Disclosing to informed traders*

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Abstract

We develop a model in which a firm's manager can voluntarily disclose to privately-informed investors. We show that, in equilibrium, the manager only discloses sufficiently favorable news. If the manager is known to be informed but disclosure is costly, the probability of disclosure increases with market liquidity and the stock trades at a discount relative to expected cash flows. However, when investors are uncertain about whether the manager is informed, disclosure can decrease with market liquidity, and the stock can trade at a premium relative to expected cash flows. Moreover, contrary to common intuition, ex-ante public information can *crowd in* more voluntary disclosure.

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Voluntary disclosures by firms account for nearly two-thirds of the return variation created by firm-level public announcements (Beyer, Cohen, Lys, and Walther (2010)). A large empirical literature studies how these disclosures relate to market outcomes, such as liquidity and average returns, which depend on investor information. Yet existing theory is largely silent on how such voluntary disclosures affect trade based on private information and, conversely, how such information affects a firm's propensity to disclose.[1] Moreover, understanding these interactions is important for policy. The impact of regulations that affect the availability and quality of public information about firms depends on how they affect firms' incentives to voluntarily disclose complementary information and investors' incentives to trade on their private information.[2]

We develop a model of voluntary disclosure in which a firm's price is determined through trade among privately-informed risk-averse investors and noise traders. With some probability, the firm's manager is informed about its cash flows, which are normally distributed. Before trading begins, the manager chooses whether to publicly disclose this information at a cost to the firm, with the goal of maximizing the firm's expected price. In addition to the manager's decision, investors use their private information and the price signal to update their beliefs when trading the stock. Importantly, the manager anticipates that even when he chooses not to disclose, the stock price reflects noisy information about cash flows as a result of informed trading.

In equilibrium, the manager follows a threshold strategy: he discloses information if and only if it is sufficiently favorable. As a result, when the manager does not disclose, investors know that he may be concealing bad news, which causes their beliefs to be asymmetric. This implies that the traditional, "linear-normal" approach (e.g., Hellwig (1980)) to solving for an equilibrium price, which is linear in investors' private information and noise trade, cannot be applied. Instead, we build on the approach developed by Breon-Drish (2015) to characterize the financial market equilibrium. We show that when there is no disclosure, the equilibrium price is a non-linear, noisy signal of fundamentals that aggregates investors' beliefs about

[1]Specifically, our focus is on voluntary, or discretionary, disclosures in which the manger chooses whether to disclose information after observing its realization, but cannot commit to a disclosure policy ex-ante. See Leuz and Verrecchia (2000a), Balakrishnan, Billings, Kelly, and Ljungqvist (2014), Boone and White (2015) and Jayaraman and Wu (2020) for the empirical relation between disclosure and liquidity, and Boone, Floros, and Johnson (2016), Lev and Penman (1990), Jiang, Xu, and Yao (2009), Zhou and Zhou (2020) for the relation between disclosure and returns.

[2]Even in the absence of regulation, managers often voluntarily provide extensive information. Firms provide qualitative discussions, forecasts, and non-GAAP earnings to help investors predict future outcomes that are not captured by mandatory financial reports. For instance, despite the absence of regulation requiring ESG reporting, among the S&P 500, 78% provide ESG reports and 36% of these are audited (Kwon, Welsh, Lukomnik, and Young (2018)). In fact, Ross (1979) argues that given firms' incentives to disclose information voluntarily, mandatory disclosure regulation is neither necessary nor desirable.

both the cash flows and the likelihood of manager being informed.

A key takeaway from our analysis is that the underlying economic friction driving non-disclosure plays a critical role in determining market outcomes.[3] We focus on two widely studied and economically important benchmarks. The "costly disclosure" benchmark considers the case where the manager is known to be informed but voluntary disclosure is costly, as in Verrecchia (1983). The "probabilistic information" benchmark assumes the manager may be uninformed with some probability but there are no disclosure costs (e.g., Dye (1985), Jung and Kwon (1988)). We show that the nature of the equilibrium, including the interactions among voluntary disclosure, liquidity and expected returns, differ radically across these benchmarks.

In the costly disclosure benchmark, we find that there always exists a unique threshold equilibrium. Moreover, the likelihood of disclosure increases with improvements in market liquidity, that is, an increase in noise trading volatility, or a decrease in investors' information precision. Conditional on non-disclosure, the firm's expected cash flows exceed its expected price, even when its per-capita supply is zero. This undervaluation is consistent with a negative relation between voluntary disclosure of idiosyncratic, proprietary information and a firm's cost of capital, as documented in, e.g., Boone et al. (2016).

In this benchmark, because the manager is known to be informed, investors know that the manager must have observed bad news when he does not disclose. Following non-disclosure, investor beliefs about future cash flows are negatively skewed so that investors are exposed to "downside" risk when holding the stock. Consistent with the documented evidence of asymmetric liquidity (e.g., Avramov, Chordia, and Goyal (2006), Johnson and So (2018)), this asymmetry in payoffs causes noise trader sales to have a larger price impact than noise trader purchases. Consequently, the firm is priced at a discount on average, even when the per-capita supply of the stock is zero. Finally, when investors' private information is noisier or noise trade is more volatile, investors face greater uncertainty following non-disclosure, which increases this price discount and, consequently, increases the manager's incentive to disclose.

In the probabilistic information benchmark, we show that a unique threshold equilibrium exists as long as investors' information is sufficiently imprecise. Furthermore, in stark contrast to the costly disclosure benchmark, the likelihood of disclosure can decrease with liquidity, and the stock can trade at a premium relative to expected cash flows conditional on non-disclosure. These differences arise because non-disclosure does not necessarily indi-

[3] In the absence of such frictions, firms would always disclose their information to investors, regardless of the news they possess. Intuitively, if they did not disclose, investors would infer that their news corresponds to the worst possible outcome – see the discussion of the disclosure principle in Dye (1985).

cate that the manager is hiding bad news – he may simply be uninformed. Thus, following non-disclosure, investors use their information to update their beliefs about both whether the manager was informed and, if he was informed, the news he observed. This causes the likelihood of disclosure to increase with the precision of investor information and, consequently, decrease with liquidity. Intuitively, when investors have more precise private (or price) information, they are better able to detect whether the manager is informed. This increases the manager's incentive to disclose when he has bad news since, otherwise, the non-disclosure price more closely reflects this news.

Moreover, investors face "upside" risk: conditional on non-disclosure, there is some probability that the firm's cash flows are very high (and the manager is uninformed) even if the price is relatively low, which makes selling the stock risky for investors. As a result, investors may demand greater price compensation to absorb noise trader buying than to absorb noise trader selling. We show that this asymmetry can lead to a price *premium*, whereby the firm's expected non-disclosure price can be higher than its expected cash flows. This offers a potential explanation for the puzzling empirical evidence showing firms that strategically refrain from certain types of disclosure, such as guidance, earn *lower* abnormal returns (Jiang et al. (2009) and Zhou and Zhou (2020)).

Finally, we explore how ex-ante public information affects voluntary disclosure and overall market informativeness. When investors lack private information, prior work typically finds that public information "crowds out" voluntary disclosure. Intuitively, more public information reduces investor uncertainty, which attenuates the negative inference they draw from non-disclosure. This, in turn, reduces the manager's incentives to disclose.

With privately-informed investors, we find that there are two additional effects. First, more public information leads investors to trade less intensely on their private signals, and can make the non-disclosure price *less* informative about fundamentals. This *substitution channel* tends to increase the manager's incentives to disclose.[4] Second, more public information decreases investor uncertainty, thereby reducing the wedge between the (non-disclosure) price and expected cash flows – we refer to this as the *valuation channel*. The valuation channel increases the average non-disclosure price in the costly disclosure benchmark, which discourages disclosure. However, it can reduce the average non-disclosure price in the probabilistic information benchmark, thereby encouraging disclosure.

We show that in the costly disclosure benchmark, the substitution channel dominates the valuation channel when disclosure costs are high and investors' private information is

[4] As we discuss in Section V, this substitution channel is consistent with the evidence in Brown and Hillegeist (2007) and Jayaraman and Wu (2019) that annual report disclosure quality and segment reporting, respectively, are negatively associated with informed trade.

precise.[5] As a result, more public information "crowds in" voluntary disclosure under these conditions. On the other hand, in the probabilistic information benchmark, more public information mitigates the overvaluation following non-disclosure. When the public signal is sufficiently precise, we show this can dominate the substitution channel, again leading public information to crowd in voluntary disclosure.

Our analysis demonstrates that one must identify the underlying friction driving non-disclosure to understand the relation between voluntary disclosure and market outcomes (e.g., liquidity, expected returns, and price informativeness). In Section VI, we propose approaches that may be useful in doing so. In some empirical settings, it is immediately apparent which friction is at play. For instance, redactions in contract disclosures, withholding information about segment-level performance, and non-disclosure of details about filed patents are instances in which the firm evidently has information, but chooses not to disclose it. On the other hand, for firms with secure market power, the proprietary costs to disclosure may be negligible, so that non-disclosure may be primarily attributable to a lack of information.

The rest of the paper is organized as follows. Section I reviews the related literature. Section II presents the model and discusses our assumptions, and Section III presents the equilibrium characterization. Section IV discusses the implications of our analysis for the likelihood of disclosure and firm valuation. Section V introduces an ex-ante public signal to the benchmark model and characterizes when public information can "crowd in" voluntary disclosure. Section VI discusses approaches for identifying which frictions drive non-disclosure in a given setting, and then discusses our model's empirical predictions and policy implications. Section VII concludes, and proofs and extensions can be found in Appendix A and B, respectively.

I. Related Literature

Our paper contributes to two strands of literature: models of voluntary disclosure and models of privately-informed investors. The literature on voluntary disclosure, starting with Jovanovic (1982), Verrecchia (1983), and Dye (1985), typically models financial markets in a stylized manner, assuming that investors are uninformed, risk neutral, or both.[6] There are some notable exceptions. Bertomeu, Beyer, and Dye (2011) and Petrov (2016) analyze set-

[5] When disclosure costs are large, the manager is indifferent between disclosing and not when his signal is very high. Thus, a reduction in the informativeness of the non-disclosure price causes a large drop in the non-disclosure price this manager expects, which increases his incentives to disclose.

[6] Examples of voluntary disclosure models with risk-averse, but uninformed traders include Verrecchia (1983), Cheynel (2013), Jorgensen and Kirschenheiter (2015), and Dye and Hughes (2018).

tings in which there is a single risk-neutral informed trader, while Einhorn (2018) considers trade based on private information only when non-disclosure is completely uninformative. Almazan, Banerji, and Motta (2008) endogenize the manager's incentives to use cheap talk communication when facing a market with risk-neutral, informed investors.

In contrast, analysis of disclosure in the context of privately-informed investors has largely focused on either non-strategic disclosure or settings in which the manager can commit, ex-ante, to a public signal with chosen precision (see Goldstein and Yang (2017) for a recent survey).[7] To the best of our knowledge, our paper is the first to study voluntary disclosure to a market of heterogeneously-informed, risk-averse investors when the manager cannot commit to a disclosure strategy. A key step is to allow investors to learn from prices in an environment where the price does not have a standard "linear-normal" form. We build on the insights of Breon-Drish (2015) to overcome this challenge: as in his paper, we show that there exists a unique equilibrium in which the price is a generalized linear function of a noisy signal about fundamentals.[8]

The common intuition in the existing literature is that prior public information and voluntary disclosure are substitutes, especially when they are about the same underlying fundamental shocks (e.g., Verrecchia (1990), Bertomeu, Vaysman, and Xue (2019)).[9] Our analysis suggests that these two types of information may instead be complementary when investors are privately informed. Existing work has documented alternative economic channels to generate a similar relation. Friedman, Hughes, and Michaeli (2020, 2022) show that these information sources may be complements when firms experience a discrete gain should investors' expectations exceed a cutoff. Einhorn (2005) find that certain correlation structures between public information and voluntary disclosure lead them to be complements. Frenkel, Guttman, and Kremer (2020) find that disclosure by an external party may crowd in firm disclosure when the external party and the firm possess information with correlated probabilities.

Our finding that the firm's expected price can differ from its expected cash flows even in the absence of "traditional" risk premia (e.g., when the aggregate supply of the asset is zero) is similar to existing results in the literature. Albagli et al. (2021) consider a setting

[7]Examples of models in which the firm can ex-ante commit to a disclosure policy include Xiong and Yang (2021), Schneemeier (2019), and Cianciaruso, Marinovic, and Smith (2020). More generally, Diamond (1985), Kurlat and Veldkamp (2015), Banerjee, Davis, and Gondhi (2018) and Goldstein and Yang (2019) study how public disclosures affect the extent to which investor information is reflected in prices.

[8]Other papers that have considered rational expectations equilibria with non-linear prices include Banerjee and Green (2015), Glebkin (2015), Albagli, Hellwig, and Tsyvinski (2020), Albagli, Hellwig, and Tsyvinski (2021), Chabakauri, Yuan, and Zachariadis (2021), Smith (2019), Lenkey (2020), and Glebkin, Malamud, and Teguia (2020).

[9]As Goldstein and Yang (2017) and Goldstein and Yang (2019) point out, this may not be the case if the two sources of information are about different components of payoffs.

with privately-informed, risk-neutral investors with position limits, while Chabakauri et al. (2021) consider an economy with privately-informed CARA investors. In both papers, when investors have non-normal priors, prices are non-linear in the asset's noisy supply. As in our model, this non-linearity implies that the expected price is generally not equal to the expected payoff. The generality of the approach in these papers allows them to explore the implications of private information for a rich set of asset classes, including stocks, bonds and options.

We complement this work by focusing on how a firm's voluntary disclosure decision endogenously leads to non-normal investor beliefs, which in turn result in a non-linear price. Conditional on non-disclosure, investors' beliefs about payoffs are given by a mixture of a normal and truncated normal distribution, where the truncation is determined by the firm's disclosure decision in equilibrium. Importantly, as our analysis highlights, the nature of the non-linearity (that is, whether or not the price is concave in underlying shocks) depends on the underlying friction driving non-disclosure.[10] Relating valuation to disclosure allows our model to speak to the large empirical literature that studies how voluntary firm disclosures affect their costs of capital (e.g., Botosan (2006)).

II. Model Setup

Our model features verifiable disclosure (e.g., Jovanovic (1982), Verrecchia (1983), Dye (1985)) in a market with privately-informed investors (e.g., Hellwig (1980)).

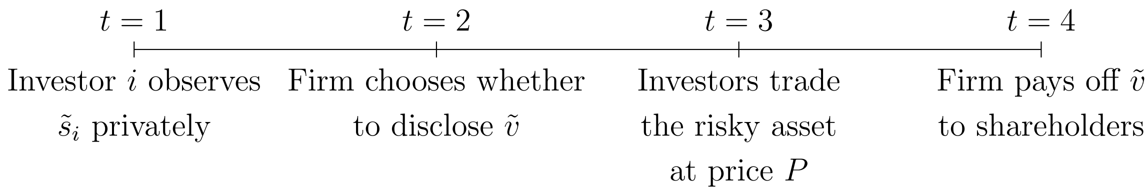
Payoffs. Investors trade in both a risky and a risk-free security. The gross return on the risk-free security is normalized to 1. The risky security is the stock of a firm, which pays terminal cash flows $\tilde{v}$ that are normally distributed with mean 0 and variance σ_v^2, that is, $\tilde{v} \sim N\left(0, \sigma_v^2\right)$. We normalize the mean of cash flows to zero without loss of generality. We assume that there are noise/liquidity traders who submit demands of $\tilde{z} \sim N\left(0, \sigma_z^2\right)$. The aggregate supply of the risky asset is $\kappa \geq 0$.

Preferences and information. There is a continuum of investors indexed by $i \in [0,1]$. Each investor i is endowed with initial wealth W_0, and exhibits CARA utility with risk-tolerance τ over terminal wealth W_i, where:

$$W_i = W_0 + D_i(\tilde{v} - P),$$

[10] Since the price is globally concave in the costly disclosure benchmark, our results on undervaluation follow from an argument in the spirit of Jensen's inequality, as in the existing literature. However, the price is neither globally concave nor convex in the probabilistic information benchmark, so our analysis of this case provides a technical contribution relative to earlier work.

Figure 1: Timeline of events



and D_i denotes his demand for the stock. Investor i observes a private signal $\tilde{s}_i$ of the form:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i. \tag{1}$$

The error terms follow the distributions $\tilde{\varepsilon}_i \sim N\left(0, \sigma_\varepsilon^2\right)$ and are independent of all other random variables.

Disclosure decision. Prior to trade, the firm's manager privately observes $\tilde{v}$ with probability $p \in (0, 1]$. Thus, as in Dye (1985), the manager's information endowment is probabilistic. Conditional on being informed, the manager can verifiably disclose this information to the market, subject to a disclosure cost of $c \geq 0$ borne by the firm (e.g., a proprietary cost). The manager aims to maximize his expectation of the equilibrium price. Moreover, if the manager does not learn $\tilde{v}$, he is unable to credibly convey this lack of information to the market.

Note that our model allows for either or both standard disclosure frictions: a disclosure cost and a random information endowment. To prevent "unravelling," we assume that at least one of these two frictions is present, that is, at least one of $c > 0$ and $p < 1$ holds. For specific results, we will focus on the two benchmarks from the literature: (i) $c > 0$ and $p = 1$, which we refer to as the costly disclosure benchmark, and (ii) $c = 0$ and $p < 1$, which we refer to as the probabilistic information benchmark.

The timing of events is summarized in Figure 1. At date $t = 1$, investor i observes his private signal $\tilde{s}_i$. At $t = 2$, if informed, the manager chooses whether to disclose $\tilde{v}$. Conditional on disclosure, the price at date $t = 3$ is completely determined by the disclosed information. Conditional on non-disclosure, investors use their private signals and the information in prices to choose their demands and the price is determined by market clearing. Finally, the firm pays off $\tilde{v}$ to shareholders at $t = 4$.

A. Discussion of assumptions

Our benchmark analysis makes a number of simplifying assumptions for analytical tractability.

Perfect verifiable disclosure. The assumption that disclosure is verifiable, as opposed to manipulable, is common in the literature. Einhorn and Ziv (2012) shows that the possibility of costly manipulation does not qualitatively affect analyses of verifiable disclosure; hence we rule it out for parsimony. Note we also assume the manager observes the value of the firm perfectly. The essential assumption that lends tractability to our analysis is that the manager has incremental information to the market; qualitatively similar results hold when the manager's signal is noisy.

Public information. Our benchmark analysis focuses on how the presence of investors' private information affects voluntary disclosure. In Section V, we extend the model to allow for an ex-ante public signal and study how the precision of public information affects the probability of voluntary disclosure in our setting. As we shall see, the presence of privately-informed investors can have important implications for whether public information crowds out or crowds in voluntary disclosure. In Appendix A, we further extend the benchmark model to allow for both an ex-ante public signal and an ex-post public signal, and show that our equilibrium characterization remains qualitatively the same.

Noise trade. As usual in models of informed trade, noise trade ensures that the equilibrium price does not fully reveal the firm's value. In our model, it also causes the manager to face uncertainty regarding the market reaction should he *refrain from disclosure*, and thus the relative payoffs to disclosing and not disclosing. This complements related work that considers situations in which managers face uncertainty about the market reaction *should they disclose* (Suijs (2007)), or audience preferences (Bond and Zeng (2021)). Note further that our model also accommodates the case in which the firm's disclosure decision influences the extent of noise trade. This might occur if, say, a fraction of liquidity traders times their trades until after uncertainty is resolved and thus avoids trading when the firm withholds its news. As we will see, noise trade only plays a pertinent role when the firm does not disclose. Thus, our model can capture this case if we interpret the level of noise trade as its level conditional on non-disclosure. Finally, in Appendix B, we show that our results are qualitatively similar if the noise in prices arises due to hedging demands of the informed investors.

III. Equilibrium

We focus on a class of equilibria that is both intuitive and commonly studied in voluntary disclosure models.

Definition 1. *A* **threshold equilibrium** *is characterized by a threshold $T \in \mathbb{R}$ such that the manager discloses if and only if he is informed and $\tilde{v} > T$.*

In classical disclosure models, any equilibrium must take this form, as the manager's payoff given non-disclosure is constant and his payoff given disclosure increases in $\tilde{v}$. However, it is less clear that all equilibria must take this form in our model: not only does the manager's payoff given disclosure depend upon $\tilde{v}$, but so too does his payoff given non-disclosure (through investors' trading behavior).

We can show that in any equilibrium, the manager discloses sufficiently large realizations and withholds sufficiently low realizations of $\tilde{v}$.[11] This rules out equilibria such as those in Clinch and Verrecchia (1997) and Kim and Verrecchia (2001), in which the manager discloses exclusively extreme or moderate values. However, we have not been able to either establish existence or rule out equilibria consisting of disjoint disclosure sets that are bounded from below.

To characterize a threshold equilibrium, our initial focus is on deriving the firm's price when the manager does not disclose; denote this event by ND. In contrast to standard models without private information, this price depends upon the firm's value through investors' private signals. As we will see, only the average investor signal, $\int s_i di$ influences price, which, given that there is a continuum of investors, simply equals v. Thus, let $P_{ND}(v, z; T)$ denote equilibrium price given non-disclosure when the firm's value is $\tilde{v} = v$, noise trade is $\tilde{z} = z$, and the market believes that the threshold above which the manager discloses is T.

A. Market pricing

Given the asymmetric nature of the manager's disclosure behavior in a threshold equilibrium, the absence of a disclosure leaves investors with a non-normal posterior. This implies that

[11] Equilibria in which the manager discloses upon observing $\tilde{v}$ *below* some threshold T are easily ruled out: if the manager followed such a strategy, the firm's price when he does not disclose would be no less than T, for otherwise there would exist an arbitrage opportunity. Moreover, in any equilibrium, the firm's price conditional on disclosure is simply $\tilde{v} - c$. Thus, the manager would prefer to deviate, refraining from disclosure when they observe $\tilde{v} < T + c$. Likewise, in any equilibrium, the manager always discloses upon observing sufficiently high $\tilde{v}$. Intuitively, if the manager did not disclose upon observing $\tilde{v} > T$, then the firm's price conditional on non-disclosure would be bounded above by $\hat{T} = \max(0, T)$ (since, conditional on the manager not being informed, the expected cash flow is zero). However, this implies that when the manager observes $\tilde{v} > \hat{T} + c$, he would prefer to deviate to disclosing.

there does not exist an equilibrium in which $P_{ND}(v, z; T)$ is a linear function of v and z. We solve for the equilibrium by applying and extending the techniques developed in Breon-Drish (2015). In particular, we conjecture and verify the existence of a "generalized" linear equilibrium in which, rather than a linear function, the price is a continuous, monotonic transformation of a linear function of the firm's value v and noise trade z:[12]

$$P_{ND}(v, z; T) = G\left(v + \beta z; T\right), \tag{2}$$

where $G\left(x; T\right)$ is a strictly increasing, smooth function of x.

The key feature of such an equilibrium is that, just as in a linear equilibrium, investor i can infer a "truth-plus-noise" signal $\tilde{s}_p = \tilde{v} + \beta\tilde{z}$ from the price, so that:

$$\tilde{s}_p|\tilde{v} \sim N(\tilde{v}, \sigma_p^2), \quad \text{where} \quad \sigma_p^2 = \beta^2\sigma_z^2. \tag{3}$$

This characterization allows for a tractable calculation of investors' posterior beliefs given their private signals and the information in price. In particular, investors' updated beliefs $\tilde{v}$ given their price and private signals are again normal with mean and variance:

$$\tilde{\mu}_i \equiv \mathbb{E}[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}\left(\frac{\tilde{s}_i}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}\right), \tag{4}$$

$$\sigma_s^2 \equiv \mathrm{var}[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}. \tag{5}$$

To complete the derivation of the equilibrium price, we follow a series of steps, which are outlined in detail in the appendix. First, taking as given the form of price in expression (2), we derive each investor's demand as a function of their information set, which includes not only their private signal $\tilde{s}_i$ and the signal contained in the price, $\tilde{s}_p$, but also the knowledge that the firm has not disclosed. Note that the manager does not disclose when either they are uninformed or they observe $\tilde{v} \leq T$. Thus, investors' beliefs given non-disclosure are the mixture of a normal distribution and truncated normal distribution, with mean and variance parameters given in expressions (4) and (5). Next, we apply the market-clearing condition to solve for the equilibrium price as a function of β. Finally, we solve for β to ensure the price is consistent with the conjecture in (2) and verify that the price is monotonic in $\tilde{s}_p$.

The following proposition characterizes the resulting equilibrium. In stating this result,

[12]In particular, we apply the results in Proposition 2.1 of the Online Appendix of Breon-Drish (2015). Note our framework fits into the exponential family of distributions that is necessary to apply these results, as we show in the proof of Proposition 1. Breon-Drish (2015) also demonstrates that the generalized linear equilibria we consider here are unique among the class of equilibria in which price is a continuous function. We abstract from equilibria with discontinuous prices as considered by Pálvölgyi and Venter (2015).

we let $\phi(x)$ and $\Phi(x)$ denote the density and distribution function of a standard normal distribution, and we let $h(x) \equiv \frac{\phi(x)}{\Phi(x)}$ denote the inverse-Mills ratio.

Proposition 1. *In a threshold equilibrium with threshold $T \in \mathcal{R}$, when the manager refrains from disclosure, there exists a unique equilibrium in the financial market. In this equilibrium, the firm's price equals:*

$$P_{ND}(v, z; T) = \frac{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) P_I(v, z; T) + (1-p) P_U(v, z)}{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) + 1 - p}, \tag{6}$$

where:

$$P_U(v, z) \equiv \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z - \kappa), \tag{7}$$

$$P_I(v, z; T) \equiv P_U(v, z) - \sigma_s h\left(\frac{T - P_U(v, z)}{\sigma_s}\right), \tag{8}$$

$\beta = \frac{\sigma_\varepsilon^2}{\tau}$, *and* $\sigma_p^2 = \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2}$. *Moreover,* $P_{ND}(v, z; T)$ *is strictly increasing in* v, z, *and* T.

To develop intuition for the equilibrium non-disclosure price, it is helpful to first consider the components P_U in equation (7) and P_I in equation (8) separately (we suppress the dependence of P_{ND}, P_U and P_I on (v, z, T) in what follows). First, note that P_U captures the non-disclosure price if the manager were known to be *uninformed* – that is, when $p = 0$. In this case, the absence of disclosure is entirely uninformative and the equilibrium price may be derived as in standard models of trade in which the firm's value is normally distributed and investors possess CARA utility (e.g., Hellwig (1980)). Specifically, the optimal demand for investor i is given by:

$$D_i(\mu_i, P) = \tau \frac{\mu_i - P}{\sigma_s^2}, \tag{9}$$

where P is the equilibrium price. Applying market clearing, the price then equals the average investor's posterior mean plus a risk-adjustment term that is proportional to noise traders' excess demand $z - \kappa$:

$$P = \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z - \kappa) \equiv P_U. \tag{10}$$

By substituting the expressions for investor beliefs in (4)-(5), one can easily verify that P_U is a linear function of the price signal $s_p \equiv v + \frac{\sigma_\varepsilon^2}{\tau} z$. Importantly, note that the price P_{ND} depends upon investors' private information signals only through P_U. As a result, the price aggregates these signals in precisely the same manner as in traditional noisy rational expectations models without voluntary disclosure. Thus, the signal investors glean from the non-disclosure price is identical to the one that arises in Hellwig (1980). Likewise, the price depends upon investors' risk tolerance τ only through P_U.

On the other hand, for a fixed T, P_I denotes the non-disclosure price when the manager is known to be *informed* – that is, when $p = 1$. A familiar special case is one in which the manager is informed and investors are risk-neutral and uninformed (as captured by letting $p = 1$ and $\sigma_\varepsilon \to \infty$). In this case, the non-disclosure price is simply equal to the firm's expected cash flows given that $\tilde{v} < T$, which reduces to (e.g., Verrecchia (1990)):

$$P_I = \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right] = \mathbb{E}\left[\tilde{v}\right] - \sigma_v h\left(\frac{T - \mathbb{E}\left[\tilde{v}\right]}{\sigma_v}\right). \tag{11}$$

Equation (8) illustrates that the non-disclosure price when both investors and the manager have information P_I combines features of expressions (10) and (11). Specifically, this price equals the firm's expected cash flows given $\tilde{v} < T$, where the mean parameter of the payoff reflects the price that would arise were the manager uninformed and the variance parameter reflects investors' variance parameter given their signals.

Finally, expression (6) shows that in the general case, the firm's price is a weighted average of the price if the manager was known to be uninformed (i.e., P_U) and the price if the manager was known to be informed but did not disclose their information (i.e., P_I). The weights reflect the perceived likelihood that the manager is informed, presuming again that the prior mean over firm value is P_U. Thus, in contrast to the Dye (1985) model, these weights depend upon the noise trader demand z and the investors' private signals: a more optimistic signal indicates that the absence of a disclosure more likely resulted from an uninformed manager, as opposed to an informed manager who observed negative news.

B. Disclosure decision

We next analyze the manager's disclosure choice. The manager who observes $\tilde{v} = v$ discloses if and only if his payoff given disclosure exceeds the expected non-disclosure price conditional on $\tilde{v} = v$, that is,

$$B(v; T) \equiv v - c - \mathbb{E}\left[P_{ND}|\tilde{v} = v\right] \geq 0. \tag{12}$$

A threshold equilibrium is incentive compatible if the manager is more inclined towards disclosure when his observed signal $\tilde{v} = v$ is greater. This would clearly be the case if the non-disclosure price was independent of the firm's value, as in voluntary disclosure models without informed trade. However, in our setting the non-disclosure price reflects the firm's value through investors' trading behavior, which may cause this condition to be violated.

An intuitive sufficient condition for there to exist a threshold equilibrium is that the non-disclosure price reacts to a marginal change in the firm's value only partially, that is, $\frac{\partial P_{ND}}{\partial v} < 1$. This ensures that the manager is more inclined towards disclosure as his signal rises, that is, $B(v;T)$ increases in v. While this condition may seem natural given that investors observe noisy signals, as explained below, it is in fact possible that the price responds more than one-for-one with a change in the value of the firm. To determine when this is the case, we next characterize $\frac{\partial P_{ND}}{\partial v}$.

Lemma 1. *In a threshold equilibrium with threshold $T \in \mathcal{R}$, when the manager does not disclose, the price response to a marginal change in the firm's value satisfies:*

$$\frac{\partial P_{ND}}{\partial v} = var\left[\tilde{v}\,|ND, \tilde{\mu}_j = P_U\right]\left(var^{-1}\left[\tilde{s}_j|\tilde{v}\right] + var^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{13}$$

The price response to a shift in $\tilde{v}$ is equal to the posterior variance perceived by an investor whose posterior mean parameter $\tilde{\mu}_j$ is equal to P_U, multiplied by the combined precision of their private signal and the signal they receive from price. To gain intuition, consider the case when the manager is known to be uninformed ($p = 0$), as in standard noisy rational expectations models with normal distributions. In this case,

$$\frac{\partial P_U}{\partial v} = \frac{\partial}{\partial v}\left[\int_0^1 \mu_i di + \frac{\sigma_s^2}{\tau} z\right].$$

Upon substituting for μ_i and applying Bayes' rule for normal distributions, this reduces to:

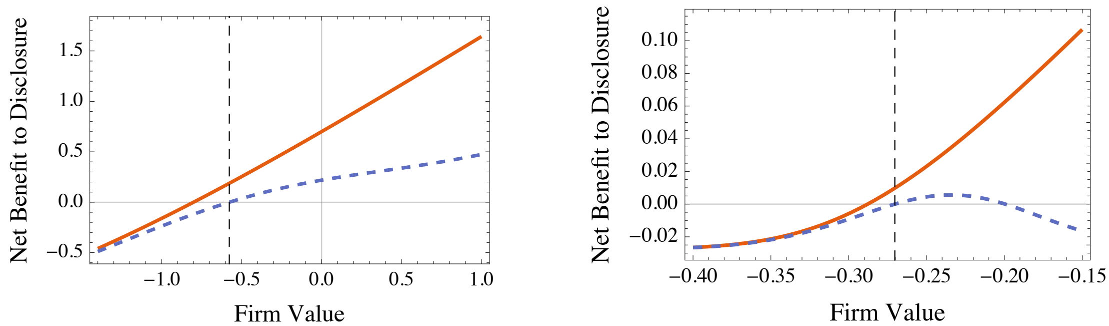
$$\text{var}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]\left(\text{var}^{-1}\left[\tilde{s}_j|\tilde{v}\right] + \text{var}^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{14}$$

One can verify that this is always less than one, and so the price responds only partially to an increase in firm value. Intuitively, the price response is driven by the product of investors' posterior uncertainty and the total precision of their information signals.

When the manager is informed with some probability, the posterior variance that appears in expression (14), $\text{var}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]$, is replaced by $\text{var}\left[\tilde{v}\,|ND, \tilde{\mu}_j = P_U\right]$, which conditions on the event of non-disclosure (for a "representative" investor whose signals $\tilde{s}_j$ and $\tilde{s}_p$ lead them to have the posterior belief $\tilde{\mu}_j = P_U$). Therefore, when the manager may be informed, the event of non-disclosure changes the marginal reaction to the firm's information by adjusting investors' posterior variance. When the manager is *always* informed (i.e., $p = 1$), observing non-disclosure reveals that $\tilde{v} < T$. Because this strictly reduces the possible outcomes for the firm's value, investors' posterior variances fall short of the prior variance, and thus the marginal price response $\frac{\partial P_{ND}}{\partial v}$ falls short of the response when the manager is uninformed.

Figure 2: Existence and non-existence of a threshold equilibrium.

The plot shows the net benefit to disclosure $B(v;T) = v - c - \mathbb{E}[P_{ND}|\tilde{v} = v]$ as a function of the observed value v for $p = 1$ (solid) and $p = 0.95$ (dashed) respectively. The vertical dashed line in each panel corresponds to the conjectured threshold T. The left panel illustrates an example of low investor information precision ($\sigma_\varepsilon = 0.75$) while the right panel illustrates the case of high information precision ($\sigma_\varepsilon = 0.2$). The remaining parameters are $c = 0.025$ and $\sigma_z = 1$.



(a) Low Private Information ($\sigma_\varepsilon = 0.75$) (b) High Private Information ($\sigma_\varepsilon = 0.2$)

Thus, this response is less than 1.

In contrast, when $p < 1$, non-disclosure may cause investors' posterior variance to *exceed* the prior variance (see Dye and Hughes (2018)). Intuitively, in this case, investors face an additional source of uncertainty given non-disclosure: they do not know whether the manager was informed with bad news or uninformed, outcomes that have very different implications for firm value. Nevertheless, we show in the next proposition that if the combined precision of investors' private and price signals are not excessively large, the sensitivity of the non-disclosure price to v is bounded above by 1, which, as previously mentioned, ensures the existence of a threshold equilibrium. Furthermore, we show that, when a threshold equilibrium exists, it is unique.

Proposition 2. *Suppose that either* $p = 1$ *or* $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1 + \frac{1}{2}p(1-p)\right)\right]^{-1}$, *where* $\sigma_p^2 = \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2}$. *Then, there exists a unique threshold equilibrium in which* T *is given by:*

$$T - c = \mathbb{E}[P_{ND}(T, \tilde{z})]. \tag{15}$$

Figure 2 illustrates this result. For a conjectured threshold equilibrium with threshold T, the figure plots the net benefit to a manager with cash flow v of disclosing relative to not disclosing, $B(v;T)$. Note that in a threshold equilibrium, we must have that $B(v;T) \geq 0$ for $v \geq T$ and $B(v;T) < 0$ for $v < T$. When investors' signals are sufficiently noisy (panel (a)), the net benefit $B(v;T)$ is always increasing in v and so there exists a threshold equilibrium.

This is characterized by the point where $B(v;T) = 0$.

However, when investors' signals are sufficiently precise and they face uncertainty about whether the manager is informed (i.e., $p < 1$), the net benefit from disclosure can decrease with v (as shown in the dashed line in panel (b)). The reason is that, in this case, investors' beliefs about whether the manager is informed, and thus their expectations of firm value, change rapidly as their signals rise above the disclosure threshold T. Intuitively, investors know that, given non-disclosure, if $v > T$, the manager could not have been informed. Thus, as investors' beliefs increase past T, they increasingly believe that the manager did not disclose because he is uninformed, and so their beliefs about cash flows improve very quickly. This implies that the net benefit to disclosure falls in v for v close to the conjectured disclosure threshold T.[13]

IV. Implications

This section analyzes our model's implications. We assume the parameters are such that the threshold equilibrium we characterize in Proposition 2 exists. Section A characterizes how the probability of disclosure depends on the underlying parameters of the model, including the precision of investor information and the volatility of noise trading. Section B analyzes the firm's valuation relative to its expected cash flows in our setting.

A. *Probability of disclosure*

We begin by providing some standard results on how the probability of disclosure depends on underlying parameters, which establish the continuity between our model and canonical models of voluntary disclosure.

Proposition 3. *The probability of disclosure decreases in the disclosure cost c, increases in the probability that the manager is informed p, and increases in the aggregate supply κ of the firm.*

The first two comparative statics (i.e., with respect to c and p) are intuitive and aligned with prior literature (Jung and Kwon (1988), Verrecchia (1983)). The net benefit of disclosure decreases in c, thereby reducing the probability of disclosure. As the probability the manager is informed p increases, the market penalizes non-disclosure more strongly, which

[13] Specifically, note that if only firms with v in the interval where $B(v;T) > 0$ in panel (b) disclose, this does not constitute a threshold equilibrium, since some firms with $v > T$ do not disclose. More generally, as discussed in footnote 11, there cannot exist equilibria in which firms only disclose if v lies in a single bounded interval.

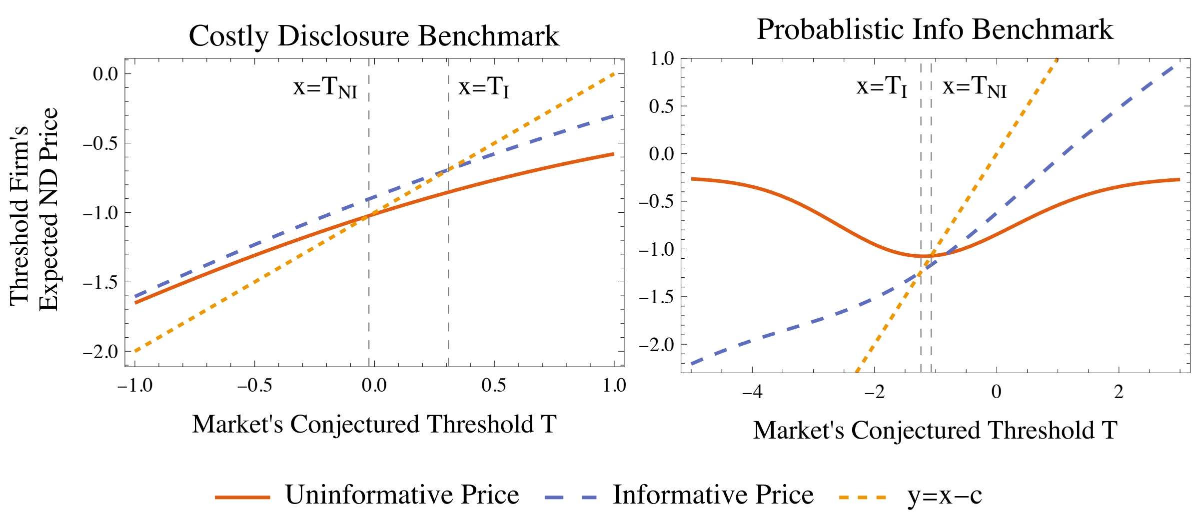
incentivizes more disclosure. Next, when the aggregate supply of the firm κ grows, the risk premium associated with non-disclosure increases, as investors must bear a larger amount of aggregate risk, on average. This lowers the non-disclosure price, which increases the manager's proclivity to disclose.

We next examine how the extent of private information and noise trade affect the frequency of disclosure. These predictions are novel to our model and speak to the empirical relationship between liquidity and firms' voluntary disclosure decisions.

Proposition 4. *The probability of disclosure can increase or decrease with noise trade volatility (σ_z) and the precision of investors' private information ($1/\sigma_\varepsilon^2$).*
(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), the probability of disclosure increases in noise trading volatility and decreases in investors' information precision, when c is sufficiently large.
(ii) In the probabilistic information benchmark (i.e., $p < 1$, $c = 0$), the probability of disclosure decreases in noise trading volatility and increases in investors' information precision, when investors' private information is sufficiently precise.

Figure 3: Information in price and the probability of disclosure

The figure depicts how information in price influences the manager's incentives to disclose. The plots compare the payoffs to disclosure and non-disclosure that accrue to the manager who observes $\tilde{v} = T$, when investors conjecture that the manager discloses when he observes $\tilde{v} > T$. The parameters are $\sigma_v = \sigma_z = \tau = 1$; $\kappa = 0.25$; $c = 1$ in the costly disclosure benchmark and $p = 0.95$ in the probabilistic info benchmark. We set $\sigma_\varepsilon = 1.5$ for the informative price case (dashed blue line) and $\sigma_\varepsilon = \infty$ for the uninformative price case (solid line).



To understand the proposition, note either a decrease in noise trading volatility or an increase in investors' private information precision raises investors' overall information precision given non-disclosure (i.e., $1/\sigma_\varepsilon^2 + 1/\sigma_p^2$), and thus makes the price more informative. As Figure 3 illustrates, an increase in price informativeness has opposing effects on disclosure incentives across the two benchmarks.

Specifically, the figure compares the expected non-disclosure price for the "threshold firm," that is, $\mathbb{E}[P_{ND}(T, \tilde{z}; T)]$, to the payoff from disclosure $T - c$ (dotted, yellow line) for each benchmark. The solid red line corresponds to the expected non-disclosure price when the price is uninformative (i.e., $\sigma_\varepsilon = \infty$), while the dashed blue line corresponds to the expected non-disclosure price when the price is informative (i.e., $\sigma_\varepsilon = 1.5$). Recall that an equilibrium requires the "threshold firm" to be indifferent between not disclosing and disclosing:

$$\mathbb{E}[P_{ND}(T, \tilde{z}; T)] = T - c.$$

As such, the respective equilibria are determined by the points of intersection, which are indicated by the gray dashed lines $x = T_I$ and $x = T_{NI}$.

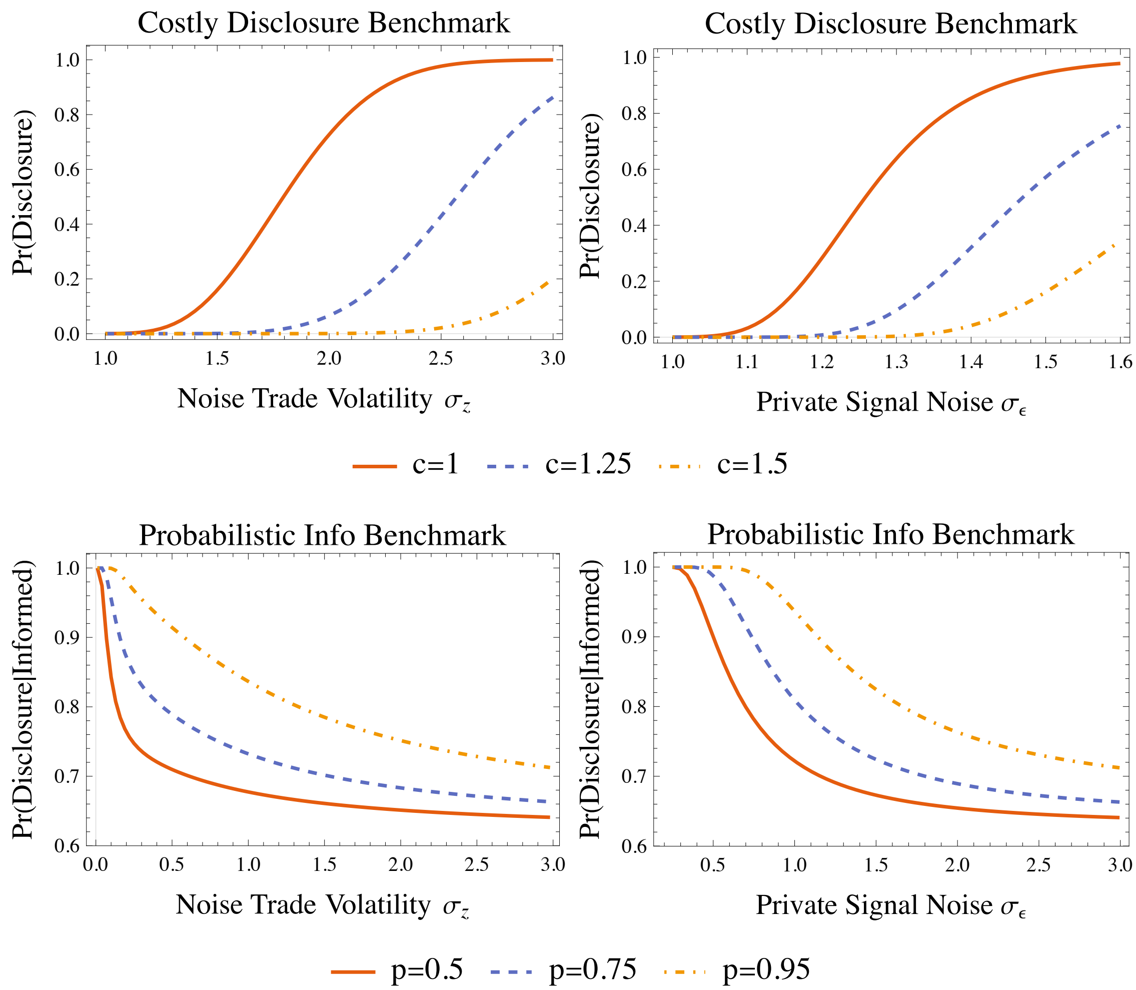
In the costly disclosure benchmark, since $c > 0$, note that the threshold firm $v = T$ has a higher value than the expected non-disclosure price $\mathbb{E}[P_{ND}(T, \tilde{z}; T)]$. Moreover, as the non-disclosure price becomes more informative, it better reflects this firm's actual value $v = T$, and so is higher on average (i.e., the dashed line is higher than the solid line). This decreases the manager's incentives to disclose, and so the equilibrium threshold increases with price informativeness, that is, $T_I > T_{NI}$.

In contrast, in the probabilistic information benchmark, more investor information increases the average non-disclosure price $\mathbb{E}[P_{ND}(T, \tilde{z}; T)]$ if and only if T is large (the blue curve single-crosses the red curve from below). This reflects a core difference between the costly disclosure and probabilistic information benchmarks. Because the non-disclosing firm may be uninformed, the threshold firm $v = T$ has a *lower* value than the average non-disclosing firm when T is low. As a result, increasing price informativeness *decreases* the expected non-disclosure price that the threshold type anticipates. This increases the manager's incentive to disclose, and so the equilibrium threshold decreases with price informativeness, that is, $T_I < T_{NI}$.

Figure 4 provides an illustration of Proposition 4. Notably, while the analytical proofs rely on limiting arguments, numerical exploration suggests that these results extend to a large range of parameter values. The result highlights that the underlying friction generating non-disclosure plays a qualitatively important role in determining the relation between disclosure and the drivers of liquidity. Existing empirical analyses of this relation typically focus on the impact that changes in disclosure have on market liquidity (e.g., Leuz and Verrecchia

Figure 4: Probability of disclosure versus determinants of liquidity

The figure plots the probability of disclosure conditional on the manager being informed as a function of noise trading volatility and investors' private information precision. The parameters in the costly disclosure (probabilistic info) benchmarks are set to $\sigma_\varepsilon = \sigma_z = 1$, $\sigma_v = 3$, $\tau = 1$ and $\kappa = 0.1$ ($\sigma_\varepsilon = \sigma_z = \sigma_v = 3$, $\tau = 1$ and $\kappa = 0.1$).



(2000b), Balakrishnan et al. (2014)). As we expand upon in Section B, our analysis suggests that, in addition, anticipated changes in liquidity (e.g., via an increase in noise trading volatility or a reduction in investors' information quality) can impact managers' incentives to disclose.

B. *Firm valuation*

We now characterize the firm's valuation, that is, its expected price, and the link between voluntary disclosure and the cost of capital. Following the prior literature that studies disclosure's impact on the cost of capital in CARA-normal models, we refer to the cost of capital as expected future dollar returns, that is, $\mathbb{E}\left[\tilde{v} - \tilde{P}\right]$ (see Goldstein and Yang (2017)). In addition to being of independent interest, this result also plays a role in understanding how changes in public information quality affect voluntary disclosure, which we discuss in the next section.

Proposition 5. *Conditional on non-disclosure, the firm's expected value generally differs from its expected price.*
(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), the firm's expected value exceeds its expected price, that is,

$$\mathbb{E}\left[P_{ND}|ND\right] < \mathbb{E}\left[\tilde{v}|ND\right].$$

Thus, voluntary disclosure is negatively associated with the firm's cost of capital.
(ii) In the probabilistic information endowment benchmark (i.e., $p < 1$, $c = 0$), when investors' private signal precision $1/\sigma_\varepsilon$ and the aggregate supply κ are sufficiently low, the firm's expected price exceeds its expected value, that is,

$$\mathbb{E}\left[P_{ND}|ND\right] > \mathbb{E}\left[\tilde{v}|ND\right].$$

Thus, voluntary disclosure can be positively associated with the firm's cost of capital.

While this result is stated in terms of relative valuation conditional on non-disclosure, it also applies to the firm's *unconditional* valuation (i.e., $\mathbb{E}\left[\tilde{v} - \tilde{P}\right]$). The reason is that there is no misvaluation if the manager discloses.

To gain intuition for Proposition 5, first consider the costly disclosure benchmark. Panel (a) of Figure 5 illustrates that the non-disclosure price in this case is concave in noise trader demand (z).[14] The concavity of the firm's price is rooted in the investors' risk preferences; the intuition is most clear when comparing noise trader purchases to sales. When noise traders purchase a sufficient quantity of the firm's shares, investors short the stock and demand a boost in price to do so. However, in the costly disclosure benchmark, non-disclosure implies that firm value cannot be too high, and so the downside from shorting is limited, that is, their payoffs are positively skewed. In contrast, when noise traders sell, investors must bear the risk of being long. In this case, their downside is unlimited, that is, their payoffs are

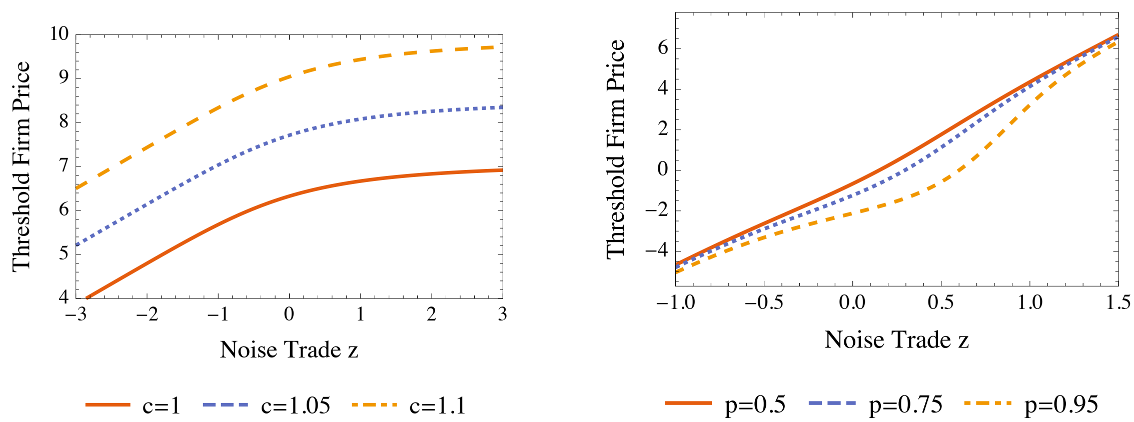
[14]For concreteness, we focus on the price given $\tilde{v} = T$, though the plot looks similar for other levels of v.

negatively skewed. Thus, investors require greater price compensation to provide liquidity to noise traders when they sell than when they purchase shares.[15]

This result is analogous to those in Albagli et al. (2021) and Chabakauri et al. (2021). As in these papers, the concavity in prices in noise trader demand implies that noise traders depress the firm's price. Note the negative impact that noise traders have on price augments the classic risk premium, so that the firm's expected price falls short of its expected cash flows.

Figure 5: Curvature of the Price

The figure plots the firm's non-disclosure price when its value is equal to the disclosure threshold, that is, $\tilde{v} = T$, as a function of noise trade z. The left (right) plot depicts the case in which $p = 1$ and $c > 0$ ($p < 1$ and $c = 0$). The parameters in the left (right) plot are set to $\sigma_v = 3$, $\sigma_\varepsilon = 1$, $\sigma_z = 1.25$, $\tau = 1$ and $\kappa = 0$ ($\sigma_v = \sigma_\varepsilon = \sigma_z = 3$, $\tau = 1$ and $\kappa = 0$).



(a) Costly Disclosure Benchmark (b) Probabilistic Endowment Benchmark

Next, consider the probabilistic information benchmark. Panel (b) of Figure 5 illustrates that, in this case, the non-disclosure price is convex in noise trader demand for intermediate levels of this demand. This result can also be traced back to investor preferences for skewness. In this case, the firm's cash flows are not bounded above: non-disclosure can arise either because the manager is informed but the cash flows are low, or because the manager is uninformed and the cash flows are unbounded. As a result, payoffs can exhibit positive skewness conditional on non-disclosure – even though the price is low, there is a possibility that the payoff is very high. This implies that investors demand a large price compensation (increase) for short positions when noise traders buy.

When the distribution of noise trade is concentrated on the region in which the price function is convex, the expected price exceeds expected cash flows given non-disclosure, so

[15]The reason is that investors with CARA preferences exhibit prudence (since $u''' > 0$), and thus have a distaste for negatively-skewed payoffs (e.g., Eeckhoudt and Schlesinger (2006)).

that voluntary disclosure is *positively* associated with the cost of capital. However, formally proving part (ii) of Proposition 5 is more nuanced, since the price is not globally concave or convex.[16] The condition that κ is sufficiently small in Proposition 5 (ii) ensures that standard risk-premium effects do not overwhelm the overvaluation that noise trade creates. As in traditional noisy rational expectations models, an increase in the aggregate supply of the risky asset lowers the expected price.

It is worth noting that the excess valuation results in Proposition 5 arise even in the absence of investor private information, since they rely only on investors' risk preferences and their (perceived) conditional distribution of payoffs, given non-disclosure, and noise trading. In fact, as we illustrate in Figure 6, the magnitude of over-/under-valuation increases with the noise in investors' private information, since this exposes investors to greater uncertainty about the firm's payoffs.

Specifically, Figure 6 plots how excess valuation conditional on non-disclosure varies with model parameters for the costly disclosure benchmark (solid line), the probabilistic information benchmark (dashed line), and a setting in which both frictions are present (dotted line). Consistent with intuition, the plots show that the magnitude of misvaluation increases with prior uncertainty σ_v, noise in investors' private information σ_ε, and noise trading volatility σ_z. Moreover, the relation between valuation and the firm's supply κ is in line with the standard risk premium effect. Since risk-averse investors have to bear more aggregate risk in equilibrium as κ increases, firm valuation decreases with κ. For sufficiently large κ, the standard risk premium ultimately dominates the overvaluation that noise trade creates in the probabilistic information benchmark.

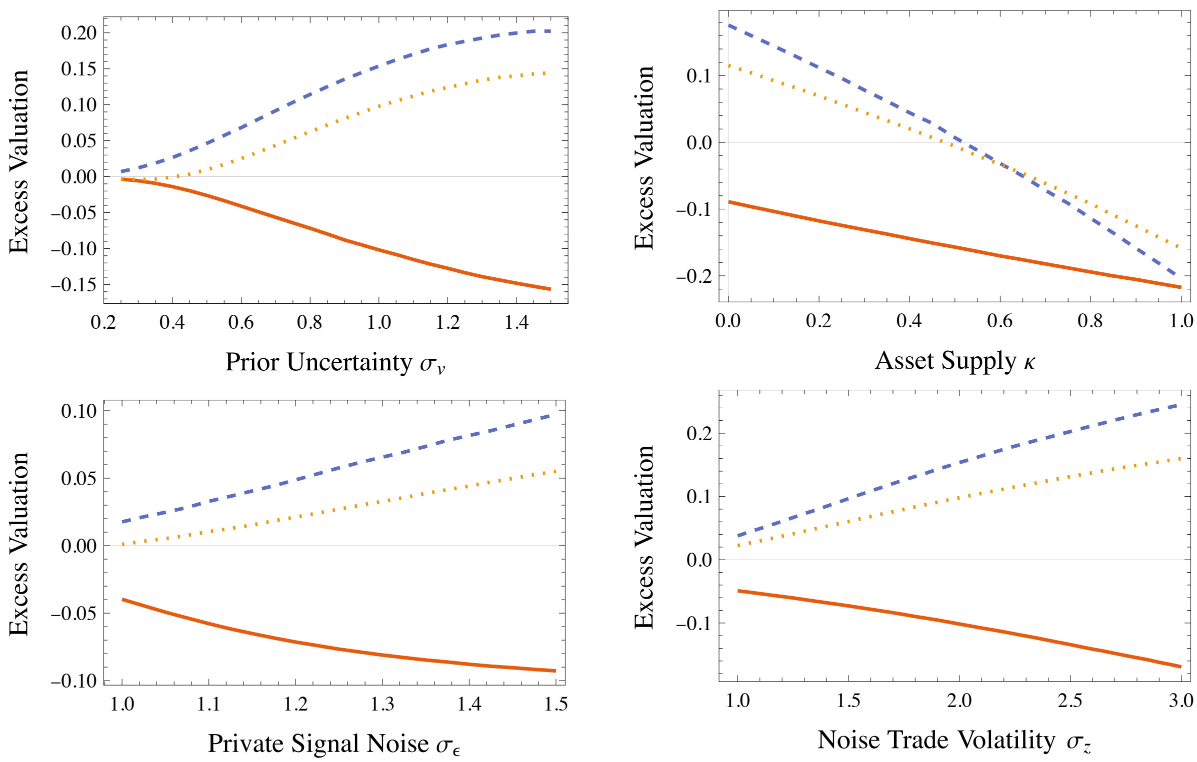
Given that the firm's price and cash flows are aligned when the firm discloses, Proposition 5 immediately implies that voluntary disclosure is negatively associated with its cost of capital in the costly disclosure benchmark, but may be positively related with its cost of capital in the probabilistic information benchmark. Importantly, these results hold even when the firm's supply is zero, so that the disclosure does not have any direct effect on the risk premium, which is consistent with the disclosure concerning a firm's idiosyncratic cash flows.[17] This result contrasts with analyses of mandatory disclosure, which find that

[16]Our proof of the above result relies on an argument based upon the "minimum principle" of Guttman, Kremer, and Skrzypacz (2014) to demonstrate that noise trade tends to raise valuations. Intuitively, the minimum principle implies that, absent private information, the equilibrium disclosure threshold minimizes the non-disclosure price over all potential thresholds. Moreover, the price expression (6) reveals that, on average, noise trade has the same effect on price as creating random variation in the disclosure threshold. This can lead to higher prices on average and, consequently, overvaluation.

[17]Intuitively, a low value of κ should apply broadly to individual stocks when (i) voluntary disclosure is about firm-specific information and (ii) investors are well-diversified, since any individual stock is a small component of an investor's portfolio. However, formally establishing this is beyond the scope of the current paper, since it involves solving a model of voluntary disclosure with multiple firms.

Figure 6: Excess Valuation

The figure plots the firm's expected price less its expected cash flows conditional on non-disclosure $\mathbb{E}[P_{ND} - \tilde{v}|ND]$ as a function of σ_v and κ. The solid line corresponds to the costly disclosure benchmark ($c = 0.75$, $p = 1$), the dashed line corresponds to probabilistic information benchmark ($c = 0$, $p = 0.95$), and the dotted line corresponds to a setting with both frictions ($c = 0.25$, $p = 0.95$). Other parameters are given by $\sigma_v = 1$, $\sigma_z = \sigma_\varepsilon = 2$, $\tau = 2$, $\kappa = 0.1$.



disclosure has no impact on the cost of capital when the firm is in zero supply (e.g., Hughes, Liu, and Liu (2007), Goldstein and Yang (2017)), and can help to reconcile mixed evidence in different empirical settings; we discuss this further in Section VI.

V. Public Information and the Probability of Disclosure

The impact of public information on voluntary disclosure is critical to assessing the efficacy of disclosure regulations, as it determines their effect on the overall level of information available to market participants. As we discuss in Section VI, an extensive empirical literature has studied this relationship, but documents mixed evidence. The ambiguous nature of this evidence, and in particular the finding that, in some cases, public information is associated with greater voluntary disclosure, is at odds with traditional models of disclosure. These models suggest that public information either crowds out disclosure (e.g., Verrecchia (1990)) or leaves it unchanged (e.g., Jung and Kwon (1988)). Moreover, in standard models with informed investors, disclosure is usually modeled as a non-discretionary commitment to release a public signal to the market. In these settings, better external information also tends to crowd out disclosure when both types of information are about the same dimension of fundamentals.[18]

We next study how public information affects voluntary disclosure when investors also have access to private information, and show that considering such private information can help to explain why public information may, in some cases, lead to more voluntary disclosure. To do so, we extend our benchmark model to allow for a mandatory (non-strategic), ex-ante public signal $\tilde{y}$ that is revealed on date $t = 1$:

$$\tilde{y} = \tilde{v} + \tilde{\eta}, \tag{16}$$

where $\tilde{\eta} \sim N(0, \sigma_\eta^2)$ is independent of all other random variables, including whether the manager is privately informed about cash flows.

Note because the disclosure arrives at date $t = 1$, the manager observes the outcome of the public signal before making his disclosure decision.[19] While this assumption is made primarily for tractability, empirical evidence suggests that this is a realistic feature of prominent voluntary disclosures. For instance, Beyer et al. (2010) find that, on average, management earnings forecasts generate significantly larger price reactions than earnings. This suggests that managers are aware of much of the information in forthcoming earnings when deciding whether to provide a forecast. In Appendix A, we consider how introducing a public signal that arrives after the voluntary disclosure decision is made affects our results. We show that our equilibrium characterization extends naturally to this case and find numerically that our results in this section are robust.

We begin by generalizing our equilibrium characterization incorporate the public signal.

Proposition 6. *Suppose that either $p = 1$ and/or $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}$ is sufficiently small, and fix a realization of $\tilde{y} = y$. Then, there exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T(y)$. The equilibrium threshold satisfies:*

$$T(y) - c = \mathbb{E}\left[P_{ND} | \tilde{v} = T(y), \tilde{y} = y\right], \tag{17}$$

[18] See, e.g., Diamond (1985). See also Goldstein and Yang (2017), which discusses when this finding might not hold in such models.

[19] Our results would not change if the manager knows the public signal's outcome when disclosing, but this signal arrives after the voluntary disclosure.

where:

$$P_{ND}(v,z,y) = \frac{p\Phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right) P_I\left(v,z,y\right) + (1-p)\, P_U\left(v,z,y\right)}{p\Phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right) + 1 - p}, \tag{18}$$

$$P_U(v,z,y) \equiv \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z-\kappa), \tag{19}$$

$$P_I(v,z,y) \equiv P_U(v,z,y) - \sigma_s h\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right), \tag{20}$$

and investor beliefs are given by:

$$\tilde{\mu}_i \equiv \mathbb{E}[\tilde{v}|\tilde{y},\tilde{s}_i,\tilde{s}_p] = \left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left(\frac{\tilde{y}}{\sigma_\eta^2}+\frac{\tilde{s}_i}{\sigma_\varepsilon^2}+\frac{\tilde{s}_p}{\sigma_p^2}\right), \tag{21}$$

$$\sigma_s^2 \equiv var[\tilde{v}|\tilde{y},\tilde{s}_i,\tilde{s}_p] = \left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}, \tag{22}$$

where $\sigma_p^2 = \frac{\sigma_\varepsilon^4\sigma_z^2}{\tau^2}$. *Moreover, the equilibrium threshold* $T(y)$ *satisfies:*

$$T(y) = T(0) + \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right], \tag{23}$$

and is increasing in y.

This proposition clarifies the public signal's impact on the equilibrium outcomes. In particular, equation (23) shows that the equilibrium threshold increases with expected cash flows given the public signal, and thus rises in the signal. Intuitively, when expected cash flows are greater, the price given non-disclosure rises, which discourages disclosure. However, we next show that, as in standard models (e.g., Einhorn (2005)), the realization of such a signal has no impact on the *probability of disclosure*. An increase in the signal not only raises the threshold, but also increases the likelihood that the firm's value exceeds a given threshold; these two forces have precisely offsetting impacts on the probability of disclosure.

Lemma 2. *Fix a realization of the public signal* $\tilde{y}=y$. *Then, the probability of disclosure in equilibrium* $\Pr(\tilde{v}<T(y)|y)$ *does not depend on the realization* y *of the public signal.*

Given the above observation, the following result characterizes the impact of public information on the probability of voluntary disclosure in our setting.

Proposition 7. *More public information can* ***crowd-in*** *voluntary disclosure:*

(i) In the costly disclosure benchmark (i.e., $p=1$, $c>0$*), an increase in the precision of the public signal increases the probability of disclosure when* $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2} > \frac{1}{var[\tilde{v}|\tilde{y}]}$ *and disclosure is sufficiently expensive.*

(ii) In the probabilistic information benchmark (i.e., $p < 1$, $c = 0$), when investors' private information is not too precise, there exists a range of values of public information precision such that an increase in the precision of the public signal increases the probability of disclosure.

To gain intuition, it is helpful to focus on the case in which $\kappa = 0$ and express the non-disclosure price as follows:[20]

$$P_{ND}(v,z,y) = P_U(v,z,y) - \frac{p\sigma_s\phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right)}{p\Phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right)+1-p}. \tag{24}$$

That is, the non-disclosure price can be written as the price were the manager uninformed, $P_U(v,z,y)$, less a "discount" that reflects investors' inference from non-disclosure. Since the probability of disclosure is independent of $\tilde{y}$, we can focus on the case in which $\tilde{y} = 0$. The effect of public information on voluntary disclosure is primarily determined by how it impacts the threshold firm's expected price when it does not disclose, $\mathbb{E}\left[P_{ND}\left(T(0), \tilde{z}, 0\right)\right]$.[21] Equation (24) demonstrates that this expected price depends on σ_η through its impact on σ_s and P_U. Let $\Pi(\sigma_s, P_U)$ denote the non-disclosure price $P_{ND}\left(T(0), \tilde{z}, 0\right)$ as a function of these two components. Moreover, let $\Pi_{\tilde{z}=0}(\sigma_s, P_U) = P_{ND}\left(T(0), 0, 0\right)$ denote the price in the hypothetical alternative in which $\tilde{z}$ were fixed at zero. Then, differentiating and adding and subtracting terms, we arrive at:

$$\begin{aligned}\frac{\partial\mathbb{E}\left[P_{ND}\left(T(0),\tilde{z},0\right)\right]}{\partial\sigma_\eta} &= \frac{d\mathbb{E}\left[\Pi(\sigma_s,P_U)\right]}{d\sigma_\eta} \\ &= \underbrace{\frac{\partial\Pi_{\tilde{z}=0}}{\partial\sigma_s}\frac{\partial\sigma_s}{\partial\sigma_\eta}}_{\text{Standard channel}} + \underbrace{\mathbb{E}\left[\frac{\partial\Pi}{\partial P_U}\frac{\partial P_U\left(T(0),\tilde{z},0\right)}{\partial\sigma_\eta}\right]}_{\text{Substitution channel}} + \underbrace{\frac{\partial\mathbb{E}\left[\Pi-\Pi_{\tilde{z}=0}\right]}{\partial\sigma_s}\frac{\partial\sigma_s}{\partial\sigma_\eta}}_{\text{Valuation channel}}.\end{aligned} \tag{25}$$

This equation reveals that better public information affects the firm's incentives to disclose via three channels.

Standard Channel. The first channel, which is captured by $\frac{\partial\Pi_{\tilde{z}=0}}{\partial\sigma_s}\frac{\partial\sigma_s}{\partial\sigma_\eta}$, is directly analogous to standard disclosure models: greater public information reduces investor uncertainty,

[20] When $\kappa > 0$, an additional channel arises: greater public information lowers the risk premium given non-disclosure, thereby raising the non-disclosure price (as in Dye and Hughes (2018)). This tends to push towards public information crowding out disclosure. This does not change any of the intuition we provide below and is accounted for in our proofs.

[21] Because $\mathbb{E}\left[P_{ND}(T,\tilde{z},0)\right]$ increases in the disclosure threshold, the implicit function theorem implies that -1 times this derivative determines how the equilibrium disclosure threshold changes with respect to σ_η. However, the complete argument in the appendix further accounts for the fact that a change in public information quality also changes the likelihood the firm's value falls below a given threshold.

which attenuates the negative inference investors draw from non-disclosure (e.g., Verrecchia (1990)). This raises the non-disclosure price, discouraging disclosure. Equation (24) shows that, holding fixed P_U, σ_s affects P_{ND} purely through the non-disclosure "discount." Thus, $\frac{\partial \Pi_{\tilde{z}=0}}{\partial \sigma_s} \frac{\partial \sigma_s}{\partial \sigma_\eta}$ captures the impact of σ_η on this discount.[22]

Substitution Channel. Second, as public information improves, investors place relatively less weight on their private signals. We refer to this as the *substitution channel*. This is reflected in the model via $\mathbb{E}\left[\frac{\partial \Pi}{\partial P_U} \frac{\partial P_U(T(0),\tilde{z},0)}{\partial \sigma_\eta}\right]$. Intuitively, $P_U\left(T\left(0\right), \tilde{z}, 0\right)$ captures the aggregation of investors' beliefs that is reflected in the non-disclosure price, and $\frac{\partial \Pi}{\partial P_U}$ reflects how strongly the non-disclosure price varies in this statistic. Note that this substitution from private to public information makes the non-disclosure price P_{ND} less informative about the firm's value. According to Proposition 4, this increases the firm's incentive to disclose in the costly disclosure benchmark and decreases the firm's incentives to disclose in the probabilistic information benchmark.

Importantly, the substitution channel cannot arise in settings where investors do not have private information and, as such, is a distinctive feature of our analysis. Recent empirical research on the feedback effect provides indirect support for this channel. For instance, Jayaraman and Wu (2019) show that the introduction of SFAS 131, which required greater disclosure of segment level information by firms, led to a substantive decrease in the probability of informed trade for affected firms. Similarly, using the staggered implementation of EDGAR, Bird, Karolyi, Ruchti, and Truong (2021) and Goldstein, Yang, and Zuo (2022) argue that greater access to public firm level information led to "crowding out" of private information acquisition by investors, which in turn, affected firms' investment decisions (as evidenced by lower investment-price sensitivity). In our setting, this substitution from private to public information by investors affects firms' voluntary disclosure choices.

Valuation Channel. Finally, better public information reduces the degree of misvaluation in equilibrium – we refer to this as the *valuation channel*. Recall that misvaluation is driven by the asymmetric risk borne by investors when taking the other side of noise trader purchases versus sales. Thus, we can think of the misvaluation expected by the threshold firm as its expected non-disclosure price less the non-disclosure price it would receive in a hypothetical alternative where noise trade were instead fixed at zero, that is, $\mathbb{E}\left[\Pi - \Pi_{\tilde{z}=0}\right]$.[23] Figure 6

[22]It is worth noting that this channel arises in irrespective of whether investors are privately informed, and can arise even when investors are risk neutral when disclosure is costly (e.g., Verrecchia (1990)). However, it does not arise when investors are risk-neutral in the probabilistic information benchmark (see Jung and Kwon (1988), who show that prior uncertainty does not influence the probability of disclosure in this setting).

[23]This is only an approximate means of isolating the "valuation" channel in our model that is useful for

shows that misvaluation tends to increase with investor uncertainty, and as a result, it tends to decrease in the precision of public information.

The effect of this channel on the likelihood of voluntary disclosure depends on whether the non-disclosure price exhibits overvaluation or undervaluation. In the costly disclosure benchmark, the valuation channel *reduces undervaluation*, which reduces the benefit from disclosure for the firm. In contrast, the valuation channel can *reduce overvaluation* in the probabilistic information benchmark, and thus increase the firm's incentive to disclose information.

The overall impact of public information depends on the interaction of these channels. In the costly disclosure benchmark, the substitution channel dominates and thus public information crowds in voluntary disclosure when investor information is precise and disclosure costs are high. The condition on signal precisions is intuitive: investors' private and price information must be sufficiently precise (relative to the uncertainty given public information) to ensure that their signals play a significant role in determining the equilibrium price. Moreover, when disclosure costs are high, the disclosure threshold is high, so that investors' beliefs given non-disclosure, $\tilde{v}|\tilde{v} < T$, are approximately normal. As a result, the non-disclosure discount (i.e., the second term in equation (24)) approaches zero, and the non-disclosure price approaches the standard, linear price P_U. This causes both the standard channel (which is driven by $P_{ND} - P_U$) and the valuation channel (which is driven by the non-linearity of P_{ND}) to approach zero. In turn, this implies that the "crowding out" effect of these two channels is attenuated.

On the other hand, in the probabilistic information benchmark, we show that "crowding in" can arise due to the valuation channel. In particular, recall from Proposition 5 that the presence of noise trade causes the firm to be over-valued, that is, $\mathbb{E}[\Pi - \Pi_{\tilde{z}=0}] > 0$. Thus, better public information reduces overvaluation, which in turn increases the marginal firm's incentive to disclose. We show that this effect can dominate the standard and substitution channels when investors' private information is not too precise. Intuitively, when investors' private information is noisy, the substitution channel is muted, and, as shown in Figure 6, the degree of overvaluation is larger.

For comparison, our next result establishes sufficient conditions for crowding out to arise.

Proposition 8. *More public information can* ***crowd out*** *voluntary disclosure.*

(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), an increase in the precision of the public signal decreases the probability of disclosure when disclosure is sufficiently cheap

conveying intuition. To fully remove noise trade from the model, we would let $\sigma_z^2 \to 0$. However, this would not only remove any misvaluation, but would also render the price perfectly informative.

so that the probability of disclosure is more than $\frac{1}{2}$ *and/or when* $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} < \frac{1}{var[\tilde{v}|\tilde{y}]}$.

(ii) In the probabilistic information benchmark (i.e., $p < 1$, $c = 0$*), when investors' private information is sufficiently precise, an increase in the precision of the public signal decreases the probability of disclosure.*

The above result provides a natural analog to the sufficient conditions for crowding in from Proposition 7. In the costly disclosure benchmark, recall that we need high disclosure costs *and* sufficiently precise private information to ensure that the substitution channel dominates the standard channel and the valuation channel, and consequently, generates crowding in. Part (i) of the above result implies that relaxing either condition yields the opposite result.

Similarly, part (ii) implies that in the probabilistic information benchmark, public information crowds out voluntary disclosure when investors' information is sufficiently precise. This is because, as illustrated in Figure 6, overvaluation is small when investors' private information is very precise, and so the valuation channel is weak. Thus, the substitution channel dominates, so that more informative public information decreases voluntary disclosure.

It is worth noting that while the standard and valuation channels arise even in the absence of investor private information, the substitution channel does not arise when all investors are uninformed. This implies the following result.

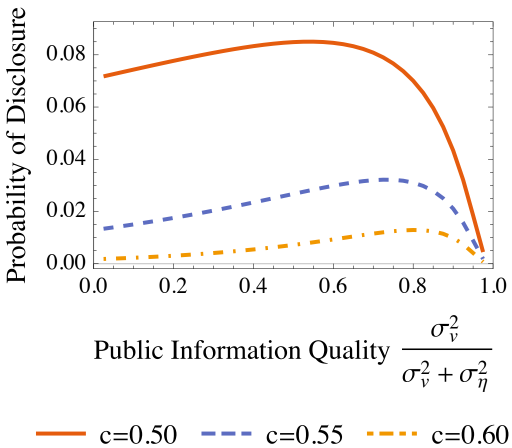
Corollary 1. *Suppose investors do not have private information, that is,* $\sigma_\varepsilon = \infty$.
(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$*), more public information always crowds out voluntary disclosure.*
(ii) In the probabilistic information benchmark (i.e., $p < 1$, $c > 0$*), more public information can crowd in or crowd out voluntary disclosure.*

In the costly disclosure benchmark, more public information reduces the manager's incentives to disclose through both the standard and valuation channels, since under-valuation decreases. This implies that in the absence of investor private information, crowding in cannot arise in this benchmark. However, in the probabilistic information benchmark, the absence of private information does not rule out either crowding in or crowding out. In this case, whether public information increases or decreases the likelihood of voluntary disclosure is primarily driven by the valuation channel. When the price exhibits overvaluation (e.g., when κ is low), public information tends to crowd in voluntary disclosure; when the price falls short of expected cash flows (e.g., when κ is relatively high), it tends to crowd out voluntary disclosure instead.

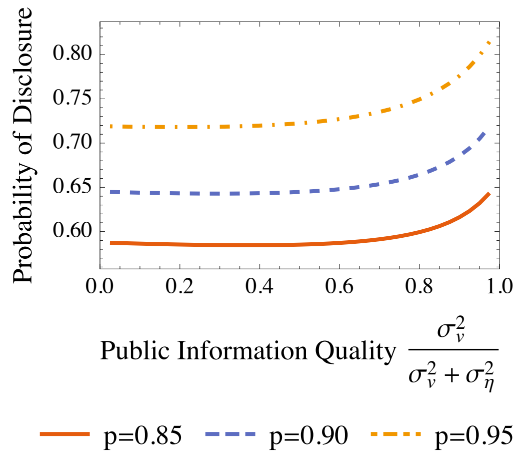
Figure 7 provides a numerical illustration of our results by plotting the probability of voluntary disclosure as a function of public information quality. Our measure of public

Figure 7: Probability of Disclosure vs. Information Quality

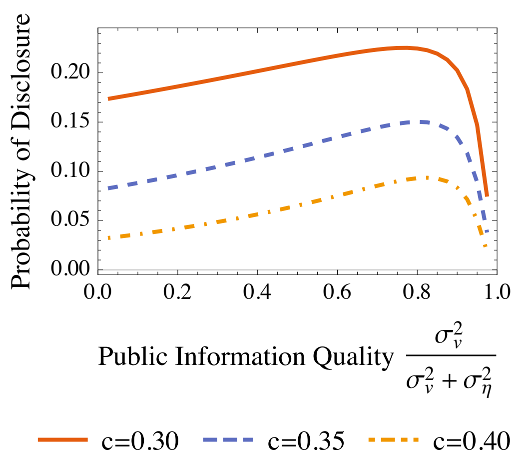
The figure plots the probability that the manager discloses in the costly disclosure and probabilistic endowment benchmarks as a function of public information quality, defined as the "signal-to-noise" ratio of the public signal $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$. Other parameters are $\tau = 1$, $\kappa = 0.1$ and $\sigma_v = 1.5$.



(a) Costly Disclosure Benchmark
($p = 1$, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)



(b) Probabilistic Endowment Benchmark
($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)



(c) Both Frictions
(p=0.85, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)

information quality, $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$ measures the "signal to noise" ratio of the public signal $\tilde{y}$ with respect to the fundamental $\tilde{v}$, that is, it measures $\frac{\text{cov}(\tilde{v},\tilde{y})}{\text{var}(\tilde{y})}$. Panel (a) illustrates that in the costly disclosure benchmark, public information crowds in voluntary disclosure when disclosure costs are sufficiently high and public information quality is relatively low, or equivalently, private information precision is relatively high. Panel (b) illustrates that, in the probabilistic information benchmark, the crowding in effect is quite robust, but strongest when public information quality is high. Finally, panel (c) suggests that when both frictions are in effect, crowding in can arise for a wide range of parameters. Together these plots suggests that public information crowding in voluntary disclosure is a robust feature of our setting, in contrast to traditional models of voluntary disclosure without privately-informed investors.

Ex-ante public information and overall market informativeness

We next consider how a change in ex-ante information quality influences overall market informativeness. We measure overall market informativeness as the posterior variance of payoffs conditional on the publicly available information, that is, $\mathbb{E}[\text{var}[\tilde{v}|\tilde{y},\tilde{P},\tilde{\Lambda}]]$, where $\tilde{\Lambda} \in \{D, ND\}$ denotes whether or not there is voluntary disclosure, and $\tilde{P} = P_{ND}$ when $\tilde{\Lambda} = ND$ and $\tilde{P} = v$ when $\tilde{\Lambda} = D$. This analysis is particularly useful from a policy perspective, because it speaks to how a change in mandatory disclosure affects the average amount of information available to an uninformed, rational investor.[24]

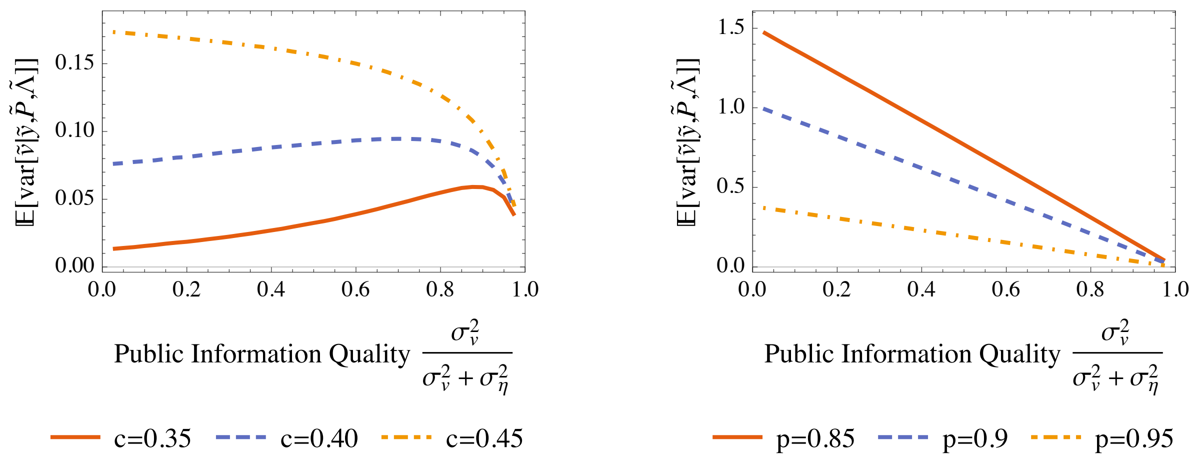
Figure 8 illustrates how overall market informativeness changes with public information quality. Panel (a) shows that better ex-ante public information *reduces* overall informativeness when disclosure costs are low and the public signal is not very precise. This implies that the crowding out effect of mandatory on voluntary disclosure can be sufficiently potent to cause disclosure mandates to be counterproductive. However, overall informativeness *increases* with public information quality in the costly disclosure benchmark when disclosure costs are sufficiently high or when public information quality is high.

In contrast, panel (b) shows that public information has a robust positive impact on overall informativeness in the probabilistic endowment benchmark, and has the largest impact when there is more uncertainty about whether the manager is informed (i.e., p is closer to 1/2). This is intuitive – the public signal is not only informative about the fundamental payoff v, but also helps reduce uncertainty about whether the manager is informed when

[24]We find similar results to those in this section when examining the information available to an *informed* investor, $\mathbb{E}[\text{var}[\tilde{v}|\tilde{s}_i,\tilde{y},\tilde{P},\tilde{\Lambda}]]$, as well as the *relative* uncertainty faced by an uninformed and informed investor, that is, $\mathbb{E}[\text{var}[\tilde{v}|\tilde{y},\tilde{P},\tilde{\Lambda}]] - \mathbb{E}[\text{var}[\tilde{v}|\tilde{s}_i,\tilde{y},\tilde{P},\tilde{\Lambda}]]$. Thus, our results speak to both regulatory objectives to reduce the uncertainty faced by investors at large, as well as to "level the playing field" among investors.

Figure 8: Overall Informativeness vs. Public Information

The figure plots the expected posterior variance $\mathbb{E}\left[\text{var}\left[\tilde{v}|\tilde{y},\tilde{P},\tilde{\Lambda}\right]\right]$ as a function of the amount of public information, defined as the "signal-to-noise" ratio of the public signal $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$. Other parameters are set to $\kappa = 0.1$, $\tau = 1$, and $\sigma_v = 1.5$.



(a) Costly Disclosure Benchmark ($p = 1$, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)

(b) Probabilistic Endowment Benchmark ($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)

there is no disclosure.[25] Taken together, these results highlight how changes in mandatory disclosure can have different effects on overall informativeness across firms, depending on the interaction between investors' private information and the firm's incentives to disclose.

VI. Empirical Predictions

In this section, we discuss the empirical predictions of our model in two steps. The key takeaway of our analysis is that the nature of the friction that drives non-disclosure is critical for understanding how voluntary disclosure interacts with liquidity, price impact, valuation and public information. Thus, we first suggest approaches to identify the underlying non-disclosure friction in Section A. Next, having identified the primary friction for a given firm, we highlight specific testable predictions of our model in Section B.

A. *Identifying the friction driving non-disclosure*

An important initial step in testing our model's implications is to identify the friction that leads a manager to refrain from disclosing in a given empirical setting. The existing empirical

[25] Recall that an informed manager discloses only when they possess sufficiently good news, and so a high realization of y together with non-disclosure is indicative of the manager not being informed.

literature has studied several settings in which it is readily apparent that managers possess information (i.e., $p = 1$), and so they fit into the costly disclosure benchmark we consider. For example, prior literature shows that firms frequently redact information from contracts that they are required to present in their SEC filings (Verrecchia and Weber (2006), Boone et al. (2016)). It is immediate that managers are aware of information they redact, and so this work argues that proprietary costs drive non-disclosure.

Berger and Hann (2007) study managers' tendency to withhold segment-level performance. As internal accounting systems enable managers to readily observe the break down of their performance into segment-level earnings, they argue that agency and proprietary costs lead managers to withhold in this setting. Relatedly, Gow, Larcker, and Zakolyukina (2021) find that managers sometimes refuse to answer questions during conference calls that ask for monetary amounts, locations, and times, and attribute this to an unwillingness to reveal proprietary information. Finally, prior literature studies firms' decisions to patent technologies that they are known to possess, as Regulation S-K requires them to disclose the presence (but not specifics) of such technologies (Glaeser (2018), Saidi and Zaldokas (2021)).

In other settings, consistent with our probabilistic information benchmark, disclosure costs may be minimal and it may be unclear to investors whether managers possess verifiable information. For instance, proprietary costs are likely to be negligible for firms who enjoy secure monopoly power, or for highly competitive industries, as these firms' performance is not relevant to their peers' production decisions. Relatedly, proprietary costs may be low for firms with highly-differentiated products. The pharmaceutical/bio-technology industry is a salient example. Firms in this industry invest in R&D, such as clinical trials, that if successful, provides them with monopoly power. Thus, disclosing positive outcomes likely does not impose competitive costs on these firms. Moreover, the outcome of such R&D often produces verifiable results with large implications for firm value that arrive at unknown times (Dobson (2000)). Hence, managers' information endowments in this industry at any point in time are likely unknown.[26]

B. Predictions conditional on non-disclosure

Having determined the primary friction driving non-disclosure (as discussed above), our analysis delivers new empirical predictions.

Market liquidity and the prevalence of voluntary disclosure. Proposition 4 predicts that the frequency of disclosure is negatively related to measures of illiquidity and price in-

[26] For example, a 2014 analysis found that 4 years after 400 randomly selected trials finished, 30% of them had not disclosed their results (Saito and Gill (2014), Reardon (2016)).

formativeness in the costly disclosure benchmark, but is positively related to these measures in the probabilistic information benchmark.[27] While most existing empirical work focuses on the impact of voluntary disclosure on liquidity, a small body of work studies how liquidity affects disclosure. Boone and White (2015) finds that index ownership, which one might interpret as raising market liquidity, leads managers to issue additional, more specific forecasts. Similarly, Jayaraman and Wu (2020) find that transitory, non-fundamental shocks are associated with more frequent capex forecasts. These studies provide suggestive evidence consistent with the costly disclosure benchmark, though our results call for additional analysis of the two-way interaction between liquidity and voluntary disclosure.

Valuation, disclosure, and the cost of capital. Proposition 5 implies that firms in which non-disclosure is driven by costs to disclosing tend to be under-valued and should generate higher expected returns. This is broadly consistent with the evidence from Boone et al. (2016), which shows that firms that redact information from their IPO filings tend to experience substantial under-pricing and higher costs of capital. As the manager is clearly known to be informed regarding the information they redact from a contract, this is clearly a case in which the manager's decision not to disclose is driven by proprietary costs.

On the other hand, firms in which non-disclosure is generated by uncertainty about whether the manager is informed may be over-valued and should generate lower average returns. This is in contrast to the common intuition from existing models that suggests *more* disclosure leads to a lower cost of capital (e.g., Dye and Hughes (2018)). However, it is consistent with existing empirical evidence showing firms that refrain from providing guidance or receive low analysts' disclosure scores earn *lower* expected returns, even after controlling for standard risk-factor exposures (Lev and Penman (1990), Jiang et al. (2009), Zhou and Zhou (2020)). Jiang et al. (2009) further shows that this helps to explain the "idiosyncratic volatility puzzle." The reason is that non-disclosing firms also experience higher volatility than disclosing firms. This finding is also consistent with our model: if the firm discloses, market liquidity sharply rises so that return volatility declines.[28]

Together with the link between disclosure and skewness discussed in Section B, these results also lead to a negative relation between idiosyncratic skewness and expected returns (e.g., Jiang et al. (2009), Conrad, Dittmar, and Ghysels (2013), Boyer and Vorkink (2014)). This is similar to the results in Albagli et al. (2021) and Chabakauri et al. (2021).

[27] Though imperfect, there are a number of empirical measures that can be used to capture illiquidity (e.g., Amihud (2002)) and price informativeness (e.g., D'avila and Parlatore (2020)).

[28] In our model, since the manager's disclosure reveals cash flows perfectly, volatility after disclosure is zero. In practice, since the manager's disclosure is likely to be noisy, we expect volatility after disclosure to be lower to the extent that disclosure reduces the uncertainty investors face about cash flows.

Impact of public information on voluntary disclosure. Regulators often motivate disclosure requirements as means to mitigate adverse selection across investors and "level the playing field." While a standard critique of such policies is that they "crowd out" voluntary disclosure by firms (e.g., Verrecchia (1990)), the empirical evidence is mixed. Some papers suggest that firms increase voluntary disclosures to mitigate reductions in external information quality (e.g., Balakrishnan et al. (2014), Guay, Samuels, and Taylor (2016), Barth, Landsman, and Taylor (2017)), but others argue that public information and voluntary disclosures are positively correlated (e.g., Francis, Nanda, and Olsson (2008), Bischof and Daske (2013), Kim and Ljungqvist (2021)).

Our analysis helps reconcile this evidence. As illustrated by Figure 7, for firms in which non-disclosure is driven by disclosure costs, mandatory disclosure complements voluntary disclosure when disclosure costs are high (so that voluntary disclosure is infrequent) and investor information is sufficiently precise. And when non-disclosure is driven by uncertainty about the manager's information, mandatory disclosure crowds in voluntary disclosure when investor information is sufficiently imprecise.

On the other hand, mandatory disclosure substitutes voluntary disclosure and can increase residual uncertainty when disclosure costs are low and managers are very likely to be informed. These settings can be readily identified: they correspond to situations in which managers are very likely to issue informative voluntary disclosures in the absence of regulation (as appears to be the present state of ESG reporting; see Kwon et al. (2018)).

VII. Conclusions

Standard voluntary disclosure models assume that investors do not have access to private information. We show that this assumption is an economically important restriction, and relaxing it has novel implications. A key takeaway of our analysis is that the friction driving non-disclosure has important implications for how investors' private information affects voluntary disclosure and the overall information content of prices. When disclosure costs drive non-disclosure, we show that probability of voluntary disclosure decreases with illiquidity and price informativeness and average prices are lower than expected cash flows, and voluntary disclosures are negatively associated with firms' costs of capital. On the other hand, when investors face uncertainty about whether the manager is informed, voluntary disclosure can increase with illiquidity and price informativeness, average prices can be higher than expected cash flows, and voluntary disclosures can be positively associated with firms' costs of capital.

Our analysis also has important implications for regulatory changes that affect the public information available to investors. Contrary to standard intuition, we show that ex-ante public information can "crowd in" more voluntary disclosure, especially when firms face high disclosure costs or when investors face substantial uncertainty about firm payoffs. As such, higher requirements for mandatory disclosures may actually increase voluntary disclosure by firms and improve overall informativeness, in contrast to the standard criticism against such regulations.

Our model is stylized and may be extended along several dimensions. For instance, investors and the manager are endowed with information in our model. It would be interesting to study how the interaction between disclosure and trade affects both parties' incentives to acquire information. In traditional models of costly disclosure, the manager usually prefers to commit not to acquire information (ex-ante) because disclosure is costly but has no impact on real decisions (e.g., investment). However, as we discuss in Section B, our analysis implies that managers may find it valuable to acquire information with some probability, since the possibility of voluntary disclosure can lead to overvaluation on average. Similarly, while a model of endogenous information acquisition by investors is not immediately tractable, we expect some of our results to extend to this setting. For instance, to the extent that more public information crowds out private information acquisition, it is likely to crowd in voluntary disclosure as in our current model.

Our model assumes that the manager cannot commit to a disclosure policy ex-ante. In a complementary paper, Cianciaruso et al. (2020) study the optimal disclosure policy with commitment. In the class of threshold strategies, they show that the firm prefers to commit to a "recognition" policy that involves disclosing bad news (below a threshold), but withholding good news. We expect the optimal disclosure policy to have a similar form in our setting with privately-informed investors so long as disclosure is not too costly.

Finally, we consider a model without real effects (e.g., production) or feedback effects. As an interesting extension, one could consider the possibility that managers use their disclosure policy to elicit information from the market and inform their investment choices, similar to Lassak (2020)'s analysis in a single-investor setting. Alternatively, one might consider how voluntary disclosure influences the incentives of managers to invest, as in Ben-Porath, Dekel, and Lipman (2018), when investors possess private information.

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A. Proofs

A. *Proof of Proposition 1*

To begin, as in the text, let $\sigma_p^2 = \text{var}[\tilde{s}_p|\tilde{v}] = \beta^2\sigma_z^2$, $\sigma_s^2 = \text{var}[\tilde{v}|\tilde{s}_j, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$, and $\tilde{\mu}_j = \mathbb{E}[\tilde{v}|\tilde{s}_j, \tilde{s}_p] = \sigma_s^2\left(\frac{\tilde{s}_j}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}\right)$. Moreover, let $g(x) \equiv \mathbb{E}[\tilde{v}|ND, \tilde{\mu}_j = x]$ denote investor j's conditional expectation of firm value given non-disclosure when $\tilde{s}_j$ and $\tilde{s}_p$ are such that $\tilde{\mu}_j = x$. This function plays a central role in the analysis and thus we begin by characterizing its properties.

Lemma A.1. *The function $g(x)$ satisfies:*

$$g(x) = \frac{\int_{-\infty}^{\infty} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\,dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\,dv}, \tag{26}$$

where $f(v|ND)$ denotes the PDF of firm value given non-disclosure. Furthermore,

$$g'(x) = \frac{var[\tilde{v}|ND, \tilde{\mu}_j = x]}{\sigma_s^2} > 0. \tag{27}$$

Proof of Lemma A.1. To start, we derive investor j's posterior distribution over $\tilde{v}$ given $\tilde{\mu}_j$ and the event of non-disclosure ND, whose density function we denote by $f(v|ND, \mu_j)$. Note that $\tilde{\mu}_j|v \sim N\left(\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v, \sigma_s^4\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)\right)$, and so:

$$\begin{aligned}
f(v|ND, \mu_j) &\propto f(v, \mu_j|ND) \\
&= f(\mu_j|v)\, f(v|ND) \\
&\propto \exp\left[-\frac{\left(\mu_j - \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v\right)^2}{2\sigma_s^4\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)}\right] f(v|ND) \\
&\propto \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{\mu_j}{\sigma_s^2}v\right] f(v|ND),
\end{aligned}$$

where the second line follows from the fact that the event ND is uninformative regarding $\tilde{\mu}_j$ conditional on $\tilde{v}$ (since, given $\tilde{v}$, variation in $\tilde{\mu}_j$ is driven only by $\tilde{\varepsilon}_j$ and $\tilde{z}$). Now, as this density function must integrate to 1, we have:

$$f(v|ND, \mu_j) = \frac{\exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{\mu_j}{\sigma_s^2}v\right] f(v|ND)}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{\mu_j}{\sigma_s^2}v\right] f(v|ND)\,dv}. \tag{28}$$

Hence, for any integer $k > 0$,

$$\mathbb{E}\left[\tilde{v}^k | ND, \tilde{\mu}_j = x\right] = \frac{\int_{-\infty}^{\infty} v^k \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv}. \tag{29}$$

Substituting $k = 1$, we obtain

$$g\left(x\right) = \frac{\int_{-\infty}^{\infty} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv},$$

which proves the first part of the lemma (see Breon-Drish (2015) for proofs that these integrals in fact exist and that derivative-integral interchange is valid below). Next, differentiating the above equation, we arrive at:

$$\begin{aligned} g'\left(x\right) &= \frac{1}{\sigma_s^2} \frac{\int_{-\infty}^{\infty} v^2 \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv} \\ &\quad - \frac{1}{\sigma_s^2} \left(\frac{\int_{-\infty}^{\infty} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \frac{x}{\sigma_s^2} v\right] f\left(v|ND\right) dv} \right)^2. \end{aligned}$$

Now, note from equation (29), this implies:

$$\begin{aligned} g'\left(x\right) &= \frac{1}{\sigma_s^2}\left\{\mathbb{E}\left[\tilde{v}^2|ND, \tilde{\mu}_j = x\right] - \mathbb{E}\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]^2\right\} \\ &= \frac{\operatorname{var}\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]}{\sigma_s^2}. \end{aligned}$$

□

We next apply this result to derive the investors' demands.

Lemma A.2. *Investor j's demand in the event of non-disclosure given the price P_{ND} equals:*

$$D_j = \frac{\tau}{\sigma_s^2}\left[\mu_j - g^{-1}\left(P_{ND}\right)\right]. \tag{30}$$

Proof of Lemma A.2. Since μ_j is a sufficient statistic for investor j's signals s_j, s_p, her demand satisfies:

$$D_j = \arg\max_y - \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{\tau}\left(y\left(\tilde{v} - P_{ND}\right)\right)\right\} f\left(v|ND, s_j, s_p\right) dv$$

$$= \quad \arg\max_{y} - \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{\tau}\left(y\left(\tilde{v}-P_{ND}\right)\right)\right\} f\left(v|ND,\mu_j\right)dv.$$

It is easily verified that this function is concave and thus the first-order condition is sufficient for a solution. Applying equation (28) and Lemma A.1, the first-order condition reduces as follows:

$$\begin{aligned}
P_{ND} &= \frac{\int_{-\infty}^{\infty} v\exp\left(-\tau^{-1}D_j v\right) f\left(v|ND,\mu_j\right)dv}{\int_{-\infty}^{\infty}\exp\left(-\tau^{-1}D_j v\right) f\left(v|ND,\mu_j\right)dv} \\
&= \frac{\int_{-\infty}^{\infty} v\exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2+\left(-\frac{D_j}{\tau}+\frac{\mu_j}{\sigma_s^2}\right)v\right] f\left(v|ND\right)dv}{\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2+\left(-\frac{D_j}{\tau}+\frac{\mu_j}{\sigma_s^2}\right)v\right] f\left(v|ND\right)dv} \\
&= g\left(\mu_j-\frac{\sigma_s^2 D_j}{\tau}\right).
\end{aligned}$$

Now, as Lemma A.1 shows that $g' > 0$, g is invertible, and so we can solve the above equation to arrive at equation (30). □

We may now derive the firm's price by applying the market-clearing condition:

$$\begin{aligned}
\kappa = z + \int_0^1 D_i di \quad &\Leftrightarrow \quad \frac{\sigma_s^2}{\tau}\left(\kappa - z\right) = \int \mu_i di - g^{-1}\left(P_{ND}\right) \\
&\Leftrightarrow \quad P_{ND} = g\left(\int \mu_i di + \frac{\sigma_s^2}{\tau}\left(z-\kappa\right)\right).
\end{aligned}$$

Substituting for μ_i and σ_s^2 and applying the law of large numbers, we arrive at:

$$\begin{aligned}
P_{ND} &= g\left(\int\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left[\frac{\tilde{s}_i}{\sigma_\varepsilon^2}+\frac{\tilde{s}_p}{\sigma_p^2}+\frac{z-\kappa}{\tau}\right]di\right) \\
&= g\left(\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left[\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v+\left(\frac{\beta}{\sigma_p^2}+\frac{1}{\tau}\right)z-\frac{\kappa}{\tau}\right]\right).
\end{aligned}$$

Observe that, as conjectured, this takes the form of a generalized linear equilibrium, that is, $P_{ND} = G\left(v+\beta z\right)$, with:

$$\begin{aligned}
G\left(x\right) &= g\left(\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left[\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)x-\frac{\kappa}{\tau}\right]\right); \\
\beta &= \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left(\frac{\beta}{\sigma_p^2}+\frac{1}{\tau}\right).
\end{aligned}$$

Solving the second equation for β yields a unique solution $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. This equilibrium solution for β in turn implies that:

$$\sigma_p^2 = \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2},$$

and $s_p = v + \frac{\sigma_\varepsilon^2}{\tau} z$. Note further that, as Lemma A.1 shows that $g'(x) > 0$, we immediately have that the price is monotonic in s_p. Substituting and simplifying, we have that the unique generalized linear equilibrium price satisfies:

$$\begin{aligned} P_{ND}(v,z) &= g(P_U(v,z)) = \mathbb{E}[\tilde{v}|ND, \tilde{\mu}_j = P_U(v,z)], && (31) \\ \text{where } P_U(v,z) &\equiv \sigma_s^2 \left[\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right) \left(v + \frac{\sigma_\varepsilon^2}{\tau} z \right) - \frac{\kappa}{\tau} \right]. && (32) \end{aligned}$$

To complete the proof, we show that the price expression (31) can be re-expressed as in the proposition. Note that the event of non-disclosure ND results either from an informed manager who observed $\tilde{v} < T$ or an uninformed manager; denote the former event by $\tilde{\Gamma} = 1$ and the latter by $\tilde{\Gamma} = 0$. Then, we have:

$$\begin{aligned} \mathbb{E}[\tilde{v}|ND, \tilde{\mu}_j = P_U(v,z)] = \Pr\left(\tilde{\Gamma} = 1 | \tilde{\mu}_j = P_U(v,z)\right) \mathbb{E}[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = P_U(v,z)] + \\ \Pr\left(\tilde{\Gamma} = 0 | \tilde{\mu}_j = P_U(v,z)\right) P_U(v,z). \quad (33) \end{aligned}$$

Note that $\Pr(\tilde{v} < T | \tilde{\mu}_j = P_U(v,z)) = \Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right)$. Therefore, we can apply Bayes' rule to arrive at:

$$\Pr\left(\tilde{\Gamma} = 1 | \tilde{\mu}_j = P_U(v,z)\right) = 1 - \Pr\left(\tilde{\Gamma} = 0 | \tilde{\mu}_j = P_U(v,z)\right) = \frac{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) + 1 - p}. \quad (34)$$

To explicitly derive $\mathbb{E}[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = P_U(v,z)]$, we may apply the formula for the mean of a truncated normal distribution, which yields:

$$\mathbb{E}[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = P_U(v,z)] = P_U(v,z) - \sigma_s h\left(\frac{T - P_U(v,z)}{\sigma_s}\right) \equiv P_I(v,z). \quad (35)$$

Substituting equations (34) and (35) into equation (33) yields the expression for price defined in the proposition:

$$P_{ND} = \frac{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right)\left(P_U(v,z) - \sigma_s h\left(\frac{T - P_U(v,z)}{\sigma_s}\right)\right) + (1-p) P_U(v,z)}{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) + 1 - p}. \quad (36)$$

□

B. Proof of Lemma 1

Observe from equations (31) and (32) that:

$$\frac{\partial P_{ND}(v,z)}{\partial v} \quad = \quad g'\left(P_U\left(v,z\right)\right) * \sigma_s^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right). \tag{37}$$

Now, applying equation (27), we have:

$$g'\left(P_U\left(v,z\right)\right) = \frac{1}{\sigma_s^2} \mathrm{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_U\left(v,z\right)\right].$$

Substituting into equation (37) and simplifying, we have:

$$\frac{\partial P_{ND}(v,z)}{\partial v} \quad = \quad \mathrm{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_U\left(v,z\right)\right]\left(\mathrm{var}^{-1}\left[\tilde{s}_j|\tilde{v}\right] + \mathrm{var}^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{38}$$

□

C. Proof of Proposition 2

Let $\Psi(T,\tilde{z}) \equiv T - P_{ND}(T,\tilde{z};T)$. Then, $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ denotes the incremental payoff to the manager who observes $\tilde{v} = T$ from disclosing, relative to not disclosing, when investors conjecture that the manager discloses if and only if $\tilde{v} > T$. We establish the proposition in three lemmas. Lemma A.3 states that, under the conditions stated in the proposition, if investors conjecture that the manager discloses if and only if $\tilde{v} > T$, then his payoff to disclosing is strictly increasing in firm value, that is, $\frac{\partial}{\partial v}\left(v - c - \mathbb{E}\left[P_{ND}\left(v,\tilde{z};T\right)\right]\right) > 0$. This implies that, if T solves $\mathbb{E}\left[\Psi(T,\tilde{z})\right] = c$, then T corresponds to a threshold equilibrium. Next, Lemma A.4 states that $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ strictly increases in T, and thus, when a threshold equilibrium exists, it is unique. Finally, Lemma A.5 states that there exists a T^* such that $\mathbb{E}\left[\Psi(T^*,\tilde{z})\right] = c$. Together, these lemmas imply that T^* corresponds to the unique threshold equilibrium.

Lemma A.3. *Suppose that* $p = 1$ *and/or* $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1 + \frac{1}{2}p\left(1-p\right)\right)\right]^{-1}$*. Then,* $\forall v, T \in \mathcal{R}$*:*

$$\frac{\partial}{\partial v}\left(v - c - \mathbb{E}\left[P_{ND}\left(v,\tilde{z};T\right)\right]\right) > 0.$$

Proof of Lemma A.3. We first argue that it is sufficient to show that:

$$\forall v,z,T \in \mathcal{R}, \quad \frac{\partial P_{ND}\left(v,z;T\right)}{\partial v} < 1. \tag{39}$$

To see why this is sufficient, note that, because $\frac{\partial P_{ND}(v,z;T)}{\partial v} = \frac{\tau}{\sigma_\varepsilon^2}\frac{\partial P_{ND}(v,z;T)}{\partial z}$, condition (39) implies that $|P_{ND}(v,z;T)|$ is sublinear in z. That is, we have $\left|\frac{1}{\sigma_z}P_{ND}(v,z;T)\,\phi\left(\frac{z}{\sigma_z}\right)\right| < \left|\frac{1}{\sigma_z}A\phi\left(\frac{z}{\sigma_z}\right)\right|$ for some $A \in \mathcal{R}$ that does not depend upon z, and, being the expectation of an absolute normal, $\int_{-\infty}^{\infty}\left|\frac{1}{\sigma_z}A\phi\left(\frac{z}{\sigma_z}\right)\right| dz$ is finite. Thus, by the dominated convergence theorem, when condition (39) holds,

$$\frac{\partial}{\partial v}\mathbb{E}\left[P_{ND}(v,\tilde{z};T)\right] = \frac{1}{\sigma_z}\int_{-\infty}^{\infty}\frac{\partial P_{ND}(v,z;T)}{\partial v}\phi\left(\frac{z}{\sigma_z}\right)dz < 1.$$

We proceed to show that condition (39) holds in each of the two cases stated in the lemma.

Case 1: $\mathbf{p = 1}$. Let:

$$\Delta_v \equiv \frac{\partial}{\partial v}P_U(v,z) = \frac{\sigma_v^2\left(\sigma_\varepsilon^2+\sigma_p^2\right)}{\sigma_\varepsilon^2\sigma_p^2+\sigma_v^2\left(\sigma_\varepsilon^2+\sigma_p^2\right)} \quad \text{and} \tag{40}$$

$$\Delta_z \equiv \frac{\partial}{\partial z}P_U(v,z) = \frac{\sigma_\varepsilon^2}{\tau}\frac{\sigma_v^2\left(\sigma_\varepsilon^2+\sigma_p^2\right)}{\sigma_\varepsilon^2\sigma_p^2+\sigma_v^2\left(\sigma_\varepsilon^2+\sigma_p^2\right)}, \tag{41}$$

and notice that $\Delta_v \in (0,1)$. Appealing to Proposition 1, we have that, when $p=1$, $P_{ND}(v,z;T)$ reduces to:

$$P_{ND}(v,z;T) = P_U(v,z) - \sigma_s h\left(\frac{T-P_U(v,z)}{\sigma_s}\right).$$

Differentiating this expression with respect to v yields:

$$\Delta_v\left[1+h'\left(\frac{T-P_U(v,z)}{\sigma_s}\right)\right]. \tag{42}$$

It may be verified that the inverse-Mills ratio satisfies $h'(x) \in (-1,0)$ and thus this belongs to $(0,1)$.

Case 2: $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1+\frac{1}{2}p(1-p)\right)\right]^{-1}$. Recall from expression (38) that we have:

$$\frac{\partial P_{ND}(v,z;T)}{\partial v} = \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\mathrm{var}\left[\tilde{v}|ND,\tilde{\mu}_j = P_U(v,z)\right] > 0. \tag{43}$$

Let $\tilde{\Gamma}=1$ when the manager is informed and 0 otherwise. Then, applying the law of total variance:

$$\begin{aligned}\mathrm{var}\left[\tilde{v}|ND,\tilde{\mu}_j = P_U(v,z)\right] &= \mathbb{E}_\Gamma\left\{\mathrm{var}_\Gamma\left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\} \\ &+\mathrm{var}_\Gamma\left\{\mathbb{E}_\Gamma\left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\},\end{aligned} \tag{44}$$

where the subscripts on the expectations and variances indicate they are taken over $\tilde{\Gamma}$ only. Now, applying the fact that the variance of a truncated normal always lies below the prior variance, we have:

$$\begin{aligned}
& \mathbb{E}_{\Gamma}\left\{\operatorname{var}_{\Gamma}\left[\tilde{v}|\tilde{\Gamma}, ND, \tilde{\mu}_j = P_U(v,z)\right]\right\} \qquad (45)\\
= & \frac{p\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)\operatorname{var}\left[\tilde{v}|\tilde{v}<T, \tilde{\mu}_j = P_U(v,z)\right] + (1-p)\sigma_s^2}{p\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)+1-p} < \sigma_s^2.
\end{aligned}$$

Next, applying the variance of a binary distribution, we have:

$$\begin{aligned}
& \operatorname{var}_{\Gamma}\left\{\mathbb{E}_{\Gamma}\left[\tilde{v}|\tilde{\Gamma}, ND, \tilde{\mu}_j = P_U(v,z)\right]\right\}\\
= & \Pr\left(\tilde{\Gamma}=1|ND, \tilde{\mu}_j = P_U(v,z)\right)\Pr\left(\tilde{\Gamma}=0|ND, \tilde{\mu}_j = P_U(v,z)\right) *\\
& \left\{\mathbb{E}\left[\tilde{v}|\tilde{\Gamma}=1, ND, \tilde{\mu}_j = P_U(v,z)\right] - \mathbb{E}\left[\tilde{v}|\tilde{\Gamma}=0, ND, \tilde{\mu}_j = P_U(v,z)\right]\right\}^2\\
= & \frac{p(1-p)\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)\sigma_s^2 h\left(\frac{T-P_U(v,z)}{\sigma_s}\right)^2}{\left(p\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)+1-p\right)^2}\\
= & \frac{p(1-p)\sigma_s^2\phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)^2}{\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)\left(p\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)+1-p\right)^2} < p(1-p)\sigma_s^2\frac{\phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)^2}{\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)}. \qquad (46)
\end{aligned}$$

It may be verified that $\frac{\phi(x)^2}{\Phi(x)}$ is bounded above by $\frac{1}{2}$ and thus the above expression is bounded over all realizations of v and z by $\frac{p(1-p)\sigma_s^2}{2}$. Combining (44), (45), and (46), we have:

$$\frac{\partial P_{ND}(v,z;T)}{\partial v} < \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\left(1+\frac{p(1-p)}{2}\right) < \sigma_v^2\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\left(1+\frac{p(1-p)}{2}\right),$$

such that, for $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1+\frac{1}{2}p(1-p)\right)\right]^{-1}$, $\frac{\partial P_{ND}(v,z;T)}{\partial v} \in (0,1)$. □

Lemma A.4. $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ *strictly increases in* T. *Thus, if a threshold equilibrium exists, it is unique.*

Proof of Lemma A.4. We first show that, fixing any $T \in \mathcal{R}$, $|\Psi(T,z)\frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is bounded above by an integrable function. Note that, because $h'(x) < 0$, when $T - P_U(T,z) > 0$, we have:

$$|\Psi(T,z)| = T - P_U(T,z) + \frac{p\sigma_s\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)+1-p}$$

$$< \quad T - P_U(T,z) + \sigma_s h\left(\frac{T - P_U(T,z)}{\sigma_s}\right) < T - P_U(T,z) + \sigma_s h(0),$$

which is linear in z, and hence its product with the PDF a normal distribution is integrable. Hence, $|\Psi\left(T,z\right)\frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is integrable on $\{z : T - P_U(T,z) > 0\}$. To see that $|\Psi\left(T,z\right)\frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is also integrable on $\{z : T - P_U(T,z) < 0\}$, note for $T - P_U(T,z) < 0$,

$$|\Psi\left(T,z\right)| \quad = \quad \left|T - P_U(T,z) + \frac{p\sigma_s\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) + 1 - p}\right| < |T - P_U(T,z)|,$$

which is also linear in z. Given these results, we may apply the dominated convergence theorem to arrive at $\frac{\partial}{\partial T}\mathbb{E}\left[\Psi\left(T,\tilde{z}\right)\right] = \mathbb{E}\left[\frac{\partial}{\partial T}\Psi\left(T,\tilde{z}\right)\right]$. Now, absorbing T into the numerator of $\mathbb{E}\left[P_{ND}(T,\tilde{z};T)\right]$ and expressing $\mathbb{E}\left[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = P_U\left(T,z\right)\right]$ in its integral form, we may write $\frac{\partial}{\partial T}\Psi\left(T,\tilde{z}\right)$ as:

$$\frac{\partial}{\partial T}\Psi\left(T,z\right) = \frac{\partial}{\partial T}\frac{p\left[\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)T - \int_{-\infty}^{T}\frac{v}{\sigma_s}\phi\left(\frac{v-P_U(T,z)}{\sigma_s}\right)dv\right] + (1-p)\left(T - P_U\left(T,z\right)\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) + 1 - p}. \tag{47}$$

Now, integration by parts yields:

$$\int_{-\infty}^{T}\frac{v}{\sigma_s}\phi\left(\frac{v - P_U\left(T,z\right)}{\sigma_s}\right)dv = T\Phi\left(\frac{T - P_U\left(T,z\right)}{\sigma_s}\right) - \int_{-\infty}^{T}\Phi\left(\frac{v - P_U\left(T,z\right)}{\sigma_s}\right)dv. \tag{48}$$

Note further that:

$$\begin{aligned}\frac{\partial}{\partial T}\int_{-\infty}^{T}\Phi\left(\frac{v - P_U\left(T,z\right)}{\sigma_s}\right)dv &= \Phi\left(\frac{T - P_U\left(T,z\right)}{\sigma_s}\right) - \int_{-\infty}^{T}\frac{\Delta_v}{\sigma_s}\phi\left(\frac{v - P_U\left(T,z\right)}{\sigma_s}\right)dv \\ &= (1-\Delta_v)\Phi\left(\frac{T - P_U\left(T,z\right)}{\sigma_s}\right),\end{aligned} \tag{49}$$

where Δ_v, as defined in (40), belongs to $(0,1)$. Applying equations (48) and (49), we may calculate the derivative in expression (47) as follows:

$$\begin{aligned}&\frac{\partial}{\partial T}\Psi\left(T,z\right) = \frac{\partial}{\partial T}\frac{p\int_{-\infty}^{T}\Phi\left(\frac{v-P_U(T,z)}{\sigma_s}\right)dv + (1-p)\left(T - P_U\left(T,z\right)\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) + 1 - p} \\ \propto \quad &(1-\Delta_v)\left[p\Phi\left(\frac{T - P_U\left(T,z\right)}{\sigma_s}\right) + 1 - p\right]^2 \\ &-\frac{1-\Delta_v}{\sigma_s}p\phi\left(\frac{T - P_U\left(T,z\right)}{\sigma_s}\right)\left[p\int_{-\infty}^{T}\Phi\left(\frac{T - P_U\left(v,z\right)}{\sigma_s}\right)dv + (1-p)\left(T - P_U\left(T,z\right)\right)\right]\end{aligned}$$

$$\propto \frac{p^2}{1-p}\left[\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)^2 - \frac{1}{\sigma_s}\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{T-P_U(v,z)}{\sigma_s}\right)dv\right]$$
$$+1-p+2p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) - p\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\frac{T-P_U(T,z)}{\sigma_s}.$$

Now, note that the normal distribution is log concave, which implies that, $\forall x \in \mathcal{R}$, $\Phi(x)^2 - \frac{1}{\sigma_s}\phi(x)\int_{-\infty}^{x}\Phi(v)\,dv > 0$ (Bagnoli and Bergstrom (2005)). Thus, we have:

$$\begin{aligned}
\frac{\partial}{\partial T}\Psi(T,z) &> 1-p+2p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) - p\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\frac{T-P_U(T,z)}{\sigma_s} \\
&\propto 1 - \frac{T-P_U(T,z)}{\sigma_s}\frac{p\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}{1-p+2p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}.
\end{aligned}$$

When $\frac{T-P_U(T,z)}{\sigma_s} < 0$, this is trivially positive. When $\frac{T-P_U(T,z)}{\sigma_s} > 0$, we have that it exceeds:

$$1 - \frac{1}{2}\frac{T-P_U(T,z)}{\sigma_s}h\left(\frac{T-P_U(T,z)}{\sigma_s}\right).$$

Now, this is positive since, $\forall x \in \mathcal{R}$, $xh(x) < 1$. $\square$

Lemma A.5. *There exists a $T^* \in \mathcal{R}$ such that $\mathbb{E}[\Psi(T^*,\tilde{z})] = c$.*

Proof of Lemma A.5. It is easily seen that $\mathbb{E}[\Psi(T,\tilde{z})]$ is a continuous function of T. Thus, to prove the lemma, it is sufficient to show that:

$$\lim_{T\to-\infty}\mathbb{E}[\Psi(T,\tilde{z})] < c < \lim_{T\to\infty}\mathbb{E}[\Psi(T,\tilde{z})]. \tag{50}$$

To see that this holds, note that, applying equation (36), we have:

$$\begin{aligned}
\Psi(T,z) &= T - \frac{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\left(P_U(T,z) - \sigma_s h\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\right) + (1-p)P_U(T,z)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right) + 1 - p} \\
&= T(1-\Delta_v) - \Delta_z z + \tau^{-1}\sigma_s^2\kappa + \frac{p\sigma_s\phi\left(\frac{T(1-\Delta_v)-\Delta_z z+\tau^{-1}\sigma_s^2\kappa}{\sigma_s}\right)}{p\Phi\left(\frac{T(1-\Delta_v)-\Delta_z z+\tau^{-1}\sigma_s^2\kappa}{\sigma_s}\right)+1-p}.
\end{aligned} \tag{51}$$

Note that $\lim_{x\to\infty}\frac{p\sigma_s\phi(x)}{p\Phi(x)+1-p} = 0$. Combining this with the fact that $\Delta_v < 1$, we have that $\Psi(T,z)$ converges pointwise in z to ∞ as $T \to \infty$. Note further that, from the proof of the previous lemma, $\Psi(T,z)$ is increasing in T. Consequently, $\Psi(T,z) - \Psi(0,z) > 0$. Thus, we can apply Fatou's lemma, and arrive at:

$$\lim_{T\to\infty} \mathbb{E}\left[\Psi(T,\tilde{z}) - \Psi(0,\tilde{z})\right] \geq \mathbb{E}\left[\lim_{T\to\infty} \Psi(T,\tilde{z}) - \Psi(0,\tilde{z})\right] = \infty.$$

This verifies the second inequality in (50).

Next, to prove the first inequality in (50), applying the fact that $h(x) \to -x$ as $x \to -\infty$, we have:

$$\lim_{x\to-\infty}\left(x + \frac{p\phi(x)}{p\Phi(x)+1-p}\right) = \begin{cases} 0 & \text{when } p = 1 \\ -\infty & \text{when } p \in (0,1). \end{cases}$$

Thus, when $p \in (0,1)$ ($p = 1$), we have that $\Psi(T,z)$ converges pointwise in z to $-\infty$ (to 0) as $T \to -\infty$. Applying Fatou's lemma as above, this immediately implies that, when $p \in (0,1)$, $\lim_{T\to-\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right] = -\infty$, which verifies the inequality holds in this case. Next, when $p = 1$, as in the proof of the previous lemma, we may apply the dominated convergence theorem to interchange limit and expectations to immediately arrive at $\lim_{T\to-\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right] = \mathbb{E}\left[\lim_{T\to-\infty}\Psi(T,\tilde{z})\right] = 0$. Moreover, given the assumption that one of the disclosure frictions is always present, we have that, if $p = 1$, then $c > 0$. Thus, we once again have verified the inequality holds. □

This completes the proof of Proposition 2. □

D. *Proof of Proposition 3*

Because the distribution of v does not depend the parameters $\{c, p, \kappa\}$, it is sufficient to show that T increases in c and decreases in p and κ. Applying the implicit function theorem, for $\Psi(v,z)$ as defined in the proof of the previous proposition, we obtain:

$$\begin{aligned}
\frac{\partial T}{\partial c} &= \mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T}\right]^{-1} > 0; \\
\frac{\partial T}{\partial p} &= \mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T}\right]^{-1}\mathbb{E}\left[\frac{\partial\Psi(T,\tilde{z})}{\partial p}\right] \\
&= -\mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T}\right]^{-1}\mathbb{E}\left[\frac{\sigma_s\phi\left(\frac{T-P_U(T,\tilde{z})}{\sigma_s}\right)}{\left(p\Phi\left(\frac{T-P_U(T,\tilde{z})}{\sigma_s}\right)+1-p\right)^2}\right] < 0; \\
\frac{\partial T}{\partial \kappa} &= \mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T}\right]^{-1}\mathbb{E}\left[\frac{\partial\Psi(T,\tilde{z})}{\partial \kappa}\right]
\end{aligned}$$

$$= \mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T}\right]^{-1}\mathbb{E}\left[\left.\frac{\partial\Psi(v,\tilde{z})}{\partial v}\right|_{v=T} * \left(-\frac{\sigma_s^2}{\tau}\frac{1}{1-\Delta_v}\right)\right] = -\frac{\sigma_s^2}{\tau}\frac{1}{1-\Delta_v} < 0.$$

$\square$

E. *Proof of Proposition 4*

Note that the probability of disclosure depends on σ_ε and σ_z only through T (and decreases in T). Thus to prove this result, we characterize how T changes with σ_ε and σ_z. We start by deriving some preliminary results. Let:

$$\begin{aligned} A &\equiv \frac{\partial}{\partial T}\frac{T-P_U(T,z)}{\sigma_s} = \frac{\sigma_v^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4}; \\ B &\equiv \frac{\partial}{\partial z}\frac{T-P_U(T,z)}{\sigma_s} = \frac{\sigma_v^2\sigma_\varepsilon^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)}, \end{aligned}$$

so that $\frac{T-P_U(T,z)}{\sigma_s} = A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+Bz$. Furthermore, let $G_p(x) \equiv x + \frac{p\phi(x)}{p\Phi(x)+1-p}$, so that we may rewrite the equilibrium condition as:

$$\mathbb{E}\left[G_p\left(A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+Bz\right)\right]-\frac{c}{\sigma_s}=0.$$

Now, the implicit function theorem yields that:

$$\begin{aligned} \frac{\partial T}{\partial\sigma_\varepsilon} &= -\frac{\mathbb{E}\left[\left(\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial\sigma_\varepsilon}+\tilde{z}\frac{\partial B}{\partial\sigma_\varepsilon}\right)G_p'\right]+\frac{c}{\sigma_s^2}\frac{\partial\sigma_s}{\partial\sigma_\varepsilon}}{A\mathbb{E}\left[G_p'\right]} \\ &= -\frac{\frac{\partial B}{\partial\sigma_\varepsilon}\mathbb{E}\left[\tilde{z}G_p'\right]+\frac{\partial A}{\partial\sigma_\varepsilon}\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\mathbb{E}\left[G_p'\right]+\frac{c}{\sigma_s^2}\frac{\partial\sigma_s}{\partial\sigma_\varepsilon}}{A\mathbb{E}\left[G_p'\right]}, \end{aligned}$$

where G_p' is evaluated at $A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+Bz$. Since $A>0$ and $\mathbb{E}\left[G_p'\right]>0$, we have:

$$\frac{\partial T}{\partial\sigma_\varepsilon} \propto -\frac{\partial B}{\partial\sigma_\varepsilon}\mathbb{E}\left[\tilde{z}G_p'\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial\sigma_\varepsilon}\mathbb{E}\left[G_p'\right]-\frac{c}{\sigma_s^2}\frac{\partial\sigma_s}{\partial\sigma_\varepsilon}.$$

Applying Stein's lemma, this simplifies to:

$$\frac{\partial T}{\partial\sigma_\varepsilon} \propto -\frac{\partial B}{\partial\sigma_\varepsilon}B\sigma_z^2\mathbb{E}\left[G_p''\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial\sigma_\varepsilon}\mathbb{E}\left[G_p'\right]-\frac{c}{\sigma_s^2}\frac{\partial\sigma_s}{\partial\sigma_\varepsilon}. \tag{52}$$

Next, observe that we can write the equilibrium condition as:

$$\mathbb{E}\left[G_p\left(A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+B\sigma_z\tilde{\vartheta}\right)\right]-\frac{c}{\sigma_s}=0,$$

where $\tilde{\vartheta} \sim N(0,1)$. So, the implicit function theorem yields that:

$$\begin{aligned}
\frac{\partial T}{\partial \sigma_z} &= -\frac{\mathbb{E}\left[\left(\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_z}+\frac{\partial(B\sigma_z)}{\partial \sigma_z}\tilde{\vartheta}\right)G_p'\right]+\frac{c}{\sigma_s^2}\frac{\partial \sigma_s}{\partial \sigma_z}}{A\mathbb{E}\left[G_p'\right]} \\
&\propto -\frac{\partial\left(B\sigma_z\right)}{\partial \sigma_z}\mathbb{E}\left[\tilde{\vartheta}G_p'\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_z}\mathbb{E}\left[G_p'\right]-\frac{c}{\sigma_s^2}\frac{\partial \sigma_s}{\partial \sigma_z},
\end{aligned}$$

so that, again applying Stein's lemma,

$$\frac{\partial T}{\partial \sigma_z} \propto -\frac{\partial\left(B\sigma_z\right)}{\partial \sigma_z}B\sigma_z\mathbb{E}\left[G_p''\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_z}\mathbb{E}\left[G_p'\right]-\frac{c}{\sigma_s^2}\frac{\partial \sigma_s}{\partial \sigma_z}. \tag{53}$$

We next use expressions (52) and (53) to prove both parts of the proposition.

Part i. Note that, for $p=1$, we have $G_p(x)=x+h(x)$ and so $\lim_{x\to\infty}G_p''(x)=\lim_{x\to\infty}h''(x)=0$. Note further that $\lim_{c\to\infty}T=\infty$. Now, as h'' is bounded, we may interchange the limit and integral to obtain:

$$\lim_{c\to\infty}\mathbb{E}\left[G_p''\right]=\mathbb{E}\left[h''\left(\lim_{T\to\infty}\left(A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+B\sigma_z\tilde{\vartheta}\right)\right)\right]=0.$$

Furthermore, it is straightforward to verify that $\frac{\partial\sigma_s}{\partial\sigma_\varepsilon},\frac{\partial\sigma_s}{\partial\sigma_z}>0$ and:

$$\begin{aligned}
\frac{\partial A}{\partial \sigma_\varepsilon} &= \frac{\sigma_v\sigma_z\sigma_\varepsilon\left(2\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\sigma_v^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)+\sigma_z^2\sigma_\varepsilon^4\right)^{3/2}}>0; \\
\frac{\partial A}{\partial \sigma_z} &= \frac{\tau^2\sigma_v\sigma_\varepsilon^2}{\left(\sigma_v^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)+\sigma_z^2\sigma_\varepsilon^4\right)^{3/2}}>0.
\end{aligned}$$

Combining these results, we can sign the limits of expressions (52) and (53) as $c\to\infty$:

$$\begin{aligned}
\lim_{c\to\infty}\frac{\partial T}{\partial \sigma_\varepsilon} &\propto -\lim_{c\to\infty}\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_\varepsilon}\mathbb{E}\left[G_p'\right]-\lim_{c\to\infty}\frac{c}{\sigma_s^2}\frac{\partial \sigma_s}{\partial \sigma_\varepsilon}<0; \\
\lim_{c\to\infty}\frac{\partial T}{\partial \sigma_z} &\propto -\lim_{c\to\infty}\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_z}\mathbb{E}\left[G_p'\right]-\lim_{c\to\infty}\frac{c}{\sigma_s^2}\frac{\partial \sigma_s}{\partial \sigma_\varepsilon}<0.
\end{aligned}$$

Part ii. Beginning with the comparative static on σ_ε, note from expression (52) that:

$$\begin{aligned}
\lim_{\sigma_\varepsilon\to 0}\frac{\partial T}{\partial \sigma_\varepsilon} &\propto \lim_{\sigma_\varepsilon\to 0}\left(-\frac{\partial B}{\partial \sigma_\varepsilon}\sigma_z^2\mathbb{E}\left[G_p''\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial \sigma_\varepsilon}\mathbb{E}\left[G_p'\right]\right) \\
&= \left(-\lim_{\sigma_\varepsilon\to 0}\frac{\partial A}{\partial \sigma_\varepsilon}\right)\left(\lim_{\sigma_\varepsilon\to 0}\left\{\left(\frac{\partial A}{\partial \sigma_\varepsilon}\right)^{-1}\frac{\partial B}{\partial \sigma_\varepsilon}B\sigma_z^2\mathbb{E}\left[G_p''\right]+\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\mathbb{E}\left[G_p'\right]\right\}\right).
\end{aligned}$$

Note that, because $\frac{\partial A}{\partial\sigma_\varepsilon}>0$, $\frac{\partial T}{\partial\sigma_\varepsilon}$ will be positive for small σ_ε if and only if the second term in the product above has a limit that is negative. Simplifying, we obtain $\lim_{\sigma_\varepsilon\to 0}\left(\frac{\partial A}{\partial\sigma_\varepsilon}\right)^{-1}\frac{\partial B}{\partial\sigma_\varepsilon}B=$

$\frac{\sigma_v^2}{2\tau\sigma_z}$ so that $\lim_{\sigma_\varepsilon\to 0}\left(\frac{\partial A}{\partial\sigma_\varepsilon}\right)^{-1}\frac{\partial B}{\partial\sigma_\varepsilon}B\sigma_z^2\mathbb{E}\left[G_p''\right]$ is finite. Next, note that that the equilibrium condition when $c=0$ yields:

$$\mathbb{E}\left[G_p\left(A\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)+B\tilde{z}\right)\right]=0\Leftrightarrow\mathbb{E}\left[\frac{T-P_U\left(T,\tilde{z}\right)}{\sigma_s}+\frac{p\phi\left(\frac{T-P_U(T,\tilde{z})}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,\tilde{z})}{\sigma_s}\right)+1-p}\right]=0.$$

Now, note that, fixing $x\in\mathcal{R}$, $\lim_{\sigma_\varepsilon\to 0}\frac{x-P_U(x,\tilde{z})}{\sigma_s}=-\frac{\tilde{z}}{\sigma_z}$. Hence, we have that:

$$\lim_{\sigma_\varepsilon\to 0}\mathbb{E}\left[\Psi\left(x,\tilde{z}\right)\right]=\mathbb{E}\left[\frac{p\phi\left(-\frac{\tilde{z}}{\sigma_z}\right)}{p\Phi\left(-\frac{\tilde{z}}{\sigma_z}\right)+1-p}\right]>0.$$

Since $\Psi\left(x,\tilde{z}\right)$ increases in x, this implies that, as $\sigma_\varepsilon\to 0$, the equilibrium threshold T that solves $\mathbb{E}\left[\Psi\left(T,\tilde{z}\right)\right]=0$ must approach $-\infty$. Thus, since $\mathbb{E}\left[G_p'\right]>0$, we have that:

$$\lim_{\sigma_\varepsilon\to 0}\left\{\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\mathbb{E}\left[G_p'\right]\right\}=-\infty, \tag{54}$$

which completes the proof that the probability of disclosure decreases in σ_ε for small σ_ε. Moving to the result on σ_z, note from expression (53) that, when $c=0$,

$$\begin{aligned}\lim_{\sigma_\varepsilon\to 0}\frac{\partial T}{\partial\sigma_z}\quad&\propto\quad\lim_{\sigma_\varepsilon\to 0}\left\{-\frac{\partial\left(B\sigma_z\right)}{\partial\sigma_z}\mathbb{E}\left[G_p''\right]-\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\frac{\partial A}{\partial\sigma_z}\mathbb{E}\left[G_p'\right]\right\}\\&=\quad\left(-\lim_{\sigma_\varepsilon\to 0}\frac{\partial A}{\partial\sigma_z}\right)\left(\lim_{\sigma_\varepsilon\to 0}\left\{\left(\frac{\partial A}{\partial\sigma_z}\right)^{-1}\frac{\partial\left(B\sigma_z\right)}{\partial\sigma_z}B\sigma_z\mathbb{E}\left[G_p''\right]+\left(T+\frac{\sigma_v^2}{\tau}\kappa\right)\mathbb{E}\left[G_p'\right]\right\}\right).\end{aligned}$$

Again, because $\frac{\partial A}{\partial\sigma_z}>0$, $\frac{\partial T}{\partial\sigma_z}$ will be positive for small σ_ε if and only if the second term in the product above has a limit that is negative. Note that $\lim_{\sigma_\varepsilon\to 0}\left(\frac{\partial A}{\partial\sigma_z}\right)^{-1}\frac{\partial(B\sigma_z)}{\partial\sigma_z}B\sigma_z=\frac{\sigma_v^2\sigma_z}{\tau}$ and that G_p'' is bounded, so that $\lim_{\sigma_\varepsilon\to 0}\left(\frac{\partial A}{\partial\sigma_z}\right)^{-1}\frac{\partial(B\sigma_z)}{\partial\sigma_z}B\sigma_z\mathbb{E}\left[G_p''\right]$ is finite. Applying equation (54) completes the proof that the probability of disclosure decreases in σ_z for small σ_ε.

□

F. Proof of Proposition 5

Part i. Since $\mathbb{E}\left[\tilde{z}\right]=0$, we can write the expected price given non-disclosure, $\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]$, as:

$$\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]\;=\;\mathbb{E}\left[P_U(\tilde{v},\tilde{z})|\tilde{v}<T\right]-\sigma_s\mathbb{E}\left[h\left(\frac{T-P_U(\tilde{v},\tilde{z})}{\sigma_s}\right)|\tilde{v}<T\right]$$

$$= \int_i \mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di - \sigma_s \mathbb{E}\left[h\left(\frac{T-\int_i \tilde{\mu}_i di - \frac{\sigma_s^2}{\tau}\left(\tilde{z}-\kappa\right)}{\sigma_s}\right)|\tilde{v}<T\right]. \quad (55)$$

Now, we may write the firm's expected value conditional on non-disclosure as:

$$\begin{aligned}
\mathbb{E}\left[\tilde{v}|\tilde{v}<T\right] &= \mathbb{E}\left\{\mathbb{E}\left[\tilde{v}|\tilde{v}<T,\tilde{s}_j,\tilde{s}_p\right]|\tilde{v}<T\right\} \\
&= \mathbb{E}\left[\tilde{\mu}_j|\tilde{v}<T\right] - \sigma_s \mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\tilde{\mu}_j\right)\right)|\tilde{v}<T\right], \quad (56)
\end{aligned}$$

for an arbitrary investor j. Given that investors' signals are homogeneously distributed, $\int_i \mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di = \mathbb{E}\left[\tilde{\mu}_j|\tilde{v}<T\right]$. Thus, combining equations (55) and (56) yields:

$$\begin{aligned}
&\mathbb{E}\left[P_{ND}|\tilde{v}<T\right] - \mathbb{E}\left[\tilde{v}|\tilde{v}<T\right] \\
\propto\; &\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\tilde{\mu}_j\right)\right)|\tilde{v}<T\right] - \mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\int_i \tilde{\mu}_i di - \frac{\sigma_s^2}{\tau}\left(\tilde{z}-\kappa\right)\right)\right)|\tilde{v}<T\right] \\
<\; &\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\tilde{\mu}_j\right)\right)|\tilde{v}<T\right] - \mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\int_i \tilde{\mu}_i di - \frac{\tilde{z}}{\tau}\sigma_s^2\right)\right)|\tilde{v}<T\right].
\end{aligned}$$

Next, note that the inverse-Mills ratio $h(\cdot)$ is convex. Thus, to show that the above expression is negative, it is sufficient to show that, conditional on $\tilde{v}<T$, $\tilde{\mu}_j \succ_{SSD} \int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$, where $\succ_{SSD}$ denotes second-order stochastic dominance. It is straightforward to verify that the coefficients on $\tilde{v}$ in $\tilde{\mu}_j$ and $\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$ are identical. Therefore, the components of variation driven by $\tilde{v}$ in both $\tilde{\mu}_j$ and $\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$ are identical. Together with the normality of the error terms $\{\tilde{\varepsilon}_i\}$ and $\tilde{z}$ and their independence of $\tilde{v}$, this implies that second-order stochastic dominance reduces to the relative variance conditional on $\tilde{v}$, that is,

$$\tilde{\mu}_j \succ_{SSD} \int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2 \Leftrightarrow \mathrm{var}\left[\tilde{\mu}_j|\tilde{v}\right] < \mathrm{var}\left[\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right].$$

Calculating these variances, we have:

$$\begin{aligned}
\mathrm{var}\left[\tilde{\mu}_j|\tilde{v}\right] &= \mathrm{var}\left[\frac{\frac{1}{\sigma_\varepsilon^2}\tilde{\varepsilon}_j + \frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau}\tilde{z}}{\frac{1}{\sigma_\varepsilon^2}+\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}+\frac{1}{\sigma_v^2}}\right] = \frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)^2}; \\
\mathrm{var}\left[\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right] &= \mathrm{var}\left[\frac{\left(\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau}+\frac{1}{\tau}\right)\tilde{z}}{\frac{1}{\sigma_\varepsilon^2}+\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}+\frac{1}{\sigma_v^2}}\right] = \frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)^2}{\left(\tau^2\sigma_v^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\right)^2}.
\end{aligned}$$

Taking the difference yields $-\frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^4\sigma_\varepsilon^6\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)^2} < 0$.

Part ii. We show that the firm is over-valued when $\sigma_\varepsilon \to \infty$; by continuity, this ensures that the firm is over-valued for σ_ε sufficiently large. Note that $\sigma_\varepsilon \to \infty$, $P_U(v,z) \to \frac{\sigma_v^2}{\tau}(z-\kappa)$ and $\sigma_s \to \sigma_v$. Thus, in this limit, the non-disclosure price given a threshold T does not depend directly on the firm's value; denote this price by $\hat{P}_{ND}(z;T)$. In this limit, we have that the equilibrium condition reduces to:

$$\begin{aligned} 0 &= \mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right] \\ \Leftrightarrow 0 &= \mathbb{E}\left\{\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa) + \frac{p\phi\left(\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa)\right)}{p\Phi\left(\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa)\right)+1-p}\right\}. \end{aligned}$$

Now, since $\frac{\phi'(x)}{\phi(x)} = -x$, note that:

$$\begin{aligned} \left[\frac{\partial}{\partial T}\hat{P}_{ND}(0;T)\right]_{T=\hat{T}} = 0 \quad &\Leftrightarrow \quad \frac{p\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{p\Phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)+1-p} = \frac{\phi'\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)} \\ &\Leftrightarrow \quad \frac{p\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{p\Phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)+1-p} = -\frac{\hat{T}}{\sigma_v} + \frac{\kappa\sigma_v}{\tau} \\ &\Leftrightarrow \quad \hat{T} - \hat{P}_{ND}(0;\hat{T}) = 0. \end{aligned}$$

Consistent with the minimum principle in Acharya, DeMarzo, and Kremer (2011), this implies that the equilibrium threshold when there is no noise trade $\hat{T}$ satisfies $\hat{T} = \arg\min_x \hat{P}_{ND}(0;x)$ (the second-order condition is straightforward to verify). Now, note that this implies:

$$\begin{aligned} \mathbb{E}\left[\hat{T} - \hat{P}_{ND}(\tilde{z};\hat{T})\right] &= \hat{T} - \mathbb{E}\left[\hat{P}_{ND}\left(0;\hat{T} - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right] \\ &< \hat{T} - \mathbb{E}\left[\hat{P}_{ND}\left(0;\hat{T}\right)\right] = 0. \end{aligned}$$

Now, as shown in Lemma A.4, $\mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right]$ strictly increases in T. Thus, the equilibrium threshold with noise trade T^* (i.e., the solution to $\mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right] = 0$), satisfies $T^* > \hat{T}$. Now, note that when $\kappa = 0$, $\hat{P}_{ND}(0;T^*) = \mathbb{E}[\tilde{v}|ND]$ (where ND here refers to the event of non-disclosure in the equilibrium in which the manager discloses when $v > T^*$), and thus:

$$\begin{aligned} \mathbb{E}\left[\hat{P}_{ND}(\tilde{z};T^*)\right] - \mathbb{E}[\tilde{v}|ND] &= \mathbb{E}\left[\hat{P}_{ND}(\tilde{z};T^*)\right] - \hat{P}_{ND}(0;T^*) \\ &= T^* - \hat{P}_{ND}(0;T^*), \end{aligned}$$

because T^* by definition satisfies the equilibrium condition $\mathbb{E}\left[\hat{P}_{ND}\left(\tilde{z};T^*\right)\right] = T^*$. Now, from the proof of Lemma A.4, $x - \hat{P}_{ND}(0;x)$ is increasing in x. Thus, since $T^* > \hat{T}$ and $\hat{T} - \hat{P}_{ND}(0;\hat{T}) = 0$, we have that $T^* - \hat{P}_{ND}\left(0;T^*\right) > 0$. Because this holds for $\kappa = 0$ and price is continuous in κ, it also holds for small positive κ. □

G. *Proof of Proposition 6*

The public signal is observable to all agents in the model prior to the disclosure and trading stages. Thus, the proofs of Propositions 1 and 2 directly extend to this case upon replacing the prior mean and variance parameters 0 and σ_v^2 with the mean and variance parameters conditional on the public signal, $\mathbb{E}\left[\tilde{v}|\tilde{y}\right]$ and $\text{var}\left[\tilde{v}|\tilde{y}\right]$. □

H. *Proof of Lemma 2*

Note we can rewrite the equilibrium condition as:

$$\begin{aligned}
0 &= T - c - \mathbb{E}\left[P_{ND}(T,\tilde{z},y)\right] \\
&= \mathbb{E}\left[T - c - \left(P_U\left(T,\tilde{z},y\right) - \sigma_s \frac{p\phi\left(\frac{T-P_U(T,\tilde{z},y)}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,\tilde{z},y)}{\sigma_s}\right)+1-p}\right)\right] \\
&\propto \mathbb{E}\left[\frac{T-P_U\left(T,\tilde{z},y\right)}{\sigma_s} + \frac{p\phi\left(\frac{T-P_U(T,\tilde{z},y)}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,\tilde{z},y)}{\sigma_s}\right)+1-p} - \frac{c}{\sigma_s}\right].
\end{aligned}$$

Now, we can manipulate equations (19), (21), and (22) to arrive at:

$$\begin{aligned}
\frac{T-P_U\left(T,z,y\right)}{\sigma_s} &= \frac{1}{\sigma_s}\left[T - \frac{\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right] + \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\left(T+\frac{\sigma_\varepsilon^2}{\tau}z\right)}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}}\right] - \frac{\sigma_s\kappa}{\tau} \\
&= \frac{1}{\sigma_s}\frac{\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\left(T-\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) - \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\frac{\sigma_\varepsilon^2}{\tau}z}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}} - \frac{\sigma_s\kappa}{\tau} \\
&\equiv A_1\left(T-\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) + A_2 z + A_3\kappa.
\end{aligned}$$

Thus, the equilibrium condition may be written:

$$\mathbb{E}\left[A_1\left(T-\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right)+A_3\kappa + \frac{p\phi\left(A_1\left(T-\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right)+A_2\tilde{z}+A_3\kappa\right)}{p\Phi\left(A_1\left(T-\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right)+A_2\tilde{z}+A_3\kappa\right)+1-p} - \frac{c}{\sigma_s}\right] = 0.$$

Now, this implies that:

$$T - \mathbb{E}\left[\tilde{v}|\tilde{y} = y\right] = t^*, \tag{57}$$

where t^* solves:

$$\mathbb{E}\left[A_1 t^* + A_3\kappa + \frac{p\phi\left(A_1 t^* + A_2\tilde{z} + A_3\kappa\right)}{p\Phi\left(A_1 t^* + A_2\tilde{z} + A_3\kappa\right) + 1 - p} - \frac{c}{\sigma_s}\right] = 0.$$

We now have that the probability of disclosure given $\tilde{y} = y$ satisfies:

$$\begin{aligned}\Pr\left(\tilde{v} > T\left(y\right)|\tilde{y} = y\right) &= \Pr\left(\tilde{v} > t^* + \mathbb{E}\left[\tilde{v}|\tilde{y} = y\right]|\tilde{y} = y\right) \\ &= \Pr\left(\tilde{v} - \mathbb{E}\left[\tilde{v}|\tilde{y} = y\right] > t^*|\tilde{y} = y\right) = \Phi\left(\frac{t^*}{\sqrt{\mathrm{var}\left(\tilde{v}|\tilde{y}\right)}}\right),\end{aligned}$$

Since t^* does not depend upon y, this is independent of y. Finally, the result that $T(y) = T(0) + \mathbb{E}\left[\tilde{v}|\tilde{y} = y\right]$ follows from equation (57) and the fact that $\mathbb{E}\left[\tilde{v}|\tilde{y} = 0\right] = 0$. $\square$

I. *Proof of Propositions 7 and 8*

Costly disclosure case.

Applying Lemma 2, the probability of disclosure equals:

$$\begin{aligned}\Pr\left(\tilde{v} > T\left(\tilde{y}\right)\right) &= \int \Pr\left(\tilde{v} > T\left(\tilde{y}\right)|\tilde{y} = x\right) dF_y\left(x\right) \\ &= \Pr\left(\tilde{v} > T\left(0\right)|\tilde{y} = 0\right) \\ &= 1 - \Phi\left(\frac{T\left(0\right)}{\sqrt{\mathrm{var}\left(\tilde{v}|\tilde{y}\right)}}\right) \\ &= 1 - \Phi\left(\sqrt{\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}}T\left(0\right)\right).\end{aligned} \tag{58}$$

Let $\Omega^{CD}\left(T, \sigma_\eta\right)$ denote net expected benefit from disclosure when $y = 0$ as a function of T and σ_η in the costly disclosure benchmark:

$$\Omega^{CD}\left(T, \sigma_\eta\right) \equiv T - c - \mathbb{E}\left[P_U\left(T, \tilde{z}, 0\right) - \sigma_s h\left(\frac{T - P_U\left(T, \tilde{z}, 0\right)}{\sigma_s}\right)\right],$$

and let Ω_1^{CD} and Ω_2^{CD} denote the derivatives of Ω^{CD} with respect to its first and second arguments. Then, note that:

$$\begin{aligned}
&\frac{\partial}{\partial \sigma_\eta}\left[1-\Phi\left(\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}}T(0)\right)\right] \\
= \ & -\phi\left(\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}}T(0)\right)\left[-\frac{\Omega_2^{CD}(T(0),\sigma_\eta)}{\Omega_1^{CD}(T(0),\sigma_\eta)}\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}}-\frac{1}{\sigma_\eta^3}\left(\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}\right)^{-\frac{1}{2}}T(0)\right] \\
\propto \ & \sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2^{CD}(T(0),\sigma_\eta)+T(0)\,\Omega_1^{CD}(T(0),\sigma_\eta).
\end{aligned}$$

Analogous arguments to those in the proof of Proposition 2 enable us to interchange the order of limits/derivatives and expectations in calculating this expression. Doing so, and simplifying, yields:

$$\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2^{CD}(T(0),\sigma_\eta)+T(0)\,\Omega_1^{CD}(T(0),\sigma_\eta)\equiv\mathbb{E}\left[\tilde{B}_0+\tilde{B}_T T(0)+\tilde{B}_z\tilde{z}\right],$$

where (suppressing the argument $\sigma_s^{-1}(T(0)-P_U(T(0),\tilde{z},0))$ of h and h'):

$$\begin{aligned}
\tilde{B}_0 &= \sigma_s^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)h+\sigma_s^4\frac{\kappa\left(\sigma_\eta^2+\sigma_v^2\right)\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau\sigma_\eta^2\sigma_v^2\sigma_z^2\sigma_\varepsilon^2}(2+h'); \\
\tilde{B}_T &= \sigma_s^4\left[\left(\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}\right)\left(\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]}-\frac{1}{\sigma_\varepsilon^2}-\frac{1}{\sigma_p^2}\right)-\frac{\left(\sigma_\eta^2+\sigma_v^2\right)\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\sigma_\eta^2\sigma_v^2\sigma_z^2\sigma_\varepsilon^4}h'\right]; \\
\tilde{B}_z &= -\sigma_s^4\frac{(2+h')\left(\sigma_\eta^2+\sigma_v^2\right)\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau\sigma_\eta^2\sigma_v^2\sigma_z^2\sigma_\varepsilon^2}.
\end{aligned}$$

In summary, $\frac{\partial}{\partial\sigma_\eta}\Pr(\tilde{v}>T(\tilde{y}))\lesseqgtr 0 \Leftrightarrow \mathbb{E}\left[\tilde{B}_0+\tilde{B}_T T(0)+\tilde{B}_z\tilde{z}\right]\lesseqgtr 0$.

Crowding out. We next establish the sufficient conditions stated in Proposition 8 for $\frac{\partial}{\partial\sigma_\eta}\Pr(\tilde{v}>T(\tilde{y}))>0$: either $\Pr(\text{Disclosure})<\frac{1}{2}$ and $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}<\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]}$, or $\Pr(\text{Disclosure})>\frac{1}{2}$. Given that $h'>-1$, we have that $\tilde{B}_0>0$. Furthermore, because $\tilde{z}$ is mean zero,

$$\begin{aligned}
\mathbb{E}\left[\tilde{B}_z\tilde{z}\right] \ \propto -\mathbb{E}\left[h'\left(\tfrac{T(0)-P_U(T(0),\tilde{z},0)}{\sigma_s}\right)\tilde{z}\right] &= -\operatorname{cov}\left[h'\left(\frac{T(0)-P_U(T(0),\tilde{z},0)}{\sigma_s}\right),\tilde{z}\right] \\
&= \Delta_z\sigma_z^2\mathbb{E}\left[h''\left(\frac{T(0)-P_U(T(0),\tilde{z},0)}{\sigma_s}\right)\right],
\end{aligned}$$

where the second line follows by Stein's lemma. As $h''>0$ and $\Delta_z>0$, this expression is positive. Note further that $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}<\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]}\Rightarrow\tilde{B}_T>0$. Combining these facts, when $T(0)>0$ and $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}<\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]}$, $\mathbb{E}\left[\tilde{B}_0+\tilde{B}_T T(0)+\tilde{B}_z\tilde{z}\right]>0$. Now, applying Lemma 2, $T(0)>0$ if and only if c is such that $\forall y, T(y) > E[\tilde{v}|y]$. This, in turn, is equivalent to $\Pr(\text{Disclosure}) < \frac{1}{2}$. Next, after performing tedious calculations, we can rewrite $\mathbb{E}\left[\tilde{B}_0 + \tilde{B}_T T(0) + \tilde{B}_z \tilde{z}\right]$ in the following form:

$$\frac{\sigma_\eta \sigma_v \sigma_z^3 \sigma_\varepsilon^6 \left(\sigma_\eta^2 + \sigma_v^2\right)}{\left(\sigma_v^2 \left(\tau^2 \sigma_\eta^2 + \sigma_z^2 \sigma_\varepsilon^2 \left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right) + \sigma_\eta^2 \sigma_z^2 \sigma_\varepsilon^4\right)^{3/2}} \mathbb{E}\left[h + \frac{T(0) - P_U(T(0), \tilde{z}, 0)}{\sigma_s} h' + 2\frac{T(0) - P_U(T(0), \tilde{z}, 0)}{\sigma_s}\right]$$
$$- \mathbb{E}\left[\frac{(1+h')\, T(0)\, \sigma_z^2 \sigma_\varepsilon^4 \left(\sigma_\eta^2 + \sigma_v^2\right)}{\sigma_v^2 \left(\tau^2 \sigma_\eta^2 + \sigma_z^2 \sigma_\varepsilon^2 \left(\sigma_\eta^2 + \sigma_\varepsilon^2\right)\right) + \sigma_\eta^2 \sigma_z^2 \sigma_\varepsilon^4}\right].$$

Now, it can be verified that the inverse-Mills ratio satisfies $h(x) + xh'(x) + 2x > 0$. Together with the fact that $h'(x) > -1$, we have $T(0) < 0 \Rightarrow \mathbb{E}\left[\tilde{B}_0 + \tilde{B}_T T(0) + \tilde{B}_z \tilde{z}\right] > 0$.

Crowding in. We next move to prove the sufficient condition stated in Proposition 7 for $\frac{\partial}{\partial \sigma_\eta} \Pr(\tilde{v} > T(\tilde{y})) < 0$: c is large and $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} > \frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]}$. Note Proposition 3 extends immediately to the case with public information so that $\frac{\partial T(0)}{\partial c} > 0$. Furthermore, since, for any finite T, $\lim_{c\to\infty} \Omega^{CD}(T, \sigma_\eta) = -\infty$, we have that $\lim_{c\to\infty} T(0) = \infty$. Therefore,

$$\lim_{c\to\infty} \mathbb{E}\left[\tilde{B}_0 + \tilde{B}_T T(0) + \tilde{B}_z \tilde{z}\right] = \lim_{T(0)\to\infty} \mathbb{E}\left[\tilde{B}_0 + \tilde{B}_T T(0) + \tilde{B}_z \tilde{z}\right].$$

Now, applying the fact that inverse-Mills ratio satisfies $h(x)$ and $h'(x) \to 0$, we obtain:

$$\begin{aligned}
\lim_{T(0)\to\infty} \mathbb{E}\left[\tilde{B}_0\right] &= \lim_{T(0)\to\infty} \mathbb{E}\left[\sigma_s^3 \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\eta^2}\right) h + \sigma_s^4 \frac{\kappa\left(\sigma_\eta^2 + \sigma_v^2\right)\left(\tau^2 + \sigma_z^2 \sigma_\varepsilon^2\right)}{\tau \sigma_\eta^2 \sigma_v^2 \sigma_z^2 \sigma_\varepsilon^2} (2 + h')\right] \\
&= 2\sigma_s^4 \frac{\kappa\left(\sigma_\eta^2 + \sigma_v^2\right)\left(\tau^2 + \sigma_z^2 \sigma_\varepsilon^2\right)}{\tau \sigma_\eta^2 \sigma_v^2 \sigma_z^2 \sigma_\varepsilon^2}; \\
\lim_{T(0)\to\infty} \mathbb{E}\left[\tilde{B}_z \tilde{z}\right] &= \lim_{T(0)\to\infty} \mathbb{E}\left[-\tilde{z} * \sigma_s^4 \frac{(2 + h')\left(\sigma_\eta^2 + \sigma_v^2\right)\left(\tau^2 + \sigma_z^2 \sigma_\varepsilon^2\right)}{\tau \sigma_\eta^2 \sigma_v^2 \sigma_z^2 \sigma_\varepsilon^2}\right] = 0; \\
\lim_{T(0)\to\infty} \tilde{B}_T &= \lim_{T(0)\to\infty} \sigma_s^4 \left[\left(\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}\right)\left(\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]} - \frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_p^2}\right) - \frac{\left(\sigma_\eta^2 + \sigma_v^2\right)\left(\tau^2 + \sigma_z^2 \sigma_\varepsilon^2\right)}{\sigma_\eta^2 \sigma_v^2 \sigma_z^2 \sigma_\varepsilon^4} h'\right] \\
&= \sigma_s^4 \left(\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}\right)\left(\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]} - \frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_p^2}\right).
\end{aligned}$$

Thus,

$$\lim_{T(0)\to\infty} \mathbb{E}\left[\tilde{B}_0 + \tilde{B}_T T(0) + \tilde{B}_z \tilde{z}\right] = \operatorname{sign}\left(\frac{1}{\operatorname{var}[\tilde{v}|\tilde{y}]} - \frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_p^2}\right) * \infty.$$

This completes the proof.

Probabilistic info case.

Crowding in. We first show that, when investors' information is not too precise, on a set of values of σ_η of positive measure, we have $\frac{\partial \Pr(\tilde{v}>T^*(\tilde{y})|\tilde{y})}{\partial \sigma_\eta} < 0$. Recall from the proof of part (ii) in Proposition 5 that, as $\sigma_\varepsilon \to \infty$, the equilibrium threshold T^* exceeds the threshold in which there is no noise trade, $\hat{T}$. This result extends to the case in which there is a public signal $\tilde{y}$ because its derivation holds for any prior mean and variance parameters. That is, letting the equilibrium thresholds in the presence and absence of noise trade be $T^*(\tilde{y})$ and $\hat{T}(\tilde{y})$, respectively, we have $T^*(\tilde{y}) > \hat{T}(\tilde{y})$, and thus $\Pr(\tilde{v} > T^*(\tilde{y})|\tilde{y}) - \Pr\left(\tilde{v} > \hat{T}(\tilde{y})|\tilde{y}\right) < 0$. To complete the proof, we show that:

$$\lim_{\sigma_\eta \to 0}\left[\Pr(\tilde{v} > T^*(\tilde{y})|\tilde{y}) - \Pr\left(\tilde{v} > \hat{T}(\tilde{y})|\tilde{y}\right)\right] = 0.$$

This immediately implies that $\frac{\partial \Pr(\tilde{v}>T^*(\tilde{y})|\tilde{y})}{\partial \sigma_\eta} < 0$ for σ_η in a set of positive measure.[29] From equation (58), we have:

$$\Pr(\tilde{v} > T(\tilde{y})|\tilde{y}) = 1 - \Phi\left(\frac{T(0)}{\sqrt{\operatorname{var}[\tilde{v}|\tilde{y}]}}\right),$$

and thus:

$$\lim_{\sigma_\eta \to 0}\left[\Pr(\tilde{v} > T^*(\tilde{y})|\tilde{y}) - \Pr\left(\tilde{v} > \hat{T}(\tilde{y})|\tilde{y}\right)\right] = \lim_{\sigma_\eta \to 0}\left[\Phi\left(\frac{\hat{T}(0)}{\sqrt{\operatorname{var}[\tilde{v}|\tilde{y}]}}\right) - \Phi\left(\frac{T^*(0)}{\sqrt{\operatorname{var}[\tilde{v}|\tilde{y}]}}\right)\right].$$

So, letting $t_n^*(\sigma_\eta) \equiv \lim_{\sigma_\varepsilon \to \infty} \frac{T^*(0)}{\sqrt{\operatorname{var}[\tilde{v}|\tilde{y}]}}$ and $\hat{t}_n(\sigma_\eta) \equiv \lim_{\sigma_\varepsilon \to \infty} \frac{\hat{T}(0)}{\sqrt{\operatorname{var}[\tilde{v}|\tilde{y}]}}$, we need to show that $\lim_{\sigma_\eta \to 0}\left[t_n^*(\sigma_\eta) - \hat{t}_n(\sigma_\eta)\right] = 0$. Note that, as $\sigma_\varepsilon \to \infty$, $P_U(\tilde{v}, \tilde{z}, 0) \to \tau^{-1}\sigma_s^2(\tilde{z} - \kappa)$ and $\sigma_s^2 \to \frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}$. Now, let:

$$\gamma^*(t, \sigma_\eta) \equiv t + \sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\kappa}{\tau} + \mathbb{E}\left[\frac{p\phi\left(t - \sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\tilde{z}-\kappa}{\tau}\right)}{p\Phi\left(t - \sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\tilde{z}-\kappa}{\tau}\right) + 1 - p}\right]$$

[29] To see why, let $q(T, \sigma_\eta)$ denote $\Pr(\tilde{v} > T^*(\tilde{y})|\tilde{y})$ as a function of T and σ_η. Suppose by contradiction that $\frac{\partial q(T^*,\sigma_\eta)}{\partial \sigma_\eta} > 0$ a.e. Then, fixing an $x > 0$, we have $q(T^*, x) - q\left(\hat{T}, x\right) = \delta_x < 0$. Thus, $\forall x' \in (0, x)$, $q(T^*, x') - q\left(\hat{T}, x'\right) < \delta_x$. This contradicts the fact that $\lim_{\sigma_\eta \to 0} q(T^*, \sigma_\eta) = \lim_{\sigma_\eta \to 0} q\left(\hat{T}, \sigma_\eta\right)$.

denote the limit of the equilibrium condition when $\tilde{y}=0$ as $\sigma_\varepsilon \to \infty$, as a function of σ_η and the "normalized" threshold $t=\frac{T}{\sqrt{\text{var}[\tilde{v}|\tilde{y}]}}$. Moreover, let

$$\hat{\gamma}\left(t,\sigma_\eta\right) \equiv t+\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\kappa}{\tau}+\frac{p\phi\left(t+\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\kappa}{\tau}\right)}{p\Phi\left(t+\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{\kappa}{\tau}\right)+1-p}$$

denote the analogous condition when there is no noise trade. Then, by definition, we have $\gamma^*\left(t_n^*\left(\sigma_\eta\right),\sigma_\eta\right)=0$ and $\hat{\gamma}\left(\hat{t}_n\left(\sigma_\eta\right),\sigma_\eta\right)=0$. Now, it is easily verified that $\frac{\partial\hat{\gamma}(t,\sigma_\eta)}{\partial t},\frac{\partial\gamma^*(t,\sigma_\eta)}{\partial t}>0$. Thus, we can apply the implicit function theorem to arrive at:

$$\gamma^*\left(\lim_{\sigma_\eta\to 0}t_n^*\left(\sigma_\eta\right),0\right)=0 \text{ and } \hat{\gamma}\left(\lim_{\sigma_\eta\to 0}\hat{t}_n\left(\sigma_\eta\right),0\right)=0.$$

Now, critically, it can be verified that $\hat{\gamma}\left(t,0\right)=\gamma^*\left(t,0\right)$. This implies that:

$$\hat{\gamma}\left(\lim_{\sigma_\eta\to 0}\hat{t}_n\left(\sigma_\eta\right),0\right)=\gamma^*\left(\lim_{\sigma_\eta\to 0}\hat{t}_n\left(\sigma_\eta\right),0\right)=\gamma^*\left(\lim_{\sigma_\eta\to 0}t_n^*\left(\sigma_\eta\right),0\right).$$

Again, applying the fact that $\frac{\partial\gamma^*(t,\sigma_\eta)}{\partial t}>0$, this yields $\lim_{\sigma_\eta\to 0}t_n^*\left(\sigma_\eta\right)=\lim_{\sigma_\eta\to 0}\hat{t}_n\left(\sigma_\eta\right)$, as desired.

Crowding out. We now show that when σ_ε is sufficiently close to zero, $\frac{\partial}{\partial\sigma_\eta}\Pr\left(\tilde{v}>T\left(\tilde{y}\right)\right)>0$. Let $\Omega^{PI}\left(T,\sigma_\eta\right)$ denote the threshold firm's net expected benefit from disclosure when $y=0$ as a function of T and σ_η in the probabilistic info benchmark:

$$\Omega^{PI}\left(T,\sigma_\eta\right)=\mathbb{E}\left[\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}+\frac{p\phi\left(\frac{T-P_U(T,\tilde{z},0)}{\sigma_s}\right)}{p\Phi\left(\frac{T-P_U(T,\tilde{z},0)}{\sigma_s}\right)+1-p}\right],$$

and let $\Omega_1^{PI}\left(T,\sigma_\eta\right)$ and $\Omega_2^{PI}\left(T,\sigma_\eta\right)$ denote the partial derivatives of Ω^{PI} with respect to its first and second arguments, respectively. From (58), we obtain that $\frac{\partial}{\partial\sigma_\eta}\Pr\left(\tilde{v}>T\left(\tilde{y}\right)\right)$ satisfies:

$$\frac{\partial}{\partial\sigma_\eta}\left[1-\Phi\left(\sqrt{\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}}T\left(0\right)\right)\right]\propto\sigma_\eta^3\left(\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}\right)\frac{\Omega_2^{PI}\left(T\left(0\right),\sigma_\eta\right)}{\Omega_1^{PI}\left(T\left(0\right),\sigma_\eta\right)}+T\left(0\right).$$

Now, we have that:

$$\lim_{\sigma_\varepsilon\to 0}\Omega_1^{PI}\left(T,\sigma_\eta\right)=\mathbb{E}\left[\lim_{\sigma_\varepsilon\to 0}G_p'\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\frac{\partial}{\partial T}\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\right];$$

$$\lim_{\sigma_\varepsilon\to 0}\Omega_2^{PI}\left(T,\sigma_\eta\right)=\mathbb{E}\left[\lim_{\sigma_\varepsilon\to 0}G_p'\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\frac{\partial}{\partial\sigma_\eta}\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\right],$$

where $G_p(\cdot)$ was defined in the proof of Proposition 4. Calculating and simplifying the derivatives in this expression, we obtain that:

$$\begin{aligned}
&\lim_{\sigma_\varepsilon\to 0}\frac{\Omega_2^{PI}(T(0),\sigma_\eta)}{\Omega_1^{PI}(T(0),\sigma_\eta)}\\
&=\lim_{\sigma_\varepsilon\to 0}\left\{-\frac{T(0)\,\sigma_v^2\left(\sigma_v^2\left(2\tau^2\sigma_\eta^2+\sigma_z^2\left(2\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\varepsilon^4\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}{\sigma_\eta\left(\sigma_\eta^2+\sigma_v^2\right)\left(\sigma_v^2\left(\tau^2\sigma_\eta^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}\right.\\
&\left.+\frac{\sigma_\eta\sigma_v^4\sigma_\varepsilon^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau\left(\sigma_\eta^2+\sigma_v^2\right)\left(\sigma_v^2\left(\tau^2\sigma_\eta^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}\frac{\mathbb{E}\left[(\tilde{z}-\kappa)\,G_p'\left(\frac{T(0)-P_U(T,\tilde{z},0)}{\sigma_s}\right)\right]}{\mathbb{E}\left[G_p'\left(\frac{T(0)-P_U(T,\tilde{z},0)}{\sigma_s}\right)\right]}\right\}.
\end{aligned}$$

Taking limits, we obtain:

$$\lim_{\sigma_\varepsilon\to 0}-\frac{\sigma_\eta\sigma_v^4\sigma_\varepsilon^2\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\tau\left(\sigma_\eta^2+\sigma_v^2\right)\left(\sigma_v^2\left(\tau^2\sigma_\eta^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}=0;$$

$$\lim_{\sigma_\varepsilon\to 0}\frac{\mathbb{E}\left[(\tilde{z}-\kappa)\,G_p'\left(\frac{T-P_U(T,\tilde{z},0)}{\sigma_s}\right)\right]}{\mathbb{E}\left[G_p'\left(\frac{T-P_U(T,\tilde{z},0)}{\sigma_s}\right)\right]}=\frac{\mathbb{E}\left[(\tilde{z}-\kappa)\,G_p'\left(-\frac{\tilde{z}-\kappa}{\sigma_z}\right)\right]}{\mathbb{E}\left[G_p'\left(-\frac{\tilde{z}-\kappa}{\sigma_z}\right)\right]}<\infty.$$

Thus, we have:

$$\begin{aligned}
&\lim_{\sigma_\varepsilon\to 0}\left[\sigma_\eta^3\left(\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}\right)\frac{\Omega_2^{PI}(T(0),\sigma_\eta)}{\Omega_1^{PI}(T(0),\sigma_\eta)}+T(0)\right]\\
&=\left[\lim_{\sigma_\varepsilon\to 0}T(0)\right]\left[\lim_{\sigma_\varepsilon\to 0}\left(1-\sigma_\eta^3\left(\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_v^2}\right)\frac{\sigma_v^2\left(\sigma_v^2\left(2\tau^2\sigma_\eta^2+\sigma_z^2\left(2\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\varepsilon^4\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}{\sigma_\eta\left(\sigma_\eta^2+\sigma_v^2\right)\left(\sigma_v^2\left(\tau^2\sigma_\eta^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_\eta^2+\sigma_\varepsilon^2\right)\right)+\sigma_\eta^2\sigma_z^2\sigma_\varepsilon^4\right)}\right)\right]\\
&=-\lim_{\sigma_\varepsilon\to 0}T(0).
\end{aligned}$$

To complete the proof, we show that $\lim_{\sigma_\varepsilon\to 0}T(0)=-\infty$. Note that:

$$\lim_{\sigma_\varepsilon\to 0}\frac{T-P_U(T,\tilde{z},0)}{\sigma_s} = -\frac{\tilde{z}-\kappa}{\sigma_z}.$$

Hence, since $\tilde{z}$ is mean zero, we have:

$$\lim_{\sigma_\varepsilon\to 0}\Omega^{PI}(T,\sigma_\eta)=\mathbb{E}\left[\frac{\kappa}{\sigma_z}+\frac{p\phi\left(-\frac{\tilde{z}-\kappa}{\sigma_z}\right)}{p\Phi\left(-\frac{\tilde{z}-\kappa}{\sigma_z}\right)+1-p}\right]>0.$$

Combined with the fact that $\Omega_1^{PI}>0$, this implies that the solution $T(0)$ to $\Omega^{PI}(T(0),\sigma_\eta)=0$ must approach $-\infty$ as σ_ε approaches 0.

□

B. Extensions

A. Post-Disclosure Public Signal

When studying the impact of public information on voluntary disclosure in Section V, we assume that the public signal arrives prior to disclosure, and is thus observable by the manager. In some contexts, managers may not be able to predict the outcome of public information.[30] Moreover, disclosure regulation likely influences both the amount of existing public information and the amount of public information that is expected to arrive in the future.

To address this issue, in this appendix, we extend our analysis to incorporate a public signal that arrives after the disclosure decision. Suppose now that the firm releases a signal both before (ex-ante) and after (ex-post) the disclosure:

$$\tilde{y}_a = \tilde{v} + \tilde{\eta}_a; \quad \tilde{y}_p = \tilde{v} + \tilde{\eta}_p,$$

respectively, where $\tilde{\eta}_a \sim N(0, \sigma^2_{\eta,a})$ and $\tilde{\eta}_p \sim N(0, \sigma^2_{\eta,p})$ are independent of all other random variables in the model. The key distinction between these signals is that $\tilde{y}_a$ is observable to the manager when disclosing, while $\tilde{y}_p$ is not. In this sense, $\tilde{y}_p$ acts similarly to investors' private information in our model. Thus, the derivation of equilibrium is a straightforward extension of our main analysis, and we summarize the results below.

Proposition B.1. *Suppose that either $p = 1$ and/or $\frac{1}{\sigma^2_{\eta,p}} + \frac{1}{\sigma^2_{\varepsilon}} + \frac{1}{\sigma^2_{p}}$ is sufficiently small, and fix a realization of $\tilde{y}_a = y_a$. Then, there exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T(y_a)$. In this equilibrium, the firm's non-disclosure price takes the same form as in Proposition 6, upon re-defining:*

$$\tilde{\mu}_i = \mathbb{E}\left[\tilde{v}|y_a, \tilde{y}_p, \tilde{s}_i, \tilde{s}_p\right]; \quad \sigma^2_s = var\left[\tilde{v}|y_a, \tilde{y}_p, \tilde{s}_i, \tilde{s}_p\right].$$

Moreover, the equilibrium disclosure threshold satisfies:

$$T(y_a) - c = \mathbb{E}\left[P_{ND}|\tilde{v} = T(y_a), \tilde{y} = y_a\right], \tag{59}$$

where the expectation is taken over $\tilde{s}_p$ and $\tilde{y}_p$.

Proof of Proposition B.1. The proof is a straightforward extension of the main analysis in

[30] See Acharya et al. (2011) and Frenkel et al. (2020) for analyses of post-disclosure public information in other settings.

our paper upon adding $\tilde{y}_p$ to investors' conditioning set. The only step that materially differs is the derivation of Lemma A.3, which establishes sufficient conditions on when the manager is more inclined to disclose as the firm's value increases. The reason is that the ex-post public signal raises the sensitivity of the non-disclosure price to the firm's value. It can be verified that the sensitivity of the price to v (conditional on the public signal $\tilde{y}_a$, which is a known constant) is now $\sigma_s^2\left(\frac{1}{\sigma_{\eta,p}^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)$, as opposed to $\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)$. Thus, the analogous argument to the one in the proof of Lemma A.3 shows that the appropriate sufficient conditions are now that $p=1$ or $\frac{1}{\sigma_{\eta,p}^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}$ sufficiently small. □

While an analytical treatment is not tractable, we next numerically study the probability of voluntary disclosure in this setting. We conduct two analyses. First, we consider how varying only ex-post information quality affects voluntary disclosure. We then consider how simultaneously varying the quality of both ex-ante and ex-post information quality affects voluntary disclosure. The goal of the latter analysis is to provide insight into the effects of persistent differences in disclosure quality, such as those driven by disclosure mandates.

Figure B.1 depicts the results. Observe first that, in the probabilistic information benchmark, ex-post public information raises the likelihood of voluntary disclosure. This is consistent with the findings in the main text. On the other hand, in the costly disclosure benchmark, ex-post information quality lowers the likelihood of voluntary disclosure. However, when jointly varying ex-ante and ex-post information quality, public information may again crowd in voluntary disclosure when c is large.

B. *Noise from Hedging Demands*

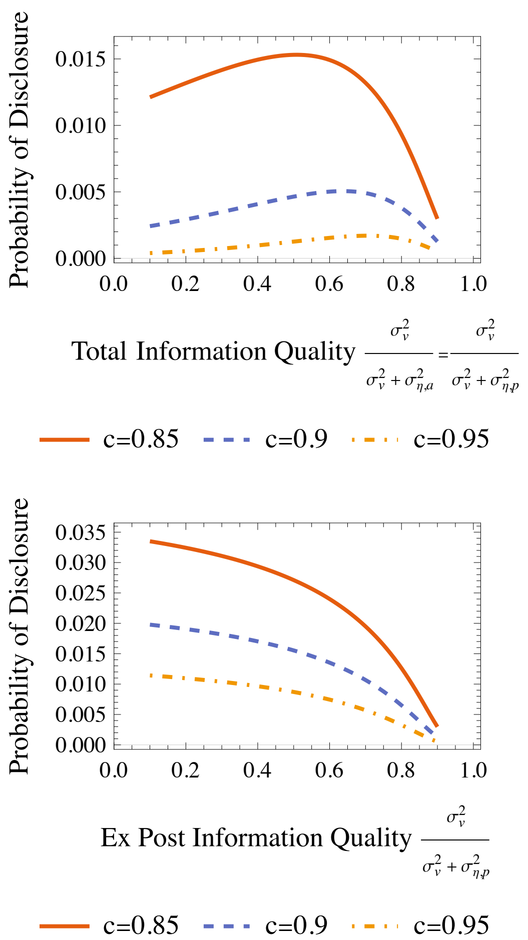
In this appendix, we show that our results continue to hold when noise is driven by investors' desire to hedge outside exposures to the asset, as in, for example, Ganguli and Yang (2009) and Bond and Garcia (2022). Such demands are commonly interpreted as stemming from investors' human capital exposures. The key difference in this setting is that the behavior of the traders who introduce noise into price is now endogenously influenced by the disclosure. Nevertheless, we will show that our results are qualitatively similar in this case.

Formally, suppose now that there is no noise trade. Instead, the investors have non-tradeable exposures of a risk $\tilde{U}$ that is correlated with the stock's payoffs:

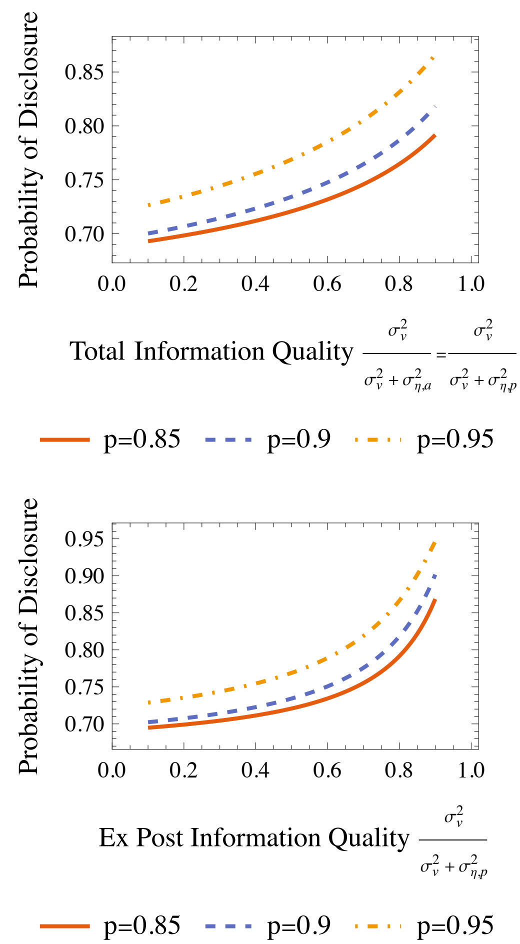
$$\tilde{U}=\tilde{v}+\tilde{\xi} \text{ and } \tilde{\xi}\sim N\left(0,\sigma_\xi^2\right).$$

Figure B.1: Probability of Disclosure vs. Information Quality (Ex-Post Signal)

The figure plots the probability of disclosure as a function of the amount of public information σ_η. The upper plots simultaneously vary the quality of both ex-ante and ex-post information, while the lower plots vary the quality of ex-post information only. Other parameters are set to $\kappa = 0$, $\tau = 1$, and $\sigma_v = 2$. In the lower left (right) plot, $\sigma_{\eta,a}$ is set to 1 (2).



(a) Costly Disclosure Benchmark
($p = 1$, $\sigma_\varepsilon = \sigma_z = 1$)



(b) Probabilistic Information Benchmark
($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)

Investor i's exposure $\tilde{Z}_i$ has both an investor-specific and a common component; the common component serves the role of ensuring the price does not fully reveal investors' private information. Given that it serves an analogous role to noise trade in the main model, we refer to the common component as $\tilde{z}$. Formally, $\tilde{Z}_i = \tilde{z} + \tilde{\zeta}_i$, where:

$$\tilde{\zeta}_i \sim N\left(0, \sigma_\zeta^2\right) \text{ and } \tilde{z} \sim N\left(0, \sigma_z^2\right).$$

We assume that the errors $\tilde{\zeta}_i$ are independent across investors and both $\tilde{z}$ and $\tilde{\zeta}_i$ are independent of all other random variables in the model. To summarize, investor i's terminal wealth given their demand, which we now refer to as $D_{i,H}$ to distinguish it from their demand in our baseline model, satisfies:

$$\tilde{W}_i = D_{i,H}\left(\tilde{v} - P\right) + \left(\tilde{z} + \tilde{\zeta}_i\right)\tilde{U}.$$

In the next proposition, we verify that the general nature of the equilibrium we study is robust to this version of the model. We again focus on equilibria in which the price is monotonic in a linear combination of v and z, which we now refer to as $\tilde{s}_{p,H} \equiv \tilde{v} + \beta_H \tilde{z}$ to distinguish from our main analysis. Similar to other models that introduce hedging demands, there are now two potential equilibria in the trading stage. However, across both equilibria, we show that the conditions to ensure the existence and uniqueness of a threshold disclosure equilibrium are unchanged.

Proposition B.2. *Suppose that $\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2 > 0$. Then, there are two equilibria in which price takes the same form as in our main text, upon replacing $P_U(v,z)$ by:*

$$P_{U,H}(v,z) = \sigma_{s,H}^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right)(v + \beta_H z) - \frac{\sigma_{s,H}^2}{\tau}\kappa,$$

where:

$$\sigma_{s,H}^2 = var\left[\tilde{v}|\tilde{s}_i, \tilde{s}_{p,H}, \tilde{Z}_i\right] = \frac{\beta_H^2\sigma_\zeta^2\sigma_v^2\sigma_z^2\sigma_\varepsilon^2}{\sigma_v^2\left(\sigma_z^2\left(\beta_H^2\sigma_\zeta^2 + \sigma_\varepsilon^2\right) + \sigma_\zeta^2\sigma_\varepsilon^2\right) + \beta_H^2\sigma_\zeta^2\sigma_z^2\sigma_\varepsilon^2},$$

and β_H satisfies:

$$\beta_H = -\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon \pm \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right).$$

Moreover, a threshold disclosure equilibrium exists and is unique when either $p = 1$ or σ_ε^2 is sufficiently large.

Proof. We start by characterizing the non-disclosure price. Note that investor j chooses their demand to solve:

$$
\begin{aligned}
& \arg\max_{D_{j,H}} \mathbb{E}_j \left[\exp\left(-\tau^{-1} D_{j,H}\left(\tilde{v} - P_{ND}\right) - \tau^{-1}\tilde{Z}_j\tilde{U}\right)\right] \\
= \; & \arg\max_{D_{j,H}} \mathbb{E}_j \left[\exp\left(-\tau^{-1}\left(D_{j,H} + \tilde{Z}_j\right)\left(\tilde{v} - P_{ND}\right) - \tau^{-1}\tilde{Z}_j\left(P_{ND} + \tilde{\xi}\right)\right)\right] \\
= \; & \arg\max_{D_{j,H}} \exp\left(-\tau^{-1}\tilde{Z}_j\left(P_{ND} + \tilde{\xi}\right)\right) \mathbb{E}_j \left[\exp\left(-\tau^{-1}\left(D_{j,H} + \tilde{Z}_j\right)\left(\tilde{v} - P_{ND}\right)\right)\right] \\
= \; & \arg\max_{D_{j,H}} \mathbb{E}_j \left[\exp\left(-\tau^{-1}\left(D_{j,H} + \tilde{Z}_j\right)\left(\tilde{v} - P_{ND}\right)\right)\right].
\end{aligned}
$$

This equation reveals that the investor's optimal demand plus their outside exposure, $D_{j,H} + \tilde{Z}_j$, satisfies precisely the same maximization problem as the investor's optimal demand in our baseline model, D_j. Consequently, following Lemma A.2, we have that investor j's demand conditional on non-disclosure satisfies:

$$
D_{j,H} + \tilde{Z}_j = \frac{\tau}{\sigma_{s,H}^2}\left[\mu_{j,H} - g^{-1}\left(P_{ND}\right)\right],
$$

and thus:

$$
\begin{aligned}
P_{ND} &= g\left(\int \mu_{j,H} dj - \frac{\sigma_{s,H}^2}{\tau}\left(\int \tilde{Z}_j dj - \kappa\right)\right) \\
&= g\left(\int \mu_{j,H} dj - \frac{\sigma_{s,H}^2}{\tau}\left(\tilde{z} - \kappa\right)\right),
\end{aligned}
$$

where:

$$
\begin{aligned}
\mu_{j,H} &\equiv \mathbb{E}\left[\tilde{v} | \tilde{s}_j, \tilde{s}_{p,H}, \tilde{Z}_j\right]; \\
\sigma_{s,H}^2 &\equiv \operatorname{var}\left[\tilde{v} | \tilde{s}_j, \tilde{s}_{p,H}, \tilde{Z}_j\right].
\end{aligned}
$$

Now, applying Bayes' rule, we obtain:

$$
\begin{aligned}
\mu_{j,H} &= \frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\tilde{s}_j + \frac{\sigma_{s,H}^2}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\tilde{s}_{p,H} - \frac{\sigma_{s,H}^2}{\beta_H \sigma_\zeta^2}\tilde{Z}_j; \\
\sigma_{s,H}^2 &= \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right)^{-1}.
\end{aligned}
$$

Hence, we have:

$$
\begin{aligned}
& \int \mu_{j,H} dj - \frac{\sigma_{s,H}^2}{\tau}\tilde{z} \\
= \; & \sigma_{s,H}^2 \int \left[\frac{1}{\sigma_\varepsilon^2}\tilde{s}_j + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\tilde{s}_{p,H} - \frac{1}{\beta_H \sigma_\zeta^2}\tilde{Z}_j\right] dj - \frac{\sigma_{s,H}^2}{\tau}\tilde{z}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\tilde{v} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)(\tilde{v} + \beta_H \tilde{z}) - \frac{1}{\beta_H \sigma_\zeta^2}\tilde{z} - \frac{\sigma_{s,H}^2}{\tau}\tilde{z} \\
&= \sigma_{s,H}^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right)\tilde{v} + \sigma_{s,H}^2\left(\frac{1}{\beta_H}\left(\frac{1}{\sigma_z^2}\right) - \frac{1}{\tau}\right)\tilde{z}.
\end{aligned}
$$

This implies that, for an equilibrium, we must have:

$$
\beta_H - \frac{\frac{1}{\beta_H}\left(\frac{1}{\sigma_z^2}\right) - \frac{1}{\tau}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)} = 0
$$

$$
\Leftrightarrow \tau\sigma_\zeta^2\beta_H^2 + \sigma_\zeta^2\sigma_\varepsilon^2\beta_H + \tau\sigma_\varepsilon^2 = 0.
$$

When $\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2 > 0$, this has two solutions, which correspond to two equilibria:

$$
\beta_H = -\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon \pm \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right).
$$

Finally, substituting for $g(\cdot)$, we obtain the result in the proposition. We next move to study the disclosure equilibrium. Recall from the proof of Proposition 2 that, in order for a threshold equilibrium exist, we need that:

$$
\forall v, z, T \in \mathcal{R},\ \frac{\partial P_{ND}(v, z; T)}{\partial v} < 1.
$$

We next show that this holds when either $p = 1$ or $p < 1$ and σ_ε is large. When $p = 1$, it is sufficient to have that $\frac{\partial P_{U,H}(v,z)}{\partial v} < 1$ (see expression (42)). This continues to hold in both equilibria in the financial market since:

$$
\frac{\partial P_{U,H}(v,z)}{\partial v} = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right)^{-1}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right).
$$

Next, when $p < 1$, we obtain:

$$
\begin{aligned}
\frac{\partial P_{ND}(v, z; T)}{\partial v} &= \frac{\partial P_{U,H}(v,z)}{\partial v} * g'(P_{U,H}(v,z)) \\
&= \sigma_{s,H}^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right) * \frac{\mathrm{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_{U,H}\right]}{\sigma_{s,H}^2} \\
&= \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right) * \mathrm{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_{U,H}\right] \\
&< \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\right) * \sigma_v^2\left(1 + \frac{p(1-p)}{2}\right).
\end{aligned}
$$

Thus, a sufficient condition is that:

$$\frac{1}{\sigma_{\varepsilon}^{2}}+\frac{1}{\beta_{H}^{2}}\left(\frac{1}{\sigma_{\zeta}^{2}}+\frac{1}{\sigma_{z}^{2}}\right)<\left[\sigma_{v}^{2}\left(1+\frac{p\left(1-p\right)}{2}\right)\right]^{-1}.$$

Note that β_H^2 is smaller, and thus this inequality is more difficult to satisfy, in the equilibrium in which $\beta_H = -\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon - \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right)$. In this case, applying L'Hopital's rule, we obtain:

$$\lim_{\sigma_\varepsilon\to\infty}\beta_H^{-2} = \lim_{\sigma_\varepsilon\to\infty}\left(-\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon - \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right)\right)^{-2} = 0.$$

Thus, it is sufficient that σ_ε is large. Finally, one can verify that the proof that a threshold equilibrium is unique when it exists depends upon the function $P_{U,H}$ only in that it requires $\frac{\partial P_{U,H}(v,z)}{\partial v} < 1$. We have shown above that this is true when either $p = 1$ or $p < 1$ and σ_ε is large. □

The next corollary replicates our two main findings regarding the nature of equilibrium in the main text: misvaluation and the potential that public information crowds in voluntary disclosure.

Corollary 2. *Conditional on non-disclosure, the firm's expected value (generally) differs from its expected price.*

(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), the firm's expected value exceeds its expected price, that is,

$$\mathbb{E}\left[P_{ND}|ND\right] < \mathbb{E}\left[\tilde{v}|ND\right].$$

(ii) In the probabilistic information endowment benchmark (i.e., $p < 1$, $c = 0$), when investors' private signal precision $1/\sigma_\varepsilon$ and the aggregate supply κ are sufficiently low, the firm's expected price exceeds its expected value, that is,

$$\mathbb{E}\left[P_{ND}|ND\right] > \mathbb{E}\left[\tilde{v}|ND\right].$$

Now, suppose that, as in Section V, the firm releases the public signal $\tilde{y} = \tilde{v} + \tilde{\eta}$ prior to the disclosure decision.

(iii) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), an increase in the precision of the public signal increases the probability of disclosure when $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{var\left[\tilde{s}_{p,H}|\tilde{v}\right]} > \frac{1}{var\left[\tilde{v}|\tilde{y}\right]}$ and disclosure is sufficiently expensive.

(iv) *In the probabilistic information benchmark, when investors' private information is not too precise, there exists a range of values of public information precision such that an increase in the precision of the public signal increases the probability of disclosure.*

Proof. As the proofs follow the same structure as those in the main text, we highlight only the steps that change in this version of the model.

Part (i) Note that the proof of Proposition 5 relied upon two features regarding $P_U(\tilde{v}, \tilde{z})$ and $\tilde{\mu}_j$: (i) the coefficient on $\tilde{v}$ in each of these expressions is the same, and (ii) $\text{var}\left[P_U(\tilde{v}, \tilde{z})|\tilde{v}\right] > \text{var}\left[\tilde{\mu}_j|\tilde{v}\right]$. It can immediately be verified that the coefficients on $\tilde{v}$ in $P_{U,H}$ and $\tilde{\mu}_{j,H}$ remain identical. Next, observe that:

$$\begin{aligned}
\text{var}\left[P_{U,H}(\tilde{v}, \tilde{z})|\tilde{v}\right] &= \left(\sigma_{s,H}^2\left(\frac{1}{\beta_H\sigma_z^2} - \frac{1}{\tau}\right)\right)^2\sigma_z^2 = \frac{1}{\beta_H^2\tau^2}\frac{\sigma_{s,H}^4}{\sigma_z^2}\left(\tau - \beta_H\sigma_z^2\right)^2 \\
\text{var}\left[\tilde{\mu}_{j,H}|\tilde{v}\right] &= \text{var}\left[\frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\tilde{s}_j + \frac{\sigma_{s,H}^2}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\tilde{s}_p - \frac{\sigma_{s,H}^2}{\beta_H\sigma_\zeta^2}\tilde{Z}_j|\tilde{v}\right] \\
&= \text{var}\left[\frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\tilde{\varepsilon}_j + \frac{\sigma_{s,H}^2}{\beta_H^2}\left(\frac{1}{\sigma_\zeta^2} + \frac{1}{\sigma_z^2}\right)\beta_H\tilde{z} - \frac{\sigma_{s,H}^2}{\beta_H\sigma_\zeta^2}\left(\tilde{z} + \tilde{\zeta}_i\right)\right] \\
&= \text{var}\left[\frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\tilde{\varepsilon}_j + \frac{\sigma_{s,H}^2}{\beta_H\sigma_z^2}\tilde{z} - \frac{\sigma_{s,H}^2}{\beta_H\sigma_\zeta^2}\tilde{\zeta}_i\right] \\
&= \left(\frac{\sigma_{s,H}^2}{\sigma_\varepsilon^2}\right)^2\sigma_\varepsilon^2 + \left(\frac{\sigma_{s,H}^2}{\beta_H\sigma_z^2}\right)^2\sigma_z^2 + \left(\frac{\sigma_{s,H}^2}{\beta_H\sigma_\zeta^2}\right)^2\sigma_\zeta^2.
\end{aligned}$$

Substituting and simplifying, in the equilibrium in which $\beta_H = -\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon - \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right)$, we have:

$$\text{var}\left[\tilde{\mu}_{j,H}|\tilde{v}\right] - \text{var}\left[P_{U,H}(\tilde{v}, \tilde{z})|\tilde{v}\right] = \frac{\sigma_{s,H}^4\left(\sigma_\zeta\left(\sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2} - \sigma_\zeta\sigma_\varepsilon\right) - 2\sigma_z^2\sigma_\varepsilon\right)}{2\tau^2\sigma_\varepsilon} < 0,$$

and in the equilibrium in which $\beta_H = -\frac{\sigma_\varepsilon}{2\sigma_\zeta\tau}\left(\sigma_\zeta\sigma_\varepsilon + \sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2}\right)$, we have:

$$\text{var}\left[\tilde{\mu}_{j,H}|\tilde{v}\right] - \text{var}\left[P_{U,H}(\tilde{v}, \tilde{z})|\tilde{v}\right] = -\frac{\sigma_{s,H}^4\left(\sigma_\zeta\left(\sqrt{\sigma_\zeta^2\sigma_\varepsilon^2 - 4\tau^2} + \sigma_\zeta\sigma_\varepsilon\right) + 2\sigma_z^2\sigma_\varepsilon\right)}{2\tau^2\sigma_\varepsilon} < 0.$$

Part (ii) Note the proof of Proposition 5 does not rely upon the specific properties of P_U, except for the fact that it remains random when $\sigma_\varepsilon \to \infty$. It is easily verified that $P_{U,H}$ also satisfies this property.

Part (iii) Following the same steps as in the proof of Proposition 7, we obtain:

$$\begin{aligned}\operatorname{sign}\left(\frac{\partial}{\partial\sigma_\eta}\Pr\left(\tilde{v}>T(\tilde{y})\right)\right) &= \operatorname{sign}\left(\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2^{CD}\left(T\left(0\right),\sigma_\eta\right)+T\left(0\right)\Omega_1^{CD}\left(T\left(0\right),\sigma_\eta\right)\right)\\ &\underset{c\to\infty}{\longrightarrow} \operatorname{sign}\left(\frac{1}{var\left[\tilde{v}|\tilde{y}\right]}-\frac{1}{\sigma_\varepsilon^2}-\frac{1}{var\left[\tilde{s}_{p,H}|\tilde{v}\right]}\right).\end{aligned}$$

Thus, if $\frac{1}{var[\tilde{v}|\tilde{y}]}-\frac{1}{\sigma_\varepsilon^2}-\frac{1}{var\left[\tilde{s}_{p,H}|\tilde{v}\right]}<0$ when c grows large, additional public information raises the probability of disclosure.

Part (iv) As discussed in the proof of part (ii), the proof of Proposition 5 immediately extends to the current model. As a result, the disclosure threshold when $\sigma_\varepsilon\to\infty$ strictly exceeds the corresponding threshold in the absence of noise in price. Thus, the same steps applied in the proof of Proposition 7 can be applied to show the stated result. □