Competition and Collusion Among Strategic Traders Who Face Uncertainty

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Abstract

Conventional wisdom suggests that informed investors benefit from coordinating their trades as if acting monopolistically. However, we show that this may not hold when investors face uncertainty about market liquidity. In a Kyle (1985) framework, we compare trading profits under monopolistic and competitive equilibria when informed investors face parameter uncertainty about liquidity trading volatility. While low uncertainty favors coordination, we show that the expected profit of an *individual* investor in the competitive equilibrium can be higher than the *total* profits for all investors in the monopolistic equilibrium when uncertainty is sufficiently high. Allowing traders to endogenously choose whether or not to collude gives rise to novel predictions: a small increase in uncertainty about liquidity trading can lead to jumps in trading volume, price volatility, and expected price informativeness.

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1 Introduction

Antitrust enforcement and the identification of coordinated behavior are some of the primary objectives of regulatory agencies worldwide. Recently such issues have come to the forefront as the implementation of machine learning, artificial intelligence, and big data in pricing and trading mechanisms has sparked concerns regarding the potential for such algorithms to learn to collude tacitly. A series of policy reports and proposed rules (e.g., OECD, 2017; SEC, 2023), legal studies (e.g., Azzutti, Ringe, and Stiehl, 2022), and experimental works (e.g., Calvano, Calzolari, Denicolò, and Pastorello, 2020; Colliard, Foucault, and Lovo, 2022; Dou, Goldstein, and Ji, 2024) have brought attention to these issues. A common theme of the latter is that tacit collusion should result in, and be identifiable via, excess profits relative to a competitive benchmark.

We show this may no longer be the case if traders face uncertainty about the volatility of liquidity trading. In a standard strategic trading model, we compare expected trading profits under two equilibria when traders face uncertainty about the volatility of liquidity trading. In the monopolistic equilibrium, traders pool their information and trade as if they were a single investor. In the competitive equilibrium, traders compete with each other by trading strategically on their private information. Consistent with standard intuition, we show that when uncertainty is sufficiently low, expected profits are higher in the monopolistic equilibrium.

However, when uncertainty is high, expected profits may be *higher* in the competitive equilibrium. In fact, as uncertainty becomes sufficiently high, the expected profit for any individual trader in the competitive equilibrium can exceed the total profits for all investors in the monopolistic equilibrium. When traders endogenously choose whether to pay a cost to coordinate, they will choose to do so only when uncertainty is sufficiently low. We show that this can lead to novel predictions: small increases in uncertainty about liquidity trading can lead to jumps in trading volume, price volatility, and expected price informativeness.

Model and intuition. Specifically, we consider a two-trader extension of Kyle (1985), where traders observe conditionally independent signals about the terminal value of the risky security and submit market orders. A risk-neutral, competitive market maker sets the price as the conditional expected value of the security, given the total order flow from informed investors and liquidity (noise) traders. Following Hong and Rady (2002), we assume that while the market maker knows the distribution of noise trading, the informed investors do not — instead, they face uncertainty about whether noise trading volatility is low or high. This reflects a realistic feature of many market settings: while investors may be better informed about security fundamentals than the market maker, they are likely to be less well-informed about market conditions, and in particular the trading behavior of other participants.[1]

We compare equilibria and trading profits under two scenarios. In the monopolistic equilib-

[1] For example, funds may outsource the evaluation of trade execution costs to third parties such as Investment Technology Group, recently acquired by Virtu Financial. Moreover, such uncertainty is particularly important for machine-learning based algorithms, which are designed under the premise of uncertain trading environments and payoffs.

rium, we assume investors coordinate their behavior and trade as a single trader by pooling their information and submitting trades to maximize aggregate profits. In the competitive equilibrium, each investor conditions only on her own signal and best responds to her conjectures about the other investors' and the market maker's strategy. In either equilibrium, the intensity with which the investors trade on their information depends on the precision of their signals and their expectation of the price impact they will face, based on whether noise trading volatility is high or low. In contrast, the market maker conditions on both the order flow and the volatility of noise trading when setting the price impact: when noise trading volatility is high (low), price impact is low (high, respectively).

The intuition for our key result – that profits can be greater in the competitive than in the monopolistic equilibrium when investors face uncertainty – relies on three observations.

First, for any level of uncertainty, investors trade more aggressively in the competitive equilibrium than in the monopolistic equilibrium. Intuitively, this is because each trader does not fully account for the impact of her trading intensity on the profits of other traders in the competitive equilibrium, and so trades too aggressively on her private information. In the absence of uncertainty, this implies that trading profits are lower in the competitive equilibrium.[2]

Second, in a setting with uncertainty, traders must choose a single trading intensity as a best response to the expected price impact, without knowing whether noise trading volatility is high or low. This leads to deviations from the profit-maximizing intensity that would arise if traders knew the realized noise trading volatility. Intuitively, traders in the monopolistic equilibrium are too aggressive in the low-liquidity state and too conservative in the high-liquidity state. Since traders are more aggressive in the competitive equilibrium than in the monopolistic equilibrium, competition leads to even lower profits when market liquidity is low. However, when market liquidity is sufficiently high, being more aggressive (or less conservative) yields higher profits for traders in the competitive equilibrium.

Finally, the impact of uncertainty depends on how it affects profits in these two states across the two equilibria. Crucially, as uncertainty increases, the liquidity states become more extreme: noise trading volatility becomes either very high or very low. In the low-liquidity state, profit opportunities shrink for both equilibria - in fact, in the limit, when there is no noise trading, profits are zero. As a result, for sufficiently high uncertainty, expected profits are dominated by the high-liquidity state, where less conservative trading under competition generates higher profits.

Endogenous collusion and implications. Our main results lead to a surprising conclusion. When speculators face uncertainty about the behavior of liquidity traders, realized profits of competitive speculators may exceed the profits of speculators who pool their information and trade monopolistically. Importantly, this implies that the focus on speculative profits alone when trying to identify monopolistic behavior in the financial markets can be misleading.

Given the difference in expected trading profits across the monopolistic and competitive equi-

[2] This is consistent with the existing literature on multi-trader strategic trading models (e.g., Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996; and Back, Cao, and Willard, 2000).

libria, we then consider the traders' incentives to collude. Specifically, we allow traders to pay a cost to coordinate on trading after they learn the level of uncertainty about noise trading (but before they observe signals about payoffs). In this case, traders endogenously choose to behave monopolistically only when uncertainty about noise trading volatility is below an (endogenous) threshold. As such, one should expect that monopolistic behavior is more prevalent during times of low market uncertainty and competition is more prevalent during times of high uncertainty.

Endogenizing collusion also generates a number of novel predictions about how market uncertainty affects observables. We show that both price volatility and expected price informativeness can increase with uncertainty about noise trading volatility. Additionally, while expected trading volume tends to decline with uncertainty in general, there can be regions in which small increases in uncertainty lead to positive jumps in volume.

To see why, note that conditional on an equilibrium, price volatility and expected price informativeness do not depend on uncertainty about noise trading, and expected trading volume decreases with it.[3] However, all else equal, we show that price volatility, expected price informativeness, and expected trading volume are all higher in the competitive equilibrium than in the monopolistic equilibrium because traders trade more aggressively on their private information under competition. With endogenous collusion, this implies that a small increase in uncertainty around the threshold can lead to a switch from the monopolistic equilibrium to the competitive equilibrium, which in turn leads to a positive jump in price volatility, expected price informativeness, and trading volume.

Robustness. Our main results extend beyond the stylized two-trader setting with noisy signals. In Section 5, we show that when traders are perfectly informed about asset values and the number of traders is arbitrary (but finite), competitive profits can still exceed monopolistic profits when uncertainty about noise trading volatility is sufficiently high. This confirms that our mechanism is not driven by noise in information but rather by the interaction between uncertainty and strategic trading behavior.

We further characterize sufficient conditions on the distribution of noise trading volatility under which competitive profits dominate. Specifically, we show that this occurs when the distribution is sufficiently left-skewed (low-liquidity states are sufficiently likely) while retaining substantial uncertainty. These conditions ensure that expected profits are not dominated by low-liquidity states where both equilibria perform poorly, while maintaining significant probability mass on high-liquidity states where competition's less conservative trading is advantageous. The analysis carries over to continuous distributions as we illustrate using the Beta distribution.

Related literature. Our analysis extends the model in Hong and Rady (2002) by allowing investors to have conditionally independent signals. As in this earlier work, we show that profits decrease with uncertainty about the volatility of liquidity trading. However, our focus is on how uncertainty affects the relative profits of investors under competition and under coordination. In

[3] Since the market maker knows the volatility of noise trading, he is able to condition on the order flow efficiently, and so price volatility and expected price informativeness are independent of noise trading volatility (as in Kyle (1985). However, higher uncertainty decreases the trading intensity of informed speculators and so leads to lower trading volume.

Section 5, we show that our results also obtain when investors are perfectly informed about asset values and when the number of investors is arbitrarily high (but fixed): in either case, expected profits are higher under competition when uncertainty about noise trading volatility is sufficiently high.

More generally, our paper is related to the broader literature in which market participants face uncertainty about other participants in the market. These include models in which traders are unsure about the presence or number of other informed traders, or the precision of their information.[4] Collin-Dufresne and Fos (2016) consider a related model in which the volatility of noise trading is stochastic — however, importantly, because their model is in continuous time, traders face no uncertainty about it. In contrast to this literature, which focuses on the impact of uncertainty on individual trader profits, our analysis is concerned with how this uncertainty affects the incentives for traders to coordinate versus compete.

Our analysis also speaks to the recent literature on artificial intelligence and machine learning that studies the potential for tacit collusion among algorithms in agent-based settings. A growing number of papers study such effects not only in the financial market settings (e.g., Dou et al., 2024; Colliard et al., 2022) but also in the more traditional product-pricing sectors (e.g., Cho and Williams, 2024; Calvano et al., 2020). Using simulations, these papers argue that algorithms appear to implicitly converge to strategies that are consistent with collusive behavior, even though they are unable to explicitly communicate or coordinate with each other.[5] Our analysis suggests a possible confound in such settings. In the presence of parameter uncertainty, as is likely faced by such algorithms, profits may be high even in the absence of tacit collusion. As such, our analysis suggests that one should account for the *direct* impact of parameter uncertainty when interpreting the evidence from such simulations as being indicative of collusive behavior.

It is worth noting that our economic mechanism does not rely on ambiguity aversion (e.g., Caballero and Krishnamurthy (2008), Condie and Ganguli (2011), Easley and O'Hara (2009)) or overconfidence (e.g., Kyle and Wang (1997), Benos (1998)). Instead, ours is a setting of two-sided private information: investors are better informed about asset values while the market maker is better informed about liquidity trading volatility.

Overview. The rest of the paper is organized as follows. Section 2 describes the model and discusses the important assumptions. Section 3 presents the main analysis of the paper. Section 4 allows traders to choose whether to coordinate or compete and investigates the effects of this choice on market observables. Section 5 explores how our results change under different assumptions about the information available to traders, the number of traders and the distribution of noise trading volatility. Section 6 discusses the implications of our analysis for delegated portfolio management and antitrust regulatory policy and presents concluding remarks. All proofs are in the Appendix.

[4] For instance, see Altı, Kaniel, and Yoeli (2012), Li (2013), Banerjee and Green (2015), Back, Crotty, and Li (2018), Dai, Wang, and Yang (2021), and Banerjee and Breon-Drish (2020).

[5] Empirically identifying such effects is inherently challenging, although Assad, Clark, Ershov, and Xu (2024) conjecture that collusive effects may be present in the German retail gasoline markets.

2 Model

Our model is a single-period variant of the multi-trader Kyle model in Hong and Rady (2002), where strategic traders face uncertainty about noise trading volatility. There are two assets: a risky asset and a risk-free asset with interest rate normalized to zero. The risky asset pays off a terminal value of $v \sim N\left(0, \sigma_v^2\right)$ at the end of the period.

We extend the setting in Hong and Rady (2002) by assuming that traders have conditionally independent, private signals about the value of the risky asset. Specifically, there are two strategic traders, indexed by $i \in \{1, 2\}$. Trader i observes a private signal of the form:

$$s_i = v + \varepsilon_i, \quad \text{where } \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right), \tag{1}$$

and submits market order x_i. The aggregate trade from noise traders is denoted by $z \sim N\left(0, \sigma_z^2\right)$, where the variance of noise trading is distributed according to:

$$\sigma_z^2 \in \left\{\bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta\right\}, \text{ where } \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta\right) = \frac{1}{2}, \tag{2}$$

and $\bar{\sigma}^2 > \delta \geq 0$. Finally, there is a risk-neutral, competitive market maker (he), who is privately informed about the realization of σ_z^2, and sets the price P of the risky asset conditional on this information and the total order flow, which we denote by $y = x_1 + x_2 + z$. We assume that v, ε_1, ε_2, and z, as governed by the realization of σ_z^2, are all mutually independent.

As in Hong and Rady (2002), we assume that the market maker knows the realization of σ_z^2 while the strategic traders do not. The above specification implies that the volatility of noise trading variance is given by δ, since

$$\mathbb{V}\left[\sigma_z^2\right] = \frac{1}{4}\left(2\delta\right)^2 = \delta^2. \tag{3}$$

For ease of exposition, and with some abuse of notation, we will refer to the parameter δ as the **uncertainty** about noise trading volatility in what follows.

We restrict attention to symmetric, linear equilibria in which (i) the equilibrium trade by investor i is given by $x_i = \beta s_i$, and (ii) the market maker's pricing rule is (conditionally) linear in the order flow $y = x_1 + x_2 + z$. Denote the pricing rule by:

$$P\left(y; \sigma_z^2\right) = \begin{cases} \mathbb{E}\left[v | y, \sigma_z^2 = \bar{\sigma}^2 - \delta\right] = \lambda_h y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \mathbb{E}\left[v | y, \sigma_z^2 = \bar{\sigma}^2 + \delta\right] = \lambda_l y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}. \tag{4}$$

2.1 Discussion of assumptions

Our goal is to explore the impact of uncertainty about noise trading volatility in a setting that deviates minimally from the traditional Kyle (1985) framework. In Section 5, we allow for more general distributions of noise trading volatility, and derive sufficient conditions under which the competitive equilibrium generates higher expected profits than the monopolistic equilibrium. It is worth distinguishing our setting from one in which investors face ambiguity about the distribution of noise trading. Importantly, investors in our setting know the true distribution of noise trading volatility (given by (2)) and are risk-neutral. As such, our results are not driven by ambiguity or ambiguity aversion.

The key assumption in our analysis is that investors are less informed about the distribution of noise trading than the market maker. The assumption that the market maker knows the volatility of noise trading perfectly is made for analytical tractability. In a setting where the market maker faces uncertainty about noise trading volatility, he would update not only on the value of the asset, but also on the volatility of noise trading from the order flow. With normally distributed payoffs and signals, the price would no longer be linear in the order flow and, consequently, the investors' strategies would no longer be linear in their signals, which makes the analysis intractable. In Appendix B, we consider a simplified version of the model in which (i) payoffs are binary, (ii) traders cannot short-sell, and (iii) market makers also face uncertainty about the distribution of noise trading volatility. We characterize the equilibrium and show (numerically) that there are parameter regions in which our main result obtains: profits under the competitive equilibrium are higher than under the monopolistic equilibrium.

Finally, it would be interesting to explore the implications of uncertainty about the distribution of noise trading in other trading environments, which allow traders to infer information about the volatility of noise trading from the price. For instance, if investors were allowed to submit limit orders (or condition trading intensity β on the price), in principle they would be able to condition on the information in prices about the distribution of noise trading. However, the analysis in such a setting is not immediately tractable, and so we leave it for future work.

3 Analysis

In what follows, we compare two scenarios. First, we consider the equilibrium in which traders perfectly coordinate their behavior and trade as a single monopolist. Then we consider the equilibrium in which traders compete against each other.

3.1 Monopolistic equilibrium

We begin by considering the equilibrium in which traders behave monopolistically, i.e., they pool their information and submit perfectly coordinated trades to maximize total profits. This is equivalent to a single strategic trader who observes both signals $\{s_1, s_2\}$ and trades x_M to maximize the following objective

$$\mathbb{E}\left[x_M\left(v-\lambda\left(x_M+z\right)\right)|s_1,s_2\right]=x_M\mathbb{E}\left[v|s_1,s_2\right]-\left(\tfrac{\lambda_h+\lambda_l}{2}\right)x_M^2, \tag{5}$$

where the equality follows from the above conjecture for the market maker's pricing rule. This implies that the optimal trading strategy is given by

$$x_M = \frac{1}{\lambda_h + \lambda_l}\mathbb{E}\left[v|s_1, s_2\right] = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\sigma_v^2}{\sigma_e^2 + 2\sigma_v^2}\right)(s_1 + s_2) \equiv \beta_M (s_1 + s_2), \tag{6}$$

since $\mathbb{E}[v|s_1, s_2] = (s_1\sigma_\varepsilon^{-2} + s_2\sigma_\varepsilon^{-2})/(\sigma_v^{-2} + 2\sigma_\varepsilon^{-2})$. Symmetry implies that the strategy puts equal weight on the two traders' signals.

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, his pricing rule can be written as

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, \beta_M (s_1 + s_2) + z\right)}{\mathbb{V}\left(\beta_M (s_1 + s_2) + z\right)} = \frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2 + 2\beta_M^2\sigma_\varepsilon^2 + \sigma_z^2}. \tag{7}$$

Solving the above system of equations for $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ gives us the following result.

Proposition 1. *When traders behave monopolistically, there exists a unique, linear equilibrium with $x_M = \beta_M (s_1 + s_2)$ and*

$$P\left(y; \sigma_z^2\right) = \begin{cases} \lambda_{M,h} y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_{M,l} y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}, \tag{8}$$

where

$$\beta_M = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \quad \lambda_{M,h} = \frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2 + 2\beta_M^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta}, \text{ and } \quad \lambda_{M,l} = \frac{2\beta_M\sigma_v^2}{4\beta_M^2\sigma_v^2 + 2\beta_M^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{9}$$

Moreover, β_M is decreasing in δ, $\lambda_{M,h}$ is increasing in δ, $\lambda_{M,l}$ is decreasing in δ, but $\frac{\lambda_{M,h}+\lambda_{M,l}}{2}$ is increasing in δ. The investors' combined expected trading profits are given by

$$\pi_M = \mathbb{E}\left[x_M (v - P)\right] = \beta_M\sigma_v^2 = \frac{\sigma_v^2\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \tag{10}$$

which is decreasing in δ.

The above result highlights that the monopolistic trading intensity β_M and trading profits π_M are decreasing in uncertainty about noise trading volatility, δ. In fact, as $\delta \to \bar{\sigma}^2$, both trading intensity and expected profits approach zero. Note that for a fixed β_M, the price impact $\lambda(\sigma_z^2)$ is a convex function of noise trade volatility σ_z^2, as highlighted by (7).[6] This implies that holding the mean $\bar{\sigma}^2$ fixed, an increase in the variance of noise trading volatility increases the *average* price impact the (monopolistic) strategic trader faces, i.e., $\frac{\lambda_{M,h}+\lambda_{M,l}}{2}$ increases with δ. As a result, the trading intensity, β, decreases with δ. Finally, note that aggregate expected profits are proportional

[6] It is worth noting that β_M does not depend on the realized σ_z^2, but $\lambda(\sigma_z^2)$ does.

to trading intensity and, therefore, are decreasing in δ, since

$$\pi_M = \mathbb{E}\left[x\left(v - \tfrac{\lambda_h+\lambda_l}{2}x\right)\right] = \beta_M \sigma_v^2, \tag{11}$$

as we show in the proof of the proposition.

3.2 Competitive equilibrium

Next we consider the equilibrium in which traders behave competitively, i.e., each trader conditions only on her own signal and trades independently, taking the strategies of the other participants as given. Specifically, investor i chooses to maximize:

$$\mathbb{E}\left[x_i\left(v - \lambda x_i - \lambda\left(\beta_j s_j + z\right)\right)|s_i\right] = x_i\left(1 - \tfrac{\lambda_h+\lambda_l}{2}\beta_j\right)\mathbb{E}\left[v|s_i\right] - \tfrac{\lambda_h+\lambda_l}{2}x_i^2. \tag{12}$$

This implies that the optimal trading strategy is given by

$$x_i = \frac{\left(1 - \frac{\lambda_h+\lambda_l}{2}\beta_j\right)}{\lambda_h+\lambda_l}\mathbb{E}\left[v|s_i\right] = \frac{\left(1 - \frac{\lambda_h+\lambda_l}{2}\beta_j\right)}{\lambda_h+\lambda_l}\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}s_i \equiv \beta_i s_i, \tag{13}$$

since $\mathbb{E}\left[v|s_i\right] = s_i\sigma_\varepsilon^{-2}/(\sigma_v^{-2}+\sigma_\varepsilon^{-2})$. Symmetry implies

$$x_j = \frac{\left(1 - \frac{\lambda_h+\lambda_l}{2}\beta_i\right)}{\lambda_h+\lambda_l}\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}s_j \equiv \beta_j s_j, \tag{14}$$

and in a symmetric equilibrium, we have:

$$\beta_1 = \beta_2 \equiv \beta_C = \frac{1}{\lambda_h+\lambda_l}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}. \tag{15}$$

Since the market maker can condition on the order flow and the noise trading volatility, σ_z^2, his pricing rule can be written as

$$\lambda\left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, \beta_1 s_1 + \beta_2 s_2 + z\right)}{\mathbb{V}\left(\beta_1 s_1 + \beta_2 s_2 + z\right)} = \frac{\left(\beta_i+\beta_j\right)\sigma_v^2}{\left(\beta_i+\beta_j\right)^2\sigma_v^2 + \left(\beta_i^2+\beta_j^2\right)\sigma_\varepsilon^2 + \sigma_z^2}. \tag{16}$$

Solving the above system of equations for $\{\beta_C, \lambda_{C,h}, \lambda_{C,l}\}$ after imposing $\beta_1 = \beta_2 \equiv \beta_C$ gives us the following result.

Proposition 2. *When traders behave competitively, there exists a unique, symmetric, linear equilibrium with $x_i = \beta_C s_i$ and*

$$P\left(y;\sigma_z^2\right) = \begin{cases} \lambda_{C,h}y & \text{if } \sigma_z^2 = \bar{\sigma}^2 - \delta \\ \lambda_{C,l}y & \text{if } \sigma_z^2 = \bar{\sigma}^2 + \delta \end{cases}, \tag{17}$$

where $\beta_C = \frac{1}{2}\sqrt{\frac{\sqrt{\bar{\sigma}^4(2\sigma_\varepsilon^2+3\sigma_v^2)^2-4\delta^2(\sigma_\varepsilon^4+3\sigma_\varepsilon^2\sigma_v^2+2\sigma_v^4)}+\bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4+3\sigma_\varepsilon^2\sigma_v^2+2\sigma_v^4}}$,

$$\lambda_{C,h} = \frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2+2\beta_C^2\sigma_\varepsilon^2+\bar{\sigma}^2-\delta}, \quad \textit{and } \lambda_{C,l} = \frac{2\beta_C\sigma_v^2}{4\beta_C^2\sigma_v^2+2\beta_C^2\sigma_\varepsilon^2+\bar{\sigma}^2+\delta}. \tag{18}$$

Moreover, β_C *is decreasing in* δ*,* $\lambda_{C,h}$ *is increasing in* δ*,* $\lambda_{C,l}$ *is decreasing in* δ*, but* $\frac{\lambda_{C,h}+\lambda_{C,l}}{2}$ *is increasing in* δ*. Each investor's expected trading profits are given by*

$$\pi_C = \mathbb{E}\left[x_C\left(v-P\right)\right] = \beta_C\sigma_v^2\tfrac{\sigma_v^2+\sigma_\varepsilon^2}{2\sigma_\varepsilon^2+3\sigma_v^2}, \tag{19}$$

which is decreasing in δ*.*

As in the monopolistic equilibrium, trading intensity β_C and profits are decreasing in uncertainty about noise trading volatility, because the average price impact that traders face (i.e., $\frac{\lambda_{C,h}+\lambda_{C,l}}{2}$) is increasing in δ. The next section characterizes how these equilibria differ in their response to uncertainty.

3.3 Comparison of expected profits

Given the characterization in the previous subsections, we now compare expected profits in the two equilibria.

Proposition 3. *There exist* $0 < \underline{\delta} < \bar{\delta} < \bar{\sigma}^2$ *such that:*

(i) if $\delta < \underline{\delta}$*, total profits are higher in the monopolistic equilibrium, i.e.,* $\pi_M > 2\pi_C$*;*

(ii) if $\underline{\delta} < \delta < \bar{\delta}$*, total profits are higher in the competitive equilibrium, but individual profits in the competitive equilibrium are not higher than total profits in the monopolistic equilibrium, i.e.,* $2\pi_C > \pi_M > \pi_C$*;*

(iii) if $\bar{\delta} < \delta$*, individual profits in the competitive equilibrium are higher than total profits in the monopolistic equilibrium, i.e.,* $\pi_C > \pi_M$*.*

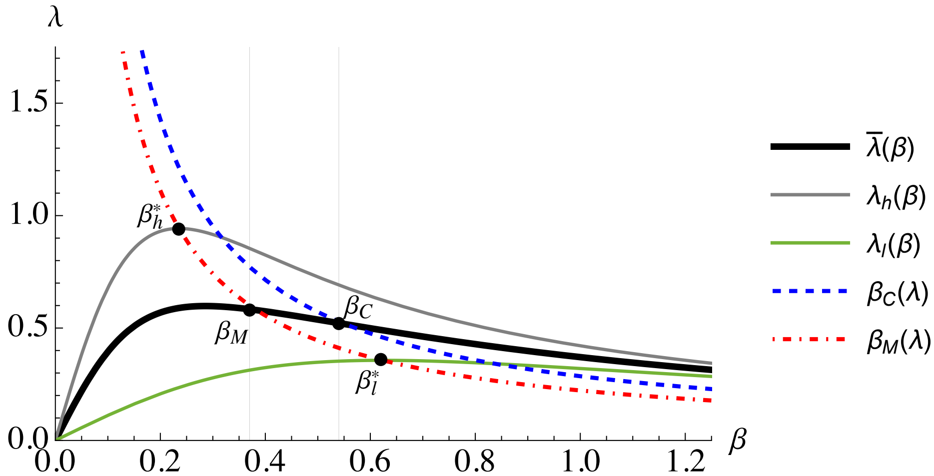
The above result highlights that the relative benefit of coordination among strategic traders depends on the uncertainty they face about the distribution of noise trading. Specifically, we show that when uncertainty about noise trading volatility is sufficiently high, expected profits for an *individual* investor in the competitive equilibrium can be higher than *total* profits for all investors in the monopolistic equilibrium.

Consider the best response functions of the market maker and the strategic traders in the two equilibria, as illustrated in Figure 1. Irrespective of the equilibrium, the market maker's best response function, given a (symmetric) trading strategy β, can be expressed as:

$$\lambda(\beta,\sigma_z^2) = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2+2\beta^2\sigma_\varepsilon^2+\sigma_z^2} = \begin{cases}\lambda_h(\beta) & \text{if} \quad \sigma_z^2 = \bar{\sigma}^2-\delta\\ \lambda_l(\beta) & \text{if} \quad \sigma_z^2 = \bar{\sigma}^2+\delta\end{cases}. \tag{20}$$

Figure 1: Best response functions

The figure plots the average price impact $\bar{\lambda} \equiv \frac{\lambda_h+\lambda_l}{2}$ (black) as a best response to the intensity β chosen by traders, and the trading intensities β_M (dot-dashed) and β_C (dashed) as best responses to the average price impact set by the market maker in the monopolistic and competitive equilibrium respectively. The figure also plots the realized price impact, conditional on realized noise trading volatility: λ_h (gray) corresponds to high price impact (i.e., $\sigma_z^2 = \bar{\sigma}^2 - \delta$), and λ_l (green) corresponds to low price impact (i.e., $\sigma_z^2 = \bar{\sigma}^2 + \delta$). Parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$, $\bar{\sigma} = 2$, and $\delta = 3$.



These best response functions correspond to the gray (λ_h) and green (λ_l) lines in Figure 1. Similarly, for a given expected price impact λ, denote the trader's best-response under the monopolistic equilibrium as $\beta_M(\lambda)$ and the competitive equilibrium as $\beta_C(\lambda)$ — these correspond to the dot-dashed red line and the dashed blue line, respectively, in Figure 1.

The main mechanism underlying Proposition 3 follows from three observations.[7]

Observation 1. *For a fixed level of noise trading volatility σ_z^2, the trading intensity under the competitive equilibrium is higher than under the monopolistic equilibrium.*

Intuitively, this is because in a competitive equilibrium, each trader does not account for the impact of her trading intensity on the behavior of others and, consequently, trades too aggressively on her private information. Formally, note that for a fixed λ,

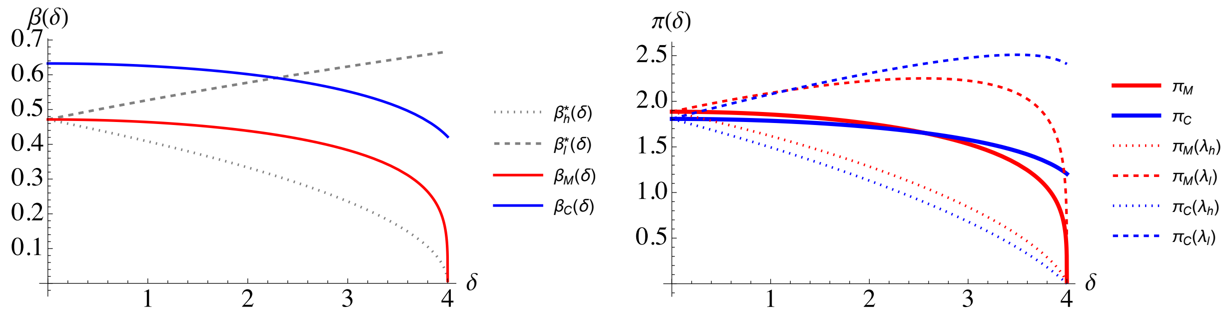
$$\beta_M(\lambda) = \frac{1}{2\lambda}\left(\frac{2\sigma_v^2}{4\sigma_v^2 + 2\sigma_\varepsilon^2}\right) < \frac{1}{2\lambda}\left(\frac{2\sigma_v^2}{3\sigma_v^2 + 2\sigma_\varepsilon^2}\right) = \beta_C(\lambda).$$

In Figure 1, this implies that the blue, dashed line (i.e., $\beta_C(\lambda)$) is always to the right of the red, dot-dashed line (i.e., $\beta_M(\lambda)$). Moreover, for a given σ_z^2, the equilibrium trading intensity of the monopolistic investor maximizes both aggregate trading profits and price impact. In Figure 1, this is illustrated as the intersection of the monopolist's best response function $\beta_M(\lambda)$ at β_h^* for the high price impact function λ_h, and at β_l^* for the low price impact function λ_l. As a result, for a

[7] We formally establish these observations in Appendix A.4.

Figure 2: Trading strategies and profits versus uncertainty

The figure plots trading strategies and profits under the monopolistic and competitive equilibria. Panel (a) plots the monopolistic ($\beta_M(\delta)$) and competitive ($\beta_C(\delta)$) equilibrium trading intensities as a function of δ. It also plots the profit maximizing trading intensities $\beta_h^*(\delta)$ and $\beta_l^*(\delta)$ that would arise if traders knew that price impact was high (i.e., $\sigma_z^2 = \bar{\sigma}^2 - \delta$) and low (i.e., $\sigma_z^2 = \bar{\sigma}^2 + \delta$), respectively. Panel (b) plots conditional (dotted and dashed) and expected (solid) aggregate profits for the monopolistic (red) and competitive (blue) equilibrium as a function of δ. Parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$, and $\bar{\sigma} = 2$.



(a) Trading strategies (b) Profits

given σ_z^2, the equilibrium trading intensity under competition is higher — for a given price impact function, the intersection with $\beta_C(\lambda)$ is to the right of the intersection with $\beta_M(\lambda)$.

An immediate consequence is that aggregate profits are lower in the competitive equilibrium than in the monopolistic equilibrium. This reflects the notion that investors with correlated information trade more aggressively under competition and drive down profits, which is consistent with the intuition from the existing literature (e.g., Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back et al. (2000)).

Observation 2. *For sufficiently high uncertainty about noise trading volatility, investors' trading intensity is "too conservative" in the high-liquidity state and "too aggressive" in the low-liquidity state in either equilibrium*

Since investors do not know whether noise trading volatility is high or low, they best respond to the *average* price impact: in Figure 1, this corresponds to the intersection of the traders' best response functions with the $\mathbb{E}[\lambda] \equiv \bar{\lambda}$ function (black line). In the monopolistic equilibrium, this implies that the best response to the average price impact trades "too aggressively" when $\lambda = \lambda_h$ and "too conservatively" when $\lambda = \lambda_l$. In Figure 1, this corresponds to $\beta_M > \beta_h^*$ and $\beta_M < \beta_l^*$, respectively.

Panel (a) of Figure 2 illustrates how trading intensities change as uncertainty increases. It plots the trading intensity for the monopolistic equilibrium β_M (solid, red), the competitive equilibrium β_C (solid, blue), and the profit-maximizing intensity conditional on knowing noise trading volatility, i.e., β_h^* (dotted, gray) and β_l^* (dashed, gray). Note that, as δ increases, noise trading volatility increases in the more liquid state and decreases in the less liquid state. This implies that the profit maximizing intensity β_h^* decreases with δ, while β_l^* increases with δ.

Note that competition leads to more aggressive trading for any level of δ (see Lemma 2 in the Appendix). For sufficiently low levels of uncertainty (when $\delta < \delta^* \equiv \frac{2\bar{\sigma}^2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}$), the competitive trading intensity is too aggressive in both the high- and low-liquidity states: $\beta_C > \beta_l^* > \beta_h^*$. As uncertainty increases, however, the competitive trading intensity first coincides with and then falls below β_l^*. For sufficiently high uncertainty ($\delta > \delta^*$), the competitive traders are also too aggressive in the low-liquidity regime, too conservative in the high-liquidity regime.

As panel (b) of Figure 2 illustrates, this implies that profits respond differently to increased uncertainty across equilibria. In the low-liquidity state, trading is too aggressive under both equilibria, and more aggressive with competition, and so competitive profits are always lower in this case, i.e., for all $\delta < \bar{\sigma}^2$, the blue dotted line is lower than the red dotted line in panel (b). In contrast, the relative ranking of expected profits across the two equilibria switch as δ increases in the high-liquidity state. When δ is sufficiently low, expected profits are higher under the monopolistic equilibrium since traders are too aggressive in the competitive equilibrium. However, when δ is sufficiently high, trading is too conservative under both equilibria, but less conservative with competition. Importantly, this ensures that for sufficiently high δ, expected profits under competition are always higher than in the monopolistic setting, i.e., the blue dashed line is higher than the red dashed line in panel (b).

Observation 3. *For sufficiently high uncertainty about noise trading volatility, differences in expected trading profits across equilibria are dominated by the corresponding profits in the high-liquidity state.*

As δ increases, noise trading volatility and, consequently, the opportunity for profits in the low-liquidity (high λ) state decreases. In the limit, when $\delta \to \bar{\sigma}^2$, the order-flow is fully revealing to the market maker, and so investors cannot make any profits in either equilibrium. As a result, with increasing uncertainty, aggregate expected profits are primarily driven by profits that investors can earn in the high-liquidity state. Given the above arguments, as δ gets sufficiently large, aggregate profits in the competitive equilibrium are higher than in the monopolistic equilibrium. The more aggressive trading induced by competition becomes particularly valuable when uncertainty is high: the incremental profits gained in the more liquid state dominate the incremental losses in the less liquid state.

As we show in the proof of Proposition 3, an even stronger result holds under the assumptions of our baseline analysis. Specifically, if σ_z^2 has a uniform distribution over $\{\bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta\}$, we demonstrate that the monopolistic trading intensity and, consequently, expected profits converge to zero as $\delta \to \bar{\sigma}^2$. This ensures that (iii) of Proposition 3 arises in our setting: when uncertainty is sufficiently high, individual profits in the competitive equilibrium are higher than total profits in the monopolistic equilibrium.

While this result need not hold more generally, in Section 5, we characterize sufficient conditions for more general distributions of σ_z^2 under which expected aggregate profits are higher under the competitive equilibrium than under the monopolistic equilibrium, i.e., (ii) of Proposition 3 holds.

We show that it is sufficient to ensure that low-liquidity (i.e., low σ_z^2) states are sufficiently likely and uncertainty about σ_z^2 is sufficiently high. Intuitively, these conditions ensure that expected aggregate profits are (i) not dominated by the low-liquidity states and (ii) sufficiently higher for the competitive equilibrium in the high-liquidity states.

4 Endogenous collusion and implications for observables

Given the results of the previous section, we extend the analysis to allow traders to optimally choose whether to compete or to collude. We then explore the model's implications for observables including return volatility, trading volume, and price informativeness.

4.1 Endogenous collusion

So far, our analysis has focused on comparing expected profits across two possible equilibria. In this section, we allow investors to endogenously choose whether or not to behave monopolistically. Specifically, we assume that before trading investors can choose to pay a cost c to pool their information and coordinate their trading. This cost reflects not only the direct costs of monitoring and enforcing coordination, and sharing information and resources, but also the indirect costs of litigation and regulatory risk.

The following result characterizes the endogenous choice of collusion in this setting.

Corollary 1. *There exists a threshold $\bar{c} > 0$, such that:*

(i) if $c > \bar{c}$, investors never choose to coordinate; and

(ii) if $c \leq \bar{c}$, there exists a threshold $\hat{\delta}(c)$ such that investors choose to coordinate if and only if $\delta < \hat{\delta}(c)$. In this case, $\hat{\delta}(c)$ is decreasing in c. Moreover, when $\sigma_\varepsilon^2 = 0$, $\hat{\delta}(\cdot)$ is increasing in σ_v and $\bar{\sigma}$.

Note that investors should choose to coordinate if and only if their net benefit from doing so exceeds the cost, i.e., if

$$\Pi(\delta) \equiv \pi_M(\delta) - 2\pi_C(\delta) \geq c.$$

The proof of Proposition 3 establishes that the difference between the expected profits $\Pi(\delta)$ in the two cases is maximized for $\delta = 0$ and decreasing in δ. This implies that if $c > \bar{c}$, where

$$\bar{c} \equiv \Pi(0) = \frac{\bar{\sigma}\sigma_v^2}{\sqrt{2}} \left(\frac{1}{\sqrt{2\sigma_v^2 + \sigma_\varepsilon^2}} - \frac{2\sqrt{\sigma_v^2 + \sigma_\varepsilon^2}}{3\sigma_v^2 + 2\sigma_\varepsilon^2} \right),$$

then investors will never choose to coordinate. Moreover, for $c < \bar{c}$, investors are indifferent between coordinating and not when $\delta = \hat{\delta}(c)$, where $\Pi(\hat{\delta}(c)) = c$. Since $\Pi(\cdot)$ is a decreasing function, the above implies $\hat{\delta}(c)$ is decreasing in c.

Furthermore, in the case when investors' private signals are perfectly informative (i.e., $\sigma_\varepsilon = 0$), we can show that collusion is more likely when the informational advantage of strategic traders is higher (i.e., σ_v^2 is higher) and when the average level of noise trading volatility is higher (i.e., $\bar{\sigma}$ is higher) since both increase the incremental benefit from collusion.

Intuitively, investors choose to coordinate when both the cost of collusion and the uncertainty about noise trading volatility are sufficiently low. As we discuss further in Section 6, this has implications for the delegated portfolio management sector, in which groups of investors (funds) can choose to compete or coordinate depending on the uncertainty that they face.

4.2 Implications for observables

Next, we characterize how uncertainty about noise trading affects observables such as price volatility, trading volume, and price informativeness. Note that since the market maker is risk-neutral, the expected return on the security is always zero. We first provide a characterization of each observable under the monopolistic and competitive equilibria. We then show that allowing for endogenous collusion generates novel predictions.

4.2.1 Volatility

We can compute the volatility of the price as the square root of the unconditional variance $\mathbb{V}(P)$, which by the law of total variance can be expressed as

$$\mathbb{V}(P) = \mathbb{E}[\mathbb{V}(P|\sigma_z^2)] + \mathbb{V}(\mathbb{E}[P|\sigma_z^2]).$$

Since $\mathbb{E}[P|\sigma_z^2] = 0$, the above simplifies to:

$$\begin{aligned}\mathbb{V}(P) &= \frac{1}{2}\left(\lambda_h^2 \mathbb{V}(y|\sigma_z^2 = \bar{\sigma}^2 - \delta) + \lambda_l^2 \mathbb{V}(y|\sigma_z^2 = \bar{\sigma}^2 + \delta)\right) \\ &= (\lambda_h + \lambda_l)\beta\sigma_v^2,\end{aligned}$$

where the second equality follows from the definition of λ in each equilibrium. This implies the following result.

Proposition 4. *(i) Under the monopolistic equilibrium, the unconditional variance of the price is given by*

$$\mathbb{V}(P) = \sigma_v^2 \times \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} \equiv V_M,$$

while under the competitive equilibrium, the unconditional variance of the price is given by

$$\mathbb{V}(P) = \sigma_v^2 \times \frac{2\sigma_v^2}{3\sigma_v^2 + 2\sigma_\varepsilon^2} \equiv V_C.$$

Moreover, $V_M < V_C$.
(ii) If collusion is endogenous and $c < \bar{c}$, then the volatility of price is weakly increasing in the uncertainty about noise trading volatility.

The result is reminiscent of the traditional Kyle (1985) setting without uncertainty about noise trading volatility. Intuitively, because the market maker knows the realized value of σ_z^2, he is able to set price impact λ to efficiently extract the information in order flow in both the low and high volatility regimes. The strategic investors anticipate this when choosing their optimal trades: even though they do not know the realized volatility of noise trading, their strategy is a best response to the average price impact. As a result, unconditional price volatility is completely unaffected by the volatility of noise trading σ_z^2, and, in particular, δ.

The variance in the price is affected by the degree of coordination, however. Since investors trade less aggressively on their information in the monopolistic equilibrium, price volatility is always lower in this case than in the competitive equilibrium. This implies that if the choice of collusion is endogenous, an increase in uncertainty about noise trading volatility makes it more likely that investors choose the competitive equilibrium, which in turn implies that price volatility is higher. While the result might appear to be immediate, the mechanism that gives rise to it is novel to our setting, and relies importantly on investors' ability to endogenously choose whether or not to coordinate their behavior in response to changes in uncertainty.

4.2.2 Volume

Following Admati and Pfleiderer (1988), we define expected volume as the sum of the expectations of the absolute value of trades by informed investors, noise traders and the market maker, i.e.,

$$\mathcal{V} = \mathbb{E}[|\sum_i x_i|] + \mathbb{E}[|z|] + \mathbb{E}[|y|].$$

Note that conditional on σ_z^2, each of these trades is normally distributed with mean zero. This implies that we can characterize the expected volume as follows.

Proposition 5. *(i) Conditional on the behavior of strategic traders, the total expected volume is:*

$$\mathcal{V}(\delta) = \left(\sigma_x + \tfrac{1}{2}\left(\sqrt{\bar{\sigma}^2+\delta} + \sqrt{\bar{\sigma}^2-\delta}\right) + \tfrac{1}{2}\left(\sqrt{\bar{\sigma}^2+\delta+\sigma_x^2} + \sqrt{\bar{\sigma}^2-\delta+\sigma_x^2}\right)\right)\frac{2\sqrt{2}}{\sqrt{\pi}},$$

where $\sigma_x \equiv \sqrt{\mathbb{V}\left(\sum_i x_i\right)} = \beta\sqrt{4\sigma_v^2 + 2\sigma_\varepsilon^2}$. *Moreover,* $\mathcal{V}(\delta)$ *is decreasing in* δ.
(ii) All else equal, total expected volume is higher under the competitive equilibrium than under the monopolistic equilibrium, i.e., $\mathcal{V}_C(\delta) > \mathcal{V}_M(\delta)$.

The above result highlights that, with the behavior of strategic traders fixed, the total expected volume decreases with uncertainty about noise trading volatility, δ. Intuitively, the trading intensity of informed investors, β, is decreasing in uncertainty, as we show in Propositions 1 and 2. Moreover, expected trading volume of noise traders is also decreasing in δ, which in turn implies that the trading volume induced by the market maker is also decreasing in δ.[8]

[8] Even though the expected *variance* of noise trading σ_z^2 is unaffected by δ, the expected *volatility* (or standard deviation) of noise trading, which determines expected volume, is a decreasing function. This is because $\mathbb{E}[\sqrt{x}] \leq \sqrt{\mathbb{E}[x]}$.

Part (ii) of the result implies that the competitive equilibrium is characterized by a higher expected volume than the monopolistic equilibrium. This is driven by a higher trading intensity under the competitive equilibrium: $\beta_M < \beta_C$. The above results also imply that if collusion is endogenous and $c < \bar{c}$, then expected volume is strictly decreasing in δ, interrupted by a discontinuous upward jump at $\hat{\delta}(c)$, after which it resumes its decreasing trend.

4.2.3 Price informativeness

Let $PI(P) = -\mathbb{V}[v|P]$ denote the price informativeness in a given equilibrium. The following lemma establishes that, as in Hong and Rady (2002), price informativeness in either equilibrium depends on the realization of the price when investors face uncertainty about the volatility of noise trading.

Lemma 1. *Price informativeness can be expressed as*

$$PI(P) \equiv -\mathbb{V}[v|P] = -\left(\bar{p}\,(P) \times \sigma_v^2\,(1-2\beta\lambda_h) + (1-\bar{p}\,(P)) \times \sigma_v^2\,(1-2\beta\lambda_l)\right),$$

where

$$\bar{p}(P) \equiv \Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta|P) = \frac{1}{1+\sqrt{\frac{\lambda_h}{\lambda_l}}e^{-\frac{P^2}{2\sigma_v^2}\frac{\lambda_h-\lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{21}$$

and $\beta = \beta_M, \lambda_h = \lambda_{M,h}$ and $\lambda_l = \lambda_{M,l}$ for the monopolistic equilibrium, and $\beta = \beta_C, \lambda_h = \lambda_{C,h}$ and $\lambda_l = \lambda_{C,l}$ for the competitive equilibrium.

Figure 3 provides an illustration of this dependence for the monopolistic and competitive equilibria. As emphasized by Hong and Rady (2002), price informativeness is higher for larger absolute realizations of P (larger $|P|$), since these realizations allow one to better distinguish the high noise trading volatility state from the low noise trading volatility state. Specifically, one can show that $\lim_{P^2\to\infty} \bar{p}(P) = 1$, i.e., for sufficiently large realizations of P, one becomes arbitrarily certain that $\sigma_z^2 = \bar{\sigma}^2 - \delta$.

While price informativeness for a given realization of P varies with uncertainty δ, as illustrated by Figure 3, expected price informativeness is independent of δ. The following result establishes how this varies across equilibria in our setting.

Proposition 6. *(i) Under the monopolistic equilibrium, expected price informativeness is given by*

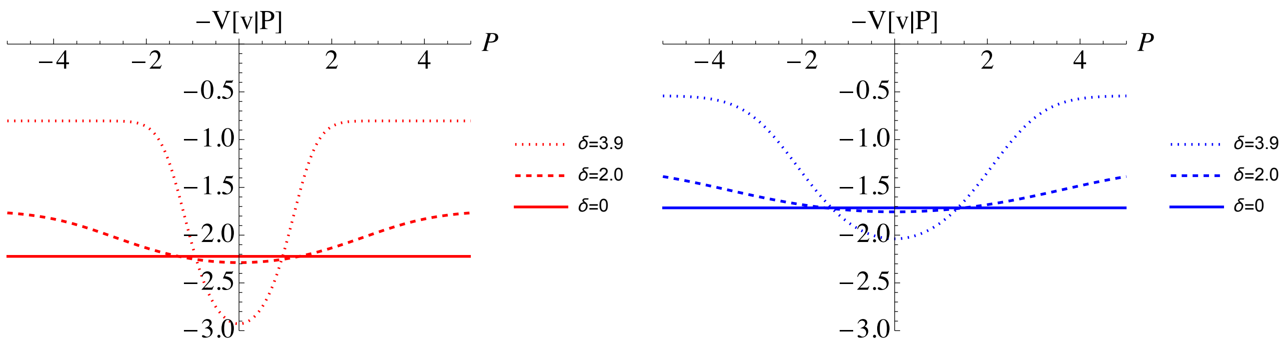
$$\mathbb{E}[PI(P)] = -\sigma_v^2\left(\frac{\sigma_\varepsilon^2+\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\right) \equiv PI_M,$$

while under the competitive equilibrium, expected price informativeness is given by

$$\mathbb{E}[PI(P)] = -\sigma_v^2\left(\frac{2\sigma_\varepsilon^2+\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}\right) \equiv PI_C.$$

Figure 3: Price informativeness $PI(P)$ under the monopolistic and competitive equilibria

The equilibrium is characterized by $\{\beta_M, \lambda_{l,M}, \lambda_{h,M}\}$ for the monopolistic equilibrium and $\{\beta_C, \lambda_{l,C}, \lambda_{h,C}\}$ for the competitive equilibrium. Other parameters are set to $\sigma_v = 2$, $\sigma_\varepsilon = 1$ and $\bar{\sigma} = 2$.



(a) $PI(P)$ for monopolistic equilibrium

(b) $PI(P)$ for competitive equilibrium

Moreover, $PI_C > PI_M$.

(ii) If collusion is endogenous and $c < \bar{c}$*, then the expected price informativeness is weakly increasing in the uncertainty about noise trading volatility.*

Part (i) of the above result establishes that expected price informativeness is higher under the competitive equilibrium, even when investors are uncertain about noise trading volatility. While this might initially appear to be at odds with our earlier results, note that one can express expected price informativeness as

$$\mathbb{E}[PI(P)] = \mathbb{E}[-\mathbb{V}[v|P]] = -\sigma_v^2(1 - \beta(\lambda_l + \lambda_h)),$$

as we verify in the proof of the above result. This implies that one can express expected profits in the monopolistic and competitive equilibria as:

$$\pi_M = \beta_M \sigma_v^2 = \frac{1}{2}(\sigma_v^2 + PI_M) \times \left(\frac{\lambda_{M,h} + \lambda_{M,l}}{2}\right)^{-1}, \quad \text{and} \tag{22}$$

$$\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2} = \frac{1}{2}(\sigma_v^2 + PI_C) \times \left(\frac{\lambda_{C,h} + \lambda_{C,l}}{2}\right)^{-1} \times \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{23}$$

respectively, which implies that profits are positively related to expected price informativeness and negatively related to expected market impact. Therefore, greater profits are generated in tandem with more informative prices and more liquid markets.

Part (ii) implies that when investors can endogenously choose to coordinate, price informativeness can actually *increase* with uncertainty about noise trading volatility. It is worth noting that contrary to existing models of "crowding in" of information, this is not driven by endogenous information acquisition. Instead, it arises even when investors' information is fixed, purely due to a change in their trading strategies. This distinguishes our analysis from existing work, and suggests novel implications for the impact of regulatory policy (which affects the cost c of collusion) for price informativeness. We discuss these further in Section 6.

5 Robustness and Extensions

Our benchmark analysis restricts attention to a simple setting with noisy signals and two investors to facilitate exposition. We now discuss how our results change when we modify these assumptions. Specifically, consider an extension of the model introduced in Section 2, with the following modifications:

(A1) There are $N > 1$ strategic traders.

(A2) Each trader i observes the value of the risky asset perfectly, i.e., $s_i = v$.

(A3) The aggregate trade from noise traders is denoted by $z \sim N\left(0, \sigma_z^2\right)$, where the variance of noise trading is distributed according to:

$$\sigma_z^2 \in \left\{\bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta\right\}, \text{ where } \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta\right) = p, \tag{24}$$

where $p \in [0,1]$ and $\bar{\sigma}^2 > \delta \geq 0$.

Assumption (A1) allows us to characterize how our main results extend to settings with more than two traders. Assumption (A2) lets us explore whether our results are driven by noise in investors' information. Finally, (A3) allows us to consider a richer set of distributions for noise trading volatility and to distinguish between the likelihood of a low-liquidity regime (i.e., p) and the level of the drop in liquidity (i.e., δ).

As before, we restrict attention to symmetric, linear equilibria in which (i) the equilibrium trade by investor i is given by $x_i = \beta s_i = \beta v$, and (ii) the market maker's pricing rule is (conditionally) linear in the order flow $y = \sum_{i=1}^N x_i + z$. Denote the pricing rule by:

$$P\left(y; \sigma_z^2\right) = \mathbb{E}\left[v | y, \sigma_z\right] = \lambda(\sigma_z)\, y \tag{25}$$

and let $\bar{\lambda} \equiv \mathbb{E}[\lambda(\sigma_z)]$. For a given N, let us denote β_N as the trading intensity in the competitive equilibrium. Each trader i maximizes

$$\max_x \ \mathbb{E}[x(v-P)|v] = \max_x \ \mathbb{E}\left[x\left(v - \lambda(\sigma_z)\left(x + \sum_{j\neq i}\beta_N v\right)\right)\middle| v\right] \tag{26}$$

$$= \max_x \ xv(1 - \bar{\lambda}(N-1)\beta_N) - \bar{\lambda}x^2, \tag{27}$$

which results in the following first order condition:

$$v\left(\beta_N\left(\bar{\lambda} - N\bar{\lambda}\right) + 1\right) = 2x\bar{\lambda}. \tag{28}$$

Imposing symmetry (i.e., $x = \beta_N v$) implies that:

$$\beta_N = \frac{1}{2\bar{\lambda}} + \frac{1}{2}(1-N)\,\beta_N \qquad \Leftrightarrow \qquad \beta_N = \frac{1}{(N+1)\bar{\lambda}}. \tag{29}$$

As before, the market maker can condition on the order flow and so price impact λ is given by:

$$\lambda(\sigma_z) = \frac{\mathbb{C}(v, N\beta_N v + z)}{\mathbb{V}(N\beta_N v + z)} = \frac{N\beta_N \sigma_v^2}{N^2\beta_N^2\sigma_v^2 + \sigma_z^2}. \tag{30}$$

Substituting (30) into (29), implies that the equilibrium β_N is the solution to:

$$\mathbb{E}\left[\frac{N^2\beta_N^2\sigma_v^2}{N^2\beta_N^2\sigma_v^2 + \sigma_z^2}\right] = \frac{N}{N+1}. \tag{31}$$

The monopolistic equilibrium, in which each investor uses trading intensity $\beta = \beta_M$, is characterized by the (aggregate) trading intensity $\beta_1 \equiv N\beta_M$. This implies β_1 is the solution to:

$$\mathbb{E}\left[\frac{\beta_1^2\sigma_v^2}{\beta_1^2\sigma_v^2 + \sigma_z^2}\right] = \frac{1}{2}. \tag{32}$$

Finally, expected profits can be expressed as:

$$N\pi_N \equiv N\mathbb{E}[x(v-p)] = N\mathbb{E}\left[\beta_N v\left(v - \lambda(\sigma_z)(N\beta_N v + z)\right)\right] = \frac{N}{N+1}\beta_N\sigma_v^2. \tag{33}$$

The following result establishes the existence and uniqueness of symmetric linear equilibria.

Proposition 7. *Suppose assumptions (A1) and (A2) hold, and the distribution of σ_z^2 is bounded. (i) When traders behave competitively, there exists a unique, symmetric, linear equilibrium with $x_i = \beta_C\, v$, where $\beta_C = \beta_N$ is the unique positive solution to (31), and total expected profits are given by*

$$N\pi_C = \frac{N}{N+1}\beta_C\sigma_v^2. \tag{34}$$

(ii) When traders behave monopolistically, there exists a unique, symmetric, linear equilibrium with $x_i = \beta_M\, v$, where $\beta_M = \beta_1/N$ and β_1 is the unique positive solution to (32), and total expected profits are given by

$$\pi_M = \frac{1}{2}\beta_1\sigma_v^2. \tag{35}$$

The above result implies that aggregate profits are higher in the competitive equilibrium than in the monopolistic equilibrium if and only if

$$\rho_N \equiv \frac{N\pi_N}{\pi_1} = \frac{2N}{N+1}\frac{\beta_N}{\beta_1} > 1. \tag{36}$$

To better understand how aggregate profits vary across the monopolistic and competitive equilibria, we begin by characterizing the ratio ρ_N for a specific distribution of noise trading.

Proposition 8. *Suppose the distribution of noise trading volatility is given by (A3), and fix* σ_v^2 *and* $\bar{\sigma}^2$*. There exist functions* $0 \leq \underline{p}(\delta, N) \leq 1/2 \leq \bar{p}(\delta, N) \leq 1$*, such that:*

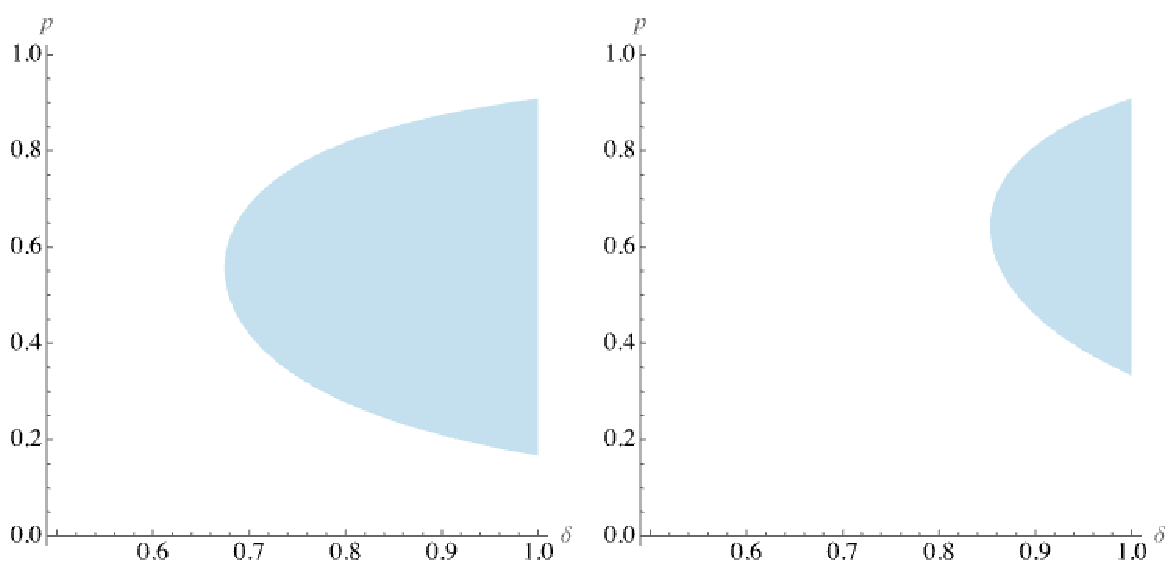
(i) For a fixed δ*, there exists* $\bar{N}(\delta)$*, such that for all* $N < \bar{N}(\delta)$ *and for all* $p \in (\underline{p}(\delta, N), \bar{p}(\delta, N))$*, total expected profits are higher under competition, i.e.,* $\rho_N(\delta, N, p) > 1$*.*

(ii) For a fixed N*, there exists a* $\bar{\delta}(N)$*, such that for all* $\delta > \bar{\delta}(N)$ *and for all* $p \in (\underline{p}(\delta, N), \bar{p}(\delta, N))$*, total expected profits are higher under competition, i.e.,* $\rho_N(\delta, N, p) > 1$*.*

Figure 4 provides an illustration of the above result. Specifically, the plots illustrate the range of parameters within which profits under the competitive equilibrium are higher than under the monopolistic equilibrium, i.e., where $\rho_N > 1$. There are a number of takeaways from Proposition 8 and Figure 4.

Figure 4: Parameter region where $\rho_N > 1$.

The shaded region corresponds to the range of the parameter space in which profits under the competitive equilibrium are higher than under the monopolistic equilibrium, i.e., where $\rho_N > 1$. Other parameters are set to $\sigma_v^2 = 1$ and $\bar{\sigma}^2 = 1$.



(a) $\rho_N > 1$ for $N = 2$

(b) $\rho_N > 1$ for $N = 5$

First, noise in the investors' information does not appear to qualitatively affect our conclusions: when uncertainty is sufficiently high, total profits in the competitive equilibrium are higher even when investors have perfect information about payoffs.

Second, the result obtains even when the distribution of σ_z^2 is not symmetric around its mean (i.e., even if $p \neq 1/2$). In fact, competitive profits are more likely to be higher when (all else equal), the low-liquidity state is more likely (i.e., $p > 1/2$). However, it is important to note that sufficiently high uncertainty about σ_z^2 is necessary. Fixing all other parameters, $\rho_N < 1$ for sufficiently high

p, even though this implies a higher likelihood of the low-liquidity state. On the other hand, for a fixed probability p, an increase in uncertainty δ increases the likelihood that competitive profits are higher.

Third, the above result highlights the competing effects of the number of investors N and the degree of uncertainty δ about noise trading volatility on the relative benefit of collusion. Part (i) implies that, for a fixed level of uncertainty, expected profits under the competitive equilibrium are higher only if the number of investors is sufficiently low. Intuitively, for a fixed δ, as N increases, investors compete more aggressively in the competitive equilibrium driving profits lower, while total profits remain unaffected in the monopolistic equilibrium.[9] As a result, when N is sufficiently high, competitive profits are lower. This is illustrated in Figure 4: the parameter region for which competitive profits are higher shrinks as N increases (from panel (a) to panel (b)).

However, in part (ii), we also show that for a fixed number N of investors, expected profits under the monopolistic equilibrium are lower than under the competitive equilibrium when uncertainty about noise trading volatility is sufficiently high. As such, the effects of uncertainty about noise trading volatility, which we highlight in our benchmark analysis, obtain even if the number of investors in the economy is large.

5.1 General distribution of σ_z^2

In this section, we provide sufficient conditions on the distribution of σ_z^2 that ensure total profits in the competitive equilibrium are higher than in the monopolistic one. This allows us to generalize our results to continuous distributions.

Proposition 9. *Fix $N > 1$. Suppose there exists $\epsilon > 0$ and $0 < a < 1$ such that $\mathbb{E}[\sigma_z^2 | \sigma_z^2 < \epsilon] \leq a\epsilon$. If*

$$a < a(N) \equiv \frac{(N-1)^2}{2(N+1)^3}, \tag{37}$$

and $\Pr(\sigma_z^2 < \epsilon) \in \left(\underline{p}(N), \bar{p}(N)\right)$, where

$$\underline{p}(N) \equiv \frac{3N+1-\sqrt{(N-1)^2 - 2a(N+1)^3}}{4(N+1)} \geq \frac{1}{2}, \text{ and} \tag{38}$$

$$\bar{p}(N) \equiv \frac{3N+1+\sqrt{(N-1)^2 - 2a(N+1)^3}}{4(N+1)} \leq \frac{N}{N+1}, \tag{39}$$

then total profits are higher under the competitive equilibrium than under the monopolistic equilibrium (i.e., $\rho_N > 1$).

The sufficient conditions to ensure that total profits are higher under competition require both a lower bound on the tail probability (i.e., $\Pr(\sigma_z^2 < \epsilon) > \underline{p}(N) \geq \frac{1}{2}$) and an upper bound on the tail expectation (i.e., $\mathbb{E}[\sigma_z^2 | \sigma_z^2 < \epsilon] \leq a\epsilon$). Intuitively, this requires that it is sufficiently likely that σ_z^2

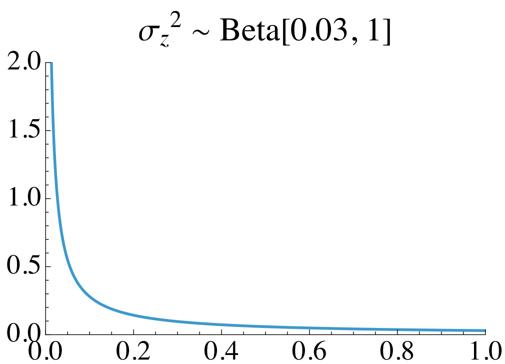
[9] This result is consistent with earlier work (e.g., Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996; and Back et al., 2000).

takes on sufficiently low values. In other words, the distribution of liquidity is skewed toward zero. At the same time, it must retain significant dispersion, so that high-liquidity realizations occur with positive probability: $\Pr(\sigma_z^2 < \epsilon) < \bar{p}(N)$. Figure 5 illustrates that a sufficiently left-skewed Beta distribution satisfies the conditions of the Proposition.

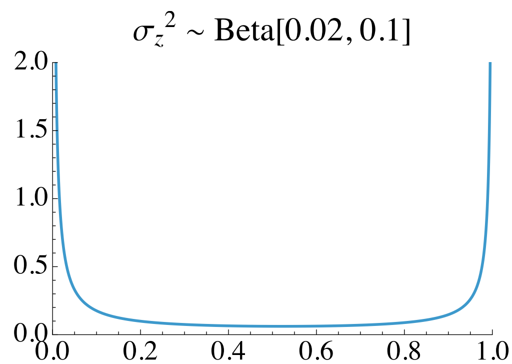
The Proposition re-emphasizes that the profit comparison between competitive and monopolistic speculators is driven by how they respond to liquidity uncertainty. Coordination amplifies exposure to adverse tail realization: "monopolistic" speculators internalize the possibility of extreme price impact in low-liquidity states and contract trading accordingly, while competitive speculators remain more robust to such events. As a result, sufficiently skewed and dispersed liquidity can overturn the standard profitability ranking.

Figure 5: Beta distributions that yield higher profits under the competitive equilibrium

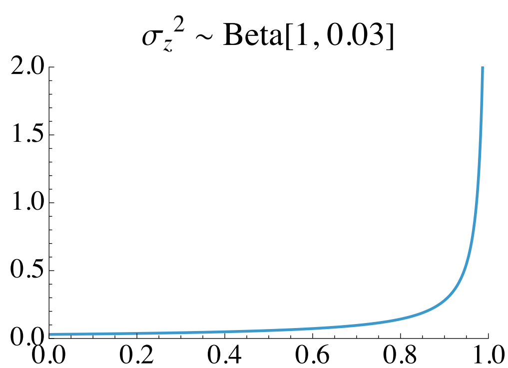
The figure plots examples of Beta distributions which satisfy the sufficient conditions in Proposition 9 (panels (a) and (b)) and that do not (panel (c)). Other parameters are set to $\sigma_v = 1$ and $N = 3$.



(a) conditions satisfied



(b) conditions satisfied



(c) conditions not satisfied

6 Concluding remarks

We consider a multi-investor extension of the Kyle (1985) model in which investors face uncertainty about the volatility of liquidity trading. We compare expected trading profits under a competitive equilibrium to those under a monopolistic equilibrium, in which all investors combine their information and coordinate perfectly. We show that when uncertainty is low, expected profits are higher under the monopolistic equilibrium, consistent with existing work. However, we find that when uncertainty increases, this may no longer be the case. In fact, when uncertainty about liquidity trading volatility is sufficiently high, we show that the expected profit for an *individual* investor in the competitive equilibrium can be higher than the *total* profit for all investors under the monopolistic equilibrium. As such, uncertainty about liquidity trading can have a substantive impact on the relative benefits of collusion among strategic investors.

Empirical relevance. A key challenge for testing our model's implications is identifying a measure of uncertainty about noise trading volatility or liquidity. One approach would be to calculate the time-series volatility of standard liquidity measures, such as bid-ask spreads, market depth, or the Amihud (2002) measure of illiquidity. Similarly, one could build on the analysis of Peress and Schmidt (2021) by using time-series volatility of their retail trading intensity measure, which they argue is a measure of noise trading volatility.

One can also use coarser measures to identify settings in which investors face increased uncertainty about liquidity. For instance, investors might be more concerned about sudden dry-ups in liquidity when trading in small stocks or during after-hours or pre-market trading. At the aggregate level, investors might be more concerned about low liquidity right after episodes with sudden changes in market liquidity, like "flash crashes."

As we outline below, our analysis has implications for the delegated portfolio management sector and antitrust regulation.

Delegated portfolio management. Mutual fund family structure has been shown to provide for strategic benefits for the top-performers within a family at the expense of the under-performers (e.g., Bhattacharya, Lee, and Pool (2013), Gaspar, Massa, and Matos (2006), Eisele, Nefedova, Parise, and Peijnenburg (2020)). Our model can be viewed through the lens of competition vs. cooperation within the fund family (e.g., Evans, Prado, and Zambrana (2020)), whereby a family may choose to have the same portfolio manager run multiple funds or assign the investment decisions in these funds to different managers. Our analysis suggests that in periods of greater uncertainty, or for investment strategies where liquidity regimes are highly variable and difficult to predict, fund families would benefit from inducing competition among its portfolio managers, even if these managers have correlated ideas and strategies. Conversely, when traders face low uncertainty about liquidity, the family should encourage cooperation among portfolio managers running different strategies (e.g., via the sharing of ideas as in Kacperczyk and Seru (2012) and Cici, Jaspersen, and Kempf (2017)) or assign the same portfolio manager to multiple funds (thereby fully internalizing the effect of her trading across the different strategies). A natural starting point for testing this implication of the model is to consider various measures of team-managed vs. solo-managed funds (e.g., Patel and Sarkissian (2017)) and interact these with variation in noise trading volatility.

Antitrust policy. Several academic and policy papers have raised concerns regarding the potential for AI/ML-based algorithms to learn to collude tacitly (e.g., Azzutti et al. (2022)). A common theme is that tacit collusion may result in either supra-competitive speculative profits (e.g., Dou et al. (2024)) or market maker mark-ups (e.g., Colliard et al. (2022)). Our analysis highlights a confound in relying solely on measures of profitability as an indicator for anti-trust regulation. Because trading algorithms inherently face parameter uncertainty, the less-aggressive trading may be an optimal response to such uncertainty as opposed to an intent to collude tacitly.

Regulations aimed at making order flow more transparent have the potential to resolve this confounding effect: if traders are endowed with a reliable estimate of noise trading volatility, one may argue with greater confidence that the supra-competitive profits are indeed outcomes of collusive behavior. However, as highlighted by Azzutti et al. (2022), greater transparency might also facilitate collusion because deviations from optimal behavior are more immediate and punishments for deviations are easier to implement.

On a related note, policies aimed at reducing the uncertainty regarding noise trading volatility for informed speculators (i.e., reducing δ) have the potential to improve speculative profits in both competitive and monopolistic settings – an immediate consequence of Propositions 1 and 2. Importantly, however, our analysis uncovers a potential drawback of such policies. Note that as uncertainty about liquidity falls, coordination becomes more attractive to traders (Proposition 3). Moreover, Proposition 6 establishes that average price informativeness is lower in the monopolistic equilibrium than under competition. As such, an increase in transparency about liquidity trading can inadvertently lead to greater collusion among sophisticated traders and, consequently, lower price informativeness.

Future work. Our model is stylized for tractability and expositional clarity, but may be extended along a number of dimensions. It would be interesting to compare the impact of competition versus coordination in a dynamic version of our model in which noise trading volatility evolves stochastically, and strategic traders learn about this over time. It would also be informative to consider the impact of investor uncertainty along other dimensions (e.g., the number of other investors in the market or their risk aversion). Finally, allowing for heterogeneous information quality and endogenous information acquisition would further test the robustness of our main result. We leave these extensions for future work.

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A Proofs

A.1 Proof of Proposition 1

The solutions $\{\beta_M, \lambda_{M,h}, \lambda_{M,l}\}$ follow from solving the system of equations:

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{40}$$

$$\lambda_h = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{41}$$

$$\lambda_l = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{42}$$

There are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields $\beta = \frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}$,

$$\lambda_h = \frac{\sigma_v^2\sqrt[4]{\bar{\sigma}^4 - \delta^2}\left(\sqrt{\bar{\sigma}^4 - \delta^2} - \bar{\sigma}^2 + \delta\right)}{\sqrt{2}\delta\left(\bar{\sigma}^2 - \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}, \text{ and } \lambda_l = \frac{\sigma_v^2\sqrt[4]{\bar{\sigma}^4 - \delta^2}\left(\bar{\sigma}^2 + \delta - \sqrt{\bar{\sigma}^4 - \delta^2}\right)}{\sqrt{2}\delta\left(\bar{\sigma}^2 + \delta\right)\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}. \tag{43}$$

Note that

$$\frac{\partial\beta}{\partial\delta} = -\frac{\delta}{2\sqrt{2}\left(\bar{\sigma}^4 - \delta^2\right)^{3/4}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}} = -\frac{\delta}{2\bar{\sigma}^4 - 2\delta^2}\beta < 0, \tag{44}$$

which together with

$$\beta = \frac{1}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right) \tag{45}$$

$$= \frac{2}{\lambda_h + \lambda_l}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{2}{\sigma_v^2} + \frac{4}{\sigma_\varepsilon^2}}\right), \tag{46}$$

implies that

$$\frac{\partial}{\partial\delta}\frac{\lambda_h + \lambda_l}{2} > 0. \tag{47}$$

Moreover, note that

$$\frac{\partial\lambda_h}{\partial\delta} = \frac{2\sigma_v^2\left(\frac{\partial\beta}{\partial\delta}\left(\bar{\sigma}^2 - \delta\right) + \beta - 2\beta^2\frac{\partial\beta}{\partial\delta}\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{48}$$

$$= \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{49}$$

$$= \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - 2\left(\frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}\right)^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{50}$$

$$= \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2}\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{51}$$

$$> \frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 - \delta\bar{\sigma}^2\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 - \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} > 0, \tag{52}$$

since $\bar{\sigma}^2 > \delta$. Similarly,

$$\frac{\partial\lambda_l}{\partial\delta} = -\frac{2\sigma_v^2\left(-\frac{\partial\beta}{\partial\delta}\left(\bar{\sigma}^2 + \delta\right) + \beta + 2\beta^2\frac{\partial\beta}{\partial\delta}\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{53}$$

$$= -\frac{\beta\sigma_v^2\left(-\delta\bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\beta^2\delta\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\delta^2 - \bar{\sigma}^4\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{54}$$

$$= -\frac{\beta\sigma_v^2\left(-\delta\bar{\sigma}^2 - 2\bar{\sigma}^4 + \delta^2 + 2\delta\left(\frac{\sqrt[4]{\bar{\sigma}^4 - \delta^2}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + 2\sigma_v^2}}\right)^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)}{\left(\delta^2 - \bar{\sigma}^4\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} \tag{55}$$

$$= -\frac{\beta\sigma_v^2\left(2\bar{\sigma}^4 - \delta^2 + \delta\left(\bar{\sigma}^2 - \sqrt{\bar{\sigma}^4 - \delta^2}\right)\right)}{\left(\bar{\sigma}^4 - \delta^2\right)\left(\bar{\sigma}^2 + \delta + 2\beta^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\right)^2} < 0 \tag{56}$$

Finally, note that expected trading profits are given by:

$$\pi_M = \mathbb{E}\left[x_M\left(v - \tfrac{\lambda_h + \lambda_l}{2}x_M\right)\right] \tag{57}$$

$$= \mathbb{E}\left[x_M\left(\mathbb{E}\left[v|s_1, s_2\right] - \tfrac{\lambda_h + \lambda_l}{2}x_M\right)\right] \tag{58}$$

$$= \tfrac{\lambda_h + \lambda_l}{2}\mathbb{E}\left[x_M^2\right] \tag{59}$$

$$= \tfrac{\lambda_h + \lambda_l}{2}\beta^2\mathbb{E}\left[\left(s_1 + s_2\right)^2\right] \tag{60}$$

$$= \frac{\beta_M}{2}\left(\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{2}{\sigma_\varepsilon^2}}\right)\left(4\sigma_v^2 + 2\sigma_\varepsilon^2\right) \tag{61}$$

$$= \beta_M\sigma_v^2, \tag{62}$$

which implies profits are decreasing in δ. □

A.2 Proof of Proposition 2

The equilibrium is characterized by the system of equations:

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l\right)\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)} \tag{63}$$

$$\lambda_h = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 - \delta} \tag{64}$$

$$\lambda_l = \frac{2\beta\sigma_v^2}{4\beta^2\sigma_v^2 + 2\beta^2\sigma_\varepsilon^2 + \bar{\sigma}^2 + \delta}. \tag{65}$$

As before, there are four sets of solutions, but we select the set with $\lambda_h, \lambda_l > 0$ to ensure that the trader's second order condition is satisfied. This yields:

$$\beta = \frac{1}{2}\sqrt{\frac{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}. \tag{66}$$

Let $\Gamma \equiv \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right)$. Note that

$$\Gamma > \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\left(\bar{\sigma}^4\right)\left(\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4\right) = \bar{\sigma}^4\sigma_v^4 > 0, \tag{67}$$

and

$$\frac{\partial\Gamma}{\partial\delta} = -8\delta\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) < 0, \tag{68}$$

and $\beta = \frac{1}{2}\sqrt{\frac{\sqrt{\Gamma} + \bar{\sigma}^2\sigma_v^2}{\sigma_\varepsilon^4 + 3\sigma_\varepsilon^2\sigma_v^2 + 2\sigma_v^4}}$, which implies

$$\frac{\partial\beta}{\partial\delta} = -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta} < 0. \tag{69}$$

This, together with

$$\beta = \frac{2\sigma_v^2}{\left(\lambda_h + \lambda_l\right)\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)} \tag{70}$$

$$= \frac{2}{\lambda_h + \lambda_l} \times \frac{\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{71}$$

implies that

$$\frac{\partial}{\partial\delta}\frac{\lambda_h + \lambda_l}{2} > 0. \tag{72}$$

Moreover, note that

$$\lim_{\delta\to 0}\beta = \frac{\bar{\sigma}}{\sqrt{2}\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}} \equiv \bar{\beta} \tag{73}$$

and

$$\lim_{\delta\to\bar{\sigma}^2}\beta = \frac{\bar{\sigma}\sigma_v}{\sqrt{2}\sqrt{\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} \equiv \underline{\beta}. \tag{74}$$

Next, note that

$$\lambda_l^{-1} = \frac{\bar{\sigma}^2 + \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0 \tag{75}$$

$$\lambda_h^{-1} = \frac{\bar{\sigma}^2 - \delta}{2\beta\sigma_v^2} + \beta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) > 0. \tag{76}$$

This implies

$$\frac{\partial\lambda_l^{-1}}{\partial\delta} = \left(2 - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\frac{\partial\beta}{\partial\delta} + \frac{1}{2\beta\sigma_v^2} \tag{77}$$

$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta}\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2}\right) + \frac{1}{2\beta\sigma_v^2} \tag{78}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(-\delta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2}\right) + \frac{\sqrt{\Gamma}}{\sigma_v^2}\right) \tag{79}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta}\left(-\delta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2} - \frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2}\right) + \frac{\sqrt{\bar{\sigma}^4\sigma_v^4}}{\sigma_v^2}\right) \tag{80}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2 + \delta\frac{\bar{\sigma}^2 + \delta}{2\beta^2\sigma_v^2} - \delta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\right) \tag{81}$$

$$> \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2 + \delta\frac{\bar{\sigma}^2 + \delta}{2\bar{\beta}^2\sigma_v^2} - \delta\left(2 + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\right) \tag{82}$$

$$= \frac{1}{2\sqrt{\Gamma}\beta}\left(\bar{\sigma}^2 + \frac{\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\bar{\sigma}^2\sigma_v^2} - \delta\right) > 0, \tag{83}$$

which implies $\frac{\partial \lambda_l}{\partial \delta} < 0$.

Similarly, note that:

$$\frac{\partial \lambda_h^{-1}}{\partial \delta} = \left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right)\frac{\partial \beta}{\partial \delta} - \frac{1}{2\beta\sigma_v^2} \tag{84}$$

$$= -\frac{\delta}{2\sqrt{\Gamma}} \times \frac{1}{\beta}\left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) - \frac{1}{2\beta\sigma_v^2} \tag{85}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta}\left(\delta\left(2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2\sigma_v^2} + \frac{\sigma_\varepsilon^2}{\sigma_v^2}\right) + \frac{\sqrt{\Gamma}}{\sigma_v^2}\right) \tag{86}$$

$$= -\frac{1}{2\sqrt{\Gamma}\beta\sigma_v^2}\left(\delta\left(2\sigma_v^2 + \sigma_\varepsilon^2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2}\right) + \sqrt{\Gamma}\right). \tag{87}$$

Let

$$f(\delta) \equiv \delta\left(2\sigma_v^2 + \sigma_\varepsilon^2 - \frac{\bar{\sigma}^2 - \delta}{2\beta^2}\right) + \sqrt{\Gamma} \tag{88}$$

$$= \frac{\sqrt{\Gamma}\left(\sigma_v^2\left(\bar{\sigma}^2 + 2\delta\right) + \delta\sigma_\varepsilon^2\right) + \delta\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\left(2\delta\left(\sigma_\varepsilon^2 + \sigma_v^2\right) - \bar{\sigma}^2\left(2\sigma_\varepsilon^2 + \sigma_v^2\right)\right) + \Gamma}{\bar{\sigma}^2\sigma_v^2 + \sqrt{\Gamma}}. \tag{89}$$

Note that $f(\delta) > 0$ if and only if

$$\sqrt{\Gamma}\left(\sigma_v^2\left(\bar{\sigma}^2 + 2\delta\right) + \delta\sigma_\varepsilon^2\right) > -\delta\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\left(2\delta\left(\sigma_\varepsilon^2 + \sigma_v^2\right) - \bar{\sigma}^2\left(2\sigma_\varepsilon^2 + \sigma_v^2\right)\right) - \Gamma \tag{90}$$

$$= \delta\bar{\sigma}^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\left(2\sigma_\varepsilon^2 + \sigma_v^2\right) + 2\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) - \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 \tag{91}$$

Next, note that RHS of (91) is given by

$$\delta\bar{\sigma}^2\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\left(2\sigma_\varepsilon^2 + \sigma_v^2\right) + 2\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) - \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 \tag{92}$$

$$< \bar{\sigma}^4\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)\left(2\sigma_\varepsilon^2 + \sigma_v^2\right) + 2\bar{\sigma}^4\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) - \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 \tag{93}$$

$$= -\bar{\sigma}^4\sigma_v^2\left(\sigma_\varepsilon^2 + 3\sigma_v^2\right) < 0 \tag{94}$$

and so $f(\delta) > 0$ always. This implies $\frac{\partial\lambda_h}{\partial\delta} > 0$.

Finally, note that expected profits for a single trader are given by:

$$\pi_{C,i} = \mathbb{E}\left[x_i\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_j\right)\mathbb{E}[v|s_i] - \frac{\lambda_h + \lambda_l}{2}x_i^2\right] \tag{95}$$

$$= \mathbb{E}\left[\beta_C\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_C\right)\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}}s_i^2 - \frac{\lambda_h + \lambda_l}{2}\beta_C^2 s_i^2\right] \tag{96}$$

$$= \beta_C\left[\left(1 - \frac{\lambda_h + \lambda_l}{2}\beta_C\right)\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}} - \frac{\lambda_h + \lambda_l}{2}\beta_C\right]\mathbb{E}\left[s_i^2\right] \tag{97}$$

$$= \beta_C\left[(\lambda_h + \lambda_l)\beta_C - \frac{\lambda_h + \lambda_l}{2}\beta_C\right]\mathbb{E}\left[s_i^2\right] \tag{98}$$

$$= \frac{\beta_C}{2}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}\mathbb{E}\left[s_i^2\right] \tag{99}$$

$$= \frac{\beta_C}{2}\frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}\left(\sigma_v^2 + \sigma_\varepsilon^2\right) \tag{100}$$

$$= \beta_C\sigma_v^2\tfrac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}, \tag{101}$$

which completes the proof. □

The following Lemma is useful for proving Proposition 3.

Lemma 2. *The trading intensity under coordination is lower than the trading intensity under competition, i.e.,* $\beta_C > \beta_M$*. Moreover,* $\lim_{\delta\to 0}\beta_M/\beta_C = \sqrt{\frac{\sigma_\varepsilon^2+\sigma_v^2}{\sigma_\varepsilon^2+2\sigma_v^2}}$*,* $\lim_{\delta\to\bar{\sigma}^2}\beta_M/\beta_C = 0$*, and* β_M/β_C *is decreasing in* δ*.*

Proof. Let $B \equiv \frac{\beta_M}{\beta_C}$. Then, given the above expressions

$$B^2 = \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)} + \bar{\sigma}^2\sigma_v^2}. \tag{102}$$

Now,

$$B^2 \leq 1 \tag{103}$$

$$\Leftrightarrow \sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)} + \bar{\sigma}^2\sigma_v^2 \geq 2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right) \tag{104}$$

$$\Leftrightarrow \sqrt{\bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)} \geq 2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right) - \bar{\sigma}^2\sigma_v^2 \tag{105}$$

$$\Leftrightarrow \bar{\sigma}^4\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right) \geq \begin{array}{c} 4\left(\bar{\sigma}^4 - \delta^2\right)\left(\sigma_\varepsilon^2 + \sigma_v^2\right)^2 + \bar{\sigma}^4\sigma_v^4 \\ -4\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\bar{\sigma}^2\sigma_v^2 \end{array} \tag{106}$$

$$4\sigma_v^2\left(\bar{\sigma}^4 - \delta^2\right)\left(\sigma_\varepsilon^2 + \sigma_v^2\right) \geq -4\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\bar{\sigma}^2\sigma_v^2, \tag{107}$$

which always holds since $\bar{\sigma}^2 > \delta$. This implies that $\beta_C > \beta_M$. Moreover,

$$\lim_{\delta \to 0} B^2 = \frac{1}{2 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}} = \frac{\sigma_\varepsilon^2 + \sigma_v^2}{\sigma_\varepsilon^2 + 2\sigma_v^2}, \tag{108}$$

$$\lim_{\delta \to \bar{\sigma}^2} B^2 = \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)} + \bar{\sigma}^2 \sigma_v^2} \tag{109}$$

$$= \lim_{\delta \to \bar{\sigma}^2} \frac{2\sqrt{\bar{\sigma}^4 - \delta^2}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)}{\sqrt{\bar{\sigma}^4 \sigma_v^4} + \bar{\sigma}^2 \sigma_v^2} = 0, \tag{110}$$

and

$$\frac{\partial B^2}{\partial \delta} = -\frac{\delta \bar{\sigma}^2 \sigma_v^2}{\left(\bar{\sigma}^4 - \delta^2\right)\sqrt{\bar{\sigma}^4 \left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2 - 4\delta^2 \left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} B^2 < 0, \tag{111}$$

which completes the proof. □

A.3 Proof of Proposition 3

Recall that in the monopolist case, we can express profits as $\pi_M = \beta_M \sigma_v^2$ and in the competitive case we have $\pi_C = \beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}$. This implies

$$\rho \equiv \frac{\pi_M}{2\pi_C} = \frac{\beta_M \sigma_v^2}{2\beta_C \sigma_v^2 \frac{\sigma_v^2 + \sigma_\varepsilon^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}} = B \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2}. \tag{112}$$

Lemma 2 implies

$$\lim_{\delta \to 0} \rho = \sqrt{\frac{\left(2\sigma_\varepsilon^2 + 3\sigma_v^2\right)^2}{4\left(\sigma_\varepsilon^2 + \sigma_v^2\right)\left(\sigma_\varepsilon^2 + 2\sigma_v^2\right)}} > 1, \tag{113}$$

and

$$\lim_{\delta \to \bar{\sigma}^2} \rho = 0, \tag{114}$$

and

$$\frac{\partial \rho}{\partial \delta} = \frac{\partial B}{\partial \delta} \frac{2\sigma_\varepsilon^2 + 3\sigma_v^2}{2\sigma_v^2 + 2\sigma_\varepsilon^2} < 0. \tag{115}$$

This implies there exists a $\underline{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\underline{\delta}\right) = 1$, and another $\bar{\delta} \in \left(0, \bar{\sigma}^2\right)$ such that $\rho\left(\bar{\delta}\right) = \frac{1}{2}$. Additionally,

$$\frac{\partial \rho}{\partial \delta} < 0 \iff 2\pi_C \frac{\partial \pi_M}{\partial \delta} - \pi_M \frac{\partial 2\pi_C}{\partial \delta} < 0 \iff \frac{\partial \pi_M}{\partial \delta}\left(\frac{\partial 2\pi_C}{\partial \delta}\right)^{-1} > \frac{\pi_M}{2\pi_C}, \tag{116}$$

which implies that if $\pi_M > 2\pi_C$ (i.e., if $\delta \in (0, \underline{\delta})$) then $-\frac{\partial \pi_M}{\partial \delta} > -\frac{\partial 2\pi_C}{\partial \delta}$. □

A.4 Proofs of Observations 1 - 3

Proof of Observation 1

For a given σ_z^2, the market maker's best response function is single-peaked in β since

$$\lambda_\beta(\beta) = \frac{2\sigma_v^2\left(\sigma_z^2 - 2\beta^2\left(2\sigma_v^2+\sigma_\varepsilon^2\right)\right)}{\left(2\beta^2\left(2\sigma_v^2+\sigma_\varepsilon^2\right)+\sigma_z^2\right)^2},$$

which is exactly zero for $\beta = \frac{\sigma_z}{\sqrt{2}\sqrt{2\sigma_v^2+\sigma_\varepsilon^2}} \equiv \beta^*$, positive for $\beta < \beta^*$ and negative for $\beta > \beta^*$. Further, aggregate trader profits,

$$\pi(\beta;\lambda) = 2\beta\sigma_v^2 - \beta^2\lambda\left(4\sigma_v^2+2\sigma_\varepsilon^2\right),$$

are maximized by the monopolistic traders to be $\beta_M = \frac{\sigma_z}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}}$, which coincides with the peak price impact, $\beta_M = \beta^*$. Additionally, profits are also single-peaked in β :

$$\pi_\beta(\beta) = \frac{2\sigma_v^2\sigma_z^2\left(\sigma_z^2 - 2\beta^2\left(2\sigma_v^2+\sigma_\varepsilon^2\right)\right)}{\left(2\beta^2\left(2\sigma_v^2+\sigma_\varepsilon^2\right)+\sigma_z^2\right)^2}.$$

Finally, note that the competitive trading intensity is too aggressive relative to β_M, since

$$\beta_C = \frac{\sigma_z}{\sqrt{2\sigma_v^2+2\sigma_\varepsilon^2}} > \frac{\sigma_z}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}} = \beta_M = \beta^*.$$

□

Proof of Observation 2

First, recall from Lemma 2, that $\beta_C > \beta_M$, i.e., the equilibrium competitive trading intensity is always greater for a given level of uncertainty δ. Next, let's consider how the trading intensity in the monopolistic equilibrium compares to the optimal trading intensity in the low-liquidity regime. From Proposition 1 and Observation 1, when $\sigma_z^2 = \bar{\sigma}^2 - \delta$ we have

$$\beta_M^2 - \beta_h^{*2} = \frac{\sqrt{\bar{\sigma}^4-\delta^2}-\bar{\sigma}^2+\delta}{4\sigma_v^2+2\sigma_\varepsilon^2} \geq 0 \iff \bar{\sigma}^4-\delta^2 \geq (\bar{\sigma}^2-\delta)^2 \iff 2\delta(\bar{\sigma}^2-\delta) \geq 0,$$

which implies that the monopolistic traders are too aggressive in the low-liquidity regime, and, since $\beta_M < \beta_C$, so are the competitive traders. As $\delta \to \bar{\sigma}^2$, the monopolistic and optimal trading intensities converge to zero.

Now let's compare the monopolistic trading intensity to the optimal trading intensity in the high-liquidity state, i.e., when $\sigma_z^2 = \bar{\sigma}^2 + \delta$,

$$\beta_l^{*2} - \beta_M^2 = \frac{\bar{\sigma}^2+\delta-\sqrt{\bar{\sigma}^4-\delta^2}}{4\sigma_v^2+2\sigma_\varepsilon^2} \geq 0 \iff (\bar{\sigma}^2+\delta)^2 \geq \bar{\sigma}^4-\delta^2 \iff 2\delta(\bar{\sigma}^2+\delta) \geq 0,$$

implying that the monopolistic trading intensity is always less than the optimal trading intensity in the high-liquidity state unless there is no uncertainty. Taken together, the monopolistic trading intensity is too aggressive in the low-liquidity regime and too conservative in the high-liquidity regime.

The competitive trading intensity is more aggressive than the optimal trading intensity in the low-liquidity state. Relative to the optimal trading intensity in the high-liquidity state, from Proposition 2 and Observation 1, the following holds:

$$\beta_C^2 - \beta_l^{*2} = \frac{-2\sigma_\varepsilon^2\left(\bar{\sigma}^2+\delta\right) + \sqrt{\bar{\sigma}^4\left(3\sigma_v^2+2\sigma_\varepsilon^2\right)^2 - 4\delta^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(2\sigma_v^2+\sigma_\varepsilon^2\right)} - \left(\sigma_v^2\left(\bar{\sigma}^2+2\delta\right)\right)}{4\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(2\sigma_v^2+\sigma_\varepsilon^2\right)},$$

which is decreasing in δ and has a single zero for $\delta \in [0, \bar{\sigma}^2]$ at $\delta^* = \frac{2\bar{\sigma}^2\sigma_v^2}{2\sigma_\varepsilon^2+3\sigma_v^2}$. This implies that for sufficiently low uncertainty, $0 \leq \delta < \delta^*$, the competitive trading intensity is too aggressive in both liquidity regimes. As uncertainty increases, when $\delta = \delta^*$, the competitive trading intensity is optimal in the high-liquidity regime ($\beta_C = \beta_l^*$) and too aggressive in the low-liquidity regime ($\beta_C > \beta_h^*$). Finally, when $\delta > \delta^*$, the competitive trading intensity is too aggressive in the low-liquidity regime and too conservative in the high-liquidity regime.

Taken together, when $\delta > \delta^*$ both the competitive and monopolistic trading intensities are too aggressive in the low-liquidity state, too conservative in the high-liquidity state, and the competitive trading intensity is greater than the monopolistic trading intensity. □

Proof of Observation 3

From Observation 1, it is immediate that

$$\beta_h^* = \frac{\sqrt{\bar{\sigma}^2-\delta}}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}} \text{ and } \beta_l^* = \frac{\sqrt{\bar{\sigma}^2+\delta}}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}},$$

which implies that maximum attainable profits in the high and low price impact states are

$$\pi_h^* = \frac{\sigma_v^2\sqrt{\bar{\sigma}^2-\delta}}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}} \text{ and } \pi_l^* = \frac{\sigma_v^2\sqrt{\bar{\sigma}^2+\delta}}{\sqrt{4\sigma_v^2+2\sigma_\varepsilon^2}}.$$

Therefore, as uncertainty increases, the profit opportunity in the high (low) liquidity state increases (decreases), and as uncertainty approaches its maximum value ($\delta \to \bar{\sigma}^2$), the profit opportunity in the low-liquidity state approaches zero.

Since both the monopolistic and competitive equilibria yield profits that are lower than π_h^* in the low-liquidity state, the ranking of overall expected profits across the two equilibria are determined by the corresponding profits in the high-liquidity state when δ is sufficiently large.

□

A.5 Proof of Corollary 1

The proof of Proposition 3 establishes that the difference between the expected profits $\Pi(\delta)$ in the two cases is maximized for $\delta = 0$ and is decreasing in δ. This leads to (i) and $\hat{\delta}(c)$ decreasing in c.

Next, consider the case $\sigma_\varepsilon = 0$. Then,

$$\Pi(\delta) = \frac{1}{2}\sigma_v \sqrt[4]{\bar{\sigma}^4 - \delta^2} - \sqrt{\frac{\sqrt{9\,\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}{2}}\sigma_v \frac{1}{3} = c.$$

Note that $\frac{\partial \Pi}{\partial \sigma_v} > 0$ for $\delta < \hat{\delta}$, and therefore $\frac{\partial \hat{\delta}}{\partial \sigma_v} = -\frac{\frac{\partial \Pi}{\partial \sigma_v}}{\frac{\partial \Pi}{\partial \hat{\delta}}} > 0$.

Similarly, note that $\frac{\partial \Pi}{\partial \bar{\sigma}} = \sigma_v \bar{\sigma} \left\{ \frac{1}{2} \frac{\bar{\sigma}^2}{\sqrt[4]{\bar{\sigma}^4 - \delta^2}^3} - \frac{1}{3}\frac{1}{2Q}\left(\frac{9\bar{\sigma}^2}{\sqrt{9\,\bar{\sigma}^4 - 8\delta^2}} + 1 \right) \right\}$ where $Q \equiv \sqrt{\frac{\sqrt{9\,\bar{\sigma}^4 - 8\delta^2} + \bar{\sigma}^2}{2}}$. We can show that $\frac{\partial \Pi}{\partial \bar{\sigma}} > 0$. First, denote $r \equiv \sqrt[4]{\bar{\sigma}^4 - \delta^2}$. Then, notice that $\sqrt{9\,\bar{\sigma}^4 - 8\delta^2} > \sqrt{9r^4} = 3r^2$. Then, note that

$$\frac{1}{2} \frac{\bar{\sigma}^2}{\sqrt[4]{\bar{\sigma}^4 - \delta^2}^3} - \frac{1}{3}\frac{1}{2Q}\left(\frac{9\bar{\sigma}^2}{\sqrt{9\,\bar{\sigma}^4 - 8\delta^2}} + 1 \right) > 0 \Leftrightarrow \frac{\bar{\sigma}^2}{r^3}Q - \frac{1}{3}\left(\frac{9\bar{\sigma}^2}{3r^2} + 1 \right) > 0.$$

Finally, note that $Q > \sqrt{\frac{3}{2}\,r^2 + \bar{\sigma}^2} > \sqrt{2}\,r$, and then

$$\frac{\bar{\sigma}^2}{r^3}Q - \frac{1}{3}\left(\frac{3\bar{\sigma}^2}{r^2} + 1 \right) > \sqrt{2}\,\frac{\bar{\sigma}^2}{r^2} - \frac{\bar{\sigma}^2}{r^2} - \frac{1}{3} > \sqrt{2} - 1 - \frac{1}{3} > 0 \Rightarrow \quad \frac{\partial \Pi}{\partial \bar{\sigma}} > 0.$$

Then, the following holds: $\frac{d\hat{\delta}}{d\bar{\sigma}} = -\frac{\frac{\partial \Pi}{\partial \bar{\sigma}}}{\frac{\partial \Pi}{\partial \hat{\delta}}} > 0$. □

A.6 Proof of Proposition 4

(i) Note that under the monopolistic equilibrium, $(\lambda_l + \lambda_h)\,\beta_M = \frac{\sigma_v^2}{\sigma_\varepsilon^2 + 2\sigma_v^2}$, and under the competitive equilibrium, $(\lambda_h + \lambda_l)\,\beta_C = \frac{2\sigma_v^2}{2\sigma_\varepsilon^2 + 3\sigma_v^2}$, which yields expressions for V_M and V_C, respectively.
(ii) By Corollary 1, investors compete if and only if $\delta > \hat{\delta}$, and thus $V(P) = V_C > V_M$ if and only if $\delta > \hat{\delta}$. □

A.7 Proof of Proposition 5

(i) Note that conditional on σ_z, the aggregate informed trading, $\sum_i x_i$, noise trading, z, and aggregate order flow, y, are normally distributed with mean zero.

Next, note that for $\mathcal{Z} \sim N(0, \sigma^2)$, it holds that $\mathbb{E}[|\mathcal{Z}|] = 2\mathbb{E}[\mathcal{Z}|\mathcal{Z} > 0] = 2\left(0 + \sigma\frac{\bar{p}(0)}{1-\Phi(0)}\right) = \frac{2\sqrt{2}}{\sqrt{\pi}}\sigma$, where $\bar{p}()$ and $\Phi()$ are the density and CDF of a standard normal random variable.

Next, the expected trading volume of informed investors is

$$\mathcal{V}_I = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\sigma_x = \tfrac{2\sqrt{2}}{\sqrt{\pi}}\beta\left(4\sigma_v^2 + 2\sigma_\varepsilon^2\right).$$

From Propositions 1 and 2, informed investors trade less aggressively when uncertainty is higher, i.e., $\frac{\partial \beta}{\partial \delta} < 0$. Thus, $\frac{\partial \mathcal{V}_I}{\partial \delta} < 0$.

Analogously, the expected trading volume of noise traders, given by

$$\mathcal{V}_N = \tfrac{2\sqrt{2}}{\sqrt{\pi}} \left(\tfrac{1}{2}\sqrt{\bar{\sigma}^2 + \delta} + \tfrac{1}{2}\sqrt{\bar{\sigma}^2 - \delta} \right),$$

and the expected aggregate trading volume,

$$\mathcal{V}_M = \tfrac{2\sqrt{2}}{\sqrt{\pi}} \left(\tfrac{1}{2}\sqrt{\bar{\sigma}^2 + \delta + \sigma_x^2} + \tfrac{1}{2}\sqrt{\bar{\sigma}^2 - \delta + \sigma_x^2} \right),$$

both decrease with δ by Jensen's inequality.

Therefore, $\mathcal{V} = \mathcal{V}_I + \mathcal{V}_N + \mathcal{V}_M$ is decreasing in δ.

(ii) This follows immediately from $\beta_M < \beta_C$. □

A.8 Proof of Lemma 1

In either equilibrium, $P = \lambda (x + z)$ where $x = \beta (s_1 + s_2)$ and $s_i = v + \varepsilon_i$. This implies:

$$\lambda \left(\sigma_z^2\right) = \frac{\mathbb{C}\left(v, x + z | \sigma_z^2\right)}{\mathbb{V}\left(x + z | \sigma_z^2\right)} = \frac{2\beta \sigma_v^2}{4\beta^2 \sigma_v^2 + 2\beta^2 \sigma_\varepsilon^2 + \sigma_z^2} \tag{117}$$

$$\mathbb{V}\left[v | P, \sigma_z^2\right] = \sigma_v^2 - \frac{\mathbb{C}\left(v, x + z | \sigma_z^2\right)^2}{\mathbb{V}\left(x + z | \sigma_z^2\right)} \tag{118}$$

$$= \sigma_v^2 \left(1 - \frac{4\beta^2 \sigma_v^2}{4\beta^2 \sigma_v^2 + 2\beta^2 \sigma_\varepsilon^2 + \sigma_z^2}\right) \tag{119}$$

$$= \sigma_v^2 \left(1 - 2\beta\lambda\right). \tag{120}$$

Moreover, this implies that conditional on σ_z^2, P is normally distributed with mean zero and variance:

$$\mathbb{V}\left[P | \sigma_z^2\right] = \lambda^2 \left(4\beta^2 \sigma_v^2 + 2\beta^2 \sigma_\varepsilon^2 + \sigma_z^2\right) = 2\beta\lambda\sigma_v^2. \tag{121}$$

Note that

$$\bar{p}\left(P\right) \equiv \Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta | P\right) \tag{122}$$

$$= \frac{\Pr\left(\sigma_z^2 = \bar{\sigma}^2 - \delta, P = p\right)}{\Pr\left(P = p\right)} \tag{123}$$

$$= \frac{\Pr(P = p | \sigma_z^2 = \bar{\sigma}^2 - \delta) \Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta)}{\Pr(P = p | \sigma_z^2 = \bar{\sigma}^2 - \delta) \Pr(\sigma_z^2 = \bar{\sigma}^2 - \delta) + \Pr(P = p | \sigma_z^2 = \bar{\sigma}^2 + \delta) \Pr(\sigma_z^2 = \bar{\sigma}^2 + \delta)} \tag{124}$$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h \sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h \sigma_v^2}}\right)}{\frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h \sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h \sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l \sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l \sigma_v^2}}\right)} \tag{125}$$

$$= \frac{1}{1+\sqrt{\frac{\lambda_h}{\lambda_l}}e^{-\frac{P^2}{2\sigma_v^2}\frac{\lambda_h-\lambda_l}{2\beta\lambda_h\lambda_l}}}, \tag{126}$$

where $f(\cdot)$ is the pdf of the standard normal distribution. The result follows from noting that:

$$\mathbb{V}[v|P] = \mathbb{E}\left[\mathbb{V}\left[v|P,\sigma_z^2\right]|P\right] + \mathbb{V}\left[\mathbb{E}\left[v|P,\sigma_z^2\right]|P\right] \tag{127}$$

$$= \mathbb{E}\left[\mathbb{V}\left[v|P,\sigma_z^2\right]|P\right] + \mathbb{V}[P|P] \tag{128}$$

$$= \bar{p}(P) \times \sigma_v^2(1-2\beta\lambda_h) + (1-\bar{p}(P)) \times \sigma_v^2(1-2\beta\lambda_l). \tag{129}$$

□

A.9 Proof of Proposition 6

Note that

$$\mathbb{E}[\mathbb{V}[v|P]] = \int_{-\infty}^{\infty} \left(\sigma_v^2(1-2\beta\lambda_l) + \bar{p}(P) \times 2\sigma_v^2\beta(\lambda_l-\lambda_h)\right) F(P)\, dP \tag{130}$$

where

$$F(P) = \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) + \frac{1}{2} \times \frac{1}{\sqrt{2\beta\lambda_l\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_l\sigma_v^2}}\right) \tag{131}$$

is the unconditional distribution of P. This implies:

$$\mathbb{E}[\mathbb{V}[v|P]] = \sigma_v^2(1-2\beta\lambda_l) + \int_{-\infty}^{\infty} \sigma_v^2\beta(\lambda_l-\lambda_h) \times \frac{1}{\sqrt{2\beta\lambda_h\sigma_v^2}} f\left(\frac{P}{\sqrt{2\beta\lambda_h\sigma_v^2}}\right) dP \tag{132}$$

$$= \sigma_v^2(1-2\beta\lambda_l) + \sigma_v^2\beta(\lambda_l-\lambda_h) \tag{133}$$

$$= \sigma_v^2(1-\beta(\lambda_l+\lambda_h)). \tag{134}$$

The result follows from plugging in the expressions for $\{\beta, \lambda_h, \lambda_l\}$ for each equilibrium and simplifying. □

A.10 Proof of Proposition 7

By the Leibniz integral rule (and the dominated convergence theorem, more generally),

$$\frac{\partial}{\partial\beta_N}\mathbb{E}\left[\frac{N^2\beta_N^2\sigma_v^2}{N^2\beta_N^2\sigma_v^2+\sigma_z^2}\right] = \mathbb{E}\left[\frac{\partial}{\partial\beta_N}\left(\frac{N^2\beta_N^2\sigma_v^2}{N^2\beta_N^2\sigma_v^2+\sigma_z^2}\right)\right] > 0$$

Then, for a fixed N, since this expectation is zero for $\beta_N = 0$ and it converges to 1 for $\beta_N \to +\infty$, the following equation has a unique root:

$$\mathbb{E}\left[\frac{N^2\beta_N^2\sigma_v^2}{N^2\beta_N^2\sigma_v^2+\sigma_z^2}\right] = \frac{N}{N+1}.$$

This establishes the existence and uniqueness of the equilibrium, since we pick the positive value of β_N which satisfies the above. $\square$

A.11 Proof of Proposition 8

Suppose $\sigma_z^2 \in \left\{\bar{\sigma}^2 - \delta, \bar{\sigma}^2 + \delta\right\}$ with probability $\{p, 1-p\}$. Denote $b_N \equiv N^2 \beta_N^2 \sigma_v^2$

$$\mathbb{E}\left[\frac{b_N}{b_N + \sigma_z^2}\right] = p\frac{b_N}{b_N + \bar{\sigma}^2 - \delta} + (1-p)\frac{b_N}{b_N + \bar{\sigma}^2 + \delta} = \frac{N}{N+1} \tag{135}$$

which implies

$$b_N = \frac{1}{2}\left((N-1)\bar{\sigma}^2 + \sqrt{4N\left(\bar{\sigma}^4 - \delta^2\right) + \left(\bar{\sigma}^2 - N\bar{\sigma}^2 + \delta(N+1)(2p-1)\right)^2} - (\delta(N+1)(2p-1))\right) \tag{136}$$

and

$$b_1 = \sqrt{\bar{\sigma}^4 + 4\delta^2(p-1)p} + \delta\left(1 - 2p\right) \tag{137}$$

The sufficient condition for the competitive equilibrium to yield higher total profits is:

$$\frac{b_N}{b_1} = \frac{(N-1)\bar{\sigma}^2 + \sqrt{4N(\bar{\sigma}^4-\delta^2) + (\bar{\sigma}^2 - N\bar{\sigma}^2 + \delta(N+1)(2p-1))^2} - \delta(N+1)(2p-1)}{2\left(\sqrt{\bar{\sigma}^4 + 4\delta^2(p-1)p} + \delta(1-2p)\right)} > \left(\frac{N+1}{2}\right)^2. \tag{138}$$

After isolating square-root terms and squaring (on the relevant region), the condition reduces to a quadratic inequality in p, which yields:

$$1 < N < \bar{N}(\delta) \quad \text{or} \quad \bar{\delta}(N) < \delta \leq \bar{\sigma}^2 \quad \text{and } \underline{p} < p < \bar{p}, \quad \text{where} \tag{139}$$

$$\bar{N}(\delta) \equiv \frac{2\delta^2 - \bar{\sigma}^4}{\bar{\sigma}^4 - \delta^2} + \sqrt{\frac{-8\delta^2\bar{\sigma}^4 + 4\bar{\sigma}^8 + 5\delta^4}{\left(\delta^2 - \bar{\sigma}^4\right)^2}} \tag{140}$$

$$\bar{\delta}(N) \equiv \sqrt{\frac{(N-1)(N+3)}{N(N+4)-1}}\bar{\sigma}^2 \tag{141}$$

$$\underline{p}(\delta, N) \equiv \frac{1}{8}\left(\frac{(1 - N(N(N+5)+11))\sqrt{2\delta^2(N(N+4)-1) - 2(N-1)(N+3)\bar{\sigma}^4} + 2(N+3)\sqrt{N+1}(N-1)^2\bar{\sigma}^2}{\delta(N+1)^{3/2}(N(N+4)-1)} + 4\right) \tag{142}$$

$$\bar{p}(\delta, N) \equiv \frac{1}{8}\left(\frac{(N(N(N+5)+11) - 1)\sqrt{2\delta^2(N(N+4)-1) - 2(N-1)(N+3)\bar{\sigma}^4} + 2(N+3)\sqrt{N+1}(N-1)^2\bar{\sigma}^2}{\delta(N+1)^{3/2}(N(N+4)-1)} + 4\right) \tag{143}$$

Note that

$$\frac{\partial \bar{N}(\delta)}{\partial \delta} = \frac{2\delta\bar{\sigma}^4\left(\frac{\delta^2}{\sqrt{-8\delta^2\bar{\sigma}^4 + 4\bar{\sigma}^8 + 5\delta^4}} + 1\right)}{\left(\bar{\sigma}^4 - \delta^2\right)^2} > 0 \tag{144}$$

$$\lim_{\delta \to \bar{\sigma}^2} \bar{N}(\delta) = \infty \tag{145}$$

Also note that $\bar{\delta}(N+1) - \bar{\delta}(N) > 0$ for $\forall N > 1$. Indeed, we can represent $\bar{\delta}(N) = \bar{\sigma}^2 \sqrt{f(N)}$ and show that

$$f(N+1) - f(N) = \frac{(N+1)^2+2(N+1)-3}{(N+1)^2+4(N+1)-1} - \frac{N^2+2N-3}{N^2+4N-1} = \frac{2N^2+6N+12}{((N+1)^2+4(N+1)-1)(N^2+4N-1)} > 0$$

$$f(N) < 1 \qquad \text{for} \quad \forall N$$

This concludes the proofs for both part (i) and part (ii) $\square$

A.12 Proof of Proposition 9

Again, use the notation $b_N \equiv N^2 \beta_N^2 \sigma_v^2$ to rewrite (31):

$$\mathbb{E}\left[\frac{b_N}{b_N+\sigma_z^2}\right] = \frac{N}{N+1}$$

Then,

$$1 - \mathbb{E}\Big[\frac{b_N}{b_N+\sigma_z^2}\Big] = \frac{1}{N+1}$$

By Markov's inequality,

$$\mathbb{P}(\sigma_z^2 \geq \epsilon)\, \frac{\epsilon}{b_N+\epsilon} \leq \mathbb{E}\left[\frac{\sigma_z^2}{b_N+\sigma_z^2}\right] = \frac{1}{N+1}$$

Denote $\mathbb{P}(\eta < \epsilon) \equiv p$. Then, we can write

$$b_N + \epsilon \geq (N+1)(1-p)\epsilon \quad \Leftrightarrow \quad b_N \geq (N-(N+1)p)\,\epsilon$$

Next, we can find an upper bound for b_1. Notice that $\mathbb{E}\big[\frac{b_1}{b_1+\sigma_z^2}\big] = \frac{1}{2}$. Also, note that

$$\mathbb{E}\left[\frac{b_1}{b_1+\sigma_z^2}\right] \geq \mathbb{E}\left[\frac{b_1}{b_1+\sigma_z^2}\mathbf{1}\{\sigma_z^2 \leq \epsilon\}\right] = \mathbb{E}\left[\frac{b_1}{b_1+\sigma_z^2}|\sigma_z^2 \leq \epsilon\right] p \geq \frac{b_1}{b_1+\mu_\epsilon}\, p, \quad \mu_\epsilon \equiv \mathbb{E}[\sigma_z^2 \,|\, \sigma_z^2 \leq \epsilon],$$

where the tail expectation $\mu_\epsilon \leq a\epsilon$, where $a < 1$. The smaller a is, the more concentrated near zero the distribution is. The inequality above implies

$$\frac{1}{2} \geq \frac{b_1}{b_1+\mu_\epsilon} p \quad \Leftrightarrow \quad \mu_\epsilon \geq b_1\,(2p-1) \tag{146}$$

so for $p > \frac{1}{2}$, we have $b_1 \leq \frac{\mu_\epsilon}{2p-1}$. This implies

$$\frac{b_N}{b_1} \geq \frac{(N-(N+1)p)\epsilon}{\frac{a\epsilon}{2p-1}} = \frac{1}{a}\,(N-(N+1)\,p)\,(2p-1) \tag{147}$$

We need the above to be higher than $\left(\frac{N+1}{2}\right)^2$. So the set of sufficient conditions is

$$\frac{1}{a}\,(N-(N+1)\,p)\,(2p-1) \geq \left(\frac{N+1}{2}\right)^2. \tag{148}$$

This implies that if

$$a < \frac{(N-1)^2}{2(N+1)^3} \tag{149}$$

and

$$p \in \left(\frac{3N+1-\sqrt{(N-1)^2-2a(N+1)^3}}{4(N+1)}, \frac{3N+1+\sqrt{(N-1)^2-2a(N+1)^3}}{4(N+1)} \right), \tag{150}$$

then profits under competition are higher than under coordination. $\square$

B Extension: Market makers do not observe the regime

In our main analysis, we assume that while traders face uncertainty about noise trading volatility, the market maker does not. In part, as we discuss in Section 2.1 this is because of tractability. Specifically, when the market maker faces uncertainty about noise trading volatility, he updates on this from the order flow, which would render the price a non-linear function of the order flow. In turn, with the assumption of normally distributed payoffs and signals, this would imply that the traders' strategies are no longer linear in their signals, and so the conjectured equilibrium unravels.

In this appendix, we consider a simplified version of the main model where we are able to relax the assumption that market makers observe the regime σ_z. Specifically, we assume that the risky asset's pay-off follows a binary distribution:

$$v \in \{0, 1\} \qquad \text{with } \mathbb{P}(v = 1) = \tfrac{1}{2}$$

Traders observe v perfectly and can only trade long. This implies, that their equilibrium strategy will be binary: buy x shares after observing $v = 1$ and 0 otherwise. This allows us to characterize the equilibrium strategy and price.

We further assume that aggregate trade from noise trading is $z \sim W(j, k)$, i.e., a Weibull distribution with scale j and shape k. To capture the uncertainty about the liquidity regime, we assume that the scale j is unknown:

$$j \in \{j_h, j_l\} \qquad \text{with } \mathbb{P}(j = j_h) = \omega.$$

In other words, z follows a mixture of Weibull distributions with density

$$m(z) = \omega f(z, j_h, k) + (1 - \omega) f(z, j_l, k),$$

where $f(z, j, k) = \frac{k}{j} \left(\frac{z}{j}\right)^{k-1} e^{-\left(\frac{z}{j}\right)^k} \times \mathbb{I}\{z \geq 0\}$. Recall that for a random variable with $z \sim W(j, k)$, the mean and variance are given by:

$$\mathbb{E}[z] = j\Gamma(1 + 1/k), \quad \text{and} \quad \mathbb{V}[z] = j^2 \left[\Gamma(1 + 2/k) - (\Gamma(1 + 1/k))^2\right].$$

The pricing rule is:

$$P(y) = \mathbb{E}[v|y] = \mathbb{P}(v = 1|y) = \frac{m\left(y - \sum_i x_i\right)}{m\left(y - \sum_i x_i\right) + m(y)}$$

where x_i is the conjectured trading quantity of trader i. In the monopolistic case, the trader's profit is given by

$$\pi_M(x_M) = x_M \mathbb{E}_z[1 - P(z + x_M)]$$

and the respective FOC is

$$\mathbb{E}_z\left[1 - P\left(z + x_M\right) - x_M \tfrac{\partial P(z + x_M)}{\partial x_M}\right] = 0.$$

In the competitive case, we can characterize the profit (after observing v) of each trader as

$$\pi_C(x_i) = x_i \mathbb{E}_z[1 - P(z + x_i + x_j)]$$

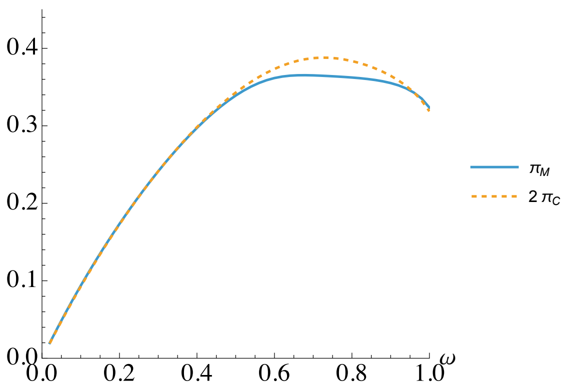
where x_j and $P(y)$ are taken as given. Then, the FOC of this optimization problem is:

$$\mathbb{E}_z\left[1 - P\left(z + 2x_C\right) - x_C \tfrac{\partial P(z + 2x_C)}{\partial x_C}\right] = 0.$$

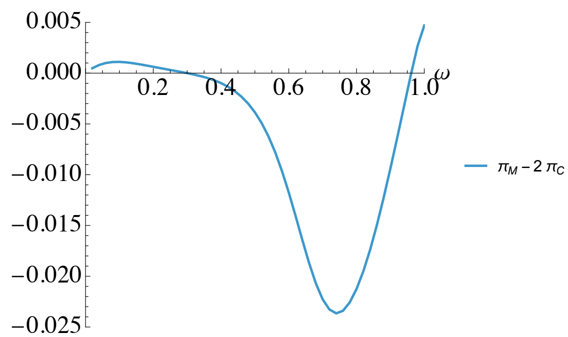
We solve for the equilibrium trading strategies and expected profits numerically for different values of ω. Note that uncertainty about noise trading volatility is high for intermediate ω, but is low when ω is close to zero or one. Figure 6 provides an illustration of this exercise. For the given set of parameters, note that expected profits under competition are higher for intermediate ω (high uncertainty about noise trading volatility), but lower for extreme ω. Moreover, as panel (c) suggests, trading in the monopolistic equilibrium is more sensitive to ω for intermediate ω. These results suggest that our main results are qualitatively robust to relaxing the assumption that the market maker is informed about noise trading volatility.

Figure 6: Two-sided uncertainty about σ_z^2

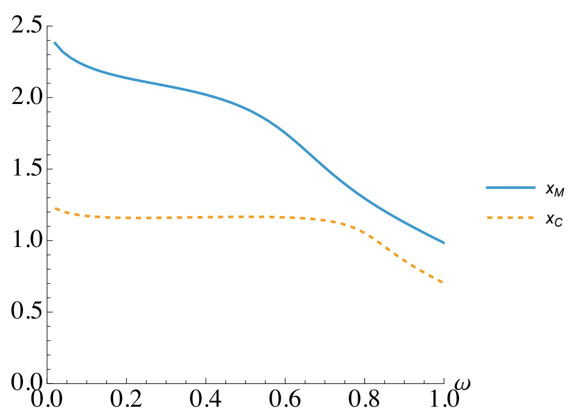
The figure plots the average total expected profits under the monopolistic (π_M) and competitive ($2\pi_C$) equilibria when neither the market maker nor the traders observe the realization of noise trading volatility σ_z^2. We assume that noise trading z follows a mixture of Weibull distributions where $k = 4$, $j_h = 2.75$ and $j_l = 0.5$, where the probability $\Pr(j = j_h) = \omega$.



(a) Expected profits vs. ω



(b) Difference in profits $\pi_M - \pi_C$ vs. ω



(c) Optimal trades vs. ω