Motivated Beliefs in Coordination Games

Snehal Banerjee, Jesse Davis and Naveen Gondhi*

February 17, 2021

Abstract

We characterize how wishful thinking affects the interpretation of information in economies with strategic and external effects. While players always choose to exhibit overconfidence in private information, their interpretation of public information depends on how non-fundamental volatility affects payoffs. When volatility increases payoffs, players may endogenously disagree: some under-react to public news, while others overreact. In contrast to rational expectations, public information can increase dispersion in actions while private information can increase aggregate volatility. Our analysis has novel implications for the social value of information and demonstrates how endogenous beliefs can reconcile recent evidence on forecast revisions and information rigidities.

JEL Classification: D62, D82, D83, D84
Keywords: anticipatory utility, wishful thinking, beauty contests, coordination games, motivated beliefs.

*Banerjee (snehalb@ucsd.edu) is at the University of California - San Diego; Davis (Jesse_Davis@kenan-flagler.unc.edu) is at the University of North Carolina - Chapel Hill; and Gondhi (naveen.gondhi@insead.edu) is at INSEAD. We thank Simone Galperti, Ben Hebert, Jennifer L'ao, Jonathan Parker, Alessandro Pavan, Isabel Trevino, and Ming Yang for valuable feedback.

1 Introduction

In most economic settings, an individual's payoffs depend not only on fundamentals but also on the behavior of others. A large literature following Morris and Shin (2002) has explored how such strategic considerations alter the use, acquisition, and social value of information, and has formed the basis for the analysis of transparency and disclosure in macroeconomic policy. A maintained assumption in these analyses is that individuals exhibit rational expectations, i.e., they know the true joint distribution of signals and fundamentals, and so correctly interpret the information available.[1]

In practice, however, individuals do not behave this way. There is extensive evidence that individuals experience direct utility flows, called *anticipatory utility*, from their beliefs about future events (e.g., excitement about an upcoming celebration).[2] In such cases, people tend to engage in "wishful thinking": they distort their interpretation of information to increase their current well-being, even when such distortions are costly.[3] Our goal is to understand how such belief distortions are shaped by strategic considerations and the economic environment and, in turn, what these subjective beliefs imply for both observables and policy.

We characterize how wishful thinking affects the interpretation of private and public information in the generalized coordination game of Angeletos and Pavan (2007). Even though it is costly to do so, we show that players always deviate from rational expectations. Surprisingly, how players distort their perception of public versus private information does not depend on whether actions exhibit strategic complementarity or substitutability, but instead on how non-fundamental volatility affects payoffs.[4] When volatility decreases pay-

[1] An exception is Dupraz (2015), who considers a setting in which players exhibit ambiguity aversion and do not know the true model of the economy.

[2] See the recent survey by Bénabou and Tirole (2016) on motivated beliefs. Anticipatory utility differs from the concept of *anticipated* utility, whereby agents hold fixed the parameters they learn about when choosing actions (e.g., Kreps (1998), Cogley and Sargent (2008)).

[3] For example, Oster, Shoulson, and Dorsey (2013) provides evidence of individuals choosing not to learn about their risk of a deadly disease, even when the test was effectively costless.

[4] This is in contrast to the literature on information acquisition in related settings (e.g., Hellwig and Veldkamp (2009), Myatt and Wallace (2012)).

offs, there exists a unique symmetric equilibrium in which all players exhibit over-confidence about both private and public information. In contrast, when non-fundamental volatility increases payoffs, such symmetric equilibria may not exist. Instead, the model generates endogenous disagreement about public information: some investors under-react to public news, while the rest over-react.

This disagreement arises even though players are homogeneous and symmetrically informed ex-ante. We show that the necessary conditions for such disagreement arise naturally in a wide range of applied settings (e.g., production economies with investment complementarities, or canonical industrial organization models with Bertrand or Cournot competition). As such, our model provides a novel explanation for how such settings can give rise to time-varying, endogenous disagreement. Moreover, consistent with a large literature in macroeconomics and finance, this disagreement varies predictably with the economic environment.[5]

Our model also generates sharp predictions that distinguish it from the rational expectations benchmark, and help reconcile observed empirical evidence. We show that more precise public information can increase disagreement and dispersion in actions, while more precise private information can increase non-fundamental volatility in the aggregate action. When the cost of distorting beliefs is sufficiently high, consensus forecasts under-react to information even when individual forecasts overreact; in contrast, when the cost of distorting beliefs is relatively low, both forecasts can exhibit overreaction. These results provide a unified explanation for the seemingly conflicting evidence recently documented in the literature on individual and consensus forecasts' reaction to information.[6]

Finally, our analysis suggests that accounting for subjective belief choice can be important from a policy perspective. In the canonical beauty-contest model

[5]See for instance, Mankiw, Reis, and Wolfers (2003), Andrade, Crump, Eusepi, and Moench (2016), Hong and Stein (2007).

[6]For instance, Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2020) show that consensus forecasts of macroeconomic variables tend to underreact to information, while Bordalo, Gennaioli, Porta, and Shleifer (2019) establishes that consensus forecasts of firm's long term earnings growth tend to exhibit overreaction. This evidence is made all the more puzzling by the observation that individual forecasts tend to exhibit overreaction even for macroeconomic variables (Bordalo, Gennaioli, Ma, and Shleifer (2020)).

of Morris and Shin (2002) (and the Angeletos and Pavan (2004) variant), we find that more precise private and public information can *harm* welfare when players exhibit wishful thinking, even when they unambiguously increase welfare under rational expectations. This suggests that macroeconomic policy that affects the transparency and availability of information must account not only for how strategic considerations affect the use and acquisition of information, but also consider the impact on the *interpretation* of information.

The key intuition for our results is that subjective beliefs affect a player's anticipatory utility through two channels: (i) uncertainty about fundamentals, and (ii) non-fundamental volatility in the aggregate action. The "fundamental uncertainty" channel implies that higher perceived precision of both public and private information increases anticipatory utility. This is because better (perceived) information about fundamentals leads to more informationally efficient actions, which increase anticipated payoffs and, hence, anticipatory utility.

In contrast, the "non-fundamental volatility" channel applies only to public information and captures whether common noise in players' actions increases payoffs. We show that, on the one hand, common noise can help facilitate coordination (e.g., if investment in the new technology has positive externalities).[7] On the other hand, excess volatility in the aggregate action usually decreases utility (e.g., if fluctuations in aggregate investment are very costly for an individual firm). The net effect of these forces determines how the "non-fundamental volatility" channel affects the player's anticipatory utility. Importantly, the intensity of this channel depends on the subjective beliefs of others: when others beliefs' place more weight on the public signal, perceived non-fundamental volatility is more sensitive to beliefs about the precision of the public signal.

When non-fundamental volatility decreases payoffs, an increase in the perceived precision of public information increases anticipatory utility unambigu-

[7] As we show in Section 4.2, the coordination benefit of common noise does not depend on the sign of the externality, but on the magnitude. Specifically, we show that common noise can be beneficial in facilitating coordination, irrespective of whether individuals' actions exhibit strategic complementarity or substitutability. See Section 5.1 for a number of applications where common noise is beneficial. In contrast, Section 6.2 highlights that in the special case of the canonical beauty contest model, common noise is beneficial only when the game exhibits strategic complementarity.

ously. In this case, the "fundamental uncertainty" and "non-fundamental volatility" channels push in the same direction and believing that the public signal is more precise makes players better off. As a result, there exists a unique symmetric equilibrium in which (i) all players overestimate the precision of both public and private information and (ii) players are more overconfident about the public signal. When players' signals are sufficiently, though not perfectly, informative, this overconfidence can even lead players to completely dismiss their prior information.

When non-fundamental volatility increases payoffs, an increase in common noise can increase anticipatory utility. In this case, the two channels work in opposite directions. In the benchmark model, the cost of distorting beliefs is proportional to the loss in the average, ex-post realized utility due to acting on subjective beliefs. When this cost is high (i.e., when the average realized loss due to under-weighting public information is large), the unique symmetric equilibrium features overestimation of the precision of both signals though now players' choose to be more overconfident about their private information.

When the cost of distorting beliefs is sufficiently low, however, this symmetric equilibrium can no longer be sustained. Intuitively, when others overweight the public signal, the non-fundamental volatility channel dominates the fundamental uncertainty channel. As a result, and as long as it is not too costly to do so, an individual has an incentive to deviate and interpret the public signal as being very noisy. On the other hand, when all other players dismiss public information, the fundamental uncertainty channel dominates for a given individual, who then deviates by over-weighting the public signal.

Instead, we show that there exists a unique, mixed-strategy equilibrium which features different interpretations of the public signal: some players choose to over-weight public information, while the rest under-weight it. Though players are ex-ante homogeneous, this equilibrium features both endogenous heterogeneity in beliefs and endogenous disagreement in response to public information.

We then study how changes in the information environment affect equilibrium outcomes. An objective increase in the precision of either signal increases the perceived precision of both signals. As a result, providing more precise public information can increase dispersion of actions across players, since they choose to increase the weight on their private information endogenously. Similarly, making private information more precise can increase non-fundamental volatility due to the increase this induces in players' perceived public precision. These predictions distinguish our model from rational expectations, and help reconcile the puzzling evidence in forecast error predictability, as discussed above.

Moreover, as is well-known, changes in these observables have important social implications: all else equal, an increase in either dispersion or non-fundamental volatility reduces welfare in the settings we analyze. As such, our model has different implications for the social provision of information than the rational expectations benchmark. We also show how such subjective beliefs can be socially beneficial: for example, in the setting of Morris and Shin (2002), we show how players' overconfidence in their private information can undo socially-inefficient coordination on the public signal.

The next section briefly discusses the related literature. Section 3 presents the model and discusses the key assumptions. Section 4 presents the benchmark analysis of the model, including a discussion of how subjective beliefs affect anticipatory utility and a characterization of the subjective belief equilibria. Section 5 discusses how the model's implications for observables like dispersion, volatility and forecast error predictability distinguish it from the rational expectations benchmark. Section 6 characterizes how information provision can affect welfare in a subjective beliefs equilibrium, using the canonical beauty contest model as an illustration. Section 7 concludes, and all proofs and additional analyses are in Appendix.

2 Related Literature

2.1 Anticipatory utility and subjective belief choice

The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905) who considers agents who derive contemporaneous utility not simply from current actions but also the anticipation of future utility flows. For example, an individual who anticipates a negative, future experience (e.g., a risky medical procedure) experiences a negative, contemporaneous utility flow (e.g., anxiety about potential bad outcomes). In contrast, beliefs about future, positive events can increase an agent's current utility (e.g., excitement about a long-awaited vacation). There is now an extensive economic literature that incorporates anticipatory utility into models of belief choice (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). When agents experience anticipatory utility, an individual's subjective beliefs affect not only his actions but also the contemporaneous utility he experiences. As Bénabou and Tirole (2016) emphasize in their survey, this generates a tension between holding "accurate" beliefs, which lead to ex-post optimal actions and "desirable" beliefs, which increase contemporaneous utility flows. Importantly, we note that individuals do not exhibit "multiple selves" but consciously hold a single set of beliefs about the world.[8]

The most closely related papers are Brunnermeier and Parker (2005) and Caplin and Leahy (2019). Brunnermeier and Parker (2005) show how subjective belief choice ("optimal expectations") can help explain a preference for skewness, portfolio under-diversification and consumption/savings patterns. Caplin and Leahy (2019) show how anticipatory utility and belief choice can generate a large number of common behavioral biases, such as confirmation bias, optimism, and the endowment effect as well as both polarization and procrastination. Our analysis builds on this earlier work, but our focus is different: we are interested in understanding how payoff externalities, strategic interaction, and the information environment, affect the interpretation of public and private information. While the cost of distorting beliefs is closely related to the optimal expectations framework of Brunnermeier, Gollier, and Parker (2007), we show that our results are robust to alternative cost functions, such as the statistical

[8]Our assumption that it is costly to deviate from the objective distribution is a modeling convenience. Our intention is to capture the idea that individuals behave as though deviating too far from accurate beliefs is costly, perhaps due to previous experience. As in the literature on robust control, our use of the objective distribution in specifying the cost does not imply that the agent "knows" the true distribution. Caplin and Leahy (2019) discuss this distinction in more detail.

distance measures utilized in Caplin and Leahy (2019).

In Banerjee, Davis, and Gondhi (2019), we explore the implications of wishful thinking in a standard financial market setting. That paper complements the analysis in this paper by characterizing how investors' subjective beliefs about the informativeness of others' signals affect the interpretation of endogenous price information. In contrast, this paper highlights the impact of players' subjective perception of both private and *exogenous* public information in a more flexible setting featuring both strategic substitutability and complementarity. Moreover, the generalized setting we analyze allows us to highlight how subjective beliefs can give rise to consistent patterns of disagreement and overconfidence across a wide range of applications.

In a related paper, Bénabou (2013) analyzes how wishful thinking and reality denial spread through organizations. As in our framework, each agent derives anticipatory utility from his future prospects; however, there is no strategic complementarity / substitutability in payoffs, which is a key determinant of optimal actions (and beliefs) in our setting.

2.2 Social Value of Information

Our paper contributes to the rich literature on coordination games by highlighting the importance of belief choice for understanding both individual and aggregate actions as well as the social value of information. On the normative front, Morris and Shin (2002) introduce their seminal "beauty contest" model to show how public information can reduce welfare in settings with strategic complementarity.[9] Their analysis was generalized by Angeletos and Pavan (2007) to a large class of quadratic-Gaussian economies, which is the framework in which we establish our main results. More recently, Colombo, Femminis, and Pavan (2014) study the welfare implications when players choose the informativeness of their private information. endogenous. In contrast, our model highlights the impact of players endogenous beliefs about the available information. In so doing, we show that the provision of information can give rise to changes in

[9]See Bond, Edmans, and Goldstein (2012), Goldstein and Sapra (2014), and Goldstein and Yang (2017) for recent surveys on implications of disclosure in financial markets.

perception, and therefore social welfare, that do not arise under rational expectations. Given the prevalence of behavioral biases in decision-making, it has been suggested that policymakers would find it valuable to de-bias individuals.[10] Our analysis suggests that, when agents choose their beliefs, this may be neither feasible nor desirable.

Our paper is closely related to the literature following Hellwig and Veldkamp (2009), Myatt and Wallace (2012), and Colombo et al. (2014) that focuses on how strategic considerations affect information acquisition. In a recent paper, Hébert and La'O (2020) explore the implications of rational inattention in a generalized coordination game. They characterize how features of the information cost function affect players' allocation of attention and the ramifications for non-fundamental volatility and the efficiency properties of equilibrium. Our analysis is complementary to this literature in its objectives and approach: we explore players' subjective interpretation of the available information and what consequences these subjective beliefs have for equilibrium outcomes.

3 Model Setup

Our analysis builds on the generalized setting formalized in Angeletos and Pavan (2007). There is a unit measure continuum of players indexed by $i \in [0, 1]$. Each player chooses an action, $k_i \in \mathbb{R}$, to maximize his expected payoff. This payoff, U_i, also depends upon the true state of the world, θ, as well as the actions of all other players, denoted by the vector k_{-i}. We assume that U_i is (i) quadratic in its arguments and (ii) symmetric across the actions of other players (i.e., $U_i\left(k_i, k_{-i}, \theta\right) = U_i\left(k_i, k'_{-i}, \theta\right)$ for any permutation k'_{-i} of k_{-i}). Let $K \equiv \int_0^1 k_j dj$ denote the average action of all other players and $\sigma_k \equiv \left(\int_0^1 \left(k_j - K\right)^2 dj\right)^{\frac{1}{2}}$ denote the dispersion of others' actions.

As Angeletos and Pavan (2007) show, the above implies that payoffs can be expressed as a function $U_i \equiv u\left(k_i, K, \sigma_k, \theta\right)$, where $u\left(\cdot\right)$ is quadratic and its partials satisfy $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$ for all (k, K, θ). This

[10] See, for example, Babcock, Loewenstein, and Issacharoff (1997), Arkes (1991), and Jolls and Sunstein (2006).

generalized functional form ensures tractability while still preserving flexibility for our analysis.[11] For instance, this payoff structure includes settings in which aggregate activity can create positive or negative externalities ($u_K \neq 0$ or $u_\sigma \neq 0$) and allows for strategic substitutability or complementarity ($u_{kK} < 0$ or $u_{kK} > 0$, respectively).

Assumption 1. *$u(k_i, K, \sigma_K, \theta)$ satisfies the following conditions: (i) $u_{kk} < 0$, (ii) $-u_{kK}/u_{kk} < 1$, (iii) $u_{kk} + 2u_{kK} + u_{KK} < 0$, and (iv) $u_{\sigma\sigma} + u_{kk} < 0$.*

Assumptions (i) and (ii) ensure that the equilibrium action is unique and bounded. Angeletos and Pavan (2007) show that assumptions (iii) and (iv) ensure the same for the socially efficient action in a rational expectations equilibrium - in their absence, the social planner could generated unbounded utility for each agent by randomizing their actions. We show below that these same conditions play an important role in determining the effect of non-fundamental volatility on anticipatory utility and the choice of subjective beliefs.

Equilibrium actions depend upon fundamentals, i.e., $u_{k\theta} \neq 0$, but players have incomplete information when choosing their action. Specifically, $\theta \sim N(0, 1/\tau)$ and each player observes both a private signal

$$s_i = \theta + \varepsilon_i, \tag{1}$$

and a public signal

$$s = \theta + \eta, \tag{2}$$

where the common error $\eta \sim N(0, 1/\tau_\eta)$, and individual errors $\varepsilon_i \sim N(0, 1/\tau_e)$ are independent of θ and across each other.

We extend this generalized setting by allowing players to exhibit subjective beliefs about the quality of their information. Specifically, we allow player i to perceive the error in her signals to be:

$$\varepsilon_i \sim_i N\left(0, \frac{1}{\delta_{e,i}\tau_e}\right), \quad \eta \sim_i N\left(0, \frac{1}{\delta_{\eta,i}\tau_\eta}\right). \tag{3}$$

[11] As noted in Angeletos and Pavan (2007), one can also interpret this setting as a second-order approximation of a much broader class of economies.

If $\delta_{e,i} = \delta_{\eta,i} = 1$, player i's beliefs coincide with the objective distribution: he exhibits **rational expectations**. When $\delta_{e,i}$ ($\delta_{\eta,i}$) is greater than one, player i overweights the private (public) signal when forming expectations: he believes the signal contains less noise. The opposite is true when $\delta_{e,i}$ or $\delta_{\eta,i}$ is less than one. While many of our results allow for arbitrarily subjective beliefs, i.e., $\delta_{e,i}, \delta_{\eta,i} \in [0, \infty)$, in some settings, equilibrium existence requires a lower bound $\underline{\delta}$, where $0 \leq \underline{\delta} < 1$.[12] In all other ways, the players are rational: in particular, they (i) take as given other players' actions and (ii) update using Bayes' rule.

If a player chooses to deviate from rational expectations, he incurs a cost,

$$C\left(\delta_{e,i}, \delta_{\eta,i}\right) = \mathbb{E}\left[u\left(k_i^*\left(1,1\right), K, \sigma_k, \theta\right)\right] - \mathbb{E}\left[u\left(k_i^*\left(\delta_{e,i}, \delta_{\eta,i}\right), K, \sigma_k, \theta\right)\right], \tag{4}$$

where $k_i^*\left(1,1\right)$ corresponds to the action under the objective distribution while $k_i^*\left(\delta_{e,i}, \delta_{\eta,i}\right)$ corresponds to the action chosen by player i, i.e.,

$$k_i^*\left(\delta_{e,i}, \delta_{\eta,i}\right) \equiv \arg\max_{k_i} \mathbb{E}_i\left[u\left(k_i, K, \sigma_k, \theta\right) \middle| s_i, s\right]. \tag{5}$$

As in Banerjee, Davis, and Gondhi (2019), we refer to this specification as the **experienced utility penalty** since it captures the expected loss in experienced utility (under the objective measure) from using subjective beliefs when choosing actions. We explore the robustness of our main results to general cost functions, such as the statistical distance between the chosen and objective distributions, in Appendix B.1.

For ease of notation, we denote the expectation and variance of random variable X, given player i's perception of his information ($\delta_{e,i}$ and $\delta_{\eta,i}$) by $\mathbb{E}_i[X]$ and $\text{var}_i[X]$, respectively. Anticipating his optimal action, each player chooses his subjective beliefs to maximize his anticipatory utility net of the cost of deviating from rational expectations. Formally, we denote player i's **anticipatory utility** by:

$$AU_i\left(\delta_{e,i}, \delta_{\eta,i}\right) \equiv \mathbb{E}_i\left[\mathbb{E}_i\left[u\left(k_i^*\left(\delta_{e,i}, \delta_{\eta,i}\right), K, \sigma_k, \theta\right) \middle| s_i, s\right]\right]. \tag{6}$$

[12] We examine the role of $\underline{\delta}$ in Section 4.3 when analyzing settings in which non-fundamental volatility can increase players' utility.

$$= \mathbb{E}_i \left[u \left(k_i^* \left(\delta_{e,i}, \delta_{\eta,i} \right), K, \sigma_k, \theta \right) \right], \tag{7}$$

and so player i's optimal subjective beliefs maximize:

$$\max_{\delta_{e,i}, \delta_{\eta,i}} AU_i \left(\delta_{e,i}, \delta_{\eta,i} \right) - \psi C \left(\delta_{e,i}, \delta_{\eta,i} \right), \tag{8}$$

where $\psi \geq 0$ scales the utility cost of distorting beliefs. Note that when $\psi \to \infty$, the cost of distorting beliefs is arbitrarily high, and the optimal choice of beliefs converges to the rational expectations benchmark i.e., $\delta_{e,i} = \delta_{\eta,i} = 1$.[13]

The penalty function in (4) need not imply that players are endowed with knowledge of the objective distribution. We interpret the experienced utility penalty (and the optimal expectations of Brunnermeier and Parker (2005)) as one in which players make their choices utilizing a single, subjective model of the world. This subjective model is "close to the truth" in the sense that the player's choices do not generate too large of a loss from the perspective of someone endowed with the objective distribution. We acknowledge that the actual process by which players form their subjective model may include experimentation, learning and experience, whereby players learn to trade off "desirable" models (that increase anticipatory utility) against "accurate" models (that increase experienced utility). We utilize the specification in (4) because it provides a tractable and, arguably, natural characterization of the outcome of this process.

Finally, we restrict our benchmark analysis to allow for subjective beliefs about private and public signals. In principle, agents can also entertain beliefs about the signals of others and we analyze such a setting in Section B.2. We show that such beliefs affect neither players' equilibrium actions nor the aggregate observables on which we focus below.

[13] An alternative interpretation of this specification is that ψ parameterizes the relative utility players enjoy from anticipatory utility relative to their realized utility.

4 Equilibrium

We first characterize the equilibrium actions, given an arbitrary set of subjective beliefs. In Section 4.2, we characterize how subjective belief choice affect anticipatory utility, given the beliefs of others and the anticipation of equilibrium actions and in Section 4.3, we establish the subjective belief choice equilibrium.

4.1 Equilibrium actions

We begin by characterizing the optimal actions of each player. If all players observed θ perfectly, then the optimal action would be $\kappa(\theta)$, where:

$$\kappa(\theta) = \underbrace{-\frac{u_k(0,0,0,0)}{u_{kk}+u_{kK}}}_{\equiv \kappa_0} - \underbrace{\frac{u_{k\theta}}{u_{kk}+u_{kK}}}_{\equiv \kappa_1}\theta \tag{9}$$

However, players have incomplete information. Given player $i's$ subjective beliefs, $\delta_{e,i}$ and $\delta_{\eta,i}$, and the realization of s_i and s, his optimal action, k_i is

$$k_i^*(\delta_{e,i},\delta_{\eta,i}) = r\mathbb{E}_i[K|s_i,s] + (1-r)\,\mathbb{E}_i[\kappa(\theta)|s_i,s], \tag{10}$$

where $r \equiv -\frac{u_{kK}}{u_{kk}}$ is the **equilibrium degree of coordination** across players. This term captures the extent to which each player chooses to align his action with his expectation of others actions, K, relative to his expectation of the full-information target, $\kappa(\theta)$. Given our assumptions on the joint distribution of fundamentals and signals, Bayesian updating implies that player i's conditional beliefs about θ are given by

$$\mathbb{E}_i[\theta|s_i,s] = A_i s_i + B_i s, \text{ and } \mathrm{var}_i[\theta|s_i,s] = \frac{1-A_i-B_i}{\tau}, \tag{11}$$

where player i's weights on the private and public signals are given by:

$$A_i \equiv \frac{\delta_{e,i}\tau_e}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}, \text{ and } B_i \equiv \frac{\delta_{\eta,i}\tau_\eta}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}. \tag{12}$$

Given these descriptions, the following lemma characterizes equilibrium actions.

Lemma 1. *Given a choice of subjective beliefs $\{\delta_{e,i}, \delta_{\eta,i}\}_i$ for each player i, there always exists a unique, linear equilibrium in which player i's optimal action is given by:*

$$k_i\left(\delta_{e,i}, \delta_{\eta,i}\right) = \kappa_0 + \kappa_1\left(\alpha_i s_i + \beta_i s\right), \tag{13}$$

and the aggregate action is given by:

$$K = \kappa_0 + \alpha\theta + \beta s, \tag{14}$$

where $\alpha_i = \frac{1-r}{1-rA}A_i$, $\beta_i = \frac{(1-r)B_i+Br}{1-rA}$, $\alpha = \frac{A(1-r)}{1-rA}\kappa_1$, $\beta = \frac{B}{1-rA}\kappa_1$, $A = \int_i \frac{\delta_{e,i}\tau_e}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}di$ *and* $B = \int_i \frac{\delta_{\eta,i}\tau_\eta}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}di$.

While the form of this solution is standard, the distinguishing feature of our setting is that the relative weights placed on the private and public signal (α_i and β_i, respectively) are distorted by each player i's subjective beliefs (through A_i and B_i). Moreover, when there is a benefit to coordination, i.e., if $r \neq 0$, these weights also depend upon the subjective beliefs of *other* players through A and B, which player i takes as given. We show below that both dimensions play a critical role in the equilibrium choice of beliefs.

4.2 Anticipatory Utility and Subjective Beliefs

In our setting, players derive contemporaneous utility from their anticipation of future payoffs, which depend upon their subjective beliefs about both fundamentals and equilibrium actions, as we characterize next.

Proposition 1. *Given player i's subjective beliefs, $\delta_{e,i}$ and $\delta_{\eta,i}$, anticipatory utility can expressed as:*

$$AU_i\left(\delta_{e,i}, \delta_{\eta,i}\right) \propto \left[\underbrace{u_{kk}\left(1-r\right)^2 var_i\left[\theta|s_i, s\right]}_{\text{fundamental uncertainty channel}} - \underbrace{\left(u_{KK} - r^2 u_{kk}\right) B^2 var_i\left[s|\theta\right]}_{\text{non-fundamental volatility channel}}\right]. \tag{15}$$

A player's subjective beliefs affect his anticipatory utility through two channels. First, when the player believes that his fundamental forecast is more precise, i.e., when $\text{var}_i[\theta|s_i, s]$ decreases, his anticipatory utility increases (since $u_{kk} < 0$ in equation (15)) - we refer to this as the **fundamental uncertainty channel**. Second, the player's anticipatory utility is affected by his beliefs about the error in the average action. We refer to this as the **non-fundamental volatility channel**, since believing that the public signal is noisier i.e., increasing $\text{var}_i[s|\theta]$, increases his expectation of volatility in the aggregate action, i.e., $\text{var}_i(K|\theta)$ is higher.

Whether non-fundamental volatility increases or decreases anticipatory utility depends on the sign of $(u_{KK} - r^2 u_{kk})$. This term reflects the direct effect of aggregate actions on player i's utility (u_{KK}) and the indirect effect due to strategic considerations ($-r^2 u_{kk}$). Since $u_{kk} < 0$, the latter term is always positive, whether actions are strategic complements or substitutes. This reflects the benefit of common noise in a strategic setting because it increases (decreases) the covariance between the player's action and the aggregate action when $r > 0$ ($r < 0$). As a result, the impact of this perceived common error depends upon whether the direct effect of aggregate volatility is sufficiently negative: if $u_{KK} < r^2 u_{kk} < 0$, then the player's anticipatory utility increases when he perceives that the public signal is more informative. Otherwise, the player benefits from believing that the public signal is noisier.

In what follows, it will be useful to define the coefficient

$$\chi \equiv \frac{(u_{KK} - r^2 u_{kk})}{-u_{kk}(1-r)^2}, \tag{16}$$

which captures the relative magnitude of the non-fundamental (i.e., $(u_{KK} - r^2 u_{kk})$) and informational (i.e., $-u_{kk}(1-r)^2$) channels.

Corollary 1. *Anticipatory utility is always increasing in $\delta_{e,i}$. If $\chi \leq 0$, then anticipatory utility is increasing in $\delta_{\eta,i}$; otherwise, it is U-shaped in $\delta_{\eta,i}$.*

The player's belief about the quality of his private signal affects only the information channel: an increase in $\delta_{e,i}$ reduce the player's uncertainty (i.e., $\text{var}_i[\theta|s_i, s]$) and so anticipatory utility increases. In contrast, a player's beliefs about the public signal affect both channels,

$$\frac{\partial AU_i}{\partial \delta_{\eta,i}} = u_{kk}\left(\kappa_1 \frac{1-r}{1-rA}\right)^2 \left(\frac{\partial \mathrm{var}_i\left[\theta|s_i,s\right]}{\partial \delta_{\eta,i}} + \frac{\partial \mathrm{var}_i\left[s|\theta\right]}{\partial \delta_{\eta,i}}\left(\chi B^2\right)\right) \tag{17}$$

$$= u_{kk}\left(\kappa_1 \frac{1-r}{1-rA}\right)^2 \left(-\frac{\tau_\eta}{\left(\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau\right)^2} - \frac{\chi B^2}{\delta_{\eta,i}^2 \tau_\eta}\right) \tag{18}$$

An increase in the perceived precision of the public signal decreases the player's perception of both fundamental uncertainty (i.e., $\frac{\partial \mathrm{var}_i[\theta|s_i,s]}{\partial \delta_{\eta,i}} < 0$) and non-fundamental volatility (i.e., $\frac{\partial \mathrm{var}_i[s|\theta]}{\partial \delta_{\eta,i}} < 0$). When $\chi < 0$, anticipatory utility decreases in non-fundamental volatility, so an increase in $\delta_{\eta,i}$ unambiguously increases anticipatory utility. When $\chi > 0$, the fundamental uncertainty and non-fundamental volatility channels operate in opposite directions. If $\delta_{\eta,i}$ is sufficiently low, the non-fundamental channel dominates (as $\delta_{\eta,i} \to 0$, $\frac{\partial \mathrm{var}_i[\theta|s_i,s]}{\partial \delta_{\eta,i}}$ is bounded above, while $\frac{\partial \mathrm{var}_i[s|\theta]}{\partial \delta_{\eta,i}}$ is not), generating a negative relation between anticipatory utility and $\delta_{\eta,i}$. When $\delta_{\eta,i}$ is sufficiently high, the informational channel dominates and anticipatory utility increases with $\delta_{\eta,i}$ since $\chi B^2 < 1$.[14]

4.3 Equilibrium Belief Choice

In what follows, we analyze how players' optimal subjective beliefs change with players' preferences as well as the information environment. We begin with a characterization of equilibria when non-fundamental volatility decreases anticipatory utility.

Proposition 2. *If non-fundamental volatility lowers anticipatory utility ($\chi \leq 0$)*

(i) If $\psi \leq \frac{\tau_e+(1-\chi)\tau_\eta}{2\tau}$, then all players choose to ignore their prior and choose $A_i = \frac{\tau_e}{\tau_e+\tau_\eta}$ and $B_i = \frac{\tau_\eta}{\tau_e+\tau_\eta}$.

(ii) If $\psi > \frac{\tau_e+(1-\chi)\tau_\eta}{2\tau}$, then there exists a unique symmetric equilibrium in

[14] We have $\chi < 1$ because of assumptions (i) and (iii) about the player's payoff. Intuitively, this implies that even when non-fundamental volatility increases anticipatory utility, the effect of fundamental uncertainty (via the informational channel) is larger in magnitude. $B \leq 1$ by Bayes rule since it reflects the weight that players on average put on the public signal.

which the optimal precision choices of each player i are given by:

$$\delta_e = \frac{2\tau\psi\left(2\psi+1\right)+\left(\chi-1\right)\chi\tau_\eta}{2\psi\left(2\tau\psi+\left(\chi-1\right)\tau_\eta-\tau_e\right)} > 1, \quad \textit{and} \quad \delta_\eta = \delta_e - \frac{\chi}{2\psi} > 1 \tag{19}$$

Moreover, δ_e and δ_η are increasing in τ_e and τ_η , and decreasing in τ, ψ and χ.

Since $\chi < 0$, players always choose to overestimate the precision of *both* signals in any symmetric equilibrium. Intuitively, this overestimation is larger when it is less costly to deviate from rational expectations (e.g., when prior information is imprecise, i.e., low τ, or the cost of distorting beliefs is small, i.e., low ψ) and when the available information is more precise (high τ_e, τ_η). When the overall costs are sufficiently low (i.e., when $\psi \leq \frac{\tau_e+(1-\chi)\tau_\eta}{2\tau}$), the distortion is sufficiently severe that players ignore their priors i.e., $A_i = \frac{\tau_e}{\tau_e+\tau_\eta}$ and $B_i = \frac{\tau_\eta}{\tau_e+\tau_\eta}$.

The relative distortion in public versus private precision depends on the relative importance of the fundamental and non-fundamental channels. With $\chi < 0$, the non-fundamental channel induces an incremental marginal benefit of increasing perceived public precision, and consequently, $\delta_\eta > \delta_e$. As χ rises (i.e., approaches zero), the relative impact of the non-fundamental volatility channel, and therefore the incentive to distort $\delta_{\eta,i}$, falls. Interestingly, this also reduces the optimal $\delta_{e,i}$ because the experienced utility penalty induces complementarity between the player's chosen beliefs. If a player chooses to increase $\delta_{\eta,i}$, which increases the weight she places on the public signal, then it is beneficial to increase $\delta_{e,i}$, since this restores balance between public and private information.

Next, we consider the case when non-fundamental volatility increases anticipatory utility ($\chi > 0$). When the choice of $\delta_{\eta,i}$ is unbounded below (i.e., if players can choose $\delta_{\eta,i} = 0$) and $\chi > 0$, an equilibrium does not exist. To see why, conjecture an equilibrium in which some players place positive weight on the public signal so that $B > 0$. Then it is optimal for player i to deviate and choose $\delta_{\eta,i} = 0$ since this increases non-fundamental volatility and maximizes his anticipatory utility. On the other hand, if all other players place zero weight on the public signal (i.e., $\delta_\eta = 0$ so that $B = 0$), then player i's beliefs about the public signal affect her anticipatory utility through the information channel, only. As a result, it is beneficial for player i to deviate by choosing $\delta_{\eta,i} > 0$.

To sustain an equilibrium with $\chi > 0$, we bound the subjective beliefs about the public signal precision, i.e., restrict $\delta_{\eta,i} \geq \underline{\delta}$ for some $\underline{\delta} > 0$.[15] The following proposition characterizes how this bound affects the nature of equilibrium.

Proposition 3. *Suppose non-fundamental volatility increases anticipatory utility (i.e., $\chi > 0$), and $\underline{\delta} > 0$. Then there exists a $\bar{\psi} > 0$ such that:*

(i) If $\psi \in \left(0, \bar{\psi}\right)$, the unique equilibrium is a mixed strategy equilibrium and is characterized by the quadruple $(\lambda, \delta_{e1}, \delta_{e2}, \delta_{\eta 2})$ which solve a system of equations (specified in the Appendix). In this equilibrium, a fraction λ of players optimally chooses $\delta_{e,i} = \delta_{e,1}$ and $\delta_{\eta,i} = \underline{\delta}$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{e,i} = \delta_{e,2}$ and $\delta_{\eta,i} = \delta_{\eta,2}$. Moreover, in this equilibrium, the players' subjective beliefs $(\delta_{e1}, \delta_{e2}, \delta_{\eta 2})$ do not depend on χ while λ decreases with χ.

(ii) If $\psi > \bar{\psi}$, then the unique equilibrium is symmetric and is given by (19).

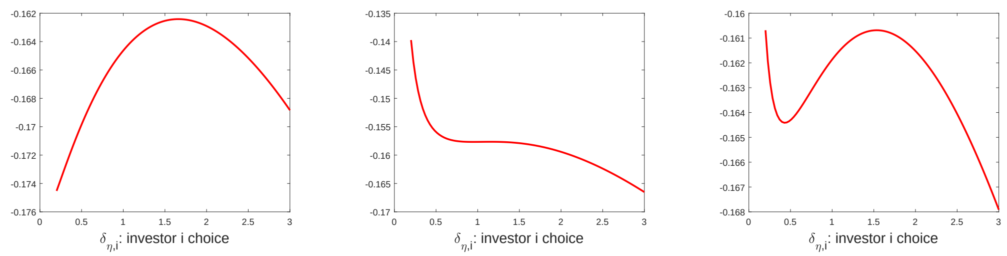
Following the intuition provided above, when $\chi > 0$ and $B > 0$, players can increase their anticipatory utility by lowering $\delta_{\eta,i}$; however, since $\underline{\delta} > 0$, the benefit of doing so is now bounded. When the cost of deviating from rational expectations is sufficiently high, i.e., when $\psi > \bar{\psi}$, the net benefit of lowering $\delta_{\eta,i}$ is too small and so the symmetric equilibrium is preserved.

When $\psi < \bar{\psi}$, this is no longer the case and a player benefits from deviating to the corner solution $(\delta_{e1}, \underline{\delta})$ when no one else is doing so. However, as the measure of players who choose $\delta_{\eta,i} = \underline{\delta}$ increases, the aggregate weight placed on the public signal, B, falls, reducing the utility from choosing $(\delta_{e1}, \underline{\delta})$ as is clear from (17). The unique equilibrium features mixing between this choice and an interior maximum, $(\delta_{e2}, \delta_{\eta 2})$. In this mixed strategy equilibrium, the optimal subjective beliefs depend on the information environment only, while the measure of those who choose each profile depends upon preferences (as captured by χ).

[15] In their survey, Epley and Gilovich (2016) discuss an alternative motive for such a bound: subjective beliefs are reached through motivated reasoning and must be "reasonable", i.e., they are naturally bound by the limits of the observable evidence.

Figure 1: Total utility versus $\delta_{\eta,i}$ when $\psi < \overline{\psi}$

The figure plots the anticipatory utility, net of costs, for player i as a function of $\delta_{\eta,i}$. Other parameters: $\tau = 10$, $\tau_e = \tau_s = 1$, $\chi = 0.5$, $\underline{\delta} = 0.2$, $\psi = 1$.

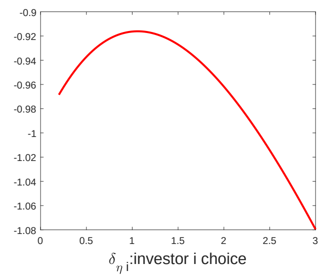


(a) $\delta_\eta = \underline{\delta}$ (b) $\delta_\eta = 1.6$ (c) Mixed eqm.

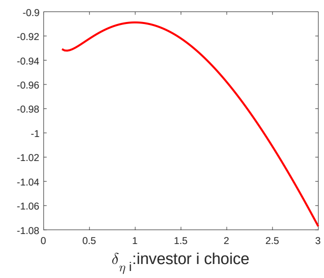
Figure 1 provides a numerical illustration of this intuition when $\psi < \overline{\psi}$. The panels show player i's anticipatory utility, net of costs, as a function of $\delta_{\eta,i}$, given the beliefs of others. In panel (a), all other agents choose $\delta_\eta = \underline{\delta} = 0.2$. Since B is sufficiently low, player i deviates by over-weighting the public signal (chooses $\delta_{\eta,i} = 1.6$). In panel (b), we consider an alternative equilibrium in which all other agents choose $\delta_\eta = 1.6$. Now, the non-fundamental channel dominates, i.e., B is sufficiently large, and player i strictly prefers to believe the public signal is (relatively) uninformative. In both cases, a symmetric equilibrium is ruled out. Panel (c) illustrates the mixed strategy equilibrium in which a fraction $\lambda = 0.36$ of players discount the public signal while the remaining fraction $1 - \lambda = 0.64$ overweight it. As is clear, given the choice of all other agents, player i is indifferent between these two (sets of) beliefs.

In Figure 2, all other parameters are the same as in Figure 1 except ψ, which now exceeds $\overline{\psi}$. In panel (a), all other agents choose $\delta_\eta = \underline{\delta} = 0.2$, but player i deviates by over-weighting the public signal. In panel (b), we consider an alternative equilibrium in which all other agents choose $\delta_\eta = 1.03$. In this case, player i chooses $\delta_{\eta,i} = \delta_\eta$, i.e., the symmetric equilibrium exists.

Figure 2: Anticipatory utility net of costs versus $\delta_{\eta,i}$ when ψ is high
The figure plots the anticipatory utility net of costs for agent i as a function of $\delta_{\eta,i}$. Other parameters: $\tau = 10$, $\tau_e = \tau_s = 1$, $\chi = 0.5$, $\underline{\delta} = 0.2$, , $\psi = 10$.



(a) $\delta_\eta = \underline{\delta}$



(b) $\delta_\eta = 1.03$

5 Observable Implications

We now characterize the impact on observables of endogenizing players' beliefs and, in so doing, derive testable predictions that distinguish our theory.

5.1 Endogenous Disagreement about Public Information

The mixed strategy equilibrium provides an illustration of how disagreement about the interpretation of public information can arise endogenously, even in a setting with ex-ante homogeneous players. Consistent with the empirical literature, the model predicts that observed disagreement will increase after the release of a public signal relative to rational expectations. Moreover, given the importance of disagreement in macroeconomic dynamics (see, e.g., Mankiw et al. (2003)), our model can help explain why and help predict how such disagreement varies over time. In this vein, in Appendix C, we apply the results of our general framework and show that such disagreement naturally arises across a wide range of applications including a competitive economy with incomplete markets (Section C.1), a production economy in which investment exhibits complementarity across firms (Section C.2), Bertrand and Cournot competition in a canonical industrial organization setting (Section C.3) and finally, a setting with information spillovers between financial markets and the real economy (Section C.4).

Across all these settings, we show that non-fundamental volatility increases players' anticipatory utility, i.e., $\chi > 0$. For instance, in the setting with investment complementarities (Section C.2), χ is positive because there is a strategic benefit from more common noise in the public signal because it facilitates coordination in investment. In addition to strategic considerations, we also show that players can benefit from volatility in the aggregate action. For example, in the competitive setting with incomplete markets (Section C.1), households benefit from volatility in aggregate production ($u_{KK} > 0$) since, in equilibrium, they are able to purchase more of the good at a lower price.

As a result, despite the differences in each setting, the optimal subjective beliefs they give rise to share a number of similarities. First, all symmetric equilibria feature more overconfidence in players' private information than in the public signal (i.e., $\delta_e > \delta_\eta$). Importantly, this relative overweighting of private information does not follow immediately from strategic substitutability in actions. For instance, relative to rational expectations, firms choose to align their action more closely with their private information not only when competing on quantity (and actions are strategic substitutes) but, somewhat surprisingly, also do so when competing on price (and actions are strategic complements). This feature differentiates our setting from models of information acquisition in which the quality of private information acquired is generally decreasing in the degree of coordination (e.g., Colombo et al. (2014)).

Second, our results imply that the mixed strategy equilibria can arise in which some players not only underweight the public signal relative to the private signal but also underweight it relative to the rational expectations benchmark. In the setting with information spillovers, one interpretation of such equilibria is that technological innovation endogenously creates disagreement among financial market participants. This disagreement can lead to large trading volume even upon public announcements, consistent with Kandel and Pearson (1995).

This suggests that our mechanism could explain why technological revolutions are often associated with higher disagreement.

5.2 Dispersion and Non-fundamental Volatility

Changes in the information environment influence not only players' subjective beliefs but also their chosen actions. Let $\sigma_k^2 \equiv \int (k_i - K)^2 \, di$ denote the dispersion in players' actions and let $\nu^2 \equiv \mathbb{E}\left[(K - \kappa(\theta))^2\right]$ denote non-fundamental volatility (i.e., the volatility in the aggregate action driven by imperfect information). In what follows, we show how players' endogenous beliefs can reverse the standard relationship between information quality and both σ_k^2 and ν^2.

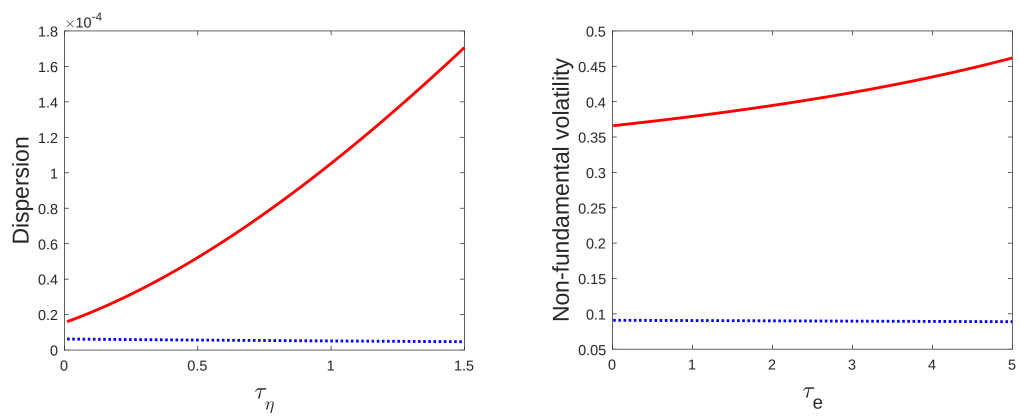
Proposition 4. *(1) If players' exhibit rational expectations (i.e., if $\psi \to \infty$), then dispersion decreases with public information quality ($\frac{\partial \sigma_k^2}{\partial \tau_\eta} < 0$) while non-fundamental volatility decreases with private information quality ($\frac{\partial \nu^2}{\partial \tau_e} < 0$).*
(2) Suppose the unique equilibrium in subjective beliefs is symmetric. Then,

- *(i) if χ is sufficiently negative, dispersion increases with public information quality (i.e., $\frac{\partial \sigma_k^2}{\partial \tau_\eta} > 0$), and*
- *(ii) if r is sufficiently close to one and private signal precision is sufficiently low, then non-fundamental volatility can increase with private information precision (i.e., $\frac{\partial \nu^2}{\partial \tau_e} > 0$).*

Under rational expectations, as public information quality improves, players choose to place relatively less weight on their private signals which reduces dispersion. Under subjective beliefs, however, an increase in the precision of either signal increases players' perceived precision of *both* signals in the symmetric equilibrium. We show that, when strategic complementarity considerations are sufficiently strong and private signals are sufficiently noisy, an increase in τ_η can even induce players to place relatively more weight on their private signals, which increases cross-sectional dispersion σ_k^2. Figure 3, panel (a), numerically illustrates that this endogenous increase in dispersion can be large and economically meaningful. Interestingly, this change in perception, and therefore dispersion, stands in contrast to models of information acquisition (e.g., Colombo et al. (2014)). In those settings, providing a more precise public signal crowds out the acquisition of private information. This provides a potentially testable prediction that distinguishes our setting from one with endogenous information acquisition.

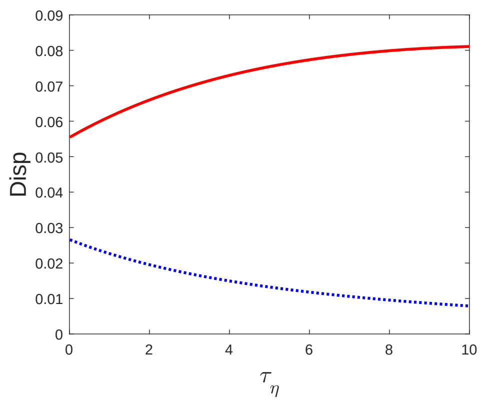
When the utility cost of non-fundamental volatility is negative, the increase in $\delta_{\eta,i}$ that follows an increase in τ_e amplifies players' over-weighting of the public signal. When this response is sufficiently large (i.e., $\chi < 0$ is sufficiently negative), it can even generate higher non-fundamental volatility ν^2. Under rational expectations, this does not arise: providing more information, whether public or private, reduces volatility. For example, as Figure 3, panel (b), illustrates, an increase in τ_e reduces the weight players place on the common noise in the public signal, lowering ν^2 under rational expectations but increasing it under subjective beliefs.

Figure 3: Observable implications in a symmetric equilibrium
The figure plots dispersion of actions and non-fundamental volatility in a symmetric equilibrium. For comparison, it also plots these observables in a rational expectations equilibrium (dotted lines). Other parameters are: $\tau = 10$, $r = 0.975$, $\chi = -10$, $\psi = 1$.

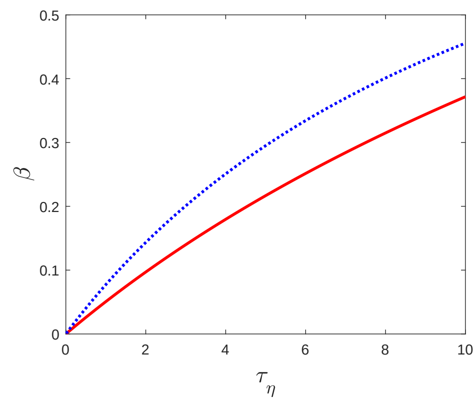


(a) Dispersion vs τ_η (b) Non-fundamental volatility vs τ_e

Figure 4: Observable implications in the mixed strategy equilibrium
The figure plots dispersion of actions and responsiveness of aggregate actions to public information (β) in a mixed strategy equilibrium. For comparison, it also plots these observables in a rational expectations equilibrium (dotted lines). Other parameters are: $\tau = 10$, $\tau_e = 1$, $r = -0.95$, $\underline{\delta} = 0.1$, $\psi = 1$.



(a) Dispersion vs τ_η



(b) β vs τ_η

When χ is positive and ψ is small, the equilibrium exhibits mixed-strategies. As Proposition 3 shows, this implies that some investors under-react to the public signal, since they choose $\delta_{\eta,i} < 1$. As panel (a) of Figure 4 demonstrates, this can lead to increased dispersion, even as public information becomes more precise since the two groups of players place different weights on the public signal. Moreover, under-weighting the public signal generates sluggishness in the aggregate action. Let β be the regression coefficient when the aggregate action K is projected on the public signal s. As panel (b) of Figure 4 highlights, while the aggregate action increases with the quality of public information under the subjective belief equilibrium, the response is muted relative to rational expectations. This novel source of endogenous under-reaction is consistent with recent survey evidence documented by Angeletos, Huo, and Sastry (2020) and, more generally, with the sluggish response of prices and quantities targeted by macroeconomic models with dispersed information (see, e.g., Lorenzoni (2011) for a survey).

5.3 Forecast Error Predictability

Our model also helps shed light on recent puzzling evidence about macroeconomic forecasts. Coibion and Gorodnichenko (2015) develop a new approach to measuring information rigidities by regressing forecast errors on forecast revisions. Under the null of rational expectations and full information, the forecast errors are unpredictable and so the regression coefficient is zero. However, the authors document that the regression coefficient for consensus forecasts is significantly positive in the data, suggesting that aggregate forecasts underreact to available information which they attribute to sticky or noisy information. Bordalo, Gennaioli, Ma, and Shleifer (2020) document that individual forecasts tend to exhibit overreaction: the analogous regression coefficients are negative.

In our model, one can measure the forecast revision of player i as the difference between his conditional and unconditional expectations of θ i.e.,

$$FR_i \equiv \mathbb{E}_i\left[\theta|s_i, s\right] - \mathbb{E}_i\left[\theta\right], \tag{20}$$

while his forecast error is given by

$$FE_i \equiv \theta - \mathbb{E}_i\left[\theta|s_i, s\right]. \tag{21}$$

Taking averages across i give us the analogous expressions for the consensus forecast revision $\bar{FR}$ and consensus forecast error $\bar{FE}$. The covariance $\text{cov}\left(FE_i, FR_i\right)$ is a measure of how player i reacts to all of his available information: under rational expectations, $\text{cov}\left(FE_i, FR_i\right) = 0$, while $\text{cov}\left(FE_i, FR_i\right) < 0$ ($\text{cov}\left(FE_i, FR_i\right) > 0$) implies that the player under-reacts (overreacts).

The following result characterizes how subjective belief choice affects this covariance for both individual and consensus forecasts.

Proposition 5. *(i) If ψ is sufficiently high, consensus forecasts underreact and individual forecasts overreact to information i.e., cov$\left(\bar{FE}, \bar{FR}\right) > 0$ and cov$(FE_i, FR_i) < 0$.*

(ii) If ψ is sufficiently low and $\chi < 0$, both consensus and individual forecasts overreact to information i.e., cov$\left(\bar{FE}, \bar{FR}\right) < 0$ and cov$(FE_i, FR_i) < 0$.

(iii) If ψ is sufficiently low and $\chi > 0$, some individuals overreact to information while others may underreact to it.

Whether individual and consensus forecasts feature overreaction or underreaction depends on both preferences, through χ, as well as on the cost of belief distortion, ψ. The proposition implies that our model can generate the general pattern of consensus underreaction and individual overreaction in macroeconomic forecasts documented by Bordalo, Gennaioli, Ma, and Shleifer (2020) when the cost of belief distortion is sufficiently high.[16] However, our model also suggests that when ψ is sufficiently low, both consensus and individual forecasts may exhibit overreaction, consistent with the evidence on firm's long term earnings growth, as documented by Bordalo, Gennaioli, Porta, and Shleifer (2019). Taken together, these results suggest that accounting for subjective belief choice may provide a unified approach for better understanding belief formation across a number of different settings.

Our analysis complements other recent approaches to resolving this puzzling evidence. For instance, Bordalo et al. (2020) show that a diagnostic expectations variant of a model with dispersed information can help reconcile this evidence, while Angeletos et al. (2020) develop a model with dispersed noisy information and over-extrapolation. Our model relies on a distinct mechanism that, importantly, is state-dependent; as such, our analysis generates novel predictions about the settings in which we should expect to see over-reaction (or underreaction) at the individual and aggregate levels in terms of variation in the cost of belief distortion and the cost of non-fundamental volatility.

6 Welfare

When choosing their subjective beliefs, players fail to internalize how these beliefs (and the resultant distortion in actions) impact the well-being of others. In what follows, we analyze the normative implications of these choices.

[16]When $\psi \to \infty$, players hold rational expectations and both covariances converge to zero.

6.1 Social Value of Subjective Beliefs

Consider a social planner who chooses to measure a weighted average of aggregate anticipatory utility and aggregate objective utility, given θ:

$$W\left(\theta,\{s_i\}_i,s;\gamma\right)=\gamma\int_i AU_i\left(\delta_{e,i},\delta_{\eta,i}\right)di+(1-\gamma)\int_i \mathbb{E}\left[u\left(k_i^*\left(\delta_{e,i},\delta_{\eta,i}\right),K,\sigma_k,\theta\right)\right]di. \tag{22}$$

This specification of the social planner's objective function nests two natural benchmarks. When $\gamma=0$, the social planner evaluates welfare by aggregating experienced utility under objective beliefs — this corresponds to the standard welfare measure under rational expectations. When $\gamma=\frac{1}{1+\psi}$, the social planner this corresponds to a setting in which the social planner respects the players' preferences and the measure fully captures both their anticipatory utility as well as the objective cost of belief distortion.[17]

We show (in the appendix) that the unconditional expectation of this measure in a symmetric equilibrium, which we denote by W^0, can be written as

$$\begin{aligned}W^0&\equiv\mathbb{E}\left[W\left(\theta,\{s_i\}_i,s;\gamma\right)\right]\\&=\begin{array}{c}\underbrace{\left(\mathbb{E}\left[u\left(\kappa(\theta),\kappa(\theta),0,\theta\right)\right]+\mathcal{A}\mathrm{var}(\theta)\right)}_{\equiv W_1}\\+\underbrace{\tfrac{1}{2}\left(u_{\sigma\sigma}+u_{kk}\right)\sigma_i^2+\frac{1}{2}\left(u_{kk}+u_{KK}+2u_{kK}\right)\nu^2}_{\equiv W_2}\\-\underbrace{\frac{1}{2}\gamma\left(\left(\delta_e-1\right)u_{kk}\frac{\alpha^2}{\delta_e\tau_e}+\left(u_{kk}+u_{KK}+2u_{kK}\right)\left(\delta_\eta-1\right)\frac{\beta^2}{\delta_s\tau_s}\right)}_{\equiv W_3}\end{array}\end{aligned}\tag{24}$$

There are three components to welfare: W_1 captures the impact of fundamental uncertainty, W_2 captures the incremental effect of cross-sectional dispersion and non-fundamental volatility, and W_3 captures the additional impact of subjective

[17]Specifically, when $\gamma=\frac{1}{1+\psi}$, we have

$$W\left(\theta,\{s_i\}_i,s;\gamma\right)=\frac{1}{1+\psi}\left(\int_i AU_i\left(\delta_{e,i},\delta_{\eta,i}\right)-\psi C\left(\delta_{e,i},\delta_{\eta,i}\right)di\right), \tag{23}$$

where $\frac{1}{1+\psi}$ is a normalization.

beliefs on aggregate anticipatory utility.[18] Under rational expectations (i.e., when $\psi \to \infty$), W_3 is zero (since $\delta_e = \delta_\eta = 1$) and the above measure converges to the welfare measure found in Angeletos and Pavan (2007).

The expression highlights the various channels through which subjective beliefs affect welfare in our setting. First, we show that $\mathcal{A}$ depends on the wedge between the sensitivity of the aggregate action to fundamentals (i.e., $\alpha + \beta$) relative to the sensitivity of the full information action (i.e., κ_1). This wedge depends on the weight placed on each signal, which is driven by both objective information quality as well as subjective beliefs.

Second, because we assume that both $u_{kk} + u_{KK} + 2u_{kK} < 0$ and $u_{\sigma\sigma} + u_{kk} < 0$ (Assumptions (iii) and (iv)), both dispersion and non-fundamental volatility decrease the second component of welfare W_2. This is consistent with the rational expectations benchmark of Angeletos and Pavan (2007), and ensures that arbitrarily large random variation in players' beliefs (and therefore actions) does not lead to unbounded utility, all else equal. However, as discussed above, both observables depend not only on the economic environment but, importantly, on players' perception of their information.

Finally, when $\gamma > 0$, the social planner also accounts for the anticipatory utility experienced by players. When players over-estimate the precision of their private and public signals (i.e., $\delta_e, \delta_\eta > 1$), $W_3 > 0$ because $u_{kk} < 0$ and $u_{kk} + u_{KK} + 2u_{kK} < 0$. Intuitively, players choose to over-estimate the precision of their information precisely because it increases anticipatory utility and this channel is captured by W_3.

In Angeletos and Pavan (2007), the social value of private and public information depends upon the social cost of dispersion ($u_{\sigma\sigma} + u_{kk}$) relative to the social cost of non-fundamental volatility ($u_{kk} + u_{KK} + 2u_{kK}$). As is clear from Proposition 4, however, when players choose their beliefs, the impact of changes in the information environment on both observables is altered. For concreteness, we illustrate the implications of these additional effects in the context of the canonical beauty contest model.

[18] Both $\mathbb{E}\left[u\left(\kappa\left(\theta\right), \kappa\left(\theta\right), 0, \theta\right)\right]$, which reflects the expected utility if all players were perfectly informed about θ, and $\mathcal{A}$ is characterized in the appendix.

6.2 Social Value of Information

We consider payoff specifications from the canonical model of Morris and Shin, 2002) (denoted by u_{MS}), as well as the modification introduced by Angeletos and Pavan (2004) (denoted by u_{AP}):

$$u_{MS} \equiv -\rho \left(k_i - K\right)^2 - (1-\rho)\left(k_i - \theta\right)^2 + \rho\sigma_k^2, \tag{25}$$

$$u_{AP} \equiv u_{MS} - \rho\sigma_k^2. \tag{26}$$

As is standard, we assume $|\rho| < 1$ and note that when $\rho > 0$ ($\rho < 0$), players' actions are complements (substitutes). First, we establish equilibrium beliefs.

Proposition 6. *Suppose ψ is sufficiently high (i.e., $\psi > \frac{\tau_e + (\chi-1)\tau_\eta}{2\tau}$). Then,*

(i) If $\rho > 0$, there exists a unique symmetric equilibrium in which the optimal precision choices are given by (19) and $\chi = \frac{-\rho}{1-\rho}$.

(ii) Suppose $\rho < 0$ and $\overline{\delta} > 0$. Then, if $\psi_{bc} < \psi$, the equilibrium solution is (19), where $\chi = \frac{-\rho}{1-\rho}$. Otherwise, a mixed strategy equilibrium exists and is characterized by the quadruple $(\lambda, \delta_{e1}, \delta_{e2}, \delta_{s2})$ which solve a system of equations (specified in the Appendix).

These results follow from recognizing that in beauty contest games of strategic complementarity (i.e., $\rho > 0$), anticipatory utility is decreasing in non-fundamental volatility (i.e., $\chi = -\frac{\rho}{1-\rho} < 0$), while in games of strategic substitutability, the reverse is true. We emphasize, however, that this arises not because of the strategic nature of players' actions but because of the direct impact of aggregate volatility: in these settings, $u_{KK} = -u_{kK}$.[19] Intuitively, when tracking the aggregate action is beneficial, aggregate errors are costly, while when trying to distance your action from others, they are beneficial.

Figure 5 illustrates the implications of subjective beliefs on the social value of information. The figure plots various welfare measures as functions of private and public signal precision (τ_e and τ_η, respectively), under the payoff of Morris

[19] The direct impact of non-fundamental volatility, u_{KK}, is simply $-\rho$, and the benefit of non-fundamental volatility which arises from increased covariance with the actions of others (i.e., $u_{kk}r^2$) is ρ^2. As a result, given that $|\rho| < 1$, the effect of u_{KK} always dominates.

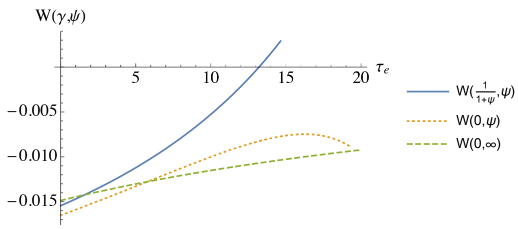
and Shin (2002) and Angeletos and Pavan (2004). Recall that the only difference between the two specifications is whether the payoff to coordination has social benefits (i.e., the impact of $u_{\sigma\sigma}$ on welfare), and so the players' choice of subjective beliefs is the same across the two specifications. The solid line plots the welfare measure W^0 when the social planner fully accounts for both the aggregate anticipatory utility and the objective cost of deviating from rational expectations (i.e., when $\gamma = \frac{1}{1+\psi}$). The dashed line corresponds to the welfare measure when $\gamma = 0$, in which case the social planner only weighs the aggregate experienced utility under objective beliefs. Finally, the dotted line corresponds to the (objective) welfare measure in the rational expectations equilibrium (i.e., when $\psi \to \infty$). Consistent with standard intuition, welfare always increases with private signal precision (i.e., τ_e) under rational expectations. Moreover, for our parameter configuration, welfare also increases with public signal precision (i.e., τ_η).

As the plots illustrate, however, neither conclusion necessarily holds when players can choose their subjective beliefs. First, panels (a) and (c) imply that welfare can decrease in private information precision. An increase in private signal precision leads players increases players' over-estimation of both signals' precisions. As discussed above, the increase in perceived private precision leads to an increase in dispersion, while the increase in perceived public precision can lead to an increase in non-fundamental volatility. When private information is sufficiently imprecise (relative to the prior, τ), these subjective beliefs can overwhelm the increase in welfare due to the objective reduction in fundamental uncertainty.[20] Similarly, panels (b) and (d) illustrate how the provision of public information can reduce welfare under subjective belief choice, even when it improves welfare under rational expectations. As with private information, an increase in the public signal precision leads to overestimation of both signals' precision, with analogous harmful welfare effects through dispersion and non-fundamental volatility.

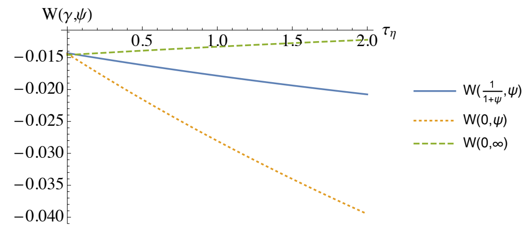
[20] Under rational expectations, the welfare benefit of a reduction in fundamental uncertainty that follows from private information provision always outweighs the (potentially) negative impact of increased dispersion. In our setting, this need not be the case, since the increase in dispersion reflects not only the quality of the information but also players' subjective beliefs.

Figure 5: Social Value of Information

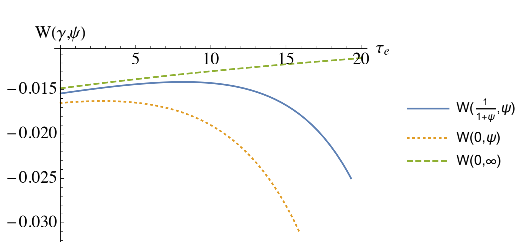
The figure plots the social planner's objective $W^0(\gamma, \psi)$ as a function of private information precision τ_e and public information precision τ_η, where γ reflects the social planner's weight on aggregate anticipatory utility and ψ parametrizes the players' cost of distorting subjective beliefs. Unless mentioned, the other parameters are $\tau = 10$, $\tau_e = 1$, $\tau_s = 0.1$, $\rho = 0.85$ and and $\psi = 1$.



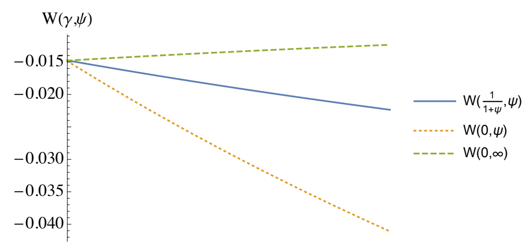
(a) W^0 versus τ_e, Morris-Shin



(b) W^0 versus τ_η, Morris-Shin



(c) W^0 versus τ_e, Angeletos-Pavan



(d) W^0 versus τ_η, Angeletos-Pavan

Notably, distorting the perceived precision of signals can improve aggregate well-being, even when the social planner only accounts for objective, experienced utility (i.e., $W(\gamma = 0, \psi)$). When $\rho > 0$, players over-weight the public signal from a normative perspective under Morris and Shin (2002). This can be undone under the subjective beliefs equilibrium because players perceive private signals to be more precise which tilts their actions back towards their private signal. This effect is highlighted in Figure 5, panel (a): welfare can be higher under subjective beliefs, even when the social planner accounts only for objective, experienced utility (i.e., $W(\gamma = 0, \psi) > W(\gamma = 0, \psi = \infty)$).[21]

Taken together, these results demonstrate that understanding the welfare

[21] In contrast, under the setting of Angeletos and Pavan (2004), $W(\gamma = 0, \psi)$ is necessarily lower, as shown in panel (c), since the benefits of coordination do not only accrue to individual players.

implications of information provision requires knowledge not only of how players utilize the information but also how they choose to perceive it. On the one hand, they highlight that the provision of more precise private or public signals can be counterproductive when individuals exhibit wishful thinking. On the other hand, in the absence of other tools, the social planner may be able to utilize players' distorted beliefs to reduce the impact of externalities.

7 Conclusions

In a standard model of externalities (Angeletos and Pavan (2007)), we allow players to choose their perceived precision of both private and public information. Our model endogenously generates overconfidence about private signals and can lead to endogenous disagreement about the interpretation of public information. We apply these general results across a series of widely-studied applications (e.g., IO models with Bertrand or Cournot competition), and show how our model can help explain observed variation in macroeconomic aggregates, including time-varying disagreement and empirically-documented patterns of over- and under-reaction in forecasts. In contrast to rational expectations equilibria, we show how more public information can lead to increased disagreement while more private information can lead to higher aggregate volatility. This suggests that players' subjective perception of information has important implications for the social value of information.

There are natural opportunities for future work. First, it would be interesting to study how information acquisition or attention interacts with players endogenous perception in the presence of externalities. Second, we hope to explore how wishful thinking interacts with robust control preferences (e.g., Hansen and Sargent (2001), Hansen and Sargent (2008)). As Caplin and Leahy (2019) point out, robust control preferences are a natural parallel to wishful thinking: individuals who exhibit robust control choose optimal actions under "worst-case" subjective beliefs while under wishful thinking players operate under "best-case" beliefs. We view these approaches as complementary - individuals are likely to exhibit wishful thinking in some settings, but robust control under others - and hope to add to a budding literature (e.g., Bhandari, Borovicka, and Ho (2019)) which allows both types to arise.

References

George A Akerlof and William T Dickens. The economic consequences of cognitive dissonance. *The American economic review*, 72(3):307–319, 1982.

Philippe Andrade, Richard K Crump, Stefano Eusepi, and Emanuel Moench. Fundamental disagreement. *Journal of Monetary Economics*, 83:106–128, 2016.

George-Marios Angeletos and Alessandro Pavan. Transparency of information and coordination in economies with investment complementarities. *American Economic Review*, 94(2):91–98, May 2004.

George-Marios Angeletos and Alessandro Pavan. Efficient use of information and social value of information. *Econometrica*, 75(4):1103–1142, 2007.

George-Marios Angeletos, Guido Lorenzoni, and Alessandro Pavan. Wall street and silicon valley: A delicate interaction. *Working Paper*, 2018.

George-Marios Angeletos, Zhen Huo, and Karthik A Sastry. Imperfect macroeconomic expectations: Evidence and theory. Technical report, National Bureau of Economic Research, 2020.

Hal Arkes. Costs and benefits of judgment errors: Implications for debiasing. *Psychological Bulletin*, 110(3):486–498, 1991.

Linda Babcock, George Loewenstein, and Samuel Issacharoff. Creating convergence: Debiasing biased litigants. *Law & Social Inquiry*, 22(4):913–925, 1997.

Snehal Banerjee, Jesse Davis, and Naveen Gondhi. Choosing to disagree in financial markets. *Available at SSRN*, 2019.

Roland Bénabou. Groupthink: Collective delusions in organizations and markets. *Review of Economic Studies*, 80(2):429–462, 2013.

Roland Bénabou and Jean Tirole. Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives*, 30(3): 141–64, 2016.

Anmol Bhandari, Jaroslav Borovicka, and Paul Ho. Survey data and subjective beliefs in business cycle models. 2019.

Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *The Annual Review of Financial Economics is*, 4:339–60, 2012.

Pedro Bordalo, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer. Diagnostic expectations and stock returns. *The Journal of Finance*, 74(6):2839–2874, 2019.

Pedro Bordalo, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. Overreaction in macroeconomic expectations. *American Economic Review*, 110(9):2748–82, 2020.

Markus K Brunnermeier and Jonathan A Parker. Optimal expectations. *American Economic Review*, 95(4):1092–1118, 2005.

Markus K Brunnermeier, Christian Gollier, and Jonathan A Parker. Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review*, 97(2):159–165, 2007.

Andrew Caplin and John Leahy. Psychological expected utility theory and anticipatory feelings. *The Quarterly Journal of Economics*, 116(1):55–79, 2001.

Andrew Caplin and John Leahy. Wishful thinking. 2019.

Timothy Cogley and Thomas J Sargent. Anticipated utility and rational expectations as approximations of bayesian decision making. *International Economic Review*, 49(1):185–221, 2008.

Olivier Coibion and Yuriy Gorodnichenko. Information rigidity and the expectations formation process: A simple framework and new facts. *The American Economic Review*, pages 2644–2678, 2015.

Luca Colombo, Gianluca Femminis, and Alessandro Pavan. Information acquisition and welfare. *Review of Economic Studies*, 81(4):1438–1483, 2014.

Stéphane Dupraz. Coordination under ambiguity. 2015.

Kfir Eliaz and Ran Spiegler. Contracting with diversely naive agents. *The Review of Economic Studies*, 73(3):689–714, 2006.

Nicholas Epley and Thomas Gilovich. The mechanics of motivated reasoning. *Journal of Economic Perspectives*, 30(3):133–140, 2016.

Itay Goldstein and Haresh Sapra. Should banks' stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends in Finance*, 8 (1):1–54, 2014.

Itay Goldstein and Liyan Yang. Information disclosure in financial markets. *Annual Review of Financial Economics*, 2017.

Lars Peter Hansen and Thomas J Sargent. *Robustness*. Princeton university press, 2008.

LarsPeter Hansen and Thomas J Sargent. Robust control and model uncertainty. *American Economic Review*, 91(2):60–66, 2001.

Benjamin M Hébert and Jennifer La'O. Information acquisition, efficiency, and non-fundamental volatility. Technical report, National Bureau of Economic Research, 2020.

Christian Hellwig and Laura Veldkamp. Knowing what others know: Coordination motives in information acquisition. *The Review of Economic Studies*, 76(1):223–251, 2009.

Harrison Hong and Jeremy C Stein. Disagreement and the stock market. *Journal of Economic Perspectives*, 21(2):109–128, 2007.

Herbert Stanley Jevons. *Essays on economics*. Macmilan, 1905.

Christine Jolls and Cass R Sunstein. Debiasing through law. *The Journal of Legal Studies*, 35(1):199–242, 2006.

E. Kandel and N.D. Pearson. Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy*, 103:831–872, 1995.

David M Kreps. Anticipated utility and dynamic choice. *Econometric Society Monographs*, 29:242–274, 1998.

George Loewenstein. Anticipation and the valuation of delayed consumption. *The Economic Journal*, 97(387):666–684, 1987.

Guido Lorenzoni. News and aggregate demand shocks. *Annu. Rev. Econ.*, 3(1): 537–557, 2011.

N Gregory Mankiw, Ricardo Reis, and Justin Wolfers. Disagreement about inflation expectations. *NBER macroeconomics annual*, 18:209–248, 2003.

Stephen Morris and Hyun Song Shin. Social value of public information. *The American Economic Review*, 92(5):1521–1534, 2002.

David P Myatt and Chris Wallace. Endogenous information acquisition in coordination games. *The Review of Economic Studies*, 79(1):340–374, 2012.

Emily Oster, Ira Shoulson, and E. Ray Dorsey. Limited life expectancy, human capital, and health investments. *American Economic Review*, 103(5):1977–2002, 2013.

A Proofs

A.1 Proof of Lemma 1

Suppose player i conjectures that the average action has the form $K = \kappa_0 + \alpha\theta + \beta s$, where α and β are determined in equilibrium. Then,

$$\begin{aligned} k_i &= \kappa_0 + (r\alpha + (1-r)\,\kappa_1)\,\mathbb{E}_i\,[\theta|s_i, s] + r\beta s && (27) \\ &= \kappa_0 + (r\alpha + (1-r)\,\kappa_1)\,A_i s_i + ((r\alpha + (1-r)\,\kappa_1)\,B_i + r\beta)\,s. && (28) \end{aligned}$$

This implies that, in equilibrium, the average action (across all players) is given by:

$$K = \int_i k_i di = \kappa_0 + (r\alpha + (1-r)\,\kappa_1)\,A\theta + ((r\alpha + (1-r)\,\kappa_1)\,B_i + r\beta)\,s, \quad (29)$$

where $A \equiv \int_i A_i di$ and $B \equiv \int_i B_i di$ reflect the average weights players put on their private and public signals, respectively.[22] Matching terms we show that:

$$\alpha = (r\alpha + (1-r)\,\kappa_1)\,A, \quad \beta = ((r\alpha + (1-r)\,\kappa_1)\,B_i + r\beta)\,, \quad (31)$$

which can be solved to yield:

$$\alpha = \frac{A\,(1-r)\,\kappa_1}{1-rA}, \quad \beta = \frac{B}{1-rA}\kappa_1. \quad (32)$$

A.2 Proof of Proposition 1

Let

$$\mathcal{K} \equiv rK + (1-r)\,\kappa$$

denote the target action for each player. Note that we can express the utility as:

$$U\,(k, K, \theta, \sigma) = \begin{array}{c} U\,(\kappa, \kappa, \theta, 0) + U_k\,(\kappa, \kappa, \theta, 0)\cdot(k-\kappa) + U_K\,(\kappa, \kappa, \theta, 0)\cdot(K-\kappa) + \\ \frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}\,(k-\kappa)^2 + U_{KK}\,(K-\kappa)^2 + 2U_{kK}\,(k-\kappa)\,(K-\kappa)\right) \end{array} \quad (33)$$

[22] Implicitly, we are assuming that the law of large numbers implies:

$$\int_i A_i s_i di = \int_i A_i di \times \int_i s_i di = A \times \theta. \quad (30)$$

Specifically, this assumes there is no cross-sectional correlation between A_i and s_i, but this is valid because $(\delta_{e,i}, \delta_{\eta,i})$ are chosen before s_i (and s) are observed.

Note that $U_k(\kappa,\kappa,\theta,0) = u_k + u_{kk}\kappa + u_{k\theta}\theta + u_{kK}\kappa = 0$ and

$$(k-\kappa)^2 = (k-\mathcal{K})^2 + r^2(K-\kappa)^2 + 2r(k-\mathcal{K})(K-\kappa)$$

Let $\mathbb{E}_j[\cdot]$ denote expectations w.r.t. arbitrary beliefs - we will later plug in subjective and objective beliefs. Then,

$$\mathbb{E}_j[U(k,K,\theta,\sigma)] = \mathbb{E}_j\left[\begin{array}{c} U(\kappa,\kappa,\theta,0) + (U_K + rU_{kk}(k-\mathcal{K}) + U_{kK}(k-\kappa))\cdot(K-\kappa) \\ +\frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}(k-\mathcal{K})^2 + \left(U_{KK}+r^2U_{kk}\right)(K-\kappa)^2\right) \end{array}\right] \tag{34}$$

Since $rU_{kk} = -U_{kK}$,we have that

$$rU_{kk}(k-\mathcal{K}) + U_{kK}(k-\kappa) = U_{kK}(\mathcal{K}-\kappa) = -r^2U_{kk}(K-\kappa)$$

which implies

$$\mathbb{E}_j[U(k,K,\theta,\sigma)] = \mathbb{E}_j\left[\begin{array}{c} U(\kappa,\kappa,\theta,0) + U_K(K-\kappa) + \\ \frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}(k-\mathcal{K})^2 + (U_{KK}-r^2U_{kk})(K-\kappa)^2\right) \end{array}\right] \tag{35}$$

Let $u_0 \equiv U(\kappa,\kappa,\theta,0)$, and $\mathcal{A} \equiv \frac{U_K(K-\kappa)}{\text{var}(\theta)}$. Then, anticipatory utility is given by:

$$AU_i = \mathbb{E}_i[u_i] = \begin{array}{c} \mathbb{E}[u_0] + \mathcal{A}\text{var}(\theta) + \frac{U_{\sigma\sigma}}{2}\sigma_i^2 \\ +\frac{1}{2}\left(U_{kk}\mathbb{E}_i\left[(k-\mathcal{K})^2\right] + (U_{KK}-r^2U_{kk})\mathbb{E}_i\left[(K-\kappa)^2\right]\right) \end{array} \tag{36}$$

Moreover,

$$\mathbb{E}_i\left[(k-\mathcal{K})^2\right] = \left(\frac{\kappa_1(1-r)}{1-rA}\right)^2 \text{var}_i[\theta|s_i,s]$$

$$\mathbb{E}_i\left[(K-\kappa)^2\right] = \left(\frac{\kappa_1(1-A-B)}{1-rA}\right)^2 \text{var}[\theta] + \left(\frac{\kappa_1 B}{1-rA}\right)^2 \text{var}_i[s|\theta]$$

Substituting all the above expressions into 36 and simplifying gives us

$$AU_i(\delta_{e,i},\delta_{\eta,i}) = \Gamma + \left(\frac{\kappa_1}{1-rA}\right)^2\left[u_{kk}(1-r)^2\text{var}_i[\theta|s_i,s] - \left(u_{KK}-r^2u_{kk}\right)B^2\text{var}_i[s|\theta]\right].$$

A.3 Proof of Corollary 1

Taking partial derivatives of equation 6, we can write

$$\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{e,i}} \propto \left(\mathrm{var}_i\left[\theta|s_i,s\right]\right)^2\tau_e > 0$$

which implies that anticipatory utility is always increasing in $\delta_{e,i}$. Moreover,

$$\begin{aligned}\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{\eta,i}} &\propto \left[\left(\mathrm{var}_i\left[\theta|s_i,s\right]\right)^2\tau_\eta - \frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{B^2}{\delta_{\eta i}^2\tau_\eta}\right] \\ &\propto \left[\left(\mathrm{var}_i\left[\theta|s_i,s\right]\right)^2 - \frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2}\right] \\ &= \left[\left(\mathrm{var}_i\left[\theta|s_i,s\right]\right)^2 - \chi\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2}\right]\end{aligned} \tag{37}$$

If $\chi \leq 0$, then anticipatory utility is always increasing in $\delta_{\eta,i}$. If $\chi > 0$, then anticipatory utility decreases in $\delta_{\eta,i}$ for low values of $\delta_{\eta,i}$and increases for high values of $\delta_{\eta,i}$ i.e., anticipatory utility is U-shaped in $\delta_{\eta,i}$.

A.4 Proof of Proposition 2

Equation (35) implies that, for any given beliefs.

$$\mathbb{E}_j\left[U\left(k,K,\theta,\sigma\right)\right] = \mathbb{E}_j\left[\begin{array}{c} U\left(\kappa,\kappa,\theta,0\right)+U_K\left(K-\kappa\right)+ \\ \frac{1}{2}\left(U_{\sigma\sigma}\sigma^2+U_{kk}\left(k-\mathcal{K}\right)^2+\left(U_{KK}-r^2U_{kk}\right)\left(K-\kappa\right)^2\right)\end{array}\right] \tag{38}$$

Using this for objective beliefs, we get

$$OU_i\left(\delta_{ei},\delta_{\eta,i}\right) = \mathbb{E}\left[U\left(k_i\left(\delta_{ei},\delta_{\eta,i}\right),K,\theta,\sigma\right)\right].$$

Note that

$$\mathbb{E}\left[\left(k-\mathcal{K}\right)^2\right] = \left(\frac{\kappa_1\left(1-r\right)}{1-rA}\right)^2\left(\frac{\tau+\delta_{ei}^2\tau_e+\delta_{\eta i}^2\tau_\eta}{\left(\tau+\delta_{ei}\tau_e+\delta_{\eta i}\tau_\eta\right)^2}\right)$$

which implies

$$OU_i\left(\delta_{ei},\delta_{\eta,i}\right) = L + \left(\frac{\kappa_1\left(1-r\right)}{1-rA}\right)^2\left(\frac{\tau+\delta_{ei}^2\tau_e+\delta_{\eta i}^2\tau_\eta}{\left(\tau+\delta_{ei}\tau_e+\delta_{\eta i}\tau_\eta\right)^2}\right).$$

The objective of player i is

$$\max_{\delta_{ei},\delta_{\eta i}} AU_i + \psi OU_i$$

The FOC with respect to $\delta_{ei}$is

$$1 - 2\psi\delta_{e,i} + \frac{2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta i}^2\tau_\eta\right)}{\tau + \delta_{ei}\tau_e + \delta_{\eta i}\tau_\eta} = 0 \tag{39}$$

The FOC with respect to $\delta_{\eta,i}$ for any interior equilibrium is

$$1 - \chi\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2 \upsilon_i^2} - 2\psi\delta_{\eta i} + \frac{2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta i}^2\tau_\eta\right)}{\tau + \delta_{ei}\tau_e + \delta_{\eta i}\tau_\eta} = 0 \tag{40}$$

In a symmetric equilibrium,$\delta_{\eta,i} = \delta_\eta \forall i$ which implies

$$B = \delta_{\eta,i}^2\tau_\eta^2\upsilon_i^2$$

and equation (40) simplifies to

$$1 - \chi - 2\psi\delta_{\eta i} + \frac{2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta i}^2\tau_\eta\right)}{\tau + \delta_{ei}\tau_e + \delta_{\eta i}\tau_\eta} = 0$$

Solving the FOCs gives us

$$\delta_e = \frac{\chi\left(\chi - 1\right)\tau_s + 2\psi\left(2\psi + 1\right)\tau}{2\psi\left(2\psi\tau - \tau_e + \left(\chi - 1\right)\tau_\eta\right)} \tag{41}$$

$$\delta_\eta = \frac{\tau_e\chi + 2\psi\tau\left(2\psi - \left(\chi - 1\right)\right)}{2\psi\left(2\psi\tau - \tau_e + \left(\chi - 1\right)\tau_\eta\right)} \tag{42}$$

which implies:

$$\delta_e = \delta_\eta + \frac{\chi}{2\psi} \tag{43}$$

These are indeed the equilibrium δ_e and δ_η when the denominator is positive i.e., $\left(2\psi\tau - \tau_e + \left(\chi - 1\right)\tau_\eta\right) > 0$. If this condition is not satisfied, then players will choose $\delta_e, \delta_\eta \to \infty$. In this case, the optimal weights A and B are given by

$$A_i = \frac{\tau_e}{\tau_e + \tau_\eta} \qquad B_i = \frac{\tau_\eta}{\tau_e + \tau_\eta}.$$

Next, we check the second order conditions. Denote $\upsilon_i = \frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}$. Recall that

$$TU = -\upsilon_i + x\frac{B^2}{\delta_{\eta i}\tau_\eta} - \psi\nu_i^2\left(\tau + \delta_{ei}^2\tau_e + \delta_{\eta i}^2\tau_\eta\right) \tag{44}$$

The SOC are given by:

$$TU_{ee} = -\frac{2\tau_e\Big(\psi\Big(\tau((3-2\delta_e)\tau_e+2\delta_\eta\tau_\eta)+\delta_\eta\tau_s(\delta_\eta(3\tau_e+\tau_\eta)-2\delta_e\tau_e)+\tau^2\Big)+\tau_e(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{45}$$

$$TU_{\eta\eta} = \frac{2B^2x}{\delta_\eta^3\tau_\eta} - \frac{2\tau_\eta\Big(\psi\Big((\delta_e\tau_e+\tau)^2+\delta_e\tau_e\tau_\eta(3\delta_e-2\delta_\eta)+\tau(3-2\delta_\eta)\tau_\eta\Big)+\tau_\eta(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{46}$$

$$TU_{e\eta} = -\frac{2\tau_e\tau_\eta\Big(\psi\delta_e^2\tau_e+\delta_e(\tau_e-2\psi\delta_\eta(\tau_e+\tau_\eta))+\tau\psi(-2\delta_e-2\delta_\eta+3)+\delta_\eta\tau_\eta(\psi\delta_\eta+1)+\tau\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{47}$$

Want to show that,

$$TU_{ee} < 0 \text{ or } TU_{\eta\eta} < 0 \tag{48}$$

$$\text{and } TU_{ee}TU_{\eta\eta} > TU_{\eta e}^2 \tag{49}$$

Note that imposing the equilibrium solutions gives us:

$$TU_{ee} = -\frac{8\psi^3\tau_e\left(\tau_e\left(\tau_e+\tau_\eta\right)-4\tau\psi\tau_e+4\tau\psi^2\left(\tau_\eta+\tau\right)\right)\left(\tau_e+\tau_\eta-\chi\tau_\eta-2\tau\psi\right)^2}{\left(4\tau\psi^2\left(\tau_e+\tau_\eta+\tau\right)+\chi^2\tau_e\tau_\eta\right)^3} \tag{50}$$

$$TU_{\eta\eta} = \frac{2\tau_\eta\Big(\psi\delta_\eta\Big(-(\delta_e\tau_e+\tau)^2+\delta_e\tau_e\tau_\eta(2\delta_\eta-3\delta_e)+\tau(2\delta_\eta-3)\tau_\eta\Big)+(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)(x(\delta_e\tau_e+\tau)+(\chi-1)\delta_\eta\tau_\eta)\Big)}{\delta_\eta(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{51}$$

Note that

$$TU_{ee} < 0 \Leftrightarrow \tau_e\left(\tau_e+\tau_\eta\right)+4\tau\psi\left(\psi\left(\tau_\eta+\tau\right)-\tau_e\right) > 0. \tag{52}$$

Treating the above equation as a quadratic in ψ, it is easy to show that the determinant is negative, which implies that the above equations holds $\forall\psi$.

$$\Delta \equiv TU_{ee}TU_{\eta\eta} - TU_{\eta e}^2$$

$$= -\frac{64\psi^6\tau_e\tau_\eta(\tau_e+\tau_\eta-\chi\tau_\eta-2\tau\psi)^6\begin{pmatrix} -2\tau\chi^3\psi\tau_e\tau_\eta+\chi^3\tau_e^2\tau_\eta+12\tau\chi^2\psi^2\tau_e\tau_\eta+6\tau\chi^2\psi\tau_e\tau_\eta \\ -12\tau\chi\psi^2\tau_e\tau_\eta-16\tau^2\psi^4\tau_e-8\tau^2\psi^3\tau_e+24\tau^2\chi\psi^3\tau_e \\ -12\tau^2\chi\psi^2\tau_e-12\tau\chi\psi^2\tau_e^2-16\tau^2\psi^4\tau_\eta-8\tau^2\psi^3\tau_\eta \\ +24\tau^2\chi\psi^3\tau_\eta-16\tau^3\psi^4-8\tau^3\psi^3+24\tau^3\chi\psi^3 \end{pmatrix}}{(\chi\tau_e+4\tau\psi^2+2\tau\psi-2\tau\chi\psi)(\chi^2\tau_e\tau_\eta+4\tau\psi^2\tau_e+4\tau\psi^2\tau_\eta+4\tau^2\psi^2)^6}$$

since $\delta_\eta = \delta_e + \frac{\chi}{2\psi}$. This implies:

$$\lim_{\psi\to\infty} \Delta = -\frac{64\psi^6 \tau_e \tau_\eta \left(2\tau\psi\right)^6 \left(-16\tau^2\psi^4\tau_e - 16\tau^2\psi^4\tau_\eta - 16\tau^3\psi^4\right)}{\left(4\tau\psi^2\right)\left(4\tau\psi^2\tau_e + 4\tau\psi^2\tau_\eta + 4\tau^2\psi^2\right)^6} > 0 \tag{53}$$

which implies that for ψ sufficiently large, the SOC holds.

A.5 Proof of Proposition 3

Denote $v_i = \frac{1}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta}$. The objective of player i is

$$\max_{\delta_{\eta,i},\delta_{e,i}} -v_i + \chi \frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi \nu_i^2 \left(\tau + \delta_{e,i}^2 \tau_e + \delta_{\eta,i}^2 \tau_\eta\right)$$

subject to $\delta_{e,i} \geq \underline{\delta}$ and $\delta_{\eta,i} \geq \underline{\delta}$. Let ω_e and ω_η denote the Lagrange multipliers for these inequalities. We define the Lagrangian $L = -v_i + \chi \frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi\nu_i^2 \left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right) + \omega_e \delta_{e,i} + \omega_\eta \delta_{\eta,i}$. The FOC with respect to $\delta_{ei}$for any equilibrium is

$$1 - 2\psi\delta_{e,i} + 2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right) v_i + \omega_e = 0 \tag{54}$$

The FOC with respect to $\delta_{\eta,i}$ for any equilibrium is

$$1 - \chi \frac{B^2}{\delta_{\eta,i}^2 \tau_\eta^2 v_i^2} - 2\psi\delta_{\eta,i} + 2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right) v_i + \omega_\eta = 0 \tag{55}$$

Step 1: Given any strategy $(\delta_{\eta,-i}, \delta_{e,-i})$, we first prove that agent i's solution is always unique. For this to be true, the above system of equations should always at-most one solution in $(\underline{\delta}, \infty) \times (\underline{\delta}, \infty)$. Substituting the first equation into second, we get

$$\underbrace{4\tau_\eta^3 \left(2\psi\tau + \tau_e - \tau_\eta\right) \delta_\eta^{*2} \left(\delta_\eta^* + \frac{2\psi\tau - \tau_e}{2\psi\tau_\eta}\right) \left[\frac{\tau\left(1+2\psi\right)}{2\psi\tau + \tau_e - \tau_\eta} - \delta_\eta^*\right]}_{\equiv g\left(\delta_\eta^*\right)} = \underbrace{4B^2\chi\left(\delta_\eta^{*2}\tau_\eta\left(\tau_e + \tau_\eta\right) + 2\delta_\eta^*\tau_\eta\tau + \tau\tau_e + \tau^2\right)^2}_{\equiv f\left(\delta_\eta^*\right)}$$

It is enough to show that the above set of equations will have at most 2 solutions. If so, one will be a minima and other will be a maxima. First, we note that $g\left(0\right) = g'\left(0\right) = 0, g''\left(0\right) > 0,$ and $g''' < 0$ while $f\left(0\right) > 0, f', f'', f''' > 0$. Let x^* be defined such that $g' > 0$ for $[0, x^*)$ and $g' < 0$ for (x^*, ∞). Finally, we denote $h\left(\delta_\eta^*\right) \equiv g\left(\delta_\eta^*\right) - f\left(\delta_\eta^*\right)$. and note that $h(0), h'(0) < 0$. There are two cases to consider.

- Case 1: Suppose $g\left(\delta_\eta^*\right) < f\left(\delta_\eta^*\right)$ for all values of $\delta_\eta^* > 0$. Then there is no solution with $\delta_\eta^* > 0$.

- Case 2: Suppose $g\left(\delta_\eta^*\right)$ is not less than $f\left(\delta_\eta^*\right)$ for all values of δ_η^*, i.e., there exists some x_1 such that $h(x_1) = 0$. Suppose that $h'(x_1) > 0$, so that for $x + e > x_1$ (where e can be arbitrarily small), $h(x + e) > 0$. That $h'(x_1) > 0$ implies that $g'(x_1) > f'(x_1) > 0$. Note that it must be the case that $x_1 < x^*$ (since $g'(x_1) > 0$) and there must exist at least one x_2 such that $x_1 < x_2 < x^*$ and $h'(x_2) = 0$ (since g' is continuous and $g'(x^*) = 0$). We want to show that only one such x_2 exists and that $h'(x) < 0$ for all $x > x_2$.
  - (1) If $g''(x_1) < f''(x_1)$, then this implies that $g''(x_1) < f''(x_1)$ for all $x > x_1$ since $g''' < 0$ and $f''' > 0$. But then this implies that $h'(x) = g'(x) - f'(x)$ is decreasing for all $x > x_1$. Thus, there exists only one $x_2 > x_1$ such that $h(x_2) = 0$ and $h'(x) < 0$ for all $x > x_2$.
  - (2) If $g''(x_1) > f''(x_1)$, then this implies that $h'(x)$ is increasing for at least some interval $[x_1, x_1 + u]$. However, note that since $g''' < 0$ and $f''' > 0$, that $g''(x) - f''(x)$ is decreasing and so there exists just one point, $x + u$, such that $g''(x_1 + u) = h''(x_1 + u)$. Furthermore, $g''(x) < f''(x)$ for all $x > x_1 + u$ and so $h'(x)$ is decreasing for all $x > x_1 + u$. But then this implies that there exists only one $x_2 > x_1 + u > x_1$ such that $h(x_2) = 0$ and $h'(x) < 0$ for all $x > x_2$.

Step 2: Given any strategy $(\delta_{\eta,-i}, \delta_{e,-i})$, player i will also never choose $\delta_{\eta,i} = \infty$ which gives agent finite anticipatory utility at infinite cost. This implies that agent i either choose a interior point or $\delta_{\eta,i} = \underline{\delta}$. If a player chooses $\delta_{si} = \underline{\delta}$, she chooses $\delta_{e,i} = \delta_{e1}$ using the FOC. (54) for any interior equilibrium can be rewritten as:

$$(\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta) - 2\psi\delta_{e1}\left(\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta\right) + 2\psi\left(\tau + \delta_{e1}^2\tau_e + \underline{\delta}^2\tau_\eta\right) = 0$$

which simplifies to

$$\delta_{e1} = \frac{\tau + \underline{\delta}\tau_\eta + 2\psi\left(\tau + \underline{\delta}^2\tau_\eta\right)}{2\psi\left(\tau + \underline{\delta}\tau_\eta\right) - \tau_e}. \tag{56}$$

Also, Corollary 1 shows that anticipatory utility is increasing in $\delta_{e,i}$ which implies that the optimal $\delta_{e,i} \geq 1$ implying that the constraint $\delta_{ei} \geq \underline{\delta}$ is never binding because $\underline{\delta} < 1$ by assumption. This implies that, the only possible mixed strategy equilibrium is one in which agent is indifferent between $(\delta_{e1}, \underline{\delta})$ and the unique interior solution which solves both the FOCs. This implies that, for any mixed strategy equilibrium, indifference equation is

$$-v_1 + \chi \frac{B^2}{\underline{\delta}\tau_\eta} - \psi \nu_1^2 \left(\tau + \delta_{e1}^2 \tau_e + \underline{\delta}^2 \tau_\eta\right) = -v^* + \chi \frac{B^2}{\delta_\eta^* \tau_\eta} - \psi \nu^{*2} \left(\tau + \delta_e^{*2} \tau_e + \delta_\eta^{*2} \tau_\eta\right)$$

where $v_1 = \frac{1}{\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta}$ and $v^* = \frac{1}{\tau + \delta_e^* \tau_e + \delta_\eta^* \tau_\eta}$.

Step 3: For high ψ, the only equilibrium is symmetric. For $\psi \to \infty$, the interior equilibrium solving 54 and 55 is $\delta_\eta^* = \delta_e^* = 1$ and in this equilibrium, agents choose not to incur any cost. If a mixed strategy equilibrium exists, agents have to be indifferent between $(\delta_{e1}, \underline{\delta})$ and $(1, 1)$. If an agent chooses $(\delta_{e1}, \underline{\delta})$, the benefit of deviation i.e., anticipatory utility, is finite (since $\underline{\delta}$ is non-zero), but the cost is infinite since ψ is very large. Hence, agents cannot be indifferent and the only equilibrium possible is symmetric in which all agents choose $(1, 1)$. Hence, by continuity, for ψ high enough, the only equilibrium is symmetric.

Step 4: For $\psi \to 0$, there is no symmetric interior equilibrium. For $\psi \to 0$, the equilibrium solving 54 and 55 is $\delta_\eta^* = \delta_e^* = \infty$. Suppose all players choose $\delta_\eta^* = \delta_e^* = \infty$, player i will deviate and choose $(\delta_{e1}, \underline{\delta})$ iff the utility she gets is higher with this deviation. Using equation 56, $\delta_{e1} = \infty$ when $\psi \to 0$. For interior symmetric equilibrium to hold, we need

$$-v_1 + \chi \frac{B^2}{\underline{\delta}\tau_\eta} - \psi \nu_1^2 \left(\tau + \delta_{e1}^2 \tau_e + \underline{\delta}^2 \tau_\eta\right) < -v^* + \chi \frac{B^2}{\delta_\eta^* \tau_\eta} - \psi \nu^{*2} \left(\tau + \delta_e^{*2} \tau_e + \delta_\eta^{*2} \tau_\eta\right).$$

In this case, $v^* = 0$, $v_1 = 0$ and $B = \frac{\tau_\eta}{\tau_\eta + \tau_e}$. The above equation translates into

$$\chi \frac{1}{\underline{\delta}\tau_\eta} \left(\frac{\tau_\eta}{\tau_\eta + \tau_e}\right)^2 < 0 \iff \chi < 0$$

which by assumption is not true. Hence a interior symmetric equilibrium is not possible. Conjecture that the equilibrium features mixing and a fraction λ of the agents choose $(\underline{\delta}, \delta_{e1} = \infty)$ and the remaining choose $\left(\delta_\eta^* = \infty, \delta_e^* = \infty\right)$. The fraction λ is pinned down by the indifference condition

$$-v_1 + \chi \frac{B^2}{\underline{\delta}\tau_\eta} - \psi \nu_1^2 \left(\tau + \delta_{e1}^2 \tau_e + \underline{\delta}^2 \tau_\eta\right) = -v^* + \chi \frac{B^2}{\delta_\eta^* \tau_\eta} - \psi \nu^{*2} \left(\tau + \delta_e^{*2} \tau_e + \delta_\eta^{*2} \tau_\eta\right).$$

which translates to $\lambda = 1$. This implies that a mixed strategy equilibrium exists. Hence, for $\psi \to 0$, a mixed strategy equilibrium exists.

Step 5: For low ψ, a mixed strategy equilibrium exists. Conjecture an equilibrium in which a fraction λ of the agents choose $(\delta_{e1}, \underline{\delta})$ and the remaining choose $\left(\delta_e^*, \delta_\eta^*\right)$. Suppose $2\psi\left(\tau + \underline{\delta}\tau_\eta\right) - \tau_e > 0$. In this equilibrium, the following equations have to hold:

$$1-2\psi\delta_e^*+2\psi\frac{\tau+\left(\delta_e^*\right)^2\tau_e+\left(\delta_\eta^*\right)^2\tau_\eta}{\tau+\delta_e^*\tau_e+\delta_\eta^*\tau_\eta}=0$$

$$1-\chi\frac{B^2\left(\tau+\delta_e^*\tau_e+\delta_\eta^*\tau_\eta\right)^2}{\left(\delta_\eta^*\right)^2\tau_\eta^2}-2\psi\delta_s^*+2\psi\frac{\tau+\left(\delta_e^*\right)^2\tau_e+\left(\delta_\eta^*\right)^2\tau_\eta}{\tau+\delta_e^*\tau_e+\delta_\eta^*\tau_\eta}=0$$

$$\delta_{e1}=\frac{\tau+\underline{\delta}\tau_\eta+2\psi\left(\tau+\underline{\delta}^2\tau_\eta\right)}{2\psi\left(\tau+\underline{\delta}\tau_\eta\right)-\tau_e}$$

$$-\frac{1}{\tau+\delta_{e,1}\tau_e+\underline{\delta}\tau_\eta}+\chi\frac{B^2}{\underline{\delta}\tau_\eta}-\psi\frac{\tau+\delta_{e1}^2\tau_e+\underline{\delta}^2\tau_\eta}{\left(\tau+\delta_{e,1}\tau_e+\underline{\delta}\tau_\eta\right)^2}=-\frac{1}{\tau+\delta_e^*\tau_e+\delta_\eta^*\tau_\eta}+\chi\frac{B^2}{\delta_\eta^*\tau_\eta}-\psi\frac{\tau+\delta_e^{*2}\tau_e+\left(\delta_\eta^*\right)^2\tau_\eta}{\left(\tau+\delta_e^*\tau_e+\delta_\eta^*\tau_\eta\right)^2}$$

Substituting the first equation into second, we get

$$4\tau_\eta^3\left(2\psi\tau+\tau_e-\tau_\eta\right)\delta_\eta^{*2}\left(\delta_\eta^*+\frac{2\psi\tau-\tau_e}{2\psi\tau_\eta}\right)\left(\frac{\tau\left(1+2\psi\right)}{2\psi\tau+\tau_e-\tau_\eta}-\delta_\eta^*\right)=4B^2x\left(\delta_\eta^{*2}\tau_\eta\left(\tau_e+\tau_\eta\right)+2\delta_\eta^*\tau_\eta\tau+\tau\tau_e+\tau^2\right)^2 \tag{57}$$

In step 1, we proved that above equation will have at most 2 solutions in $\delta_\eta^*\in(0,\infty)$. If so, the lower one will be minima and the higher one will be maxima. We are interested in the higher one. It is easy to see that, as B increases, the optimal δ_η^* decreases. Moreover, substituting optimal δ_{e1} and δ_e^* into the indifference condition, it reduces to

$$\frac{\tau_e-4\psi\left(\underline{\delta}\tau_\eta\left(\psi\underline{\delta}+1\right)+\tau(\psi+1)\right)}{4\psi\left(\left(\underline{\delta}\tau_\eta+\tau\right)^2+\tau_e\left(\underline{\delta}^2\tau_\eta+\tau\right)\right)}+\chi\frac{B^2}{\underline{\delta}\tau_\eta}=\frac{\tau_e-4\psi\left(\delta_\eta^*\tau_\eta\left(\psi\delta_\eta^*+1\right)+\tau\left(\psi+1\right)\right)}{4\psi\left(\left(\delta_\eta^*\tau_\eta+\tau\right)^2+\tau_e\left(\delta_\eta^{*2}\tau_\eta+\tau\right)\right)}+\chi\frac{B^2}{\delta_\eta^*\tau_\eta} \tag{58}$$

Solving for a mixed strategy equilibrium finally boils down to solving 57 and 58 for B and δ_η^*. Suppose ψ is low enough such that $2\psi\left(\tau+\underline{\delta}\tau_\eta\right)-\tau_e<0$. In this case, $\delta_{e1}\to\infty$ and $\delta_e^*,\delta_\eta^*\to\infty$ which implies $B=(1-\lambda)\frac{\tau_\eta}{\tau_e+\tau_\eta}$. The indifference equation reduces to

$$\chi\frac{B^2}{\underline{\delta}\tau_\eta}-\frac{\psi}{\tau_e}=-\frac{\psi}{\tau_e+\tau_\eta}$$

which implies that

$$(1-\lambda)^2=\psi\underline{\delta}\frac{\tau_e+\tau_\eta}{\chi\tau_e}$$

which implies that, for low ψ , $\lambda\in(0,1)$ and a mixed strategy equilibrium exists.

A.6 Proof of Proposition 4

Note that dispersion in actions is given by

$$\begin{aligned}\sigma_i^2 &= \frac{(\alpha_i+\beta_i-\alpha-\beta)^2}{\tau}+\frac{\alpha_i^2}{\tau_e}+\frac{(\beta_i-\beta)^2}{\tau_\eta}\\ &= \frac{\kappa_1^2(r-1)^2\left(\tau_\eta\left(\tau_e\left(-A_i+A-B_i+B\right)^2+\tau A_i^2\right)+\tau\left(B-B_i\right)^2\tau_e\right)}{\tau\tau_e\tau_\eta\left(1-rA\right)^2}\end{aligned}$$

In a symmetric equilibrium, $A_i=A$ and $B_i=B$ so that:

$$\sigma_i^2=\frac{\kappa_1^2(r-1)^2\tau_e\left(2\psi(2\psi+1)\tau+(\chi-1)\chi\tau_\eta\right)^2}{\left(-2\psi r\tau\tau_e+4\psi^2\tau\left(-r\tau_e+\tau_e+\tau_\eta+\tau\right)+\chi\tau_e\left(-r\chi+r+\chi\right)\tau_\eta\right)^2} \tag{59}$$

Note that non-fundamental volatility is given by

$$\nu^2=\frac{(\alpha+\beta-\kappa_1)^2}{\tau}+\frac{\beta^2}{\tau_\eta} \tag{60}$$

$$=\frac{\kappa_1^2\left((A+B-1)^2\tau_\eta+B^2\tau\right)}{\tau\tau_\eta\left(1-rA\right)^2} \tag{61}$$

$$=\frac{\kappa_1^2\Big(-16\psi^3\tau^2\tau_e+\chi^2\tau_e^2\tau_\eta+4\psi^2\tau\Big(2\tau_e\tau_\eta+\tau_e^2+(\chi-1)^2\tau_\eta(\tau_\eta+\tau)\Big)-4\psi\tau(\chi-1)\chi\tau_e\tau_\eta+16\psi^4\tau^2(\tau_\eta+\tau)\Big)}{(-2\psi r\tau\tau_e+4\psi^2\tau(\tau_e(1-r)+\tau_\eta+\tau)+\chi\tau_e(r+\chi-r\chi)\tau_\eta)^2} \tag{62}$$

For the rational expectations equilibrium ($\psi\to\infty$), we have:

$$\lim_{\psi\to\infty}\frac{\partial\nu^2}{\partial\tau_e}=-\frac{2\kappa_1^2\left(1-r\right)\left(\tau_\eta+\tau\right)}{\left(\tau_e\left(1-r\right)+\tau_\eta+\tau\right)^3}\leq 0.$$

$$\lim_{\psi\to\infty}\frac{\partial\sigma_i^2}{\partial\tau_\eta}=-\frac{2\kappa_1^2(r-1)^2\tau_e}{\left(\tau_e\left(1-r\right)+\tau_\eta+\tau\right)^3}\leq 0$$

Assume $\psi=1$. For the subjective expectations equilibrium, we have:

$$\frac{\partial\sigma_i^2}{\partial\tau_\eta}=\frac{4\kappa_1^2(r-1)^2\tau(\chi+2)\tau_e\left(2\tau(\chi-3)-\chi\tau_e\right)\left((\chi-1)\chi\tau_\eta+6\tau\right)}{\left(\chi\tau_e(-r\chi+r+\chi)\tau_\eta+(4-6r)\tau\tau_e+4\tau\tau_\eta+4\tau^2\right)^3}$$

This implies:

$$\frac{\partial\sigma_i^2}{\partial\tau_\eta}\geq 0\Leftrightarrow\chi\leq -2 \tag{63}$$

This implies that when χ is sufficiently negative (and δ's are well-defined), increasing public precision can **increase** dispersion. For non-fundamental volatility, we can show that

$$\frac{\partial \nu^2}{\partial \tau_e} \geq 0 \Leftrightarrow \frac{-\chi^2\tau_e\tau_\eta - 4\tau\tau_e + 8\tau\tau_\eta + 2\chi^2\tau_\eta^2 + 2\tau\chi^2\tau_\eta - 2\chi\tau_\eta^2 - 2\tau\chi\tau_\eta + 8\tau^2}{2\tau_e\tau_\eta - \chi^2\tau_e\tau_\eta + \chi\tau_e\tau_\eta - 4\tau\tau_e + 2\tau_\eta^2 + 10\tau\tau_\eta + 2\chi^2\tau_\eta^2 + 2\tau\chi^2\tau_\eta - 4\chi\tau_\eta^2 - 4\tau\chi\tau_\eta + 8\tau^2} < r < 1$$

This implies that, for r high enough, increasing private information can increase non-fundamental volatility.

A.7 Proof of Proposition 5

Forecast revision of player i is

$$FR_i \equiv \mathbb{E}_i\left[\theta | s_i, s\right] - \mathbb{E}_i\left[\theta\right], \tag{64}$$

while his forecast error is

$$FE_i \equiv \theta - \mathbb{E}_i\left[\theta | s_i, s\right]. \tag{65}$$

At the aggregate level, the regression coefficient between ex-post mean forecast errors and ex-ante mean forecast revisions is

$$CG_a \propto \text{Cov}\left(\mathbb{E}\left(\theta - \mathbb{E}_i\left[\theta | s_i, s\right]\right), \mathbb{E}\left(\mathbb{E}_i\left[\theta | s_i, s\right] - \mathbb{E}_i\left[\theta\right]\right)\right) \propto \frac{\left(1 - A - B\right)\left(A + B\right)}{\tau} - \frac{B^2}{\tau_s}.$$

At the individual level, the regression coefficient between ex-post forecast errors and ex-ante forecast revisions is

$$CG_i \propto \text{Cov}\left(\theta - \mathbb{E}_i\left[\theta | s_i, s\right], \mathbb{E}_i\left[\theta | s_i, s\right] - \mathbb{E}_i\left[\theta\right]\right) = \frac{\left(1 - A_i - B_i\right)\left(A_i + B_i\right)}{\tau} - \frac{A_i^2}{\tau_e} - \frac{B_i^2}{\tau_s}$$

(i) When ψ is sufficiently high, we will always have symmetric equilibrium and the optimal beliefs are given by 19. Substituting in the regression coefficient above, we get

$$CG_a \propto \frac{\delta_e\tau_e + \delta_s\tau_s - \delta_s^2\tau_s}{\left(\tau + \delta_e\tau_e + \delta_s\tau_s\right)^2}$$

Taking the limit as $\psi \to \infty$, we get

$$\lim_{\psi\to\infty} CG_a = \frac{\tau_e}{\left(\tau_e + \tau_s + \tau\right)^2} > 0$$

which implies that the regression coefficient is positive. At the individual level, the regression coefficient is

$$CG_i \propto \frac{\delta_e (1-\delta_e)\tau_e + \delta_s (1-\delta_s)\tau_s}{(\tau + \delta_e\tau_e + \delta_s\tau_s)^2}$$

In a symmetric equilibrium, $\delta_e, \delta_s > 1$ which implies that CG_i is negative.

(ii) When ψ is low and $\chi < 0$, we will always have symmetric equilibrium and the optimal beliefs are given by 19. Taking the limit as $\psi \to 0$, we get

$$\lim_{\psi\to 0} CG_a = \lim_{\psi\to 0} \frac{\delta_e\tau_e + \delta_s\tau_s - \delta_s^2\tau_s}{(\tau + \delta_e\tau_e + \delta_s\tau_s)^2} = -\frac{\tau_s}{(\tau_e + \tau_s)^2}$$

which implies that the regression coefficient is negative. At the individual level, the regression coefficient is

$$CG_i \propto \frac{\delta_e (1-\delta_e)\tau_e + \delta_s (1-\delta_s)\tau_s}{(\tau + \delta_e\tau_e + \delta_s\tau_s)^2}$$

In a symmetric equilibrium, $\delta_e, \delta_s > 1$ which implies that CG_i is negative.

(iii) When ψ is low and $\chi > 0$, the equilibrium features mixing (by proposition 3). This implies that some players overreact to information while others may underreact to it.

A.8 Proof of Proposition 6

Note that, in both utility specifications u_{MS} and u_{AP}, $r = \rho$ and χ is given by

$$\chi = \frac{u_{KK} - u_{kk}r^2}{-u_{kk}(1-r)^2} = -\frac{r}{1-r}.$$

The nature of the equilibrium comes directly from Propositions 2 and 3 .

B Extensions

B.1 Generalized Cost function

While our benchmark analysis focuses on the experienced utility penalty, we show that some our results extend to a general class of cost functions that are well-behaved, as defined below.

Definition 1. A cost function $C(\delta_{e,i}, \delta_{\eta,i})$ is **well-behaved** if $C(1,1) = \frac{\partial C}{\delta_{e,i}}(1,1) = \frac{\partial C}{\delta_{\eta,i}}(1,1) = 0$, and C is strictly convex (i.e., its global minimum is at $(1,1)$).

The following result characterizes symmetric equilibria with any well-behaved cost functions in our setting.

Proposition 7. *In any symmetric equilibrium, with any well-behaved cost function, all players are over-confident about both their private signals and the public signal (i.e., $\delta_{e,i} = \delta_e > 1$ and $\delta_{\eta,i} = \delta_\eta > 1$ for all i). Moreover, it must be the case that $A_i = A > A_{RE}$ and $B_i = B > B_{RE}$, i.e., players overweight both private and public signals relative to the rational expectations equilibrium.*

As discussed above, regardless of the choice made by others, player i benefits from believing that his private signal is more informative: $\frac{\partial AU_i}{\delta_{e,i}} \geq 0$. Since any deviation from rational expectations is costly, this implies that players must exhibit (weak) over-confidence in a symmetric equilibrium. When $\chi < 0$, a similar argument leads to over-confidence about public information as well. Somewhat surprisingly, however, we show that symmetric equilibria feature overconfidence in the public signal even when aggregate volatility is beneficial to each player, i.e., $\chi > 0$. This arises because the fundamental uncertainty channel always dominates the non-fundamental volatility channel when $\delta_{\eta,i} = \delta_\eta$.[23]

The second half of Proposition 7 extends this result further. Note that a player's weights on her private and public signal, i.e., A_i and B_i, respectively, depend on the subjective beliefs about the relative precision of the signals. As such, a higher subjective precision of a given signal does not necessarily imply that the player overweights this information (relative to rational expectations) when forming his conditional expectation. For instance, holding fixed $\delta_e > 1$, there exists sufficiently large δ_η such that players begin to underweight their private signal, despite their overconfidence in it. However, we show that the gap between δ_e and δ_η is bounded in equilibrium so that both (subjective) weights, A and B, are above their rational expectations counterparts.

Proof of Proposition 7

Corollary 1 argues that anticipatory utility is monotonic in δ_{ei} which implies $\delta_e \geq 1$. Moreover, equation (37) implies

$$\frac{\partial AU\left(\delta_{e,i}, \delta_{\eta,i}\right)}{\partial \delta_{\eta,i}} \propto \left[\left(\mathrm{var}_i\left[\theta|s_i, s\right]\right)^2 - \frac{\left(U_{KK} - U_{kk} r^2\right)}{-U_{kk}\left(1-r\right)^2} \frac{B^2}{\delta_{\eta i}^2 \tau_\eta^2}\right].$$

In a symmetric equilibrium,

$$B = \bar{\delta_\eta} \bar{\mathrm{var}}\left[\theta|s_i, s\right] \tau_\eta.$$

[23] On the margin, player i always benefits from believing that the public signal is more informative than others do i.e., $\frac{\partial AU_i}{\delta_{\eta,i}} \geq 0$ when evaluated at $\delta_{\eta,i} = \delta_\eta$. As a result, the equilibrium choice of δ_η cannot be below one. If it were, player i would benefit from choosing $\delta_{\eta,i} > \delta_\eta$ because such a deviation increases anticipatory utility and lowers the cost of distorting her beliefs (i.e., she would move $\delta_{\eta,i}$ closer to 1).

Using this equation, we can write

$$\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{\eta,i}}>0 \iff \frac{\left(\text{var}_i\left[\theta|s_i,s\right]\right)}{\bar{\text{var}}\left[\theta|s_i,s\right]}>\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{\left(\bar{\delta}_\eta\right)^2}{\delta_{\eta,i}^2}$$

In any symmetric equilibrium, this inequality holds if

$$\underbrace{\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}}_{\equiv\chi}<1 \tag{66}$$

which translates to $U_{kk}+U_{KK}+2U_{kK}<0$, which we assume is always true. This condition guarantees that the first-best allocation is unique and bounded. This implies that, in any symmetric equilibrium, $\delta_\eta>1$. Note that

$$\frac{\partial AU}{\partial A_i}=-\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\begin{pmatrix}\left(1-r\right)\left(U_{kk}\left(1+r-2rA\right)+2\left(1-A\right)U_{kK}\right)\\ +\frac{B^2\left(r^2U_{kk}+2rU_{kK}+U_{KK}\right)}{B_i}\end{pmatrix} \tag{67}$$

$$=-\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\frac{\left(U_{kk}+U_{kK}\right)^2}{U_{kk}}+\left(\frac{U_{kk}U_{KK}-U_{kK}^2}{U_{kk}}\right)\frac{B^2}{B_i}\right) \tag{68}$$

$$\frac{\partial AU}{\partial B_i}=-\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\begin{pmatrix}\left(1-r\right)\left(U_{kk}\left(1+r-2rA\right)+2\left(1-A\right)U_{kK}\right)\\ +\frac{B^2\left(r^2U_{kk}+2rU_{kK}+U_{KK}\right)}{B_i^2}\left(1-A_i\right)\end{pmatrix} \tag{69}$$

$$=-\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\frac{\left(U_{kk}+U_{kK}\right)^2}{U_{kk}}+\left(\frac{U_{kk}U_{KK}-U_{kK}^2}{U_{kk}}\right)\frac{B^2}{B_i^2}\left(1-A_i\right)\right) \tag{70}$$

Note that if $U_{kk}U_{KK}-U_{kK}^2>0$, then the above implies we should have both $\frac{\partial AU}{\partial A_i}>0$ and $\frac{\partial AU}{\partial B_i}>0$, since $U_{kk}<0$. □

B.2 Subjective beliefs about others' signals

In this section, we modify our benchmark model to allow players to hold subjective beliefs about the private signals observed by other players. As in the benchmark model, each individual observes

$$s_i=\theta+\varepsilon_i \quad \varepsilon_i\sim N\left(0,1/\tau_e\right), \tag{71}$$

but player i believes that

$$\varepsilon_j =_i \sqrt{1-\rho_i^2}\eta + \rho_i \varepsilon_j \quad \varepsilon_j \sim N\left(0, \tfrac{1}{\delta_i \tau_e}\right), \tag{72}$$

where ρ_i and δ_i are chosen by player i to maximize his anticipatory utility, net of costs. To highlight the role of beliefs about others, we shut down the public signal, s, and we do not allow players to hold subjective beliefs about the precision of their own signal. All other features of the benchmark model are unchanged.

Suppose player i conjectures that $K = \kappa_0 + \alpha\left(\theta + \sqrt{1-\rho_i^2}\eta\right)$, where α is determined in equilibrium. Then,

$$\begin{aligned} k_i &= \mathbb{E}_i\left[r\left(\kappa_0 + \alpha\left(\theta+\eta\right)\right) + \left(1-r\right)\left(\kappa_0 + \kappa_1\theta\right)|s_i\right] && (73)\\ &= \kappa_0 + \left(r\alpha + \left(1-r\right)\kappa_1\right)\mathbb{E}_i\left[\theta|s_i\right] && (74)\end{aligned}$$

This implies that, in equilibrium, player i believes that the average action (across all players) is given by

$$K = \int_i k_i di = \kappa_0 + \left(r\alpha + \left(1-r\right)\kappa_1\right) A\left(\theta + \sqrt{1-\rho_i^2}\eta\right), \tag{75}$$

Matching terms we show that α is unchanged from our benchmark model.

We can rewrite anticipated utility as

$$AU_i\left(\rho_i, \delta_i\right) = \begin{array}{c} \mathbb{E}\left[u_0\right] + \mathcal{A}\mathrm{var}\left(\theta\right) + \frac{1}{2}u_{\sigma\sigma}\mathbb{E}_i\left[\sigma_k^2\right] \\ +\frac{1}{2}\left(u_{kk}\mathbb{E}_i\left[\left(k-\mathcal{K}\right)^2\right] + \left(u_{KK} - r^2 u_{kk}\right)\mathbb{E}_i\left[\left(K-\kappa\right)^2\right]\right), \end{array} \tag{76}$$

where the key distinction from our benchmark is that players' subjective beliefs about other's signals impact their expectation of the dispersion in their actions, σ_k^2. Given player i's subjective beliefs,

$$\mathbb{E}_i\left[\sigma^2\right] = \int \left(k_i - K\right)^2 \, di = \frac{\rho_i^2\alpha^2}{\delta_i\tau_e} \tag{77}$$

$$\mathbb{E}_i\left[\left(k-\mathcal{K}\right)^2\right] = \frac{\left(r\alpha + \left(1-r\right)\kappa_1\right)^2}{\tau} + \frac{r^2\alpha^2\left(1-\rho_i^2\right)}{\delta_i\tau_e} \tag{78}$$

$$\mathbb{E}_i\left[\left(K-\kappa\right)^2\right] = \frac{\left(\frac{\kappa_1(A-1)}{1-rA}\right)^2}{\tau} + \frac{\alpha^2\left(1-\rho_i^2\right)}{\delta_i\tau_e} \tag{79}$$

This implies that

$$\frac{\partial AU_i}{\partial \delta_i} = -u_{\sigma\sigma}\left(\frac{\rho_i^2\alpha^2}{2\delta_i^2\tau_e}\right) - u_{KK}\left(\frac{\alpha^2\left(1-\rho_i^2\right)}{2\delta_i^2\tau_e}\right) \tag{80}$$

$$= -\frac{\alpha^2}{2\delta_i^2\tau_e}\left(\rho_i^2 u_{\sigma\sigma} + \left(1-\rho_i^2\right)u_{KK}\right) \tag{81}$$

$$\frac{\partial AU_i}{\partial \rho_i} = u_{\sigma\sigma}\left(\frac{\rho_i\alpha^2}{\delta_i\tau_e}\right) - \left(u_{KK} - r^2 u_{kk}\right)\frac{\alpha^2\rho_i}{\delta_i\tau_e} \tag{82}$$

$$= \frac{\rho_i\alpha^2}{\delta_i\tau_e}\left(u_{\sigma\sigma} - u_{KK}\right) \tag{83}$$

Note that k_i does *not* depend upon player i's subjective beliefs and, as a result, aggregate observables, such as realized dispersion and non-fundamental volatility, are also unaffected. Moreover, this implies that, under the experienced utility penalty, there is no cost to holding such subjective beliefs. As a result, the optimal ρ_i and δ_i depend only on the above expressions.

Note that when $u_{\sigma\sigma} > u_{KK}$, player i has an incentive to decrease the correlation in others signals. Intuitively, when dispersion is preferable to aggregate volatility, he prefers higher ρ_i, i.e. others' signals are more independent. Moreover, if $u_{\sigma\sigma}$ is positive, then he wants to believe that δ_i is lower, since this further increases the dispersion in players' actions; if $u_{\sigma\sigma}$ is negative, he chooses to increase δ_i.

If aggregate volatility is relatively better than dispersion ($u_{\sigma\sigma} > u_{KK}$), however, he prefers to believe that others information is more correlated, i.e., lower ρ_i. Analogous to the intuition above, the choice of δ_i then depends upon the sign of u_{KK}: if it's positive, player i lowers δ_i to increase aggregate volatility, whereas if it's negative, he has in incentive to believe that there is less common noise, i.e., increase δ_i.

C Appendix B: Alternative Applications

In this section, we consider the implications of our analysis for a wide range of applied models.

C.1 Efficient Competitive Economies

Consider an incomplete-market competitive economy in which agents' choices are strategic substitutes. There are two goods and a continuum of households (who act as consumers and producers). q_{1i} and q_{2i} are the quantities of each good purchased by consumer i, and his preferences are

$$u_i = \theta q_{1i} - \frac{bq_{1i}^2}{2} + q_{2i}$$

while his budget constraint is

$$pq_{1i} + q_{2i} = e + \pi_i$$

where θ is a relative demand shock, p is the price of good one and, good two serves as the numeraire, e is an exogenous endowment of good two, and π_i are the profits of producer i. Profits, therefore, are given by $\pi_i = pk_i - \frac{k_i^2}{2}$.

Consumer i chooses the optimal bundle of goods (q_{1i}, q_{2i}) to maximize his utility and so in equilibrium $p = \theta - bq_{1i}$. Households are ex-ante identical which, together with market clearing, implies that $q_{1i} = K$ for all i and therefore $p = \theta - bK$. This example is thus captured by the generalized framework with the utility

$$U(k, K, \sigma_k, \theta) = (\theta - bK)\, k - k^2/2 + bK^2/2 + e$$

which implies that

$$u_{kk} = -1, u_{kK} = -b, u_{k\theta} = 1, u_{KK} = b$$

and so $\kappa_0 = 0$,$\kappa_1 = \frac{1}{1+b}$ and

$$\chi \equiv \frac{u_{KK} - u_{kk} r^2}{-u_{kk}\,(1-r)^2} = \frac{b}{1+b} > 0.$$

C.2 Investment complementarities

Consider a setting in which the terminal value of the firm is given by $V \equiv V(R, k)$ where R measures the return on investment, or productivity, of the project available and k represents the scale of investment in the project. For analytical tractability, let

$$V(R, k) = Rk - \tfrac{1}{2}k^2, \tag{84}$$

$$R = (1-a)\,\theta + aK, \tag{85}$$

with $a \in (0, 1/2)$, and $\theta \in \mathbb{R}$ represents the firm's exogenous productivity. This implies that firm i's utility is

$$U_i(k_i, K, \theta) = (1-a)\,\theta k_i + aKk_i - \tfrac{1}{2}k_i^2$$

which falls into the class of general objective functions analyzed. This implies that

$$u_{kk} = -1, u_{kK} = a, u_{k\theta} = 1 - a.$$

which implies that $r = a$ and $\kappa_1 = 1$ and so

$$\chi = -\frac{\left(u_{KK} - r^2 u_{kk}\right)}{u_{kk}\left(1-r\right)^2} = \frac{a^2}{\left(1-a\right)^2} > 0.$$

C.3 Cournot versus Bertrand Competition

First, consider a Cournot setting in which firms compete on quantity. For any firm, consumer demand is $p = a_0 + a_1\theta - a_2 q - a_3 Q$ (with $a_0, a_1, a_2, a_3 > 0$), where p denotes the price at which the firm sells each unit, q is the number of units the firm produces , Q is the average quantity produced across all firms, and θ is a fundamental demand shock/shifter. Each firm's profits are $u = pq - C\left(q\right)$, where $C(q) = c_1 q + c_2 q^2$ is cost of production (with $c_1, c_2 > 0$). This model is contained within the generalized framework with $k \equiv q, K \equiv Q$, and

$$U(k, K, \sigma_k, \theta) = \left(a_0 - c_1 + a_1\theta - a_3 K\right) k - \left(a_2 + c_2\right) k^2$$

This implies that

$$U_{kk} = -2\left(a_2 + c_2\right), U_{kK} = -a_3, U_{k\theta} = a_1, U_k = a_0 - c_1.$$

which implies that $r = \frac{-a_3}{2(a_2+c_2)}$ and $\kappa_1 = -\frac{a_1}{-2(a_2+c_2)-a_3}$ and

$$\chi = -\frac{\left(u_{KK} - r^2 u_{kk}\right)}{u_{kk}\left(1-r\right)^2} = \frac{\frac{a_3^2}{4(a_2+c_2)^2}}{\left(1 + \frac{a_3}{2(a_2+c_2)}\right)^2} > 0.$$

Next, consider a Bertrand setting in which firms compete on price. Consumer demand for each firm is $q = b_0 + b_1\theta' - b_2 p + b_3 P$ (with $b_0, b_1, b_2, b_3 > 0$), where q denotes the quantity produced by a given firm, p is the price set by a given firm, P is the average price across all firms, and θ is, again, an exogenous demand shock. As in Angeletos and Pavan (2007), we assume $b_3 < b_2$, and so an equal increase in p and P reduces q. Firm profits are as above. This model is contained within the generalized framework where $k \equiv p - c_1, K \equiv P - c_1$ (actions are now prices), and

$$\begin{aligned}
U\left(k, K, \sigma_k, \theta\right) &= \left[\left(\theta - k + bK\right) k - c\left(\theta - k + bK\right)^2\right] \\
&= \left[\left(\theta k - k^2 + bKk\right) - c\left(\theta^2 + k^2 + b^2 K^2 - 2\theta k - 2bkK + 2\theta bK\right)\right]
\end{aligned}$$

$$= \theta k (1+2c) - k^2 (1+c) + Kk (b+2bc) - c\theta^2 - cb^2K^2 - 2bc\theta K$$

This implies that

$$U_{kk} = -2(1+c), U_{kK} = b(1+2c), U_{k\theta} = 1+2c, U_{KK} = -2cb^2$$

$$U_{K\theta} = -2bc, U_{\theta\theta} = -2c.$$

which implies that $r = \frac{b(1+2c)}{2(1+c)}$ and $\kappa_1 = \frac{1+2c}{2(1+c)-b(1+2c)}$ and

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}(1-r)^2} = \frac{-2cb^2 + 2(1+c)\left(\frac{b(1+2c)}{2(1+c)}\right)^2}{2(1+c)\left(1 - \frac{b(1+2c)}{2(1+c)}\right)^2}.$$

Since the denominator is always positive, this implies that

$$sign(\chi) = sign\left(\frac{-4(1+c)cb^2 + b^2(1+2c)^2}{2(1+c)}\right) > 0.$$

C.4 Information Spillovers

The model of Angeletos, Lorenzoni, and Pavan (2018) considers a novel channel through which the information in the real sector affects behavior in the financial sector. The real sector is comprised of entrepreneurs making investment decisions, and the financial sector is comprised of investors who provide liquidity to the "real" economy. All players are risk-neutral and the discount rate is zero. There are three dates, $t \in \{1, 2, 3\}$. At $t = 1$, a new investment opportunity becomes available with productivity θ. This investment pays off at $t = 3$. There is a continuum of entrepreneurs who can choose how much to invest in the new technology. Let k_i denote the investment of entrepreneur i, and let the cost of this investment be $\frac{k_i^2}{2}$. Entrepreneurs have access to an information technology that generates both a private and a "public" signal that they utilize when making their investment decision.[24] The joint distribution of fundamentals and signals follows the specification detailed in Section 3.

At $t = 2$, an entrepreneur is hit by an idiosyncratic liquidity shock with probability $l \in [0, 1]$. Entrepreneurs hit by this shock do not value consumption at $t = 3$ and so strictly prefer to sell their capital, at a price p, to investors.[25]

[24] It is public in the sense that all entrepreneurs observe the same signal; however, as we discuss below, it is assumed that investors do not observe the signal and can only learn about θ through the aggregate investment level.

[25] It is assumed in ALP (2018) that the entrepreneurs not hit by the shock are precluded

Thus, entrepreneur i's payoff is $u_i = c_{i1} + c_{i2} + s_i c_{i3}$, where c_{it} denotes player i's consumption in period t and $s_i \in \{0, 1\}$ equals zero if he is hit by a liquidity shock. Thus, taken together, this implies that an entrepreneur's expected utility at the time of investment is given by

$$\mathbb{E}_i \left[u_i | s_i, s\right] = \mathbb{E}_i \left[\left(1 - l\right)\theta k_i + lpk_i - \frac{1}{2}k_i^2 | s_i, s\right]. \tag{86}$$

The financial market is competitive and the price p is determined through market clearing. Investors do not have access to their own information technology but, given the assumptions above, update their beliefs about the productivity of the technology utilizing the information contained in the supply of capital to be liquidated. It can be shown that, given the distributional assumptions and the risk-neutrality of traders, that $p = \mathbb{E}\left[\theta | K\right] = \alpha_1 K$, where α_1 is pinned down in equilibrium, shown below.[26] This implies that expected utility of entrepreneur is given by

$$E\left[u_i\left(k_i, K, \theta\right) | s_i, s\right] = E_i \left[\left(1 - l\right)\theta k_i + lk_i\left(\alpha_1 K\right) - \frac{1}{2}k_i^2 | s_i, s\right], \tag{87}$$

and, therefore, each entrepreneur's optimal action is $k_i = E_i\left[\left(1 - l\right)\theta + l\left(\alpha_1 K\right)\right]$. Note that this specification follows the generalized model analyzed above, which allows us to utilize the fixed-point solution for k_i found in (13). Aggregating across all entrepreneurs, this implies that aggregate investment can be written as

$$\begin{aligned} K &= \frac{1 - l}{1 - lA}\kappa_1 A\theta + \frac{B}{1 - lA}\kappa_1 s \\ &= \frac{\kappa_1\left(\left(1 - l\right)A + B\right)}{1 - lA}\left(\theta + \frac{B}{\left(1 - l\right)A + B}\eta\right). \end{aligned} \tag{88}$$

It is straightforward to see that the aggregate level of capital reveals a signal of the form $\xi = \theta + \frac{B}{\left(\left(1 - r\right)A + B\right)}\eta$ to investors which, given the linear-normal structure, verifies the conjectured functional form for the price of capital. On the other hand, however, note that

$$\chi \equiv \frac{u_{KK} - u_{kk}r^2}{-u_{kk}\left(1 - r\right)^2} = \frac{l^2\alpha_1^2}{\left(1 - l\alpha_1\right)^2} > 0. \tag{89}$$

---

from trading.

[26] Investors are able to back out the aggregate level of capital utilizing l since liquidity shocks are independent.