# Asymmetric Information, Disagreement, and the Valuation of Debt and Equity

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**Abstract**

We study the prices of a firm's debt and equity in a market where investors have private information and may exhibit differences of opinion. We show how debt and equity valuations, and the impact of public information and distress risk on these valuations, depend upon disagreement and the intensity of liquidity trading. Moreover, debt and equity prices exhibit drift and reversals in response to news, the strength of which depend upon the firm's leverage. Finally, the firm's capital structure can influence its valuation, and the optimal capital structure depends on the relative amount of liquidity trade in debt versus equity.

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# 1 Introduction

The impact of public and private information on valuation is central to accounting and finance research. Economic models of this relationship typically focus on how such information affects the pricing of all-equity firms (e.g., Grossman and Stiglitz (1980)). However, the corporate bond market is of similar scale, and reflects the same fundamentals, as the stock market. Moreover, corporate leverage has been growing over time, and firms access debt markets for capital more frequently than the equity market (Bai, Bali, and Wen (2019)). In addition, researchers have often linked "abnormal" equity returns to leverage and distress risk, and recent empirical work shows that, similar to stocks, corporate bond prices exhibit predictable patterns such as drift and disagreement premia.[1]

In light of this evidence, it is important to understand how public and private information influence the valuation of a firm's public debt, and to assess how the impact of this information on stock valuation depends upon financial leverage. To address these questions, we develop a model in which risk-averse investors trade in both the equity and public debt of a levered firm. These investors observe diverse private information signals and trade alongside liquidity/noise traders in both the equity and debt markets. We allow for a flexible specification of investor beliefs regarding the information content in price that nests two natural benchmarks: investors may either exhibit rational expectations (RE) and correctly interpret the information in prices, or exhibit differences of opinion (DO), and completely dismiss the information in prices.

We show that the interaction between trade on private information and financial leverage generates several novel patterns in debt and equity valuation. For instance, dispersion in investor beliefs leads to lower expected returns on debt but higher expected returns on equity. Firm-specific public information affects the valuation of both equity and debt, but the direction and magnitude of these effects depend on whether investors exhibit DO or RE, the intensity of liquidity trade, and the firm's default risk. Equity and debt prices can exhibit drift and reversals in response to past news, even when all investors have rational expectations. Finally, a firm's capital structure can influence its valuation even in the absence of traditional frictions (e.g., tax shields of debt, distress costs), and the optimal choice of leverage depends on the relative amount of liquidity trading in debt versus equity.

To characterize the equilibrium in our model, we overcome a key challenge: levered

[1]See e.g., Bonsall and Miller (2017), Choi and Kim (2018), Nozawa, Qiu, and Xiong (2021) for recent analyses of abnormal returns in the debt market. The notion that distress risk may explain abnormal equity returns dates to Fama and French (1993) and has been explored in a number of papers since including Penman, Richardson, and Tuna (2007), Campbell, Hilscher, and Szilagyi (2008), Chava and Purnanandam (2010), Caskey, Hughes, and Liu (2012).

equity and debt have option-like, and thus non-linear, payoffs as a function of the firm's underlying cash-flows. As a result, unlike standard models of trade in an unlevered firm (e.g., Grossman and Stiglitz (1980), Hellwig (1980)), there does not exist an equilibrium in which the security prices are linear in the fundamental and liquidity-trader demand. Instead, applying techniques from Breon-Drish (2015) and Chabakauri, Yuan, and Zachariadis (2021), we show that there exists an equilibrium in which the equity and debt prices depend non-linearly on fundamentals and liquidity trade.

To gain intuition for the drivers of equity and debt valuations in this equilibrium, we start by focusing on a benchmark setting in which liquidity-trader demands in the stock and bond markets are identical. In this case, debt and equity prices convey the same information signal to investors, and the information content of this signal does not depend on the firm's capital structure. We find that equity and debt valuations depend crucially on how investors update from this price signal. Specifically, after controlling for systematic risk, the expected return on equity is negative when investors exhibit RE, but is positive when investors exhibit DO and liquidity-trading volatility is sufficiently low.[2] Analogously, the expected return on debt is positive under RE, but negative under DO when liquidity-trading volatility is low.

These results are driven by how the security prices respond to investors' private information and liquidity-trader demand in equilibrium. Consider the pricing of equity, which, similar to a call option, has a payoff that is convex in the firm's underlying cash-flows (the case for debt, which has a concave payoff, is analogous). First, an investor's demand for equity responds more strongly to a decrease in her conditional expectation of equity payoffs than to a corresponding increase. Intuitively, when an investor receives good news about firm cash-flows, this increases not only her expectation of equity payoffs, but also increases her perceived risk – these offsetting forces dampen the impact of such news on her demand. In contrast, when she observes bad news about cash-flows, both expected payoffs and the perceived risk fall: when the news is extremely bad, she is very certain the equity is almost worthless. This implies that pessimistic signals are disproportionately reflected in the equity price, and as a result, the average price is lower as disagreement across investors increases.

Second, liquidity-trader demand also has an asymmetric impact on prices. Since equity payoffs are bounded below but unbounded above, the risk for investors from being long is limited, while the risk from being short is substantial. As a result, investors charge a larger price compensation when they sell to liquidity traders than when they have to buy from them. This implies that the average equity price is higher when the volatility of liquidity

[2] The source of systematic risk in our model is the aggregate supply of each security that investors have to hold, and so our results should be interpreted as predictions about "alphas" once we have controlled appropriately for "betas." In contrast to the case of an unlevered firm, we show that even when the aggregate supply of the securities is zero, the expected return, or "alpha" on debt and equity can be non-zero.

trading increases. Because debt payoffs are concave and bounded above, the implications are reversed: more disagreement leads to higher debt prices, while higher volatility of liquidity trading leads to lower debt prices.

The relative impact of these forces depends crucially on how investors use the information in prices. When investors exhibit RE, they update efficiently from the common price signal when forming their beliefs. This reduces equilibrium disagreement and amplifies the impact of liquidity trading, and so equity (debt) prices are high (low) on average. In contrast, when investors exhibit DO, they do not update from the price signal and so disagreement is higher. As a result, when the volatility of liquidity trading is sufficiently low, the first channel dominates and so equity (debt) prices are low (high) on average.

**Implications.** We next show that this economic mechanism leads to a number of implications that are relevant to empirical research on the valuation of debt and equity. First, it implies that more disagreement about firm-specific cash-flows leads to an increase in equity returns and a decrease in debt returns, after controlling for systematic risk and liquidity-trading volatility. On the other hand, an increase in liquidity-trading volatility leads to a decrease in expected equity returns and an increase in expected debt returns, after controlling for systematic risk and disagreement. Moreover, these effects strengthen with leverage, so that the relation between default risk and expected returns depends on the prevalence of investor disagreement and liquidity trade.

Next, we consider the impact of introducing public information (e.g., an earnings announcement) before trading in our model. We show that when investors exhibit RE, an increase in the quality of public information increases expected equity returns but decreases debt returns, and vice versa when investors exhibit DO and liquidity-trading volatility is sufficiently low. Intuitively, this is because higher-quality public information reduces both the extent of disagreement and the impact of liquidity trading on prices.

Interestingly, our model implies that leverage can give rise to post-announcement drift and reversals even when such effects are absent with unlevered equity. For instance, we find that even when investors exhibit RE, there is post-announcement drift in equity and reversal in debt if the likelihood of default after the announcement is below 50%. In this case, the more positive the public news, the lower the likelihood of default, which increases the expected return on equity and decreases the expected return on debt.

Finally, our general model considers a setting in which liquidity trading in debt and equity markets can have arbitrary correlation. In this case, investors update their beliefs about cash-flows from both the debt and equity prices, which now provide distinct signals about fundamentals. We show that this generates a spillover across markets: liquidity-trader demand in the equity market increases debt prices and vice versa. Moreover, our analysis implies that the correlation between equity and debt prices is hump-shaped in leverage: it is high for intermediate levels of leverage, but low when the likelihood of default is extremely low or extremely high.

The key insights from our baseline analysis extend to this setting: expected equity prices increase and expected debt prices decrease in the volatility of liquidity trading in each market. However, this effect is stronger within a market, e.g., liquidity trading in equity has a stronger positive impact on expected equity prices than does liquidity trading in debt. Thus, the overall effect of liquidity trading in equity (debt) is to raise (lower) the combined price of the firm's equity and debt. In our baseline analysis, because liquidity traders' demands in both markets are identical, their impact on the combined valuation of debt and equity cancel out exactly. As a result, the total value of the levered firm is equal to the value of the unlevered firm i.e., Modigliani and Miller's irrelevance result obtains. However, when liquidity trading differs across the two markets, this is no longer true. Specifically, we find that when the volatility of liquidity trading in equity is higher than that in debt, the total market value of the firm is hump-shaped in leverage. This suggests that an interior level of leverage is optimal for the firm, even in the absence of traditional frictions associated with debt financing (e.g., tax shields, distress costs).

The rest of the paper is as follows. The next section discusses the related literature and our incremental contribution. Section 3 presents the model, and section 4 characterizes the equilibrium in the baseline case. Section 5 characterizes how the expected return on debt and equity depend on the features of the model. Section 6 studies the impact of a public signal about cashflows, and Section 7 presents the characterization of the equilibrium when the liquidity trading in the two markets are not identical. Section 8 presents the empirical implications of the model, including a discussion of existing empirical research that speaks to our results. Finally, Section 9 concludes. Unless noted otherwise, proofs are in the Appendix.

# 2 Related Literature

One contribution of our analysis is to develop a tractable model of trade in equity and debt that allows for studying both differences in opinion and rational expectations. In addition to allowing investors to "agree to disagree", our model differs from the standard rational expectations framework of Hellwig (1980) in only one way: it allows for both equity and debt, issued by the same firm. We show that this leads to readily interpretable closed-form solutions for demands and prices in the two securities.

A growing literature explores the implications of allowing investors to "agree to disagree" about the informativeness of others' signals (e.g., Miller (1977), Morris (1994), Kandel and

Pearson (1995), Scheinkman and Xiong (2003), Banerjee and Kremer (2010)). Building on the approach in Banerjee (2011), our model allows for flexible subjective beliefs regarding this informativeness, and includes as limits the cases in which investors fully update and do not update from prices. Our results highlight that how investors condition on the information in prices has qualitatively important effects on expected returns.[3] Related to our model, Bloomfield and Fischer (2011) study how disagreement impacts returns in a single-firm, equity-only setting. They find that, when investors think that the market ignores valuable information, expected returns go down, while when investors think that the market places weight on irrelevant information, expected returns go up.

On the technical side, Chabakauri et al. (2021) offers the closest model to ours, analyzing private information in a contingent-claims framework that allows for trade in debt and equity. When applying their model to study debt and equity prices, their focus is on showing that the informativeness of these prices does not depend on the firm's capital structure (a result that also holds in our model). We complement their work by allowing investors to potentially disregard the information in prices, by analyzing expected debt and equity returns, and by considering the joint effects of public and private information.[4]

The economic questions on which we focus are more closely related to noisy rational expectations models of debt and equity markets that study the impact of non-linearity in security prices. Albagli, Hellwig, and Tsyvinski (2021) consider a setting in which risk-neutral, informed investors have position limits and trade in a bond with binary payoffs, and find that the bond price overweights risk. Davis (2017) extends their analysis to consider the firm's issuance decision over time and across markets, in a setting where investors choose how much information to acquire about fundamentals. Back and Crotty (2015) considers the pricing of debt and equity in a continuous-time, Kyle model in which a strategic, informed investor can trade in both debt and equity markets, and market making is integrated. They show that the stock-bond correlation depends on the cross-market lambda, and is positive (negative) when the strategic trader is informed about the mean (risk) of firm's assets. Finally, Pasquariello and Sandulescu (2021) consider a single period Kyle model of debt and equity with segmentation in market making to characterize how stock-bond correlation varies with the probability of default.

[3]As such, our analysis also has implications for settings where investors dismiss the information in prices due to other reasons, including "cursedness" (e.g., Eyster, Rabin, and Vayanos (2018)), costly price information (e.g., Mondria, Vives, and Yang (2022)) and "wishful thinking" (e.g., Banerjee, Davis, and Gondhi (2019)).

[4]Chabakauri et al. (2021) do not study the expected returns on debt and equity. However, they do study the relationship between skewness in payoffs and expected returns. Our results show that the relationship between skewness and prices they document are reversed when investors ignore the information in prices and liquidity-trade volatility is low.

We view our analysis as complementary to this earlier work. While these papers consider settings in which the price is determined by risk-neutral investors / market makers, investors in our model are risk-averse. Moreover, while these models only consider the rational expectations equilibrium, our model allows investors to disagree about the information content of prices. We show that this has important implications for how the non-linearity in payoffs affects expected returns.

Our study of public information and expected returns relates to the literature on information quality, information asymmetry, and the cost of capital. This literature studies how these constructs influence the magnitude of an unlevered firm's risk premium in settings with competitive (Hughes, Liu, and Liu (2007), Lambert, Leuz, and Verrecchia (2007), Dutta and Nezlobin (2017)) and strategic investors (Lambert, Leuz, and Verrecchia (2012), Caskey, Hughes, and Liu (2015)). In a competitive market setting, our results contribute to this work by showing that the impact of these constructs on expected returns depends upon the firm's capital structure and the extent to which investors learn from prices. Finally, our analysis of public information and capital structure contributes to past work on disclosure's impact on the prices of levered firms, such as Fischer and Verrecchia (1997), Bertomeu, Beyer, and Dye (2011), and Beyer and Dye (2021), by considering risk-averse, diversely-informed investors.

# 3 Model Setup

We consider a model of trade among informed investors in the spirit of Hellwig (1980), with two modifications: we allow the firm to be levered and for investors to potentially ignore the information in price.

**Payoffs.** Investors trade in the risky debt and equity of a firm alongside a risk-free security. The gross return on the risk-free security is normalized to 1. The firm's total cash-flows are $\mathcal{V} \equiv \mu + \theta$, where $\theta \sim N(0, \sigma_\theta^2)$.[5] The firm has debt with a face value of $K$, i.e., equity payoffs are $V_E = \max(\mathcal{V} - K, 0)$ and debt payoffs are $V_D = \min(\mathcal{V}, K)$, so that $\mathcal{V} = V_E + V_D$. We assume that there are liquidity traders who submit demands of $z \sim N(0, \sigma_z^2)$ in both the equity and debt markets.

The assumption that the liquidity-trader demands in the debt and equity markets are perfectly correlated is made for analytical tractability and expositional clarity. In Section 7, we explore the implications of our analysis in a setting where the liquidity trading in the two markets follows a general bivariate normal distribution, which allows for imperfect

[5]We assume that $\mathcal{V}$ is unconditionally normally distributed in order to keep traders' updating problem simple and transparent. One can extend our results to general distributions for $\mathcal{V}$ using the approach of Breon-Drish (2015).

correlation and/or different variances across the markets.

**Preferences and Information.** There is a continuum of investors indexed by $i \in [0,1]$. Let $x_{E,i}$ and $x_{D,i}$ denote investor $i$'s demands for the equity and debt, respectively, and let $P_E$ denote the equity and $P_D$ the debt price. Each investor $i$ is endowed with initial wealth $W_0$ and $\kappa$ shares of the stock, and exhibits CARA utility with risk-tolerance $\tau$ over terminal wealth $W_i$, where:

$$W_i = W_0 + x_{E,i}(V_E - P_E) + x_{D,i}(V_D - P_D).$$

Investor $i$ observes a private signal $s_i$ of the form:

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where the error terms $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$ are independent of all other random variables.

**Subjective Beliefs.** We allow for a flexible specification of subjective beliefs about the private information of others. Following Banerjee (2011), we assume that investor $i$'s beliefs about her own signal are given by (1), but her beliefs about investor $j$'s signal are given by:

$$s_j =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + \varepsilon_j, \tag{2}$$

where the random variables $\xi_i \sim_i N(0, \sigma_\theta^2)$ and $\varepsilon_j \sim_i N\left(0, \sigma_\varepsilon^2\right)$ are independent of all other random variables and each other, and $\rho \in [0,1]$ parameterizes the difference in opinions.[6] We use a subscript $i$ on expectations, variances, and distributions to refer to investor $i$'s subjective beliefs.

The above specification provides a tractable way to nest two natural benchmarks. When $\rho = 1$, investors exhibit rational expectations (as in Hellwig (1980)): this is equivalent to assuming that all investors share common priors about the joint distribution of fundamentals and signals. In this case, investors fully condition on the information in prices (in addition to their private information) when updating their beliefs about fundamentals. When $\rho = 0$, investors exhibit "pure" differences of opinion (as in Miller (1977)): each investor believes no other investor has payoff relevant information, and so prices are not incrementally informative about payoffs.[7] In this case, investors do not place any weight on prices when updating their beliefs. Moreover, when $\rho \in (0,1)$, investors are partially dismissive of the information content of others' private signals; hence, investors only partially account for the information

[6]The assumption that $\xi_i$ has the same distribution as $\theta$ ensures that investor $i$ cannot detect the error in her subjective beliefs based on the unconditional mean and variance of others' signals.

[7]Note that investor $i$ believes that $\xi_i$ is the common "error" in all other investors' signals. This is analogous to the subjective beliefs of investors in other difference of opinions models (e.g., Scheinkman and Xiong (2003)) and in the "cursed equilibrium" of Eyster et al. (2018).

contained in prices when determining their demands.

# 4 Analysis

Because the equity and debt securities are effectively options on the underlying cash-flows, their payoffs are not normally distributed. As a result, the equilibrium in which prices are linear in the fundamental and liquidity trade, which is common in traditional rational expectations models, does not exist. Instead, we focus on the following notion of equilibrium, which is a two-asset version of the equilibrium studied in Breon-Drish (2015).

**Definition 1.** *A "generalized linear equilibrium" is one in which there exist monotonic functions $h_E(\cdot)$, $h_D(\cdot)$ and a coefficient $\beta \in \mathbb{R}$ such that the equity and debt prices are given by:*

$$P_E = h_E(\bar{s} + \beta z); \tag{3}$$

$$P_D = h_D(\bar{s} + \beta z). \tag{4}$$

*where $\bar{s} = \int s_j dj$ is the average private signal.*

The key feature of such an equilibrium is that each investor can infer identical linear statistics from the debt and equity prices:

$$s_E = s_D \equiv \bar{s} + \beta z =_i \rho\theta + \sqrt{1-\rho^2}\,\xi_i + \beta z. \tag{5}$$

In particular, when $\rho = 0$, investors perceive $s_D$ and $s_E$ to be uninformative about $\theta$. On the other hand, when $\rho > 0$, the linear structure in (5) ensures that Bayesian updating from prices continues to take on a tractable form. To solve for an equilibrium, we derive investors' demands given these beliefs, apply market clearing, and verify that the resulting price indeed takes the "generalized linear" form in (4).

## 4.1 Benchmarks

To provide intuition for the equilibrium that arises in the general case, we start by characterizing the equilibrium in two natural benchmarks.

### 4.1.1 Unlevered firm benchmark

First, consider the case in which the firm issues only equity (i.e., when $K \to -\infty$). In this case, the payoff to equity holders is normally distributed, as in traditional models, and so we recover the standard, linear equilibrium. Moreover, since the firm only issues one type of security, investor $i$ infers a single linear statistic from the unlevered equity price of the form:

$$s_U \equiv \overline{s} + \beta z =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + \beta z, \tag{6}$$

where $\beta$ is determined in equilibrium. It is worth noting that the objective distribution of the signal is given by $s_U = \theta + \beta z$, which coincides with investors' beliefs when $\rho = 1$.

Given this signal, investor $i$'s conditional beliefs about cash-flows $\mathcal{V}$ are Normal with moments given by

$$\mu_i \equiv \mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right) \text{ and} \tag{7}$$

$$\sigma_s^2 \equiv \mathbb{V}_i\left[\mathcal{V}|s_i, P_U\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}, \tag{8}$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2\sigma_z^2}{\rho^2}$, and where it is understood that when $\rho = 0$, we take $\frac{1}{\sigma_p^2} = \frac{1}{\rho\sigma_p^2} = 0$ in the above expressions. Standard calculations imply that investor $i$'s optimal demand for the security is given by

$$x_i = \tau\left(\frac{\mu_i - P_U}{\sigma_s^2}\right), \tag{9}$$

and market clearing implies that the equilibrium price is given by:

$$P_U = \int \mu_i di + \frac{\sigma_s^2}{\tau}\left(z - \kappa\right).$$

This implies the following result.

**Lemma 1.** ***Unlevered firm benchmark.*** *Suppose that the firm only issues equity (i.e., $K \to -\infty$). Then, there is a unique linear equilibrium in which the firm's price satisfies:*

$$P_U(\cdot) = \mu + \sigma_s^2\left(\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)(\overline{s} + \beta z) - \frac{\kappa}{\tau}\right), \tag{10}$$

*where $\beta = \frac{\sigma_\varepsilon^2}{\tau}$, $\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2\sigma_z^2}{\rho^2}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$.*

Notably, the above equilibrium coincides with the rational expectations equilibrium in Hellwig (1980) when $\rho = 1$. On the other hand, when $\rho = 0$, investors ignore the information in prices (since the weight they put on $s_U$ in (7) is zero), even though it conveys information about cash-flows.

### 4.1.2 Risk-neutral, uninformed benchmark

As a second benchmark, consider the setting in which investors are risk neutral (i.e., $\tau \to \infty$) and completely uninformed (i.e., $\sigma_\varepsilon^2 \to \infty$). In this case, the price of each security is given by the unconditional expectation of its payoff i.e.,

$$P_E = \mathbb{E}\left[\max\left(\mathcal{V} - K, 0\right)\right] \text{ and } \quad P_D = \mathbb{E}\left[\min\left(\mathcal{V}, K\right)\right].$$

In what follows, the definition below will be convenient.

**Definition 2.** *Suppose $x \sim N(\mu_x, \sigma_x^2)$. Let $M_E(\mu_x, \sigma_x^2, K)$ and $M_D(\mu_x, \sigma_x^2, K)$ denote:*

$$M_E(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\max\left(x - K, 0\right)\right] = \left[1 - \Phi\left(\frac{K-\mu_x}{\sigma_x}\right)\right]\left[\mu_x - K + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{1-\Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right], \quad (11)$$

$$M_D(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\min\left(x, K\right)\right] = K - \Phi\left(\frac{K-\mu_x}{\sigma_x}\right)\left[K - \mu_x + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{\Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right]. \quad (12)$$

It is worth noting that since $\max(x - K, 0)$ is an increasing, convex function of $x - K$, we immediately have that $M_E(\mu_x, \sigma_x^2, K)$ is increasing in $\mu_x$ and $\sigma_x^2$, but decreasing in $K$. Similarly, since $\min(x, K) = K + \min(x - K, 0)$ is increasing and concave in $x$, we have that $M_D(\mu_x, \sigma_x^2, K)$ is increasing in $\mu_x$ and $K$, but decreasing in $\sigma_x^2$.

Given the above definition, we can characterize the equilibrium in this benchmark as follows.

**Lemma 2.** ***Risk-neutral, uninformed benchmark.*** *Suppose that investors are risk neutral and uninformed (i.e., $\tau \to \infty, \sigma_\varepsilon^2 \to \infty$). Then, there is a unique equilibrium in which the firm's equity and debt prices are given by $P_E = M_E\left(\mu, \sigma_\theta^2, K\right)$ and $P_D = M_D\left(\mu, \sigma_\theta^2, K\right)$. Moreover, the total value of the firm is given by $P_E + P_D = \mu$.*

The above results are intuitive. Note that $\Pr(\mathcal{V} < K) = \Phi\left(\frac{K-\mu}{\sigma_\theta}\right)$ reflects the probability that the firm defaults on its debt. Given this, the price of equity is given by the probability of no default times the conditional expected cash-flows, given no default i.e.,

$$P_E = \Pr(\mathcal{V} > K) \times \mathbb{E}[\mathcal{V} - K | \mathcal{V} > K],$$

which corresponds to the expression for $M_E$ in (11), evaluated at the firm's cash-flow mean and variance. Similarly, the price of debt is given by the face value of debt, $K$, minus the probability of default times the loss given default i.e.,

$$P_D = K - \Pr(\mathcal{V} < K) \times \mathbb{E}[K - \mathcal{V} | \mathcal{V} < K],$$

which corresponds to the expression for $M_D$ in (12). Not surprisingly, since investors are uninformed and risk-neutral, the total value of the firm reflects the unconditional expected cash-flows. In the following subsection, we show that the equilibrium prices when investors are risk averse and privately informed are natural generalizations of the above expressions.

## 4.2 Equilibrium

To start, we study investors' demands holding fixed the equity and debt prices. We then show that the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, which lends tractability to our model.

**Lemma 3.** *Given equity and debt prices $P_E$ and $P_D$, investors' demands take the form:*

$$\begin{pmatrix} x_{E,i} \\ x_{D,i} \end{pmatrix} = \frac{\tau}{\sigma_s^2} \left[ \begin{pmatrix} \mu_i \\ \mu_i \end{pmatrix} - G \begin{pmatrix} P_E \\ P_D \end{pmatrix} \right], \tag{13}$$

*for a function $G(\cdot)$ defined in the appendix. As a result, the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, i.e., they depend upon $\{s_i\}$ and $z$ only through the statistic $s_U$.*

The first part of the lemma illustrates that investors' demands are additively separable in investors' beliefs about the firm's total cash flow, $\mu_i = \mathbb{E}_i [\mathcal{V}]$, and the prices $P_E, P_D$. Moreover, each investor speculates on her beliefs in the same direction in both markets, and exhibits the same trading aggressiveness in the two markets:

$$\frac{\partial x_{E,i}}{\partial \mu_i} = \frac{\partial x_{D,i}}{\partial \mu_i} = \frac{\tau}{\sigma_s^2} = \frac{\text{risk tolerance}}{\text{posterior uncertainty}}. \tag{14}$$

Intuitively, both securities are exposed to the firm's underlying cash-flows in the same direction. One might posit that investors would trade more aggressively in the security that is more exposed to a shift in the firm's cash-flows. For instance, when the firm's expected cash-flows $\mu$ are large, the debt almost certainly pays off $K$, and so the equity is considerably more sensitive to a change in $\mu$. Thus, one might expect an investor with a positive signal to take a larger position in the equity than the debt. However, while the expected payoffs to trading on private information are greater in the security that is more exposed to $\theta$, so too is the risk, and these two effects precisely offset. As we will see, this feature of investors' demands has important consequences for the expected returns on the securities.

Equation (14) holds regardless of the firm's debt level $K$, which implies that the firm's capital structure does not influence the investors' trading aggressiveness. In addition, the total supplies of the equity and debt to be absorbed by the investors, $\kappa - z$, are identical. Together, these results imply that the equity and debt prices contain the same information and that this information is the same as in the case where the firm is unlevered, as in Chabakauri et al. (2021). Therefore, investors' expectations and variances of total firm cash-flows in equilibrium are identical to those in equations (7) and (8).

Building on these findings, the next proposition characterizes the equilibrium price and investor demands.

**Proposition 1.** *There exists a generalized linear equilibrium in which the equity and debt prices satisfy:*

$$P_E = M_E\left(P_U, \sigma_s^2, K\right) \text{ and } P_D = M_D\left(P_U, \sigma_s^2, K\right). \tag{15}$$

*Moreover, the total value of the equity and debt is equal to $P_U$ i.e., $P_U = P_E + P_D$, and investors' equilibrium equity and debt demands satisfy:*

$$x_{E,i} = x_{D,i} = \tau \frac{\mu_i - \int \mu_j dj}{\sigma_s^2} - z + \kappa. \tag{16}$$

This proposition demonstrates that the firm's equity and debt prices are equal to their expected payoffs under the beliefs of a representative investor who views the firm's unlevered cash-flows to be distributed as $\mathcal{V} \sim N(P_U, \sigma_s^2)$. Such an investor would be exactly indifferent between holding units of either security or not, and so the price of each security must coincide with her subjective conditional expectation of the security payoff. Moreover, the total market price of the firm's equity and debt, $P_E + P_D$, is independent of the firm's capital structure and equal to the price were the firm unlevered $P_U$. This implies the Modigliani-Miller theorem holds in this setting, even though investors have private information. Finally, consistent with the finding in Lemma 3 that investors speculate on their beliefs equally in both markets, their equilibrium demands in the two markets are identical and coincide with their demands in the unlevered firm case.

Because the securities' prices can be expressed as their expected payoffs under the beliefs of a representative investor, they satisfy a number of intuitive features. For instance, any feature that shifts up the price were the firm unlevered, $P_U$, while holding fixed posterior uncertainty $\sigma_s^2$, will also cause the prices of the debt and equity to increase. This yields the following result.

**Corollary 1.** *The firm's equity and debt prices:*

*(i) increase in mean cash-flows, $\mu$,*

*(ii) increase in liquidity-trader demand, $z$, and*

*(iii) decrease in per-capita supply of the stock, $\kappa$.*

*Moreover, the firm's equity (debt) price decreases (increases) in the face value of debt, $K$.*

Figure 1: Price Function

This figure plots the equilibrium price of equity, debt, and a claim to the total cash-flow of the firm as a function of fundamentals $\theta$ and liquidity trade $z$. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \sigma_z^2 = \rho = \mu = \tau = K = 1; \kappa = 0.1$.

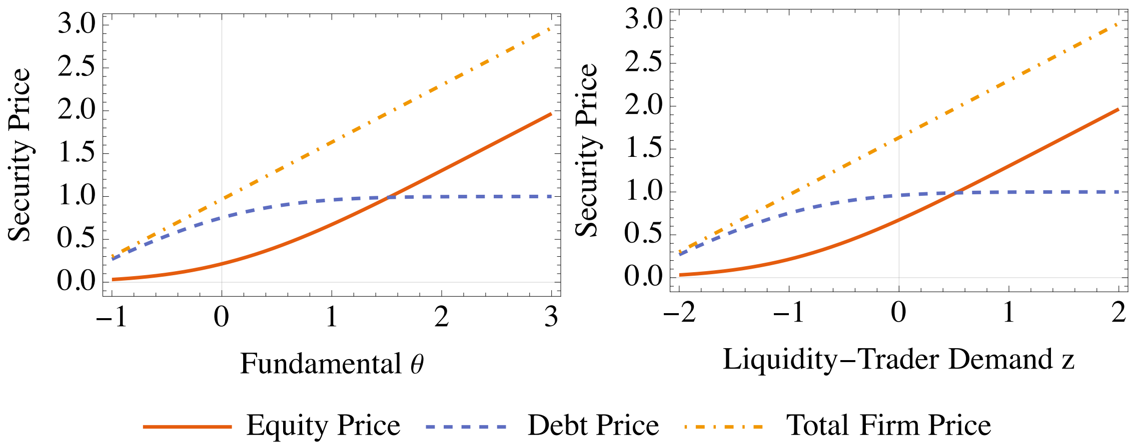



Figure 1 illustrates the equity and debt price functions. The two plots show that the debt and equity prices co-move in our model, which occurs because both investors and liquidity traders speculate on their private information in the same direction in both markets. However, as the fundamental $\theta$ or liquidity trade $z$ rise, the reaction in the equity market grows relative to that in the debt market. This reflects the upper and lower bounds on debt and equity payoffs, respectively.

# 5 Expected Return on Debt and Equity

In our setting the (dollar) return on debt and equity can be expressed as $R_D = V_D - P_D$ and $R_E = V_E - P_E$, respectively.[8] When the security payoff is linear in fundamental shocks and liquidity trade, the expected return typically increases with the per-capita supply of the

[8]The main results in this section extend immediately to percent returns $\frac{V_x - P_x}{P_x}$. The reason is that our results speak primarily to the sign of expected returns on each security, which is unchanged upon dividing by $P_x$ (assuming the mean cash flows are sufficiently high to ensure that the intuitive condition $P_x > 0$ holds).

asset. For instance, note that the price of the unlevered firm from Lemma 1 implies that the expected return on unlevered equity is given by

$$\mathbb{E}[R_U] = \mathbb{E}[\mathcal{V} - P_U] = \frac{\sigma_s^2}{\tau}\kappa.$$

This implies that if the per-capita supply of the firm is zero (i.e., $\kappa = 0$), so is the expected return, because the firm does not expose investors, on average, to any risk. The firm's price under zero net supply corresponds to the price of the idiosyncratic cash-flows of a typical firm in the economy. The reason is that, as the typical firm is a small part of the overall economy, its idiosyncratic cash-flows exhibit negligible correlation with the average investor's wealth.

In contrast, when security payoffs are non-linear, this is no longer true. The following result illustrates that in general, the expected return on debt and equity systematically differ from zero, even when the per-capita supply of shares is zero. This implies that idiosyncratic risk is priced in our model.

**Proposition 2.** *Let $\Omega = \mathbb{V}[P_U] + \sigma_s^2$. The firm's expected equity and debt prices are:*

$$\mathbb{E}[P_E(P_U)] = M_E\left(\mu - \frac{\sigma_s^2}{\tau}\kappa,\, \Omega,\, K\right);$$
$$\mathbb{E}[P_D(P_U)] = M_D\left(\mu - \frac{\sigma_s^2}{\tau}\kappa,\, \Omega,\, K\right).$$

*When the per-capita endowment of shares is zero (i.e., $\kappa = 0$), we have that:*

*(i) The expected return on equity is positive (i.e., $\mathbb{E}[R_E] > 0$) if and only if $\Omega < \sigma_\theta^2$.*

*(ii) The expected return on debt is positive (i.e., $\mathbb{E}[R_D] > 0$) if and only if $\Omega > \sigma_\theta^2$.*

The proof of the above result builds on the observation that the price of each security can be expressed as the subjective conditional expectation of the payoff for the security from the perspective of the representative investor, who believes $\mathcal{V} \sim N(P_U, \sigma_s^2)$ (see Proposition 1). The expressions for $\mathbb{E}[P_E(P_U)]$ and $\mathbb{E}[P_D(P_U)]$ then follow from evaluating the expectations of the security prices over different realizations of $P_U$. Finally, the claims about the expected security returns follow from convexity (concavity) of payoffs to equity (debt) securities. For instance, note that when $\kappa = 0$, Proposition 2 implies that the expected equity price is given by $\mathbb{E}[P_E] = M_E(\mu, \Omega, K)$, while the expected payoff to equity is given by $\mathbb{E}[V_E] = M_E(\mu, \sigma_\theta^2, K)$. Since $M_E$ reflects the expectation of a convex function of $\theta$, it is increasing in the variance, and so $\mathbb{E}[V_E - P_E] > 0$ when $\Omega < \sigma_\theta^2$.

To understand the economic intuition underlying these results, it is useful to consider the characterization in the following corollary.

**Corollary 2.** *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$).*

*(i) When $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, then for any value of $\rho \in [0,1]$, the expected return on equity is negative, and the expected return on debt is positive i.e., $\mathbb{E}[R_E] < 0$ and $\mathbb{E}[R_D] > 0$.*

*(ii) When $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists $\rho^* \in (0,1)$ such that the expected return on equity is positive when $\rho < \rho^*$ and negative otherwise, and the expected return on debt is negative when $\rho < \rho^*$ and positive otherwise.*

*In particular, the expected return on equity (debt) is always negative (positive) when investors exhibit RE ($\rho = 1$), but can be positive (negative) when investors exhibit DO ($\rho = 0$).*

To see the intuition behind this result, note that we can rewrite the key quantity that determines the signs of the expected returns, $\Omega - \sigma_\theta^2$, using the law of total variance, as:[9]

$$\Omega - \sigma_\theta^2 = \mathbb{V}\left[P_U\right] - \mathbb{V}_i\left\{\mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right]\right\} \tag{17}$$

$$= \mathbb{V}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau} z\right] - \mathbb{V}_i\left[\mu_i\right]. \tag{18}$$

One can interpret this expression as the difference between the variance of the conditional expectation of the representative investor, who has subjective (conditional) beliefs $\mathcal{V} \sim N(P_U, \sigma_s^2)$, and that of an arbitrary investor, who has subjective (conditional) beliefs $\mathcal{V} \sim (\mu_i, \sigma_s^2)$. The difference in variances is driven by two countervailing effects. On the one hand, $P_U$ is less variable than the expectation $\mu_i$ of the average investor because it reflects the aggregate (or average) valuation (i.e., $\mathbb{V}[\int \mu_j dj] < \mathbb{V}_i[\mu_i]$). On the other hand, $P_U$ is more variable because it is more sensitive to liquidity-trading shocks via the "risk compensation" term $\frac{\sigma_s^2}{\tau} z$.

We show in the appendix that we can re-express equation (18) as follows:

$$\Omega - \sigma_\theta^2 = \underbrace{-\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right]}_{\text{(1) Belief dispersion}} + \underbrace{\mathbb{V}\left[\frac{\sigma_s^2}{\tau} z\right]}_{\text{(2) Liquidity-trading variability}} \tag{19}$$

$$+ \underbrace{2 \times \mathbb{C}\left[\int \mu_j dj, \frac{\sigma_s^2}{\tau} z\right]}_{\text{(3) Mean–liquidity trade correlation}} + \underbrace{\mathbb{V}\left[\mu_i\right] - \mathbb{V}_i\left[\mu_i\right]}_{\text{(4) Subjective variance difference}} \tag{20}$$

[9]In particular, the law of total variance yields $\sigma_\theta^2 = \sigma_s^2 + \mathbb{V}_i\left\{\mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right]\right\}$.

The above decomposition helps clarify the economic forces that drive expected returns. We focus on the case of equity; the intuition is precisely reversed in the case of debt.

(1) **Belief dispersion.** First, equilibrium disagreement across investors leads to a reduction in $\Omega - \sigma_\theta^2$, and consequently, an increase in equity returns. Recall from Lemma 3 that an investor's demand for equity is linear in her expectation of firm cash-flows, $\mathbb{E}_i[\mathcal{V}]$. However, because the *equity payoff* depends non-linearly on $\mathcal{V}$, this implies that the investor's demand responds non-linearly to her expectation of this payoff, $\mathbb{E}_i[V_E]$.

    Specifically, an investor responds more strongly to a unit decrease in her conditional expectation than a unit increase i.e., her demand is *concave* in her expectation of equity cash-flows. Intuitively, this is driven by the fact that investors become more uncertain about the equity's payoffs and so perceive more equity risk when they receive a higher signal (i.e., $\frac{\partial \mathbb{V}_i[V_E|s_i,P_U]}{\partial s_i} < 0$).[10] As a result, more pessimistic signals are disproportionately reflected in the price, which pushes it lower. Note that this feature is absent in traditional settings where the payoff is conditionally normal and so investors' demands (and prices) are linear in their conditional expectations.

(2) **Liquidity-trading variability.** The impact of liquidity trading on prices is also non-linear. When liquidity traders sell the firm's equity, investors must hold larger long positions and demand a drop in price to do so. However, since the equity payoffs are truncated from below (i.e., equity payoffs are positively skewed), the downside from being long is limited, and the price compensation is relatively small.[11] On the other hand, when liquidity traders buy equity, informed investors bear the risk of being short. In this case, their downside is unlimited and so they charge a large increase in the price for bearing the risk. On average, this pushes prices up and thus reduces expected returns.[12]

(3) **Mean–liquidity trade correlation.** The third term in the decomposition reflects the fact that the investors' average valuation $\int \mu_j dj$ is positively correlated with liquidity-trader demand $z$ because investors condition on the information in prices when forming

[10]Formally, the observation follows from the fact that an investor's expectation of the equity payoff, $M_E(\mathbb{E}_i[\mathcal{V}], \sigma_s^2, K)$, is an increasing and convex function of her expectation of total cash-flows, $\mathbb{E}_i[\mathcal{V}]$, and so its inverse is concave.

[11]Investors with CARA utility exhibit a preference for such skewness, see e.g., Eeckhoudt and Schlesinger (2006).

[12]This asymmetric risk-compensation effect is absent in traditional models with linear prices because the value is symmetric and unbounded (usually normal). However, it is analogous to the "skewness effect" discussed in Albagli et al. (2021), Chabakauri et al. (2021), Cianciaruso, Marinovic, and Smith (2022), and Banerjee, Marinovic, and Smith (2022).

their beliefs (when $\rho > 0$).[13] An increase in this correlation reduces expected returns because it amplifies the forces above. Specifically, an increase in buying from liquidity traders not only exposes informed investors to more downside risk (as in (2) above), but also increases their conditional expectation of cash-flows, which increases their perceived risk (as in (1) above). The resulting increase in price that investors demand as compensation is large relative to the drop in prices they demand when liquidity traders sell. On average, this leads to higher expected prices and lower expected returns.

(4) **Subjective variance difference.** The fourth, and final, term in the decomposition reflects the fact that the actual variance of investor beliefs is (weakly) higher than their subjective variance. This term arises because each investor perceives the price signal as being less correlated with her private signal than it truly is. Consequently, investors underestimate how strongly their beliefs will vary with $\theta$. This term is zero if and only if $\rho \in \{0, 1\}$. For $\rho = 1$, investors correctly condition on price and therefore their subjective variance of beliefs is equal to the objective one. On the other hand, for $\rho = 0$, investors condition only on their private signals, for which they know the correct variance.

Importantly, the combined effect of these forces depends on how investors learn from the price. When investors ignore the information in price (i.e., $\rho = 0$), the mean–liquidity trade correlation term is zero. Moreover, the belief dispersion channel dominates the liquidity-trading variability channel if and only if $\sigma_z^2$ is sufficiently low relative to investors' risk tolerance $\tau$ and private information quality $1/\sigma_\varepsilon^2$; in this case, the expected return on equity is positive.

In contrast, when investors have rational expectations and fully incorporate the information in prices (i.e., $\rho = 1$), the mean–liquidity trade correlation term is maximized. Moreover, the belief dispersion channel is relatively weak since disagreement tends to be low when investors condition on a common, public (price) signal. Thus, belief dispersion tends to be dominated by the other forces so that the equity earns negative expected returns. Surprisingly, this implies the difference between expected cash-flows and equity prices may be larger under rational expectations than under difference of opinions.

We next characterize how expected returns on the two securities relate to the model's parameters.

**Corollary 3.** *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$).*

[13] Specifically, when liquidity traders purchase shares, this pushes the price up. However, since investors cannot readily detect whether price changes are driven by liquidity trade or information, this increases investors' conditional expectations of cash-flows.

*(i) The magnitudes of the expected returns in the debt and equity,* $|\mathbb{E}[R_E]|, |\mathbb{E}[R_D]|$, *are hump-shaped in* $K$ *and maximized at* $K = \mu$.

*(ii) Expected equity returns decrease and expected debt returns increase with liquidity-trading volatility* $\sigma_z$.

Figure 2: Expected Return Comparative Statics

This figure plots expected returns on the equity and debt as a function of the model parameters. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \mu = \tau = K = 1; \sigma_z^2 = 0.75^2; \kappa = 0; \rho = 0.5$.

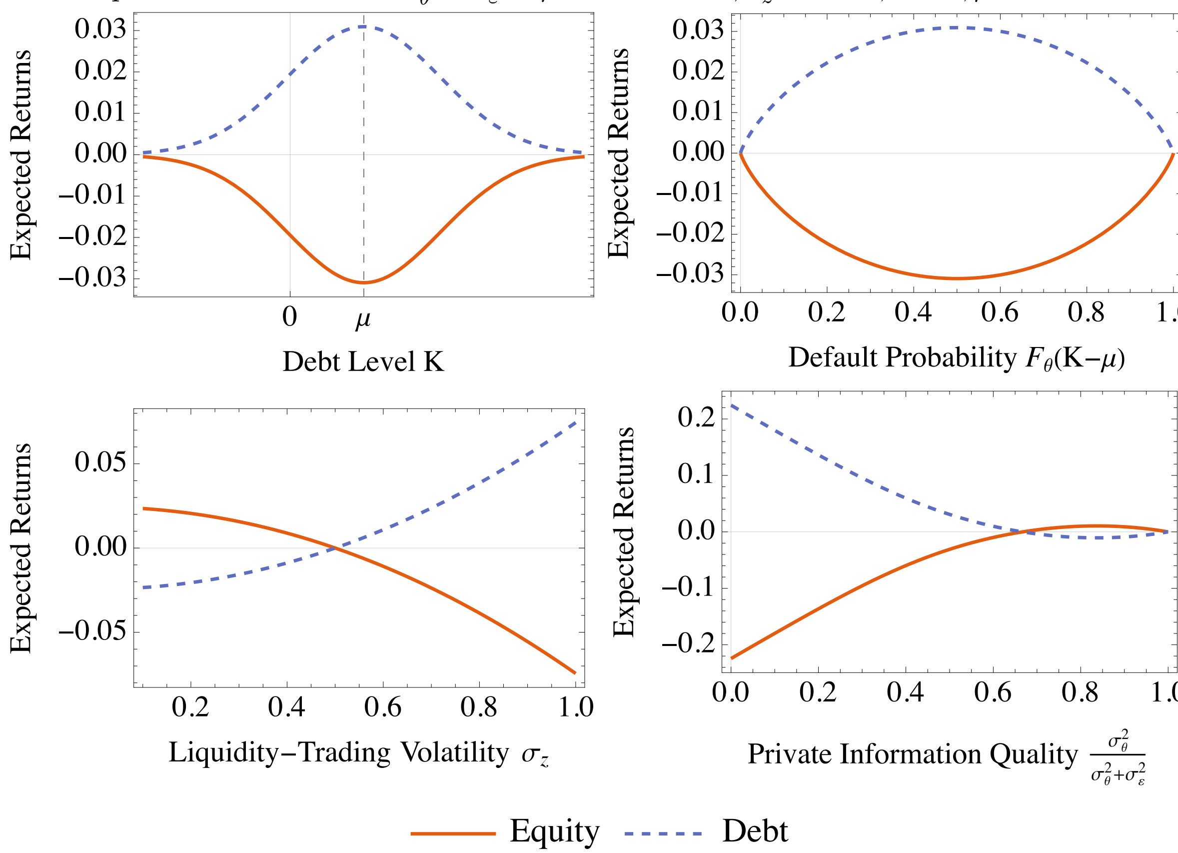



Figure 2 illustrates the corollary. The upper panels demonstrate how expected returns vary with the firm's leverage; the upper-left (upper-right) panel demonstrates this in terms of the level of debt (default probability). They show that these returns are maximized when the debt level is equal to the firm's expected cash-flows, corresponding to a 50% default probability. Intuitively, this reflects the fact that the concavity (convexity) of debt (equity, respectively) payoffs is maximized given a 50% probability of default, similar to the notion that an option's convexity is maximized when it is at-the-money. As most firms have default probabilities much lower than 50% (see, e.g., Chava and Purnanandam (2010)), this result suggests that empirically, expected returns on debt increase, and expected returns on equity fall, with default risk. We return to this point when discussing the empirical implications of our model in Section 8.

Next, when liquidity-trading volatility rises, the variance of $P_U$ rises, which reduces equity returns and increases debt returns. Finally, prior uncertainty and private information quality, which we can jointly capture via the signal to noise ratio $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, have a non-monotonic impact on expected returns. This follows from the observation that these parameters have a non-monotonic impact on disagreement. When investors' private information quality is very high, their beliefs converge towards the true value of the firm, and hence there is no disagreement. Likewise, when investors' private information quality is very low, they rely on their common priors and so do not disagree.

# 6 Public Information

In this section, we introduce a public signal and study its effects on prices and expected returns. This enables us to connect our model to the empirical literature on how earnings and other sources of public news influence returns. Formally, we now assume that there is a public signal released prior to trade:

$$y = \theta + \eta;\ \eta \sim N\left(0, \sigma_\eta^2\right),$$

where $\eta$ is independent of all other random variables. Because the public signal is jointly normally distributed with the rest of the random variables in the economy, the derivation of the equilibrium characterized in Proposition 1 continues to hold upon updating investors' beliefs to reflect the public signal. Hence, the equilibrium prices and demands are equal to those stated in Proposition 1 after replacing $\mathbb{E}[\mathcal{V}] = \mu$ with $\mathbb{E}\left[\mathcal{V}|y\right] = \mu + \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}\right)^{-1} \frac{1}{\sigma_\eta^2} y$ and the prior variance of $\mathcal{V}$, $\mathbb{V}[\mathcal{V}] = \sigma_\theta^2$, with $\mathbb{V}\left[\mathcal{V}|y\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}\right)^{-1}$, and adjusting the updating accordingly. We build on this observation to study the impact of public information quality and the direction of public news on returns in the following subsections.

## 6.1 Public Information Quality and Expected Returns

We begin by characterizing the impact that public information quality has on expected equity and debt returns.

**Proposition 3.** *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$). An increase in public information quality (i.e., higher $1/\sigma_\eta^2$) reduces the magnitude of expected returns in equity and debt. Thus,* *(i) If $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, then for any $\rho \in [0,1]$, an increase in public information quality raises expected equity returns and lowers expected debt returns.*

*(ii) If $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists $\rho^* \in (0,1)$ such that an increase in public information quality lowers expected equity returns and raises expected debt returns when $\rho < \rho^*$, and vice versa when $\rho > \rho^*$.*

*In particular, when investors exhibit RE ($\rho = 1$), an increase in public information quality raises expected equity returns and lowers expected debt returns. However, when investors exhibit pure DO ($\rho = 0$), an increase in public information quality lowers expected equity returns and raises expected debt returns when $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, and vice versa when $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$.*

Note that when the aggregate endowment of shares is zero, the expected return on unlevered equity (i.e., $\mathbb{E}[R_V]$) is zero, and so public information quality has no effect on expected returns. This reflects the fact that, when $\kappa = 0$, information about the firm's cash-flows $\mathcal{V}$ are purely idiosyncratic and can be diversified away.

However, when the firm has leverage, this no longer applies. Interestingly, public information attenuates the magnitude of expected returns in the securities in the model, regardless of whether these returns are positive or negative. Intuitively, recall that expected returns are driven by the four effects outlined in expression (19). Applying (7), this expression reduces as follows:

$$\Omega - \sigma_\theta^2 = \sigma_s^4 \left( -\mathbb{E}\left[\frac{1}{\sigma_\varepsilon^4}\left(s_i - \int s_j dj\right)^2\right] + \mathbb{V}\left[\frac{1}{\tau} z\right] + 2 \times \mathbb{C}\left[\frac{1}{\rho\sigma_p^2} s_U, \frac{1}{\tau} z\right] \right.$$
$$\left. + \mathbb{V}\left[\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right] - \mathbb{V}_i\left[\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right]\right),$$

i.e., each of these four effects is proportional to the square of investors' posterior variances, $\sigma_s^4$. As public information reduces investors' posterior uncertainty, it has a proportional, negative impact on each of the three drivers of expected returns. This implies that, in contrast to standard intuition, firm-specific public information always *raises* expected returns on one of the two securities, while *decreasing* the expected return on the other.

## 6.2 Post-Announcement Drift

We next study the relationship between the sign of the news provided on the release of the public signal and future returns. Formally, we say that security $x \in \{E, D\}$ exhibits *post-announcement drift* when the security expected return, conditional on the public signal $y$, satisfies

$$\frac{\partial}{\partial y}\mathbb{E}\left[V_x - P_x\left(y\right)\middle|\, y\right] > 0, \tag{21}$$

and exhibits *post-announcement reversal* when the conditional expected return satisfies

$$\frac{\partial}{\partial y}\mathbb{E}\left[V_x - P_x\left(y\right)\middle|\, y\right] < 0. \tag{22}$$

The following result characterizes how the incidence of drift / reversal depends on the security payoff and the extent to which investors condition on prices.

**Proposition 4.** *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$). Consider a firm that, after the disclosure, is left with less than a 50% chance of default (i.e., $\mathbb{E}\left[\theta|y\right] - K > 0$).*

*(i) When investors exhibit rational expectations (i.e., $\rho = 1$), there is neither drift nor reversal in unlevered equity. However, there is positive drift in levered equity and reversal in the debt.*

*(ii) When investors exhibit pure difference of opinions (i.e., $\rho = 0$), there is neither drift nor reversal in unlevered equity. However, levered equity exhibits reversal when $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, but drift otherwise. Moreover, the debt exhibits drift when $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, but reversal otherwise.*

This result is a consequence of Part (i) of Corollary 3. The magnitudes of expected debt and equity returns are maximized when the equity is 50% probability of default. Thus, for the typical firm, which has a default probability considerably less than 50%, the more positive the public news $y$, the lower the magnitude of returns in both securities following the announcement. Thus, a security that earns negative (positive) expected returns will have higher (lower) expected returns following a more positive announcement, generating drift (reversal).

# 7 Imperfectly correlated liquidity trade

In this section, we consider a generalization of the baseline setting in which liquidity-trader demand differs across the debt and equity markets, but is potentially correlated. Specifically, in contrast to the baseline setting described in Section 3, demand shocks $z = (z_D, z_E) \in \mathbb{R}^2$ follow a general bivariate normal distribution $z \sim N(\mathbf{0}, \Sigma_z)$ with $\Sigma_z$ an arbitrary $2 \times 2$ positive definite covariance matrix.[14] As in the baseline setting, we let $x_{D,i}$ and $x_{E,i}$ denote

[14]The proofs of all results in this section allow for an arbitrary mean vector $\mu_z \in \mathbb{R}^2$ and allow for a covariance matrix $\Sigma_z$ that is only positive semi-definite. In the text, we normalize the means to zero and consider only strictly positive definite $\Sigma_z$ for expositional clarity.

the investor's demand for debt and equity respectively, with $x_i = (x_{D,i}, x_{E,i})$ the vector of demands.

The definition of a generalized linear equilibrium is analogous to that above, but generalized to account for the fact that in this setting the the debt and equity prices generally depend on two non-identical linear statistics.

**Definition 3.** *A "generalized linear equilibrium" is one in which there exists an injective function $P(\cdot,\cdot) = (P_D(\cdot), P_E(\cdot))$ mapping $\mathbb{R}^2$ into $\mathbb{R}^2$ and linear statistics of the form*

$$s_{p1} = \int s_j dj + \beta_{1D} z_D + \beta_{1E} z_E$$
$$s_{p2} = \int s_j dj + \beta_{2D} z_D + \beta_{2E} z_E$$

*such that the equilibrium price is*

$$P(s_{p1}, s_{p2}) = \begin{pmatrix} P_D(s_{p1}, s_{p2}) \\ P_E(s_{p1}, s_{p2}) \end{pmatrix}.$$

Let $\overline{s} \equiv \int s_j dj$ denote the cross-sectional average signal and let

$$s_p = \mathbf{1}\overline{s} + Bz$$

concisely denote the stacked vector of price-signals, with $\mathbf{1}$ a conformable vector of ones and $B = \left(\begin{smallmatrix} \beta_{1D} & \beta_{1E} \\ \beta_{2D} & \beta_{2E} \end{smallmatrix}\right)$ the $2 \times 2$ matrix of coefficients on $z$. In the main text we will focus on the case in which $\Sigma_z$ is invertible (i.e., strictly positive definite).[15]

Given $s_i$ and the conjectured $s_p$, investor $i$'s beliefs about the firm cash flow $\mathcal{V}$ are normal with conditional moments

$$\mu_i \equiv \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = \mu + \sigma_s^2 \left( \frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1} \frac{1}{\rho} s_p \right), \text{ and} \tag{23}$$

$$\sigma_s^2 \equiv \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1} \right)^{-1}, \tag{24}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 \mathbf{1}\mathbf{1}' + \frac{1}{\rho^2} B\Sigma_z B'$.[16] These are the analogues to Equations (7)-(8) in the benchmark analysis.

[15] The case in which $\Sigma_z$ is singular (e.g., perfectly correlated liquidity trade across both markets, or one of the liquidity trades constant) is considered in the formal derivations in the appendix.

[16] Because $\Sigma_z$ is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, $\Sigma_p$, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho = 0$.

We next extend our characterization of the investor's optimal demand in Lemma 3 to this case.

**Lemma 4.** *Fix any $P = (P_D, P_E) \in \mathbb{R}^2$. The optimal demand of trader $i$ is given by*

$$x_i = \frac{\tau}{\sigma_s^2} \left( \mathbf{1}\mu_i - G(P) \right),$$

*where $G : \mathbb{R}^2 \to \mathbb{R}^2$ is a function defined in the proof.*

As before, investor $i$'s optimal demand is additively separable in her beliefs $\mu_i$ and the prices, and her trading aggressiveness again remains the same in each security. The equilibrium debt and equity prices follow from imposing market clearing and matching coefficients on the price-signal vector $s_p$.

**Proposition 5.** *There exists an equilibrium in the financial market. The vector of equilibrium asset prices takes the form*

$$P = g' \left( \mathbf{1} \frac{\int \mu_j \, dj}{\sigma_s^2} - \frac{1}{\tau} \left( \kappa \mathbf{1} - z \right) \right) \tag{25}$$

$$= g' \left( \frac{1}{\sigma_s^2} \left( \mathbf{1}\mu + \sigma_s^2 \left( I \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}' \Sigma_p^{-1} \frac{1}{\rho} \right) s_p - \frac{\sigma_s^2}{\tau} \mathbf{1}\kappa \right) \right) \tag{26}$$

*where the equilibrium coefficient matrix is $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, and $g' : \mathbb{R}^2 \to \mathbb{R}^2$ is the gradient of a function $g : \mathbb{R}^2 \to \mathbb{R}$, both given in closed-form the Appendix.*

The above result extends the generalized linear equilibrium characterized in Proposition 1. Combining the expression for the optimal demand from Lemma 6 and the equilibrium price in this proposition immediately yields the equilibrium quantity demanded by each investor, which we record in the following corollary.

**Corollary 4.** *The equilibrium demand of investor $i$ is*

$$x_i = \tau \frac{\mu_i - \int \mu_j dj}{\sigma_s^2} \mathbf{1} + \kappa \mathbf{1} - z. \tag{27}$$

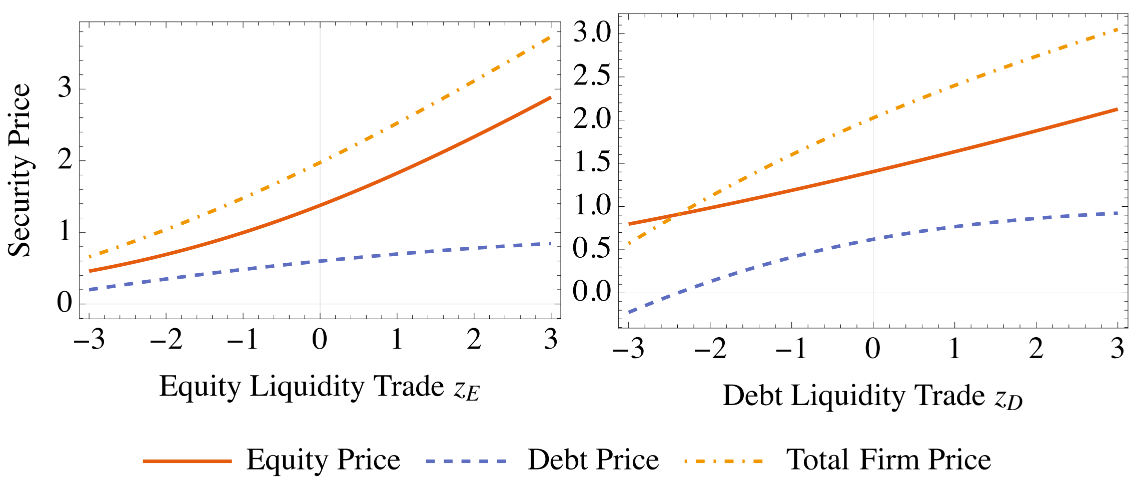
This result shows that the speculative portion of each investor's holdings are equal across the debt and equity markets, and, as in our benchmark model, are given by $\tau \frac{\mu_i - \int \mu_j dj}{\sigma_s^2}$. Thus, investors' debt and equity demands differ if and only if the liquidity trade in the debt and equity markets differs.

## 7.1 Cross-Market Demand Spillovers

Figure 3 illustrates how the equity and debt prices respond to liquidity-trader demand in each market. Specifically, liquidity-trader demand for a given security affects not only the price of that security, but also the price of the other security. Intuitively, this is driven by both information and risk effects. Since demand in either security may be perceived as informed, it raises investors' expectations of cash flows, and consequently, the price of both securities. In addition, holding debt (equity) exposes an investor to the risk of the firm's underlying cash flows, which also makes them view the equity (debt) as riskier. Thus, equity demand also raises the price of debt, and vice versa, via investor risk aversion. However, demand for equity has a much stronger effect on the equity price than on the debt price through this risk aversion effect, and vice versa. As a result, the demand spillover between the two markets is incomplete in the sense that $z_E$ has a stronger impact on the equity price than $z_D$, and $z_D$ has a stronger impact on the debt price than $z_E$.

Figure 3: Cross-Market Demand Spillovers

This figure plots the expected security prices conditional on equity liquidity trade $z_E$ (left panel) and debt liquidity trade $z_D$ (right panel). The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.

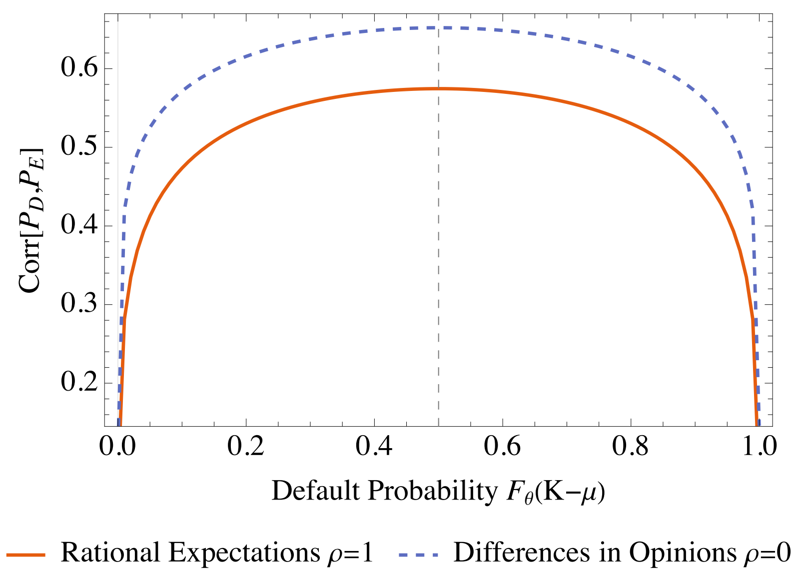



As in the baseline model (e.g., see Figure 1), the equity (debt) price is a convex (concave) function of demand in *each* market. This, in turn, implies that our main results regarding expected returns continue to hold in this case. Interestingly, however, the total price of the firm (i.e., $P_E + P_D$) is convex in equity liquidity-trader demand, but concave in debt liquidity-trader demand. As we discuss further in the next subsection, this implies that the Modigliani-Miller theorem no longer holds in this setting. Instead, the sum of the firm's debt and equity prices is greater (lower) than the price of an unlevered firm, on average, when the volatility of equity liquidity trading is higher (lower) than that of debt liquidity trading.

The incomplete spillover of demand shocks across securities also affects the correlation between equity and debt prices. As Figure 4 illustrates, the correlation between debt and equity prices is maximized when the likelihood of default is 50%.[17] Intuitively, when the default probability approaches zero, the payoff to debt is almost risk-free, and so demand shocks in either security have little impact on the debt price but significant impact on the equity price. Similarly, when the probability of default approaches one, the value of equity approaches zero and is relatively insensitive to demand shocks, but the price of debt remains responsive to such shocks. As a result, the correlation in prices approaches zero in both extremes. In contrast, for intermediate levels of distress, both security prices are sensitive to demand shocks, and so price correlation is high.

Figure 4: Leverage and Debt-Equity Price Correlation

This figure plots the correlation between the debt and equity prices as a function of the firm's default risk. The parameters are set to: $\mu = 3; K = 2; \sigma_\theta^2 = 1; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \tau = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.




[17] Pasquariello and Sandulescu (2021) derive a similar result in a Kyle setting with a single, informed investor and risk-neutral market makers in equity and debt markets, who only condition on the order-flow in their own market. As such, the market making in their model is segmented. In contrast, markets are integrated in our setting: investors can update their beliefs from equity and debt prices and can trade in both markets.

## 7.2 Capital Structure and Total Firm Valuation

We next show that, in contrast to our baseline specification, the Modigliani-Miller theorem does not hold, i.e., the firm's equity and debt prices do not, in general, sum to the price of the unlevered firm: $\mathbb{E}[P_E + P_D] \neq \mathbb{E}[P_U]$. As a result, the firm's capital structure can meaningfully impact its value.

In Figure 5, we show that the expected price of the debt plus equity relative to the price of the unlevered firm depends upon the relative amount of liquidity trade in the two markets. For instance, the left panel of Figure 5 plots $\mathbb{E}[P_E + P_D]$ as a function of $\sqrt{\mathbb{V}[z_E]}$, holding fixed the volatility of debt liquidity trade (i.e., $\sqrt{\mathbb{V}[z_D]}$). The plot illustrates that the expected value of debt plus equity is higher than the expected value of the unlevered firm (i.e., $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U] = 1$) if and only if the volatility of equity liquidity trading is higher than that of debt liquidity trading (i.e., $\mathbb{V}[z_E] > \mathbb{V}[z_D]$). Intuitively, this is because liquidity trade in the equity market does not fully spill over into the debt market. As such, the price-increasing effect that equity liquidity-trading volatility has on equity prices tends to raise the overall value of the firm.

This result holds irrespective of whether investors use the information in prices (i.e., whether $\rho = 1$ or $\rho = 0$), but is stronger when investors do not condition on prices (i.e., when $\rho = 0$). This is because, even though the effect of belief dispersion on expected prices in the two securities precisely offset, investors face more uncertainty when they do not condition on prices, and this increases the sensitivity of prices to liquidity trading shocks. The right panel of Figure 5 shows the same result by plotting the expected value of debt plus equity as a function of debt liquidity-trading volatility, holding fixed the equity liquidity-trading volatility.

Next, in Figure 6, we study how the firm's capital structure influences its valuation. The left-hand panel considers the case in which equity liquidity-trading volatility exceeds that in the debt, so that $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U]$. In this case, there is an interior optimal capital structure that maximizes the firm's valuation. Intuitively, an all equity or all debt firm is suboptimal as, in either case, the firm has linear payoffs, and so $\mathbb{E}[P_E + P_D] \to \mathbb{E}[P_U]$. For the same reason, as shown in the right-hand panel, when liquidity-trading volatility in debt exceeds that in equity so that $\mathbb{E}[P_E + P_D] < \mathbb{E}[P_U]$, the optimal capital structure is either all equity or all debt.

Figure 5: Violations of Modigliani-Miller

This figure plots the total expected firm price, $\mathbb{E}[P_E + P_D]$, as a function of liquidity trade volatility in the two markets. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$. Note that $\mathbb{E}[P_U] = \mu = 1$.

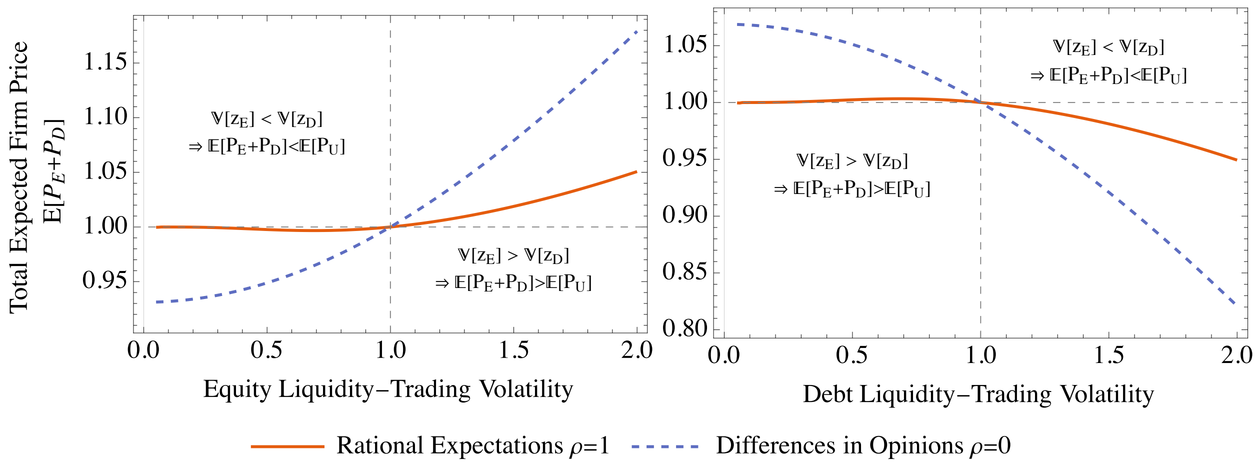
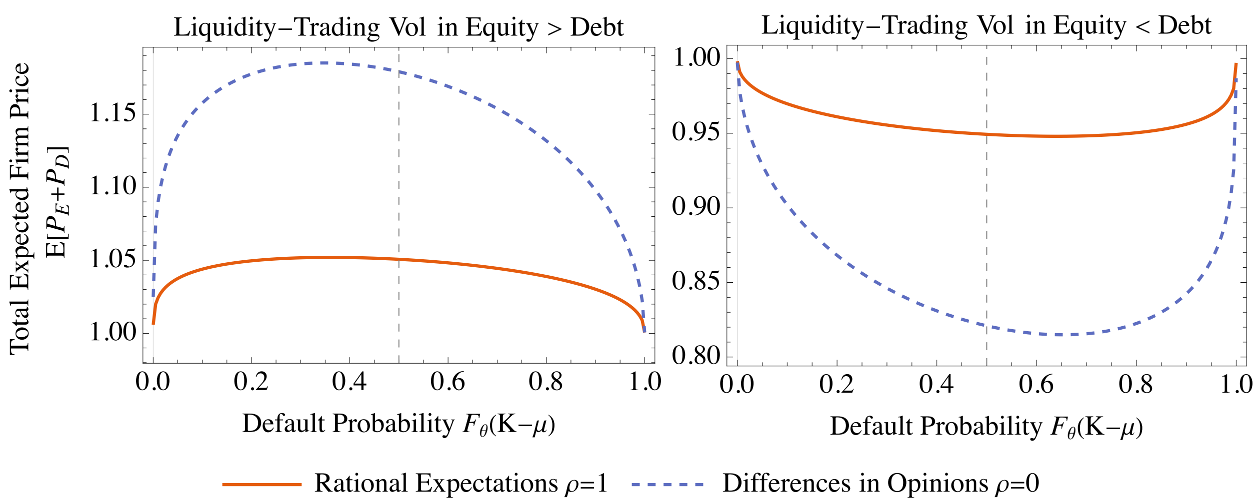



Figure 6: Optimal Capital Structure

This figure plots the total expected firm price, $\mathbb{E}[P_E + P_D]$, as a function of the firm's leverage, paramaterized in terms of its ex-ante probability of default, $F_\theta(K - \mu)$. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$. In the left-hand panel, $\mathbb{V}[z_E] = 4$ and $\mathbb{V}[z_D] = 1$ and in the right-hand panel, $\mathbb{V}[z_E] = 1$ and $\mathbb{V}[z_D] = 4$. Note that $\mathbb{E}[P_U] = \mu = 1$.




# 8 Empirical Implications

Our model generates predictions on expected debt and equity returns and how these returns vary with financial distress, the quality of public information, the sign of past news, disagreement, and the prevalence of liquidity trade. We summarize these predictions and their relation to existing empirical work below; several are consistent with existing empirical analyses while others have yet to be tested. Given that a key feature of our model is the ability of both diversely-informed investors and liquidity traders to take positions in debt, our results apply most clearly to public debt markets. As such, when referencing debt markets, we focus on the literature on public bond markets. Moreover, our findings speak specifically to expected returns after controlling for systematic risk exposures, i.e., the predictions are about "alphas" for debt and equity, after accounting for the appropriate "betas" that capture systematic risk exposures. Finally, we focus on our predictions for firms with less than 50% probability of default, as such firms represent the vast majority of publicly-traded stocks.

**Distress risk and expected equity returns.** Several empirical studies examine the relationship between distress risk and equity returns, controlling for standard systematic risk exposures. However, such a relationship is difficult to reconcile with representative-agent asset-pricing theory due to the forces of diversification, leading the literature to propose that distress risk is mispriced (e.g., Campbell et al. (2008)). In addition, the findings in these studies vary with the proxy for distress risk and the sample period examined. For instance, applying different metrics of financial distress, Campbell et al. (2008) find a negative relation and Chava and Purnanandam (2010) a positive relation between distress risk and returns. Relatedly, Penman et al. (2007) and Caskey et al. (2012) find a negative relationship between accounting-based metrics of leverage and expected returns.

Our model offers a potential explanation for such mixed results. It suggests that, when investors possess private information, the relationship between distress risk and expected stock returns depends on (i) the extent to which investors disagree and (ii) the prevalence of liquidity trade in a firm's stock and bonds. Existing work proposes several proxies for disagreement, such as volume and forecast dispersion, but few proxies for the prevalence of liquidity trade. Potential proxies for liquidity-trader activity in our model are the variability of returns relative to the variability of the firm's fundamentals or the variability of holdings by unsophisticated investors over time, or the frequency with which a stock is included or dropped from indices. Thus, our analysis motivates the following regressions, where main effects are omitted for brevity and the predicted signs are presented below the coefficients:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{>0} Distress_t \times Disagreement_t + \underbrace{\beta_{2,E}}_{<0} Distress_t \times LiqTradeVol_t;$$

$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{<0} Distress_t \times Disagreement_t + \underbrace{\beta_{2,D}}_{>0} Distress_t \times LiqTradeVol_t.$$

These predictions follow from the observation that the expected return on equity (debt) is decreasing (increasing) in the difference in variance, $\Omega - \sigma_\theta^2$, which is decreasing in investor disagreement and increasing in liquidity-trading volatility (e.g., see (19)).

**Disagreement and expected equity / debt returns.** Considerable prior literature studies the relationship between disagreement and expected equity returns. Despite the intuitive theory that disagreement together with short-sale constraints should lead to overpricing, this literature finds results that depend on the empirical proxy, firm size, and time period considered (e.g., Diether, Malloy, and Scherbina (2002), Johnson (2004), Banerjee (2011), Cen, Wei, and Yang (2017), Hou, Xue, and Zhang (2020), Chang, Hsiao, Ljungqvist, and Tseng (2022)).[18] Our model, which does not feature short-sale constraints, predicts that an increase in the extent to which investors agree to disagree *increases* expected equity returns, which may help to reconcile this evidence. Notably, the economic forces that drive our findings strengthen with leverage. This is roughly consistent with the evidence in Buraschi, Trojani, and Vedolin (2014) that the relation between *economy-wide* disagreement and expected stock returns increases with leverage, though our model predicts this relationship should arise for firm-specific disagreement.

**Public information quality and expected equity / debt returns.** Our model also speaks to the longstanding literature that studies public information quality and expected stock returns (see the reviews by Dechow, Ge, and Schrand (2010) and Bertomeu and Cheynel (2015)), and the more sporadic work that considers public information quality and expected bond returns. Our model predicts that there is cross-sectional and time-series variation in the relation between public information quality and expected bond and equity returns. Specifically, our analysis in Section 6.1 motivates the following regressions:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{<0} InfoQuality_t \times Disagreement_t + \underbrace{\beta_{2,E}}_{>0} InfoQuality_t \times LiqTradeVol_t;$$

$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{>0} InfoQuality_t \times Disagreement_t + \underbrace{\beta_{2,D}}_{<0} InfoQuality_t \times LiqTradeVol_t,$$

where disagreement is measured *after* the release of the public information under consideration (but prior to the period over which future returns are measured).

The existing literature on accounting information and bond returns provides some evidence consistent with our predictions. Bharath, Sunder, and Sunder (2008) find that higher-

[18] Johnson (2004) shows equity valuation should increase in a firm's idiosyncratic volatility since equity payoffs are a call option on the firm's assets. To the extent that disagreement proxies are related to idiosyncratic volatility, he argues that this can justify the observed negative relation between equity returns and disagreement proxies. In our model, disagreement and idiosyncratic volatility (driven by liquidity trading) have different effects on equity valuation, and consequently, expected returns.

quality accounting is associated with lower interest spreads in the public bond market, consistent with our model's predictions when liquidity traders tend to be inactive and disagreement is high. Bonsall and Miller (2017) find that lower quality accounting information (as proxied by financial statement readability) increases disagreement and tends to be associated a higher cost of debt. Chang et al. (2022) exploits staggered implementation of EDGAR to show that for firms with better public information (i.e., firms included in EDGAR), disagreement around earnings announcements resolves more quickly, and so returns are more negative. The findings in these two papers are consistent with our model's predictions when $\rho$ and liquidity-trading volatility (i.e., $\sigma_z$) are sufficiently low – see Proposition 3 (ii).

**Post-announcement drift in equity / debt.** Our analysis of announcement drift speaks to returns following the release of any public news and, as such, to the empirical literature on post-earnings announcement drift in stocks and bonds. Similar to the predictions discussed in the previous applications, our analysis in Section 6.2 motivates the following regression analysis:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{<0} PublicNews_t \times Disagreement_t + \underbrace{\beta_{2,E}}_{>0} PublicNews_t \times LiqTradeVol_t;$$
$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{>0} PublicNews_t \times Disagreement_t + \underbrace{\beta_{2,D}}_{<0} PublicNews_t \times LiqTradeVol_t,$$

Public news may be measured using either the direction of a past earnings announcement or analyst forecast revision, or a return-based proxy such as equity returns around prior earnings or equity returns over the past year.

Existing research provides evidence consistent with these predictions in both the equity and debt markets. Garfinkel and Sokobin (2006) find that post-earnings announcement drift in stocks goes *down* with disagreement when measured using forecast dispersion.[19] Moreover, using a regression specification to the one above, Nozawa et al. (2021) finds that price drift in the bond market goes *up* with disagreement, using a number of proxies for disagreement. Our model suggests that these effects are likely to be amplified for firms in financial distress.

**Covariance between expected equity and debt returns.** A central prediction of our model is that a firm's expected equity and debt returns are inversely related. The key intuition is that equity and debt payoffs are skewed in opposite directions. Again, our predictions concern "alphas" for debt and equity, after accounting for the appropriate "betas" that capture systematic risk exposures. Such exposures likely lead to common sources of

[19]While they find the opposite result when disagreement is measured using trading volume, in our model, trading volume can also represent liquidity trade. Thus, our model does not make a clear prediction on how post-earnings announcement drift varies with volume.

variation in expected debt and equity returns that counteract the source of return variation we study. Past empirical evidence showing that several of the factors that predict equity returns do not predict debt returns is consistent with this finding (Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017), Choi and Kim (2018), Bali, Subrahmanyam, and Wen (2021)).

**Co-movement in equity and bond prices.** The analysis in Section 7.1 implies that shocks to demand in either security impact both debt and equity prices, and so induce correlation in these securities' prices. Our results are broadly consistent with the evidence in Back and Crotty (2015), who show that while the unconditional correlation between stock and bond returns is low, the correlation in the parts driven by order flow is quite large. Our analysis suggests that the stock-bond correlation is higher when liquidity trading in the two markets is more correlated. Our results are also consistent with the evidence of Pasquariello and Sandulescu (2021), who document that the stock-bond correlation is low when the firm-level default probability is either very high or very low, but higher otherwise.

**Capital structure and firm valuation.** When liquidity traders' demands in equity and bond markets are not identical, our model implies that the capital structure of the firm affects its total valuation, even in the absence of traditional frictions (e.g., tax shields of debt, distress costs). Since we expect that for most firms, the probability of default is lower than 50% and the volatility of liquidity trading in equity is higher than that in debt, our model predicts that an increase in leverage leads to an increase in firm value. Moreover, the model predicts that, ceteris paribus, the impact of an increase in leverage is larger when investors dismiss the information in prices.

# 9 Conclusion

We develop a model where privately-informed, risk-averse investors trade alongside liquidity traders in the debt and equity of a firm. We show that the impact of private and public information on security valuation depends on the firm's likelihood of default, the intensity of liquidity trading in each market, and the extent to which investors learn from prices. We find that security prices tend to exhibit post-announcement drift or reversals even when investors exhibit rational expectations. Finally, we show that a firm's capital structure can affect its total valuation even in the absence of traditional frictions associated with debt issuance (e.g., tax shields, distress costs).

As the previous section highlights, our model generates a number of novel empirical predictions about the relation among disagreement, liquidity trading, distress risk and debt and equity valuation. Moreover, our model serves as a useful benchmark for future theoretical analysis. For instance, it would be interesting to explore the incentives of investors to acquire information (e.g., Davis (2017)) in our setting when the liquidity trading in debt and equity are not identical, as well as to study the effects of segmentation across debt and equity markets. It would also be interesting to study how joint trade in equity and debt influence managers' investment decisions, both through their costs of capital and through managerial learning from debt and equity prices (see Davis and Gondhi (2019) for a model of the latter).

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# A Proofs

## A.1 Proof of Lemmas 1 and 2

These results are limiting cases of Proposition 1 below.

## A.2 Proof of Lemma 3

This result is a special case of Lemma 6, where we define $G(P) \equiv \sigma_s^2 \times (g')^{-1}(P)$ to condense notation in the statement of the Lemma.

## A.3 Proof of Proposition 1

The existence of a generalized linear equilibrium is a special case of Proposition 6 and the representation of the equilibrium demands is a special case of Corollary 4. It remains to show that the expression for the equilibrium price from Proposition 6 can be represented in terms of the $M_E$ and $M_D$ functions and that the equilibrium debt and equity prices sum to $P_U$. From Proposition 6, we have that the equilibrium price vector satisfies

$$\begin{aligned} P &= g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_U - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \\ &= g'\left(\mathbf{1}\frac{P_U(\theta, z)}{\sigma_s^2}\right), \end{aligned}$$

where the second line uses the definition of $P_U$ (from Lemma 1) to simplify the argument of the gradient $g'$ and where the function $g : \mathbb{R}^2 \to \mathbb{R}$ satisfies

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right).$$

Computing the two partial derivatives that make up the gradient $g'$ yields

$$\begin{aligned} \frac{\partial g}{\partial y_1} &= \left(\sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}\right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \\ &\quad + K\frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}; \\ \frac{\partial g}{\partial y_2} &= \left(\sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)}{1 - \Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)} - K\right) \frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}. \end{aligned}$$

Evaluating these expressions at $y_1 = y_2 = \frac{P_U}{\sigma_s^2}$ gives the debt and equity prices, respectively:

$$\begin{aligned} P_D = \left.\frac{\partial g}{\partial y_1}\right|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U - \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{\Phi\left(\frac{K-P_U}{\sigma_s}\right)}\right)\Phi\left(\frac{K-P_U}{\sigma_s}\right) + K\left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_D(P_U, \sigma_s^2, K), \end{aligned}$$

and

$$\begin{aligned} P_E = \left.\frac{\partial g}{\partial y_2}\right|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U + \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)} - K\right)\left(1-\Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_E(P_U, \sigma_s^2, K), \end{aligned}$$

as claimed. Adding the expressions above immediately yields that the overall firm value is:

$$P_D + P_E = M_E\left(P_U, \sigma_s^2, K\right) + M_D\left(P_U, \sigma_s^2, K\right) = P_U.$$

## A.4 Proof of Corollary 1

It is straightforward to verify that $M_E(x, \cdot, \cdot)$ and $M_D(x, \cdot, \cdot)$ increase in $x$. Hence, results (i)-(iii) follow from the fact that, as can be seen in Lemma 1, $P_U$ increases in $\theta$ and $z$ and decreases in $\kappa$. To verify that the equity and debt prices decrease and increase in $K$, respectively, note:

$$\begin{aligned} \frac{\partial}{\partial K} M_D(P_U, \sigma_s^2, K) &= -\frac{\partial}{\partial K} M_E(P_U, \sigma_s^2, K) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right) - \frac{K-P_U}{\sigma_s}\phi\left(\frac{K-P_U}{\sigma_s}\right) - \phi'\left(\frac{K-P_U}{\sigma_s}\right) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s^2}\right) > 0. \end{aligned}$$

## A.5 Proof of Proposition 2

Observe that $P_U$ is unconditionally normally distributed with mean

$$\begin{aligned} \mathbb{E}\left[P_U\right] &= \mathbb{E}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau}(z-\kappa)\right] \\ &= \int \mathbb{E}\left[\mu_j\right] dj - \frac{\sigma_s^2}{\tau}\kappa \\ &= \mu - \frac{\sigma_s^2}{\tau}\kappa. \end{aligned} \tag{28}$$

Thus, we have:

$$\begin{aligned}
\mathbb{E}\left[P_E\left(P_U\right)\right] &= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(P_U,\sigma_s^2\right)\right]\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x+P_U-K,0\right)|x\sim N\left(0,\sigma_s^2\right)\right]\right\} \\
&= \mathbb{E}\left[\max\left(x+y-K,0\right)|x\sim N\left(0,\sigma_s^2\right),y\sim N\left(\mathbb{E}\left[P_U\right],\mathbb{V}\left[P_U\right]\right)\right] \\
&= \mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(\mu-\frac{\sigma_s^2}{\tau}\kappa,\sigma_s^2+\mathbb{V}\left[P_U\right]\right)\right] \\
&= M_E\left(\mu-\frac{\sigma_s^2}{\tau}\kappa,\Omega,K\right).
\end{aligned}$$

The debt result follows analogously.

We next show how the equity payoffs compare to equity expected cash-flows; the result for debt follows analogously. Observe that, as $\kappa\to 0$, the expected equity price approaches $M_E\left(\mu,\Omega,K\right)$. Now, the expected equity payoff equals:

$$\mathbb{E}\left[\max\left(\theta-K,0\right)\right] = M_E\left(\mu,\sigma_\theta^2,K\right).$$

Thus, equity earns negative expected returns if and only if $M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)>0$ and earns positive expected returns if and only if $M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)<0$. Now, note that the derivative of $M_E$ with respect to its second argument is:

$$\begin{aligned}
\frac{\partial M_E\left(\mu,x,K\right)}{\partial x} &= \frac{\partial}{\partial x}\left\{x^{\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)-\left[1-\Phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)\right]\left(K-\mu\right)\right\} \\
&= \frac{1}{2}x^{-\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)>0.
\end{aligned}$$

Thus, we have that:

$$M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)\gtrless 0\Leftrightarrow\Omega-\sigma_\theta^2\gtrless 0,$$

which completes the proof of statements (i) and (ii) in the proposition.

## A.6 Proof of Corollary 2

We would like to write down an explicit expression for $\Omega-\sigma_\theta^2=\mathbb{V}(P_U)+\sigma_s^2-\sigma_\theta^2$ in terms of the deep parameters of the model. We have

$$\begin{aligned}
\mathbb{V}\left(P_U\right) &= \left(\sigma_s^2\right)^2\mathbb{V}\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)\left(\overline{s}+\beta z\right)\right) \\
&= \left(\sigma_s^2\right)^2\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2\sigma_\theta^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2\beta^2\sigma_z^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= \left(\sigma_s^2\right)^2 \left( \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right)^2 \sigma_\theta^2 - 2 \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) \right. \\
&\qquad \left. + \frac{1}{\sigma_\theta^2} + \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right)^2 \beta^2 \sigma_z^2 \right) \\
&= \sigma_\theta^2 - \sigma_s^2 + \left(\sigma_s^2\right)^2 \left( -\frac{1}{\sigma_\varepsilon^2} - \frac{1}{\rho \sigma_p^2} + \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right)^2 \beta^2 \sigma_z^2 \right) \\
&= \sigma_\theta^2 - \sigma_s^2 + \left(\sigma_s^2\right)^2 \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) \left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{\left(1 - \rho^2\right) \sigma_\theta^2 + \beta^2 \sigma_z^2} \right),
\end{aligned}
$$

where the fourth equality uses the definition of $\sigma_s^2 = \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$ to simplify and collect terms, and the final line substitutes in for $\sigma_p^2 = \frac{1-\rho^2}{\rho^2} \sigma_\theta^2 + \frac{\beta^2 \sigma_z^2}{\rho^2}$ and groups terms. Hence,

$$
\begin{aligned}
\Omega - \sigma_\theta^2 &= \left(\sigma_s^2\right)^2 \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) \left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{\left(1 - \rho^2\right) \sigma_\theta^2 + \beta^2 \sigma_z^2} \right) \\
&\propto \left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{\left(1 - \rho^2\right) \sigma_\theta^2 + \beta^2 \sigma_z^2} \right),
\end{aligned}
$$

and therefore

$$
\operatorname{sgn}\left(\Omega - \sigma_\theta^2\right) = \operatorname{sgn}\left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{\left(1 - \rho^2\right) \sigma_\theta^2 + \beta^2 \sigma_z^2} \right). \tag{29}
$$

It is immediate that the expression in the sgn function in eq. (29) is strictly increasing in $\rho$ and takes value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1$ at $\rho = 0$ and value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 0$ at $\rho = 1$. It follows that if $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 1 \Leftrightarrow \sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$ then $\Omega - \sigma_\theta^2 > 0$ for all $\rho \in [0, 1]$. On the other hand, if $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists a $\rho^* \in (0, 1)$ such that $\Omega - \sigma_\theta^2 < 0$ for $\rho \in [0, \rho^*)$ and $\Omega - \sigma_\theta^2 > 0$ for $\rho \in (\rho^*, 1]$.

## A.7 Derivation of Expression 19

We have

$$
\begin{aligned}
&\mathbb{V}\left[ \int \mu_j dj + \frac{\sigma_s^2}{\tau} z \right] - \mathbb{V}_i\left[\mu_i\right] \\
&= \mathbb{V}\left[ \int \mu_j dj \right] - \mathbb{V}_i\left[\mu_i\right] + \mathbb{V}\left[ \frac{\sigma_s^2}{\tau} z \right] + 2\mathbb{C}\left[ \int \mu_j dj, \frac{\sigma_s^2}{\tau} z \right] \\
&= \mathbb{V}\left[ \int \mu_j dj \right] - \mathbb{V}[\mu_i] + \mathbb{V}[\mu_i] - \mathbb{V}_i\left[\mu_i\right] + \mathbb{V}\left[ \frac{\sigma_s^2}{\tau} z \right] + 2\mathbb{C}\left[ \int \mu_j dj, \frac{\sigma_s^2}{\tau} z \right],
\end{aligned} \tag{30}
$$

where the first equality uses the standard result for expressing the variance of a sum, and the second equality adds and subtracts $\mathbb{V}[\mu_i]$. The first two terms in eq. (30) can be written as

$$\begin{aligned}\mathbb{V}\left[\int \mu_j dj\right] - \mathbb{V}[\mu_i] &= \mathbb{V}\left[\sigma_s^2\left(\frac{\int s_j dj}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] - \mathbb{V}\left[\sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] \\ &= \mathbb{V}\left[\sigma_s^2\left(\frac{\theta}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] - \mathbb{V}\left[\sigma_s^2\left(\frac{\theta+\varepsilon_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] \\ &= -\left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2. \end{aligned} \tag{31}$$

Expression (19) now follows immediately upon noting that

$$\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right] = \mathbb{E}\left[\left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\varepsilon_i\right)^2\right] = \left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2, \tag{32}$$

which allows one to express eq. (31) as $\mathbb{V}\left[\int \mu_j dj\right] - \mathbb{V}[\mu_i] = -\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right]$

## A.8 Proof of Corollary 3

**Part (i)** We consider equity returns; the proof for debt returns is analogous. We have

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) &= \operatorname{sgn}\left(\operatorname{sgn}\left(\mathbb{E}[R_E]\right)\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\ &= \operatorname{sgn}(\mathbb{E}[R_E])\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\ &= -\operatorname{sgn}\left(\Omega - \sigma_\theta^2\right)\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right). \end{aligned}$$

Differentiating the expected return with respect to $K$ yields

$$\begin{aligned}\frac{\partial}{\partial K}\mathbb{E}[R_E] &= \frac{\partial}{\partial K}\left(M_E(\mu, \sigma_\theta^2, K) - M_E(\mu, \Omega, K)\right) \\ &= \frac{\partial}{\partial K}\int_\Omega^{\sigma_\theta^2}\frac{\partial}{\partial x}M_E(\mu, x, K)dx \\ &= \int_\Omega^{\sigma_\theta^2}\frac{\partial^2}{\partial K \partial x}M_E(\mu, x, K)dx, \end{aligned}$$

where the second equality uses the fundamental theorem of calculus to express the difference in the $M_E$ function an integral. Computing the cross-partial derivative of $M_E$ yields

$$\frac{\partial^2}{\partial K \partial x}M_E(\mu, x, K) = \frac{\partial}{\partial K}\frac{1}{2}\frac{1}{\sqrt{x}}\phi\left(\frac{K-\mu}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\phi'\left(\frac{K-\mu}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\frac{\mu-K}{\sqrt{x}}\phi\left(\frac{K-\mu}{\sqrt{x}}\right).$$

Hence, for $K < \mu$, we have

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_{\Omega}^{\sigma_\theta^2} \underbrace{\frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K)}_{>0} dx\right) \\ &= -\operatorname{sgn}(\Omega - \sigma_\theta^2),\end{aligned}$$

and therefore

$$\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = \operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) > 0.$$

On the other hand for $K > \mu$,

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_{\Omega}^{\sigma_\theta^2} \underbrace{\frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K)}_{<0} dx\right) \\ &= \operatorname{sgn}(\Omega - \sigma_\theta^2),\end{aligned}$$

and therefore

$$\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = -\operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) < 0.$$

Because $|\mathbb{E}[R_E]|$ is strictly increasing in $K$ for $K < \mu$ and strictly decreasing in $K$ for $K > \mu$, it follows that $|\mathbb{E}[R_E]|$ is hump-shaped in $K$ and achieves its maximum at $K = \mu$.

**Part (ii)** Consider debt returns. We have that:

$$\begin{aligned}\frac{\partial \mathbb{E}\left[R_D\right]}{\partial \sigma_z} &= \frac{\partial M_D\left(\mu, \sigma_\theta^2, K\right)}{\partial \sigma_z} - \frac{\partial M_D\left(\mu, \Omega, K\right)}{\partial \sigma_z} \\ &= -\frac{\partial M_D\left(\mu, \Omega, K\right)}{\partial \Omega}\frac{\partial \Omega}{\partial \sigma_z} \propto \frac{\partial \Omega}{\partial \sigma_z}.\end{aligned}$$

Similarly, for equity returns, we obtain $\frac{\partial \mathbb{E}[R_E]}{\partial \sigma_z} \propto -\frac{\partial \Omega}{\partial \sigma_z}$. Now,

$$\frac{\partial \Omega}{\partial \sigma_z} = \frac{2\sigma_\theta^4 \sigma_z \sigma_\varepsilon^4 \begin{pmatrix} \tau^4 \sigma_\theta^2 \sigma_z^2 \sigma_\varepsilon^4 \left(3\left(1-\rho^2\right)^2 \sigma_\theta^4 + \left(3 - \rho^3(\rho+2)\right)\sigma_\theta^2 \sigma_\varepsilon^2 + 2\rho^3 \sigma_\varepsilon^4\right) \\ +(1-\rho)\tau^6 \sigma_\theta^4 \left((1-\rho)^2(\rho+1)^3 \sigma_\theta^4 + (1-\rho)(\rho+1)\left(2\rho^2+\rho+1\right)\sigma_\theta^2 \sigma_\varepsilon^2 + 2\rho^3 \sigma_\varepsilon^4\right) \\ 3\tau^2 \sigma_\theta^2 \sigma_z^4 \sigma_\varepsilon^8 \left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right) + \sigma_z^6 \sigma_\varepsilon^{12}\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) \end{pmatrix}}{\tau^2\left(\sigma_z^2 \sigma_\varepsilon^4\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) + \tau^2 \sigma_\theta^2\left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right)\right)^3} > 0.$$

## A.9 Proof of Proposition 3

As stated in the text, the derivations of all previous results continue to hold upon incorporating the public signal $y$ into the investors' belief updates. Doing so, and grouping terms appropriately, we obtain that the unlevered firm price satisfies:

$$P_U(\cdot) = \mu + \sigma_s^2 \left( \frac{1}{\sigma_\eta^2} y + \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) (\bar{s} + \beta z) - \frac{\kappa}{\tau} \right),$$
$$\text{where} \quad \beta = \frac{\sigma_\varepsilon^2}{\tau},\ \sigma_p^2 = \frac{1-\rho^2}{\rho^2} \sigma_\theta + \frac{\beta^2 \sigma_z^2}{\rho^2},\ \sigma_s^2 = \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}.$$

By iterated expectations, we again obtain that, for $\kappa = 0$, $\mathbb{E}[P_U] = \mu$. So, for $x \in \{E, D\}$:

$$\begin{aligned} \mathbb{E}[P_x] &= \mathbb{E}\left[ M_x(P_U(\theta, z, y), \sigma_s^2, K) \right] \\ &= M_x(\mu, \Omega, K), \end{aligned}$$

where we again define $\Omega = \mathbb{V}[P_U] + \sigma_s^2$. Thus, public information quality increases expected equity (debt) returns if and only if it decreases (increases) $\Omega - \sigma_\theta^2$. As in the proof of Corollary 2 we can write

$$\Omega - \sigma_\theta^2 = \left(\sigma_s^2\right)^2 \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) \left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\sigma_\theta^2 + \beta^2 \sigma_z^2} \right), \tag{33}$$

where here $\sigma_s^2 = \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$.

Note that, as established in Proposition 2 in the baseline model, the sign of $\Omega - \sigma_\theta^2$ determines the signs of expected returns on debt and equity. Note furthermore that the only object in eq. (33) that depends on $\sigma_\eta^2$ is $\sigma_s^2$, which is trivially increasing in $\sigma_\eta^2$. Hence, $\Omega - \sigma_\theta$ increases in $\sigma_\eta^2$ if and only if $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\sigma_\theta^2 + \beta^2 \sigma_z^2} > 0$. That is,

$$\operatorname{sgn}\left( \frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} \right) = \operatorname{sgn}\left( \frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\sigma_\theta^2 + \beta^2 \sigma_z^2} \right).$$

It is immediate that the expression in the sgn function in eq. (**??**) is strictly increasing in $\rho$ and takes value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1$ at $\rho = 0$ and value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 0$ at $\rho = 1$. It follows that if $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 1 \Leftrightarrow \sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$ then $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} > 0$ for all $\rho \in [0, 1]$. On the other hand, if $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists a $\rho^* \in (0, 1)$ such that $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} < 0$ for $\rho \in [0, \rho^*)$ and $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} > 0$ for $\rho \in [\rho^*, 1]$.

## A.10 Proof of Proposition 4

Fix $\kappa = 0$. Consider first the unlevered firm. Computing the expected return given the public signal $y$ yields

$$\begin{aligned}
\mathbb{E}[\mathcal{V} - P_U|y] &= \mathbb{E}[\mathcal{V}|y] - \mathbb{E}[P_U|y] \\
&= \mu + \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}} y - \mathbb{E}\left[ \mu + \sigma_s^2 \left( \frac{1}{\sigma_\eta^2} y + \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) (\overline{s} + \beta z) \right) \middle| y \right] \\
&= \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}} y - \sigma_s^2 \left( \frac{1}{\sigma_\eta^2} y + \left( \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}} y \right) \\
&= \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}} y - \sigma_s^2 \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}} y \\
&= 0.
\end{aligned}$$

This implies that conditional on the public signal, the unlevered firm always has expected return zero, regardless of the value of $\rho$. This also establishes that the conditional expected firm cash flow is equal to the conditional expectation of $P_U$, $\mathbb{E}[\mathcal{V}|y] = \mathbb{E}[P_U|y]$, which will be used in the next step of the proof.

Now, consider the equity in the levered firm. The result for debt is analogous. Using the notation from Definition 2 we can write

$$\mathbb{E}[V_E|y] = \mathbb{E}[\max\{\mathcal{V} - K, 0\}|y] = M_E(\mathbb{E}[\mathcal{V}|y], \mathbb{V}[\mathcal{V}|y], K) = M_E(\mathbb{E}[P_U|y], \mathbb{V}[\mathcal{V}|y], K)$$

where the final equality uses the previous result. Similarly,

$$\mathbb{E}[P_E|y] = \mathbb{E}[M_E(P_U, \sigma_s^2, K)|y] = M_E(\mathbb{E}[P_U|y], \mathbb{V}[P_U|y] + \sigma_s^2, K) = M_E(\mathbb{E}[P_U|y], \hat{\Omega}, K),$$

where the final equality defines $\hat{\Omega} \equiv \mathbb{V}[P_U|y] + \sigma_s^2$, which we note is constant with respect to $y$ due to the fact that the conditional variance $\mathbb{V}[P_U|y]$ does not depend on the realization of $y$.

Now, differentiating the conditional expected return with respect to $y$ yields

$$\frac{\partial}{\partial y}\mathbb{E}[V_E - P_E|y] = \frac{\partial}{\partial u}\left( M_E(u, \mathbb{V}[\mathcal{V}|y], K) - M_E(u, \hat{\Omega}, K) \right)\Bigg|_{u=\mathbb{E}[P_U|y]} \times \frac{\partial}{\partial y}\mathbb{E}[P_U|y]$$

It is immediate that $\frac{\partial}{\partial y}\mathbb{E}[P_U|y] = \frac{\partial}{\partial y}\mathbb{E}[\mathcal{V}|y] = \frac{\frac{1}{\sigma_\eta^2}}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\eta^2}} > 0$. Hence, we have

$$\operatorname{sgn}\frac{\partial}{\partial y}\mathbb{E}[V_E - P_E|y] = \operatorname{sgn}\frac{\partial}{\partial u}\left(M_E(u,\mathbb{V}[\mathcal{V}|y],K) - M_E(u,\hat{\Omega},K)\right)\Bigg|_{u=\mathbb{E}[P_U|y]}.$$

To sign this, note that by the fundamental theorem of calculus, we can express the difference in the $M_E$ function as an integral:

$$M_E(u,\mathbb{V}[\mathcal{V}|y],K) - M_E(u,\hat{\Omega},K) = \int_{\hat{\Omega}}^{\mathbb{V}[\mathcal{V}|y]} \frac{\partial}{\partial x} M_E(u,x,K)dx,$$

so that the object we desire to sign is

$$\frac{\partial}{\partial u}\int_{\hat{\Omega}}^{\mathbb{V}[\mathcal{V}|y]} \frac{\partial}{\partial x} M_E(u,x,K)dx\Bigg|_{u=\mathbb{E}[P_U|y]} = \int_{\hat{\Omega}}^{\mathbb{V}[\mathcal{V}|y]} \frac{\partial^2}{\partial u \partial x} M_E(u,x,K)\Bigg|_{u=\mathbb{E}[P_U|y]} dx. \tag{34}$$

We have

$$\frac{\partial^2}{\partial u\partial x}M_E(u,x,K) = \frac{\partial}{\partial u}\frac{1}{2}\frac{1}{\sqrt{x}}\phi\left(\frac{K-u}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\frac{K-u}{\sqrt{x}}\phi\left(\frac{K-u}{\sqrt{x}}\right).$$

Hence,

$$\frac{\partial^2}{\partial u\partial x}M_E(u,x,K)\Bigg|_{u=\mathbb{E}[P_U|y]} < 0$$

if and only if $\mathbb{E}[P_U|y] - K > 0 \Leftrightarrow \mathbb{E}[\mathcal{V}|y] - K > 0$.

Finally, returning to eq. (34), we therefore have that when $\mathbb{E}[\mathcal{V}|y] - K > 0$ the expression $\int_{\hat{\Omega}}^{\mathbb{V}[\mathcal{V}|y]} \frac{\partial^2}{\partial u\partial x}M_E(u,x,K)dx\Big|_{u=\mathbb{E}[P_U|y]}$ is positive if and only if $\hat{\Omega} > \mathbb{V}[\mathcal{V}|y]$ (i.e., if and only if the lower limit of integration is greater than the upper one). Following analogous steps to those in the proof of Corollary 2 to group terms, we can write

$$\begin{aligned}\hat{\Omega} - \mathbb{V}[\mathcal{V}|y] &= \left(\sigma_s^2\right)^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho\frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2}\right) \\ &\propto \left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho\frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2}\right).\end{aligned}$$

For $\rho = 1$, we therefore have

$$\operatorname{sgn}\left(\hat{\Omega} - \mathbb{V}[\mathcal{V}|y]\right) = \operatorname{sgn}\left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2}\right) > 0.$$

Hence, $\frac{\partial}{\partial u}\int_{\hat{\Omega}}^{\mathbb{V}[\mathcal{V}|y]}\frac{\partial}{\partial x}M_E(u,x,K)dx\Big|_{u=\mathbb{E}[P_U|y]} > 0$, so there is positive drift in the levered equity, as claimed in the Proposition.

Next, for $\rho = 0$, we have

$$\operatorname{sgn}\left(\hat{\Omega} - \mathbb{V}[\mathcal{V}|y]\right) = \operatorname{sgn}\left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1\right),$$

which is positive if and only if $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, so that there is drift in the levered equity if $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$ and reversal if $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, as claimed in the Proposition.

### A.11 Proof of Lemma 4

This is a special case of Lemma 6, where we define the function $G(P) = \sigma_s^2 \times (g')^{-1}(P)$ in the text.

### A.12 Proof of Proposition 5

This is a special case of Proposition 6 in which $\mu_z = (0,0)$ and $\Sigma_z$ is positive definite.

## B Equilibrium with arbitrary, correlated liquidity trading

In this section, we characterize the equilibrium in the fully-general version of the model in which liquidity trading $z = (z_D, z_E)$ follows a general bivariate normal distribution $N(\mu_z, \Sigma_z)$ where $\mu_z \in \mathbb{R}$ is an arbitrary vector of means, and $\Sigma_z$ is an arbitrary positive semi-definite covariance matrix. As in the text, we consider equilibria of the "generalized linear" form specified in Definition 3 where the endogenous price statistics take the form

$$s_p = \mathbf{1}\bar{s} + B\left(z - \mu_z\right).$$

with $B = \left(\begin{smallmatrix}\beta_{1D} & \beta_{1E}\\ \beta_{2D} & \beta_{2E}\end{smallmatrix}\right)$ the $2\times 2$ matrix of coefficients to be determined.

We begin by characterizing an arbitrary investor $i$'s conditional distribution of the vector of debt and equity payoffs, $V = (V_D, V_E)$, given arbitrary $N(\mu_i, \sigma_s^2)$ beliefs about the underlying firm cash flow $\mathcal{V}$.

**Lemma 5.** *Suppose that $\mathcal{V}$ is conditionally normally distributed with mean $\mu_i$ and variance $\sigma_s^2$. Then the vector $V = (\min(\mathcal{V}, K), \max(\mathcal{V} - K, 0))$ follows a bivariate exponential family* *with moment-generating function (MGF) that is finite for any $u \in \mathbb{R}^2$, and is given explicitly by*

$$\mathbb{E}_i\left[\exp\left\{u'V\right\}\right] = \exp\left\{g\left(u + \mathbf{1}\frac{\mu_i}{\sigma_s^2}\right) - g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2}\right)\right\}$$

*where the function $g : \mathbb{R}^2 \to \mathbb{R}$ is defined as*

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{\left(y_1 - y_2\right)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right). \tag{35}$$

*Proof.* (Lemma 5) The claim about finiteness follows immediately once we have proven that the MGF takes the given form since, by inspection, the function $g$ is finite on all of $\mathbb{R}^2$. The claim that the distribution is an exponential family also follows immediately from the functional form (see e.g., Sampson (1975), Hoffmann and Schmidt (1982)). To establish the expression for the MGF, write

$$\begin{aligned}
&\mathbb{E}_i\left[\exp\left\{u'V\right\}\right] \\
&= \int_{-\infty}^{\infty} \exp\left\{u_1 \min\left\{t, K\right\} + u_2 \max\left\{t - K, 0\right\}\right\} dF_\theta(t|s_i, s_p) \\
&= \int_{-\infty}^{K} \exp\left\{u_1 t\right\} dF_\theta(t|s_i, s_p) + \int_{K}^{\infty} \exp\left\{u_1 K + u_2\left(t - K\right)\right\} dF_\theta(t|s_i, s_p) \\
&= \exp\left\{\mu_i u_1 + \frac{1}{2}\sigma_s^2 u_1^2\right\}\Phi\left(\frac{K - \mu_i - \sigma_s^2 u_1}{\sigma_s}\right) \\
&\qquad + \exp\left\{\left(u_1 - u_2\right)K + \mu_i u_2 + \frac{1}{2}\sigma_s^2 u_2^2\right\}\left(1 - \Phi\left(\frac{K - \mu_i - \sigma_s^2 u_2}{\sigma_s}\right)\right) \\
&= \exp\left\{\frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s}\right) \\
&\qquad + \exp\left\{\left(u_1 - u_2\right)K + \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s}\right)\right) \\
&= \left(\exp\left\{\tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2} + u_1\right)^2\right\}\Phi\left(\tfrac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s}\right)\right. \\
&\qquad \left. + \exp\left\{\left(u_1 - u_2\right)K + \tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2} + u_2\right)^2\right\}\left(1 - \Phi\left(\tfrac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s}\right)\right)\right)\exp\left\{-\tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2}\right)^2\right\}.
\end{aligned}$$

Taking the logarithm, this expression is identical to that in the Lemma after recognizing that

$g$ as defined in the statement of the Lemma satisfies $g\left(\begin{smallmatrix} y \\ y \end{smallmatrix}\right) = \frac{1}{2}\sigma_s^2 y^2$ when both arguments are identical. $\square$

With trader beliefs pinned down, we next characterize the optimal demand.

**Lemma 6.** *Fix any $P = (P_D, P_E) \in \mathbb{R}^2$. The optimal demand of trader $i$ is given by*

$$x_i = \tau \left( \mathbf{1}\frac{\mu_i}{\sigma_s^2} - (g')^{-1}(P) \right)$$

*where $(g')^{-1} : \mathbb{R}^2 \to \mathbb{R}^2$ is the inverse of the gradient $g'\left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right) \equiv \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$.*

*Proof.* (Lemma 6) From Lemma 5, we can compute the trader's conditional expected utility given an arbitrary demand $x_i$ as

$$\mathbb{E}\left[ -\exp\left\{ -\frac{1}{\tau} x_i' (V - P) \right\} \right] = -\exp\left\{ \frac{1}{\tau} x_i' P + g\left( \mathbf{1}\frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau} x_i \right) - g\left( \mathbf{1}\frac{\mu_i}{\sigma_s^2} \right) \right\}.$$

Letting $g' = \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$ denote the gradient of $g$, the FOC is

$$0 = g'\left( \mathbf{1}\frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau} x_i \right) - P.$$

Note that the Hessian matrix $g'' \equiv \begin{pmatrix} \frac{\partial^2}{\partial y_1^2} g & \frac{\partial^2}{\partial y_1 \partial y_2} g \\ \frac{\partial^2}{\partial y_1 \partial y_2} g & \frac{\partial^2}{\partial y_2^2} g \end{pmatrix}$ is necessarily positive definite, owing to the fact that it is the matrix of 2nd derivatives of the cumulant generating function of $V$, which is strictly convex. It follows that the optimum, if it exists, is unique and the FOC is sufficient to characterize it. Hence, it suffices to show that there exists a demand $x_i \in \mathbb{R}^2$ that satisfies the FOC.

Due to the positive-definiteness of $g''$ it follows that the gradient $g' : \mathbb{R}^2 \to \mathbb{R}^2$ is injective and therefore invertible on its range. Furthermore, the range of $g'$ is all of $\mathbb{R}^2$ (i.e., $g'$ is surjective). This follows from the following. First, we have already established that $g$ is strictly convex. Second, it is easily verified from the definition of $g$ that we have $\|g'(u_n)\| \to \infty$ for any sequence $u_n \in \mathbb{R}^2$ with $\|u_n\| \to \infty$. These two observations allow us to apply Theorem 26.6 in Rockafellar (1970) to conclude that $g'$ maps $\mathbb{R}^2$ onto itself. Now, rearranging the FOC and performing the inversion of $g'$ delivers the expression for $x_i$ in the Lemma. $\square$

**Proposition 6.** *There exists an equilibrium in the financial market. The vector of equilib-*

*rium asset prices takes the form*

$$P = g'\left(\mathbf{1}\frac{\int \mu_j\, dj}{\sigma_s^2} - \frac{1}{\tau}\left(\kappa\mathbf{1} - z\right)\right). \tag{36}$$

*where the function $g' : \mathbb{R}^2 \to \mathbb{R}^2$ is given in closed-form in eqs. (42)–(43) the proof.*

1. *If $\Sigma_z$ is invertible, then the equilibrium price vector is*

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{37}$$

*where the equilibrium price signals coefficient matrix is diagonal $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$*

2. *If $\Sigma_z$ is singular and of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is identical in the two markets, $z_E = z_D = \zeta$ for $\zeta \sim N(0, \sigma_z^2)$), then the equilibrium price vector is*

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{38}$$

*where $s_p = \overline{s} + \beta\zeta$ is one-dimensional, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2}{\rho^2}\sigma_z^2$ with $\beta = \frac{\sigma_\varepsilon^2}{\tau}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$.*

3. *If $\rho < 1$, and $\Sigma_z$ is singular and not of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated but with different variances, or is perfectly negatively correlated, or at least one of the $z_j$ is constant), then the equilibrium price vector is given by*

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{39}$$

*where $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$, $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$, and the vector $a \in \mathbb{R}^2$ is defined in the proof.*

4. *If $\rho = 1$, and $\Sigma_z$ is singular and not of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$, then there exists a fully-revealing equilibrium in which $P_D = \min\{\mathcal{V}, K\}$ and $P_E = \max\{\mathcal{V} - K, 0\}$.*

*Proof.* (Proposition 6) Using the expression for trader demand from Lemma 6, the market clearing condition yields

$$\int x_j dj + z = \mathbf{1}\kappa$$

$$\Leftrightarrow \int x_j dj + z - \mu_z = \mathbf{1}\kappa - \mu_z$$

$$\Leftrightarrow \tau \left( \mathbf{1} \frac{\int \mu_j dj}{\sigma_s^2} - (g')^{-1}(P) \right) + z - \mu_z = \mathbf{1}\kappa - \mu_z$$

$$\Leftrightarrow P = g' \left( \mathbf{1} \frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \right). \tag{40}$$

Because the vector of the liquidity trade $z$ enters explicitly multiplied only by a scalar, we can conclude that in any equilibrium it suffices to consider only diagonal coefficient matrices $B$ with identical elements on the diagonal. That is, $B = \beta I$ for $\beta \in \mathbb{R}$ still to be determined.

A closed-form expression for the gradient $g'\left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right)$ follows from computing the partial derivatives of the function $g$ as defined in Lemma 5:

$$\frac{\partial g}{\partial y_1} = \left( \sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right)} \right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\} \Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\} \Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{41}$$

$$+ K \frac{\exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\} \Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{42}$$

$$\frac{\partial g}{\partial y_2} = \left( \sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)}{1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)} - K \right) \frac{\exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\} \Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)}. \tag{43}$$

To complete the proof and derive the explicit expressions in the Proposition, it is convenient to separately consider the cases of positive definite $\Sigma_z$ and singular $\Sigma_z$.

If $\Sigma_z$ is positive definite, then we can write the conditional moments explicitly as

$$\mu_i = \mathbb{E}\left[\mathcal{V} | s_i, s_p\right] = \mu + \sigma_s^2 \left( \frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1} \frac{1}{\rho} s_p \right), \text{ and} \tag{44}$$

$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V} | s_i, s_p\right) = \left( \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1} \right)^{-1} \tag{45}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 \mathbf{1}\mathbf{1}' + \frac{1}{\rho^2} B \Sigma_z B'$. Because $\Sigma_z$ is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, $\Sigma_p$, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho \to 0$.

Substituting the explicit expression for $\mu_i$ in the argument of $g'$ in eq. (40) and grouping terms yields

$$
\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2\frac{1}{\sigma_\varepsilon^2}\bar{s} + \sigma_s^2\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right) + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(\mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\bar{s} + \frac{\sigma_\varepsilon^2}{\tau}(z-\mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}
$$

Matching coefficients on the initial conjecture $s_p = \mathbf{1}\bar{s} + B(z - \mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$
\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)
$$

which, upon plugging back into $g'$, matches the expression in the Proposition.

If $\Sigma_z$ is singular, then the matrix $\Sigma_p$ that appears above is not invertible and the above expressions for beliefs do not apply directly.[20] Intuitively, in this case there is only a single shock to liquidity trading and so the vector of price-signals $s_p$ collapse to an informationally-equivalent one-dimensional signal.

If $\Sigma_z$ is of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated, with identical variance in both markets, as in the baseline model), then the price statistics themselves are necessarily identical across both markets (i.e., $s_{p1} = s_{p2}$). Abusing notation to let $s_p \in \mathbb{R}$ denote this common price statistic and $\zeta = z_D - \mu_{zD} = z_E - \mu_{zE} \in \mathbb{R}$ denote the common liquidity trade shock realization, the expressions for the conditional moments become

$$
\mu_i = \mathbb{E}[\mathcal{V}|s_i, s_p] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}s_p\right), \text{ and} \tag{46}
$$

$$
\sigma_s^2 = \mathbb{V}(\mathcal{V}|s_i, s_p) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1} \tag{47}
$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{1}{\rho^2}\beta^2\sigma_z^2$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for $\mu_i$ in the argument of $g'$ in eq. (40) (recalling that $\zeta \in \mathbb{R}$ denotes the common liquidity trade realization in this case) and grouping terms

[20]The cases can be handled in a unified way by re-representing the above expressions for the conditional moments in forms involving pseudo-inverses of $\Sigma_p$. However, to avoid tedious technical complications, we choose to treat the case of singular $\Sigma_z$ separately. Details of the unified treatment are available on request.

yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}\mathbf{1}\zeta - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2 \frac{1}{\sigma_\varepsilon^2}\overline{s} + \sigma_s^2 \frac{1}{\rho\sigma_p^2} s_p\right) + \mathbf{1}\frac{1}{\tau}\zeta - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\rho\sigma_p^2}s_p + \frac{1}{\sigma_\varepsilon^2}\left(\overline{s} + \frac{\sigma_\varepsilon^2}{\tau}\zeta\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

Matching coefficients on the initial conjecture $s_p = \overline{s} + \beta(z - \mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)$$

which, upon plugging back into $g'$, matches the expression in the Proposition.

If $\Sigma_z$ is singular but not of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., the liquidity trade is perfectly positively correlated but has different variances in the two markets, or is perfectly negatively correlated, or is constant in at least one of the markets), then price statistics $s_p = (s_{p1}, s_{p2})$ can be combined to solve for $\overline{s}$. That is, there exists a vector $a \in \mathbb{R}^2$ such that $\overline{s} = a's_p$.[21] Hence, the conditional moments for trader $i$ are

$$\begin{aligned}
\mu_i &= \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}a's_p\right), \text{ and} \\
\sigma_s^2 &= \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1},
\end{aligned}$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$ is strictly positive since $\rho < 1$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for $\mu_i$ in the argument of $g'$ in eq. (40) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}a's_p\right)\right) + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}\left(\mathbf{1}\kappa - \mu_z\right) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(\mathbf{1}\frac{1}{\rho\sigma_p^2}a's_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\overline{s} + \frac{\sigma_\varepsilon^2}{\tau}(z - \mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

[21] It can be shown that $a = \left(\frac{\mathbb{V}(z_E)}{\mathbb{V}(z_E) - \mathbb{C}(z_D, z_E)} \; \frac{\mathbb{V}(z_D)}{\mathbb{V}(z_D) - \mathbb{C}(z_D, z_E)}\right)$ when the correlation is $\pm 1$, which is finite given the form of $\Sigma_z$ under consideration in this case. If $\mathbb{V}(z_D) = 0$ or $\mathbb{V}(z_E) = 0$, one can take $a = (1, 0)$ or $a = (0, 1)$, respectively.

Matching coefficients on the initial conjecture $s_p = \overline{s} + B(z - \mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right) \tag{48}$$

which, upon plugging back into $g'$, matches the expression in the Proposition.

Finally, if $\rho = 1$ in the previous case, then in equilibrium traders can directly infer $\theta = \overline{s} = a's_p$ from the vector of asset prices. Because payoffs are riskless given observation of $\theta$, the equilibrium prices must then be $P_D = \min\{\mu + \theta, K\}$ and $P_E = \max\{\mu + \theta - K, 0\}$ to preclude arbitrage. This set of prices is not of the posited generalized linear form, but it is now easily confirmed that such fully-revealing prices constitute an equilibrium. $\square$