Costly Disclosure of Redundant Information*

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Abstract

Why do firms engage in costly, voluntary disclosure of information which is eventually subsumed by a later announcement? We consider a two-period model in which the firm's manager can choose to disclose short-term information which becomes redundant before the second period. When disclosure costs are sufficiently low, we show the manager discloses redundant information even if she only cares about the long-term (second period) price. Intuitively, by doing so, she decreases information acquisition by early investors but increases acquisition by late investors. We show that the increase in second-period acquisition more than offsets the initial decrease and, consequently, improves long term prices.

Keywords: costly voluntary disclosure, information acquisition, redundant information

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1 Introduction

Will voluntary disclosure of information (e.g., the existence of ongoing projects), which will be completely subsumed by a later announcement (e.g., the ultimate cash flows from the projects), have an incremental impact on a firm's stock price at the time of the later announcement? A Bayesian economist might be tempted to respond that the answer is no. After all, the early information becomes redundant at the later date.

This reaction reflects a broader attitude about the impact on long-term valuations of trading on short-term information. Stiglitz (1989) considers the example of investors acquiring information today that becomes publicly available tomorrow. While there may be a private benefit to trading on such information in the short-term, there is no long-term effect on prices: the eventual change in the firm's value is the same.[1] In this case, what are the firm's incentives to disclose such short-term information?

Importantly, the types of settings we have in mind are ones in which disclosure is truly redundant. It does not lead to feedback effects that would impact firm investment decisions, nor does it convey any information about persistent unobservable factors that could impact firm value beyond the subsequent disclosure at the later date. Trueman (1986) forcefully argues that in such a setting:

> "*the disclosure would simply advance the time at which investors learn something about the firm's earnings. The market value of the firm at the end of the period, after the actual earnings had been reported, however, would be unaffected by the forecast release (since the estimated earnings becomes irrelevant for valuation at that time).*"

While the above argument is intuitive, in this paper we show that strategic early disclosure can increase firm market value at future dates even after the disclosure becomes redundant. Our insight is that disclosure affects investors' information acquisition decisions, and consequently, influences the information environment at later dates. Disclosure directly impacts early investors' information choices, which changes the public information available to late investors (via the information revealed by short-term prices), and consequently affects their information acquisition and demand for the stock.

To highlight the economic channel, we restrict attention to a stylized setting. Specifically, we study a model with two trading periods in which the firm's terminal value depends on the payoffs to a long term project (i.e., the assets-in-place of the firm) and, possibly, a short term project. If it exists, the short-term project's payoff is publicly revealed before trade in

[1] In fact, Stiglitz (1989) argues that it may be desirable to tax short-term turnover to make prices less volatile, even though this reduces or eliminates the incorporation of short-term information into prices.

the second period. The long-term project's payoff is revealed only after trade in the second period, when it is paid out as part of the terminal value.

We assume that the manager knows with certainty whether the short-term project exists, and can disclose this information truthfully before the first round of trade by paying a cost (as in Verrecchia (1983)). The manager's objective at the disclosure stage is to maximize the long-run (second-period) market value. The short-term project's payoff is publicly revealed before the second round of trading, independently of the manager's disclosure decision, and makes the manager's disclosure completely redundant. Before trading in each round, investors choose whether or not to acquire costly information about the long-term project (as in Grossman and Stiglitz (1980)).

Our main result is that, provided that the cost of disclosure is not too high, the manager will voluntarily disclose the existence of the short-term project. By disclosing, the manager affects information acquisition and trading by investors. When early (first-period) investors learn that there is a short-term project, they face more uncertainty about the second-period price. As a result, informed investors in the first period trade less aggressively, which reduces first-period price informativeness.[2] Following the revelation of the short-term project's payoff prior to the second round of trade, this leads more second-period investors to become informed about the long-term project. We show that the impact of more information acquisition at the later date dominates the impact of higher uncertainty in the short term. The resulting long term prices more precisely reflect the firm's true value and hence are higher on average, due to a lower risk premium.

There is an extensive literature on understanding the rationales for disclosure. Diamond (1985) shows how pre-commitment to publicly disclosing information can improve welfare by improving risk sharing and saving real resources which would otherwise be devoted to private information acquisition. In the presence of proprietary disclosure costs, sufficiently good news is disclosed and bad news is withheld (Verrecchia (1983)), and improvements in the quality of managers' information increases disclosure (Verrecchia (1990)). A similar threshold disclosure characterization exists if investors are uncertain as to whether the manager has information (Dye (1985), Jung and Kwon (1988)). Litigation risk induces an asymmetric impact on incentives to disclose, tilting managers to disclose negative earnings news earlier (Skinner (1994)). Diamond and Verrecchia (1991) show disclosure changes risk for market

[2] The disclosure of the project has two effects on first-period price informativeness. First, it affects the fraction of first-period traders that acquire information. Second, it lowers the trading intensity of those traders who choose to become informed. While the fraction of informed traders can, in general, either increase or decrease, the trading intensity effect always dominates, leading to an unambiguous negative relation between disclosure and first-period price informativeness.

makers, which affects their willingness to provide liquidity.[3]

Our paper is related to the literature on earnings guidance, which focuses on the manager's incentives to influence investors' expectations about future earnings. In Trueman (1986), early voluntary disclosure that is later validated by a mandatory disclosure helps the manager signal to investors about persistent skill in identifying optimal investment decisions.[4] In contrast, our model is designed so that at the time of the mandatory disclosure, any earlier voluntary disclosures become completely redundant. Yet, our analysis highlights a channel whereby the earlier disclosure still increases firm value at the later date.

The literature on feedback effects highlights a related complementarity between disclosure and informed trading by investors.[5] In such models, the manager chooses to strategically disclose information to encourage investors to trade more aggressively on their private information, or acquire more information. This results in more informative prices and, consequently, better informed real decisions by the manager. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein, Schneemeier, and Yang (2020) highlight how disclosure along one dimension of fundamentals can crowd in more informed trading along another dimension. Yang (2020) considers an oligopoly setting where firms disclose information about consumer demand to encourage investors to trade on their private information about an orthogonal component. Smith (2020) shows that disclosure about a firm's riskiness can induce investors to acquire more information about fundamentals. Lassak (2020) studies a setting where disclosure about cash flows can increase uncertainty, and shows that the firm discloses information only if it crowds in more learning by investors.

The economic mechanism in our model is distinct from this literature. First, the manager's motivation for disclosure does not rely on any feedback effects: the manager does not learn from, or make investment decisions based on, the equilibrium price. Second, the direct effect of the firm's disclosure about the short-term project is to *discourage* informed trading by early investors and so make short-term prices less informative. However, through the endogenous information acquisition choices of long-term investors, we show that this results in more informative prices in the long term. Finally, in contrast to related papers, it is key that the relevant disclosure is rendered completely redundant in the long term, and yet has an incremental effect on the long term price.

Our analysis also highlights the importance of considering the consequences of risk and investor risk-aversion for voluntary disclosure, and of studying the interaction between differ-

[3] See also surveys in Verrecchia (2001), Dye (2001) and Beyer, Cohen, Lys, and Walther (2010).

[4] In his model, the manager's ability to predict the firm's future optimal production level is the unobservable characteristic. By releasing a forecast that is subsequently validated, the manager signals to investors that he has that skill, which improves subsequent firm investment decisions, and hence increases firm value.

[5] See Bond, Edmans, and Goldstein (2012) and Goldstein and Yang (2017) for recent surveys.

ent sources of endogenous information in financial markets. Two closely related papers along these dimensions are Dye and Hughes (2018) and Banerjee, Marinovic, and Smith (2021). Dye and Hughes (2018) study a firms' voluntary disclosure decisions to uninformed, risk averse investors, and explore how disclosure is affected by systematic risk. Banerjee, Marinovic, and Smith (2021) study how a firm's disclosure decision is affected by the presence of dispersed private information across risk-averse investors. In these papers, importantly there is only one trading date (i.e., there is no notion of redundancy), and the manager's disclosure and the information that investors are endowed with are both about the firm's total cash flow. Our analysis, which we view as complementary, features endogenous information acquisition by investors over multiple periods.

2 Model

Payoffs. There are four dates $t \in \{0, 1, 2, 3\}$ and two securities. The gross return on the risk-free security is normalized to one. The risky security is a claim to a public firm with terminal value V, which will be realized at $t = 3$. The value V is given by

$$V = \bar{V} + x\eta + \theta + u, \tag{1}$$

where $\bar{V}$ is a known constant, η, θ, and u are independently normally distributed and $x \in \{0, 1\}$ is independent with prior probability p on $x = 1$. The aggregate supply of the risky security is given by $Z_t = \bar{Z} + z_t$ where z_t are normally distributed, and are independent of each other and other random variables. We denote the date t price of the risky security by P_t, and note that $P_3 = V$. We normalize the means of η, θ, u and z_t to zero without loss of generality, and denote the variance (precision) of these shocks by $\sigma^2_{(\cdot)}$ ($\tau_{(\cdot)}$, respectively).[6]

The event where $x = 1$ corresponds to the case where the secondary project exists and $x = 0$ to the case it does not. The payoff to the secondary project $x\eta$ is publicly revealed at date $t = 2$, and the payoff $\theta + u$ is publicly revealed at date $t = 3$ when the asset pays off.

Investors. There are overlapping generations (OLG) of investors. Generation t consists of a continuum of investors, indexed by $i \in [0, 1]$ with CARA utility and risk-aversion γ. Investor i in generation t can pay a cost c to observe θ immediately before trading at date

[6] In particular, note that since $x\eta$ is revealed at date 2, and the managers' disclosure decision is based on the date 2 price, the prior mean of η is irrelevant for her decision. If the manager's payoff also depends on the date 1 price, then a positive mean for η would strengthen our result by increasing the manager's incentive to disclose when the project exists ($x = 1$), in order for the positive mean to be fully reflected in P_1 as an additional $x\mu_\eta$ term, and to not disclose when the project does not exist ($x = 0$), in which case the prior mean is irrelevant for P_1.

t, and submits demand schedule X_{it} to maximize her expected utility over wealth at date $t+1$. We denote investors who choose to acquire information about θ by $i = I$, those who choose to remain uninformed by $i = U$, and the fraction who choose to become informed at date t by λ_t. We will use $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$ to denote the relevant conditional expectation and conditional variance operators for investor types $i \in \{I, U\}$.

Manager. The firm's manager knows x at date $t = 0$ and chooses whether or not to disclose it at a cost $c_D > 0$ (e.g., see Verrecchia (1983)).[7] Let $d = D$ and $d = ND$ correspond to the choice of disclosure and non-disclosure, respectively. The manager optimally chooses her disclosure strategy to maximize the expected date 2 price. Let

$$U_d(x) = \mathbb{E}\left[P_2 \middle| d, x\right]$$

denote the expected price conditional on the realized value of x and the disclosure decision d.[8] Formally, a type x manager's problem is

$$U(x) \equiv \max_{d \in D, ND} U_d(x) - c_D \mathbf{1}_{\{d=D\}}. \tag{2}$$

Figure 1: Timeline of events

$t=0$	$t=1$	$t=2$	$t=3$
Manager observes x, chooses D or ND	Frac. λ_1 observe θ Inv. demand X_{i1} Asset trades at P_1	$x\eta$ publicly revealed Frac. λ_2 observe θ Inv. demand X_{i2} Asset trades at P_2	θ, u revealed Asset pays V

Figure 1 summarizes the timing of events.

2.1 Discussion of assumptions

We consider an OLG setting in order to transparently and tractably illustrate the important economic forces. The key mechanism, that disclosing higher x increases information collec-

[7] Importantly, note that the manager does not observe the realization of η until date 2.

[8] Note that because $\mathbb{E}[x\eta|x] = 0$, and because ηx is publicly-disclosed before the $t = 2$ trading round and therefore enters any price function linearly, the expected price in the event of no disclosure is identical across values of x, $U_{ND}(0) = U_{ND}(1)$.

tion at $t = 2$, is robust to settings in which traders are fully dynamic and choose whether to observe (noisy) signals before each trading round.[9] What is important is that the existence of the project reduces price informativeness in the first trading round by making the asset riskier (and thereby inducing informed traders to trade less aggressively), which increases the value of acquiring information prior to the $t = 2$ trading round. All of our results go through as stated for the more general setting $x \in \{x_L, x_H\}$ for any nonnegative $x_L < x_H$, and where we interpret the manager's disclosure as being directly about the riskiness of the short-term project. Similarly, the assumption that asset supply is iid in our OLG setting is for simplicity. Numerical investigation shows that our results are robust to correlated asset supply (i.e., $Z_t = \overline{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t$ for $\phi \neq 0$).

We focus on a manger who is concerned with the "long run", post-disclosure price P_2 so that the disclosure is unambiguously redundant by the time her "utility" is realized. This is the starkest setting in which to illustrate that, despite the redundancy, she may still find it optimal to engage in costly disclosure. Clearly, if the manager also cares about the "short run" price P_1, which is directly affected by disclosure, then she may have an incentive to do so (e.g., if η had a sufficiently positive mean). One could also allow the manager's objective to depend on the terminal price $P_3 = \bar{V} + x\eta + \theta + u$, but because this quantity is exogenous, excluding it is without loss of generality. Moreover, it is not necessary for the payoff $x\eta$ to be perfectly revealed at date 2 for our mechanism to operate – a noisy signal about $x\eta$ would have qualitatively similar implications, although the analysis would be more cumbersome. While it would be interesting to extend our analysis to solve for the manager's objective as part of an optimal contract, such an extension is beyond the scope of the current paper. However, we expect that our mechanism will be present, qualitatively, in any situation in which the optimal contract places positive weight on the long-run price.

3 Analysis

Our focus is to show that there exists an equilibrium in which a manager with $x = 1$ discloses this information at date 0, while a manager with $x = 0$ does not disclose any information. Importantly, in this equilibrium, investors at date 1 and 2 infer $x = 0$ (with probability 1) in the event that the manager does not disclose.

We shall establish this by working backwards. First, taking a disclosure d and investor's information acquisition choices λ_1 and λ_2 as given, we solve for the equilibrium prices P_1 and P_2 in Section 3.1. Next, given a disclosure policy d, we solve for the optimal information

[9] Indeed, because the asset supply is iid, optimal trading strategies would endogenously remain myopic in such a setting.

acquisition choices at dates 1 and 2 in Section 3.2. Finally, we establish sufficient conditions under which our conjectured disclosure policy is the unique equilibrium policy in Section 3.3.

3.1 Financial market equilibrium

For given disclosure and information choices, the derivation of the financial market equilibrium is standard. Fix the fraction λ_t of investors in generation t who acquire information about θ. We conjecture that prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{3}$$

where the $s_{pt} \equiv \theta + b_t z_t$ for $t \in \{1, 2\}$. In particular, the date t price provides a noisy, linear signal s_{pt} about θ to the uninformed investors of that generation. Moreover, the uninformed investors at date 2 can condition on the date 1 price to infer s_{p1}. This implies that the conditional beliefs of an uninformed investor at date $t = 1$ are given by:

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}, \quad \text{where } \tau_{p1} \equiv \tau_z / b_1^2. \tag{4}$$

Similarly, the conditional beliefs of an uninformed investor at date $t = 2$ are given by:

$$\mathbb{E}_{U2}[\theta] = \frac{\tau_{p1} s_{p1} + \tau_{p2} s_{p2}}{\tau_\theta + \tau_{p1} + \tau_{p2}}, \quad \mathbb{V}_{U2}[\theta] = \frac{1}{\tau_\theta + \tau_{p1} + \tau_{p2}} \equiv \frac{1}{\tau_{U2}}, \quad \text{where } \tau_{p2} \equiv \tau_z / b_2^2. \tag{5}$$

Note that investor i in generation t chooses optimal demand X_{it} to maximize CARA utility over wealth at date $t + 1$ i.e.,

$$\begin{aligned} X_{it} &\equiv \arg\max_x \mathbb{E}_{it}[-e^{-\gamma\{W_t + x(P_{t+1} - P_t)\}}] \qquad (6) \\ &= \frac{\mathbb{E}_{it}[P_{t+1}] - P_t}{\gamma \mathbb{V}_{it}[P_{t+1}]} \qquad (7) \end{aligned}$$

where $P_3 = V$. This implies that the optimal demand for date 2 informed and uninformed investors are given by

$$X_{I2} = \frac{1}{\gamma} \frac{\bar{V} + x\eta + \theta - P_2}{1/\tau_u}, \quad \text{and} \quad X_{U2} = \frac{1}{\gamma} \frac{\bar{V} + x\eta + \mathbb{E}_{U,2}[\theta] - P_2}{1/\tau_u + 1/\tau_{U2}}, \tag{8}$$

respectively. The market clearing condition at date 2 is:

$$\lambda_2 X_{I2} + (1-\lambda_2) X_{U2} = \bar{Z} + z_2. \tag{9}$$

Re-arranging terms, we see that the market clearing price verifies the conjecture in (3). Similarly, the optimal demand for date 1 informed and uninformed investors are given by

$$X_{I1} = \frac{1}{\gamma}\frac{A_2 + B_2\theta + C_2 s_{p1} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2}}, \quad \text{and} \quad X_{U1} = \frac{1}{\gamma}\frac{A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}}, \tag{10}$$

respectively. Again, the date 1 market clearing condition, which is given by:

$$\lambda_1 X_{I1} + (1-\lambda_1) X_{U1} = \bar{Z} + z_1, \tag{11}$$

implies that the market clearing price verifies the conjecture in (3).

The following result characterizes the financial market equilibrium.

Lemma 1. *Fix $\lambda_1, \lambda_2 \in [0,1]$. There exists an equilibrium in which date 1 and 2 equilibrium prices are given by (3), where the price signals $s_{pt} \equiv \theta + b_t z_t$,*

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{B_2 \lambda_1}\left(\frac{B_2^2\gamma^2}{\lambda_2^2 \tau_u^2 \tau_z} + \frac{x^2}{\tau_\eta}\right), \tag{12}$$

and the price coefficients A_1, A_2, B_1, B_2 and C_2 are characterized in the appendix.

3.2 Information acquisition choices

Given the characterization of the financial market equilibrium in the previous section, one can characterize the optimal information acquisition choices for generation t investors by comparing their expected utility with and without private information.

Specifically, note that the expected utility from acquiring information is given by:[10]

$$U_{I,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{It}[-e^{-\gamma\{W_t + X_{It}(P_{t+1} - P_t) - c\}}]], \tag{13}$$

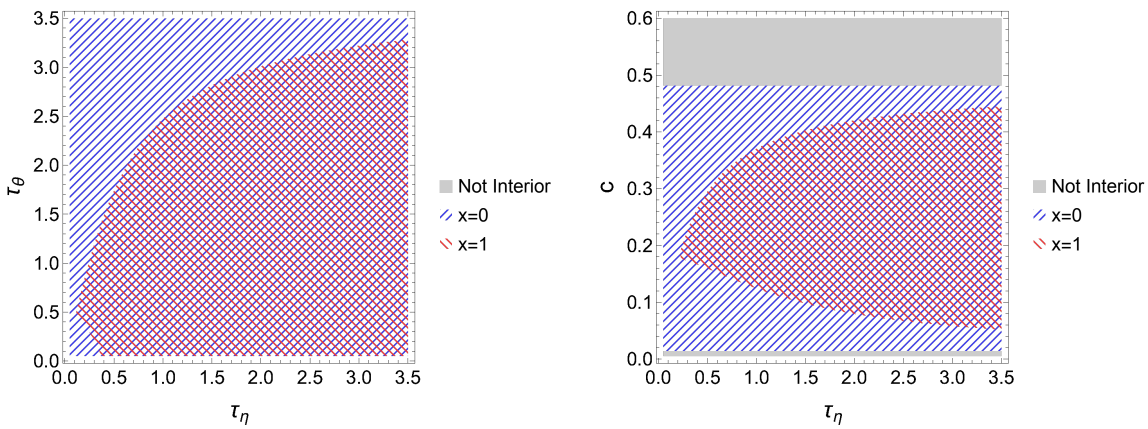
while the expected utility from not acquiring information is given by:

$$U_{U,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{Ut}[-e^{-\gamma\{W_t + X_{Ut}(P_{t+1} - P_t)\}}]]. \tag{14}$$

[10]Note that $\mathbb{E}_{t^-}[\cdot]$ refers to the expectation of generation t investors before they have acquired any information or observed the date t price.

Figure 2: Parameter regions in which $\lambda_t \in (0, 1)$

The figures plot the region of the parameter space in which $\lambda_1, \lambda_2 \in (0, 1)$ for $x = 0$ and $x = 1$. Unless specified, the other parameters are set to $\gamma = 0.5$, $c = 0.2$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_\eta = 1$ and $\tau_z = 1$.



(a) τ_θ versus τ_η (b) τ_η versus c

Standard calculations show that the relative expected utility can be expressed as:

$$\Gamma_t(\lambda_1, \lambda_2) \equiv \frac{U_{I,t}}{U_{U,t}} = e^{\gamma c} \sqrt{\frac{\mathbb{V}_{It}[P_{t+1}]}{\mathbb{V}_{Ut}[P_{t+1}]}} \tag{15}$$

just as in Grossman and Stiglitz (1980). Note that if $\Gamma_t(\lambda_t = 1) < 1$, then all investors in generation t choose to become informed (i.e., $\lambda_t = 1$), while if $\Gamma_t(\lambda_t = 0) > 1$, then no investors acquire information (i.e., $\lambda_t = 0$). The following result gives conditions under which the λ_t's are interior.

Lemma 2. *Fix $x \in \{0, 1\}$. If there exist $\lambda_1 \in (0, 1)$ and $\lambda_2 \in (0, 1)$ that solve the following system of two equations, where the coefficients $B_2(\lambda_1, \lambda_2)$ and $b_1(\lambda_1, \lambda_2)$ are as defined in Lemma 1, then there exists an interior equilibrium in the information market.*

$$\frac{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_\theta + \tau_z / b_1^2(\lambda_1,\lambda_2)}}{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_z\left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1 \tag{16}$$

$$\frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2(\lambda_1,\lambda)} + \tau_z\left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} = e^{2\gamma c} - 1 \tag{17}$$

Due to the highly nonlinear nature of the information market equilibrium conditions, the equilibrium λ_t's are not generally available in closed-form and it is difficult to pin down analytical conditions on primitives that ensure that the equilibrium is interior. However, it is straightforward to numerically solve for equilibrium. Figure 2 provides illustrations of regions of the parameter space in which λ_t 's are interior. Panel (a) illustrates how the region varies as with the prior precisions of the long-term project, τ_θ, and the short-term project, τ_η. For the displayed parameter region, the equilibrium is always interior for $x = 0$. Naturally, when the short-term project does not exist ($x = 0$), the region does not vary with τ_η. When the short-term project exists, the region of interior equilibria smaller because, when τ_η is sufficiently small (i.e., the short-term project is sufficiently risky) then no traders acquire information at $t = 1$ ($\lambda_1 = 0$). On the other hand, when τ_η grows without bound and the short-term project becomes risk-less, the $x = 1$ equilibrium is isomorphic to the $x = 0$ equilibrium (in which the project does not exist) and therefore the interior regions must coincide.

Panel (b) illustrates how the region of interior equilibria varies with the cost of information c and the prior precision of the short-term project, τ_η. Again, we see that when $x = 0$, the value of τ_η naturally has no effect on the equilibrium. For both $x = 0$ and $x = 1$, an interior equilibrium (if one exists) holds for an intermediate region of costs. If the cost is "too high" then zero traders acquire information in at least one of the periods ($\lambda_t = 0$), while if the cost is "too low", then all traders acquire information in at least one of the periods ($\lambda_t = 1$). On the other hand, for any fixed cost c, in the $x = 1$ case, we again have $\lambda_1 \to 0$ as τ_η shrinks, while the equilibria coincide when τ_η becomes sufficiently large.

3.3 Disclosure decision

We begin by showing that when the equilibrium features "interior" information choices i.e., $\lambda_1, \lambda_2 \in (0, 1)$, the fraction of investors who acquire information at date 2 (i.e., λ_2) is higher for $x = 1$ than $x = 0$.

Lemma 3. *Suppose $\lambda_1(x), \lambda_2(x) \in (0, 1)$ for both $x \in \{0, 1\}$. Then, $\lambda_2(1) > \lambda_2(0)$.*

This result is intuitive. When $x = 1$, date 1 investors face higher unlearnable uncertainty. This leads informed investors to trade less aggressively on their information and makes the date 1 price less informative.[11] In turn, this implies that prior to acquiring information, date 2 investors have a higher conditional variance about θ, which leads to more information acquisition prior to the date 2 trading round.[12] Moreover, this implies that the unconditional

[11] While it has an ambiguous effect on date 1 information acquisition itself, λ_1, we show that the net effect is always to reduce date 1 price informativeness.

[12] Note that while we focus on interior equilibria to keep the analysis transparent and avoid cumbersome enumeration and treatment of corner cases, this mechanism is robust to allowing such equilibria. For instance, when $\lambda_2(0) \in (0, 1)$ and $\lambda_2(1) = 1$ then it is trivially true that we have $\lambda_2(0) < \lambda_2(1)$.

expectation of the date 2 price is higher when $x = 1$, which we formalize next. Recall that in the conjectured equilibrium, traders perfectly infer $x = 0$ in the event of non-disclosure so that $\mathbb{E}[P_2|d = D, x = 0] = \mathbb{E}[P_2|d = ND, x = 0]$, and therefore that is suffices to characterize the expected price for the two types $x \in \{0, 1\}$ only in the event that the manager discloses, $d = D$.

Lemma 4. *Consider any equilibrium in which traders observe (or infer) the value of x, and information choices are interior, $\lambda_1(x), \lambda_2(x) \in (0, 1)$. We have*

$$\mathbb{E}[P_2\big|d = D, x = 1] > \mathbb{E}[P_2\big|d = D, x = 0]$$

Proof. Note that

$$\mathbb{E}[P_2\big|d = D, x] = \bar{V} - \frac{\gamma}{\frac{\lambda_2(x)}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2(x))}{\mathbb{V}_{U2}[P_3]}}\bar{Z}, \tag{18}$$

Furthermore, when λ_2 is interior, it is pinned down by the information acquisition condition $\Gamma_2(\lambda_2) = 1$, which implies

$$\mathbb{V}_{U2}[P_3] = e^{2\gamma c}V_{I2}[P_3] = \frac{e^{2\gamma c}}{\tau_u}. \tag{19}$$

Hence, the weighted average precision can be expressed as:

$$\frac{\lambda_2(x)}{\mathbb{V}_{I2}[P_3]} + \frac{(1 - \lambda_2(x))}{\mathbb{V}_{U2}[P_3]} = \tau_u \left(\lambda_2(x) \left(1 - e^{-2c\gamma}\right) + e^{-2c\gamma}\right). \tag{20}$$

This implies that the expected price $\mathbb{E}[P_2\big|d = D, x]$ is an increasing function of λ_2. Finally, Lemma 3 shows that $\lambda_2(1) > \lambda_2(0)$, which establishes that $\mathbb{E}[P_2\big|d = D, x = 1] > \mathbb{E}[P_2\big|d = D, x = 0]$. □

We are now ready to establish the existence of the conjectured equilibrium.

Proposition 1. *Suppose the cost of disclosure $c_D > 0$ is not too large. Then, there exists an equilibrium in which a manager with $x = 1$ discloses this information, but a manager with $x = 0$ does not.*

Proof. To establish that the conjectured equilibrium is in fact an equilibrium, it suffices to show that each manager type prefers to play her conjectured strategy given the strategy of the other type. Consider first the $x = 0$ manager. Taking the $x = 1$ manager's disclosure strategy as given, then if the $x = 0$ manager follows her conjectured strategy and does not disclose she is inferred to be the low type. If she deviates and discloses $x = 0$ then she is still identified as the low type and also pays the disclosure cost. Hence, playing her conjectured strategy is optimal.

Consider now the $x = 1$ manager. Taking the $x = 0$ manager's strategy of non-disclosure as given, we need to establish that the $x = 1$ manager prefers to disclose. Supposing that she instead deviates and refrains from disclosing, then in the conjectured equilibrium the market will infer her to be an $x = 0$ type. The incremental benefit from not disclosing versus disclosing is

$$U_{ND}(1) - (U_D(1) - c_D) = \mathbb{E}[P_2|x = 0, d = ND] - (\mathbb{E}[P_2|x = 1, d = D] - c_D) \tag{21}$$

$$= \mathbb{E}[P_2|x = 0, d = D] - (\mathbb{E}[P_2|x = 1, d = D] - c_D) \tag{22}$$

$$= c_D - (\mathbb{E}[P_2|x = 1, d = D] - \mathbb{E}[P_2|x = 0, d = D]) \tag{23}$$

where the second equality uses the fact that in the conjectured equilibrium, $\mathbb{E}[P_2|d = D, x = 0] = \mathbb{E}[P_2|d = ND, x = 0]$ and the final equality rearranges. Lemma 4 shows that $\mathbb{E}[P_2|x, d = D]$ is strictly increasing in x. Hence, as long as the cost of disclosure c_D is not too high, this incremental benefit will be negative and hence it is optimal for the $x = 1$ manager to disclose. □

This proposition establishes the existence of an equilibrium of our conjectured form when the disclosure cost is sufficiently small; however, it does not speak to the existence of other equilibria in which, e.g., both types do not disclose with positive probability. We would like to establish conditions under which the equilibrium is unique. To do so, we must entertain the possibility of managers following mixed disclosure strategies, so it is helpful to make explicit the dependence of expected P_2 on the market's belief about the manager's type. Hence, let $U_{ND}(x; q) = \mathbb{E}[P_2|d = ND, x]$ denote the expected price as a function of x in the event that the manger does not disclose and the market assigns probability q to $x = 1$ in the event of no disclosure. Letting r_0 denote the probability that an $x = 0$ manager discloses and r_1 the probability that an $x = 1$ manager discloses, the market assigns probability

$$q(r_0, r_1) = \frac{p(1 - r_1)}{(1 - p)(1 - r_0) + p(1 - r_1)}$$

that $x = 1$ in the event of no disclosure. The following Lemma establishes that we can rule out any equilibria with interior λ in which an $x = 0$ manager discloses with positive probability.

Lemma 5. *There do not exist equilibria with $\lambda_1(x), \lambda_2(x) \in (0, 1)$ in which an $x = 0$ manager* *discloses with strictly positive probability,* $r_0 > 0$.

Intuitively, in any conjectured equilibrium in which an $x = 0$ manager discloses, she can make herself strictly better off by refraining from disclosing, saving the disclosure cost, and, at worst, still being perfectly identified at an $x = 0$ type. Owing to Lemma 5, the only remaining candidate equilibria (with interior λ's) are those in which the $x = 0$ manager never discloses and the $x = 1$ manager discloses with probability less than one, $r_1 \in [0, 1)$.

In general, when the $x = 1$ manager mixes between disclosure and not, there does not exist a linear equilibrium in the financial market. Moreover, due to the nonlinearity of the problem, it is unclear whether or not any noisy rational equilibrium exists in this case, and if so, what its properties are.[13] However, under an additional continuity condition, the following proposition rules out such equilibria when disclosure costs are sufficiently low and the prior probability p is sufficiently low.

Proposition 2. *Suppose that* $U_{ND}(1; q)$ *is continuous in* q *at* $q = 0$*. Then if the disclosure cost* c_D *and the prior probability* p *that* $x = 1$ *are sufficiently small, there does not exist an equilibrium with* $\lambda_1(x), \lambda_2(x) \in (0, 1)$ *in which an* $x = 1$ *manager discloses with probability* $r_1 < 1$.

Intuitively, when p is very low, if an $x = 1$ manager does not disclose the market assigns probability close to zero that she is the $x = 1$ type. We know from Proposition 1 that, as long as costs are sufficiently small, if the market assigns probability *equal to* zero that $x = 1$, then an $x = 1$ manager finds it optimal to disclose and thereby identify herself to the market. Hence, under continuity of non-disclosure expected utility U_{ND} at $q = 0$, it remains optimal for the $x = 1$ manager to disclose when this probability is positive but small.

We expect our results to be qualitatively similar if x follows a discrete distribution with more than two states and the disclosure cost is sufficiently small. In this case, we conjecture that all managers above the lowest possible realization of x will find it optimal to disclose and the lowest possible type will refrain from disclosing. Moreover, while we expect that our mechanism is qualitatively robust to even more general distributions and cost functions, unfortunately, such settings are generally intractable.[14]

[13] Unfortunately, such a setting does not meet any known conditions for characterizing equilibria outside of the linear class (e.g., the "exponential family" condition of Breon-Drish (2015)).

[14] In particular, as we have discussed above, if, in equilibrium, traders place positive probability on more than one value of x, then there is no longer a linear equilibrium in the financial market, and we are unable to characterize equilibrium, or demonstrate existence.

4 Conclusions

We study how voluntary disclosure affects information acquisition in a dynamic model of trading. We show that a firm manager may be willing to disclose information that becomes redundant at a later date, even if she intends to maximize long-term share prices. By disclosing information about the presence of a short-term risky project, the manager increases perceived risk and reduces price informativeness in early periods. Once the payoffs of this project are revealed, later investors acquire information more aggressively. We show that increased information acquisition by later investors can dominate the short-term increase in uncertainty, and lead to long-term prices that are more informative and higher on average.

Our analysis suggests a number of natural extensions. For instance, it would be interesting to consider the strategic timing of voluntary disclosure (as in Guttman, Kremer, and Skrzypacz (2014)) in a setting with dynamic information acquisition. It might also be interesting to study how voluntary disclosure is affected by dynamic information acquisition in the presence of real investment and feedback effects. Finally, one could endogenize the manager's objective as part of an optimal contracting problem and study disclosure policies in the resulting equilibrium. We leave these questions for future work.

References

Snehal Banerjee, Iván Marinovic, and Kevin Smith. Disclosing to informed traders. *Working paper*, 2021.

Anne Beyer, Daniel A Cohen, Thomas Z Lys, and Beverly R Walther. The financial reporting environment: Review of the recent literature. *Journal of Accounting and Economics*, 50 (2-3):296–343, 2010.

Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *The Annual Review of Financial Economics is*, 4:339–60, 2012.

Bradyn Breon-Drish. On existence and uniqueness of equilibrium in a class of noisy rational expectations models. *The Review of Economic Studies*, 82(3):868–921, 2015.

Douglas W Diamond. Optimal release of information by firms. *The Journal of Finance*, 40 (4):1071–1094, 1985.

D.W. Diamond and R.E. Verrecchia. Disclosure, liquidity, and the cost of capital. *Journal of Finance*, 46(4):1325–1359, 1991.

Ronald A Dye. Disclosure of nonproprietary information. *Journal of Accounting Research*, pages 123–145, 1985.

Ronald A Dye. An evaluation of "essays on disclosure" and the disclosure literature in accounting. *Journal of accounting and economics*, 32(1-3):181–235, 2001.

Ronald A. Dye and John S. Hughes. Equilibrium voluntary disclosures, asset pricing, and information transfers. *Journal of Accounting and Economics*, 66(1):1 – 24, 2018. ISSN 0165-4101.

Itay Goldstein and Liyan Yang. Information diversity and complementarities in trading and information acquisition. *Journal of Finance*, 70(4):1723–1765, 2015.

Itay Goldstein and Liyan Yang. Information disclosure in financial markets. *Annual Review of Financial Economics*, 9:101–125, 2017.

Itay Goldstein and Liyan Yang. Good disclosure, bad disclosure. *Journal of Financial Economics*, 131(1):118–138, 2019.

Itay Goldstein, Jan Schneemeier, and Liyan Yang. Market feedback: Who learns what? 2020.

S.J. Grossman and J.E. Stiglitz. On the impossibility of informationally efficient markets. *The American Economic Review*, 70(3):393–408, 1980.

Ilan Guttman, Ilan Kremer, and Andrzej Skrzypacz. Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review*, 104(8):2400–2420, 2014.

Woon-Oh Jung and Young K Kwon. Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research*, pages 146–153, 1988.

Matthias Lassak. Voluntary disclosure, price informativeness, and efficient investment. *Price Informativeness, and Efficient Investment (November 16, 2020)*, 2020.

D.J. Skinner. Why firms voluntarily disclose bad news. *Journal of Accounting Research*, 32: 38–60, 1994.

Kevin Smith. Risk information, investor learning, and informational feedback. 2020.

Joseph E Stiglitz. Using tax policy to curb speculative short-term trading. *Journal of financial services research*, 3(2):101–115, 1989.

Brett Trueman. Why do managers voluntarily release earnings forecasts? *Journal of accounting and economics*, 8(1):53–71, 1986.

Robert E Verrecchia. Discretionary disclosure. *Journal of Accounting and Economics*, 5: 179–194, 1983.

Robert E Verrecchia. Information quality and discretionary disclosure. *Journal of accounting and Economics*, 12(4):365–380, 1990.

Robert E Verrecchia. Essays on disclosure. *Journal of accounting and economics*, 32(1-3): 97–180, 2001.

Liyan Yang. Disclosure, competition and learning from asset prices. *Rotman School of Management Working Paper*, (3095970), 2020.

A Proofs

A.1 Proof of Lemma 1

We can solve for the coefficients b_1 and b_2 by observing that $s_{p,t}$ is a linear transformation of the informed residual demand $\lambda_t X_{It} - z_t$. This implies:

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{\lambda_1} \frac{x^2/\tau_\eta + B_2^2/\tau_{p2}}{B_2}. \tag{24}$$

Next, we solve for the price coefficients by imposing market clearing and matching coefficients. Specifically, note that B_2 is given by:

$$B_2 = \frac{\lambda_2 \left(\tau_{U1} + \tau_u\right) + \tau_{p2}}{\tau_{U1} + \lambda_2 \tau_u + \tau_{p2}}, \tag{25}$$

where $\tau_{U1} = \tau_\theta + \tau_{p1}$, and $\tau_{pt} = \tau_z / b_t^2$. Substituting, this implies that B_2 is a solution to $H(B_2) = 0$, where

$$H(B_2) = \frac{\lambda_2 \left(\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \tau_u \right) + \lambda_2 \tau_u^2 \tau_z \right)}{\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \lambda_2 \tau_u \right) + \lambda_2^2 \tau_u^2 \tau_z} - B_2 \tag{26}$$

Note that

$$\begin{aligned} H(0) &= \frac{\lambda_2 \left(\gamma^2 \left(\tau_\theta + \tau_u\right) + \lambda_2 \tau_u^2 \tau_z\right)}{\gamma^2 \left(\tau_\theta + \lambda_2 \tau_u\right) + \lambda_2^2 \tau_u^2 \tau_z} > 0 \\ H(1) &= \frac{(\lambda_2 - 1)\left(\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z\right)^2 + \lambda_1^2 \lambda_2^4 \tau_\eta^2 \tau_u^4 \tau_z^3\right)}{\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z\right)^2 + 2\gamma^2 \lambda_2^3 x^2 \tau_\eta \tau_u^3 \tau_z (\gamma^2 + \lambda_2 \tau_u \tau_z) + \lambda_2^5 x^4 \tau_u^5 \tau_z^2 (\gamma^2 + \lambda_2 \tau_u \tau_z) + \lambda_2 \tau_\eta^2 \tau_u \left(\gamma^6 + \gamma^4 \lambda_2 \tau_u \tau_z + \lambda_1^2 \lambda_2^3 \tau_u^3 \tau_z^3\right)} \\ &\leq 0, \end{aligned}$$

since $\lambda_2 \leq 1$, which implies there exists a solution to $H(B_2) = 0$ for $B_2 \in (0, 1)$.

Given B_2, we can solve for (b_1, b_2), and then solve for the other coefficients using the following system:

$$A_2 = \bar{V} - \frac{\gamma \bar{Z} \left(\gamma^2 \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \tau_u \right) + \lambda_2^2 \tau_u^2 \tau_z \right)}{\gamma^2 \tau_u \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u \right) + \lambda_2^2 \tau_u^3 \tau_z} \tag{27}$$

$$A_1 = A_2 - \frac{\gamma \bar{Z}}{\frac{1-\lambda_1}{B_2^2 \left(\frac{1}{\frac{\tau_z}{b_1^2} + \tau_\theta} + \frac{b_2^2}{\tau_z} \right) + \frac{x^2}{\tau_\eta}} + \frac{\lambda_1 \tau_\eta \tau_z}{b_2^2 B_2^2 \tau_\eta + x^2 \tau_z}} \tag{28}$$

$$C_2 = \frac{b_2^2 (1 - \lambda_2) \tau_z}{b_1^2 \left(b_2^2 \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u \right) + \tau_z \right)} \tag{29}$$

$$B_1 = \frac{b_1^2 \tau_\theta (B_2 \lambda_1 + C_2)(b_2^2 B_2^2 \tau_\eta + x^2 \tau_z) + (B_2 + C_2) \tau_z (B_2^2 (b_1^2 \lambda_1 + b_2^2) \tau_\eta + x^2 \tau_z)}{B_2^2 \tau_\eta (b_2^2 (b_1^2 \tau_\theta + \tau_z) + b_1^2 \lambda_1 \tau_z) + x^2 \tau_z (b_1^2 \tau_\theta + \tau_z)}. \tag{30}$$

□

A.2 Proof of Lemma (2)

In an interior equilibrium, λ_1 and λ_2 are characterized by the conditions $\Gamma_t(\lambda_1, \lambda_2) = 1$ for $t \in \{1, 2\}$, where Γ_t is defined in eq. (32). Plugging in to the $t = 1$ condition and rearranging yields

$$\frac{\mathbb{V}_{U1}[P_2]}{\mathbb{V}_{I1}[P_2]} = e^{2\gamma c} \Leftrightarrow \frac{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta} + B_2^2 \mathbb{V}_{U1}(\theta)}{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} \Leftrightarrow \frac{\frac{B_2^2}{\tau_\theta + \tau_z / b_1^2}}{B_2^2 \left(\frac{\gamma}{\lambda_2 \tau_u} \right)^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting the price function from eq. (3), and the second equivalence follows from rearranging and substituting in for the equilibrium values of $\mathbb{V}_{U1}(\theta)$ and b_2. Similarly, plugging in to the $t = 2$ condition and rearranging yields

$$\frac{\mathbb{V}_{U2}[V]}{\mathbb{V}_{I2}[V]} = e^{2\gamma c} \Leftrightarrow \frac{\frac{1}{\tau_u} + \frac{1}{\tau_{U2}}}{\frac{1}{\tau_u}} = e^{2\gamma c} \Leftrightarrow \frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma} \right)^2} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting in for the variances in terms of precision, and the second equivalence follows from rearranging and substituting in the equilibrium value of τ_{U2}. □

A.3 Proof of Lemma 3

For an interior λ_1, λ_2, note that the information equilibrium conditions imply

$$W = \frac{\mathbb{V}_{Ut}[\mathbb{E}_{It}[P_{t+1}]]}{\mathbb{V}_{It}[P_{t+1}]} \tag{31}$$

The conditions for date 2 and 1 are given by:

$$W = \frac{1/\tau_{U2}}{1/\tau_e}, \quad \text{and} \quad W = \frac{B_2^2/\tau_{U1} + x^2/\tau_\eta}{B_2^2/\tau_{p2} + x^2/\tau_\eta}, \tag{32}$$

respectively. Plugging in $\tau_{U2} = \tau_{U1} + \tau_{p2}$ and rearranging terms gives:

$$\tau_e = W\left(\tau_{U1} + \tau_{p2}\right) \tag{33}$$

$$\frac{B_2^2}{\left(B_2^2/\tau_{p2} + x^2/\tau_\eta\right)} = W\tau_{U1} \tag{34}$$

Next, recall that since B_2 is given by

$$B_2 = \frac{\lambda_2\left(\tau_{U1} + \tau_u\right) + \tau_{p2}}{\tau_{U1} + \lambda_2\tau_u + \tau_{p2}} \tag{35}$$

$$\Rightarrow B_2 = \frac{\lambda_2\tau_e\left(1 + W\right) + W\tau_{p2}\left(1 - \lambda_2\right)}{\left(1 + \lambda_2 W\right)\tau_e} \tag{36}$$

$$= \frac{\lambda_2\left(\gamma^2(W+1) + \left(1 - \lambda_2\right)\lambda_2\tau_z W\tau_e\right)}{\gamma^2\left(\lambda_2 W + 1\right)} \tag{37}$$

Combining (33) and (34), and then plugging in (37) yields

$$\frac{x^2}{\tau_\eta} = \frac{B_2^2}{\tau_{p2}}\frac{\left(W+1\right)\tau_{p2} - \tau_e}{\tau_e - W\tau_{p2}} \tag{38}$$

$$= \frac{\lambda_2^2\left(\gamma^2\left(1+W\right) + \left(1-\lambda_2\right)\lambda_2 W\tau_e\tau_z\right)^2}{\gamma^4\left(1+\lambda_2 W\right)^2}\frac{\left(W+1\right)\tau_{p2} - \tau_e}{\tau_{p2}\left(\tau_e - W\tau_{p2}\right)} \tag{39}$$

$$= \frac{\left(\gamma^2(W+1) + \left(1-\lambda_2\right)\lambda_2 W\tau_e\tau_z\right)^2\left(\gamma^2 - \lambda_2^2(W+1)\tau_e\tau_z\right)}{\gamma^2\tau_e^2\left(\lambda_2 W + 1\right)^2\tau_z\left(\lambda_2^2 W\tau_e\tau_z - \gamma^2\right)} \equiv G\left(\lambda_2\right) \tag{40}$$

Note that $G(0) = -\frac{\gamma^2(W+1)^2}{\tau_e^2\tau_z} < 0$ and $G(1) = -\frac{\gamma^2\left(\gamma^2 - (W+1)\tau_e\tau_z\right)}{\tau_e^2\tau_z\left(\gamma^2 - W\tau_e\tau_z\right)}$. For the equilibrium $\lambda_2 \in (0,1)$ to exist, we need to have: $G(1) > \frac{x^2}{\tau_\eta} > 0 > G(0)$, which is equivalent to restricting

$$\frac{\gamma^2}{(1+W)\,\tau_z} < \tau_e < \frac{\gamma^2}{W\tau_z} \tag{41}$$

$$\Leftrightarrow \gamma^2 > W\tau_e\tau_z, \tag{42}$$

$$\gamma^2 < (1+W)\,\tau_e\tau_z \tag{43}$$

Moreover, tedious algebra establishes that

$$G(\lambda_2) = \frac{\left(\frac{W+1}{W} - \frac{\gamma^2}{\lambda_2^2 W\tau_e\tau_z}\right)\left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2\tau_e\tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2}{\tau_e\left(\frac{1}{W} - \frac{\lambda_2^2\tau_e\tau_z}{\gamma^2}\right)} \equiv g_1(\lambda_2)\, g_2(\lambda_2)\, g_3(\lambda_2) \tag{44}$$

We want to show that the above is increasing in λ_2. Now,

$$g_3(\lambda_2) = \frac{1}{\tau_e\left(\frac{1}{W} - \frac{\lambda_2^2\tau_e\tau_z}{\gamma^2}\right)} > 0 \tag{45}$$

from (41), and $g_3(\lambda_2)$ is increasing in λ_2. Next,

$$g_2(\lambda_2) = \left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2\tau_e\tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2 > 0 \tag{46}$$

and since $\frac{1}{W} - \frac{\lambda_2^2\tau_e\tau_z}{\gamma^2} > 0$, $g_2(\lambda_2)$ is increasing in λ_2. Finally, $g_1(\lambda_2)$ is increasing in λ_2. Moreover, it must be positive since $G(\lambda_2) = \frac{x^2}{\tau_\eta} > 0$. This implies

$$\frac{dG}{d\lambda_2} = g_1' g_2 g_3 + g_1 g_2' g_3 + g_1 g_2 g_3' > 0. \tag{47}$$

To summarize this shows that $G(\lambda_2)$ is increasing in λ_2. Since $G(\lambda_2) = \frac{x^2}{\tau_\eta}$, this implies that the equilibrium λ_2 is increasing in x^2 as long as λ_1 and λ_2 are interior.

□

A.4 Proof of Lemma 5

Suppose to the contrary that there exists an interior equilibrium in which an $x = 0$ manager discloses with probability $r_0 \in (0, 1]$, and an $x = 1$ manager discloses with probability

$r_1 \in [0,1]$. In such an equilibrium, the market assigns probability

$$q(r_0, r_1) = \frac{p(1-r_1)}{(1-p)(1-r_0) + p(1-r_1)}$$

that $x = 1$ in the event of no disclosure. Consider first the case $r_1 = 1$. In this case the market assigns probability $q = 1$ and we therefore have

$$U_{ND}(0; q) = U_D(0) \Rightarrow U_D(0) - c_D < U_{ND}(0; q),$$

which implies that the $x = 0$ manager strictly prefers not disclosing. Consider next the case in which $r_1 \in (0,1)$. In this case, because the $x = 1$ manager does not disclose with positive probability, then we know that she is either indifferent (in the case $r_1 \in (0,1)$) or strictly prefers not disclosing (in the case $r_1 = 0$), which implies:

$$\begin{aligned} U_D(1) - c_D - U_{ND}(1; q) &\leq 0 \\ \Rightarrow U_D(0) - c_D - U_{ND}(0; q) &< 0 \end{aligned}$$

where the second line follows from Lemma 4, which establishes that $U_D(1) > U_D(0)$. This implies that the $x = 0$ manager strictly prefers not disclosing. Finally, consider the case $r_1 = 0$. In this case, the $x = 1$ manager strictly prefers to not disclose, which implies

$$U_D(1) - c_D - U_{ND}(1; q) < 0 \Rightarrow U_D(0) - c_D - U_{ND}(0; q)$$

where the second inequality again follows from Lemma 4 and implies that the $x = 0$ manager strictly prefers to not disclose. □

A.5 Proof of Proposition 2

We know from Proposition 1 that for sufficiently small $c_D > 0$ a high-type manager strictly prefers disclosing to not disclosing and being assigned probability 0 of being the $x = 1$ type:

$$U_{ND}(1; 0) < U_D(1) - c_D \Rightarrow U_D(1) - c_D - U_{ND}(1; 0) > 0 \tag{48}$$

Fix such a sufficiently small c_D and suppose that there also exists an equilibrium in which the $x = 1$ manager discloses with probability $r_1 < 1$ and the $x = 0$ manager never discloses,

$r_0 = 0$. In such an equilibrium, the market assigns probability

$$q(0, r_1) = \frac{p(1 - r_1)}{(1 - p) + p(1 - r_1)} < p$$

that $x = 1$ in the event of no disclosure. Under the assumed continuity of U_{ND}, for every $\varepsilon > 0$ there exists $q_\varepsilon > 0$ such that for $q \in [0, q_\varepsilon)$ we have

$$-\varepsilon < U_{ND}(1, q) - U_{ND}(1, 0) < \varepsilon.$$

Now, pick any ε such that $0 < \varepsilon < U_D(1) - c_D - U_{ND}(1; 0)$, which is guaranteed to exist owing to eq. (48). For any $q \in [0, q_\varepsilon)$ we have

$$\begin{aligned} U_{ND}(1; q) &< U_{ND}(1; q) + \varepsilon \\ &< U_D(1) - c_D \end{aligned}$$

where the first line follows from the continuity of U_{ND} and the second line follows from the choice of ε. Because $q(0, r_1) < p$ for any value of r_1, this implies that as long as $p < q_\varepsilon$ we have

$$U_{ND}(1; q(0, r_1)) < U_D(1) - c_D$$

which implies that the $x = 1$ manager strictly prefers to disclose, which is a contradiction. $\square$