Feedback Effects and Systematic Risk Exposures*

Snehal Banerjee[†] Bradyn Breon-Drish[‡] Kevin Smith[§]

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Abstract

We study the implications of the "feedback effect" when a firm's investment decision affects its exposure to a systematic risk factor. As a leading example, we consider a manager's decision to invest in a "green" project based on her firm's stock price. The firm's price conveys information regarding both the project's cash flows and its discount rate, which depends on its factor exposure. The interaction between the firm's investment and its factor exposure yields novel predictions about the likelihood of investment, expected returns, and future profitability. Moreover, while feedback makes investment more informationally efficient, it can reduce investor welfare.

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†Email: snehalb@ucsd.edu. Rady School of Management, UC San Diego, 9500 Gilman Drive, La Jolla, CA 92093, United States.

‡Email: bbreondrish@ucsd.edu. Rady School of Management, UC San Diego, 9500 Gilman Drive, La Jolla, CA 92093, United States.

§Email: kevinsm@stanford.edu. Graduate School of Business, Stanford University, 655 Knight Way, Stanford, CA 94305, United States.

1 Introduction

Financial markets play two main roles in the economy. First, they aggregate and convey information to managers about investment opportunities, as emphasized by the "feedback effects" literature. Second, markets allow investors to share and hedge risks. We show that the interaction of these two roles has important consequences for firms' investment decisions and investor welfare. In particular, a firm's investment choices, which depend on the information in its stock price, endogenously change its risk exposures. In turn, these exposures drive investors' hedging demands for the stock, which affect the information that the stock price conveys to the manager, and ultimately, investor welfare.

To study this interaction, we develop a model of feedback effects in which a firm's manager decides whether to invest in a project that is exposed to a systematic risk factor. The firm's stock is traded by risk-averse investors who are informed about the project's expected cash flows and have background risks that are correlated with the risk factor. The manager's objective is to maximize the firm's stock price, and so she chooses to invest when the project has a positive net present value (NPV). Moreover, she learns about investors' cash flow information and their aggregate risk exposure, which drives the project's discount rate, from the stock price.

The systematic risk factor we consider is normalized to be negatively correlated to investor wealth, and so positive (negative) exposures correspond to higher (lower) valuations. Moreover, the *magnitude* of the project's risk exposure determines the extent to which its NPV is driven by variation in its discount rate. In the presence of market feedback, we show that this has novel implications for investment decisions. First, controlling for its ex-ante profitability, the likelihood of investment in a project is non-monotonic in its risk exposure. For instance, for projects that are ex-ante unattractive, the likelihood of investment for positively- and negatively-exposed projects can be higher than for projects with no exposure. Second, while investment in positively-exposed projects leads to lower expected stock returns than that in unexposed projects, investment in negatively-exposed projects can also lead to lower returns. Third, investment in negatively-exposed projects is associated with a greater increase in the firm's expected cash flows than investment in positively-exposed projects.[1]

Our analysis also has implications for investor welfare. When investors' exposures to the systematic risk factor are identical, we show that a manager who maximizes price tends

[1] As we discuss in Section 6, the first and third predictions distinguish our model from a feedback setting where the manager maximizes expected cash flows, while the second prediction distinguishes our model from a setting in which the manager learns from an exogenous public signal, but there is no feedback from the price.

to under-invest relative to the welfare-maximizing investment rule, while a manager who maximizes expected cash flows tends to under-invest in positively-exposed projects but over-invest in profitable, negatively-exposed projects. More generally, while feedback necessarily increases the firm's share price, we show that it can decrease investor welfare when the project has a positive (ex-ante) expected NPV and investors' per-capita ownership of the firm is sufficiently small, or their exposures to the risk factor are sufficiently diverse. This is because the manager fails to appropriately account for the externality that her investment has on investors' ability to hedge and share risk.

Application to Climate-Sensitive Investment. Our analysis applies quite broadly to investment in risky projects when market feedback plays an important role. However, a particularly salient application is to climate-sensitive investment, and so we use it to describe our model's economic forces and predictions. Consistent with our model's key assumptions, there is substantial evidence that investors have time-varying exposures to climate risk that affect their demands for green and brown stocks and alter these stocks' discount rates (e.g., Choi, Gao, and Jiang (2020), Bolton and Kacperczyk (2021b), Pástor, Stambaugh, and Taylor (2022)).

Moreover, consistent with managers responding to the information that prices contain about cash flows and discount rates, empirical evidence shows that firms' investment in climate-exposed projects respond to changes in their stock prices, even when driven by shocks to investor demand for green exposure rather than cash flow news (e.g., Li, Shan, Tang, and Yao (2020), Bai, Chu, Shen, and Wan (2021), Briere and Ramelli (2021)). Finally, investors' climate exposure differs with age, geography, and adaptability (Giglio, Kelly, and Stroebel (2021)), and, as evidenced by the swath of actively-traded climate-based ETFs, investors appear to use financial markets as a means to hedge and share such risk exposures.[2]

Intuition. We show that the manager's investment decision follows a threshold strategy: she only invests when the stock price is sufficiently high, because this implies that the project's NPV, conditional on the price information, is positive. The price information depends not only on investors' information about the project's future cash flows, but also on the project's climate risk exposure, or "greenness". A "green" ("brown") project is defined as one that pays higher (lower) cash flows when climate outcomes are worse, while a "neutral" project's cash flows are uncorrelated with climate outcomes. As such, green projects are negatively exposed to the climate risk factor, while brown projects are positively exposed.[3]

[2] For example, see Ilhan (2020), Krueger, Sautner, and Starks (2020), Ilhan, Krueger, Sautner, and Starks (2021), Giglio et al. (2021) and our discussion in Section 3.1. Moreover, total assets under management in sustainability-focused funds roughly doubled from Q4 2019 to Q3 2022, concurrent with over 200 sustainability fund launches per year (Morningstar (2022)).

[3] Our definitions of "green" versus "brown" projects are consistent with the labels in the empirical litera-

For example, consider a consumer electronics firm deciding whether to invest in electric vehicle (EV) technology, such as batteries or semiconductors. Such a green investment is negatively exposed to climate risk: for instance, shifts in regulatory policy in response to climate change may lead to more favorable treatment of electric vehicles relative to traditional vehicles.[4] Thus, the manager's decision depends, in part, on the fact that such investments are likely to perform better when aggregate climate outcomes are worse.

A project's risk exposure influences this decision via two channels. First, it affects the conditional expectation of the project's NPV. A green project provides a hedge against bad climate outcomes and so, all else equal, has a higher valuation, while a brown project does worse when climate outcomes are bad and so has a lower valuation. This tends to make the likelihood of investment higher for green projects than for neutral ones, and higher for neutral projects than for brown ones.

Second, because the aggregate hedging demand is stochastic, the higher the magnitude of the project's risk exposure (irrespective of whether it is brown or green), the higher the volatility of the conditional NPV (i.e. the more volatile the "discount rate" of the project). For an ex-ante unattractive project, this increases the likelihood that the project will have a positive NPV, and so increases the likelihood of investment.[5] When this second channel dominates, the manager may be more likely to invest in brown projects that are ex-ante unattractive than in comparable neutral projects.

Because green projects hedge adverse climate outcomes, they tend to have higher valuations and thus lower expected returns. In contrast, because brown projects perform poorly when climate outcomes are worse, they have lower valuations on average. Therefore, if a manager chooses to invest in a brown project, it must be because either: (i) the price information about cash flows is sufficiently positive to overcome the discount due to its climate exposure, or (ii) its discount rate is unexpectedly low relative to the average brown project. This implies that, conditional on investment, brown projects tend to have higher future cash

ture (e.g., Engle, Giglio, Kelly, Lee, and Stroebel (2020), Bolton and Kacperczyk (2021a)), as we discuss in Section 3.1.

[4]Panasonic, historically associated with consumer electronics, is now also a leading manufacturer of rechargeable batteries for electric vehicle companies (e.g., Tesla supplier Panasonic plans additional $4 bln U.S. EV battery plant, Reuters, Aug 26, 2022). Such investments are likely to benefit from regulatory changes that provide tax subsidies to encourage the purchase of electric vehicles, which is an example of climate transition risk. More generally, Giglio et al. (2021) distinguish two types of climate risk: transition risk (i.e., "risks to cash flows arising from a possible transition to a low- carbon economy") versus physical risks (i.e., "direct impairment of productive assets resulting from climate change").

[5]For an ex-ante attractive project, the increased variance in NPV decreases the likelihood of investment. As we discuss further in Section 6, the increase in the magnitude of the project's risk exposure has the opposite effect when the manager maximizes expected cash flows. In this case, higher risk-exposures make the price a noisier signal about the project's cash flows and so the manager is more likely to follow her priors i.e., invest more in ex-ante attractive projects, but not invest in ex-ante unattractive ones.

flows (or profitability) than green projects. Moreover, a brown project that the manager chooses to invest in is likely valued higher than the (unconditional) average brown project, and consequently has a lower expected return. As a result, the negative relation between expected returns and "greenness" is less prominent for brown firms than for green firms.

Welfare. We also characterize how the firm's investment in climate-exposed projects impacts investor welfare. We first consider a benchmark in which all investors have identical exposures to the climate risk factor. In this case, we show that the price-maximizing investment rule tends to under-invest relative to the welfare-maximizing price-contingent investment rule. Intuitively, this is because the price reflects the marginal risk of an additional share, but welfare depends on the average risk borne by investors. Because the marginal risk of the last share is higher than the average risk of all shares held in equilibrium, the price-maximizing rule tends to under-invest. We also show that a manager who maximizes expected cash flows tends to under-invest in green projects, and over-invest in brown projects, relative to welfare maximization. This is because she ignores the fact that green projects improve welfare by reducing investors' exposure to adverse climate shocks, and vice versa for brown projects.

We then consider the general setting in which investors have heterogeneous exposures to climate risk. This is a realistic feature: an investor who lives in coastal California is more exposed to climate risk due to rising sea levels, and so has a different demand for green stocks, than an investor who lives in central Kansas.[6] In such settings, a firm's investment in a climate-sensitive project has an additional impact on welfare because it allows investors to use the firm's stock to help share risk: all else equal, both investors are better off when the Kansas investor sells some shares of a firm that invests in green EV projects to the California investor. However, the welfare improvement as a result of this "risk-sharing" channel is not captured by the stock price, which reflects the average of investors' marginal risk exposures, and not the heterogeneity in their exposures.

As a result, while feedback necessarily increases the firm's share price, we show that it can decrease investor welfare. Intuitively, without feedback, the manager would always invest in an ex-ante attractive project, but with feedback, she would not invest in such a project if the equilibrium price was sufficiently low. This lower investment increases welfare due to higher stock valuations, but decreases welfare due to the risk-sharing channel. When investors' exposures to climate risk are sufficiently diverse or per-capita ownership of the firm is sufficiently small, the latter effect dominates and welfare is higher without feedback

[6] Consistent with this, Ilhan (2020) documents that households with differential exposures to sea level rise have different participation in equity markets, and consequently, different portfolios. More generally, as we discuss in Section 3.1, investors have different exposures to climate risk and find this risk difficult to hedge.

than with. In such settings, our analysis suggests that providing additional incentives for managers to invest in green projects (e.g., by linking their compensation to climate scores) can increase investor welfare, even though it may lead to lower valuations and lower future profitability.

Overview. The rest of the paper is organized as follows. The next section discusses related literature. Section 3 presents the model and discusses key assumptions. Section 4 characterizes the equilibrium in our setting. Section 5 characterizes the model's implications for market outcomes. Section 6 characterizes how our results change when the manager maximizes expected cash flows or uses an exogenous public signal (instead of the price) to make her investment decision. Section 7 presents our results on investor welfare. Section 8 summarizes the empirical predictions from our analysis, and Section 9 concludes. Proofs of our results are in Appendix A, and additional analysis is presented in the Internet Appendix.

2 Related Literature

Our paper is related to two strands of literature. First, it adds to the literature on feedback effects (see Bond, Edmans, and Goldstein (2012) and Goldstein and Yang (2017) for recent surveys and early work by Khanna, Slezak, and Bradley (1994), Subrahmanyam and Titman (2001) and others). In contrast to our setting, much of this existing literature focuses on economies in which (i) investors are either risk-neutral or the stock price is set by a risk-neutral market maker, (ii) the noise in prices arises due to noise traders with unmodeled utility functions, and (iii) the manager's investment choice maximizes the firm's expected terminal cash flow. As a result, such models are not well suited to study how discount rate variation affects investment decisions or how feedback affects investor welfare.[7] To our knowledge, our paper is the first model of feedback effects in which managers learn not only about cash flows but also about discount rates from prices, even though the importance of this channel has long been recognized (e.g., Diamond (1967)).

The most closely related papers in this literature are Dow and Rahi (2003), Hapnes (2020), and Gervais and Strobl (2021). Dow and Rahi (2003) explore how increases in informed trading affect investment efficiency and risk sharing in a setting where investors are risk averse but prices are set by a risk-neutral market maker. They argue that investment efficiency always improves with more informed trading, but risk sharing may either worsen due to the Hirshleifer (1971) effect, or improve when information decreases uncertainty over

[7]See Diamond and Verrecchia (1981), Wang (1994), Schneider (2009), Ganguli and Yang (2009), Manzano and Vives (2011), and Bond and Garcia (2022) for models in which noise is driven by hedging needs as in our model. Existing feedback models with risk-neutral pricing include Dow, Goldstein, and Guembel (2017), Davis and Gondhi (2019), and Goldstein, Schneemeier, and Yang (2020).

the component of the asset's payoffs that are unrelated to the component that investors wish to hedge. Hapnes (2020) characterizes managerial investment behavior and investor information acquisition in a Grossman and Stiglitz (1980)-type model with feedback; however, the analysis does not study the effect of feedback on welfare. Gervais and Strobl (2021) consider the impact of informed, active money management on investment decisions in a setting with feedback. They study how the gross and net performance of the actively managed fund compares with the market portfolio and study how the presence of an informed money manager affects welfare.

We view our analysis as complementary. We focus on how investment in a project affects the risk exposure of a firm's cash flows, which in turn, affects how useful the stock is for hedging. This highlights a novel channel through which feedback affects welfare: intuitively, firms' investment decisions *endogenously* affect the degree of market completeness in the economy.[8] Also, since investors are identically informed in our analysis, the traditional Hirshleifer (1971) effect is turned off, which allows us to clearly distinguish our novel channel from earlier work.[9]

Our focus on welfare is also complementary to recent work by Bond and Garcia (2022), who show that while indexing may reduce price efficiency, it improves retail investor welfare due to improvements in risk sharing. Tension between notions of firm profitability and welfare also appears in Goldstein and Yang (2022), who show that improvements in price informativeness increase producer profits due to better-informed real investment, but may harm welfare by destroying risk-sharing opportunities, similar to the Hirshleifer (1971) effect. Similar to our findings, other papers studying discrete investment choice also emphasize the importance of the firm's "default" investment decision in the absence of feedback.[10] Our analysis complements this earlier work by identifying a novel tension between managerial investment choices and welfare that is driven by how investment affects the ability of investors to use the stock to hedge risk.

Our paper is also related to the growing theoretical literature on ESG investing and

[8]This also distinguishes our analysis from Marín and Rahi (1999), Marín and Rahi (2000), and Eckwert and Zilcha (2003), who consider how exogenous differences in market completeness influence investor welfare.

[9]While the Hirshleifer (1971) effect and our risk-sharing channel both affect the ability of investors to share risk, the two mechanisms are distinct. The Hirshleifer (1971) effect refers to the phenomenon where the introduction of public information destroys risk-sharing opportunities. In contrast, our risk-sharing channel captures the fact that endogenous investment decisions can affect the effective completeness of the market by directly changing the risk exposures of traded securities.

[10]For instance, Dow et al. (2017) show that investors' equilibrium information acquisition hinges on whether the firm defaults to a risky or a riskless project. Davis and Gondhi (2019) show that complementarity in learning depends on both the default investment decision and on the correlation between the investment and assets in place. Goldstein et al. (2020) study information acquisition in a feedback model with multiple sources of uncertainty. They show that investors seek to acquire the same information as management for positive NPV projects, but different information for negative NPV projects.

climate risk.[11] Our work is most closely related to Pástor, Stambaugh, and Taylor (2021) and Goldstein, Kopytov, Shen, and Xiang (2021). Pástor et al. (2021) show that green assets have lower costs of capital because investors enjoy holding them and they hedge climate risk. Goldstein et al. (2021) consider a model where traditional and green investors are informed about a firm's financial and ESG output, and demonstrate that this can lead to multiple equilibria. Our setting generates distinct predictions for green investment decisions and welfare by incorporating the feedback effect and considering green investment's impact on risk-sharing efficiency.

3 Model

We consider a model of feedback effects where the investment is exposed to a systematic risk. We present the model in the context of climate risk as it is a significant and direct application, but, as we discuss in the conclusion, our analysis has other applications.

Payoffs. There are four dates $t \in \{1, 2, 3, 4\}$ and two securities. The risk-free security is normalized to the numeraire. A share of the risky security is a claim to terminal per-share cash flows V generated by the firm at date four, and trades on dates one and three at prices P_1 and P_3, respectively.

Investors. There is a continuum of investors, indexed by $i \in [0, 1]$, with CARA utility over terminal wealth with risk aversion γ. Investor i has initial endowment of n shares of the risky asset and, $z_i = Z + \zeta_i$ units of exposure to a non-tradeable source of income that has payoff of $-\eta_C$, where $Z \sim N\left(\mu_Z, \tau_Z^{-1}\right)$, $\zeta_i \sim N\left(0, \tau_\zeta^{-1}\right)$ and $\eta_C \sim N\left(0, \tau_\eta^{-1}\right)$ are independent of each other and all other random variables.[12] Investor i chooses trades X_{it}, $t \in \{1, 3\}$ to maximize her expected utility over terminal wealth, which is given by

$$W_i = (n + X_{i1} + X_{i3}) V - X_{i3} P_3 - X_{i1} P_1 - z_i \eta_C. \tag{1}$$

We interpret η_C as climate risk shocks, which reduce investor wealth and, consequently, utility.[13] Furthermore, Z captures investors' aggregate exposure to climate risk shocks, and μ_Z is the average exposure to climate risk. The natural restriction for this interpretation is $\mu_Z > 0$ i.e., shocks to the climate (i.e., positive innovations to η_C) have, in expectation, a negative impact on the average investor. In our analysis, we will focus on this restriction to

[11] Additional studies include Heinkel, Kraus, and Zechner (2001), Friedman and Heinle (2016), Chowdhry, Davies, and Waters (2019), Oehmke and Opp (2020), and Pedersen, Fitzgibbons, and Pomorski (2021).

[12] We let $\tau_{(\cdot)}$ denote the unconditional precision and $\sigma^2_{(\cdot)}$ the unconditional variance of all random variables.

[13] While, for concreteness, we refer to η_C as a non-tradeable payoff, it is equivalent to interpret it as a non-monetary climate shock to which investors are differentially exposed and that affects their utility directly.

clearly distinguish between projects that are positively vs. negatively exposed to the climate.

We further require the parameter restriction $\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)^{-1}-\gamma^2\frac{1}{\tau_\eta}>0$ in order to ensure that the unconditional expected utility is finite. Intuitively, if this condition is violated, the climate payoffs $z_i\eta_C$ are sufficiently uncertain ex-ante that the expected utility diverges to $-\infty$. We summarize these restrictions in the following assumption, which is maintained throughout our analysis.

Assumption 1. *(i) The average exposure to climate risk μ_Z is positive, i.e., $\mu_Z>0$.*
(ii) Uncertainty about overall climate payoffs is sufficiently small i.e., $\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)^{-1}>\gamma^2\frac{1}{\tau_\eta}$.

The firm. The firm generates cash flows per share $A\sim N\left(\mu_A,\tau_A^{-1}\right)$ from assets in place. In addition, the firm's manager decides whether to invest in a new project. The investment decision is binary and denoted by $k\in\{0,1\}$. The firm's cash flow per share, given an investment choice k, equals

$$V(k)=A+k\left(\theta+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I-c\right), \tag{2}$$

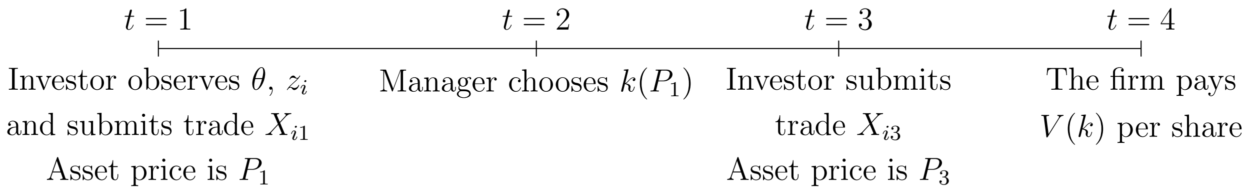
where $\theta\sim N\left(\mu_\theta,\tau_\theta^{-1}\right)$ and $\eta_C,\ \eta_I\sim N\left(0,\tau_\eta^{-1}\right)$ are independent of each other and other random variables, $\alpha\in[-1,1]$ and $c\geq 0$. The component θ reflects the learnable component of cash flows for the investment opportunity, η_C reflects shocks to the "climate" component of cash flows, and η_I reflects shocks to the "idiosyncratic" component of cash flows. The cost of investment is c, which is assumed to be non-negative.

The parameter α captures the extent to which the project's cash flows are correlated with climate risk shocks. When $\alpha=0$, the new project's cash flows are uncorrelated with climate risk and so are not useful for hedging – we refer to such projects as "neutral" projects. When $\alpha>0$, the project's cash flows are *higher* when climate outcomes are worse (η_C is higher), and so we refer to these projects as "green" projects. This increase in cash flows may be due to higher demand for the product (e.g., electric vehicles) or regulatory changes (e.g., higher taxes on greenhouse gas emissions) driven by adverse changes in the climate. Analogously, when $\alpha<0$, the project's cash flows are *lower* when climate outcomes are worse, and so we refer to these projects as "brown" projects.[14]

Information and timing of events. Figure 1 summarizes the timing of events. At date one, all investors observe θ perfectly. Let $\mathcal{F}_{i1}=\sigma\left(\theta,z_i,P_1\right)$ and $\mathcal{F}_{i3}=\sigma\left(\theta,z_i,P_1,P_3,k\right)$ denote investor i's information set at the trading stages, with associated expectation, covariance, and variance operators, $\mathbb{E}_{it}[\cdot]$, $\mathbb{C}_{it}[\cdot]$ and $\mathbb{V}_{it}[\cdot]$, respectively. Then, investor i chooses

[14]Note that since positive realizations of η_C shocks increase marginal utility, green projects are *negatively* exposed to climate risk, while brown projects are *positively* exposed.

Figure 1: Timeline of events



trade X_i to maximize her expected utility i.e.,

$$\mathcal{W}_i \equiv \sup_{x \in \mathbb{R}} \mathbb{E}_{i1} \left[-e^{-\gamma W_i} \right]. \tag{3}$$

The date one price is determined by the market clearing condition

$$\int_i X_{i1} di = 0. \tag{4}$$

At date two, the manager invests to maximize the date three price. The manager does not observe θ directly, but can condition on the information in the stock price P_1. Hence, her information set is the public signal $\mathcal{F}_m = \sigma(P_1)$.

The date three price is again determined by the market clearing condition (4), evaluated at the $t = 3$ trades X_{i3} that maximize trader expected utilities at that date. Note, however, that since the manager's investment decision is perfectly anticipated by investors at date one, and there are no additional shocks or information, we show that *in equilibrium* the date three price is equal to the date one price. At date four, the firm's terminal cash flows per share V are realized and paid to the investors.

Equilibrium. An equilibrium consists of trades $\{X_{i1}, X_{i3}\}$, prices $\{P_1, P_3\}$, and an investment rule $k(P_1)$ such that (i) the trades X_{it} maximize investor i's expected utility, given her information $\mathcal{F}_{it}$ and the investment rule $k(P_1)$, (ii) the investment rule $k(P_1)$ maximizes the expected date three price $\mathbb{E}[P_3|\mathcal{F}_m]$, and (iii) the equilibrium prices $\{P_1, P_3\}$ are determined by market clearing at dates one and three, respectively.

3.1 Discussion of Assumptions

Price maximization. The manager in our model invests to maximize price, which is equivalent to investing in projects with positive NPV. This is consistent with what managers reportedly do in practice: Graham and Harvey (2001) document that around 75% of CFOs they survey use NPV analysis when evaluating projects. Moreover, it is consistent with prior work that builds upon the investment CAPM and q theory of investment, which typically assumes that the firm invests to maximize its market capitalization (e.g., Zhang (2005), Liu, Whited, and Zhang (2009)). Existing papers on feedback effects typically assume that the firm maximizes expected terminal cash flow. However, in these papers, the marginal investor is typically risk neutral and so this objective is essentially equivalent to maximizing the firm's price. As we discuss further in Section 6.1, the two objectives are different in our setting because of the impact of the risk premium on the price.

Two trading dates. The manager in our model both learns from the stock price and seeks to maximize the stock price. This requires a well-defined market price prior to the investment decision, from which the manager can learn, and a well-defined market price after the investment decision, over which we can specify the manager's maximization problem. However, our results are not an artifact of the two-date setting. As will be seen, because the trading dates are otherwise identical, *in equilibrium* the price is identical at both dates. In principle, one could capture similar forces with a single trading date if the manager could simultaneously commit to a real investment schedule $k(P)$ at the same time that traders trade. However, because of the conceptual complications and questionable practical relevance of such a model, we instead work with the two trading date setting.

Green and brown projects. There is some disagreement in the literature regarding how different types of stocks' returns correlate with climate outcomes (e.g., see Giglio et al. (2021)). In our model, we simply define green and brown projects as those which perform better and worse when given adverse climate shocks, respectively. As we shall see, green stocks carry a price premium, while brown stocks carry a discount, as a result of their exposure to climate risk. Thus, given the evidence in Bolton and Kacperczyk (2021a) and Hsu, Li, and Tsou (2022), green (brown) firms can, for instance, be thought of as those with low (high) emissions. For tractability, we abstract from other sources of systematic risk and only focus on exposure to climate risk. However, we expect our results will be qualitatively similar in a multi-factor model in which the project's discount rate depends on its exposure to the relevant risk factors.

Homogeneous investor information. Since our primary focus is on *managerial* learning from prices, we shut down *investor* learning from prices by assuming that all investors share a common signal about fundamentals. The assumption simplifies the analysis and ensures that the financial market equilibrium does not exhibit multiplicity of the type studied by Ganguli and Yang (2009). Moreover, this assumption ensures that the traditional Hirshleifer (1971) effect does not arise in our setting, in contrast to results from the existing literature (e.g., Marín and Rahi (2000), Dow and Rahi (2003)). Finally, we have confirmed that our main results are qualitatively similar when investors have private signals and learn from the price.

Assets in place. The presence of assets in place is not qualitatively important for our results, but aids tractability by ensuring the firm's cash flows remain uncertain in the absence of investment. However, the assumption that assets in place are uncorrelated with climate risk is made for expositional clarity and can be relaxed.[15]

Divestment decisions. Since the investment decision is binary, one can equivalently apply our analysis to study divestment decisions. For instance, a firm with $k = 1$ and $\alpha < 0$ is a firm with an existing negative climate exposure (e.g., a traditional car manufacturer). In this case, a decision of $k = 0$ corresponds to divesting brown technology, or equivalently, investing in green technology that mitigates the firm's existing exposure (e.g., by transitioning to electric vehicle technology). However, the leading application we have in mind is a neutral firm (i.e., a firm with status quo $k = 0$) deciding whether to invest in a climate-exposed project.

Aggregate demand for hedging. In our model, the average investor's exposure to climate shocks, and, consequently, their desire for hedging (as captured by Z) is stochastic. This is consistent with the empirical evidence that aggregate demand for climate hedges varies across time and with economic conditions. For instance, Bolton and Kacperczyk (2021a) shows that the pricing of carbon-transition risk varies across countries and has risen over time. Moreover, Choi et al. (2020) show that the price premium applied to green vs. brown stocks varies with weather patterns, and Alekseev, Giglio, Maingi, Selgrad, and Stroebel (2021) shows that weather patterns influence mutual-fund demand for climate-exposed stocks. As we discuss below, this variation generates changes in the discount rate that the manager applies to the project when making her investment decision.

Market incompleteness and hedging ability. Our model is one of incomplete markets. The firm's investment decision endogenously changes the completeness of the market by allowing investors to trade the climate risk factor (we refer to this as the "risk-sharing" channel; see Section 7). The starkness of this result is a consequence of discrete investment choice, but the economic mechanism arises more generally. Under a continuous investment choice, as the firm invests more, its cash flows are more sensitive to the risk investors seek to hedge vs. the assets in place. All else equal, this makes it less costly for investors to hedge their exposures using the stock, in the sense that they are exposed to less extraneous risk.[16]

A potential concern is that this channel would disappear if markets were complete and

[15] For instance, if A is positively correlated with η_C, one can decompose A as $A = \lambda \eta_C + \varepsilon_A$ for $\lambda > 0$ and $\mathbb{C}(\varepsilon_A, \eta_C) = 0$. In this case, the investment decision still changes the overall exposure of the firm to climate risk (i.e., λ with no investment vs. $\lambda + \alpha$ with investment), and the economic forces underlying our analysis continue to operate.

[16] An earlier version of the paper considered more general investment decisions and found that the key economic forces that drive our results obtain in this more general setting.

investors could trade η_C directly. In practice, markets appear to be far from complete: investors have different exposures to climate risk due to differences in their demographic characteristics and risk preferences (e.g., Ilhan et al. (2021)), and find this risk difficult to hedge (e.g., Pástor et al. (2021), Giglio et al. (2021), Krueger et al. (2020)).[17] Indeed, Engle et al. (2020) find that a dynamic equity portfolio optimized to hedge climate risk is at most 30% correlated with news on such risk.[18]

Another potential concern is that the investment decision of a single firm will not have a meaningful impact on market completeness. A multi-firm model with discount rate variation and feedback effects is not analytically tractable. However, we expect that the impact of climate investment on market completeness would aggregate across firms and thus continue to be relevant in such a setting. That is, one can interpret our model as that of a representative firm in an industry or sector with correlated shocks to profitability and climate exposures. In practice, we expect that correlated investment choices (e.g., several automakers investing in EV technology) should affect investors' ability to hedge climate risk. Moreover, since stock prices do not fully reflect the risk-sharing benefit of climate-sensitive investment, our observation that managers fail to internalize this welfare externality would continue to hold in a multi-firm economy.

4 Equilibrium

In general, solving for an equilibrium with feedback effects is complicated by the fact that the asset price must simultaneously clear the market, be consistent with manager and trader beliefs, and be consistent with the anticipated real investment decision. We focus on equilibria of the following form.

Definition 1. *A* **threshold equilibrium** *is one in which:*
(i) the price at both dates depends on the underlying random variables through a linear statistic, $s_p = \theta + \frac{1}{\beta} Z$, where β is an endogenous constant,

[17]As Pástor et al. (2021) point out "[u]nanticipated climate changes present investors with an additional source of risk, which is non-traded and only partially insurable." Similarly, Giglio et al. (2021) state "... many of the effects of climate change are sufficiently far in the future that neither financial derivatives nor specialized insurance markets are available to directly hedge those long-horizon risks. Instead, investors are largely forced to insure against realizations of climate risk by building hedging portfolios on their own." Finally, based on their survey evidence, Krueger et al. (2020) state that "... many market participants, including institutional investors, find climate risks difficult to price and hedge, possibly because of their systematic nature, [...] and challenges in finding suitable hedging instruments."

[18]The multidimensional nature of climate risk may also contribute to market incompleteness. Different types of investments may be necessary to hedge the various dimensions of climate risk. For instance, green energy may serve as a hedge of carbon-transition risk, while green real estate may better hedge the potential for sea-level rise.

(ii) the manager invests in the project if and only if $P_1(s_p) > P_1(\bar{s})$ for an endogenous threshold $\bar{s}$, and
(iii) the price is continuous and weakly increasing in s_p and takes an identical piecewise linear form at both dates

$$P_3 = P_1 = \begin{cases} A_1 + B_1 s_p & \text{when } s_p > \bar{s} \\ A_0 & \text{when } s_p \leq \bar{s} \end{cases}, \tag{5}$$

where the price coefficients A_0, A_1, and B_1 are endogenous.

This type of equilibrium has an intuitive structure and several desirable properties. First, the equilibrium price is a generalized linear function of fundamentals, i.e., it is monotonic in $s_p = \theta + \frac{1}{\beta} Z$. Second, the manager invests when the price is sufficiently high, which in equilibrium is equivalent to investing when the price signal s_p is sufficiently high, i.e., when it exceeds a cutoff $\bar{s}$. Thus, the price naturally is piecewise linear in s_p, increasing in s_p when the manager invests, and constant when she does not. These properties ensure the analysis is tractable and facilitate comparison to existing work.

In the appendix, we formally solve the model by working backwards. We sketch the approach here. Given an investment decision $k \in \{0, 1\}$ at date 2, investor i's beliefs about the asset payoff at $t = 3$ are conditionally normal, with

$$\mathbb{E}_{i3}[V(k)] = \mu_A + k(\theta - c), \ \mathbb{C}_{i3}(V(k), \eta_C) = \frac{k\alpha}{\tau_\eta}, \text{ and } \mathbb{V}_{i3}(V(k)) = \frac{1}{\tau_A} + \frac{k^2}{\tau_\eta}, \tag{6}$$

and hence her optimal trade is

$$X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k), \eta_C) z_i - P_3}{\gamma \mathbb{V}_{i3}(V(k))} - (n + X_{i1}). \tag{7}$$

In turn, market clearing implies that

$$P_3 = \begin{cases} \mu_A - \frac{\gamma}{\tau_A} n & \text{when } k = 0 \\ \mu_A - \frac{\gamma}{\tau_A} n + (\theta - c) - \frac{\gamma}{\tau_\eta}(n - \alpha Z) & \text{when } k = 1 \end{cases}. \tag{8}$$

Comparing this expression to the conjectured threshold equilibrium, we must have $s_p = \theta + \frac{\gamma}{\tau_\eta} \alpha Z$ and $\bar{s} = c + \frac{\gamma}{\tau_\eta} n$.

At date two, the manager optimally chooses whether to invest to maximize the date three price, i.e.,

$$k^* = \underset{k \in \{0,1\}}{\arg\max}\, \mathbb{E}[P_3 | \mathcal{F}_m]. \tag{9}$$

In a threshold equilibrium, the manager infers s_p from the date one price when $P_1 > A_0$, and, when $P_1 \leq A_0$, she is only able to infer that $s_p \leq \bar{s}$. Moreover, expression (8) shows that the date three price given investment exceeds that given no investment if and only if $s_p > \bar{s}$. Thus, the manager invests upon observing $P_1 > A_0$, and does not invest upon observing $P_1 \leq A_0$.

Finally, stepping back to $t = 1$, observe from the conjectured form of price in a threshold equilibrium that, when $s_p > \bar{s}$, P_1 precisely reveals s_p to investors, while when $s_p \leq \bar{s}$, P_1 reveals only that $s_p \leq \bar{s}$. Importantly, this knowledge is sufficient to perfectly anticipate the manager's investment decision. As a result, *in equilibrium* investors can perfectly anticipate P_3 and therefore the equilibrium price at $t = 1$ must satisfy $P_1 = P_3$ in order to for the market to clear.

The following proposition establishes the existence and uniqueness of a threshold equilibrium.

Proposition 1. *There exists a unique threshold equilibrium in which the equilibrium prices are*

$$P_1 = P_3 = \mu_A - \frac{\gamma}{\tau_A} n + k \left(s_p - c - \frac{\gamma}{\tau_\eta} n \right), \tag{10}$$

and the manager's investment decision is

$$k = \mathbf{1} \left\{ P_1 > \mu_A - \tfrac{\gamma}{\tau_A} n \right\} = \mathbf{1} \left\{ s_p > \bar{s} \right\}, \tag{11}$$

where $s_p = \theta + \frac{1}{\beta} Z$*, with* $\frac{1}{\beta} = \frac{\gamma}{\tau_\eta} \alpha$*, and* $\bar{s} \equiv c + \frac{\gamma}{\tau_\eta} n$*.*

A couple of observations about the equilibrium are in order. First, and key for the tractability of our setting, in this equilibrium the traders' beliefs about the asset payoff remain *conditionally* normal in all states of the world since the manager's investment decision is P_1-measurable.

Second, the manager's optimal investment takes the form of a NPV rule, whereby she invests if and only the statistic

$$NPV \equiv s_p - \bar{s} = \underbrace{\theta - c}_{\text{cash flows}} \underbrace{- \tfrac{\gamma}{\tau_\eta} (n - \alpha Z)}_{\text{discount rate}} \tag{12}$$

is greater than zero. The first term, $\theta - c$, reflects the expected cash flows from the project, net of investment costs – this captures the "cash flow news" contained in the price. The second term, $-\frac{\gamma}{\tau_\eta} (n - \alpha Z)$, reflects a discount due to the risk premium investors demand for holding shares of the stock. We refer to this as "discount rate news" because it reflects variation in the project's impact on price that is driven by factors other than its expected cash flows. Consistent with intuition, the discount is higher (the NPV is lower) when the firm is larger (i.e., n is higher) because investors have to bear more aggregate risk. Moreover, the discount is lower (higher) for green (brown, respectively) projects when $Z > 0$.[19] This is because green projects reduce investors' exposure to (negative) climate shocks, while brown projects exacerbate it. We note that, while the cash flow and discount rate news in price are not separately observable to the manager, because they both influence how the project will impact price, they both factor into her decision of whether to invest.

5 Implications

In this section, we present the model's implications for observable quantities.

5.1 Probability of investment

We begin by characterizing how the probability of investment depends on model parameters.

Proposition 2. *In equilibrium, the unconditional probability of investment is given by*

$$\Pr(s_p > \bar{s}) = \Phi\left(\frac{\mathbb{E}[s_p] - \bar{s}}{\sqrt{\mathbb{V}[s_p]}}\right), \tag{13}$$

where $\mathbb{E}[s_p] = \mu_\theta + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$, $\mathbb{V}[s_p] = \frac{1}{\tau_\theta} + \frac{1}{\tau_Z}\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2$, *and* $\Phi(\cdot)$ *denotes the CDF of a standard normal random variable. The probability of investment:*
(i) increases with $\mu_\theta - c$*;*
(ii) decreases with n*;*
(iii) increases with μ_Z *for green firms (i.e.,* $\alpha > 0$*), but decreases with* μ_Z *for brown firms (i.e.,* $\alpha < 0$*);*
(iv) increases with τ_θ *and* τ_Z *if and only if* $\mathbb{E}[s_p] - \bar{s} = \mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta} > 0$*; and*
(v) decreases with α *if and only if* $\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\tau_\eta\tau_Z}{\gamma\alpha}\frac{1}{\tau_\theta}\mu_Z\right)\operatorname{sgn}(\alpha) > 0$.

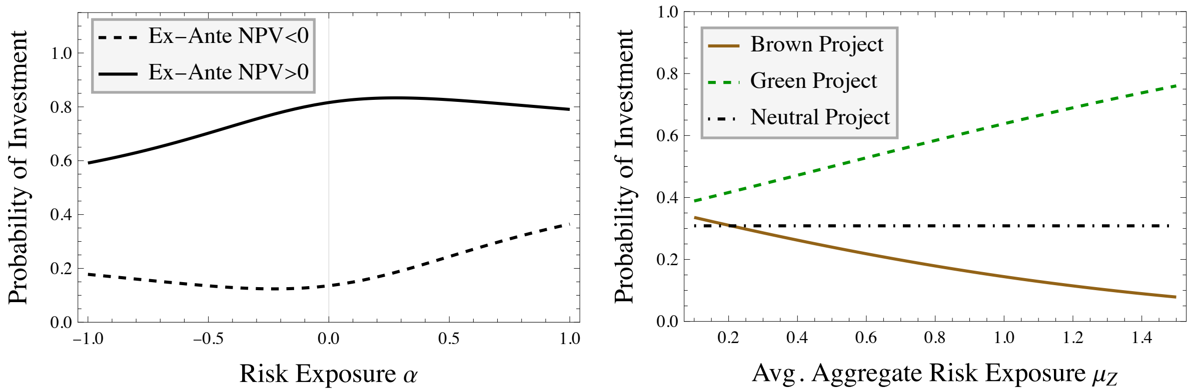
Consistent with intuition, the proposition establishes that the probability of investment increases in the expected NPV of the project $\mathbb{E}[s_p] - \bar{s}$ and decreases (increases) with the variance of the price signal $\mathbb{V}[s_p]$ when $\mathbb{E}[s_p] - \bar{s} > 0$ ($\mathbb{E}[s_p] - \bar{s} < 0$, respectively). This directly implies parts (i)-(iv) of the proposition. From equation (12), we know that the expected NPV increases with expected profitability $\mu_\theta - c$, decreases with firm size n, and increases with μ_Z if and only if $\alpha > 0$, which implies (i)-(iii). Similarly, an increase in τ_θ

[19] It is possible that $Z < 0$ in our model so that brown projects are priced at a premium. However, the probability of this outcome can be made arbitrarily small setting μ_Z and τ_Z appropriately.

or τ_Z leads to a reduction in the variance of the price signal $\mathbb{V}[s_p]$, which leads to more investment when the expected NPV is positive (i.e., $\mathbb{E}[s_p] - \bar{s} > 0$), but less investment when it is negative.

Figure 2: Probability of Investment

This figure plots the probability that the firm invests investment as a function of α and μ_Z. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = \tau_A = \gamma = 1$; $\tau_Z = 0.5$; $\mu_Z = 0.5$; $n = 0.1$. The left-hand plot depicts results for both projects that have positive and negative ex-ante NPV. In the ex-ante positive (negative) NPV project case, we set $\mu_\theta - c = -1$ ($\mu_\theta - c = 1$), which implies that, $\forall \alpha \in [-1, 1]$, $\mathbb{E}[s_p] - \bar{s} > 0$ ($\mathbb{E}[s_p] - \bar{s} < 0$).



(a) Prob(Invest) vs. α (b) Prob(Invest) vs. μ_Z

Part (v) of the proposition shows that the project's sensitivity to the risk factor, α, has a nuanced impact on the likelihood that the manager invests. An increase in α has two, potentially offsetting, effects. First, an increase in α increases the expected NPV $\mathbb{E}[s_p] - \bar{s}$ because it reduces the on-average discount due to climate risk. Since the manager's objective is to maximize the share price, this implies that all else equal, investment in green projects is likelier than brown projects. We refer to this as the "expected NPV" channel.

Second, an increase in the magnitude $|\alpha|$ increases the variance of the price signal $\mathbb{V}[s_p]$, which in turn makes the conditional NPV of the project more variable. All else equal, this makes it more likely that a project with negative expected NPV will be desirable ex-post (i.e., increases the likelihood that the investment option will be "in the money"), and so increases the likelihood of investment of such a project. Similarly, it reduces the likelihood that a project with positive expected NPV will be ex-post desirable, and so decreases the likelihood of investment in such a project. We refer to this as the "variance of NPV" channel. The overall effect of α depends on the relative magnitude of these two channels.

We next derive the implications of this result for the likelihood that a manager invests in a green (brown) project relative to a climate-neutral project.

Corollary 1. *(i) The firm is more likely to invest in a green than a neutral project, i.e.,* $\Pr\left(s_p > \bar{s}; \alpha = 1\right) > \Pr\left(s_p > \bar{s}; \alpha = 0\right)$ *if and only if*

$$\frac{\gamma \mu_Z}{\tau_\eta} > \left(\sqrt{1 + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{\tau_\theta}{\tau_Z}} - 1\right)\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta}\right). \tag{14}$$

(ii) The firm is more likely to invest in a neutral than a brown project, i.e., $\Pr\left(s_p > \bar{s}; \alpha = 0\right) > \Pr\left(s_p > \bar{s}; \alpha = -1\right)$ *if and only if*

$$\frac{\gamma \mu_Z}{\tau_\eta} > -\left(\sqrt{1 + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{\tau_\theta}{\tau_Z}} - 1\right)\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta}\right). \tag{15}$$

The corollary shows that for projects that are sufficiently profitable ex ante, the manager is less likely to invest in a green than a neutral project despite the fact that it has, in expectation, a reduced (and potentially negative) risk premium. The manager is more likely to invest in a green project only when the average exposure to climate risk μ_Z is sufficiently large relative to the project's ex-ante profitability $\mu_\theta - c$. Again, this stems from the variance of NPV channel, which reduces the likelihood of investment for highly profitable projects because the manager's investment decision in such projects is effectively a real option that is in the money ex ante. When the project's ex-ante profitability is sufficiently large, this effect dominates and so the firm is more likely to invest in a neutral project.

In contrast, for sufficiently profitable projects (i.e., when $\mu_\theta - c > \frac{\gamma n}{\tau_\eta}$), the firm is always more likely to invest in a neutral project than a brown one. This is because expected NPV and variance of NPV channels reinforce each other. Relative to a neutral project, a brown project has a lower expected NPV *and* gives rise to a more variable price signal. For profitable projects, both effects tend to lower the likelihood of investment.

5.2 Expected return conditional on investment

The impact of investment on expected returns is widely studied empirically, and existing evidence typically focuses on the average relationship between these variables in the cross-section. Our results highlight that the relation can depend critically on the nature of projects that firms invest in and on the information environment.

The next proposition establishes our main results on expected returns.

Proposition 3. *In equilibrium, the expected return conditional on no investment is*

$$\mathbb{E}\left[V - P_3 | k = 0\right] = \frac{\gamma n}{\tau_A}, \tag{16}$$

and the expected return conditional on investment is given by

$$\mathbb{E}\left[V - P_3 | k = 1\right] = \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma \alpha}{\tau_\eta} \mathbb{E}\left[Z | s_p > \bar{s}\right]. \tag{17}$$

Conditional on investment, the expected return:
(i) increases in $\mu_\theta - c$, τ_Z, and n,
(ii) decreases in μ_Z for $\alpha > 0$ and increases in μ_Z for $\alpha < 0$, and,
(iii) decreases in α if $\mu_Z > 0$ and $\alpha \geq 0$.

When the manager does not invest, the expected return reflects the standard risk premium that investors demand for owning the stock i.e., $\frac{\gamma}{\tau_A} n$. Conditional on investment, the expected return is driven by two components: (i) the standard risk premium $\gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right)$, and (ii) the investors' expected aggregate exposure to climate risk conditional on investment: $-\frac{\gamma\alpha}{\tau_\eta} \mathbb{E}\left[Z | s_p > \bar{s}\right]$. Intuitively, as previously discussed, $-\frac{\gamma\alpha}{\tau_\eta} Z$ captures the portion of the project's discount rate that is driven by its exposure to climate risk, and so $-\frac{\gamma\alpha}{\tau_\eta} \mathbb{E}\left[Z | s_p > \bar{s}\right]$ captures the expectation of this discount rate conditional on investment.

To understand the determinants of this expectation, note that we can write:

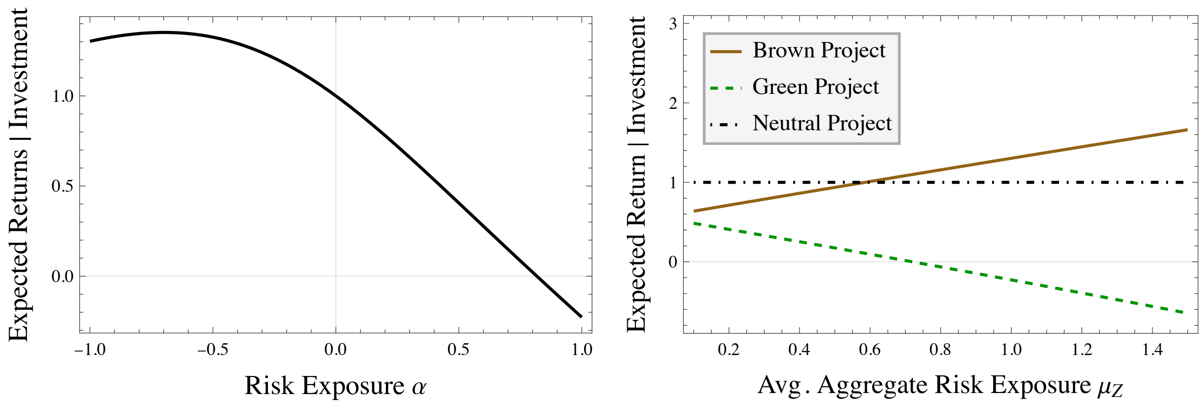
$$-\frac{\gamma\alpha}{\tau_\eta} \mathbb{E}\left[Z | s_p > \bar{s}\right] = \underbrace{-\frac{\gamma\alpha}{\tau_\eta} \mu_Z}_{\text{avg. risk exposure}} - \overbrace{\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2 \frac{\frac{1}{\tau_Z}}{\sqrt{\mathbb{V}\left[s_p\right]}} H\left(\frac{\bar{s} - \mathbb{E}[s_p]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)}^{\text{adjustment for manager's investment strategy}}, \tag{18}$$

where $H(\cdot) = \frac{\phi(\cdot)}{1 - \Phi(\cdot)}$ is the hazard ratio of the standard normal distribution. This reveals two channels through which investment relates to expected returns. First, investment unconditionally raises the magnitude of the covariance between the firm's cash flows and the climate risk shock, which leads to the portion of the risk premium driven by the average risk exposure term μ_Z. Second, and novel to our model, the manager chooses to invest only when price is high, and thus their expectation of the discount rate, $\mathbb{E}\left[Z | s_p\right]$, is low. Consequently, conditional on investment, the discount rate is, in expectation, below its unconditional mean; this channel is captured by the hazard-rate term above.

This decomposition provides intuition for the above proposition. An increase in profitability $\mu_\theta - c$ increases the likelihood of investment and, therefore, increases the expected return via the second term in (18). Similarly, when the risk factor generates less variation

Figure 3: Expected Returns Given Investment

This figure plots the firm's expected returns given investment as a function of the model's parameters. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = \tau_A = \mu_Z = \mu_\theta = c = \gamma = 1; \tau_Z = 2; n = 0.5$.



(a) Expected Returns vs. α (b) Expected Returns vs. μ_Z

in the discount rate (i.e., τ_Z is greater), $\mathbb{V}[s_p]$ tends to fall, which raises expected returns. While an increase in firm size n tends to lower the likelihood of investment and so decreases expected return via the second term in (18), this channel is dominated by the direct effect of the standard risk premium channel (i.e., $\gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right)$), and so expected returns increase with n.

Moreover, it is immediate that expression (18) decreases in μ_Z when $\alpha > 0$, but increases otherwise. This is intuitive: when the average exposure to climate risk increases, expected returns for green projects decrease while those for brown projects increase. Similarly, for green projects ($\alpha > 0$), an increase in α leads to a direct decrease in the risk premium term (since $\mu_Z > 0$), and an increase in the variance of the price signal $\mathbb{V}[s_p]$. Both these effects tend to lower the expected return.

We next characterize how expected returns, conditional on investment, vary with the type of project the firm is investing in.

Corollary 2. *Conditional on investment, a green firm always has a lower expected return than a neutral firm i.e.,*

$$\mathbb{E}[V - P_3 | k = 1; \alpha = 1] \leq \mathbb{E}[V - P_3 | k = 1; \alpha = 0]. \tag{19}$$

However, a neutral firm has a higher expected return than a brown firm i.e.,

$$\mathbb{E}[V - P_3 | k = 1; \alpha = -1] \leq \mathbb{E}[V - P_3 | k = 1; \alpha = 0] \tag{20}$$

if and only if μ_Z is sufficiently small.

As discussed above, in the case of green firms ($\alpha > 0$), the two components in expression (18) both serve to decrease expected returns as $|\alpha|$ increases. More interestingly, expression (18) also implies that for brown firms ($\alpha < 0$), the two components can move in opposite directions as α changes. As a result, when the aggregate expected exposure to climate risk μ_Z is sufficiently small, brown firms may have lower expected returns than neutral firms. This is a consequence of the manager's reliance on price when investing. All else equal, brown projects are ex-ante less attractive than green projects due to a higher average discount due to climate risk. Hence, in order for a manager to find a brown project sufficiently attractive to invest in, the first-period price must be particularly high, reflecting, in part, a lower than expected discount rate.

Panel (a) in Figure 3 provides an illustration of this result: it plots the expected return, conditional on investment, as a function of α, demonstrating that this relationship is single-peaked. Panel (b) illustrates that this non-monotonicity arises when μ_Z is sufficiently small. This suggests that the extent of the non-monotonicity depends on the aggregate expected exposure to climate risk, and so, in general, can vary over time and across markets – which may be captured by measures of climate risk news such as those proposed by Engle et al. (2020), Choi et al. (2020), and Alekseev et al. (2021). Moreover, even when the relationship between greenness and expected returns is monotonic, it is concave (i.e., the difference in expected returns between brown and neutral project is smaller than that between neutral and green projects).

5.3 Future profitability

Standard models of feedback effects typically imply that more informative prices lead to more profitable investment decisions. A number of empirical papers find evidence consistent with this prediction (e.g., Chen, Goldstein, and Jiang (2007)). In our model, however, investment need not always raise the firm's expected cash flows, as summarized by the following proposition.

Proposition 4. *Let $\Delta V \equiv \mathbb{E}[V|k=1] - \mathbb{E}[V|k=0]$ denote the change in expected cash flows due to investment. Then,*

(i) ΔV is always positive when $\alpha = 0$.

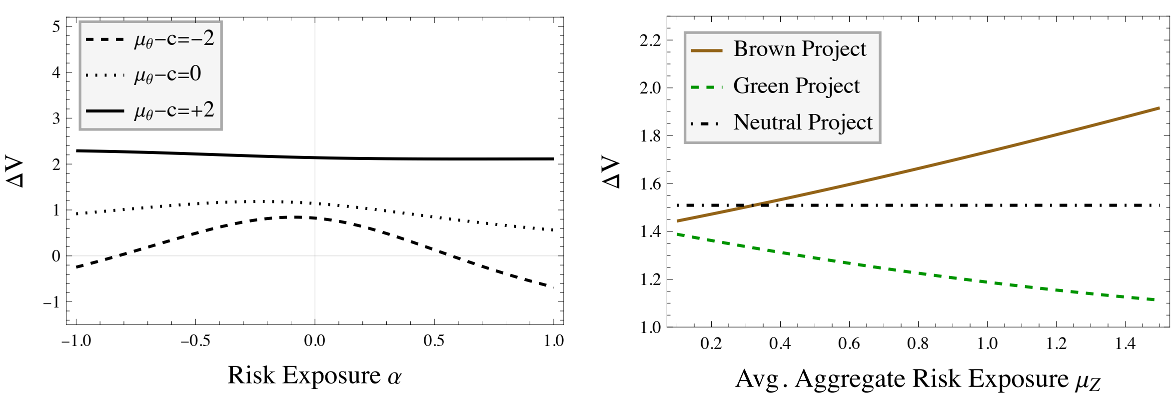
(ii) ΔV is negative when $\alpha \neq 0$ and $\mu_\theta - c$ is sufficiently negative.

(iii) ΔV is increasing in $\mu_\theta - c$ and n, decreasing in μ_Z when $\alpha > 0$, and increasing in μ_Z when $\alpha < 0$.

Figure 4 provides an illustration of this result. As before, the result follows from the observation that the manager's investment decision depends not only on the project's profitability, but also its discount rate. Part (i) of the above proposition corresponds to the standard intuition from the existing literature – when the project does not have a climate risk exposure, feedback-based investment increases the firm's expected profitability. However, part (ii) implies that when the project has a climate risk exposure, the manager may still invest even when it has low or even negative expected profitability because it is sufficiently valuable to investors as a climate risk hedge. This occurs when the project's ex-ante profitability is low. In this case, the manager invests in the project because of good discount rate news, as opposed to cash flow news. That is, the price signal is more likely to be sufficiently high for investment ($s_p > \bar{s}$) because of high hedging benefits (driven by a realization of Z) as opposed to high future cash flows (θ).

Figure 4: Change in Expected Cash Flows Due to Investment

This figure plots the impact of the investment on the firm's expected cash flows as a function of α and μ_Z. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = \tau_\eta = c = \tau_A = \tau_Z = \mu_Z = \gamma = 1$; $n = 0.5$. Panel (b) focuses on the case in which $\mu_\theta - c = 1$.



(a) ΔV vs. α

(b) ΔV vs. μ_Z

Consistent with intuition, the expected change in future cash flows as a result of investment increases with the project's expected profitability. Moreover, because the threshold for investment increases with n, conditional on investment, expected future cash flows increase with n. Finally, the dependence on μ_Z follows from the fact that holding cash flows fixed, the expected pricing of the project increases with μ_Z for green projects, but decreases with μ_Z for brown projects. As a result, conditional on investment, expected cash flows are decreasing in μ_Z for green projects, but increasing in μ_Z for brown projects.

6 Robustness

In this section, we clarify which economic mechanisms and implications distinguish our results from alternative specifications.

6.1 Value maximization

When investors anticipate that the manager will invest, the price reveals s_p to the manager, which is a sufficient statistic for the NPV of the project. The price does not separately reveal the cash flow, θ, or discount rate, $\frac{\gamma\alpha}{\tau_\eta}Z$, components in s_p, which effectively serve as "noise" for one another. As discussed in Section 3.1, much earlier work on feedback effects assumes that managers maximize expected cash flows or terminal value, in which case the fact that the discount rate shock serves as noise for the cash flow plays a nontrivial role.[20]

Indeed, as we show in Internet Appendix B, the two objectives lead to different investment behavior in our model. To see why, observe that the manager's conditional expectation of cash flows, given s_p, is

$$\mathbb{E}\left[V\left(k\right)|s_p\right] = \mu_A + k\left(\mathbb{E}\left[\theta|s_p\right] - c\right), \text{ where} \tag{21}$$

$$\mathbb{E}\left[\theta|s_p\right] = \frac{\tau_\theta\mu_\theta + \tau_p\left(s_p - \frac{\gamma}{\tau_\eta}\alpha\mu_Z\right)}{\tau_\theta + \tau_p}, \text{ and } \tau_p = \tau_Z\left(\frac{\tau_\eta}{\gamma\alpha}\right)^2. \tag{22}$$

This implies that the equilibrium in the case of value maximization features investment only when $s_p > \bar{s}_V$, where

$$\bar{s}_V \equiv c - \frac{\tau_\theta\left(\mu_\theta - c\right)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z. \tag{23}$$

The key difference between price maximization and value maximization is that the latter does not incorporate the discount rate component of the NPV. With value maximization, an increase in the magnitude of the project's risk exposure (i.e., an increase in $|\alpha|$) makes the price a noisier signal about cash flows, which implies that the manager's investment decision depends more heavily on the ex-ante profitability of the project and less heavily on the stock price. Consequently, we show in Internet Appendix B that the probability of investment is increasing in $|\alpha|$ if the project is ex-ante profitable i.e., $\mu_\theta - c > 0$, but is decreasing otherwise. In contrast, with price maximization, recall that an increase in α affects both the conditional mean and variance of the project's NPV, and so has a more nuanced effect on the probability of investment (see Proposition 2). We also show that the probability of

[20] In much of this work, the price is set by risk-neutral investors or market makers, and so there is no risk premium.

investment does not depend on aggregate risk exposures, n or μ_Z, with value maximization in contrast to the case with price maximization. Finally, in contrast to Proposition 4, expected future profitability conditional on investment is *always* positive (i.e., $\Delta V > 0$) with value maximization since the manager's investment decision is chosen to maximize expected cash flows.

Comparing the investment thresholds, note that value maximization leads to more investment than price maximization (i.e., $\bar{s} > \bar{s}_V$) if and only if:

$$\frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z) > -\frac{\tau_\theta}{\tau_p}\left(\mu_\theta - c\right). \tag{24}$$

In particular, there is "over-investment" with value maximization (relative to price maximization) when projects are sufficiently profitable ex-ante (i.e., $\mu_\theta - c$ is sufficiently high), investors' expected climate exposures are small (i.e., μ_Z is low), or the project is sufficiently brown (i.e., α is small or negative).[21] This implies that existing models which ignore the discount rate channel (i.e., assume value maximization) predict that investment in profitable, climate-neutral (or brown) projects should be higher than it actually is. Moreover, as we discuss further in the next section, value maximization tends to lead to lower investor welfare than price maximization.

6.2 Learning from non-price information

An important feature of our analysis is that the manager conditions on the information in prices when choosing whether to invest in the project. To clarify the role of such "feedback effects," we consider a setting where the manager learns from non-price information in Internet Appendix C. To eliminate learning from prices, we assume now that investors do not trade at date 1, and instead trade only on date 3, after the investment is made, which implies that the manager does not see a price signal when investing. Instead, we assume that the manager observes a public signal $y = \theta + \varepsilon$ at date 1, where $\varepsilon \sim N(0, \tau_\varepsilon^{-1})$ is independent of θ and Z, and then optimally chooses investment to maximize the expected stock price. We show that the manager invests if and only if $y > \bar{y}$, where

$$\bar{y} \equiv \mu_\theta - \tau_\theta\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon}\right)\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}\left(n - \alpha\mu_Z\right)\right). \tag{25}$$

The key distinction between learning from a public versus a price signal stems from the fact that shocks to investors' hedging demands, i.e., Z, have a persistent impact on a firm's

[21]Note that when investors are risk-neutral (i.e., $\gamma = 0$), $\bar{s} = \bar{s}_V = c$ and so the investment rules coincide.

stock price. As a result, the manager does not treat these shocks as noise in the way that she treats the error, ε, in a public signal. This implies that, while a project's risk exposure (α) still affects the conditional expected NPV of the project, it does not affect the volatility of the NPV. In contrast to Proposition 2, we show that the probability of investment increases in α so long as $\mu_Z > 0$: intuitively, greener projects have higher NPV in these cases, and so the manager is more likely to invest in them. Moreover, we show that the profitability of the project does not affect expected returns conditional on investment: all else equal, green projects always have lower returns than neutral projects, which have lower returns than brown projects. This is in contrast to the conclusions of Proposition 3 and Corollary 2.

7 Welfare

In this section, we explore the relationship between feedback, investment, and investor welfare. We begin by characterizing the channels through which investment affects investor welfare in Section 7.1. In Section 7.2, we consider a special case in which investors have homogeneous climate exposures (i.e., $\tau_\zeta \to \infty$). This allows us to explicitly characterize the welfare-maximizing price-contingent investment rule and compare it to the price-maximizing rule. We show that even though price maximization does not generally maximize welfare. In Section 7.3, we re-introduce heterogeneity in risk-exposures and show how the manager's use of the information in price may harm investor welfare.

7.1 The impact of investment on welfare

Existing models of feedback effects focus on the impact that feedback has on a firm's expected cash flows. In many such models, investors are risk neutral so that maximizing expected cash flows aligns with welfare maximization.[22] However, in our model, investor risk aversion implies that investment has multiple, potentially offsetting effects on investor welfare, due to the riskiness of the project and the stock's usefulness as a hedge.

Because traders are ex-ante symmetric, the ex-ante expected utility of an arbitrary trader is an unambiguous measure of welfare:

$$\mathcal{W} \equiv \mathbb{E}\left[-e^{-\gamma W_i(k(s_p))}\right] \tag{26}$$

$$= \Pr(k=1)\,\mathbb{E}\left[-e^{-\gamma W_i(1)}|k=1\right] + \Pr(k=0)\,\mathbb{E}\left[-e^{-\gamma W_i(0)}|k=0\right], \tag{27}$$

[22]Section 2 discusses notable exceptions, like Dow and Rahi (2003).

where

$$W_i(k) = \begin{cases} X_i\left(V(1) - P\right) + nV(1) - z_i\eta_C & k = 1 \\ nV(0) - z_i\eta_C & k = 0 \end{cases}. \tag{28}$$

Proposition IA3 in Internet Appendix A characterizes this expression in closed form. However, to understand the relevant economic forces, it is helpful to study the simpler special case in which investment is *fixed* at arbitrary level k, in which case the model reduces to a standard unconditionally linear-normal form. In this case, we have

$$\mathbb{E}\left[-e^{-\gamma W_i(k)}\right] = -e^{-\gamma\, CE(k)}, \tag{29}$$

where the certainty equivalent $CE(k)$ can be expressed, after grouping terms, as

$$\begin{aligned} CE(k) = {} & \underbrace{\mathbb{E}[V(k)]n}_{\text{Cash flow channel}} \underbrace{- \frac{\gamma}{2}\left(\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta} + \frac{1-\alpha^2}{\tau_\eta}\right)\right)n^2}_{\text{Non-climate risk channel}} \\ & \underbrace{- \frac{\gamma}{2}\frac{1}{\tau_\eta}\left(\mu_Z - k\alpha n\right)^2}_{\text{Climate risk channel}}(1+\Gamma) - \frac{1}{\gamma}\log(D(k)) \end{aligned} \tag{30}$$

where

$$D(k) = \underbrace{\sqrt{\frac{\frac{1}{\tau_A} + k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\eta} + \frac{1}{\beta^2\left(\tau_Z + \tau_\zeta\right)}\right)}}}_{\text{Value of information}} \sqrt{\frac{\Gamma}{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}}, \tag{31}$$

and

$$\Gamma \equiv \frac{\gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)}{1 - \gamma^2\frac{1}{\tau_\eta}\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)\left(1 - \underbrace{\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)} + \frac{1}{\tau_\eta}\right)} \times \left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}}\right)^2}_{\text{Risk-sharing channel}}\right)}. \tag{32}$$

We have explicitly labeled all five terms in these expressions that depend on the investment choice k and will discuss them in turn.

- The **cash flow channel** reflects that investment affects the investor's expected wealth via their ownership of n shares. Investment increases (decreases) welfare through this channel when the project's expected cash flows are positive (negative).

- The **non-climate risk channel** reflects that the investment increases investors' exposure to non-climate risks via the θ and η_I shocks.

- The **climate risk channel** captures the fact that the investment affects investors' aggregate exposure to climate shocks. The average investor's total climate exposure is $\mu_Z - k\alpha n$, reflecting both the direct exposure and the exposure through ownership of the stock. When the direct exposure is sufficiently large (i.e., $\mu_Z > n$), investment in green ($\alpha > 0$) projects mitigates aggregate climate exposure and consequently raises welfare, while investment in brown ($\alpha < 0$) projects amplifies aggregate climate exposure and reduces welfare. This channel is further scaled by the term $1 + \Gamma$, which reflects *uncertainty* about the exposure to climate risk. When investors' total exposure to climate risk $Z + \zeta_i$ is constant (i.e., $\tau_Z, \tau_\zeta \to \infty$), we have $\Gamma = 0$. However, when investors face uncertainty about their exposure from either source, $\Gamma > 0$, which amplifies the disutility of climate exposure.

- The **risk-sharing channel** reflects that the project enables investors to share their idiosyncratic exposures to climate risk, ζ_i, by trading the stock. All else equal, investment improves welfare through this channel. By sharing risk, investors reduce the dispersion in their climate exposures, reducing the effect of uncertainty about exposures, Γ.

  The overall amount of risk-sharing reflects both the effectiveness of the stock as a hedging instrument (i.e., the correlation of the stock return with climate risk), and the proportion of climate exposures that are hedgeable (i.e., the proportion of climate exposures that are idiosyncratic, ζ_i):

$$\text{Risk-sharing channel} = \underbrace{\frac{k^2\alpha^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}}_{\substack{\text{Hedging effectiveness of stock}\\ =\mathrm{Corr}^2(V-P,\eta_C|z_i)}} \times \underbrace{\left(\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^2}_{\%\text{ hedgeable climate exposure}}. \tag{33}$$

- The **value of information channel** captures the fact that investors' information about cash flows renders the stock more useful in hedging. Observing θ increases the conditional correlation between the stock's payoff and η_C. Moreover, this effect is only relevant when the project is undertaken, and so disappears when $k = 0$. This takes a familiar form of the ratio of investors' conditional variance of the asset return with and without conditioning on θ.[23]

[23] In our model, investors are endowed with information. However, this term still captures the improvement in utility as a result of observing θ relative to being uninformed. Specifically, given fixed k, this ratio can be

Importantly, when the manager chooses investment to maximize the expected price, she fails to appropriately account for the impact of her decision on the other components of welfare, as we discuss next.

7.2 Homogeneous risk exposures

We begin with a special case of our model in which all investors have homogeneous exposures to climate risk. In this case, we can explicitly characterize the welfare-maximizing price-contingent investment rule, as we show in the following Proposition.[24]

Proposition 5. *Suppose that agents have identical exposures to climate risk (i.e., $\tau_\zeta \to \infty$). Then, the welfare-maximizing s_p-dependent investment policy is*

$$\arg\max_{k\in\{0,1\}} \mathcal{W}\left(k; s_p\right) = \mathbf{1}\left\{s_p > \bar{s}_W\right\}, \tag{34}$$

where $\bar{s}_W \equiv c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n$. Moreover,

(i) price maximization always leads to under-investment relative to welfare maximization, since $\bar{s} - \bar{s}_W = \frac{1}{2}\frac{\gamma}{\tau_\eta}n$, and

(ii) value maximization leads to under-investment relative to welfare maximization if and only if

$$\bar{s}_V - \bar{s}_W = \frac{\gamma}{\tau_\eta}\alpha\mu_Z - \frac{\tau_\theta\left(\mu_\theta - c\right)}{\tau_p} - \frac{1}{2}\frac{\gamma}{\tau_\eta}n > 0, \tag{35}$$

but over-investment otherwise.

As we show in the appendix, the expressions for welfare simplify considerably when investors have homogeneous exposures to climate risk because there is no risk-sharing trade in equilibrium. Consequently, neither the risk-sharing channel nor the value of information channel are operational. As a result, the difference between the welfare-maximizing and

represented as $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V-P|z_i)}$, which reflects the proportional improvement in expected utility from conditioning on θ, z_i, and P vs. z_i alone. The welfare expressions in Bond and Garcia (2022) include a similar term, which they further decompose into a product of the classic value of cash flow information, $\frac{\mathbb{V}(V|\theta,z_i,P)}{\mathbb{V}(V|z_i,P)}$, and the value of providing vs. demanding liquidity (i.e., using a price-contingent schedule vs. not), $\frac{\mathbb{V}(V|z_i,P)}{\mathbb{V}(V-P|z_i)}$. Because these effects are not a primary focus of our analysis we choose to concisely represent them in a single term.

[24] To streamline the presentation and derivation, we formulate the investment rule as s_p-contingent. However, as in the baseline model, it can be implemented as a price-contingent rule. Intuitively, the equilibrium price reveals s_p in any states in which traders anticipate that the investment is undertaken and does not reveal s_p otherwise. This allows one to directly map the s_p-contingent investment rule to an equivalent price-contingent rule in which the manager does not invest if she observes a price realization that anticipates no investment, $P_1 = \mu_A - \frac{\gamma}{\tau_A}n$, and invests otherwise.

price-maximizing rules stems from the fact that, while welfare depends on the average risk borne by investors, the price reflects the marginal risk of holding an additional share of the firm. Since the marginal risk of the last share is higher than the average risk of all shares investors hold, the price-maximizing rule leads to under-investment relative to the welfare-maximizing rule.[25]

Proposition 5 also clarifies that the value-maximizing investment rule does not maximize welfare even in this special case. The value-maximizing rule over-weights expected cash flows, but under-weights both non-climate risk and climate risk, compared to the welfare-maximizing rule. Hence, value maximization tends to lead to *under-investment* in green projects, but *over-investment* in brown projects that are ex-ante profitable, relative to welfare maximization.

7.3 Heterogeneous risk exposures

The previous discussion illustrates that even when investors have identical climate exposures, price maximization is not equivalent to welfare maximization, unless the per-capita endowment of shares is arbitrarily small (i.e., $n \to 0$). These differences are further amplified when investors have heterogeneous climate exposures.

To gain intuition, note that the share price $P(k)$ can be expressed as

$$P(k) = \mathbb{E}_i \left[V\right] + \gamma Z \mathbb{C}_i \left[V, \eta_C\right] - \gamma n \mathbb{V}_i \left[V\right]. \tag{36}$$

This expression reveals that the price reflects the aggregate climate exposure, Z, but does not reflect the diversity in climate exposures (i.e., τ_ζ^{-1}), which determines the gains from sharing climate risk (i.e., the risk-sharing channel). Similarly, the price does not reflect the value of information channel, because it does not capture the additional hedging benefit that investors gain from having observed θ in the event that the manager invests (i.e., when $k = 1$). Because each of these channels improves welfare, this implies that a price-maximizing manager tends to under-invest in climate-sensitive projects relative to a welfare-maximizing rule.

Finally, the expression for the certainty equivalent in (30) implies that heterogeneity in exposures amplifies the welfare effect of the climate risk channel. Specifically, one can show that the amplification factor Γ (weakly) increases in τ_ζ^{-1}, and so the project's impact on welfare via aggregate climate risk rises with τ_ζ^{-1}. Thus, the price also does not fully account

[25]An analogous difference is also highlighted by Levit, Malenko, and Maug (2022) who show that while prices are determined by the valuation of the marginal investor, valuation is determined by the valuation of the average (post-trade) shareholder in their setting. Bernhardt, Liu, and Marquez (2018) highlight a similar difference in the context of takeovers.

for the climate risk channel, leading to under-investment in green projects, which reduce aggregate climate risk, and over-investment in brown projects, which increase it.

The misalignment between the manager's objective and investor welfare implies that feedback from prices need not necessarily improve welfare. To formalize this intuition, we compare investor welfare to a benchmark in which the manager ignores the information in price and instead chooses k to maximize the *ex-ante expectation* of the share price. Recall from Proposition 1 that the investment's effect on price is the NPV, $s_p - \bar{s}$, and so, in this benchmark, the manager invests if and only if $\mathbb{E}[s_p] - \bar{s} > 0$.

The next proposition characterizes sufficient conditions under which feedback reduces welfare.

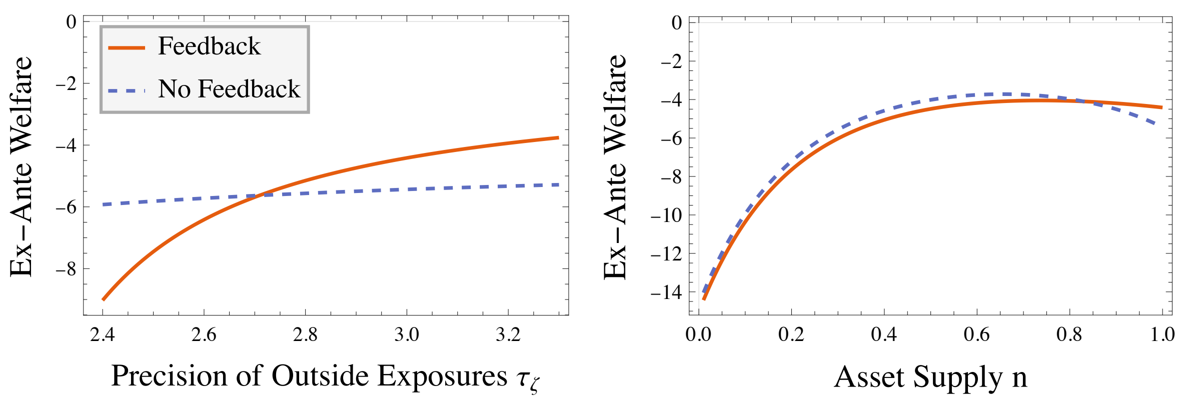
Proposition 6. *Feedback reduces welfare if the expected NPV (i.e., $\mathbb{E}[s_p] - \bar{s}$) is positive, and*

(i) n is sufficiently small, or

(ii) risk-sharing needs are sufficiently large (i.e., τ_ζ is sufficiently small).

Figure 5: Ex-ante welfare: Feedback vs. No feedback

This figure plots the ex-ante welfare (i.e., ex-ante expected utility) as a function of α and μ_Z for a project with positive expected NPV. Unless otherwise mentioned, the parameters employed are: $\tau_\theta = 0.5$; $\tau_\zeta = 3$; $\tau_Z = 2$; $\mu_A = 0$; $\tau_A = 5$; $\mu_\theta = c = \tau_\eta = \gamma = \mu_Z = n = 1$. These parameters ensure the expected NPV of the project is positive.



(a) Welfare vs. τ_ζ

(b) Welfare vs. n

Figure 5 provides an illustration of these results, and the intuition is as follows. When the ex-ante NPV of the project is positive, in the no-feedback benchmark, the manager always invests. In contrast, given feedback, the firm does not invest for $s_p \leq \bar{s}$. On the one hand, feedback improves the expected price of the stock, which tends to improve welfare through the cash flow channel. On the other hand, because it leads to no-investment in some states, feedback reduces the ability of investors to use the stock as a hedge, and so reduces welfare via the risk-sharing channel. It also affects aggregate exposure to non-climate and climate risk (with the direction depending on the sign of α).

When the per-capita endowment of shares n is small, the cash flow and non-climate risk channels are relatively small. Moreover, the firm's investment decision has a small effect on the aggregate climate exposure, and so the climate risk channel is muted. However, the risk-sharing channel remains important since it is unaffected by n: regardless of the firm's size, its stock remains a useful hedge in the event of investment. Consequently, the risk-sharing channel dominates, and investors are better off with a rule that always invests, yielding hedging benefits in all states of the world. Analogously, when τ_ζ is small, investors' exposures to the climate are highly diverse, so that the ability to share risk provides them with large welfare gains. Hence, the risk-sharing channel dominates in the limit, once again leading investors to prefer an investment rule that ensures that the asset is always useful for hedging.

As an aside, note that the manager's use of price information *always* enhances the firm's expected price: because the manager's objective is to maximize price, any additional information that she conditions on can only increase the price, in expectation. Hence, Proposition 6 implies that an increase in the firm's share price need not align with an improvement in investor welfare.

8 Empirical Predictions

Our model offers predictions that are relevant to several streams of empirical work, which we summarize in this section.

Climate investment. Proposition 2 sheds light on how firm-level investment in green projects varies with market conditions. Consistent with intuition, when aggregate climate exposures (μ_Z) are high, the amount of green (brown) investment increases (decreases). Importantly, this is not due to any explicit preference for green projects by investors but instead works through the fact that green projects enjoy a lower cost of capital in the event of high climate exposures. Aggregate climate exposures may be proxied for using news on climate change, as measured in, e.g., Engle et al. (2020) and Choi et al. (2020). Intuitively, negative climate news likely raises investors' expectations of their exposure to the climate.

Result (iv) in Proposition 2 speaks to how the price informativeness affects the prevalence of green investment. The model predicts that for projects that are already ex-ante attractive (i.e., have positive expected NPV $\mathbb{E}[s_p] - \overline{s} > 0$), investment is less likely when the price contains more information about cash flows or discount rates (i.e., τ_θ or τ_Z are lower).

However, if the project is marginal (i.e., $\mathbb{E}[s_p] - \overline{s} = 0$), then additional information has no effect. Importantly, the result also implies that more information in the price can reduce investment in green projects if they have a positive expected NPV.

Finally, Proposition 2 implies that for ex-ante unattractive projects, the likelihood of investment increases with the magnitude of climate exposure, especially for green firms. In such cases, a policy maker needs to provide stronger incentives to discourage investment in brown projects than to encourage investment in green projects.

Climate exposure and expected returns. Significant prior literature studies how firms' climate exposures affect their expected returns (i.e., whether there is a "climate risk premium"; see Gillan, Koch, and Starks (2021) for a review). Our model predicts that the impact of a project's climate exposure on expected future returns depends upon whether the project is green or brown. Specifically, while green projects earn unambiguously lower expected returns the greener they are, the relation between expected returns and "brownness" is relatively flat. This implies one must exercise caution in interpreting cross-sectional evidence about how average returns vary with measures of climate risk exposure (e.g., environmental scores, carbon emission intensity), and may help reconcile the mixed empirical evidence in the literature. For instance, while Chava (2014), Bolton and Kacperczyk (2021a), and Hsu et al. (2022) find that greener firms earn lower returns (or have lower costs of equity and debt capital), Larcker and Watts (2020) and Berk and van Binsbergen (2021) argue there does not appear to be a large difference in returns across green vs. non-green bonds and stocks, respectively.

Proposition 3 also predicts that all else equal, expected returns conditional on investment are *higher* when there is (i) *less uncertainty* about the aggregate climate risk exposure (i.e., τ_Z is higher) and when (ii) the ex-ante profitability of the project is higher (i.e., $\mu_\theta - c$ is higher). These predictions are a consequence of feedback from prices. Projects with higher ex-ante profitability $\mu_\theta - c$ have positive NPV even for relatively high discount rates. Consequently, conditional on investment, highly profitable projects tend to have higher expected returns compared to less profitable projects.[26] Similarly, less uncertainty about climate risk exposures implies that the manager's investment decision is driven more by cash flow news. As a result, conditioning on investment is a better indicator that cash flows are high than that the discount rate is low.

Climate investment and future profitability. Considerable prior work studies

[26]These findings are related but distinct from Fama and French (2015)'s argument for why more profitable firms will experience higher returns. Fama and French (2015)'s explanation is of an econometric nature: controlling for firms' valuations, in the cross-section, more profitable firms must earn lower returns because they ultimately pay higher dividends. In our model, this relationship instead arises due to the manager's optimal investment choice.

whether ESG-related investments are, in general, more profitable according to metrics that capture expected cash flows such as ROA, ROE, and revenue growth (e.g., Gillan et al. (2021)). Proposition 4 implies that, when climate risk exposure is high, one should expect to see a negative cross-sectional relation between project "greenness" and standard measures of ex-post profitability. This is consistent with the evidence documented by Di Giuli and Kostovetsky (2014), who show that an improvement in CSR policies is associated with future under-performance and decline in future ROA, and Cheema-Fox, Serafeim, and Wang (2022), who show that pure-play green stocks experience high returns but low profitability. Notably, this is a novel consequence of managerial learning from prices and not of, e.g., agency problems, which have been argued to generate such a relation (Cheng, Hong, and Shue (2013), Buchanan, Cao, and Chen (2018)).

Executive compensation and climate-risk metrics. Our welfare results speak to the recent debate on the effectiveness of the use of climate-risk metrics in executive compensation. On the one hand, there has been a rapid increase in the use of such measures. Edmans (2021) cites that "51% of large U.S. companies and 45% of leading U.K. firms use ESG metrics in their incentive plans," and Hill (2021) cites a survey conducted by Deloitte in September 2021, which suggests that "24 per cent of companies polled expected to link their long-term incentive plans for executives to net zero or climate measures over the next two years."[27]

On the other hand, there is ample skepticism about the effectiveness of such incentives. In addition to issues around measurement and monitoring of such objectives and the possibility of unintended consequences, Edmans (2021) argues that incentivizing ESG performance may not necessarily lead to better financial performance. Instead, he advocates for the use of long-term stock-based compensation, arguing that "[s]ince material ESG factors ultimately improve the long-term stock price, this holds CEOs accountable for material ESG issues – even if they aren't directly measurable."

Our analysis suggests that this may not be true because the stock price (even in the long term) does not fully account for the benefit of investing in climate-exposed projects. As such, providing additional incentives based on climate metrics (e.g., bonuses linked to climate targets) can improve overall investor welfare. This is despite the fact that such incentives may decrease stock prices and future profitability on average by leading to inefficient over-investment (from the perspective of a price-maximization objective) in green projects. Yet, when investors have diverse climate risk exposures and find it difficult to hedge these exposures, such incentives improve their ability to hedge risks and, consequently, can improve

[27]More broadly, Edmans, Gosling, and Jenter (2021) find that over 50% of surveyed directors and investors report that offering variable pay to CEO is in part useful to "motivate the CEO to improve outcomes other than long-term shareholder value."

overall welfare.

9 Conclusions

In this paper, we develop a model of informational feedback effects in which a firm's investment alters its exposure to an aggregate risk, and discuss its application to climate-exposed investment. When a firm invests in a project that is exposed to climate risk, it affects how useful the asset is as a hedge for climate risk. As a result, the firm's stock price reflects information about investors' climate exposures and the project's expected cash flows, which are both relevant to the manager's investment choice. We show that this has novel implications for how a project's greenness affects the likelihood of investment, conditional expected returns and future profitability. Moreover, we show that because the price does not fully reflect the welfare externality generated by investment in climate-sensitive projects, price-maximization tends to lead to under-investment in green projects.

In addition to climate-exposed investments, our model's predictions on investment, returns, and profitability apply broadly to investments that are exposed to systematic risks with variable risk premia. For instance, investments that are exposed to commodity prices may serve as inflation hedges and thus may have discount rates that vary with investors' aggregate inflation concerns. Moreover, investments in emerging markets are exposed to aggregate demand in those markets, and so are likely to have discount rates that vary with uncertainty over this demand. Our model's implications for welfare also apply more generally, whenever the market is incomplete with respect to the investment's risk exposure.

A notable contribution of our analysis is to provide a tractable feedback effects framework with investor risk aversion and priced risk factors. Immediate extensions include generalizations to the structure of cash flows and information. For instance, allowing for both public and private information signals would enable future research to assess the merits of disclosure regarding firms' climate risk exposures. Similarly, introducing multiple dimensions of fundamentals as in Goldstein and Yang (2019), Goldstein et al. (2020) could enable future work to assess how climate-exposed investments interact with the other risks that firms face. Finally, it may be interesting to consider how information acquisition by investors and managers interact in our setting.

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A Proofs

A.1 Proof of Proposition 1

Begin by conjecturing an equilibrium of the form posited in the text. Suppose that there is a random variable of the form $s_p = \theta + \frac{1}{\beta} Z$ and threshold $\overline{s} \in \mathbb{R}$ such that the asset price at the two trading dates is identical and takes the form

$$P_3 = P_1 = \begin{cases} A_1 + B_1 s_p & s_p > \overline{s} \\ A_0 & s_p \leq \overline{s} \end{cases} \tag{37}$$

for constants A_t and B_t such that this function is strictly increasing in the investment region, $B_1 > 0$ weakly increasing overall, $A_1 + B_1 \overline{s}_p \geq A_0$.

We can now derive the equilibrium, and confirm the above conjecture, by working backwards. At date $t = 3$, traders can observe the actual investment decision made at $t = 2$. Hence, they perceive the asset payoff as conditionally normally distributed with conditional moments

$$\mathbb{E}_{i3}[V(k)] = \mathbb{E}_{i3}[A + k(\theta + \alpha \eta_C + \sqrt{1-\alpha^2}\eta_I - c)] = \mu_A + k(\theta - c) \tag{38}$$

$$\mathbb{C}_{i3}(V(k), \eta_C) = k\alpha \frac{1}{\tau_\eta} \tag{39}$$

$$\mathbb{V}_{i3}(V(k)) = \frac{1}{\tau_A} + k^2 \frac{1}{\tau_\eta} \tag{40}$$

The problem of an arbitrary trader at this date is a static one,

$$\max_{x \in \mathbb{R}} \mathbb{E}_{i3}[-e^{-\gamma W_{i4}}] \tag{41}$$

where the terminal wealth is

$$W_{i4} = (n + X_{i1} + x)\, V - x P_3 - X_{i1} P_1 - z_i \eta_C. \tag{42}$$

where X_{i1}, the trade from the $t = 1$ trading round, is taken as given.

It is immediate that this problem leads to a standard mean-variance demand function

$$X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k), \eta_C) z_i - P_3}{\gamma \mathbb{V}_{i3}(V(k))} - (n + X_{i1}). \tag{43}$$

Plugging in for the conditional moments from above and enforcing market clearing yields equilibrium price

$$P_3 = \mu_A + k(\theta - c) + \gamma k \alpha \frac{1}{\tau_\eta} Z - \gamma \left(\frac{1}{\tau_A} + k^2 \frac{1}{\tau_\eta} \right) n \tag{44}$$

$$= \mu_A - \gamma \frac{1}{\tau_A} n + k \left(\theta + \gamma \alpha \frac{1}{\tau_\eta} Z - c - \gamma \frac{1}{\tau_\eta} n \right) \tag{45}$$

where the second line collects terms and uses the fact that $k \in \{0, 1\}$ implies $k = k^2$ to simplify. Hence, to be consistent with our initial conjecture, the endogenous signal s_p has coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z.

Stepping back to $t = 2$, the manager's problem is to solve

$$\max_{k\in\{0,1\}} \mathbb{E}[P_3|\mathcal{F}_m] \tag{46}$$

where she can condition on the first period asset price, $\mathcal{F}_m = \sigma(P_1)$. Using the expression for P_3 derived above, the manager's problem reduces to

$$\max_{k\in\{0,1\}} k\mathbb{E}\left[s_p - c - \gamma\tfrac{1}{\tau_\eta}n\middle|\mathcal{F}_m\right] \tag{47}$$

The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[s_p|\mathcal{F}_m] > c + \gamma\frac{1}{\tau_\eta}n \\ 0 & \mathbb{E}[s_p|\mathcal{F}_m] \leq c + \gamma\frac{1}{\tau_\eta}n \end{cases} \tag{48}$$

Given the conjectured price function, if the manager observes $P_1 = A_0$, she infers $s_p \leq \overline{s}$, while if she observes any $P_1 > A_0$, she infers the realized value of s_p, necessarily strictly greater than $\overline{s}$. Hence, a threshold value $\overline{s} = c+\gamma\frac{1}{\tau_\eta}n$ is consistent with the initial conjecture.

In principle, any threshold $\overline{s}'$ such that (i) $\overline{s}' \geq c+\gamma\frac{1}{\tau_\eta}n$ and (ii) $\mathbb{E}[s_p|s_p \leq \overline{s}'] \leq c+\gamma\frac{1}{\tau_\eta}n$ is consistent with the conjecture. This is because (i) ensures that if the manager infers the realized value of s_p, necessarily strictly greater than $\overline{s}'$, her optimal action is $k = 1$, and (ii) ensures that if the manager infers $s_p \leq \overline{s}'$, her optimal action is $k = 0$. Our particular choice of $\overline{s} = c + \gamma\frac{1}{\tau_\eta}n$ is the unique such threshold that leads to a price function that is continuous at the threshold. Furthermore, with the equilibrium investment function pinned down, returning to the expression for P_3 derived above, to be consistent with our conjecture, the price coefficients must satisfy

$$A_0 = \mu_A - \gamma\tfrac{1}{\tau_A}n \tag{49}$$

$$A_1 = \mu_A - \gamma\tfrac{1}{\tau_A}n - c - \gamma\tfrac{1}{\tau_\eta}n \tag{50}$$

$$B_1 = 1. \tag{51}$$

Finally, stepping back to $t = 1$, the problem of an arbitrary trader is

$$\max_{x\in\mathbb{R}} \mathbb{E}_{i1}[-e^{-\gamma W_{i4}}] \tag{52}$$

where the terminal wealth is

$$W_{i4} = (n + x + X_{i3})\, V - X_{i3}P_3 - xP_1 - z_i\eta_C. \tag{53}$$

and where the optimal $t = 3$ demand X_{i3} was derived above. Given the functional form for P_3, the realization of P_3 is perfectly anticipated under the trader information set $\mathcal{F}_{i1} = \sigma(\theta, z_i, P_1)$. Hence, to rule out arbitrage, the price must satisfy $P_1 = P_3$, and consequently all traders are indifferent to trading at $t = 1$ at this equilibrium price. This completes the construction of equilibrium. □

A.2 Proof of Proposition 2

The probability of investment is given by

$$\Pr(s_p > \bar{s}) = 1 - \Phi\left(\frac{\overline{s} - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}[s_p] - \bar{s}}{\sqrt{\mathbb{V}[s_p]}}\right) \tag{54}$$

$$= \Phi\left(\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right) \tag{55}$$

This immediately implies probability of investment is increasing in $\mu_\theta - c$, decreasing in n, increasing in μ_Z. Moreover, given that Φ is strictly increasing, for any arbitrary parameter b we have, after applying the monotonic transformation $\Phi^{-1}(\cdot)$ and using the definition $NPV = \theta - c - \gamma\frac{1}{\tau_\eta}(n - \alpha Z)$ from the text to condense notation

$$\frac{\partial}{\partial b}\Pr(s_p > \bar{s}) \propto \frac{\partial}{\partial b}\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \tag{56}$$

$$= \frac{\partial}{\partial b}\frac{\mathbb{E}[NPV]}{\sqrt{\mathbb{V}(NPV)}} \tag{57}$$

$$= \frac{\sqrt{\mathbb{V}(NPV)}\frac{\partial}{\partial b}\mathbb{E}[NPV] - \mathbb{E}[NPV]\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\mathbb{V}(NPV)} \tag{58}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}}\left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \operatorname{sgn}(\mathbb{E}[NPV])\frac{\frac{\partial}{\partial b}\sqrt{\mathbb{V}(NPV)}}{\sqrt{\mathbb{V}(NPV)}}\right) \tag{59}$$

$$= \frac{|\mathbb{E}[NPV]|}{\sqrt{\mathbb{V}(NPV)}}\left(\frac{\frac{\partial}{\partial b}\mathbb{E}[NPV]}{|\mathbb{E}[NPV]|} - \frac{1}{2}\operatorname{sgn}(\mathbb{E}[NPV])\frac{\frac{\partial}{\partial b}\mathbb{V}(NPV)}{\mathbb{V}(NPV)}\right). \tag{60}$$

For α we have

$$\frac{\partial}{\partial \alpha}\Pr(s_p > \bar{s}) \propto \left(\frac{\partial}{\partial \alpha}\mathbb{E}[NPV] - \frac{1}{2}\mathbb{E}[NPV]\frac{\frac{\partial}{\partial \alpha}\mathbb{V}(NPV)}{\mathbb{V}(NPV)}\right) \tag{61}$$

$$= \frac{\gamma\mu_Z}{\tau_\eta} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} \tag{62}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\gamma\mu_Z}{\tau_\eta}\frac{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{63}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\alpha\gamma\mu_Z}{\tau_\eta}\frac{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}} - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{64}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\frac{\alpha\gamma\mu_Z}{\tau_\eta}\left(1 + \frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right) - \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha\gamma\mu_Z}{\tau_\eta}\right)\right) \tag{65}$$

$$= \frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\left(\frac{\frac{1}{\tau_\theta}}{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}\right)\mu_Z-\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}\right)\right) \tag{66}$$

$$= -\frac{\frac{1}{\alpha}\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}-\frac{\tau_\eta\tau_Z}{\alpha\gamma}\frac{1}{\tau_\theta}\mu_Z\right) \tag{67}$$

which implies

$$\frac{\partial}{\partial\alpha}\Pr\left(s_p>\bar{s}\right)<0\Leftrightarrow \operatorname{sgn}(\alpha)\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}-\frac{\tau_\eta\tau_Z}{\alpha\gamma}\frac{1}{\tau_\theta}\mu_Z\right)>0. \tag{68}$$

Moreover, note that because the parameters $\tau\in\{\tau_Z,\tau_\theta\}$ do not enter the expected NPV and increases in these τ strictly decrease the variance of the NPV, we have

$$\frac{\partial}{\partial\tau_Z}\Pr\left(s_p>\bar{s}\right),\frac{\partial}{\partial\tau_\theta}\Pr\left(s_p\geq\bar{s}\right)\propto-\tfrac{1}{2}\operatorname{sgn}\left(\mathbb{E}\left[NPV\right]\right)\frac{\frac{\partial}{\partial\tau}\mathbb{V}(NPV)}{\mathbb{V}(NPV)} \tag{69}$$

$$\propto \operatorname{sgn}\left(\mathbb{E}\left[NPV\right]\right) \tag{70}$$

so that the dependence is pinned down by the sign of the expected NPV, which immediately establishes the claimed result. □

A.3 Proof of Corollary 1

The first inequality follows from comparing the probabilities from Proposition 1, evaluated at $\alpha=1$ vs. $\alpha=0$:

$$\Phi\left(\frac{\mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right)>\Phi\left(\frac{\mu_\theta-c-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}}}\right) \tag{71}$$

$$\Leftrightarrow \frac{\mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\gamma\mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}>\frac{\mu_\theta-c-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}}} \tag{72}$$

$$\Leftrightarrow \mu_\theta-c-\frac{\gamma n}{\tau_\eta}+\frac{\gamma\mu_Z}{\tau_\eta}>\sqrt{1+\left(\frac{\gamma}{\tau_\eta}\right)^2\frac{\tau_\theta}{\tau_Z}}\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}\right) \tag{73}$$

$$\Leftrightarrow \frac{\gamma\mu_Z}{\tau_\eta}>\left(\sqrt{1+\left(\frac{\gamma}{\tau_\eta}\right)^2\frac{\tau_\theta}{\tau_Z}}-1\right)\left(\mu_\theta-c-\frac{\gamma n}{\tau_\eta}\right). \tag{74}$$

Similarly, the second inequality follows from comparing the probabilities from Proposition 1, evaluated at $\alpha = 0$ vs. $\alpha = -1$:

$$\Phi\left(\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}}}\right) > \Phi\left(\frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\gamma \mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right) \tag{75}$$

$$\Leftrightarrow \frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}}} > \frac{\mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\gamma \mu_Z}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \tag{76}$$

$$\Leftrightarrow \sqrt{1 + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{\tau_\theta}{\tau_Z}} \left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta}\right) > \mu_\theta - c - \frac{\gamma n}{\tau_\eta} - \frac{\gamma \mu_Z}{\tau_\eta} \tag{77}$$

$$\Leftrightarrow \frac{\gamma \mu_Z}{\tau_\eta} > -\left(\sqrt{1 + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{\tau_\theta}{\tau_Z}} - 1\right)\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta}\right). \tag{78}$$

A.4 Statement and Proof of Lemma A.1

The following Lemma will be useful for proving some of the results from the body of the paper.

Lemma A.1. *Define*

$$\Gamma = \mathbb{E}[s_p | s_p > \bar{s}]. \tag{79}$$

We have

$$\Gamma = \mathbb{E}\left[s_p\right] + \sqrt{\mathbb{V}\left[s_p\right]} H\left(\frac{\bar{s} - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right), \tag{80}$$

where $H(x) = \frac{\phi(x)}{1 - \Phi(x)}$ is the hazard ratio for the standard normal distribution. Moreover, Γ is increasing with μ_θ, c, μ_Z and n, and is increasing in α for $\alpha \geq 0$ if $\mu_Z > 0$.

The expression for Γ follows from standard results for the expectation of a truncated normal random variable. To derive the comparative statics results, note that by plugging in the explicit expressions for the threshold $\bar{s}$ and the moments of s_p, we can express Γ as

$$\Gamma = \mu_\theta + \frac{\alpha \gamma \mu_Z}{\tau_\eta} + \sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha \gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} H\left(-\frac{\mu_\theta - c + \frac{\alpha \gamma \mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha \gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right). \tag{81}$$

Note that $H(x) > 0$ and $H'(x) \in (0, 1)$. This immediately implies that Γ is increasing in μ_θ, c, μ_Z, n. To prove the claim for α, let $A \equiv \mathbb{E}\left[s_p\right] = \mu_\theta + \frac{\alpha \gamma \mu_Z}{\tau_\eta}$ and $B \equiv \sqrt{\mathbb{V}\left[s_p\right]} =$

$\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}$. Then,

$$\Gamma = A + BH\left(\frac{\bar{s}-A}{B}\right) \tag{82}$$

$$\Leftrightarrow \tfrac{\partial}{\partial\alpha}\Gamma = \tfrac{\partial}{\partial\alpha}A + \left(\tfrac{\partial}{\partial\alpha}B\right)H\left(\frac{\bar{s}-A}{B}\right) + BH'\left(\frac{\bar{s}-A}{B}\right)\times\left(\frac{-B\frac{\partial}{\partial\alpha}A-(\bar{s}-A)\frac{\partial}{\partial\alpha}B}{B^2}\right) \tag{83}$$

$$= \left(\tfrac{\partial}{\partial\alpha}A\right)\left(1-H'\left(\frac{\bar{s}-A}{B}\right)\right)+\left(\tfrac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\bar{s}-A}{B}\right)-\left(\frac{\bar{s}-A}{B}\right)H'\left(\frac{\bar{s}-A}{B}\right)\right). \tag{84}$$

Note that $H'(x)\in(0,1)$ and $H(x)-xH'(x)>0$, and that

$$\tfrac{\partial}{\partial\alpha}A = \tfrac{\gamma\mu_Z}{\tau_\eta} \tag{85}$$

$$\tfrac{\partial}{\partial\alpha}B = \tfrac{1}{2}\frac{2\alpha\left(\frac{\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}, \tag{86}$$

which are (weakly) positive if $\mu_Z\geq 0$ and $\alpha\geq 0$, respectively. We conclude that $\frac{\partial}{\partial\alpha}\Gamma>0$ for $\mu_Z>0$ and $\alpha\geq 0$. $\square$

A.5 Proof of Proposition 3

Using the expression for the equilibrium asset price in eq. (10), the expressions for conditional expected return given no investment and investment are straightforward.

To derive the comparative statics results for the expected return conditional on investment, note that we can write:

$$\mathbb{E}\left[V-P_3|k=1\right] = \mathbb{E}\left[V-P_3|s_p>\bar{s}\right] \tag{87}$$

$$= \gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)-\frac{\gamma\alpha}{\tau_\eta}\mathbb{E}\left[Z|s_p>\bar{s}\right] \tag{88}$$

$$= \gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)-\mathbb{E}\left[s_p-\theta|s_p>\bar{s}\right] \tag{89}$$

$$= \gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)-\mathbb{E}\left[s_p-\left(\mu_\theta+\frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\left(s_p-\mu_\theta-\frac{\gamma\alpha}{\tau_\eta}\mu_Z\right)\right)\middle| s_p>\bar{s}\right] \tag{90}$$

$$= \gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)+\frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\mu_\theta-\frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\frac{\gamma\alpha}{\tau_\eta}\mu_Z-\frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\Gamma, \tag{91}$$

where the third equality uses the law of iterated expectations to write $\mathbb{E}[\theta|s_p>\bar{s}]=\mathbb{E}[\mathbb{E}[\theta|s_p]|s_p>\bar{s}]$, and where the last line collects terms and uses where $\Gamma=\mathbb{E}\left[s_p|s_p>\bar{s}\right]$ as in Lemma A.1.

Now, plugging in for Γ from Lemma A.1 and grouping terms further yields

$$\mathbb{E}\left[V-P_3|k=1\right]=\gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)-\frac{\gamma\alpha}{\tau_\eta}\left(\mu_Z+\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}H\left(-\frac{\mu_\theta-c+\frac{\alpha\gamma\mu_Z}{\tau_\eta}-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right)\right). \tag{92}$$

This immediately implies $\mathbb{E}\left[V-P_3|k=1\right]$ is increasing in $\mu_\theta-c$. Further, it is decreasing in μ_Z for $\alpha>0$ and increasing for $\alpha<0$ since

$$\frac{\partial}{\partial\mu_Z}\mathbb{E}\left[V-P_3|k=1\right]=-\frac{\gamma\alpha}{\tau_\eta}\left(1-\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}H'\left(-\frac{\mu_\theta-c+\frac{\alpha\gamma\mu_Z}{\tau_\eta}-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right)\right)<0 \tag{93}$$

because $0<H'<1$.

Now, consider n and note that

$$\begin{aligned}\frac{\partial}{\partial n}\mathbb{E}\left[V-P_3|k=1\right]&=\gamma\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right)-\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\frac{\gamma}{\tau_\eta}H'\left(-\frac{\mu_\theta-c+\frac{\alpha\gamma\mu_Z}{\tau_\eta}-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right) &(94)\\ &=\gamma\frac{1}{\tau_A}+\gamma\frac{1}{\tau_\eta}\left(1-\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}H'\left(-\frac{\mu_\theta-c+\frac{\alpha\gamma\mu_Z}{\tau_\eta}-\frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}}\right)\right) &(95)\\ &\geq 0 &(96)\end{aligned}$$

again because $0<H'<1$.

Considering α, following the proof of Lemma A.1, let $A\equiv\mathbb{E}\left[s_p\right]=\mu_\theta+\frac{\alpha\gamma\mu_Z}{\tau_\eta}$ and $B\equiv\sqrt{\mathbb{V}\left[s_p\right]}=\sqrt{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}$. We can write

$$\begin{aligned}\frac{\partial}{\partial\alpha}\mathbb{E}\left[V-P_3|k=1\right]&=\frac{\partial}{\partial\alpha}\left(-\frac{\gamma\alpha}{\tau_\eta}\left(\mu_Z+\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{B}H\left(\frac{\bar{s}-A}{B}\right)\right)\right) &(97)\\ &=\frac{\partial}{\partial\alpha}\left(\left(-\frac{\gamma\alpha}{\tau_\eta}\mu_Z-\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}BH\left(\frac{\bar{s}-A}{B}\right)\right)\right) &(98)\end{aligned}$$

$$= -\frac{\gamma}{\tau_\eta}\mu_Z - \frac{\partial}{\partial\alpha}\left(\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right) BH\left(\frac{\overline{s}-A}{B}\right) \tag{99}$$

$$- \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\frac{\partial}{\partial\alpha}\left(BH\left(\frac{\overline{s}-A}{B}\right)\right). \tag{100}$$

We clearly have $-\frac{\partial}{\partial\alpha}\left(\frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\right) BH\left(\frac{\overline{s}-A}{B}\right) < 0$ as long as $\alpha > 0$, so it remains to establish that the remaining terms are, collectively, negative. Note that we can express $-\frac{\gamma}{\tau_\eta}\mu_Z = -\frac{\partial}{\partial\alpha}A$, so we want to sign

$$-\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\frac{\partial}{\partial\alpha}\left(BH\left(\frac{\overline{s}-A}{B}\right)\right). \tag{101}$$

Note that we have

$$\begin{aligned}
\frac{\partial}{\partial\alpha}BH\left(\frac{\overline{s}-A}{B}\right) &= \left(\frac{\partial}{\partial\alpha}B\right)H\left(\frac{\overline{s}-A}{B}\right) + BH'\left(\frac{\overline{s}-A}{B}\right)\frac{\partial}{\partial\alpha}\frac{\overline{s}-A}{B}\\
&= \left(\frac{\partial}{\partial\alpha}B\right)H\left(\frac{\overline{s}-A}{B}\right) + BH'\left(\frac{\overline{s}-A}{B}\right)\left[\frac{-B\frac{\partial}{\partial\alpha}A-(\overline{s}-A)\frac{\partial}{\partial\alpha}B}{B^2}\right]\\
&= \left(\frac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right) - \left(\frac{\partial}{\partial\alpha}A\right)H'\left(\frac{\overline{s}-A}{B}\right)
\end{aligned}$$

and plugging in to eq. (101) now yields

$$-\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(\left(\frac{\partial}{\partial\alpha}B\right)\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right) - \left(\frac{\partial}{\partial\alpha}A\right)H'\left(\frac{\overline{s}-A}{B}\right)\right) \tag{102}$$

$$= -\left(1 - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}H'\right)\frac{\partial}{\partial\alpha}A - \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2\frac{1}{\tau_Z}}\left(H\left(\frac{\overline{s}-A}{B}\right) - \frac{\overline{s}-A}{B}H'\left(\frac{\overline{s}-A}{B}\right)\right)\left(\frac{\partial}{\partial\alpha}B\right). \tag{103}$$

We know from the proof of Lemma A.1 that $\frac{\partial}{\partial\alpha}A > 0$ if $\mu_Z > 0$ and $\frac{\partial}{\partial\alpha}B \geq 0$ for $\alpha \geq 0$. Furthermore, we always have $H' \in (0,1)$ and $H(x) - xH'(x) \geq 0$. It follows therefore that the expression in eq. (101) is negative and hence that $\frac{\partial}{\partial\alpha}\mathbb{E}[V - P_3|k=1] < 0$ for $\mu_Z > 0$ and $\alpha \geq 0$.

Finally, consider τ_θ. It will be more convenient to study dependence on the variance $1/\tau_\theta$.

We have

$$\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[V-P_3|k=1\right] = -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[Z|s_p\geq\bar{s}\right]$$

$$= -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\mu_Z+\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\frac{1}{\beta^2}\frac{1}{\tau_Z}}\left(\mathbb{E}\left[s_p|s_p\geq\bar{s}\right]-\mathbb{E}\left[s_p\right]\right)\right)$$

$$= -\frac{\gamma\alpha}{\tau_\eta}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\mu_Z+\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\frac{1}{\tau_\theta}+\frac{1}{\beta^2}\frac{1}{\tau_Z}}\sqrt{\mathbb{V}\left[s_p\right]}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right)$$

$$= -\left(\frac{\gamma\alpha}{\tau_\eta}\right)^2\frac{1}{\tau_Z}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right).$$

Hence, the derivative of expected returns with respect to $1/\tau_\theta$ has the opposite sign of $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)\right)$. Applying the monotonic transformation $\log(\cdot)$ to $\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)$ and differentiating yields

$$\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(\frac{1}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right) = -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}+\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(H\left(\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right)\right)$$

$$= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}+\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}$$

$$= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}-\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}$$

$$= -\frac{1}{2}\frac{1}{\mathbb{V}[s_p]}\left(1+\frac{H'\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}{H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)}\frac{\bar{s}-\mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right).$$

Let $K<0$ be the unique root of the function $1+\frac{H'(K)}{H(K)}K$ and note that this function crosses zero from below as its argument increases, so that it is strictly negative for points below K and strictly positive for points above K.

We conclude that $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\log\left(\frac{\frac{1}{\beta}\frac{1}{\tau_Z}}{\sqrt{\mathbb{V}[s_p]}}H\left(\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right)\right)>0$ if and only if $\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}<K$. From this, it follows that $\frac{\partial}{\partial\left(\frac{1}{\tau_\theta}\right)}\mathbb{E}\left[V-P_3|k=1\right]<0$ and hence $\frac{\partial}{\partial\tau_\theta}\mathbb{E}\left[V-P_3|k=1\right]>0$. □

A.6 Proof of Corollary 2

The difference between expected returns in the case $\alpha = 1$ vs. $\alpha = 0$ follows from plugging in the expressions from Proposition 3:

$$\Delta_{R+} = \mathbb{E}\left[V - P_3 | k = 1; \alpha = 1\right] - \mathbb{E}\left[V - P_3 | k = 1; \alpha = 0\right] \tag{104}$$

$$= \mathbb{E}\left[V - P_3 | k = 1; \alpha = 1\right] - \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) \tag{105}$$

$$= -\frac{\gamma\alpha}{\tau_\eta}\left(\mu_Z + \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right)\Bigg|_{\alpha=1} \tag{106}$$

$$= -\frac{\gamma}{\tau_\eta}\left(\mu_Z + \frac{\left(\frac{\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c + \frac{\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right) \tag{107}$$

where the next-to-last line follows from substituting from eq. (92). It is now immediate from inspection that if $\mu_Z > 0$ then $\Delta_{R+} < 0$. The claimed comparative static results follow directly from Proposition 3, applied to $\mathbb{E}\left[V - P_3 | k = 1; \alpha = 1\right]$.

The difference between expected returns in the case $\alpha = 0$ vs. $\alpha = -1$ also follows from plugging in the expressions from Proposition 3:

$$\Delta_{R-} = \mathbb{E}\left[V - P_3 | k = 1; \alpha = 0\right] - \mathbb{E}\left[V - P_3 | k = 1; \alpha = -1\right] \tag{108}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[V - P_3 | k = 1; \alpha = -1\right] \tag{109}$$

$$= \frac{\gamma\alpha}{\tau_\eta}\left(\mu_Z + \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c + \frac{\alpha\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right)\Bigg|_{\alpha=-1} \tag{110}$$

$$= -\frac{\gamma}{\tau_\eta}\left(\mu_Z + \frac{-\left(\frac{\gamma}{\tau_\eta}\right)\frac{1}{\tau_Z}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c - \frac{\gamma\mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right) \tag{111}$$

where the next-to-last line follows from substituting from eq. (92). Note that only the first μ_Z in the parentheses has the potential to make this overall expression negative, if μ_Z is sufficiently large. Hence, to establish the claim in the Corollary that $\Delta_{R-} > 0$ if and only if μ_Z is sufficiently small, it suffices to show that this expression (i) is monotonically decreasing in μ_Z and (ii) is strictly positive for $\mu_Z \to 0$. Differentiating with respect to μ_Z, we obtain:

$$\frac{\partial \Delta_{R-}}{\partial \mu_Z} = -\frac{\gamma^3 \tau_\theta \left(1 - H'\left(-\frac{\mu_\theta - c - \frac{\gamma \mu_Z}{\tau_\eta} - \frac{\gamma n}{\tau_\eta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right)\right) + \gamma \tau_\eta^2 \tau_Z}{\gamma^2 \tau_\eta \tau_\theta + \tau_\eta^3 \tau_Z}, \tag{112}$$

which, since $H' \in (0,1)$, is strictly negative. Furthermore, it is immediate from inspection that $\lim_{\mu_Z \to 0} \Delta_{R-} > 0$. This establishes implies the claimed result. $\square$

A.7 Proof of Proposition 4

$$\begin{aligned}
\Delta V &= \mathbb{E}\left[V | k = 1\right] - \mathbb{E}\left[V | k = 0\right] && (113)\\
&= \mathbb{E}\left[V | s_p > \bar{s}\right] - \mathbb{E}\left[V | s_p \le \bar{s}\right] && (114)\\
&= \mathbb{E}\left[\theta - c | s_p > \bar{s}\right] && (115)\\
&= \mathbb{E}\left[\mu_\theta + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\left(s_p - \mathbb{E}\left[s_p\right]\right) \middle| s_p > \bar{s}\right] - c && (116)\\
&= \mu_\theta - c + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\sqrt{\mathbb{V}\left[s_p\right]} H\left(\frac{\bar{s} - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) && (117)\\
&= \mu_\theta - c + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}\left(n - \alpha \mu_Z\right)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}}\right) && (118)
\end{aligned}$$

where the fourth equality follows from the law of iterated expectations after conditioning on s_p, and the fifth equality substitutes in for $\mathbb{E}[s_p | s_p > \bar{s}]$ from Lemma A.1.

When $\alpha = 0$, we have

$$\begin{aligned}
\Delta V &= \mu_\theta - c + \frac{1}{\sqrt{\tau_\theta}} H\left(-\sqrt{\tau_\theta}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta} n\right)\right) && (119)\\
&= \frac{1}{\sqrt{\tau_\theta}}\left(\sqrt{\tau_\theta}\left(\mu_\theta - c\right) + H\left(-\sqrt{\tau_\theta}\left(\mu_\theta - c\right) + \sqrt{\tau_\theta}\frac{\gamma}{\tau_\eta} n\right)\right) && (120)\\
&\ge \frac{1}{\sqrt{\tau_\theta}}\left(\sqrt{\tau_\theta}\left(\mu_\theta - c\right) + H\left(-\sqrt{\tau_\theta}\left(\mu_\theta - c\right)\right)\right) && (121)\\
&> 0 && (122)
\end{aligned}$$

since $x + H(-x) > 0$ for all x.

Next, supposing that $\alpha \neq 0$, consider the behavior of ΔV as $\mu_\theta - c$ becomes arbitrarily negative. We have

$$\lim_{\mu_\theta - c \to -\infty} \Delta V \tag{123}$$

$$= \lim_{\mu_\theta - c \to -\infty} \left(\mu_\theta - c + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} H\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \right) \right) \tag{124}$$

$$= \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \left(1 + \frac{\frac{1}{\tau_\theta}}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \frac{H\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \right)}{\mu_\theta - c} \right) \tag{125}$$

$$= \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \left(1 - \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \right) \tag{126}$$

$$= \frac{\left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}} \lim_{\mu_\theta - c \to -\infty} (\mu_\theta - c) \tag{127}$$

$$= -\infty \tag{128}$$

where the next-to-last equality follows from continuity and the fact that $\lim_{x\to-\infty} \frac{H(-a(x-b))}{x} = -\lim_{x\to\infty} \frac{H(a(x+b))}{x} = -a$ for any $a > 0, b \in \mathbb{R}$. Hence, if $\alpha \neq 0$, then for $\mu_\theta - c$ sufficiently negative, we have $\Delta V < 0$.

Further, differentiating yields

$$\frac{\partial}{\partial(\mu_\theta - c)} \Delta V = 1 - \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\gamma\alpha}{\tau_\eta}\right)^2 \frac{1}{\tau_z}} H'\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \right) \tag{129}$$

and since $H'(x) \in (0,1)$, we have $\frac{\partial}{\partial(\mu_\theta - c)} \Delta V \in (0,1)$.

Moreover,

$$\frac{\partial}{\partial\mu_Z} \Delta V = -\frac{\gamma\alpha}{\tau_\eta} \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\gamma\alpha}{\tau_\eta}\right)^2 \frac{1}{\tau_z}} H'\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \right), \tag{130}$$

which is negative for $\alpha > 0$ and positive for $\alpha < 0$.

Finally,

$$\frac{\partial}{\partial n} \Delta V = \frac{\gamma}{\tau_\eta} \frac{\frac{1}{\tau_\theta}}{\frac{1}{\tau_\theta} + \left(\frac{\gamma\alpha}{\tau_\eta}\right)^2 \frac{1}{\tau_z}} H'\left(-\frac{\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n - \alpha\mu_Z)}{\sqrt{\frac{1}{\tau_\theta} + \left(\frac{\alpha\gamma}{\tau_\eta}\right)^2 \frac{1}{\tau_Z}}} \right) > 0. \tag{131}$$

$\square$

A.8 Proof of Proposition 5

To establish the welfare-maximizing rule, note that the s_p-conditional expected utility is a special case of Proposition IA2 in which $\tau_\zeta \to \infty$. In this limit, the functions Q and D that characterize the expected utility $\mathcal{W}(k; s_p) = -D(k; s_p)\exp\{Q(k; s_p)\}$ are

$$\begin{aligned} Q\left(k; s_p\right) = &-\gamma\left(\mu_A + k\left(\mathbb{E}_p[\theta] - c\right)\right)n + \tfrac{1}{2}\gamma^2\left(\left(\tfrac{1}{\tau_A} + k^2\tfrac{1}{\tau_\eta}\right)n^2 - 2k\alpha\tfrac{1}{\tau_\eta}n\mathbb{E}_p[Z] + \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\right) \\ &+ \tfrac{1}{2}\gamma^2\left(\gamma\tfrac{1}{\tau_\eta}\mathbb{E}_p[Z]\right)'\left(\tfrac{1}{\beta^2\mathbb{V}_p(\theta)} - \gamma^2\tfrac{1}{\tau_\eta}\right)^{-1}\left(\gamma\tfrac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \end{aligned}$$

and

$$D(k; s_p) = \sqrt{\frac{1}{\beta^2\mathbb{V}_p\left(\theta\right)}}\left(\tfrac{1}{\beta^2\mathbb{V}_p(\theta)} - \gamma^2\tfrac{1}{\tau_\eta}\right)^{-1/2}.$$

Only the function Q in the above expression depends on k. Further using the fact that $k^2 = k$ for $k \in \{0, 1\}$, it follows that the welfare-maximizing s_p-dependent investment rule is

$$\begin{aligned} k(s_p) &= \arg\max_{k\in\{0,1\}} \mathcal{W}\left(k; s_p\right) \\ &= \arg\max_{k\in\{0,1\}} \left(\mu_A + k\left(\mathbb{E}_p[\theta] - c\right)\right)n - \tfrac{1}{2}\gamma\left(\left(\tfrac{1}{\tau_A} + k\tfrac{1}{\tau_\eta}\right)n^2 - 2k\alpha\tfrac{1}{\tau_\eta}n\mathbb{E}_p[Z] + \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\right) \\ &= \mathbf{1}\left\{\mathbb{E}_p[\theta] - c + \alpha\frac{\gamma}{\tau_\eta}\mathbb{E}_p[Z] - \frac{1}{2}\frac{\gamma}{\tau_\eta}n > 0\right\} \\ &= \mathbf{1}\left\{s_p - c - \frac{1}{2}\frac{\gamma}{\tau_\eta}n > 0\right\}. \end{aligned}$$

Defining $\bar{s}_W \equiv c + \frac{1}{2}\frac{\gamma}{\tau_\eta}n$ delivers the investment rule in the Proposition.

Furthermore, using the expressions for the price-maximizing threshold, $\bar{s} = c + \frac{\gamma}{\tau_\eta}n$, from Proposition 1, and the value-maximizing threshold, $\bar{s}_V = c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$, from eq. (23), yields the expressions for $\bar{s} - \bar{s}_W$ and $\bar{s}_V - \bar{s}_W$.

A.9 Proof of Proposition 6

If the unconditional NPV is positive $\mathbb{E}[s_p] - \bar{s} > 0$, then a manager who does not condition on price optimally invests in all states of the world, leading to 'no feedback' investment $k_{NF} = 1$. Hence, if $\mathbb{E}[s_p] > \bar{s}$, then to establish that feedback reduces welfare, it suffices to show that welfare is higher with $k_{NF} = 1$ than with the equilibrium investment rule $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$.

The small n limit in the Proposition is easier to establish by considering the limit of the s_p-conditional welfare in Proposition IA2 state-by-state, while the small τ_ζ limit is easier to establish using the unconditional welfare expression in Proposition IA3 directly.

We proceed first with the claim for $n \to 0$. Note that the difference in unconditional welfare can be written as the difference in the expected values of the conditional welfare expressions:

$$\begin{aligned}
&\mathcal{W}(\text{No Feedback}) - \mathcal{W}(\text{Feedback}) \\
&= \mathbb{E}\left[\mathcal{W}(k_{NF}; s_p) - \mathcal{W}(k(s_p); s_p)\right] \\
&= \mathbb{E}\left[\mathcal{W}(1; s_p) - \mathcal{W}(k(s_p); s_p)\right] \\
&= \mathbb{E}\left[\mathbf{1}_{\{s_p \le \bar{s}\}}\left(\mathcal{W}(1; s_p) - \mathcal{W}(k(s_p); s_p)\right)\right] \qquad (132)
\end{aligned}$$

where the next-to-last line substitutes in $k_{NF} = 1$, and the last line uses the fact that $k(s_p) = k_{NF} = 1$ in states $s_p > \bar{s}$, which leads to identical conditional expected utilities in such states.

Now, for any state $s_p \le \bar{s}$, consider

$$\begin{aligned}
\mathcal{W}(1; s_p) - \mathcal{W}(0; s_p) &= -D(1; s_p) \exp\{Q(1; s_p)\} - (-D(0; s_p) \exp\{Q(0; s_p)\}) \\
&= D(0; s_p) \exp\{Q(0; s_p)\} - D(1; s_p) \exp\{Q(1; s_p)\}.
\end{aligned}$$

If we can show that in the $n \to 0$ limit, this expression is positive, and can justify passing the limit through the expectation in eq.(132), this will establish that welfare is higher with no feedback than with feedback.

Because n does not enter $D(k; s_p)$ and $D(k; s_p)$ is strictly decreasing in k, it suffices to show that $Q(0; s_p) > Q(1; s_p)$ in the limit. We have

$$\begin{aligned}
&\lim_{n \to 0} Q(k; s_p) \\
&= \lim_{n \to 0} \Bigg\{ -\gamma\left(\mu_A + k\left(\mathbb{E}_p[\theta] - c\right)\right) n \\
&\quad + \tfrac{1}{2}\gamma^2 \left(\left(\tfrac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \tfrac{1}{\tau_\eta}\right)\right) n^2 - 2k\alpha \tfrac{1}{\tau_\eta} n \mathbb{E}_p[Z] + \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\right) \\
&\quad + \tfrac{1}{2}\gamma^2 \left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} n + \gamma k\alpha \frac{1}{\tau_\eta} n - \gamma \frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right)' \\
&\quad \times \left(\frac{1}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} - \gamma^2 \frac{1}{\tau_\eta} + \gamma^2 \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta + \beta^2(\tau_Z + \tau_\zeta)} + \frac{1}{\tau_\eta}\right)} \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}}\right)^{-1} \\
&\quad \times \left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} n + \gamma k\alpha \frac{1}{\tau_\eta} n - \gamma \frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \Bigg\} \\
&= \tfrac{1}{2}\gamma^2 \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z] + \tfrac{1}{2}\gamma^2 \left(\gamma \frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \left(\frac{1}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} - \gamma^2 \frac{1}{\tau_\eta} + \gamma^2 \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}} \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A} + k^2\left(\frac{1}{\tau_\theta + \beta^2(\tau_Z + \tau_\zeta)} + \frac{1}{\tau_\eta}\right)} \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta) + \frac{1}{\tau_\zeta}}\right)^{-1} \left(\gamma \frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right).
\end{aligned}$$

It is immediate from inspection of this expression that $\lim_{n \to 0} Q(1; s_p) < \lim_{n \to 0} Q(0; s_p)$. Interchanging the limit and expectation in eq. (132) follows from the dominated convergence theorem. To see this, note that

$$\begin{aligned}
&\left|\mathbf{1}_{\{s_p \le \bar{s}\}} D(0; s_p) \exp\{Q(0; s_p)\} - \mathbf{1}_{\{s_p \le \bar{s}\}} D(1; s_p) \exp\{Q(1; s_p)\}\right| \\
&\le \mathbf{1}_{\{s_p \le \bar{s}\}} D(0; s_p) \exp\{Q(0; s_p)\} + \mathbf{1}_{\{s_p \le \bar{s}\}} D(1; s_p) \exp\{Q(1; s_p)\}
\end{aligned}$$

and for $s_p \leq \overline{s}$ and all $n < \varepsilon$ we have

$$
\begin{aligned}
Q\left(k; s_p\right) = & -\gamma\left(\mu_A + k\left(\mathbb{E}_p[\theta] - c\right)\right) n \\
& + \tfrac{1}{2}\gamma^2\left(\left(\tfrac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \tfrac{1}{\tau_\eta}\right)\right) n^2 - 2k\alpha\tfrac{1}{\tau_\eta} n \mathbb{E}_p[Z] + \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\right) \\
& + \tfrac{1}{2}\gamma^2\left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} n + \gamma k\alpha\frac{1}{\tau_\eta} n - \gamma\frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right)' \\
& \times\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1} \\
& \times\left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} n + \gamma k\alpha\frac{1}{\tau_\eta} n - \gamma\frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \\
= & -\gamma\mu_A + \tfrac{1}{2}\gamma^2\tfrac{1}{\tau_A}n^2 + \tfrac{1}{2}\gamma^2\left(\tfrac{1}{\tau_\theta} - \tfrac{1}{\tau_\eta}\right)n^2 + \tfrac{1}{2}\gamma^2\tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z] - \gamma k\left(s_p - \overline{s}\right) n \\
& + \tfrac{1}{2}\gamma^2\left(\gamma^2\left(\frac{1}{\tau_\eta}\right)^2\mathbb{E}_p^2[Z] - 2\gamma\frac{1}{\tau_\eta}\frac{k\frac{1}{\beta}\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} n\mathbb{E}_p[Z] + \left(\frac{k\frac{1}{\beta}\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^2 n^2\right) \\
& \times\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1} \\
\leq & -\gamma\mu_A + \tfrac{1}{2}\gamma^2\tfrac{1}{\tau_A}\varepsilon^2 + \tfrac{1}{2}\gamma^2\left|\tfrac{1}{\tau_\theta} - \tfrac{1}{\tau_\eta}\right|\varepsilon^2 + \tfrac{1}{2}\gamma^2\tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z] - \gamma k\left(s_p - \overline{s}\right)\varepsilon \\
& + \tfrac{1}{2}\gamma^2\left(\gamma^2\left(\frac{1}{\tau_\eta}\right)^2\mathbb{E}_p^2[Z] + 2\gamma\frac{1}{\tau_\eta}\frac{k\frac{1}{\beta}\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\varepsilon|\mathbb{E}[Z]| + \left(\frac{k\frac{1}{\beta}\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^2\varepsilon^2\right) \\
& \times\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1} \\
\equiv & \hat{Q}(k; s_p, \varepsilon)
\end{aligned}
$$

and therefore $\mathbb{E}\left[\mathbf{1}_{\{s_p \leq \overline{s}\}} D(k; s_p)\exp\left\{Q(k; s_p)\right\}\right] \leq \mathbb{E}\left[\mathbf{1}_{\{s_p \leq \overline{s}\}} D(k; s_p)\exp\left\{\hat{Q}(k; s_p, \varepsilon)\right\}\right] < \infty$. Hence, we have bounded $\mathbf{1}_{\{s_p \leq \overline{s}\}} D(k; s_p)\exp\left\{Q(k; s_p)\right\}$, for n sufficiently small, by an integrable function that does not depend on n, which allows us to apply the dominated convergence theorem.

Next, consider $\tau_\zeta \downarrow$, equivalently $1/\tau_\zeta \uparrow$. Note that in order for unconditional expected utility to exist, we must have $\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} > 0 \Leftrightarrow 0 \leq \frac{1}{\tau_\zeta} < \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Hence, the relevant limit is $\frac{1}{\tau_\zeta} \uparrow \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$. Using the unconditional welfare expression, welfare under the no-feedback investment level $k_{NF} = 1$ is higher than under the feedback policy $k(s_p) = \mathbf{1}_{\{s_p > \overline{s}\}}$ if and only if

$$
\begin{aligned}
-D(1)\exp\{Q(1)\} &> -\Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} \\
&\quad -\left(1-\Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)\right)D(1)\exp\{Q(1)\} \\
\Leftrightarrow \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} &> \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\}.
\end{aligned}
$$

Hence, to establish the claimed result, it suffices to show

$$
\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} > \lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\}.
$$

We will show this by establishing that $\lim_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\{Q(0)\} = \infty$, while $\limsup_{\frac{1}{\tau_\zeta}\uparrow\frac{\tau_\eta}{\gamma^2}-\frac{1}{\tau_Z}} \Phi\left(\tfrac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)D(1)\exp\{Q(1)\} < \infty$. Letting $a = \frac{\tau_\eta}{\gamma^2} - \frac{1}{\tau_Z}$ to reduce clutter, we have

$$
\begin{aligned}
\lim_{\frac{1}{\tau_\zeta}\uparrow a} D(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}\right)}}\sqrt{\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}}\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1/2} \\
&= \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+1/a)}\right)}}\sqrt{\frac{1}{\frac{1}{\tau_Z}+a}}\lim_{\frac{1}{\tau_\zeta}\uparrow a}\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1/2} \\
&= \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2(\tau_Z+1/a)}\right)}}\sqrt{\frac{1}{\frac{1}{\tau_Z}+a}}\begin{cases}\infty & k=0 \\ \left(\frac{1}{\frac{1}{\tau_Z}+a}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{a}{\frac{1}{\tau_Z}+a}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+1/a)}+\frac{1}{\tau_\eta}\right)}\frac{a}{\frac{1}{\tau_Z}+a}\right)^{-1/2} & k>0\end{cases}.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\lim_{\frac{1}{\tau_\zeta}\uparrow a} Q(k) &= \lim_{\frac{1}{\tau_\zeta}\uparrow a}\Bigg\{-\gamma\left(\mu_A+k(\mu_\theta-c)\right)n+\tfrac{1}{2}\gamma^2\binom{n}{\mu_Z}'\begin{pmatrix}\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}\right) & -k\alpha\frac{1}{\tau_\eta} \\ -k\alpha\frac{1}{\tau_\eta} & \frac{1}{\tau_\eta}\end{pmatrix}\binom{n}{\mu_Z} \\
&\quad \tfrac{1}{2}\gamma^2\left(\gamma\tfrac{1}{\tau_\eta}\mu_Z-\gamma k\alpha\tfrac{1}{\tau_\eta}n\right)'\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1}\left(\gamma\tfrac{1}{\tau_\eta}\mu_Z-\gamma k\alpha\tfrac{1}{\tau_\eta}n\right)\Bigg\} \\
&= \begin{cases}\infty & k=0 \\ \text{Finite} & k>0\end{cases}.
\end{aligned}
$$

Because the function Φ is bounded, together these results imply that

$$\limsup_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) D(1)\exp\{Q(1)\} < \infty.$$

It remains to show that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \infty$. Considering $\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}$, if $1/\beta = 0$,then $\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}$ is constant in τ_ζ and we are done. Considering $1/\beta \neq 0$, we have

$$\begin{aligned}
\lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}} &= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{m(0)}{\sqrt{v(0)}} \\
&= \lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{-\gamma^2\frac{1}{\beta}\frac{1}{\tau_Z}\frac{1}{\tau_\eta}\mu_Z-\gamma^2\left(\frac{1}{\beta}\frac{1}{\tau_Z}\gamma\frac{1}{\tau_\eta}\right)\times\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}\right)^{-1}\gamma\frac{1}{\tau_\eta}\mu_Z}{\sqrt{\mathbb{V}(s_p)+\gamma^2\frac{1}{\tau_\eta}\frac{1}{\beta^2}\frac{1}{\tau_Z^2}+\gamma^2\frac{1}{\beta}\frac{1}{\tau_Z}\frac{1}{\tau_\eta}\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}\right)^{-1}\gamma^2\frac{1}{\beta}\frac{1}{\tau_Z}\frac{1}{\tau_\eta}}} \\
&= \begin{cases} -\infty & \frac{1}{\beta} > 0 \\ \infty & \frac{1}{\beta} < 0 \end{cases}
\end{aligned}$$

where the first equality follows from $v(0)\to\infty$ so $\frac{\bar{s}-\mathbb{E}[s_p]}{\sqrt{v(0)}}\to 0$, the second equality substitutes for $m(0)$ and $v(0)$, and the final equality evaluates the limit.

If $1/\beta < 0$, the proof is complete, since $Q(0)\to\infty$, $D(0)\to\infty$ and in this case $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) > 0$, so that $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \infty$. If $1/\beta > 0$, then $\lim_{\frac{1}{\tau_\zeta}\uparrow a} \Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right) = 0$, so the limit is still indeterminate. Write $\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\}$ as

$$\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right) D(0)\exp\{Q(0)\} = \frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}}$$

and note that the relevant limit ultimately depends on the relative rate at which the various terms grow as $x \equiv \left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}\right)^{-1}$ approaches ∞ so that we can write

$$\lim_{\frac{1}{\tau_\zeta}\uparrow a} \frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}} = \lim_{x\to\infty} \frac{\Phi\left(-\sqrt{x}\right)}{\frac{1}{\sqrt{x}}\exp\{-x\}}.$$

Using L'Hospital's rule yields

$$
\begin{aligned}
\lim_{x\to\infty}\frac{\Phi\left(-\sqrt{x}\right)}{\frac{1}{\sqrt{x}}\exp\left\{-x\right\}} &= \lim_{x\to\infty}\frac{-\frac{1}{2}x^{-1/2}\phi\left(-\sqrt{x}\right)}{-\frac{1}{\sqrt{x}}\exp\left\{-x\right\}-\frac{1}{2}x^{-3/2}\exp\left\{-x\right\}} \\
&= \lim_{x\to\infty}\frac{\phi\left(-\sqrt{x}\right)}{2\exp\left\{-x\right\}+x^{-1}\exp\left\{-x\right\}} \\
&= \lim_{x\to\infty}\frac{\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}x\right\}}{2\exp\left\{-x\right\}+x^{-1}\exp\left\{-x\right\}} \\
&= \infty,
\end{aligned}
$$

which establishes $\lim_{\frac{1}{\tau_\zeta}\uparrow a}\frac{\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)}{\frac{1}{D(0)}\exp\{-Q(0)\}}=\infty$ and completes the proof. □

Feedback Effects and Systematic Risk Exposures: Internet Appendix

Table of Contents

A Investor welfare

In this section, we characterize traders' expected utilities, which is a key step in establishing the welfare results in the text. Because it poses no additional difficulty, we characterize the expected utility for an arbitrary s_p-dependent investment rule and associated equilibrium asset price. In the material that follows, we will let I_n denote an $n \times n$ identity matrix and will follow the convention that all vectors are column vectors, with row vectors indicated explicitly, using $'$ to denote transposes.

The proof of Proposition 1 established that, in equilibrium, we have $P_1 \equiv P_3$ and traders are indifferent to any trading strategy with aggregate trade

$$X_{i3} + X_{i1} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k), \eta_C) z_i - P_3}{\gamma \mathbb{V}_{i3}(V(k))} - n \tag{IA1}$$

across the two rounds. Consequently, the following material proceeds, without loss of generality, under the assumption that $X_{i1} = 0$ and consequently only the $t = 3$ trading round contributes to the utility. Hence, to reduce notational clutter, we suppress the $t = 3$ dependence of the price function and other equilibrium objects where no confusion will result.

Proposition IA1. *Consider an arbitrary s_p-dependent investment rule $k(s_p)$, with associated asset value $V = V(k(s_p))$ and pricing rule $P(s_p)$. Let $U = -\eta_C$ concisely denote the non-tradeable payoff. Consider an arbitrary investor i and define the 5×1 random vector*

$$Y = \begin{pmatrix} V - P \\ \vec{V} \\ \vec{Z}_i \end{pmatrix}, \tag{IA2}$$

where $\vec{V} = \begin{pmatrix} V(k) \\ U \end{pmatrix}$ and $\vec{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ are the sub-vectors of the tradeable and non-tradeable payoffs and the endowments, respectively. Let $\mathcal{F}_p = \sigma(s_p)$ be the information set given s_p. Finally, define the 5×5 matrix $\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma} I_2 \\ 0 & \frac{1}{\gamma} I_2 & 0 \end{pmatrix}$. The conditional expected utility of investor i given $\mathcal{F}_p$ is

$$-\left|\mathbb{V}_i(V-P)\mathbb{V}_p^{-1}(V-P)\right|^{1/2}\left|I_4+\begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix}\mathbb{V}_p((\vec{V},\vec{Z}_i)|V-P)\right|^{-1/2} \exp\left\{-\tfrac{1}{2}\mathbb{E}_p[Y]'(\mathbb{V}_p(Y)+\mathcal{I})^{-1}\mathbb{E}_p[Y]\right\} \tag{IA3}$$

The following result expresses the s_p-conditional expected utility in a form that is more amenable to economic interpretation.

Proposition IA2. *Consider an arbitrary s_p-dependent investment rule $k(s_p)$, with associated asset value $V = V(k(s_p))$ and pricing rule $P(s_p)$. The conditional expected utility of investor i given $\mathcal{F}_p = \sigma(s_p)$ can be written as*

$$\mathcal{W}(k; s_p) \equiv -D(k; s_p) \exp\{Q(k; s_p)\} \tag{IA4}$$

where the quadratic form Q is

$$
\begin{aligned}
Q\left(k;s_p\right) = & -\gamma\left(\mu_A + k\left(\mathbb{E}_p[\theta]-c\right)\right)n \\
& + \tfrac{1}{2}\gamma^2\left(\left(\tfrac{1}{\tau_A} + k^2\left(\mathbb{V}_p(\theta|z_i) + \tfrac{1}{\tau_\eta}\right)\right)n^2 - 2k\alpha\tfrac{1}{\tau_\eta}n\mathbb{E}_p[Z] + \tfrac{1}{\tau_\eta}\mathbb{E}_p^2[Z]\right) \\
& + \tfrac{1}{2}\gamma^2\left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}n + \gamma k\alpha\frac{1}{\tau_\eta}n - \gamma\frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right)' \\
& \times\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1} \\
& \times\left(\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}n + \gamma k\alpha\frac{1}{\tau_\eta}n - \gamma\frac{1}{\tau_\eta}\mathbb{E}_p[Z]\right)
\end{aligned}
$$

and the determinant term D is

$$
\begin{aligned}
D(k;s_p) = & \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\mathbb{V}_p\left(\theta|z_i\right)\right)}}\sqrt{\frac{1}{\beta^2\mathbb{V}_p\left(\theta\right)+\frac{1}{\tau_\zeta}}} \\
& \times\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1/2}.
\end{aligned}
$$

Taking the expectation of the s_p-conditional utility with respect to s_p delivers the unconditional welfare, which we record in the following Proposition.

Proposition IA3. *The unconditional expected utility can be written as*

$$
\begin{aligned}
\mathcal{W} = & -\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(0)}{\sqrt{v(0)}}\right)D(0)\exp\left\{Q(0)\right\} \\
& - \left(1-\Phi\left(\frac{\bar{s}-\mathbb{E}[s_p]+m(1)}{\sqrt{v(1)}}\right)\right)D(1)\exp\left\{Q(1)\right\}
\end{aligned}
$$

where

$$
D(k) = \sqrt{\frac{\frac{1}{\tau_A}+k^2\frac{1}{\tau_\eta}}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\eta}+\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}\right)}}\sqrt{\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}}\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1/2},
$$

$$
\begin{aligned}
Q(k) = & -\gamma\left(\mu_A + k(\mu_\theta - c)\right)n + \tfrac{1}{2}\gamma^2\begin{pmatrix} n \\ \mu_Z \end{pmatrix}'\begin{pmatrix} \frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_\eta}\right) & -k\alpha\frac{1}{\tau_\eta} \\ -k\alpha\frac{1}{\tau_\eta} & \frac{1}{\tau_\eta} \end{pmatrix}\begin{pmatrix} n \\ \mu_Z \end{pmatrix} \\
& + \tfrac{1}{2}\gamma^2\left(\gamma\frac{1}{\tau_\eta}\mu_Z - \gamma k\alpha\frac{1}{\tau_\eta}n\right)'
\end{aligned}
$$

$$\times\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1}$$
$$\times\left(\gamma\frac{1}{\tau_\eta}\mu_Z-\gamma k\alpha\frac{1}{\tau_\eta}n\right)$$

and

$$\begin{aligned}m(k)=\gamma k\tfrac{1}{\tau_\theta}n-\gamma\tfrac{1}{\beta}\tfrac{1}{\tau_Z}\left(\gamma\tfrac{1}{\tau_\eta}\mu_Z-\gamma k\alpha\tfrac{1}{\tau_\eta}n\right)\\ -\gamma^2\left(\tfrac{1}{\beta}\tfrac{1}{\tau_Z}\gamma\tfrac{1}{\tau_\eta}\right)\times\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1}\\ \times\left(\gamma\tfrac{1}{\tau_\eta}\mu_Z-\gamma k\alpha\tfrac{1}{\tau_\eta}n\right);\end{aligned}$$

$$v(k)=\mathbb{V}(s_p)+\gamma^2\tfrac{1}{\tau_\eta}\tfrac{1}{\beta^2}\tfrac{1}{\tau_Z^2}+\gamma^2\tfrac{1}{\beta}\tfrac{1}{\tau_Z}\tfrac{1}{\tau_\eta}\left(\frac{1}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}-\gamma^2\frac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\beta^2\left(\tau_Z+\tau_\zeta\right)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}}\right)^{-1}\gamma^2\tfrac{1}{\beta}\tfrac{1}{\tau_Z}\tfrac{1}{\tau_\eta}.$$

A.1 Proof of Proposition IA1

To compute the expected utility, we will use the law of iterated expectations, first computing the expectation conditional on $\mathcal{F}_{i^+}=\sigma(\{\theta,z_i,s_p\})$, which is the trader information set augmented with s_p, and then conditional on $\mathcal{F}_p$. We emphasize that the initial step is, in principle, not identical to computing the expectation given the trader's information set $\mathcal{F}_i$ itself since s_p is only inferred by the trader in states in which investment is positive and the asset price has non-trivial dependence on s_p. However, as will be seen, the conditional expected utilities given $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ are identical.

Note that because $V=A$ in any state with zero investment (i.e., in any state in which the trader does not infer s_p from the price) and because the $\mathcal{F}_i$ and $\mathcal{F}_{i^+}$ information sets coincide in any states with positive investment (i.e., in any state in which the trader is able to infer s_p from the price), we necessarily have that $\vec{V}$ is conditionally jointly normally distributed under both $\mathcal{F}_{i^+}$ and $\mathcal{F}_i$ with conditional means

$$\begin{aligned}\mathbb{E}_{i^+}[V]&=\begin{cases}\mathbb{E}[A|\theta,z_i,s_p] & k=0\\ \mathbb{E}[A+\theta-c+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I|\theta,z_i,s_p] & k>0\end{cases} && \text{(IA5)}\\ &=\begin{cases}\mathbb{E}[A] & k=0\\ \mathbb{E}[A+\theta-c|\theta,z_i,s_p] & k>0\end{cases} && \text{(IA6)}\\ &=\mathbb{E}_i[V] && \text{(IA7)}\end{aligned}$$

and

$$\mathbb{E}_{i^+}[U]=\mathbb{E}_{i^+}[U|\theta,z_i,s_p]=0=\mathbb{E}_i[U], \quad \text{(IA8)}$$

and with conditional variances and covariances

$$\mathbb{V}_{i^+}[V] = \begin{cases} \mathbb{V}[A|\theta, z_i, s_p] & k = 0 \\ \mathbb{V}[A + \theta - c + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I|\theta, z_i, s_p] & k > 0 \end{cases} \tag{IA9}$$

$$= \begin{cases} \mathbb{V}[A] & k = 0 \\ \mathbb{V}[A + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I|\theta, z_i, s_p] & k > 0 \end{cases} \tag{IA10}$$

$$= \mathbb{V}_i[V], \tag{IA11}$$

$$\mathbb{V}_{i^+}[U] = \mathbb{V}[U|\theta, z_i, s_p] = \tfrac{1}{\tau_\eta} = \mathbb{V}_i[U] \tag{IA12}$$

and

$$\mathbb{C}_{i^+}(V, U) = \begin{cases} \mathbb{C}(A, U|\theta, z_i, s_p) & k = 0 \\ \mathbb{C}(A + \theta - c + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I, U|\theta, z_i, s_p) & k > 0 \end{cases} \tag{IA13}$$

$$= \begin{cases} 0 & k = 0 \\ \mathbb{C}(\alpha\eta_C, U|\theta, z_i, s_p) & k > 0 \end{cases} \tag{IA14}$$

$$= \mathbb{C}_i(V, U). \tag{IA15}$$

Because the conditional distributions of payoffs are identical under both information sets, it follows that the expected utility given $\mathcal{F}_{i^+}$ and $\mathcal{F}_i$ are identical and given by the expresssion derived for the expected utility given $\mathcal{F}_i$ in Proposition 1.

$$\mathbb{E}_i[-e^{-\gamma W_{i4}^*}] = -e^{-\gamma n \mathbb{E}_i[V] + \gamma\left(n + z_i \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}\right)\mathbb{E}_i[V-P] - \frac{1}{2}\frac{\mathbb{E}_i^2[V-P]}{\mathbb{V}_i(V-P)} + \frac{1}{2}\gamma^2 \mathbb{V}_i(U|V-P) z_i^2} \tag{IA16}$$

where $h_i = \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}$ is the conditional regression coefficient of the endowment payoff U on the asset return $V - P$.

To complete the proof, we need to compute the conditional expectation of this quantity given $\mathcal{F}_p$. Let $\vec{h}_i = \left(1, \frac{\mathbb{C}_i(V-P,U)}{\mathbb{V}_i(V-P)}\right)$ be the 2×1 vector of conditional regression coefficients of (V, U) on $V - P$ and define the 5×5 block matrix

$$a_i = \begin{pmatrix} \mathbb{V}_i^{-1}(V-P) & 0 & -\gamma \vec{h}_i' \\ 0 & 0 & \gamma I_2 \\ -\gamma \vec{h}_i & \gamma I_2 & -\gamma^2 \mathbb{V}_i(\vec{V}|V-P) \end{pmatrix}. \tag{IA17}$$

With this notation, we can concisely write the $\mathcal{F}_{i^+}$ expected utility above as

$$\mathbb{E}_{i^+}\left[-e^{-\gamma\left(\left(\frac{\mathbb{E}_i[V-P] - \gamma \mathbb{C}_i(V,U) z_i}{\gamma \mathbb{V}_i(V)} - n\right)(V-P) + z_i U + nV\right)}\right] = -e^{-\frac{1}{2}\mathbb{E}_i[Y]' a_i \mathbb{E}_i[Y]}. \tag{IA18}$$

The random vector $\mathbb{E}_i[Y]$ is conditionally jointly normally distributed given $\mathcal{F}_p$ since the investment decision $k(s_p)$ is known given $\mathcal{F}_p$.[28]

We can now use standard formulas for expected exponential-quadratic forms of normal random vectors to compute

$$\mathbb{E}_R\left[-e^{-\frac{1}{2}\mathbb{E}_p[Y]'a_i\mathbb{E}_p[Y]}\right] \tag{IA19}$$

$$= -\left|a_i\right|^{-1/2}\left|\mathbb{V}_p(\mathbb{E}_i[Y])+a_i^{-1}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mathbb{E}_p[Y]'\left(\mathbb{V}_p(\mathbb{E}_i[Y])+a_i^{-1}\right)^{-1}\mathbb{E}_p[Y]\right\} \tag{IA20}$$

where we use the law of iterated expectations to write $\mathbb{E}_p[\mathbb{E}_i[Y]] = \mathbb{E}_p[Y]$. This expression requires that the matrix a_i is invertible. However, using standard formulas for determinants of partitioned matrices (e.g., eq. (5) in Henderson and Searle (1981)) we can compute

$$|a_i| = \left|\mathbb{V}_i^{-1}(V-P)\right|\left|-\gamma^2 I_2\right| = \gamma^4\left|\mathbb{V}_i^{-1}(V-P)\right| > 0 \tag{IA21}$$

so that a_i is invertible and using standard formulas for inverses of partitioned matrices (e.g., eq. (8) in Henderson and Searle (1981)) we have

$$a_i^{-1} = \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,\vec{V}) & 0 \\ \mathbb{C}_i\left(\vec{V},V-P\right) & \mathbb{V}_i\left(\vec{V}\right) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}. \tag{IA22}$$

It now follows from the law of total variance, noting that $\vec{Z}_i$ is $\mathcal{F}_i$-measurable and so $\mathbb{V}_i(\vec{Z}_i) = 0$, that

$$\mathbb{V}_p(\mathbb{E}_i[Y]) + a_i^{-1} = \mathbb{V}_p(\mathbb{E}_i[Y]) + \begin{pmatrix} \mathbb{V}_i(V-P) & \mathbb{C}_i(V-P,\vec{V}) & 0 \\ \mathbb{C}_i\left(\vec{V},V-P\right) & \mathbb{V}_i\left(\vec{V}\right) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{IA23}$$

$$= \mathbb{V}_p(Y) + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix} \tag{IA24}$$

$$\equiv \mathbb{V}_p(Y) + \mathcal{I} \tag{IA25}$$

where the final equality defines the 5×5 matrix $\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}$.

Putting together everything above, the conditional expected utility can be written

$$-\left|a_i\right|^{-1/2}\left|\mathbb{V}_p(\mathbb{E}_i[Y])+a_i^{-1}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mathbb{E}_p[Y]'\left(\mathbb{V}_p(\mathbb{E}_i[Y])+a_i^{-1}\right)^{-1}\mathbb{E}_p[Y]\right\} \tag{IA26}$$

[28]Note that $\mathbb{E}_i[Y]$ follows a singular normal distribution since n is a constant. That is, the conditional variance matrix of $\mathbb{E}_i[Y]$ is only positive semidefinite. However, defining the random vector in this way causes no difficulties in the derivation below and simplifies the algebra by treating the endowment of shares and the non-tradeable in a unified way.

$$= -\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\mathbb{V}_p(Y)+\mathcal{I}\right|^{-1/2}\exp\left\{-\frac{1}{2}\mathbb{E}_p(Y)'\left(\mathbb{V}_p(Y)+\mathcal{I}\right)^{-1}\mathbb{E}_p(Y)\right\}. \tag{IA27}$$

Finally applying eq. (5) of Henderson and Searle (1981) with $A = \mathbb{V}_p(V-P)$, $V = U' = \begin{pmatrix} \mathbb{C}_p(\vec{V},V-P) \\ \mathbb{C}_p(\vec{Z}_i,V-P) \end{pmatrix}$ and $D = \begin{pmatrix} \mathbb{V}_p(\vec{V}) & \mathbb{C}_p(\vec{V},\vec{Z}_i)+\frac{1}{\gamma}I_2 \\ \mathbb{C}_p(\vec{Z}_i,\vec{V})+\frac{1}{\gamma}I_2 & \mathbb{V}_p(\vec{Z}_i) \end{pmatrix}$ to compute the determinant of $|\mathbb{V}_p(Y)+\mathcal{I}|$ yields

$$\begin{aligned}
|\mathbb{V}_p(Y)+\mathcal{I}| &= |A|\left|D - VA^{-1}U\right| && \text{(IA28)}\\
&= |\mathbb{V}_p(V-P)|\left|\mathbb{V}_p((\vec{V},\vec{Z}_i)|V-P) + \begin{pmatrix} 0 & \frac{1}{\gamma}I_2 \\ \frac{1}{\gamma}I_2 & 0 \end{pmatrix}\right| && \text{(IA29)}\\
&= \frac{1}{\gamma^2}|\mathbb{V}_p(V-P)|\left|I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix}\mathbb{V}_p((\vec{V},\vec{Z}_i)|V-P)\right|. && \text{(IA30)}
\end{aligned}$$

Plugging this expression into the most recent expression for the expected utility now yields the expression in the statement of the Proposition. □

A.2 Proof of Proposition IA2

From Proposition IA1, we know that the expected utility is

$$-\left|\mathbb{V}_i(V-P)\mathbb{V}_p^{-1}(V-P)\right|^{1/2}\left|I_4+\begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix}\mathbb{V}_p((\vec{V},\vec{Z}_i)|V-P)\right|^{-1/2}\exp\left\{-\tfrac{1}{2}\mathbb{E}_p[Y]'(\mathbb{V}_p(Y)+\mathcal{I})^{-1}\mathbb{E}_p[Y]\right\}. \tag{IA31}$$

Focus first on the quadratic form in the exponential, which we will denote $Q(s_p)$. Use the law of total variance, conditioning on z_i, to decompose the matrix in the quadratic form as

$$\mathbb{V}_p(Y)+\mathcal{I} = \mathbb{V}_p(Y|z_i)+\mathcal{I}+\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y),$$

which allows us to use standard matrix inversion results (e.g., eq. (17) in Henderson and Searle (1981)) to express the matrix inverse as

$$\begin{aligned}
&(\mathbb{V}_p(Y)+\mathcal{I})^{-1}\\
&= (\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\\
&\qquad - (\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\left(\mathbb{V}_p(z_i)+\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\right)^{-1}\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\\
&= (\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\\
&\qquad - (\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\\
&\qquad\qquad \times\left(\mathbb{V}_p^{-1}(z_i)+\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\right)^{-1}\\
&\qquad\qquad\quad \times \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1},
\end{aligned}$$

so that the overall quadratic form is

$$-\frac{1}{2}\mathbb{E}_p[Y]'(\mathbb{V}_p(Y)+\mathcal{I})^{-1}\mathbb{E}_p[Y] \tag{IA32}$$

$$= -\frac{1}{2}\mathbb{E}_p[Y]' \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{E}_p[Y] \tag{IA33}$$

$$+ \frac{1}{2}\mathbb{E}_p[Y]'(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i) \tag{IA34}$$

$$\times \left(\mathbb{V}_p^{-1}(z_i)+\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\right)^{-1} \tag{IA35}$$

$$\times \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)(\mathbb{V}_p(Y|z_i)+\mathcal{I})^{-1}\mathbb{E}_p[Y]. \tag{IA36}$$

Let us first focus on the term $-\frac{1}{2}\mathbb{E}_p[Y]' \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{E}_p[Y]$. We have

$$\begin{aligned}
&\left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \\
&= \begin{pmatrix} \mathbb{V}_p(V-P|z_i) & \mathbb{C}_p(V-P,\vec{V}|z_i) & 0 \\ \mathbb{C}_p(\vec{V},V-P|z_i) & \mathbb{V}_p(\vec{V}|z_i) & \frac{1}{\gamma}I_2 \\ 0 & \frac{1}{\gamma}I_2 & 0 \end{pmatrix}^{-1} \\
&= \begin{pmatrix} \mathbb{V}_p^{-1}(V-P|z_i) & 0 & -\gamma\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i) \\ 0 & 0 & \gamma I_2 \\ -\gamma\mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i) & \gamma I_2 & -\gamma^2\mathbb{V}_p(\vec{V}|V-P,z_i) \end{pmatrix}.
\end{aligned}$$

Furthermore,

$$\mathbb{E}_p[Y] = \begin{pmatrix} \mathbb{E}_p[V-P] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}_i] \end{pmatrix} \tag{IA37}$$

$$= \begin{pmatrix} \gamma\mathbb{C}_i(V,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix} \tag{IA38}$$

$$= \begin{pmatrix} \gamma\mathbb{C}_p(V-P,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix} \tag{IA39}$$

where the final equality uses the fact that the law of total covariance implies

$$\mathbb{C}_p(V-P,\vec{V}) = \mathbb{C}_p\left(V - \mathbb{E}_i[V] + \gamma\mathbb{C}_i(V,\vec{V})\vec{Z},\vec{V}\right) \tag{IA40}$$

$$= \mathbb{C}_p\left(V - \mathbb{E}_i[V],\vec{V}\right) \tag{IA41}$$

$$= \mathbb{C}_i(V,\vec{V}). \tag{IA42}$$

It follows that the first term in eq. (IA36) is

$$-\frac{1}{2}\mathbb{E}_p[Y]' \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{E}_p[Y] \tag{IA43}$$

$$= -\frac{1}{2}\begin{pmatrix} \gamma\mathbb{C}_p(V-P,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix}' \tag{IA44}$$

$$\times \begin{pmatrix} \mathbb{V}_p^{-1}(V-P|z_i) & 0 & -\gamma\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i) \\ 0 & 0 & \gamma I_2 \\ -\gamma\mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i) & \gamma I_2 & -\gamma^2\mathbb{V}_p(\vec{V}|V-P,z_i) \end{pmatrix} \begin{pmatrix} \gamma\mathbb{C}_p(V-P,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix} \tag{IA45}$$

$$= -\frac{1}{2}\begin{pmatrix} 0 \\ \gamma\mathbb{E}_p[\vec{Z}] \\ -\gamma^2\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}]+\gamma\mathbb{E}_p[\vec{V}] \end{pmatrix}' \begin{pmatrix} \gamma\mathbb{C}_p(V-P,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix} \tag{IA46}$$

$$= -\gamma\mathbb{E}_p[\vec{Z}]'\mathbb{E}[\vec{V}] + \tfrac{1}{2}\gamma^2\mathbb{E}_p[\vec{Z}]'\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}] \tag{IA47}$$

where the second equality uses the relation $\mathbb{V}_p(\vec{V}|V-P,z_i) = \mathbb{V}_p(\vec{V}|z_i) - \mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i)$ to simplify the third element of the first vector.

Now consider the second term in eq. (IA36). We have that

$$\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i) = \begin{pmatrix} \mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{V},z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i) \end{pmatrix} \tag{IA48}$$

is a vector of conditional regression coefficients.

It follows that

$$\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\left(\mathbb{V}_p(Y|z_i)+\mathcal{I}\right)^{-1}\mathbb{E}_p[Y] \tag{IA49}$$

$$= \begin{pmatrix} \mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{V},z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i) \end{pmatrix}' \begin{pmatrix} \mathbb{V}_p^{-1}(V-P|z_i) & 0 & -\gamma\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i) \\ 0 & 0 & \gamma I_2 \\ -\gamma\mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i) & \gamma I_2 & -\gamma^2\mathbb{V}_p(\vec{V}|V-P,z_i) \end{pmatrix} \begin{pmatrix} \gamma\mathbb{C}_p(V-P,\vec{V})\mathbb{E}_p[\vec{Z}] \\ \mathbb{E}_p[\vec{V}] \\ \mathbb{E}_p[\vec{Z}] \end{pmatrix} \tag{IA50}$$

$$= \begin{pmatrix} \mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{V},z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i) \end{pmatrix}' \begin{pmatrix} 0 \\ \gamma\mathbb{E}_p[\vec{Z}] \\ -\gamma^2\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}]+\gamma\mathbb{E}_p[\vec{V}] \end{pmatrix} \tag{IA51}$$

$$= \gamma\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,\vec{V})\mathbb{E}_p[\vec{Z}] + \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,\vec{Z}_i)\left(\gamma\mathbb{E}_p[\vec{V}] - \gamma^2\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}]\right) \tag{IA52}$$

$$= \gamma\begin{pmatrix} \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,k\theta) \\ 0 \end{pmatrix}'\mathbb{E}_p\left[\vec{Z}_i\right] + \begin{pmatrix} 0 \\ 1 \end{pmatrix}'\left(\gamma\mathbb{E}_p\left[\vec{V}\right] - \gamma^2\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}]\right) \tag{IA53}$$

$$= \gamma\frac{k\mathbb{C}_p(Z,\theta)}{\mathbb{V}_p(z_i)}n + \gamma\mathbb{E}_p[U] - \gamma^2\mathbb{C}_p(U,V)n - \gamma^2\mathbb{V}_p(U)\mathbb{E}_p[z_i] \tag{IA54}$$

$$= \gamma\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}n + \gamma^2 k\alpha\tfrac{1}{\tau_\eta}n - \gamma^2\tfrac{1}{\tau_\eta}\mathbb{E}_p[Z] \tag{IA55}$$

where the next-to-last line performs the matrix multiplication explicitly, and the final line simplifies using $\mathbb{C}_p(Z,\theta) = \beta\mathbb{C}_p(\theta+\frac{1}{\beta}Z-\theta,\theta) = \beta\mathbb{C}_p(s_p-\theta,\theta) = -\beta\mathbb{V}_p(\theta)$, and similarly for $\mathbb{V}_p(z_i)$, and uses $\mathbb{E}_p[U] = \mathbb{E}_p[-\eta_C] = 0$.

To complete the simplification of the second term in eq. (IA36), we need to compute

$$\left(\mathbb{V}_p^{-1}(z_i) + \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\left(\mathbb{V}_p(Y|z_i)+\mathcal{I}\right)^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\right)^{-1}. \tag{IA56}$$

We have

$$
\begin{aligned}
&\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, Y)\left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1}\mathbb{C}_p(Y, z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA57)} \\
&= \begin{pmatrix} \mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{V},z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i) \end{pmatrix}' \begin{pmatrix} \mathbb{V}_p^{-1}(V-P|z_i) & 0 & -\gamma\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i) \\ 0 & 0 & \gamma I_2 \\ -\gamma\mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i) & \gamma I_2 & -\gamma^2\mathbb{V}_p(\vec{V}|V-P,z_i) \end{pmatrix} \begin{pmatrix} \mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{V},z_i)\mathbb{V}_p^{-1}(z_i) \\ \mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i) \end{pmatrix} && \text{(IA58)} \\
&= \left(\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,V-P)\mathbb{V}_p^{-1}(V-P|z_i) - \gamma\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,\vec{Z}_i)\mathbb{C}_p(\vec{V},V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\right)\mathbb{C}_p(V-P,z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA59)} \\
&\quad + 2\gamma\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, \vec{Z}_i)\mathbb{C}_p(\vec{V}, z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA60)} \\
&\quad + \left(-\gamma\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,\vec{V}|z_i) - \gamma^2\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,\vec{Z}_i)\mathbb{V}_p(\vec{V}|V-P,z_i)\right)\mathbb{C}_p(\vec{Z}_i,z_i)\mathbb{V}_p^{-1}(z_i). && \text{(IA61)}
\end{aligned}
$$

Note that

$$
\begin{aligned}
\mathbb{C}_p(V-P, z_i) &= \mathbb{C}_p(V - \mathbb{E}_i[V] + \gamma\mathbb{C}_i(V, \vec{V})\vec{Z}, z_i) && \text{(IA62)} \\
&= \gamma\mathbb{C}_i(V, \vec{V})\mathbb{C}_p(\vec{Z}, z_i) && \text{(IA63)} \\
&= \gamma\mathbb{C}_p(V-P, \vec{V})\mathbb{C}_p(\vec{Z}, z_i) && \text{(IA64)}
\end{aligned}
$$

and furthermore, $\mathbb{C}_p(\vec{Z}_i, z_i) = (0, \mathbb{V}_p(z_i))$ and $\mathbb{C}_p(\vec{V}, z_i) = (\mathbb{C}_p(V, z_i), 0)$, so that the term $\mathbb{C}_p(z_i, \vec{Z}_i)\mathbb{C}_p(\vec{V}, z_i) = 0$.

It follows that the previous displayed equation can be written, after grouping terms, as

$$
\begin{aligned}
&\gamma^2\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, \vec{Z})\mathbb{C}_p(\vec{V}, V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, \vec{V})\mathbb{C}_p(\vec{Z}, z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA65)} \\
&- \gamma^2\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, \vec{Z}_i)\mathbb{C}_p(\vec{V}, V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, \vec{V})\mathbb{C}_p(\vec{Z}, z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA66)} \\
&- \gamma^2\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, \vec{Z})\mathbb{C}_p(\vec{V}, V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, \vec{V})\mathbb{C}_p(\vec{Z}_i, z_i)\mathbb{V}_p^{-1}(z_i) && \text{(IA67)} \\
&- \gamma^2\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, \vec{Z}_i)\mathbb{V}_p(\vec{V}|V-P, z_i)\mathbb{C}_p(\vec{Z}_i, z_i)\mathbb{V}_p^{-1}(z_i). && \text{(IA68)}
\end{aligned}
$$

Furthermore, $\mathbb{C}(\vec{Z}_i, z_i) = \left(\begin{smallmatrix} 0 \\ \mathbb{V}_p(z_i) \end{smallmatrix}\right)$ and $\mathbb{C}(\vec{Z}, z_i) = \left(\begin{smallmatrix} 0 \\ \mathbb{V}_p(Z) \end{smallmatrix}\right)$ which further simplifies the previous expression to

$$
\begin{aligned}
&\gamma^2\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)}\mathbb{C}_p(U, V-P)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, U)\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)} && \text{(IA69)} \\
&- \gamma^2\mathbb{C}_p(U, V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, U)\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)} && \text{(IA70)} \\
&- \gamma^2\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)}\mathbb{C}_p(U, V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, U) && \text{(IA71)} \\
&- \gamma^2\mathbb{V}_p(U|V-P, z_i). && \text{(IA72)}
\end{aligned}
$$

We can write

$$
\mathbb{V}_p(U|V-P, z_i) = \mathbb{V}_p(U|z_i) - \mathbb{C}_p(U, V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P, U|z_i), \quad \text{(IA73)}
$$

which after plugging in and simplifying yields

$$-\gamma^2\mathbb{V}_p(U|z_i)+\gamma^2\left(1-\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)}\right)\mathbb{C}_p(U,V-P|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,U|z_i)\left(1-\tfrac{\mathbb{V}_p(Z)}{\mathbb{V}_p(z_i)}\right) \tag{IA74}$$

$$=-\gamma^2\mathbb{V}(U)+\gamma^2\tfrac{\mathbb{V}_p(\zeta_i)}{\mathbb{V}_p(z_i)}\mathbb{C}_p(V-P,U|z_i)\mathbb{V}_p^{-1}(V-P|z_i)\mathbb{C}_p(V-P,U|z_i)\tfrac{\mathbb{V}_p(\zeta_i)}{\mathbb{V}_p(z_i)} \tag{IA75}$$

$$=-\gamma^2\tfrac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}. \tag{IA76}$$

Putting everything together above, we have established that the quadratic form in the exponential is

$$Q(k;s_p)=-\gamma\mathbb{E}_p[\vec{Z}]'\mathbb{E}[\vec{V}]+\tfrac{1}{2}\gamma^2\mathbb{E}_p[\vec{Z}]'\mathbb{V}_p(\vec{V}|z_i)\mathbb{E}_p[\vec{Z}] \tag{IA77}$$

$$-\left(\gamma\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}n+\gamma^2k\alpha\tfrac{1}{\tau_\eta}n-\gamma^2\tfrac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \tag{IA78}$$

$$\left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}-\gamma^2\tfrac{1}{\tau_\eta}+\gamma^2\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)}\frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1} \tag{IA79}$$

$$\left(\gamma\frac{-k\beta\mathbb{V}_p(\theta)}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}n+\gamma^2k\alpha\tfrac{1}{\tau_\eta}n-\gamma^2\tfrac{1}{\tau_\eta}\mathbb{E}_p[Z]\right) \tag{IA80}$$

where the inverse term is guaranteed to be finite for $k\in\{0,1\}$ under the maintained parameter restriction $\left(\frac{1}{\tau_Z}+\frac{1}{\tau_\zeta}\right)^{-1}-\gamma^2\frac{1}{\tau_\eta}>0$ After performing the matrix multiplication in the first line and factoring γ^2 out of the final term, this yields the expression for Q in the Corollary.

Now, consider the determinant term in the expected utility. From eq. (IA27) in the proof of Proposition IA1, we can write the determinant term as

$$-\frac{1}{\gamma^2}\left|\mathbb{V}_i^{-1}(V-P)\right|^{-1/2}\left|\mathbb{V}_p(Y)+\mathcal{I}\right|^{-1/2}. \tag{IA81}$$

Note that

$$|\mathbb{V}_p(Y)+\mathcal{I}| \tag{IA82}$$

$$=\left|\mathbb{V}_p(Y|z_i)+\mathcal{I}+\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\right| \tag{IA83}$$

$$=|\mathbb{V}_p(Y|z_i)+\mathcal{I}|\left|I+\left(\mathbb{V}_p(Y|z_i)+\mathcal{I}\right)^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\right| \tag{IA84}$$

$$=|\mathbb{V}_p(Y|z_i)+\mathcal{I}|\left|I+\left(\mathbb{V}_p(Y|z_i)+\mathcal{I}\right)^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{V}_p(z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\right| \tag{IA85}$$

$$=|\mathbb{V}_p(Y|z_i)+\mathcal{I}|\left|I+\mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i,Y)\left(\mathbb{V}_p(Y|z_i)+\mathcal{I}\right)^{-1}\mathbb{C}_p(Y,z_i)\mathbb{V}_p^{-1}(z_i)\mathbb{V}_p(z_i)\right| \tag{IA86}$$

$$= |\mathbb{V}_p(Y|z_i) + \mathcal{I}|\, |\mathbb{V}_p(z_i)| \left|\mathbb{V}_p^{-1}(z_i) + \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, Y) \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{C}_p(Y, z_i)\mathbb{V}_p^{-1}(z_i)\right| \tag{IA87}$$

$$= \tfrac{1}{\gamma^4} |\mathbb{V}_p(V - P|z_i)|\, |\mathbb{V}_p(z_i)| \left|\mathbb{V}_p^{-1}(z_i) + \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, Y) \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{C}_p(Y, z_i)\mathbb{V}_p^{-1}(z_i)\right| \tag{IA88}$$

where the first equality follows from the law of total variance, the second equality uses the multiplicative property of the determinant, the third equality multiplies and divides by $\mathbb{V}_p(z_i)$ in the second determinant, the fourth equality uses the Matrix Determinant Lemma (e.g., eq. (6) in Henderson and Searle (1981)), and the fourth line computes $|\mathbb{V}_p(Y|z_i) + \mathcal{I}|$.

Since we established

$$\mathbb{V}_p^{-1}(z_i) + \mathbb{V}_p^{-1}(z_i)\mathbb{C}_p(z_i, Y) \left(\mathbb{V}_p(Y|z_i) + \mathcal{I}\right)^{-1} \mathbb{C}_p(Y, z_i)\mathbb{V}_p^{-1}(z_i) \tag{IA89}$$

$$= \frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)} \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} \tag{IA90}$$

as part of deriving the expression for $Q(k; s_p)$ above, we can now plug back into eq. (IA81) to yield the overall determinant term

$$-D(k; s_p) = -\sqrt{\frac{\mathbb{V}_i(V-P)}{\mathbb{V}_p(V-P|z_i)}}\sqrt{\frac{1}{\mathbb{V}_p(z_i)}} \tag{IA91}$$

$$\times \left(\frac{1}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} - \gamma^2\frac{1}{\tau_\eta} + \gamma^2 \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}} \frac{k^2\alpha^2\left(\frac{1}{\tau_\eta}\right)^2}{\frac{1}{\tau_A}+k^2\left(\frac{1}{\tau_\theta+\beta^2(\tau_Z+\tau_\zeta)}+\frac{1}{\tau_\eta}\right)} \frac{\frac{1}{\tau_\zeta}}{\beta^2\mathbb{V}_p(\theta)+\frac{1}{\tau_\zeta}}\right)^{-1/2}, \tag{IA92}$$

which matches the expression in the statement of the Corollary after substituting the explicit expressions for the conditional variances in the first two square root terms. □

A.3 Proof of Proposition IA3

Using the law of iterated expectations, the unconditional expected utility can be represented as the unconditional expectation as the s_p-conditional expected utility from Proposition IA1, evaluated at the equilibrium investment rule $k(s_p) = \mathbf{1}_{\{s_p > \bar{s}\}}$:

$$\mathcal{W} = \mathbb{E}\left[\mathcal{W}(s_p)\right] \tag{IA93}$$

$$= \mathbb{P}(s_p > \bar{s})\mathbb{E}[\mathcal{W}(s_p)|s_p > \bar{s}] + \mathbb{P}(s_p \le \bar{s})\mathbb{E}[\mathcal{W}(s_p)|s_p \le \bar{s}]. \tag{IA94}$$

From Proposition IA1, we have that the conditional expected utilities are of the form

$$\mathcal{W}(s_p) = -\left|\mathbb{V}_i(V-P)\mathbb{V}_p^{-1}(V-P)\right|^{1/2} \left|I_4 + \begin{pmatrix} 0 & \gamma I_2 \\ \gamma I_2 & 0 \end{pmatrix} \mathbb{V}_p((\vec{V}, \vec{Z}_i)|V-P)\right|^{-1/2} \tag{IA95}$$

$$\times \exp\left\{-\tfrac{1}{2}\mathbb{E}_p[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}_p[Y(k)]\right\} \tag{IA96}$$

where the asset payoff $V = V(k)$ and price function $P = P(k)$ are those associated with the a particular investment decision $k \in \{0, 1\}$, and where the vector

$$Y(k) = \begin{pmatrix} V(k) - P \\ \vec{V} \\ \vec{Z}_i \end{pmatrix}, \tag{IA97}$$

with $\vec{V} = \begin{pmatrix} V(k) \\ -\eta_C \end{pmatrix}$ and $\vec{Z}_i = \begin{pmatrix} n \\ z_i \end{pmatrix}$ the sub-vectors of the tradeable and non-tradeable payoffs and the endowments, respectively.

To evaluate the expected utility, eq. (IA94), it is straightforward to calculate the probabilities of the two regions. It remains to to calculate the conditional expectation of $\mathcal{W}(s_p)$ given $s_p > \bar{s}$ and $s_p \leq \bar{s}$. Given that the determinant terms in eq. (IA96) are constant within each region, this reduces to computing the expectation of the exponential term.

To proceed, note that using standard normal-normal updating we can represent

$$\mathbb{E}_p[Y(k)] = \mathbb{E}[Y(k)] + \mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]). \tag{IA98}$$

Hence, the expression in the exponential in eq. (IA96) can be written as

$$-\frac{1}{2}\mathbb{E}_p[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}_p[Y(k)] \tag{IA99}$$

$$= -\frac{1}{2}\mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA100}$$

$$- \mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]) \tag{IA101}$$

$$- \tfrac{1}{2}(s_p - \mathbb{E}[s_p])\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p)(s_p - \mathbb{E}[s_p]), \tag{IA102}$$

which is in the quadratic form $d + a'X + X'AX$ with $X = s_p - \mathbb{E}[s_p]$ and

$$d = -\frac{1}{2}\mathbb{E}[Y(k)]'(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA103}$$

$$a = -\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{E}[Y(k)] \tag{IA104}$$

$$A = -\tfrac{1}{2}\mathbb{V}^{-1}(s_p)\mathbb{C}(s_p, Y(k))(\mathbb{V}_p(Y(k)) + \mathcal{I})^{-1}\mathbb{C}(Y(k), s_p)\mathbb{V}^{-1}(s_p). \tag{IA105}$$

We can now compute the two conditional expectations, corresponding to the investment and no-investment regions, using Lemma IA1 from Appendix A.4, which provides a closed-form expression for the expected exponential-quadratic of a truncated normally distributed random variable.

A large amount of tedious algebra, analogous to that in the proof of the conditional welfare expression in Proposition IA2, then delivers the expression in the Proposition. From inspection of the determinant term in the expression, the maintained parameter restriction $\left(\frac{1}{\tau_Z} + \frac{1}{\tau_\zeta}\right)^{-1} - \gamma^2 \frac{1}{\tau_\eta} > 0$ further further ensures that the expected utility is finite. $\square$

A.4 Exponential-quadratic form of truncated normal random vector

The following result computes the unconditional expectation of an exponential-quadratic form of a truncated normal random variable, which is used to characterize the unconditional welfare under the equilibrium investment rule.

Lemma IA1. *Suppose $X \in \mathbb{R}^n$ is distributed $N(\mu, \Sigma)$ with positive definite variance matrix Σ. Consider the quadratic form $d + a'X + X'AX$, for conformable d, a and A. Let $\mathcal{C} \subseteq \mathbb{R}^n$ be an arbitrary set.*

Suppose that $I - 2\Sigma A$ is positive definite and define the composite parameters

$$\hat{\mu} = (I - 2\Sigma A)^{-1}(\mu + \Sigma a) \tag{IA106}$$

$$\hat{\Sigma} = (\Sigma^{-1} - 2A)^{-1} = (I - 2\Sigma A)^{-1}\Sigma \tag{IA107}$$

$$\widehat{\mathcal{C}} = \{B\,(x - \hat{\mu}) : x \in \mathcal{C}\}, \tag{IA108}$$

where B is the invertible, symmetric $n \times n$ matrix square root that factorizes the positive definite $\hat{\Sigma}^{-1}$ as $BB = \hat{\Sigma}^{-1}$.

We have

$$\mathbb{E}\left[\exp\{d + a'X + X'AX\}\middle| X \in \mathcal{C}\right] \tag{IA109}$$

$$= \frac{\int_{\widehat{\mathcal{C}}}\phi(y)dy}{\int_{\mathcal{C}}\phi(y)dy}\frac{1}{|I-2\Sigma A|^{1/2}}\exp\left\{d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}(\mu+\Sigma a)'\Sigma^{-1}(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right\}. \tag{IA110}$$

A.5 Proof of Lemma IA1

Writing the expectation explicitly as an integral, we have

$$\mathbb{E}\left[\exp\{d + a'X + X'AX\}\middle| X \in \mathcal{C}\right] \tag{IA111}$$

$$= \int_{\mathbb{R}^n} \exp\{d + a'x + x'Ax\}\,\mathbb{P}(X \in dx | X \in \mathcal{C}) \tag{IA112}$$

$$= \int_{\mathcal{C}} \exp\{d + a'x + x'Ax\}\,\frac{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left\{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\right\}}{\mathbb{P}(X \in \mathcal{C})}dx. \tag{IA113}$$

By completing the square, we can group the terms in the exponentials as

$$d + a'x + x'Ax - \tfrac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu) \tag{IA114}$$

$$= d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}\left[(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right]'(\Sigma^{-1}-2A)\left[(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right] \tag{IA115}$$

$$-\tfrac{1}{2}\left(x - (\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x - (\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right) \tag{IA116}$$

$$= d - \tfrac{1}{2}\mu'\Sigma^{-1}\mu + \tfrac{1}{2}(\mu+\Sigma a)'\Sigma^{-1}(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a) \tag{IA117}$$

$$-\tfrac{1}{2}\left(x - (I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x - (I-2\Sigma A)^{-1}(\mu+\Sigma a)\right). \tag{IA118}$$

So, plugging back in to eq. (IA113) yields

$$= \int_{\mathcal{C}} \exp\{d + a'x + x'Ax\} \frac{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\right\}}{\mathbb{P}(X \in \mathcal{C})} dx \tag{IA119}$$

$$= \exp\left\{d - \frac{1}{2}\mu'\Sigma^{-1}\mu + \frac{1}{2}(\mu+\Sigma a)'\Sigma^{-1}(\Sigma^{-1}-2A)^{-1}\Sigma^{-1}(\mu+\Sigma a)\right\} \tag{IA120}$$

$$\times \frac{1}{\mathbb{P}(X\in\mathcal{C})} \int_{\mathcal{C}} \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)'(\Sigma^{-1}-2A)\left(x-(I-2\Sigma A)^{-1}(\mu+\Sigma a)\right)\right\} .dx \tag{IA121}$$

Letting

$$\hat{\mu} = (I - 2\Sigma A)^{-1}(\mu + \Sigma a) \tag{IA122}$$

$$\hat{\Sigma} = (\Sigma^{-1} - 2A)^{-1} = (I - 2\Sigma A)^{-1}\Sigma \tag{IA123}$$

$$\hat{\mathcal{C}} = \left\{B\left(x - (I - 2\Sigma A)^{-1}(\mu + \Sigma a)\right) : x \in \mathcal{C}\right\} \tag{IA124}$$

where B is the invertible, symmetric $n \times n$ matrix square root that factorizes the positive definite matrix $\hat{\Sigma}^{-1}$ as $\hat{\Sigma}^{-1} = BB$, we can further express the integral in eq. (IA121) as

$$\int_{\mathcal{C}} \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(x-\hat{\mu})'\hat{\Sigma}^{-1}(x-\hat{\mu})\right\} dx \tag{IA125}$$

$$= \int_{\hat{\mathcal{C}}} \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \frac{1}{|B|} \exp\left\{-\frac{1}{2}y'y\right\} dy \tag{IA126}$$

$$= \frac{1}{|I-2\Sigma A|^{1/2}} \int_{\hat{\mathcal{C}}} \phi(y) dy \tag{IA127}$$

where the first equality changes variables $y = B^{-1}\left(x - (I - 2\Sigma A)^{-1}(\mu + \Sigma a)\right)$ and the final line uses $|B| = |BB|^{1/2} = \left|\hat{\Sigma}^{-1}\right|^{1/2} = \left|\Sigma^{-1}(I - 2\Sigma A)\right|^{1/2}$ and simplifies notation using the n-dimensional standard normal cdf $\phi(x) = \frac{1}{(2\pi)^{n/2}} \exp\left\{-\frac{1}{2}y'y\right\}$. Plugging this expression for the integral back into eq. (IA121) delivers the expression in the Lemma. □

B Value-maximizing manager

In this section we consider a setting in which the manager seeks to maximize the expected terminal value $V(k)$ rather than the share price P_3. Formally, we continue to follow the same model setup as in the main text, with one change. We assume now that, when making her investment decision at $t = 2$, the manager chooses the investment $k \in \{0, 1\}$, conditional on $\mathcal{F}_m = \sigma(P_1)$, to maximize the expected terminal value of the firm,

$$\max_{k\in\{0,1\}} \mathbb{E}[V(k)|\mathcal{F}_m].$$

Because the manager cares only about the terminal value and not the share price, for simplicity we assume that there is only a single round of trade, which takes place prior to the real investment decision (i.e., there is not an additional round of trade between the real investment decision and the terminal date). Incorporating a second round of trade does not affect any results below, and in equilibrium we would necessarily have $P_1 = P_3$, as in the main model. Because there is only one trading round, to eliminate notational clutter we suppress the time subscript on the price function and write P instead of P_1 below.

The key difference between price and value maximization is that the risk premium attached to the project by the market is no longer relevant to the manager when deciding whether to take it. This leads to two changes in the economic forces that drove our results, which in turn cause value maximization to lead to significantly different empirical and welfare implications than in our baseline model. First, project greenness no longer directly affects the project's desirability by reducing the expected discount rate. Second, project greenness still influences the probability of investment by altering the volatility of prices, but this effect operates in the opposite direction as in the case where the manager maximizes price. The reason is that, while the price variation created by climate exposure causes the project's NPV to be more volatile, it causes the manager's expectation of its cash flows to be *less* volatile. This is because the variation in the price that investors' concerns for the climate create cause price to a noisier signal of project cash flows.

B.1 Equilibrium derivation

As in the main model in the body of the paper, we search for a threshold equilibrium in which the asset price at date 1 depends on the underlying random variables through a linear statistic of the form $s_p = \theta + \frac{1}{\beta} Z$, (ii) the manager invests in the project if and only if $s_p > \overline{s}_V$ for endogenous threshold $\overline{s}_V$, and the asset price takes the piecewise linear form

$$P = \begin{cases} A_1 + B_1 s_p & s_p > \overline{s}_V \\ A_0 & s_p \leq \overline{s}_V \end{cases}.$$

We proceed by working backwards. Again, at $t = 2$, the manager's problem is to solve

$$\max_{k \in \{0,1\}} \mathbb{E}[V(k)|\mathcal{F}_m] \tag{IA128}$$

where she can condition on the first period price, $\mathcal{F}_m = \sigma(P)$. The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[\theta|\mathcal{F}_m] > c \\ 0 & \mathbb{E}[\theta|\mathcal{F}_m] \leq c. \end{cases} \tag{IA129}$$

Given the conjectured price function, if the manager observes $P = A_0$, she infers that, with probability 1, $s_p \leq \overline{s}_V$, while if she observes any $P > A_0$, she infers the realized value of s_p, necessarily strictly greater than $\overline{s}_V$. Hence, a threshold value $\overline{s}_V$ such that $\mathbb{E}[\theta|s_p = \overline{s}_V] = c$ is consistent with the initial conjecture. Since $\mathbb{E}[\theta|s_p] = \mu_\theta + \frac{\tau_p}{\tau_\theta + \tau_p}\left(s_p - \mu_\theta - \frac{1}{\beta}\mu_Z\right)$, with $\tau_p \equiv \beta^2 \tau_Z$, substituting into this optimal investment rule and grouping terms implies that the manager optimally invests if and only if

$$
\begin{aligned}
s_p &> \mu_\theta + \frac{1}{\beta}\mu_Z - \frac{\tau_\theta + \tau_p}{\tau_p}(\mu_\theta - c) \\
&= c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z,
\end{aligned}
$$

where the second line substitutes in the equilibrium value $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ derived above and rearranges. This yields the equilibrium investment threshold is $\overline{s}_V = c - \frac{\tau_\theta(\mu_\theta - c)}{\tau_p} + \frac{\gamma}{\tau_\eta}\alpha\mu_Z$.

At date $t = 1$, traders can condition on P and anticipate that the manager's investment decision $k = k(P)$ depends on P as derived above. Hence, they perceive the asset payoff as conditionally normally distributed with moments

$$\mathbb{E}_{i1}[V(k)] = \mathbb{E}_{i1}[A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c)] = \mu_A + k(\theta - c) \tag{IA130}$$

$$\mathbb{C}_{i1}(V(k), \eta_C) = k\alpha\tfrac{1}{\tau_\eta} \tag{IA131}$$

$$\mathbb{V}_{i1}(V(k)) = \tfrac{1}{\tau_A} + k^2\tfrac{1}{\tau_\eta}. \tag{IA132}$$

The problem of an arbitrary trader is a static one,

$$\max_{x\in\mathbb{R}} \mathbb{E}_i[-e^{-\gamma W_i}] \tag{IA133}$$

where the terminal wealth is

$$W_i = (n + x)V - xP - z_i\eta_C. \tag{IA134}$$

It is immediate that this problem leads to a standard mean-variance demand function

$$X_i = \frac{\mathbb{E}_{i1}[V(k)] + \gamma\mathbb{C}_{i1}(V(k), \eta_C)z_i - P}{\gamma\mathbb{V}_{i1}(V(k))} - n. \tag{IA135}$$

Plugging in for the conditional moments from above and enforcing market clearing yields equilibrium price

$$P = \mu_A + k(\theta - c) + \gamma k\alpha\tfrac{1}{\tau_\eta}Z - \gamma\left(\tfrac{1}{\tau_A} + k^2\tfrac{1}{\tau_\eta}\right)n \tag{IA136}$$

$$= \mu_A - \gamma\tfrac{1}{\tau_A}n + k\left(\theta + \gamma\alpha\tfrac{1}{\tau_\eta}Z - c - \gamma\tfrac{1}{\tau_\eta}n\right) \tag{IA137}$$

where the second line collects terms and uses the fact that $k \in \{0, 1\}$ implies $k = k^2$ to simplify.

Returning to the initial conjecture, to be consistent with this expression the endogenous signal s_p must have coefficient $\frac{1}{\beta} = \frac{\gamma\alpha}{\tau_\eta}$ on Z and the price function coefficients must satisfy

$$A_0 = \mu_A - \gamma\tfrac{1}{\tau_A}n \tag{IA138}$$

$$A_1 = \mu_A - \gamma\tfrac{1}{\tau_A}n - c - \gamma\tfrac{1}{\tau_\eta}n \tag{IA139}$$

$$B_1 = 1. \tag{IA140}$$

This completes the construction of equilibrium.

The following proposition collects the above results and formalizes the existence and uniqueness of a threshold equilibrium.

Proposition IA1. *There exists a unique threshold equilibrium in which the equilibrium price is*

$$P = \mu_A - \frac{\gamma n}{\tau_A} + k\left(s_p - c - \frac{\gamma}{\tau_\eta} n\right), \tag{IA141}$$

and the manager's investment decision is

$$k = \mathbf{1}\left\{s_p > \bar{s}_V\right\}, \tag{IA142}$$

where $s_p \equiv \theta + \frac{\gamma\alpha}{\tau_\eta} Z$, $\tau_p \equiv \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$, *and* $\bar{s}_V \equiv \mu_\theta + \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \tau_\theta\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_p}\right)(\mu_\theta - c)$.

B.2 Implications

This section derives the analogous versions of the implications in the body of the paper in the value maximization setting.

B.2.1 Probability of investment

Proposition IA2. *In equilibrium, the unconditional probability of investment is given by*

$$\Pr\left(s_p > \bar{s}_V\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_V}{\sqrt{\mathbb{V}\left[s_p\right]}}\right), \tag{IA143}$$

where $\mathbb{E}\left[s_p\right] = \mu_\theta + \frac{1}{\beta}\mu_Z$, $\mathbb{V}\left[s_p\right] = \frac{1}{\tau_\theta} + \frac{1}{\tau_p}$, *and* $\tau_p \equiv \left(\frac{\tau_\eta}{\gamma\alpha}\right)^2 \tau_Z$. *The probability of investment:*
(i) increases with $\mu_\theta - c$*;*
(ii) does not depend on n *or* μ_Z*;*
(iii) increases (decreases) with τ_θ *and* $|\alpha|$ *and decreases (increases) with* τ_Z *if and only if* $\mu_\theta - c > 0$ $(\mu_\theta - c < 0)$.

Proof. The probability of investment is given by

$$\Pr\left(s_p > \bar{s}_V\right) = 1 - \Phi\left(\frac{\bar{s}_V - \mathbb{E}[s_p]}{\mathbb{V}[s_p]}\right) = \Phi\left(\frac{\mathbb{E}\left[s_p\right] - \bar{s}_V}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) \tag{IA144}$$

$$= \Phi\left(\frac{\tau_\theta\left(\frac{1}{\tau_\theta} + \frac{1}{\tau_p}\right)(\mu_\theta - c)}{\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}}\right) \tag{IA145}$$

$$= \Phi\left(\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\,(\mu_\theta - c)\right). \tag{IA146}$$

Direct inspection now immediately yields the claimed results. $\square$

Corollary IA1. *If $\mu_\theta - c > 0$, the firm is more likely to invest in a green or brown project than a neutral project, i.e.,* $\Pr(s_p > \bar{s}_V; \alpha = 1) = \Pr(s_p > \bar{s}_V; \alpha = -1) > \Pr(s_p > \bar{s}_V; \alpha = 0)$. *If $\mu_\theta - c < 0$, the firm is less likely to invest in a green or brown project than a neutral project, i.e.,* $\Pr(s_p > \bar{s}_V; \alpha = 1) = \Pr(s_p > \bar{s}_V; \alpha = -1) < \Pr(s_p > \bar{s}_V; \alpha = 0)$.

B.2.2 Expected return conditional on investment

Proposition IA3. *In equilibrium, the expected return conditional on no investment is*

$$\mathbb{E}[V - P|k = 0] = \frac{\gamma n}{\tau_A}, \tag{IA147}$$

and the expected return conditional on investment is given by

$$\mathbb{E}[V - P|k = 1] = \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mathbb{E}[Z|s_p > \bar{s}_V]. \tag{IA148}$$

Conditional on investment, the expected return:
(i) increases in $\mu_\theta - c$, and n,
(ii) decreases in μ_Z for $\alpha > 0$ and increases in μ_Z for $\alpha < 0$, and,
(iii) may either increase or decrease in α, even when $\alpha > 0 and \mu_Z > 0$.

Proof. Using the expression for the equilibrium asset price in eq. (IA141), the expression for the conditional expected return given no investment is straightforward. The expression in the case of investment follows from using the asset price in eq. (IA141) to write:

$$\begin{aligned}
\mathbb{E}[V - P|k = 1] &= \mathbb{E}[V - P|s_p > \bar{s}_V] && \text{(IA149)}\\
&= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta} Z \middle| s_p > \bar{s}_V\right] && \text{(IA150)}\\
&= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta} Z \middle| s_p\right] \middle| s_p > \bar{s}_V\right] && \text{(IA151)}\\
&= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[\frac{\gamma\alpha}{\tau_\eta}\mu_Z + \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(s_p - \mathbb{E}[s_p]\right) \middle| s_p > \bar{s}_V\right] && \text{(IA152)}\\
&= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{\frac{1}{\tau_p}}{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\sqrt{\mathbb{V}[s_p]} H\left(\frac{\bar{s}_V - \mathbb{E}[s_p]}{\sqrt{\mathbb{V}[s_p]}}\right) && \text{(IA153)}\\
&= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma\alpha}{\tau_\eta}\mu_Z - \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}} H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right). && \text{(IA154)}
\end{aligned}$$

This immediately implies that $\mathbb{E}[V - P|k = 1]$ is increasing in $\mu_\theta - c$ and n, and is decreasing in μ_Z for $\alpha > 0$ and increasing in μ_Z for $\alpha < 0$. Now, considering α, we have

$$\frac{\partial}{\partial\alpha}\mathbb{E}[V - P|k = 1]$$

$$
\begin{aligned}
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial \alpha}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right) H\left(\cdot\right) - \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\frac{\partial}{\partial \alpha} H\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\left(\mu_\theta - c\right)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial \tau_p}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right)\left(\frac{\partial}{\partial \alpha}\tau_p\right) H\left(\cdot\right) \\
&\quad + \frac{\tau_\theta\left(\mu_\theta - c\right)\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial \tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\right)\left(\frac{\partial}{\partial \alpha}\tau_p\right) H'\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\left(\mu_\theta - c\right)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial \alpha}\tau_p\right)\left(\left(\frac{\partial}{\partial \tau_p}\frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right) H\left(\cdot\right) - \frac{\tau_\theta\left(\mu_\theta - c\right)\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial \tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\right) H'\left(\cdot\right)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \left(\frac{\partial}{\partial \alpha}\tau_p\right)\left(\left(-\frac{1}{\tau_p}\frac{\frac{\partial}{\partial \tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}} - \frac{1}{\tau_p^2}\frac{1}{\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}}\right) H(\cdot) + \frac{-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}b\left(\mu_\theta - c\right)}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(\frac{\partial}{\partial \tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\right) H'(\cdot)\right) \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z + \left(\frac{\partial}{\partial \alpha}\tau_p\right)\frac{1}{\tau_p^2}\frac{1}{\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}} H\left(\cdot\right) \\
&\quad + \left(\frac{\partial}{\partial \alpha}\tau_p\right)\left(\frac{\partial}{\partial \tau_p}\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}\right)\frac{1}{\tau_p\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_p}\right)}\left(H\left(\cdot\right) - \left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\left(\mu_\theta - c\right)\right) H'\left(\cdot\right)\right).
\end{aligned}
$$

The first term is clearly negative for $\mu_Z > 0$. For $\alpha > 0$, the second term is negative, but the final term is positive. □

Corollary IA2. *Suppose $\mu_Z > 0$. Conditional on investment, a green firm always has a lower expected return than a neutral firm i.e.,*

$$\mathbb{E}\left[V - P|k = 1; \alpha = 1\right] \leq \mathbb{E}\left[V - P|k = 1; \alpha = 0\right].$$

Moreover, a neutral firm has a higher expected return than a brown firm i.e.,

$$\mathbb{E}\left[V - P|k = 1; \alpha = -1\right] \leq \mathbb{E}\left[V - P|k = 1; \alpha = 0\right]$$

if μ_Z is sufficiently small.

Proof. The difference between expected returns in the case where $\alpha = 1$ vs. $\alpha = 0$ follows from plugging in the expressions from Proposition IA3:

$$
\begin{aligned}
\Delta_{R+} &= \mathbb{E}\left[V - P|k = 1; \alpha = 1\right] - \mathbb{E}\left[V - P|k = 1; \alpha = 0\right] && \text{(IA155)} \\
&= \mathbb{E}\left[V - P|k = 1; \alpha = 1\right] - \gamma n\left(\frac{1}{\tau_A}+\frac{1}{\tau_\eta}\right) && \text{(IA156)} \\
&= -\frac{\gamma}{\tau_\eta}\mu_Z - \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_p}}} H\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta}+\tfrac{1}{\tau_p}}\left(\mu_\theta - c\right)\right)\Bigg|_{\alpha=1}. && \text{(IA157)}
\end{aligned}
$$

It is now immediate from inspection that if $\mu_Z > 0$ then $\Delta_{R+} < 0$.

The difference between expected returns in the case $\alpha = 0$ vs. $\alpha = -1$ also follows from plugging in the expressions from Proposition IA3:

$$\Delta_{R-} = \mathbb{E}\left[V - P|k = 1; \alpha = 0\right] - \mathbb{E}\left[V - P|k = 1; \alpha = -1\right] \tag{IA158}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \mathbb{E}\left[V - P|k = 1; \alpha = -1\right] \tag{IA159}$$

$$= -\frac{\gamma}{\tau_\eta}\mu_Z + \frac{1}{\tau_p\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}} H\left(-\tau_\theta\sqrt{\tfrac{1}{\tau_\theta} + \tfrac{1}{\tau_p}}\left(\mu_\theta - c\right)\right)\Bigg|_{\alpha=-1}. \tag{IA160}$$

Clearly, this expression is positive if $\mu_Z > 0$ is sufficiently small. □

B.2.3 Future profitability

Proposition IA4. *Let $\Delta V \equiv \mathbb{E}\left[V|k = 1\right] - \mathbb{E}\left[V|k = 0\right]$ denote the change in expected cash flows due to investment. Then, ΔV is always positive.*

Proof. We have

$$\Delta V = \mathbb{E}\left[V|k = 1\right] - \mathbb{E}\left[V|k = 0\right] \tag{IA161}$$

$$= \mathbb{E}\left[V|s_p > \bar{s}_V\right] - \mathbb{E}\left[V|s_p \leq \bar{s}_V\right] \tag{IA162}$$

$$= \mathbb{E}\left[A + \theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c|s_p > \bar{s}_V\right] - \mathbb{E}\left[A|s_p \leq \bar{s}_V\right] \tag{IA163}$$

$$= \mathbb{E}\left[\theta - c|s_p > \bar{s}_V\right] \tag{IA164}$$

$$= \mathbb{E}\left[\mu_\theta + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\left(s_p - \mathbb{E}\left[s_p\right]\right)\Bigg|s_p > \bar{s}_V\right] - c \tag{IA165}$$

$$= \mu_\theta - c + \frac{\tau_\theta^{-1}}{\mathbb{V}\left[s_p\right]}\sqrt{\mathbb{V}\left[s_p\right]} H\left(\frac{\bar{s}_V - \mathbb{E}\left[s_p\right]}{\sqrt{\mathbb{V}\left[s_p\right]}}\right) \tag{IA166}$$

$$= \mu_\theta - c + \frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}} H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right) \tag{IA167}$$

$$= \frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}}\left(\tau_\theta\sqrt{\tfrac{1}{\tau_\theta} + \tfrac{1}{\tau_p}}\left(\mu_\theta - c\right) + H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_p}}\left(\mu_\theta - c\right)\right)\right) \tag{IA168}$$

$$> 0, \tag{IA169}$$

since $x + H\left(-x\right) > 0$ for all x. □

C Managerial learning from public signal

In this section we consider a setting in which the manager conditions her investment decision on an explicit "truth plus noise" signal about the fundamental θ instead of conditioning on the asset price. Our goal is to show that, in our setting where managers maximize price and the noise in price as a signal of cash flows is driven by investors' hedging demands, information in prices has a distinctive impact on managerial decisions relative to public information more generally. Intuitively, investors' hedging demands have a persistent impact on a firm's stock price. Thus, the variation in the price signal that they create is relevant to the how the project will impact this price. This implies that the manager does not view this variation as noise when deciding whether to invest in the same way that she views the noise in a public signal. As we will see, this implies that several of our empirical predictions are specific to market feedback, i.e., they do not apply to managerial learning more generally.

Formally, we maintain all the assumptions regarding investor preferences and exposures, and project payoffs. However, we replace managerial learning from prices with learning from a public signal as follows. First, to eliminate learning from prices, we assume now that investors do not trade at date 1, and instead trade only after the investment is made on date 3, which implies that the manager does not see a price signal when investing. (Because there is only one round of trade, to eliminate notational clutter we suppress the subscript on the price function and write P instead of P_3 below.) We can think of this alternative setting as one in which investors do not yet have private information on project payoffs at the time the manager decides whether to invest. Second, at date 1, we introduce an exogenous, "truth-plus-noise" public signal that is observable to investors and the manager. This signal is given by $y = \theta + \varepsilon$, where ε is an independent $N(0, \tau_\varepsilon^{-1})$ random variable. Given y, the manager chooses the optimal investment $k(y) \in \{0, 1\}$ at $t = 2$ to maximize the expected time-3 price, $\mathbb{E}[P]$.

C.1 Equilibrium derivation

As in our baseline model, we search for a threshold equilibrium in which (i) the asset price at date 3 depends on the underlying random variables through a linear statistic of the form $s_p = \theta + \frac{1}{\beta} Z$, (ii) the manager invests in the project if and only if $y > \overline{y}$ for endogenous threshold $\overline{y}$, and the asset price takes the piecewise linear form

$$P = \begin{cases} A_1 + B_1 s_p & y > \overline{y} \\ A_0 & y \leq \overline{y} \end{cases}.$$

We proceed by working backwards. At date $t = 3$, traders can observe the actual investment decision $k = k(y)$ made at $t = 2$. Hence, they perceive the asset payoff as conditionally normally distributed with conditional moments

$$\mathbb{E}_{i3}[V(k)] = \mathbb{E}_{i3}[A + k(\theta + \alpha\eta_C + \sqrt{1-\alpha^2}\eta_I - c)] = \mu_A + k(\theta - c) \tag{IA170}$$

$$\mathbb{C}_{i3}(V(k), \eta_C) = k\alpha \tfrac{1}{\tau_\eta} \tag{IA171}$$

$$\mathbb{V}_{i3}(V(k)) = \tfrac{1}{\tau_A} + k^2 \tfrac{1}{\tau_\eta}. \tag{IA172}$$

The problem of an arbitrary trader at this date is a static one,

$$\max_{x \in \mathbb{R}} \mathbb{E}_{i3}[-e^{-\gamma W_i}] \tag{IA173}$$

where the terminal wealth is

$$W_i = (n + x) V - xP - z_i \eta_C. \tag{IA174}$$

It is immediate that this problem leads to a standard mean-variance demand function

$$X_{i3} = \frac{\mathbb{E}_{i3}[V(k)] + \gamma \mathbb{C}_{i3}(V(k), \eta_C) z_i - P}{\gamma \mathbb{V}_{i3}(V(k))} - n. \tag{IA175}$$

Plugging in for the conditional moments from above and enforcing market clearing yields equilibrium price

$$P = \mu_A + k(\theta - c) + \gamma k \alpha \tfrac{1}{\tau_\eta} Z - \gamma \left(\tfrac{1}{\tau_A} + k^2 \tfrac{1}{\tau_\eta} \right) n \tag{IA176}$$

$$= \mu_A - \gamma \tfrac{1}{\tau_A} n + k \left(\theta + \gamma \alpha \tfrac{1}{\tau_\eta} Z - c - \gamma \tfrac{1}{\tau_\eta} n \right), \tag{IA177}$$

where the second line collects terms and uses the fact that $k \in \{0, 1\}$ implies $k = k^2$ to simplify. Hence, to be consistent with our initial conjecture, the endogenous signal s_p must have coefficient $\frac{1}{\beta} = \frac{\gamma \alpha}{\tau_\eta}$ on Z.

Stepping back to $t = 2$, the manager's problem is to solve

$$\max_{k \in \{0,1\}} \mathbb{E}[P | \mathcal{F}_m] \tag{IA178}$$

where she can condition on the public signal, $\mathcal{F}_m = \sigma(y)$. Using the expression for P derived above, the manager's problem reduces to

$$\max_{k \in \{0,1\}} k \mathbb{E} \left[s_p - c - \gamma \tfrac{1}{\tau_\eta} n \middle| \mathcal{F}_m \right]. \tag{IA179}$$

The optimal investment is therefore

$$k = \begin{cases} 1 & \mathbb{E}[s_p | \mathcal{F}_m] > c + \gamma \frac{1}{\tau_\eta} n \\ 0 & \mathbb{E}[s_p | \mathcal{F}_m] \leq c + \gamma \frac{1}{\tau_\eta} n. \end{cases} \tag{IA180}$$

Since $\mathbb{E}[s_p | \mathcal{F}_m] = \mathbb{E}[\theta | \mathcal{F}_m] + \frac{1}{\beta} \mu_Z = \mu_\theta + \frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon} (y - \mu_\theta) + \frac{1}{\beta} \mu_Z$, substituting into this optimal investment rule and grouping terms implies that the manager optimally invests if and only if

$$y > \mu_\theta - \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \left(\mu_\theta - c - \gamma \tfrac{1}{\tau_\eta} n + \frac{1}{\beta} \mu_Z \right)$$

so that, after substituting in the equilibrium value $\frac{1}{\beta} = \frac{\gamma \alpha}{\tau_\eta}$ derived above and rearranging, the equilibrium investment threshold is $\overline{y} = \mu_\theta - \tau_\theta \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon} \right) \left(\mu_\theta - c - \frac{\gamma}{\tau_\eta} (n - \alpha \mu_Z) \right)$.

The following proposition collects the above results and establishes the existence and uniqueness of a threshold equilibrium.

Proposition IA1. *There exists a unique threshold equilibrium in which the equilibrium price is*

$$P = \mu_A - \frac{\gamma n}{\tau_A} + k \left(s_p - c - \frac{\gamma}{\tau_\eta} n \right), \tag{IA181}$$

and the manager's investment decision is

$$k = \mathbf{1} \{ y > \bar{y} \}, \tag{IA182}$$

where $s_p \equiv \theta + \frac{\gamma \alpha}{\tau_\eta} Z$ *and* $\bar{y} \equiv \mu_\theta - \tau_\theta \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon} \right) \left(\mu_\theta - c - \frac{\gamma}{\tau_\eta} (n - \alpha \mu_Z) \right)$.

C.2 Implications

This section derives the analogous versions of the implications in the body of the paper, but applied to the public signal model.

C.2.1 Probability of investment

Proposition IA2. *In equilibrium, the unconditional probability of investment is given by*

$$\Pr (y > \bar{y}) = \Phi \left(\frac{\mathbb{E}[y] - \bar{y}}{\sqrt{\mathbb{V}[y]}} \right), \tag{IA183}$$

where $\mathbb{E}[y] = \mu_\theta$, $\mathbb{V}[y] = \frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon}$. *The probability of investment:*
(i) increases with $\mu_\theta - c$*;*
(ii) decreases with n*;*
(iii) increases with μ_Z *for green firms (i.e.,* $\alpha > 0$*), but decreases with* μ_Z *for brown firms (i.e.,* $\alpha < 0$*);*
(iv) increases (decreases) with τ_θ *and decreases (increases) with* τ_ε *if and only if* $\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha \gamma \mu_Z}{\tau_\eta} > 0$ $\left(\mu_\theta - c - \frac{\gamma n}{\tau_\eta} + \frac{\alpha \gamma \mu_Z}{\tau_\eta} < 0\right)$*;*
(v) does not depend on τ_Z*;*
(vi) increases with α *if* $\mu_Z > 0$ *and decreases with* α *if* $\mu_Z < 0$

Proof. The probability of investment is given by

$$\Pr (y > \bar{y}) = 1 - \Phi \left(\frac{\bar{y} - \mathbb{E}[y]}{\mathbb{V}[y]} \right) = \Phi \left(\frac{\mathbb{E}[y] - \bar{y}}{\sqrt{\mathbb{V}[y]}} \right) \tag{IA184}$$

$$= \Phi \left(\frac{\tau_\theta \left(\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon} \right) \left(\mu_\theta - c - \frac{\gamma}{\tau_\eta} (n - \alpha \mu_Z) \right)}{\sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon}}} \right) \tag{IA185}$$

$$= \Phi \left(\tau_\theta \sqrt{\frac{1}{\tau_\theta} + \frac{1}{\tau_\varepsilon}} \left(\mu_\theta - c - \frac{\gamma}{\tau_\eta} (n - \alpha \mu_Z) \right) \right). \tag{IA186}$$

Direct inspection now immediately yields the claimed results. □

Corollary IA3. *Suppose $\mu_Z > 0$. The firm is more likely to invest in a green than a neutral project, i.e.,* $\Pr(y > \bar{y}; \alpha = 1) > \Pr(y > \bar{y}; \alpha = 0)$*, and is more likely to invest in a neutral than a brown project, i.e.,* $\Pr(y > \bar{y}; \alpha = 0) > \Pr(y > \bar{y}; \alpha = -1)$.

C.2.2 Expected return conditional on investment

Proposition IA3. *In equilibrium, the expected return conditional on no investment is*

$$\mathbb{E}\left[V - P | k = 0\right] = \frac{\gamma n}{\tau_A}, \tag{IA187}$$

and the expected return conditional on investment is given by

$$\mathbb{E}\left[V - P | k = 1\right] = \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma \alpha}{\tau_\eta} \mu_Z. \tag{IA188}$$

Conditional on investment, the expected return:
(i) increases in n,
(ii) decreases in μ_Z if $\alpha > 0$ and increases in μ_Z if $\alpha < 0$, and,
(iii) decreases in α if $\mu_Z > 0$ and increases in α if $\mu_Z < 0$.

Proof. Using the expression for the equilibrium asset price in eq. (IA181), it is straightforward to derive the expression for the conditional expected return given no investment. The expression in the case of investment follows from using the asset price in eq. (IA181) to write:

$$\mathbb{E}\left[V - P | k = 1\right] = \mathbb{E}\left[V - P | y > \overline{y}\right] \tag{IA189}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma \alpha}{\tau_\eta} \mathbb{E}\left[Z | y > \overline{y}\right] \tag{IA190}$$

$$= \gamma n \left(\frac{1}{\tau_A} + \frac{1}{\tau_\eta}\right) - \frac{\gamma \alpha}{\tau_\eta} \mu_Z \tag{IA191}$$

where the final line follows because Z is independent of y. The claimed comparative statics results now follow from inspection. □

Corollary IA4. *Suppose $\mu_Z > 0$. Conditional on investment, a green firm always has a lower expected return than a neutral firm i.e.,*

$$\mathbb{E}\left[V - P | k = 1; \alpha = 1\right] \leq \mathbb{E}\left[V - P | k = 1; \alpha = 0\right].$$

Moreover, a neutral firm has a lower expected return than a brown firm i.e.,

$$\mathbb{E}\left[V - P | k = 1; \alpha = -1\right] \geq \mathbb{E}\left[V - P | k = 1; \alpha = 0\right].$$

C.2.3 Future profitability

Proposition IA4. *Let $\Delta V \equiv \mathbb{E}\left[V | k = 1\right] - \mathbb{E}\left[V | k = 0\right]$ denote the change in expected cash flows due to investment. Then,*

(i) ΔV is always positive when $n - \alpha \mu_Z \geq 0$.

(ii) ΔV is negative when $n-\alpha\mu_Z<0$ and $\mu_\theta - c$ is sufficiently negative.

(iii) ΔV is increasing in $\mu_\theta - c$ and n, decreasing in μ_Z when $\alpha>0$, and increasing in μ_Z when $\alpha<0$.

(iv) ΔV is decreasing in α when $\mu_Z>0$ and increasing in α when $\mu_Z<0$.

Proof. We have

$$\begin{aligned}
\Delta V &= \mathbb{E}\left[V|k=1\right]-\mathbb{E}\left[V|k=0\right] && \text{(IA192)}\\
&= \mathbb{E}\left[V|y>\overline{y}\right]-\mathbb{E}\left[V|y\le\overline{y}\right] && \text{(IA193)}\\
&= \mathbb{E}\left[A+\theta+\alpha\eta_C+\sqrt{1-\alpha^2}\eta_I - c|y>\overline{y}\right]-\mathbb{E}\left[A|y\le\overline{y}\right] && \text{(IA194)}\\
&= \mathbb{E}\left[\theta - c|y>\overline{y}\right] && \text{(IA195)}\\
&= \mathbb{E}\left[\mu_\theta+\frac{\tau_\theta^{-1}}{\mathbb{V}\left[y\right]}\left(y-\mathbb{E}\left[y\right]\right)\middle|y>\bar{y}\right]-c && \text{(IA196)}\\
&= \mu_\theta - c+\frac{\tau_\theta^{-1}}{\mathbb{V}\left[y\right]}\sqrt{\mathbb{V}\left[y\right]}H\left(\frac{\bar{y}-\mathbb{E}\left[y\right]}{\sqrt{\mathbb{V}\left[y\right]}}\right) && \text{(IA197)}\\
&= \mu_\theta - c+\frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}}H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\right). && \text{(IA198)}
\end{aligned}$$

First, when $n-\alpha\mu_Z\ge 0$, we have

$$\begin{aligned}
\Delta V &= \mu_\theta - c+\frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}}H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\right) && \text{(IA199)}\\
&\ge \frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}}\left(\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c\right)+H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c\right)\right)\right) && \text{(IA200)}\\
&> 0, && \text{(IA201)}
\end{aligned}$$

since $-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\ge -\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c\right)$ for $n-\alpha\mu_Z\ge 0$ and $x+H\left(-x\right)>0$ for all x. Next, for $n-\alpha\mu_Z<0$, we have

$$\begin{aligned}
&\lim_{\mu_\theta-c\to-\infty}\Delta V\\
&= \lim_{\mu_\theta-c\to-\infty}\left(\mu_\theta - c+\frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}}H\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\right)\right)\\
&= \lim_{\mu_\theta-c\to-\infty}\left(\frac{\left(\mu_\theta-c\right)\left(1-\Phi\left(\cdot\right)\right)+\frac{1}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}}\phi\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\right)}{1-\Phi\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta-c-\frac{\gamma}{\tau_\eta}\left(n-\alpha\mu_Z\right)\right)\right)}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \lim_{\mu_\theta - c \to -\infty} \left(\frac{(1-\Phi(\cdot)) + (\mu_\theta - c)\,\tau_\theta \sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\phi(\cdot) - \phi'\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\phi\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)} \right) \\
&= \lim_{\mu_\theta - c \to -\infty} \left(\frac{(1-\Phi(\cdot)) + \tau_\theta \sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\,\phi\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)}{\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\phi\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)} \right) \\
&= \lim_{\mu_\theta - c \to -\infty} \left(\frac{-\tau_\theta \sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\phi(\cdot) + \left(\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\right)^2 \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\,\phi'\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)}{\left(\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\right)^2\phi'\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right)} \right) \\
&= \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z) \\
&< 0,
\end{aligned}
$$

where the second equality groups terms, the third equality applies L'Hospital's rule, the fourth equality groups terms, the fifth equality applies L'Hospital's rule again, and the final equality evaluates the limit.

To establish the claimed result for $\mu_\theta - c$, observe that:

$$
\frac{\partial}{\partial(\mu_\theta - c)}\Delta V = 1 - H'\left(-\tau_\theta\sqrt{\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}}\left(\mu_\theta - c - \frac{\gamma}{\tau_\eta}(n-\alpha\mu_Z)\right)\right) \tag{IA202}
$$

and since $H'(x) \in (0,1)$, we have $\frac{\partial}{\partial(\mu_\theta - c)}\Delta V \in (0,1)$. The remaining comparative statics in (iii) and (iv) follow from direct inspection of ΔV, using the fact that $H(\cdot)$ is strictly increasing. □

The following Corollary specializes the results in the previous Proposition to explicit statements about neutral vs. brown vs. green firms.

Corollary IA5. *Let $\Delta V \equiv \mathbb{E}[V|k=1] - \mathbb{E}[V|k=0]$ denote the change in expected cash flows due to investment. Then,*
(i) ΔV is always positive for neutral firms, $\alpha = 0$
(ii) ΔV is always positive for brown firms, $\alpha < 0$, if $\mu_Z > 0$
(iii) ΔV is always negative for ex-ante unattractive ($\mu_\theta - c$ sufficiently low) green firms, $\alpha > 0$, if $\mu_Z > 0$ is sufficiently large.