Motivated Beliefs in Coordination Games

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Abstract

We characterize the impact of anticipatory utility on players' subjective interpretation of information in a general coordination game. In any symmetric equilibrium, players choose to over-estimate the precision of their information. Players' perception of public information quality relative to private depends on the utility cost of non-fundamental volatility, *not* whether actions are strategic complements or substitutes. Moreover, when non-fundamental volatility increases utility, players may endogenously choose to disagree: some underweight public news, while others overreact to it. In contrast to rational expectations equilibria, public information provision can increase disagreement while private information can increase aggregate volatility. Such belief distortions can improve social welfare and potentially explain puzzling patterns in individual/consensus responsiveness to news.

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1 Introduction

In most equilibrium settings, an individual's payoffs depend not only on economic fundamentals, but on the behavior of others. A large literature following Morris and Shin (2002) has focused on how an individual's actions, and their use of information, depends on the interaction of these forces. This work has highlighted how strategic considerations, or "the nature of the game," can have important consequences for the use, acquisition and social provision of information, and has formed the basis for the analysis of transparency and disclosure in macroeconomic policy.[1] However, a central assumption in this analysis is that individuals exhibit "rational expectations" i.e., they know the true joint distribution of signals and fundamentals, and so correctly interpret the information available to them.[2]

Yet in practice, individuals often deviate from rational expectations and exhibit motivated reasoning instead (e.g., see Bénabou and Tirole (2016) for a recent survey on this topic). There is extensive evidence that individuals experience direct utility flows (anticipatory utility) from their *beliefs* about future events (e.g, excitement about a future vacation, distress about an upcoming medical procedure). Such individuals may distort their interpretation of information in order to increase their current well-being, e.g., by choosing not to learn about the risk of deadly disease (Oster, Shoulson, and Dorsey, 2013), updating in ways consistent with their political beliefs (Kahan, 2013) or interpreting uninformative signals of ability as positive indicators (Exley and Kessler, 2019). Importantly, such deviations from rational expectations are not fixed or "hard-wired" but, instead, depend upon the specific nature of the economic environment. In particular, Bénabou and Tirole (2016) point out that "individuals will overestimate or underestimate their own abilities depending upon which distortion is advantageous."

We study how the nature of the game affects how players *perceive* the information available to them. In a standard model of externalities (as in Angeletos and Pavan (2007)), we allow players to choose their perceived precision of both private and public information. Even though deviating from rational expectations is costly, we show that all players choose to do so in equilibrium. First, in any symmetric equilibrium, all players overestimate the precision of both the private and public signal. Surprisingly, whether players overweight the private or public signal more does *not* depend on whether actions exhibit strategic complementarity or substitutability per se, but depends on the *the utility cost of non-fundamental volatility.*

[1] Morris and Shin (2002) and Angeletos and Pavan (2007) explore how strategic considerations affect the relative use of private and public information in equilibrium, and how these equilibrium outcomes differ from the socially optimal (or welfare maximizing) allocations. Relatedly, Hellwig and Veldkamp (2009) and Myatt and Wallace (2012) focus on how strategic considerations affect information acquisition.

[2] An exception is Dupraz (2015), who considers a setting in which players exhibit ambiguity aversion and do not know the true model of the economy.

We show that players overweight the public signal more when non-fundamental volatility decreases utility; otherwise, they are relatively more overconfident about their private information. Second, when non-fundamental volatility increases utility, symmetric equilibria may not exist. Instead, we show that asymmetric subjective beliefs can arise when the cost of belief distortion is sufficiently low. These equilibria feature *endogenous* disagreement in the interpretation of public information: some players choose to overestimate the precision of public information, while the others under-react to it.

Such endogenous distortions in subjective beliefs can have important implications for observables and policy. In contrast to rational expectations equilibria, we show that increasing the true precision of private information can *increase* non-fundamental volatility in the aggregate action, while increasing the true precision of public information can *increase* dispersion across individuals' actions. Since an increase in either aggregate observable can decrease welfare, our analysis suggests that regulators should be cautious when proposing increased provision of public or private information. Moreover, we show that a social planner who only cares about average ex-post utility may prefer that players distort their beliefs, rather than exhibit rational expectations. Specifically, such subjective belief distortions can reduce the externalities individuals impose on others in the beauty contest setting of Morris and Shin (2002), suggesting that debiasing individuals is not necessarily in policymakers' best interest. Finally, our model can also explain when and why we might observe under-reaction of aggregate variables to public news.

Formally, we extend the generalized quadratic-Gaussian model of Angeletos and Pavan (2007) by allowing for subjective belief choice. There is a continuum of players whose payoffs depend not only upon their action and economic fundamentals but also the actions of others. The setting allows not only for both strategic complementarity and substitutability, but also accommodates potential externalities imposed by aggregate actions, including aggregate volatility. Each player has access to private and public information about fundamentals which they utilize when choosing their optimal action. While this generalized setting is necessarily abstract, it allows us to more precisely identify the economics underlying our results without the restrictions imposed by specific parameterizations.[3] In Section 6, we discuss how these results apply across a wide range of specific applications.

In contrast to the existing literature, we do not assume that each player exhibits rational expectations. Instead, each player's interpretation of information is influenced by his anticipatory utility: he experiences a direct utility flow today which arises from his beliefs

[3]For example, in the canonical beauty contest model of Morris and Shin (2002), the same parameter drives the degree of strategic complementarity and the net impact of aggregate volatility on players' utility. This need not be the case more generally and, as we discuss below, separating the two effects is important.

about future outcomes (e.g., his apprehension about making the "wrong call", or his excitement about making the right one). As a result, he engages in "wishful thinking" (Caplin and Leahy, 2019). Specifically, each player chooses his subjective beliefs about the precision of private and public information to maximize his anticipatory utility, subject to a cost of deviating from rational expectations. The cost parametrizes how far subjective beliefs can be from rational expectations, and depends on the average loss in realized payoffs as a result of sub-optimal actions.[4]

Given subjective beliefs, equilibrium actions are standard. Each player's equilibrium action depends on his beliefs about fundamentals and the actions of others, and on whether actions exhibit strategic complementarity or substitutability. Given this optimal action (and taking others' beliefs and actions as given), we show that subjective beliefs affect a player's anticipatory utility through two channels: (i) uncertainty about fundamentals, and (ii) non-fundamental volatility in the aggregate action. The "fundamental uncertainty" channel implies that higher perceived precision of both public and private information increases anticipatory utility. This is because lower (perceived) uncertainty about fundamentals leads to more informationally efficient actions, which leads to higher payoffs (and hence higher anticipatory utility).

The "non-fundamental volatility" channel captures whether common noise in actions increases players' payoffs. On the one hand, common noise can help facilitate coordination (e.g., if investment in the new technology has positive externalities).[5] On the other hand, excess volatility in the aggregate action usually decreases utility (e.g., if fluctuations in aggregate investment are very costly for an individual firm). How the perceived precision of public information affects anticipatory utility through the non-fundamental volatility channel depends on the net effect of these forces on payoffs, which we refer to as the *utility cost of non-fundamental volatility*.

When non-fundamental volatility decreases payoffs, then an increase in common noise reduces anticipatory utility unambiguously. In this case, "fundamental uncertainty" channel and "non-fundamental volatility" channel work in the same direction. This implies that,

[4]Many of our results hold for more general cost specifications e.g., those based on the Kullback-Leibler distance between subjective and objective beliefs (as in Caplin and Leahy (2019)). However, we focus on the case where the cost of belief distortion depends on the loss in realized utility due to the sub-optimal choice of beliefs by the player for analytical tractability. Finally, the rational expectations benchmark is nested as a special case of our model, where the cost parameter (scaling the utility loss from deviation) is set to infinity.

[5]As we show in Section 4.2, the coordination benefit of common noise does not depend on the sign of the externality, but on the magnitude. Specifically, we show that common noise can be beneficial in facilitating coordination, irrespective of whether individuals' actions exhibit strategic complementarity *or* substitutability. See Section 6.2 and Appendix B for a number of applications where common noise is beneficial. In contrast, Section 6.1 highlights that in the special case of the canonical beauty contest model, common noise is beneficial only when the game exhibits strategic complementarity.

believing that public signal is less noisy leads to lower non-fundamental volatility, which makes agent happier. As a result, there exists a unique symmetric equilibrium in which (i) all players overestimate the precision of *both* signals and (ii) players are more overconfident about the public signal. When players' priors are sufficiently diffuse, this overconfidence leads players to ignore their prior information in choosing their optimal actions.

When non-fundamental volatility increases payoffs, an increase in common noise increases anticipatory utility. In this case, the "fundamental uncertainty" channel and "non-fundamental volatility" channel work in opposite directions for the public signal. If the cost of deviating from rational expectations is sufficiently high, this leads to a symmetric equilibrium in which, again, players overestimate the precision of both signals but now choose to be more overconfident about their private information. However, if the cost of deviating is low, however, an asymmetric equilibrium arises in which a positive measure of the players choose to *underestimate* the precision of the public signal relative to rational expectations while the others *overestimate* it. Though players are ex-ante homogenous, this asymmetric equilibrium gives rise to both endogenous heterogeneity in beliefs and endogenous disagreement in response to public information.

We then study how changes in the information environment affect equilibrium outcomes when subjective beliefs are endogenous. In our benchmark analysis, we show that an objective increase in the precision of either signal increases the perceived precision of *both* signals. As a result, providing more precise public information can increase dispersion of actions across players, since they choose to increase the weight on their private information endogenously. Similarly, making private information more precise can increase non-fundamental volatility due to the increase this induces in players' perceived public precision. These predictions run counter to rational expectations and provide potentially testable implications of wishful thinking. Moreover, as is well-known, such changes can have important social implications: all else equal, an increase in either dispersion or non-fundamental volatility reduces welfare in these settings.

For concreteness, we apply our results to the canonical beauty contest model of Morris and Shin (2002) as well as the modification introduced by Angeletos and Pavan (2004). In these settings, the impact of aggregate volatility depends only on the nature of the coordination game. As a result, in games of complementarity, non-fundamental volatility is costly and players choose to be more overconfident about the public signal. In games of substitutability, either a symmetric equilibrium arises (in which players are are more confident about their private information) or there is an asymmetric equilibrium in which some players believe the public signal is more noisy than it objectively is. We illustrate settings in which the provision of information (either private or public) can reduce aggregate well-being when players choose their subjective beliefs but not under rational expectations. We also derive the welfare-maximizing beliefs and show that these do not necessarily correspond to rational expectations, even if the social planner only cares about average realized utility. For instance, under strategic complementarity, players coordinate "too much" from the social planner's perspective. This excessive coordination can be undone under subjective beliefs, because players are overconfident about their private information. As a result, paradoxically, average realized utility maybe higher in the subjective beliefs equilibrium than under the rational expectations equilibrium.

We also consider several alternative settings which nest within the generalized framework we analyze. These include the canonical industrial organization models with both Bertrand and Cournot competition, a production setting with investment complementarities and an efficient economy with incomplete markets. Somewhat surprisingly, we show that in each of these disparate settings, the utility cost of non-fundamental volatility is negative which leads to (i) more overconfidence in private information and (ii) can potentially lead to asymmetric equilibria. Taken together, this analysis suggests that excessive disagreement can arise endogenously in setting where players subjectively choose how to interpret the information available to them. Moreover, we show that in the asymmetric equilibria, while some players over-react to public information, the responsiveness of aggregate variables to public news can be lower than in rational expectations in these equilibria.

Our model also provides a rich set of predictions about whether individual and consensus forecasts underreact or overreact to information. Using the methodology introduced by Coibion and Gorodnichenko (2015), the literature documents evidence of both underreaction and overreaction in consensus forecasts. For instance, Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2018) show that consensus forecasts of macroeconomic variables tend to underreact to information, while Bordalo, Gennaioli, Porta, and Shleifer (2019) establishes that consensus forecasts of firm's long term earnings growth tend to exhibit overreaction. This evidence is made all the more puzzling by the observation that individual forecasts tend to exhibit overreaction even for macroeconomic variables (Bordalo, Gennaioli, Ma, and Shleifer (2018)).

Our model provides a unified framework that explains how these apparently conflicting patterns may arise. We show that when the cost of distorting beliefs is sufficiently high, consensus forecasts underreact to information even when individual forecasts overreact. On the other hand, when the cost of distorting beliefs is relatively low, both consensus and individual forecasts exhibit overreaction when volatility reduces utility (as is arguably the case with long-term earnings growth).

The next section discusses the related literature and our contribution. Section 3 presents the model and discusses the key assumptions. Section 4 presents the benchmark analysis of the model, including a discussion of how subjective beliefs affect anticipatory utility and a characterization of the symmetric and asymmetric subjective belief equilibria. Section 5 discusses how the model's implications for observables like dispersion and aggregate volatility can help distinguish it from the rational expectations benchmark, and how information provision can affect welfare in a subjective beliefs equilibrium. Section 6 presents an application of the general analysis to the canonical beauty contest model and discusses implications for a broader set of applications, and Section 7 concludes. All proofs are in Appendix A, and applications are presented in B.

2 Related Literature

2.1 Anticipatory utility and subjective belief choice

The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905) who considers agents who derive contemporaneous utility not simply from current actions but also the anticipation of future utility flows. For example, an individual who anticipates a negative, *future* experience (e.g., a risky medical procedure) experiences a negative, *contemporaneous* utility flow (e.g., anxiety about potential bad outcomes). In contrast, beliefs about future, positive events can increase an agent's current utility (e.g., excitement about a long-awaited vacation). There is now an extensive economic literature that incorporates anticipatory utility into models of belief choice (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). When agents experience anticipatory utility, an individual's subjective beliefs affect not only his actions but also the current utility he experiences. As Bénabou and Tirole (2016) emphasize in their survey, this generates a tension between holding "accurate" beliefs, which lead to ex-post optimal actions and "desirable" beliefs, which increase contemporaneous utility flows. Importantly, we note that individuals do not exhibit "multiple selves" but consciously hold a *single* set of beliefs about the world.[6]

Caplin and Leahy (2019) utilize a similar framework in which agents choose to engage in "wishful thinking" - as in our model, individuals choose their subjective beliefs to maximize anticipatory utility subject to a cost of deviating from the objective distribution. They

[6]Our assumption that it is costly to deviate from the objective distribution is a modeling convenience. Our intention is to capture the idea that individuals behave as though deviating too far from accurate beliefs is costly, perhaps due to previous experience. As in the literature on robust control, our use of the objective distribution in specifying the cost does not imply that the agent "knows" the true distribution. Caplin and Leahy (2019) discuss this distinction in more detail.

choose to model this cost as a function of the distance between the objective and subjective distributions. Their analysis shows how anticipatory utility and belief choice can generate a number of common behavioral biases, such as confirmation bias, optimism, and the endowment effect as well as both polarization and procrastination. Our analysis includes general cost functions and show how both overconfidence and under/over-reaction to public information can arise endogenously in coordination games.

Brunnermeier and Parker (2005) allow both anticipatory utility and "memory" utility to affect an individuals' well-being. In their model, individuals choose both their ex-ante beliefs and subsequently, optimal actions, in order to maximize their expected well-being. In Brunnermeier and Parker (2005), the cost of holding optimal beliefs is the objective loss in experienced utility which arises from choosing actions utilizing distorted beliefs, unlike the "statistical distance" measure of Caplin and Leahy (2019). Both Brunnermeier and Parker (2005) and Brunnermeier, Gollier, and Parker (2007)) utilize their framework to help explain observed preference for skewness, portfolio under-diversification and consumption/savings patterns. We utilize a similar cost function in deriving individual's optimal beliefs but in a setting with asymmetric information.[7]

In a related paper, Bénabou (2013) analyzes how wishful thinking and reality denial spread through organizations such as firms, bureaucracies, and markets. As in our framework, each agent derives anticipatory utility from his future prospects; however, there is no strategic complementarity / substitutability in payoffs, which is a key determinant of optimal actions (and beliefs) in our setting.

Given the prevalence of behavioral biases in decision-making, it has been suggested that policymakers would find it valuable to de-bias individuals.[8] Our analysis suggests that, when agents choose their beliefs, this may be neither feasible nor desirable. In particular, we show that the subjective beliefs equilibrium can give rise to higher social welfare than in identical settings where individuals are constrained to exhibit rational expectations.

2.2 Social Value of Information

Our paper contributes to the rich literature on coordination games by highlighting the importance of belief choice for understanding both individual and aggregate actions as well as

[7]Though the objective function we utilize resembles that of Brunnermeier and Parker (2005), our interpretation is more similar to Caplin and Leahy (2019). One interpretation of Brunnermeier and Parker (2005), suggested by Caplin and Leahy (2019),is that agents exhibit divided selves since the agent chooses subjective beliefs (at date zero) by evaluating outcomes under the objective distribution. Agents in our model, in contrast, evaluate payoffs under their subjective beliefs only, but their choice of subjective beliefs is restricted so as not to deviate "too far" from the objective distribution (as in Caplin and Leahy (2019)).

[8]See, for example, Babcock, Loewenstein, and Issacharoff (1997), Arkes (1991), and Jolls and Sunstein (2006).

the social value of information. On the normative front, Morris and Shin (2002) introduce their seminal "beauty contest" model to show how public information can reduce welfare in settings with strategic complementarity.[9] Their analysis was generalized by Angeletos and Pavan (2007) to a large class of quadratic-Gaussian economies, which is the framework in which we establish our main results. More recently, Colombo, Femminis, and Pavan (2014) study the welfare implications when players information choice is endogenous. In contrast, our model highlights the impact of players choice of *beliefs*. We show that the provision of information can lead individuals to revise their interpretation of *all* available information, which can give rise to changes in social welfare that do not arise under rational expectations.

A growing literature suggests that investors "agree to disagree" about the interpretation of public information (e.g., Kandel and Pearson (1995) , Banerjee and Kremer (2010)). Our model provides a formal foundation for when and why players who observe same public information can end up becoming increasingly polarized in their beliefs. Moreover, in our setting with belief choice, the provision of private information can increase non-fundamental volatility while an increase in the precision of the public signal can increase dispersion in players' actions.

3 Model Setup

Our analysis builds on the generalized setting formalized in Angeletos and Pavan (2007). There is a unit measure continuum of players indexed by $i \in [0, 1]$. Each player chooses an action, $k_i \in \mathbb{R}$, to maximize his expected payoff. This payoff, U_i, also depends upon the true state of the world, θ, as well as the actions of all other players, denoted by the vector k_{-i}. We assume that U_i is (i) quadratic in its arguments and (ii) symmetric across the actions of other players (i.e., $U_i(k_i, k_{-i}, \theta) = U_i\left(k_i, k'_{-i}, \theta\right)$ for any permutation k'_{-i} of k_{-i}). Let $K \equiv \int_0^1 k_j dj$ denote the average action of all other players and $\sigma_k \equiv \left(\int_0^1 (k_j - K)^2 dj\right)^{\frac{1}{2}}$ denote the dispersion of others' actions. Then, as Angeletos and Pavan (2007) show, the above implies that payoffs can be expressed as a function

$$U_i \equiv u(k_i, K, \sigma_k, \theta), \tag{1}$$

where $u(\cdot)$ is quadratic and its partials satisfy $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$

[9]See Bond, Edmans, and Goldstein (2012), Goldstein and Sapra (2014), and Goldstein and Yang (2017) for recent surveys on implications of disclosure in financial markets.

for all (k, K, θ). This generalized functional form ensures tractability while still preserving flexibility for our analysis.[10] We make the following assumptions about the payoff function u.

Assumption 1. *The payoff function $U_i = u(k_i, K, \sigma_K, \theta)$ satisfies the following:*

(i) $u_{kk} < 0$

(ii) $-u_{kK}/u_{kk} < 1$.

(iii) $u_{kk} + 2u_{kK} + u_{KK} < 0$

(iiv)$u_{\sigma\sigma} + u_{kk} < 0$.

Assumptions (i) and (ii) ensure that the equilibrium action is unique and bounded. Angeletos and Pavan (2007) show that assumptions (iii) and (iv) ensure the same for the socially efficient action in a rational expectations equilibrium - in their absence, the social planner could generated unbounded utility for each agent by randomizing their actions. As we shall show, the same conditions play an important role in determining the effect of non-fundamental volatility on anticipatory utility and the choice of subjective beliefs in our setting. This payoff structure includes settings in which aggregate activity can create positive or negative externalities ($u_K \neq 0$ or $u_\sigma \neq 0$) and allows for strategic substitutability or complementarity ($u_{kK} < 0$ or $u_{kK} > 0$, respectively).

Equilibrium actions depend upon fundamentals, i.e., $u_{k\theta} \neq 0$, but players have incomplete information when choosing their action. Specifically, θ is drawn from a mean-zero normal distribution with variance $\frac{1}{\tau}$ and each player observes both

- a private signal, $s_i = \theta + \varepsilon_i$ where $\varepsilon_i \sim N(0, 1/\tau_e)$ with $\int_0^1 \varepsilon_j \, dj = 0$, and
- a public signal, $s = \theta + \eta$ where $\eta \sim N(0, 1/\tau_\eta)$.

We extend this generalized setting by allowing players to exhibit subjective beliefs about the quality of their information. In all other ways, the players are rational: in particular, they (i) take as given other players' actions and (ii) update using Bayes' rule. In what follows, we allow player i to perceive the error in her signals to be:

$$\varepsilon_i \sim_i N\left(0, \frac{1}{\delta_{e,i}\tau_e}\right), \quad \eta \sim_i N\left(0, \frac{1}{\delta_{\eta,i}\tau_\eta}\right). \tag{2}$$

While many of our results allow for arbitrarily subjective beliefs, i.e., $\delta_{e,i}, \delta_{\eta,i} \in [0, \infty]$, in some settings, equilibrium existence requires a lower bound $\underline{\delta}$, where $0 \leq \underline{\delta} < 1$.[11] If

[10] As noted in Angeletos and Pavan (2007), one can also regard our setting as a second-order approximation of a much broader class of economies.

[11] We show this in detail in Section 4.3.1 when analyzing settings in which non-fundamental volatility can increase players' utility.

$\delta_{e,i} = \delta_{\eta,i} = 1$, player i's beliefs coincide with the objective distribution: he exhibits **rational expectations**. On the other hand, when $\delta_{e,i}$ $(\delta_{s,i})$ is greater than one, player i overweights the private (public) signal when forming expectations: he believes the signal contains less noise. The opposite is true when $\delta_{e,i}$ $(\delta_{s,i})$ is less than one: the player underweights the signal because he believes that it is noisier than it objectively is.For ease of expression, we denote the expectation and variance of random variable X, given player i's perception of his information ($\delta_{e,i}$ and $\delta_{\eta,i}$) by $\mathbb{E}_i [X]$ and $\mathrm{var}_i [X]$, respectively. Under the objective distribution, we follow standard notation and denote the expectation and variance of X by $\mathbb{E}[X]$ and $\mathrm{var}[X]$, respectively.

When a player chooses to deviate from the objective distribution (i.e., from $\delta_{e,i} = \delta_{\eta,i} = 1$), he incurs a cost, denoted by $C(\delta_{e,i}, \delta_{\eta,i})$. We assume that this cost is well-behaved as defined below.

Definition 1. A cost function $C(\delta_{e,i}, \delta_{\eta,i})$ is **well-behaved** if $C(1,1) = \frac{\partial C}{\delta_{e,i}}(1,1) = \frac{\partial C}{\delta_{\eta,i}}(1,1) = 0$, and C is strictly convex (i.e., its global minimum is at $(1,1)$).

While many of our results apply to general, well-behaved cost functions, we focus on a special case in which the cost of holding distorted beliefs is the loss in expected utility under the objective distribution due to the distorted action. As in Banerjee, Davis, and Gondhi (2019), we refer to this specification as **experienced utility penalty**. Specifically, the cost of choosing $(\delta_{e,i}, \delta_{\eta,i})$ is given by:

$$C(\delta_{e,i}, \delta_{\eta,i}) = \mathbb{E}[u(k_i^*(1,1), K, \sigma_k, \theta)] - \mathbb{E}[u(k_i^*(\delta_{e,i}, \delta_{\eta,i}), K, \sigma_k, \theta)], \tag{3}$$

where $k_i^*(1,1)$ corresponds to the action under rational expectations (or objective beliefs), and $k_i^*(\delta_{e,i}, \delta_{\eta,i})$ corresponds to the action that maximizes player i's subjective beliefs, conditional on the choice of $\{\delta_{e,i}, \delta_{\eta,i}\}$ i.e.,

$$k_i^*(\delta_{e,i}, \delta_{\eta,i}) \equiv \arg\max_{k_i} \mathbb{E}_i \left[u(k_i, K, \sigma_k, \theta) \middle| s_i, s\right]. \tag{4}$$

Anticipating this choice of actions, each player chooses his subjective beliefs to maximize his anticipatory subjective utility net of costs. Formally, we denote player i's **anticipatory utility** by:

$$AU_i(\delta_{e,i}, \delta_{\eta,i}) \equiv \mathbb{E}_i \left[\mathbb{E}_i \left[u(k_i^*(\delta_{e,i}, \delta_{\eta,i}), K, \sigma_k, \theta) \middle| s_i, s\right]\right]. \tag{5}$$

$$= \mathbb{E}_i [u(k_i^*(\delta_{e,i}, \delta_{\eta,i}), K, \sigma_k, \theta)]. \tag{6}$$

The optimal subjective beliefs maximize:

$$\max_{\delta_{e,i},\delta_{\eta,i}} AU_i\left(\delta_{e,i},\delta_{\eta,i}\right)-\psi C\left(\delta_{e,i},\delta_{\eta,i}\right), \tag{7}$$

where $\psi \geq 0$ scales the utility cost of distorting beliefs. In particular, note that when $\psi \to \infty$, the cost of distorting beliefs is arbitrarily high, and the optimal choice of beliefs converges to the rational expectations benchmark i.e., $\delta_{e,i} = \delta_{\eta,i} = 1$.[12]

3.1 Discussion of Assumptions

Caplin and Leahy (2019) emphasizes that the wishful thinking approach employed here (and in their work) has a parallel in the robust control literature (e.g., Hansen and Sargent (2001), Hansen and Sargent (2008)). When faced with model uncertainty, players who exhibit robust control choose their action utilizing "worst-case" subjective beliefs. As in our setting, the set of subjective beliefs considered must be plausible, i.e., such beliefs are commonly restricted to be "close" to the objective distribution.[13] More specifically, a robust control player chooses action a and subjective beliefs μ to solve

$$\min_{\mu}\max_{k}\mathbb{E}_{\mu}\left[u\left(k\right)\right]+C\left(\mu\right). \tag{8}$$

As in our setting, $C(\mu)$ reflects the penalty of choosing subjective beliefs, μ, that differ from the reference distribution, while $\mathbb{E}_{\mu}\left[u\left(k\right)\right]$ reflects the *subjective* expected utility from action k under "worst case" beliefs μ. In contrast, an player engaging in wishful thinking chooses action k and subjective beliefs μ to solve:

$$\max_{\mu}\max_{k}\mathbb{E}_{\mu}\left[u\left(k\right)\right]-C\left(\mu\right). \tag{9}$$

In this case, the choice of μ reflects "wishful thinking" that the player engages in so that anticipatory utility, $\mathbb{E}_{\mu}\left[u\left(a\right)\right]$, is maximized.

There is a large psychology literature documenting behavior consistent with both approaches. Robust control is often motivated by evidence of ambiguity aversion, and there is a large economics literature which has shown that it can help explain a number of stylized facts (e.g., the equity premium puzzle). However, there is also substantial evidence for both optimism and motivated beliefs, as suggested by the papers discussed above, and emphasized

[12] An alternative interpretation of this specification is that ψ parameterizes the relative utility players enjoy from anticipatory utility.

[13] This is usually done using a statistical penalty function like the Kullback-Leibler divergence.

by Caplin and Leahy (2019). We believe that these differing behaviors may arise in different contexts, and our analysis suggests that accounting for wishful thinking may be an important step in understanding both belief formation and socially-optimal policy in coordination games.

We emphasize that the penalty function in (3) need not imply that players are endowed with knowledge of the objective distribution. This specification provides a tractable and, arguably, natural functional form that captures the utility cost of subjective beliefs. This interpretation contrasts with the description found in Brunnermeier and Parker (2005), in which players choose beliefs with knowledge of the objective distribution and then choose their actions under their chosen subjective model. We interpret the experienced utility penalty (and the optimal expectations of Brunnermeier and Parker (2005)) as one in which players make there choices utilizing a single, subjective model of the world. This subjective model is "close to the truth" in the sense that the player's choices do not generate too large of a loss from the perspective of someone endowed with the objective distribution. Finally, we acknowledge that the actual process by which players form their subjective model may include experimentation, learning and experience, processes through which players learn to trade off "desirable" models (that increase anticipatory utility) against "accurate" models (that increase experienced utility). We utilize the specification in (3) because it provides a tractable characterization of this process.

4 Equilibrium

In this section, we characterize the existence and uniqueness of equilibrium by working backwards. The first subsection characterizes the equilibrium actions, given an arbitrary set of subjective beliefs. In Section 4.2, we characterize how a player's subjective belief choice affects his anticipatory utility, given the beliefs of others and the anticipation of equilibrium actions. Finally, in Section 4.3, we characterize the equilibrium and then establish conditions for existence and uniqueness under the experienced utility penalty.

4.1 Equilibrium actions

We begin this section by characterizing the optimal actions of each player. If all players observed θ perfectly, then the optimal action would be $\kappa(\theta)$, where:

$$\kappa(\theta) = -\frac{u_k(0,0,0,0)}{u_{kk}+u_{kK}} - \frac{u_{k\theta}}{u_{kk}+u_{kK}}\theta \quad (10)$$

$$\equiv \kappa_0 + \kappa_1\theta. \quad (11)$$

However, players have incomplete information. Given player $i's$ subjective beliefs, $\delta_{e,i}$ and $\delta_{\eta,i}$, and the realization of s_i and s, his optimal action, k_i is

$$k_i^*(\delta_{e,i}, \delta_{\eta,i}) = r\mathbb{E}_i[K|s_i, s] + (1-r)\mathbb{E}_i[\kappa(\theta)|s_i, s], \quad (12)$$

where $r \equiv -\frac{u_{kK}}{u_{kk}}$ is the **equilibrium degree of coordination** across players. This term captures the extent to which each player chooses to align his action with his expectation of others choices, K, relative to his expectation of the full-information target, $\kappa(\theta)$. Specifically, if we let $\mathcal{K} \equiv rK + (1-r)\kappa(\theta)$ denote the players' optimal target under incomplete information, then we can express

$$k_i^*(\delta_{e,i}, \delta_{\eta,i}) = \mathbb{E}_i[\mathcal{K}|s_i, s]. \quad (13)$$

This solution is standard in this general class of models; the distinction, in our setting, is the player's use of subjective beliefs when forming expectations of $\mathcal{K}$. In particular, a player's optimal action is distorted by his subjective expectation of θ. Given our assumptions on the joint distribution of fundamentals and signals, Bayesian updating implies that player i's conditional beliefs about θ are given by

$$\mathbb{E}_i[\theta|s_i, s] = A_i s_i + B_i s, \text{ and } \mathrm{var}_i[\theta|s_i, s] = \frac{1-A_i-B_i}{\tau}, \quad (14)$$

where player i's weights on the private and public signals are given by:

$$A_i \equiv \frac{\delta_{e,i}\tau_e}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta}, \text{ and } B_i \equiv \frac{\delta_{\eta,i}\tau_\eta}{\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau_\eta}. \quad (15)$$

Note that in the rational expectations benchmark (i.e., when $\delta_{e,i} = \delta_{\eta,i} = 1$), the corresponding weights are given by:

$$A_{RE} \equiv \frac{\tau_e}{\tau + \tau_e + \tau_\eta}, \text{ and } B_{RE} = \frac{\tau_e}{\tau + \tau_e + \tau_\eta}. \quad (16)$$

Given these descriptions, the following lemma characterizes equilibrium actions.

Lemma 1. *Given a choice of subjective beliefs $\{\delta_{e,i}, \delta_{\eta,i}\}_i$ for each player i, there always exists a unique, linear equilibrium in which player i's optimal action is given by:*

$$k_i\left(\delta_{e,i}, \delta_{\eta,i}\right) = \kappa_0 + \kappa_1\left(\alpha_i s_i + \beta_i s\right), \tag{17}$$

and the aggregate action is given by:

$$K = \kappa_0 + \alpha\theta + \beta s, \tag{18}$$

where $\alpha_i = \frac{1-r}{1-rA}A_i$, $\beta_i = \frac{(1-r)B_i + Br}{1-rA}$, $\alpha = \frac{A(1-r)}{1-rA}\kappa_1$, $\beta = \frac{B}{1-rA}\kappa_1$, $A = \int_i \frac{\delta_{e,i}\tau_e}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}di$ *and* $B = \int_i \frac{\delta_{\eta,i}\tau_\eta}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}di$.

Player i's optimal action puts weight α_i and β_i on his private and public information, respectively. These weights depend on both the subjective beliefs and strategic considerations. To see this note that in the absence of strategic considerations (i.e., $r = 0$) and under rational expectations (i.e., $\delta_{e,i} = \delta_{\eta,i} = 1$), the optimal weights are given by

$$\alpha_i = A_{RE} \equiv \frac{\tau_e}{\tau + \tau_e + \tau_\eta} \text{ and } \beta_i = B_{RE} = \frac{\tau_e}{\tau + \tau_e + \tau_\eta}. \tag{19}$$

When either $\delta_{e,i}$ or $\delta_{\eta,i}$ differ from one, the player distorts the weights on his information i.e., $A_i \neq A_{RE}$ and $B_i \neq B_{RE}$. Moreover, when $r \neq 0$ (so that the player derives direct utility from his alignment with others' actions), the player's optimal action is further distorted by the subjective beliefs of the other players, which he takes as given. This is reflected in the average weight they place on their information, A and B. As we show below, both channels play a critical role in the equilibrium choice of beliefs.

4.2 Anticipatory Utility and Subjective Beliefs

In our setting, players derive contemporaneous utility from their anticipation of future payoffs, which depend on (i) their anticipation of equilibrium actions, and (ii) their subjective beliefs about fundamentals and signals. The following result characterizes how player i's anticipatory utility depends on his beliefs about actions and fundamentals.

Proposition 1. *Given player i's subjective beliefs, $\delta_{e,i}$ and $\delta_{\eta,i}$, anticipatory utility can be written*

$$AU_i\left(\delta_{e,i}, \delta_{\eta,i}\right) = \Gamma + \left(\frac{\kappa_1}{1-rA}\right)^2 \left[\underbrace{u_{kk}\left(1-r\right)^2 var_i\left[\theta|s_i, s\right]}_{\textit{fundamental uncertainty channel}} - \underbrace{\left(u_{KK} - r^2 u_{kk}\right) B^2 var_i\left[s|\theta\right]}_{\textit{non-fundamental volatility channel}}\right], \tag{20}$$

where Γ, defined in the Appendix, is independent of player i's subjective beliefs.

A player's subjective beliefs affect his anticipatory utility through two channels. First, when the player believes that this fundamental forecast is more precise, i.e., when $\text{var}_i\left[\theta|s_i, s\right]$ decreases, his anticipatory utility increases- we refer to this as the **fundamental uncertainty channel**. Second, the player's anticipatory utility is affected by his subjective beliefs about the error in the average action, which is captured by the perceived error in the public signal i.e., $\text{var}_i\left[s|\theta\right]$. We refer to this as the **non-fundamental volatility channel**, since higher noise in the public signal leads to more volatility in the aggregate action relative to the full information benchmark i.e., $\text{var}_i\left(K|\theta\right)$ is higher.

Whether non-fundamental volatility increases or decreases anticipatory utility depends on the sign of $u_{KK} - r^2 u_{kk}$. This term reflects the direct effect of aggregate actions on player i's utility (u_{KK}) and the indirect effect due to strategic considerations ($-r^2 u_{kk}$). Since $u_{kk} < 0$, the latter term is always positive. This reflects the benefit of common noise in a strategic setting because it increases (decreases) the covariance between the player's action and the aggregate action when $r > 0$ ($r < 0$). Taken together, the impact of this perceived common error depends upon whether the direct effect is sufficiently negative: if $u_{KK} - r^2 u_{kk} < 0$, then the player's anticipatory utility necessarily increases when he perceives that the public signal is more informative. If $u_{KK} - r^2 u_{kk} > 0$, however, perceiving that the public signal is noisier can be beneficial.

In what follows, it will be useful to define the coefficient

$$\chi \equiv \frac{\left(u_{KK} - r^2 u_{kk}\right)}{-u_{kk}\left(1-r\right)^2}, \tag{21}$$

which captures the relative magnitude of the non-fundamental (i.e., $\left(u_{KK} - r^2 u_{kk}\right)$) and informational (i.e., $-u_{kk}\left(1-r\right)^2$) channels. Given our discussion above, $\chi < 0$ when non-fundamental volatility reduces anticipatory utility while $\chi > 0$ when non-fundamental volatility increases anticipatory utility. The assumptions about the coefficients of the utility function, in particular, assumption (iii) which ensures that random variation in the aggregate action does not increase welfare, ensure that $\chi \leq 1$. Intuitively, this implies that even when non-fundamental volatility increases anticipatory utility, the effect of fundamental uncertainty (via the informational channel) is larger in magnitude.

The next corollary follows directly.

Corollary 1. *Anticipatory utility is always increasing in $\delta_{e,i}$. If $\chi \leq 0$, then anticipatory utility is increasing in $\delta_{\eta,i}$; otherwise, it is U-shaped in $\delta_{\eta,i}$.*

The player's belief about the quality of his private signal affects only the information channel: an increase in $\delta_{e,i}$ reduce the player's uncertainty (i.e., $\text{var}_i[\theta|s_i, s]$) and so anticipatory utility increases. In contrast, a player's beliefs about the public signal affect both the information channel and the non-fundamental volatility channel, as highlighted by the following expression:

$$\frac{\partial AU_i}{\partial \delta_{\eta,i}} = u_{kk} \left(\kappa_1 \frac{1-r}{1-rA}\right)^2 \left(\frac{\partial \text{var}_i[\theta|s_i, s]}{\partial \delta_{\eta,i}} + \frac{\partial \text{var}_i[s|\theta]}{\partial \delta_{\eta,i}} \left(\chi B^2\right)\right) \tag{22}$$

$$= u_{kk} \left(\kappa_1 \frac{1-r}{1-rA}\right)^2 \left(-\frac{\tau_\eta}{\left(\tau + \delta_{e,i}\tau_e + \delta_{\eta,i}\tau\right)^2} - \frac{1}{\delta_{\eta,i}^2 \tau_\eta} \left(\chi B^2\right)\right) \tag{23}$$

An increase in the perceived precision of the public signal decreases the player's perception of both fundamental uncertainty (i.e., $\frac{\partial \text{var}_i[\theta|s_i,s]}{\partial \delta_{\eta,i}} < 0$) and non-fundamental volatility (i.e., $\frac{\partial \text{var}_i[s|\theta]}{\partial \delta_{\eta,i}} < 0$). When $\chi < 0$, anticipatory utility decreases in non-fundamental volatility, so an increase in $\delta_{\eta,i}$ unambiguously increases anticipatory utility. When $\chi > 0$, however, the fundamental uncertainty and non-fundamental volatility channels operate in opposite directions. When $\delta_{\eta,i}$ is sufficiently low, the non-fundamental channel dominates (as $\delta_{\eta,i} \to 0$, $\frac{\partial \text{var}_i[\theta|s_i,s]}{\partial \delta_{\eta,i}}$ is bounded above, while $\frac{\partial \text{var}_i[s|\theta]}{\partial \delta_{\eta,i}}$ is not), which leads to a negative relation between anticipatory utility and $\delta_{\eta,i}$. When $\delta_{\eta,i}$ is sufficiently high, the informational channel dominates and anticipatory utility increases with $\delta_{\eta,i}$ since $\chi B^2 < 1$.[14]

4.3 Equilibrium Subjective Belief Choice

The above results characterize how a player's subjective beliefs affect his anticipatory utility, taking others' beliefs and actions as given. Next, we characterize the features of any symmetric equilibrium in our setting.

Proposition 2. *In any symmetric equilibrium, with any well-behaved cost function, all players are over-confident about both their private signals and the public signal (i.e., $\delta_{e,i} = \delta_e > 1$ and $\delta_{\eta,i} = \delta_\eta > 1$ for all i). Moreover, it must be the case that $A_i = A > A_{RE}$ and $B_i = B > B_{RE}$, i.e., players overweight both private and public signals relative to the rational expectations equilibrium.*

As discussed above, regardless of the choice made by others, player i benefits from believing that his private signal is more informative: $\frac{\partial AU_i}{\delta_{e,i}} \geq 0$. Since any deviation from rational expectations is costly, this implies that players must exhibit (weak) over-confidence in a symmetric equilibrium. When $\chi < 0$, a similar argument leads to over-confidence about public information as well.

[14] We have $\chi < 1$ because of assumptions (i) and (iii) about the player's payoff. $B \leq 1$ by Bayes rule since it reflects the weight that players on average put on the public signal.

Somewhat surprisingly, however, we show that symmetric equilibria feature overconfidence in the public signal even when $\chi > 0$. This arises because the fundamental uncertainty channel always dominates the non-fundamental volatility channel when $\delta_{\eta,i} = \delta_\eta$. On the margin, player i always benefits from believing that the public signal is more informative than others do i.e., $\frac{\partial AU_i}{\delta_{\eta,i}} \geq 0$ when evaluated at $\delta_{\eta,i} = \delta_\eta$. As a result, the equilibrium choice of δ_η cannot be below one. If it were, player i would benefit from choosing $\delta_{\eta,i} > \delta_\eta$ because such a deviation increases anticipatory utility and lowers the cost of distorting her beliefs (i.e., she would move $\delta_{\eta,i}$ closer to 1).

The second half of Proposition 2 extends this result further. Note that a player's weights on her private and public signal, i.e., A_i and B_i, respectively, depend on the subjective beliefs about the *relative* precision of the signals. As such, a higher subjective precision of a given signal does *not* necessarily imply that the player overweights this information (relative to rational expectations) when forming his conditional expectation. For instance, holding fixed $\delta_e > 1$, there exists sufficiently large δ_η such that players begin to *underweight* their private signal, despite their overconfidence in it. However, we show that the gap between δ_e and δ_η is bounded in equilibrium so that both (subjective) weights, A and B, are above their rational expectations counterparts.

4.3.1 Belief Choice: Experienced Utility Penalty

In what follows, we utilize the experienced utility penalty defined in (3) to derive conditions for equilibrium existence. This exercise also allows us to characterize how players' optimal subjective beliefs change with players' preferences as well as the information environment using closed-form expressions for δ_e and δ_η.

We begin with a characterization of equilibria when non-fundamental volatility decreases anticipatory utility.

Proposition 3. *Suppose players incur the experienced utility penalty (3) and non-fundamental volatility decreases anticipatory utility (i.e., $\chi \leq 0$).*

(i) If ψ is sufficiently small, then all players choose to ignore their prior and choose $A_i = \frac{\tau_e}{\tau_e+\tau_\eta}$ *and* $B_i = \frac{\tau_\eta}{\tau_e+\tau_\eta}$.

(ii) If ψ is sufficiently large, then there exists a unique symmetric equilibrium in which the optimal precision choices of each player i are given by:

$$\delta_e = \frac{2\tau\psi\left(2\psi+1\right)+\left(\chi-1\right)\chi\tau_\eta}{2\psi\left(2\tau\psi+\left(\chi-1\right)\tau_\eta-\tau_e\right)} > 1, \quad \textit{and} \quad \delta_\eta = \delta_e - \frac{\chi}{2\psi} > 1 \tag{24}$$

Moreover, δ_e and δ_η are increasing in τ_e and τ_η , and decreasing in τ, ψ and χ.

Since $\chi < 0$, anticipatory utility is decreasing in both fundamental uncertainty and non-fundamental volatility. Given the preceding discussion, this naturally leads players to overestimate the precision of both signals in any symmetric equilibrium. The extent to which a player distorts his beliefs depends on how costly the distortions are. Intuitively, the player finds it less costly to deviate from rational expectations when he either has imprecise prior information (i.e., low τ) or the cost of distorting beliefs is small (i.e., low ψ). When the overall costs are sufficiently low (i.e., when $\psi \leq \frac{\tau_e+(1-\chi)\tau_\eta}{2\tau}$), the players choose to behave as if they are simply ignoring their priors i.e., $A_i = \frac{\tau_e}{\tau_e+\tau_\eta}$ and $B_i = \frac{\tau_\eta}{\tau_e+\tau_\eta}$. When the costs are sufficiently large, the distortion is not as extreme; however, consistent with the above discussion, the magnitude of distortions decrease with ψ and τ. Analogously, the cost of perceiving that the signals are more precise than they actually are is lower when the signals are, in fact, more precise. This implies that the magnitude of the distortions increases with τ_e and τ_η.[15]

The relative distortion in public versus private precision depends on the relative importance of the fundamental and non-fundamental channels, as captured by χ. With $\chi < 0$, the non-fundamental channel induces an incremental marginal benefit of increasing perceived public precision, and consequently, $\delta_\eta > \delta_e$. As χ rises (i.e., approaches zero), the relative impact of the non-fundamental volatility channel falls, and so each player has a weaker incentive to distort $\delta_{\eta,i}$ upward. Interestingly, this also reduces the optimal choice of $\delta_{e,i}$. This is because the experienced utility penalty induces complementarity between the choice of $\delta_{e,i}$ and $\delta_{\eta,i}$ for player i. This is because of the nature of the experienced utility penalty. If a player chooses to increase $\delta_{\eta,i}$, which increases the weight she places on the public signal, then it is beneficial to increase $\delta_{e,i}$, since this can help restore balance between public and private sources of information. As a result, when χ increases both δ_η and δ_e fall.

Next, we consider the case when non-fundamental volatility increases anticipatory utility ($\chi > 0$). Note that, in this each player may have an incentive to *decrease* the perceived precision of the public signal, as this increases non-fundamental volatility. When the choice of $\delta_{\eta,i}$ is unbounded below (i.e., if players can choose to set $\delta_{\eta,i} = 0$), this leads to non-existence of equilibria. To see why, conjecture an equilibrium in which some players place positive weight on the public signal so that $B > 0$. Then it is optimal for player i to deviate and choose $\delta_{\eta,i} = 0$ since this maximizes her anticipatory utility through the non-fundamental volatility channel. On the other hand, if all other players place zero weight on the public signal (i.e., they choose $\delta_\eta = 0$, so that $B = 0$), then player i's beliefs about

[15] This is also why, when τ_e and τ_η are sufficiently high, players ignore their prior. We note that the additional weight on the public precision, $-\chi > 0$, reflects the additional benefit of belief distortion generated by the non-fundamental volatility channel.

the public signal affect her anticipatory utility through the information channel, only. As a result, it is beneficial for player i to deviate by choosing $\delta_{\eta,i} > 0$.

This suggests that in order to sustain an equilibrium with $\chi > 0$, we need to bound the subjective beliefs about the public signal precision $\delta_{\eta,i}$ away from zero i.e., restrict $\delta_{\eta,i} \geq \underline{\delta}$ for some $\underline{\delta} > 0$. The following proposition characterizes how this bound affects the nature of equilibrium.

Proposition 4. *Suppose players incur the experienced utility penalty (3), non-fundamental volatility increases anticipatory utility (i.e., $\chi > 0$), and $\underline{\delta} > 0$. Then there exists a $\bar{\psi} > 0$ such that:*

(i) If $\psi \in (0, \underline{\psi})$, the unique equilibrium is asymmetric and is characterized by the quadruple $(\lambda, \delta_{e1}, \delta_{e2}, \delta_{\eta 2})$ which solve a system of equations (specified in the Appendix). In this equilibrium, a fraction λ of players optimally chooses $\delta_{e,i} = \delta_{e,1}$ and $\delta_{\eta,i} = \underline{\delta}$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{e,i} = \delta_{e,2}$ and $\delta_{\eta,i} = \delta_{\eta,2}$. Moreover, in this equilibrium, the players' subjective beliefs $(\delta_{e1}, \delta_{e2}, \delta_{\eta 2})$ do not depend on χ while λ decreases with χ.

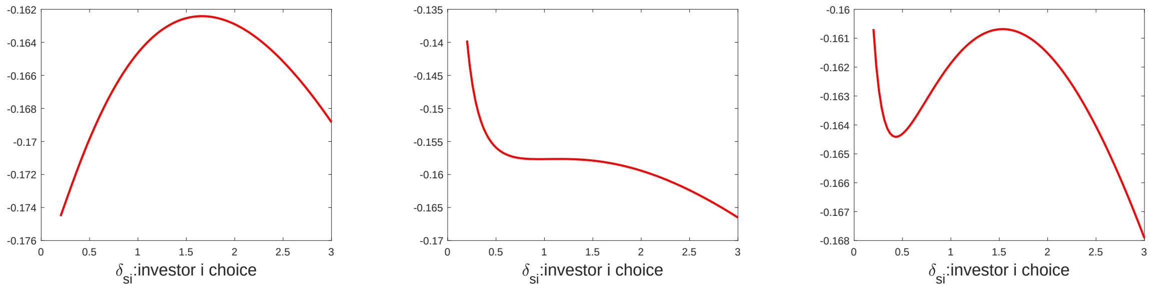
(ii) If $\psi > \bar{\psi}$, then the unique equilibrium is symmetric and is given by (24).

Following the intuition provided above, when $\chi > 0$ and $B > 0$, players can increase their anticipatory utility by lowering $\delta_{\eta,i}$; however, since $\underline{\delta} > 0$, the benefit of doing so is now bounded. We consider first the asymmetric equilibrium. If $\lambda = 0$, a player has an incentive to deviate to the corner solution $(\delta_{e1}, \underline{\delta})$ since the cost of deviating from rational expectations is relatively low (i.e., $\psi < \bar{\psi}$). However, as the measure of players who choose $\delta_{\eta,i} = \underline{\delta}$ increases, the aggregate weight placed on the public signal falls, reducing the utility from choosing $(\delta_{e1}, \underline{\delta})$. The equilibrium λ makes players indifferent between this choice and an interior maximum, $(\delta_{e2}, \delta_{\eta 2})$. In this asymmetric equilibrium, the optimal subjective beliefs depend on the information environment only, while the measure of those who choose each profile depends upon preferences (as captured by χ). When the cost of deviating from rational expectations is sufficiently high, i.e., when $\psi > \bar{\psi}$, the net benefit lowering $\delta_{\eta,i}$ is too small and so the symmetric equilibrium is preserved.

The plots in Figure 1 provide a numerical illustration of the case when $\psi < \overline{\psi}$. The panels show player i's anticipatory utility, net of costs, as a function of $\delta_{\eta,i}$, given the beliefs of others. In panel (a), we take as given that all other agents choose $\delta_\eta = \underline{\delta}$. In response, player i has an incentive to deviate by over-weighting the public information (i.e., by setting $\delta_{\eta i} \approx 1.6$) since B is sufficiently low. In panel (b), we consider an alternative symmetric equilibrium in which all other agents choose $\delta_\eta = 1.6$. Now, the non-fundamental channel dominates, i.e., B is sufficiently large, and player i strictly prefers to believe the public signal is (relatively) uninformative. In both cases, a symmetric equilibrium is ruled out because an

Figure 1: Total utility versus $\delta_{\eta,i}$ when $\psi < \overline{\psi}$

The figure plots the anticipatory utility, net of costs, for player i as a function of her choice $\delta_{\eta,i}$. Other parameters are: $\tau = 10$, $\tau_e = \tau_s = 1$, $\chi = 0.5$, $\underline{\delta} = 0.2$, $\psi = 1$.



(a) $\delta_\eta = \underline{\delta}$ (b) $\delta_\eta = 1.6$ (c) Mixed eqm.

individual agent has an incentive to deviate. Given the non-existence of symmetric equilibria, we explore the existence of asymmetric equilibria in which agents mix between two sets of beliefs. Panel (c) of Figure 1 illustrates the asymmetric equilibrium described above. As is clear, given the choice of all other agents, player i is indifferent between two (sets of) beliefs. In equilibrium, a fraction $\lambda = 0.36$ of players discount the public signal while the remaining fraction $1 - \lambda = 0.64$ overweight it.

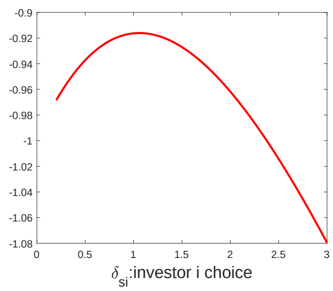
The plots in Figure 2 provide a numerical example when $\psi > \overline{\psi}$. Note that all other parameters are the same as in the figure above. The panels show agent i's anticipatory utility, net of costs, as a function of $\delta_{\eta,i}$, given the behavior of others. In panel (a), all other agents choose $\delta_\eta = \underline{\delta}$. In this case, agent i has an incentive to deviate by over-weighting the public information (i.e., by setting $\delta_{\eta i} > 1.03$). In panel (b), we consider an alternative symmetric equilibrium in which all other agents choose $\delta_\eta = 1.03$. In this case, player i will also choose $\delta_{\eta,i} = 1.03$, i.e., the symmetric equilibrium exists.

The existence of the asymmetric equilibrium provides an illustration of how disagreement about the interpretation of public information can arise endogenously, even with ex-ante homogenous players. Moreover, in such equilibria, disagreement will increase (relative to the rational expectations benchmark) after the release of a public signal. This is consistent with the existing literature, e.g. Kandel and Pearson (1995).

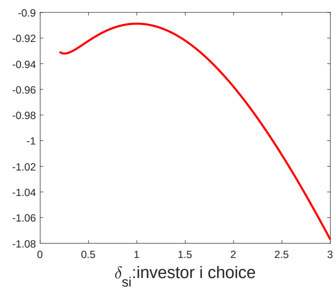
5 Observable Implications and Welfare

As Angeletos and Pavan (2007) demonstrate, information quality is a critical determinant of the cross-sectional dispersion in players' actions and non-fundamental volatility in the

Figure 2: Anticipatory utility net of costs versus $\delta_{\eta,i}$ when ψ is high
The figure plots the anticipatory utility net of costs for agent i as a function of her choice $\delta_{\eta,i}$. Other parameters are: $\tau = 10$, $\tau_e = \tau_s = 1$, $\chi = 0.5$, $\underline{\delta} = 0.2$, , $\psi = 10$.



(a) $\delta_\eta = \underline{\delta}$



(b) $\delta_\eta = 1.03$

aggregate action. In this section, we characterize how subjective belief choice can affect these aggregate observables. This allows us to derive testable predictions that distinguish our theory from the standard, rational expectations approach.

Let $\sigma_k^2 \equiv \int (k_i - K)^2 \, di$ denote the dispersion in players' actions and let $\nu^2 \equiv \mathbb{E}\left[(K - \kappa(\theta))^2\right]$ denote non-fundamental volatility (i.e., the volatility in the aggregate action driven by imperfect information). We begin by confirming well-known comparative statics under rational expectations.

Lemma 2. *If players' exhibit rational expectations (i.e., if $\psi \to \infty$), then dispersion decreases with public information quality ($\frac{\partial \sigma_k^2}{\partial \tau_\eta} < 0$) while non-fundamental volatility decreases with private information quality ($\frac{\partial \nu^2}{\partial \tau_e} < 0$).*

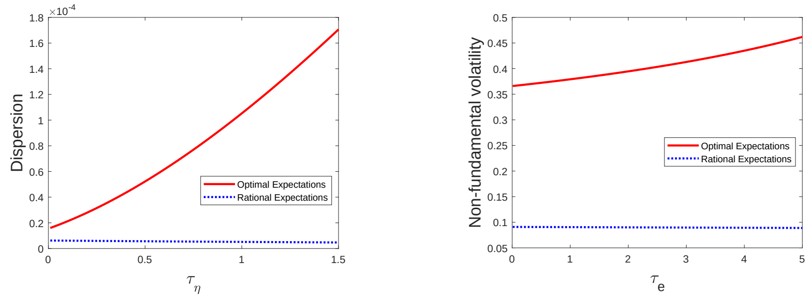
The intuition for both results is straightforward. As public information quality improves, players choose to place relatively more weight on the public signal. This reduces the weight they place on their private signals which reduces dispersion. Similarly, players place relatively more weight on their private information when its precision increases. This reduces the weight they place on the common noise found in the public signal which reduces non-fundamental volatility.

As highlighted by the previous section, changes in the information environment are important determinants of players' choice of subjective beliefs. These changes in perception, however, also affect their actions which then feed into both σ_k^2 and ν^2. As the following proposition highlights, subjective belief choice can lead to very different predictions on the relationship between information quality and observables.

Proposition 5. *Suppose players incur the experienced utility penalty (3), and non-fundamental uncertainty decreases anticipatory utility (i.e., $\chi < 0$). Then,*

Figure 3: Observable implications in a symmetric equilibrium

The figure plots dispersion of actions and non-fundamental volatility in a symmetric equilibrium. For comparison, it also plots these observables in a rational expectations equilibrium (dotted lines). Other parameters are: $\tau = 10$, $r = -0.975$, $\chi = -10$, $\psi = 1$.



(a) Dispersion vs τ_η (b) non-fundamental volatility vs τ_e

(i) if χ is sufficiently negative, dispersion increases with public information quality (i.e., $\frac{\partial \sigma_k^2}{\partial \tau_\eta} > 0$), and

(ii) if r is sufficiently close to one and private signal precision is sufficiently low, then non-fundamental volatility can increase with private information precision (i.e., $\frac{\partial \nu^2}{\partial \tau_e} > 0$).

Under the experienced utility penalty, an increase in the precision of *either* signal increases players' perceived precision of *both* signals in the symmetric equilibrium. For instance, an increase in private information precision, τ_e, leads to an increase in both $\delta_{e,i}$ and $\delta_{\eta,i}$. When the utility cost of non-fundamental volatility is sufficiently large (i.e., $\chi < 0$ is sufficiently negative), the increase in $\delta_{\eta,i}$ amplifies players' over-weighting of the public signal, and consequently, generates higher non-fundamental volatility ν^2. Similarly, when strategic complementarity considerations are sufficiently strong and private signals are sufficiently noisy, players choose to place relatively more weight on their private signals in response to an increase in *public* information, leading to higher cross-sectional dispersion σ_k^2.

So far, we have considered the implications of subjective beliefs in a symmetric equilibrium i.e., when χ is negative or when χ is positive and ψ is large. Next we consider a setting when χ is positive and ψ is small. In this case, as Proposition 4 shows, the equilibrium type is asymmetric. We consider two observables in this case: (i) dispersion of actions, and (ii) responsiveness of aggregate actions to news. As we see in Figure 4 panel (a), dispersion of actions increases with public news. This is because, in an asymmetric equilibrium, the two groups of agents place different weights on the public signals, which leads to dispersed actions.

More generally, our model helps shed light on recent puzzling evidence about macroeconomic forecasts. In an influential paper, Coibion and Gorodnichenko (2015) develop a new approach to measuring whether forecasts exhibit rational expectations by regressing forecast errors on forecast revisions. Under the null of rational expectations and full information, the forecast errors are unpredictable and so the regression coefficient is zero. However, the authors document that the regression coefficient for consensus forecasts is significantly positive in the data, suggesting that aggregate forecasts underreact to available information. The authors attribute this underreaction to informational rigidities (sticky or noisy information). In follow-up work, Bordalo, Gennaioli, Ma, and Shleifer (2018) document that while underreaction in consensus forecasts is a robust feature of a larger set of macroeconomic series, individual forecasts tend to exhibit overreaction: the regression coefficients are significantly negative.

In our model, one can measure the forecast revision of player i as the difference between his conditional and unconditional expectations of θ i.e.,

$$FR_i \equiv \mathbb{E}_i \left[\theta | s_i, s\right] - \mathbb{E}_i \left[\theta\right], \tag{25}$$

while his forecast error is given by

$$FE_i \equiv \theta - \mathbb{E}_i \left[\theta | s_i, s\right]. \tag{26}$$

Taking averages across i give us the analogous expressions for the consensus forecast revision $\bar{FR}$ and consensus forecast error $\bar{FE}$. Note that the covariance $\text{cov}\left(FE_i, FR_i\right)$ is a measure of how player i reacts to all of his available information: under rational expectations, we have $\text{cov}\left(FE_i, FR_i\right) = 0$, while $\text{cov}\left(FE_i, FR_i\right) < 0$ ($\text{cov}\left(FE_i, FR_i\right) > 0$) implies that the player under-reacts (overreacts, respectively) to his information.

The following result establishes characterizes how subjective belief choice affects this covariance for both individual and consensus forecasts.

Proposition 6. *Suppose players incur the experienced utility penalty (3). Then*

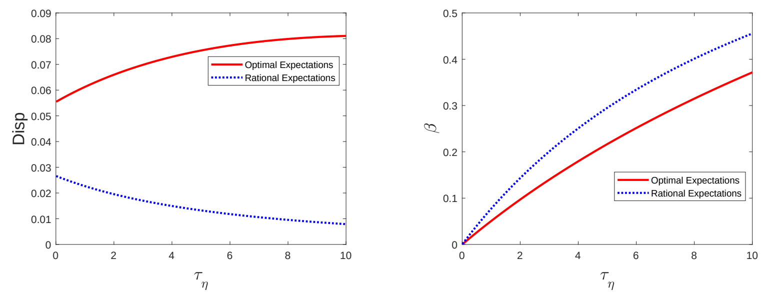
(i) If ψ is sufficiently high, consensus forecasts underreact and individual forecasts overreact to information i.e., cov$\left(\bar{FE}, \bar{FR}\right) > 0$ and cov$\left(FE_i, FR_i\right) < 0$.

(ii) If ψ is sufficiently low and $\chi < 0$, both consensus and individual forecasts overreact to information i.e., cov$\left(\bar{FE}, \bar{FR}\right) < 0$ and cov$\left(FE_i, FR_i\right) < 0$.

(iii) If ψ is sufficiently low and $\chi > 0$, some individuals overreact to information while others may underreact to it.

The above results suggest that whether individual and consensus forecasts feature overreaction or underreaction depends on the utility cost of non-fundamental volatility χ and on the utility cost of belief distortion ψ. Specifically, our model can generate the general

Figure 4: Observable implications in an asymmetric equilibrium
The figure plots dispersion of actions and responsiveness of aggregate actions to public information (β) in an asymmetric equilibrium. For comparison, it also plots these observables in a rational expectations equilibrium (dotted lines). Other parameters are: $\tau = 10$, $\tau_e = 1$, $r = -0.95$, $\underline{\delta} = 0.1$, $\psi = 1$.



(a) Dispersion vs τ_η (b) β vs τ_η

pattern of consensus underreaction and individual overreaction in macroeconomic forecasts documented by Bordalo, Gennaioli, Ma, and Shleifer (2018) when the cost of belief distortion is sufficiently high (but not infinite).[16] However, our model also suggests that when ψ is sufficiently low, both consensus and individual forecasts may exhibit overreaction, consistent with the evidence on firm's long term earnings growth, as documented by Bordalo, Gennaioli, Porta, and Shleifer (2019). Together, the results suggest that accounting for subjective belief choice may provide a unified approach for better understanding belief formation across a number of different settings.

These qualitatively different predictions on observables, relative to rational expectations, are potentially useful in identifying wishful thinking in observed behavior. Moreover, cross-sectional dispersion and non-fundamental volatility are critical determinants of aggregate well-being, or welfare. Consider a social planner who chooses to measure a weighted average of aggregate anticipatory utility and aggregate objective utility, given θ:

$$W\left(\theta, \{s_i\}_i, s; \gamma\right) = \gamma \int_i AU_i\left(\delta_{e,i}, \delta_{\eta,i}\right) di + (1-\gamma) \int_i \mathbb{E}\left[u\left(k_i^*\left(\delta_{e,i}, \delta_{\eta,i}\right), K, \sigma_k, \theta\right)\right] di. \quad (27)$$

This specification of the social planner's objective function nests two natural benchmarks. First, when $\gamma = 0$, the social planner evaluates welfare by aggregating experienced utility under objective beliefs — this corresponds to the standard welfare measure under rational expectations. Second, when $\gamma = \frac{1}{1+\psi}$, the social planner fully accounts for the aggregate anticipatory utility experienced by the players — this corresponds to a setting in which the

[16]When $\psi = \infty$, we are back to the rational expectations benchmark and both covariances are zero.

social planner respects the players' preferences, weighing both their anticipatory utility as well as the objective cost of subjective belief distortion. [17]

Under the experienced utility penalty, we show (in the appendix) that the *unconditional* expectation of this measure in a symmetric equilibrium, which we denote by W^0, can be expressed as

$$W^0 \equiv \mathbb{E}\left[W\left(\theta,\{s_i\}_i,s;\gamma\right)\right] = \begin{array}{c} \underbrace{\left(\mathbb{E}\left[u\left(\kappa\left(\theta\right),\kappa\left(\theta\right),0,\theta\right)\right]+\mathcal{A}\mathrm{var}\left(\theta\right)\right)}_{\equiv W_1} \\ +\underbrace{\tfrac{1}{2}\left(u_{\sigma\sigma}+u_{kk}\right)\sigma_i^2+\frac{1}{2}\left(u_{kk}+u_{KK}+2u_{kK}\right)\nu^2}_{\equiv W_2} \\ \underbrace{-\frac{1}{2}\gamma\left(\left(\delta_e-1\right)u_{kk}\frac{\alpha^2}{\delta_e\tau_e}+\left(u_{kk}+u_{KK}+2u_{kK}\right)\left(\delta_\eta-1\right)\frac{\beta^2}{\delta_s\tau_s}\right)}_{\equiv W_3} \end{array}, \tag{29}$$

where $\mathbb{E}\left[u\left(\kappa\left(\theta\right),\kappa\left(\theta\right),0,\theta\right)\right]$ reflects the expected utility if all players were perfectly informed about θ and $\mathcal{A}$ is characterized in the appendix. There are three components to welfare: (i) W_1 captures the impact of fundamental uncertainty, (ii) W_2 captures the incremental effect of cross-sectional dispersion and non-fundamental volatility, and (iii) W_3 captures the additional impact of subjective beliefs on aggregate anticipatory utility. Importantly, note that in the rational expectations benchmark (i.e., when $\psi\to\infty$), the contribution of the last term is zero (since $\delta_e=\delta_\eta=1$) and the above measure converges to the welfare measure found in Angeletos and Pavan (2007) (i.e., $W^0=W_1+W_2$).

The expression highlights the various channels through which subjective beliefs affect welfare in our setting. First, note that $\mathcal{A}$ is proportional to $\alpha+\beta-\kappa_1$, and so depends on the wedge between the sensitivity of the aggregate equilibrium action to fundamentals (i.e., $\alpha+\beta$) and the sensitivity of the full information action to fundamentals (i.e., κ_1). This wedge depends not only on the noise in players' information but also their chosen subjective beliefs.

Second, because we assume that both $u_{kk}+u_{KK}+2u_{kK}<0$ and $u_{\sigma\sigma}+u_{kk}<0$ (Assumptions (iii) and (iv)), both dispersion and non-fundamental volatility decrease the second

[17] Specifically, when $\gamma=\frac{1}{1+\psi}$, we have

$$W\left(\theta,\{s_i\}_i,s;\gamma\right)=\frac{1}{1+\psi}\left(\int_i AU_i\left(\delta_{e,i},\delta_{\eta,i}\right)-\psi C\left(\delta_{e,i},\delta_{\eta,i}\right)di\right), \tag{28}$$

where $\frac{1}{1+\psi}$ is a normalization.

component of welfare W_2. This is consistent with the rational expectations benchmark of Angeletos and Pavan (2007), and ensures that arbitrarily large random variation in players' beliefs (and therefore actions) does not lead to unbounded utility, all else equal. As discussed above, both observables depend not only on the economic environment but, importantly, on players' perception of their information.

Finally, when $\gamma > 0$, the social planner also accounts for the anticipatory utility experienced by players. When players over-estimate the precision of their private and public signals (i.e., $\delta_e, \delta_\eta > 1$), $W_3 > 0$ because $u_{kk} < 0$ and $u_{kk} + u_{KK} + 2u_{kK} < 0$. Intuitively, players choose to over-estimate the precision of their information precisely because it increases anticipatory utility and this channel is captured by W_3.

In Angeletos and Pavan (2007), the social value of private and public information depends upon the social cost of dispersion $(u_{\sigma\sigma} + u_{kk})$ relative to the social cost of non-fundamental volatility $(u_{kk} + u_{KK} + 2u_{kK})$. As is clear from Proposition 5, however, when players choose their beliefs, the impact of changes in the information environment on both observables is altered. For example, in a setting where an increase in τ_e is always beneficial under rational expectations (because it lowers non-fundamental volatility), when players can choose their beliefs, an increase in τ_e can lower W. We consider the implications of this indirect effect of information provision in the applications that follow.

6 Applications

In what follows, we consider the impact of subjective belief choice across representative settings that follow the generalized framework analyzed above. In Section 6.1, we analyze the classic beauty contest model. After deriving equilibrium beliefs, we show how the value of both private and public information differ relative to the rational expectations benchmark and examine the impact of subjective beliefs on welfare. In Section 6.2, we consider a set of applications and demonstrate how players' preference for non-fundamental volatility affects subjective beliefs equilibria.

6.1 Beauty Contest

We consider the payoff found in both the canonical model (Morris and Shin, 2002), denoted by u_{MS}, as well as the modification introduced by Angeletos and Pavan (2004), which we denote by u_{AP}. The payoff to player i is given by

$$u_{MS} \equiv -\rho\left(k_i - K\right)^2 - \left(1-\rho\right)\left(k_i - \theta\right)^2 + \rho\sigma_k^2, \tag{30}$$

$$u_{AP} \equiv u_{MS} - \rho\sigma_k^2. \tag{31}$$

As is standard, we assume $|\rho| < 1$ and note that when $\rho > 0$ ($\rho < 0$), players' actions are complements (substitutes). First, we establish players' equilibrium beliefs.

Proposition 7. *Suppose players incur the experienced utility penalty and the prior precision τ is sufficiently high (i.e., $2\psi\tau > \tau_e + (x-1)\tau_\eta$).*

(i) If $\rho > 0$, then there exists a unique symmetric equilibrium in which the optimal precision choices are given by (24) and $\chi = \frac{-\rho}{1-\rho}$.

(ii) Suppose $\rho < 0$ and $\overline{\delta} > 0$. Then, if $\psi > \psi_{bc}$, then the equilibrium solution is (24), where $\chi = \frac{-\rho}{1-\rho}$. If $\psi < \psi_{bc}$then an asymmetric equilibrium exists and is characterized by the quadruple $(\lambda, \delta_{e1}, \delta_{e2}, \delta_{s2})$ which solve a system of equations (specified in the Appendix).

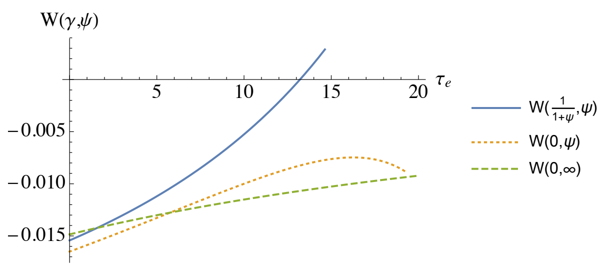
The above result follows from recognizing that in beauty contest games of strategic complementarity (i.e., $\rho > 0$), anticipatory utility is decreasing in non-fundamental volatility (i.e., $\chi = -\frac{\rho}{1-\rho} < 0$), while in games of strategic substitutability, the reverse is true. This arises, however, not because of the strategic nature of players' actions but because of the direct impact of aggregate volatility: it is straightforward to see that in these settings $u_{KK} = -u_{kK}$.[18] Intuitively, when tracking the aggregate action is beneficial, aggregate errors are costly, while when trying to distance your action from others, they are beneficial.

With strategic complementarity, players distort their perception of public information precision more than their perception of private information precision, i.e., $\delta_\eta > \delta_e$. Intuitively, this is beneficial because it increases players' perception of the equilibrium degree of coordination across player's actions. In contrast, and by similar logic, with strategic substitutability players choose to set $\delta_\eta < \delta_e$ in any symmetric equilibrium. This result is reminiscent of but distinct from the findings of Hellwig and Veldkamp (2009). Their analysis focuses on the optimal information *acquisition* in a closely-related setting. They show that when players' actions exhibit complementarity so does the information acquisition, irrespective of whether the signal is public or private. In our framework, when actions are strategic complements, players choose to *perceive* that both private and public signals are more precise than they actually are.

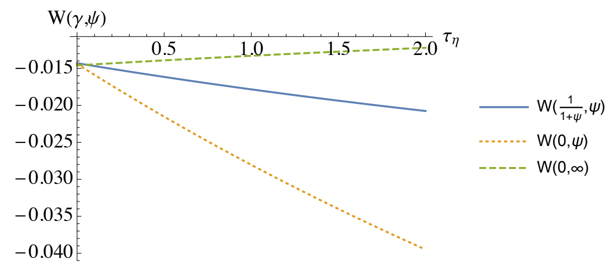
[18]The direct impact of non-fundamental volatility, u_{KK}, is simply $-\rho$, and the benefit of non-fundamental volatility which arises from increased covariance with the actions of others (i.e., $u_{kk}r^2$) is ρ^2. As a result, given that $|\rho| < 1$, the effect of u_{KK} always dominates.

Figure 5: Social Value of Information

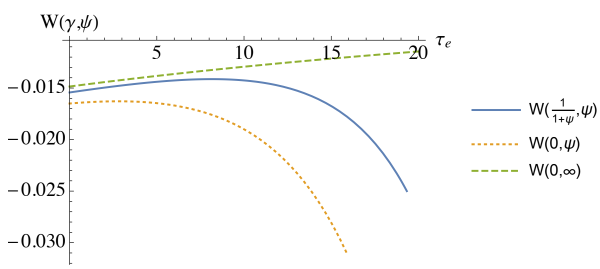
The figure plots the social planner's objective $W^0(\gamma,\psi)$ as a function of private information precision τ_e and public information precision τ_η, where γ reflects the social planner's weight on aggregate anticipatory utility and ψ parametrizes the players' cost of distorting subjective beliefs. The solid line plots $W\left(\gamma=\frac{1}{1+\psi},\psi\right)$, which accounts for the aggregate anticipatory utility in the subjective beliefs equilibrium; the dashed line plots $W(\gamma=0,\psi)$ which measures the aggregate experienced utility in the subjective beliefs equilibrium; and the dotted line plots $W(\gamma=0,\psi=\infty)$, which measures aggregate experienced utility in the rational expectations equilibrium. Unless mentioned, the other parameters are $\tau=10$, $\tau_e=1$, $\tau_s=0.1$, $\rho=0.85$ and and $\psi=1$.



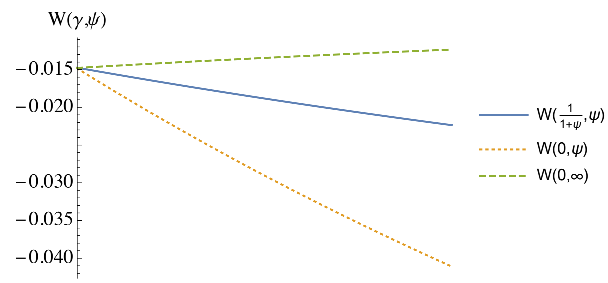
(a) W^0 versus τ_e, Morris-Shin



(b) W^0 versus τ_η, Morris-Shin



(c) W^0 versus τ_e, Angeletos-Pavan



(d) W^0 versus τ_η, Angeletos-Pavan

Figure 5 highlights the implications of subjective beliefs on the social value of information. The figure plots various welfare measures as functions of private and public signal precision (τ_e and τ_η, respectively), under the payoff of Morris and Shin (2002) and Angeletos and Pavan (2004). Recall that the only difference between the two specifications is whether the payoff to coordination has social benefits (i.e., the impact of $u_{\sigma\sigma}$ on welfare), and so the players' choice of subjective beliefs is the same across the two specifications. The solid line plots the welfare measure W^0 when $\gamma=\frac{1}{1+\psi}$ — this corresponds to the benchmark when the social planner fully accounts for both the aggregate anticipatory utility and the objective cost of deviating from rational expectations. The dashed line corresponds to the welfare measure when $\gamma=0$, in which case the social planner only weighs the aggregate experienced utility under objective beliefs, but ignores the benefits of belief distortion due to anticipatory utility. Finally, the dotted line corresponds to the (objective) welfare measure in the rational expectations equilibrium (i.e., when $\psi \to \infty$). Consistent with standard intuition, welfare always increases with private signal precision (i.e., τ_e) under rational expectations. Moreover, for our parameter configuration, welfare also increases with public signal precision (i.e., τ_η).

As the plots illustrate, however, neither conclusion necessarily holds when players can choose their subjective beliefs. First, panels (a) and (c) imply that welfare can decrease in private information precision. An increase in private signal precision leads players to over-estimate the precision of both signals more, as detailed in Proposition 3 and the discussion which follows. When the *true* precision of the private signal is not very high, an increase in *perceived* private precision can lead to an increase in dispersion.[19] Moreover, all else equal, the increase in perceived *public* precision can lead to an increase in non-fundamental volatility. When private information is sufficiently imprecise (relative to the prior, τ), these subjective beliefs can overwhelm the increase in welfare due to the objective reduction in fundamental uncertainty.

For our choice of parameters, welfare can decrease with private information precision when the social planner focuses solely on experienced utility (i.e., sets $\gamma = 0$) for both the Morris-Shin and Angeletos-Pavan specifications. Moreover, we find that for the Angeletos-Pavan specification, welfare can also decrease with more private information even when the social planner fully weighs the players' anticipatory utility (i.e., sets $\gamma = \frac{1}{1+\psi}$). The difference stems from the fact that under the Angeletos-Pavan specification, the welfare measure accounts for the social value of coordination. This exacerbates the negative impact of players' overconfidence in their private information.

Similarly, panels (b) and (d) illustrate how the provision of public information can reduce welfare under subjective belief choice, even when it improves welfare under rational expectations. As with private information, an increase in the public signal precision leads to overestimation of both signals' precision, with analogous harmful welfare effects through dispersion and non-fundamental volatility. As a result, both welfare measures in the subjective beliefs equilibrium (i.e., $W(\gamma = 0, \psi)$ and $W\left(\gamma = \frac{1}{1+\psi}, \psi\right)$ can decrease with public information precision, τ_η, for both the Morris-Shin and Angeletos-Pavan specifications.

These results clearly demonstrate that understanding the welfare implications of information provision requires knowledge not only of how players utilize the information but also how they choose to perceive it. To better understand how players' subjective beliefs affect aggregate well-being, we characterize the welfare-maximizing beliefs.

[19]Under rational expectations, the welfare benefit of a reduction in fundamental uncertainty that follows from private information provision always outweighs the (potentially) negative impact of increased dispersion. In our setting, this need not be the case, since the increase in dispersion reflects not only the quality of the information but also players' subjective beliefs.

Proposition 8. *(1) When payoffs to player i are given by $u_i = u_{AP}$, then $W(\gamma = 0, \psi)$ is maximized when $\delta_e = \delta_\eta = 1$ and $W\left(\gamma = \frac{1}{1+\psi}, \psi\right)$ is maximized when*

$$\delta_e = \delta_\eta = \frac{2\tau\psi + \tau}{2\tau\psi - \tau_e(1-\rho) - \tau_\eta}. \tag{32}$$

(2)When payoffs to player i are given by $u_i = u_{MS}$, then $W(\gamma = 0, \psi)$ is maximized when $\delta_\eta = 1$ and $\delta_e = \frac{1}{1-\rho}$ and $W\left(\gamma = \frac{1}{1+\psi}, \psi\right)$ is maximized when

$$\delta_e = \frac{\psi}{\psi(1-\rho) - \rho}\delta_\eta, \quad \delta_\eta = \frac{\tau(2\psi+1)(\psi(1-\rho)-\rho)}{\psi(2\tau\psi - \tau_e + \tau_\eta) + \rho(\psi(\tau_e - 2\tau(\psi+1)) + (\psi+1)\tau_\eta)}. \tag{33}$$

Recall that with the Angeletos and Pavan (2004) specification (i.e., $u_i = u_{AP}$), the payoffs to coordination accrue to the individuals and at the aggregate level. For instance, when $\rho > 0$, coordination is individually and socially beneficial. In this case, aggregate realized utility is maximized when individual realized utility is. But this implies that when the social planner ignores the impact of anticipatory utility (i.e., $W(\gamma = 0, \psi)$) welfare is maximized under rational expectations (i.e., when $\delta_e = \delta_\eta = 1$). This symmetry is preserved under $W\left(\gamma = \frac{1}{1+\psi}, \psi\right)$; however, even though the social planner accounts for players' anticipatory utility, when $\rho \neq 0$,welfare-maximizing and equilibrium beliefs do not align. In this case, players fail to internalize how their subjective beliefs affect others. For instance, one can show that players' equilibrium subjective beliefs about the precision of public and private information exceed what is socially optimal (from equation (32)) when $\psi = 1$ and $\rho > 0$.

In contrast, under the Morris-Shin specification (i.e., $u_i = u_{MS}$) , the payoff to coordination accrues only to players, but not at the aggregate level. Intuitively, while coordination is individually beneficial, it has no social benefit. As a result, equilibrium strategies that are optimal from an individual's perspective are not usually optimal from an aggregate perspective. For instance, when $\rho > 0$, strategic complementarity induces players to "overweight" their public information relative to the social optimum. The above result suggests that distorting the *perceived* precision of signals may improve aggregate well-being, even when the social planner only accounts for objective, experienced utility (i.e., $W(\gamma = 0, \psi)$). Specifically, when $\rho > 0$, the over-weighting of the public information is partially undone if the players perceive private signals to be more precise than they are (i.e., if $\delta_e = \frac{1}{1-\rho} > 1$), because this tilts the players' actions back towards their private information. This tilt remains socially optimal under $W\left(\gamma = \frac{1}{1+\psi}, \psi\right)$; when $\rho > 0$, the welfare-maximizing δ_e always exceeds the corresponding δ_η. This effect is highlighted in panel (a) of Figure 5: under the setting of Morris and Shin (2002), welfare can be higher under subjective beliefs, even when the social planner accounts *only* for objective, experienced utility (i.e., $W(\gamma = 0, \psi) > W(\gamma = 0, \psi = \infty)$). In contrast, under the setting of Angeletos and Pavan (2004), $W(\gamma = 0, \psi)$ is necessarily lower, as shown in panel (c).

6.2 Endogenous disagreement

In Appendix B, we apply the results of our general framework across a number of applications. Specifically, we analyze subjective belief choice in a competitive economy with incomplete markets (Section B.1), a production economy in which investment exhibits complementarity across firms (Section B.2), Bertrand and Cournot competition in a canonical industrial organization setting (Section B.3) and finally, a setting with information spillovers between financial markets and the real economy (Section B.4).

Across all these settings, we show that non-fundamental volatility can increase players' anticipatory utility, i.e., $\chi > 0$. For instance, in the setting with investment complementarities (Section B.2), χ is positive because the strategic benefit of more common noise in the public signal (which facilitates more coordination in investment) can outweigh the perceived information loss. In addition to this strategic consideration, players can also benefit from volatility in the aggregate action. For example, in-the competitive setting with incomplete markets (Section B.1), we show that households benefit from volatility in aggregate production ($u_{KK} > 0$) since, in equilibrium, they are able to purchase more of the good at a lower price.

As a result, despite their differences, the choice of subjective beliefs share a number of similarities across all these settings. First, all symmetric equilibria feature more overconfidence in players' private information than in the public signal (i.e., $\delta_e > \delta_\eta$). Such symmetric equilibria arise whenever the cost of deviating from rational expectations is sufficiently high. Importantly, this relative overweighting of private information does not follow immediately from strategic substitutability in actions. For instance, relative to rational expectations, firms choose to align their action more closely with their private information when they compete on quantity (where actions are strategic substitutes). Somewhat surprisingly, however, our results imply that this also occurs when firms compete on price (when actions are strategic complements).

Second, our results imply that when the cost of deviating from rational expectations is relatively low, asymmetric equilibria arise in which some investors not only underweight the public signal relative to the private signal but also underweight it relative to the rational expectations benchmark. One interpretation of such equilibria in the setting with information spillovers is that technological innovation endogenously creates disagreement among financial market participants. This disagreement can lead to large trading volume even upon public announcements, consistent with Kandel and Pearson (1995). Hence, our mechanism can explain why technological revolutions are often associated with higher disagreement.

7 Conclusions

In a standard model of externalities (Angeletos and Pavan (2007)), we allow players to choose their perceived precision of both private and public information. While players always choose to exhibit overconfidence about private signals, we show that their subjective interpretation of public information, and the nature of equilibrium itself, depends on the utility cost of non-fundamental volatility. When non-fundamental volatility decreases utility, there exists a unique, symmetric equilibrium where players optimally choose to exhibit overconfidence in their public information. However, when non-fundamental volatility increases utility, such symmetric equilibria may not exist. Instead, players endogenously choose different interpretations of public information: some players choose to underreact to such information, while the rest overweight it.

Our model provides a formal foundation for how ex-ante homogeneous individuals who observe same public information can become increasingly polarized in their beliefs and actions. Such endogenous distortions in subjective beliefs have important implications for observables and policy. In contrast to rational expectations equilibria, we show that increasing the true precision of private information can *increase* non-fundamental volatility in the aggregate action, while increasing the true precision of public information can *increase* dispersion across individuals' actions. Since an increase in either aggregate observable can decrease welfare, our analysis suggests that regulators should be cautious when proposing increased provision of public or private information. Moreover, we show that a social planner may prefer that players distort their beliefs even when her welfare measure does not account for their anticipatory utility. Specifically, such belief distortions can reduce the externalities individuals impose on others in the beauty contest setting of Morris and Shin (2002), suggesting that debiasing individuals is not necessarily in policymakers' best interest.

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A Proofs

A.1 Proof of Lemma 1

Suppose player i conjectures that the average action has the form $K = \kappa_0 + \alpha\theta + \beta s$, where α and β are determined in equilibrium. Then,

$$\begin{aligned} k_i &= \kappa_0 + (r\alpha + (1-r)\kappa_1)\,\mathbb{E}_i[\theta|s_i, s] + r\beta s & (34)\\ &= \kappa_0 + (r\alpha + (1-r)\kappa_1)\,A_i s_i + ((r\alpha + (1-r)\kappa_1)\,B_i + r\beta)\,s. & (35)\end{aligned}$$

This implies that, in equilibrium, the average action (across all players) is given by:

$$K = \int_i k_i di = \kappa_0 + (r\alpha + (1-r)\kappa_1)\,A\theta + ((r\alpha + (1-r)\kappa_1)\,B_i + r\beta)\,s, \tag{36}$$

where $A \equiv \int_i A_i di$ and $B \equiv \int_i B_i di$ reflect the average weights players put on their private and public signals, respectively.[20] Matching terms we show that:

$$\alpha = (r\alpha + (1-r)\kappa_1)\,A, \quad \beta = ((r\alpha + (1-r)\kappa_1)\,B_i + r\beta)\,, \tag{38}$$

which can be solved to yield:

$$\alpha = \frac{A\,(1-r)\,\kappa_1}{1-rA}, \quad \beta = \frac{B}{1-rA}\kappa_1. \tag{39}$$

A.2 Proof of Proposition 1

Let

$$\mathcal{K} \equiv rK + (1-r)\,\kappa$$

denote the target action for each player. Note that we can express the utility as:

$$U(k, K, \theta, \sigma) = \begin{array}{l} U(\kappa, \kappa, \theta, 0) + U_k(\kappa, \kappa, \theta, 0)\cdot(k-\kappa) + U_K(\kappa, \kappa, \theta, 0)\cdot(K-\kappa) \\ +\frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}(k-\kappa)^2 + U_{KK}(K-\kappa)^2 + 2U_{kK}(k-\kappa)(K-\kappa)\right) \end{array} \tag{40}$$

[20] Implicitly, we are assuming that the law of large numbers implies:

$$\int_i A_i s_i di = \int_i A_i di \times \int_i s_i di = A \times \theta. \tag{37}$$

Specifically, this assumes there is no cross-sectional correlation between A_i and s_i, but this is valid because $(\delta_{e,i}, \delta_{\eta,i})$ are chosen before s_i (and s) are observed.

Note that $U_k(\kappa,\kappa,\theta,0) = u_k + u_{kk}\kappa + u_{k\theta}\theta + u_{kK}\kappa = 0$ and

$$(k-\kappa)^2 = (k-\mathcal{K})^2 + r^2(K-\kappa)^2 + 2r(k-\mathcal{K})(K-\kappa)$$

Let $\mathbb{E}_j[\cdot]$ denote expectations w.r.t. arbitrary beliefs - we will later plug in subjective and objective beliefs. Then,

$$\mathbb{E}_j[U(k,K,\theta,\sigma)] = \mathbb{E}_j\left[\begin{array}{c} U(\kappa,\kappa,\theta,0) + (U_K + rU_{kk}(k-\mathcal{K}) + U_{kK}(k-\kappa))\cdot(K-\kappa) \\ +\frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}(k-\mathcal{K})^2 + (U_{KK}+r^2U_{kk})(K-\kappa)^2\right) \end{array}\right] \tag{41}$$

Since $rU_{kk} = -U_{kK}$,we have that

$$rU_{kk}(k-\mathcal{K}) + U_{kK}(k-\kappa) = U_{kK}(\mathcal{K}-\kappa) = -r^2U_{kk}(K-\kappa)$$

which implies

$$\mathbb{E}_j[U(k,K,\theta,\sigma)] = \mathbb{E}_j\left[\begin{array}{c} U(\kappa,\kappa,\theta,0) + U_K(K-\kappa) \\ +\frac{1}{2}\left(U_{\sigma\sigma}\sigma^2 + U_{kk}(k-\mathcal{K})^2 + (U_{KK}-r^2U_{kk})(K-\kappa)^2\right) \end{array}\right] \tag{42}$$

Let $u_0 \equiv U(\kappa,\kappa,\theta,0)$, and $\mathcal{A} \equiv \frac{U_K(K-\kappa)}{\text{var}(\theta)}$. Then, anticipatory utility is given by:

$$AU_i = \mathbb{E}_i[u_i] = \begin{array}{c} \mathbb{E}[u_0] + \mathcal{A}\text{var}(\theta) + \frac{U_{\sigma\sigma}}{2}\sigma_i^2 \\ +\frac{1}{2}\left(U_{kk}\mathbb{E}_i\left[(k-\mathcal{K})^2\right] + (U_{KK}-r^2U_{kk})\mathbb{E}_i\left[(K-\kappa)^2\right]\right) \end{array} \tag{43}$$

Moreover,

$$\mathbb{E}_i\left[(k-\mathcal{K})^2\right] = \left(\frac{\kappa_1(1-r)}{1-rA}\right)^2 \text{var}_i[\theta|s_i,s]$$

$$\mathbb{E}_i\left[(K-\kappa)^2\right] = \left(\frac{\kappa_1(1-A-B)}{1-rA}\right)^2 \text{var}[\theta] + \left(\frac{\kappa_1 B}{1-rA}\right)^2 \text{var}_i[s|\theta]$$

Substituting all the above expressions into 43 and simplifying gives us

$$AU_i(\delta_{e,i},\delta_{\eta,i}) = \Gamma + \left(\frac{\kappa_1}{1-rA}\right)^2\left[u_{kk}(1-r)^2\text{var}_i[\theta|s_i,s] - \left(u_{KK}-r^2u_{kk}\right)B^2\text{var}_i[s|\theta]\right].$$

A.3 Proof of Corollary 1

Taking partial derivatives of equation 5, we can write

$$\frac{\partial AU(\delta_{e,i},\delta_{\eta,i})}{\partial\delta_{e,i}} \propto (\text{var}_i[\theta|s_i,s])^2\,\tau_e > 0$$

which implies that anticipatory utility is always increasing in $\delta_{e,i}$. Moreover,

$$\begin{aligned}\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{\eta,i}} &\propto \left[\left(\text{var}_i\left[\theta|s_i,s\right]\right)^2\tau_\eta-\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{B^2}{\delta_{\eta i}^2\tau_\eta}\right]\\ &\propto \left[\left(\text{var}_i\left[\theta|s_i,s\right]\right)^2-\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2}\right]\\ &= \left[\left(\text{var}_i\left[\theta|s_i,s\right]\right)^2-\chi\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2}\right]\end{aligned} \tag{44}$$

If $\chi\leq 0$, then anticipatory utility is always increasing in $\delta_{\eta,i}$. If $\chi>0$, then anticipatory utility decreases in $\delta_{\eta,i}$ for low values of $\delta_{\eta,i}$and increases for high values of $\delta_{\eta,i}$ i.e., anticipatory utility is U-shaped in $\delta_{\eta,i}$.

A.4 Proof of proposition 2

Corollary 1 argues that anticipatory utility is monotonic in δ_{ei} which implies $\delta_e\geq 1$. Moreover, Equation 44 implies

$$\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{\eta,i}} \propto \left[\left(\text{var}_i\left[\theta|s_i,s\right]\right)^2-\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{B^2}{\delta_{\eta i}^2\tau_\eta^2}\right].$$

In a symmetric equilibrium,

$$B=\bar{\delta_\eta}\bar{\text{var}}\left[\theta|s_i,s\right]\tau_\eta.$$

Using this equation, we can write

$$\frac{\partial AU\left(\delta_{e,i},\delta_{\eta,i}\right)}{\partial\delta_{\eta,i}}>0 \iff \frac{\left(\text{var}_i\left[\theta|s_i,s\right]\right)}{\bar{\text{var}}\left[\theta|s_i,s\right]}>\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}\frac{\left(\bar{\delta_\eta}\right)^2}{\delta_{\eta,i}^2}$$

In any symmetric equilibrium, this inequality holds if

$$\underbrace{\frac{\left(U_{KK}-U_{kk}r^2\right)}{-U_{kk}\left(1-r\right)^2}}_{\equiv\chi}<1 \tag{45}$$

which translates to $U_{kk}+U_{KK}+2U_{kK}<0$, which we assume is always true. This condition guarantees that the first-best allocation is unique and bounded. This implies that, in any symmetric equilibrium, $\delta_\eta > 1$. Note that

$$\frac{\partial AU}{\partial A_i} = -\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\begin{array}{c}\left(1-r\right)\left(U_{kk}\left(1+r-2rA\right)+2\left(1-A\right)U_{kK}\right)\\ +\frac{B^2\left(r^2U_{kk}+2rU_{kK}+U_{KK}\right)}{B_i}\end{array}\right) \tag{46}$$

$$= -\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\frac{\left(U_{kk}+U_{kK}\right)^2}{U_{kk}}+\left(\frac{U_{kk}U_{KK}-U_{kK}^2}{U_{kk}}\right)\frac{B^2}{B_i}\right) \tag{47}$$

$$\frac{\partial AU}{\partial B_i} = -\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\begin{array}{c}\left(1-r\right)\left(U_{kk}\left(1+r-2rA\right)+2\left(1-A\right)U_{kK}\right)\\ +\frac{B^2\left(r^2U_{kk}+2rU_{kK}+U_{KK}\right)}{B_i^2}\left(1-A_i\right)\end{array}\right) \tag{48}$$

$$= -\frac{\kappa_1^2}{2\left(1-rA\right)^2\tau}\left(\frac{\left(U_{kk}+U_{kK}\right)^2}{U_{kk}}+\left(\frac{U_{kk}U_{KK}-U_{kK}^2}{U_{kk}}\right)\frac{B^2}{B_i^2}\left(1-A_i\right)\right) \tag{49}$$

Note that if $U_{kk}U_{KK}-U_{kK}^2 > 0$, then the above implies we should have both $\frac{\partial AU}{\partial A_i} > 0$ and $\frac{\partial AU}{\partial B_i} > 0$, since $U_{kk} < 0$.

A.5 Proof of Proposition 3

Equation (42) implies that, for any given beliefs.

$$\mathbb{E}_j\left[U\left(k,K,\theta,\sigma\right)\right] = \mathbb{E}_j\left[\begin{array}{c}U\left(\kappa,\kappa,\theta,0\right)+U_K\left(K-\kappa\right)\\ +\frac{1}{2}\left(U_{\sigma\sigma}\sigma^2+U_{kk}\left(k-\mathcal{K}\right)^2+\left(U_{KK}-r^2U_{kk}\right)\left(K-\kappa\right)^2\right)\end{array}\right] \tag{50}$$

Using this for objective beliefs, we get

$$OU_i\left(\delta_{ei},\delta_{\eta,i}\right) = \mathbb{E}\left[U\left(k_i\left(\delta_{ei},\delta_{\eta,i}\right),K,\theta,\sigma\right)\right].$$

Note that

$$\mathbb{E}\left[\left(k-\mathcal{K}\right)^2\right] = \left(\frac{\kappa_1\left(1-r\right)}{1-rA}\right)^2\left(\frac{\tau+\delta_{ei}^2\tau_e+\delta_{\eta i}^2\tau_\eta}{\left(\tau+\delta_{ei}\tau_e+\delta_{\eta i}\tau_\eta\right)^2}\right)$$

which implies

$$OU_i\left(\delta_{ei},\delta_{\eta,i}\right) = L+\left(\frac{\kappa_1\left(1-r\right)}{1-rA}\right)^2\left(\frac{\tau+\delta_{ei}^2\tau_e+\delta_{\eta i}^2\tau_\eta}{\left(\tau+\delta_{ei}\tau_e+\delta_{\eta i}\tau_\eta\right)^2}\right).$$

The objective of player i is

$$\max_{\delta_{ei},\delta_{\eta i}} AU_i+\psi OU_i$$

The FOC with respect to $\delta_{ei}$is

$$1-2\psi\delta_{e,i}+\frac{2\psi\left(\tau+\delta_{e,i}^{2}\tau_{e}+\delta_{\eta i}^{2}\tau_{\eta}\right)}{\tau+\delta_{ei}\tau_{e}+\delta_{\eta i}\tau_{\eta}}=0 \tag{51}$$

The FOC with respect to $\delta_{s,i}$ for any interior equilibrium is

$$1-\chi\frac{B^{2}}{\delta_{\eta i}^{2}\tau_{\eta}^{2}\upsilon_{i}^{2}}-2\psi\delta_{\eta i}+\frac{2\psi\left(\tau+\delta_{e,i}^{2}\tau_{e}+\delta_{\eta i}^{2}\tau_{\eta}\right)}{\tau+\delta_{ei}\tau_{e}+\delta_{\eta i}\tau_{\eta}}=0 \tag{52}$$

In a symmetric equilibrium,$\delta_{\eta,i}=\delta_{\eta}\forall i$ which implies

$$B=\delta_{\eta,i}^{2}\tau_{\eta}^{2}\upsilon_{i}^{2}$$

and equation (52) simplifies to

$$1-\chi-2\psi\delta_{\eta i}+\frac{2\psi\left(\tau+\delta_{e,i}^{2}\tau_{e}+\delta_{\eta i}^{2}\tau_{\eta}\right)}{\tau+\delta_{ei}\tau_{e}+\delta_{\eta i}\tau_{\eta}}=0$$

Solving the FOCs gives us

$$\delta_{e}=\frac{\chi\left(\chi-1\right)\tau_{s}+2\psi\left(2\psi+1\right)\tau}{2\psi\left(2\psi\tau-\tau_{e}+\left(\chi-1\right)\tau_{\eta}\right)} \tag{53}$$

$$\delta_{\eta}=\frac{\tau_{e}\chi+2\psi\tau\left(2\psi-\left(\chi-1\right)\right)}{2\psi\left(2\psi\tau-\tau_{e}+\left(\chi-1\right)\tau_{\eta}\right)} \tag{54}$$

which implies:

$$\delta_{e}=\delta_{\eta}+\frac{\chi}{2\psi} \tag{55}$$

These are indeed the equilibrium δ_e and δ_η when the denominator is positive i.e., $\left(2\psi\tau-\tau_{e}+\left(\chi-1\right)\tau_{\eta}\right)>0$. If this condition is not satisfied, then players will choose $\delta_e,\delta_\eta\to\infty$. In this case, the optimal weights A and B are given by

$$A_{i}=\frac{\tau_{e}}{\tau_{e}+\tau_{\eta}}\qquad B_{i}=\frac{\tau_{\eta}}{\tau_{e}+\tau_{\eta}}.$$

Next, we check the second order conditions. Denote $\upsilon_{i}=\frac{1}{\tau+\delta_{e,i}\tau_{e}+\delta_{\eta,i}\tau_{\eta}}$. Recall that

$$TU=-\upsilon_{i}+x\frac{B^{2}}{\delta_{\eta i}\tau_{\eta}}-\psi\nu_{i}^{2}\left(\tau+\delta_{ei}^{2}\tau_{e}+\delta_{\eta i}^{2}\tau_{\eta}\right) \tag{56}$$

The SOC are given by:

$$TU_{ee} = -\frac{2\tau_e\Big(\psi\Big(\tau((3-2\delta_e)\tau_e+2\delta_\eta\tau_\eta)+\delta_\eta\tau_s(\delta_\eta(3\tau_e+\tau_\eta)-2\delta_e\tau_e)+\tau^2\Big)+\tau_e(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{57}$$

$$TU_{\eta\eta} = \frac{2B^2x}{\delta_\eta^3\tau_\eta} - \frac{2\tau_\eta\Big(\psi\Big((\delta_e\tau_e+\tau)^2+\delta_e\tau_e\tau_\eta(3\delta_e-2\delta_\eta)+\tau(3-2\delta_\eta)\tau_\eta\Big)+\tau_\eta(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{58}$$

$$TU_{e\eta} = -\frac{2\tau_e\tau_\eta\Big(\psi\delta_e^2\tau_e+\delta_e(\tau_e-2\psi\delta_\eta(\tau_e+\tau_\eta))+\tau\psi(-2\delta_e-2\delta_\eta+3)+\delta_\eta\tau_\eta(\psi\delta_\eta+1)+\tau\Big)}{(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{59}$$

Want to show that,

$$TU_{ee} < 0 \text{ or } TU_{\eta\eta} < 0 \tag{60}$$

$$\text{and } TU_{ee}TU_{\eta\eta} > TU_{\eta e}^2 \tag{61}$$

Note that imposing the equilibrium solutions gives us:

$$TU_{ee} = -\frac{8\psi^3\tau_e\left(\tau_e\left(\tau_e+\tau_\eta\right)-4\tau\psi\tau_e+4\tau\psi^2\left(\tau_\eta+\tau\right)\right)\left(\tau_e+\tau_\eta-\chi\tau_\eta-2\tau\psi\right)^2}{\left(4\tau\psi^2\left(\tau_e+\tau_\eta+\tau\right)+\chi^2\tau_e\tau_\eta\right)^3} \tag{62}$$

$$TU_{\eta\eta} = \frac{2\tau_\eta\Big(\psi\delta_\eta\Big(-(\delta_e\tau_e+\tau)^2+\delta_e\tau_e\tau_\eta(2\delta_\eta-3\delta_e)+\tau(2\delta_\eta-3)\tau_\eta\Big)+(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)(x(\delta_e\tau_e+\tau)+(\chi-1)\delta_\eta\tau_\eta)\Big)}{\delta_\eta(\delta_e\tau_e+\delta_\eta\tau_\eta+\tau)^4} \tag{63}$$

Note that

$$TU_{ee} < 0 \Leftrightarrow \tau_e\left(\tau_e+\tau_\eta\right)+4\tau\psi\left(\psi\left(\tau_\eta+\tau\right)-\tau_e\right) > 0. \tag{64}$$

Treating the above equation as a quadratic in ψ, it is easy to show that the determinant is negative, which implies that the above equations holds $\forall\psi$.

$$\Delta \equiv TU_{ee}TU_{\eta\eta} - TU_{\eta e}^2$$

$$= -\frac{64\psi^6\tau_e\tau_\eta(\tau_e+\tau_\eta-\chi\tau_\eta-2\tau\psi)^6\begin{pmatrix} -2\tau\chi^3\psi\tau_e\tau_\eta+\chi^3\tau_e^2\tau_\eta+12\tau\chi^2\psi^2\tau_e\tau_\eta+6\tau\chi^2\psi\tau_e\tau_\eta \\ -12\tau\chi\psi^2\tau_e\tau_\eta-16\tau^2\psi^4\tau_e-8\tau^2\psi^3\tau_e+24\tau^2\chi\psi^3\tau_e \\ -12\tau^2\chi\psi^2\tau_e-12\tau\chi\psi^2\tau_e^2-16\tau^2\psi^4\tau_\eta-8\tau^2\psi^3\tau_\eta \\ +24\tau^2\chi\psi^3\tau_\eta-16\tau^3\psi^4-8\tau^3\psi^3+24\tau^3\chi\psi^3 \end{pmatrix}}{(\chi\tau_e+4\tau\psi^2+2\tau\psi-2\tau\chi\psi)(\chi^2\tau_e\tau_\eta+4\tau\psi^2\tau_e+4\tau\psi^2\tau_\eta+4\tau^2\psi^2)^6}$$

since $\delta_\eta = \delta_e + \frac{\chi}{2\psi}$.

This implies:

$$\lim_{\psi\to\infty}\Delta = -\frac{64\psi^6\tau_e\tau_\eta\left(2\tau\psi\right)^6\left(-16\tau^2\psi^4\tau_e-16\tau^2\psi^4\tau_\eta-16\tau^3\psi^4\right)}{\left(4\tau\psi^2\right)\left(4\tau\psi^2\tau_e+4\tau\psi^2\tau_\eta+4\tau^2\psi^2\right)^6} > 0 \tag{65}$$

which implies that for ψ sufficiently large, the SOC holds.

A.6 Proof of Proposition 4

Denote $\upsilon_i = \frac{1}{\tau+\delta_{e,i}\tau_e+\delta_{\eta,i}\tau_\eta}$. The objective of player i is

$$\max_{\delta_{\eta,i},\delta_{e,i}} -\upsilon_i + \chi \frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi\nu_i^2 \left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right)$$

subject to $\delta_{e,i} \geq \underline{\delta}$ and $\delta_{\eta,i} \geq \underline{\delta}$. Let ω_e and ω_η denote the Lagrange multipliers for these inequalities. We define the Lagrangian $L = -\upsilon_i + \chi\frac{B^2}{\delta_{\eta,i}\tau_\eta} - \psi\nu_i^2\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right) + \omega_e\delta_{e,i} + \omega_\eta\delta_{\eta,i}$. The FOC with respect to $\delta_{ei}$for any equilibrium is

$$1 - 2\psi\delta_{e,i} + 2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right)\upsilon_i + \omega_e = 0 \tag{66}$$

The FOC with respect to $\delta_{\eta,i}$ for any equilibrium is

$$1 - \chi\frac{B^2}{\delta_{\eta,i}^2\tau_\eta^2\upsilon_i^2} - 2\psi\delta_{\eta,i} + 2\psi\left(\tau + \delta_{e,i}^2\tau_e + \delta_{\eta,i}^2\tau_\eta\right)\upsilon_i + \omega_\eta = 0 \tag{67}$$

- Step 1: Given any strategy $(\delta_{\eta,-i}, \delta_{e,-i})$, we first prove that agent i's solution is always unique. For this to be true, the above system of equations should always at-most one solution in $(\underline{\delta}, \infty) \times (\underline{\delta}, \infty)$. Substituting the first equation into second, we get

$$\underbrace{4\tau_\eta^3\left(2\psi\tau + \tau_e - \tau_\eta\right)\delta_\eta^{*2}\left(\delta_\eta^* + \frac{2\psi\tau - \tau_e}{2\psi\tau_\eta}\right)\left[\frac{\tau\left(1+2\psi\right)}{2\psi\tau + \tau_e - \tau_\eta} - \delta_\eta^*\right]}_{\equiv g\left(\delta_\eta^*\right)} = \underbrace{4B^2\chi\left(\delta_\eta^{*2}\tau_\eta\left(\tau_e+\tau_\eta\right) + 2\delta_\eta^*\tau_\eta\tau + \tau\tau_e + \tau^2\right)^2}_{\equiv f\left(\delta_\eta^*\right)}$$

  It is enough to show that the above set of equations will have at most 2 solutions. If so, one will be a minima and other will be a maxima. First, we note that $g(0) = g'(0) = 0, g''(0) > 0,$ and $g''' < 0$ while $f(0) > 0, f', f'', f''' > 0$. Let x^* be defined such that $g' > 0$ for $[0, x^*)$ and $g' < 0$ for (x^*, ∞). Finally, we denote $h\left(\delta_\eta^*\right) \equiv g\left(\delta_\eta^*\right) - f\left(\delta_\eta^*\right)$. and note that $h(0), h'(0) < 0$. There are two cases to consider.

  - Case 1: Suppose $g\left(\delta_\eta^*\right) < f\left(\delta_\eta^*\right)$ for all values of $\delta_\eta^* > 0$. Then there is no solution with $\delta_\eta^* > 0$.
  - Case 2: Suppose $g\left(\delta_\eta^*\right)$ is not less than $f\left(\delta_\eta^*\right)$ for all values of δ_η^*, i.e., there exists some x_1 such that $h(x_1) = 0$. Suppose that h'$(x_1) > 0$, so that for $x + e > x_1$ (where e can be arbitrarily small), $h(x+e) > 0$. That h'$(x_1) > 0$ implies that g'$(x_1) > f$'$(x_1) > 0$. Note that it must be the case that $x_1 < x^*$ (since $g'(x_1) > 0$) and there must exist at least one x_2 such that $x_1 < x_2 < x^*$ and h'$(x_2) = 0$ (since g' is continuous and g'$(x^*) = 0$). We want to show that only one such x_2 exists and that h'$(x) < 0$ for all $x > x_2$.

- * (1) If $g"(x_1) < f"(x_1)$, then this implies that $g"(x_1) < f"(x_1)$ for all $x > x_1$ since $g"' < 0$ and $f"' > 0$. But then this implies that $h'(x) = g'(x) - f'(x)$ is decreasing for all $x > x_1$. Thus, there exists only one $x_2 > x_1$ such that $h(x_2) = 0$ and $h'(x) < 0$ for all $x > x_2$.
  * (2) If $g"(x_1) > f"(x_1)$, then this implies that $h'(x)$ is increasing for at least some interval $[x_1, x_1 + u]$. However, note that since $g"' < 0$ and $f"' > 0$, that $g"(x) - f"(x)$ is decreasing and so there exists just one point, $x + u$, such that $g"(x_1 + u) = h"(x_1 + u)$. Furthermore, $g"(x) < f"(x)$ for all $x > x_1 + u$ and so $h'(x)$ is decreasing for all $x > x_1 + u$. But then this implies that there exists only one $x_2 > x_1 + u > x_1$ such that $h(x_2) = 0$ and $h'(x) < 0$ for all $x > x_2$.

- Step 2: Given any strategy $(\delta_{\eta,-i}, \delta_{e,-i})$, player i will also never choose $\delta_{\eta,i} = \infty$ which gives agent finite anticipatory utility at infinite cost. This implies that agent i either choose a interior point or $\delta_{\eta,i} = \underline{\delta}$. If a player chooses $\delta_{si} = \underline{\delta}$, she chooses$\delta_{e,i} = \delta_{e1}$ using the FOC 66 for any interior equilibrium can be rewritten as:

$$(\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta) - 2\psi\delta_{e1}(\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta) + 2\psi\left(\tau + \delta_{e1}^2\tau_e + \underline{\delta}^2\tau_\eta\right) = 0$$

which simplifies to

$$\delta_{e1} = \frac{\tau + \underline{\delta}\tau_\eta + 2\psi\left(\tau + \underline{\delta}^2\tau_\eta\right)}{2\psi(\tau + \underline{\delta}\tau_\eta) - \tau_e}. \tag{68}$$

Also, Corollary 1 shows that anticipatory utility is increasing in $\delta_{e,i}$ which implies that the optimal $\delta_{e,i} \geq 1$ implying that the constraint $\delta_{ei} \geq \underline{\delta}$ is never binding because $\underline{\delta} < 1$ by assumption. This implies that, the only possible asymmetric equilibrium is one in which agent is indifferent between $(\delta_{e1}, \underline{\delta})$ and the unique interior solution which solves both the FOCs. This implies that, for any asymmetric equilibrium, indifference equation is

$$-v_1 + \chi\frac{B^2}{\underline{\delta}\tau_\eta} - \psi\nu_1^2\left(\tau + \delta_{e1}^2\tau_e + \underline{\delta}^2\tau_\eta\right) = -v^* + \chi\frac{B^2}{\delta_\eta^*\tau_\eta} - \psi\nu^{*2}\left(\tau + \delta_e^{*2}\tau_e + \delta_\eta^{*2}\tau_\eta\right)$$

where $v_1 = \frac{1}{\tau + \delta_{e1}\tau_e + \underline{\delta}\tau_\eta}$ and $v^* = \frac{1}{\tau + \delta_e^*\tau_e + \delta_\eta^*\tau_\eta}$.

- Step 3: For high ψ, the only equilibrium is symmetric. For $\psi \to \infty$, the interior equilibrium solving 66 and 67 is $\delta_\eta^* = \delta_e^* = 1$ and in this equilibrium, agents choose not to incur any cost. If an asymmetric equilibrium exists, agents have to be indifferent between $(\delta_{e1}, \underline{\delta})$ and $(1, 1)$. If an agent chooses $(\delta_{e1}, \underline{\delta})$, the benefit of deviation i.e., anticipatory utility, is finite (since $\underline{\delta}$ is non-zero), but the cost is infinite since ψ is very large. Hence, agents cannot be indifferent and the only equilibrium possible is symmetric in which all agents choose $(1,1)$. Hence, by continuity, for ψ high enough, the only equilibrium is symmetric.

- Step 4: For $\psi \to 0$, there is no symmetric interior equilibrium. For $\psi \to 0$, the equilibrium solving 66 and 67 is $\delta_\eta^* = \delta_e^* = \infty$. Suppose all players choose $\delta_\eta^* = \delta_e^* = \infty$, player i will deviate and choose $(\delta_{e1},\underline{\delta})$ iff the utility she gets is higher with this deviation. Using equation 68, $\delta_{e1} = \infty$ when $\psi \to 0$. For interior symmetric equilibrium to hold, we need

$$-v_1 + \chi \frac{B^2}{\underline{\delta}\tau_\eta} - \psi \nu_1^2 \left(\tau + \delta_{e1}^2 \tau_e + \underline{\delta}^2 \tau_\eta\right) < -v^* + \chi \frac{B^2}{\delta_\eta^* \tau_\eta} - \psi \nu^{*2} \left(\tau + \delta_e^{*2} \tau_e + \delta_\eta^{*2} \tau_\eta\right).$$

In this case, $v^* = 0$, $v_1 = 0$ and $B = \frac{\tau_\eta}{\tau_\eta + \tau_e}$. The above equation translates into

$$\chi \frac{1}{\underline{\delta}\tau_\eta} \left(\frac{\tau_\eta}{\tau_\eta + \tau_e}\right)^2 < 0 \iff \chi < 0$$

which by assumption is not true. Hence a interior symmetric equilibrium is not possible. Conjecture that the equilibrium is asymmetric and a fraction λ of the agents choose $(\underline{\delta}, \delta_{e1} = \infty)$ and the remaining choose $\left(\delta_\eta^* = \infty, \delta_e^* = \infty\right)$. The fraction λ is pinned down by the indifference condition

$$-v_1 + \chi \frac{B^2}{\underline{\delta}\tau_\eta} - \psi \nu_1^2 \left(\tau + \delta_{e1}^2 \tau_e + \underline{\delta}^2 \tau_\eta\right) = -v^* + \chi \frac{B^2}{\delta_\eta^* \tau_\eta} - \psi \nu^{*2} \left(\tau + \delta_e^{*2} \tau_e + \delta_\eta^{*2} \tau_\eta\right).$$

which translates to $\lambda = 1$. This implies that an asymmetric equilibrium exists. Hence, for $\psi \to 0$, an asymmetric equilibrium exists.

- Step 5: For low ψ, an asymmetric equilibrium exists. Conjecture an asymmetric equilibrium in which a fraction λ of the agents choose $(\delta_{e1}, \underline{\delta})$ and the remaining choose $\left(\delta_e^*, \delta_\eta^*\right)$. Suppose $2\psi\left(\tau + \underline{\delta}\tau_\eta\right) - \tau_e > 0$. In this equilibrium, the following equations have to hold:

$$1 - 2\psi\delta_e^* + 2\psi \frac{\tau + \left(\delta_e^*\right)^2 \tau_e + \left(\delta_\eta^*\right)^2 \tau_\eta}{\tau + \delta_e^* \tau_e + \delta_\eta^* \tau_\eta} = 0$$

$$1 - \chi \frac{B^2 \left(\tau + \delta_e^* \tau_e + \delta_\eta^* \tau_\eta\right)^2}{\left(\delta_\eta^*\right)^2 \tau_\eta^2} - 2\psi\delta_s^* + 2\psi \frac{\tau + \left(\delta_e^*\right)^2 \tau_e + \left(\delta_\eta^*\right)^2 \tau_\eta}{\tau + \delta_e^* \tau_e + \delta_\eta^* \tau_\eta} = 0$$

$$\delta_{e1} = \frac{\tau + \underline{\delta}\tau_\eta + 2\psi\left(\tau + \underline{\delta}^2\tau_\eta\right)}{2\psi\left(\tau + \underline{\delta}\tau_\eta\right) - \tau_e}$$

$$-\frac{1}{\tau + \delta_{e,1}\tau_e + \underline{\delta}\tau_\eta} + \chi\frac{B^2}{\underline{\delta}\tau_\eta} - \psi\frac{\tau + \delta_{e1}^2\tau_e + \underline{\delta}^2\tau_\eta}{\left(\tau + \delta_{e,1}\tau_e + \underline{\delta}\tau_\eta\right)^2} = -\frac{1}{\tau + \delta_e^*\tau_e + \delta_\eta^*\tau_\eta} + \chi\frac{B^2}{\delta_\eta^*\tau_\eta} - \psi\frac{\tau + \delta_e^{*2}\tau_e + \left(\delta_\eta^*\right)^2\tau_\eta}{\left(\tau + \delta_e^*\tau_e + \delta_\eta^*\tau_\eta\right)^2}$$

Substituting the first equation into second, we get

$$4\tau_\eta^3\left(2\psi\tau + \tau_e - \tau_\eta\right)\delta_\eta^{*2}\left(\delta_\eta^* + \frac{2\psi\tau - \tau_e}{2\psi\tau_\eta}\right)\left(\frac{\tau\left(1 + 2\psi\right)}{2\psi\tau + \tau_e - \tau_\eta} - \delta_\eta^*\right) = 4B^2x\left(\delta_\eta^{*2}\tau_\eta\left(\tau_e + \tau_\eta\right) + 2\delta_\eta^*\tau_\eta\tau + \tau\tau_e + \tau^2\right)^2 \tag{69}$$

In step 1, we proved that above equation will have at most 2 solutions in $\delta_\eta^* \in (0, \infty)$. If so, the lower one will be minima and the higher one will be maxima. We are interested in the higher one. It is easy to see that, as B increases, the optimal δ_η^* decreases. Moreover, substituting optimal δ_{e1} and δ_e^* into the indifference condition, it reduces to

$$\frac{\tau_e - 4\psi\left(\underline{\delta}\tau_\eta\left(\psi\underline{\delta} + 1\right) + \tau(\psi + 1)\right)}{4\psi\left(\left(\underline{\delta}\tau_\eta + \tau\right)^2 + \tau_e\left(\underline{\delta}^2\tau_\eta + \tau\right)\right)} + \chi\frac{B^2}{\underline{\delta}\tau_\eta} = \frac{\tau_e - 4\psi\left(\delta_\eta^*\tau_\eta\left(\psi\delta_\eta^* + 1\right) + \tau\left(\psi + 1\right)\right)}{4\psi\left(\left(\delta_\eta^*\tau_\eta + \tau\right)^2 + \tau_e\left(\delta_\eta^{*2}\tau_\eta + \tau\right)\right)} + \chi\frac{B^2}{\delta_\eta^*\tau_\eta} \tag{70}$$

Solving for an asymmetric equilibrium finally boils down to solving 69 and 70 for B and δ_η^*. Suppose ψ is low enough such that $2\psi\left(\tau + \underline{\delta}\tau_\eta\right) - \tau_e < 0$. In this case, $\delta_{e1} \to \infty$ and $\delta_e^*, \delta_\eta^* \to \infty$ which implies $B = (1 - \lambda)\frac{\tau_\eta}{\tau_e + \tau_\eta}$. The indifference equation reduces to

$$\chi\frac{B^2}{\underline{\delta}\tau_\eta} - \frac{\psi}{\tau_e} = -\frac{\psi}{\tau_e + \tau_\eta}$$

which implies that

$$(1 - \lambda)^2 = \psi\underline{\delta}\frac{\tau_e + \tau_\eta}{\chi\tau_e}$$

which implies that, for low ψ , $\lambda \in (0, 1)$ and an asymmetric equilibrium exists.

A.7 Proof of Lemma 2

Note that dispersion in actions is given by

$$\begin{aligned}\sigma_i^2 &= \frac{\left(\alpha_i + \beta_i - \alpha - \beta\right)^2}{\tau} + \frac{\alpha_i^2}{\tau_e} + \frac{\left(\beta_i - \beta\right)^2}{\tau_\eta} \\ &= \frac{\kappa_1^2(r-1)^2\left(\tau_\eta\left(\tau_e\left(-A_i + A - B_i + B\right)^2 + \tau A_i^2\right) + \tau\left(B - B_i\right)^2\tau_e\right)}{\tau\tau_e\tau_\eta\left(1 - rA\right)^2}\end{aligned}$$

In a symmetric equilibrium, $A_i = A$ and $B_i = B$ so that:

$$\sigma_i^2 = \frac{\kappa_1^2(r-1)^2\tau_e\left(2\psi(2\psi + 1)\tau + (\chi - 1)\chi\tau_\eta\right)^2}{\left(-2\psi r\tau\tau_e + 4\psi^2\tau\left(-r\tau_e + \tau_e + \tau_\eta + \tau\right) + \chi\tau_e\left(-r\chi + r + \chi\right)\tau_\eta\right)^2} \tag{71}$$

Note that non-fundamental volatility is given by

$$\nu^2 = \frac{(\alpha+\beta-\kappa_1)^2}{\tau} + \frac{\beta^2}{\tau_\eta} \tag{72}$$
$$= \frac{\kappa_1^2\left((A+B-1)^2\tau_\eta + B^2\tau\right)}{\tau\tau_\eta\left(1-rA\right)^2} \tag{73}$$
$$= \frac{\kappa_1^2\left(-16\psi^3\tau^2\tau_e+\chi^2\tau_e^2\tau_\eta+4\psi^2\tau\left(2\tau_e\tau_\eta+\tau_e^2+(\chi-1)^2\tau_\eta(\tau_\eta+\tau)\right)-4\psi\tau(\chi-1)\chi\tau_e\tau_\eta+16\psi^4\tau^2(\tau_\eta+\tau)\right)}{(-2\psi r\tau\tau_e+4\psi^2\tau(\tau_e(1-r)+\tau_\eta+\tau)+\chi\tau_e(r+\chi-r\chi)\tau_\eta)^2} \tag{74}$$

For the rational expectations equilibrium ($\psi \to \infty$), we have:

$$\lim_{\psi\to\infty}\frac{\partial\nu^2}{\partial\tau_e} = -\frac{2\kappa_1^2\left(1-r\right)\left(\tau_\eta+\tau\right)}{\left(\tau_e\left(1-r\right)+\tau_\eta+\tau\right)^3} \leq 0.$$

$$\lim_{\psi\to\infty}\frac{\partial\sigma_i^2}{\partial\tau_\eta} = -\frac{2\kappa_1^2(r-1)^2\tau_e}{\left(\tau_e\left(1-r\right)+\tau_\eta+\tau\right)^3} \leq 0$$

A.8 Proof of Proposition 5

Note that dispersion of actions in a symmetric equilibrium is given by

$$\sigma_i^2 = \frac{\kappa_1^2(r-1)^2\tau_e\left(2\psi(2\psi+1)\tau+(\chi-1)\chi\tau_s\right)^2}{\left(-2\psi r\tau\tau_e+4\psi^2\tau\left(-r\tau_e+\tau_e+\tau_\eta+\tau\right)+\chi\tau_e(-r\chi+r+\chi)\tau_\eta\right)^2}$$

Assume $\psi = 1$. For the subjective expectations equilibrium, we have:

$$\frac{\partial\sigma_i^2}{\partial\tau_\eta} = \frac{4\kappa_1^2(r-1)^2\tau(\chi+2)\tau_e\left(2\tau(\chi-3)-\chi\tau_e\right)\left((\chi-1)\chi\tau_\eta+6\tau\right)}{\left(\chi\tau_e(-r\chi+r+\chi)\tau_\eta+(4-6r)\tau\tau_e+4\tau\tau_\eta+4\tau^2\right)^3}$$

This implies:

$$\frac{\partial\sigma_i^2}{\partial\tau_\eta} \geq 0 \Leftrightarrow \chi \leq -2 \tag{75}$$

This implies that when χ is sufficiently negative (and δ's are well-defined), increasing public precision can **increase** dispersion.

Note that non-fundamental volatility is given by

$$\nu^2 = \frac{\kappa_1^2\left(-16\psi^3\tau^2\tau_e+\chi^2\tau_e^2\tau_\eta+4\psi^2\tau\left(2\tau_e\tau_\eta+\tau_e^2+(\chi-1)^2\tau_\eta\left(\tau_\eta+\tau\right)\right)-4\psi\tau(\chi-1)\chi\tau_e\tau_\eta+16\psi^4\tau^2\left(\tau_\eta+\tau\right)\right)}{\left(-2\psi r\tau\tau_e+4\psi^2\tau\left(\tau_e\left(1-r\right)+\tau_\eta+\tau\right)+\chi\tau_e\left(r+\chi-r\chi\right)\tau_\eta\right)^2}.$$

We can show that

$$\frac{\partial\nu^2}{\partial\tau_e} \geq 0 \Leftrightarrow \frac{-\chi^2\tau_e\tau_\eta-4\tau\tau_e+8\tau\tau_\eta+2\chi^2\tau_\eta^2+2\tau\chi^2\tau_\eta-2\chi\tau_\eta^2-2\tau\chi\tau_\eta+8\tau^2}{2\tau_e\tau_\eta-\chi^2\tau_e\tau_\eta+\chi\tau_e\tau_\eta-4\tau\tau_e+2\tau_\eta^2+10\tau\tau_\eta+2\chi^2\tau_\eta^2+2\tau\chi^2\tau_\eta-4\chi\tau_\eta^2-4\tau\chi\tau_\eta+8\tau^2} < r < 1$$

This implies that, for r high enough, increasing private information can increase non-fundamental volatility.

A.9 Proof of Proposition 6

Forecast revision of player i is

$$FR_i \equiv \mathbb{E}_i\left[\theta|s_i, s\right] - \mathbb{E}_i\left[\theta\right], \tag{76}$$

while his forecast error is

$$FE_i \equiv \theta - \mathbb{E}_i\left[\theta|s_i, s\right]. \tag{77}$$

At the aggregate level, the regression coefficient between ex-post mean forecast errors and ex-ante mean forecast revisions is

$$CG_a \propto \mathrm{Cov}\left(\mathbb{E}\left(\theta - \mathbb{E}_i\left[\theta|s_i,s\right]\right), \mathbb{E}\left(\mathbb{E}_i\left[\theta|s_i,s\right] - \mathbb{E}_i\left[\theta\right]\right)\right) \propto \frac{\left(1-A-B\right)\left(A+B\right)}{\tau} - \frac{B^2}{\tau_s}.$$

At the individual level, the regression coefficient between ex-post forecast errors and ex-ante forecast revisions is

$$CG_i \propto \mathrm{Cov}\left(\theta - \mathbb{E}_i\left[\theta|s_i,s\right], \mathbb{E}_i\left[\theta|s_i,s\right] - \mathbb{E}_i\left[\theta\right]\right) = \frac{\left(1-A_i-B_i\right)\left(A_i+B_i\right)}{\tau} - \frac{A_i^2}{\tau_e} - \frac{B_i^2}{\tau_s}$$

(i) When ψ is sufficiently high, we will always have symmetric equilibrium and the optimal beliefs are given by 24. Substituting in the regression coefficient above, we get

$$\begin{aligned} CG_a &\propto \frac{\delta_e\tau_e + \delta_s\tau_s - \delta_s^2\tau_s}{\left(\tau + \delta_e\tau_e + \delta_s\tau_s\right)^2} \\ &= \frac{2\psi\tau_e\,\big((\chi-1)\chi^2\tau_s^2 + 2\tau\tau_s(\chi+2\psi)((\chi-2)\psi+\chi-1) + 4\tau^2(2\psi+1)\psi^2\big) - (2\psi+1)\tau_e^2\left(\chi^2\tau_s + 4\tau\psi^2\right) - 4\tau(\chi-1)\psi^2\tau_s}{\left(\tau_e\left(\chi^2\tau_s + 4\tau\psi^2\right) + 4\tau\psi^2\left(\tau_s+\tau\right)\right)^2} \end{aligned}$$

Taking the limit as $\psi \to \infty$, we get

$$\lim_{\psi\to\infty} CG_a = \frac{\tau_e}{\left(\tau_e + \tau_s + \tau\right)^2} > 0$$

which implies that the regression coefficient is positive. At the individual level, the regression coefficient is

$$CG_i \propto \frac{\delta_e\left(1-\delta_e\right)\tau_e + \delta_s\left(1-\delta_s\right)\tau_s}{\left(\tau + \delta_e\tau_e + \delta_s\tau_s\right)^2}$$

In a symmetric equilibrium, $\delta_e, \delta_s > 1$ which implies that CG_i is negative.

(ii) When ψ is low and $\chi < 0$, we will always have symmetric equilibrium and the optimal beliefs are given by 24. Taking the limit as $\psi \to 0$, we get

$$\lim_{\psi\to 0} CG_a = \lim_{\psi\to 0} \frac{\delta_e\tau_e + \delta_s\tau_s - \delta_s^2\tau_s}{\left(\tau + \delta_e\tau_e + \delta_s\tau_s\right)^2} = -\frac{\tau_s}{\left(\tau_e + \tau_s\right)^2}$$

which implies that the regression coefficient is negative. At the individual level, the regression coefficient is

$$CG_i \propto \frac{\delta_e (1-\delta_e) \tau_e + \delta_s (1-\delta_s) \tau_s}{(\tau + \delta_e \tau_e + \delta_s \tau_s)^2}$$

In a symmetric equilibrium, $\delta_e, \delta_s > 1$ which implies that CG_i is negative.

(iii) When ψ is low and $\chi > 0$, the equilibrium is asymmetric (by proposition 4). This implies that some players overreact to information while others may underreact to it.

A.10 Proof of Proposition 7

Note that, in both utility specifications u_{MS} and u_{AP}, $r = \rho$ and χ is given by

$$\chi = \frac{u_{KK} - u_{kk} r^2}{-u_{kk} (1-r)^2} = -\frac{r}{1-r}.$$

The nature of the equilibrium comes directly from propositions (4) and (3).

A.11 Proof of Proposition 8

Let $\xi_i \equiv k_i - \kappa$ and $\xi \equiv K - k$. Note that $\mathbb{E}_j [\xi_i] = \mathbb{E}_j [\xi] = 0$. Let $u_0 \equiv U(\kappa, \kappa, \theta, 0)$, and $\mathcal{A} \equiv (u_{K\theta} + (u_{KK} + u_{kK}) \kappa_1)(\alpha + \beta - \kappa_1)$. Then,

1. Anticipatory utility is given by:

$$AU_i = \mathbb{E}_i [u_i] = \begin{array}{c} \mathbb{E}_i [u_0] + \mathcal{A} \mathrm{var}_i (\theta) + \frac{U_{\sigma\sigma}}{2} \sigma_i^2 \\ + \frac{1}{2} (U_{kk} \mathrm{var}_i (\xi_i) + U_{KK} \mathrm{var}_i (\xi) + 2U_{kK} \mathrm{cov}_i (\xi_i, \xi)) \end{array} \tag{78}$$

2. Objective utility is given by:

$$OU_i = \mathbb{E} [u_i] = \begin{array}{c} \mathbb{E} [u_0] + \mathcal{A} \mathrm{var} (\theta) + \frac{U_{\sigma\sigma}}{2} \sigma_i^2 \\ + \frac{1}{2} (U_{kk} \mathrm{var} (\xi_i) + U_{KK} \mathrm{var} (\xi) + 2U_{kK} \mathrm{cov} (\xi_i, \xi)) \end{array} \tag{79}$$

After plugging in all the variances and covariances into the above expressions and simplifying, we can write welfare as

$$W \equiv \int_i \gamma AU_i + (1-\gamma) OU_i di \tag{80}$$

$$= \begin{array}{c} \left(\mathbb{E} [u_0] + \mathcal{A} \mathrm{var} (\theta) + \frac{U_{\sigma\sigma}}{2} \sigma_i^2 + \frac{(U_{kk} + U_{KK} + 2U_{kK})}{2} \frac{(\alpha+\beta-\kappa_1)^2}{\tau} \right) \\ + \frac{\gamma}{2} \left(U_{kk} \left(\frac{\alpha^2}{\delta_e \tau_e} (1 + \psi \delta_e) \right) + (U_{kk} + U_{KK} + 2U_{kK}) \left(\frac{\beta^2}{\delta_\eta \tau_\eta} (1 + \psi \delta_\eta) \right) \right) \end{array} \tag{81}$$

In the Morris and Shin (2002) utility specification, $U_{kk} = -1$, $U_{KK} = -\rho$, $U_{k\theta} = 1 - \rho$, $U_{kK} = \rho$,$U_{\sigma\sigma} = \rho$ which implies that $\kappa_1 = 1$, $\mathcal{A} = 0$, $\chi = \frac{-\rho}{1-\rho}$, $\alpha = \frac{A(1-\rho)}{1-\rho A}$, $\beta = \frac{B}{1-\rho A}$. This implies that welfare is given by

$$W_{MS}(\gamma,\psi)=\tfrac{1}{2}\frac{\alpha^2}{\tau_e}\left((\rho-1)+(1-\gamma)\frac{\delta_e-1}{\delta_e}\right)+\tfrac{(\rho-1)}{2}\frac{(\alpha+\beta-1)^2}{\tau}+\tfrac{(\rho-1)}{2}\frac{\beta^2}{\tau_\eta}\left(1-(1-\gamma)\frac{(\delta_\eta-1)}{\delta_\eta}\right).$$

$W_{MS}(\gamma=0,\psi)$ is maximized when $\delta_\eta=1$ and $\delta_e=\frac{1}{1-\rho}$ and $W\left(\gamma=\frac{1}{1+\psi},\psi\right)$ is maximized when

$$\delta_e=\frac{\psi}{\psi(1-\rho)-\rho}\delta_\eta,\quad \delta_\eta=\frac{\tau(2\psi+1)(\psi(1-\rho)-\rho)}{\psi(2\tau\psi-\tau_e+\tau_\eta)+\rho(\psi(\tau_e-2\tau(\psi+1))+(\psi+1)\tau_\eta)}. \tag{82}$$

In the Angeletos and Pavan (2004) utility specification, $U_{kk}=-1$, $U_{KK}=-\rho$, $U_{k\theta}=1-\rho$, $U_{kK}=\rho$,$U_{\sigma\sigma}=0$ which implies that welfare is given by

$$W_{AP}(\gamma,\psi)=\tfrac{1}{2}\frac{\alpha^2}{\tau_e}\left(-1+(1-\gamma)\frac{\delta_e-1}{\delta_e}\right)+\tfrac{(\rho-1)}{2}\frac{(\alpha+\beta-1)^2}{\tau}+\tfrac{(\rho-1)}{2}\frac{\beta^2}{\tau_\eta}\left(1-(1-\gamma)\frac{(\delta_\eta-1)}{\delta_\eta}\right).$$

$W_{AP}(\gamma=0,\psi)$ is maximized when $\delta_e=\delta_\eta=1$ and $W\left(\gamma=\frac{1}{1+\psi},\psi\right)$ is maximized when

$$\delta_e=\delta_\eta=\frac{2\tau\psi+\tau}{2\tau\psi-\tau_e(1-\rho)-\tau_\eta}. \tag{83}$$

B Appendix B: Alternative Applications

In this section, we show how the results can be put to work in a few applications of interest.

B.1 Efficient Competitive Economies

Consider an incomplete-market competitive economy in which agents' choices are strategic substitutes. There are two goods and a continuum of households (who act as consumers and producers). q_{1i} and q_{2i} are the quantities of each good purchased by consumer i, and his preferences are

$$u_i=\theta q_{1i}-\frac{bq_{1i}^2}{2}+q_{2i}$$

while his budget constraint is

$$pq_{1i}+q_{2i}=e+\pi_i$$

where θ is a relative demand shock, p is the price of good one and, good two serves as the numeraire, e is an exogenous endowment of good two, and π_i are the profits of producer i. Profits, therefore, are given by $\pi_i=pk_i-\frac{k_i^2}{2}$.

Consumer i chooses the optimal bundle of goods (q_{1i},q_{2i}) to maximize his utility and so in equilibrium $p=\theta-bq_{1i}$. Households are ex-ante identical which, together with market clearing, implies that $q_{1i} = K$ for all i and therefore $p = \theta - bK$. This example is thus captured by the generalized framework with the utility

$$U(k, K, \sigma_k, \theta) = (\theta - bK)\, k - k^2/2 + bK^2/2 + e$$

which implies that

$$u_{kk} = -1, u_{kK} = -b, u_{k\theta} = 1, u_{KK} = b$$

and so $\kappa_0 = 0$,$\kappa_1 = \frac{1}{1+b}$ and

$$\chi \equiv \frac{u_{KK} - u_{kk} r^2}{-u_{kk}\,(1-r)^2} = \frac{b}{1+b} > 0.$$

B.2 Investment complementarities

Consider a setting in which the terminal value of the firm is given by $V \equiv V(R, k)$ where R measures the return on investment, or productivity, of the project available and k represents the scale of investment in the project. For analytical tractability, let

$$V(R, k) = Rk - \tfrac{1}{2}k^2, \tag{84}$$

$$R = (1-a)\,\theta + aK, \tag{85}$$

with $a \in (0, 1/2)$, and $\theta \in \mathbb{R}$ represents the firm's exogenous productivity. This implies that firm i's utility is

$$U_i(k_i, K, \theta) = (1-a)\,\theta k_i + aKk_i - \tfrac{1}{2}k_i^2$$

which falls into the class of general objective functions analyzed. This implies that

$$u_{kk} = -1, u_{kK} = a, u_{k\theta} = 1 - a.$$

which implies that $r = a$ and $\kappa_1 = 1$ and so

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}\,(1-r)^2} = \frac{a^2}{(1-a)^2} > 0.$$

B.3 Cournot versus Bertrand Competition

First, consider a Cournot setting in which firms compete on quantity. For any firm, consumer demand is $p = a_0 + a_1\theta - a_2 q - a_3 Q$ (with $a_0, a_1, a_2, a_3 > 0$), where p denotes the price at which the firm sells each unit, q is the number of units the firm produces , Q is the average quantity produced across all firms, and θ is a fundamental demand shock/shifter. Each firm's profits are $u = pq - C(q)$, where $C(q) = c_1 q + c_2 q^2$ is cost of production (with $c_1, c_2 > 0$). This model is contained within the generalized framework with $k \equiv q, K \equiv Q$, and

$$U(k, K, \sigma_k, \theta) = (a_0 - c_1 + a_1\theta - a_3 K)\, k - (a_2 + c_2)\, k^2$$

This implies that

$$U_{kk} = -2\,(a_2 + c_2)\,, U_{kK} = -a_3, U_{k\theta} = a_1, U_k = a_0 - c_1.$$

which implies that $r = \frac{-a_3}{2(a_2+c_2)}$ and $\kappa_1 = -\frac{a_1}{-2(a_2+c_2)-a_3}$ and

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}\,(1-r)^2} = \frac{\frac{a_3^2}{4(a_2+c_2)^2}}{\left(1 + \frac{a_3}{2(a_2+c_2)}\right)^2} > 0.$$

Next, consider a Bertrand setting in which firms compete on price. Consumer demand for each firm is $q = b_0 + b_1\theta' - b_2 p + b_3 P$ (with $b_0, b_1, b_2, b_3 > 0$), where q denotes the quantity produced by a given firm, p is the price set by a given firm, P is the average price across all firms, and θ is, again, an exogenous demand shock. As in Angeletos and Pavan (2007), we assume $b_3 < b_2$, and so an equal increase in p and P reduces q. Firm profits are as above. This model is contained within the generalized framework where $k \equiv p - c_1, K \equiv P - c_1$ (actions are now prices), and

$$\begin{aligned}
U\,(k, K, \sigma_k, \theta) &= \left[(\theta - k + bK)\, k - c\,(\theta - k + bK)^2\right] \\
&= \left[\left(\theta k - k^2 + bKk\right) - c\left(\theta^2 + k^2 + b^2K^2 - 2\theta k - 2bkK + 2\theta bK\right)\right] \\
&= \theta k\,(1 + 2c) - k^2\,(1 + c) + Kk\,(b + 2bc) - c\theta^2 - cb^2K^2 - 2bc\theta K
\end{aligned}$$

This implies that

$$U_{kk} = -2\,(1 + c)\,, U_{kK} = b\,(1 + 2c)\,, U_{k\theta} = 1 + 2c, U_{KK} = -2cb^2, U_{K\theta} = -2bc, U_{\theta\theta} = -2c.$$

which implies that $r = \frac{b(1+2c)}{2(1+c)}$ and $\kappa_1 = \frac{1+2c}{2(1+c)-b(1+2c)}$ and

$$\chi = -\frac{(u_{KK} - r^2 u_{kk})}{u_{kk}\,(1-r)^2} = \frac{-2cb^2 + 2\,(1+c)\left(\frac{b(1+2c)}{2(1+c)}\right)^2}{2\,(1+c)\left(1 - \frac{b(1+2c)}{2(1+c)}\right)^2}.$$

Since the denominator is always positive, this implies that

$$sign\left(\chi\right) = sign\left(\frac{-4\left(1+c\right)cb^2 + b^2\left(1+2c\right)^2}{2\left(1+c\right)}\right) > 0.$$

B.4 Information Spillovers

The model of Angeletos, Lorenzoni, and Pavan (2018) considers a novel channel through which the information in the real sector affects behavior in the financial sector. The real sector is comprised of entrepreneurs making investment decisions, and the financial sector is comprised of investors who provide liquidity to the "real" economy. All players are risk-neutral and the discount rate is zero. There are three dates, $t \in \{1, 2, 3\}$. At $t = 1$, a new investment opportunity becomes available with productivity θ. This investment pays off at $t = 3$. There is a continuum of entrepreneurs who can choose how much to invest in the new technology. Let k_i denote the investment of entrepreneur i, and let the cost of this investment be $\frac{k_i^2}{2}$. Entrepreneurs have access to an information technology that generates both a private and a "public" signal that they utilize when making their investment decision.[21] The joint distribution of fundamentals and signals follows the specification detailed in Section 3.

At $t = 2$, an entrepreneur is hit by an idiosyncratic liquidity shock with probability $l \in [0, 1]$. Entrepreneurs hit by this shock do not value consumption at $t = 3$ and so strictly prefer to sell their capital, at a price p, to investors.[22] Thus, entrepreneur i's payoff is $u_i = c_{i1} + c_{i2} + s_i c_{i3}$, where c_{it} denotes player i's consumption in period t and $s_i \in \{0, 1\}$ equals zero if he is hit by a liquidity shock. Thus, taken together, this implies that an entrepreneur's expected utility at the time of investment is given by

$$\mathbb{E}_i\left[u_i|s_i, s\right] = \mathbb{E}_i\left[\left(1-l\right)\theta k_i + lpk_i - \frac{1}{2}k_i^2|s_i, s\right]. \tag{86}$$

The financial market is competitive and the price p is determined through market clearing. Investors do not have access to their own information technology but, given the assumptions above, update their beliefs about the productivity of the technology utilizing the information contained in the supply of capital to be liquidated. It can be shown that, given the distributional assumptions and the risk-neutrality of traders, that $p = \mathbb{E}\left[\theta|K\right] = \alpha_1 K$, where α_1 is pinned down in equilibrium, shown below.[23] This implies that expected utility

[21] It is public in the sense that all *entrepreneurs* observe the same signal; however, as we discuss below, it is assumed that investors do not observe the signal and can only learn about θ through the aggregate investment level.

[22] It is assumed in ALP (2018) that the entrepreneurs not hit by the shock are precluded from trading.

[23] Investors are able to back out the aggregate level of capital utilizing l since liquidity shocks are independent.

of entrepreneur is given by

$$E\left[u_i\left(k_i, K, \theta\right)|s_i, s\right] = E_i\left[\left(1-l\right)\theta k_i + lk_i\left(\alpha_1 K\right) - \frac{1}{2}k_i^2|s_i, s\right], \tag{87}$$

and, therefore, each entrepreneur's optimal action is $k_i = E_i\left[\left(1-l\right)\theta + l\left(\alpha_1 K\right)\right]$. Note that this specification follows the generalized model analyzed above, which allows us to utilize the fixed-point solution for k_i found in (17). Aggregating across all entrepreneurs, this implies that aggregate investment can be written as

$$\begin{aligned} K &= \frac{1-l}{1-lA}\kappa_1 A\theta + \frac{B}{1-lA}\kappa_1 s \\ &= \frac{\kappa_1\left(\left(1-l\right)A + B\right)}{1-lA}\left(\theta + \frac{B}{\left(1-l\right)A+B}\eta\right). \end{aligned} \tag{88}$$

It is straightforward to see that the aggregate level of capital reveals a signal of the form $\xi = \theta + \frac{B}{\left(\left(1-r\right)A+B\right)}\eta$ to investors which, given the linear-normal structure, verifies the conjectured functional form for the price of capital. On the other hand, however, note that

$$\chi \equiv \frac{u_{KK} - u_{kk}r^2}{-u_{kk}\left(1-r\right)^2} = \frac{l^2\alpha_1^2}{\left(1-l\alpha_1\right)^2} > 0. \tag{89}$$