Information Control and Firm Value[*]

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Abstract

Information control governs how easily internal communication leaks to the market. A manager exerts costly effort and privately communicates about fundamentals to an employee, who aligns her action accordingly. Looser control makes leaks more precise, increasing price informativeness and managerial effort through stronger market discipline. However, it induces the manager to distort internal communication, generating misalignment within the firm. When alignment is sufficiently important, looser control reduces firm value despite increasing price informativeness. Optimal control tightness increases with the importance of alignment, cash-flow volatility, and disclosure quality. In contrast, mandating greater public disclosure improves both price informativeness and real efficiency.

JEL: G10, G12, G14, G32
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1 Introduction

Publicity is justly commended as a remedy for social and industrial diseases. Sunlight is said to be the best of disinfectants; electric light the most efficient policeman.

— Louis Brandeis, Other People's Money – and How Bankers Use It (1914)

Transparency is often viewed as unambiguously good for markets. However, the relationship between information flows and firm performance is more nuanced than the above quote suggests. Firms face a fundamental tension. A manager must communicate transparently with employees to coordinate actions and create value. However, this internal communication can leak to the market when information control is imperfect. While such leaks can strengthen market discipline on managerial decisions, they can also induce the manager to strategically distort internal communication in the first place, thereby worsening internal coordination and decreasing firm value.

Information leakage is pervasive in practice. Corporate insiders routinely share information with family, friends, and professional contacts, sometimes intentionally and sometimes inadvertently.[1] Regulatory programs such as the SEC's Dodd-Frank Whistleblower Program explicitly incentivize employees to report information externally.[2] Financial and reputational incentives can lead employees to leak information about upcoming products or features to the media. Even without explicit incentives, confidentiality agreements are imperfectly enforced, electronic communications can be hacked or intercepted, and employees may discuss internal matters in social settings.

We analyze how the extent to which firms can restrict internal information from becoming available to the public – the *tightness of information control* – affects internal communication, managerial decisions, price informativeness, and real efficiency. Because looser information control makes market participants better informed, it can improve managerial behavior and increase price informativeness. However, when internal alignment and effective internal communication are sufficiently important, looser control can *reduce* firm value and harm real efficiency by distorting internal communication. Our mechanism highlights a novel trade-off between *internal* and *external* information efficiency: policies that increase market discipline by improving investor information can lead to worse internal communication and lower coordination within the firm.

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[1]For example, see Ahern (2017) for evidence on the prevalence and motivations of information tipping.

[2]Since its inception, the SEC Whistleblower Program has awarded more than $1.9 billion to 397 whistleblowers and has led to enforcement actions with remedies exceeding $6 billion. In fiscal year 2023 alone, the SEC's Office of the Whistleblower received over 18,000 tips.

In practice, firms may engage in costly investments to ensure tighter controls: secure communication systems, compartmentalized information flows, restrictions and monitoring of employee communications, and enforcement of non-disclosure agreements. Our analysis suggests that the impact of such investments on firm value depends systematically on firm characteristics and on regulatory policies that affect the firm's mandatory disclosures. Specifically, we show that the optimal tightness of information control increases with the relative costs of misalignment, the firm's cash-flow volatility, and the precision of public disclosures.

Model Overview and Intuition. The firm consists of a manager (he) and an employee (she). The manager chooses how much costly effort to exert, which increases the mean of interim cash-flows. To incentivize effort, his compensation increases with the firm's stock price. There is a mandatory public disclosure (e.g., earnings) that provides a noisy public signal about the interim cash-flows. In addition, the manager privately communicates with the employee about the realization of cash-flows – and crucially, he can bias this internal report at a private cost. The employee's objective is to maximize the alignment between her action and interim cash-flows, based on the public disclosure and the manager's internal message.

A key feature of our analysis is that internal communication may leak to the market. We parameterize the extent of this permeability by the *tightness of information control*: tighter (looser) controls lead to less (more) informative leaks. The firm's terminal value depends both on the realized interim cash flows and on the extent of internal alignment, and the stock price reflects the market's conditional expectation of this terminal value given the public disclosure and the leaked information.

In this setting, we characterize how public disclosure quality and the tightness of information control (which governs the informativeness of the leaked information) affect managerial effort, price informativeness and firm value (real efficiency). We show that both higher disclosure quality and looser information control lead to more managerial effort and higher price informativeness. However, while higher disclosure quality always increases firm value, looser information control can lower real efficiency when internal alignment is sufficiently important for firm value.

To see why, note that an increase in public disclosure quality or a loosening of information control both make the price more informative about interim cash-flows, and thus lead to greater managerial effort and higher firm value. Yet these changes have opposite effects on the relative weights the market places on the public disclosure versus the leak. When information control is looser, the market relies more on the leaked signal when valuing the firm. This strengthens the manager's incentive to distort his internal communication to the employee because these messages (indirectly) influence his stock-based compensation more.

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However, this bias makes the internal communication less informative and reduces the quality of internal coordination.

When the importance of internal alignment is low, the benefit from increased effort dominates, so loosening information control increases firm value and real efficiency due to enhanced market discipline. When misalignment is sufficiently costly, however, the coordination loss from worsening internal communication can dominate, and real efficiency becomes hump-shaped in the looseness of information control. In this case, we show that there exists an optimal, interior level of information control that maximizes firm value. Moreover, this optimal level of information control tightness increases with the relative importance of alignment, the volatility of interim cash-flows, and the precision of public disclosure.

In an extension, we allow the manager to manipulate the public disclosure as well as internal communication. This analysis yields an additional insight: looser information control reduces the market's reliance on public disclosures, weakening the manager's incentives to bias these disclosures. Thus, loosening information control can *improve* disclosure quality even as it worsens internal communication. The net effect on firm value continues to depend on the relative importance of internal alignment.

Contributions and Implications. Our analysis makes several contributions. First, we provide a framework for analyzing the trade-off between market discipline and internal coordination when internal communication can leak externally. This highlights a novel organizational cost of transparency which stems from the strategic distortion of internal communication, rather than from standard proprietary or competition-based costs. This trade-off is central to evaluating policies that affect the permeability of internal information, including whistleblower programs, data protection regulations, and corporate confidentiality policies.

Second, our analysis provides predictions for how information control policies vary across firms. All else equal, we show that firms with higher costs of misalignment and with higher cash-flow volatility should optimally choose tighter information controls. These include firms that are vertically integrated, have high organizational complexity and cross-functional dependencies, and operate in industries that require greater specialization. This helps explain why firms in the same industry can have divergent responses to leaks, such as Apple's strict policies compared to Google's more relaxed approach.[3] More generally, our results imply that

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[3]Apple is famously secretive about its upcoming products and aggressively pursues leakers (see "Here's why Apple says it hates leaks", July 2021, and "Apple sues leaker Jon Prosser for stealing iOS secrets," July 2025, from The Verge), while Google has adopted a more relaxed policy (see "Google solves its Pixel 10 leaks by just showing us the phone," July 2025, from The Verge). This is consistent with our model's predictions to the extent that misalignment is more costly for Apple than for Google: while Apple's success relies heavily on vertically integrating its software and hardware offerings, Google's operations are more decentralized and

variation in organization, operating environment, and the public disclosure regime should map into systematic cross-sectional differences in optimal information control tightness.

Third, we show that policies with similar effects on price informativeness can have qualitatively different implications for firm value. While both improving public disclosure and loosening information control increase informativeness and managerial effort, only the former robustly enhances firm value. This result highlights a fundamental measurement issue: price-based metrics alone are insufficient for assessing real efficiency.[4]

Because of this distinction, different regulatory instruments, those targeting information control versus those targeting disclosure, have systematically different welfare effects. Regulatory policies that relax information control (e.g., stronger whistleblower protections, weaker enforcement of confidentiality agreements) can improve real efficiency for some firms while harming others in which internal coordination is critical. In contrast, disclosure-based interventions, such as mandating more informative public disclosures or increasing scrutiny of public reports, enhance both price informativeness and real efficiency without distorting internal communication.

Outline. The rest of the paper is organized as follows. The next section briefly discusses the related literature and the paper's contribution. Section 3 presents the model and discusses the key assumptions, while Section 4 characterizes the equilibrium. Section 5 presents the analysis of how information control and public disclosure quality affect price informativeness and real efficiency in our setting. Section 6 extends the analysis to allow the manager to manipulate the public disclosure. Section 7 concludes with a discussion of empirical predictions and policy implications. All proofs and additional analysis are in the Appendix.

2 Related Literature

Our model builds on the seminal work by Dye (1988) and Fischer and Verrecchia (2000), and on subsequent work by Frankel and Kartik (2019) and Ball (2025) that studies a sender's incentives to engage in costly manipulation when communicating with a receiver. We contribute to this line of research by embedding manipulable communication inside the firm: the manager in our model not only communicates with an employee, but also chooses costly effort that affects the distribution of fundamentals. As a result, the incentives for distorting communication and for exerting effort are jointly determined and depend endogenously on

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less coordinated across its product lines.

[4]This echoes the distinction between forecasting and revelatory price efficiency highlighted by Bond, Edmans, and Goldstein (2012).

the tightness of information control.

A central feature of our model is that internal communication serves a coordination role within the firm: the employee's action should be aligned with firm fundamentals, and internal communication helps achieve this alignment. This builds on the broader literature on strategic communication in organizations (e.g., Crawford and Sobel, 1982; Dessein, 2002; Alonso, Dessein, and Matouschek, 2008; and Rantakari, 2008). Our setting differs from this literature in an important way: in our model, the manager and the employee have aligned preferences over the employee's action, but the manager's communication is distorted by the possibility that his message will be leaked to the market. In this sense, our analysis provides a capital-market foundation for the "transparency trade-offs" emphasized in economics (e.g., Morris and Shin, 2002; and Prat, 2005): making internal discourse more externally observable can strengthen discipline, but can also degrade the quality of information transmitted inside the firm. This perspective connects to Bolton, Brunnermeier, and Veldkamp (2013) and Dessein and Santos (2021), who study how alignment and coordination shape corporate culture and organizational structure. Our analysis further shows that policies affecting information permeability can have important effects on internal coordination.

Our paper contributes to the growing literature on information leakage and its consequences for firm behavior. While much of the existing work focuses on intentional disclosures, we emphasize the role of *unintended* information flows. In our setting, leaks occur despite the firm's attempts to maintain confidentiality. Several empirical papers document the prevalence and consequences of information leakage in practice (e.g., Keown and Pinkerton, 1981; Meulbroek, 1992; Campbell, Gordon, Loeb, and Zhou, 2003; Ahern, 2017; Green, Huang, Wen, and Zhou, 2019; and Akey, Grégoire, and Martineau, 2022). These papers highlight that information control is imperfect in practice, motivating our theoretical analysis of how firms should respond. More closely related is the theoretical work on strategic disclosure and information control (e.g., Admati and Pfleiderer, 2000; Ebert, Schäfer, and Schneider, 2019; and Frenkel, Guttman, and Kremer, 2020), as well as the broader literature about disclosure regulations (see, e.g., Leuz and Wysocki, 2016; and Goldstein and Yang, 2017). Our paper differs by focusing on *internal* communication that directly affects firm value but may inadvertently leak to external markets, rather than on strategic disclosure choices. This distinction allows us to separate (i) the quality of public disclosure and (ii) the tightness of information control governing informal leakage, and to study how these two levers interact.

Our paper is also related to the literature on whistleblowing, which provides an institutionally important channel through which internal information leaks to outsiders. Nan and Zheng (2023) study a setting where a manager who detects a product defect can share

this information with an employee, who may then blow the whistle. They show that strong whistleblower incentives can discourage managers from reporting defects internally. Nan, Tang, and Zhang (2024) and Nan, Tang, and Ye (2025) study how whistleblower programs affect information quality and regulatory enforcement. Our analysis abstracts from explicitly modeling the costs and incentives for whistleblowing. Instead, our notion of information control broadly captures policies and institutions that affect how much internal information is leaked to the public, which includes whistleblower programs.

The extension in Section 6 relates to the literature on communication with multiple receivers that compares the effectiveness of private versus public communication.[5] In this setting, the manager communicates differently with the market and the employee, since he wishes to induce different behavior from these parties. The manager's incentives to communicate are influenced by the fact that while his disclosure to the market is public, his private communication with the employee may also be leaked to the market. This is in contrast to much of the existing literature which assumes that receivers cannot learn each others' private signals.[6]

Finally, our paper highlights how changes in information control can have opposite effects on measures of price efficiency and real efficiency, connecting to the broader literature on the real effects of financial markets (see the recent surveys by Bond et al. (2012) and Kanodia and Sapra (2016)). Our paper highlights a novel channel through which greater information permeability can increase price informativeness and strengthen market discipline, yet reduce firm value by distorting internal communication, a cost of transparency distinct from standard proprietary costs.

3 Model

We consider a setting in which a firm's manager and employee take actions that affect firm value. The overall value of the firm depends on both the manager's costly effort and the extent to which the employee's action is aligned with fundamentals, which depends on the manager's internal communication.

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[5]See Farrell and Gibbons (1989); Levy and Razin (2004); Johns (2007); Koessler (2008); and Goltsman and Pavlov (2011) for general communication games. Newman and Sansing (1993) and Gigler (1994) consider the problem that a firm's disclosure of information about product demand may be simultaneously observed by the capital market, shareholders and competitors. Gertner, Gibbons, and Scharfstein (1988) examine the choice of financial structure when the financing contract is observed by both the capital market and a competing firm in a signaling model. Spiegel and Spulber (1997) consider the audiences as the capital market and a regulator.

[6]An exception is Hagenbach and Koessler (2010), who study formation of a communication network where the agents can exchange private messages.

A central feature of the environment is that this internal communication may be (imperfectly) inferred by outsiders through leaks. We model the **tightness** of the firm's **information control** by a policy variable $z \geq 0$: a higher z corresponds to tighter information control and makes any public signal about internal communication noisier. As we shall show, this creates a trade-off between *internal coordination*, which benefits from more truthful internal communication, versus *external discipline / price informativeness*, which benefits from more permeable information flows.

Specifically, there are four dates $\{1, 2, 3, 4\}$ and three participants: (i) the firm manager M (he), (ii) the firm employee E (she), and (iii) a representative risk-neutral investor. The terminal (date four) value of the firm $V(\omega, a)$ depends on the manager's action through intermediate cash flows, ω, and on the employee's action, a, as described below.

Timing. The timing of events is summarized in Figure 1.

- At date one, the manager exerts effort x at cost $x^2/2$, which generates intermediate cash flows ω, where $\omega|x \sim N(x, \sigma_\omega^2)$. The realization of ω is privately observed by the manager.

- At date two, a public disclosure $d = \omega + \xi$ about the intermediate payoffs ω is realized, where $\xi \sim N(\mu_\xi, \sigma_\xi^2)$. Moreover, the manager engages in internal communication with the employee by sending a signal t about ω that he can manipulate at a cost $\left(\frac{\tau}{2}\right) \frac{(t-\omega)^2}{\varepsilon}$, where $\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. The manager privately observes his cost parameter ε, but this is unobserved by the employee and the market.

- At date three, the employee takes an action a that is aligned with the intermediate cash flows of the firm, i.e., to minimize $(a - \omega)^2$. Moreover, a noisy public signal about the internal communication is revealed:

$$m = t + \eta, \qquad \eta \sim N(0, z\sigma_\eta^2). \tag{1}$$

The parameter z captures the **tightness** of information control. A higher z corresponds to tighter control and hence a noisier (less informative) public signal about the internal message.

The date three price, P, reflects the investor's expectation of the firm's terminal payoffs $V(\omega, a)$, conditional on the public signals d and m, i.e.,

$$P(d, m) = \mathbb{E}[V(\omega, a)|d, m]. \tag{2}$$

Figure 1: Timeline of events

$t = 1$	$t = 2$	$t = 3$	$t = 4$
M chooses effort x and observes ω	Pub. disclosure $d(\omega)$; M privately sends message $t(\omega)$ to E	E chooses action a; Market sets price P	$V(\omega, a)$ is realized
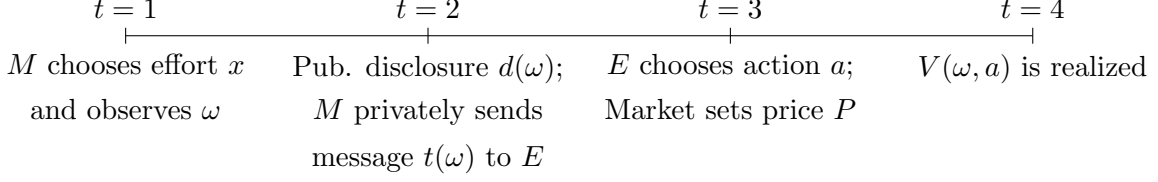
- At date four, the firm's terminal cash flows, given by

$$V(\omega, a) = \omega - \beta(a - \omega)^2, \tag{3}$$

are realized. Here $\beta > 0$ measures the relative importance of alignment between the employee's action and the firm's fundamentals.

Payoffs. The manager chooses x at date one and t at date two to maximize the conditional expectation of his payoff:

$$u_M = P(d, m) - \frac{x^2}{2} - \frac{\tau}{2} \frac{(t - \omega)^2}{\varepsilon}. \tag{4}$$

The employee chooses a at date three to maximize the conditional expectation of her payoff:

$$u_E = \mathbb{E}[\omega - \beta(a - \omega)^2 | d, t]. \tag{5}$$

We assume that the vector $\theta \equiv (\omega, \varepsilon, \xi, \eta)$ is jointly normal and mutually independent.

Equilibrium. We focus on pure strategy, subgame perfect equilibrium in which the price is a linear function of the public signals. In particular, a **linear equilibrium** is characterized by: (i) an optimal choice of effort x and internal message $t(\omega)$ by the manager which maximize (conditional) expectation of u_M in (4); (ii) an optimal choice of action a by the employee which maximizes (conditional) expectation of u_E in (5); (iii) a price of the form $P = b_0 + b_d d + b_m m$ which satisfies (2); and (iv) participants' beliefs that satisfy Bayes' rule wherever it is well-defined.

3.1 Discussion of Assumptions

Our setting is different from traditional settings in which the sender provides a signal about exogenous fundamentals. Instead, the manager engages in communication about *endogenous* cash flows, the distribution of which he influences by exerting costly effort. The key distortion

is that the manager cannot ensure that the signal he sends to the employee remains within the firm, and thus the communication environment feeds back into the manager's real decision-making. Importantly, we distinguish between the intended public **disclosure** d and the unintended **revealed** or **leaked** message m.

Our "truth-plus-noise" specification for the leaked information is made for tractability and expositional clarity. One can interpret m as an aggregate signal which captures the (payoff relevant) summary of all leaked information. In this case, the precision of this aggregate signal is driven by the information tightness parameter z, which measures the degree of information control. Stronger access controls, monitoring, or restrictions (e.g., compartmentalization or enforcement of confidentiality) imply that information control is tight (i.e., z is high) and so less information is leaked to the market. In this case, the aggregate signal m is less informative about cash-flows.[7] Similarly, the precision or quality of the public disclosure is parameterized by σ_ξ^2: the higher the required level of public disclosure, the lower σ_ξ^2 (and vice versa) — this is a standard specification in the literature. As we show below, while disclosure quality and the looseness of information control have similar implications for the manager's effort choice and price informativeness, they can have qualitatively different implications for real efficiency.

The specifications for the manager and employee payoffs are primarily for tractability. The fact that the manager's payoff depends on the price reflects that most firm executives receive stock-based compensation in practice and implies that the manager has an incentive to distort his communication to the employee. If instead, the manager's payoff only depended on the terminal cash-flow V, then the manager would have no incentive to distort his communication (i.e., choose $t = \omega$) and would invest efficiently a constant amount $x^* = 1$. We expect our results to be qualitatively similar if, instead, the manager's payoff was driven by a weighted average of P and V, although the analysis would be more cumbersome.

Similarly, the fact that the employee's payoff is sensitive to the terminal value ensures that she has an incentive to align her action with the fundamentals, i.e., choose a as close to ω as possible. Making her payoff also depend on the price P would not qualitatively change our results.[8] A key assumption for our analysis is that firm value depends, in part, on the alignment of the employee's action with interim cash-flows. This misalignment gives rise to a real cost associated with the distortion in internal communication. Existing work, including

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[7]One could alternatively assume that the market observes the internal message perfectly with some probability that decreases in z (a "truth-or-no-information" structure). We adopt the truth-plus-noise formulation because the joint assumption of additive normal noise and normal shocks yields linear updating rules. A truth-or-no-information structure delivers qualitatively similar results; detailed derivations are available upon request.

[8]Given that the variables ω and η are independent, the linear equilibrium as we characterized continues to exist. Moreover, since she takes the price coefficients as given, the employee's choice of a remains unchanged.

Bolton et al. (2013) and Dessein and Santos (2021), has explored the role of such alignment components of firm value in shaping corporate culture and organizational structure.

The assumption that the vector of shocks θ follows a multivariate normal distribution implies that the conditional distribution of ω, given the signals, is also normal, which ensures that characterizing agents' updating of beliefs is tractable. As in Fischer and Verrecchia (2000), this implies that the cost of manipulating signals can sometimes be negative (since ε can take on negative values). However, by setting μ_ε appropriately, the likelihood of this can be made arbitrarily small. One could alternatively ensure non-negativity by assuming that the shocks are elliptically distributed (as in Frankel and Kartik (2019) and Ball (2025)), but this makes the characterization of real efficiency in our analysis less tractable.[9]

4 Analysis

We begin by characterizing the first-best outcome as a benchmark. In the absence of moral hazard, the first-best level of effort is $x_{FB} = 1$ and the manager engages in no distortion of the internal communication i.e., $t_{FB}(\omega) = \omega$.

Next, we solve the model by working backwards.

Date three. Given her objective in (5), the employee's optimal choice of action a is given by

$$a^* = \arg\max_a \mathbb{E}\left[\omega - \beta\left(a - \omega\right)^2 | d, t\right] = \mathbb{E}\left[\omega | d, t\right]. \tag{6}$$

Next, note that the price P can be expressed as

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\begin{aligned}
P &= \mathbb{E}\left[V(\omega, a^*)|d, m\right] \\
&= \mathbb{E}\left[\omega| d, m\right] - \beta\mathbb{E}\left[\mathbb{E}\left[\left(a^* - \omega\right)^2 | d, t\right] | d, m\right] \\
&= \mathbb{E}\left[\omega| d, m\right] - \beta\mathbb{E}\left[\mathbb{V}\left[\omega| d, t\right] | d, m\right],
\end{aligned}
$$

where the final equality follows from the observation that $a^* = \mathbb{E}[\omega|d, t]$. Intuitively, the more informative the disclosure d and the internal message t are about cash flows ω, the better aligned the action a^* is. In turn, this translates to a higher valuation P for the firm, since in expectation, $\mathbb{V}[\omega|d, t]$ is smaller.

We shall conjecture, and then verify, that the price is linear in the public signals d and m, i.e.,

$$P = b_0 + b_d d + b_m m.$$

[9]Specifically, unless the shocks are normally distributed, the conditional variance of ω depends on the realization of the signals (e.g., see Foster and Viswanathan (1993)), which implies that the expected firm value and real efficiency are no longer analytically tractable.

Date two. The manager conditions on the realization of ω and ε when choosing his internal communication to the employee. Specifically, he chooses $t(\omega, \varepsilon)$ to maximize:

$$\mathbb{E}[u_M|\omega, \varepsilon] = \mathbb{E}[P|\omega, \varepsilon] - \frac{x^2}{2} - \frac{\tau}{2}\frac{(t - \omega)^2}{\varepsilon}$$

$$= b_0 + b_d (\omega + \mu_\xi) + b_m t - \frac{x^2}{2} - \frac{\tau}{2}\frac{(t - \omega)^2}{\varepsilon},$$

since $\mathbb{E}[d|\omega, \varepsilon] = \omega + \mu_\xi$ and $\mathbb{E}[m|\omega, \varepsilon] = t(\omega, \varepsilon)$. This implies that the manager's optimal choice is given by

$$t^*(\omega, \varepsilon) = \omega + \frac{b_m}{\tau}\varepsilon. \tag{7}$$

This is intuitive. The higher the weight, b_m, the market puts on the leak m, the stronger the incentive of the manager to distort the signal t away from ω. The extent of manipulation decreases in the manager's cost of doing so, which is captured by τ.

Let the market's conjecture about x be given by $\hat{x}$. Given the above characterization of t and the signal structure in (1), we have that the market's beliefs can be expressed as:

$$\mathbb{E}[\omega|m, d] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d - \mu_\xi}{\sigma_\xi^2} + \frac{m - \frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}$$

and

$$\mathbb{E}\left[\mathbb{V}\left[\omega|d, t\right]\big| d, m\right] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}},$$

which implies

$$P = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d - \mu_\xi}{\sigma_\xi^2} + \frac{m - \frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}}.$$

Note that this verifies the conjecture that the price is linear in the signals, d and m.

Date one. Given these coefficients, the manager's effort choice x maximizes:

$$\mathbb{E}[u_M] = b_0 + b_m\mathbb{E}[m] + b_d\mathbb{E}[d] - \frac{x^2}{2} - \frac{\tau}{2}\mathbb{E}\left[\frac{(t - \omega)^2}{\varepsilon}\right]$$

$$= b_0 + (b_m + b_d) x + \frac{b_m^2}{\tau}\mu_\varepsilon + b_d\mu_\xi - \frac{x^2}{2} - \frac{\tau}{2}\mathbb{E}\left[\left(\frac{b_m}{\tau}\right)^2 \varepsilon\right],$$

so the equilibrium effort is

$$x^* = b_m + b_d. \tag{8}$$

Consistent with intuition, the manager's effort is increasing in the market's total weights on the public signal (i.e., b_d) and on the leaked signal (i.e., b_m). The following proposition establishes that there exists a unique equilibrium in our setting (conditional on z).

Proposition 1. *There exists a unique linear equilibrium characterized by the optimal choices in (6), (7), and (8), and an equilibrium price $P = b_0 + b_m m + b_d d$, where the price coefficients are pinned down by*

$$b_0 = \frac{\frac{b_m+b_d}{\sigma_\omega^2} - \frac{\mu_\xi}{\sigma_\xi^2} - \frac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}},$$

$$b_d = \frac{\frac{1}{\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}},$$

and $b_m \in (0,1)$ is the unique solution to

$$b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}. \tag{9}$$

The proof establishes that there is a unique solution $b_m \in (0,1)$ to the fixed point characterized by (9) which pins down the equilibrium. This expression reflects the fact that the weight b_m that investors put on the leaked signal m inversely depends on the variance of the error in m. This, in turn, depends on both the tightness of information control (through z) and the extent to which the manager distorts his internal communication (as given by equation (7)), which in turn, increases with the weight b_m that investors put on the leaked signal.

5   Measures of Efficiency

In this section, we characterize how changes in information control tightness (z) and public disclosure quality ($1/\sigma_\xi^2$) affect two different measures of efficiency: price informativeness and real efficiency.

We define **price informativeness**, denoted by PI, as the conditional precision of interim

cash-flows ω, given the price P, i.e., $PI = (\mathbb{V}[\omega|P])^{-1}$. This is analogous to the notion of forecasting price efficiency in Bond et al. (2012) and captures the extent to which the security price is informative about firm fundamentals.[10]

The following result characterizes how changing information control and enhancing public disclosure quality influence the price coefficients b_d and b_m, equilibrium effort, and price informativeness.

Proposition 2. *In equilibrium, a loosening of information control (i.e., higher $1/z$) leads to: (i) an increase in the price coefficient on the leaked signal, b_m; (ii) a decrease in the price coefficient on the public disclosure, b_d; and (iii) an increase in the manager's equilibrium effort, $x^* = b_d + b_m$, as well as in price informativeness, PI.*
An increase in the quality of public disclosures (i.e., higher $1/\sigma_\xi^2$) leads to: (i) a decrease in the price coefficient on the leaked signal, b_m; (ii) an increase in the price coefficient on the public disclosure, b_d; and (iii) an increase in the manager's equilibrium effort, $x^* = b_d + b_m$, and price informativeness, PI.

Figure 2 provides an illustration of the above result. Intuitively, loosening information control (higher $1/z$) increases the precision of the leaked signal m, which leads to an increase in the weight b_m investors put on this signal. All else equal, this leads to a decrease in the weight b_d investors put on the public disclosure. Similarly, an increase in public disclosure quality leads to a more precise public disclosure d, which leads to an increase in its weight b_d, but a decrease in the weight b_m investors put on the leaked signal.

In both cases, the informativeness of the total information available to investors increases, as suggested by an increase in $b_d + b_m$. This immediately implies that optimal effort x^* changes with either policy. Finally, as we show in the proof, one can express price informativeness as:

$$PI = (\mathbb{V}[\omega|P])^{-1} = \frac{1}{\sigma_\omega^2(1 - (b_d + b_m))},$$

and so price informativeness moves in the same direction as $b_d + b_m$.

While loosening information control (increasing $1/z$) and improving public disclosure quality have qualitatively similar effects on the informativeness of market price, they have

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[10]Note that in contrast to settings where investors have heterogeneous information and price informativeness measures the extent to which the price aggregates this information, in our setting investors are symmetrically informed. However, price informativeness captures the extent to which the price reflects information about fundamentals to an econometrician. An alternate measure of price informativeness would be how informative the price is about terminal cash flows, i.e., $(\mathbb{V}[V|P])^{-1}$. We prefer our measure of price informativeness because it is more analytically tractable and intuitive, and because it maps more closely to empirical measures of price informativeness proposed in the literature (e.g., Bai, Philippon, and Savov (2016); Dávila and Parlatore (2018)).

Figure 2: Price coefficients as a function of z and σ_ξ
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = z = 1$.
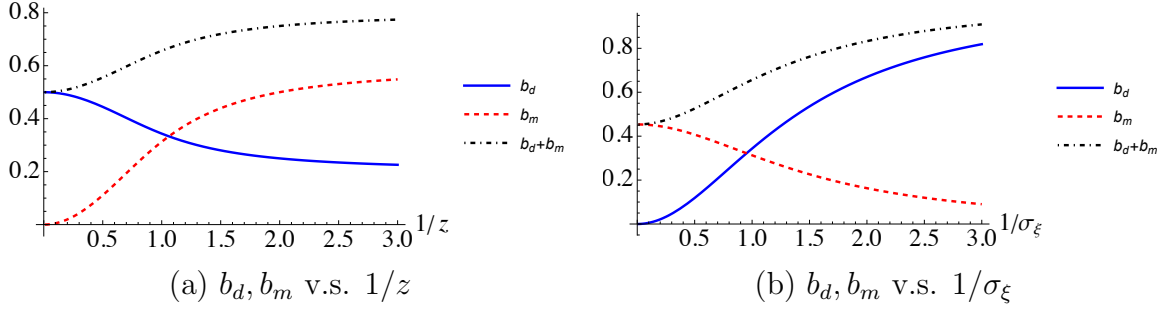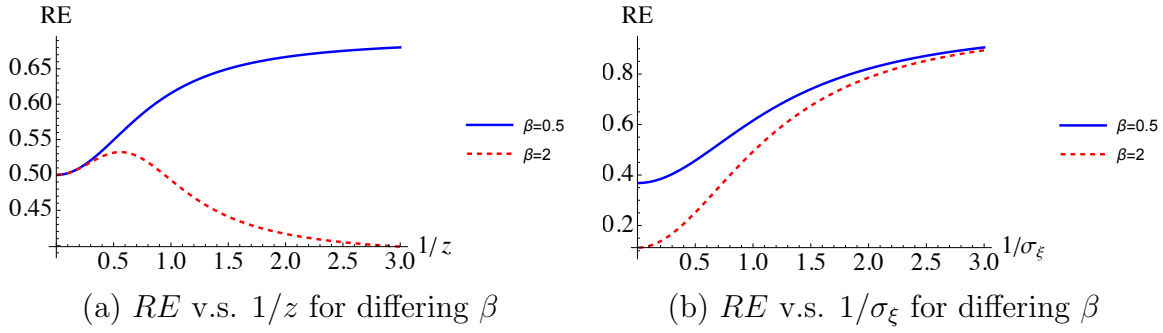

(a) b_d, b_m v.s. $1/z$

(b) b_d, b_m v.s. $1/\sigma_\xi$

Figure 3: Real efficiency as a function of z, σ_ξ, and β.
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = z = 1$.

(a) RE v.s. $1/z$ for differing β

(b) RE v.s. $1/\sigma_\xi$ for differing β

different implications for *real* efficiency. Specifically, define **real efficiency**, denoted by RE, as the unconditional expected value of the firm, i.e., $RE = \mathbb{E}[V]$. The following result shows how policy changes affect real efficiency in our setting.

Proposition 3. *An increase in public disclosure quality (i.e., higher $1/\sigma_\xi^2$) always leads to an increase in real efficiency RE. In contrast, there exists $0 < \underline{\beta} \leq \overline{\beta}$, such that:*
(i) when $\beta \leq \underline{\beta}$, real efficiency RE increases in the looseness of information control $1/z$, and
(ii) when $\beta > \overline{\beta}$, real efficiency RE is hump-shaped in the looseness of information control $1/z$.

Figure 3 provides an illustration of the above result. To gain some intuition, note that

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one can express real efficiency as the sum of two components:

$$RE = \mathbb{E}[V] = \mathbb{E}[\omega] - \beta\mathbb{E}[\mathbb{V}[\omega|d,t]].$$

The first component, $\mathbb{E}[\omega]$, is determined by the manager's equilibrium effort i.e., $\mathbb{E}[\omega] = x^* = b_d + b_m$, which is increasing in the looseness of information control $1/z$ and increasing in public disclosure quality.

The second component, $\beta\mathbb{E}[\mathbb{V}[\omega|d,t]]$, reflects the loss in firm value due to the misalignment between fundamentals and the employee's action. This loss is proportional to the average posterior uncertainty that the employee faces after observing the public disclosure d and the internal communication t. An increase in public disclosure quality $(1/\sigma_\xi^2)$ leads to a decrease in the employee's uncertainty, and so reduces the loss in value due to misalignment. As a result, firm value and real efficiency always increase in public disclosure quality, as illustrated by panel (b) of Figure 3.

In contrast, loosening information control (increasing $1/z$) increases the market's reliance on the leaked signal (a higher b_m), which increases the manager's incentive to distort internal communication. As a result, the employee's posterior uncertainty $\mathbb{V}[\omega|d,t]$ increases, which increases the loss in firm value due to misalignment. This cost is larger when internal alignment is more important (i.e., when β is high).

When β is sufficiently low, the cost from worsened internal communication is small and the increase in effort (due to stronger market discipline) dominates, so RE increases in $1/z$. When β is sufficiently high, the gains from strengthened market discipline and higher effort dominate for moderate increases in $1/z$; however, when information control becomes sufficiently loose (i.e., $1/z$ is very large), the losses from deteriorated internal alignment dominate. As a result, RE is hump-shaped in $1/z$.

This implies that when β is sufficiently high, there exists an interior optimum level of tightness, denoted by z^*, which maximizes real efficiency. One can interpret this as the tightness of information control a firm's board (or a regulator) should target, if their goal is to maximize firm value. The next result characterizes how this optimum level changes with model parameters.

Proposition 4. *Suppose $\beta > \bar{\beta}$, so that real efficiency is hump-shaped in z and maximized at z^*. Then, the optimal tightness of information control z^* is increasing in β, σ_ω and $1/\sigma_\xi$, but decreasing in σ_η.*

Figure 4 provides an illustration of these results. In each of these cases, a change in the underlying parameter differentially affects the marginal benefit (i.e., $-\frac{\partial\beta\mathbb{E}[\mathbb{V}[\omega|d,t]]}{\partial z}$) and the marginal cost (i.e., $\frac{\partial\mathbb{E}[\omega]}{\partial z}$) of increasing z. For instance, an increase in the sensitivity to

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Figure 4: Optimal level of information control z^* as a function of model parameters. Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = z = 1$ and $\beta = 2$.
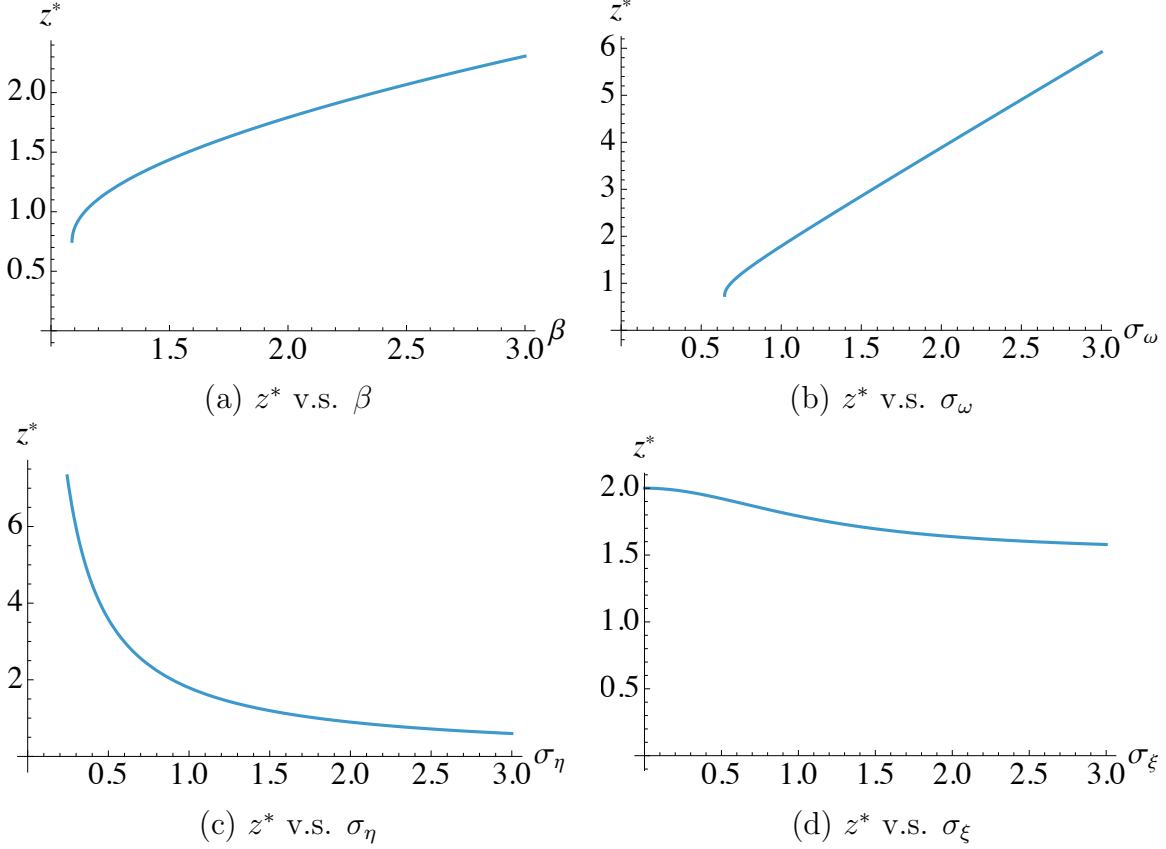
misalignment, β, raises the marginal benefit of increasing z while leaving the marginal cost unchanged; as a result, the optimal choice z^* increases. Similarly, an improvement in public disclosure quality (i.e., a higher $1/\sigma_\xi$) reduces the weight market participants place on the leak and, consequently, lowers the marginal cost of tighter information control, leading to a higher optimal z^*.

An increase in cash flow volatility, σ_ω, increases both the marginal benefit and the marginal cost. On the one hand, when higher cash flow volatility leads investors to place greater weight on the leak (i.e., b_m increases), thereby raising the marginal cost of tighter information control. On the other hand, a higher b_m strengthens the manager's incentive to distort the message, which increases the marginal benefit of increasing z. When β is sufficiently large, the increase in the marginal benefit from improved alignment outweighs the increase in the marginal cost from reduced effort, and the optimal level of information control, z^*, increases.

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In contrast, an increase in σ_η reduces the quality of the signal leaked to the market, which, holding all else equal, lowers b_m. This reduction decreases both the marginal benefit and the marginal cost of increasing z. When β is sufficiently large, the decline in the marginal benefit dominates, leading to a lower optimal choice of z^*.

Importantly, the above results imply that one must be cautious about evaluating the impact of information-control policies on the cross-section of firms, since they can have qualitatively different effects across different firms. Specifically, information control should be tighter for firms in which internal alignment is more important (i.e., β is higher) and cash flows are more volatile (i.e., σ_ω is higher), but looser when leaked information is more likely to be noisy (i.e., σ_η is higher). Moreover, the optimal tightness of information control increases with an increase in the quality of public disclosure (i.e., increases when $1/\sigma_\xi$ increases).

The above results also highlight an important difference across the two types of information. While tightening information control and improving public disclosure quality can both reduce the market's reliance on leaked internal information, they can have opposite effects on firm value and real efficiency when the relative value loss from within-firm misalignment is sufficiently high. One might conjecture that this difference arises because the manager can manipulate his internal communication, but cannot distort the public signal. In the next section, we show that our results remain qualitatively similar when the manager can manipulate both internal communication and public disclosure.

6 Endogenous public disclosure

In the main analysis, we take public disclosure d as an exogenous (verifiable) signal about fundamentals ω. In practice, managers can influence the content and timing of public disclosures, subject to legal, regulatory, and reputational frictions. This section extends the existing analysis by allowing the manager to *endogenously* choose public disclosure and by studying how the intensity of public disclosure scrutiny interacts with the tightness of information control.

At date two, after observing $(\omega, \varepsilon, \xi)$, the manager chooses both an internal message t and a public disclosure d. As before, distorting internal communication is privately costly with parameter τ and shock ε. In addition, we assume that biasing public disclosure away from ω is costly. Specifically, the manager incurs the quadratic cost

$$\left(\frac{\delta}{2}\right)\frac{(d-\omega)^2}{\xi},$$

where $\delta > 0$ captures *public disclosure scrutiny*. A higher δ makes it more costly to deviate

from truthful disclosure (e.g., due to tighter audit/regulatory scrutiny, higher litigation risk, or stronger reputational penalties). The random variable $\xi \sim N(\mu_\xi, \sigma_\xi^2)$ captures discretion in public disclosure and is independent of all other shocks.

It is immediate that the date three choice of the employee is given by (6), as before. Moreover, given the linear conjecture for the price $P = b_0 + b_d d + b_m m$, the manager's date two objective naturally generalizes to maximize:

$$\mathbb{E}[u_M | \omega, \varepsilon, \xi] = \mathbb{E}[P | \omega, \varepsilon, \xi] - \frac{x^2}{2} - \frac{\tau}{2} \frac{(t - \omega)^2}{\varepsilon} - \frac{\delta}{2} \frac{(d - \omega)^2}{\xi}$$

$$= b_0 + b_d d + b_m t - \frac{x^2}{2} - \frac{\tau}{2} \frac{(t - \omega)^2}{\varepsilon} - \frac{\delta}{2} \frac{(d - \omega)^2}{\xi},$$

which implies that his optimal choices are given by:

$$t^*(\omega, \varepsilon) = \omega + \frac{b_m}{\tau} \varepsilon, \quad \text{and} \quad d^*(\omega, \xi) = \omega + \frac{b_d}{\delta} \xi. \tag{10}$$

This is also intuitive. The greater the weight the price places on public disclosure (i.e., the higher b_d), the stronger the incentive to distort the signal, and the larger the weight placed on the "error" ξ in the optimal disclosure. In turn, this implies that the leaked message is given by:

$$m = \omega + \frac{b_m}{\tau} \varepsilon + \eta, \qquad \eta \sim N(0, z\sigma_\eta^2).$$

Let $\hat{x}$ denote the market's conjecture about the manager's date-one effort x. Standard Gaussian updating yields

$$\mathbb{E}[\omega | d, m] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d - \frac{b_d}{\delta} \mu_\xi}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{m - \frac{b_m}{\tau} \mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

Moreover,

$$\mathbb{E}[\mathbb{V}[\omega | d, t] | d, m] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}},$$

which does not depend on (d, m). Therefore,

$$P = \mathbb{E}[\omega | d, m] - \beta \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}}. \tag{11}$$

Matching coefficients in (11), the equilibrium coefficients (b_d, b_m) solve the system:

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}} \quad \text{and} \quad b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}. \tag{12}$$

The intercept b_0 is

$$b_0 = \frac{\frac{\hat{x}}{\sigma_\omega^2} - \frac{\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} - \frac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}}. \tag{13}$$

At date one, conditional on linear pricing, the manager again chooses effort x to maximize the expected price net of the quadratic effort cost. Hence, the equilibrium effort is

$$x^* = b_d + b_m. \tag{14}$$

Proposition 5. *There exists a unique linear equilibrium. In equilibrium, the manager chooses t^* and d^* as in (10), the employee chooses $a^* = \mathbb{E}[\omega|d, t]$, the market prices $P = b_0 + b_d d + b_m m$, where (b_d, b_m) solve (12) and b_0 is given by (13), and the manager exerts x^* as in (14).*

We compare two levers that shape the informational environment: the looseness of information control, $1/z$, and the intensity of public disclosure scrutiny, δ. Looser information control (i.e., a higher $1/z$) makes the leaked message m more informative and shifts informational weight toward the leaked signal m. Higher disclosure scrutiny (i.e., a higher δ) reduces the manager's ability to bias public disclosure and strengthens the informativeness of d.

Importantly, these levers interact via the manager's endogenous distortion incentives. When information control is tight (high z), investors rely more heavily on d, which increases the marginal benefit of biasing public disclosure and makes d endogenously noisier. When information control is loose (low z), investors place more weight on m, which reduces the marginal benefit of distorting d and thereby improves the informativeness of public disclosure. In this sense, looser information control not only strengthens market discipline through a more informative m, but also indirectly disciplines public reporting by weakening incentives to manipulate d. Higher scrutiny δ complements information control by directly curbing manipulation and improving the informativeness of d.

Proposition 6. *Fix parameters other than (z, δ). In equilibrium:*

1. *Increasing public disclosure scrutiny δ increases effort x^*, increases price informativeness, and increases real efficiency.*

2. *Tightening information control (increasing z) decreases effort x^* and decreases price informativeness.*

3. *Real efficiency is decreasing in z when β is sufficiently small, and is hump-shaped in z when β is sufficiently large.*

Panels (a) and (b) of Figure 5 illustrate how the market weights b_d and b_m change with the tightness of internal information control, z, and the intensity of public disclosure scrutiny, δ. As in the main model, a decrease in the tightness leads to more informative leaks. In turn, this leads to a higher market weight on the leaked information (b_m) and a lower market weight on public disclosure (b_d).

Figure 5: Price coefficients and real efficiency under endogenous public disclosure
Unless otherwise specified, other coefficients are set to $\sigma_\omega = \sigma_\varepsilon = \sigma_\eta = \sigma_\xi = \tau = \beta = \delta = z = 1$.

(a) b_d, b_m v.s. $1/z$

(b) b_d, b_m v.s. δ

(c) RE v.s. $1/z$ for differing β

(d) RE v.s. δ for differing β
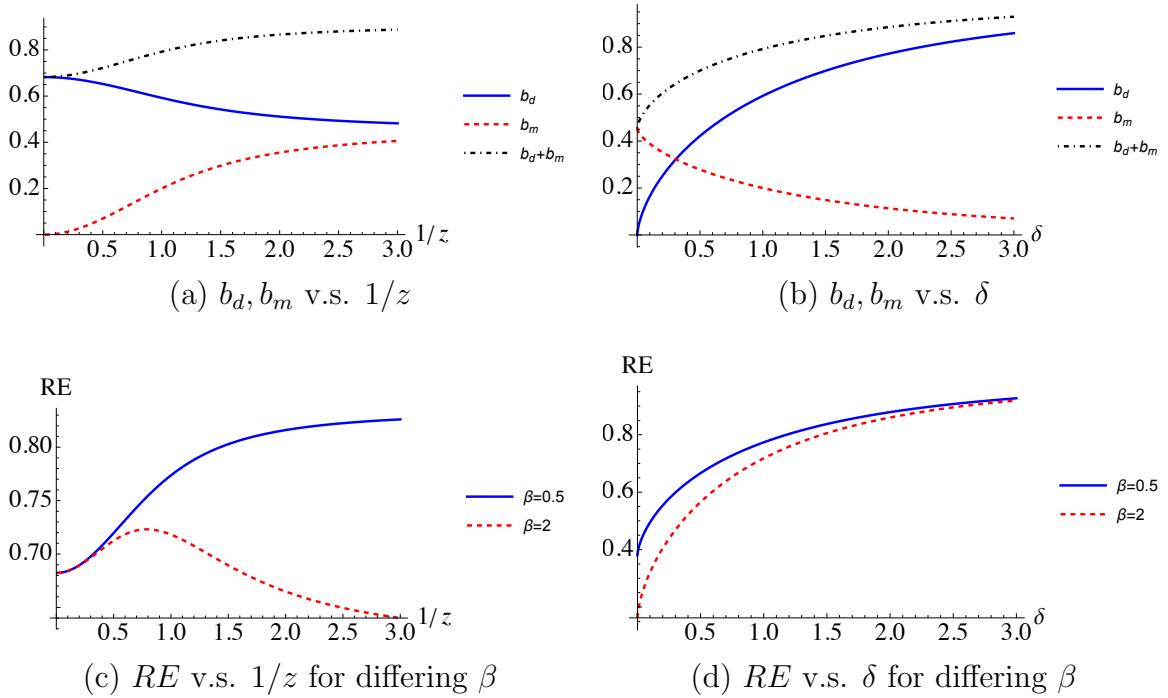
In contrast to the main analysis, this latter effect is muted: a lower weight on public disclosure also reduces the manager's incentive to distort such reports, making d a more

informative signal about ω as a result. This uncovers a novel implication of loosening information control: by reducing the market's reliance on public disclosures, such policies weaken managerial incentives to manipulate these reports, thereby making them more informative.

Similarly, an increase in public disclosure scrutiny δ makes it costlier for the manager to distort the public report. This leads to an increase in the market weight b_d on the public disclosure,[11] and a corresponding decrease in the weight b_m on the leaked information. In turn, this reduces the manager's incentive to distort his internal communication.

The proposition establishes that a decrease in the tightness of information control or an increase in public disclosure scrutiny leads to higher effort and higher price informativeness, as in the main model. Moreover, as panels (c) and (d) illustrate, while greater scrutiny of public disclosure unambiguously increases firm value and real efficiency, excessively loose or excessively tight information control can reduce real efficiency when the relative importance of alignment is sufficiently high (i.e., when β is large).

7 Conclusion

We develop a model to study how the tightness of information control, which reflects the extent to which internal communication becomes available to market participants, affects managerial decisions, internal communication, price informativeness, and real efficiency. Our analysis reveals a fundamental trade-off: looser information control strengthens market discipline and increases price informativeness, but can reduce real efficiency by worsening the quality of internal communication and weakening internal coordination.

The key mechanism is as follows. When information control is looser, signals about internal communication that reach the market are more informative, leading investors to place greater weight on these signals when valuing the firm. This strengthens market discipline, inducing the manager to exert more costly effort. At the same time, because internal messages become more price-relevant, the manager has stronger incentives to strategically distort internal communication. When internal alignment is sufficiently important for firm value, the resulting deterioration in internal communication can outweigh the benefits of improved market discipline, lowering real efficiency. Our analysis generates novel testable empirical predictions and has implications for policy design, as we discuss next.

Empirical predictions. Our model generates two sets of predictions. First, firm policies regarding information control should vary systematically with firm characteristics. For instance, firms where internal alignment and coordination are more important should opti-

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[11]It has been well documented empirically that greater public scrutiny increases the weight that investors place on accounting disclosures in valuing the firm (Collins and Kothari, 1989; Teoh and Wong, 1993).

mally have tighter information controls. This includes firms that are vertically integrated, have high organizational complexity and cross-functional dependencies, and operate in industries that require greater specialization.[12] Although we are unaware of existing research that speaks directly to this prediction, it is broadly consistent with the prevalence of tighter information controls in certain industries (e.g., technology, pharmaceuticals, defense). The model also predicts tighter controls for firms with more volatile cash-flows and for firms with higher public disclosure quality (e.g., due to stronger auditing or regulatory scrutiny).

Second, our model generates predictions about responses to changes in information control tightness. When information control becomes looser (e.g., after expansions of external reporting channels for employees or weakening of confidentiality protections), our model predicts an increase in price informativeness and a reduction in managerial misconduct (e.g., Wilde, 2017; Call, Martin, Sharp, and Wilde, 2018), but also a worsening of internal communication. While internal communication is difficult to observe directly, proxies include employee surveys about information sharing, measures of coordination failures, and usage intensity of internal reporting systems. Bowen, Call, and Rajgopal (2010) provide preliminary evidence: targets of whistleblowing allegations are more likely to have unclear internal communication channels. Moreover, the model predicts a differential impact on real efficiency: valuations should increase more following a loosening of information control for firms in which internal coordination is less important. Finally, to the extent that firms can affect the quality of their public disclosures, our extension further predicts that looser information controls would improve disclosure quality (e.g., higher earnings response coefficients or improved analyst forecast accuracy).

A key challenge in testing these predictions is identifying measures of the tightness of information control. At the firm level, one could proxy for information control tightness using measures of firms' confidentiality policies, use of non-disclosure agreements (e.g., Sockin, Sojourner, and Starr, 2024), investment in information security (e.g., Chai, Kim, and Rao, 2011), and organizational structure. At the aggregate level, changes in regulatory policy can provide natural settings to explore the differential impact of changes in information control across firms. These include the introduction of additional whistleblower protections (e.g., the inception of the SEC Whistleblower Program as part of the 2010 Dodd-Frank Act), the strengthening of laws protecting trade secrets (e.g., the 2016 Defend Trade Secrets Act and the staggered implementation of the Uniform Trade Secrets Act), and state-level variation in the adoption of laws restricting the use of non-compete clauses or non-disclosure agreements (e.g., deHaan, Wang, Zhou, and Zhou, 2026).

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[12]Atalay, Hortaçsu, and Syverson (2014) provide a measure of vertical integration across firms, while Bushman, Chen, Engel, and Smith (2004) provide a measure of organizational complexity.

Policy Implications. Our analysis has implications for regulatory policy which affects firms' informational control tightness. For instance, whistleblower programs that incentivize employees to report information externally effectively loosen information control. Our model predicts that such policies are more likely to be beneficial in settings where internal coordination is less important (e.g., simple production processes, standardized products), but can be counterproductive when coordination is crucial (e.g., complex organizations, knowledge-intensive industries), because the coordination losses from distorted internal communication can outweigh the governance gains from stronger market discipline. Conversely, regulations that strengthen corporate confidentiality (e.g., trade secret protections, non-disclosure agreement enforcement, and data security requirements) can improve real efficiency for firms where internal coordination is important, even though they may reduce price informativeness.

Another important implication of our analysis is that policies affecting public disclosure quality and policies affecting information control have different implications. While loosening information control can decrease firm value and real efficiency when alignment costs are high, improving disclosure quality always improves real efficiency in our setting. This suggests that regulators seeking to improve market functioning should prioritize *direct* improvements in public disclosure (e.g., through more stringent reporting requirements, better auditing standards, or enhanced enforcement of disclosure rules), rather than relying on *indirect* improvements through policies that affect information leakage.

Future work. Our model is stylized for tractability and expositional clarity, but can be naturally extended along several directions. While we focus on a firm with a representative employee to isolate the key mechanism, it would be interesting to study how information control policies interact with coordination among multiple employees and across different organizational structures. More explicit modeling of the instruments of information control (e.g., implementing confidentiality policies, building information silos) might uncover additional interactions between information control choices, effort provision, price efficiency and firm value. We leave these explorations for future work.

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A   Proofs

Proof of Proposition 1. At date three, given (d, t), the employee chooses a to maximize $\mathbb{E}[\omega - \beta(a - \omega)^2 | d, t]$, which yields

$$a^*(d, t) = \mathbb{E}[\omega | d, t].$$

Conjecture a linear equilibrium price

$$P = b_0 + b_d d + b_m m, \tag{15}$$

where (b_0, b_d, b_m) are constants. Since $\mathbb{E}[m | t] = t$, at date two the manager chooses t to maximize $b_0 + b_d d + b_m \mathbb{E}[m | t] - \left(\frac{\tau}{2}\right) \frac{(t - \omega)^2}{\varepsilon}$, which implies the first-order condition

$$b_m - \frac{\tau}{\varepsilon}(t - \omega) = 0.$$

Thus the manager's optimal internal message is

$$t^* = \omega + \frac{b_m}{\tau}\varepsilon. \tag{16}$$

Substituting (16) into the information-control technology $m = t + \eta$ with $\eta \sim N(0, z\sigma_\eta^2)$ yields

$$m = \omega + \frac{b_m}{\tau}\varepsilon + \eta, \qquad \eta \sim N(0, z\sigma_\eta^2).$$

Given conjectured effort $\hat{x}$, Gaussian updating implies

$$\mathbb{E}[\omega | d, m] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d - \mu_\varepsilon}{\sigma_\xi^2} + \frac{m - \frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

Moreover,

$$\mathbb{E}\left[\mathbb{V}[\omega | d, t] | d, m\right] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}}, \tag{17}$$

which does not depend on (d, m). Therefore the price

$$P = \mathbb{E}[\omega | d, m] - \beta \, \mathbb{E}\left[\mathbb{V}[\omega | d, t] | d, m\right]$$

is linear in (d, m). Matching coefficients between this expression and (15) yields the stated

formulas for b_0 and b_d, and the fixed-point equation for b_m:

$$b_m = \frac{1}{1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)}. \tag{18}$$

Let

$$H(b_m) \equiv b_m - \frac{1}{1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)}.$$

Then $H(0) = -\dfrac{1}{1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)z\sigma_\eta^2} < 0$, $H(1) = 1 - \dfrac{1}{1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\frac{\sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)} > 0$, and

$$H'(b_m) = 1 + \frac{2\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\sigma_\varepsilon^2 b_m/\tau^2}{\left(1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)^2} > 0.$$

Hence H is strictly increasing and admits a unique root $b_m \in (0, 1)$. Given b_m, the equilibrium b_d is uniquely determined by the coefficient-matching condition

$$b_d = \frac{1 - b_m}{1 + \sigma_\xi^2/\sigma_\omega^2} = \frac{(1 - b_m)\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}.$$

Finally, at date one the manager chooses x to maximize $\mathbb{E}[P] + \frac{1}{2}(1 - x^2)$. Since $\mathbb{E}[P]$ is affine in x with slope $(b_d + b_m)$, the optimal effort is $x^* = b_d + b_m$ as stated. $\square$

Proof of Proposition 2. Consider the fixed-point equation for b_m in (18). Write it as

$$H(b_m; z) = b_m - \frac{1}{1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)} = 0.$$

As shown in the proof of Proposition 1, $H_{b_m} > 0$ on $(0, 1)$, so the implicit function theorem applies. Differentiating,

$$\frac{\partial b_m}{\partial z} = -\frac{H_z}{H_{b_m}} = -\frac{\dfrac{\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\sigma_\eta^2}{\left(1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)^2}}{1 + \dfrac{2\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\sigma_\varepsilon^2 b_m/\tau^2}{\left(1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)^2}} < 0.$$

Hence looser information control (higher $1/z$) increases the weight b_m.

Next, as shown in the proof of Proposition 1,

$$b_d = \frac{(1 - b_m)\sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}. \tag{19}$$

So b_d decreases with $1/z$. Since $x^* = b_d + b_m$, we have

$$\frac{\partial x^*}{\partial z} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \frac{\partial b_m}{\partial z} < 0.$$

So the equilibrium effort increases with $1/z$.

For public disclosure quality, differentiating the fixed point with respect to q yields

$$\frac{\partial b_m}{\partial \left(1/\sigma_\xi^2\right)} = -\frac{H_q}{H_{b_m}} = -\frac{\dfrac{\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)}{\left(1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)^2}}{1 + \dfrac{2\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\sigma_\varepsilon^2 b_m/\tau^2}{\left(1 + \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\left(\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)^2}} < 0.$$

Thus improving public disclosure quality (increasing $1/\sigma_\xi^2$) reduces b_m. Using (19), we have $\partial b_d/\partial\left(1/\sigma_\xi^2\right) > 0$ and

$$\frac{\partial x^*}{\partial\left(1/\sigma_\xi^2\right)} = \frac{\sigma_\omega^2\left(1 - b_m\right)}{\left(1 + \sigma_\omega^2/\sigma_\xi^2\right)^2} > 0.$$

Finally, price informativeness is

$$PI \equiv \frac{1}{\mathbb{V}[\omega|P]}.$$

Because P is an affine function of $\mathbb{E}[\omega|d, m]$ (the remaining term is the constant $-\beta\,\mathbb{E}[\mathbb{V}[\omega|d, t]|d, m]$), conditioning on P is equivalent to conditioning on (d, m) for the purpose of forecasting ω. Hence

$$\mathbb{V}[\omega|P] = \mathbb{V}[\omega|d, m] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

Moreover, it follows from Proposition 1 that

$$\mathbb{V}[\omega|d, m] = \sigma_\omega^2\left(1 - (b_d + b_m)\right) = \sigma_\omega^2(1 - x^*).$$

Therefore,

$$PI = \frac{1}{\sigma_\omega^2(1 - x^*)},$$

which is increasing in x^*. Since x^* increases with $1/z$ and with $1/\sigma_\xi^2$, the comparative statics for PI follow. $\qquad\square$

Proof of Proposition 3. Real efficiency can be written as

$$RE = b_m + b_d - \beta\, \mathbb{E}\left[\mathbb{V}[\omega|d,t]\right] = x^* - \beta\, \mathbb{E}\left[\mathbb{V}[\omega|d,t]\right]. \tag{20}$$

From (17),

$$\mathbb{E}\left[\mathbb{V}[\omega|d,t]\right] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{\tau^2}{b_m^2\sigma_\varepsilon^2}}. \tag{21}$$

Moreover, using (19),

$$x^* = b_d + b_m = \frac{b_m\sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2}, \tag{22}$$

which is increasing in b_m.

An increase in public disclosure quality corresponds to an increase in $1/\sigma_\xi^2$. From (21), the internal forecast error $\mathbb{E}[\mathbb{V}[\omega|d,t]]$ decreases when $1/\sigma_\xi^2$ increases, because b_m decreases with $1/\sigma_\xi^2$ by Proposition 2. Also, Proposition 2 shows that x^* increases with disclosure quality. Therefore RE increases with disclosure quality.

Next, we examine the impact of information control z. Proposition 2 implies that b_m is strictly decreasing in z. Hence the shape of RE in z is the reverse of the shape of RE in b_m. Differentiating (20) with respect to b_m and using (22) yields

$$\frac{dRE}{db_m} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} - \beta \cdot \frac{2\frac{\tau^2}{\sigma_\varepsilon^2}b_m}{\left(\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)b_m^2 + \frac{\tau^2}{\sigma_\varepsilon^2}\right)^2}. \tag{23}$$

As $b_m \downarrow 0$, the second term vanishes, so $\lim_{b_m\downarrow 0}\frac{dRE}{db_m} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} > 0$.

Let

$$G(b_m) \equiv \frac{2\frac{\tau^2}{\sigma_\varepsilon^2}b_m}{\left(\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)b_m^2 + \frac{\tau^2}{\sigma_\varepsilon^2}\right)^2}.$$

A direct calculation shows that G is single-peaked on $\mathbb{R}_+$, attaining its maximum at

$$b^\dagger = \sqrt{\frac{\frac{\tau^2}{\sigma_\varepsilon^2}}{3\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)}}. \tag{24}$$

Let $\bar{b}_m \equiv \lim_{z\downarrow 0} b_m(z) \in (0,1)$ denote the maximal equilibrium weight on m (under the

loosest information control). Define thresholds

$$\underline{\beta} \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \cdot \frac{1}{\max_{b \in (0,\bar{b}_m]} G(b)}, \qquad \overline{\beta} \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \cdot \frac{1}{G(\bar{b}_m)}.$$

If $\beta \leq \underline{\beta}$, then $\frac{dRE}{db_m} \geq 0$ for all $b_m \in (0, \bar{b}_m]$, so RE is increasing in b_m and therefore decreasing in z (increasing in $1/z$). If $\beta > \overline{\beta}$, then $\frac{dRE}{db_m} < 0$ at $b_m = \bar{b}_m$ while it is positive near 0. By single-peakedness of G, RE is hump-shaped in b_m on $(0, \bar{b}_m]$ and hence hump-shaped in $1/z$. □

Proof of Proposition 4. Recall that

$$\overline{\beta} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \cdot \frac{1}{G(\bar{b}_m)} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} \frac{\bar{b}_m^3 \sigma_\varepsilon^2}{2\tau^2} \left(\frac{\tau^2}{\bar{b}_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2,$$

where $\bar{b}_m$ is the equilibrium solution to $H(b_m) = 0$ as $z \to 0$. As shown in Proposition 3, when $\beta \leq \underline{\beta}$, real efficiency RE decreases in the tightness of information control z. When $\beta > \overline{\beta}$, nevertheless, real efficiency RE is hump-shaped in the tightness of information control and maximized at some z^*. We show how z^* varies with the parameters.

Let b_m^* be the value of b_m that maximize

$$RE = \frac{b_m \sigma_\xi^2 + \sigma_\omega^2}{\sigma_\xi^2 + \sigma_\omega^2} - \beta \left(\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}} \right).$$

Given that RE is a continuously differentiable function of b_m, we have

$$\frac{dRE}{db_m} \bigg|_{b_m = b_m^*} = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\omega^2} - \frac{2\beta \frac{\tau^2}{b_m^3 \sigma_\varepsilon^2}}{\left(\frac{\tau^2}{b_m^2 \sigma_\varepsilon^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_\omega^2} \right)^2} \bigg|_{b_m = b_m^*} = 0. \tag{25}$$

Let $L \equiv dRE/db_m$. It is clear from the proof of Proposition 3 that $b_m^* < b_m^\dagger = \sqrt{\frac{\tau^2 \sigma_\xi^2 \sigma_\omega^2}{3\sigma_\varepsilon^2 (\sigma_\xi^2 + \sigma_\omega^2)}}$ which is defined to be the minimizer of L. By the definition of $b_m^\dagger$, $\partial L/\partial b_m < 0$ for $b_m < b_m^\dagger$. It follows that

$$\frac{\partial L}{\partial b_m} \bigg|_{b_m = b_m^*} = \frac{6\beta\tau^2 b_m^2 \sigma_\xi^4 \sigma_\omega^4 \sigma_\varepsilon^4 \left(\sigma_\xi^2 + \sigma_\omega^2 \right) - 2\beta\tau^4 \sigma_\xi^6 \sigma_\omega^6 \sigma_\varepsilon^2}{\left(b_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2 \right) + \tau^2 \sigma_\xi^2 \sigma_\omega^2 \right)^3} \bigg|_{b_m = b_m^*} < 0. \tag{26}$$

Moreover, note that the optimal z is given by

$$z = \frac{1 - b_m - \left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\frac{\sigma_\varepsilon^2}{\tau^2} b_m^3}{\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_\xi^2}\right)\sigma_\eta^2 b_m}, \tag{27}$$

where b_m is evaluated at b_m^*. We also have

$$\frac{\partial z}{\partial b_m} = -\frac{\tau^2 \sigma_\xi^2 \sigma_\omega^2 + 2b_m^3 \sigma_\varepsilon^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)}{\tau^2 \sigma_\eta^2 b_m^2 \left(\sigma_\xi^2 + \sigma_\omega^2\right)} < 0. \tag{28}$$

Eq (26), (27) and (28) will be used to prove the following comparative statics.

Note that the optimal z satisfies

$$\frac{dz}{d\beta} = \frac{\partial z}{\partial \beta} + \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \beta}\right|_{b_m=b_m^*}\right).$$

Because $\partial z/\partial \beta = 0$ by (27) and

$$\left.\frac{\partial b_m}{\partial \beta}\right|_{b_m=b_m^*} = -\left.\frac{\partial L/\partial \beta}{\partial L/\partial b_m}\right|_{b_m=b_m^*} = -\left.\frac{-\frac{2\tau^2 b_m \sigma_\xi^4 \sigma_\omega^4 \sigma_\varepsilon^2}{\left(b_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2 \sigma_\xi^2 \sigma_\omega^2\right)^2}}{\frac{6\beta\tau^2 b_m^2 \sigma_\xi^4 \sigma_\omega^4 \sigma_\varepsilon^4 \left(\sigma_\xi^2+\sigma_\omega^2\right)-2\beta\tau^4 \sigma_\xi^6 \sigma_\omega^6 \sigma_\varepsilon^2}{\left(b_m^2 \sigma_\varepsilon^2 \left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2 \sigma_\xi^2 \sigma_\omega^2\right)^3}}\right|_{b_m=b_m^*} < 0$$

by the Implicit function theorem, we have

$$\frac{dz}{d\beta} = \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \beta}\right|_{b_m=b_m^*}\right) > 0$$

by (26) and (28). It follows that the optimal z^* is increasing in β.

Note that the optimal z satisfies

$$\frac{dz}{d\sigma_\eta} = \frac{\partial z}{\partial \sigma_\eta} + \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \sigma_\eta}\right|_{b_m=b_m^*}\right).$$

Because $\partial z/\partial \sigma_\eta = -\frac{2z}{\sigma_\eta} < 0$ by (27) and

$$\left.\frac{\partial b_m}{\partial \sigma_\eta}\right|_{b_m=b_m^*} = -\left.\frac{\partial L/\partial \sigma_\eta}{\partial L/\partial b_m}\right|_{b_m=b_m^*} = 0,$$

we have $dz/d\sigma_\eta < 0$. It follows that the optimal z^* is decreasing in σ_η.

Note that the optimal z satisfies

$$\frac{dz}{d\sigma_\omega} = \frac{\partial z}{\partial \sigma_\omega} + \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \sigma_\omega}\right|_{b_m=b_m^*}\right).$$

By the Implicit function theorem, we have

$$\left.\frac{\partial b_m}{\partial \sigma_\omega}\right|_{b_m=b_m^*} = -\left.\frac{\partial L/\partial \sigma_\omega}{\partial L/\partial b_m}\right|_{b_m=b_m^*} = -\left.\frac{-\frac{8\beta\tau^2 b_m^3 \sigma_\xi^6 \sigma_\omega^3 \sigma_\varepsilon^4}{\left(b_m^2\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3} - \frac{2\sigma_\xi^2\sigma_\omega}{\left(\sigma_\xi^2+\sigma_\omega^2\right)^2}}{\frac{6\beta\tau^2 b_m^2 \sigma_\xi^4\sigma_\omega^4\sigma_\varepsilon^4\left(\sigma_\xi^2+\sigma_\omega^2\right)-2\beta\tau^4\sigma_\xi^6\sigma_\omega^6\sigma_\varepsilon^2}{\left(b_m^2\sigma_\varepsilon^2\left(\sigma_\xi^2+\sigma_\omega^2\right)+\tau^2\sigma_\xi^2\sigma_\omega^2\right)^3}}\right|_{b_m=b_m^*} < 0$$

by (26). Moreover, since

$$\frac{\partial z}{\partial \sigma_\omega} = \frac{2(1-b_m)\sigma_\omega\sigma_\xi^4}{\sigma_\eta^2 b_m\left(\sigma_\xi^2+\sigma_\omega^2\right)^2} > 0$$

by (27) and $\partial z/\partial b_m|_{b_m=b_m^*} < 0$ by (28), it follows that

$$\frac{dz}{d\sigma_\omega} = \frac{\partial z}{\partial \sigma_\omega} + \frac{\partial z}{\partial b_m^*}\frac{\partial b_m^*}{\partial \sigma_\omega} > 0.$$

It follows that the optimal z^* is increasing in σ_ω.

Note that the optimal z satisfies

$$\frac{dz}{d\sigma_\xi} = \frac{\partial z}{\partial \sigma_\xi} + \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*}\right).$$

By the Implicit function theorem, we get

$$\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*} = -\left.\frac{\partial L/\partial \sigma_\xi}{\partial L/\partial b_m}\right|_{b_m=b_m^*}$$

$$= \left.\frac{\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^3 - 4(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2 b_m^3\sigma_\varepsilon^4\sigma_\omega^4\sigma_\xi^2}{(\sigma_\omega^2+\sigma_\xi^2)^2\beta\tau^2\sigma_\varepsilon^2\sigma_\xi^3\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2-3b_m^2\sigma_\varepsilon^2(\sigma_\xi^2+\sigma_\omega^2)\right)}\right|_{b_m=b_m^*}.$$

It follows from (25) that

$$\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2+\sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^2 = 2\beta\tau^2 b_m\sigma_\varepsilon^2\sigma_\xi^2\sigma_\omega^4(\sigma_\xi^2+\sigma_\omega^2).$$

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By substituting the term $\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)^2$ into the equation, we get

$$\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*}$$

$$=\frac{\left(\tau^2\sigma_\xi^2\sigma_\omega^2 + (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)2\beta\tau^2 b_m\sigma_\varepsilon^2\sigma_\xi^2\sigma_\omega^4(\sigma_\xi^2 + \sigma_\omega^2) - 4(\sigma_\omega^2 + \sigma_\xi^2)^2\beta\tau^2 b_m^3\sigma_\varepsilon^4\sigma_\omega^4\sigma_\xi^2}{(\sigma_\omega^2 + \sigma_\xi^2)^2\beta\tau^2\sigma_\varepsilon^2\sigma_\xi^3\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\Bigg|_{b_m=b_m^*}$$

$$=\frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2 + \sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\Bigg|_{b_m=b_m^*}.$$

It follows from $b_m^* < b_m^\dagger = \sqrt{\frac{\tau^2\sigma_\xi^2\sigma_\omega^2}{3\sigma_\varepsilon^2(\sigma_\xi^2+\sigma_\omega^2)}}$ that

$$[\tau^2\sigma_\xi^2\sigma_\omega^2 - 3\left(\sigma_\xi^2 + \sigma_\omega^2\right)b_m^2\sigma_\varepsilon^2]|_{b_m=b_m^*} > 0. \tag{29}$$

So we have

$$\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*} = \frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2 + \sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\Bigg|_{b_m=b_m^*}$$

$$> \frac{2b_m\sigma_\omega^2\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{(\sigma_\omega^2 + \sigma_\xi^2)\sigma_\xi\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}\Bigg|_{b_m=b_m^*}$$

$$= \frac{2b_m^*\sigma_\omega^2}{(\sigma_\omega^2 + \sigma_\xi^2)\sigma_\xi}.$$

It then follows from (27) and (28) that

$$\frac{dz}{d\sigma_\xi} = \frac{\partial z}{\partial \sigma_\xi} + \left(\left.\frac{\partial z}{\partial b_m}\right|_{b_m=b_m^*}\right)\left(\left.\frac{\partial b_m}{\partial \sigma_\xi}\right|_{b_m=b_m^*}\right)$$

$$< \frac{2(1-b_m)\sigma_\omega^4\sigma_\xi}{\sigma_\eta^2 b_m\left(\sigma_\xi^2 + \sigma_\omega^2\right)^2} - \frac{\sigma_\xi^2\sigma_\omega^2}{\sigma_\eta^2 b_m^2\left(\sigma_\xi^2 + \sigma_\omega^2\right)}\left(\frac{2b_m\sigma_\omega^2}{(\sigma_\omega^2 + \sigma_\xi^2)\sigma_\xi}\right)\Bigg|_{b_m=b_m^*}$$

$$= \frac{2\sigma_\omega^4\sigma_\xi}{\sigma_\eta^2\left(\sigma_\xi^2 + \sigma_\omega^2\right)^2}\left(\frac{1-b_m}{b_m} - \frac{1}{b_m}\right)\Bigg|_{b_m=b_m^*}$$

$$< 0.$$

Hence, the optimal z^* is decreasing in σ_ξ (equivalently, increasing in public disclosure quality $1/\sigma_\xi$).

Finally, note that the optimal z satisfies

$$\frac{dz}{d\sigma_\varepsilon} = \frac{\partial z}{\partial \sigma_\varepsilon} + \left(\frac{\partial z}{\partial b_m}\bigg|_{b_m=b_m^*}\right)\left(\frac{\partial b_m}{\partial \sigma_\varepsilon}\bigg|_{b_m=b_m^*}\right).$$

By the Implicit function theorem, we get

$$\frac{\partial b_m}{\partial \sigma_\varepsilon}\bigg|_{b_m=b_m^*} = -\frac{\partial L/\partial \sigma_\varepsilon}{\partial L/\partial b_m}\bigg|_{b_m=b_m^*} = -\frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}.$$

It follows from (29) that

$$\frac{\partial b_m}{\partial \sigma_\varepsilon}\bigg|_{b_m=b_m^*} = -\frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - (\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)} < -\frac{2b_m\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)}{\sigma_\varepsilon\left(\tau^2\sigma_\xi^2\sigma_\omega^2 - 3(\sigma_\xi^2 + \sigma_\omega^2)b_m^2\sigma_\varepsilon^2\right)} = -\frac{2b_m}{\sigma_\varepsilon}.$$

By (27) and (28), we have

$$\begin{aligned}
\frac{dz}{d\sigma_\varepsilon} &= \frac{\partial z}{\partial \sigma_\varepsilon} + \left(\frac{\partial z}{\partial b_m}\bigg|_{b_m=b_m^*}\right)\left(\frac{\partial b_m}{\partial \sigma_\varepsilon}\bigg|_{b_m=b_m^*}\right)\\
&> -\frac{2b_m^2\sigma_\varepsilon}{\tau^2\sigma_\eta^2} + \left(-\frac{2b_m\sigma_\varepsilon^2}{\tau^2\sigma_\eta^2}\right)\left(-\frac{2b_m}{\sigma_\varepsilon}\right)\bigg|_{b_m=b_m^*}\\
&= \frac{2b_m^2\sigma_\varepsilon}{\tau^2\sigma_\eta^2}\bigg|_{b_m=b_m^*}\\
&> 0.
\end{aligned}$$

Hence, the optimal z^* is increasing in σ_ε. $\square$

Proof of Proposition 5. Fix a tightness of information control z. The action a^* is the same as derived in Proposition 1. Similarly, the price is equal to

$$P = \mathbb{E}\left[V|d,m\right] = \mathbb{E}\left[\omega|\,d,m\right] - \beta\mathbb{E}\left[\mathbb{V}\left[\omega|\,d,t\right]|\,d,m\right].$$

We conjecture that there is an equilibrium in which the price takes the form of $P = b_0 + b_d d + b_m m$, where b_0, b_d, b_m are constants such that $b_d, b_m \geq 0$. Since $\mathbb{E}\left[\eta\right] = 0$, we have that $\mathbb{E}\left[m|t\right] = t$. This implies that given the realizations of $(\omega, \varepsilon, \xi)$, the manager chooses d

and t to maximize:

$$\mathbb{E}\left[P|\omega,\varepsilon,\xi\right] - \left(\frac{\delta}{2}\right)\frac{(d-\omega)^2}{\xi} - \left(\frac{\tau}{2}\right)\frac{(t-\omega)^2}{\varepsilon}$$

$$=b_0 + b_d d + b_m \mathbb{E}\left[m|t\right] - \left(\frac{\delta}{2}\right)\frac{(d-\omega)^2}{\xi} - \left(\frac{\tau}{2}\right)\frac{(t-\omega)^2}{\varepsilon}.$$

The FOC imply that

$$b_d = \frac{\delta}{\xi}(d-\omega) \Rightarrow d(\omega,\xi) = \omega + \frac{b_d}{\delta}\xi$$

and

$$b_m = \frac{\tau}{\varepsilon}(t-\omega) \Rightarrow t(\omega,\varepsilon) = \omega + \frac{b_m}{\tau}\varepsilon.$$

Let $\hat{x}$ be the market's conjecture about x. Given the above characterizations of d and t, and the employee's optimal choice of z, the market's beliefs can be expressed as:

$$\mathbb{E}\left[\omega|m,d\right] = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d-\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}$$

and

$$\mathbb{E}\left[\mathbb{V}\left[\omega|d,t\right]|d,m\right] = \mathbb{E}\left[\left.\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}\right|d,m\right]$$

$$= \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.$$

which implies

$$P = \frac{\frac{\hat{x}}{\sigma_\omega^2} + \frac{d-\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{m-\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}} - \beta\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}.$$

Note that this verifies the conjecture that the price is linear in the signals d and m. Matching

terms, we have:

$$b_d = \cfrac{\cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2}}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}$$

$$b_m = \cfrac{\cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}} \tag{30}$$

$$b_0 = \cfrac{\cfrac{\hat{x}}{\sigma_\omega^2} - \cfrac{\frac{b_d}{\delta}\mu_\xi}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} - \cfrac{\frac{b_m}{\tau}\mu_\varepsilon}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}} - \beta \cfrac{1}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}}.$$

The equilibrium b_d and b_m are given by the solution to the first two equations.

Next, we show that there is a unique solution (b_d, b_m), which implies existence and uniqueness of the equilibrium market price. First, it is clear that the solution, if it exists, must satisfy $b_d, b_m \in [0, 1]$. Then we show that there is a unique solution $b_d(b_m) \in (0, 1)$ for any b_m. Define

$$Q(b_d; b_m) \equiv b_d - \cfrac{\cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2}}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

For any given b_m, b_d is the solution to $Q(b_d; b_m) = 0$ by the first equation of (30). Note that

$$\lim_{b_d \downarrow 0} Q(b_d; b_m) = -1 < 0$$

$$Q(1; b_m) = 1 - \cfrac{\cfrac{\delta^2}{\sigma_\xi^2}}{\left(\cfrac{1}{\frac{b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2} + \cfrac{\delta^2}{\sigma_\xi^2} + \cfrac{1}{\sigma_\omega^2}\right)} > 0$$

and

$$\frac{\partial Q(b_d; b_m)}{\partial b_d} = 1 + \cfrac{2\delta^2 \left(\cfrac{1}{\frac{b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2} + \cfrac{1}{\sigma_\omega^2}\right)}{b_d^3 \sigma_\xi^2 \left(\cfrac{\delta^2}{b_d^2 \sigma_\xi^2} + \cfrac{1}{\frac{b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2} + \cfrac{1}{\sigma_\omega^2}\right)^2} > 0,$$

which implies for every b_m, there exists a unique $b_d(b_m) \in (0, 1)$ by the intermediate value theorem.

Given $b_d(b_m)$, we show that there is a unique solution b_m as follows. By (30), we have

$$1 - (b_d + b_m) = \cfrac{\cfrac{1}{\sigma_\omega^2}}{\cfrac{1}{\sigma_\omega^2} + \cfrac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \cfrac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

Moreover,

$$\frac{b_d^3}{\delta^2}\sigma_\xi^2 = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}.$$

It follows that

$$b_m = 1 - \left(b_d + \frac{b_d^3\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right). \tag{31}$$

Substituting the solution $b_d(b_m)$ into (31), we solve for the equilibrium b_m as the solution to $F = 0$, where

$$F\left(b_m, b_d(b_m)\right) \equiv b_m + b_d + \frac{b_d^3\sigma_\xi^2}{\delta^2\sigma_\omega^2} - 1.$$

Since $b_d\left(b_m\right) \in (0,1)$, we have

$$F\left(1, b_d(1)\right) = 1 + b_d\left(1\right) + \frac{b_d^3\left(1\right)\sigma_\xi^2}{\delta^2\sigma_\omega^2} - 1 > 0$$

$$F\left(0, b_d(0)\right) = -\left(1 - b_d\left(0\right)\right) + \frac{b_d^3\left(0\right)\sigma_\xi^2}{\delta^2\sigma_\omega^2}$$

$$= -\left(1 - b_d\left(0\right)\right) + \frac{1}{\sigma_\omega^2}\frac{1 - b_d\left(0\right)}{\frac{1}{\sigma_\omega^2} + \frac{1}{z\sigma_\eta^2}}$$

$$= \left(1 - b_d\left(0\right)\right)\left(\frac{1}{1 + \frac{\sigma_\omega^2}{z\sigma_\eta^2}} - 1\right)$$

$$< 0,$$

where the second equation of $F(0, b_d(0))$ follows from $Q\left(b_d; b_m = 0\right) = 0$ that

$$\frac{b_d^3\left(0\right)\sigma_\xi^2}{\delta^2} = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d(0)}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{z\sigma_\eta^2}} = \frac{1 - b_d\left(0\right)}{\frac{1}{\sigma_\omega^2} + \frac{1}{z\sigma_\eta^2}}.$$

Further, note that

$$\frac{dF}{db_m} = 1 + \left(1 + \frac{3\sigma_\xi^2 b_d^2}{\delta^2\sigma_\omega^2}\right)\frac{db_d}{db_m} > 0,$$

which follows from the implicit function theorem that

$$\frac{db_d}{db_m} = -\frac{\partial Q/\partial b_m}{\partial Q/\partial b_d} > 0,$$

where $\partial Q/\partial b_m < 0$ and $\partial Q/\partial b_d > 0$ as shown above. Hence, there exists a unique solution $b_m, b_d \in (0,1)$ by the intermediate value theorem.

Finally, we solve for the equilibrium effort x^*. Given the coefficients b_d and b_m, the

manager's effort choice maximizes:

$$b_0 + b_m \mathbb{E}\left[m\right] + b_d \mathbb{E}\left[d\right] + \frac{1}{2}\left(1 - x^2\right) - \left(\frac{\delta}{2}\right) \mathbb{E}\left[\left(\frac{b_d}{\delta}\right)^2 \xi\right] - \left(\frac{\tau}{2}\right) \mathbb{E}\left[\left(\frac{b_m}{\tau}\right)^2 \varepsilon\right].$$

So the equilibrium effort maximizes $(b_m + b_d)\, x + \frac{1}{2}\left(1 - x^2\right)$ and is given by

$$x^* = b_m + b_d.$$

This characterizes the unique linear equilibrium as stated in the proposition. $\qquad\square$

Proof of Proposition 6. It follows from (30) that

$$\frac{b_d^3 \sigma_\xi^2}{\delta^2} = \frac{b_m^3 \sigma_\varepsilon^2}{\tau^2} + z b_m \sigma_\eta^2. \tag{32}$$

Differentiating (32) with respect to δ, we get

$$\left(\frac{3 b_d^2 \sigma_\xi^2}{\delta^2}\right)\left(\frac{db_d}{d\delta}\right) - \frac{2 b_d^3 \sigma_\xi^2}{\delta^3} = \left(\frac{3 b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(\frac{db_m}{d\delta}\right). \tag{33}$$

Differentiating (31) with respect to δ, we get

$$\frac{db_m}{d\delta} = -\left(1 + \frac{3 b_d^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2}\right)\left(\frac{db_d}{d\delta}\right) + \frac{2 b_d^3 \sigma_\xi^2}{\delta^3 \sigma_\omega^2}. \tag{34}$$

Equations (33) and (34) imply that

$$\left(\frac{3 b_d^2 \sigma_\xi^2}{\delta^2} + \left(\frac{3 b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(1 + \frac{3 b_d^2 \sigma_\xi^2}{\delta^2 \sigma_\omega^2}\right)\right)\left(\frac{db_d}{d\delta}\right) = \frac{2 b_d^3 \sigma_\xi^2}{\delta^3} + \left(\frac{3 b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(\frac{2 b_d^3 \sigma_\xi^2}{\delta^3 \sigma_\omega^2}\right).$$

So $db_d/d\delta > 0$. Eliminating $(b_d^3 \sigma_\xi^2)/\delta^2$ in (31) and (32), we get

$$(1 - (b_d + b_m))\sigma_\omega^2 = \frac{b_m^3 \sigma_\varepsilon^2}{\tau^2} + z b_m \sigma_\eta^2. \tag{35}$$

Differentiating (35) with respect to δ yields

$$-\sigma_\omega^2 \frac{db_d}{d\delta} = \left(\frac{3 b_m^2 \sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2 + \sigma_\omega^2\right)\frac{db_m}{d\delta}.$$

So $db_m/d\delta < 0$. We also observe that

$$\frac{dx^*}{d\delta} = \frac{d(b_m + b_d)}{d\delta} = -\frac{1}{\sigma_\omega^2}\left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\frac{db_m}{d\delta} > 0.$$

Hence, an increase in public disclosure scrutiny δ leads to an increase in effort x^*.

Next, we examine how public disclosure scrutiny δ affects price informativeness and real efficiency. Since $P = b_0 + b_m m + b_d d$, where $m = \omega + \frac{b_m}{\tau}\varepsilon + \eta$, $\eta \sim N(0, z\sigma_\eta^2)$ and $d = \omega + \frac{b_d}{\delta}\xi$, we can write

$$\mathbb{V}[\omega|P] = \mathbb{V}[\omega|y_P] = \frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\sigma_y^2}},$$

where $y_P \equiv \omega + \frac{1}{b_m + b_d}\left(b_m\left(\frac{b_m}{\tau}\varepsilon + \eta\right) + b_d\left(\frac{b_d}{\delta}\xi\right)\right)$ and

$$\sigma_y^2 \equiv \frac{\left(b_d^2\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2 + b_m^2\left(\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)}{(b_m + b_d)^2}.$$

It follows from (30) that

$$\frac{1}{\sigma_y^2} = \frac{(b_m + b_d)^2}{\left(b_d^2\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2 + b_m^2\left(\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2\right)\right)}$$

$$= \frac{(b_m + b_d)^2}{\frac{b_d}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}} + \frac{b_m}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}}$$

$$= \frac{b_m + b_d}{\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2 + z\sigma_\eta^2}}}$$

$$= \frac{b_m + b_d}{\sigma_\omega^2(1 - (b_d + b_m))}.$$

So we have

$$\mathbb{V}[\omega|P] = \frac{1}{\frac{1}{\sigma_y^2} + \frac{1}{\sigma_\omega^2}}$$

$$= \frac{1}{\frac{b_m + b_d}{\sigma_\omega^2(1 - (b_d + b_m))} + \frac{1}{\sigma_\omega^2}} \tag{36}$$

$$= \sigma_\omega^2(1 - (b_d + b_m))$$

$$= \sigma_\omega^2(1 - x^*),$$

implying that $\mathbb{V}[\omega|P]$ is decreasing with δ. Hence, $PI = (\mathbb{V}[\omega|P])^{-1}$ is increasing with δ.

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For real efficiency, recall that

$$
\begin{aligned}
RE &= \mathbb{E}\left[V^*\right] \\
&= \mathbb{E}\left[\omega - \beta\left(a - \omega\right)^2\right] \\
&= x^* - \beta\mathbb{E}[\mathbb{V}[\omega|d,t]] \\
&= x^* - \beta\left(\frac{1}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}\right).
\end{aligned}
$$

Taking derivative with respect to δ, we get

$$
\frac{dRE}{d\delta} = \frac{dx^*}{d\delta} + 2\beta\left(\frac{-\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(\frac{db_d}{d\delta}\right) + \frac{\delta}{b_d^2\sigma_\xi^2} - \left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right)}{\left(\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}\right)^2}\right).
$$

By (33), we have

$$
\frac{b_d^2\sigma_\xi^2}{\delta^2}\left(3\frac{db_d}{d\delta} - \frac{2b_d}{\delta}\right) = \left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(\frac{db_m}{d\delta}\right) < 0.
$$

It follows that

$$
\begin{aligned}
-\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(\frac{db_d}{d\delta}\right) + \frac{\delta}{b_d^2\sigma_\xi^2} - \left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right) &= \left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(-\frac{db_d}{d\delta} + \frac{b_d}{\delta}\right) - \left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right) \\
&> -\frac{1}{3}\left(\frac{\delta^2}{b_d^3\sigma_\xi^2}\right)\left(3\frac{db_d}{d\delta} - 2\frac{b_d}{\delta}\right) - \left(\frac{\tau^2}{\sigma_\varepsilon^2 b_m^3}\right)\left(\frac{db_m}{d\delta}\right) \\
&> 0,
\end{aligned}
$$

since $3\frac{db_d}{d\delta} - \frac{2b_d}{\delta} < 0$ from the equation above and $db_m/d\delta < 0$. Because $dx^*/d\delta > 0$, we conclude that $dRE/d\delta > 0$, i.e., real efficiency is increasing with public disclosure scrutiny.

We examine how the tightness of information control z affects effort, price informativeness and real efficiency as follows. Recall that the equilibrium values of b_d and b_m are given by (31) and (32). Differentiating (31) with respect to z, we get

$$
\frac{db_m}{dz} = -\frac{db_d}{dz}\left(1 + \frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right). \tag{37}
$$

Differentiating (32) with respect to z, we get

$$\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2}\right)\left(\frac{db_d}{dz}\right) = \left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(\frac{db_m}{dz}\right) + \sigma_\eta^2 b_m. \tag{38}$$

It follows from (37) and (38) that

$$\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2} + \left(\frac{3b_m^2\sigma_\varepsilon^2}{\tau^2} + z\sigma_\eta^2\right)\left(1 + \frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right)\right)\left(\frac{db_d}{dz}\right) = \sigma_\eta^2 b_m > 0.$$

So $db_d/dz > 0$. By (37), we also obtain $db_m/dz < 0$ and

$$\frac{dx^*}{dz} = \frac{d(b_m + b_d)}{dz} = -\left(\frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2}\right)\left(\frac{db_d}{dz}\right) < 0.$$

Hence, tighter information control (larger z) leads to a decrease in effort x^*. Further, it follows from (36) that

$$\frac{d\mathbb{V}[\omega|P]}{dz} = -\sigma_\omega^2\frac{dx^*}{dz} > 0.$$

So $PI = (\mathbb{V}[\omega|P])^{-1}$ is decreasing with z.

It is useful in the remaining proof to define the limits of b_m and b_d as $z \to 0$ and $z \to \infty$. When $z \to \infty$, (30) converges to

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}$$

$$b_m = 0.$$

Thus, $b_{d\infty} \equiv \lim_{z\to\infty} b_d > 0$ and $\lim_{z\to\infty} b_m = 0$. Note that b_{d0} is the unique root of the above cubic equation and the supremum of b_d. When $z \to 0$, (30) converges to

$$b_d = \frac{\frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}} \tag{39}$$

$$b_m = \frac{\frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}}{\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}} \tag{40}$$

implying

$$\frac{\sigma_\xi^2}{\delta^2}b_d^3 = \frac{\sigma_\varepsilon^2}{\tau^2}b_m^3.$$

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With this relation, (39) and (40) imply a cubic equation for each of b_m and b_d with a unique positive root. The roots are denoted by $b_{m\infty} > 0$ and $b_{d\infty} > 0$. To summarize, as z increases, b_m decreases starting from b_{m0} to 0, while b_d increases from b_{d0} to $b_{d\infty}$.

Turning to real efficiency, (31) implies

$$t \equiv \frac{db_d}{db_m} = -\frac{\delta^2 \sigma_\omega^2}{\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2}$$

as we change z. Here, we view b_d as a function of a single variable b_m, which is uniquely determined by z implicitly. It follows that

$$\frac{dt}{db_m} = \frac{6 b_d \delta^2 \sigma_\omega^2 \sigma_\xi^2}{\left(\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2\right)^2} t$$

$$= -\frac{6 b_d \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2} t^2$$

$$= -\frac{2}{b_d} (1 + t) t^2.$$

Note that

$$t \in \left(-\frac{\delta^2 \sigma_\omega^2}{\delta^2 \sigma_\omega^2 + 3b_{d0}^2 \sigma_\xi^2}, -\frac{\delta^2 \sigma_\omega^2}{\delta^2 \sigma_\omega^2 + 3b_{d\infty}^2 \sigma_\xi^2} \right),$$

and $1 + t$ has a strictly positive infimum. Define

$$A(b_m) \equiv \frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2 \sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2 \sigma_\varepsilon^2}.$$

Then,

$$\frac{dA}{db_m} = -\frac{2\delta^2}{b_d^3 \sigma_\xi^2} t - \frac{2\tau^2}{b_m^3 \sigma_\varepsilon^2}$$

and

$$\frac{d^2 A}{db_m^2} = \frac{6\delta^2}{b_d^4 \sigma_\xi^2} t^2 - \frac{2\delta^2}{b_d^3 \sigma_\xi^2} \frac{dt}{db_m} + \frac{6\tau^2}{b_m^4 \sigma_\varepsilon^2}$$

$$= \frac{6\delta^2}{b_d^4 \sigma_\xi^2} t^2 + \frac{2\delta^2}{b_d^3 \sigma_\xi^2} \frac{6 b_d \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2} t^2 + \frac{6\tau^2}{b_m^4 \sigma_\varepsilon^2}$$

$$= \frac{6\delta^2}{b_d^3 \sigma_\xi^2} \left[\frac{1}{b_d} + \frac{2 b_d \sigma_\xi^2}{\delta^2 \sigma_\omega^2 + 3b_d^2 \sigma_\xi^2} \right] t^2 + \frac{6\tau^2}{b_m^4 \sigma_\varepsilon^2}$$

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Note that

$$\frac{dA}{db_m} = -\frac{2\delta^2}{b_d^3\sigma_\xi^2}t - \frac{2\tau^2}{b_m^3\sigma_\varepsilon^2}$$

$$\leq -\frac{2\delta^2}{b_d^3\sigma_\xi^2}t - \frac{2\delta^2}{b_d^3\sigma_\xi^2}$$

$$= -\frac{2\delta^2}{b_d^3\sigma_\xi^2}(1+t)$$

$$< 0,$$

because (32) implies that

$$\frac{b_d^3\sigma_\xi^2}{\delta^2} \geq \frac{b_m^3\sigma_\varepsilon^2}{\tau^2}.$$

As

$$RE = b_m + b_d - \frac{\beta}{A},$$

we have

$$\frac{dRE}{db_m} = 1 + t + \frac{\beta}{A^2}\frac{dA}{db_m}.$$

Therefore,

$$\frac{dRE}{db_m} \gtrless 0 \iff \frac{A^2(1+t)}{(-dA/db_m)} \gtrless \beta.$$

Define from above that

$$h(b_m) = \frac{A(b_m)^2(1+t(b_m))}{(-dA/db_m)}.$$

Because $\frac{db_m}{dz} < 0$, RE is increasing (decreasing) in z if and only if $\frac{dRE}{db_m} < 0$ (> 0), or equivalently, if and only if $h(b_m) \lessgtr \beta$.

First, we consider the behavior of $h(b_m)$ when z is large. As noted above, $1+t$ is bounded in $\left(\frac{3b_{d0}^2\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_{d0}^2\sigma_\xi^2}, \frac{3b_{d\infty}^2\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_{d\infty}^2\sigma_\xi^2}\right)$. Since $b_m \to 0$ and $b_d \to b_{d\infty} > 0$ as $z \to \infty$,

$$\lim_{z\to\infty} h(b_m) = \lim_{z\to\infty}\left[\frac{1}{\sigma_\omega^2} + \frac{1}{\left(\frac{b_d}{\delta}\right)^2\sigma_\xi^2} + \frac{1}{\left(\frac{b_m}{\tau}\right)^2\sigma_\varepsilon^2}\right]^2 \frac{3b_d^2\sigma_\xi^2}{\delta^2\sigma_\omega^2 + 3b_d^2\sigma_\xi^2}\frac{1}{\frac{2\delta^2}{b_d^3\sigma_\xi^2}t + \frac{2\tau^2}{b_m^3\sigma_\varepsilon^2}},$$

$$= \infty.$$

Thus, when z is sufficiently large, $h(b_m)$ becomes arbitrarily large. It implies that, for any value of β, $h(b_m) > \beta$ for sufficiently large z and RE is decreasing in z.

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Next, we consider $\frac{dh}{db_m}$. Denote $\frac{dA}{db_m}$ and $\frac{dt}{db_m}$ by A' and t', respectively. Then,

$$\frac{dh(b_m)}{db_m} = \frac{[2AA'(1+t) + A^2 t'](-A') + A^2 A''(1+t)}{A'^2}, \tag{41}$$

the sign of which is driven by $\frac{1}{b_m}$ terms because $\lim_{z\to\infty} b_m = 0$ and $\lim_{z\to\infty} b_d = b_{d\infty} > 0$. If we collect the highest-order terms of $(1/b_m)$ in the numerator of (41), which is a polynomial of $(1/b_m)$, and drop all other terms, we are left with

$$A(1+t)\left(AA'' - 2A'^2\right) \approx A(1+t)\left(\frac{\tau^2}{b_m^2 \sigma_\varepsilon^2}\frac{6\tau^2}{b_m^4 \sigma_\varepsilon^2} - 2\left(\frac{2\tau^2}{b_m^3 \sigma_\varepsilon^2}\right)^2\right)$$

$$= A(1+t)\left(\frac{-2\tau^4}{b_m^6 \sigma_\varepsilon^4}\right)$$

$$< 0,$$

implying that $\frac{dh(b_m)}{db_m} < 0$ for large z. Since $\frac{db_m}{dz} < 0$, $\frac{dh(b_m(z))}{dz} = \frac{dh}{db_m}\frac{db_m}{dz} > 0$ for sufficiently large z. In other words, there exists a $\hat{z}$ such that, for all $z \in [\hat{z}, \infty)$, $\frac{dh(b_m(z))}{dz} > 0$.

As $h(b_m)$ is continuous and $\lim_{z\to 0} h(b_m)$ is finite, $h(b_m(z))$ attains a maximum on $[0, \hat{z}]$ by the extreme value theorem. Call the maximum of $h(b_m)$ M. Since $\lim_{z\to\infty} h(b_m) = \infty$, there exists a $\bar{z} > \hat{z}$ such that $h(b_m(\bar{z})) > M$. If we choose β to be

$$\bar{\beta} \equiv h(b_m(\bar{z})),$$

- For all $z \in [0, \bar{z})$, $h(b_m(z)) < \bar{\beta}$, because $h(b_m(z)) \le M < \bar{\beta}$ on $[0, \hat{z}]$ and $h(b_m(z))$ is increasing on $[\hat{z}, \bar{z})$.

- For all $z \in (\bar{z}, \infty)$, $h(b_m(z)) > \bar{\beta}$, because $h(b_m(z))$ is increasing on $[\hat{z}, \infty)$ and $h(b_m(\bar{z})) = \bar{\beta}$.

All combined, RE is hump-shaped when $\beta = \bar{\beta}$. It can be easily seen that RE is hump-shaped for all $\beta > \bar{\beta}$.

As noted above, $1 + t$ and $-\frac{dA}{db_m}$ are bounded above and below away from 0, while A is bounded below away from 0 as it is always larger than $\frac{1}{\sigma_\omega^2}$. It follows that $\frac{A^2(1+t)}{(-dA/db_m)}$ is bounded below away from 0. Thus, if β is sufficiently small, $\frac{A^2(1+t)}{(-dA/db_m)} > \beta$ for all z, so $\frac{dRE}{db_m} > 0$ for all z. Since $\frac{db_m}{dz} < 0$, RE is decreasing everywhere in z. $\qquad\square$