Choosing to Disagree in Financial Markets

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How do investors interpret information?

Rational expectations implies learning is efficient

- **Assumes** subjective beliefs agree with objective distribution
- Why? Objective beliefs are *accurate,* forward looking

Overwhelming evidence that people do not behave this way!
e.g., excess predictability, volatility and volume

Behavioral literature explores role of cognitive frictions and biases (e.g., over-confidence, dismissiveness),

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Behavioral literature explores role of cognitive frictions and biases (e.g., over-confidence, dismissiveness), but is silent on **when / why** such distortions arise

Given a choice, how do investors interpret information?

We allow investors to choose how to interpret information in a standard, Hellwig (1980) setting

- Observe conditionally i.i.d. private signals and (noisy) price

- Well-being *also* depends on anticipation of future outcomes

- Investors choose precision of private / price signals ex-ante

Subjective beliefs trade off:

Desirability higher anticipatory utility

versus

Accuracy higher experienced utility

Summary of Results: Investors choose to disagree

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Return predictability, volume, volatility and welfare can be higher under chosen beliefs than under rational expectations

Related Literature

Belief Choice See survey by Benabou and Tirole (2016)

- Caplin and Leahy (2019)'s model of "wishful thinking"
- Brunnermeier and Parker (2005)'s model of "optimal expectations"

Deviations from Rational Expectations

- Overconfidence: Odean (1998); Daniel et al. (1998); Daniel, Hirshleifer, and Subrahmanyam (2001); Gervais and Odean (2001)

- Under-weighting price information:
  Difference of opinions (e.g., Banerjee, Kaniel and Kremer, 2009)
  Rational inattention (e.g., Kacperczyk et. al. 2016)
  Cursedness (e.g., Eyster, Vayanos and Rabin, 2018)
  Costly learning from prices (e.g., Vives and Yang, 2018)

What drives choice of beliefs?
(a.k.a. motivating motivated beliefs)

Choice of subjective beliefs depends on overall goal

Discounted expected utility: Goal is to maximize future, experienced (ex-post) utility

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- Accurate beliefs $\Rightarrow$ accurate decisions

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Robust control: Goal is to optimize against bad (worse-case) scenarios

- Prefer to choose *pessimistic* subjective beliefs
- But, distortion in beliefs $\Rightarrow$ distorted actions, lower ex-post utility

Trade-off: accuracy vs. robustness (down-side protection)

$$\min_{\mu} \max_{a} \mathbb{E}_{\mu}[u(a)] + C(\mu, \mu_0)$$

where $C(\mu, \mu_0)$ is **cost** of choosing beliefs $\mu \neq$ objective beliefs μ_0

Anticipatory Utility and Wishful thinking

Anticipatory Utility Well-being *also* directly depends on subjective beliefs through anticipation of future outcomes

- E.g., Dreaming about winning the lottery, excitement about an upcoming vacation, anxiety about a big presentation

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Wishful thinking and different interpretations: an example

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Figure 1: Balcetis & Dunning (2006)

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When desirable, 72% saw "B" and 61% saw "13"

Wishful thinking and motivated reasoning affects the acquisition and interpretation of information in many settings

- Oster, Shoulson & Dorsey (2013): Don't want to learn if at risk for Huntington's even if test is cheap and perfectly predictive

- Ganguly and Tasoff (2016): Pay to avoid getting tested for HSV-1 / HSV-2

- Eli & Rao (2011): People under-react to negative feedback on intelligence / beauty, but respond to good news

- Karlsson, Loewenstein & Seppi (2009): Investors monitor their portfolios more in rising markets

- Babcock and Loewenstein (1997): Randomly assigned "prosecutors" interpret the same evidence to be more consistent with defendant's guilt than assigned "defense attorneys"

- Exley and Kessler (2019): Interpret uninformative signals about ability as favorable

Moreover, expertise / cognitive ability can exacerbate the biases e.g., political bias in Kahan (2013), Kahan, Peters, Dawson & Slovic (2014)

Model Setup

Payoffs, Signals and Preferences

There are three dates $t = 0, 1, 2$ and two assets:

- Risk-free asset is normalized to numeraire
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Continuum of investors with CARA (γ) utility over terminal ($t = 2$) wealth
Normalize initial wealth to $W_0 = 0$ for presentation.

At date $t = 1$, investor i

(i) observes private signal $s_i = F + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, 1/\tau_e)$ is i.i.d.

(ii) observes equilibrium price P infers a signal $s_p = F + \beta z$

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Aggregate supply of the asset is $z \sim \mathcal{N}(0, 1/\tau_z)$, so market clearing:

$$\int_i x_i(s_i, P) di = z$$

Subjective Beliefs

Investor i's subjective beliefs about:

- error in private signal: $\varepsilon_i \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{e,i}\tau_e}\right)$

- aggregate supply shock: $z \sim_i \mathcal{N}\left(0, \frac{1}{\delta_{z,i}\tau_z}\right)$

where $\delta_{e,i}, \delta_{z,i} \in [0, \infty)$ parameterize the degree to which the investor **over**- or **under-estimates** info from s_i and P, respectively

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Remarks:

- **Rational Expectations** is a special case: $\delta_{e,i} = \delta_{z,i} = 1$

- Beliefs about supply noise $\Leftrightarrow$ Beliefs about others

Anticipated Utility

Each investor adopts her chosen beliefs as her "true" model.

- At date $t = 1$, optimal demand is

$$x_i(s_i, P; \delta_{e,i}, \delta_{z.i}) = \frac{\mathbb{E}_i[F] - P}{\gamma \text{var}_i[F]}$$

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- At date $t = 0$, anticipated utility is

$$AU(\delta_{e,i}, \delta_{z,i}) = \mathbb{E}_i \left[\mathbb{E}_i \left[-e^{-\gamma x_i(s_i, P) \times (F - P)} \big| s_i, P \right] \right]$$

Anticipated utility is *current utility* derived from expectation of the future.

Cost of Belief Distortion

Deviations from objective distribution impose a cost $C(\delta_{e,i}, \delta_{z,i})$, so investor i chooses $\delta_{e,i}$ and $\delta_{z,i}$ to maximize:

$$AU(\delta_{e,i}, \delta_{z,i}) - C(\delta_{e,i}, \delta_{z,i})$$

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Experienced utility penalty: The cost reflects objective utility loss from distorted actions i.e.,

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- Similar to Brunnermeier and Parker (2005)'s optimal expectations

Well-behaved cost function: $C(\cdot)$ is increasing, strictly convex, and

$$C(1,1) = \frac{\partial C(1,1)}{\partial \delta_{e,i}} = \frac{\partial C(1,1)}{\partial \delta_{z,i}} = 0$$

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Note: Investor need not "know" objective distribution, convenience / discipline for economist

Solving the Model

"Standard" Financial Market Equilibrium

Lemma: Given investors' subjective beliefs $\delta_{e,i}$ and $\delta_{z,i}$ $\forall i \in [0,1]$, there always exists a unique, linear equilibrium with

$$P = \Lambda s_p, \text{ where } \Lambda = \frac{\int_i \delta_{e,i}\tau_e + \delta_{z,i}\tau_p \, di}{\int_i \tau + \delta_{e,i}\tau_e + \delta_{z,i}\tau_p \, di}, \quad s_p = F + \beta z$$

and with $\tau_p \equiv \tau_z/\beta^2$, and $\beta \equiv -\dfrac{\gamma}{\tau_e \int_i \delta_{e,i} \, di}$.

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Subjective beliefs affect prices through two channels:

- Higher private signal precision $\delta_{e,i}$ increases signal to noise ratio of s_p
  increases $|\beta|$

- Higher precision of either signal increases price sensitivity to shocks
  i.e., increases Λ

Subjective Beliefs and Anticipated Utility

Anticipated utility increases in the volatility of conditional Sharpe Ratio:

$$AU(\delta_{e,i}, \delta_{z,i}) = -\sqrt{\frac{1}{\text{var}_i(SR_i)}} = -\sqrt{\frac{\text{var}_i[F|s_i, P]}{\text{var}_i[F - P]}}$$

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Lemma: Anticipated utility increases in perceived private precision ($\delta_{e,i}$), but is U-shaped in perceived price precision ($\delta_{z,i}$)

Results

Benchmark: Overconfidence in private information

Suppose investors have **objective** beliefs about **prices** i.e., $\delta_{z,i} = 1$.

Theorem: There exists a unique symmetric equilibrium in which the investors are overconfident about private information i.e., $\delta_{e,i} > 1$.

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With experienced utility penalty, δ_e increases with τ and τ_z, decreases with risk aversion γ.

Intuition: More informative prior or price $\Rightarrow$ less costly to distort $\delta_{e,i}$

General Case: Subjective beliefs about price information

Key: Strength of speculative effect depends on **equilibrium behavior**

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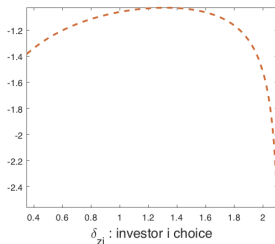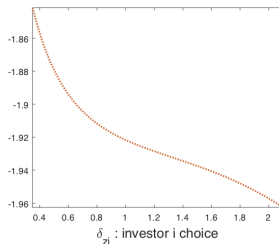
(a) $\delta_z = 0$ (b) $\delta_z = 1$

Figure 2: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

Dismissiveness in symmetric equilibria

**Theorem:** In **any** symmetric equilibrium, all investors are:
(i) (weakly) over-confident about their private info i.e., $\delta_{e,i} \geq 1$
(ii) dismissive of price info i.e., $\delta_{z,i} < 1$

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Intuition: I refuse to learn from prices when others are doing so.
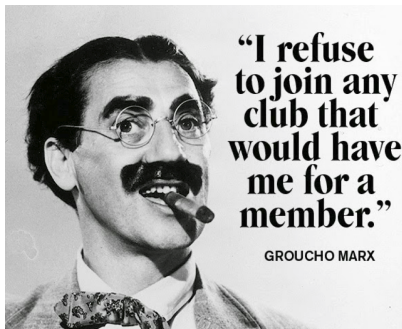
"I refuse to join any club that would have me for a member."

GROUCHO MARX

Risk aversion and symmetric equilibria

Note that in a symmetric equilibrium, the price is

$$P = \bar{\mathbb{E}}_i[F|s_i, P] - \gamma \mathrm{var}_i[F|s_i, P]z$$

$\Rightarrow$ All else equal, price is less informative as risk aversion γ increases

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Theorem: With exp. utility penalty, there exist cutoffs $\underline{\gamma} < \bar{\gamma}$ such that

(i) For $\gamma \geq \bar{\gamma}$, there exists a unique, symmetric equilibria in which all investors **ignore** price information and correctly interpret private information (i.e., $\delta_{z,i} = 0$ and $\delta_{e,i} = 1$).

(ii) For $\gamma \leq \underline{\gamma}$, there does **not** exist a symmetric equilibrium.

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(ii) For $\gamma \leq \underline{\gamma}$, there does **not** exist a symmetric equilibrium.

Intuition: When prices are sufficiently uninformative ($\gamma \geq \bar{\gamma}$), ignoring prices is not *too* costly, so symmetric equilibrium can be sustained

More generally, we have $\delta_{z,i} < 1$ and $\delta_{e,i} > 1$

Risk tolerance and asymmetric equilibria

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There exist **asymmetric** equilibria characterized by $(\lambda, \delta_e, \delta_z)$ where

(i) fraction λ optimally chooses $\delta_{e,i} = 1$ and $\delta_{z,i} = 0$

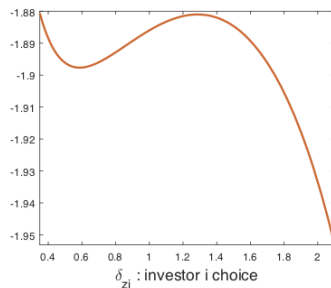(ii) fraction $1 - \lambda$ optimally chooses $\delta_e, \delta_z > 1$

Figure 3: $AU(\cdot) - C(\cdot)$ versus $\delta_{z,i}$

Implications of Asymmetric Equilibria

Observed heterogeneity in investment styles arise endogenously:

- **Fundamental traders** who find *mispriced securities* using their private info, but *dismiss* the information in prices

- **Technical traders** use price trends, reminiscent of overweighting price information

This is not a difference in degrees, but **in kind**: bias in opposite directions

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Model predicts such heterogeneity arises when risk tolerance and price informativeness are high:

- in more developed financial markets

- in larger (more widely held) assets

Return and Volume Characteristics

We compare predictions a number of return-volume observables

Focus for the talk:

(i) Price informativeness

(ii) Return predictability

(iii) Return volatility

(1) Price informativeness is higher under subjective beliefs

Recall that $P = \Lambda s_p$, where $s_p = F + \beta z$

Price informativeness is precision of price signal s_p i.e.,

$$\tau_p = \tau_z / \beta^2, \quad \text{where} \quad \beta = -\frac{\gamma}{\tau_e \int_i \delta_{e,i} di}$$

Relative to RE, investors (weakly) overweight private information

$\Rightarrow$ Lower β^2

$\Rightarrow$ Higher signal to noise ratio

This has implications for real / allocative efficiency in a richer environment

(2) Return predictability is higher and can be positive

Return predictability $\theta = \frac{\text{cov}(F-P,P)}{\text{var}(P)}$ regress return on lagged return

With no noise, investors with

- **RE:** Condition on prices correctly $\Rightarrow$ No predictability
- **Wishful thinking:** Under-react to prices $\Rightarrow$ Excess predictability

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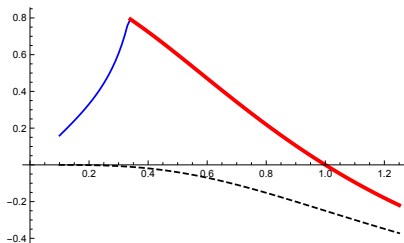Adding noise generates reversals (negative correlation)

Figure 4: Predictability (θ) versus Risk aversion (γ)

(3) Return volatility can be higher or lower than RE

$$\sigma_R^2 \equiv \text{var}(F - P) = (1 - \Lambda)^2 \text{var}(F) + \Lambda^2 \beta^2 \text{var}(z)$$

Compared to RE, wishful thinking implies:

- Prices are (weakly) more informative lower β^2
- Prices are less sensitive to shocks lower Λ

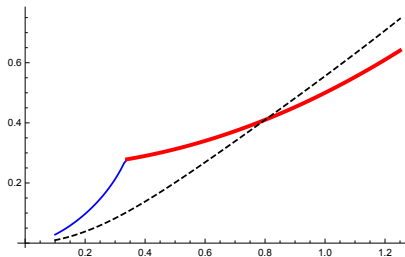Overall effect depends on the relative strength of the two

Figure 5: Volatility (σ_R) versus Risk aversion (γ)

Expected Utility of Investors

How do subjective beliefs affect **investor** utility?

- Under the subjective measure, investors are better off
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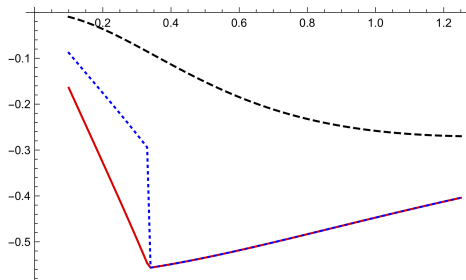
Technical traders are better off than Fundamental traders

Figure 6: RE (dashed), Technical (dotted), Fundamental (solid)

How do investor's subjective beliefs affect **other participants?**

"Utility" for liquidity traders:

$$U_z(z) \equiv \mathbb{E}\left[-e^{-\gamma_z(W_0 - z(F-P))}\right]$$

Compared to RE, liquidity traders face:

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- lower price impact

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Liquidity trader utility is higher when the second effect dominates

Welfare is higher subjective beliefs when γ is high

Theorem: Suppose $\gamma_z \leq \gamma$. Then,

(i) liquidity traders **always** have higher expected utility under symmetric equilibrium that in RE

(ii) when $\Lambda_{AE} < \Lambda_{RE}$, liquidity traders have higher expected utility under the asymmetric equilibrium than in RE
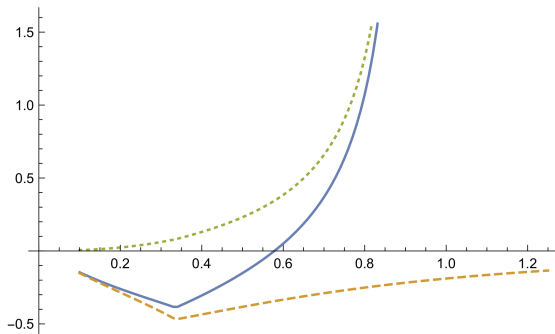

Figure 7: $U - U_{RE}$ vs γ: Investors (dashed), Noise traders (dotted), Sum (solid)

Extensions: Public signals and Ex-post belief choice

Public Signals: Consider signals of the type $s = F + \eta$

- Tradeoff between information effect vs. speculative effect

- Anticipated utility is U-shaped in perceived precision

- But, in any symmetric equilibrium, investors **overweight** public signal

Ex-post belief choice: Choose perceived precision *after* observing signal

- Not tractable to solve for general equilibrium prices are not linear

- Taking others actions as fixed, partial equilibrium analysis suggest results are robust:

  When private signal realizations are sufficiently far from priors, investors are over-confident in their private info, but dismissive of prices

Conclusions and Future Work

Subjective belief choice tells us **when** investors exhibit biases:

- Naturally gives rise to over-confidence and dismissiveness
- Can generate **endogenous** differences in behavior

Fruitful approach to explore how different biases arise in different settings

- How do results change in settings with strategic complementarity (e.g., coordination games)
- How do incentives affect the interpretation of information (e.g., fund managers, entrepreneurs)

Land or Sea?

Land or Sea?

66.7% horse vs. 72.7% seal