Disclosing to Informed Traders

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Prices reflect information from multiple sources

Prices reflect

- private information dispersed across investors
  Miller (1977), Hellwig (1980), Grossman Stiglitz (1980)
- strategic, public disclosures by firms
  Verrecchia (1983), Dye (1985)

Most analysis focuses on one or the other, but misses the **interaction**:

- How do strategic disclosures depend on investor information?
- How well do prices reflect fundamentals given interaction?

Important for empirical and policy analysis of financial markets

Dispersed Private Information + Voluntary Disclosure

We model an economy in which

- Risk-averse investors have private signals about fundamentals
- Investors can exhibit rational expectations or difference of opinions
- Firm strategically chooses whether to disclose verifiable information (at a cost) before trading

Challenge: Voluntary disclosure breaks the "linearity" of standard CARA-normal setting

Key Results

There **exists** a **unique** threshold equilibrium in which firm discloses good news, but withholds bad news

(1) More public information can **"crowd in"** voluntary disclosure
In contrast to common intuition that implies "crowding out"

(2) Firm is "mis-valued" relative to expected cash flows
Rational expectations $\Rightarrow$ **under-valuation** always
Difference of opinions $\Rightarrow$ **over-valuation** sometimes

Key Results - Why do we care?

There **exists** a **unique** threshold equilibrium in which firm discloses good news, but withholds bad news

(1) More public information can **"crowd in"** voluntary disclosure

In contrast to common intuition that implies "crowding out"

Important for regulatory disclosure policy

(2) Firm is "mis-valued" relative to expected cash flows

Rational expectations $\Rightarrow$ **under-valuation** always

Difference of opinions $\Rightarrow$ **over-valuation** sometimes

Pricing errors can be larger under RE, so may be a misleading metric

Under RE: negative relation between skewness and expected returns

Related Literature

Voluntary disclosure: Jovanovic (1982), Verrecchia (1983), and Dye (1985) - investors are uninformed, risk neutral, or both

- Risk-averse and uninformed: Verrecchia (1983), Cheynel (2013), Jorgensen and Kirschenheiter (2015), and Dye and Hughes (2018)
- Risk neutral and informed: Bertomeu, Beyer, and Dye (2011), Petrov (2016); Einhorn (2018) - Kyle model

Dispersed information models: Disclosure is either exogenous or nondiscretionary i.e., firm commits to disclosure policy

- Rational expectations: Hellwig (1980), Admati (1985)
- Difference of opinions: Miller (1977), Morris (1994)
- Commitment to disclosure: Goldstein and Yang (2019), Yang (2020), Schneemeier (2019), Cianciaruso, Marinovic, and Smith (2020)

Non-linear noisy REE techniques: Breon-Drish (2015)

- Banerjee Green (2015), Albagli Hellwig Tsyvinski (2015), Chabakauri Yuan Zachariadis (2017), Glebkin (2015), Smith (2019)...

Model

Payoffs, Preferences and Information

Risk-free asset is numeraire

Risky asset pays $v \sim N(m, \sigma_v^2)$ WLOG set $m = 0$

- Noise traders demand $z \sim N(0, \sigma_z^2)$
- Asset has zero net supply

  Shuts down risk-premium effects - see Dye and Hughes (2018)

Continuum of investors $i \in [0, 1]$ with CARA utility over wealth:

$$W_i = W_0 + D_i(v - P)$$

and risk tolerance τ.

Investor i observes "truth plus noise" signal

$$s_i = v + \varepsilon_i, \quad \text{where} \quad \varepsilon_i \sim N(0, \sigma_e^2), \text{ i.i.d.}$$

Subjective beliefs and interpreting price information

Investor i correctly infers distribution of her own signal, but has subjective beliefs about investor j's signal:

$$s_j =_i \rho v + \xi_i \sqrt{1 - \rho^2} + \varepsilon_j$$

where $\xi_i \sim_i N(m, \sigma_v^2)$ and $\varepsilon_j \sim_i N(0, \sigma_e^2)$ are independent of v, ε_i and each other

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Parameter ρ captures beliefs about others' signals Banerjee (2011)

- $\rho = 1$: rational expectations $\Rightarrow$ correctly condition on prices
- $\rho = 0$: difference of opinions $\Rightarrow$ dismiss price information completely
- $\rho \in (0, 1)$: partial dismissiveness of others

We will see that how investors interpret prices plays an important role!

Firm's disclosure decision

The firm's manager observes v before trading begins

Manager can pay a cost $c > 0$ and verifiably disclose this information

- Benchmark: It is common knowledge that manager is informed, as in Verrecchia (1983)
- Extension: Manager is informed with prob $p \in [0, 1]$ e.g., Dye (1985)

Manager's objective is to maximize next period's price (net of costs)

Timeline and Equilibrium

$t = 1$	$t = 2$	$t = 3$
Manager observes v	Investors observe s_i	Firm pays v
Chooses $d \in \{D, ND\}$	Submit $D(s_i, P, d)$	
	Asset price is P_D or P_{ND}	

Equilibrium:

- Disclosure decision: Disclose iff $P_D - c \geq \mathbb{E}[P_{ND}]$
- Given disclosure choice d, signal s_i and price P, $D(s_i, P, d)$ maximizes investor i's expected utility
- The price P clears the market

$$\int_i D(s_i, P, d)di + z = 0$$

Analysis

Financial Market Equilibrium

Conjecture: firm discloses if and only if $v > T$.

- If firm discloses,

$$P_D = v.$$

- If firm does not disclose, investors learn that $v < T$

Conditional on $d = ND$, v is a **truncated normal**

- Standard approach: Normal $v \Rightarrow P$ is a linear signal of v
- But, with truncated-normal v, this is no longer possible!

Generalized Linear Equilibrium

We extend the analysis in Breon-Drish (2015)

Conjecture P_{ND} is a "generalized" linear signal i.e., for some $G' > 0$,

$$P_{ND} = G(\bar{s} + \beta z), \quad \text{where} \quad \bar{s} = \int_i s_i di =_i \rho v + \xi \sqrt{1 - \rho^2}$$

- When $\rho = 0$, P_{ND} is irrelevant for updating beliefs
- When $\rho > 0$, can invert P_{ND} into a noisy, linear signal about v:

$$s_p = \frac{1}{\rho} G^{-1}(P_{ND}) \sim_i N(v, \sigma_p^2)$$

Note: When $\rho = 1$ (i.e., RE), $s_p = v + \beta z$.

Updating beliefs

Conditional on s_p and private signal s_i, cash flows are

$$v | s_i, s_p \sim_i N(\mu_i, \sigma_s^2), \quad \text{where}$$

$$\mu_i = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1} \left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_p}{\sigma_p^2}\right) \quad \text{and} \quad \sigma_s^2 = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$$

So expected cash flows, conditional on no disclosure:

$$\mathbb{E}_i[v | v < T, \mu_i, \sigma_s^2] = \mu_i - \sigma_s h\left(\frac{T - \mu_i}{\sigma_s}\right)$$

where $h(x) \equiv \frac{\phi(x)}{\Phi(x)}$ is the inverse Mills ratio for the normal distribution

Lemma. *Suppose the firm does not disclose if $v < T$. Then, the no disclosure price is given by:*

$$P_{ND} = G(s_p), \quad \textit{where} \quad s_p = \bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z$$

where $G(\cdot)$ is an increasing, concave function.

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where $G(\cdot)$ is an increasing, concave function.

Price is conditional expectation of investor for who $s_i = s_p = \bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z$ i.e.,

$$P_{ND} = \mathbb{E}_i \left[v \middle| v < T, s_i = s_p = \bar{s} + \tfrac{\sigma_\varepsilon^2}{\tau} z \right]$$

Note: Noisy signal s_p is same as in standard noisy RE (Hellwig) model

$$P_H = \bar{\mu} + \tfrac{\sigma_s^2}{\tau} z = \mathbb{E}_i \left[v \middle| s_i = s_p = \bar{s} + \tfrac{\sigma_\varepsilon^2}{\tau} z \right]$$

Result: Threshold Disclosure Equilibrium

Proposition. *There exists a unique threshold equilibrium in which the manager discloses if and only if $v \geq T$. The threshold is characterized by:*

$$\underbrace{\mathbb{E}\left[P_{ND} | v = T\right]}_{\textit{Don't disclose}} \quad = \quad \underbrace{T - c}_{\textit{Disclose}}$$

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Existence / Uniqueness is (slightly) trickier than usual:

Threshold T is the root of

$$H(v) \equiv \mathbb{E}[P_{ND}|v] - (v - c)$$

- P_{ND} partially reveals the v, unlike models without informed investors
- Need to ensure $\mathbb{E}[P_{ND}|v]$ not increase too quickly

  problematic in Dye extension

Implications

Question: Does public information decrease disclosure?

Empirical evidence is mixed

Very important from a policy perspective

- Regulators propose more public disclosure to "level the playing field"
- Firms / academics argue this can crowd out discretionary disclosure

Standard intuition: more ex-ante public info (lower σ_v)

$\Rightarrow$ More informative P_{ND}

$\Rightarrow$ *Costly* disclosure less attractive

Public info **crowds out** voluntary disclosure

Public Information can **crowd in** disclosure

Our model features an offsetting effect:

More ex-ante public info

$\Rightarrow$ Investors put less weight on private info

$\Rightarrow$ Less informative P_{ND} esp when private info is more precise

$\Rightarrow$ Disclosure becomes more attractive

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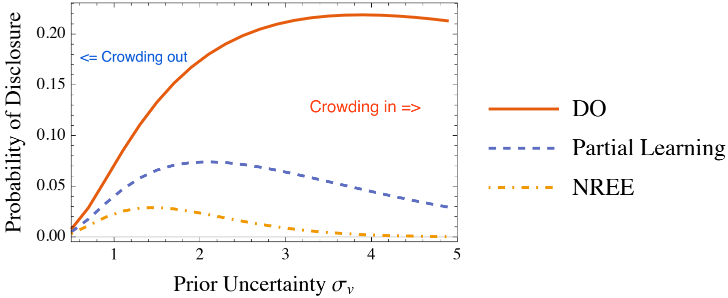
Public info can **crowd in** voluntary disclosure

Proposition. *When disclosure is sufficiently expensive and private info is sufficiently precise (relative to public info / prior), ex-ante public info crowds in more disclosure*

Crowding out vs. crowding in



Crowding in is likely to benefit firms that “need it” the most

High uncertainty, ex-ante public info is noisy, high costs of disclosure

Price informativeness (e.g., $\mathbb{E}[\text{var}(v|P)]$) can be non-monotonic in public information and depends on cost of disclosure

Question: How well do prices reflect fundamentals?

Standard Intuition: Aggregate supply is zero i.e., no systematic risk

$\Rightarrow$ On average, price reflects expected values

- Standard noisy RE models without aggregate risk (net zero supply)
- Standard disclosure models, since $P = \mathbb{E}[v|v < T]$

Mispricing is interpreted as evidence of behavioral biases / frictions

Undervaluation and Overvaluation

Proposition. *Conditional on no disclosure, the average price systematically deviates from expected cashflows:*

- **Rational expectations:** *always undervaluation*

$$\mathbb{E}\left[P_{ND} | v < T\right] < \mathbb{E}\left[v | v < T\right].$$

- **Differences of opinion:** *overvaluation (undervaluation) when noise trading volatility is low (high):*

$$\mathbb{E}\left[P_{ND} | v < T\right] \lesseqgtr \mathbb{E}\left[v | v < T\right] \quad \Leftrightarrow \quad \frac{\tau^2}{\sigma_\varepsilon^2} \lesseqgtr \sigma_z^2.$$

This translates into ex-ante mis-valuation since the firm is correctly valued when it discloses

Over-vs-Under valuation

$$P_{ND} = \mathbb{E}_i \left[v \middle| v < T, s_i = s_p = \bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z \right]$$

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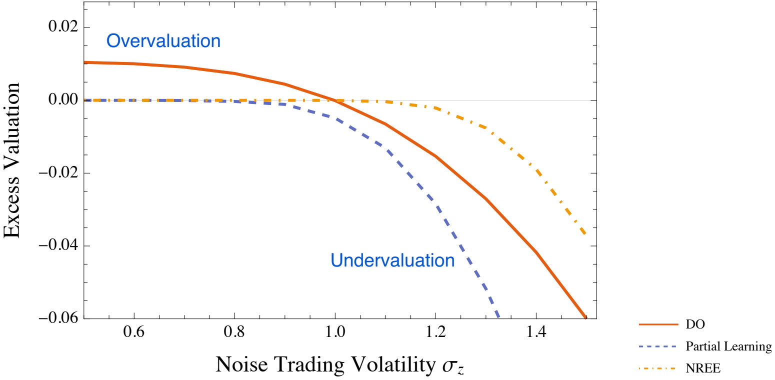
$$P_{ND} = \mathbb{E}_i \left[v \middle| v < T, s_i = s_p = \bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z \right]$$

Concavity implies $\mathbb{E}[P_{ND}|v < T] > \mathbb{E}[v|v < T]$ if average investor's beliefs are more volatile than marginal investor's beliefs i.e.,

$$\mathbb{E}\left[P_{ND}|v < T\right] \lesseqgtr \mathbb{E}\left[v|v < T\right] \quad \Leftrightarrow \quad \text{var}[\mu_i] \lesseqgtr \text{var}\left[\bar{\mu} + \frac{\sigma_s^2}{\tau} z\right]$$

- For RE, $\text{var}[\mu_i] < \text{var}\left[\bar{\mu} + \frac{\sigma_s^2}{\tau} z\right]$ always because investors condition on prices so amplify effect of noise
- For DO, $\text{var}[\mu_i] > \text{var}\left[\bar{\mu} + \frac{\sigma_s^2}{\tau} z\right]$ when noise vol is relatively small

$\mathbb{E}[P_{ND} - v | v < T]$ versus noise trading volatility



RE: Always under-valuation similar to Albagli, Hellwig, Tsyvinski (2015)

DO: Over-valuation when noise trading volatility is low

Valuation Implications

- Under/overvaluation can arise without frictions / biases
- Avg. pricing error $\mathbb{E}[(v - P)^2]$ can be higher with RE than with DO
- (DO model) Firm can have **lower** cost of capital / expected return when it **does not** disclose contrast to standard intuition / models
- (RE model) Negative relation between average returns and skewness
  - Returns are negatively skewed with no disclosure
  - Expected returns are positive in this case

Extension: Randomly Informed Manager

Randomly Informed Manager

Suppose the manager is informed with prob $p \in [0, 1]$ as in Dye (1985)

Conditional on no disclosure, price is weighted average:

$$P_{ND,p} = \frac{p \Pr(v < T | s_p) P_{ND}(s_p) + (1 - p) P_H(s_p)}{p \Pr(v < T | s_p) + (1 - p)}$$

where

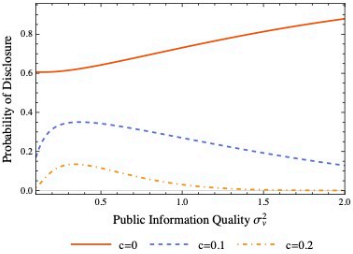
- $P_{ND}(s_p)$ is the non-disclosure price from earlier
- $P_H(s_p) = \bar{\mu} - \frac{\sigma_s^2}{\tau} z$ is the Hellwig price (i.e., if manager is uninformed)

Threshold disclosure is characterized by:

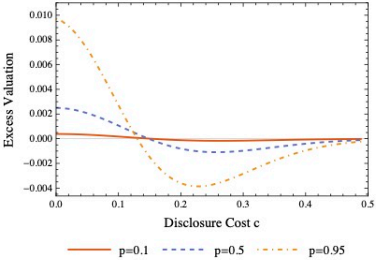
$$\mathbb{E}[P_{ND,p} | v = T] = T - c$$

Equilibrium exists if σ_ε is sufficiently large Otherwise LHS increases too quickly

Disclosure and Valuation



(a) Crowding In Vs. Crowding Out



(b) Under- Vs. Over-valuation

- Public info **crowds in** disclosure when cost c is sufficiently large
- Can generate over-valuation even with RE P_{ND} is not always concave

Conclusions

We develop a model to study how diverse, private information across investors affects voluntary disclosure by firms

- Public info *crowds in* disclosure when disclosure costs are high
- Under- (RE) vs. over-valuation (DO) relative to expected cashflows
- Negative relation between expected returns and skewness in RE

Opportunities for future work:

- Endogenous information acquisition by manager / investors
- Timing of disclosure (pre- vs. post-disclosure public info)
- Endogenize cashflows via investment decisions (feedback effects)

Appendix

Dye Extension: Existence of Threshold Equilibrium

The most significant difference comes in the magnitude of the price response. We show that:

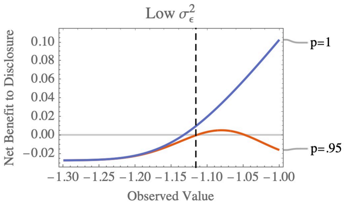
$$\frac{\partial P_{ND}(v,z)}{\partial v} = \begin{array}{l} var\left[\tilde{v}\,\middle|\, ND, \tilde{s}_i = \tilde{s}_p = v + \frac{\sigma_\varepsilon^2}{\tau} z\right] \\ * \left(var^{-1}\left[\tilde{s}_i | \tilde{v}\right] + var^{-1}\left[\tilde{s}_p | \tilde{v}\right]\right). \end{array}$$

- This formula generalizes the canonical Bayesian updating formula with normal prior/likelihood to an arbitrary prior.
- Dye and Hughes (2018) show that it is possible in a disclosure equilibrium that $var\left[\tilde{v} | ND\right] > var\left[\tilde{v}\right]$.
- This manifests as a price reaction that can exceed 1; for $v \approx T$,

$$\frac{\partial P_{ND}(v,z)}{\partial v} > 1.$$

Dye Extension: Existence of Threshold Equilibrium

- A marginal price reaction that exceeds 1 can break down the disclosure equilibrium.
- Higher firm types are *less* inclined towards disclosure.
- A threshold equilibrium exists when σ_ε^2 is not too small.



Dye Extension: Valuation

P_{ND} may no longer be concave in the marginal investor's expectation.

- Recall:

$$\frac{\partial P_{ND}}{\partial v} \propto var\left[\tilde{v}\,\middle|ND, \tilde{s}_i = \tilde{s}_p = v + \frac{\sigma_\varepsilon^2}{\tau}z\right]$$

$$\implies \frac{\partial^2 P_{ND}}{\partial^2 v} \propto \frac{\partial var\left[\tilde{v}\,\middle|ND, \tilde{s}_i = \tilde{s}_p = v + \frac{\sigma_\varepsilon^2}{\tau}z\right]}{\partial v}.$$

- The conditional variance can *increase* in v as v approaches the T:
  Reflects more uncertainty about whether the manager was informed.
- This can lead to overvaluation even with RE.