Asymmetric Information, Disagreement, and the Valuation of Debt and Equity*

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Abstract

We study debt and equity valuation when investors have private information and may exhibit differences of opinion. We find that idiosyncratic distress risk impacts expected debt and equity returns. Expected debt returns increase and expected equity returns decrease with this risk for typical firms, but these relations weaken, and can even reverse, when (i) firms are close to default, (ii) disagreement is high, or (iii) liquidity is low. Furthermore, firms' capital structures affect their valuation even without classical capital structure frictions (e.g., tax shields, distress costs): when liquidity in equity is higher than in debt, leverage can raise firm value.

JEL: G10, G12, G14, G32
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1 Introduction

We develop a model of debt and equity valuations where investors have private information and may exhibit differences of opinions about firm cash flows. Our model generates predictions that are consistent with existing evidence but are difficult to derive from traditional models where investors have identical beliefs. First, we show that idiosyncratic distress risk raises the expected return on investment-grade debt, as in Huang and Huang (2012) and Bai, Goldstein, and Yang (2020). Second, distress risk lowers expected equity returns for firms that are far from bankruptcy, as documented by Campbell, Hilscher, and Szilagyi (2008), even when investors rationally account for the impact of such risk on firm value. However, this relationship reverses for firms close to bankruptcy, and thus the overall relation between distress risk and equity returns is hump-shaped, consistent with Garlappi, Shu, and Yan (2008). Finally, firm-specific public information lowers expected debt returns, in line with the evidence in Bharath, Sunder, and Sunder (2008) and Jaskowski and Rettl (2023).

Our model also generates novel predictions about how the relation between firm-specific default risk and expected returns varies across firms. We find that this relation weakens, and can even reverse, when disagreement among investors is sufficiently high or when liquidity trading is very low. Moreover, considering the prices of equity and debt jointly, we show that a firm's capital structure affects its valuation even in the absence of traditional frictions such as tax shields of debt or distress costs. Specifically, the optimal choice of leverage depends on the relative amount of liquidity trading in each security. When liquidity trading is higher in equity, the total value of a levered firm can be higher than that of an unlevered firm.

Overview of Model and Intuition. In our model, privately-informed, risk-averse investors trade the debt and equity of a levered firm alongside liquidity traders. We allow, but do not require, investors to agree to disagree about the quality of others' information and, consequently, dismiss the information in prices. Our model nests two natural benchmarks as special cases: investors may either exhibit rational expectations (RE) and correctly interpret the information in prices, or exhibit pure differences of opinion (DO) and completely ignore price information. A key challenge in characterizing the equilibrium is that the payoffs to levered equity and debt are option-like, and depend non-linearly on the firm's underlying cash flows, and so standard approaches (e.g., Hellwig (1980)) cannot be employed. Instead, we apply recent work on non-linear equilibria by Breon-Drish (2015), and especially the multi-asset generalization by Chabakauri, Yuan, and Zachariadis (2021), to characterize an equilibrium in which security prices depend non-linearly on beliefs about fundamentals and liquidity trading.

The option-like nature of the equity and debt payoffs affects how the prices of these securities aggregate investor information and respond to liquidity shocks. To gain intuition, we start with a benchmark setting where liquidity-trader demands in the equity and debt markets are identical. In this case, debt and equity prices each convey the same information signal to investors. We find that equity and debt valuations depend crucially on how investors update from this price signal. Specifically, after controlling for systematic risk, the expected return on equity is negative and the expected return on debt is positive unless investors are sufficiently dismissive of price information and liquidity-trading volatility is sufficiently low.[1]

These results are driven by how the security prices respond to investors' private information and liquidity-trader demand in equilibrium. Consider the pricing of equity, which has a payoff that is convex in the firm's underlying cash flows (similar to a call option). The case for debt, which has a concave payoff, is analogous. First, an investor's demand for equity responds more strongly to a decrease in her conditional expectation of equity payoffs than to a corresponding increase. Intuitively, when she observes bad news about cash flows, both expected equity payoffs and the perceived risk fall: when the news is extremely bad, she is very certain the equity is almost worthless. In contrast, because equity payoffs are unbounded above, good news about a firm's cash flows increase both her expected payoffs and her perceived risk. These offsetting forces dampen the impact of such news on her demand. As a result, the equity price places disproportionate weight on pessimistic signals and, consequently, the average price declines when investor disagreement increases.

Second, liquidity-trader demand also has an asymmetric impact on prices. Since equity payoffs are bounded below but unbounded above, the risk for investors from being long is limited, while the risk from being short is substantial. As a result, investors charge a larger price compensation when they sell to liquidity traders than when they have to buy from them. This implies that the average equity price is higher when the volatility of liquidity trading increases. Because debt payoffs are concave and bounded above, the implications are reversed: more disagreement leads to higher debt prices, while higher liquidity-trading volatility leads to lower debt prices.

We show that the relative impact of these forces depends crucially on how investors use the information in prices. When investors interpret the price as being informative (e.g., when they exhibit RE), they put more weight on the common (price) signal. This reduces equilibrium disagreement and amplifies the impact of liquidity trading, and so equity (debt)

[1]The source of systematic risk in our model is the aggregate supply of each security that investors have to hold. As such, our results should be interpreted as predictions about "alphas" from the perspective of an outside econometrician who is controlling for systematic risk (or "betas") of the securities. However, these alphas do not reflect mispricing from the perspective of investors in the model. In contrast to the case of an unlevered firm, we show that even when the aggregate supply of the securities is zero, the expected return, or "alpha" on debt and equity can be non-zero.

prices are high (low) on average. In contrast, when investors dismiss the information in prices, disagreement is higher. As a result, when the liquidity-trading volatility is sufficiently low, the first channel dominates and so equity (debt) prices are low (high) on average.

Implications. The above economic mechanisms have several empirically-relevant implications. First, for firms with low default risk (below 50%), we find that higher leverage is associated with lower debt prices, even after controlling for systematic risk. Thus, consistent with the credit-spread puzzle, an increase in firm-specific default risk raises the discount rate on the firm's debt by more than the expected losses conditional on default (e.g., Huang and Huang (2012)). Similarly, equity returns are negatively related to leverage when default probabilities are low (consistent with Campbell et al. (2008)), but increase with leverage for highly distressed firms.[2]

Second, the above intuition implies that these relations weaken, and can even reverse, when disagreement across investors is sufficiently high and when the volatility of liquidity trading is sufficiently low. As such, our model provides novel predictions on how the relation between expected returns and distress risk varies across firms. Specifically, the expected return on debt is positively related to the interaction between distress risk and liquidity-trading volatility, but negatively related to the interaction between distress risk and disagreement; the predictions for the expected return on equity are reversed.

Third, for a fixed probability of default, we show that expected return on debt (equity) increases (decreases) with liquidity-trading volatility. Since higher liquidity-trading volatility corresponds to higher return volatility in our setting, this may help to explain the idiosyncratic volatility puzzle for the cross-section of equity returns (e.g., Ang, Hodrick, Xing, and Zhang (2006)).[3]

Next, we consider the impact of introducing public information (e.g., an earnings announcement) before trading in our model. An increase in the quality of public information decreases debt returns (consistent with the evidence in Jaskowski and Rettl (2023)) and increases equity returns, unless investors are very dismissive of price information and liquidity-trading volatility is sufficiently low. Intuitively, this is because higher-quality public information reduces both the extent of disagreement and the impact of liquidity trading on prices.

Finally, we generalize our benchmark model to consider a setting in which liquidity

[2]Note this overall hump shape is consistent with the results in Garlappi et al. (2008) and Garlappi and Yan (2011). We note, however, that our model specifically predicts that expected equity returns increase in default risk only for firms with default probabilities greater than 50%.

[3]Consistent with liquidity trade as a potential driver of a positive relationship between idiosyncratic volatility and expected equity returns, Hou and Loh (2016) show that market frictions explain a significant portion of this puzzle.

trading in debt and equity markets can have arbitrary correlation. Investors update their beliefs about cash flows from both the debt and equity prices, which generates a spillover across markets: liquidity-trader demand in the equity market increases debt prices and vice versa. Our analysis implies that the correlation between equity and debt prices increases with leverage when the likelihood of default is sufficiently low, consistent with the empirical evidence in Pasquariello and Sandulescu (2021).

In our baseline analysis, because liquidity traders' demands in both markets are identical, their impact on the combined valuation of debt and equity cancel out exactly. As a result, the total value of the levered firm is equal to the value of the unlevered firm i.e., Modigliani and Miller's irrelevance result obtains. However, when liquidity trading differs across the two markets, this is no longer true. Instead, we find that when the volatility of liquidity trading in equity is higher than that in debt, the positive effect on equity prices dominates the negative effect on debt prices and so the total market value of the levered firm is hump-shaped in leverage. This suggests that an interior level of leverage is optimal for the firm, even in the absence of traditional frictions associated with debt financing (e.g., tax shields, distress costs).

The rest of the paper is as follows. The next section discusses the related literature and our incremental contribution. Section 3 presents the model, and Section 4 characterizes the equilibrium in the baseline case. Section 5 characterizes how the expected returns on debt and equity depend on the features of the model, and how they depend on the quality of public information about cash flows. Section 6 presents the characterization of the equilibrium when the liquidity trading in the two markets are not identical. Section 7 presents the empirical implications of the model and discusses existing empirical research that relates to our results. Finally, Section 8 concludes. Unless noted otherwise, proofs are in the Appendix.

2 Related Literature

One contribution of our analysis is to study trade in equity and debt in a setting that allows for both heterogeneity in investor information and differences of opinion. That is, in addition to jointly considering both equity and debt issued by the same firm (as in Chabakauri et al. (2021)), our model differs from the standard noisy rational expectations framework by allowing investors to "agree to disagree". We show that this leads to readily-interpretable closed-form solutions for demands and prices in the two securities and novel predictions on how non-linearity in payoffs affects expected return

Chabakauri et al. (2021) offers the closest model to ours, analyzing private information in a general multi-asset noisy rational expectations framework with CARA investors. When applying their model to study debt and equity prices, their focus is on showing that the informativeness of these prices does not depend on the firm's capital structure (a result that also holds in our model). We complement their work by allowing investors to potentially disregard the information in prices, by analyzing expected debt and equity returns, and by considering the joint effects of public and private information. Moreover, while Chabakauri et al. (2021) do not focus on the expected returns on debt and equity, they do characterize the relationship between payoff skewness and expected returns when investors exhibit rational expectations. Our results show that the relationship between skewness and prices they document can reverse when investors dismiss the information in prices and liquidity-trading volatility is low.

Our analysis is related to noisy rational expectations models of debt and equity markets in which non-linearity in security prices plays a key role. To study the credit spread puzzle, Albagli, Hellwig, and Tsyvinski (2021) consider a setting in which risk-neutral, informed investors have position limits and trade in a bond with binary payoffs, and find that the bond price overweights risk. Davis (2017) extends their analysis to consider the firm's issuance decision over time and across markets, in a setting where investors choose how much information to acquire about fundamentals. Back and Crotty (2015) consider the pricing of debt and equity in a continuous-time Kyle model in which a strategic, informed investor can trade in both debt and equity markets, and market making is integrated. They show that the stock-bond correlation depends on the cross-market lambda, and is positive (negative) when the strategic trader is informed about the mean (risk) of firm's assets.

In a single-period Kyle model of debt and equity with segmentation in market making, Pasquariello and Sandulescu (2021) study how changes in leverage affect the sensitivity of debt and equity to firm value, and consequently, affect the intensity of informed speculation in each security. This gives rise to variation in liquidity across debt and equity, and non-monotonicity in the co-movement of their prices. Chaigneau (2022) considers capital structure when investors have information on both upside and downside risks. Finally, Frenkel (2023) considers a Glosten-Milgrom model of debt trading to characterize how negative news for firms that are close to default can trigger more information acquisition, and subsequently, lead to liquidity dry-ups.

We view our analysis as complementary to this earlier work. While these papers largely consider settings in which the price is determined by risk-neutral investors / market makers, investors in our model are risk-averse. Moreover, while these models focus on rational expectations equilibrium, our model allows investors to "agree to disagree" about the informativeness of others' signals, and consequently dismiss the information in prices. We show that this has important implications for how non-linearity in payoffs affects expected returns.[4]

Finally, our study of public information and expected returns relates to the literature on information quality, information asymmetry, and the cost of capital.[5] Our results contribute to this work by showing that the impact of these constructs on expected returns depends upon the firm's capital structure and the extent to which investors learn from prices.

3 Model Setup

We consider a model of trade among informed investors in the spirit of Hellwig (1980), with two modifications: we allow the firm to be levered and for investors to potentially ignore the information in price.

Payoffs. Investors trade in the risky debt and equity of a firm alongside a risk-free security. The gross return on the risk-free security is normalized to 1. The firm's total cash flows are $\mathcal{V} \equiv \mu + \theta$, where $\theta \sim N\left(0, \sigma_\theta^2\right)$. The assumption that $\mathcal{V}$ is unconditionally normally distributed keeps the traders' updating problem simple and transparent, but can be relaxed using the approach of Breon-Drish (2015) and Chabakauri et al. (2021). The firm has debt with a face value of K, i.e., equity payoffs are $V_E = \max\left(\mathcal{V} - K, 0\right)$ and debt payoffs are $V_D = \min\left(\mathcal{V}, K\right)$, so that $\mathcal{V} = V_E + V_D$. We assume that there are liquidity traders who submit identical demand $z \sim N\left(0, \sigma_z^2\right)$ in both the equity and debt markets. The supply of the firm's securities is $\kappa \geq 0$ (i.e., there are κ units of debt and κ shares outstanding).

The assumption that the liquidity-trader demands in the debt and equity markets are perfectly correlated is made for analytical tractability and expositional clarity in our initial analysis. In Section 6, we explore a setting where liquidity trading in the two markets follows a general bivariate normal distribution, which allows for imperfect correlation and/or different variances across the markets.

Preferences and Information. There is a unit mass of investors indexed by $i \in [0, 1]$. Each investor i is endowed with κ shares of the stock and bond, and exhibits CARA utility with risk-tolerance τ over terminal wealth W_i. Let $x_{E,i}$ and $x_{D,i}$ denote investor i's demands for the equity and debt, respectively (so that $x_{k,i} - \kappa$ is her net trade in security $k \in \{D, E\}$),and let P_E denote the equity and P_D the debt price per share/unit. The terminal

[4]Models in which investors "agree to disagree" about others' information include Miller (1977), Morris (1994), Kandel and Pearson (1995), Scheinkman and Xiong (2003), and Banerjee (2011). Our analysis also has implications for settings where investors dismiss the information in prices due to other reasons, including "cursedness" (e.g., Eyster, Rabin, and Vayanos (2018)), costly price information (e.g., Mondria, Vives, and Yang (2022)) and "wishful thinking" (e.g., Banerjee, Davis, and Gondhi (2019)).

[5]See, e.g., Hughes, Liu, and Liu (2007), Lambert, Leuz, and Verrecchia (2007), and Dutta and Nezlobin (2017) for analyses of competitive markets and Lambert, Leuz, and Verrecchia (2012) and Caskey, Hughes, and Liu (2015) for analyses of imperfectly competitive markets.

wealth W_i of investor i is therefore:

$$W_i = \kappa(P_D + P_E) + x_{E,i}(V_E - P_E) + x_{D,i}(V_D - P_D).$$

Investor i observes a private signal s_i of the form:

$$s_i = \theta + \varepsilon_i, \tag{1}$$

where the error terms $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ are independent of all other random variables.

Subjective Beliefs. We allow for a flexible specification of subjective beliefs about the private information of others. Following Banerjee (2011), we assume that investor i's beliefs about her own signal are given by (1), but her beliefs about investor j's signal are given by:

$$s_j =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + \varepsilon_j, \tag{2}$$

where the random variables $\xi_i \sim_i N(0, \sigma_\theta^2)$ and $\varepsilon_j \sim_i N(0, \sigma_\varepsilon^2)$ are independent of all other random variables and each other, and $\rho \in [0,1]$ parameterizes the difference in opinions. The assumption that ξ_i has the same distribution as θ ensures that investor i cannot detect the error in her subjective beliefs based on the unconditional mean and variance of others' signals. We use a subscript i on expectations, variances, and distributions to refer to investor i's subjective beliefs.

The above specification provides a tractable way to nest two natural benchmarks. When $\rho = 1$, investors exhibit rational expectations (as in Hellwig (1980)): this is equivalent to assuming that all investors share common priors about the joint distribution of fundamentals and signals. In this case, investors fully condition on the information in prices (in addition to their private information) when updating their beliefs about fundamentals. When $\rho = 0$, investors exhibit "pure" differences of opinion (as in Miller (1977)): each investor believes no other investor has payoff relevant information, and so prices are not incrementally informative about payoffs.[6] In this case, investors do not place any weight on prices when updating their beliefs. Moreover, when $\rho \in (0,1)$, investors are partially dismissive of the information content of others' private signals; hence, investors only partially account for the information contained in prices when determining their demands.

The assumption that all investors can trade in both markets is made for tractability, but also serves as a useful benchmark. It allows us to focus on the implications of belief heterogeneity on debt and equity valuations without introducing differences in clienteles,

[6] Note that investor i believes that ξ_i is the common "error" in all other investors' signals. This is analogous to the subjective beliefs of investors in other difference of opinions models (e.g., Scheinkman and Xiong (2003)) and in the "cursed equilibrium" of Eyster et al. (2018).

investor information or risk aversion across these securities. In practice, one might argue that bond markets are more specialized and have less participation than equities. Although we expect the economic mechanisms that we study to operate in richer settings, explicitly accounting for different groups of investors in each security (e.g., investor specialization) is intractable in our framework.[7]

Equilibrium. An equilibrium consists of demands $\{x_{E,i}, x_{D,i}\}_{i\in[0,1]}$ and prices (P_D, P_E) such that (i) the demands $(x_{D,i}, x_{E,i})$ maximize investor i's expected utility, given her information $\mathcal{F}_i = \sigma(s_i, P_D, P_E)$ and subjective belief formation mechanism described above, and (ii) the equilibrium prices (P_D, P_E) are determined by market clearing

$$\int x_{k,i}di + z = \kappa; \qquad k \in \{D, E\}. \tag{3}$$

4 Analysis

Because the equity and debt securities are effectively options on the underlying cash flows, their payoffs are not normally distributed. As a result, the equilibrium in which prices are linear in the fundamental and liquidity trade, which is common in traditional rational expectations models, does not exist. Instead, we focus on the following notion of equilibrium, which is a two-asset version of the equilibrium studied in Breon-Drish (2015) and Chabakauri et al. (2021).

Definition 1. *A "generalized linear equilibrium" is one in which there exist monotonic functions $h_E(\cdot)$, $h_D(\cdot)$ and a coefficient $\beta \in \mathbb{R}$ such that the equity and debt prices are given by:*

$$P_E = h_E(\overline{s} + \beta z); \tag{4}$$

$$P_D = h_D(\overline{s} + \beta z), \tag{5}$$

where $\overline{s} = \int s_j dj$ is the average private signal.

The key feature of such an equilibrium is that each investor can infer identical linear statistics from the debt and equity prices:

$$s_E = s_D \equiv \overline{s} + \beta z =_i \rho\theta + \sqrt{1-\rho^2}\,\xi_i + \beta z. \tag{6}$$

[7] As Section 6 illustrates, we can allow for differences in the informativeness of debt and equity prices by assuming that liquidity trading in the two markets have different variances.

In particular, when $\rho = 0$, investors perceive s_D and s_E to be uninformative about θ. On the other hand, when $\rho > 0$, the linear structure in (6) ensures that Bayesian updating from prices continues to take on a tractable form. To solve for an equilibrium, we derive investors' demands given these beliefs, apply market clearing, and verify that the resulting price indeed takes the "generalized linear" form in (5).

4.1 Benchmarks

To provide intuition for the equilibrium that arises in the general case, we start by characterizing the equilibrium in two natural benchmarks.

4.1.1 Unlevered firm benchmark

First, consider the case in which the firm issues only equity (i.e., when $K \to -\infty$). In this case, the payoff to equity holders is normally distributed, as in traditional models, and so we recover the standard, linear equilibrium. Moreover, since the firm only issues one type of security, investor i infers a single linear statistic from the unlevered equity price of the form:

$$s_U \equiv \overline{s} + \beta z =_i \rho\,\theta + \sqrt{1-\rho^2}\,\xi_i + \beta z, \tag{7}$$

where β is determined in equilibrium. It is worth noting that the objective distribution of the signal is given by $s_U = \theta + \beta z$, which coincides with investors' beliefs when $\rho = 1$.

Given this signal, investor i's conditional beliefs about cash flows $\mathcal{V}$ are normal with moments given by

$$\mu_i \equiv \mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right) \text{ and} \tag{8}$$

$$\sigma_s^2 \equiv \mathbb{V}_i\left[\mathcal{V}|s_i, P_U\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}, \text{where} \tag{9}$$

$$\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2\sigma_z^2}{\rho^2} \tag{10}$$

and where it is understood that when $\rho = 0$, we take $\frac{1}{\sigma_p^2} = \frac{1}{\rho\sigma_p^2} = 0$ in the above expressions. Standard calculations imply that investor i's optimal demand for the security is given by

$$x_i = \tau\left(\frac{\mu_i - P_U}{\sigma_s^2}\right), \tag{11}$$

and market clearing implies that the equilibrium price is given by:

$$P_U = \int \mu_i di + \frac{\sigma_s^2}{\tau}(z - \kappa).$$

This implies the following result.

Lemma 1. ***Unlevered firm benchmark.*** *Suppose that the firm only issues equity (i.e., $K \to -\infty$). Then, there is a unique linear equilibrium in which the firm's price satisfies:*

$$P_U(\cdot) = \mu + \sigma_s^2 \left(\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) (\bar{s} + \beta z) - \frac{\kappa}{\tau} \right), \tag{12}$$

where $\beta = \frac{\sigma_\varepsilon^2}{\tau}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$.

Notably, the above equilibrium coincides with the rational expectations equilibrium in Hellwig (1980) when $\rho = 1$. On the other hand, when $\rho = 0$, investors ignore the information in prices (since the weight they put on s_U in (8) is zero), even though it conveys information about cash flows.

4.1.2 Risk-neutral, uninformed benchmark

As a second benchmark, consider the setting in which investors are risk neutral (i.e., $\tau \to \infty$) and completely uninformed (i.e., $\sigma_\varepsilon^2 \to \infty$). In this case, the price of each security is given by the unconditional expectation of its payoff i.e.,

$$P_E = \mathbb{E}\left[\max\left(\mathcal{V} - K, 0\right)\right] \text{ and } \quad P_D = \mathbb{E}\left[\min\left(\mathcal{V}, K\right)\right].$$

In what follows, the definition below will be convenient.

Definition 2. *Suppose $x \sim N(\mu_x, \sigma_x^2)$. Let $M_E(\mu_x, \sigma_x^2, K)$ and $M_D(\mu_x, \sigma_x^2, K)$ denote:*

$$M_E(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\max\left(x - K, 0\right)\right] = \left[1 - \Phi\left(\frac{K - \mu_x}{\sigma_x}\right)\right] \left[\mu_x - K + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{1 - \Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right], \tag{13}$$

$$M_D(\mu_x, \sigma_x^2, K) \equiv \mathbb{E}\left[\min\left(x, K\right)\right] = K - \Phi\left(\frac{K - \mu_x}{\sigma_x}\right) \left[K - \mu_x + \sigma_x \frac{\phi\left(\frac{K-\mu_x}{\sigma_x}\right)}{\Phi\left(\frac{K-\mu_x}{\sigma_x}\right)}\right]. \tag{14}$$

It is worth noting that since $\max(x - K, 0)$ is an increasing, convex function of $x - K$, we immediately have that $M_E(\mu_x, \sigma_x^2, K)$ is increasing in μ_x and σ_x^2, but decreasing in K. Similarly, since $\min(x, K) = K + \min(x - K, 0)$ is increasing and concave in x, we have that $M_D(\mu_x, \sigma_x^2, K)$ is increasing in μ_x and K, but decreasing in σ_x^2.

Given the above definition, we can characterize the equilibrium in this benchmark as follows.

Lemma 2. ***Risk-neutral, uninformed benchmark.*** *Suppose that investors are risk neutral and uninformed (i.e., $\tau \to \infty, \sigma_\varepsilon^2 \to \infty$). Then, there is a unique equilibrium in which the firm's equity and debt prices are given by $P_E = M_E\left(\mu, \sigma_\theta^2, K\right)$ and $P_D = M_D\left(\mu, \sigma_\theta^2, K\right)$. Moreover, the total value of the firm is given by $P_E + P_D = \mu$.*

The above results are intuitive. Note that $\Pr(\mathcal{V} < K) = \Phi\left(\frac{K-\mu}{\sigma_\theta}\right)$ reflects the probability that the firm defaults on its debt. Given this, the price of equity is given by the probability of no default times the conditional expected cash flows, given no default i.e.,

$$P_E = \Pr(\mathcal{V} > K) \times \mathbb{E}[\mathcal{V} - K | \mathcal{V} > K],$$

which corresponds to the expression for M_E in (13), evaluated at the firm's cash flow mean and variance. Similarly, the price of debt is given by the face value of debt, K, minus the probability of default times the loss given default i.e.,

$$P_D = K - \Pr(\mathcal{V} < K) \times \mathbb{E}[K - \mathcal{V} | \mathcal{V} < K],$$

which corresponds to the expression for M_D in (14). Not surprisingly, since investors are uninformed and risk-neutral, the total value of the firm reflects the unconditional expected cash flows. In the following subsection, we show that the equilibrium prices when investors are risk averse and privately informed are natural generalizations of the above expressions.

4.2 Equilibrium

To start, we study investors' demands holding fixed the equity and debt prices. We then show that the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, which lends tractability to our model.

Lemma 3. *Given equity and debt prices P_E and P_D, investors' demands take the form:*

$$\begin{pmatrix} x_{E,i} \\ x_{D,i} \end{pmatrix} = \frac{\tau}{\sigma_s^2}\left[\begin{pmatrix} \mu_i \\ \mu_i \end{pmatrix} - G\begin{pmatrix} P_E \\ P_D \end{pmatrix}\right], \tag{15}$$

for a function $G(\cdot)$ defined in the appendix. As a result, the firm's equity and debt prices contain the same information as in the unlevered firm benchmark, i.e., they depend upon $\{s_i\}$ and z only through the statistic s_U.

The first part of the lemma illustrates that investors' demands are additively separable in investors' beliefs about the firm's total cash flow, $\mu_i = \mathbb{E}_i[\mathcal{V}]$, and the prices P_E, P_D. Moreover, each investor speculates on her beliefs in the same direction in both markets, and exhibits the same trading aggressiveness in the two markets:

$$\frac{\partial x_{E,i}}{\partial \mu_i} = \frac{\partial x_{D,i}}{\partial \mu_i} = \frac{\tau}{\sigma_s^2} = \frac{\text{risk tolerance}}{\text{posterior uncertainty}}. \tag{16}$$

Intuitively, both securities are exposed to the firm's underlying cash flows in the same direction. One might posit that investors would trade more aggressively in the security that is more exposed to a shift in the firm's cash flows. For instance, when the firm's expected cash flows μ are large, the debt almost certainly pays off K, and so the equity is considerably more sensitive to a change in μ. Thus, one might expect an investor with a positive signal to take a larger position in the equity than the debt. However, while the expected payoffs to trading on private information are greater in the security that is more exposed to θ, so too is the risk, and these two effects precisely offset. As we will see, this feature of investors' demands has important consequences for the expected returns on the securities.[8]

Equation (16) holds regardless of the firm's debt level K, which implies that the firm's capital structure does not influence the investors' trading aggressiveness. In addition, the total supplies of the equity and debt to be absorbed by the investors, $\kappa - z$, are identical. Together, these results imply that the equity and debt prices contain the same information and that this information is the same as in the case where the firm is unlevered, as in Chabakauri et al. (2021). Therefore, investors' expectations and variances of total firm cash flows in equilibrium are identical to those in equations (8) and (9).

Building on these findings, the next proposition characterizes the equilibrium price and investor demands.

Proposition 1. *There exists a generalized linear equilibrium in which the equity and debt prices satisfy:*

$$P_E = M_E\left(P_U, \sigma_s^2, K\right) \text{ and } P_D = M_D\left(P_U, \sigma_s^2, K\right). \tag{17}$$

Moreover, the total value of the equity and debt is equal to P_U i.e., $P_U = P_E + P_D$, and investors' equilibrium equity and debt demands satisfy:

$$x_{E,i} = x_{D,i} = \tau \frac{\mu_i - \int \mu_j dj}{\sigma_s^2} - z + \kappa. \tag{18}$$

[8] Note we have verified that this result holds even when investors' have arbitrarily-distributed priors, i.e., it does not depend upon normally-distributed priors.

This proposition demonstrates that the firm's equity and debt prices are equal to their expected payoffs under the beliefs of a representative investor who views the firm's unlevered cash flows to be distributed as $\mathcal{V} \sim N(P_U, \sigma_s^2)$. Such an investor would be exactly indifferent between holding a marginal unit of either security or not, and so the price of each security must coincide with her subjective conditional expectation of the security payoff. Importantly, note that the risk premium associated with bearing the aggregate supply of κ units of equity and debt are reflected in the mean P_U – this is analogous to the adjustment for risk in the traditional "risk-neutral" pricing approach.

The total market price of the firm's equity and debt, $P_E + P_D$, is independent of the firm's capital structure and equal to the price were the firm unlevered P_U. This implies the Modigliani-Miller theorem holds in this setting, even though investors have private information. Finally, consistent with the finding in Lemma 3 that investors speculate on their beliefs equally in both markets, their equilibrium demands in the two markets are identical and coincide with their demands in the unlevered firm case.

Because the securities' prices can be expressed as their expected payoffs under the beliefs of a representative investor, they satisfy a number of intuitive features. For instance, any feature that shifts up the unlevered price, P_U, while holding fixed posterior uncertainty σ_s^2, will also cause the prices of the debt and equity to increase. This yields the following result.

Corollary 1. *The firm's equity and debt prices:*

(i) increase in mean cash flows, μ,

(ii) increase in liquidity-trader demand, z, and

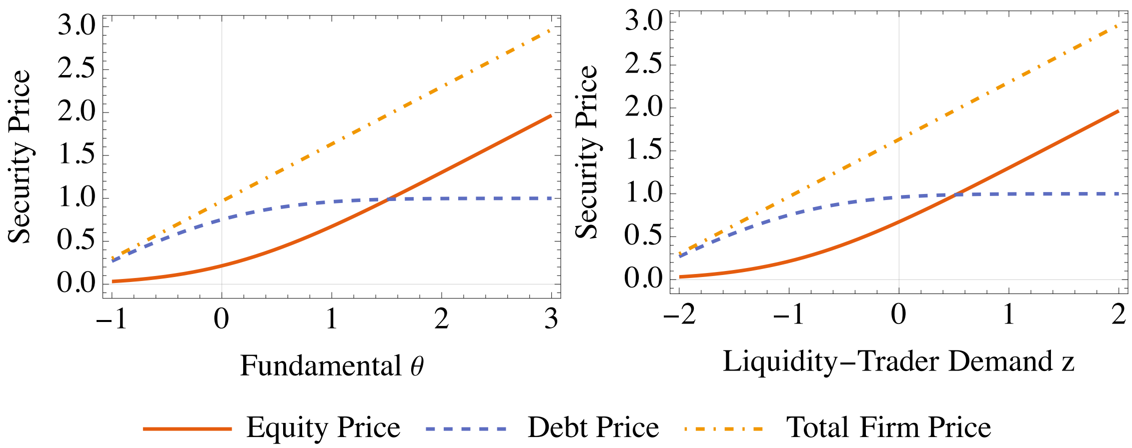
(iii) decrease in per-capita supply of the stock, κ.

Moreover, the firm's equity (debt) price decreases (increases) in the face value of debt, K.

Figure 1 illustrates the equity and debt price functions. The two plots show that the debt and equity prices co-move in our model, which occurs because both investors and liquidity traders speculate on their private information in the same direction in both markets. However, as the fundamental θ or liquidity trade z rise, the reaction in the equity market grows relative to that in the debt market. This reflects the upper and lower bounds on debt and equity payoffs, respectively.

Figure 1: Price Function

This figure plots the equilibrium price of equity, debt, and a claim to the total cash flow of the firm as a function of fundamentals θ and liquidity trade z. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \sigma_z^2 = \rho = \mu = \tau = K = 1; \kappa = 0.1$.



5 Expected Return on Debt and Equity

In our setting the (dollar) return on debt and equity can be expressed as $R_D = V_D - P_D$ and $R_E = V_E - P_E$, respectively.[9] When the security payoff is linear in fundamental shocks and liquidity trade, the expected return typically increases with the per-capita supply of the asset. For instance, note that the price of the unlevered firm from Lemma 1 implies that the expected return on unlevered equity is given by

$$\mathbb{E}[R_U] = \mathbb{E}[\mathcal{V} - P_U] = \frac{\sigma_s^2}{\tau}\kappa.$$

This implies that if the per-capita supply of the firm is zero (i.e., $\kappa = 0$), so is the expected return, because the firm does not expose investors, on average, to any risk. The firm's price under zero net supply corresponds to the price of the idiosyncratic cash flows of a typical firm in the economy. The reason is that, as the typical firm is a small part of the overall economy, its idiosyncratic cash flows exhibit negligible correlation with the average investor's wealth.

In contrast, when security payoffs are non-linear, this is no longer true. The following result illustrates that in general, the expected return on debt and equity systematically differ

[9]The main results in this section extend immediately to percent returns $\frac{V_x - P_x}{P_x}$. The reason is that our results speak primarily to the sign of expected returns on each security, which is unchanged upon dividing by P_x (assuming the mean cash flows are sufficiently high to ensure that the intuitive condition $P_x > 0$ holds). Deriving analytical expressions for percent returns is intractable, however.

from zero, even when the per-capita supply of shares is zero. This implies that idiosyncratic risk is priced in our model.

Proposition 2. *Let $\Omega = \mathbb{V}[P_U] + \sigma_s^2$. The firm's expected equity and debt prices are:*

$$\mathbb{E}[P_E(P_U)] = M_E\left(\mu - \frac{\sigma_s^2}{\tau}\kappa,\, \Omega,\, K\right);$$
$$\mathbb{E}[P_D(P_U)] = M_D\left(\mu - \frac{\sigma_s^2}{\tau}\kappa,\, \Omega,\, K\right).$$

When the per-capita endowment of shares is zero (i.e., $\kappa = 0$), we have that:

(i) The expected return on equity is positive (i.e., $\mathbb{E}[R_E] > 0$) if and only if $\Omega < \sigma_\theta^2$.

(ii) The expected return on debt is positive (i.e., $\mathbb{E}[R_D] > 0$) if and only if $\Omega > \sigma_\theta^2$.

The proof of the above result builds on the observation that the price of each security can be expressed as the subjective conditional expectation of the payoff for the security from the perspective of the representative investor, who believes $\mathcal{V} \sim N(P_U, \sigma_s^2)$ (see Proposition 1). The expressions for $\mathbb{E}[P_E(P_U)]$ and $\mathbb{E}[P_D(P_U)]$ then follow from evaluating the expectations of the security prices over different realizations of P_U. Finally, the claims about the expected security returns follow from convexity (concavity) of payoffs to equity (debt) securities. For instance, note that when $\kappa = 0$, Proposition 2 implies that the expected equity price is given by $\mathbb{E}[P_E] = M_E(\mu, \Omega, K)$, while the expected payoff to equity is given by $\mathbb{E}[V_E] = M_E(\mu, \sigma_\theta^2, K)$. Since M_E reflects the expectation of a convex function of θ, it is increasing in the variance, and so $\mathbb{E}[V_E - P_E] > 0$ when $\Omega < \sigma_\theta^2$.

To understand the economic intuition underlying these results, it is useful to consider the characterization in the following corollary.

Corollary 2. *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$).*

(i) When $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, then for any value of $\rho \in [0, 1]$, the expected return on equity is negative, and the expected return on debt is positive i.e., $\mathbb{E}[R_E] < 0$ and $\mathbb{E}[R_D] > 0$.

(ii) When $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists $\rho^* \in (0, 1)$ such that the expected return on equity is positive when $\rho < \rho^*$ and negative otherwise, and the expected return on debt is negative when $\rho < \rho^*$ and positive otherwise.

To see the intuition behind this result, note that we can rewrite the key quantity that determines the signs of the expected returns, $\Omega - \sigma_\theta^2$, using the law of total variance, as:[10]

$$\Omega - \sigma_\theta^2 = \mathbb{V}\left[P_U\right] - \mathbb{V}_i\left\{\mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right]\right\} \tag{19}$$

$$= \mathbb{V}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau} z\right] - \mathbb{V}_i\left[\mu_i\right]. \tag{20}$$

One can interpret this expression as the difference between the variance of the conditional expectation of the representative investor, who has subjective (conditional) beliefs $\mathcal{V} \sim N(P_U, \sigma_s^2)$, and that of an arbitrary investor, who has subjective (conditional) beliefs $\mathcal{V} \sim (\mu_i, \sigma_s^2)$. The difference in variances is driven by two countervailing effects. On the one hand, P_U is less variable than the expectation μ_i of the average investor because it reflects the aggregate (or average) valuation (i.e., $\mathbb{V}[\int \mu_j dj] < \mathbb{V}_i[\mu_i]$). On the other hand, P_U is more variable because it is more sensitive to liquidity-trading shocks via the "risk compensation" term $\frac{\sigma_s^2}{\tau} z$.

We show in the appendix that we can re-express equation (20) as follows:

$$\Omega - \sigma_\theta^2 = -\underbrace{\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right]}_{\text{(1) Belief dispersion}} + \underbrace{\mathbb{V}\left[\frac{\sigma_s^2}{\tau} z\right]}_{\text{(2) Liquidity-trading variability}} \tag{21}$$

$$+ \underbrace{2 \times \mathbb{C}\left[\int \mu_j dj, \frac{\sigma_s^2}{\tau} z\right]}_{\text{(3) Mean-liquidity trade correlation}} + \underbrace{\mathbb{V}\left[\mu_i\right] - \mathbb{V}_i\left[\mu_i\right]}_{\text{(4) Subjective variance difference}} \tag{22}$$

The above decomposition helps clarify the economic forces that drive expected returns. We focus on the case of equity; the intuition is precisely reversed in the case of debt.

(1) **Belief dispersion.** First, equilibrium disagreement across investors leads to a reduction in $\Omega - \sigma_\theta^2$, and consequently, an increase in equity returns. Recall from Lemma 3 that an investor's demand for equity is linear in her expectation of firm cash flows, $\mathbb{E}_i[\mathcal{V}]$. However, because the *equity payoff* depends non-linearly on $\mathcal{V}$, this implies that the investor's demand responds non-linearly to her expectation of this payoff, $\mathbb{E}_i[V_E]$.

Specifically, an investor responds more strongly to a unit decrease in her conditional expectation than a unit increase i.e., her demand is *concave* in her expectation of equity cash flows. Intuitively, this is driven by the fact that investors become more uncertain about the equity's payoffs and so perceive more equity risk when they receive a higher signal (i.e., $\frac{\partial \mathbb{V}_i[V_E|s_i,P_U]}{\partial s_i} > 0$).[11] As a result, more pessimistic signals are disproportion-

[10] In particular, the law of total variance yields $\sigma_\theta^2 = \sigma_s^2 + \mathbb{V}_i\left\{\mathbb{E}_i\left[\mathcal{V}|s_i, P_U\right]\right\}$.

[11] Formally, the observation follows from the fact that an investor's expectation of the equity payoff,

ately reflected in the price, which pushes it lower. Note that this feature is absent in traditional settings where the payoff is conditionally normal and so investors' demands (and prices) are linear in their conditional expectations.

(2) **Liquidity-trading variability.** The impact of liquidity trading on prices is also non-linear. When liquidity traders sell the firm's equity, investors must hold larger long positions and demand a drop in price to do so. However, since the equity payoffs are truncated from below (i.e., equity payoffs are positively skewed), the downside from being long is limited, and the price compensation is relatively small.[12] On the other hand, when liquidity traders buy equity, informed investors bear the risk of being short. In this case, their downside is unlimited and so they charge a large increase in the price for bearing the risk. On average, this pushes prices up and thus reduces expected returns.[13]

(3) **Mean-liquidity trade correlation.** The third term in the decomposition reflects the fact that the investors' average valuation $\int \mu_j dj$ is positively correlated with liquidity-trader demand z because investors condition on the information in prices when forming their beliefs (when $\rho > 0$). Specifically, when liquidity traders purchase shares, this pushes the price up. However, since investors cannot readily detect whether price changes are driven by liquidity trade or information, this increases investors' conditional expectations of cash flows.

    An increase in this correlation reduces expected returns because it amplifies the forces above. Specifically, an increase in buying from liquidity traders not only exposes informed investors to more downside risk (as in (2) above), but also increases their conditional expectation of cash flows, which increases their perceived risk (as in (1) above). The resulting increase in price that investors demand as compensation is large relative to the drop in prices they demand when liquidity traders sell. On average, this leads to higher expected prices and lower expected returns.

(4) **Subjective variance difference.** The fourth, and final, term in the decomposition reflects the fact that the actual variance of investor beliefs is (weakly) higher than their

$M_E(\mathbb{E}_i[\mathcal{V}], \sigma_s^2, K)$, is an increasing and convex function of her expectation of total cash flows, $\mathbb{E}_i[\mathcal{V}]$, and so its inverse is concave.

[12]Investors with CARA utility exhibit a preference for such skewness, see e.g., Eeckhoudt and Schlesinger (2006).

[13]This asymmetric risk-compensation effect is absent in traditional models with linear prices because the value is symmetric and unbounded (usually normal). However, it is analogous to the "skewness effect" discussed in Albagli et al. (2021), Chabakauri et al. (2021), Cianciaruso, Marinovic, and Smith (2022), and Banerjee, Marinovic, and Smith (2022).

subjective variance. This term arises because each investor perceives the price signal as being less correlated with her private signal than it truly is. Consequently, investors underestimate how strongly their beliefs will vary with θ. This term is zero if and only if $\rho \in \{0, 1\}$. For $\rho = 1$, investors correctly condition on price and therefore their subjective variance of beliefs is equal to the objective one. On the other hand, for $\rho = 0$, investors condition only on their private signals, for which they know the correct variance.

Importantly, the combined effect of these forces depends on how investors learn from the price. When investors ignore the information in price (i.e., $\rho = 0$), the Mean-liquidity trade correlation term is zero. Moreover, the belief dispersion channel dominates the liquidity-trading variability channel if and only if σ_z^2 is sufficiently low relative to investors' risk tolerance τ and private information quality $1/\sigma_\varepsilon^2$; in this case, the expected return on equity is positive.

In contrast, when investors have rational expectations and fully incorporate the information in prices (i.e., $\rho = 1$), the mean-liquidity trade correlation term is maximized. Moreover, the belief dispersion channel is relatively weak since disagreement tends to be low when investors condition on a common, public (price) signal. Thus, belief dispersion tends to be dominated by the other forces so that the equity earns negative expected returns. Surprisingly, this implies the difference between expected cash flows and equity prices may be larger under rational expectations than under difference of opinions.

We next characterize how expected returns on the two securities relate to the model's parameters.

Corollary 3. *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$).*

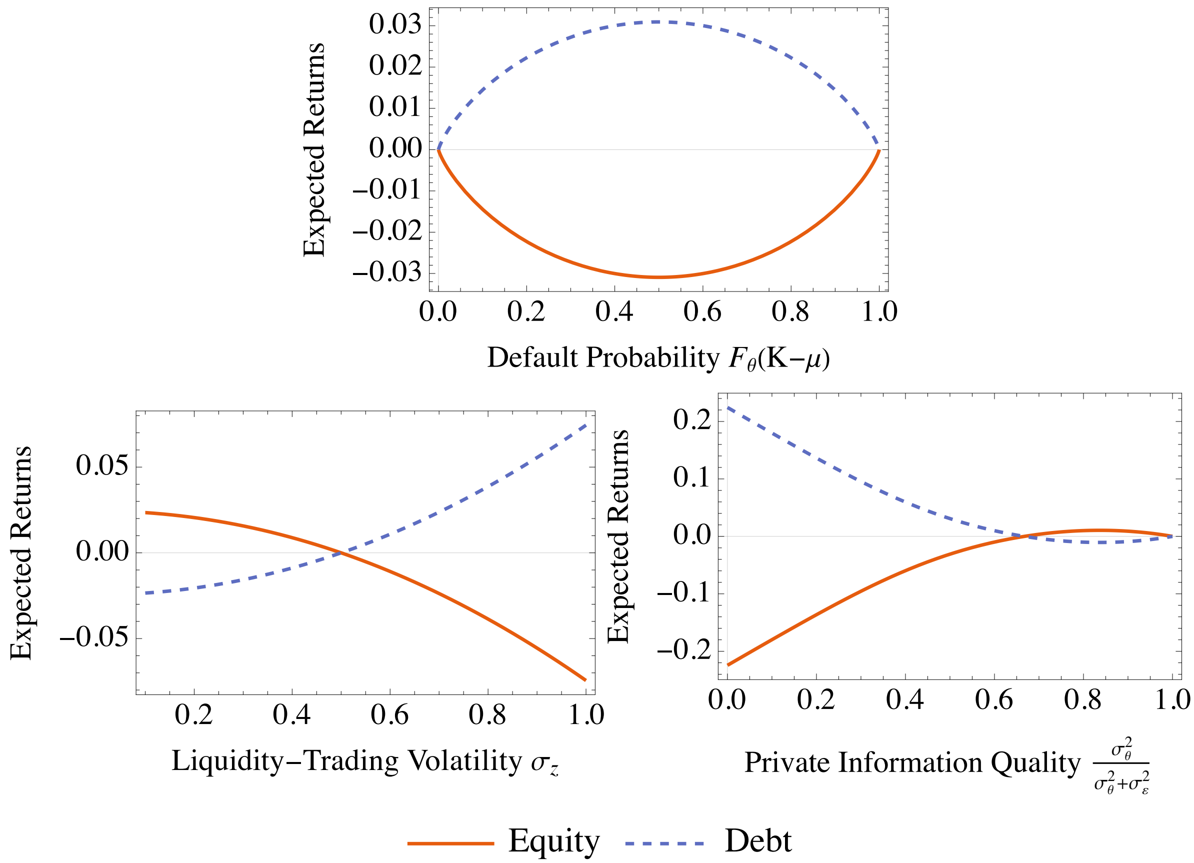
(i) The magnitudes of the expected returns in the debt and equity, $|\mathbb{E}[R_E]|, |\mathbb{E}[R_D]|$, are hump-shaped in K and maximized at $K = \mu$.

(ii) Expected equity returns decrease and expected debt returns increase with liquidity-trading volatility σ_z.

Figure 2 illustrates the corollary. The upper panel demonstrates how expected returns vary with the firm's leverage, in terms of the probability of default. The plot shows that the magnitude of these returns are maximized when the debt level is equal to the firm's expected cash flows, corresponding to a 50% default probability. Intuitively, this reflects the fact that the concavity (convexity) of debt (equity, respectively) payoffs is maximized given a 50% probability of default, similar to the notion that an option's convexity is maximized when it is at-the-money.

Figure 2: Expected Return Comparative Statics

This figure plots expected returns on the equity and debt as a function of the model parameters. The parameters are set to: $\sigma_\theta^2 = \sigma_\varepsilon^2 = \mu = \tau = K = 1; \sigma_z^2 = 0.75^2; \kappa = 0; \rho = 0.5$.



These results suggest that for the typical firm with default probability well below 50%, expected returns on debt increase, and expected returns on equity fall with default risk (even when this risk is idiosyncratic). This is consistent with the credit-spread puzzle, i.e., the finding that expected debt returns increase with default risk. The plot also demonstrates that the marginal impact of distress risk on expected debt returns is strongest for firms that have a low degree of default risk, which we have verified analytically.

Next, when liquidity-trading volatility rises, the variance of P_U rises, which reduces equity returns and increases debt returns. Finally, prior uncertainty and private information quality, which we can jointly capture via the signal to noise ratio $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, have a non-monotonic impact on expected returns. This follows from the observation that these parameters have a non-monotonic impact on disagreement. When investors' private information quality is very high, their beliefs converge towards the true value of the firm, and hence there is no disagreement. Likewise, when investors' private information quality is very low, they rely on their common priors and so do not disagree.

5.1 Public Information

Next, we characterize the impact of introducing a public signal, which enables us to connect our analysis to the empirical literature on how earnings and other sources of public news influence returns. Formally, we now assume that there is a public signal released prior to trade:

$$y = \theta + \eta;\ \eta \sim N\left(0, \sigma_\eta^2\right),$$

where η is independent of all other random variables. Because the public signal is jointly normally distributed with the rest of the random variables in the economy, the derivation of the equilibrium characterized in Proposition 1 continues to hold upon updating investors' beliefs to reflect the public signal. Hence, the equilibrium prices and demands are equal to those stated in Proposition 1 after replacing $\mathbb{E}[\mathcal{V}] = \mu$ with $\mathbb{E}\left[\mathcal{V}|y\right] = \mu + \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}\right)^{-1} \frac{1}{\sigma_\eta^2} y$ and the prior variance of $\mathcal{V}$, $\mathbb{V}[\mathcal{V}] = \sigma_\theta^2$, with $\mathbb{V}\left[\mathcal{V}|y\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2}\right)^{-1}$, and adjusting the updating accordingly.

The following result builds on this observation.

Proposition 3. *Suppose the per-capita endowment of shares is zero (i.e., $\kappa = 0$). An increase in public information quality (i.e., higher $1/\sigma_\eta^2$) reduces the magnitude of expected returns in equity and debt. Thus,*

(i) If $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$, then for any $\rho \in [0, 1]$, an increase in public information quality raises expected equity returns and lowers expected debt returns.

(ii) If $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists $\rho^* \in (0, 1)$ such that an increase in public information quality lowers expected equity returns and raises expected debt returns when $\rho < \rho^*$, and vice versa when $\rho > \rho^*$.

Note that when the aggregate endowment of shares is zero, the expected return on unlevered equity (i.e., $\mathbb{E}[R_V]$) is zero, and so public information quality has no effect on expected returns. This reflects the fact that, when $\kappa = 0$, information about the firm's cash flows $\mathcal{V}$ is purely idiosyncratic and can be diversified away.

However, when the firm has leverage, this no longer applies. Interestingly, public information attenuates the magnitude of expected returns in the securities in the model, regardless of whether these returns are positive or negative. Intuitively, recall that expected returns are driven by the four effects outlined in expression (21). Applying (8), this expression reduces as follows:

$$\Omega - \sigma_\theta^2 = \sigma_s^4 \left(-\mathbb{E}\left[\frac{1}{\sigma_\varepsilon^4} \left(s_i - \int s_j dj \right)^2 \right] + \mathbb{V}\left[\frac{1}{\tau} z \right] + 2 \times \mathbb{C}\left[\frac{1}{\rho \sigma_p^2} s_U, \frac{1}{\tau} z \right] \right.$$
$$\left. + \mathbb{V}\left[\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho \sigma_p^2} \right] - \mathbb{V}_i \left[\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho \sigma_p^2} \right] \right),$$

i.e., each of these four effects is proportional to the square of investors' posterior variances, σ_s^4. As public information reduces investors' posterior uncertainty, it has a proportional, negative impact on each of the three drivers of expected returns. This implies that, in contrast to standard intuition, firm-specific public information should have the opposite effect on debt and equity returns. If the increase in public information quality increases equity returns, it should reduce debt returns, and vice versa. We discuss testable implications of this result in Section 7.

6 Imperfectly-Correlated Liquidity Trade

In this section, we consider a generalization of the baseline setting in which liquidity-traders' demands in the debt and equity markets differ, but are potentially correlated. Specifically, in contrast to the baseline setting described in Section 3, demand shocks $z = (z_D, z_E) \in \mathbb{R}^2$ follow a general bivariate normal distribution $z \sim N(\mathbf{0}, \Sigma_z)$ with Σ_z an arbitrary 2×2 positive definite covariance matrix.[14] As in the baseline setting, we let $x_{D,i}$ and $x_{E,i}$ denote the investor's demand for debt and equity respectively, with $x_i = (x_{D,i}, x_{E,i})$ the vector of demands.

The definition of a generalized linear equilibrium is analogous to that above, but generalized to account for the fact that in this setting the debt and equity prices generally depend on two non-identical linear statistics.

Definition 3. *A "generalized linear equilibrium" is one in which there exists an injective function $P(\cdot,\cdot) = (P_D(\cdot), P_E(\cdot))$ mapping $\mathbb{R}^2$ into $\mathbb{R}^2$ and linear statistics of the form*

$$s_{p1} = \int s_j dj + \beta_{1D} z_D + \beta_{1E} z_E$$
$$s_{p2} = \int s_j dj + \beta_{2D} z_D + \beta_{2E} z_E$$

[14] The proofs of all results in this section allow for an arbitrary mean vector $\mu_z \in \mathbb{R}^2$ and allow for a covariance matrix Σ_z that is only positive semi-definite. In the text, we normalize the means to zero and consider only strictly positive definite Σ_z for expositional clarity.

such that the equilibrium price is

$$P(s_{p1}, s_{p2}) = \begin{pmatrix} P_D(s_{p1}, s_{p2}) \\ P_E(s_{p1}, s_{p2}) \end{pmatrix}.$$

Let $\overline{s} \equiv \int s_j dj$ denote the cross-sectional average signal and let

$$s_p = \mathbf{1}\overline{s} + Bz$$

concisely denote the stacked vector of price-signals, with $\mathbf{1}$ a conformable vector of ones and $B = \left(\begin{smallmatrix} \beta_{1D} & \beta_{1E} \\ \beta_{2D} & \beta_{2E} \end{smallmatrix}\right)$ the 2×2 matrix of coefficients on z. In the main text we will focus on the case in which Σ_z is invertible (i.e., strictly positive definite).[15]

Given s_i and the conjectured s_p, investor i's beliefs about the firm cash flow $\mathcal{V}$ are normal with conditional moments

$$\mu_i \equiv \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = \mu + \sigma_s^2 \left(\frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right), \text{ and} \tag{23}$$

$$\sigma_s^2 \equiv \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1}\right)^{-1}, \tag{24}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2\mathbf{1}\mathbf{1}' + \frac{1}{\rho^2}B\Sigma_z B'$.[16] These are the analogues to Equations (8)-(9) in the benchmark analysis.

We next extend our characterization of the investor's optimal demand in Lemma 3 to this case.

Lemma 4. *Fix any $P = (P_D, P_E) \in \mathbb{R}^2$. The optimal demand of trader i is given by*

$$x_i = \frac{\tau}{\sigma_s^2}\left(\mathbf{1}\mu_i - G\left(P\right)\right),$$

where $G : \mathbb{R}^2 \to \mathbb{R}^2$ is a function defined in the proof.

As before, investor i's optimal demand is additively separable in her beliefs μ_i and the prices, and her trading aggressiveness again remains the same in each security. The equilibrium debt and equity prices follow from imposing market clearing and matching coefficients on the price-signal vector s_p.

[15]The case in which Σ_z is singular (e.g., perfectly correlated liquidity trade across both markets, or one of the liquidity trades constant) is considered in the formal derivations in the appendix.

[16]Because Σ_z is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, Σ_p, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho = 0$.

Proposition 4. *There exists an equilibrium in the financial market. The vector of equilibrium asset prices takes the form*

$$P = g'\left(\mathbf{1}\frac{\int \mu_j \, dj}{\sigma_s^2} - \frac{1}{\tau}\left(\kappa\mathbf{1} - z\right)\right) \tag{25}$$

$$= g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{26}$$

where the equilibrium coefficient matrix is $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, *and* $g' : \mathbb{R}^2 \to \mathbb{R}^2$ *is the gradient of a function* $g : \mathbb{R}^2 \to \mathbb{R}$, *both given in closed-form in the Appendix.*

The above result extends the generalized linear equilibrium characterized in Proposition 1. Combining the expression for the optimal demand from Lemma 6 and the equilibrium price in this proposition immediately yields the equilibrium quantity demanded by each investor, which we record in the following corollary.

Corollary 4. *The equilibrium demand of investor* i *is*

$$x_i = \tau\frac{\mu_i - \int \mu_j dj}{\sigma_s^2}\mathbf{1} + \kappa\mathbf{1} - z. \tag{27}$$

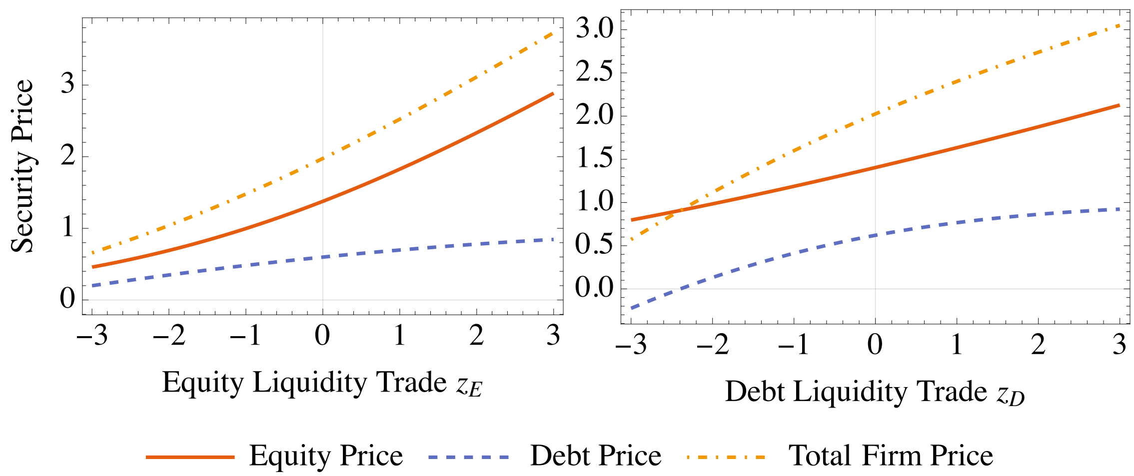
This result shows that the speculative portion of each investor's holdings are equal across the debt and equity markets, and, as in our benchmark model, are given by $\tau\frac{\mu_i - \int \mu_j dj}{\sigma_s^2}$. Thus, investors' debt and equity demands differ if and only if the liquidity trade in the debt and equity markets differ.

6.1 Cross-Market Demand Spillovers

Figure 3 illustrates how the equity and debt prices respond to liquidity-trader demand in each market. Specifically, liquidity-trader demand for a given security affects not only the price of that security, but also the price of the other security. Intuitively, this is driven by both information and risk effects. Since demand in either security may be perceived as informed, it raises investors' expectations of cash flows, and consequently, the price of both securities. In addition, holding debt (equity) exposes an investor to the risk of the firm's underlying cash flows, which also makes them view the equity (debt) as riskier. Thus, equity demand also raises the price of debt, and vice versa, via investor risk aversion. However, demand for equity has a much stronger effect on the equity price than on the debt price through this risk aversion effect, and vice versa. As a result, the demand spillover between the two markets is incomplete in the sense that z_E has a stronger impact on the equity price than z_D, and z_D has a stronger impact on the debt price than z_E.

Figure 3: Cross-Market Demand Spillovers

This figure plots the expected security prices conditional on equity liquidity trade z_E (left panel) and debt liquidity trade z_D (right panel). The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.



As in the baseline model (e.g., see Figure 1), the equity (debt) price is a convex (concave) function of demand in *each* market. This, in turn, implies that our main results regarding expected returns continue to hold in this case. Interestingly, however, the total price of the firm (i.e., $P_E + P_D$) is convex in equity liquidity-trader demand, but concave in debt liquidity-trader demand. As we discuss further in the next subsection, this implies that the Modigliani-Miller theorem no longer holds in this setting. Instead, the sum of the firm's debt and equity prices is greater (lower) than the price of an unlevered firm, on average, when the volatility of equity liquidity trading is higher (lower) than that of debt liquidity trading.

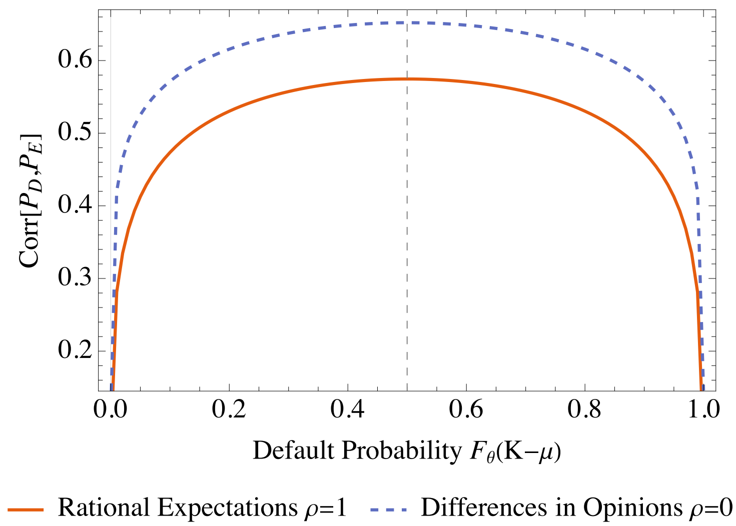
The incomplete spillover of demand shocks across securities also affects the correlation between equity and debt prices. As Figure 4 illustrates, the correlation between debt and equity prices is maximized when the likelihood of default is 50%.[17] Intuitively, when the default probability approaches zero, the payoff to debt is almost risk-free, and so demand shocks in either security have little impact on the debt price but significant impact on the equity price. Similarly, when the probability of default approaches one, the value of equity

[17]Pasquariello and Sandulescu (2021) derive a similar result in a Kyle model where risk-neutral market makers specialize in either debt or equity, while investors can trade both securities. As such, the market making in their model is segmented: the price in each market depends only on the order flow in that market. In contrast, markets are integrated in our setting: investors can update their beliefs from equity and debt prices and can trade in both markets.

approaches zero and is relatively insensitive to demand shocks, but the price of debt remains responsive to such shocks. As a result, the correlation in prices approaches zero in both extremes. In contrast, for intermediate levels of distress, both security prices are sensitive to demand shocks, and so price correlation is high.

Figure 4: Leverage and Debt-Equity Price Correlation

This figure plots the correlation between the debt and equity prices as a function of the firm's default risk. The parameters are set to: $\mu = 3; K = 2; \sigma_\theta^2 = 1; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \tau = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$.



6.2 Capital Structure and Total Firm Valuation

We next show that, in contrast to our baseline specification, the Modigliani-Miller theorem does not hold, i.e., the firm's equity and debt prices do not, in general, sum to the price of the unlevered firm: $\mathbb{E}[P_E + P_D] \neq \mathbb{E}[P_U]$. As a result, the firm's capital structure can meaningfully impact its value, even in the absence of traditional frictions (e.g., tax shields of debt, costs of financial distress).

In Figure 5, we show that the expected price of the debt plus equity relative to the price of the unlevered firm depends upon the relative amount of liquidity trade in the two markets. For instance, the left panel of Figure 5 plots $\mathbb{E}[P_E + P_D]$ as a function of $\sqrt{\mathbb{V}[z_E]}$, holding fixed the volatility of debt liquidity trade (i.e., $\sqrt{\mathbb{V}[z_D]}$). The plot illustrates that the expected value of debt plus equity is higher than the expected value of the unlevered firm (i.e., $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U] = 1$) if and only if the volatility of equity liquidity trading is higher than that of debt liquidity trading (i.e., $\mathbb{V}[z_E] > \mathbb{V}[z_D]$). Intuitively, this is because liquidity trade in the equity market does not fully spill over into the debt market. As such, the price-increasing effect that equity liquidity-trading volatility has on equity prices tends to raise the overall value of the firm.

This result holds irrespective of whether investors use the information in prices (i.e., whether $\rho = 1$ or $\rho = 0$), but is stronger when investors do not condition on prices (i.e., when $\rho = 0$). This is because, even though the effect of belief dispersion on expected prices in the two securities precisely offset, investors face more uncertainty when they do not condition on prices, and this increases the sensitivity of prices to liquidity trading shocks. The right panel of Figure 5 shows the same result by plotting the expected value of debt plus equity as a function of debt liquidity-trading volatility, holding fixed the equity liquidity-trading volatility.

Figure 5: Violations of Modigliani-Miller

This figure plots the total expected firm price, $\mathbb{E}[P_E + P_D]$, as a function of liquidity trade volatility in the two markets. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mathbb{V}[z_E] = \mathbb{V}[z_D] = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0$. Note that $\mathbb{E}[P_U] = \mu = 1$.

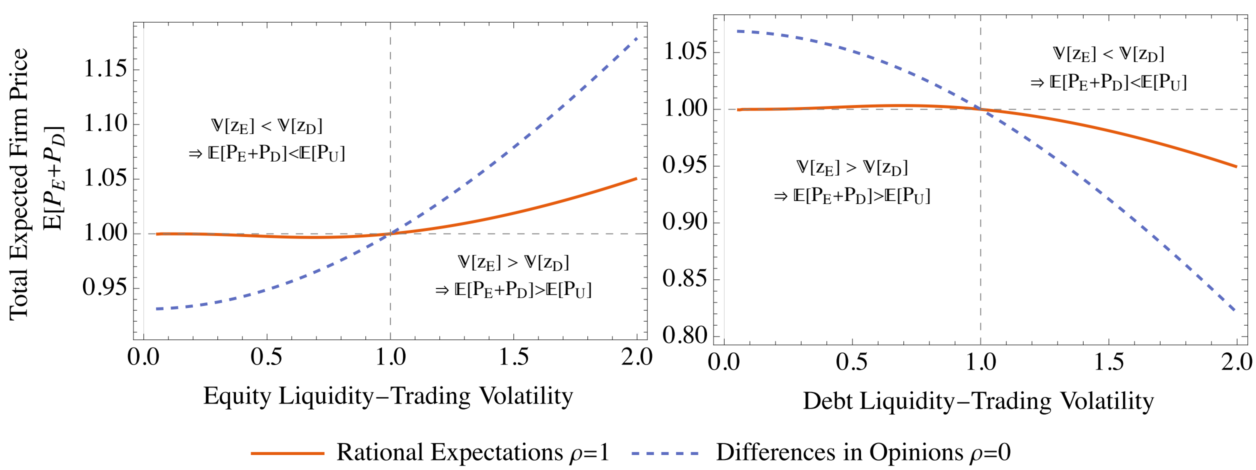
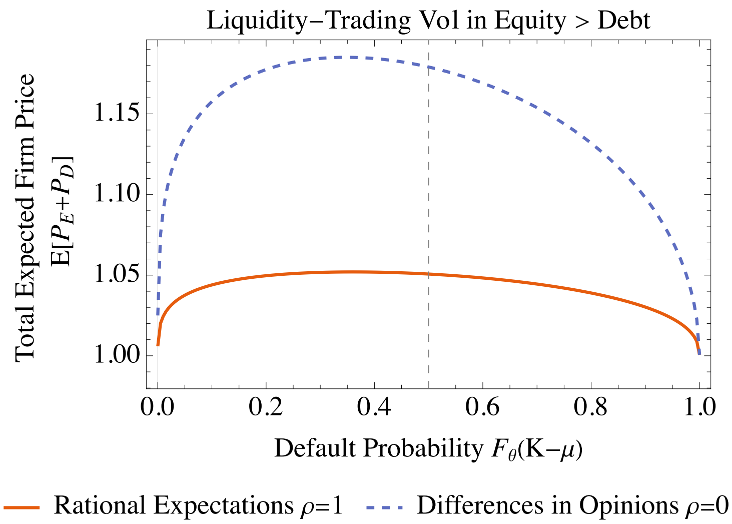


Figure 6 illustrates how the firm's capital structure influences its valuation. When equity liquidity-trading volatility exceeds that in the debt, the value of the levered firm is higher than an unlevered firm $\mathbb{E}[P_E + P_D] > \mathbb{E}[P_U]$. In this case, there is an interior optimal capital structure that maximizes the firm's valuation. Intuitively, an all equity or all debt firm is suboptimal as, in either case, the firm has linear payoffs, and so $\mathbb{E}[P_E + P_D] \to \mathbb{E}[P_U]$. This implies that even in the absence of traditional frictions associated with leverage, heterogeneity in information, beliefs, and liquidity trading across debt and equity markets causes the value of the firm to be maximized at an interior level of debt.

Figure 6: Optimal Capital Structure

This figure plots the total expected firm price, $\mathbb{E}[P_E + P_D]$, as a function of the firm's leverage, paramaterized in terms of its ex-ante probability of default, $F_\theta(K-\mu)$. The parameters are set to: $\sigma_\theta^2 = 1.5^2; \sigma_\varepsilon^2 = \mu = \tau = K = 1; \kappa = 0; \mathbb{C}[z_E, z_D] = 0; \mathbb{V}[z_E] = 4$; and $\mathbb{V}[z_D] = 1$. Note that $\mathbb{E}[P_U] = \mu = 1$.



7 Empirical Implications

Our model generates predictions on expected debt and equity returns and how these returns vary with financial distress, the quality of public information, disagreement, and the intensity of liquidity trade. Existing work proposes several proxies for these constructs that render our predictions directly testable. For instance, considerable work proxies for disagreement using volume and analyst forecast dispersion (Diether, Malloy, and Scherbina (2002), Banerjee (2011)), while other research proxies for liquidity-trader volatility based upon the concentration of a firm's ownership and the correlation in the liquidity shocks its owners face (Greenwood and Thesmar (2011), Friberg, Goldstein, and Hankins (2022)). We summarize our model's predictions and their relation to existing empirical work below; several are consistent with existing empirical analyses while others have yet to be tested.

Given that a key feature of our model is the ability of both diversely-informed investors and liquidity traders to take positions in debt, our results apply most clearly to public debt markets. As such, when referencing debt markets, we focus on the literature on public bond markets. Note our findings speak specifically to expected returns after controlling for systematic risk exposures. Specifically, the predictions are about "alphas" from the perspective of an econometrician who accounts for systematic risk (or "betas"), even though these returns do not reflect any mispricing from the perspective of investors in the model.

Finally, we focus on our predictions for firms with less than 50% probability of default, as such firms represent the vast majority of publicly-traded stocks.

Decomposing the dispersion in beliefs. In our model, dispersion in beliefs increases with (i) the extent to which investors disagree about the informativeness of others' signals (a decrease in ρ) and (ii) the volatility of liquidity trading (an increase in σ_z), all else equal. However, these have opposing effects on expected returns: holding all else fixed, an increase in liquidity-trading volatility decreases equity returns (increases debt returns), but an increase in disagreement (decrease in ρ) can increase equity returns (decrease debt returns). As such, our analysis emphasizes the importance of distinguishing between belief dispersion driven by *disagreement* (lower ρ) versus *liquidity-trading volatility* (higher σ_z).

This also sheds light on the mixed empirical evidence on the relation between belief dispersion and equity returns. The existing empirical literature finds that the sign and magnitude of this relation varies with the empirical proxy for disagreement, firm size, and time period considered (e.g., Diether et al. (2002), Johnson (2004), Banerjee (2011), Cen, Wei, and Yang (2017), Hou, Xue, and Zhang (2020), Chang, Hsiao, Ljungqvist, and Tseng (2022)). Our model predicts that in samples (or for proxies) where dispersion in beliefs is primarily driven by disagreement, the relation between expected returns and dispersion should be positive. On the other hand, when it is driven by liquidity-trading volatility, the relation should be negative.

Our analysis suggests the following regressions, where the predicted signs are presented below the coefficients:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{>0} Disagreement_t + \underbrace{\beta_{2,E}}_{<0} LiqTradeVol_t;$$

$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{<0} Disagreement_t + \underbrace{\beta_{2,D}}_{>0} LiqTradeVol_t.$$

In other words, controlling for variation in liquidity-trading volatility, the expected return on equity should be positively related to disagreement, while the expected return on debt should be negatively related to disagreement.

Distress risk and expected returns. Several empirical studies examine the relationship between firm-specific distress risk and equity returns, controlling for standard systematic risk exposures. However, such a relationship is difficult to reconcile with traditional models due to the forces of diversification, leading the literature to propose that distress risk is mispriced (e.g., Campbell et al. (2008)). Our analysis provides a rational explanation of these results in an equilibrium model without mispricing. Moreover, our model predicts that distress risk is *negatively* associated with equity returns among firms far from bankruptcy, which is consistent with the empirical evidence (e.g., Dichev (1998), Campbell et al. (2008), Penman, Richardson, and Tuna (2007), and Caskey, Hughes, and Liu (2012)).

In the debt market, several studies find that default risk has an excessive impact on credit spreads, commonly termed the credit-risk puzzle (e.g., Huang and Huang (2012), Bai et al. (2020)). Our analysis implies that expected debt returns increase with firm-specific distress risk for firms with less than a 50% probability of default is further consistent with this finding. Note while other studies offer explanations in terms of systematic distress risk (e.g., Chen, Hackbarth, and Strebulaev (2022)), our findings are novel in that they speak to firm-specific distress risk. Moreover, our model predicts that the marginal impact of distress risk on expected debt returns is strongest for firms that have a low degree of default risk.

Our model further predicts that the relationship between distress risk and expected stock returns depends on (i) the extent to which investors disagree and (ii) the prevalence of liquidity trade in a firm's stock and bonds. Thus, our analysis motivates the following regressions for firms with less than 50% probability of default, where main effects are omitted for brevity and the predicted signs are presented below the coefficients:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{>0} Distress_t \times Disagreement_t + \underbrace{\beta_{2,E}}_{<0} Distress_t \times LiqTradeVol_t;$$
$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{<0} Distress_t \times Disagreement_t + \underbrace{\beta_{2,D}}_{>0} Distress_t \times LiqTradeVol_t.$$

These predictions follow from the observation that the expected return on equity (debt) is decreasing (increasing) in the difference in variance, $\Omega - \sigma_\theta^2$, which is decreasing in investor disagreement and increasing in liquidity-trading volatility (e.g., see (21)). Moreover, our analysis suggests that for extremely distressed firms, distress risk has the opposite impact on expected returns. Note that some work finds non-monotonic or positive relationships between distress risk and returns (e.g., Chava and Purnanandam (2010), Garlappi et al. (2008)), and these cross-sectional predictions may be useful in future work to reconcile these mixed results.

Public information quality and expected equity / debt returns. Our model also speaks to the longstanding literature that studies public information quality and expected stock returns (see the reviews by Dechow, Ge, and Schrand (2010) and Bertomeu and Cheynel (2015)), and the more sporadic work that considers public information quality and expected bond returns. Our model predicts that there is cross-sectional and time-series variation in the relation between public information quality and expected bond and equity returns.

Specifically, our analysis in Section 5.1 motivates the following regressions:

$$R_{E,t+1} = \beta_{0,E} + \underbrace{\beta_{1,E}}_{<0} InfoQuality_t \times Disagreement_t + \underbrace{\beta_{2,E}}_{>0} InfoQuality_t \times LiqTradeVol_t;$$

$$R_{D,t+1} = \beta_{0,D} + \underbrace{\beta_{1,D}}_{>0} InfoQuality_t \times Disagreement_t + \underbrace{\beta_{2,D}}_{<0} InfoQuality_t \times LiqTradeVol_t,$$

where disagreement is measured *after* the release of the public information under consideration (but prior to the period over which future returns are measured).

The existing literature on accounting information and bond returns provides some evidence consistent with our predictions. Bharath et al. (2008) find that higher-quality accounting is associated with lower interest spreads in the public bond market, consistent with our model's predictions when liquidity traders tend to be inactive and disagreement is high. Bonsall and Miller (2017) find that lower quality accounting information (as proxied by financial statement readability) increases disagreement and tends to be associated a higher cost of debt. Jaskowski and Rettl (2023) exploits staggered implementation of EDGAR to show that lower information acquisition costs are associated with lower credit spreads. Chang et al. (2022) exploits the same shock to show that for firms with better public information, disagreement around earnings announcements resolves more quickly, and so equity returns are more negative. The findings in these two papers are consistent with our model's predictions when ρ and liquidity-trading volatility (i.e., σ_z) are sufficiently low – see Proposition 3 (ii).

Covariance between expected equity and debt returns. A central prediction of our model is that a firm's expected equity and debt returns are inversely related. The key intuition is that equity and debt payoffs are skewed in opposite directions. Again, our predictions concern "alphas" for debt and equity, after accounting for the appropriate "betas" that capture systematic risk exposures. Such exposures likely lead to common sources of variation in expected debt and equity returns that counteract the source of return variation we study. Past empirical evidence showing that several of the factors that predict equity returns do not predict debt returns is consistent with this finding (Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017), Choi and Kim (2018), Bali, Subrahmanyam, and Wen (2021)).

Co-movement in equity and bond prices. The analysis in Section 6.1 implies that shocks to demand in either security impact both debt and equity prices, and so induce correlation in these securities' prices. Our results are broadly consistent with the evidence in Back and Crotty (2015), who show that while the unconditional correlation between stock and bond returns is low, the correlation in the parts driven by order flow is quite large. Our analysis suggests that the stock-bond correlation is higher when liquidity trading in the two markets is more correlated. Our results are also consistent with the evidence of Pasquariello and Sandulescu (2021), who document that the stock-bond correlation is low when the firm-level default probability is either very high or very low, but higher otherwise.

Capital structure and firm valuation. When liquidity traders' demands in equity and bond markets are not identical, our model implies that the capital structure of the firm affects its total valuation, even in the absence of traditional frictions (e.g., tax shields of debt, distress costs). Since we expect that for most firms, the probability of default is lower than 50% and the volatility of liquidity trading in equity is higher than that in debt, our model predicts that an increase in leverage leads to an increase in firm value. Moreover, the model predicts that, ceteris paribus, the impact of an increase in leverage is larger when investors dismiss the information in prices.

8 Conclusion

We develop a model where privately-informed, risk-averse investors trade alongside liquidity traders in the debt and equity of a firm. We show that the impact of private and public information on security valuation depends on the firm's likelihood of default, the intensity of liquidity trading in each market, and the extent to which investors learn from prices. Finally, we show that a firm's capital structure can affect its total valuation even in the absence of traditional frictions associated with debt issuance (e.g., tax shields, distress costs).

Our model generates a number of novel empirical predictions about the relation among disagreement, liquidity trading, distress risk, and debt and equity valuation. Moreover, our model serves as a useful benchmark for future theoretical analysis. For instance, it would be interesting to explore the incentives of investors to acquire information (e.g., Davis (2017)) in our setting when the liquidity trading in debt and equity are not identical, as well as to study the effects of segmentation across debt and equity markets. It would also be interesting to study how joint trade in equity and debt influence managers' investment decisions, both through their costs of capital and through managerial learning from debt and equity prices (as explored by Davis and Gondhi (2019)).

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A Proofs

A.1 Proof of Lemmas 1 and 2

These results are limiting cases of Proposition 1 below.

A.2 Proof of Lemma 3

This result is a special case of Lemma 6, where we define $G(P) \equiv \sigma_s^2 \times (g')^{-1}(P)$ to condense notation in the statement of the Lemma.

A.3 Proof of Proposition 1

The existence of a generalized linear equilibrium is a special case of Proposition 5 and the representation of the equilibrium demands is a special case of Corollary 4. It remains to show that the expression for the equilibrium price from Proposition 5 can be represented in terms of the M_E and M_D functions and that the equilibrium debt and equity prices sum to P_U. From Proposition 5, we have that the equilibrium price vector satisfies

$$\begin{aligned} P &= g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_U - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \\ &= g'\left(\mathbf{1}\frac{P_U(\theta, z)}{\sigma_s^2}\right), \end{aligned}$$

where the second line uses the definition of P_U (from Lemma 1) to simplify the argument of the gradient g' and where the function $g : \mathbb{R}^2 \to \mathbb{R}$ satisfies

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right).$$

Computing the two partial derivatives that make up the gradient g' yields

$$\begin{aligned} \frac{\partial g}{\partial y_1} = &\left(\sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}\right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \\ &+ K\frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}; \\ \frac{\partial g}{\partial y_2} = &\left(\sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)}{1 - \Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)} - K\right) \frac{\exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1-y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}. \end{aligned}$$

Evaluating these expressions at $y_1 = y_2 = \frac{P_U}{\sigma_s^2}$ gives the debt and equity prices, respectively:

$$\begin{aligned} P_D = \left.\frac{\partial g}{\partial y_1}\right|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U - \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{\Phi\left(\frac{K-P_U}{\sigma_s}\right)}\right) \Phi\left(\frac{K-P_U}{\sigma_s}\right) + K\left(1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_D(P_U, \sigma_s^2, K), \end{aligned}$$

and

$$\begin{aligned} P_E = \left.\frac{\partial g}{\partial y_2}\right|_{y_1=y_2=\frac{P_U}{\sigma_s^2}} &= \left(P_U + \sigma_s \frac{\phi\left(\frac{K-P_U}{\sigma_s}\right)}{1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right)} - K\right) \left(1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right)\right) \\ &= M_E(P_U, \sigma_s^2, K), \end{aligned}$$

as claimed. Adding the expressions above immediately yields that the overall firm value is:

$$P_D + P_E = M_E\left(P_U, \sigma_s^2, K\right) + M_D\left(P_U, \sigma_s^2, K\right) = P_U.$$

A.4 Proof of Corollary 1

It is straightforward to verify that $M_E(x, \cdot, \cdot)$ and $M_D(x, \cdot, \cdot)$ increase in x. Hence, results (i)-(iii) follow from the fact that, as can be seen in Lemma 1, P_U increases in θ and z and decreases in κ. To verify that the equity and debt prices decrease and increase in K, respectively, note:

$$\begin{aligned} \frac{\partial}{\partial K} M_D(P_U, \sigma_s^2, K) &= -\frac{\partial}{\partial K} M_E(P_U, \sigma_s^2, K) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right) - \frac{K-P_U}{\sigma_s}\phi\left(\frac{K-P_U}{\sigma_s}\right) - \phi'\left(\frac{K-P_U}{\sigma_s}\right) \\ &= 1 - \Phi\left(\frac{K-P_U}{\sigma_s}\right) > 0. \end{aligned}$$

A.5 Proof of Proposition 2

Observe that P_U is unconditionally normally distributed with mean

$$\begin{aligned} \mathbb{E}\left[P_U\right] &= \mathbb{E}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau}(z - \kappa)\right] \\ &= \int \mathbb{E}\left[\mu_j\right] dj - \frac{\sigma_s^2}{\tau}\kappa \\ &= \mu - \frac{\sigma_s^2}{\tau}\kappa. \end{aligned} \tag{28}$$

Thus, we have:

$$\begin{aligned}
\mathbb{E}\left[P_E\left(P_U\right)\right] &= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(P_U,\sigma_s^2\right)\right]\right\}\\
&= \mathbb{E}\left\{\mathbb{E}\left[\max\left(x+P_U-K,0\right)|x\sim N\left(0,\sigma_s^2\right)\right]\right\}\\
&= \mathbb{E}\left[\max\left(x+y-K,0\right)|x\sim N\left(0,\sigma_s^2\right),y\sim N\left(\mathbb{E}\left[P_U\right],\mathbb{V}\left[P_U\right]\right)\right]\\
&= \mathbb{E}\left[\max\left(x-K,0\right)|x\sim N\left(\mu-\frac{\sigma_s^2}{\tau}\kappa,\sigma_s^2+\mathbb{V}\left[P_U\right]\right)\right]\\
&= M_E\left(\mu-\frac{\sigma_s^2}{\tau}\kappa,\Omega,K\right).
\end{aligned}$$

The debt result follows analogously.

We next show how the equity payoffs compare to equity expected cash flows; the result for debt follows analogously. Observe that, as $\kappa\to 0$, the expected equity price approaches $M_E\left(\mu,\Omega,K\right)$. Now, the expected equity payoff equals:

$$\mathbb{E}\left[\max\left(\theta-K,0\right)\right]=M_E\left(\mu,\sigma_\theta^2,K\right).$$

Thus, equity earns negative expected returns if and only if $M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)>0$ and earns positive expected returns if and only if $M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)<0$. Now, note that the derivative of M_E with respect to its second argument is:

$$\begin{aligned}
\frac{\partial M_E\left(\mu,x,K\right)}{\partial x} &= \frac{\partial}{\partial x}\left\{x^{\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)-\left[1-\Phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)\right]\left(K-\mu\right)\right\}\\
&= \frac{1}{2}x^{-\frac{1}{2}}\phi\left(x^{-\frac{1}{2}}\left(K-\mu\right)\right)>0.
\end{aligned}$$

Thus, we have that:

$$M_E\left(\mu,\Omega,K\right)-M_E\left(\mu,\sigma_\theta^2,K\right)\gtrless 0\Leftrightarrow\Omega-\sigma_\theta^2\gtrless 0,$$

which completes the proof of statements (i) and (ii) in the proposition.

A.6 Proof of Corollary 2

We would like to write down an explicit expression for $\Omega-\sigma_\theta^2=\mathbb{V}(P_U)+\sigma_s^2-\sigma_\theta^2$ in terms of the deep parameters of the model. We have

$$\begin{aligned}
\mathbb{V}\left(P_U\right) &= \left(\sigma_s^2\right)^2\mathbb{V}\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)\left(\overline{s}+\beta z\right)\right)\\
&= \left(\sigma_s^2\right)^2\left(\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2\sigma_\theta^2+\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)^2\beta^2\sigma_z^2\right)
\end{aligned}$$

$$= \left(\sigma_s^2\right)^2 \left(\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right)^2 \sigma_\theta^2 - 2\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) + \frac{1}{\sigma_\theta^2} + \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right)^2 \beta^2\sigma_z^2 \right)$$

$$= \sigma_\theta^2 - \sigma_s^2 + \left(\sigma_s^2\right)^2 \left(-\frac{1}{\sigma_\varepsilon^2} - \frac{1}{\rho\sigma_p^2} + \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right)^2 \beta^2\sigma_z^2 \right)$$

$$= \sigma_\theta^2 - \sigma_s^2 + \left(\sigma_s^2\right)^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) \left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2} \right),$$

where the fourth equality uses the definition of $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$ to simplify and collect terms, and the final line substitutes in for $\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2\sigma_z^2}{\rho^2}$ and groups terms. Hence,

$$\Omega - \sigma_\theta^2 = \left(\sigma_s^2\right)^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} \right) \left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2} \right)$$
$$\propto \left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2} \right),$$

and therefore

$$\operatorname{sgn}\left(\Omega - \sigma_\theta^2\right) = \operatorname{sgn}\left(\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2\sigma_z^2}{\left(1-\rho^2\right)\sigma_\theta^2 + \beta^2\sigma_z^2} \right). \tag{29}$$

It is immediate that the expression in the sgn function in eq. (29) is strictly increasing in ρ and takes value $\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} - 1$ at $\rho = 0$ and value $\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} > 0$ at $\rho = 1$. It follows that if $\frac{\beta^2\sigma_z^2}{\sigma_\varepsilon^2} > 1 \Leftrightarrow \sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$ then $\Omega - \sigma_\theta^2 > 0$ for all $\rho \in [0,1]$. On the other hand, if $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists a $\rho^* \in (0,1)$ such that $\Omega - \sigma_\theta^2 < 0$ for $\rho \in [0, \rho^*)$ and $\Omega - \sigma_\theta^2 > 0$ for $\rho \in (\rho^*, 1]$.

A.7 Derivation of Expression 21

We have

$$\mathbb{V}\left[\int \mu_j dj + \frac{\sigma_s^2}{\tau} z \right] - \mathbb{V}_i\left[\mu_i\right]$$
$$= \mathbb{V}\left[\int \mu_j dj \right] - \mathbb{V}_i\left[\mu_i\right] + \mathbb{V}\left[\frac{\sigma_s^2}{\tau} z \right] + 2\mathbb{C}\left[\int \mu_j dj, \frac{\sigma_s^2}{\tau} z \right]$$
$$= \mathbb{V}\left[\int \mu_j dj \right] - \mathbb{V}[\mu_i] + \mathbb{V}[\mu_i] - \mathbb{V}_i\left[\mu_i\right] + \mathbb{V}\left[\frac{\sigma_s^2}{\tau} z \right] + 2\mathbb{C}\left[\int \mu_j dj, \frac{\sigma_s^2}{\tau} z \right], \tag{30}$$

where the first equality uses the standard result for expressing the variance of a sum, and the second equality adds and subtracts $\mathbb{V}[\mu_i]$. The first two terms in eq. (30) can be written as

$$\begin{aligned}\mathbb{V}\left[\int \mu_j dj\right] - \mathbb{V}[\mu_i] &= \mathbb{V}\left[\sigma_s^2\left(\frac{\int s_j dj}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] - \mathbb{V}\left[\sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] \\ &= \mathbb{V}\left[\sigma_s^2\left(\frac{\theta}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] - \mathbb{V}\left[\sigma_s^2\left(\frac{\theta + \varepsilon_i}{\sigma_\varepsilon^2} + \frac{s_U}{\rho\sigma_p^2}\right)\right] \\ &= -\left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2. \end{aligned} \tag{31}$$

Expression (21) now follows immediately upon noting that

$$\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right] = \mathbb{E}\left[\left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\varepsilon_i\right)^2\right] = \left(\frac{\sigma_s^2}{\sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2, \tag{32}$$

which allows one to express eq. (31) as $\mathbb{V}\left[\int \mu_j dj\right] - \mathbb{V}[\mu_i] = -\mathbb{E}\left[\left(\mu_i - \int \mu_j dj\right)^2\right]$

A.8 Proof of Corollary 3

Part (i) We consider equity returns; the proof for debt returns is analogous. We have

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) &= \operatorname{sgn}\left(\operatorname{sgn}\left(\mathbb{E}[R_E]\right)\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\ &= \operatorname{sgn}(\mathbb{E}[R_E])\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) \\ &= -\operatorname{sgn}\left(\Omega - \sigma_\theta^2\right)\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right). \end{aligned}$$

Differentiating the expected return with respect to K yields

$$\begin{aligned}\frac{\partial}{\partial K}\mathbb{E}[R_E] &= \frac{\partial}{\partial K}\left(M_E(\mu, \sigma_\theta^2, K) - M_E(\mu, \Omega, K)\right) \\ &= \frac{\partial}{\partial K}\int_\Omega^{\sigma_\theta^2} \frac{\partial}{\partial x} M_E(\mu, x, K) dx \\ &= \int_\Omega^{\sigma_\theta^2} \frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K) dx, \end{aligned}$$

where the second equality uses the fundamental theorem of calculus to express the difference in the M_E function an integral. Computing the cross-partial derivative of M_E yields

$$\frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K) = \frac{\partial}{\partial K}\frac{1}{2}\frac{1}{\sqrt{x}}\phi\left(\frac{K-\mu}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\phi'\left(\frac{K-\mu}{\sqrt{x}}\right) = \frac{1}{2}\frac{1}{x}\frac{\mu - K}{\sqrt{x}}\phi\left(\frac{K-\mu}{\sqrt{x}}\right).$$

Hence, for $K < \mu$, we have

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_{\Omega}^{\sigma_\theta^2} \underbrace{\frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K)}_{>0} dx\right) \\ &= -\operatorname{sgn}(\Omega - \sigma_\theta^2),\end{aligned}$$

and therefore

$$\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = \operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) > 0.$$

On the other hand for $K > \mu$,

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial}{\partial K}\mathbb{E}[R_E]\right) &= \operatorname{sgn}\left(\int_{\Omega}^{\sigma_\theta^2} \underbrace{\frac{\partial^2}{\partial K \partial x} M_E(\mu, x, K)}_{<0} dx\right) \\ &= \operatorname{sgn}(\Omega - \sigma_\theta^2),\end{aligned}$$

and therefore

$$\operatorname{sgn}\left(\frac{\partial}{\partial K}\left|\mathbb{E}[R_E]\right|\right) = -\operatorname{sgn}^2\left(\Omega - \sigma_\theta^2\right) < 0.$$

Because $|\mathbb{E}[R_E]|$ is strictly increasing in K for $K < \mu$ and strictly decreasing in K for $K > \mu$, it follows that $|\mathbb{E}[R_E]|$ is hump-shaped in K and achieves its maximum at $K = \mu$.

Part (ii) Consider debt returns. We have that:

$$\begin{aligned}\frac{\partial \mathbb{E}\left[R_D\right]}{\partial \sigma_z} &= \frac{\partial M_D\left(\mu, \sigma_\theta^2, K\right)}{\partial \sigma_z} - \frac{\partial M_D\left(\mu, \Omega, K\right)}{\partial \sigma_z} \\ &= -\frac{\partial M_D\left(\mu, \Omega, K\right)}{\partial \Omega}\frac{\partial \Omega}{\partial \sigma_z} \propto \frac{\partial \Omega}{\partial \sigma_z}.\end{aligned}$$

Similarly, for equity returns, we obtain $\frac{\partial \mathbb{E}[R_E]}{\partial \sigma_z} \propto -\frac{\partial \Omega}{\partial \sigma_z}$. Now,

$$\frac{\partial \Omega}{\partial \sigma_z} = \frac{2\sigma_\theta^4 \sigma_z \sigma_\varepsilon^4 \begin{pmatrix} \tau^4 \sigma_\theta^2 \sigma_z^2 \sigma_\varepsilon^4 \left(3\left(1-\rho^2\right)^2 \sigma_\theta^4 + \left(3 - \rho^3(\rho+2)\right)\sigma_\theta^2 \sigma_\varepsilon^2 + 2\rho^3 \sigma_\varepsilon^4\right) \\ +(1-\rho)\tau^6 \sigma_\theta^4 \left((1-\rho)^2(\rho+1)^3 \sigma_\theta^4 + (1-\rho)(\rho+1)\left(2\rho^2+\rho+1\right)\sigma_\theta^2\sigma_\varepsilon^2 + 2\rho^3\sigma_\varepsilon^4\right) \\ 3\tau^2 \sigma_\theta^2 \sigma_z^4 \sigma_\varepsilon^8 \left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right) + \sigma_z^6 \sigma_\varepsilon^{12}\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)\end{pmatrix}}{\tau^2 \left(\sigma_z^2 \sigma_\varepsilon^4 \left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) + \tau^2 \sigma_\theta^2 \left(\sigma_\varepsilon^2 + \left(1-\rho^2\right)\sigma_\theta^2\right)\right)^3} > 0.$$

A.9 Proof of Proposition 3

As stated in the text, the derivations of all previous results continue to hold upon incorporating the public signal y into the investors' belief updates. Doing so, and grouping terms appropriately, we obtain that the unlevered firm price satisfies:

$$P_U(\cdot) = \mu + \sigma_s^2 \left(\frac{1}{\sigma_\eta^2} y + \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) (\overline{s} + \beta z) - \frac{\kappa}{\tau} \right),$$

$$\text{where} \quad \beta = \frac{\sigma_\varepsilon^2}{\tau},\ \sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_\theta + \frac{\beta^2 \sigma_z^2}{\rho^2},\ \sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}.$$

By iterated expectations, we again obtain that, for $\kappa = 0$, $\mathbb{E}[P_U] = \mu$. So, for $x \in \{E, D\}$:

$$\begin{aligned} \mathbb{E}[P_x] &= \mathbb{E}\left[M_x(P_U(\theta, z, y), \sigma_s^2, K) \right] \\ &= M_x(\mu, \Omega, K), \end{aligned}$$

where we again define $\Omega = \mathbb{V}[P_U] + \sigma_s^2$. Thus, public information quality increases expected equity (debt) returns if and only if it decreases (increases) $\Omega - \sigma_\theta^2$. As in the proof of Corollary 2 we can write

$$\Omega - \sigma_\theta^2 = \left(\sigma_s^2\right)^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho \sigma_p^2} \right) \left(\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\,\sigma_\theta^2 + \beta^2 \sigma_z^2} \right), \tag{33}$$

where here $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} \right)^{-1}$.

Note that, as established in Proposition 2 in the baseline model, the sign of $\Omega - \sigma_\theta^2$ determines the signs of expected returns on debt and equity. Note furthermore that the only object in eq. (33) that depends on σ_η^2 is σ_s^2, which is trivially increasing in σ_η^2. Hence, $\Omega - \sigma_\theta$ increases in σ_η^2 if and only if $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\sigma_\theta^2 + \beta^2 \sigma_z^2} > 0$. That is,

$$\operatorname{sgn}\left(\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} \right) = \operatorname{sgn}\left(\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1 + \rho \frac{\beta^2 \sigma_z^2}{(1-\rho^2)\,\sigma_\theta^2 + \beta^2 \sigma_z^2} \right). \tag{34}$$

It is immediate that the expression in the sgn function in eq. (34) is strictly increasing in ρ and takes value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} - 1$ at $\rho = 0$ and value $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 0$ at $\rho = 1$. It follows that if $\frac{\beta^2 \sigma_z^2}{\sigma_\varepsilon^2} > 1 \Leftrightarrow \sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$ then $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} > 0$ for all $\rho \in [0, 1]$. On the other hand, if $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$, then there exists a $\rho^* \in (0, 1)$ such that $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} < 0$ for $\rho \in [0, \rho^*)$ and $\frac{\partial(\Omega - \sigma_\theta^2)}{\partial \sigma_\eta^2} > 0$ for $\rho \in [\rho^*, 1]$.

A.10 Proof of Lemma 4

This is a special case of Lemma 6, where we define the function $G(P) = \sigma_s^2 \times (g')^{-1}(P)$ in the text.

A.11 Proof of Proposition 4

This is a special case of Proposition 5 in which $\mu_z = (0,0)$ and Σ_z is positive definite.

B Equilibrium with arbitrary, correlated liquidity trading

In this section, we characterize the equilibrium in the fully-general version of the model in which liquidity trading $z = (z_D, z_E)$ follows a general bivariate normal distribution $N(\mu_z, \Sigma_z)$ where $\mu_z \in \mathbb{R}$ is an arbitrary vector of means, and Σ_z is an arbitrary positive semi-definite covariance matrix. As in the text, we consider equilibria of the "generalized linear" form specified in Definition 3 where the endogenous price statistics take the form

$$s_p = \mathbf{1}\bar{s} + B\left(z - \mu_z\right).$$

with $B = \left(\begin{smallmatrix} \beta_{1D} & \beta_{1E} \\ \beta_{2D} & \beta_{2E} \end{smallmatrix}\right)$ the 2×2 matrix of coefficients to be determined.

We begin by characterizing an arbitrary investor i's conditional distribution of the vector of debt and equity payoffs, $V = (V_D, V_E)$, given arbitrary $N(\mu_i, \sigma_s^2)$ beliefs about the underlying firm cash flow $\mathcal{V}$.

Lemma 5. *Suppose that $\mathcal{V}$ is conditionally normally distributed with mean μ_i and variance σ_s^2. Then the vector $V = (\min(\mathcal{V}, K), \max(\mathcal{V} - K, 0))$ follows a bivariate exponential family with moment-generating function (MGF) that is finite for any $u \in \mathbb{R}^2$, and is given explicitly by*

$$\mathbb{E}_i\left[\exp\left\{u'V\right\}\right] = \exp\left\{g\left(u + \mathbf{1}\frac{\mu_i}{\sigma_s^2}\right) - g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2}\right)\right\}$$

where the function $g : \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$g\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \log\left(\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K - \sigma_s^2 y_1}{\sigma_s}\right) + \exp\left\{(y_1 - y_2)K + \frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2 y_2}{\sigma_s}\right)\right)\right). \tag{35}$$

Proof. (Lemma 5) The claim about finiteness follows immediately once we have proven that the MGF takes the given form since, by inspection, the function g is finite on all of $\mathbb{R}^2$. The claim that the distribution is an exponential family also follows immediately from the functional form (see e.g., Sampson (1975), Hoffmann and Schmidt (1982)). To establish the expression for the MGF, write

$$
\begin{aligned}
&\mathbb{E}_i\left[\exp\left\{u'V\right\}\right] \\
&= \int_{-\infty}^{\infty} \exp\left\{u_1 \min\left\{t, K\right\} + u_2 \max\left\{t - K, 0\right\}\right\} dF_\theta(t|s_i, s_p) \\
&= \int_{-\infty}^{K} \exp\left\{u_1 t\right\} dF_\theta(t|s_i, s_p) + \int_{K}^{\infty} \exp\left\{u_1 K + u_2\left(t - K\right)\right\} dF_\theta(t|s_i, s_p) \\
&= \exp\left\{\mu_i u_1 + \frac{1}{2}\sigma_s^2 u_1^2\right\} \Phi\left(\frac{K - \mu_i - \sigma_s^2 u_1}{\sigma_s}\right) \\
&\qquad + \exp\left\{\left(u_1 - u_2\right) K + \mu_i u_2 + \frac{1}{2}\sigma_s^2 u_2^2\right\}\left(1 - \Phi\left(\frac{K - \mu_i - \sigma_s^2 u_2}{\sigma_s}\right)\right) \\
&= \exp\left\{\frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\} \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s}\right) \\
&\qquad + \exp\left\{\left(u_1 - u_2\right) K + \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)^2 - \frac{1}{2}\sigma_s^2\left(\frac{\mu_i}{\sigma_s^2}\right)^2\right\}\left(1 - \Phi\left(\frac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s}\right)\right) \\
&= \left(\exp\left\{\tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2} + u_1\right)^2\right\} \Phi\left(\tfrac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_1\right)}{\sigma_s}\right)\right. \\
&\qquad \left. + \exp\left\{\left(u_1 - u_2\right) K + \tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2} + u_2\right)^2\right\}\left(1 - \Phi\left(\tfrac{K - \sigma_s^2\left(\frac{\mu_i}{\sigma_s^2} + u_2\right)}{\sigma_s}\right)\right)\right) \exp\left\{-\tfrac{1}{2}\sigma_s^2\left(\tfrac{\mu_i}{\sigma_s^2}\right)^2\right\}.
\end{aligned}
$$

Taking the logarithm, this expression is identical to that in the Lemma after recognizing that g as defined in the statement of the Lemma satisfies $g\left(\begin{smallmatrix} y \\ y \end{smallmatrix}\right) = \frac{1}{2}\sigma_s^2 y^2$ when both arguments are identical. □

With trader beliefs pinned down, we next characterize the optimal demand.

Lemma 6. *Fix any $P = (P_D, P_E)$ in set of no-arbitrage prices $\{(p_D, p_E) : p_D < K, p_E > 0\}$. There is a unique optimal demand for trader i, given by*

$$
x_i = \tau\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2} - \left(g'\right)^{-1}(P)\right)
$$

where $\left(g'\right)^{-1} : \mathbb{R}^2 \to \mathbb{R}^2$ is the inverse of the gradient $g'\left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right) \equiv \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$.

Proof. (Lemma 6) From Lemma 5, we can compute the trader's conditional expected utility given an arbitrary demand x_i as

$$\mathbb{E}\left[-\exp\left\{-\frac{1}{\tau}x_i'(V-P)\right\}\right] = -\exp\left\{\frac{1}{\tau}x_i'P + g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau}x_i\right) - g\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2}\right)\right\}.$$

Letting $g' = \begin{pmatrix} \frac{\partial}{\partial y_1} g \\ \frac{\partial}{\partial y_2} g \end{pmatrix}$ denote the gradient of g, the FOC is

$$0 = g'\left(\mathbf{1}\frac{\mu_i}{\sigma_s^2} - \frac{1}{\tau}x_i\right) - P. \tag{36}$$

Note that the Hessian matrix $g'' \equiv \begin{pmatrix} \frac{\partial^2}{\partial y_1^2} g & \frac{\partial^2}{\partial y_1 \partial y_2} g \\ \frac{\partial^2}{\partial y_1 \partial y_2} g & \frac{\partial^2}{\partial y_2^2} g \end{pmatrix}$ is necessarily positive definite, owing to the fact that it is the matrix of 2nd derivatives of the cumulant generating function of V, which is strictly convex. It follows that the optimum, if it exists, is unique and the FOC in (36) is sufficient to characterize it. Hence, it suffices to show that there exists a demand $x_i \in \mathbb{R}^2$ that satisfies eq. (36).

Due to the positive-definiteness of g'' it follows that the gradient g' is injective and therefore invertible on its range. Hence, if we can establish that the range is the set of no-arbitrage prices $\{(p_D, p_E) : p_D < K, p_E > 0\}$, the existence and characterization of the optimal demand will follow immediately from rearranging the FOC in eq. (36).

Let $S = \{(v_D, v_E) : v_D < K, v_E = 0\} \cup \{(v_D, v_E) : v_D = K, v_E > 0\}$ denote the support of the payoff vector (V_D, V_E). We claim that the range of g' is the closed convex hull of S, which is precisely the set of no-arbitrage prices. This follows from the following. First, because the cgf g is defined on all of $\mathbb{R}^2$, and furthermore because $\mathbb{R}^2$ is open, the exponential family described by the cgf is necessarily "regular" as defined by Barndorff-Nielsen (2014). It follows from Theorem 8.2 of Barndorff-Nielsen (2014) that the exponential family is "steep" and therefore from Theorem 9.2 in Barndorff-Nielsen (2014) that the gradient g' maps $\mathbb{R}^2$ onto the interior of the closed convex hull of the support S. This set, $\operatorname{int}\operatorname{conv}(S) = \{(x, y) : x < K, y > 0\}$, is the set of candidate prices in which the debt price is less than the face value K and the equity price is greater than zero, which is precisely the set of prices that do not admit arbitrage. □

Proposition 5. *There exists an equilibrium in the financial market. The vector of equilibrium asset prices takes the form*

$$P = g'\left(\mathbf{1}\frac{\int \mu_j \, dj}{\sigma_s^2} - \frac{1}{\tau}(\kappa\mathbf{1} - z)\right). \tag{37}$$

where the function $g' : \mathbb{R}^2 \to \mathbb{R}^2$ is given in closed-form in eqs. (43)–(44) the proof.

1. If Σ_z is invertible, then the equilibrium price vector is

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{38}$$

where the equilibrium price signals coefficient matrix is diagonal $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$

2. If Σ_z is singular and of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is identical in the two markets, $z_E = z_D = \zeta$ for $\zeta \sim N(0, \sigma_z^2)$), then the equilibrium price vector is

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{39}$$

where $s_p = \overline{s} + \beta\zeta$ is one-dimensional, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{\beta^2}{\rho^2}\sigma_z^2$ with $\beta = \frac{\sigma_\varepsilon^2}{\tau}$, and $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$.

3. If $\rho < 1$, and Σ_z is singular and not of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated but with different variances, or is perfectly negatively correlated, or at least one of the z_j is constant), then the equilibrium price vector is given by

$$P = g'\left(\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}\mathbf{1}\kappa\right)\right) \tag{40}$$

where $B = \begin{pmatrix} \frac{\sigma_\varepsilon^2}{\tau} & 0 \\ 0 & \frac{\sigma_\varepsilon^2}{\tau} \end{pmatrix}$, $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$, $\sigma_s^2 = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$, and the vector $a \in \mathbb{R}^2$ is defined in the proof.

4. If $\rho = 1$, and Σ_z is singular and not of the form $\Sigma_z = \mathbf{1}\mathbf{1}'\sigma_z^2$, then there exists a fully-revealing equilibrium in which $P_D = \min\{\mathcal{V}, K\}$ and $P_E = \max\{\mathcal{V} - K, 0\}$.

Proof. (Proposition 5) Using the expression for trader demand from Lemma 6, the market clearing condition yields

$$\begin{aligned}
&\int x_j dj + z = \mathbf{1}\kappa \\
\Leftrightarrow &\int x_j dj + z - \mu_z = \mathbf{1}\kappa - \mu_z \\
\Leftrightarrow &\tau\left(\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} - (g')^{-1}(P)\right) + z - \mu_z = \mathbf{1}\kappa - \mu_z
\end{aligned}$$

$$\Leftrightarrow P = g'\left(\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z)\right). \tag{41}$$

Because the vector of the liquidity trade z enters explicitly multiplied only by a scalar, we can conclude that in any equilibrium it suffices to consider only diagonal coefficient matrices B with identical elements on the diagonal. That is, $B = \beta I$ for $\beta \in \mathbb{R}$ still to be determined.

A closed-form expression for the gradient $g'\left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right)$ follows from computing the partial derivatives of the function g as defined in Lemma 5:

$$\frac{\partial g}{\partial y_1} = \left(\sigma_s^2 y_1 - \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}\right) \frac{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)+\exp\left\{(y_1-y_2)K+\frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{42}$$

$$+ K \frac{\exp\left\{(y_1-y_2)K+\frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)+\exp\left\{(y_1-y_2)K+\frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)} \tag{43}$$

$$\frac{\partial g}{\partial y_2} = \left(\sigma_s^2 y_2 + \sigma_s \frac{\phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)}{1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)} - K\right) \frac{\exp\left\{(y_1-y_2)K+\frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}{\exp\left\{\frac{1}{2}\sigma_s^2 y_1^2\right\}\Phi\left(\frac{K-\sigma_s^2 y_1}{\sigma_s}\right)+\exp\left\{(y_1-y_2)K+\frac{1}{2}\sigma_s^2 y_2^2\right\}\left(1-\Phi\left(\frac{K-\sigma_s^2 y_2}{\sigma_s}\right)\right)}. \tag{44}$$

To complete the proof and derive the explicit expressions in the Proposition, it is convenient to separately consider the cases of positive definite Σ_z and singular Σ_z.

If Σ_z is positive definite, then we can write the conditional moments explicitly as

$$\mu_i = \mathbb{E}[\mathcal{V}|s_i, s_p] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right), \text{ and} \tag{45}$$

$$\sigma_s^2 = \mathbb{V}(\mathcal{V}|s_i, s_p) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \mathbf{1}'\Sigma_p^{-1}\mathbf{1}\right)^{-1} \tag{46}$$

where $\Sigma_p \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2\mathbf{1}\mathbf{1}' + \frac{1}{\rho^2}B\Sigma_z B'$. Because Σ_z is assumed positive definite, it follows that $B\Sigma_z B'$ is positive definite. Furthermore, Σ_p, being a sum of a positive definite and positive semidefinite matrix is itself positive definite and therefore invertible, where it is understood that we take $\Sigma_p^{-1} = \mathbf{0}$ and $\Sigma_p^{-1}\frac{1}{\rho} = (\rho\Sigma_p)^{-1} = \mathbf{0}$ in the above expressions when $\rho \to 0$.

Substituting the explicit expression for μ_i in the argument of g' in eq. (41) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2\frac{1}{\sigma_\varepsilon^2}\bar{s} + \sigma_s^2\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p\right) + \frac{1}{\tau}(z-\mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z)
\end{aligned}$$

$$= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(\mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}s_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\bar{s} + \frac{\sigma_\varepsilon^2}{\tau}(z-\mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).$$

Matching coefficients on the initial conjecture $s_p = \mathbf{1}\bar{s} + B(z-\mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \mathbf{1}\mathbf{1}'\Sigma_p^{-1}\frac{1}{\rho}\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)$$

which, upon plugging back into g', matches the expression in the Proposition.

If Σ_z is singular, then the matrix Σ_p that appears above is not invertible and the above expressions for beliefs do not apply directly.[18] Intuitively, in this case there is only a single shock to liquidity trading and so the vector of price-signals s_p collapse to an informationally-equivalent one-dimensional signal.

If Σ_z is of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., liquidity trade is perfectly positively correlated, with identical variance in both markets, as in the baseline model), then the price statistics themselves are necessarily identical across both markets (i.e., $s_{p1} = s_{p2}$). Abusing notation to let $s_p \in \mathbb{R}$ denote this common price statistic and $\zeta = z_D - \mu_{zD} = z_E - \mu_{zE} \in \mathbb{R}$ denote the common liquidity trade shock realization, the expressions for the conditional moments become

$$\mu_i = \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}s_p\right), \text{ and} \tag{47}$$

$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1} \tag{48}$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2 + \frac{1}{\rho^2}\beta^2\sigma_z^2$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for μ_i in the argument of g' in eq. (41) (recalling that $\zeta \in \mathbb{R}$ denotes the common liquidity trade realization in this case) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}\mathbf{1}\zeta - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2\frac{1}{\sigma_\varepsilon^2}\bar{s} + \sigma_s^2\frac{1}{\rho\sigma_p^2}s_p\right) + \mathbf{1}\frac{1}{\tau}\zeta - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\rho\sigma_p^2}s_p + \frac{1}{\sigma_\varepsilon^2}\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau}\zeta\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

[18] The cases can be handled in a unified way by re-representing the above expressions for the conditional moments in forms involving pseudo-inverses of Σ_p. However, to avoid tedious technical complications, we choose to treat the case of singular Σ_z separately. Details of the unified treatment are available on request.

Matching coefficients on the initial conjecture $s_p = \overline{s} + \beta(z - \mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \mathbf{1}\sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right)$$

which, upon plugging back into g', matches the expression in the Proposition.

If Σ_z is singular but not of the form $\mathbf{1}\mathbf{1}'\sigma_z^2$ (i.e., the liquidity trade is perfectly positively correlated but has different variances in the two markets, or is perfectly negatively correlated, or is constant in at least one of the markets), then price statistics $s_p = (s_{p1}, s_{p2})$ can be combined to solve for $\overline{s}$. That is, there exists a vector $a \in \mathbb{R}^2$ such that $\overline{s} = a's_p$.[19] Hence, the conditional moments for trader i are

$$\mu_i = \mathbb{E}\left[\mathcal{V}|s_i, s_p\right] = \mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}a's_p\right), \text{ and}$$
$$\sigma_s^2 = \mathbb{V}\left(\mathcal{V}|s_i, s_p\right) = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1},$$

where $\sigma_p^2 \equiv \frac{1-\rho^2}{\rho^2}\sigma_\theta^2$ is strictly positive since $\rho < 1$ and it is understood that we take $\frac{1}{\sigma_p^2} = 0$ and $\frac{1}{\rho\sigma_p^2} = 0$ in the above expressions when $\rho = 0$.

Substituting this explicit expression for μ_i in the argument of g' in eq. (41) and grouping terms yields

$$\begin{aligned}
&\mathbf{1}\frac{\int \mu_j dj}{\sigma_s^2} + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \mathbf{1}\frac{1}{\sigma_s^2}\left(\mu + \sigma_s^2\left(\frac{s_i}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}a's_p\right)\right) + \frac{1}{\tau}(z - \mu_z) - \frac{1}{\tau}(\mathbf{1}\kappa - \mu_z) \\
&= \frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(\mathbf{1}\frac{1}{\rho\sigma_p^2}a's_p + \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{1}\overline{s} + \frac{\sigma_\varepsilon^2}{\tau}(z - \mu_z)\right)\right) - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right).
\end{aligned}$$

Matching coefficients on the initial conjecture $s_p = \overline{s} + B(z - \mu_z)$, with $B = \beta I$ as derived above, requires $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The previous expression now simplifies to

$$\frac{1}{\sigma_s^2}\left(\mathbf{1}\mu + \sigma_s^2\left(I\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\mathbf{1}a'\right)s_p - \frac{\sigma_s^2}{\tau}(\mathbf{1}\kappa - \mu_z)\right) \tag{49}$$

which, upon plugging back into g', matches the expression in the Proposition.

Finally, if $\rho = 1$ in the previous case, then in equilibrium traders can directly infer

[19] It can be shown that $a = \left(\frac{\mathbb{V}(z_E)}{\mathbb{V}(z_E)-\mathbb{C}(z_D,z_E)}\ \frac{\mathbb{V}(z_D)}{\mathbb{V}(z_D)-\mathbb{C}(z_D,z_E)}\right)$ when the correlation is ± 1, which is finite given the form of Σ_z under consideration in this case. If $\mathbb{V}(z_D) = 0$ or $\mathbb{V}(z_E) = 0$, one can take $a = (1, 0)$ or $a = (0, 1)$, respectively.

$\theta = \overline{s} = a's_p$ from the vector of asset prices. Because payoffs are riskless given observation of θ, the equilibrium prices must then be $P_D = \min\{\mu + \theta, K\}$ and $P_E = \max\{\mu + \theta - K, 0\}$ to preclude arbitrage. This set of prices is not of the posited generalized linear form, but it is now easily confirmed that such fully-revealing prices constitute an equilibrium. $\square$