies that the cost of incentivizing the manager increases with price informativeness when the limited liability condition binds. As such, our main tradeoff can also obtain when the manager is risk-neutral.

Proof of Lemma 2. Note that price is given by (25) and the expected price is given by

$$E\left[P\right] = \frac{\lambda}{2}\left(p_L + p_H\right) + (1-\lambda)\, p_U$$

and hence

$$\begin{aligned}\frac{\partial E\left[P\right]}{\partial \lambda} &= \frac{1}{2}\left(p_L + p_H - 2p_U\right)\\ &= \frac{1}{2(1+\beta)}\left(\tilde{\mu}(p_L) + \tilde{\mu}(p_H) - 2\tilde{\mu}(p_U)\right)\end{aligned}$$

If the manager never invests or always invests, the price is symmetric and so $\frac{\partial E[P]}{\partial \lambda} = 0$.

If $\frac{1}{2} > K > 1 - \rho$, the manager invests if $P \in \{p_u, p_H\}$ and

$$\begin{aligned}\frac{\partial E[P]}{\partial \lambda} &= \frac{1}{2(1+\beta)}\left(-y_L - (1-\tilde{\rho})(y_H - y_L)\right) \\ &= \frac{(y_H - y_L)}{2(1+\beta)}\left(K - (1-\tilde{\rho})\right) > 0\end{aligned}$$

where the last inequality holds, because $K > 1 - \rho > 1 - \tilde{\rho}$ in this case.

If $\rho > K > \frac{1}{2}$, the manager invests if $P = \{p_H\}$ and

$$\begin{aligned}\frac{\partial E[P]}{\partial \lambda} &= \frac{1}{2(1+\beta)}\left(\tilde{\rho}(y_H - y_L) + y_L\right) \\ &= \frac{(y_H - y_L)}{2(1+\beta)}\left(\tilde{\rho} - K\right) > 0\end{aligned}$$

where the last inequality holds, because $\tilde{\rho} > \rho > K$ in this case. □