Choosing to Disagree in Financial Markets

Snehal Banerjee, Jesse Davis and Naveen Gondhi[*]

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Abstract

The psychology literature documents that individuals derive current utility from their beliefs about future events. We show that, as a result, investors in financial markets choose to disagree about both private and price information. When objective price informativeness is low, each investor dismisses the private signals of others and ignores price information. In contrast, when prices are sufficiently informative, heterogeneous interpretations arise endogenously: most investors ignore prices, while the rest condition on it. Our analysis demonstrates how observed deviations from rational expectations (e.g., dismissiveness, overconfidence) arise endogenously, interact with each other, and vary with economic conditions.

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1 Introduction

The standard approach in economics assumes that market participants have rational expectations and learn efficiently from the information in prices. Yet there is ample evidence that people do not behave this way: returns exhibit excess predictability and volatility, investors are overconfident and trade too often, and individuals appear to under-react to prices in some settings but over-react in others.[1] To explain this evidence, the existing literature has explored how informational frictions, endowed behavioral biases, and cognitive limits affect how investors interpret information. However, these models are usually unable to explain the variation in interpretations across investors and over economic conditions without assuming exogenous heterogeneity across investors.[2]

To study how the interpretation of information evolves endogenously, we require a model of subjective belief choice. We build on the large literature in psychology and behavioral economics that documents that individuals experience anticipatory utility from their beliefs about future events.[3] For instance, the anticipation of a positive *future* experience generates a positive, *contemporaneous* utility flow (e.g., excitement about an upcoming vacation). On the other hand, the prospect of negative future outcomes may lower current utility (e.g., anxiety about an annual medical checkup). In such cases, individuals often distort their beliefs by engaging in "wishful thinking": they choose subjective beliefs to make themselves happier about the future, even when such distortions come at a cost.

In an economy with symmetrically informed, homogeneous investors, we show that wishful thinking leads to endogenous disagreement that varies predictably with economic conditions. We consider a setting in which a continuum of symmetrically informed investors trade a risky asset against noise traders (as in Hellwig (1980)). We allow each investor to entertain subjective beliefs about the informativeness of her own private signal as well as the private information of others. The equilibrium price aggregates investors' private information and

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[1]For instance, Shiller (1981) documents that stock returns exhibit excess volatility relative to fundamentals, and Jegadeesh and Titman (1993) document that stocks exhibit momentum. Odean (1999) documents that individual investors exhibit over-confidence as evidenced by excessive trading. Recent work by Coibion and Gorodnichenko (2012) suggests professional forecasters are slow to update their beliefs about macroeconomic variables, while Greenwood and Shleifer (2014) document that investor expectations of future returns exhibit extrapolation.

[2]The empirical evidence in Banerjee (2011) suggests that the extent to which investors condition on prices varies substantially across stocks, while there is evidence that both disagreement (e.g., Andrade, Crump, Eusepi, and Moench (2016), Fischer, Kim, and Zhou (2020)) and overconfidence (e.g., Merkle (2017)) vary over time.

[3]We follow Bénabou and Tirole (2016) in using the term "anticipatory utility" to refer to the contemporaneous utility that an individual derives from the anticipation of future outcomes. As such, anticipatory utility is distinct from the notion of *anticipated* utility, which refers to settings in which agents treat parameters they learn about as constant when formulating decisions (e.g., Kreps (1998), Cogley and Sargent (2008)).

so provides an endogenous (noisy) signal about asset payoffs. The cost of belief distortion is given by the loss in the average *ex-post*, experienced utility due to trading on subjective beliefs.[4]

In the rational expectations benchmark, all investors are constrained to agree on the interpretation of signals, and so efficiently condition on both their private signal and on the price when submitting their demand for the risky asset. With wishful thinking, however, we show that investors always choose to disagree about the interpretation of these signals. Moreover, the nature of this disagreement depends endogenously on the information environment. When prices are not very informative, there exists a unique, symmetric equilibrium in which each investor believes her own signal is informative but dismisses the information of others. As a result, investors choose to ignore the information in prices. However, when prices are sufficiently informative, the model yields a novel source of endogenous heterogeneity: while the majority of investors continue to treat the price as uninformative, the remaining investors use the information in prices to update their beliefs.

To highlight the key intuition for our results, we begin with a benchmark in which each investor is constrained to correctly interpret her own signal. Believing that others are less informed has two opposing effects on an investor's anticipatory utility. On the one hand, this implies that the price is less informative about payoffs, which increases the investor's perceived uncertainty and reduces her anticipatory utility - we refer to this as the **information effect**. On the other hand, when others are less informed, the perceived trading gains from speculating against them is higher, which increases anticipatory utility. We term this the **speculative effect**. Importantly, the speculative effect generates a type of strategic substitutability across investors' chosen beliefs. When others condition on prices more heavily, the price is more sensitive to their private information. This increases the relative benefit from perceiving others to be less informed, which leads the investor to under-react to price information more. We first show that, when all other investors are constrained to hold objective beliefs (i.e., exhibit rational expectations), a single investor strictly prefers to dismiss the information of others and completely ignores the price. In this case, the speculative effect dominates the information effect, and the utility cost of ignoring price information is not very high.[5]

Next, we characterize the equilibrium when all investors choose their subjective beliefs. When the objective price informativeness is sufficiently low (e.g., aggregate risk tolerance

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[4]We refer to this as the *experienced utility penalty*. As we discuss in Section 5.3, many of our results are qualitatively robust to other cost specifications (e.g., the Kullback Leibler distance).

[5]Since all other investors exhibit rational expectations, the price is informationally efficient and sufficiently "close" to fundamentals. As we discuss in Section 4.2, this implies the experienced utility loss from taking a sub-optimal position in the risky asset is smaller than the corresponding gain in anticipatory utility.

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is low, or noise trading volatility is high), there exists a unique, symmetric equilibrium in which *all* investors choose to dismiss the information of others. We refer to this as the *dismissive* equilibrium. The speculative effect dominates even though all investors ignore price information because prices are objectively not very informative, and so the information effect is relatively small.

However, when prices are sufficiently informative, such equilibria cannot be sustained. This is a consequence of the strategic substitutability in belief choice: when all others ignore the information in prices, the speculative effect is relatively small. Furthermore, since the price information is very precise, the information effect dominates. This leads an individual investor to deviate and choose to condition on price information.

Instead, we show that there exists a unique mixed-strategy equilibrium in which the majority of investors ignore price information while a minority condition efficiently on it. Moreover, we show that the fraction of investors who ignore price information initially decreases but then increases in price informativeness. In fact, we show that as prices become arbitrarily informative (e.g., noise trading volatility goes to zero), the fraction of investors who ignore price information approaches one because the cost of doing so is very small.[6]

We extend our benchmark analysis to allow for subjective beliefs about the volatility of supply shocks. Since subjective beliefs about others' signals and noise trading affect anticipatory utility only through their effect on the perceived precision of the price signal, our results remain qualitatively unchanged. However, the mixed-strategy equilibrium which arises features under-reaction to price information by some investors and *over-reaction* by others. Existing models generate such differences in interpretation and investment strategies by assuming that investors are ex-ante heterogeneous (e.g., in the quality of their private information, or their ability to process such information). In contrast, our model endogenously generates these opposing interpretations with ex-ante homogeneous investors, and therefore helps us better understand how such disagreement varies with underlying economic conditions.

We then explore the implications of subjective belief choice about private information. Increasing the perceived precision about one's own signal unambiguously increases anticipatory utility by reducing uncertainty i.e., only the information effect is in force. As a result, investors generically exhibit over-confidence with respect to their private information. When investors only choose subjective beliefs about their own private signal, the equilibrium degree of over-confidence decreases with prior uncertainty about payoffs, volatility of noise trading and risk aversion, and is U-shaped in the objective precision of private signals.[7] When in-

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[6]As we discuss further in Section 4.3, this is in sharp contrast to noisy rational expectations models where the fraction of investors who condition on prices usually *increases* with price informativeness.

[7]As we discuss in Section 5.2, these comparative statics distinguish our model's implications from a setting

vestors choose subjective beliefs about their own signals and those of others, then symmetric equilibria are characterized by over-confidence in private information *and* dismissiveness of others' signals. Moreover, we show that investors trade off belief distortion along these two dimensions: when an investor's subjective belief about price information is closer to the objective distribution, the degree of over-confidence increases. This negative relation between dismissiveness and over-confidence arises endogenously as a result of wishful thinking, and distinguishes our model from those which consider these biases separately.

We then characterize predictions for market observables that help distinguish our model from noisy rational expectations equilibria. For instance, the subjective beliefs equilibrium exhibits higher expected returns and higher volume than the corresponding rational expectations equilibrium. Such equilibria can even exhibit positive serial correlation in returns under conditions where the rational expectations equilibrium always leads to reversals. A key takeaway of our analysis is that investors' choice of subjective beliefs, and the resulting "behavioral bias", depend crucially on economic conditions. For example, to the extent that bull markets and macroeconomic expansions are associated with more informative prices, our model predicts these episodes feature higher diversity in investment strategies, lower volatility, and a positive relation between return predictability and volatility. In contrast, periods of high market stress and high volatility are more likely to be associated with higher correlation in investment styles, low (or even negative) serial correlation in returns, and a negative relation between predictability and volatility.[8] As discussed further in Section 6, a number of our predictions are broadly consistent with the existing empirical evidence on time variation in momentum returns and crashes (e.g., Cooper, Gutierrez Jr, and Hameed (2004), Moskowitz, Ooi, and Pedersen (2012), Daniel and Moskowitz (2016)) and price informativeness (e.g., Bai, Philippon, and Savov (2016), and Dávila and Parlatore (2019)), while others offer novel implications for future empirical work.

Our model can also help reconcile the recent evidence on forecast error predictability (e.g., Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Porta, and Shleifer (2020b)). Specifically, we show that in subjective belief equilibria where investors over-react to private information but under-react to the information in prices, consensus forecast revisions exhibit under-reaction while individual forecast revisions exhibit over-reaction. To assess our model's predictions more directly, we propose a novel empirical test: regressing individual forecast errors on lagged returns. In particular, our analysis predicts that the distribution of the regression coefficient across investors should be state-

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in which investors acquire private information endogenously (e.g., Verrecchia (1982)).

[8]It is worth noting that while other behavioral models also generate some of these predictions (e.g., dismissiveness / difference of opinions can generate drift), the state dependence of these predictions are a distinguishing feature of our model.

dependent.[9]

The rest of the paper is as follows. Section 2 briefly discusses the phenomenon of anticipatory utility and reviews the related literature. Section 3 introduces the benchmark model, discusses our assumptions, and characterizes the financial market equilibrium, given investor beliefs. Section 4 characterizes the tradeoffs associated with subjective belief choice and characterizes the equilibrium in the benchmark model. Section 5 generalizes the analysis to accommodate subjective beliefs about noise trading volatility and private signals, and explores how our results change with general cost functions. Section 6 characterizes the empirical implications of our analysis. Section 7 concludes, and proofs and extensions can be found in Appendices A and B, respectively.

2   Background and Related Literature

2.1   Anticipatory Utility and Subjective Belief Choice

Bénabou and Tirole (2016) survey the now extensive literature on motivated beliefs (e.g., Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Eliaz and Spiegler (2006)). The concept of anticipatory utility, or current subjective expected utility, dates to at least Jevons (1905) who considers agents who derive contemporaneous utility not simply from current actions but also the anticipation of future utility flows. As a result, an agent's subjective beliefs about future events will affect not just an agent's actions but also her current utility. This creates a tension between holding beliefs that are "accurate' (and therefore will lead to optimal actions) and beliefs that are "desirable" (and therefore will increase current utility). We emphasize that agents do not suffer from a "multiple selves" problem but instead choose to hold a *single* set of beliefs which accounts for this implicit tradeoff.[10]

The most closely related papers are Brunnermeier and Parker (2005) and Caplin and Leahy (2019). Brunnermeier and Parker (2005) show how subjective belief choice ("optimal expectations") is useful in understanding risk-taking, preference for skewness, optimism/pessimism, portfolio under-diversification and consumption/savings patterns. Simi-

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[9]When price informativeness is low, the regression coefficient for all investors should be positive since investors under-react to price information. When price informativeness is high, the regression coefficient is positive for some investors but negative for others.

[10]In particular, our assumption that deviations from the objective distribution are costly is a modeling convenience — we do not interpret the use of the objective distribution in specifying the cost as the agent "knowing" the true distribution. Instead, we wish to capture the notion that the agent behaves as if deviating too far from accurate beliefs is costly, either due to experience or because it reflects a departure from consensus. See Caplin and Leahy (2019) for a further discussion.

larly, Caplin and Leahy (2019) show that wishful thinking can help explain a number of behavioral biases, including optimism, procrastination, confirmation bias, polarization and the endowment effect.

We view our work as complementary, both building on these insights while offering a new perspective focused on understanding how wishful thinking affects the interpretation of endogenous information in a market setting. As such, our model derives novel predictions not only about how overconfidence and dismissiveness can arise endogenously among investors, but also how these subjective belief choices interact and the way in which they depend on market conditions. These aspects are missing from the earlier literature and have important consequences for our understanding of financial markets.

Note that subjective belief choice is not just of theoretical interest — there is substantial, direct empirical evidence that suggests individuals experience anticipatory utility, and as a result, distort their subjective beliefs in systematic ways. Individuals engage in information avoidance, for instance, by choosing not to learn about the risk of deadly disease even if the test is approximately costless (Oster, Shoulson, and Dorsey (2013)). At the same time, individuals may actively seek (and pay) to learn about potential good news, such as the outcome of a lottery-like event (Ganguly and Tasoff (2017)) or the performance of their portfolios on days when the market has done well (Karlsson, Loewenstein, and Seppi (2009)). Individuals also update asymmetrically when information is revealed: more weight is placed on good news (e.g., a positive signal about one's IQ in Mobius, Niederle, Niehaus, and Rosenblat (2014)) than bad news (e.g. a negative signal about one's attractiveness in Eil and Rao (2011)). Finally, many individuals interpret information in ways which are favorable to their current well-being, updating in ways consistent with their political beliefs (Kahan (2013)) or interpreting uninformative signals of ability as positive indicators (Exley and Kessler (2019)). This literature suggests that such wishful thinking is not generated by individuals inability to understand their environment; in fact, both Kahan (2013) and Kahan, Peters, Dawson, and Slovic (2017) show that cognitive ability can even exacerbate the effect because more sophisticated individuals can better "rationalize" their beliefs and interpretations. Given this evidence, we expect our analysis to apply, to varying degrees, to both retail and institutional investors, and believe it is important to understand the impact of such behavior on market outcomes.[11]

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[11]For instance, it is likely that sophisticated traders are particularly adept at justifying their favored bets, attributing their successes to skill and ability, while blaming losses to bad luck.

2.2 Distorted Beliefs in Financial Markets

Our paper contributes to three strands of the literature studying the impact of deviations from rational expectations, which help explain stylized facts about financial markets that are difficult to reconcile in the standard framework (e.g., excess trading volume and return predictability). The first strand focuses on differences of opinion, whereby investors "agree to disagree" about the joint distribution of payoffs and signals and therefore, incorrectly condition on the information in prices (e.g., Harrison and Kreps (1978), Kandel and Pearson (1995), Banerjee, Kaniel, and Kremer (2009) and Banerjee (2011)). The second strand focuses on the impact of overconfidence: specifically, settings in which agents believe their private information is more precise than it objectively is (e.g., Odean (1998); Daniel, Hirshleifer, and Subrahmanyam (1998, 2001); Gervais and Odean (2001); Scheinkman and Xiong (2003)). While our equilibria feature investors who are dismissive of price information, disagree about the interpretation of a public signal, and over-estimate the precision of their private signal, our focus is on understanding why and under what conditions such beliefs arise endogenously.

The final strand includes some of the alternative settings in which investors do not fully condition on the information in prices including models that feature rational inattention (e.g., Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)), cursedness (e.g., Eyster, Rabin, and Vayanos (2018)) and costly learning from prices (e.g., Mondria, Vives, and Yang (2020)).[12] Notably, in the models of Kacperczyk et al. (2016) and Mondria et al. (2020), updating one's beliefs using prices is costly (in either an attention or pecuniary sense) and, as such, investors *choose* to discount this information. We view our analysis of endogenous belief choice as complementary to this earlier work. In particular, even when learning (efficiently) from prices is costless, we show that investors may choose to dismiss price information when they experience anticipatory utility.

Our analysis also highlights that the benefits and costs of using the information in prices depends on how other investors use this information. This is reminiscent of, but distinct from, the channel discussed by Mondria et al. (2020), who show that there can exist complementarity in learning from prices: an investor may choose to learn more from prices when others become more sophisticated about learning from prices because prices become more informative. In contrast, we show endogenous beliefs can give rise to substitutability: when others are learning from prices, one's incentive to do so decreases.

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[12]While Eyster et al. (2018) show that cursedness can generate distinct predictions from a model of differences of opinions (which they term dismissiveness) when there is imperfect competition and no noise trading, our setting features perfectly competitive markets and noise in prices, and so cursedness and differences of opinions are effectively isomorphic.

More generally, unlike specific models of individual frictions or biases, our analysis suggests how a rich set of behavior (e.g., overconfidence, dismissiveness, heterogeneity in interpretations) can arise from a single, psychologically motivated feature of individual decision making (i.e., wishful thinking). This help us understand how such behaviors interact with each other (e.g., the negative relation between overconfidence and dismissiveness in our model), and how they vary with economic conditions.

3   Benchmark Model

This section introduces the model setup, provides some preliminary analysis and discusses the key assumptions of our setting.

3.1   Model setup

Asset payoffs. There are two securities. The gross return on the risk-free security is normalized to one. The terminal payoff (fundamental value) of the risky security is F, which is normally distributed with mean m and prior precision τ, i.e.,

$$F \sim \mathcal{N}\left(m, \frac{1}{\tau}\right).\tag{1}$$

We denote the market-determined price of the risky security by P, and the aggregate supply of the risky asset by $Z + z$, where

$$z \sim \mathcal{N}\left(0, \frac{1}{\tau_z}\right),\tag{2}$$

and we normalize the mean aggregate supply to $Z = 0$. We interpret shocks to the aggregate supply (i.e., z) as resulting from trades by liquidity or noise traders who trade the risky asset for non-informative reasons.

Information. There is a continuum of investors, indexed by $i \in [0, 1]$. Before trading, each investor is endowed with a private signal s_i, where

$$s_i = F + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, \frac{1}{\tau_e}\right)\tag{3}$$

and ε_i is independent and identically distributed across investors so that $\int \varepsilon_i \, di = 0$. Moreover, investors can update their beliefs about F by conditioning on the information in the price P.

Beliefs and preferences. Each investor i is endowed with initial wealth W_0 and zero shares of the risky security, and exhibits CARA utility with coefficient of absolute risk aversion γ over terminal wealth W_i:

$$W_i = W_0 + x_i (F - P), \tag{4}$$

where x_i denotes her demand for the risky security. In our benchmark model, we assume each investor has correct beliefs about her own signal, but allow for subjective beliefs about the private signals of others.[13] Specifically, we assume that investor i believes that other investors observe

$$s_j =_i F + \sqrt{1 - \rho_i^2}\eta + \rho_i \epsilon_j, \quad \eta, \epsilon_j \sim_i \mathcal{N}\left(0, \frac{1}{\pi_i \tau_e}\right), \tag{5}$$

where $=_i$ and $\sim_i$ denotes investor i's subjective beliefs. Similarly, we will denote the expectation and variance of random variable X under investor i's subjective beliefs by $\mathbb{E}_i[X]$ and $\text{var}_i[X]$, respectively.

Intuitively, equation (5) captures the idea that investor i can distort her beliefs about both the amount of noise in others' signals ($\pi_i \in [0, \infty]$) as well as the average correlation in this noise ($\rho_i \in [0, 1]$). When $\rho_i = \pi_i = 1$, investor i's beliefs satisfy **rational expectations**: her beliefs coincide with the objective distribution of the underlying shocks. When $\rho_i < 1$, investor i over-estimates the correlation in others' signals, and when $\pi_i < 1$ ($\pi_i > 1$) she over-estimates (under-estimates) the noise in their signals. As such, the parameters π_i and ρ_i reflect the degree to which investor i's distorts her subjective beliefs. We assume that such deviations from the objective distribution impose a utility cost, denoted by $C(\pi_i, \rho_i)$.

Given her choice of subjective beliefs, each investor optimally chooses her position in the risky security. Thus, optimally chosen subjective beliefs maximize her anticipatory utility, net of cost $C(\cdot)$. Formally, denote investor i's optimal demand, given her beliefs, by:

$$x_i^*(\pi_i, \rho_i) = \arg \max_{x_i} \mathbb{E}_i \left[-\gamma \exp\{-\gamma x_i (F - P) - \gamma W_0\} \,\big|\, s_i, P\right]. \tag{6}$$

and denote investor i's **anticipatory utility** by

$$AU_i(\pi_i, \rho_i) \equiv \mathbb{E}_i \left[\mathbb{E}_i \left[-\gamma \exp\{-\gamma x_i^* (F - P) - \gamma W_0\} \,\big|\, s_i, P\right]\right]. \tag{7}$$

Then, investor i optimally chooses subjective beliefs ρ_i and π_i to maximize:

$$\max_{\pi_i, \rho_i} AU_i(\pi_i, \rho_i) - C(\pi_i, \rho_i). \tag{8}$$

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[13]In Section 5.2, we also allow each investor to have subjective beliefs about her own signal and study how this choice interacts with her beliefs about others' information.

In our benchmark analysis, the cost each investor incurs by distorting her subjective beliefs is the reduction in expected utility (under the *objective* distribution) when her position in the risky asset, $x_i^* (\pi_i, \rho_i)$, is determined under her chosen *subjective* distribution. As is well-established, any deviation from the rational expectations benchmark ($\pi_i = \rho_i = 1$) is objectively inefficient: the investor is over- or under-weighting the information she receives. We refer to this cost specification as the "experienced utility" penalty.

Definition 1. Investor i incurs the **experienced utility penalty** if the cost of choosing π_i, ρ_i is given by:

$$C_{obj} (\pi_i, \rho_i) \equiv \begin{array}{l} \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^* (1,1) \times (F - P) - \gamma W_0\right\}\right] \\ -\mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^* (\pi_i, \rho_i) \times (F - P) - \gamma W_0\right\}\right] \end{array} \tag{9}$$

When investors incur the experienced utility penalty, we can show that their subjective belief choice problem can be represented as:

$$\max_{\pi_i, \rho_i} AU_i (\pi_i, \rho_i) + \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^* (\pi_i, \rho_i) \times (F - P) - \gamma W_0\right\}\right]. \tag{10}$$

This is closely related to the objective function in the "optimal expectations" approach of Brunnermeier and Parker (2005).[14] We utilize this cost function as our benchmark because of its clear interpretation, intuitive appeal and direct quantitative implications. As we discuss below, we explore how our analysis changes for more general cost functions in Section 5.3.

3.2 Discussion of assumptions

The penalty function in (9) does not necessarily imply that the investor knows the objective distribution; instead, it should be interpreted as a tractable specification for the utility cost of subjective beliefs, from the perspective of the modeler (or observer). Intuitively, we interpret the above specification (and that of Brunnermeier and Parker (2005)) as one in which investors evaluate their actions and outcomes under a single, subjective model of the world, which is "close to the truth" in the sense that the distortions in behavior do not

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[14]Under their approach, an investor optimally chooses actions under subjective beliefs $\mathbb{E}_i[\cdot]$, and the optimal choice of beliefs maximizes the investor's well-being under the objective distribution i.e.,

$$\max_{\pi_i, \rho_i} \mathbb{E}\left[\begin{array}{l} \mathbb{E}_i\left[-\gamma \exp\left\{-\gamma x_i^* (\pi_i, \rho_i) (F - P) - \gamma W_0\right\} \big| s_i, P\right] \\ -\gamma \exp\left\{-\gamma x_i^* (\pi_i, \rho_i) \times (F - P) - \gamma W_0\right\} \end{array}\right] \tag{11}$$

$$= \max_{\pi_i, \rho_i} \mathbb{E}\left[AU_i (\pi_i, \rho_i)\right] + \mathbb{E}\left[-\gamma \exp\left\{-\gamma x_i^* (\pi_i, \rho_i) \times (F - P) - \gamma W_0\right\}\right]. \tag{12}$$

In our setting, $AU_i (\pi_i, \rho_i) = \mathbb{E}\left[AU_i (\pi_i, \rho_i)\right]$ and so the two objectives coincide.

generate too large of a loss in experienced utility.[15] The subjective model that is chosen may result from a more complicated process of experimentation, learning and experience, which trades off "desirable" models (that increase anticipatory utility) against "accurate" models (that increase experienced utility). The specification in (9) provides a tractable characterization of this process from the perspective of economic modeling.

While our benchmark analysis focuses on subjective beliefs about others' signals, in Section 5.1 we extend the analysis to allow for subjective beliefs about the noise trading volatility. As we discuss in the next section, this allows us to characterize optimal beliefs in a setting where investors can choose to over-react to price information, instead of only under-reacting to it. In Section 5.2, we explore how these choices interact with investors' subjective interpretation of their own private signals.

Throughout our analysis, we restrict investors to choose subjective beliefs about the precision of these signals, and assume that they make these choices *before* observing the realizations of their signals. These assumptions allow us to tractably model how investors interpret different types of information: we can explicitly characterize the financial market equilibrium since the resulting equilibrium price is a linear signal about fundamentals. Moreover, this specification is naturally interpreted as the stage game of a dynamic, repeated setting in which investors experiment with (and update about) different models of the world.[16] While allowing for more flexibility in subjective belief choice may lead to additional implications (e.g., a preference for skewness as in Brunnermeier and Parker (2005)), we expect our main results to be qualitatively similar in these settings. However, a formal analysis is beyond the scope of this paper and left for future work.

We assume that the average aggregate supply of the risky asset is zero for analytic tractability. In Appendix B.2, we consider an extension of the model in which the expected aggregate supply of the risky asset is $Z > 0$. While this extension is not as analytically tractable, we can solve it numerically and find that the resulting equilibria are very similar to the case with $Z = 0$. Moreover, as we discuss in Section 6, relaxing this assumption also allows us to characterize the implications of subjective beliefs on unconditional expected

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[15]As Caplin and Leahy (2019) suggest, the Brunnermeier and Parker (2005) approach may be interpreted as one with divided selves: the agent chooses subjective beliefs by evaluating outcomes under the objective distribution (at date zero), and chooses actions in the following periods by evaluating outcomes under the chosen subjective distribution. In contrast, agents in our model (as in Caplin and Leahy (2019)) evaluate outcomes only under the subjective beliefs.

[16]In Appendix B.1, we explore how our results are affected when investors choose their interpretations *after* observing the signals. Unfortunately, solving for the general equilibrium in which all investors choose their beliefs is not feasible in this setting: the perceived precision depends upon the realizations of signals and the linearity of the market-clearing price is not preserved so that "closing the model" is intractable. However, the partial equilibrium analysis of a single investor's interpretation suggests that similar biases can arise even under this alternative timing.

returns.

The experienced utility penalty depends on the equilibrium subjective beliefs of other investors via investor i's beliefs about the distribution of the equilibrium price. This is in contrast models of subjective belief choice (e.g., Caplin and Leahy (2019)) that use a statistical, distance-based cost function (e.g., the K-L divergence). However, our exploration of the alternative cost functions in Section 5.3 and Appendix B.3 suggests that this dependence does not play a critical role for our main results. Even in settings where the beliefs of others do not directly affect the cost of belief distortion, they have an effect on the subjective belief choice of investor i through her anticipatory utility.

3.3   Preliminary analysis

We first solve for the financial market equilibrium, taking investors' chosen subjective beliefs as given. We will conjecture (and then verify) that the price P is a linear combination of the average signal $\bar{s}$ across agents and the aggregate supply shock z. In this case, observing the price is equivalent to observing a linear signal s_p of the form

$$s_p = \bar{s} + \beta z, \quad \text{where} \quad \bar{s} \equiv \int_i s_i di. \tag{13}$$

This implies that the objective conditional distribution of s_p is given by

$$s_p | F \sim \mathcal{N}\left(F, \frac{1}{\tau_p}\right), \quad \text{where} \quad \tau_p = \frac{\tau_z}{\beta^2}. \tag{14}$$

However, given her subjective beliefs in (5), investor i's beliefs about s_p are given by:

$$s_p | F \sim_i \mathcal{N}\left(F, \frac{1}{\delta_{p,i} \tau_p}\right), \quad \text{where} \quad \delta_{p,i} \equiv \frac{\frac{\beta^2}{\tau_z}}{\frac{\beta^2}{\tau_z} + \frac{1-\rho_i^2}{\pi_i \tau_\eta}}. \tag{15}$$

Since $\rho_i \in [0,1]$ and $\pi_i \in [0,\infty)$, it is easy to see that $\delta_{p,i} \in [0,1]$.

This representation (15) highlights that investor i's subjective beliefs about others' information (π_i, ρ_i) are only relevant to the extent that they distort her perception of the precision of the price signal, s_p. When $\delta_{p,i} < 1$, investor i under-reacts to the information in prices, either because she believes others' signals contain more noise (i.e., $\pi_i > 1$) or because they are more correlated (i.e., $\rho_i < 1$).[17] Under rational expectations, $\delta_{p,i} = 1$; moreover,

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[17]Given the objective distribution of signals, the above expression implies that investors cannot over-react to the information in prices (i.e., $\delta_{p,i} \leq 1$). To allow for this possibility, in Section 5.1 we allow investors to have subjective beliefs about the volatility of noise trading, or equivalently, the precision of supply shocks

we note that $\delta_{p,i} = 1$ any time $\rho_i = 1$, since in the absence of any perceived correlation, the noise in others' signals would be aggregated away under market clearing.

Given investor i's subjective beliefs, $\delta_{p,i}$, and conditional on her observed signals, s_i and s_p, investor i's posterior subjective beliefs are given by

$$F|s_i, s_p \sim_i \mathcal{N}\left(\mu_i, \frac{1}{\omega_i}\right), \text{ where} \tag{16}$$

$$\mu_i \equiv \mathbb{E}_i[F|s_i, s_p] = m + A_i(s_i - m) + B_i(s_p - m), \tag{17}$$

$$\omega_i \equiv \frac{1}{\text{var}_i[F|s_i, s_p]} = \frac{\tau}{1 - A_i - B_i}, \text{ and} \tag{18}$$

$$A_i \equiv \frac{\tau_e}{\tau + \tau_e + \delta_{p,i}\tau_p}, \text{ and } B_i \equiv \frac{\delta_{p,i}\tau_p}{\tau + \tau_e + \delta_{p,i}\tau_p}. \tag{19}$$

Given these subjective beliefs, the equilibrium price is characterized by (i) computing the optimal demand $x_i^*(s_i, P)$ for each investor, and then (ii) imposing market clearing. Noting that under the *objective* distribution $\bar{s} = F$, the following result characterizes the financial market equilibrium.

Lemma 1. *Given investor i's subjective beliefs $\delta_{p,i}$, there always exists a unique, linear, financial market equilibrium in which*

$$P = m + \Lambda(s_p - m), \text{ where } \Lambda = \frac{\tau_e + \bar{\delta}_{p,i}\tau_p}{\tau + \tau_e + \bar{\delta}_{p,i}\tau_p}, \tag{20}$$

$s_p = F + \beta z$, $\tau_p = \tau_z/\beta^2$, $\beta = -\frac{\gamma}{\tau_e}$, *and* $\bar{\delta}_{p,i} = \int_i \delta_{p,i} di$ *reflects the average subjective beliefs across investors.*

The financial market equilibrium is standard. Investors' subjective beliefs about the informativeness of others' signals affects the *objective* price sensitivity, Λ, to the asset's fundamental value, F. However, the objective informativeness of the price signal is unchanged. When investors deviate from rational expectations, they trade less aggressively on any perceived mis-pricing (i.e., ω_i decreases), reducing Λ. But they also place relatively more weight on their private information (A_i increases while B_i decreases), so that τ_p remains constant.[18]

In what follows, we characterize the subjective beliefs of investor i using $\delta_{p,i}$ instead of $\{\rho_i, \pi_i\}$. The definition of the experienced utility penalty $C_{obj}(\delta_{p,i})$ is modified to correspond to the analogous function when we replace $x_i^*(\pi_i, \rho_i)$ by $x_i^*(\delta_{p,i})$.

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(i.e., τ_z) — in this case, $\delta_{p,i} > 1$ corresponds to subjective beliefs that over-estimate the precision of aggregate supply shocks.

[18]In Section 5.2, we show how subjective beliefs about *private* signals can affect τ_p through β.

4   Subjective Belief Choice

In this section, we characterize the equilibrium subjective belief choice of investors, given the financial market equilibrium characterized above. We do this in steps. First, we characterize the effects of subjective belief choice on anticipatory utility. In Section 4.2, we characterize an individual investor's choice of subjective beliefs when all other investors exhibit rational expectations, highlighting the partial equilibrium implications of subjective belief choice. Finally, in Section 4.3, we allow all investors to choose their subjective beliefs, taking as given the behavior of others. Importantly, each investor also takes as given the subjective beliefs of other investors: she does not assume they hold rational expectations. Comparing these results to the partial equilibrium analysis allows us to highlight how general equilibrium considerations can give rise to endogenously different responses to price information.

4.1   Anticipatory utility

Given the optimal demand for the risky asset, anticipatory utility is given by

$$AU_i\left(\delta_{p,i}\right) = \mathbb{E}_i\left[-\exp\left\{-\frac{\left(\mathbb{E}_i\left[F|s_i, P\right] - P\right)^2}{2\mathrm{var}_i\left[F|s_i, P\right]}\right\}\right].$$

Moreover, given the characterization of the equilibrium price in Lemma 1, investor i's beliefs about the conditional return are given by:

$$\mathbb{E}_i\left[\mathbb{E}_i\left[F|s_i, P\right] - P\right] = m - m = 0, \text{ and} \tag{21}$$
$$\mathrm{var}_i\left[\mathbb{E}_i\left[F|s_i, P\right] - P\right] = \mathrm{var}_i\left[F - P\right] - \mathrm{var}_i\left[F|s_i, P\right], \tag{22}$$

where the first equality follows from the law of iterated expectations and the second equality follows from the law of total variance.[19] As a result, anticipatory utility reduces to

$$AU_i\left(\delta_{p,i}\right) = -\sqrt{\frac{\mathrm{var}_i\left[F|s_i, P\right]}{\mathrm{var}_i\left[F - P\right]}}. \tag{23}$$

From this, we derive the following result.

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[19]The law of total variance implies

$$\mathrm{var}_i\left[F - P\right] = \mathbb{E}_i\left[\mathrm{var}_i\left[F - P|s_i, P\right]\right] + \mathrm{var}_i\left[\mathbb{E}_i\left[F - P|s_i, P\right]\right],$$

which in turn, implies the above expression.

Lemma 2. *Anticipatory utility is non-monotonic in $\delta_{p,i}$: there exists some $\bar{\bar{\delta}} > 0$ such that for all $\delta_{p,i} < \bar{\bar{\delta}}$ anticipatory utility is decreasing in $\delta_{p,i}$ while for all $\delta_{p,i} > \bar{\bar{\delta}}$ it is increasing.*

Increasing the perceived precision of the price signal (i.e., increasing $\delta_{p,i}$) has two competing effects, as highlighted by the following expression:

$$\frac{\partial AU_i}{\partial \delta_{p,i}} \propto \left(\underbrace{\frac{1}{\text{var}_i[F-P]} \frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}}}_{\text{speculative effect}} - \underbrace{\frac{1}{\text{var}_i[F|s_i, s_p]} \frac{\partial \text{var}_i[F|s_i, P]}{\partial \delta_{p,i}}}_{\text{information effect}} \right), \qquad (24)$$

First, the **information effect** of learning from prices reduces the conditional variance $\text{var}_i[F|s_i, P]$: the investor has better information about the asset's value which increases anticipatory utility. This information effect reduces the volatility of the perceived return on the risky security, a benefit in and of itself, but it also allows the investor to scale up her trading position. Second, the **speculative effect** of believing prices are more informative decreases the perceived variance of the conditional expected return (i.e., $\text{var}_i(\mathbb{E}_i[F|s_i, P] - P)$), which lowers anticipatory utility. Intuitively, when the price is more informative, it tracks fundamentals more closely and, as a result, the trading opportunity is less profitable.

The overall effect on anticipatory utility of changing $\delta_{p,i}$ depends on the relative magnitude of these two effects. When $\delta_{p,i}$ is low, the speculative effect dominates, while the information effect dominates when $\delta_{p,i}$ is high. As a result, anticipatory utility first decreases and then increases in $\delta_{p,i}$. Moreover, note that the information effect can be expressed as

$$-\frac{1}{\text{var}_i[F|s_i, s_p]} \frac{\partial \text{var}_i[F|s_i, P]}{\partial \delta_{p,i}} = \frac{\tau_p}{\tau + \tau_e + \delta_{p,i}\tau_p} > 0, \qquad (25)$$

which does not depend on the behavior of others. In contrast, the speculative effect is

$$\frac{1}{\text{var}_i[F-P]} \frac{\partial \text{var}_i[F-P]}{\partial \delta_{p,i}} = -\frac{\Lambda^2 \tau}{\delta_{p,i}(\Lambda^2 \tau + (1-\Lambda)^2 \delta_{p,i}\tau_p)} < 0, \qquad (26)$$

Recall that Λ captures the sensitivity of the price to fundamentals, which is increasing in $\bar{\delta}_{p,i} = \int_i \delta_{p,i} di$. Thus, the above expression implies that the speculative effect is relatively more important when other investors interpret prices to be more informative (i.e., $\bar{\delta}_{p,i}$ is high).[20] Intuitively, when others condition on prices more heavily, the perceived loss of speculative opportunities is larger for investor i. This source of *relative strategic substitutability* will play an important role in determining the nature of equilibrium, as we demonstrate in

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[20]The term capturing the proportionality in (24) also depends on Λ, but does not affect the relative magnitude of the two effects.

the following sections.[21]

It is worth noting that these competing effects may generalize beyond our specific setting. It can be shown that anticipatory utility is a monotonic transformation of:

$$\frac{\text{var}_i\left[F-P\right]}{\text{var}_i\left[F|s_i,P\right]} = \text{var}_i\left(\frac{\mathbb{E}_i\left[F-P|s_i,P\right]}{\sqrt{\text{var}_i\left[F-P|s_i,P\right]}}\right) \equiv \text{var}_i\left(SR_i\right) \tag{27}$$

where

$$SR_i \equiv \frac{\mathbb{E}_i\left[F-P|s_i,P\right]}{\sqrt{\text{var}_i\left[F-P|s_i,P\right]}} \tag{28}$$

is investor i's conditional Sharpe ratio, given her beliefs. As shown by the partial equilibrium analysis of Van Nieuwerburgh and Veldkamp (2010), investor utility is increasing in the squared Sharpe ratio under more general preference and payoff assumptions. As such, we expect that the key effects of belief distortion on anticipatory utility (the information and speculative effects) should be qualitatively robust. Our focus is the CARA-normal setting, however, because this allows us to characterize in closed form the (general) equilibrium effects of subjective belief choice.

In the analysis that follows, we characterize equilibria using conditions on the price informativeness measure τ_p, which is a "derived" parameter (i.e., it depends on model primitives τ_e, τ_z and γ). Objective price informativeness is the model-relevant measure that determines equilibrium beliefs, and we present our results in this manner to highlight the economic intuition. However, we establish these conditions while keeping other parameters fixed, and so one should interpret changes in τ_p as corresponding to changes in either τ_z or γ, while keeping τ_e and τ constant.

4.2 Belief choice when others exhibit rational expectations

Before characterizing the equilibrium subjective belief choice across all investors, we begin by characterizing the optimal belief choice of investor i when all other investors exhibit rational expectations. The price faced by investor i is the same as in a rational expectations equilibrium: taking others' beliefs as given, she sets $\bar{\delta}_{p,i} = 1$ and so $P = m + \Lambda(s_p - m)$ where $\Lambda = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p}$. We show that, because Λ is relatively high so is the speculative effect. As a result, strategic substitutability in subjective belief choice implies that investor i chooses to underweight the information in prices.

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[21]Note that subjective belief choice is not always a strategic substitute in the standard sense, since $\frac{\partial^2 AU_i}{\partial \delta_{p,i} \partial \delta_p}$ is not negative everywhere. However, the above expressions suggest that the relative effect characterized in (24) is decreasing in Λ and, more formally, we can show that $\frac{\partial}{\partial \Lambda}\left(\frac{1}{AU_i}\frac{\partial AU_i}{\partial \delta_{p,i}}\right) < 0$.

The following proposition characterizes the optimal beliefs of investor i in this setting.

Proposition 1. *Suppose all other investors exhibit rational expectations. If investor i is subject to the experienced utility penalty, she chooses to completely ignore price information i.e., $\delta_{p,i} = 0$, when price informativeness, τ_p, is sufficiently high or sufficiently low.*

While we are able to analytically prove the result for τ_p sufficiently high and sufficiently low, numerical simulation suggest that the result holds generally for all τ_p. Intuitively, investor i's anticipatory utility is high when she believes that others are uninformed, i.e., when $\delta_{p,i} = 0$. While this implies that investor i faces more uncertainty about fundamentals and so trades less aggressively (the information effect), she expects her trades to be more profitable because others are trading on private and price signals that are essentially noise (the speculative effect). Moreover, the cost of belief distortion under the experienced utility penalty is relatively small when others exhibit rational expectations. Because the price is (objectively) informationally efficient, choosing an objectively inefficient position in the risky asset ($x_i^*(\delta_{p,i})$ instead of $x_i^*(1)$) is less costly. Together this implies that the investor chooses to completely dismiss the information in prices (set $\delta_{p,i} = 0$) when others exhibit rational expectations.

Standard intuition suggests that behavioral investors are worse off in an environment with rational investors. With wishful thinking, however, it is the presence of the rational investors which allows the investor to deviate from rational expectations. The key insight is that it is not as costly to dismiss the information in prices as long as others trade on it efficiently because the loss (per dollar of trade) is minimized in this case (i.e., $|F - P|$ is minimized). This suggests that the increased presence of rational investors need not drive out investors who engage in wishful thinking but may, instead, encourage such investors to distort their beliefs further. This provides a distinctive prediction of our model relative to settings in which investor biases are exogenously specified.

Ignoring price information when others condition on it is a result of strategic substitutability. In the next section, we utilize this same channel to show that when others dismiss the information in prices, investor i may choose to condition on this information efficiently. Morever, we show how this desire to deviate from the beliefs of others can lead to endogenous heterogeneity in the interpretation of price information.

4.3   General Equilibrium

We now turn to the general setting in which all investors optimally choose their beliefs about the quality of the information contained in prices. The following result characterizes the equilibrium when all investors choose their subjective beliefs about price information.

Proposition 2. *Suppose all investors incur the experienced utility penalty. Then there exists both $\bar{\tau}_p \geq \underline{\tau}_p > 0$, such that:*

1. *For all $\tau_p \leq \underline{\tau}_p$, there exists a unique equilibrium in which all investors ignore the information in prices (i.e., $\delta_{p,i} = \delta_p = 0$ for all i).*

2. *For all $\tau_p \geq \bar{\tau}_p$, there does not exist a pure symmetric equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$. The unique equilibrium is one in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i} = 0$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{p,i} = 1$, where λ is given by*

$$\lambda = 1 + \frac{\tau_e}{\tau_p} - \sqrt{\frac{\tau}{3\tau_p}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)} \tag{29}$$

The above result establishes sufficient conditions for the existence and uniqueness of two types of equilibria. When price informativeness is sufficiently low, the unique equilibrium is one in which all investors dismiss the information in prices *completely*. We refer to this as the **dismissive** equilibrium. On the other hand, when price informativeness is sufficiently high, we show that there cannot exist an equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ for all i).[22] We refer to this as the **mixed** equilibrium.

The intuition for this result builds on our earlier observations. When price informativeness is low, the information effect is small since dismissing the information in prices has little impact on investors' perceived uncertainty (and their resultant position in the risky asset). However, there is still a meaningful speculative effect regardless of the information others infer from the price. This is because each investor believes others are trading on uninformative *private* signals, which introduces more opportunities for profitable trade. In fact, in the limit, as $\tau_p \to 0$, note that we have:

$$\underbrace{-\frac{1}{\mathrm{var}_i[F|s_i, s_p]}\frac{\partial \mathrm{var}_i[F|s_i, P]}{\partial \delta_{p,i}} \to 0}_{\text{information effect}}, \quad \underbrace{\frac{1}{\mathrm{var}_i[F - P]}\frac{\partial \mathrm{var}_i[F - P]}{\partial \delta_{p,i}} \to \frac{1}{\delta_{p,i}}}_{\text{speculative effect}}. \tag{30}$$

Moreover, the experienced utility cost of dismissing the information in prices is not too large because the price is not very informative to begin with. Taken together, these effects lead to an equilibrium in which everyone dismisses the information in the price.

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[22]Technically, the first part establishes sufficient conditions for the existence of a pure strategy, symmetric equilibrium in pure strategies, while the second part establishes the non-existence of such equilibria and the existence of mixed-strategy, symmetric equilibria.

On the other hand, when price informativeness is objectively high, whether or not the speculative effect dominates the information effect depends on the equilibrium behavior of others. For instance, suppose all other investors choose not to efficiently condition on the price so that $\int_j \delta_{p,j} di = \delta_p < 1$ for all i. Then, as $\tau_p \to \infty$, we have:

$$-\frac{1}{\text{var}_i[F|s_i, s_p]}\frac{\partial \text{var}_i[F|s_i, P]}{\partial \delta_{p,i}} \to \frac{1}{\delta_{p,i}}, \quad \frac{1}{\text{var}_i[F - P]}\frac{\partial \text{var}_i[F - P]}{\partial \delta_{p,i}} \to 0. \tag{31}$$

This implies that in the limit, because others under-react to the information in prices, the information effect dominates and investor i has an incentive to choose a *higher* $\delta_{p,i}$. Moreover, since the cost of holding rational expectations (i.e., setting $\delta_{p,i} = 1$) is zero, investor i would choose to deviate to this point. However, Proposition 1 implies that there cannot exist an equilibrium in which all investors choose $\delta_{p,i} = 1$. As we show in the proof of Proposition 2, similar reasoning implies that for any sufficiently high τ_p, there cannot exist an equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$.

Figure 1: Anticipatory utility net of costs versus $\delta_{p,i}$

The figure plots the anticipatory utility net of costs for investor i as a function of her choice $\delta_{p,i}$. Other parameters are: $\tau = \tau_e = \tau_z = 1$, $\gamma = 0.3$. The solid blue dots in each panel indicate the maxima of the objective function.

(a) $\delta_p = 0$ (b) $\delta_p = 1$ (c) Mixed eqm.
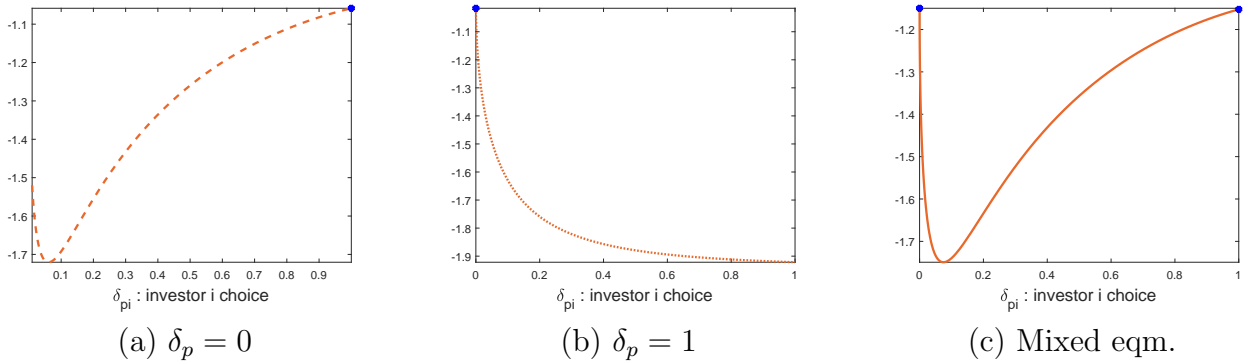
Panels (a) and (b) of Figure 1 provide a numerical illustration of this non-existence argument. The panels show investor i's anticipatory utility, net of costs, as a function of $\delta_{p,i}$, given the behavior of others. In panel (a), all other investors choose $\delta_p = 0$. In this case, investor i has an incentive to deviate by setting $\delta_{p,i} = 1$, since the information effect dominates the speculative effect. In panel (b), all other investors choose $\delta_p = 1$. Now, the speculative effect dominates and investor i strictly prefers to ignore the information in prices. In both cases, a symmetric equilibrium is ruled out because an individual investor has an incentive to deviate from the equilibrium behavior of others.

Panel (c) of Figure 1 illustrates an instance of the mixed equilibrium. In this case, each investor is indifferent between two beliefs. In equilibrium, one set of investors (a fraction $\lambda = 0.95$) ignore the information in prices completely (i.e., choose $\delta_{p,i} = 0$) while the remaining fraction ($1 - \lambda = 0.05$) interpret the information in prices correctly (i.e., set $\delta_{p,i} = 1$). Both choices are maxima, given the beliefs of others, and so the mixed equilibrium arises.

Figure 2: Composition of equilibrium beliefs: λ versus underlying parameters
The figure plots the equilibrium fraction of investors choosing $\delta_{p,i} = 0$, λ, as a function of the model primitives. Other parameters are: $\tau = \tau_e = \tau_z = 1$, $\gamma = 0.3$. The blue regions indicate the parameter combinations where we have the dismissive equilibrium in which all the agents ignore price information (i.e., $\delta_{p,i} = 0$).
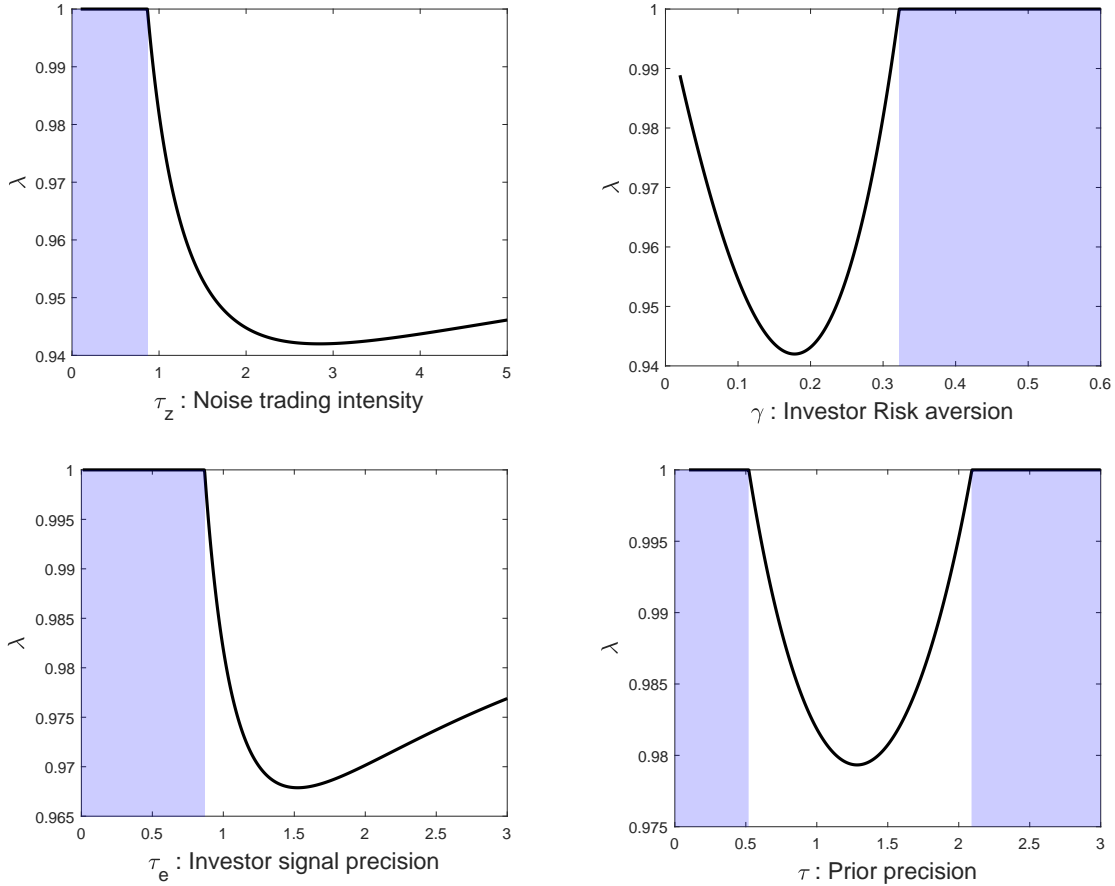
Proposition 2 also provides a characterization of how the composition of equilibrium beliefs (i.e., λ) depends on the underlying parameters of the model, which we illustrate numerically in Figure 2. The shaded region of each panel corresponds to the dismissive equilibrium (i.e., $\lambda = 1$), while the unshaded region corresponds to the mixed equilibrium (i.e., $\lambda < 1$). Recall that price informativeness τ_p is increasing in τ_e and τ_z, but decreasing

in γ, since $\tau_p = \frac{\tau_e^2 \tau_z}{\gamma^2}$. This naturally implies that the dismissive equilibrium obtains when γ is sufficiently high, or when τ_e or τ_z are sufficiently low.

Within the mixed equilibrium parameter space, however, λ is U-shaped in τ_e, τ_z and γ. To understand this non-monotonicity, consider the case of τ_z (the other parameters are analogous). When τ_z is sufficiently large (i.e., τ_p is sufficiently large), the dismissive equilibrium cannot be sustained. For initial increases in τ_z in this region, the mass of investors who condition on the price increases (i.e., λ falls). This is because the information effect dominates the speculative effect and it is relatively costly to dismiss price information. As τ_z increases, however, and more investors condition on the price, Λ increases making the speculative channel more relevant. When τ_z is sufficiently high, this swamps the information effect and, consequently, λ increases again. This is because when prices are sufficiently informative *and* a sufficient measure of other investors are incorporating this information into the price, it is no longer as costly to deviate from rational expectations.

The relation between λ and the precision of investors' prior beliefs, τ, is qualitatively and economically distinct from that found with the parameters that drive τ_p. Specifically, the dismissive equilibrium obtains when prior precision is sufficiently high or sufficiently low, while the intermediate values give rise to the mixed equilibrium. When prior precision is extremely high, the information effect is relatively small and the cost of distorting beliefs is low. Intuitively, when investor i faces little uncertainty about fundamentals, dismissing the information in prices is not very costly in terms of uncertainty reduction or inefficient investment. On the other hand, when the prior precision is low, it is costly to dismiss the information in prices; however, the speculative effect is also large, and it dominates the information effect. To see this, it is useful to return to (25)-(26), the expressions that characterize the marginal impact of the information and speculative effects. As prior precision decreases (i.e., $\tau \to 0$), these expressions show that the information effect is bounded above, but the speculative effect continues to decrease with $\delta_{p,i}$ i.e.,

$$ -\frac{1}{\operatorname{var}_i[F|s_i, s_p]} \frac{\partial \operatorname{var}_i[F|s_i, P]}{\partial \delta_{p,i}} \to \frac{\tau_p}{\tau_e + \delta_{p,i} \tau_p}, \quad \frac{1}{\operatorname{var}_i[F - P]} \frac{\partial \operatorname{var}_i[F - P]}{\partial \delta_{p,i}} \to \frac{1}{\delta_{p,i}}. \tag{32} $$

Thus, for sufficiently low $\delta_{p,i}$, the speculative effect dominates irrespective of what other investors choose, and the resulting equilibrium features dismissiveness (i.e., $\delta_{p,i} = 0$ for all i). For intermediate levels of prior precision, the relative magnitude of the speculative effect depends on the behavior of other investors, and so the mixed equilibrium obtains.

The following is a notable corollary of Proposition 2.

Corollary 1. *As $\tau_z \to \infty$, $\lambda \to 1$.*

This result implies that even as the noise in prices become arbitrarily small, the fraction

of investors who utilize price information must be zero. Intuitively, this follows from the fact that as prices become arbitrarily informative, the cost of distorting beliefs (i.e., ignoring the price) becomes smaller.[23]

This is in sharp contrast to the implications of a rational expectations equilibrium. In noisy RE models (e.g., Grossman and Stiglitz (1980)), the fraction of uninformed investors who condition on prices *increases* as the informativeness of prices increases. In the limit, as $\tau_z \to \infty$, rational expectations investors ignore their private signals and only condition on the price (i.e., $A_i \to 0$, $B_i \to 1$). In contrast, under wishful thinking, as $\tau_z \to \infty$, more investors ignore the price and condition only on their private information (i.e., $A_i > 0$, $B_i = 0$). This is true even though the cost of choosing rational expectations is zero in our setting. This qualitative difference suggests that allowing for subjective belief choice about how investors interpret information is likely to play an important role in understanding how markets aggregate and reflect information.

Finally, it is worth noting that there are two, closely-related but distinct, types of disagreement that arise in our model. In the dismissive equilibrium, each investor believes that their own signal is informative but all other investors disagree. As a result, however, all investors *agree* that the price is uninformative and choose to ignore it. In the mixed equilibrium, positive measures of investors disagree about the quality of others' signals and so also disagree about the quality of price information. In models with difference of opinions or cursedness, investors also underweight or dismiss the information in prices, but the extent to which they do so is determined as an exogenous parameter (e.g., see Banerjee (2011) or Eyster et al. (2018)).[24] In contrast, our model generates the fraction of dismissive investors λ as an *endogenous* outcome which depends on other primitive parameters of the model.

5 Generalized Model and Extensions

5.1 Beliefs about supply shocks and unconstrained $\delta_{p,i}$

In the benchmark model, investors are assumed to have correct beliefs about the precision of supply shocks which constrains $\delta_{p,i} \leq 1$, i.e., investors cannot over-estimate the quality of price information. However, if we allow investors to choose their beliefs about the volatility of noise trading (i.e., τ_z), this constraint can be relaxed. Suppose investor i's subjective

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[23]Note that the limit of the equilibria is not an equilibrium of the limit. If there is no aggregate noise in price (i.e., $\tau_z = \infty$), then we revert to a version of the Grossman (1977) paradox, since investors ignore their private information when forming demands.

[24]In other words, disagreement about private signals is driven by exogenous parameters. In Mondria et al. (2020), under-reaction to price information does not stem from disagreement about private information.

beliefs about z are given by

$$z \sim_i \mathcal{N}\left(0, \frac{1}{\psi_i \tau_z}\right),\tag{33}$$

where $\psi_i \in [0, \infty)$ is chosen along with $\{\pi_i, \rho_i\}$ to maximize (10). Then, it is straightforward to show that we can redefine $\delta_{p,i}$ as

$$\delta_{p,i} \equiv \frac{\frac{\beta^2}{\tau_z}}{\frac{\beta^2}{\psi_i \tau_z} + \frac{1 - \rho_i^2}{\pi_i \tau_\eta}} \in [0, \infty].\tag{34}$$

Then, for sufficiently large ψ_i, investor i chooses to overestimate the precision of the price signal (i.e., $\delta_{p,i} > 1$). The following result shows how Proposition 2 generalizes to this setting.

Proposition 3. *Suppose all investors incur the experienced utility penalty. Then there exists some $\bar{\bar{\tau}}_p \geq \underline{\underline{\tau}}_p > 0$, such that:*

1. *For all $\tau_p \leq \underline{\underline{\tau}}_p$, there exists a unique equilibrium in which all investors ignore the information in prices (i.e., $\delta_{p,i} = \delta_p = 0$ for all i).*

2. *For all $\tau_p \geq \bar{\bar{\tau}}_p$, there does not exist a pure symmetric equilibrium in which all investors choose the same subjective beliefs $\delta_{p,i} = \delta_p$. If there exists a solution (λ, δ_p^*) to equations (74)-(75) in the Appendix, then there exists an equilibrium in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i} = 0$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{p,i} = \delta_p^* > 1$.*

When $\delta_{p,i}$ is unconstrained, the above proposition implies that any mixed equilibrium will involve a mass of $1 - \lambda$ of investors who overweight the information in prices (instead of setting $\delta_{p,i} = 1$). The intuition parallels that of Proposition 2, but the characterization of the mixed equilibrium is less explicit since it is characterized by the pair $\{\lambda, \delta_p^*\}$ which solve a pair of non-linear equations.

It is worth noting that similar heterogeneity in investment strategies can arise in models where investors are heterogeneously informed by assumption.[25] Our model is distinct in that such heterogeneity arises in otherwise ex-ante identical traders. This implies that the observation of conflicting biases or investment styles does not require that investors are endowed with differential ability, preferences or information. Moreover, our model provides predictions about when we are more likely to see such heterogeneity arise endogenously as a result of market conditions, a feature we explore in Section 6.

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[25]For example, in noisy RE models with heterogeneously informed investors (e.g., Wang (1993)), better informed investors have as fundamental traders or contrarians, while less well informed traders condition on price information and behave like momentum traders (see also Brown and Jennings (1989)).

5.2 Beliefs about Private Signals

In this section, we explore investors' subjective beliefs about their *own* private information and how it interacts with their subjective beliefs of others' signals. Specifically, suppose investor i's subjective beliefs about her private signal is given by:

$$s_i | F \sim_i \mathcal{N} \left(F, \frac{1}{\delta_{e,i} \tau_e} \right), \tag{35}$$

where $\delta_{e,i} \in [0, \infty)$ denotes the extent to which investor i under- or over-estimates the precision of her private signal. As before, $\delta_{e,i} = 1$ corresponds to rational expectations and deviations from this generate a cost due to the experienced utility penalty (appropriately modified).

The key difference from our benchmark analysis is that subjective beliefs about the private signal only affect anticipatory utility via the information effect. The perceived precision of one's own signal affects each investor's posterior uncertainty about fundamentals (i.e., $\text{var}_i[F | s_i, s_p]$), but does not impact their perception of the unconditional uncertainty about returns (i.e., $\text{var}_i[F - P]$). Specifically,

$$\frac{\partial AU_i}{\partial \delta_{e,i}} \propto -\frac{1}{\text{var}_i[F | s_i, s_p]} \frac{\partial \text{var}_i[F | s_i, s_p]}{\partial \delta_{e,i}} = \frac{\tau_e}{\tau + \delta_{e,i} \tau_e + \delta_{p,i} \tau_p} \geq 0 \tag{36}$$

The absence of any speculative effect implies that investors never choose to underweight their private information in equilibrium.

To illustrate this effect, we consider a special case in which investors are constrained to have objective beliefs about the price signal (i.e., we assume $\delta_{p,i} = 1$ for all i), but can choose their beliefs about private signals. The following result characterizes the equilibrium.

Proposition 4. *Suppose all investors incur the experienced utility penalty, and exhibit objective beliefs about the price signal, i.e., $\delta_{p,i} = 1$ for all i. Then, there exists a unique equilibrium in which the optimal choice of $\delta_{e,i} = \delta_e$ satisfies:*

$$\frac{(\tau + \tau_p + \tau_e \delta_e (2 - \delta_e))^{\frac{3}{2}}}{(\tau + \tau_p + \tau_e \delta_e)^{\frac{3}{2}}} = 2 (\delta_e - 1). \tag{37}$$

All investors exhibit overconfidence (i.e., $\delta_e > 1$) and the equilibrium degree of overconfidence, δ_e, (i) increases with τ and τ_z, (ii) decreases with risk-aversion γ, and (iii) is U-shaped in τ_e.

While the observation that investors choose to exhibit overconfidence about their private information is intuitive given the preceding discussion, our characterization also allows us

to explore how overconfidence is shaped by the economic environment. As prior uncertainty falls (τ increases) and as the quality of the information in prices rises (τ_z increases or γ decreases), the marginal benefit of overconfidence falls: it provides a smaller increase in the investor's perceived information advantage. Interestingly, however, the cost of overconfidence falls even faster when investors have access to better outside information, and so equilibrium overconfidence is higher.[26] Similar logic applies with respect to the quality of the investor's private signal when τ_e is high (as it, too, increases the quality of price information). However, when τ_e is low, the cost of increasing overconfidence in a relatively noisy private signal outweighs the benefit and, as a result, δ_e is non-monotonic with respect to τ_e.

Overconfidence leads investors to trade more aggressively which, at first glance, is analogous to a setting in which investors have objectively more precise information. However, when we compare our results to a setting in which investors acquire information endogenously, distinct predictions arise. For instance, Verrecchia (1982) shows that investors choose to acquire more precise information when (i) prior uncertainty is higher (i.e., τ is lower) and (ii) noise in prices is higher (i.e., τ_z is lower). Intuitively, the benefit of acquiring private information is larger when investors' given information sources are of lower quality. In contrast, the opposite comparative statics arise for endogenous beliefs: it is less costly for investors to exhibit overconfidence when (i) prior uncertainty is lower, and (ii) prices are more informative. As such, one could distinguish our model's predictions from one based on costly information acquisition by comparing how dispersion in expectations or trading positions vary with prior uncertainty or price informativeness.

Next, we consider the case in which each investor optimally chooses subjective beliefs about both her private signal and the price signal. The partial derivative of equation (36) with respect to $\delta_{p,i}$ yields

$$\frac{\partial^2 AU}{\partial \delta_{e,i} \partial \delta_{p,i}} \propto \left(\frac{\kappa \left(\delta_{e,i}\tau_e + \tau \right) - 2\delta_{p,i}\tau_p\kappa - 3\delta_{p,i}^2 \tau_p}{2\delta_{p,i} \left(\kappa + \delta_{p,i} \right)} \right), \text{where} \tag{38}$$

$$\kappa \equiv \left(\frac{\Lambda}{1-\Lambda} \right)^2 \frac{\tau}{\tau_p}, \quad \text{and} \quad \Lambda = \frac{\bar{\bar{\delta}}_{e,i}\tau_e + \bar{\bar{\delta}}_{p,i}\tau_p}{\tau + \bar{\bar{\delta}}_{e,i}\tau_e + \bar{\bar{\delta}}_{p,i}\tau_p} \tag{39}$$

In general, this implies that the marginal benefit of overconfidence depends non-monotonically on the investor's subjective interpretation of the price signal $\delta_{p,i}$. For instance, when $\delta_{p,i}$ is sufficiently small, $\frac{\partial^2 AU}{\partial \delta_{e,i} \delta_{p,i}} > 0$: a decrease in $\delta_{p,i}$ lowers the marginal utility of increasing $\delta_{e,i}$. Intuitively, when $\delta_{p,i}$ is low (close to zero), choosing to be increasingly dismissive of the price (i.e., moving further away from rational expectations) lowers the marginal value of overconfidence about one's private signal. On the other hand, when $\delta_{p,i}$ is sufficiently large,

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[26]This can be seen by evaluating the numerator of equation (37).

the effect is reversed. For instance, when $\delta_{p,i}$ is higher than 1, a decrease in $\delta_{p,i}$ implies *more* objective beliefs about price information which increases the marginal benefit of distorting beliefs about private information.

Taken together, the above suggests that an investor gains more from distorting her subjective beliefs about private signals when her beliefs about the price information are less distorted. We show that this leads to the following characterization in any pure-strategy, symmetric equilibrium.

Proposition 5. *Suppose all investors incur the experienced utility penalty. There exists a* $\underline{\tau}_p > 0$, *such that for all* $\tau_p \leq \underline{\tau}_p$, *there exists a symmetric equilibrium in which* $\delta_{p,i} = \bar{\delta}_{p,i} \equiv \delta_p$ *and* $\delta_{e,i} = \bar{\delta}_{e,i} \equiv \delta_e$ *for all* i.
(i) If subjective belief choice is unconstrained (i.e., $\delta_{p,i}, \delta_{e,i} \in [0, \infty)$*), then the equilibrium choices are* $\delta_e = 1$ *and* $\delta_p = 0$.
(ii) If subjective belief choices about the price signal are bounded below (i.e., $\delta_{p,i} \geq \underline{\delta} > 0$*), then the symmetric equilibrium choices are* $\delta_p = \underline{\delta}$ *and* δ_e *satisfies*

$$\frac{1}{2\left(1 + \frac{\kappa}{\underline{\delta}}\right)^{\frac{1}{2}} (\tau_e \delta_e + \tau_p \underline{\delta} + \tau)^{\frac{3}{2}}} = \frac{(1 + \kappa)(\delta_e - 1)}{\left[(1 + \kappa)\left(\tau + \tau_e + \tau_p - \tau_e(\delta_e - 1)^2\right) - \tau_p(\underline{\delta} - 1)^2\right]^{\frac{3}{2}}} \quad (40)$$

where $\kappa = \frac{(\delta_e \tau_e + \underline{\delta}\tau_p)^2}{\tau \tau_p}$.

Proposition 5 extends our benchmark analysis to allow for subjective beliefs about private signals. As in Propositions 2 and 3, we show that there exists a unique symmetric equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ and $\delta_{e,i} = \delta_e$ for all i) when the price is not too informative.[27] Our earlier analysis implied that in a symmetric equilibrium, investors optimally choose to set $\delta_{p,i}$ as low as possible because the speculative effect always dominates the information effect. Proposition 5 shows that this remains the case when investors also choose how to interpret their own private signal.

As a result, when beliefs about price informativeness are unconstrained, investors choose to completely dismiss price information i.e., set $\delta_{p,i} = 0$. However, this also implies that investors condition on private information efficiently, i.e., $\delta_{e,i} = 1$. This is because when $\delta_{p,i} = 0$, investor i does not benefit from distorting her beliefs about the private signal (i.e., $\frac{\partial AU}{\partial \delta_{e,i}}|_{\delta_{p,i}=0} = 0$). When beliefs about price informativeness are constrained below at $\underline{\delta}$, investors still choose to maximally distort their beliefs about price informativeness (i.e., $\delta_{p,i} = \underline{\delta}$) but now also exhibit over-confidence in their private signal (i.e., $\delta_{e,i} = \delta_e > 1$).

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[27]While numerical analysis shows that analogous mixed equilibria exist when τ_p is sufficiently high, and we can characterize the equations which pin down such equilbria, we are unable to analytically establish their existence or uniqueness.

Figure 3 provides an illustration of this result: as the lower bound $\underline{\delta}$ increases, the equilibrium choice of δ_e also increases. This is consistent with the intuition above: each investor gains more from distorting her beliefs about private information when her beliefs about prices are closer to the objective distribution.

Figure 3: Plot of optimal δ_e vs the lower bound $\underline{\delta}$

The figure plots the equilibrium choice of δ_e in a symmetric equilibrium as a function of the lower bound on $\delta_{p,i}$, given by $\underline{\delta}$. Other parameters are: $\tau = \tau_e = \tau_z = 1$, $\gamma = 3$.
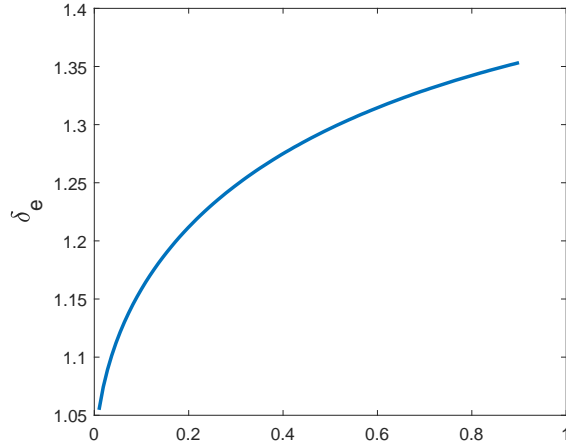
The state dependence of these predictions distinguish our model not only from standard rational expectations models but also other behavioral models. Moreover, the endogenous interaction of these biases in our setting provides a distinctive prediction relative to other models which separately consider over-confidence (e.g., Odean (1998)) or dismissiveness of price information (e.g., Eyster et al. (2018), Mondria et al. (2020)). Some aspects of these settings lead to superficially similar behavior (e.g., overconfidence implies that investors place less weight on price information). In contrast, in our model both types of biases arise endogenously as distinct phenomena as a consequence of a single assumption: individuals experience anticipatory utility. Moreover, our analysis predicts a negative relation between price dismissiveness and overconfidence in private information, which may be testable empirically. We discuss this further in Section 6.

5.3 Generalized cost functions

Our benchmark analysis shows that investors are generally dismissive of price information when the investor's cost of distorting beliefs is measured utilizing the experienced utility penalty. In this section, we show that under-reacting to price information is a robust con-

sequence of wishful thinking that arises more generally for a broad class of cost functions. Specifically, suppose that the cost function $C(\delta_{p,i})$ is well-behaved as defined below.

Definition 2. A cost function $C(\delta_{p,i})$ is **well-behaved** if $C(1) = \frac{\partial C}{\partial \delta}(1) = 0$, and $C(\cdot)$ is strictly convex (i.e., its global minimum is at $\delta_{p,i} = 1$).

The following result establishes that if a pure strategy equilibrium exists, it must feature under-reaction to price information. Notably, this result applies even in settings where $\delta_{p,i}$ is unconstrained (i.e., $\delta_{p,i} \in [0, \infty]$) and when investors choose beliefs about their private signals as well as the price.

Proposition 6. *Suppose the cost function is well-behaved. If there exists an equilibrium in which all investors choose the same subjective beliefs (i.e., $\delta_{p,i} = \delta_p$ for all i), then investors must discount the information in prices i.e., $\delta_{p,i} = \delta_p < 1$.*

The equilibrium under-reaction to price information is a consequence of the strategic substitutability in subjective belief choice we discussed in Section 4.1. Consider the optimal choice for investor i in a symmetric equilibrium where all other investors choose δ_p. As Lemma 2 establishes, anticipatory utility is U-shaped in $\delta_{p,i}$. In the proof of Proposition 6, we take this result a step further, showing that investor i's anticipatory utility is always decreasing in $\delta_{p,i}$ at $\delta_{p,i} = \delta_p$. Intuitively, investor i can improve her ability to speculate against others by decreasing the perceived precision of price information, thereby decreasing the correlation between her conditional valuation (μ_i) and those of others ($\int_j \mu_j dj$).

Next, recall that for a well-behaved cost function, deviations away from rational expectations (i.e., $\delta_{p,i} = 1$) are penalized i.e., the cost function is decreasing below one and increasing above one. But this implies that the equilibrium choice of $\delta_{p,i}$ cannot be higher than one, since if it were, investor i could increase anticipatory utility and decrease costs by lowering $\delta_{p,i}$. As such, in any symmetric equilibrium, investors must set $\delta_{p,i} < 1$.

In general, whether or not a symmetric equilibrium exists will depend on the characteristics of the cost function. In our benchmark analysis, the experienced utility penalty is depends on equilibrium choices Λ. In other models of subjective belief choice (e.g., robust control), the cost function is often specified in terms of a statistical distance measure (e.g., the K-L distance), and usually does not depend on equilibrium choices. In Appendix B.3, we numerically explore the impact of using the K-L distance as the cost function in the benchmark model of Section 3. When τ_p is sufficiently low, there exists a pure symmetric equilibrium in which all investors choose a $\delta_{p,i} = \delta_p \in (0, 1)$.[28] When τ_p is sufficiently high,

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[28]This is because the KL distance based cost function is more convex than the experienced utility penalty when $\delta_{p,i}$ is small. Hence, the optimal subjective belief choice is interior, and not $\delta_{p,i} = 0$ as with the experienced utility penalty.

there exists a mixed equilibrium in which a fraction λ optimally choose $\delta_{p,i} = \delta_p \in (0, 1)$, while the remaining $1 - \lambda$ investors choose to hold rational expectations. While a complete analysis with general cost functions is beyond the scope of this paper, this numerical illustration suggests that our results are qualitatively robust to alternative specifications of cost functions: while the dependence on the equilibrium price sensitivity Λ is a distinctive feature of the experienced utility penalty, it is not essential in generating the main results.

6   Implications

When investors experience anticipatory utility, our model predicts that they will systematically and predictably deviate from rational expectations. In this section, we explore how these deviations lead to distinctive predictions for observables. Section 6.1 provides predictions for return and volume moments. Section 6.2 provides implications for investor forecasts.

Our primary focus is to distinguish our model's predictions from the rational expectations benchmark. To be clear, a number of our predictions appear to be similar to those that would arise in existing behavioral models. For instance, as in models of difference of opinions, cursedness, or costly price information, returns can exhibit positive predictability in our setting. However, the key distinction of our model is that the predictability coefficient is state-dependent, since investors' bias is endogenous, and its relation with other observables will depend on economic conditions.

6.1   Return and volume moments

Since the risk-free security is the numeraire, the (net) return on it is zero and, consequently, the (dollar) return on the risky security is given by $R \equiv F - P$. We begin by characterizing implications for return predictability and return volatility, since these help distinguish our model from others. We then characterize predictions for expected returns and expected volume.

6.1.1 Return predictability and volatility

Return predictability. While the unconditional return on the risky asset is zero, the expected return conditional on the price is[29]

$$\mathbb{E}\left[F - P \middle| P\right] = m + \theta\left(P - m\right), \text{ where} \tag{41}$$

$$\theta \equiv \frac{\text{cov}\left(F - P, P\right)}{\text{var}\left(P\right)} = \frac{1}{\Lambda}\left(\frac{\tau_p}{\tau + \tau_p} - \Lambda\right) \tag{42}$$

The return predictability coefficient, θ, reflects the degree to which changes in the price predict changes in the risky asset's return. When $\theta < 0$, prices exhibit *reversals*, while $\theta > 0$ implies prices exhibit *drift*.

Return volatility. The unconditional variance in returns, which we also refer to as return volatility, is given by

$$\sigma_R^2 = \text{var}\left(F - P\right) = \frac{\left(1 - \Lambda\right)^2}{\tau} + \frac{\Lambda^2}{\tau_p}. \tag{43}$$

To gain some intuition for the above characterizations, we make a few observations. First, when investors choose how to interpret both their and others' private signals, the equilibrium price sensitivity to s_p can be expressed as:

$$\Lambda \equiv \frac{\bar{\delta}_{e,i}\tau_e + \bar{\delta}_{p,i}\tau_p}{\tau + \bar{\delta}_{e,i}\tau_e + \bar{\delta}_{p,i}\tau_p}. \tag{44}$$

All else equal, believing that others are less well-informed in aggregate (i.e., $\delta_{p,i} < 1$) decreases Λ relative to rational expectations, while over-confidence in private information (i.e., $\delta_{e,i} > 1$) increases Λ.

Second, prices exhibit reversals if and only if $\Lambda > \frac{\tau_p}{\tau + \tau_p}$, but exhibit drift otherwise. Thus, equation (44) implies that (i) prices always exhibit reversals under rational expectations (since $\delta_{e,i} = \delta_{p,i} = 1$), and (ii) prices cannot exhibit drift unless investors under-react to price information (i.e., $\delta_{p,i} < 1$).

Finally, return volatility increases with Λ if and only if $\Lambda > \frac{\tau_p}{\tau + \tau_p}$. Price sensitivity has two offsetting effects on volatility. When Λ is higher, prices reflect fundamentals more closely and this reduces volatility in returns (via the $\frac{(1-\Lambda)^2}{\tau}$ term in (43)). On the other hand, prices are also more sensitive to the noise in prices which increases volatility (via the $\frac{\Lambda^2}{\tau_p}$ term). The first effect dominates when Λ is low, but the second dominates when Λ is sufficiently

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[29]Unconditionally, the risky security in zero net supply: there is no aggregate risk and so $\mathbb{E}\left[R\right] = 0$. Below, we discuss an extension in which the expected aggregate supply of the asset is positive.

high.

These observations lead to the following predictions.

Proposition 7. *Suppose all investors incur the experienced utility penalty, and choose subjective beliefs about price information and private signals. Then,*
(i) Return predictability is always higher than under rational expectations and can exhibit drift.
(ii) Return volatility is higher than under rational expectations when prices are sufficiently informative.

Figure 4 provides a numerical illustration of these results when investors choose subjective beliefs about both their information as well as the signals of others. Specifically, the figure plots volatility and predictability for the rational expectations (dashed) and subjective beliefs equilibria (solid) as a function of risk aversion, γ. Recall that an increase in risk aversion makes prices objectively less informative (i.e., τ_p is decreasing in γ). The kink in the solid lines corresponds to the value of γ at which the subjective beliefs equilibrium switches from the mixed equilibrium (low γ, high τ_p) to the symmetric equilibrium (high γ, low τ_p). Consistent with Proposition 7, predictability is always higher under subjective beliefs than under rational expectations. Moreover, volatility is higher under subjective beliefs when prices are sufficiently informative (i.e., γ is sufficiently low), but lower otherwise.

Figure 4: Comparison of return and volume characteristics

The figure plots return volatility (variance) and return predictability as a function of risk aversion for subjective beliefs (solid line) and rational expectations (dotted line). Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $Z = 0$. The blue, shaded region corresponds to the symmetric equilibrium.
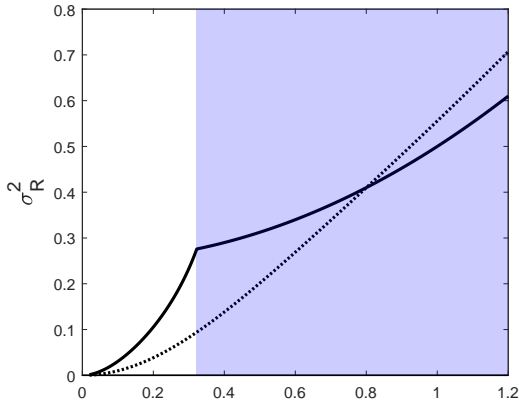
(a) Volatility vs. γ (b) Predictability θ vs. γ
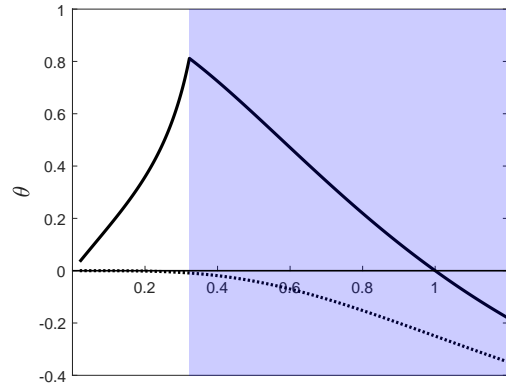
Overall, our results suggest that return predictability is a key distinguishing feature between the "wishful thinking" equilibria and the rational expectations equilibrium. In the traditional RE setting with exogenous, transient noise trading (e.g., Hellwig (1980)), returns exhibit reversals and so θ is always negative. Intuitively, an aggregate demand (supply) shock temporarily pushes the current price up (down, respectively). However, because the shock is not persistent, prices revert in the future. In our model, because some investors underweight the information in prices, prices are less responsive to these aggregate shocks. As a result, the return predictability coefficient is higher with wishful thinking than in the corresponding rational expectations equilibrium. Moreover, this under-reaction to price information can even lead to price drift when the noise trading volatility or risk-aversion is sufficiently high.[30]

The plots also illustrate how return volatility and predictability change endogenously in our model as a function of price informativeness. This yields qualitatively different predictions for the rational expectations an subjective beliefs equilibria, as summarized in the following result.

Proposition 8. *(1) In the rational expectations equilibrium, return volatility is decreasing in price informativeness while return predictability is increasing in price informativeness (i.e., $\frac{\partial}{\partial \tau_p} \sigma_R^2 < 0$ and $\frac{\partial}{\partial \tau_p} \theta > 0$).*
(2) Suppose all investors incur the experienced utility penalty, and choose subjective beliefs about the price signal only.

   (i) When τ_p is sufficiently low, return volatility is decreasing in price informativeness while return predictability is increasing in price informativeness (i.e., $\frac{\partial}{\partial \tau_p} \sigma_R^2 < 0$ and $\frac{\partial}{\partial \tau_p} \theta > 0$).

   (ii) When τ_p is sufficiently high, both return volatility and predictability decrease with price informativeness (i.e., $\frac{\partial}{\partial \tau_p} \sigma_R^2 < 0$ and $\frac{\partial}{\partial \tau_p} \theta < 0$).

Note that volatility is negatively related to price informativeness for both types of models, consistent with the empirical evidence documented by Dávila and Parlatore (2019). However, while predictability always increases with price informativeness for the rational expectations, it *decreases* with price informativeness in the mixed equilibrium with subjective beliefs. This suggests a useful approach for empirically identifying "wishful thinking". Specifically, while a positive relationship between volatility and predictability can arise in our subjective beliefs equilibria, it is inconsistent with rational expectations.[31]

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[30]This mechanism is similar to the one found in Hong and Stein (1999) and Banerjee et al. (2009).

[31]While a negative relation arises in rational expectations it can also follow from the symmetric equilibrium and so does not necessarily rule out "wishful thinking" equilibria.

More generally, to the extent that higher price informativeness is associated with bull markets and economic booms, our model predicts such periods are more likely to feature (i) lower volatility and (ii) a positive relation between return predictability (or time-series momentum) and volatility. In contrast, periods of high market stress, characterized by low risk tolerance, high uncertainty, and low price informativeness, are more likely to be associated with (i) higher volatility and (ii) a negative relation between predictability and volatility. Moreover, as Figure 4 illustrates, such periods are also associated with lower, or even negative, return predictability (i.e., reversals). These predictions appear broadly consistent with the evidence on time variation in momentum returns and crashes (e.g., see Cooper et al. (2004), Moskowitz et al. (2012), Daniel and Moskowitz (2016)).[32]

6.1.2 Expected Returns and Volume

Our benchmark analysis restricts the mean aggregate supply of the risky asset to $Z = 0$, which implies that the unconditional expected return is zero. In Appendix B.2, we consider an extension in which the aggregate supply of the risky asset is $Z > 0$. While this extension is not as analytically tractable, we solve it numerically and find that the resulting equilibria are qualitatively very similar to our benchmark.

The unconditional expected return is given by:

$$\mathbb{E}\left[R\right] = \frac{\gamma}{\int_i \omega_i di} Z, \tag{45}$$

where ω_i is investor i's posterior precision about F (i.e., $\omega_i = (\text{var}_i\left[F|s_i, s_p\right])^{-1}$), γ is the coefficient of risk aversion and Z is the aggregate supply of the risky asset. In general, subjective belief choice has two, potentially offsetting, effects on the posterior precision (ω_i) relative to the rational expectations equilibrium: over-confidence in private information increases ω_i, while dismissing price information decreases it.

Note that in the dismissive equilibrium with unconstrained subjective beliefs, all investors put zero weight on prices and correctly interpret their private information (see Proposition 5, part (i)). As a result, only the second effect is relevant, and expected returns are higher than under rational expectations. In the mixed equilibrium, we cannot characterize the net effect analytically but, as Figure 5 illustrates, we numerically find that the second effect dominates the first, and as a result expected returns are higher in the subjective beliefs equilibrium. Moreover, the difference in the risk premium relative to rational expectations falls within

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[32]It is important to note that return predictability in our model corresponds to what is described as time-series momentum in the literature, and not cross-sectional momentum. However, as Banerjee et al. (2009) show in a similar setting with multiple assets, the two notions of momentum may be related.

the mixed equilibrium as price informativeness increases since a positive measure of investors choose to condition on the price.

Figure 5: Expected returns

The figure plots the unconditional expected return as a function of risk aversion for subjective beliefs (solid line) and rational expectations (dotted line). Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $Z = 1$. The blue, shaded region corresponds to the symmetric equilibrium.
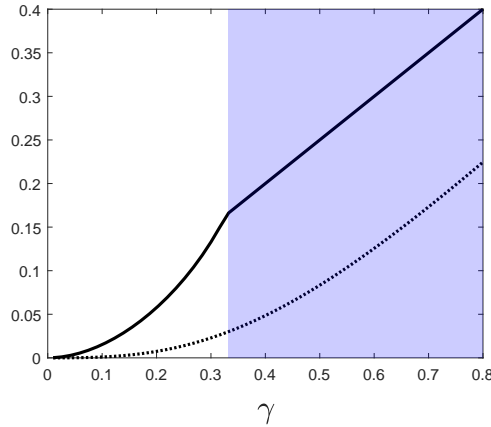
Expected Return vs. γ

In Appendix B.2, we establish that expected volume is generally higher under subjective beliefs than under rational expectations. First, when investors choose subjective beliefs about private information, they generally exhibit over-confidence. This increases disagreement relative to the rational expectations benchmark, which leads to higher volume on average. Second, when prices are sufficiently informative, the mixed equilibrium leads to an additional source of disagreement: the difference in interpretation of price information. This generates an additional source of trading volume, which is novel to our model.

6.2   Investor forecasts

The recent empirical literature has focused on the use of survey forecasts to test models of learning (e.g., Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma, and Shleifer (2020a)). In this section, we explore the implications of our model for these regressions and propose a new, simple test that distinguishes our model from both the rational expectations benchmark as well as the standard difference of opinions models.

Let $R = F - P$ denote the return on the stock and let $\mathbb{E}_i[R|s_i, P]$ denote investor i's forecast of this return. Since investors start with a common prior that $\mathbb{E}_i[R] = \mathbb{E}_i[F - P] = 0$,

the forecast revision for investor i is:

$$FR_i \equiv \mathbb{E}_i[R|s_i, P] - E_i[R]. \tag{46}$$

and the forecast error for investor i is:

$$FE_i \equiv R - \mathbb{E}_i[R|s_i, P]. \tag{47}$$

Taking averages across i gives us analogous expressions for the consensus forecast revision, $\bar{FR}$, and consensus forecast error, $\bar{FE}$.

The earliest papers in this literature (e.g., Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015)) focus on regressing consensus forecast errors on consensus forecast revisions, and document under-reaction, or equivalently, $\text{cov}(\bar{FE}, \bar{FR}) > 0$.[33] Note that this evidence does not help distinguish the class of models we consider, since:

$$\text{cov}(\bar{FE}, \bar{FR}) = \text{cov}(F - \bar{\mu}, \bar{\mu}) = \frac{\bar{\delta}_{e,i}\tau_e + \bar{\delta}_{p,i}(1 - \bar{\delta}_{p,i})\tau_p}{(\tau + \bar{\delta}_{e,i}\tau_e + \bar{\delta}_{p,i}\tau_p)^2}. \tag{48}$$

Notably, the above is positive in any model where $\bar{\delta}_{e,i} > 0$ and $\bar{\delta}_{p,i} \leq 1$, which includes rational expectations equilibria (where $\bar{\delta}_{e,i} = \bar{\delta}_{p,i} = 1$), symmetric difference of opinions or cursedness equilibria, and our symmetric "wishful thinking" equilibria.

Bordalo et al. (2020a) document that individual forecasts tend to exhibit over-reaction i.e., $\text{cov}(FE_i, FR_i) < 0$ even though consensus forecasts exhibit under-reaction. In our setting,

$$\text{cov}(FE_i, FR_i) = \text{cov}(F - \mu_i, \mu_i) = \frac{\delta_{e,i}(1 - \delta_{e,i})\tau_e + \delta_{p,i}(1 - \delta_{p,i})\tau_p}{(\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p)^2}. \tag{49}$$

This expression is zero both under rational expectations (since $\delta_{e,i} = \delta_{p,i} = 1$) and in pure difference of opinions models (since $\delta_{e,i} = 1$ while $\delta_{p,i} = 0$). However, the above coefficient can be negative in our model when investors choose to over-react to private signals (i.e., set $\delta_{e,i} > 1$) while discounting price information (i.e., set $\delta_{p,i} < 1$) (as in Section 5.2).[34] However, a negative coefficient also arises in models where investors simply exhibit over-confidence in their private signals.

To test our model's predictions more directly, we propose a regression of *individual fore-*

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[33]Bordalo, Gennaioli, Porta, and Shleifer (2019) show that consensus forecasts of firm's long term growth exhibit over-reaction. Since our model is static, we cannot distinguish between long term and short-term growth, and so cannot speak to this empirical evidence.

[34]Note that it is necessary to assume that $\delta_{p,i}$ is constrained below since otherwise the symmetric equilibrium implies $\delta_{e,i} = 1$, $\delta_{p,i} = 0$ as discussed in Section 5.2.

cast errors on lagged returns, i.e.,

$$FE_i = \alpha_i + \beta_i R_{-1} + u, \qquad (50)$$

where β_i measures the sensitivity of investor i's forecast error to lagged returns.[35] The regression coefficient β_i is proportional to

$$\beta_i \propto \text{cov}(FE_i, P) = \text{cov}(F - \mu_i, P) \propto \frac{1 - \delta_{p,i}}{\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p}. \qquad (51)$$

Under rational expectations, $\beta_i = 0$ for all investors: investors efficiently utilize their information and so ex-post forecast errors are unpredictable given date-t information. In models with difference of opinions, dismissiveness or cursedness, $\beta_i > 0$ for all investors since each investor under-reacts to the information in prices (i.e., $\delta_{p,i} < 1$).

However, in our model, the distribution of β_i across investors is state-dependent. Specifically, when price informativeness is low, all investors symmetrically under-react to the information in prices (i.e., $\delta_{p,i} = \delta_p < 1$), and so our model predicts a common, positive regression coefficient i.e., $\beta_i = \bar{\beta} > 0$. However, when price informativeness is high, the model predicts that investors exhibit heterogeneous reactions to price information. Specifically, we should observe that while some investors under-react to prices, (so that $\beta_i > 0$), the rest (weakly) over-react to prices and so $\beta_i \leq 0$.[36] As such, running regressions of the form (50) may help identify the presence of wishful thinking in financial markets. While an empirical analysis of this type is beyond the scope of the current paper, we hope future work explores such implications in more detail.

6.3   Investor Heterogeneity and Disagreement

Our model provides a mechanism through which investors, who are ex-ante identical and symmetrically informed, endogenously choose to exhibit different interpretation of public and private information in response to changing economic conditions. In contrast to settings in which investors are endowed with heterogeneous beliefs, our model predicts that the extent to which investors disagree about the interpretation of price information depends non-monotonically on objective price informativeness (see Figure 2). Specifically, when prices are extremely noisy (e.g., τ_z is very low) or very precise (e.g., τ_z is very high), most investors

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[35]In our model "lagged" returns are given by $R_{-1} = P - 0$. The advantage of running the regression at the investor level is that it controls for persistent investor-level differences in forecaster optimism (e.g., due to different priors).

[36]If we constrain $\delta_{p,i} \leq 1$ as in the benchmark analysis of Section 4, then $\beta_i = 0$. However, if we allow $\delta_{p,i} > 1$, then the mixed equilibrium of Section 5.1 implies $\beta_i < 0$.

choose to dismiss the information in prices, and so effectively agree on the interpretation of this information. However, for intermediate levels of price informativeness, many investors dismiss prices but others condition on it — this is where disagreement about the interpretation of prices is highest.

We note that the extent to which investors disagree also depends upon the behavior of other individuals. For instance, as Section 4.2 illustrates, when all other investors exhibit rational expectations, wishful thinking leads an individual investor to ignore the information in prices completely. On the other hand, when most other investors dismiss price information (and prices are sufficiently informative), some investors will choose to over-react to this information (e.g., see Section 5.1).

Finally, our model provides sharp predictions about how heterogeneity in investment strategies varies with economic conditions. For instance, the dismissive investors in our model resemble "fundamentals-based" or value investors who identify mis-priced securities using their private information. On the other hand, the investors who choose to overweight the information found in prices engage in behavior that (arguably) resembles technical or momentum trading, where investors over-extrapolate from past price changes.[37] To the extent that market booms are associated with higher price informativeness, our model suggests that such periods exhibit greater heterogeneity in investment strategies and, in particular, popularity of price / return based strategies. On the other hand, periods of market stress are associated with less diversity in investment strategies, and greater incidence of value investing.

7　Concluding Remarks

We develop a model in which investors who experience anticipatory utility choose how to interpret the information available to them before trading in financial markets. We show that wishful thinking endogenously gives rise to a rich set of behavior that is consistent with existing empirical evidence, while providing new insight on how such behavior varies with economic conditions and context. We view this as a promising approach to understanding observed behavior and briefly discuss potential areas for future work.

Public information. Our analysis focuses on the subjective interpretation of price and private information in financial markets. However, public signals (e.g., regulatory disclosures) are an important part of the economic information environment. As a first step

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[37]In our static model, there is only one round of trade and therefore, no lagged returns per se. In a multi-period model, we anticipate such over-reaction to price information may translate into over-reaction to lagged price changes (since beliefs are persistent).

to understanding the subjective interpretation of public information, Banerjee, Davis, and Gondhi (2020) study how individuals who exhibit wishful thinking interpret private and exogenous public information in a generalized economic framework with externalities.[38] In future work, we hope to study how the interpretation of "exogenous" public information (e.g., disclosures) and "endogenous" public information (e.g., prices) interact as a result of wishful thinking.

Policy implications and welfare. The notion of welfare in settings with heterogeneous beliefs and subjective interpretations is nuanced (see Brunnermeier, Simsek, and Xiong (2014) for a recent discussion and an alternative approach). In Appendix B.4, we present some preliminary analysis using a measure of welfare that is conservative in that it ignores the gain in anticipatory utility that investors experience by distorting their beliefs. First, we show that in mixed-strategy equilibria, investors who condition on prices have higher objective expected utility (and so lower anticipatory utility). Second, we show that noise traders, who are responsible for aggregate supply shocks in the risky asset, can be better off in the presence of wishful thinking investors. Moreover, overall (objective) experienced utility may be higher with wishful thinking investors than under rational expectations.[39] These results suggest that understanding the role of wishful thinking has potentially important implications for regulatory policy (e.g., disclosure regulations).

Relation to robust control. As discussed by Caplin and Leahy (2019), wishful thinking has a parallel in the robust control literature (e.g., Hansen and Sargent (2001), Hansen and Sargent (2008)). Agents who exhibit robust control are unsure about their model of the world, and choose actions optimally under the "worst-case" subjective beliefs.[40] Robust control is also motivated by a large literature in psychology and economics (which documents evidence of ambiguity aversion), and is useful in understanding a number of stylized facts

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[38]We consider a generalized coordination game (e.g., Angeletos and Pavan (2007)) and show that the interpretation of public information depends on how non-fundamental aggregate volatility affects an individual's payoffs.

[39]Because wishful thinking leads investors to dismiss price information on average, it has two potentially offsetting effects: (i) it makes the price less informative (relative to rational expectations) and (ii) it decreases the price impact of trades. We show that for noise traders, the second effect dominates the first.

[40]More concretely, a robust control agent chooses action a and subjective beliefs μ to solve

$$\min_{\mu} \max_{a} \mathbb{E}_{\mu} \left[u\left(a\right) \right] + C\left(\mu\right), \tag{52}$$

where $\mathbb{E}_{\mu} \left[u\left(a\right) \right]$ reflects the subjective expected utility from action a under "worst case" beliefs μ and $C\left(\mu\right)$ reflects the penalty of choosing subjective beliefs μ that differ from the reference distribution. Analogously, a wishful thinking agent chooses action a and subjective beliefs μ to solve:

$$\max_{\mu} \max_{a} \mathbb{E}_{\mu} \left[u\left(a\right) \right] - C\left(\mu\right), \tag{53}$$

where μ reflects the "wishful thinking" that the agent engages in to maximize anticipatory utility $\mathbb{E}_{\mu} \left[u\left(a\right) \right]$.

about aggregate financial markets (e.g., limited participation, the equity premium puzzle). We view these approaches as complementary since each type of behavior is likely to arise in different contexts (e.g., asset allocation vs. stock picking, portfolio decisions during booms vs. busts). While beyond the scope of the current paper, it would be interesting to explore the implications of investors who endogenously choose to exhibit "wishful thinking" in some domains and "robust control" preferences in others.

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A Proofs

A.1 Proof of Lemma 1

The optimal demand of investor i, given her subjective beliefs (as in equations 16, 17, 18), is given by

$$x_i^* = \frac{\mathbb{E}_i\left[F|s_i, P\right] - P}{\gamma \text{var}_i\left[F|s_i, P\right]} = \frac{\omega_i}{\gamma}\left(\mu_i - P\right). \tag{54}$$

Equilibrium prices are determined by market clearing: $\int_i x_i^* di = z$, which implies:

$$P = \frac{\int_i \omega_i\left\{m + A_i\left(F - m\right) + B_i\left(s_p - m\right)\right\} di}{\int_i \omega_i di} - \frac{\gamma}{\int_i \omega_i di} z \tag{55}$$

This verifies our conjecture for functional form of the price. □

A.2 Proof of Lemma 2

Lemma 1 implies that the price is of the form: $P = m + \Lambda\left(s_p - m\right)$. Substituting this into the anticipatory utility expression (in equation 23), we get

$$AU\left(\delta_{p,i}\right) = -\sqrt{\frac{\frac{1}{\tau + \tau_e + \delta_{p,i}\tau_p}}{\left(1 - \Lambda\right)^2\frac{1}{\tau} + \Lambda^2\frac{1}{\delta_{p,i}\tau_p}}}. \tag{56}$$

Note that given other investors' choices, investor i's marginal anticipatory utility is

$$\frac{\partial}{\partial\delta_{p,i}}AU = \frac{\left(1 - \Lambda\right)^2\delta_{p,i}^2\tau_p^2 - \Lambda^2\tau\left(\tau_e + \tau\right)}{2\delta_{p,i}\left(\Lambda^2\tau + \left(1 - \Lambda\right)^2\delta_{p,i}\tau_p\right)\left(\tau_e + \delta_{p,i}\tau_p + \tau\right)} \times \sqrt{\frac{\frac{1}{\tau + \tau_e + \delta_{p,i}\tau_p}}{\left(1 - \Lambda\right)^2\frac{1}{\tau} + \Lambda^2\frac{1}{\delta_{p,i}\tau_p}}} \tag{57}$$

This implies anticipatory utility is increasing in $\delta_{p,i}$ when

$$\frac{\delta_{p,i}^2}{\tau_e + \tau} > \frac{\Lambda^2}{\left(1 - \Lambda\right)^2}\frac{\tau}{\tau_p^2}, \tag{58}$$

i.e., it is initially decreasing and then increasing in $\delta_{p,i}$. Moreover, note that

$$\lim_{\delta_{p,i}\to 0}\frac{\partial}{\partial\delta_{p,i}}AU = -\infty \tag{59}$$

and $\frac{\partial}{\partial\delta_{p,i}}AU$ equals zero at:

$$\bar{\delta} = \frac{1}{\tau_p}\left(\frac{\Lambda}{1 - \Lambda}\right)\sqrt{\tau\left(\tau_e + \tau\right)} \tag{60}$$

□

A.3 Lemma 3 and its Proof

Lemma 3. *With experienced utility penalty, the cost function is the disutility that the investor incurs under the objective distribution and is given by*

$$C\left(\delta_{p,i}\right) = \frac{1}{\sqrt{\Lambda^2\left(\delta_{p,i}-1\right)^2 + var\left(F-P\right)\left(\tau+\tau_e+\tau_p\delta_{p,i}\left(2-\delta_{p,i}\right)\right)}}. \tag{61}$$

Proof. Based on Definition 1 and ignoring the first term (which is constant) in it, the cost function is

$$C\left(\delta_{p,i}\right) = -\mathbb{E}\left[-\gamma\exp\left\{-\gamma x_i^*\left(\delta_{p,i}\right)\times\left(F-P\right)\right\}\right]$$
$$= \mathbb{E}\left[\gamma\exp\left\{-\omega_i\left(\mu_i-P\right)\times\left(F-P\right)\right\}\right]$$

Suppose we have:

$$\begin{pmatrix}\mu_i - P \\ F - P\end{pmatrix} \sim N\left(\begin{pmatrix} m \\ m \end{pmatrix}, \begin{pmatrix} \sigma_{ERi}^2 & \sigma_{ERi,ER} \\ \sigma_{ERi,ER} & \sigma_{ER}^2 \end{pmatrix}\right). \tag{62}$$

In this case, the cost function simplifies to

$$C\left(\delta_{p,i}\right) = \sqrt{\frac{\omega_i^{-2}}{\left(\omega_i^{-1}+\sigma_{ERi,ER}\right)^2 - \sigma_{ER}^2\sigma_{ER,i}^2}}. \tag{63}$$

Note that

$$\sigma_{ER,i}^2 = var(\mu_i - P) = var\left(A_i\left(s_i-m\right)+B_i\left(s_p-m\right)-\Lambda\left(s_p-m\right)\right)$$
$$= \frac{\left(A_i+B_i-\Lambda\right)^2}{\tau} + \frac{A_i^2}{\tau_e} + \frac{\left(B_i-\Lambda\right)^2}{\tau_p}$$

$$\sigma_{ER}^2 = var(F-P) = var\left(F-m-\Lambda(s_p-m)\right) = \frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p}$$

$$\sigma_{ERi,ER} = \text{cov}\left(\mu_i-P,F-P\right) = \text{cov}\left(A_i s_i+B_i s_p-\Lambda s_p, F-\Lambda s_p\right)$$
$$= \frac{\left(A_i+B_i-\Lambda\right)\left(1-\Lambda\right)}{\tau} - \frac{\left(B_i-\Lambda\right)\Lambda}{\tau_p}.$$

Substituting these coefficients into the cost function given in equation (63) and simplifying, we get

$$C\left(\delta_{p,i}\right) = \frac{1}{\sqrt{\Lambda^2\left(\delta_{p,i}-1\right)^2 + var\left(F-P\right)\left(\tau+\tau_e+\tau_p\delta_{p,i}\left(2-\delta_{p,i}\right)\right)}}.$$

$\square$

A.4 Proof of Proposition 1

The objective of investor i is given by

$$\max_{\delta_{p,i}} AU\left(\delta_{p,i}\right) - C\left(\delta_{p,i}\right)$$

which translates into

$$\max_{\delta_{p,i}} -\sqrt{\frac{1}{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)\left(\tau + \tau_e + \delta_{p,i}\tau_p\right)}} - \sqrt{\frac{1}{\left(1 - \delta_{p,i}\right)^2 \kappa\tau_p + \left(1 + \kappa\right)\left(\tau + \tau_e + \tau_p\delta_{p,i}\left(2 - \delta_{p,i}\right)\right)}}$$

where $\kappa = \left(\frac{\Lambda}{1-\Lambda}\right)^2 \frac{\tau}{\tau_p}$. Since all other investors are rational, $\Lambda = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p}$ and κ reduces to $\kappa = \frac{\gamma^2 (\tau_e + \tau_p)^2}{\tau \tau_e^2 \tau_z}$. Investor i chooses $\delta_{p,i} = 0$ iff

$$AU\left(0\right) - C\left(0\right) > AU\left(\delta_{p,i}\right) - C\left(\delta_{p,i}\right)$$

for all $\delta_{p,i} \in (0, 1]$, or equivalently

$$1 + \frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)} - \frac{C\left(0\right)}{C\left(\delta_{p,i}\right)} > 0. \tag{64}$$

Let $R \equiv \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$ and $L \equiv \frac{C(0)}{C(\delta_{p,i})}$. First, we examine the case when $\tau_p \to 0$. Note that

$$\lim_{\tau_p \to 0} R = \sqrt{\delta_{p,i}}, \quad \lim_{\tau_p \to 0} L = 1.$$

This implies that, as $\tau_p \to 0$, condition (64) reduces to

$$\lim_{\tau_p \to 0} 1 + \frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)} - \frac{C\left(0\right)}{C\left(\delta_{p,i}\right)} = 1 + \sqrt{\delta_{p,i}} - 1 > 0$$

which implies that, for τ_p sufficiently low, investor i chooses $\delta_{p,i} = 0$.

Next, we examine the case when $\tau_p \to \infty$. Note that, $\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} L = 1$, so that

$$\lim_{\tau_p \to \infty} 1 + \frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)} - \frac{C\left(0\right)}{C\left(\delta_{p,i}\right)} > 0$$

which implies that, for τ_p high enough, investor i chooses $\delta_{p,i} = 0$.

While we prove this partial equilibrium result analytically for low and high τ_p, numerical simulations show that the result holds for all τ_p.

$\square$

A.5 Proof of Proposition 2

A.5.1 Proof of part 1

Note that $\delta_{p,i} = 0 \ \forall \ i$ is a symmetric equilibrium iff

$$AU(0) - C(0) > AU(\delta_{p,i}) - C(\delta_{p,i}) \tag{65}$$

for all $\delta_{p,i}$, or equivalently,

$$1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} > 0.$$

Let $R \equiv \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$ and $L \equiv \frac{C(0)}{C(\delta_{p,i})}$. Note that

$$\lim_{\tau_p \to 0} R = \sqrt{\delta_{p,i}}, \quad \lim_{\tau_p \to 0} L = 1 \tag{66}$$

which implies that, as $\tau_p \to 0$, condition 65 reduces to

$$\lim_{\tau_p \to 0} 1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} = \sqrt{\delta_{p,i}} > 0$$

which implies that, for τ_p low enough, the only equilibrium is $\delta_{p,i} = 0 \ \forall \ i$.

A.5.2 Proof of part 2

Step 1: For τ_p high, there is no pure strategy symmetric equilibrium.

Suppose all other investors choose $\bar{\delta}_p \neq 0$. Note that,

$$\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} L = 1, \tag{67}$$

so that

$$\lim_{\tau_p \to \infty} \left(1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} \right) > 0 \tag{68}$$

which implies that for sufficiently high τ_p, an investor prefers to choose $\delta_{p,i} = 0$ for any $\bar{\delta}_p \neq 0$. Next, suppose all other investors choose $\bar{\delta}_p = 0$. In this case,

$$\lim_{\tau_p \to \infty} R = \lim_{\tau_p \to \infty} \sqrt{2 - \bar{\delta}_p} \tag{69}$$

$$\lim_{\tau_p \to \infty} L = \lim_{\tau_p \to \infty} \frac{1}{\gamma} \sqrt{\left(2 - \bar{\delta}_p \right)} = \infty \tag{70}$$

which suggests that

$$\lim_{\tau_p \to \infty} \left(1 + \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})} - \frac{C(0)}{C(\delta_{p,i})} \right) < 0 \tag{71}$$

which implies that investor i will not choose $\delta_{p,i} = 0$. Put together, this analysis implies that there is no pure strategy symmetric equilibrium for sufficiently high τ_p.

Step 2: For τ_p high enough, the objective function of investor i is either U shaped or downward sloping in $\delta_{p,i}$ for $\delta_{p,i} \in [0,1]$.

Given that $\delta_{p,i} = 0 \ \forall \ i$ is not a equilibrium (from step 1), assume that the rest of investors choose an average $\delta_{p,i}$ of $\bar{\delta}_p \neq 0$. The objective of investor i given by

$$\max_{\delta_{p,i}} -\sqrt{\frac{1}{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)(\tau + \tau_e + \delta_{p,i}\tau_p)}} - \sqrt{\frac{1}{(1 - \delta_{p,i})^2 \kappa\tau_p + (1+\kappa)(\tau + \tau_e + \tau_p\delta_{p,i}(2 - \delta_{p,i}))}}$$

where $\kappa = \frac{\left(\tau_e + \bar{\delta}_p\tau_p\right)^2}{\tau\tau_p}$. The FOC for this objective is

$$\frac{1 - \frac{\kappa(\tau+\tau_e)}{\delta_{p,i}^2\tau_p}}{\sqrt{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)^3 (\tau + \tau_e + \delta_{p,i}\tau_p)^3}} - \frac{2(\delta_{p,i} - 1)}{\sqrt{\left((1+\kappa)(\tau + \tau_e + \tau_p) - (1 - \delta_{p,i})^2 \tau_p\right)^3}} = 0. \qquad (72)$$

Note that $\lim_{\tau_p \to \infty} \frac{\kappa}{\tau_p} = \frac{\left(\tau_e + \bar{\delta}_p\tau_p\right)^2}{\tau\tau_p^2} \to \frac{\bar{\delta}_p^2}{\tau}$. Dividing the FOC by τ_p and taking limit as $\tau_p \to \infty$, the FOC reduces to

$$1 - \frac{\bar{\delta}_p^2(\tau + \tau_e)}{\tau\delta_{p,i}^2} = 2(\delta_{p,i} - 1)$$

which simplifies to

$$3\delta_{p,i}^2 - 2\delta_{p,i}^3 = \frac{\bar{\delta}_p^2(\tau + \tau_e)}{\tau} \qquad (73)$$

In the range $\delta_{p,i} \in (0,1)$, the LHS of above equation is increasing and RHS is constant. This implies that, there is at-most one solution to the FOC . Moreover, at $\delta_{p,i} = 0$, the LHS of the FOC (72) is negative which implies that the objective function is downward sloping. All of these imply that the objective function is either U-shaped or downward sloping.

Step 3: In equilibrium, the objective function of investor i is U-shaped. This is because, if the objective function were downward sloping for all investors, all investors would choose $\delta_{p,i} = 0$ and step 1 implies that this cannot be an equilibrium.

If the objective function is U-shaped and since there cannot be a pure strategy symmetric equilibrium, the only other possible equilibrium is a mixed equilibrium where a fraction λ of agents choose $\delta_{p,i} = 0$ and the remaining choose $\delta_{p,i} = 1$. For this to be the case, all investors have to be indifferent between $\delta_{p,i} = 0$ and $\delta_{p,i} = 1$ and this indifference equation pins down equilibrium λ. The indifference condition is

$$AU(0) - C(0) = AU(1) - C(1).$$

This simplifies to

$$\sqrt{\frac{1}{(1+\kappa)(\tau + \tau_e + \tau_p) - \tau_p}} = \frac{2}{\sqrt{(1+\kappa)(\tau + \tau_e + \tau_p)}}$$

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which implies that

$$\lambda = 1 + \frac{\tau_e}{\tau_p} - \sqrt{\frac{\tau}{3\tau_p}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}$$

$\square$

A.6 Proof of Proposition 3

The proof of this proposition follows the same steps as the proof of Proposition 2 until equation (73). In equation (73), suppose we allow $\delta_{p,i} \in (0, \infty)$, the LHS of this equation increases in $\delta_{p,i}$ up to $\delta_{p,i} = 1$ and decreases thereafter. This implies that the FOC (i.e., equation 73) will either have no solutions or two solutions, the first less than one and the other greater than one. If the RHS is greater than one i.e., $\frac{\bar{\delta}_p^2(\tau + \tau_e)}{\tau} > 1$, the equation has no solution. This implies that the objective is downward sloping and all investors choose $\delta_{p,i} = 0$ which cannot be an equilibrium (from step 1). This implies that, in equilibrium, the FOC will have two solutions. Moreover, at $\delta_{p,i} = 0$, the LHS of the FOC (72) is negative which implies that the objective function is downward sloping. This implies that the solution of the FOC in $\delta_{p,i} \in (0, 1)$ is a minima and the solution of the FOC in $\delta_{p,i} \in (1, \infty)$ is a maxima. This implies that the objective function has a local maxima at $\delta_{p,i} = 0$ and another local maxima at $\delta_{p,i} > 1$.

Given the shape of the objective function and ruling out any pure strategy symmetric equilibrium (as in step 2), the only possible equilibria is a mixed equilibrium in which investors mix between two sets of beliefs: a fraction λ optimally chooses $\delta_{p,i} = 0$, while the remaining fraction $1 - \lambda$ optimally chooses $\delta_{p,i} = \delta_p^* > 1$. Finally, δ_p^* and λ solve the FOC condition and the indifference condition, given by

$$\frac{1 - \frac{\kappa(\tau + \tau_e)}{\delta_p^{*2}\tau_p}}{\sqrt{\left(1 + \frac{\kappa}{\delta_p^*}\right)^3 \left(\tau + \tau_e + \delta_p^*\tau_p\right)^3}} - \frac{2\left(\delta_p^* - 1\right)}{\sqrt{\left((1 + \kappa)\left(\tau + \tau_e + \tau_p\right) - \left(1 - \delta_p^*\right)^2 \tau_p\right)^3}} = 0. \quad (74)$$

$$AU(0) - C(0) = AU(\delta_p^*) - C(\delta_p^*). \quad (75)$$

$\square$

A.7 Proof of Proposition 4

Following similar steps as in Lemma 3, the cost function with private signal choice is given by

$$C\left(\delta_{e,i}\right) = \frac{1}{\sqrt{\left(\frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p}\right)\left(\tau + \tau_p + \tau_e\delta_{e,i}\left(2 - \delta_{e,i}\right)\right)}}$$

The FOC is given by

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}} (\tau_e \delta_{e,i} + \tau_p + \tau)^{\frac{3}{2}}} = \frac{\tau_e (\delta_{e,i} - 1)}{\left(\frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\tau_p}\right)^{\frac{1}{2}} [(\tau + \tau_p + \tau_e \delta_{e,i} (2 - \delta_{e,i}))]^{\frac{3}{2}}} \tag{76}$$

which simplifies to

$$\frac{(\tau + \tau_p + \tau_e \delta_{e,i} (2 - \delta_{e,i}))^{\frac{3}{2}}}{(\tau_p + \tau + \tau_e \delta_{e,i})^{\frac{3}{2}}} = 2 (\delta_{e,i} - 1) \tag{77}$$

which establishes the result. It is straightforward to see that the second-order conditions are satisfied and the comparative statics of δ_e follows directly from equation (77). $\qquad\square$

A.8 Proof of Proposition 5

A.8.1 Proof of part 1:

For an investor incurring the experienced utility penalty, choosing $(\delta_{e,i}, \delta_{p,i})$ yields anticipatory utility and costs:

$$AU(\delta_{e,i}, \delta_{p,i}) = -\sqrt{\frac{\tau}{(1-\Lambda)^2}} \sqrt{\frac{1}{\left(1 + \frac{\kappa}{\delta_{p,i}}\right)(\tau + \delta_{e,i}\tau_e + \delta_{p,i}\tau_p)}} \tag{78}$$

$$C(\delta_{e,i}, \delta_{p,i}) = \sqrt{\frac{\tau}{(1-\Lambda)^2}} \sqrt{\frac{1}{(1 - \delta_{p,i})^2 \kappa \tau_p + (1 + \kappa)(\tau + \tau_e \delta_{e,i}(2 - \delta_{e,i}) + \tau_p \delta_{p,i}(2 - \delta_{p,i}))}} \tag{79}$$

where $\kappa \equiv \left(\frac{\Lambda}{1-\Lambda}\right)^2 \frac{\tau}{\tau_p}$. Suppose the average action across all other players is $\bar{\delta}_e, \bar{\delta}_p$. Then, $\Lambda = \frac{\tau_e \bar{\delta}_e + \tau_p \bar{\delta}_p}{\tau + \tau_e \bar{\delta}_e + \tau_p \bar{\delta}_p}$ and so

$$\kappa = \left(\frac{\Lambda}{1-\Lambda}\right)^2 \frac{\tau}{\tau_p} = \frac{\gamma^2 \left(\tau_e \bar{\delta}_e + \tau_p \bar{\delta}_p\right)^2}{\tau \tau_e^2 \tau_z \bar{\delta}_e^2}. \tag{80}$$

Then, $(1,0)$ is a symmetric equilibrium iff all investors prefer $(1,0)$ over all other $(\delta_{e,i}, \delta_{p,i})$:

$$AU(1,0) - C(1,0) > AU(\delta_{e,i}, \delta_{p,i}) - C(\delta_{e,i}, \delta_{p,i}) \tag{81}$$

or equivalently,

$$H \equiv 1 + R - L > 0$$

where $R \equiv \frac{AU(\delta_{e,i}, \delta_{p,i})}{-C(\delta_{e,i}, \delta_{p,i})}$, $L \equiv \frac{C(1,0)}{C(\delta_{e,i}, \delta_{p,i})}$. Note that

$$\lim_{\tau_p \to 0} R = \sqrt{\frac{((2 - \delta_{e,i}) \delta_{e,i} \tau_e + \tau) \delta_{p,i}}{\delta_{e,i} \tau_e + \tau}}, \quad \lim_{\tau_p \to 0} L = \sqrt{\frac{(2 - \delta_{e,i}) \delta_{e,i} \tau_e + \tau}{\tau_e + \tau}} \tag{82}$$

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which implies that

$$\lim_{\tau_p \to 0} H = 1 + \sqrt{\frac{\left(\left(2 - \delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e + \tau}} - \sqrt{\frac{\left(2 - \delta_{e,i}\right)\delta_{e,i}\tau_e + \tau}{\tau_e + \tau}} \tag{83}$$

$$\geq 1 + \sqrt{\frac{\left(\left(2 - \delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e + \tau}} - \sqrt{\frac{\tau_e + \tau}{\tau_e + \tau}} \geq 0 \tag{84}$$

which implies $(1, 0)$ is an equilibrium for τ_p sufficiently low.

A.8.2 Proof of part 2:

The objective of investor i is given by

$$\max_{\delta_{e,i}, \delta_{p,i} \in (\underline{\delta}, \infty)} AU\left(\delta_{e,i}, \delta_{p,i}\right) - C\left(\delta_{e,i}, \delta_{p,i}\right).$$

This objective can be rewritten as

$$\max_{\delta_{p,i} \in (\underline{\delta}, \infty)} \left[\max_{\delta_{e,i} \in (\underline{\delta}, \infty)} AU\left(\delta_{e,i}, \delta_{p,i}\right) - C\left(\delta_{e,i}, \delta_{p,i}\right) \right].$$

Lets focus on the maximization inside the square bracket. The FOC is given by

$$\frac{\tau_e}{2\left(\frac{(1-\Lambda)^2}{\tau} + \frac{\Lambda^2}{\delta_{p,i}\tau_p}\right)^{\frac{1}{2}}\left(\tau_e\delta_{e,i} + \tau_p\delta_{p,i} + \tau\right)^{\frac{3}{2}}} = \frac{\operatorname{var}\left(F - P\right)\tau_e\left(\delta_{e,i} - 1\right)}{\sqrt{\Lambda^2\left(\delta_{p,i} - 1\right)^2 + \operatorname{var}\left(F - P\right)\left(\tau + \tau_e\delta_{e,i}\left(2 - \delta_{e,i}\right) + \tau_p\delta_{p,i}\left(2 - \delta_{p,i}\right)\right)}} \tag{85}$$

The SOC is also satisfied. This implies that the solution to the above equation is the global maximum. This implies that the optimal $\delta_{e,i}$ chosen will always be interior and greater than one. From now on, we will denote the solution to the above FOC as $\delta_{e,i}(\delta_{p,i})$.

Next, we prove that, for optimization with respect to $\delta_{p,i}$, agent i will always choose $\delta_{p,i} = \underline{\delta}$. Note that $\delta_{p,i} = \underline{\delta}$ is an equilibrium iff

$$AU\left(\underline{\delta}\right) - C\left(\underline{\delta}\right) > AU\left(\delta_{p,i}\right) - C\left(\delta_{p,i}\right) \tag{86}$$

for all $\delta_{p,i}$, or equivalently,

$$1 + \frac{AU\left(\delta_{p,i}\right)}{-C\left(\delta_{p,i}\right)} - \frac{C\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)} + \frac{AU\left(\underline{\delta}\right)}{C\left(\delta_{p,i}\right)} > 0. \tag{87}$$

First, we will examine what happens to $\delta_{e,i}(\delta_{p,i})$ as $\tau_p \to 0$. Taking the limit of equation 85, we can show that $\lim_{\tau_p \to 0} \delta_{e,i}(\delta_{p,i}) = 1$. Let $R \equiv \frac{AU(\delta_{p,i})}{-C(\delta_{p,i})}$, $L \equiv \frac{C(\underline{\delta})}{C(\delta_{p,i})}$, $G \equiv \frac{AU(\underline{\delta})}{C(\delta_{p,i})}$. Note that

$$\lim_{\tau_p \to 0} R = \lim_{\tau_p \to 0} \sqrt{\frac{\left(\left(2 - \delta_{e,i}\right)\delta_{e,i}\tau_e + \tau\right)\delta_{p,i}}{\delta_{e,i}\tau_e + \tau}} = \sqrt{\delta_{p,i}},$$

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$$\lim_{\gamma \to \infty} L = \lim_{\gamma \to \infty} \sqrt{\frac{\tau + \tau_e \delta_{ei} \left(2 - \delta_{ei}\right)}{\tau + \tau_e \delta_{ei} \left(\underline{\delta}\right) \left(2 - \delta_{ei} \left(\underline{\delta}\right)\right)}} = 1$$

$$\lim_{\gamma \to \infty} G = -\sqrt{\underline{\delta} \frac{\tau + \tau_e \delta_{ei} \left(2 - \delta_{ei}\right)}{\tau + \delta_{e,i} \left(\underline{\delta}\right) \tau_e}} = -\sqrt{\underline{\delta}}.$$

Substituting these limits into inequality (87), we get

$$\lim_{\tau_p \to \infty} 1 + \frac{AU \left(\delta_{p,i}\right)}{-C \left(\delta_{p,i}\right)} - \frac{C \left(\underline{\delta}\right)}{C \left(\delta_{p,i}\right)} + \frac{AU \left(\underline{\delta}\right)}{C \left(\delta_{p,i}\right)} = 1 + \sqrt{\delta_{p,i}} - 1 - \sqrt{\underline{\delta}} > 0$$

which implies that $\delta_{p,i} = \underline{\delta}$ is an equilibrium for τ_p sufficiently low.

A.9   Proof of Proposition 6

Lemma 1 implies that in any symmetric equilibrium (i.e., $\delta_{p,i} = \delta_p \ \forall \ i$), we have $\Lambda = \frac{\tau_e + \delta_p \tau_p}{\tau + \tau_e + \delta_p \tau_p}$. Moreover, note that $\frac{\partial}{\partial \delta_{p,i}} AU = 0$ at

$$\bar{\delta}_{p,i} = \frac{1}{\tau_p} \left(\frac{\Lambda}{1 - \Lambda}\right) \sqrt{\tau \left(\tau_e + \tau\right)} \tag{88}$$

$$= \sqrt{1 + \frac{\tau_e}{\tau}} \left(\delta_p + \frac{\tau_e}{\tau_p}\right) > \delta_p \tag{89}$$

But this implies $\frac{\partial}{\partial \delta_{p,i}} AU \left(\delta_{p,i} = \delta_p\right) < 0$ since $\frac{\partial AU}{\partial \delta_{p,i}} < (>)0$ for all $\delta_{p,i} < (>)\bar{\delta}_{p,i}$. Next, note that if $\delta_{p,i} = \delta_p \geq 1$, then $C'(\delta_{p,i}) > 0$. Taken together, this proves that at any proposed symmetric equilibrium where $\delta_p > 1$, investor i has an incentive to deviate and choose $\delta_{p,i} < 1$. Thus, the only possible symmetric equilibrium is one in which each investor chooses $\delta_{p,i} < 1$. This proves the proposition. $\qquad \square$

A.10   Proof of Proposition 7

Denote the return characteristics in the rational expectations equilibrium, symmetric equilibrium and mixed equilibrium be subscripted by RE, SE and ME respectively.

A.10.1   Proof of part (i)

Note that $\theta_{RE} = \frac{\tau_p(\tau + \tau_e + \tau_p)}{(\tau + \tau_p)(\tau_e + \tau_p)} - 1$ and $\theta_{SE} = \frac{\tau_p(\tau + \tau_e)}{(\tau + \tau_p)(\tau_e) + 1} - 1$ which implies that

$$\theta_{RE} - \theta_{SE} = -\frac{\tau \tau_e^2 \tau_z^2}{\left(\gamma^2 + \tau_e \tau_z\right) \left(\gamma^2 \tau + \tau_e^2 \tau_z\right)} < 0.$$

Let $\bar{\delta}_p = (1 - \lambda)$ denote the average beliefs about the precision of price signal. Note that $\Lambda_{ME} = \frac{\tau_e + (1-\lambda)\tau_p}{\tau + \tau_e + (1-\lambda)\tau_p}$ and

$$\theta_{ME} - \theta_{RE} = \frac{\tau_z}{\Lambda_{ME}\left(\beta^2\tau + \tau_z\right)} - \frac{\tau_z}{\Lambda_{RE}\left(\beta^2\tau + \tau_z\right)} \tag{90}$$

Given that $\Lambda_{ME} < \Lambda_{RE}$, it is clear that $\theta_{ME} > \theta_{RE}$.

A.10.2 Proof of part (ii)

Note that

$$\sigma^2_{R,RE} - \sigma^2_{R,SE} = \frac{\tau\left(\tau_p^2 + \tau\tau_p - 2\tau_e\left(\tau_e + \tau\right)\right)}{\left(\tau_e + \tau\right)^2\left(\tau_e + \tau_p + \tau\right)^2}, \tag{91}$$

which is positive iff $\tau_p > \frac{1}{2}\sqrt{8\tau\tau_e + 8\tau_e^2 + \tau^2} - \frac{\tau}{2}$. Moreover,

$$\sigma^2_{R,ME} = \frac{\tau\tau_p + \left((1-\lambda)\tau_p + \tau_e\right)^2}{\tau_p\left((1-\lambda)\tau_p + \tau_e + \tau\right)^2}$$

In the mixed equilibrium,

$$(1 - \lambda)\tau_p + \tau_e = \sqrt{\frac{\tau\tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}.$$

Substituting this into the expression for $\sigma^2_{R,ME}$, we get that

$$\lim_{\tau_p \to \infty} \sigma^2_{R,ME} > \sigma^2_{R,SE}.$$

This implies that our results hold for τ_p high enough which is the required condition for mixed equilibrium. $\qquad\square$

A.11 Proof of Proposition 8

A.11.1 Proof of Part 1:

Note that $\theta_{RE} = \frac{\tau_p(\tau + \tau_e + \tau_p)}{(\tau + \tau_p)(\tau_e + \tau_p)} - 1$ and hence

$$\frac{\partial\theta_{RE}}{\partial\tau_p} = (\tau + \tau_e + 2\tau_p)\frac{(\tau + \tau_p)(\tau_e + \tau_p) - \tau_p(\tau + \tau_e + \tau_p)}{(\tau + \tau_p)^2(\tau_e + \tau_p)^2} > 0.$$

Note that $\sigma^2_{R,RE} = \frac{\tau\tau_p + (\tau_p + \tau_e)^2}{\tau_p(\tau_p + \tau_e + \tau)^2}$ and hence

$$\frac{\partial\sigma^2_{R,RE}}{\partial\tau_p} = -\frac{\tau_e^2\left(3\tau_p + \tau\right) + 3\tau_e\tau_p^2 + \tau_e^3 + \tau_p^2\left(\tau_p + \tau\right)}{\tau_p^2\left(\tau_e + \tau_p + \tau\right)^3} < 0.$$

A.11.2 Proof of Part 2:

When τ_p is sufficiently low, the equilibrium is symmetric in which $\delta_{p,i} = 0 \ \forall \ i$. Note that $\theta_{SE} = \frac{\tau_p(\tau + \tau_e)}{(\tau + \tau_p)\tau_e} - 1$ and hence $\frac{\partial \theta_{SE}}{\partial \tau_p} > 0$. Note that $\sigma_{R,SE}^2 = \frac{\tau \tau_p + (\tau_e)^2}{\tau_p(\tau_e + \tau)^2}$ and hence

$$\frac{\partial \sigma_{R,SE}^2}{\partial \tau_p} = -\frac{\tau_e^2}{(\tau_e + \tau)^2 \tau_p^2} < 0.$$

When τ_p is sufficiently high, the equilibrium is mixed and $\theta_{ME} = \frac{\tau_p(\tau + \tau_e + (1-\lambda)\tau_p)}{(\tau + \tau_p)(\tau_e + (1-\lambda)\tau_p)} - 1$ and in the mixed equilibrium,

$$(1 - \lambda)\tau_p + \tau_e = \sqrt{\frac{\tau \tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}$$

which implies that

$$\theta_{ME} = \frac{\tau_p(\tau + \sqrt{\frac{\tau \tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)})}{(\tau + \tau_p)(\sqrt{\frac{\tau \tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)})} - 1$$

and $\lim_{\tau_p \to \infty} \frac{\partial \theta_{ME}}{\partial \tau_p} < 0$. Note that

$$\sigma_{R,ME}^2 = \frac{\tau \tau_p + ((1 - \lambda)\tau_p + \tau_e)^2}{\tau_p((1 - \lambda)\tau_p + \tau_e + \tau)^2} = \frac{\tau \tau_p + \left(\sqrt{\frac{\tau \tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)}\right)^2}{\tau_p\left(\sqrt{\frac{\tau \tau_p}{3}\left(\frac{\tau_p - 3\tau - 3\tau_e}{\tau + \tau_e + \tau_p}\right)} + \tau\right)^2}. \tag{92}$$

This implies that $\lim_{\tau_p \to \infty} \frac{\partial \sigma_{R,ME}^2}{\partial \tau_p} < 0$. $\qquad \square$

B Internet Appendix: Extensions

B.1 Ex-post Belief Choice

In the benchmark model, each investor chooses her subjective beliefs $(\delta_{p,i})$ before she observes the realization of her signals. In this section, we consider the implications when investor i chooses her beliefs ex-post.

Anticipatory utility is now conditional on the realizations of P and s_i, i.e., it can be written as

$$AU_i\left(\delta_{p,i}; s_i, P\right) = -\exp\left\{-\frac{1}{2}\omega_i\left(\mu_i - P\right)^2\right\}.$$

This implies

$$\frac{\partial}{\partial \delta_{p,i}} AU_i \propto (\mu_i - P) \times \{2\tau\left(s_p - m\right) + \omega_i\left(\mu_i - P\right)\}$$

$$\propto \left[s_i + \frac{B_i - \Lambda}{A_i}s_p\right]\left[(B_i\omega_i + 2\tau)s_p + \omega_i\left(A_i - \Lambda\right)s_i\right]$$

It can be shown (by rewriting the terms in brackets) that anticipatory utility is decreasing in $\delta_{p,i}$ when investor i's private signal is sufficiently distant from the price signal. As a result, and similar to our benchmark model, such an investor would dismiss others' information. On the other hand, when investor i's private signal is sufficiently close to s_p, she has no incentive to deviate from rational expectations; under the extension allowing for more general beliefs about the price (found in Section 5.1), such realizations would induce her to overweight the price, as in the mixed equilibrium we discuss in that section.

B.2 Expected Returns and Volume

Since investors start without an endowment of the risky security, realized trading volume in our economy can be characterized as

$$\mathcal{V} \equiv \int_i |x_i^*|\, di. \tag{93}$$

This implies that expected volume is given by

$$\mathbb{E}\left[\mathcal{V}\right] = \int_i \frac{\omega_i}{\gamma}|\mu_i - P|di = \int_i \frac{\tau}{\gamma(1 - A_i - B_i)}\sqrt{\frac{2}{\pi}\left(\frac{A_i^2}{\tau_e} + \frac{(B_i - \Lambda)^2}{\tau_p} + \frac{(A_i + B_i - \Lambda)^2}{\tau}\right)}di, \tag{94}$$

where A_i, B_i are appropriately redefined to reflect any subjective belief distortions about investors' own signals (as in Section 5.2). Note that volume reflects the cross-sectional variation across investor valuations (i.e., μ_i), scaled by their posterior variance (i.e., ω_i^{-1}). Such variation is driven by three channels: (i) the weight each investor places on her private signal (i.e., the $\frac{1}{\tau_e}$ term), (ii) the weight she places on price information, relative to others (i.e., the $\frac{1}{\tau_p}$ term) and (iii) the relative weight placed on her prior belief (i.e., the $\frac{1}{\tau}$ term). These observations give rise to the following result.

Proposition 9. *(i) Fixing parameters, expected volume is higher in a symmetric equilibrium than in the corresponding rational expectations equilibrium if $\delta_{e,i} > 1$.*
(i) Fixing parameters, expected volume is higher in a mixed equilibrium than in the corresponding rational expectations equilibrium.

If investors exhibit over-confidence (i.e., $\delta_{e,i} > 1$), then they (i) place more weight on their private signal, i.e., A_i increases and (ii) place relatively less weight on the price. Together, the first two terms imply that volume is higher under the symmetric equilibrium with overconfidence than under rational expectations. However, note that the last term is absent in symmetric equilibria, since $A_i + B_i = \Lambda$ in this case. In mixed equilibria, this final term reflects the variation in valuations due to the relative difference in weights each "type" of investor places on private signals and the information in prices. This difference across types generates increased trade amongst investors, even in the absence of over-confidence.

Proof of Proposition 9

Proof of Part (i): In a symmetric equilibrium, $\delta_p = \bar{\delta}$ and δ_e solves equation (40). In this equilibrium, volume under the symmetric equilibrium ($\mathcal{V}_{SE}$) is given by

$$\mathbb{E}\left[\mathcal{V}_{SE}\right] = \frac{\omega}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{A^2}{\tau_e} + \frac{(B-\Lambda)^2}{\tau_p}\right)} \tag{95}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\delta_e^2\tau_e + \frac{\delta_e^2\tau_e^2}{\tau_p}\right)} \tag{96}$$

$$> \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tau_e + \frac{\tau_e^2}{\tau_p}\right)} = \mathbb{E}\left[\mathcal{V}_{RE}\right], \tag{97}$$

where $\mathcal{V}_{RE}$ denotes volume under rational expectations. □

Proof of Part (ii): Let $\mathcal{V}_{ME}$ denote volume under the mixed equilibrium of Proposition 2. Then,

$$\mathbb{E}\left[\mathcal{V}_{ME}\right] = \lambda V_1 + (1-\lambda)V_2, \text{ where} \tag{98}$$

$$V_1 \equiv \frac{(\tau_e + \tau)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}\left(\frac{\tau_e}{\tau+\tau_e} - \Lambda_{ME}\right)^2 + \frac{1}{\tau_e}\left(\frac{\tau_e}{\tau+\tau_e}\right)^2 + \frac{1}{\tau_p}\Lambda_{ME}^2\right)} \tag{99}$$

$$V_2 \equiv \frac{\tau+\tau_e+\tau_p}{\gamma}\sqrt{\frac{2}{\pi}\left(\begin{array}{c}\frac{1}{\tau}\left(\frac{\tau_e+\tau_p}{\tau+\tau_e+\tau_p} - \Lambda_{ME}\right)^2 + \frac{1}{\tau_e}\left(\frac{\tau_e}{\tau+\tau_e+\tau_p}\right)^2 \\ + \frac{1}{\tau_p}\left(\frac{\tau_p}{\tau+\tau_e+\tau_p} - \Lambda_{ME}\right)^2\end{array}\right)} \tag{100}$$

Let

$$A\left(x\right) = \frac{x\tau_e + \left(1 - x\right)\tau_e}{x\left(\tau + \tau_e\right) + \left(1 - x\right)\left(\tau + \tau_e + \tau_p\right)} \tag{101}$$

$$B\left(x\right) = \frac{\left(1 - x\right)\left(\tau_p\right)}{x\left(\tau + \tau_e\right) + \left(1 - x\right)\left(\tau + \tau_e + \tau_p\right)} \tag{102}$$

$$\omega\left(x\right) = x\left(\tau_e + \tau\right) + \left(1 - x\right)\left(\tau + \tau_e + \tau_p\right) \tag{103}$$

$$V\left(x\right) = \frac{\omega\left(x\right)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}\left(A\left(x\right) + B\left(x\right) - \Lambda\right)^2 + \frac{1}{\tau_e}A\left(x\right)^2 + \frac{1}{\tau_p}\left(B\left(x\right) - \Lambda\right)^2\right)} \tag{104}$$

and so we can rewrite the expected mixed equilibrium volume as

$$\mathbb{E}\left[\mathcal{V}_{ME}\right] = \lambda V\left(1\right) + \left(1 - \lambda\right)V\left(0\right) \tag{105}$$

Note that

$$V\left(\lambda\right) = \frac{\omega\left(\lambda\right)}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau_e}A\left(\lambda\right)^2 + \frac{1}{\tau_p}\left(B\left(\lambda\right) - \Lambda\right)^2\right)} \tag{106}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{\left(\lambda\tau_e + \left(1 - \lambda\right)\tau_e\right)^2}{\tau_e} + \frac{\left(\left(1 - \lambda\right)\left(\tau_p\right) - \lambda\left(\tau_e\right) - \left(1 - \lambda\right)\left(\tau_e + \tau_p\right)\right)^2}{\tau_p}\right)} \tag{107}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{\left(\lambda\tau_e + \left(1 - \lambda\right)\tau_e\right)^2}{\tau_e} + \frac{\left(\lambda\tau_e + \left(1 - \lambda\right)\tau_e\right)^2}{\tau_p}\right)} \tag{108}$$

$$= \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\tau_e + \frac{\tau_e^2}{\tau_p}\right)} = \mathbb{E}\left[\mathcal{V}_{RE}\right]. \tag{109}$$

It remains to be shown that:

$$\lambda V\left(1\right) + \left(1 - \lambda\right)V\left(0\right) \geq V\left(\lambda\right) \tag{110}$$

Note that

$$V\left(x\right) = \frac{1}{\gamma}\sqrt{\frac{2}{\pi}\left(\frac{1}{\tau}\left(\alpha\left(x\right) + \beta\left(x\right) - \Lambda\omega\left(x\right)\right)^2 + \frac{1}{\tau_e}\alpha\left(x\right)^2 + \frac{1}{\tau_p}\left(\beta\left(x\right) - \Lambda\omega\left(x\right)\right)^2\right)} \tag{111}$$

where

$$\alpha\left(x\right) = x\tau_e + \left(1 - x\right)\tau_e \equiv a_0 + a_1 x \tag{112}$$

$$\beta\left(x\right) = \left(1 - x\right)\tau_p \equiv b_0 + b_1 x \tag{113}$$

$$\omega\left(x\right) = x\left(\tau_e + \tau\right) + \left(1 - x\right)\left(\tau + \tau_e + \tau_p\right) \equiv w_0 + w_1 x \tag{114}$$

$$\frac{V_{xx}}{V^3} = 4\frac{\tau + \tau_e + \tau_p}{\pi^2\gamma^4\tau\tau_e\tau_p}\left(-a_0 b_1 + a_0\Lambda w_1 + a_1 b_0 - a_1\Lambda w_0\right)^2 > 0 \tag{115}$$

which implies $V(x)$ is convex, which implies:

$$\mathbb{E}\left[\mathcal{V}_{ME}\right] = \lambda V(1) + (1 - \lambda) V(0) \geq V(\lambda) \geq \mathbb{E}\left[\mathcal{V}_{RE}\right] \tag{116}$$

This completes the proof. $\qquad\square$

B.3   K-L Distance

In our benchmark analysis, the cost function (the experienced utility penalty) is endogenous and depends on equilibrium choices of others through Λ. In other models of subjective belief choice (e.g., Caplin and Leahy (2019)), the cost function is often specified in terms of a statistical distance measure and does not depend on equilibrium choices. In this extension, we explore (numerically) the impact of using a common measure, the Kullback-Leibler distance. Given our distributional assumptions, the cost of choosing $\delta_{p,i}$ is given by

$$C(\delta_{p,i}) = \frac{1}{2}\left(\log(\delta_{p,i}) + \frac{1}{\delta_{p,i}} - 1\right). \tag{117}$$

Figure 6: Best response functions of investors

The figure plots the best response of investor i (in solid blue) as a function of average δ_p chosen by other investors. The dotted black line shows the 45 degree line. Panel (a) corresponds to high γ and panel (b) corresponds to low γ. Other parameters are set to $\tau = \tau_e = \tau_z = 1$.

(a) $\gamma = 1$ $\qquad\qquad\qquad\qquad\qquad$ (b) $\gamma = 0.3$
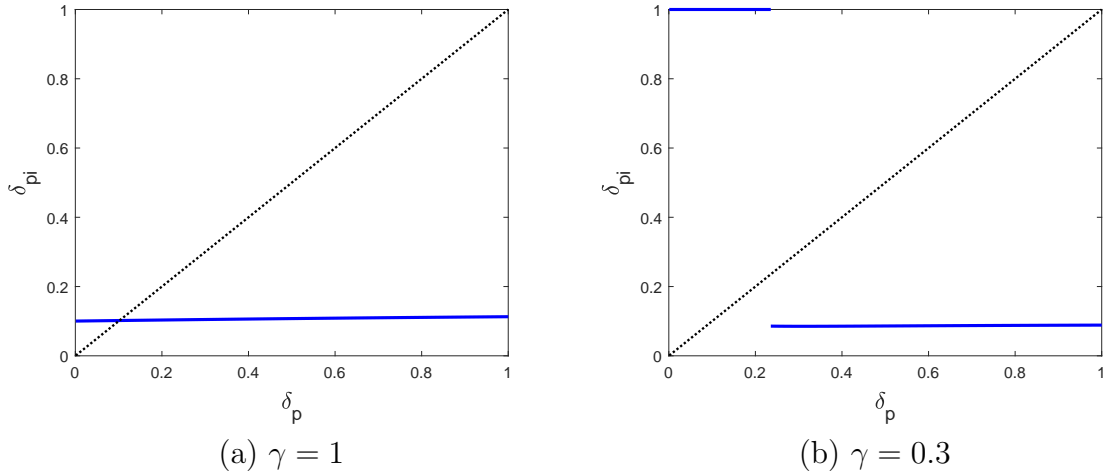
   Figure 6 plots investor i's best response function. Note that, at the intersection of the best response function and the 45-degree line, there is a symmetric equilibrium. Panel (a) shows that, for relatively high risk aversion (i.e., low price informativeness), the equilibrium is symmetric: all investors choose $\delta_{p,i} \approx 0.1$, and so (partially) dismiss the information in prices. Investors do not completely dismiss the price as it is infinitely to choose $\delta_{p,i} = 0$.

   However, panel (b) shows that when risk aversion is low (i.e., high price informativeness), a symmetric equilibrium does not exist: there is no intersection between the best response

function and the 45-degree line. Numerically, the only sustainable equilibrium is mixed: some investors discount the price while others condition on it efficiently. The intuition mirrors the relative substitutability found in the baseline model: when other investors condition on (respectively, dismiss) the price, investor i chooses to dismiss (respectively, condition on) it.

Figure 7: Composition of equilibrium beliefs with KL distance

The figure plots the equilibrium fraction of investors under-weighting the information in prices, λ as a function of γ. Other parameters are set to $\tau = \tau_e = \tau_z = 1$. The blue regions indicate the parameter combinations where we have the symmetric equilibrium in which all agents underweight the information in prices.
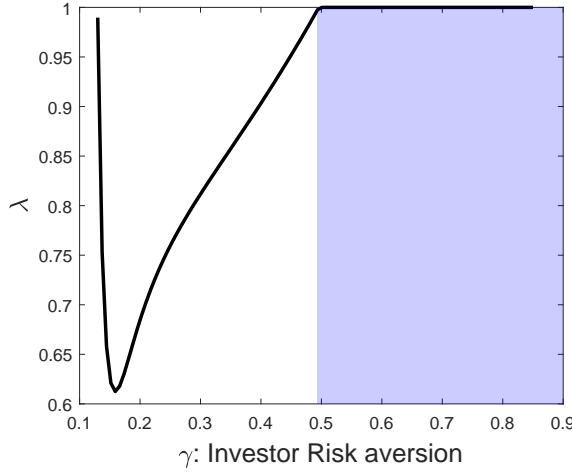
Finally, we examine how the fraction of investors who underweight the information in prices, λ, changes with risk aversion. Figure 7 plots the equilibrium fraction of investors who choose "low" $\delta_{p,i}$ as a function of risk aversion. Reassuringly, this plot is U-shaped, as in our benchmark (see Figure 2). This suggests that the comparative statics described in the main text are robust to alternative specifications of the cost function.

B.4 Welfare

In this section, we explore the welfare implications of motivated beliefs. Investors choose to deviate from rational expectations and so, under their chosen subjective beliefs, they are always better off. However, from the perspective of a social planner who holds objective beliefs and accounts only for expected utility, informed investors are strictly worse off when they deviate - their demand for the risky asset is sub-optimal given their information sets.

In what follows, we use the objective distribution as the reference beliefs and define expected utility for an informed investor as

$$U_i \equiv \mathbb{E}\left[-\exp\left\{-\gamma x_i^*\left(\delta_{p,i}^*\right) \times (F - P) - \gamma W_0\right\}\right], \tag{118}$$

where $x_i^*\left(\delta_{p,i}^*\right)$ is her optimal demand under her optimally chosen beliefs $\delta_{p,i}^*$. We emphasize that this is a conservative measure of expected utility as it only accounts for the costs

of deviating from rational expectations and does not include any gains from anticipatory utility. Figure 8 provides an illustration of the relative levels of expected utility across both the rational expectations and subjective beliefs equilibria, focusing on the generalized model found in Section 5.1.

Figure 8: Expected utility with subjective beliefs

The figure plots expected utility (y-axis) as a function of risk aversion, γ (x-axis). The dashed line plots U_i in the rational expectations equilibrium, the solid line plots U_i for the investors who dismiss price information, the dotted line plots U_i for investors who overweight price information (when the mixed equilibrium exists), and the dot-dashed line plots U_i for a hypothetical, rational expectations investor in the subjective beliefs equilibrium. Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $\gamma_z = 0.75$.
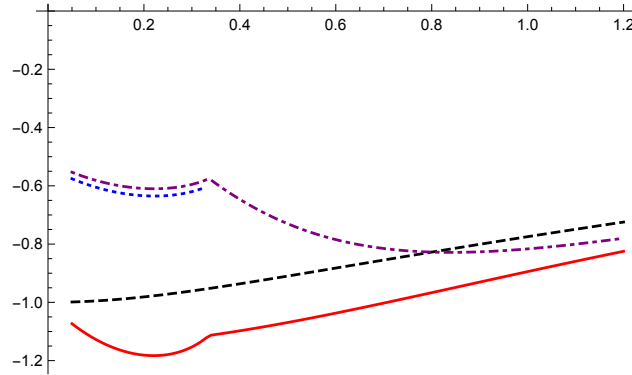
Unsurprisingly, a hypothetical rational expectations investor in the subjective beliefs equilibrium (dot-dashed) would experience a higher expected utility than those investors who hold subjective beliefs — she is optimally using all the information available to her and exploiting the behavior of the other investors and the noise traders. However, it is interesting to note that (i) in the mixed equilibrium, investors who ignore the price are significantly worse off than those that overweight it, and (ii) investors in a rational expectations equilibrium (dashed) may be worse off than some investors in the mixed equilibrium. The first result follows from the fact over-reacting to the price is more efficient than ignoring it, in equilibrium. The second result follows from the observation that expected speculative gains are lower in the rational expectations equilibrium since all investors use information efficiently. In contrast, the investors that overweight the price are able to exploit those that ignore it in the mixed equilibrium.

Next, we consider the effect of informed investors' deviations from rational expectations on the welfare of liquidity (or noise) traders. Recall that the aggregate supply, z, is noisy. Suppose that this reflects the sale of the risky asset by a liquidity trader, who has CARA utility with risk aversion γ_z and who is endowed with initial wealth W_0. Then, her expected utility is given by

$$U_z \equiv \mathbb{E}\left[-\exp\left\{-\gamma_z\left(-z\right) \times \left(F - P\right) - \gamma_z W_0\right\}\right]. \tag{119}$$

The following result characterizes the impact of motivated beliefs on both the expected utility of liquidity traders as well as total welfare ($\int U_i + U_z$).

Proposition 10. *In equilibrium, the expected utility of a liquidity trader is given by:*

$$U_z = -\sqrt{\frac{\tau_z}{\tau_z + 2\gamma_z\left(\beta\Lambda - \frac{1}{2\tau}\gamma_z(1-\Lambda)^2\right)}} \exp\left\{-\gamma_z W_0\right\} \tag{120}$$

Suppose $\gamma_z \leq \gamma$. Then:

(i) Liquidity traders have higher expected utility in the symmetric equilibrium than in the rational expectations equilibrium.

(ii) In any mixed equilibrium in which Λ is less than its rational-expectations counterpart, liquidity traders have higher expected utility in the mixed equilibrium.

(iii) There exists $\underline{\gamma} \geq 0$ such that for all $\gamma \geq \underline{\gamma}$, total welfare is higher under the subjective beliefs equilibrium than under the rational expectations equilibrium.

Expected utility for a liquidity trader depends on the equilibrium parameters through a term

$$\beta\Lambda - \frac{1}{2\tau}\gamma_z(1-\Lambda)^2. \tag{121}$$

A liquidity trader's utility is driven by two components. The first component ($\beta\Lambda$) reflects her disutility from price impact — for instance, a larger sale (higher z) pushes prices downward, which reduces her proceeds. The second term ($-\frac{1}{2\tau}\gamma_z(1-\Lambda)^2$) reflects a standard risk-aversion channel — when prices are less informative about fundamentals, the liquidity trader faces more uncertainty about her payoff, which reduces utility.[41]

Price sensitivity, Λ, is generally higher when investors exhibit rational expectations. This has offsetting effects on the liquidity trader's utility. On the one hand, a lower Λ implies that the price is less sensitive to her trade and so utility increases through the price impact channel. On the other hand, a lower Λ implies prices track fundamentals less closely which increases the risk in the liquidity trader's payoff. As we show in the proof of Proposition 10, the price impact effect always dominates the risk-aversion effect if the risk aversion of the investors is weakly higher than that of the liquidity traders (i.e., $\gamma_z \leq \gamma$). In this case, liquidity traders are always better off when informed investors choose to deviate from rational expectations.

---

[41]It is important to note that expected utility is finite only when

$$\tau_z + 2\gamma_z\left(\beta\Lambda - \frac{1}{2\tau}\gamma_z(1-\Lambda)^2\right) > 0. \tag{122}$$

Intuitively, if the combined disutility from the price impact and risk aversion terms are too large, the liquidity trader's expected utility from being forced to trade z units approaches negative infinity — she would rather exit the market and not trade if she could.

Figure 9: Difference in U_i under rational and subjective expectations equilibria
The figure plots the difference in expected utility (y-axis) as a function of γ (x-axis). The dashed line plots the difference in expected utility of informed investors (under the objective distribution) (i.e., $\int U_{i,SE} - \int U_{i,RE}$), the dotted line plots the difference in expected utility for the noise traders (i.e., $U_{z,SE} - U_{z,RE}$), and the solid line plots the difference in utility across both groups (i.e., $\int U_{i,SE} + U_{z,SE} - \left(\int U_{i,RE} + U_{z,RE} \right)$). Other parameters are set to $\tau = \tau_e = \tau_z = 1$, and $\gamma_z = 0.75$.
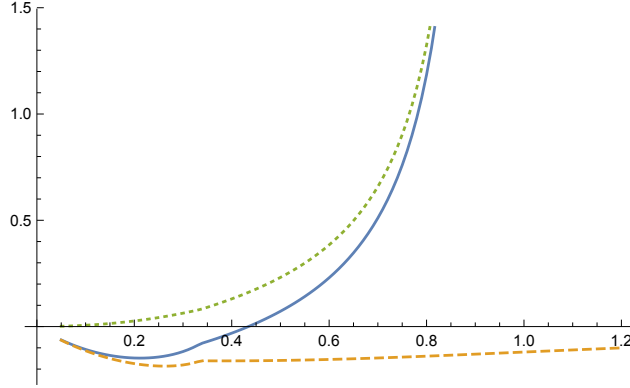
ΔU for investors (dashed), noise traders (dotted), both (solid)

Note that $\gamma_z \leq \gamma$ is a sufficient condition, but it is not necessary for liquidity traders to be better off under the subjective beliefs equilibrium. Figure 9 plots the difference in expected utility between the subjective beliefs equilibrium and the rational expectations equilibrium as a function of investor risk aversion γ for each group separately, and for both groups as a whole. The plot illustrates that for this set of parameters, liquidity traders are always better off under subjective beliefs — the dotted line is always above zero — irrespective of whether informed investors are more or less risk averse than them. In particular, note that noise trader risk aversion γ_z is fixed at 0.75, but informed investor risk aversion γ ranges from 0.1 to 1.2. Not surprisingly, under the objective distribution, the informed investors are worse off under the subjective beliefs equilibrium — the dashed line is always below zero. The solid line in Figure 9 illustrates the aggregate welfare ranking in Proposition 10, part (iii): aggregate welfare is higher in the rational expectations equilibrium when informed investor risk aversion is low, but higher under subjective beliefs otherwise. These results suggest that deviations from rational expectations may make liquidity traders better off.

Proof of Proposition 10

The utility of noise traders is

$$
\begin{aligned}
U_z &= -E\left(\gamma_z \exp\{+\gamma_z z\left(F - P\right)\}\right) \\
&= -E\left(\gamma_z \exp\{\gamma_z z F\left(1 - \Lambda\right) - \gamma \Lambda \beta z^2\}\right) \\
&= -E\left(\gamma_z \exp\left\{\left(\frac{\gamma_z^2 \left(1 - \Lambda\right)^2}{2\tau} - \gamma_z \Lambda \beta\right) z^2\right\}\right) \\
&= -\gamma_z \frac{1}{\sqrt{1 - 2\frac{1}{\tau_z}\left(\left(\frac{\gamma_z^2(1-\Lambda)^2}{2\tau} - \gamma_z \Lambda \beta\right)\right)}} \\
&= -\gamma_z \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z \Lambda \beta}}
\end{aligned}
$$

where we used the fact that $E\left(e^{a\epsilon^2}\right) = \frac{1}{\sqrt{1-2a\sigma_\epsilon^2}}$. This implies that utility of noise traders is monotonically decreasing in $\frac{\gamma_z(1-\Lambda)^2}{2\tau} - \Lambda \beta$.

Proof of Part (i) In this case, all investors dismiss the price and so

$$
\Lambda = \frac{\tau_e}{\tau + \tau_e}, \quad \Lambda_{RE} = \frac{\tau_e + \tau_p}{\tau + \tau_e + \tau_p} \tag{123}
$$

so

$$
U_{z,SE} - U_{z,RE} > 0 \tag{124}
$$

$$
\Leftrightarrow \frac{\gamma \tau \tau_p}{\tau_e(\tau_e+\tau)(\tau_e+\tau_p+\tau)} > \frac{\tau \gamma_z}{2} \frac{\tau_p(2\tau_e+\tau_p+2\tau)}{(\tau_e+\tau)^2(\tau_e+\tau_p+\tau)^2} \tag{125}
$$

$$
\Leftrightarrow \frac{\gamma}{\gamma_z} > \frac{\tau_e}{\tau+\tau_e} \frac{2(\tau_e+\tau)+\tau_p}{2(\tau_e+\tau_p+\tau)} \tag{126}
$$

which implies if $\gamma \geq \gamma_z$, then $U_{z,SE} > U_{z,RE}$. $\qquad \square$

Proof of Part (ii): In the mixed equilibrium, some investors overweight the price. Let $\int \delta_{p,i} \equiv \delta_p$, and so we write

$$
\Lambda = \frac{\delta_e + \delta_p \tau_p}{\tau + \tau_e + \delta_p \tau_p} \tag{127}
$$

This implies that

$$
U_{z,SE} - U_{RE} > 0 \tag{128}
$$

$$
\Leftrightarrow \gamma\left(\frac{\Lambda_{RE}}{\tau_e} - \frac{\Lambda}{\tau_e}\right) > \gamma_z\left(\frac{(1-\Lambda)^2}{2\tau} - \frac{(1-\Lambda_{RE})^2}{2\tau}\right) \tag{129}
$$

$$
\Leftrightarrow \frac{\gamma}{\tau_e}\left(\Lambda_{RE} - \Lambda\right) > \frac{\gamma_z}{2\tau}\left(\Lambda_{RE} - \Lambda\right)\left(2 - \left(\Lambda + \Lambda_{RE}\right)\right) \tag{130}
$$

When $\Lambda_{RE} > \Lambda$, this is equivalent to:

$$\frac{\gamma}{\gamma_z} > \frac{\tau_e}{2}\left(\frac{1}{\tau+\tau_e+\tau_p} + \frac{1}{\tau+\tau_e+\delta_p\tau_p}\right) \tag{131}$$

which is always true if $\gamma \geq \gamma_z$. □

Proof of Part (iii): Total welfare is given by:

$$W(\delta_p) = -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} - \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z\Lambda\beta}} \tag{132}$$

Moreover, for the rational expectations equilibrium, we have $\delta_p = 1$. This implies that the difference in welfare is:

$$W(\delta_p) - W_{RE} = \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} + \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}+2\gamma_z\Lambda_{RE}\beta}} \\ -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} \\ -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau}+2\gamma_z\Lambda\beta}} \end{array} \tag{133}$$

Above, we have established that when $\gamma_z \leq \gamma$ and $\Lambda < \Lambda_{RE}$, we have:

$$U_{z,SE} = -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma^2(1-\Lambda)^2}{\tau} + 2\gamma\Lambda\beta}} > -\sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta}} = U_{z,RE} \tag{134}$$

$$\Leftrightarrow \tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau} + 2\gamma_z\Lambda\beta \geq \tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta > 0 \tag{135}$$

Let

$$\bar{\gamma} \equiv \frac{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau}}{\frac{2\gamma_z\Lambda_{RE}}{\tau_e}} = \frac{\tau_e\left(\tau_z(\tau_e+\tau_p+\tau)^2-\tau\gamma_z^2\right)}{2\gamma_z(\tau_e+\tau_p)(\tau_e+\tau_p+\tau)}. \tag{136}$$

Note that

$$\lim_{\gamma\uparrow\bar{\gamma}} \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda_{RE})^2}{\tau} + 2\gamma_z\Lambda_{RE}\beta}} = \infty, \tag{137}$$

but

$$\lim_{\gamma\uparrow\bar{\gamma}} \begin{array}{c} \frac{1}{\sqrt{\left(\frac{(1-\Lambda_{RE})^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p)}} - \sqrt{\frac{\tau_z}{\tau_z - \frac{\gamma_z^2(1-\Lambda)^2}{\tau}+2\gamma_z\Lambda\beta}} \\ -\frac{1}{\sqrt{\Lambda^2(\delta_p-1)^2+\left(\frac{(1-\Lambda)^2}{\tau}+\frac{\Lambda^2}{\tau_p}\right)(\tau+\tau_e+\tau_p\delta_p(2-\delta_p))}} \end{array} \geq -c \tag{138}$$

for some $c \leq \infty$. This implies

$$\lim_{\gamma\to\bar{\gamma}} W(\delta_p) - W_{RE} > 0, \tag{139}$$

or equivalently, $\exists\underline{\gamma} \leq \bar{\gamma}$, such that for all $\gamma > \underline{\gamma}$, $W(\delta_p) > W_{RE}$. □

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