Incentivizing Effort and Informing Investment: The Dual Role of Stock Prices

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Abstract

Stock prices reflect managerial performance and aggregate investor information about investment opportunities. These dual roles are often in tension: when prices are more informative about future opportunities, they may be less effective at incentivizing managerial effort. This intuitive tradeoff has novel consequences. Higher transparency, which leads to more efficient investment, can decrease firm value. The principal may strictly prefer to delegate investment to a manager who has no informational advantage and makes ex-post inefficient choices. Investment in diversifying and (ex-ante) negative NPV projects mitigate agency problems. Finally, standard empirical measures of price efficiency provide an incomplete picture of firm value.

JEL Classification: D8, G1

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1 Introduction

Stock prices are inherently multi-dimensional. On the one hand, they play an important role in *incentive provision* via compensation contracts because they reflect information about firm performance and managerial decisions. On the other hand, they aggregate investor information about future opportunities and affect real decisions via *feedback effects*. The existing literature has largely focused on exploring each of these roles in isolation.[1] We show that their interaction is key to understanding firm investments and managerial compensation jointly. Policies that appear optimal along one of these dimensions may lead to worse overall outcomes, while choices that appear sub-optimal in isolation may, in fact, be efficient once we account for both channels.

We develop a model in which the principal of a firm must compensate her risk-averse manager for costly effort and choose whether to pursue a new investment opportunity. In equilibrium, the price provides a contractable signal of managerial effort *and* aggregates investors' information about the value of the potential investment. The model captures the tension between these roles: in equilibrium, managerial effort decreases with the quality of investors' aggregate information about future opportunities. Intuitively, when the price is more responsive to information about investment opportunities, it can become a noisier measure of managerial effort, which makes it more difficult for the principal to incentivize the manager.

While this tradeoff is intuitive, it has surprising implications. First, higher transparency, which makes prices more informative about future investment opportunities, can lead to a *decrease* in firm value even though it leads to more informationally efficient investment decisions. Second, we show that the principal may strictly prefer to delegate the investment decision to the manager even though he has no information advantage and follows an ex-post inefficient investment rule. Finally, we characterize how project characteristics can exacerbate or mitigate the managerial moral hazard problem. For instance, when managerial effort enhances assets-in-place, the principal prefers diversifying investment opportunities that are negatively correlated with existing assets. On the other hand, if managerial effort enhances the value of the investment opportunity, the principal may pursue an ex-ante negative NPV investment to alleviate the managerial moral hazard problem.

The interaction of these dual roles also has novel implications for empirical analysis. Specifically, our results highlight the importance of accounting for the contribution of new investment opportunities to the firm's cash-flows *relative* to the contribution of managerial

[1] The former role is the basis for the large literature following Holmström and Tirole (1993), which studies the role of stock price liquidity on compensation contracts. The latter role is explored by the growing literature on feedback effects – see the surveys by Bond, Edmans, and Goldstein (2012) and Goldstein (2023).

effort — we denote this by δ. We show that firm value increases with price informativeness (about future opportunities) only when feedback effects are sufficiently important (i.e., δ is high). On the other hand, existing empirical measures of real efficiency (e.g., revelatory price efficiency) may be misleading because they do not account for the impact of managerial effort and contracting on firm value. In fact, when δ is sufficiently low, *higher* revelatory price efficiency corresponds to *lower* firm value and overall real efficiency.

Finally, our analysis generates novel, testable predictions for how managerial compensation contracts depend on future investment opportunities. First, pay-for-performance sensitivity (PFP) of managerial compensation should be higher for firms in which feedback effects are relatively less important (i.e., firms with lower δ). Second, for a given firm, PFP should decrease as the price becomes more informative about future investment opportunities. Third, there is a positive relation between liquidity and PFP, but a negative relation between liquidity and price-investment sensitivity.[2]

Model Overview and Main Trade-off. Section 3 presents the model. There is a single firm with a risk-neutral principal (she) and a risk-averse manager (he). The manager can exert costly effort to increase the terminal cash flows of the firm. This effort is incentivized by an endogenous contract set by the principal to maximize her expectation of the terminal cash flows, net of managerial compensation. The principal also chooses whether or not to invest in a new project, given the information available to her. The firm's equity is a claim to the terminal cash flows, net of compensation, and is traded by a group of investors, some of who are privately informed about the profitability of the new project.

In Section 4, we solve for the financial market equilibrium and the contract offered by the principal, which depend on each other in equilibrium. Our benchmark analysis focuses on compensation contracts that are linear in price for analytical tractability.[3] In equilibrium, the stock price contains information about managerial effort and aggregates investors' private information about the new project. As a result, the principal compensates the manager using a fraction of the firm's equity — this is the **incentive provision** role — while also conditioning on the price when making her investment decision, which captures the **feedback effect** from prices to real investment.

[2]The negative relation between price informativeness and PFP is in line with the empirical evidence in Lin, Liu, and Sun (2019), while the positive relation between liquidity and PFP is consistent with the evidence in Fang, Noe, and Tice (2009) and Jayaraman and Milbourn (2012).

[3]While a general characterization of the optimal (possibly non-linear) contract is not analytically tractable in our setting, we can view our results as characterizing the first-order approximation to this contract. Our main results obtain when we allow the contract to depend on both the short-term price and the (long-term) terminal value (Appendix B.3) and when we allow the contract to be quadratic in the price (Appendix B.6). More generally, we show in the latter appendix that when the manager's contract is a strictly increasing function of the price, it is more costly to incentivize his effort when investors have more information about the investment.

In our benchmark analysis, the dual roles push firm value in opposing directions. On the one hand, when the price is more informative about the new project, the principal makes a more informed investment decision, increasing the firm value. On the other hand, when prices reflect more firm-specific private information, they are also more volatile, *conditional* on a given level of effort. Since the manager is risk-averse, this makes compensating him for effort more costly and so the principal optimally reduces the price-sensitivity of the manager's compensation in equilibrium. This, however, reduces his effort and, consequently, firm value.

As a result, the relation between price informativeness and expected firm value depends on the relative importance of feedback effects, or δ. When δ is high (low), the relative impact of learning about future investment opportunities is higher (lower) than the impact of incentivizing managerial effort. We show that expected firm value unambiguously increases (decreases) with price informativeness when δ is sufficiently high (low) but is U-shaped for intermediate levels of δ.

The tension between the two roles arises because the price is more volatile when investors are better informed about future opportunities. This is consistent with the large empirical literature on price non-synchronicity, which establishes that price informativeness and firm-specific volatility are positively related.[4] Furthermore, Chen, Goldstein, and Jiang (2007) show that price volatility (non-synchronicity) is strongly correlated with the sensitivity of investment to price, which is a measure of the feedback effect. Together, this evidence implies that the trade-off is likely to be of first-order importance in practice.

Implications. Section 5 characterizes the novel implications of the trade-off generated by the dual role of stock prices. In Section 5.1, we endogenize the information content of prices by allowing investors to choose whether or not to acquire costly information about the new project (as in Grossman and Stiglitz (1980)). Intuitively, an increase in transparency (i.e., a decrease in information costs) leads to increased investment efficiency but can lower firm value when contracting is relatively important (i.e., if δ is sufficiently low). To the extent that firms can affect transparency by changing the clarity of their disclosures, our results suggest that firms should be more opaque when the incentive provision role of prices is more important, even if it leads to less efficient investment decisions.[5]

[4] As we discuss further in Section 2 and Appendix B.9, the positive relation between price informativeness and volatility arises quite generally, especially when investors are (close to) risk-neutral. This is consistent with our interpretation that investors' information is firm-specific, and so does not affect systematic risk exposures. See Morck, Yeung, and Yu (2000), Wurgler (2000), Durnev, Morck, Yeung, and Zarowin (2003), Durnev, Morck, and Yeung (2004) for evidence consistent with the positive relation. Finally, while Dávila and Parlatore (2020) argue that the relation between volatility and price informativeness is mixed, their claim is based on a structural estimation of a model without feedback effects and so difficult to interpret in the context of our analysis.

[5] This is consistent with the evidence in Bebchuk and Fried (2004), who find that hard-to-value and

In Section 5.2, we characterize how our results change if the principal *delegates* the decision to the manager. The manager maximizes his expected payoff, and as a result, his investment rule maximizes the short-term price. Relative to the principal's investment policy, the manager over-invests in negative ex-ante NPV projects, but under-invests in positive ex-ante NPV projects. While the manager's policy is inefficient ex-post, we show that delegation can increase overall firm value when δ is sufficiently low. In particular, when the manager under-invests in high profitability projects, the price is less volatile and, consequently, less costly to use for incentive provision. As a result, the principal offers a higher-powered contract to the manager, which induces more effort. This analysis highlights the importance of accounting for the dual role of prices in understanding the allocation of control rights.

Our benchmark analysis assumes that the impact of managerial effort is independent of project's cash flows. Section 5.3 relaxes this assumption and characterizes our model's implications for project choice. First, we consider the case where the manager's effort only affects firm value if the new project is undertaken. We show that the negative relation between managerial effort and price informativeness survives when the project has a positive ex-ante NPV, but reverses when the project has a negative ex-ante NPV. Intuitively, this is because, for a positive (negative) NPV project, more informative prices lead to a lower (higher) likelihood of investment, which leads to lower (higher) effort provision.

Next, we consider the case where the impact of managerial effort is state-dependent and, therefore, correlated with the investment payoffs. Intuitively, when the project is positively (negatively) correlated with managerial output, price volatility is higher (lower), which leads to lower (higher) effort provision. Together these results suggest that project choices which may appear sub-optimal in isolation (e.g., negative ex-ante NPV projects with very different payoffs from assets in place) may, in fact, be desirable and value-enhancing because they help mitigate the moral hazard problem faced by the principal.

Section 6 summarizes the empirical predictions of the model. A broad takeaway is that it is important to condition on a measure of δ, the relative importance of feedback effects, when trying to understand the relationship among price informativeness, managerial compensation, and firm value. While there is no direct measure of δ that is widely agreed upon (to the best of our knowledge), we build on the existing empirical literature to suggest some approaches to capturing this variation. This includes using indirect measures (e.g., insider trading, index inclusion, managerial overconfidence) that have previously been shown to be related to the intensity of the feedback effect, as well as direct, survey evidence (e.g., Goldstein, Liu, and Yang (2021b)).

opaque firms tend to use more aggressive stock-based pay for managers.

Another takeaway is that one should be cautious in interpreting standard empirical measures of price efficiency. While the existing literature has focused on revelatory price efficiency (RPE) as the relevant measure of real efficiency,[6] our analysis suggests that this may be misleading. Specifically, we show that when the relative impact of the investment opportunity, δ, is sufficiently low, firm value can decrease even as RPE increases if more informative prices exacerbate the manager's agency problem.

The rest of the paper is as follows. The next section briefly discusses the related literature and the paper's contribution. Section 3 presents the model and discusses the key assumptions while Section 4 characterizes the equilibrium. Section 5 characterizes the novel implications for firm policy, while Section 6 summarizes the model's empirical predictions. Section 7 concludes. All proofs are in Appendix A and the supplemental analysis is in Appendix B.

2 Related Literature

As discussed above, our paper is at the intersection of two existing literatures: one on incentive provision using stock-based compensation, and the other on feedback effects from stock prices to investment decisions.[7] However, this work generally considers one of the roles in isolation, while abstracting from the other.[8] Our analysis implies that the interaction between these dual roles is crucial for understanding the impact of investors' information on firm value. Our model builds on the framework of Davis and Gondhi (2022). While both papers focus on the impact of price information on agency conflicts, their work is primarily focused on the extent to which endogenous learning affects investment efficiency and the observation of debt overhang and risk-shifting. In contrast, we analyze how investors' information amplifies the impact of moral hazard on firm value and consider several policies the firm can adopt to lessen its impact.

The work of Lin, Liu, and Sun (2019) and Dow and Gorton (1997) are closest to our own analysis, but differ along an important dimension. In both, the manager can utilize either their own private information to make an investment decision or, alternatively, rely on the price. Price information is a *substitute* for managerial information and so when investors are better-informed, the manager's optimal contract is lower-powered. Effectively, higher price informativeness makes contracting with the manager *easier*. Our benchmark analysis

[6]For example, Bai, Philippon, and Savov (2016) estimate a related measure of RPE which captures the response of both R&D and CAPX to prices and show that, over time, prices can explain more of the variation in firms' investment decisions.

[7]See Bond et al. (2012), Goldstein and Yang (2017), and Goldstein (2023) for surveys and Scholes (1991), Paul (1992), Holmström and Tirole (1993), Calcagno and Heider (2021) for examples of the former.

[8]For instance, while Bresnahan, Milgrom, Paul, et al. (1992) analyze both roles for prices, they shut down one dimension when analyzing the impact of change in investors' information on the other dimension.

focuses on a setting in which the manager's effort does not affect the investment decision and so contracting becomes *harder* when investors are better-informed.[9] Given the multi-faceted nature of executives' daily responsibilities, the separation we model captures an important and natural feature of managerial actions in practice.

Our results highlight that investors' information can affect managerial actions even when the two dimensions independently affect firm value. In this sense, our paper is related to the theoretical literature which considers the implications when firm value is multi-dimensional. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein, Kopytov, Shen, and Xiang (2021a) consider settings where investors are heterogeneously informed about two components of fundamentals. Similarly, in our model, we can view managerial effort and information about future investment as different components of fundamentals. One key difference, however, is that in our setting the first component depends endogenously on the equilibrium contract offered to the manager.

Gjesdal (1981) shows that, in a generalized information environment, the information system which is preferable for compensating an agent may not be preferable for decision-making. Our paper demonstrates that this tension naturally arises in settings where the signal of interest is the traded stock price. Paul (1992), Bushman and Indjejikian (1993), and Kim and Suh (1993) emphasize that the weight investors place on their information reflects the contribution of the project to firm value instead of the contribution of the manager to the project (which would be preferable for contracting purposes). Relative to these papers, our analysis highlights how investor information affects both investment efficiency and contracting efficiency, and how the interaction between these affects firm value and decisions. This interaction is also absent from Chaigneau, Edmans, and Gottlieb (2018), who study an alternative channel through which more precise information reduces effort.

Our analysis focuses on a setting in which the price is more volatile when investors are better informed about (future) investment opportunities. While some recent theoretical papers feature models in which a non-monotonic relationship arises (e.g., Brunnermeier, Sockin, and Xiong (2020)) and Dávila and Parlatore (2020)), we expect the positive relation in our model to arise more generally given the environment we analyze. In particular, since the fundamental risk (and information asymmetry) in our model is firm-specific, we expect the marginal trader to be effectively risk-neutral with respect to this risk (as in our model) since it will be largely diversified away. As we show in Appendix B.9, this implies the relation between volatility and price-informativeness is generally positive.

Our model also relates to the literature focused on how changes in transparency (e.g., due to public disclosures or a reduction in the cost of information acquisition) can have

[9] In Section 5.3 we consider how relaxing this independence assumption affects our results.

unintended consequences for price informativeness and real efficiency (see Goldstein and Sapra (2014), and Goldstein and Yang (2017) for recent surveys). We show how greater transparency can have a negative impact on contracting efficiency and, thus, reduce firm value. This is distinct from, but complements, existing analyses that have focused on how increases in disclosure or transparency can lead to "crowding out" of information that is relevant for investment decisions.[10]

Finally, our paper is related to the literature which argues that commitment to ex-post inefficient rules can be optimal. For instance, in Bond and Goldstein (2015) the benefit of this ex-ante commitment to rely less on prices is that, in equilibrium, the stock price is more informative. In a setting without feedback effects, Strobl (2014) shows that by committing to over-invest in negative NPV projects, a manager can induce investors to acquire more information, and thereby improve contracting efficiency. In our setting it can be optimal to delegate to the manager (thereby committing to an ex-post inefficient investment rule) because the resultant under-investment reduces the volatility of the price which increases contracting efficiency.[11]

3 Model

Payoffs. There are three dates $t \in \{1, 2, 3\}$, and two equally-likely states of the world, denoted by $\omega \in \{H, L\}$. The state is realized at date one, but is unobservable. The firm pays terminal cash flows V at date three, which consist of two components: (i) assets-in-place that generate x_ω and (ii) a zero-cost investment opportunity that generates δy_ω.[12] We assume that $x_H > x_L > 0$, $y_H > 0 > y_L$.[13] The parameter $\delta \geq 0$ captures the importance of the new investment opportunity relative to the firm's existing assets. There are two traded securities. The risk-free security is normalized to the numeraire. The risky security is a claim to the

[10] Goldstein and Yang (2019) show how disclosure about one component of fundamentals affects price informativeness about other components. Banerjee, Davis, and Gondhi (2018) show that an increase in transparency can encourage learning about fundamentals, but also about the behavior of other investors, and thereby make prices less informationally efficient.

[11] There are two notable distinctions between our setting and Strobl (2014). First, in Strobl (2014), the manager is better informed about the investment opportunity than the investors, and so there is no feedback from prices. Second, the manager's effort choice affects the likelihood of success of the new project in Strobl (2014), while the two are not directly related in our benchmark analysis.

[12] The assumption of zero cost is isomorphic to a setting in which the required investment is non-zero but can be made using the firm's existing cash (contained in the assets-in-place), i.e., it does not require equity holders to contribute additional capital.

[13] The assumption that assets-in-place and the investment opportunity are positively correlated does not qualitatively affect our results but eases the exposition. It is sufficient to assume that cash flows in the high state exhibit first-order stochastic dominance, with and without investment, under which the project can be either independent of or negatively correlated with assets-in-place. See Davis and Gondhi (2022) for a setting which constructs the analogous equilibrium under this more general assumption.

net cash flows of the firm, and trades at a price P at date two.

The firm. The firm consists of a principal (she) and a manager (he). The principal, indexed by p, is risk neutral and offers a take it or leave it contract to the manager at date one to maximize expected firm value, net of payments. We restrict the principal to offering the manager a *linear* compensation contract of the form:

$$W(P) = \alpha + \beta P, \tag{1}$$

where α denotes the manager's fixed wage and β denotes the sensitivity of her compensation to the market price at date two.

The manager, indexed by m, has mean-variance preferences over his payoff W, with risk aversion coefficient γ, and an outside option u_0, which we normalize to zero. At date one, the manager can exert costly effort e to increase the cash flows from assets-in-place from x_ω to $x_\omega + e$. We assume that the effort is observable but not verifiable, and requires the manager to incur a private cost, $c(e)$, where $c' \geq 0, c'' > 0, c(0) = c'(0) = 0$.[14] The manager chooses his effort level to maximize his utility from compensation net of effort costs i.e.,

$$u_m(e; \alpha, \beta) \equiv \mathbb{E}[W(P)] - \frac{\gamma}{2}\mathbb{V}[W(P)] - c(e), \tag{2}$$

subject to $u_m(e; \alpha, \beta) \geq u_0 = 0$.

At date two, the principal chooses whether or not to invest in the new opportunity in order to maximize the expected value of the terminal cash flows, given the security price. We denote this investment decision by $I(P) \in \{0, 1\}$, where $I(P) = 1$ indicates that the investment is made. Together, this implies that the terminal cash flows of the firm are given by

$$V(\omega, e, I) \equiv x_\omega + e + \delta y_\omega \times I. \tag{3}$$

The principal's optimal linear contract, (α^*, β^*), maximizes the terminal value of the firm net of compensation, subject to the manager's incentive compatibility constraint, the principal's investment rule, and the manager's participation constraint, i.e., she solves

$$\max_{\alpha,\beta} \mathbb{E}[V(\omega, e^*, I^*) - W(P)], \text{ subject to} \tag{4}$$

$$e^* = \arg\max_e u_m(e; \alpha, \beta), \tag{5}$$

$$I^* = \arg\max_I \mathbb{E}[V(\omega, e^*, I) | P], \text{ and} \tag{6}$$

[14]In Appendix B.4, we show that our results are robust to the alternative in which investors instead observed a *noisy* signal of managerial effort, as in Holmström and Tirole (1993).

$$u_m\left(e^*;\alpha,\beta\right)\geq u_0. \tag{7}$$

Investors. At date two, a unit-measure continuum of risk-neutral investors indexed by $i\in[0,1]$ trade the claim to the firm's net cash flows, i.e., $V-W$.[15] While all investors' condition on the information in the equilibrium price, P, a fraction $\lambda<1$ of these investors also observe a private, conditionally-independent signal, $s_i\in\{s_H,s_L\}$, where

$$\mathbb{P}\left[s_i=s_H|\omega=H\right]=\mathbb{P}\left[s_i=s_L|\omega\in L\right]=\rho>\tfrac{1}{2}. \tag{8}$$

The remaining fraction, $1-\lambda$, are uninformed, which we denote by $s_i=\emptyset$ with some abuse of notation. Investors can take both long and short positions in the risky security, but they are subject to position limits – specifically, we normalize the positions so that $d_i\in[-1,1]$. This implies that the optimal demand is given by

$$d\left(s_i,P\right)\equiv\arg\max_{d_i\in[-1,1]}\mathbb{E}\left[d_i\times\left(V-W-P\right)|s_i,P\right]. \tag{9}$$

To ensure that the equilibrium price is not fully revealing, the aggregate supply of the risky security available for trade is stochastic and given by $u\sim U\left[-1,1\right]$, where u is independent of all other random variables in the economy.[16] The equilibrium price, P, is determined by market clearing, i.e.,

$$\int_i d(s_i,P)di=u. \tag{10}$$

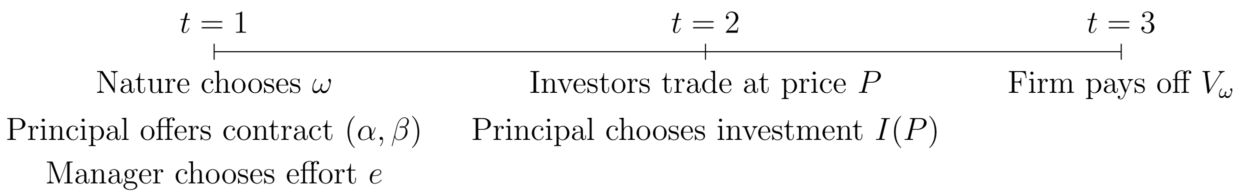
Timing of events. The timeline of events is summarized in Figure 1. At date one, nature chooses $\omega\in\{H,L\}$. The principal offers the manager a linear contract (α,β), and the manager chooses costly effort e to maximize his expected compensation. At date two, investors observe their signals (if informed) and the price, and trade the risky security. The principal observes the equilibrium price, P, and chooses whether or not to invest in the new opportunity. At date three, the firm's assets pay off cash flows V_ω given by (3).

Equilibrium. An equilibrium consists of a linear contract (α,β), effort level e, investment

[15]In Appendix B.7, we consider a setting in which investors trade a claim to the firm's terminal cash flow *without* netting out the manager's payoff.

[16]Our analysis generalizes to a setting in which the distribution of noise trading is $U[-u_b,u_b]$ where $u_b\leq 1$. As the amount of noise trading falls (i.e., as u_b shrinks towards zero), the price signal becomes (unconditionally) more informative and prices become more volatile since the marginal investor is risk neutral. Our main trade-off also obtains if the price is set by a risk-neutral market maker, as in Kyle (1985).

Figure 1: The timeline of the economy



rule $I(P)$, equilibrium demands $\{d(s_i, P)\}_i$ and equilibrium price P, such that:

(i) $d(s_i, P)$ maximizes the investor's objective in (9),

(ii) the equilibrium price P clears the risky security market (i.e., (10) holds),

(iii) the investment rule maximizes the expected firm cash flows (i.e., solves (6)),

(iv) the optimal effort level e maximizes the manager's expected utility (i.e., solves (5)),

(v) the optimal linear contract (α, β) solves the principal's problem, characterized by (4)-(7), and

(vi) posterior beliefs satisfy Bayes' rule whenever applicable.

3.1 Discussion of Assumptions

Our benchmark analysis assumes that some investors are exogenously endowed with private information about the state of the world, ω. In Section 5.1, we extend the analysis to allow for costly information acquisition. Specifically, we assume that before date one, investors choose whether or not to pay a cost, c_0, to become informed. This allows us to characterize how changes in transparency (driven by changes in c_0) affect the optimal linear contract, the expected value of the firm, and social surplus.

The principal's investment policy maximizes the firm's terminal cash flows given the information in the price in our benchmark model. We interpret the principal as representing long-term shareholders or the board of directors. In Section 5.2, we relax this constraint and allow the *manager* to choose whether or to invest to maximize his expected payoff. While this can lead to inefficient investment decisions, the main results are qualitatively similar. Moreover, we show that delegation to the manager can be optimal, even though manager's investment policy is sub-optimal, due to its positive impact on managerial effort.[17]

We refer to the positive impact of the manager's decision as arising through "effort" but this need not be interpreted literally. For instance, the private cost borne by the manager can reflect his disutility from completing value-enhancing but monotonous tasks or making difficult but valuable decisions. Given the many dimensions through which managerial actions impact firm value, we follow the existing literature (e.g., Holmström and Tirole (1993)) and assume that effort increases the value of the firm irrespective of whether or not the investment is made. Allowing for the impact of effort to be correlated with the investment payoff introduces additional forces which obscure the economic mechanism we want to focus on and reduces analytical tractability. Our setting allows us to highlight how contracting and feedback affect each other even when the two components do not mechanically depend on each other. In Section 5.3, we examine how our results extend to settings (i) in which the impact of managerial effort is state-dependent, i.e., $e \in \{e_L, e_H\}$ and (ii) in which managerial effort increases the payoffs of the new opportunity.

For our benchmark model, we assume that the principal cannot contract on the firm's realized value. This is a standard assumption in the literature, commonly motivated by the observation that the manager's tenure at the firm may be shorter than the life of the project. In Appendix B.3, we relax this assumption and allow the equilibrium contract to linearly depend on both the date two price as well as the terminal value, $V(\omega, e, I)$. We show that the principal always chooses to include the price and terminal cash flow in the equilibrium contract and the effect of investors' information about the investment on managerial effort survives.[18] Moreover, we show how the dual rule of prices generates a new channel through which heterogeneity in short- and long-term compensation can arise.

We assume that the principal and investors can observe the effort level chosen by the manager but cannot directly contract on this information. This allows us to introduce the trade-off between the allocative and contracting roles of prices in a transparent manner. In Appendix B.4, we consider a setting in which investors have access to a noisy signal of the manager's effort. We show that our key trade-off still obtains in this setting: when the price is more informative about future investment opportunities, it is less effective from a contracting perspective.

We assume that investors are risk-neutral whereas the manager is risk-averse. This is

[17]This result suggests it could be optimal for the principal to be able to commit to an ex-post inefficient investment rule if it were feasible. We rule out such commitment in the benchmark model to focus on the economic mechanism we wish to highlight and view delegation as a tool for circumventing the practical difficulties faced by the principal in committing to such a rule in practice.

[18]In our setting, this is because of a hedging benefit which reduces the volatility of the manger's compensation, while in Holmström and Tirole (1993) this arises due to the noise in investors' information about managerial effort.

natural: generally, investors are able to diversify their exposure to firm-specific risk (e.g., by holding a large portfolio of securities), while managers generally have concentrated exposure to firm-specific risk via their human capital and due to equity-based compensation. Moreover, as Albagli, Hellwig, and Tsyvinski (2021) emphasize, focusing on a setting with risk-neutral investors with position limits allows us to interpret the results as being about firm-specific information and risks. We show in Appendix B.5 that our main trade-off also obtains when the manager is risk-neutral when the optimal contract satisfies a limited liability constraint.

Finally, we restrict attention to linear contracts because a complete analytical characterization of the optimal contract is otherwise intractable. In Appendix B.6, we show numerically that our main results also arise when we allow the principal to offer a contract that is quadratic in price. As such, we interpret the contract characterized in our benchmark analysis as a first-order approximation to the more general, optimal contract (and the contract in Appendix B.6 as a second-order approximation). We expect that our key trade-off arises more generally, so long as the optimal compensation to the manager becomes more volatile when investors are more informed about future opportunities. In particular, we show that if the contract offered is a strictly increasing function of the price, it is more costly to incentivize the manager when investors have more information about the investment.

4 Equilibrium

We solve the model by working backwards.

4.1 Investment and financial market equilibrium (Date Two)

Let $q(P) \equiv \mathbb{P}[\omega = H|P]$ denote the principal's posterior beliefs after conditioning on the information in the equilibrium price. She optimally chooses to invest if and only if doing so increases her expectation of the terminal cash flow, and so follows a threshold strategy:

$$I(P) = \begin{cases} 1 & \text{if } q(P) \geq \frac{-y_L}{y_H - y_L} \equiv K \\ 0 & \text{if } q(P) < K. \end{cases} \tag{11}$$

Intuitively, the principal invests only if she is sufficiently optimistic that the investment will pay off positively, i.e., if $q(P)$ is sufficiently large. The investment threshold, K, increases with the size of the potential loss ($-y_L$) but decreases with the potential gain (y_H) and captures the ex-ante profitability of the project. A low threshold (specifically, $K < \frac{1}{2}$)

corresponds to an ex-ante profitable investment while $K > \frac{1}{2}$ implies that the project is ex-ante unprofitable.

Given investor i's objective function in Eq.(9), he also optimally adopts a threshold strategy when choosing his position in the firm's traded claim:

$$d(s_i, P) = \begin{cases} 1 & \text{if } P < \mathbb{E}[V - W|s_i, P] \\ [-1, 1] & \text{if } P = \mathbb{E}[V - W|s_i, P] \\ -1 & \text{if } P > \mathbb{E}[V - W|s_i, P]. \end{cases} \tag{12}$$

Investors can observe the manager's effort choice, e, and perfectly infer the principal's investment choice, $I(P)$, since neither the manager nor the principal possess any private information and condition only on public information, i.e., P. As such, conditional on (ω, P), all investors agree on the *state-dependent* value of the firm, $V(\omega, e^*, I(P))$, defined in (3).

However, since they are privately informed, investors have different beliefs about the likelihood of each state. Let $q(s_i, P) \equiv \mathbb{P}[\omega = H|s_i, P]$ denote investor i's beliefs conditional on observing the price P and the private signal $s_i \in \{s_H, s_L, \emptyset\}$. To emphasize the role of the traded price and to simplify our notation, let us denote $V_\omega(P) \equiv V(\omega, e^*, I(P))$. Then investor i's conditional expectation of V is given by

$$\mathbb{E}[V|s_i, P] = V_L(P) + q(s_i, P) \times \left[\underbrace{V_H(P) - V_L(P)}_{\equiv \Delta V(P)} \right], \tag{13}$$

where $\Delta V(P)$ reflects the information sensitivity of cash flows. Note that $\Delta V(P) > 0$ for any P, and so investor i's valuation $\mathbb{E}[V|s_i, P]$ is increasing in $q(s_i, P)$.[19]

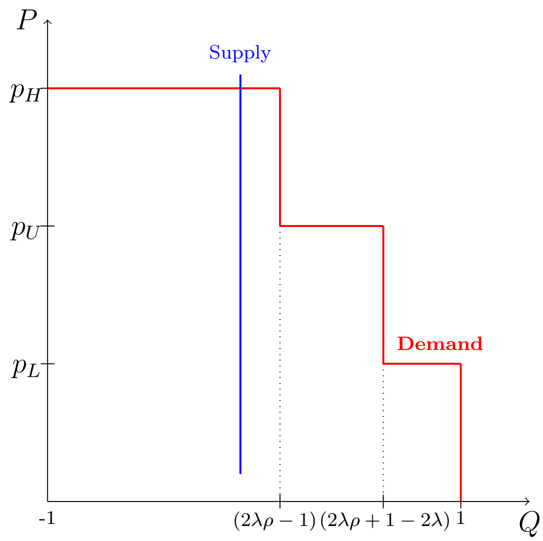
Figure 2 illustrates the market-clearing process. We conjecture and verify that, in equilibrium, there are three, distinct price levels $p_H > p_U > p_L$. For instance, consider the price at which all "pessimistic" investors (i.e., those who observed $s_i = s_L$) are indifferent between buying and selling, i.e., $p_L \equiv \mathbb{E}[V - W|s_L, p_L]$. By (13), both the uninformed and the "optimistic" investors (i.e., those who observed $s_i = s_H$) would take long positions since

$$q(s_H, p_L) > q(\emptyset, p_L) > q(s_L, p_L). \tag{14}$$

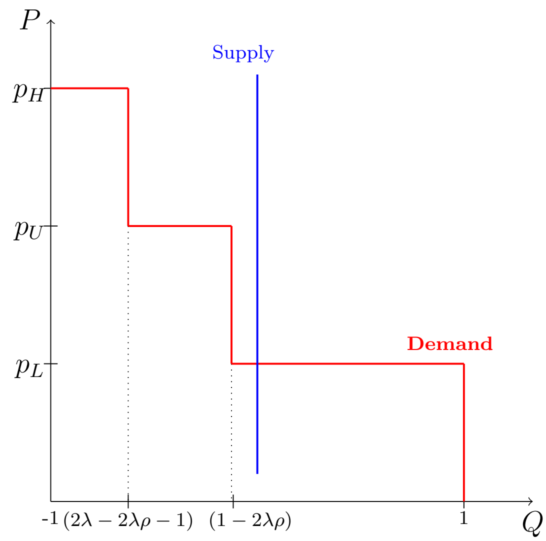
Whether this price clears the market (given the supply, u) depends on the underlying state,

[19]This is because $\Delta V(P) = x_H - x_L + \delta(y_H - y_L) \times I(P) > 0$.

Figure 2: Market-clearing



(a) Equilibrium price when $\omega = H$



(b) Equilibrium price when $\omega = L$

ω. For instance, if $\omega = H$, this is the unique equilibrium price as long as

$$u > \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 1 - 2\lambda(1-\rho). \tag{15}$$

If the supply of the risky asset, u, is any lower, then the market can only clear if one of the uninformed investors is indifferent between buying and selling. But for this to be the case, the market-clearing price must reflect the valuation of the uninformed, i.e., $P > p_L$. If $\omega = L$, the measure of informed investors who observe $s_i = s_L$ is higher and the measure that observe $s_i = s_H$ is lower. As a result, the threshold above which $P = p_L$ falls to

$$u > \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} + \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda\rho}_{\text{observed } s_i = s_L} = 1 - 2\lambda\rho., \tag{16}$$

as can be seen by comparing panels (a) and (b). Following similar steps yields the analogous market-clearing thresholds for both $p_U \equiv \mathbb{E}[V - W|\emptyset, p_U]$ and $p_H \equiv \mathbb{E}[V - W|s_H, p_H]$.

These thresholds determine the information contained in the equilibrium price. For instance, given the distribution of u and the thresholds found in (15) and (16), we know that

$$\mathbb{P}[P = p_L|\omega = L] = \lambda\rho \quad \mathbb{P}[P = p_L|\omega = H] = \lambda(1-\rho). \tag{17}$$

By Bayes' rule, this implies that $\mathbb{P}[\omega = H|P = p_L] = 1 - \rho$. Following similar steps yields the corresponding belief when the price is "high": $\mathbb{P}[\omega = H | P = p_H] = \rho$. However, in either state, the equilibrium price is p_U with probability $1-\lambda$. Thus, p_U is "uninformative" as it reveals no additional information about the true state of the world.

From this, we can derive the equilibrium beliefs of the principal and investors. For the principal, who observes only the price, we have:

$$q(p_L) = 1-\rho, \quad q(p_U) = \frac{1}{2}, \text{ and } \quad q(p_H) = \rho. \tag{18}$$

Plugging these beliefs into (11) gives the equilibrium investment rule. Then, knowing whether or not the principal invests, given the price, yields the equilibrium state- *and* price-dependent cash flows, e.g., $V_L(p_L)$. To solve for the equilibrium price, then, requires only the belief of the "marginal" investor (i.e., one who is indifferent between buying and selling the traded claim) which follows from standard updating rules:

$$q(s_H, p_H) = \tfrac{\rho^2}{\rho^2+(1-\rho)^2} \equiv \tilde{\rho}, \quad q(\emptyset, p_U) = \tfrac{1}{2}, \quad \text{ and } q(s_L, p_L) = 1-\tilde{\rho}. \tag{19}$$

With this, we formally establish the existence of the date two equilibrium.

Proposition 1. *There exists a unique financial market equilibrium with price P and investment rule $I(P)$, where the equilibrium price $P(\omega, u)$ is given by*

$$P(\omega, u) = \begin{cases} \tfrac{1}{1+\beta}\left(\mathbb{E}[V|s_L, p_L] - \alpha\right) \equiv p_L & \text{if } u < u_\omega - (1-\lambda) \\ \tfrac{1}{1+\beta}\left(\mathbb{E}[V|\emptyset, p_U] - \alpha\right) \equiv p_U & \text{if } u_\omega - (1-\lambda) \le u \le u_\omega + (1-\lambda) \\ \tfrac{1}{1+\beta}\left(\mathbb{E}[V|s_H, p_H] - \alpha\right) \equiv p_H & \text{if } u > u_\omega + (1-\lambda) \end{cases}, \tag{20}$$

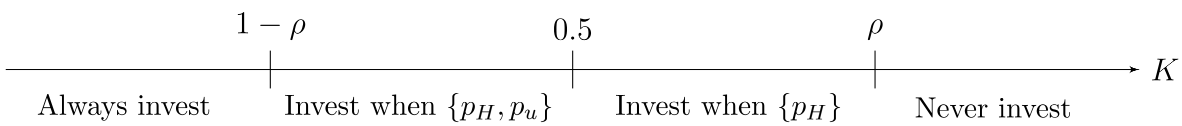
where $u_H \equiv \lambda(2\rho-1)$, $u_L = \lambda(1-2\rho)$, $p_H > p_U > p_L$, and where the marginal investor's conditional expectations are given by (13) and (19).

The equilibrium investment rule is to invest (i.e., $I(P) = 1$) if and only if (i) $1-\rho > K$ or if (ii) $\frac{1}{2} > K > 1-\rho$ and $P \in \{p_U, p_H\}$, or if (iii) $\rho > K > \frac{1}{2}$ and $P = p_H$.

In equilibrium, there exists a feedback loop between firm investment and the financial market. The principal's decision to invest depends on the price through the beliefs of the marginal investor while the realized price level is determined by the investment policy through the state-dependent firm value, $V_\omega(P)$. The equilibrium price level accounts for the fact that investors trade a claim to the firm's cash flow *net* of the manager's compensation, $W = \alpha + \beta P$.

Figure 3 illustrates the equilibrium investment rule. The optimal policy is intuitive given the ordering of her beliefs (see equation (18)) and noting that the principal only invests if she is sufficiently optimistic that the true state is high. When the investment threshold K is sufficiently high (low), the principal never invests (always invests, respectively). When the threshold is in the intermediate region, however, the principal conditions on the information in prices: she never invests when the price is low (i.e., $P = p_L$), always invests when the price is high (i.e., $P = p_H$), and only invests when the price is uninformative if K is below 1/2 (which corresponds to an investment opportunity with an ex-ante positive return.)

Figure 3: Equilibrium investment rule



Finally, we emphasize that investors' information impacts not only the principal's investment policy but also its efficiency. It is clear that an increase in the precision of their signal, ρ, necessarily increases the precision of the principal's information, improving the expected value of the investment. However, it is the measure of informed investors, λ, which determines the likelihood of observing an informative price since

$$\mathbb{P}\left[P = p_H\right] = \mathbb{P}\left[P = p_L\right] = \tfrac{\lambda}{2}, \mathbb{P}\left[P = p_U\right] = 1 - \lambda. \tag{21}$$

As a result, increases in λ also increase investment efficiency.

4.2 Managerial effort and the equilibrium contract (Date One)

Given the terms of the principal's offered contract (α, β), we can characterize the manager's optimal effort choice at date one. The manager maximizes his expected utility over compensation, $W = \alpha + \beta P$, net of effort costs i.e.,

$$\max_e \mathbb{E}\left[\alpha + \beta P\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c\left(e\right), \tag{22}$$

where P is the equilibrium price of the traded claim, characterized in Proposition 1.

The cash-flows $V_\omega\left(P\right)$ shift uniformly upward in each state with an increase in the manager's effort, e, and consequently, so does the price P. This implies that the variance of the price, and hence, the variance of the manager's payoff does not depend upon his effort level, given β. As a result, the first-order condition for the manager's problem is

$$\frac{\beta}{1+\beta} = c'\left(e\right). \tag{23}$$

Given the convexity of the cost function, the second-order condition always holds, and so the optimal choice of effort is characterized by (23). Moreover, the manager's optimal level of effort increases with the sensitivity of his compensation to the market price, β.[20]

Given the date two equilibrium characterized in Proposition 1, and the manager's optimal effort choice given by (23), the principal chooses the linear contract (α, β) that maximizes the (unconditional) expected firm value, net of the manager's payoff. Specifically, the principal's problem can be re-written as

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{24}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{25}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - \frac{\gamma}{2}\mathbb{V}\left[\alpha + \beta P\right] - c(e) \geq 0. \tag{26}$$

Since α does not affect the manager's effort choice, the principal can adjust it, given β, to ensure that the manager's participation constraint (26) binds. Moreover, the price is a linear transformation of the "marginal" investor's conditional expectation of the firm's terminal cash flow, and so

$$\mathbb{V}\left[P\right] = \frac{1}{(1+\beta)^2}\mathbb{V}\left[\tilde{\mu}(P)\right], \tag{27}$$

where

$$\tilde{\mu}(p_L) \equiv \mathbb{E}\left[V|s_L, p_L\right], \quad \tilde{\mu}(p_U) \equiv \mathbb{E}\left[V|\emptyset, p_U\right], \quad \tilde{\mu}(p_H) \equiv \mathbb{E}\left[V|s_H, p_H\right]. \tag{28}$$

With these observations, the principal's objective simplifies to

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{29}$$

The following proposition provides a characterization of the optimal linear contract.

Proposition 2. *Suppose the financial equilibrium, (P, I), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (23). Then the principal's optimal linear contract is given by (α, β), where*

$$\alpha = c(e) + \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - \beta\mathbb{E}\left[P\right], \textit{ and} \tag{30}$$

[20] The first-order condition is not linear in β because the risky security price accounts for the manager's compensation (i.e., they trade a claim to $V - W$) and so is scaled by $\frac{1}{1+\beta}$. In Appendix B.7 we consider a variant of the model in which the traded security is a claim to V and show that because the principal is able to recover the same signal of managerial effort from the price, equilibrium effort is the same in both settings.

$$\beta = \frac{1}{c''(e)\,\gamma\mathbb{V}\left[\tilde{\mu}(P)\right]}. \tag{31}$$

The principal's payoff in equation (29) has three components. The first component, $\mathbb{E}\left[x_\omega + e + \delta y_\omega I\right]$, is the expected value of the firm, which increases with the effort choice of the manager. The second term, $\frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right]$, captures the reduction in the manager's expected utility because his payoff is tied to the price of the risky security. The third term, $c(e)$, is the manager's private cost of providing effort.

The principal must compensate the manager for both of these costs in order for the contract to be accepted. As a result, she faces a trade-off: while an increase in β increases the effort exerted by the manager and, hence, firm value, it also increases the compensation he demands. Intuitively, when the manager is more risk-averse (higher γ), the price is more variable (higher $\mathbb{V}\left[\tilde{\mu}(P)\right]$), or the cost of effort increases quickly ($c''(e)$), the optimal β is lower. This is seen clearly in the expression for the optimal β, i.e., equation (31).

The offered contract and, hence, *equilibrium* managerial effort, depend upon the informativeness of the price. In particular, increases in either ρ or λ increase the volatility of the marginal investor's conditional valuation, $\tilde{\mu}(P)$, which makes the price more volatile, all else equal. But this increases the principal's cost of incentivizing effort and so, in equilibrium, the principal responds to an increase in investors' private information by reducing the sensitivity of the manager's compensation to the price (i.e., she reduces β). This, in turn, reduces the effort, e, exerted by the manager. We show this formally in the proof of the following proposition.

Proposition 3. *The optimal choice of β and e decrease with δ, λ and ρ.*

Importantly, even though the information investors impound into the price about the underlying state of the world is *independent* of managerial effort, it *amplifies* the underlying moral hazard problem. Moreover, the impact of this agency conflict increases with the importance of the investment project (i.e., e decreases with δ), even though the payoff from the manager's action does not depend upon the payoff from investment.[21]

4.3 Equilibrium value of investor information

The expected value of the firm's terminal cash flow is

$$EV \equiv \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I\right],$$

[21]These results do not rely on the state-independence of effort. We show in Section 5.3 that both can arise even when the effort is negatively or positively correlated with investors' information.

and increases in both the manager's effort choice and the efficiency of the investment decision. As a result, an increase in the quality of investors' private information introduces two, countervailing effects. Specifically, as the price becomes more informative about investment opportunities, it increases investment efficiency, but it reduces the performance-sensitivity of the manager's contract and, hence, equilibrium effort. In the next proposition, we characterize how the impact of changes in λ depend on both (i) the quality of investors' information, ρ, and (ii) the relative importance of the investment project, δ.

Proposition 4. *If $\rho < K$ or $1-\rho > K$, then EV is decreasing in λ. If $\rho > K > 1-\rho$ and the cost of effort satisfies equation (49), found in the Appendix, then*
(i) EV is increasing in λ if $\delta > \bar{\delta}$,
(ii) EV is U-shaped in λ if $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$
(iii) EV is decreasing in λ if $\delta \leq \underline{\delta}$.

Intuitively, the impact of introducing more informed investors (increasing λ) depends on the relative strength of the two channels highlighted above. In particular, it is straightforward to show that

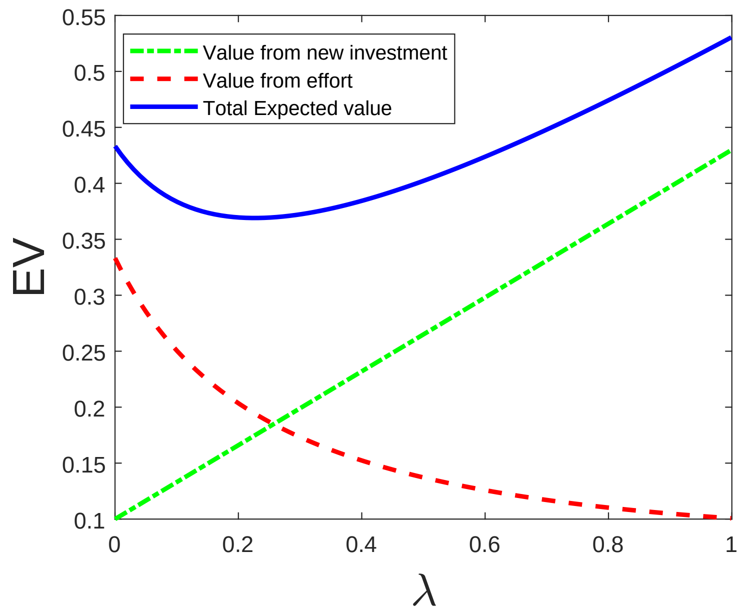
$$\frac{\partial EV}{\partial \lambda} = \underbrace{\frac{\partial e}{\partial \lambda}}_{<0} + \underbrace{\delta \times \begin{cases} 0 & \text{if } \rho < K, \\ \frac{y_H - y_L}{2}(\rho - K) & \text{if } \rho > K > \frac{1}{2}, \\ \frac{y_H - y_L}{2}(K - (1-\rho)) & \text{if } \frac{1}{2} > K > 1-\rho \\ 0 & \text{if } 1-\rho > K. \end{cases}}_{\text{marginal investment efficiency, } \geq 0} \tag{32}$$

When the principal never invests (i.e., if $\rho < K$) or always invests (i.e., if $1-\rho > K$), increasing λ has no impact on investment efficiency. As a result, more informative prices necessarily decrease the expected value because they lead to a decrease in managerial effort. Moreover, one can show that an increase in ρ give rise to the same effect in these regions.

When $\rho > K > 1-\rho$, the impact depends upon δ, which parameterizes the relative importance of the investment project. While the impact of λ on investment efficiency increases linearly, equation (49) in the Appendix provides a sufficient condition which ensures that the manager's effort choice is convex and decreasing in λ (this is satisfied, e.g., if the cost function is quadratic in managerial effort.) Intuitively, when the investment project is sufficiently important (i.e., δ is sufficiently high), the expected value increases with λ. On the other hand, if the investment project is sufficiently small relative to the firm's assets-in-place, the effect of λ on effort choice dominates and the expected value decreases even for large λ.

Figure 4: Expected value versus fraction of informed investors

The figure plots the expected value of the firm as a function of λ. The cost of effort is $\frac{c_e}{2}e^2$. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -0.9$.



For $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, however, the effect is no longer monotonic. The decline in effort for "low" λ causes the expected value to decrease initially, then increase. The plots in Figure 4 provide a numerical illustration for this scenario. The figure plots expected value (solid blue line) and its components: the manager's equilibrium effort level (red dashed line) and the expected value of the investment opportunity (green dotted line). Since the manager's effort is decreasing and convex in λ, while the value from the new investment opportunity increases at a constant rate, the expected value is U-shaped in λ.

Proposition 4 highlights how the presence of moral hazard reduces the value of investors' information. It establishes that expected value need not increase with price informativeness and, in fact, the relation between the two is either decreasing or non-monotonic unless managerial learning from the price is sufficiently important. Notably, this negative relation arises even though the information is incrementally informative for the real decision-maker. In Section 5, we explore how this novel channel through which price information exacerbates the moral hazard problem alters firm policies.

The result is also of particular importance for empirical tests of the relation between the quality of investors' aggregated information and expected value, that is, measures of real efficiency. This is true both in the time-series and the cross-section, since the nature of this relation can vary across firms (with different δ's) and within firms (e.g., for different levels of λ over time). We examine this in more detail in Section 6.

5 Implications for Firm Policy

In this section, we characterize the implications of the key trade-off we identified above: while more informative prices lead to more efficient investment decisions, they also increase the cost of incentivizing managerial effort.

5.1 Information acquisition and transparency

Our benchmark analysis shows that while an increase in λ increases the extent to which prices reveal information about the investment payoff, it can decrease firm value by making it costlier for the principal to incentivize the manager to exert effort. In this section, we endogenize λ, exploring the incentives of investors to acquire costly information, and consider how this affects the firm's optimal level of transparency.

Suppose that before date one, each investor can choose whether or not to become informed by paying a cost c_0 to obtain an informative private signal $s_i \in \{s_H, s_L\}$. Each investor takes as given the manager's optimal effort level (i.e., the solution to equation (23)) as well as the information acquisition decision of all other investors. Acquiring a private signal is only valuable if the incremental increase in his expected profits exceeds the fixed cost, c_0, given the nature of the financial market equilibrium. This implies that in any interior equilibrium (i.e., when $\lambda \in (0,1)$) where investors are indifferent between being informed and uninformed, it must be the case that

$$\mathbb{E}\left[d\left(s_i, P\right)\left(V-P\right)\right] - \mathbb{E}\left[d\left(\emptyset, P\right)\left(V-P\right)\right] = c_0. \tag{33}$$

This indifference condition yields the following equilibrium characterization.

Proposition 5. *There exists a $\hat{c}$, defined in the appendix, such that:*

(i) If $c_0 \geq \hat{c}$, then $\lambda = 0$.

(ii) If $c_0 < \hat{c}$, the measure of informed investors in equilibrium is given by

$$\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\Delta V\left(p_U\right)} < 1,$$

and is decreasing in the cost of information c_0, increasing in the precision of the signal ρ, and the information sensitivity of the risky security when the price is uninformative ($\Delta V\left(p_U\right)$).

Intuitively, the measure of informed investors is increasing in the expected payoff from being informed. In our setting, the incremental benefit from acquiring private information arises when the price is uninformative (i.e., $P = p_U$).[22] This is the reason why there are always uninformed investors in equilibrium: if $\lambda = 1$, the price is never uninformative and thus, there is no benefit to being informed. Since investors differ only in their beliefs about the payoff across the two states, the benefit of trading at the uninformed price is $\Delta V(p_U) = V_H(p_U) - V_L(p_U)$. This payoff differential is scaled by their information advantage relative to the uninformed, $\rho - \frac{1}{2}$. When information acquisition costs are sufficiently high (i.e., $c_0 > \hat{c}$), the net benefit from acquiring information is too low, even when all other investors are uninformed. As a result, all investors optimally choose not to acquire any information. Otherwise, a fraction of investors choose to acquire information, and this fraction is decreasing in c_0.

It has been suggested that one potential solution to the trade-off we highlight would be to sell the investment opportunity since the resultant decline in the volatility of the price would lead to a higher equilibrium effort. First, note that in many settings such a sale may not be feasible, e.g., if the payoff of the investment derives from the firm's existing assets, including its human capital or intellectual property. Second, Proposition 5 implies that the expected value of the investment is increasing in the information-sensitivity of the firm's traded security when λ is endogenous. This information-sensitivity depends not only on the investment payoff but the covariance of the investment payoff with the firm's assets-in-place (which is increasing in $x_H - x_L$). As such, if a potential buyer anticipated being able to learn less from the financial market due to the characteristics of its existing assets, its willingness-to-pay would decline. In such cases, it is straightforward to establish conditions under which divestment could reduce firm value even if it led to higher managerial effort in equilibrium.

Proposition 5 also suggests a natural role for policy changes that increase transparency (i.e., reduce the cost of information acquisition, c_0) given the impact of λ on firm value highlighted in Section 4.3. We characterize the impact of transparency on net value, NV, which captures the expected value of the firm net of compensation to the manager, i.e.,

$$NV = \text{Expected Terminal Cash Flow} - \text{Managerial Compensation}. \tag{34}$$

Since the optimal linear contract sets the manager's compensation to ensure that he is at

[22] As we show in the proof, the expected trading gains when the price is high or low (i.e., $P = p_H$ and $P = p_L$, respectively) are the same for both informed and uninformed investors. This is a consequence of our assumption that investors are risk-neutral and face position limits.

his reservation utility, this implies that firm value can be expressed as

$$NV = \mathbb{E}\left[x_{\omega} + e + \delta y_{\omega} \times I\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right). \tag{35}$$

The following result characterizes how an increase in transparency affects firm value.

Proposition 6. *The firm's net value (NV) increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large; NV decreases with transparency (increases with c_0) if either (i) $1-\rho > K$, or (ii) $\rho < K$, or (iii) $1-\rho < K < \rho$ and δ is sufficiently small.*

The proposition provides sufficient conditions for when an increase in transparency increases firm value, and when it decreases firm value. To gain some intuition, note that a change in information acquisition costs affects firm value via two channels:

$$\frac{d}{dc_0}NV = \delta \times \underbrace{\frac{\partial}{\partial\lambda}\mathbb{E}\left[y_{\omega} \times I\right]\frac{d\lambda}{dc_0}}_{\text{investment channel}} + \underbrace{\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right)\frac{d\lambda}{dc_0}}_{\text{incentive channel}} \tag{36}$$

The first channel, which we call the **investment channel**, captures the impact of transparency on the efficiency of the investment decision. Since an increase in λ increases the expected value of the investment, a reduction in transparency (i.e., an increase in c_0) reduces the firm's net value (NV) through this term. The second channel, which we refer to as the **incentive channel**, captures the impact of transparency on equilibrium effort provision net of costs. Since

$$\frac{\partial}{\partial\lambda}\left(e - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right)\right) = -\frac{\gamma}{2}\left(\frac{1}{1 + c''\left(e\right)\gamma\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}\right)^2\frac{\partial}{\partial\lambda}\mathbb{V}\left(\tilde{\mu}\left(P\right)\right) < 0, \tag{37}$$

and an increase in c_0 reduces λ, this term is always positive. As highlighted above, if the price is less informative about the investment project, the optimal contract is more performance-sensitive, increasing the effort the manager exerts in equilibrium. As such, a reduction in transparency increases firm net value (NV) through this channel.

Proposition 6 provides intuitive sufficient conditions for when the effect of one channel dominates the other. For the investment channel to prevail, the manager must utilize the information in the price (i.e., $1-\rho < K < \rho$) and the scale of the investment opportunity must be sufficiently large (i.e., high δ). Under these conditions, an increase in transparency leads to an increase in firm value. On the other hand, when there is no feedback effect (i.e., if $1-\rho > K$ or $\rho < K$) *or* the relative importance of the new investment is sufficiently low, the incentive channel dominates and firm value decreases with transparency.

In Appendix B.8, we characterize the aggregate impact of transparency on a measure of social surplus that accounts for not only the expected value of the firm net of compensation (i.e., NV), but also the manager's expected utility net of effort costs, and the aggregate trading gains and losses net of information acquisition costs. We find that social value increases with transparency when the feedback effect is relatively more important in determining firm value, but can decrease with transparency otherwise. This result cautions against an "one-size-fits-all" approach to regulatory policy that affects transparency in financial markets.[23] In particular, the dual role of stock prices highlighted by our analysis implies that higher transparency may not be socially desirable, even when it lowers investors' information costs and improves investment outcomes, because of its negative impact on contracting.

5.2 Delegation

The principal's investment rule, $I(P)$, maximizes her expectation of the firm's terminal cash flows, given the price. Such an investment rule, however, may not be *ex ante* optimal, given the impact of such investment on the volatility of the price and, hence, the optimal contract. Committing, however, to an alternative investment policy may be neither feasible nor credible as long as the principal is responsible for the investment decision at date two.

One solution to this commitment problem is to delegate the investment decision to the *manager*. Specifically, in what follows, we consider the implications when investors conjecture that the manager chooses whether to invest (i.e., he chooses $I_m(P) \in \{0,1\}$) to maximize the price. This is equivalent to following a threshold policy whereby the manager invests (i.e., $I_m(P) = 1$), as long as

$$q_m(P) > \frac{-y_L}{y_H - y_L} \equiv K,$$

where $q_m(P)$ denotes the beliefs of the marginal investor (i.e., the investor whose conditional valuation is reflected in the price.) given in equation (19). This change in the investment policy does not alter the financial market equilibrium given in Proposition 1 but the equilibrium investment rule is modified to reflect the manager's use of $q_m(P)$.

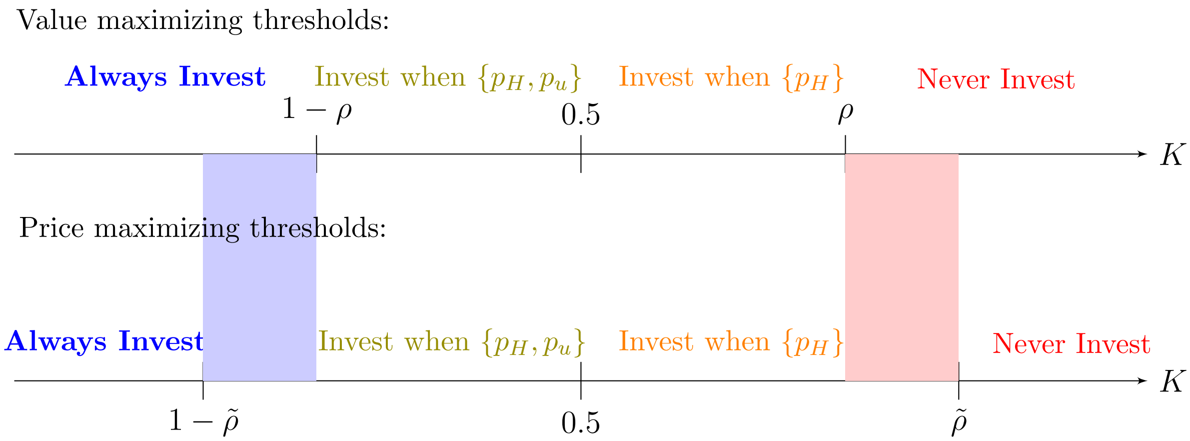
In Figure 5, we compare the manager's (price-maximizing) investment thresholds to the principal's (value-maximizing) thresholds. While their investment rules generally align, in the two shaded regions, delegating to the manager reverses the action the principal would have taken. Specifically, if $K \in (\rho, \tilde{\rho})$ the principal never invests while the manager invests if $P = p_H$; if $K \in (1-\tilde{\rho}, 1-\rho)$, the principal always invests while the manager refrains from investing when $P = p_L$.[24] As the principal's investment rule is ex post efficient, an increase

[23] Notably our results are not driven by the Hirshleifer (1971) effect, whereby more public information reduces opportunities for risk sharing, and are also distinct from the "crowding out" channel highlighted by other papers in the feedback literature (e.g, Goldstein and Yang (2019)).

[24] This reflects the fact that, as in other rational expectations models with a continuum of investors, the

Figure 5: Equilibrium Investment Thresholds

The figure characterizes the investment rule when the principal maximizes terminal value (top) to the one when the manager maximizes the price (bottom), as a function of $K = \frac{-y_L}{y_H - y_L}$.



in the fraction of informed investors (λ) increases investment efficiency when following $I(P)$. Deviations from this rule, as found in the manager's investment rule, $I_m(P)$, can therefore reduce the expected value of the investment opportunity.

Proposition 7. *Holding e fixed, when the manager follows his investment rule, $I_m(P)$, firm value decreases in λ if and only if $\rho < K < \tilde{\rho}$ or $1 - \tilde{\rho} < K < 1 - \rho$.*

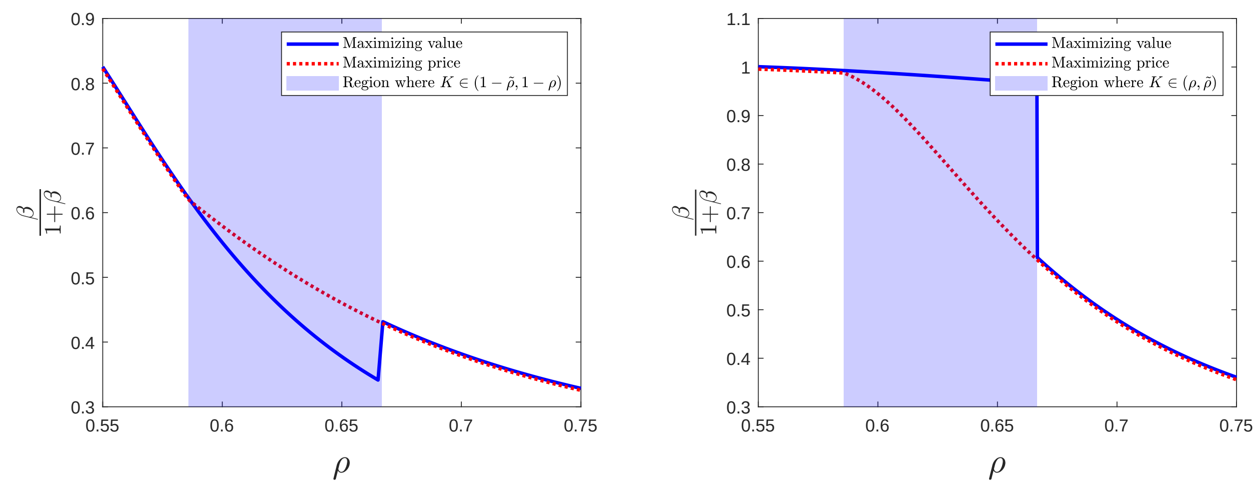
Intuitively, an increase in λ makes it more likely that the manager observes an informative price. This increases investment efficiency when the manager's investment rule coincides with the principal's (the un-shaded region). But when the two rules do not coincide (i.e., the investment threshold K lies in the shaded region), a higher λ increases the likelihood that the manager's decision is ex-post inefficient, reducing firm value. This decrease in investment efficiency is *amplified* by the presence of more informed investors (i.e., as λ increases).

Despite this source of inefficiency, however, following the manager's investment rule may be desirable because of its impact on managerial effort. Note that the manager's effort and the offered contract are the same as in the baseline model. However, the variance in prices $\mathbb{V}[\tilde{\mu}(P)]$, which affects β and e in equilibrium, depends upon the investment policy. Figure 6 plots the (scaled) optimal pay-for-performance component (i.e., $\frac{\beta}{1+\beta}$) when the investment policy maximizes the terminal value (blue, solid line) and when it maximizes the

marginal investors' equilibrium expectation of cash flows (reflected in the price) need not coincide with expected cash flows conditional on the information in the price. As a result, an investment policy that maximizes the expected price need not maximize expected value. This wedge between price maximization and value maximization arises more generally (e.g., Albagli et al. (2021), Banerjee, Breon-Drish, and Smith (2021a)).

Figure 6: Optimal pay-for-performance versus ρ

The figure plots the optimal linear contract chosen by the principal as a function of ρ. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$. Panel (a) corresponds to a setting where the principal always invests in the shaded region, but the manager does not invest for $P = p_L$. Panel (b) corresponds to a setting where the principal never invests in the shaded region, but the manager invests for $P = p_H$.



(a) $K = 0.33$ (b) $K = 0.67$

price (red, dashed line). In the shaded region of panel (a), $K \in (1 - \tilde{\rho}, 1 - \rho)$, so that the principal's value-maximizing rules always invests, while the manager's price-maximizing rule refrains from investing when $P = p_L$. This reduction in investment lowers the unconditional variance of the price which allows the principal to choose a higher β in equilibrium. As a result, the effort chosen by the manager will also be higher under the manager's investment rule. In the shaded region of panel (b), $K \in (\rho, \tilde{\rho})$, so that the principal's investment rule never invests, while the manager's policy invests when $P = p_H$. The higher propensity to invest increases the unconditional variance of the price which leads the principal to choose a lower β, which lowers managerial effort.

These results suggest that allowing the manager to determine the firm's investment policy when the project's unconditional net present value is high (i.e., when K is low) can be optimal even if the manager's preferred investment rule is ex post inefficient. For instance, suppose that the conditional value of the investment when the price is low ($P = p_L$) is close to zero, i.e., the difference between $1 - \rho$ and K is arbitrarily small. If the manager maximizes the price, he will not invest in this project (since $1 - \tilde{\rho} < K$). This has an arbitrarily small effect on the value of the investment project but, as panel (a) makes clear, can lead to a non-trivial increase in β and, hence, managerial effort. When the latter outweighs the former, it is optimal for the principal to delegate the investment decision to the manager. The following proposition formalizes this intuition.

Proposition 8. *If $K < 0.5$, there exists a $\bar{\rho} < 1 - K$ such that the principal strictly prefers to delegate investment to the manager for all $\rho \in (\bar{\rho}, 1 - K)$.*

5.3 Project Choice

When the output of the manager's effort is independent of the investment project (as in the baseline model), investors' private information exacerbates the underlying moral hazard problem. Firms, however, may have some ability to modify the project on which the manager works or the type of investment they pursue. In the appendix, we consider two distinct alternatives: allowing the manager's effort to impact the value of the investment and allowing the manager's effort to be correlated with the investment payoff.

In Appendix B.1, the manager's effort is directed towards increasing the payoff from the investment, so that the state-dependent cash flow is

$$V(\omega, e, I) \equiv x_\omega + \delta (y_\omega + e) \times I(P). \tag{38}$$

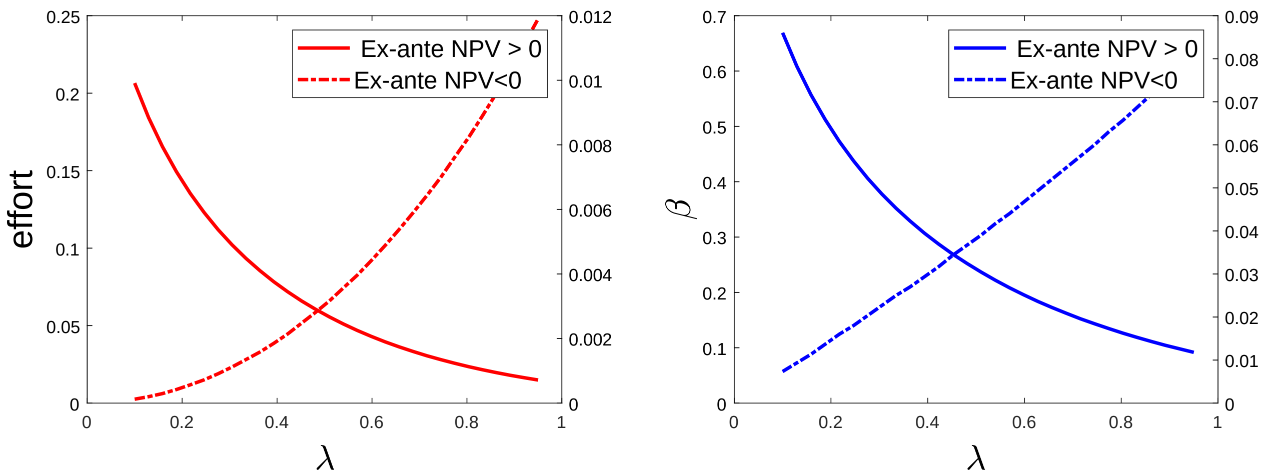
Crucially, the payoff to his effort is only realized when the investment is made. There are two cases of interest: in the first, the project is ex ante positive NPV and so the principal invests unless the price is p_L; in the second, the project is ex ante negative NPV and so she only invests if the price is p_H.[25] This implies that managerial effort is not only squandered with some probability but also that it increases the variability of his compensation. Both channels deter the manager from exerting effort relative to our baseline model.

However, the effect of investors' private information on the performance-sensitivity is more nuanced. As the number of informed investors grows, it is more likely that the price is informative. In the first case (when the project is ex ante positive NPV), an increase in price informativeness increases the likelihood that the principal will *not* invest. This lowers the optimal β and equilibrium effort level, as shown in Figure 7, found in the appendix. In the second case (when the project is ex ante negative NPV), however, an increase in λ makes it *more* likely that the principal invests, increasing the likelihood that the manager's effort pays off. In this case, the contract can be *more* performance-sensitive as the fraction of informed investors grow so that equilibrium effort is increasing in λ.

[25] There are two other cases; however, if the principal always invests, this setting is equivalent to our baseline model and if she never invests, then there is no reason to contract with the manager since the impact of his effort is never realized.

Figure 7: Optimal effort and pay-for-performance when effort affects investment payoffs

The figure plots the equilibrium effort and the optimal contract chosen by the principal as a function of λ when managerial effort affects investment payoffs (as in (38)). Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$ and $\rho = 0.7$. Moreover, we choose $y_L = -0.9$ for the positive ex-ante NPV project and $y_L = -1.1$ for the negative ex-ante NPV project.



(a) effort (b) β

These results suggest that understanding both (i) the manager's role within the firm as well as (ii) the types of investments the firm is pursuing is important for understanding how price information affects managerial compensation. They also demonstrate a novel channel through which project choice can alleviate the moral hazard problem. In particular, Figure 7 suggests that when the manager's effort affects the value of the investment, pursuing ex ante negative NPV investments may be preferable when prices are more informative.

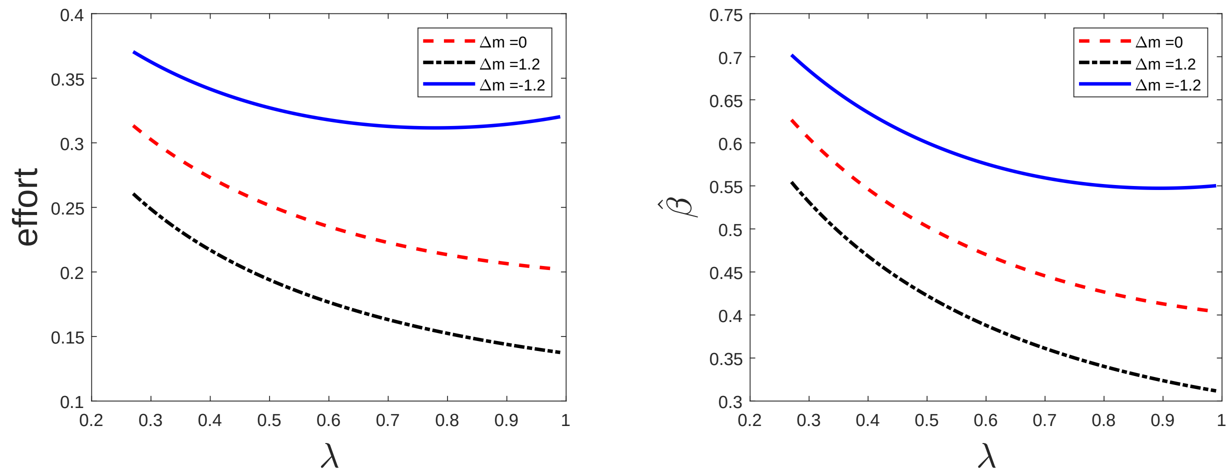
In Appendix B.2, the manager's effort affects the value of assets-in-place but the impact is state-dependent and, hence, correlated with the investment payoff. Specifically, we let

$$V(\omega, e, I) \equiv x_\omega + m_\omega e + \delta y_\omega \times I(P), \tag{39}$$

where both m_ω are commonly-known and $m_H + m_L > 0$, so that managerial effort increases firm value, on average. In this case, increasing effort can increase the variance of the price when managerial output positively covaries with the investment payoff, i.e., when $\Delta m \equiv m_H - m_L > 0$. As above, this reduces mangerial effort all else equal. On the other hand, when the two are negatively correlated (i.e., if $\Delta m < 0$), the manager finds it beneficial to exert more effort since it further reduces the variance of his compensation. Effectively, his effort serves as a hedge against the risk induced by the investment project.

Figure 8: Optimal effort and pay-for-performance when effort affects assets in place

The figure plots the equilibrium effort and the optimal contract chosen by the principal as a function of λ when managerial effort affects payoffs from assets in place (as in (39)). Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1.5, x_L = 1.05$, $y_H = 1$, $y_L = -0.9$, $\rho = 0.7$, $m_H = 1 + 0.5 * \Delta m$ and $m_L = 1 - 0.5 * \Delta m$.



(a) effort (b) $\hat{\beta}$

As we show in Figure 8, this implies that holding fixed the ex-ante value of managerial effort (i.e., holding $\frac{m_H+m_L}{2}$ constant), the principal would prefer he pursues projects which are negatively correlated with potential investments: both the optimal β and equilibrium effort increase as Δm falls. As in our baseline setting, β and effort decline with λ when the project is positively correlated. In contrast, for a negatively correlated project, we illustrate how equilibrium effort and β can increase in the measure of informed investors for λ sufficiently high. By choosing projects which allow the manager to reduce the risk he faces, the moral hazard problem is attenuated.

Figure 8 also illustrates how pursuing investments that are positively-correlated with the manager's effort can magnify the impact of moral hazard. In contrast, Davis and Gondhi (2022) show that positively-correlated projects increase endogenous information acquisition and hence, price informativeness, which reduces the likelihood of risk-shifting. Together, these results suggest careful consideration of the underlying agency conflict when deciding whether to pursue diversifying or focusing investments.

6 Empirical Predictions

6.1 Efficiency measures

In what follows, we characterize how several empirical measures of price efficiency, proposed in the literature, depend on the fraction of informed investors and the precision of their signals in our setting. To begin, we define our measures of interest.

1. **Revelatory price efficiency (*RPE*):** Revelatory price efficiency measures the extent to which prices reveal information which is relevant for real investment decisions. In our model, this can be measured as

$$RPE = \frac{\mathbb{V}\left[y_\omega\right] - \mathbb{V}\left[y_\omega | P\right]}{\mathbb{V}\left[y_\omega\right]}$$

2. **Contracting price efficiency (*CPE*):** Given the manager's risk-aversion, the price is more useful for contracting if it tracks the manager's effort more precisely. This contracting price efficiency is captured by the noise-to-signal ratio of the price about the manager's action, which is a common measure of contracting efficiency in the literature (e.g., Holmström and Tirole (1993)). Since the signal which the principal can extract is $\tilde{\mu}\left(P\right)$, which moves one-for-one with managerial effort, this measure is simply the inverse of this signal:

$$CPE = \left[\mathbb{V}\left(\tilde{\mu}\left(P\right)|e\right)\right]^{-1}.$$

3. **Forecasting price efficiency (*FPE*):** Forecasting price efficiency captures the extent to which the information in the price forecasts future firm cash flows. A common measure of forecasting price efficiency is the inverse of the variance of cash flows conditional on observing the price. In our model, this measure is given by

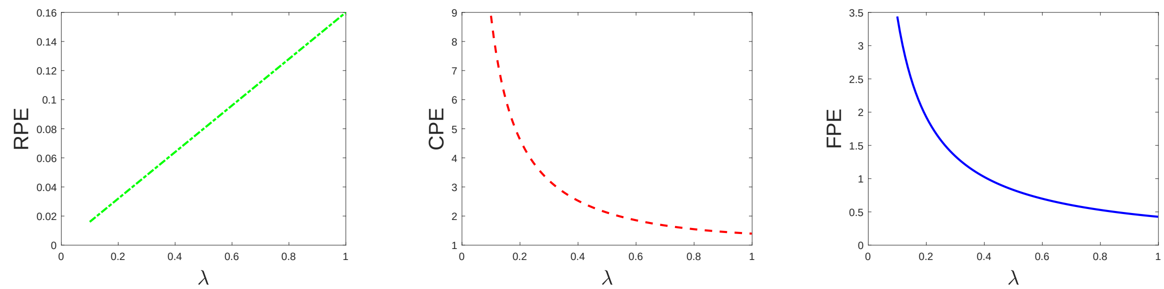
$$FPE = \left[\mathbb{V}\left(x_\omega + e + \delta y_\omega I\left(P\right)|P\right)\right]^{-1}.$$

With these definitions in mind, we formally characterize how each type of efficiency varies when investors, in the aggregate, have more information about the investment opportunity.

Proposition 9. *In the equilibrium characterized by Propositions 1 and 2, we have:*
(i) RPE increases in λ and ρ,
(ii) CPE decreases in λ and ρ,

Figure 9: Efficiency metrics versus fraction of informed

The figure plots the RPE, CPE and FPE as a function of λ. The other parameters of the model are set to: $x_H = 1$, $x_L = 0.5$, $c_e = 2$, $\rho = 0.7$ and $y_H = 1, y_L = -1.1$.



(iii) FPE decreases in λ iff $\rho > K > \frac{1}{2}$ and δ is sufficiently high i.e., $\left(1 + \delta \frac{y_H - y_L}{x_H - x_L}\right)^2 > \left(\frac{\rho^2 + (1-\rho)^2}{2\rho(1-\rho)}\right)$. FPE decreases in ρ iff $K > 0.5$ at $\rho = K$.

Figure 9 provides an illustration of this result by plotting RPE, CPE and FPE as a function of λ. As ρ increases, the signal which the principal can extract from the price is more precise. As λ increases, it is more likely that the price provides an informative signal to the principal. This implies that revelatory price efficiency increases in both ρ and λ (e.g., Panel (a) of Figure 9). On the other hand, as the price becomes more informative about investors' information, it becomes relatively less informative about the manager's action, that is, the signal to noise ratio of the price signal about e decreases. This leads to lower contracting price efficiency, as illustrated by Panel (b) of Figure 9.

The link between investors' information and forecasting price efficiency is more nuanced because the variance of the firm's cash flows is endogenous. Holding fixed the likelihood of investment, an increase in either λ or ρ increases the precision of the marginal investor's forecast, on average, which increases *FPE*. However, λ and ρ also affect the likelihood of investment, which can increase the variance in the firm's cash flows and push *FPE* downward. For example, when $\rho < K$, the manager never invests; once ρ crosses the threshold K, there is a discontinuous drop in *FPE* due to the likelihood of investment. Once in this region (i.e., when $\rho > K > \frac{1}{2}$), an increase in λ makes investment more likely. When the relative importance of the investment is sufficiently large (i.e., δ is sufficiently high) relative to the precision of investors' information, ρ, such increases in λ can lead to a decrease in *FPE*, as we see in Figure 9 panel (c).[26]

The recent literature on feedback effects has emphasized the tension between forecasting

[26]Note that the right-hand side of the inequality found in the proposition is increasing in ρ.

price efficiency (FPE) and revelatory price efficiency (RPE), for example, see Bond et al. (2012). In these models, what matters for real efficiency is whether the price reveals information necessary for decision-makers to take value-maximizing actions. As a result, the empirical literature has focused on RPE as the relevant measure of incremental real efficiency introduced through the information found in security prices (e.g., Bai et al. (2016)). Our analysis highlights that RPE is an incomplete, and potentially misleading, measure of incremental real efficiency when stock prices are used to provide managerial incentives. In equilibrium, RPE and CPE move in opposite directions and so focusing on either measure in isolation as a proxy for real efficiency can be misleading. Similarly, Proposition 9 suggests that FPE can be a misleading measure of investors' private information when firms use such information to guide real investment decisions.

6.2 Additional implications

In this section, we summarize a number of novel, testable predictions based on our analysis.

Our analysis emphasizes that the impact of price information depends upon the relative importance of feedback effects, or δ, for a given firm. While we are not aware of any agreed-upon proxies for δ, we build on the existing literature to propose some potential approaches before discussing our model-specific predictions. For example, broadly speaking, the feedback effect is likely to be more important for firms for which price sensitivity to investment is higher (e.g., Chen et al. (2007)). This recommends the use of the investment-price sensitivity as a potential proxy for δ.[27] Edmans, Jayaraman, and Schneemeier (2017) argue that the feedback effect is weaker for firms with a greater incidence of insider trading, suggesting that all else equal, insider trading intensity and δ are negatively related. As recently demonstrated by Goldstein et al. (2021b), a particularly promising approach may be to use surveys to directly solicit managers' views of how important the feedback effect is for their firm. Finally, market-to-book ratio, R&D, and intangible assets, are likely to be higher for firms in which future investment opportunities play an important role (i.e., δ is high), though one must be careful since each is also affected by other firm characteristics.

Given these potential proxies for δ, our model predicts that an increase in investors' private information (e.g., as measured by price non-synchronicity, PIN, or other measures) is positively related to firm value for those firms where the feedback effect is most important (Proposition 4). On the other hand, when the feedback effect is absent or dampened (e.g., overconfident managers or high incidence of insider trading), more informative prices should

[27]We acknowledge that since the investment-price sensitivity depends not only on the relative importance of feedback effects, but also on the informativeness of prices, liquidity and other (deeper) parameters, it provides a noisy measure.

be associated with lower firm value. One potential proxy for the quality of investor information for this feedback channel is index inclusion – as Billett, Diep-Nguyen, and Garfinkel (2020) argues, index inclusion leads to a decline in investment price sensitivity. Moreover, Banerjee, Huang, Nanda, and Xiao (2021b) show that over-confident managers are more likely to ignore information reflected in stock prices. Our model predicts that, in contrast to rational managers, more investor information should always lead to a decrease in firm value for such managers.[28]

Our model predicts that firms for which the feedback effect is relatively less important should have higher pay-for-performance sensitivity for compensation (Proposition 3). This suggests that, all else equal, firms with lower investment-price sensitivity should be associated with higher powered executive compensation contracts. Similarly, an increase in investors' information should lead to a lower pay-for-performance sensitivity for executives, all else equal (Proposition 3). Lin et al. (2019) provide evidence for this prediction using a natural experiment, namely, the introduction of the Regulation SHO Pilot Program.[29] They show that pay-for-performance decreases and price sensitivity of investment increases for such firms, consistent with the predictions of our model.[30]

A large literature has focused on the impact of stock market liquidity on firm performance through its effect on price informativeness. In our model, stock price liquidity is inversely related to price informativeness. As such, our model predicts that higher liquidity is associated with higher pay-for-performance sensitivity, consistent with both Fang et al. (2009) and Jayaraman and Milbourn (2012). On the other hand, our model predicts that higher liquidity should be associated with lower revelatory price efficiency. In this vein, Fang, Tian, and Tice (2014) show that an increase in liquidity is associated with less innovation.

To the extent that firms influence transparency, e.g., by changing the clarity of their financial reporting or disclosures[31] , our results in Section 5.1 predict that the level of transparency should predictably vary with both firm and investor characteristics (see Li (2008); You and Zhang (2009); Miller (2010)). In particular, firms should be more opaque when price-based managerial compensation plays an important role, even when feedback from market prices can improve the firm's investment decisions. This is consistent with empir-

[28]This is because the benefit of more informative prices does not accrue when the manager ignores the price: instead, the only effect is the decline in contracting price efficiency and, hence, firm value.

[29]The Reg SHO program removed short-sale restrictions for randomly selected (pilot) firms from May 2005 to August 2007, which lowered trading frictions and improved price informativeness.

[30]One could use additional shocks to price informativeness, such as a decline in analyst coverage due to brokerage mergers or closures (e.g., Hong and Kacperczyk (2010), Kelly and Ljungqvist (2012)).

[31]For example, such transparency can make it easier for investors to learn about synergies between the firm and a potential target. Similarly, providing such information can also make it cheaper for investors to estimate the likelihood that such investments will pay off positively.

ical evidence that documents hard-to-value and opaque firms tend to use more aggressive stock-based pay for their managers (Bebchuk and Fried (2004)).

In Section 5.2. we show that, relative to the setting in which the principal (e.g., the board of directors) is responsible for the investment decision, there should be higher (lower) pay-for-performance when the firm has access to better (worse) investment opportunities. Moreover, we expect to see voluntary delegation of investment by the principal when the investment is ex-ante more valuable.

The results of Section 6.1 emphasize the difficulty in interpreting existing empirical measures of real efficiency. For instance, in the absence of contracting considerations, Bai et al. (2016) interpret any increase in revelatory price efficiency as necessarily associated with an increase in aggregate efficiency (expected firm value). However, Proposition 9 implies that when contracting is sufficiently important (δ is low), aggregate real efficiency is *negatively* related to revelatory price efficiency in equilibrium. Similarly, while Fang et al. (2009) demonstrate how an increase in contracting efficiency can increase firm value, our results highlight that this is not always the case. Specifically, if the increase in contracting efficiency is accompanied by a reduction in revelatory price efficiency, the overall firm value may fall instead.

7 Conclusions

We develop a model which highlights the dual role of a firm's stock price. First, it provides the principal a contractable signal about managerial effort, and so is used for managerial compensation. Second, it aggregates investor information about a new project and so affects real investment via feedback effects. We show that these roles are generally at odds: when the price is more informative about future investment opportunities, it is more volatile and, therefore, less effective for incentive provision. Our analysis highlights why it is critical for researchers to account for the relative contribution of new investments and managerial effort when determining the impact of investor information on both firm value and its policies.

Our model is stylized for tractability and clarity of exposition, but naturally lends itself to further study. For instance, our analysis suggests that the dual role of the stock price should influence the manager's incentive to acquire, and publicly disclose, information about assets in place and future investment opportunities. We leave this analysis for future research.

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A Proofs

A.1 Proof of Proposition 1

First, we establish the market-clearing price as a function of u and ω. In equations (15) and (16), we establish the thresholds for $P = p_L$. We now establish analogous thresholds for when the price is p_u and p_H.

If $\omega = H$, then $P = p_H$ as long as

$$u < \underbrace{\lambda\rho}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_L} = 2\lambda\rho - 1, \tag{40}$$

while if $\omega = L$, then $P = p_H$ as long as

$$u < \underbrace{\lambda(1-\rho)}_{\text{observed } s_i = s_H} - \underbrace{(1-\lambda)}_{\text{uninformed}} - \underbrace{\lambda(\rho)}_{\text{observed } s_i = s_L} = 2\lambda(1-\rho) - 1. \tag{41}$$

As a result, if $\omega = H$, then $P = p_u$ as long as

$$2\lambda\rho - 1 \leq u \leq 1 - 2\lambda(1-\rho), \tag{42}$$

while if $\omega = L$, then $P = p_u$ as long as

$$2\lambda(1-\rho) - 1 \leq u \leq 1 - 2\lambda\rho. \tag{43}$$

Note, then, that if we define $u_H \equiv \lambda(2\rho - 1)$, $u_L = \lambda(1 - 2\rho)$, the thresholds above correspond to those found in the proposition.

With these market-clearing conditions, we confirm the signal that both investors and the principal can extract from the price and, in so doing, confirm the beliefs of the marginal investor at each price level. Specifically,

$$\begin{aligned}
\mathbb{P}[\omega = H|p_h] &= \frac{\mathbb{P}[\omega = H \cap P = p_H]}{\mathbb{P}[P = p_h]} = \frac{\lambda\rho}{\lambda\rho + \lambda(1-\rho)} = \rho \\
\mathbb{P}[\omega = H|p_l] &= \frac{\mathbb{P}[\omega = H \cap P = p_l]}{\mathbb{P}[P = p_l]} = \frac{\lambda(1-\rho)}{\lambda\rho + \lambda(1-\rho)} = 1 - \rho \\
\mathbb{P}[\omega = H|p_u] &= \frac{\mathbb{P}[\omega = H \cap P = p_u]}{\mathbb{P}[P = p_u]} = \frac{1-\lambda}{1-\lambda+1-\lambda} = \tfrac{1}{2}.
\end{aligned}$$

Note that, given these beliefs, the optimal investment rule, (6), yields the investment rule specified in the proposition.

Finally, in order for the conjectured equilibrium to exist, the equilibrium price levels must be distinct. To do so, we show that it is always the case that $p_H > p_u > p_L$. If the principal's investment decision is the same across any two prices, then this ordering holds trivially: V_H and V_L are the same across the two prices (given the investment policy) while $\tilde{\rho} > \frac{1}{2} > 1 - \tilde{\rho}$. Suppose instead that the principal's investment decision differs across two adjacent prices. There are two cases to consider.

(1) Suppose that the principal only invests if she observes $P = p_H$. Then, given the logic above, $p_u > p_L$ and it remains to be shown that $p_H > p_u$. Note that

$$\begin{aligned} p_H &= \frac{1}{1+\beta}\left[(1-\tilde{\rho})\,V_L(p_H) + \tilde{\rho}V_H(p_H) - \alpha\right] \\ &> \frac{1}{1+\beta}\left[(1-\rho)\,V_L(p_H) + \rho V_H(p_H) - \alpha\right] \\ &> \frac{1}{1+\beta}\left[(1-\rho)\,V_L(p_u) + \rho V_H(p_u) - \alpha\right] \\ &> \frac{1}{1+\beta}\left[\tfrac{1}{2}V_L(p_u) + \tfrac{1}{2}V_H(p_u) - \alpha\right] \\ &= p_u, \end{aligned}$$

where the first and third inequalities follows from $\tilde{\rho} > \rho > \frac{1}{2}$, while the second inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows.

(2) Suppose that the principal only invests if she observes $P \in \{p_H, p_u\}$. Then, given the logic above, $p_H > p_u$ and it remains to be shown that $p_u > p_L$. Note that

$$\begin{aligned} p_u &= \frac{1}{1+\beta}\left[\tfrac{1}{2}V_L(p_u) + \tfrac{1}{2}V_H(p_u) - \alpha\right] \\ &> \frac{1}{1+\beta}\left[\tfrac{1}{2}V_L(p_L) + \tfrac{1}{2}V_H(p_L) - \alpha\right] \\ &> \frac{1}{1+\beta}\left[\tilde{\rho}V_L(p_L) + (1-\tilde{\rho})\,V_H(p_L) - \alpha\right] \\ &= p_L, \end{aligned}$$

where the second inequality follows from $\frac{1}{2} > 1-\tilde{\rho}$, while the first inequality follows from the fact that if the principal invests, it increases the value of the firm's expected cash flows. $\square$

A.2 Proof of Proposition 2

Principal's objective is

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2 \mathbb{V}\left[\tilde{\mu}(P)\right] - c(e) \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{44}$$

The FOC is

$$\frac{\partial e}{\partial \beta}\left(1 - c'(e)\right) - \gamma \frac{\beta}{(1+\beta)^3}\mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which implies

$$\frac{1}{(1+\beta)^2}\left(\frac{1-c'(e)}{c''(e)}\right) - \gamma\frac{\beta}{(1+\beta)^3}\mathbb{V}\left[\tilde{\mu}(P)\right] = 0$$

which in turn implies

$$\beta = \frac{1}{\gamma c''(e)\,\mathbb{V}\left[\tilde{\mu}(P)\right]}.$$

Let $B = \frac{\beta}{1+\beta}$. The second order condition is

$$\frac{\partial^2 e}{\partial B^2}\left(1 - c'(e)\right) - \frac{\partial e}{\partial B}\frac{\partial e}{\partial B}c''(e) - \gamma\mathbb{V}\left[\tilde{\mu}(P)\right]$$

Note that

$$B = c'(e) \implies 1 = c''(e)\frac{\partial e}{\partial B} \implies 0 = c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B} + c''(e)\frac{\partial^2 e}{\partial B^2} = 0.$$

Substituting these into SOC, we get

$$-\frac{c'''(e)\frac{\partial e}{\partial B}\frac{\partial e}{\partial B}}{c''(e)}\left(1 - c'(e)\right) - \frac{1}{c''(e)} - \gamma\mathbb{V}\left[\tilde{\mu}(P)\right]$$

which simplifies to

$$-\frac{c'''(e)}{\left[c''(e)\right]^2}c'(e)\left(1 - c'(e)\right) - 1.$$

For the SOC to hold, we need

$$\frac{c'''(e)}{\left[c''(e)\right]^2}c'(e)\left(1 - c'(e)\right) + 1 > 0$$

□

A.3 Proof of Proposition 3

Note that

$$\beta = \frac{1}{\gamma c''(e)\,\mathbb{V}\left[\tilde{\mu}(P)\right]}$$

which implies that

$$\begin{aligned}
\frac{\partial\beta}{\partial\lambda} &= -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\lambda} \\
\frac{\partial\beta}{\partial\rho} &= -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\rho}. \\
\frac{\partial\beta}{\partial\delta} &= -\frac{1}{\gamma c''(e)\left(\mathbb{V}\left[\tilde{\mu}(P)\right]\right)^2}\frac{\partial\mathbb{V}\left[\tilde{\mu}(P)\right]}{\partial\delta}.
\end{aligned}$$

$$\mathbb{V}\left[\tilde{\mu}(P)\right] = \frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\,\tilde{\mu}(p_u)^2 - \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)\right) + (1-\lambda)\,\tilde{\mu}(p_u)\right]^2 \tag{45}$$

which implies

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \lambda}=\left(\tilde{\mu}\left(p_H\right)-\tilde{\mu}\left(p_u\right)\right)\left(\tilde{\mu}\left(p_u\right)-\tilde{\mu}\left(p_L\right)\right)+\frac{1-\lambda}{2}\left(\tilde{\mu}\left(p_H\right)+\tilde{\mu}\left(p_L\right)-2\tilde{\mu}\left(p_u\right)\right)^2>0.$$

Similarly, since $\frac{\partial \tilde{\mu}(p_L)}{\partial \rho}<0$ and $\tilde{\mu}\left(p_L\right)<\mathbb{E}\left[\tilde{\mu}\left(P\right)\right]$, then

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \rho}=\lambda\left[\frac{\partial \tilde{\mu}\left(p_H\right)}{\partial \rho}\left(\tilde{\mu}\left(p_H\right)-\mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right)+\frac{\partial \tilde{\mu}\left(p_L\right)}{\partial \rho}\left(\tilde{\mu}\left(p_L\right)-\mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right)\right]>0 \tag{46}$$

Moreover,

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \delta}>0 \tag{47}$$

This implies that

$$\frac{\partial \beta}{\partial \lambda}<0 \qquad \frac{\partial \beta}{\partial \rho}<0 \qquad \frac{\partial \beta}{\partial \delta}<0$$

Note that effort solves

$$\frac{\beta}{1+\beta}=c'\left(e\right)$$

which implies

$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \delta}=c''\left(e\right)\frac{\partial e}{\partial \delta}$$

$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \lambda}=c''\left(e\right)\frac{\partial e}{\partial \lambda}$$

$$\frac{1}{\left(1+\beta\right)^2}\frac{\partial \beta}{\partial \rho}=c''\left(e\right)\frac{\partial e}{\partial \rho}$$

which implies that effort decreases with δ, λ and ρ. □

A.4 Proof of Proposition 4

Let $B=\frac{\beta}{1+\beta}$. Note that B and e solves

$$B=c'\left(e\right) \qquad B=\frac{1}{1+\gamma c''\left(e\right)\mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}$$

Differentiating with respect to λ, we get

$$c''\left(e\right)\frac{\partial e}{\partial \lambda}=\frac{\partial B}{\partial \lambda} \qquad \frac{\partial B}{\partial \lambda}=-\gamma B^2\left[c''\left(e\right)\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}+\mathbb{V}\left[\tilde{\mu}\right]c'''\left(e\right)\frac{\partial e}{\partial \lambda}\right]$$

This implies that

$$\frac{\partial e}{\partial \lambda}=-\gamma B^2\left[\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}+\mathbb{V}\left[\tilde{\mu}\right]\frac{c'''\left(e\right)}{c''\left(e\right)}\frac{\partial e}{\partial \lambda}\right]$$

$$\frac{\partial e}{\partial \lambda}\left(\underbrace{1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}}_{>0}\right) = -\gamma B^2 \frac{\partial \mathbb{V}\left[P\right]}{\partial \lambda}$$

The term in the underlying brace in the above equation is

$$1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)} = 1 + c'(e)\left(1-c'(e)\right)\frac{c'''(e)}{\left[c''(e)\right]^2} > 0$$

because of the second order condition. This implies

$$\frac{\partial e}{\partial \lambda} = -\frac{\gamma B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}}{1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}} < 0. \tag{48}$$

Note that firm value is given by

$$\frac{\partial FV}{\partial \lambda} = \frac{\partial e}{\partial \lambda} + \delta \frac{\partial E\left[\mathbb{I}_{d_m = I} y_\omega\right]}{\partial \lambda}$$

If $\rho < K$ or $1-\rho > K$, then firm value is decreasing in λ. For the rest of the proof, assume that $\rho > K > 1-\rho$. Differentiating equation 48 , we get

$$\frac{\partial^2 e}{\partial \lambda^2} = -\frac{\gamma B^2\left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}\right)\frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} + 2\gamma B \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\frac{\partial B}{\partial \lambda} - \gamma^2 B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\left(B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + B^2 \frac{\partial \mathbb{V}[\tilde{\mu}]}{\partial \lambda}\frac{c'''(e)}{c''(e)}\right)}{\left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}\right)^2}$$

Note that $\frac{\partial^2 \mathbb{V}[\tilde{\mu}]}{\partial \lambda^2} = -\frac{1}{2}\left(p_H + p_L - 2p_u\right)^2 \leq 0$. This implies

$$\frac{\partial^2 e}{\partial \lambda^2} = \gamma B^2\left(1+\gamma B^2 \mathbb{V}\left[\tilde{\mu}\right]\frac{c'''(e)}{c''(e)}\right)\frac{-\partial^2 \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda^2} + 2\gamma B c''(e)\frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{-\partial e}{\partial \lambda} + \gamma^2 B^4 \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\left(\mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{c'''(e)}{c''(e)}\right)$$

The first two terms are positive. So, the function is convex iff the third term is also positive

$$\mathbb{V}\left[\tilde{\mu}\right]\frac{\partial \frac{c'''(e)}{c''(e)}}{\partial \lambda} + \frac{\partial \mathbb{V}\left[\tilde{\mu}\right]}{\partial \lambda}\frac{c'''(e)}{c''(e)} > 0. \tag{49}$$

If the above condition is true, effort is convex in λ, which implies that firm value is also convex in λ. This implies that there are only three possible shapes for FV: increasing, decreasing, U shaped. There exists $\bar{\delta} > \underline{\delta} > 0$ such that, for $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, FV is U shaped in λ. If $\delta \leq \underline{\delta}$, FV is decreasing in λ. If $\delta > \bar{\delta}$, FV is increasing in λ. □

A.5 Proof of Proposition 5

Recall that investors differ only in their belief about the relative likelihood of each state. This implies that

$$\mathbb{E}\left[d\left(\{s_i, P\}\right)(x_\omega + e + y_\omega I(P) - P)\right] = \left[\tfrac{\rho\lambda(1-\rho)}{2} + \tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i = s_H, p_H\right) - q\left(s_i = s_L, p_H\right)\right)\Delta V\left(p_H\right) \tag{50}$$

$$+ \left[\tfrac{\rho\lambda(1-\rho)}{2} + \tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i = s_H, p_L\right) - q\left(s_i = s_L, p_L\right)\right)\Delta V\left(p_L\right) \tag{51}$$

$$+ \left[\tfrac{(1-\lambda)\rho}{2} + \tfrac{(1-\lambda)(1-\rho)}{2}\right]\left(q\left(s_i = s_H, p_U\right) - q\left(p_U\right)\right)\Delta V\left(p_U\right) \tag{52}$$

$$+ \left[\tfrac{(1-\lambda)\rho}{2} + \tfrac{(1-\lambda)(1-\rho)}{2}\right]\left(q\left(p_U\right) - q\left(s_i = s_L, p_U\right)\right)\Delta V\left(p_U\right) \tag{53}$$

while

$$\mathbb{E}\left[d\left(\{P\}\right)(x_\omega + e + y_\omega I(P) - P)\right] = \left[\tfrac{\rho\lambda}{2} + \tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(s_i = s_H, p_H\right) - q\left(p_H\right)\right)\Delta V\left(p_H\right) \tag{54}$$

$$+ \left[\tfrac{\rho\lambda}{2} + \tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(p_L\right) - q\left(s_i = s_L, p_L\right)\right)\Delta V\left(p_L\right) \tag{55}$$

We can simplify by noting that

$$\left[\tfrac{\rho\lambda(1-\rho)}{2} + \tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i = s_H, p_H\right) - q\left(s_i = s_L, p_H\right)\right) = \rho\lambda\left(1-\rho\right)\left(\tilde{\rho} - \tfrac{1}{2}\right) \tag{56}$$

$$\left[\tfrac{\rho\lambda}{2} + \tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(s_i = s_H, p_H\right) - q\left(p_H\right)\right) = \tfrac{\lambda}{2}\left(\tilde{\rho} - \rho\right) \tag{57}$$

$$\left[\tfrac{\rho\lambda(1-\rho)}{2} + \tfrac{(1-\rho)\lambda\rho}{2}\right]\left(q\left(s_i = s_H, p_L\right) - q\left(s_i = s_L, p_L\right)\right) = \rho\lambda\left(1-\rho\right)\left(\tfrac{1}{2} - (1-\tilde{\rho})\right) \tag{58}$$

$$\left[\tfrac{\rho\lambda}{2} + \tfrac{(1-\rho)\lambda}{2}\right]\left(q\left(p_L\right) - q\left(s_i = s_L, p_L\right)\right) = \tfrac{\lambda}{2}\left((1-\rho) - (1-\tilde{\rho})\right) \tag{59}$$

Substituting in the expression for $\tilde{\rho}$ reveals that all four expressions are equal to

$$\tfrac{\rho\lambda(1-\rho)}{2}\left(\tfrac{2\rho-1}{\rho^2+(1-\rho)^2}\right). \tag{60}$$

As a result, the indifference condition reduces to

$$\tfrac{1-\lambda}{2}\left(\rho - \tfrac{1}{2}\right)\Delta V\left(p_U\right) + \tfrac{1-\lambda}{2}\left(\tfrac{1}{2} - (1-\rho)\right)\Delta V\left(p_U\right) = c \tag{61}$$

and so in an interior equilibrium, the measure of informed investors is

$$\lambda = 1 - \tfrac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}. \tag{62}$$

$$= 1 - \tfrac{c}{\left(\rho-\frac{1}{2}\right)(x_H - x_L + I(p_U)(y_H - y_L))} \tag{63}$$

More generally,

$$\lambda = \max\left\{0, 1 - \tfrac{c}{\left(\rho-\frac{1}{2}\right)\Delta V(p_U)}\right\}. \tag{64}$$

Thus, in any setting, the measure of informed investors is increasing in ρ but does *not* depend upon e. $\square$

A.6 Proof of Proposition 6

Let $B = \frac{\beta}{1+\beta}$ and note that $c'(e) = B$ and $\beta = \frac{1}{c''(e)\gamma\mathbb{V}(\tilde{\mu}(P))}$, which implies $c''(e)\frac{\partial e}{\partial \lambda} = \frac{\partial B}{\partial \lambda}$ and

$$B = \frac{\beta}{1+\beta} = \frac{1}{1+c''(e)\gamma\mathbb{V}(\tilde{\mu}(P))} \tag{65}$$

$$\Rightarrow \frac{\partial B}{\partial \lambda} = -B^2\gamma\left(c''(e)\frac{\partial}{\partial \lambda}\mathbb{V}(\tilde{\mu}(P)) + \mathbb{V}(\tilde{\mu}(P))c'''(e)\frac{\partial e}{\partial \lambda}\right). \tag{66}$$

This implies

$$\begin{aligned}
\frac{\partial}{\partial \lambda}\left(e - c(e) - \frac{\gamma}{2}B^2\mathbb{V}(\tilde{\mu}(P))\right) &= (1-c'(e))\frac{\partial e}{\partial \lambda} - \gamma\mathbb{V}(\tilde{\mu}(P))B\frac{\partial B}{\partial \lambda} - \frac{\gamma}{2}B^2\frac{\partial}{\partial \lambda}\mathbb{V}(\tilde{\mu}(P)) && (67)\\
&= (1-B-\gamma B\mathbb{V}(\tilde{\mu}(P))c''(e))\frac{\partial e}{\partial \lambda} - \frac{\gamma}{2}B^2\frac{\partial}{\partial \lambda}\mathbb{V}(\tilde{\mu}(P)) && (68)\\
&= (1-B-\gamma\mathbb{B}\mathbb{V}(\tilde{\mu}(P))c''(e))\frac{\partial e}{\partial \lambda} + \frac{1}{2}\left(1+\gamma B^2\mathbb{V}(\tilde{\mu}(P))\frac{c'''(e)}{c''(e)}\right)\frac{\partial e}{\partial \lambda} && (69)\\
&= \left(\frac{1}{2} + \frac{\gamma}{2}B^2\mathbb{V}(\tilde{\mu}(P))\frac{c'''(e)}{c''(e)}\right)\frac{\partial e}{\partial \lambda} && (70)\\
&= -\frac{\gamma}{2}B^2\frac{\partial}{\partial \lambda}\mathbb{V}(\tilde{\mu}(P)) < 0 && (71)
\end{aligned}$$

and so we can express the change in firm value as

$$\begin{aligned}
\frac{\partial}{\partial c_0}FV &= \frac{\partial}{\partial \lambda}\mathbb{E}[\delta y_\omega \times I]\frac{d\lambda}{dc_0} + \frac{\partial}{\partial \lambda}\left(e - \frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2\mathbb{V}(\tilde{\mu}(P)) - c(e)\right)\frac{d\lambda}{dc_0} && (72)\\
&= \left(\delta\frac{\partial}{\partial \lambda}\mathbb{E}[y_\omega \times I] - \frac{\gamma}{2}B^2\frac{\partial}{\partial \lambda}\mathbb{V}(\tilde{\mu}(P))\right)\frac{d\lambda}{dc_0}, && (73)
\end{aligned}$$

Next, note that

$$\frac{\partial \mathbb{E}[\delta y_\omega \times I]}{\partial \lambda} = \begin{cases} 0 & \text{if } 1-\rho > K \\ \frac{\delta(y_H - y_L)}{2}(\rho - 1 + K) & \text{if } \frac{1}{2} > K > 1-\rho \\ \frac{\delta(y_H - y_L)}{2}(\rho - K) & \text{if } \rho > K > \frac{1}{2}, \\ 0 & \text{if } \rho < K \end{cases}$$

and

$$\begin{aligned}
\frac{\partial \mathbb{V}[\tilde{\mu}(P)]}{\partial \lambda} &= \frac{1}{2}\left(\mu_H^2 + \mu_L^2 - 2\mu_U^2\right) - \mu_U(\mu_H + \mu_L) - \frac{1}{2}\lambda(\mu_H + \mu_L - 2\mu_U)^2 && (74)\\
&= \frac{1}{2}\left((\mu_H - \mu_U)^2 + (\mu_L - \mu_U)^2\right) - \frac{1}{2}\lambda(\mu_H + \mu_L - 2\mu_U)^2 && (75)\\
&\geq (\mu_H - \mu_U)(\mu_U - \mu_L) > 0 && (76)
\end{aligned}$$

So that

$$\frac{\partial}{\partial c_0}FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{77}$$

Next, note that if $1-\rho > K$ or $\frac{1}{2} > K > 1-\rho$:

$$\lim_{\delta\to 0}\lambda = 1-\frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L\right)}, \tag{78}$$

$$\lim_{\delta\to\infty}\lambda = 1 \tag{79}$$

$$\lim_{\delta\to\infty}\left(1-\lambda\right)\delta = \lim_{\delta\to\infty}\frac{c_0\delta}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L+\delta\left(y_H-y_L\right)\right)} \tag{80}$$

$$= \frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H-y_L\right)} \tag{81}$$

and if $\rho > K > \frac{1}{2}$ or $\rho < K$, then

$$\lambda = 1-\frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(x_H-x_L\right)}. \tag{82}$$

We have to consider four cases:

(1) If $1-\rho > K$, then

$$\frac{\partial}{\partial c_0}FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right)\right), \tag{83}$$

which is positive.

(2) If $\frac{1}{2} > K > 1-\rho$, then

$$\frac{\partial}{\partial c_0}FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{84}$$

$$\Rightarrow \lim_{\delta\to\infty}\frac{\partial}{\partial c_0}FV = \lim_{\delta\to\infty}-\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{85}$$

$$= -\frac{c_0}{\left(\rho-\frac{1}{2}\right)\left(y_H-y_L\right)}\mathbb{E}\left[y_\omega \times I\right]-1 < 0 \tag{86}$$

On the other hand, as delta shrinks, the following is positive:

$$\lim_{\delta\to 0}\frac{\partial}{\partial c_0}FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H-\mu_U\right)^2+\left(\mu_L-\mu_U\right)^2 \\ -\lambda\left(\mu_H+\mu_L-2\mu_U\right)^2 \end{array}\right)\right). \tag{87}$$

(3) If $\rho > K > \frac{1}{2}$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$, and

$$\frac{\partial}{\partial c_0} FV = -\left(\frac{1-\lambda}{c_0}\right)\left(\begin{array}{c} \frac{\partial}{\partial \lambda}\mathbb{E}\left[\delta y_\omega \times I\right] \\ -\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right) \end{array}\right) \tag{88}$$

$$\Rightarrow \lim_{\delta \to 0} \frac{\partial}{\partial c_0} FV = \left(\frac{1-\lambda}{c_0}\right)\left(\frac{\gamma}{4}B^2\left(\begin{array}{c} \left(\mu_H - \mu_U\right)^2 + \left(\mu_L - \mu_U\right)^2 \\ -\lambda\left(\mu_H + \mu_L - 2\mu_U\right)^2 \end{array}\right)\right), \tag{89}$$

which is positive.

(4) If $\rho < K$, then $\lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)(x_H - x_L)}$ and following analogous steps to Case 1 shows that FV increases in c_0: the only impact of increased transparency is the reduction in managerial effort since the firm never invests in this case. □

A.7 Proof of Proposition 7

Note that firm value (ignoring assets in place) is given by

$$\begin{aligned} FV &= E\left[e + \delta y_\omega \times I\right] \\ &= \begin{cases} e + \delta \frac{y_H + y_L}{2} & \text{if } 1 - \tilde{\rho} > K \\ e + (1-\lambda)\,\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)\, y_L\right) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ e + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho)\, y_L\right) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ e & \text{if } \tilde{\rho} < K \end{cases} \end{aligned}$$

Note that,

$$\frac{\partial FV}{\partial \lambda} = \begin{cases} \frac{\partial e}{\partial \lambda} & \text{if } 1 - \tilde{\rho} > K \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}\left(\rho - 1 + K\right) & \text{if } \frac{1}{2} > K > 1 - \tilde{\rho} \\ \frac{\partial e}{\partial \lambda} + \delta \frac{(y_H - y_L)}{2}\left(\rho - K\right) & \text{if } \tilde{\rho} > K > \frac{1}{2}, \\ \frac{\partial e}{\partial \lambda} & \text{if } \tilde{\rho} < K \end{cases}$$

In the first and fourth cases, firm value decreases with λ since effort decreases with λ. Next consider the second case. As λ increases, the second term is positive only when $K > 1 - \rho$. In the region where $1 - \hat{\rho} < K < 1 - \rho$, the second term is negative. High λ can be bad for firm value in this region. Finally, consider the third case. As λ increases, the second term is positive only when $\rho > K$. In the region where $\rho < K < \hat{\rho}$, the second term is negative. Again, high λ can be bad for firm value in this region. □

A.8 Proof of Proposition 8

Let the principal's utility given the optimal linear contract be denoted by

$$U_p = \underbrace{e - \frac{\gamma}{2}B^2\mathbb{V}\left[\tilde{\mu}(P)\right] - c\,(e)}_{\equiv U_{p,e}} + \delta\mathbb{E}\left[y_\omega \times I\right]. \tag{90}$$

U_p does not depend upon delegation unless the manager's investment decision differs from her own; this occurs when $\rho < K < \hat{\rho}$ and $1-\hat{\rho} < K < 1-\rho$.

First, note that in either case, the manager's investment decision is inefficient (ex-ante and as a result, $\delta\mathbb{E}\left[y_\omega \times I\right]$ is lower under delegation. However, by the envelope theorem, $\frac{\partial U_{p,e}}{\partial \mathbb{V}[\tilde{\mu}(P)]} = -\frac{\gamma}{2}B^2 < 0$: if delegation reduces the variance of $\tilde{\mu}\left(P\right)$, then the principal can be better off letting the manager invest.

If $\rho < K < \hat{\rho}$, the manager invests when the principal would not (when $P = p_H$) which increases $\tilde{\mu}\left(p_H\right)$. Since

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \tilde{\mu}\left(p_H\right)} = \lambda\left(\tilde{\mu}\left(p_H\right) - \mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right) > 0, \tag{91}$$

this implies that the principal is necessarily worse off in this region. On the other hand, if $1-\hat{\rho} < K < 1-\rho$, the manager does not invest when $P = p_L$ which increases $\tilde{\mu}\left(p_L\right)$. But note that

$$\frac{\partial \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}{\partial \tilde{\mu}\left(p_L\right)} = \lambda\left(\tilde{\mu}\left(p_L\right) - \mathbb{E}\left[\tilde{\mu}\left(P\right)\right]\right) < 0, \tag{92}$$

which implies that principal can improve his net utility in this region.

Second, note that when $1-\rho = K$, $\mathbb{E}\left[y_\omega \times I_p\right] = \mathbb{E}\left[y_\omega \times I_m\right]$, where I_p (I_m) denotes the investment rule under the principal (manager). While the investment return under either investment rule is increasing in ρ, the difference between them is strictly decreasing in ρ when $1-\hat{\rho} < K < 1-\rho$ since

$$\mathbb{E}\left[y_\omega \times I_p\right] - \mathbb{E}\left[y_\omega \times I_m\right] = \frac{\lambda}{2}\left((1-\rho)y_H + \rho y_L\right) \tag{93}$$

which decreases in ρ. On the other hand, $U_{p,e}\left(I_m\right) - U_{p,e}\left(I_p\right)$ is strictly bounded above zero for any ρ in this interval. Since $U_{p,e}\left(I_m\right) - U_{p,e}\left(I_p\right)$ and $\mathbb{E}\left[y_\omega \times I_p\right] - \mathbb{E}\left[y_\omega \times I_m\right]$ are continuous in ρ, there exists a $\bar{\rho}$ such that for all ρ such that $1-K > \rho > \bar{\rho}$ the principal strictly prefers delegation. Finally, note that in order for $\rho < 1-K$ it must be the case that $K < 0.5$. This completes the proof of the statement. □

A.9 Proof of Proposition 9

We begin by characterizing how the efficiency measures depend on the underlying parameters, given the financial market equilibrium and the optimal contract.

Lemma 1. *Consider the unique financial market equilibrium described in proposition 1 and the optimal contract described in proposition 2. Then,*

(i) Revelatory price efficiency is

$$RPE = \lambda\left(2\rho - 1\right)^2$$

(ii) Forecasting price efficiency is

$$FPE^{-1} = \begin{cases} (x_H - x_L + \delta y_H - \delta y_L)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \text{if } 1-\rho > K \\ (x_H - x_L + \delta y_H - \delta y_L)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) + (x_H - x_L)^2 \frac{\rho(1-\rho)\lambda}{2} & \text{if } \frac{1}{2} > K > 1-\rho \\ (x_H - x_L + \delta y_H - \delta y_L)^2 \frac{\rho(1-\rho)\lambda}{2} + (x_H - x_L)^2 \left(\frac{\rho(1-\rho)\lambda}{2} + \frac{1-\lambda}{4}\right) & \text{if } \rho > K > \frac{1}{2}, \\ (x_H - x_L)^2 \left[\frac{1-\lambda(2\rho-1)^2}{4}\right] & \text{if } \rho < K \end{cases} \tag{94}$$

(iii) Contracting price efficiency is

$$CPE = \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\tilde{\mu}(p_U)^2 - \left(\frac{\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)\right) + (1-\lambda)\tilde{\mu}(p_U)\right)^2\right]^{-1}$$

(iv) Firm value is

$$FV = \frac{x_H + x_L}{2} + e + \begin{cases} \delta \frac{y_H + y_L}{2} & \text{if } 1-\rho > K \\ (1-\lambda)\delta \frac{y_H + y_L}{2} + \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho) y_L\right) & \text{if } \frac{1}{2} > K > 1-\rho \\ \frac{\lambda}{2}\delta\left(\rho y_H + (1-\rho) y_L\right) & \text{if } \rho > K > \frac{1}{2}, \\ 0 & \text{if } \rho < K \end{cases}$$

where $\tilde{\mu}(p_H)$,$\tilde{\mu}(p_U)$ and $\tilde{\mu}(p_L)$ are defined in proposition (1).

Note that

$$\begin{aligned} RPE &= \frac{V[y_\omega] - V[y_\omega|p]}{V[y_\omega]} \\ &= \frac{\frac{(y_H - y_L)^2}{4} - \left[\frac{(y_H - y_L)^2}{4}(1-\lambda) + \lambda\rho(1-\rho)(y_H - y_L)^2\right]}{\frac{(y_H - y_L)^2}{4}} \\ &= \lambda(2\rho - 1)^2 \end{aligned} \tag{95}$$

Moreover, by definition, $FPE^{-1} = \mathbb{V}(x_\omega + y_\omega \mathbb{I}_{d_m = I}|p)$ and this simplifies to 94. Contracting price efficiency is captured by

$$\begin{aligned} CPE &= \left[\mathbb{V}\left[\tilde{\mu}(p)|e\right]\right]^{-1} \\ &= \left[\frac{\lambda}{2}\left(\tilde{\mu}(p_H)^2 + \tilde{\mu}(p_L)^2\right) + (1-\lambda)\tilde{\mu}(p_U)^2 - \left(\frac{\lambda}{2}\left(\tilde{\mu}(p_H) + \tilde{\mu}(p_L)\right) + (1-\lambda)\tilde{\mu}(p_U)\right)^2\right]^{-1} \end{aligned}$$

□

Appendix B - Robustness and Additional Analysis

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B.1 Effort Impacts Investment Payoff

In our benchmark model, managerial effort affects the payoff of assets-in-place. In some settings, managerial effort may improve the value of a potential investment, if undertaken. For example, this may reflect ex-ante work which increases potential synergies between the firm's existing assets and the new investment. To examine the effect of such an interaction, in this section we consider a setting in which managerial effort does not affect the value of assets-in-place but instead increases the payoff of the investment from y_ω to $y_\omega + e$. For tractability, we also assume that investors trade the firms' terminal cash flow, i.e., they do not net out the manager's compensation. All other features of the model are unchanged.

In this case, the date-one expected value of the firm is maximized when the principal invests as long as

$$q(P) > \frac{-y_L - e}{y_H - y_L} \equiv \hat{K}(e). \tag{B.1}$$

The construction of the financial market equilibrium is unchanged. Investors share common beliefs about the state-dependent value of the firm and their valuation of the traded claim can be ranked by their respective beliefs about the likelihood that $\omega = H$.

The novel feature which arises in this setting is that the value of the manager's effort is only realized if the principal chooses to invest. As a result, it is now the case that $V_L(P) = x_L + 1_{I(P)=1}(y_L + e)$. If the manager always invests ($1 - \rho > \hat{K}(e)$), the analysis is as in our benchmark model since the manager's effort always increases the value of the firm's terminal cash flow. In contrast, when the principal never invests ($\rho < \hat{K}(e)$), the manager's effort has no impact on the price level. In the other two cases, managerial effort only impacts the price with probability less than one and, as a result, the volatility of the price is also impacted:

- If $\frac{1}{2} > \hat{K}(e) > 1 - \rho$, the manager only invests if $P \in \{p_u, p_H\}$ and the marginal investors' expectation of the firms terminal cash flows are

$$\tilde{\mu}(P) = \begin{cases} \tilde{\mu}(p_H) = x_L + y_L + e + \tilde{\rho}\left[x_H - x_L + y_H - y_L\right] & \text{if } u < u_\omega - (1-\lambda) \\ \tilde{\mu}(p_u) = x_L + y_L + e + \frac{1}{2}\left[x_H - x_L + y_H - y_L\right] & \text{if } u_\omega - (1-\lambda) \le u \le u_\omega + (1-\lambda) \\ \tilde{\mu}(p_L) = x_L + (1-\tilde{\rho})\left[x_H - x_L\right] & \text{if } u > u_\omega + (1-\lambda) \end{cases} \tag{B.2}$$

- If $\rho > \hat{K}(e) > \frac{1}{2}$, the manager only invests if $P = p_H$ and the marginal investors' expectation of the firms terminal cash flows are

$$\tilde{\mu}(P) = \begin{cases} \tilde{\mu}(p_H) = x_L + y_L + e + \tilde{\rho}\left[x_H - x_L + y_H - y_L\right] & \text{if } u < u_\omega - (1-\lambda) \\ \tilde{\mu}(p_u) = x_L + \frac{1}{2}\left[x_H - x_L + y_H - y_L\right] & \text{if } u_\omega - (1-\lambda) \le u \le u_\omega + (1-\lambda) \\ \tilde{\mu}(p_L) = x_L + (1-\tilde{\rho})\left[x_H - x_L\right] & \text{if } u > u_\omega + (1-\lambda) \end{cases} \tag{B.3}$$

Given this, the manager's first-order condition now implies that his optimal effort is the solution to

$$\beta \frac{\partial \mathbb{E}\left[P\right]}{\partial e} - \frac{\gamma \beta^2}{2} \frac{\partial \mathbb{V}\left[P\right]}{\partial e} = c'\left(e\right). \tag{B.4}$$

The principal's objective reflects this new incentive-compatibility constraint:

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + \left(y_\omega + e\right) \mathbb{I}_{d_m = I} - \beta P - \alpha\right] \tag{B.5}$$

$$\text{subject to} \tag{B.6}$$

$$\beta \frac{\partial \mathbb{E}\left[P\right]}{\partial e} - \tfrac{\gamma \beta^2}{2} \frac{\partial \mathbb{V}\left[P\right]}{\partial e} = c'\left(e\right) \tag{B.7}$$

$$\mathbb{E}\left[\beta P + \alpha\right] - \tfrac{\gamma}{2} \mathbb{V}\left[\beta P + \alpha\right] - c\left(e\right) \geq 0 \tag{B.8}$$

Since the manager's participation constraint always binds, this can be expressed as

$$\max_\beta \begin{cases} e - \frac{\gamma}{2}\beta^2 \mathbb{V}\left[P\right] - c\left(e\right) & \text{if } 1-\rho > \hat{K}\left(e\right) \\ e\left(1 - \frac{\lambda}{2}\right) - \frac{\gamma}{2}\beta^2 \mathbb{V}\left[P\right] - c\left(e\right) & \text{if } \frac{1}{2} > \hat{K}\left(e\right) > 1-\rho \\ e\frac{\lambda}{2} - \frac{\gamma}{2}\beta^2 \mathbb{V}\left[P\right] - c\left(e\right) & \text{if } \rho > \hat{K}\left(e\right) > \frac{1}{2} \\ -\frac{\gamma}{2}\beta^2 \mathbb{V}\left[P\right] - c\left(e\right) & \text{if } \rho < \hat{K}\left(e\right) \end{cases} \tag{B.9}$$

subject to

$$c'\left(e\right) = \begin{cases} \beta & \text{if } 1-\rho > \hat{K}\left(e\right) \\ \beta\left(1 - \frac{\lambda}{2}\right) - \gamma\beta^2 \frac{\lambda}{2}\left[\frac{\lambda}{2} p_H + \left(1-\lambda\right) p_u - \left(1 - \frac{\lambda}{2}\right) p_L\right] & \text{if } \frac{1}{2} > \hat{K}\left(e\right) > 1-\rho \\ \beta\frac{\lambda}{2} - \gamma\beta^2 \frac{\lambda}{2}\left[p_H\left(1 - \frac{\lambda}{2}\right) - \frac{\lambda}{2} p_L - \left(1-\lambda\right) p_u\right] & \text{if } \rho > \hat{K}\left(e\right) > \frac{1}{2} \\ 0 & \text{if } \rho < \hat{K}\left(e\right) \end{cases} \tag{B.10}$$

Thus, when $1-\rho > \hat{K}\left(e\right)$, the optimal β is the same as in Appendix B.7 and equilibrium effort is the same as in our benchmark model. When $\rho < \hat{K}\left(e\right)$, the manager never exerts effort and the optimal β is zero. In the other two cases, an increase in the measure of informed investors, λ, affects the likelihood of investment which alters both (i) the probability that the investment is chosen and (ii) the volatility of the price. In both cases, the volatility of the price is increasing in λ, as in our benchmark analysis, which reduces the optimal β and equilibrium managerial effort.

However, a more informative price also impacts the likelihood of investment when $\rho > \hat{K}\left(e\right) > 1-\rho$. If the principal does not invest when the price is low (i.e., if $\frac{1}{2} > \hat{K}\left(e\right) > 1-\rho$), then an increase in λ reduces the value of managerial effort further since it is less likely that it is realized. Figure 7 plots the equilibrium effort and optimal contract as a function of the fraction of informed, λ. In the case when $\frac{1}{2} > \hat{K}\left(e\right) > 1-\rho$, an increase in λ decreases the optimal effort i.e., the tension between information role and incentive role holds in the setting. In contrast, if the manger only invests when the price is high (i.e., if $\rho > \hat{K}\left(e\right) > \frac{1}{2}$, the manager's effort is more likely to be pay off: when λ increases, it is more likely that $P = p_H$ and, holding fixed the impact of increased volatility, this channel leads to an increase in managerial effort. In this case, an increase in λ can increase both the optimal β

and equilibrium effort, as Figure 7 shows.

B.2 State-Dependent Effort

In the baseline model, we assumed that the effort is state-independent. In what follows, we consider a setting in which the impact of the manager's effort is state-dependent. Specifically, given ω, the firm's assets-in-place are worth

$$x_\omega = x_\omega^0 + m_\omega e, \tag{B.11}$$

where $x_H^0 > x_L^0$, both m_ω are constants known by all agents, and $m_H + m_L > 0$. All other features of the model are unchanged.

At date two, the principal and investors follow the same decision rules found in the main model. The key difference, however, is that the state-dependent value of the firm's assets is now given by

$$V_\omega (P) \equiv x_\omega + m_\omega e + \delta y_\omega \times I(P). \tag{B.12}$$

Otherwise, the date two equilibrium is exactly as described in Proposition 1.

At date one, the manager chooses his effort to maximize his expected utility, subject to the cost of effort as in equation (22). The manager has no private information about the likelihood of each state and so believes that the likelihood that the price is uninformative is $1-\lambda$ while the likelihood of either the high or low price is $\frac{\lambda}{2}$. Thus,

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[V|P\right]\right] &= x_L + m_L e + \left[(1-\lambda) I(p_U) + \tfrac{\lambda}{2}\left(I(p_L) + I(p_H)\right)\right] \delta y_L && \text{(B.13)}\\
&+ \tfrac{1-\lambda}{2}\left(\Delta x + \Delta m e + I(p_U)\,\delta\Delta y\right) && \text{(B.14)}\\
&+ \tfrac{\lambda}{2}\tilde{\rho}\left(\Delta x + \Delta m e + I(p_H)\,\delta\Delta y\right) && \text{(B.15)}\\
&+ \tfrac{\lambda}{2}(1-\tilde{\rho})\left(\Delta x + \Delta m e + I(p_L)\,\delta\Delta y\right) && \text{(B.16)}\\
&= \frac{m_H + m_L}{2} e + \psi && \text{(B.17)}
\end{aligned}$$

where ψ is independent of e and so

$$\frac{\partial \mathbb{E}\left[\beta\left(\frac{\mathbb{E}[V|P]-\alpha}{1+\beta}\right)+\alpha\right]}{\partial e} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{m_H + m_L}{2}\right). \tag{B.18}$$

Again, given the manager's belief about the likelihood of each price, we can also write

$$\mathbb{V}\left[\mathbb{E}\left[V|P\right]\right] = (1-\lambda)\left[x_L + m_L e + I(p_U)\,\delta y_L + \tfrac{1}{2}\left(\Delta x + \Delta m e + I(p_U)\,\delta\Delta y\right) - \mathbb{E}\left[\mathbb{E}\left[V|P\right]\right]\right]^2 \tag{B.19}$$

$$+ \frac{\lambda}{2}\left[x_L + m_L e + I(p_L)\,\delta y_L + (1-\tilde{\rho})\left(\Delta x + \Delta m e + I(p_L)\,\delta\Delta y\right) - \mathbb{E}\left[\mathbb{E}\left[V|P\right]\right]\right]^2 \tag{B.20}$$

$$+ \frac{\lambda}{2}\left[x_L + m_L e + I(p_H)\,\delta y_L + \tilde{\rho}\left(\Delta x + \Delta m e + I(p_H)\,\delta\Delta y\right) - \mathbb{E}\left[\mathbb{E}\left[V|P\right]\right]\right]^2 \tag{B.21}$$

$$= (1-\lambda)\left[x_L + I(p_U)\delta y_L + \tfrac{1}{2}(\Delta x + I(p_U)\delta\Delta y) - \psi\right]^2 \tag{B.22}$$

$$+ \tfrac{\lambda}{2}\left[\left(\tfrac{1}{2}-\tilde{\rho}\right)\Delta m e + x_L + I(p_L)\delta y_L + (1-\tilde{\rho})(\Delta x + I(p_L)\delta\Delta y) - \psi\right]^2 \tag{B.23}$$

$$+ \tfrac{\lambda}{2}\left[\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m e + x_L + I(p_H)\delta y_L + \tilde{\rho}(\Delta x + I(p_H)\delta\Delta y) - \psi\right]^2 \tag{B.24}$$

and so

$$\frac{\partial \mathbb{V}\left[\beta\left(\frac{\mathbb{E}[V|P]-\alpha}{1+\beta}\right)+\alpha\right]}{\partial e} = \left(\frac{\beta}{1+\beta}\right)^2\left[\lambda\left[\mathbb{E}[V|p_H] - \mathbb{E}[\mathbb{E}[V|P]]\right]\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m - \lambda\left[\mathbb{E}[V|p_L] - \mathbb{E}[\mathbb{E}[V|P]]\right]\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m\right] \tag{B.25}$$

$$= \left(\frac{\beta}{1+\beta}\right)^2 \lambda\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m\left(\mathbb{E}[V|p_H] - \mathbb{E}[V|p_L]\right) \tag{B.26}$$

The first-order condition of the managerial effort problem (with objective as in equation 22) is

$$\frac{\beta}{1+\beta}\left[\frac{\partial \mathbb{E}[\mathbb{E}[V|P]]}{\partial e} - \frac{\beta}{1+\beta}\frac{\gamma}{2}\frac{\partial \mathbb{V}[\mathbb{E}[V|P]]}{\partial e}\right] = c'(e) \tag{B.27}$$

$$\frac{\beta}{1+\beta}\left[\frac{m_H+m_L}{2} - \frac{\beta}{1+\beta}\frac{\gamma}{2}\lambda\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m\left(\mathbb{E}[V|p_H] - \mathbb{E}[V|p_L]\right)\right] = c'(e). \tag{B.28}$$

Note that this reduces to (23) if we set $m_H = m_L = 1$. In contrast to the benchmark model, when the impact of the manager's effort is state-dependent (i.e., if $\Delta m \neq 0$), managerial effort now affects the variance of the price and, hence, the variance of his payoff. As can be seen in (B.26), effort reduces this variance when its impact covaries negatively with the investment payoff, i.e., if $\Delta m < 0$ but increases it when it covaries positive, i.e., if $\Delta m > 0$. This is reflected in the manager's first-order condition: since $\mathbb{E}[V|p_H] > \mathbb{E}[V|p_L]$, projects which covary negatively increase equilibrium effort, all else equal, while those which positively covary reduce his effort and, hence, firm value.

Because the participation constraint binds, the principal's objective is to maximize

$$\max_{\beta} \mathbb{E}\left[x_\omega + m_\omega e + \delta y_\omega I(P)\right] - \tfrac{\gamma}{2}\beta^2\mathbb{V}[P] - c(e) \tag{B.29}$$

subject to (B.28), the new IC constraint, holding. Note that we can substitute in $\hat{\beta} \equiv \frac{\beta}{1+\beta}$, and then, solving for the principal's first-order condition, we see that

$$\left(\frac{\partial e}{\partial \hat{\beta}}\right)\frac{\frac{m_H+m_L}{2}}{\gamma\mathbb{V}[\mathbb{E}[V|P]]} = \beta, \tag{B.30}$$

which is identical to the expression derived in the main model. The difference, however, is that in the main model $\frac{\partial e}{\partial \hat{\beta}} = \frac{1}{c''(e)}$, while in this setting, it can be shown that

$$\left(\frac{\frac{m_H+m_L}{2} - \hat{\beta}\lambda\gamma\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m\left(\mathbb{E}[V|p_H] - \mathbb{E}[V|p_L]\right)}{c''(e) + \left(\hat{\beta}\right)^2\lambda\gamma\left[\left(\tilde{\rho}-\tfrac{1}{2}\right)\Delta m\right]^2}\right) = \frac{\partial e}{\partial \hat{\beta}}. \tag{B.31}$$

The difference between the two expressions lies in the state-dependence of the manager's effort, i.e., $\Delta m \neq 0$. For instance, it is straightforward to see that if $\Delta m > 0$, the manager responds less aggressively (i.e., increases his effort by less) as β increases. This is because exerting more effort increases the variance when $\Delta m > 0$. Intuitively, this suggests that the optimal β will be lower when $\Delta m > 0$, all else equal, but higher when $\Delta m < 0$.

Unfortunately, the optimal β in this setting cannot be solved in closed form. In what follows, we explore numerically the characteristics of the optimal contract and the equilibrium effort choice as a function of both λ as well as Δm.

Figure 8 plots the equilibrium effort and optimal contract as a function of the fraction of informed, λ. When Δm is zero or positive, the results of the baseline model continue to hold i.e., the incentive role and information role of prices are in tension. When Δm is sufficiently negative, however, the relationship between λ and β can flip. Intuitively, this is because the negative correlation implies that when prices are more informative, the manager benefits from exerting more effort since this reduces the variance of his compensation i.e., reduces the variance of P.

B.3 Contracting on Price and Value

In this section, we allow the principal to offer a linear contract that depends on both the short-term (date-one) price and long-term value. Suppose the manager's compensation is

$$W(P, V) = \alpha + \beta(\pi P + (1 - \pi) Z) \tag{B.32}$$

where $Z \equiv V - W$ denotes the firm's net cash flows and α, β, and π are chosen optimally by the principal. Given the terms of the principal's offered contract (α, β, π), the manager maximizes his expected utility over his compensation, $W(P, V)$, net of the cost of effort, that is, he solves

$$\max_e \mathbb{E}[\alpha + \beta(\pi P + (1 - \pi) Z)] - \frac{\gamma}{2}\mathbb{V}[\alpha + \beta(\pi P + (1 - \pi) Z)] - c(e), \tag{B.33}$$

As before, the manager optimally chooses his effort so that

$$\frac{\beta}{1 + \beta} = c'(e),$$

since both the price and the firm's terminal value increase one-for-one with effort. The principal chooses the contract (α, β, π) that maximizes the (unconditional) expected firm value, net of the manager's compensation. Specifically, she now solves

$$\max_{\alpha,\beta,\pi} \mathbb{E}[x_\omega + e + \delta y_\omega I - (\alpha + \beta(\pi P + (1 - \pi) Z))], \qquad \text{subject to :} \tag{B.34}$$

$$\frac{\beta}{1 + \beta} = c'(e), \tag{B.35}$$

$$\mathbb{E}[\alpha + \beta(\pi P + (1 - \pi) Z)] - \frac{\gamma}{2}\mathbb{V}[\alpha + \beta(\pi P + (1 - \pi) Z)] - c(e) \geq 0. \tag{B.36}$$

The following proposition provides a characterization of the optimal linear contract.

Proposition 10. *Suppose the financial equilibrium, (P, I), is given by Proposition 1 and the manager's optimal effort choice, e, is given by (B.35). Then the principal's optimal linear contract is given by (α, β, π), where*

$$\pi^* = 1 + \frac{cov\left(V - \tilde{\mu}\left(P\right), \tilde{\mu}\left(P\right)\right)}{\left(1 + \beta^*\right) \mathbb{V}\left(V - \tilde{\mu}\right) + \beta^* cov\left(V - \tilde{\mu}, \tilde{\mu}\right)} < 1, \text{ and} \tag{B.37}$$

$$\beta = \frac{1}{c''\left(e\right) \gamma \left(\mathbb{V}\left[\tilde{\mu}\left(P\right)\right] - \frac{cov\left(V - \tilde{\mu}\left(P\right), \tilde{\mu}\left(P\right)\right)^2}{\mathbb{V}\left(V - \tilde{\mu}\left(P\right)\right)}\right)}. \tag{B.38}$$

Since the manager's participation constraint is always binding, the principal's objective simplifies to

$$\max_{\beta, \pi} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \left(\frac{\beta^2 \gamma}{2} \mathbb{V}\left[\pi P + \left(1 - \pi\right) Z\right] + c\left(e\right)\right). \tag{B.39}$$

Moreover, as before, the manager's choice of effort depends upon β, only. As a result, the optimal π is chosen to minimize

$$\mathbb{V}\left[\pi P + \left(1 - \pi\right) Z\right] \tag{B.40}$$

which, it is straightforward to show, yields equation (B.37). Given this choice of π, the optimal level of β is then given by equation (B.38). In this setting, the manager's contract is more sensitive to her effort relative to the baseline model (i.e, the optimal β is always higher, see equation (31)). This increase in β, and the resultant increase in managerial effort and firm value, arises because $\mathbb{V}\left[P\right] \geq \mathbb{V}\left[\pi P + \left(1 - \pi\right) Z\right]$ at the optimal π. Intuitively, this is because $\text{cov}\left(V - \tilde{\mu}\left(P\right), \tilde{\mu}\left(P\right)\right) < 0$, and this implies $\pi < 1$: the manager always receives a positive loading on both price and value. This arises in our model because prices exhibit reversal, a feature which arises across a wide range of rational expectations equilibria models (e.g., Hellwig (1980)) and which is empirically relevant over the longer horizons over which investment payoffs are realized (e.g., De Bondt and Thaler (1985)). As a result, putting positive weights on the price and the terminal cash flow provides diversification, lowering the total risk borne by the manager. In contrast, if the price were set by a risk-neutral market maker (as in Vives (1995)), π would be one in our setting. That is, the principal would only contract on the price since $\text{cov}\left(V - \tilde{\mu}\left(P\right), \tilde{\mu}\left(P\right)\right) = 0$. This is because the price is a less noisy signal of the manager's effort relative to the terminal value.[32]

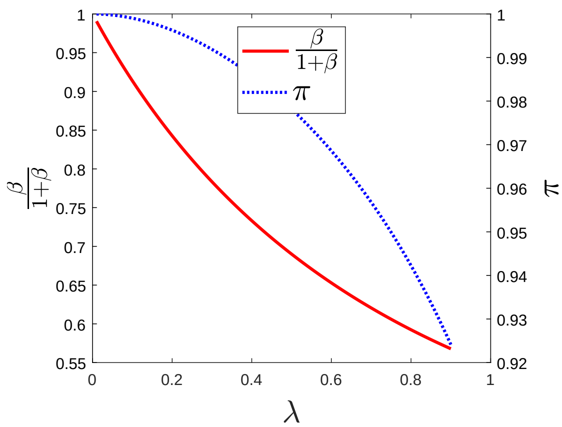
In Figure 10, we plot the optimal π and a scaled measure of performance-sensitivity, $\frac{\beta}{1+\beta}$, as a function of the fraction of informed investors, λ, and the relative importance of investment opportunities, δ. When the measure of informed investors increases, the volatility of both the price and the realized cash flow increases. As panels (a) and (b) illustrate, this leads to lower-powered incentives, i.e., β falls with λ, as in our benchmark model.

The impact on the weight the principal places on the price, however, may be non-monotonic. Recall that when the investment threshold, K, is lower than $1/2$ (panel (a)), the

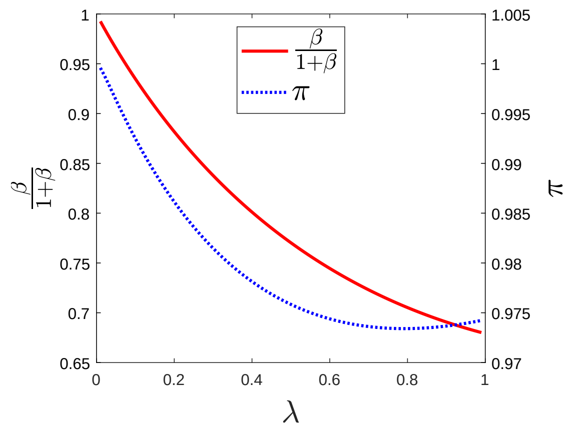
[32] If $\text{cov}\left(V - P, P\right) \geq 0$, conditioning on the terminal value adds volatility (since investors only observe a noisy signal of the true state, ω) and provides no additional incentive to exert effort.

Figure 10: Optimal pay-for-performance versus λ and δ

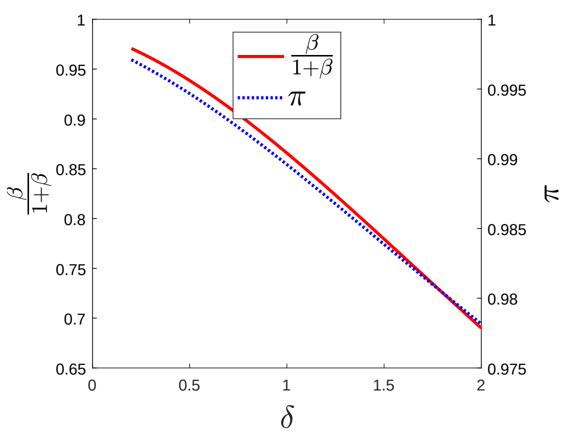
The figure plots the optimal linear contract chosen (β, π) by the principal as a function of λ and δ. The cost of effort is $\frac{e^2}{2}$. Other parameters are: $\gamma = 1$, $x_H = 1, x_L = 0.5$, $y_H = 1$, $\rho = 0.7$.



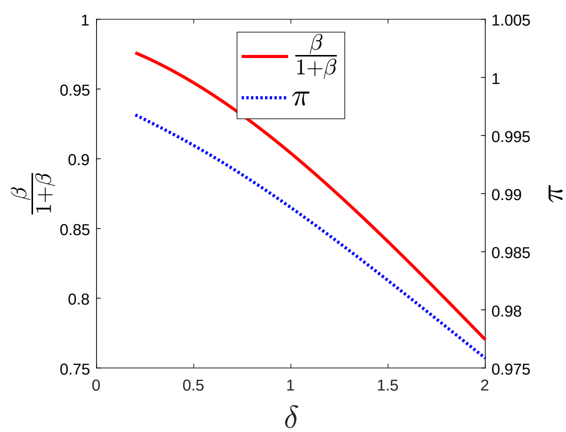
(a) $K = 0.33$, $\delta = 2$



(b) $K = 0.67$, $\delta = 2$



(c) $K = 0.33$, $\lambda = 0.5$



(d) $K = 0.67$, $\lambda = 0.5$

principal invests in the project as long as the price is not $P = p_L$. In this case, the relative weight on the price, π, decreases with λ. On the other hand, when the investment threshold is higher than 1/2 (panel (b)), the relative weight is U-shaped in λ: the principal optimally puts relatively more weight on short-term signals (the price) relative to long term signals (V) when the mass of informed investors is sufficiently low or sufficiently high. These results suggest a novel channel through which the manager's optimal short-term compensation varies as a function of investors' information.[33]

In panels (c) and (d), we plot the optimal π and β as a function of δ. As in the benchmark model, β decreases with δ since price and realized cash-flows become more volatile as the scale of the investment payoff grows. However, as δ increases, principal places relatively more weight on the long-term value, i.e., the duration of managerial compensation increases with the size of the investment opportunity. When δ is zero, the manager is paid utilizing only the short-term price which is less volatile. Intuitively, as δ grows, there is an increased benefit from hedging which leads the principal to utilize more long-term compensation. This is consistent with the results of Gopalan, Milbourn, Song, and Thakor (2014) who document that increased pay duration is associated with more growth opportunities and R&D intensity.

To the extent that λ is negatively related to liquidity, these results also speak to the literature on the relation between liquidity and stock-based compensation. Our results imply that the relative weight on short-term price compensation (i.e., π) increases with market liquidity (decreases with λ) when investment opportunities are ex-ante positive NPV (i.e., K is low), such compensation may decrease with market liquidity when investment opportunities are ex-ante negative NPV. This suggests that the impact of market liquidity on stock-based compensation and market liquidity depends in part on the nature of investment opportunities available to firms and, as a result, should vary across industries and over market conditions.

B.4 Noisy signal about effort

In the baseline model, we assumed that investors can perfectly observe the manager's equilibrium effort. In this section, we relax this restriction and assume that investors can only condition on a noisy signal of his chosen effort level.

Specifically, suppose that the terminal value of the firm's cash flows, V, is given by

$$V(\omega, e, I, \theta) = x_\omega + (e + \theta) + \delta y_\omega \times I, \tag{B.41}$$

where $\theta \sim N(0, \sigma_\theta^2)$ is independent of ω. Further suppose that in addition to observing private signals s_i about ω, all investors observe a *non-contractible* signal $s_\theta = e + \theta + \eta$, where $\eta \sim N(0, \sigma_\eta^2)$ is independent of all other random variables.

Let $V_\omega(P; s_\theta)$ denote the investors' expected cash-flow in state ω given the price P and signal s_θ. Then, we can express

$$V_\omega(P, s_\theta) = \mathbb{E}\left[V(\omega, e, I, \theta) \,|\, \omega, P, s_\theta\right] = x_\omega + \hat{e} + \kappa(s_\theta - \hat{e}) + \delta y_\omega \times I(P), \tag{B.42}$$

[33]For instance, in Peng and Röell (2014), changes in short-term compensation reflect changes in the manager's ability to manipulate the firm's value.

where $\hat{e}$ denotes the investors' inference of the manager's choice of effort, and $\kappa \equiv \frac{\mathbb{C}(s_\theta,\theta)}{\mathbb{V}(s_\theta)} = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2} \in [0,1]$ is a measure of the informativeness of s_θ. As in Holmström and Tirole (1993), investors correctly infer the manager's effort level in equilibrium. The public signal, however, provides information about θ and so investors utilize it when forming beliefs about the firm's value. Since the public signal is observed by all investors, its inclusion does not change the nature of the financial market equilibrium but it does alter the equilibrium price. Specifically, using steps analogous to those in the benchmark analysis, one can show that the equilibrium price in this setting is given by

$$P(\omega,u,s_\theta) = \begin{cases} \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_L;s_\theta\right)-\alpha\right) \equiv p_L\left(s_\theta\right) & \text{if } u < u_\omega-(1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_U;s_\theta\right)-\alpha\right) \equiv p_U\left(s_\theta\right) & \text{if } u_\omega-(1-\lambda) \le u \le u_\omega+(1-\lambda) \\ \frac{1}{1+\beta}\left(\tilde{\mu}\left(p_H;s_\theta\right)-\alpha\right) \equiv p_H\left(s_\theta\right) & \text{if } u > u_\omega+(1-\lambda) \end{cases}, \tag{B.43}$$

where the marginal investor's beliefs are given by

$$\tilde{\mu}\left(p_H;s_\theta\right) \equiv \mathbb{E}\left[V|s_H,p_H,s_\theta\right] = V_L\left(p_H,s_\theta\right)+\tilde{\rho}\Delta V\left(p_H,s_\theta\right), \tag{B.44}$$
$$\tilde{\mu}\left(p_L;s_\theta\right) \equiv \mathbb{E}\left[V|s_H,p_H,s_\theta\right] = V_L\left(p_L,s_\theta\right)+\left(1-\tilde{\rho}\right)\Delta V\left(p_L,s_\theta\right), \text{ and} \tag{B.45}$$
$$\tilde{\mu}\left(p_U;s_\theta\right) \equiv \mathbb{E}\left[V|s_U,p_U,s_\theta\right] = V_L\left(p_U,s_\theta\right)+\frac{1}{2}\Delta V\left(p_U,s_\theta\right) \tag{B.46}$$

and the equilibrium investment rule is equation (**??**).

Given this characterization of the financial market equilibrium, the manager's optimal choice of effort is given by the first order condition

$$\frac{\beta}{1+\beta}\kappa = c'\left(e\right). \tag{B.47}$$

While investors' inference of the manager's effort level, $\hat{e}$, is correct in equilibrium, changes in the manager's effort affect the price only through the public signal. Since this public signal of managerial effort is noisy (i.e., $\kappa \le 1$), the equilibrium level of effort is lower (holding fixed β) than in the benchmark (see equation (23)). Holding fixed the contract, as the investors' signal becomes more precise, κ increases which leads to an increase in the manager's effort as well.

The following proposition characterizes the optimal linear contract in this setting.

Proposition 11. *Suppose the financial market equilibrium is characterized by the price in* (B.43) *and beliefs* (B.44)-(B.46)*, and the manager's optimal choice of effort, e, is given by* (B.47)*. Then, the principal's optimal linear contract is given by (α,β) where*

$$\alpha = c\left(e\right)+\frac{\gamma}{2}\left(\frac{\beta}{1+\beta}\right)^2\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)+\kappa\sigma_\theta^2\right)-\beta\mathbb{E}\left[P\right], \text{ and} \tag{B.48}$$
$$\beta = \frac{\kappa}{c''\left(e\right)\gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)+\kappa\sigma_\theta^2\right)-\kappa\left(1-\kappa\right)}. \tag{B.49}$$

Moreover, the optimal choice of β:

(i) decreases with both λ and ρ, and

(ii) increases with κ if and only if $\gamma c''(e)\,\mathbb{V}(\tilde{\mu}(P;s_\theta)\,|s_\theta) > \kappa^2$.

Finally, the equilibrium effort decreases with both λ and ρ, and increases with κ.

The optimal linear contract now depends on the two sources of investor information reflected in the price. To gain some intuition, note that the variance in the manager's compensation can be expressed as

$$\mathbb{V}(\alpha + \beta P) = \beta^2 \mathbb{V}(P) = \left(\frac{\beta}{1+\beta}\right)^2 \left(\mathbb{V}(\tilde{\mu}(P;s_\theta)\,|s_\theta) + \kappa\sigma_\theta^2\right). \tag{B.50}$$

As in the benchmark model, for a fixed β, when the price is more informative about the true state of the world, ω, (i.e., if either λ or ρ are higher), the volatility of the price too, (captured by the $\mathbb{V}(\tilde{\mu}(P;s_\theta)\,|s_\theta)$ term). This makes the manager's compensation more risky, and hence it becomes costlier for the principal to induce the manager to exert effort. As in our benchmark analysis, the principal optimally responds to such an increase by reducing the optimal β.

When the price is more informative about effort (i.e., as κ increases), there are two opposing effects on the contract. First, as (B.47) highlights, when the signal is more informative increasing β induces more effort, on the margin. However, this increase in κ also increases the volatility of the price (as captured by the $\kappa\sigma_\theta^2$) term, which in turn, makes it more costly to incentivize effort provision. When κ is sufficiently low, Proposition 11 shows that β is increasing in κ — in this case, the positive impact on the equilibrium effort choice dominates the negative impact of more costly risk compensation. As a result, the principal optimally chooses to increase the price-sensitivity of the contract. On the other hand, when κ is sufficiently high, the former effect dominates, and the principal optimally chooses to decrease β when κ increases. Despite this non-monotonicity in the offered contract, the optimal effort chosen by the manager always increases with κ as in, e.g., Fishman and Hagerty (1989) and Holmström and Tirole (1993).

In short, our key trade-off obtains even when investors cannot observe the manager's effort perfectly: all else equal, when the price becomes more informative about investment opportunities, the price-sensitivity of the offered contract and managerial effort decrease. In contrast, when the price becomes more informative about managerial effort, the price-sensitivity of the contract can increase or decrease, depending upon on the relative informativeness of the two signals.[34]

B.5 Risk-neutral Managers

The key trade-off in our analysis is that when the price is more informative about future investment opportunities, it becomes more volatile signal about effort and, consequently, it is more costly for the principal to incentivize managerial effort. While the assumption

[34] This result differs superficially from the findings of Holmström and Tirole (1993) due to the signals they utilize in their optimal contract. Specifically, the manager's compensation depends upon a transformation of the price so that the only impact of an increase in κ is on the informativeness of this signal. As a result, more information about managerial effort always leads to a more price-sensitive contract in equilibrium.

of managerial risk aversion makes this trade-off particularly transparent in our benchmark analysis, the analysis in this section establishes that it is not necessary. Specifically, we show that the same trade-off can obtain when the manager is risk neutral, but the optimal contract satisfies a limited liability constraint, defined as follows.

Definition 1. *The optimal contract satisfies limited liability if for all $P \in \{p_H, p_U, p_L\}$, the contract satisfies $\alpha + \beta P \geq 0$.*

Note that the manager's effort choice is still given by (23). However, the objective of the principal is now given by:

$$\max_{\alpha,\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I - (\alpha + \beta P)\right], \qquad \text{subject to :} \tag{B.51}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{B.52}$$

$$\mathbb{E}\left[\alpha + \beta P\right] - c(e) \geq 0, \tag{B.53}$$

$$\alpha + \beta p_L \geq 0. \tag{B.54}$$

Given that $p_H > p_u > p_L$, condition (B.54) ensures that the limited liability constraint holds for all price levels. The above problem implies that the principal can always decrease α to improve her payoff till the manager's participation constraint (B.53) or (B.54) binds. This implies that the principal's objective can be rewritten as

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \max\left\{\beta E\left[P\right] - \beta p_L, c(e)\right\} \text{ subject to } \frac{\beta}{1+\beta} = c'(e). \tag{B.55}$$

The following lemma provides a characterization of how more information in prices affects the contract offered in equilibrium.

Lemma 2. *$E[P]$ increases in λ. Moreover, suppose at the optimal contract, the limited liability constraint (B.54) binds. Then, an increase in λ leads to a decrease in β, and consequently, the equilibrium level of effort e.*

Intuitively, the above result implies that the cost of incentivizing the manager increases with price informativeness when the limited liability condition binds. As such, our main trade-off can also obtain when the manager is risk-neutral.

B.6 Non-linear Contracts

For a general contract, $W(P)$, the manager's first-order condition can be written as[35]

$$\frac{\partial \mathbb{E}\left[W(P)\right]}{\partial e} - \frac{\gamma}{2}\frac{\partial \mathbb{V}\left[W(P)\right]}{\partial e} = c'(e). \tag{B.56}$$

[35]Note that this reduces to (23) when the contract is linear in the price since, in that setting, effort has no impact on the variance of the manager's payoff.

Since the manager's participation constraint is always binding, this implies that, for a general contract, $W(P)$, the principal's problem is

$$\max_{W(P)} \mathbb{E}\left[e + \delta y_{\omega} I\left(P\right)\right] - \tfrac{\gamma}{2}\mathbb{V}\left[W\left(P\right)\right] - c\left(e\right) \tag{B.57}$$

subject to the incentive-compatibility constraint, equation (B.56).

To impose some structure, we first consider a quadratic approximation, i.e., let $W\left(P\right) = \alpha + \beta P + \theta P^2$. In practice, such a contract could be approximately implemented utilizing a combination of equity and options. Given the potential for non-linearity in the manager's contract, we will focus on the setting in which investors trade the terminal cash flow (as in Section B.7) for tractability. It is straightforward to see that

$$\frac{\partial \mathbb{E}\left[W\left(P\right)\right]}{\partial e} = \beta + 2\theta \mathbb{E}\left[P\right], \tag{B.58}$$

and, after noting that

$$\mathbb{V}\left[W\left(P\right)\right] = \theta^2 \mathbb{V}\left[P^2\right] + 2\beta\theta Cov\left(P, P^2\right) + \beta^2 \mathbb{V}\left(P\right), \tag{B.59}$$

it can be shown that

$$\frac{\partial \mathbb{V}\left[W\left(P\right)\right]}{\partial e} = 4\left\{\theta^2 Cov\left(P, P^2\right) + \beta\theta \mathbb{V}\left(P\right)\right\}. \tag{B.60}$$

We solve for the optimal contract numerically. In contrast to our baseline model, when the price becomes more volatile (due to an increase in either ρ or λ), the principal can shift either β or θ. Figure 11 provides an illustration of how changes in investors' aggregate information affects equilibrium outcomes. While we find that the optimal β in this case is always one, the optimal θ is always negative: introducing concavity into his payoff reduces the variance of the manager's compensation. Moreover, as more investors become informed (an increase in λ) or if their signal becomes more precise (e.g., shifting from $\rho = 0.7$ to $\rho = 0.9$), theta becomes more negative to reduce the disutility experienced by the manager (panel (a)). This reduction in θ, however, comes at a cost: as the manager's first-order condition makes clear, as θ becomes more negative, equilibrium effort decreases as well (panel (b)).

This numerical illustration suggests that our key result — that managerial effort can decrease as investors become better informed about fundamentals — is not a consequence of linearity in the manager's contract.

More generally, consider a contract $W(P)$ which is a strictly increasing function of the price. It can be shown that,

$$\frac{\partial \mathbb{V}\left[W\left(P\right)\right]}{\partial \lambda} = \frac{\left(W(p_H) - W(p_U)\right)^2 + \left(W(p_L) - W(p_U)\right)^2}{2} - \frac{\lambda\left(W(p_H) + W(p_L) - 2W(p_U)\right)^2}{2}. \tag{B.61}$$

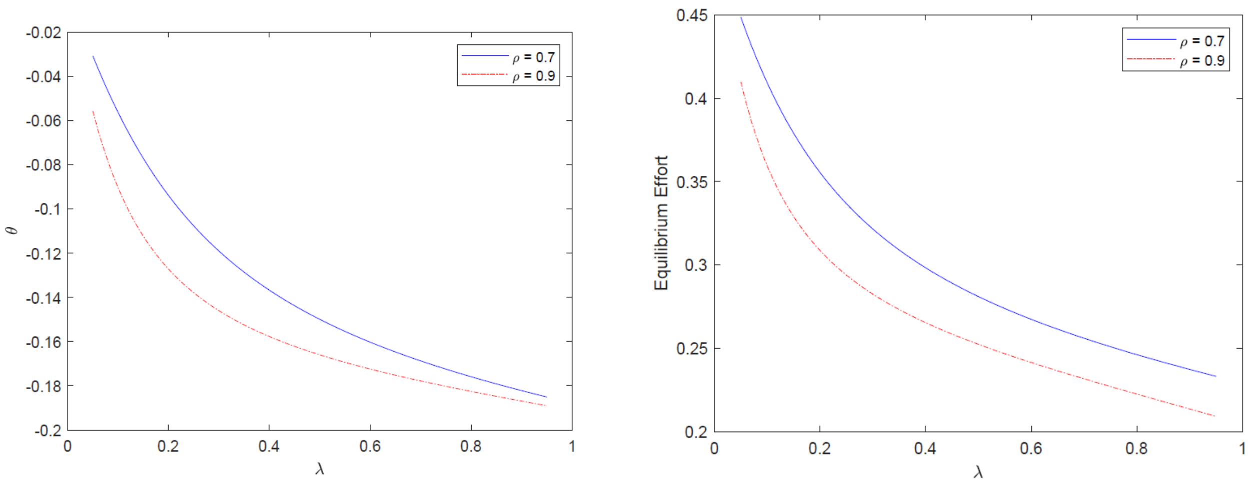
Note that this implies that

$$\frac{\partial \mathbb{V}\left[W\left(P\right)\right]}{\partial \lambda} \geq \left(W(p_H) - W(p_U)\right)\left(W(p_U) - W(p_L)\right) \tag{B.62}$$

$$> 0. \tag{B.63}$$

Figure 11: Optimal Theta and Equilibrium Effort

The figure plots the optimal θ chosen by the principal and the manager's equilibrium effort as a function of λ. The cost of effort is quadratic, $c(e) = e^2$. Other parameters are: $\gamma = 1$, $\delta = 2$, $x_H = 1, x_L = 0.5$, $y_H = 1$, $y_L = -0.5$.



(a) Optimal θ (b) Managerial Effort

This implies that it is more costly for the principal to incentivize the manager when more investors have information about the investment. As a result, a monotonic contract is sufficient to ensure that the main channel in the baseline model continues to hold.

B.7 Traded Cash Flows

In what follows, we assume that investors trade a claim to the firm's terminal cash flows *without* netting out the compensation to the manager. This could correspond to a setting in which the principal, who owns the firm, pays the manager directly instead of utilizing the firm's cash flows. In practice, this arrangement is akin to a private equity or venture capital firm paying one of its employee to improve and oversee a firm in its portfolio. We denote the price of this claim as $\hat{P}$. Note that

$$\hat{P} = \tilde{\mu}(P) \tag{B.64}$$
$$= (1+\beta) P + \alpha \tag{B.65}$$

The proof of the existence of the financial market equilibrium is unchanged since α and β are scalars and do not play a role in the proof of Proposition 1.

As a result of this change, the manager's optimal level of effort now solves

$$\beta = c'(e), \tag{B.66}$$

which becomes the principal's new IC constraint. Specifically, we can rewrite the principal's objective as

$$\max_{\beta} \mathbb{E}\left[x_\omega + e + \delta y_\omega I\right] - \frac{\gamma}{2}\beta^2 \mathbb{V}\left[\tilde{\mu}\left(P\right)\right] - c\left(e\right) \text{ subject to } \beta = c'\left(e\right). \tag{B.67}$$

Then the principal's optimal contract is given by $\left(\hat{\alpha}, \hat{\beta}\right)$, where

$$\hat{\alpha} = c\left(e\right) + \frac{\gamma}{2}\hat{\beta}^2 \mathbb{V}\left[\tilde{\mu}\left(P\right)\right] - \hat{\beta}\mathbb{E}\left[P\right], \text{ and} \tag{B.68}$$

$$\hat{\beta} = \frac{1}{1 + c''\left(e\right)\gamma \mathbb{V}\left[\tilde{\mu}\left(P\right)\right]}. \tag{B.69}$$

Thus, the contract is "lower-powered", i.e., less sensitive to the stock price. Note that, relative to the baseline model, this has no impact on the manager's equilibrium level of effort since $\hat{\beta} = \frac{\beta}{1+\beta}$, i.e., in equilibrium the manager's first-order condition is unchanged.

B.8 Social Value

In what follows, we characterize how an increase in transparency, or equivalently a decrease in c_0, affects social value (SV), where , i.e.,

$$SV = \begin{array}{c} \text{Expected Value } - \text{Managerial Compensation} \\ +\text{Managerial Utility} - \text{Manager's Cost of Effort} \\ +\text{Net Trading Gains / Losses} \\ -\text{Investor Cost of Information} \end{array} \tag{B.70}$$

Note that since (i) aggregate trading gains / losses sum to zero and (ii) the manager is exactly indifferent given his compensation (i.e., Manager utility - Manager's Cost of Effort matches his reservation utility, $u_0 = 0$), this simplifies to

$$SV = \mathbb{E}\left[x_\omega + e + \delta y_\omega \times I\right] - \frac{\gamma\beta^2}{2}\mathbb{V}\left[P\right] - c\left(e\right) - \lambda c_0 \; = \; FV - \lambda c_0. \tag{B.71}$$

This leads directly to the next proposition.

Proposition 12. *Social Value increases with transparency (decreases with c_0) if $1-\rho < K < \rho$ and δ is sufficiently large. Social Value decreases with transparency (increases with c_0) if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)\left(x_H - x_L\right)$ and if either (i) $1 - \rho > K$, or (ii) $\rho < K$, or (iii) $1 - \rho < K < \rho$ and δ is sufficiently small.*

The above result parallels Proposition 6 in that it provides sufficient conditions when an increase in transparency has an unambiguous effect on social value. Given the expression for social value in (B.71), one can express the impact of a change in c_0 on social value as:

$$\frac{d}{dc_0}SV = \frac{d}{dc_0}FV - \left(\lambda + c_0\frac{d\lambda}{dc_0}\right). \tag{B.72}$$

In addition to the investment and incentive channels, transparency has a direct effect on the SV through the aggregate cost of information acquisition. Whether this increases or decreases the social value depends on the equilibrium level of λ, since

$$\lambda + c_0 \frac{d\lambda}{dc_0} = 2\lambda - 1. \tag{B.73}$$

When information acquisition costs c_0 are sufficiently high (i.e., $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$), the equilibrium λ is lower than $\frac{1}{2}$. As a result, any further increase in the cost of learning improves social value by decreasing the fraction of investors who become informed, thereby reducing aggregate expenditure on private information acquisition. On the other hand, when information acquisition costs are low (so that $\lambda > \frac{1}{2}$), the resultant decrease in λ is not sufficiently large to offset the increase in c_0 paid by those investors who still choose to become informed. In this case, overall expenditure on information acquisition is higher and, consequently, social value is lower. We note that this is more likely to be the case when investors have access to more precise information (i.e., if ρ increases) and if the information sensitivity of assets in place is high (i.e., if $x_H - x_L$ increases). The overall impact of transparency then follows from how this channel interacts with the impact on firm value.

B.9 Price Informativeness and Volatility

Recent work (e.g., Dávila and Parlatore (2020) and Brunnermeier et al. (2020)) has highlighted the potential for non-monotonicity in the relation between price volatility and informativeness. The specific nature of this relation, however, depends upon the primitives of the model including the marginal investor's risk preferences. This is of particular relevance to our analysis given that we expect investors to be well-diversified with respect to the firm-specific risks we analyze.

In what follows, we demonstrate in a canonical setting that when the marginal investor's risk-aversion tends to zero, this relation is always positive, as in our benchmark analysis. Suppose the terminal payoff (fundamental value) of a risky security is $F \sim \mathcal{N}\left(0, \frac{1}{\tau}\right)$. We denote the market-determined price of the risky security by P, and the aggregate supply of the risky asset by $z \sim \mathcal{N}\left(0, \frac{1}{\tau_z}\right)$.

There is a continuum of investors, indexed by $i \in [0,1]$ who observe a private signal s_i, where

$$s_i = F + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right) \tag{B.74}$$

and ε_i is independent and identically distributed across investors so that $\int \varepsilon_i di = 0$. Investors submit limit orders which condition not only their private signals but alsothe price, P. Each investor i exhibits CARA utility with coefficient of absolute risk aversion γ over terminal wealth W_i: $W_i = W_0 + x_i (F - P)$, where x_i denotes her demand for the risky security.

The price is set by a competitive, risk neutral market maker (as in e.g., Kyle (1985), Vives (1995)) who conditions on all observable public information, including investors' aggregated demand. Following the usual steps, it is straightforward to show that there always exists a unique, linear, financial market equilibrium in which

$$P = \Lambda s_p, \text{ where } \Lambda = \frac{\tau_p}{\tau + \tau_p}, \tag{B.75}$$

where

$$s_p = F + \beta z, \quad \tau_p = \tau_z/\beta^2, \text{ and } \beta = -\frac{\gamma}{\tau_e}. \tag{B.76}$$

In this setting, price informativeness is denoted by τ_p; we denote price volatility by $\mathcal{V} \equiv \mathbb{V}(P)$. Then, given the equilibrium price,

$$\mathcal{V} = \Lambda^2 \left(\frac{1}{\tau} + \frac{1}{\tau_p}\right)$$

We can decompose the equilibrium elasticity of price volatility to price informativeness in a manner analogous to the approach taken by Dávila and Parlatore (2020):

$$\frac{d \log \mathcal{V}}{d \log \tau_p} = \underbrace{2\frac{d \log \Lambda}{d \log \tau_p}}_{\text{Equilibrium Learning}} - \underbrace{\frac{\tau}{\tau + \tau_p}}_{\text{Noise reduction}}$$

Since the marginal investor (i.e., the market maker in this setting) is risk-neutral, the equilibrium learning channel always dominates the noise reduction channel:

$$\frac{d \log \mathcal{V}}{d \log \tau_p} = \underbrace{2\frac{\tau}{\tau + \tau_p}}_{\text{Equilibrium Learning}} - \underbrace{\frac{\tau}{\tau + \tau_p}}_{\text{Noise reduction}} = \frac{\tau}{\tau + \tau_p} > 0.$$

This implies that price volatility and informativeness are positively related which, in turn, suggests that our key result — that managerial effort will decrease as investors become better informed about firm-specific risk — is a robust observation.

B.10 Proofs

In what follows, we provide the proofs for those statements found in the Appendix B.

Proof of Proposition 10

Note that

$$W = \alpha + \beta (\pi P + (1 - \pi)(V - W)) \implies W = \frac{\alpha}{(1 + \beta (1 - \pi))} + \frac{\beta}{(1 + \beta (1 - \pi))} (\pi P + (1 - \pi) V)$$

Moreover, the market clearing condition implies

$$P = \mathbb{E}[V - W]$$

$$= \mathbb{E}\left[V - \frac{\alpha}{(1+\beta(1-\pi))} - \frac{\beta}{(1+\beta(1-\pi))}(\pi P + (1-\pi)V)\right]$$

which implies that

$$P\left(1 + \frac{\beta\pi}{(1+\beta(1-\pi))}\right) = \mathbb{E}[V]\left(1 - \frac{\beta(1-\pi)}{(1+\beta(1-\pi))}\right) - \frac{\alpha}{(1+\beta(1-\pi))}$$

$$P = \frac{\mathbb{E}[V] - \alpha}{1+\beta} \equiv \frac{\tilde{\mu}(P) - \alpha}{1+\beta}$$

Manager's effort problem is

$$\max_{e} \mathbb{E}\left[\frac{\alpha}{(1+\beta(1-\pi))} + \frac{\beta(\pi P + (1-\pi)V)}{(1+\beta(1-\pi))}\right] - \frac{\gamma}{2}\mathbb{V}[\alpha + \beta(\pi P + (1-\pi)Z)] - c(e),$$

The FOC is

$$\frac{\beta}{1+\beta} = c'(e).$$

The optimal contract solves

$$\max_{\alpha,\beta,\pi} \mathbb{E}[x_\omega + e + \delta y_\omega I - (\alpha + \beta(\pi P + (1-\pi)Z))], \qquad \text{subject to :} \tag{B.77}$$

$$\frac{\beta}{1+\beta} = c'(e), \tag{B.78}$$

$$\mathbb{E}[\alpha + \beta(\pi P + (1-\pi)Z)] - \frac{\gamma}{2}\mathbb{V}[\alpha + \beta(\pi P + (1-\pi)Z)] - c(e) \geq 0. \tag{B.79}$$

This simplifies to

$$\max_{\beta,\pi} \mathbb{E}[x_\omega + e + \delta y_\omega I] - \frac{\gamma}{2}\beta^2\mathbb{V}[(\pi P + (1-\pi)Z)] - c(e) \quad \text{subject to } \frac{\beta}{1+\beta} = c'(e).$$

Moreover

$$\begin{aligned}
\beta^2\mathbb{V}[(\pi P + (1-\pi)Z)] &= \frac{\beta^2\mathbb{V}[(\pi P + (1-\pi)V)]}{(1+\beta(1-\pi))^2} \\
&= \frac{\beta^2\mathbb{V}\left[\left(\pi\frac{\tilde{\mu}}{1+\beta} + (1-\pi)V\right)\right]}{(1+\beta(1-\pi))^2} \\
&= \frac{\beta^2}{(1+\beta)^2}\mathbb{V}\left[\left(\tilde{\mu} + \frac{(1-\pi)(1+\beta)}{(1+\beta(1-\pi))}(V-\tilde{\mu})\right)\right]
\end{aligned}$$

Note that $\tilde{\mu}$ is independent of π and the optimal π is characterized by minimizing

$$\min_{e} \ \mathbb{V}\left[\left(\tilde{\mu} + \frac{(1-\pi)(1+\beta)}{(1+\beta(1-\pi))}(V-\tilde{\mu})\right)\right]$$

The FOC is

$$\pi = 1 + \frac{\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}{\left(1+\beta\right)\mathbb{V}\left(V-\tilde{\mu}\right)+\beta\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)}$$

Substituting this into the principal's objective, we get

$$\begin{aligned}
\mathbb{V}\left[W\right] &= \frac{\beta^2}{\left(1+\beta\right)^2}\left[\left(\mathbb{V}\left[\tilde{\mu}\right]+\left(\frac{\left(1-\pi\right)\left(1+\beta\right)}{\left(1+\beta\left(1-\pi\right)\right)}\right)^2\mathbb{V}\left(V-\tilde{\mu}\right)+2\frac{\left(1-\pi\right)\left(1+\beta\right)}{\left(1+\beta\left(1-\pi\right)\right)}\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right)\right] \\
&= \frac{\beta^2}{\left(1+\beta\right)^2}\left[\left(\mathbb{V}\left[\tilde{\mu}\right]-\frac{\left[\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\mathbb{V}\left(V-\tilde{\mu}\right)}\right)\right]
\end{aligned}$$

The objective is

$$\max_{\beta}\mathbb{E}\left[x_\omega+e+\delta y_\omega I_m\right]-\frac{\gamma}{2}\frac{\beta^2}{\left(1+\beta\right)^2}\left[\left(\mathbb{V}\left[\tilde{\mu}\right]-\frac{\left[\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\mathbb{V}\left(V-\tilde{\mu}\right)}\right)\right]-c\left(e\right)\quad\text{subject to }\frac{\beta}{1+\beta}=c'\left(e\right).$$

The FOC is

$$\begin{aligned}
\left(1-c'\left(e\right)\right)\frac{\partial e}{\partial\beta} &= \gamma\frac{\beta}{\left(1+\beta\right)^3}\left(\mathbb{V}\left[\tilde{\mu}\right]-\frac{\left[\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\mathbb{V}\left(V-\tilde{\mu}\right)}\right) \\
\beta &= \frac{1}{\gamma c''\left(e\right)\left[\left(\mathbb{V}\left[\tilde{\mu}\right]-\frac{\left[\mathrm{cov}\left(V-\tilde{\mu},\tilde{\mu}\right)\right]^2}{\mathbb{V}\left(V-\tilde{\mu}\right)}\right)\right]}.
\end{aligned}$$

$\square$

Proof of Proposition 11

Let $B\equiv\frac{\beta}{1+\beta}$, and note that choosing β optimally is equivalent to choosing B optimally. First, note that α can be chosen to ensure that the manager's participation constraint binds. Then, one can rewrite the principal's objective as

$$\max_{\beta}\mathbb{E}\left[x_\omega+\left(e+\theta\right)+\delta y_\omega\times I\right]-\frac{\gamma}{2}\mathbb{V}\left[\alpha+\beta P\right]-c\left(e\right) \tag{B.80}$$

subject to $B\kappa=c'\left(e\right)$, which implies:

$$\kappa=c''\left(e\right)e_B,\quad\text{ and }0=c'''\left(e\right)e_B^2+c''\left(e\right)e_{BB}. \tag{B.81}$$

The FOC w.r.t. B for the principal is given by

$$e_B\left(1-c'\left(e\right)\right)-\gamma B\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)+\kappa\sigma_\theta^2\right)=0 \tag{B.82}$$

$$\Leftrightarrow\frac{\kappa}{c''\left(e\right)}\left(1-\kappa B\right)-\gamma B\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)+\kappa\sigma_\theta^2\right)=0 \tag{B.83}$$

which follows from plugging in $e_B=\frac{\kappa}{c''(e)}$. This implies that the optimal choice of B is given by:

$$B=\frac{\kappa}{\gamma c''\left(e\right)\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right)+\kappa\sigma_\theta^2\right)+\kappa^2} \tag{B.84}$$

or equivalently, the optimal choice of β is given by:

$$\beta = \frac{\kappa}{\gamma c''(e)\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) - \kappa\left(1-\kappa\right)}. \tag{B.85}$$

The SOC is given by $\mathcal{S} < 0$, where

$$\mathcal{S} = e_{BB}\left(1 - c'(e)\right) - e_B^2 c''(e) - \gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) \tag{B.86}$$

Since $e_{BB} = -\frac{c'''(e)}{c''(e)}e_B^2$ and

$$\begin{aligned}
\gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) &= \frac{e_B\left(1-c'(e)\right)}{B} &\text{(B.87)}\\
&= \frac{\kappa e_B\left(1-c'(e)\right)}{c'(e)} &\text{(B.88)}\\
&= \frac{e_B^2 c''(e)\left(1-c'(e)\right)}{c'(e)} &\text{(B.89)}
\end{aligned}$$

we have

$$\begin{aligned}
\mathcal{S} &= -\frac{c'''(e)}{c''(e)}e_B^2\left(1-c'(e)\right) - e_B^2 c''(e) - \gamma\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) &\text{(B.90)}\\
&= -e_B^2\left(\frac{c'''(e)}{c''(e)}\left(1-c'(e)\right) + c''(e) + \frac{c''(e)\left(1-c'(e)\right)}{c'(e)}\right) &\text{(B.91)}\\
&= -e_B^2\left(\frac{c''(e)}{c'(e)} + \frac{c'''(e)\left(1-c'(e)\right)}{c''(e)}\right) &\text{(B.92)}
\end{aligned}$$

So we need

$$\frac{c''(e)}{c'(e)} + \frac{c'''(e)\left(1-c'(e)\right)}{c''(e)} > 0. \tag{B.93}$$

Finally, the optimal effort satisfies

$$c'(e) = B\kappa = \frac{\kappa^2}{\gamma c''(e)\left(\mathbb{V}\left(\tilde{\mu}\left(P;s_\theta\right)|s_\theta\right) + \kappa\sigma_\theta^2\right) + \kappa^2} \tag{B.94}$$

which implies that effort increases in κ. □

Proof of Lemma 2

Note that price is given by (20) and the expected price is given by

$$E[P] = \frac{\lambda}{2}\left(p_L + p_H\right) + \left(1-\lambda\right)p_U$$

and hence

$$\frac{\partial E[P]}{\partial\lambda} = \frac{1}{2}\left(p_L + p_H - 2p_U\right)$$

$$= \frac{1}{2(1+\beta)} \left(\tilde{\mu}(p_L) + \tilde{\mu}(p_H) - 2\tilde{\mu}(p_U)\right)$$

If the manager never invests or always invests, the price is symmetric and so $\frac{\partial E[P]}{\partial \lambda} = 0$.

If $\frac{1}{2} > K > 1 - \rho$, the manager invests if $P \in \{p_u, p_H\}$ and

$$\begin{aligned}
\frac{\partial E[P]}{\partial \lambda} &= \frac{1}{2(1+\beta)} \left(-y_L - (1-\tilde{\rho})(y_H - y_L)\right) \\
&= \frac{(y_H - y_L)}{2(1+\beta)} \left(K - (1-\tilde{\rho})\right) > 0
\end{aligned}$$

where the last inequality holds, because $K > 1 - \rho > 1 - \tilde{\rho}$ in this case.

If $\rho > K > \frac{1}{2}$, the manager invests if $P = \{p_H\}$ and

$$\begin{aligned}
\frac{\partial E[P]}{\partial \lambda} &= \frac{1}{2(1+\beta)} \left(\tilde{\rho}(y_H - y_L) + y_L\right) \\
&= \frac{(y_H - y_L)}{2(1+\beta)} \left(\tilde{\rho} - K\right) > 0
\end{aligned}$$

where the last inequality holds, because $\tilde{\rho} > \rho > K$ in this case. □

Proof of Proposition 12

The proof of this follows from the observation that

$$\begin{aligned}
\frac{\partial}{\partial c_0} SV &= \frac{\partial}{\partial c_0} FV - \left(\lambda + c_0 \frac{d\lambda}{dc_0}\right) && \text{(B.95)} \\
&= \frac{\partial}{\partial c_0} FV - (2\lambda - 1) && \text{(B.96)}
\end{aligned}$$

since

$$\lambda = \begin{cases} 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\left(x_H - x_L + \delta(y_H - y_L)\right)} & \text{if } 1 - \rho > K \text{ or } \frac{1}{2} > K > 1 - \rho \\ 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\left(x_H - x_L\right)} & \text{if } \rho > K > \frac{1}{2} \text{ or } \rho < K \end{cases}, \tag{B.97}$$

and so

$$\lim_{\delta \to 0} \lambda = 1 - \frac{c_0}{\left(\rho - \frac{1}{2}\right)\left(x_H - x_L\right)}. \tag{B.98}$$

This implies that $2\lambda - 1 < 0$ if and only if $c_0 > \frac{1}{2}\left(\rho - \frac{1}{2}\right)(x_H - x_L)$. □