Question 4

Numerical solution of Diffusion Equation

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Diffusion equation for 1D:

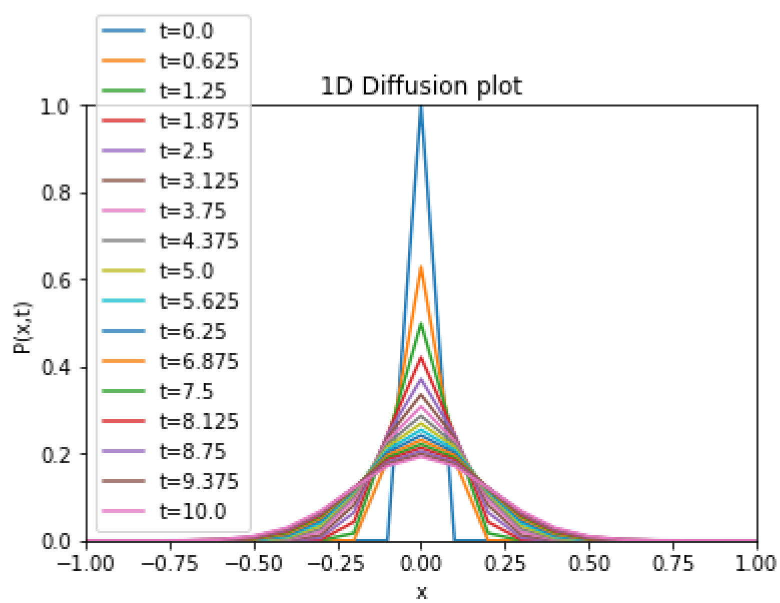
$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2}$$

Initial conditions: P(x,0) = 1 when x=0, 0 otherwise
Boundary conditions: P(-L,t) = P(L,t) = 0, t>0

Where x is the position, t is the time
Using Forward Euler Scheme: method for solving ordinary differential equations using the formula

y_(n+1) = y_n + hf(x_n,y_n),
which advances a solution from x_n to x_(n+1) = x_n + h

Plotting probability(x,t) versus position we get the distribution of random particle in 1D:



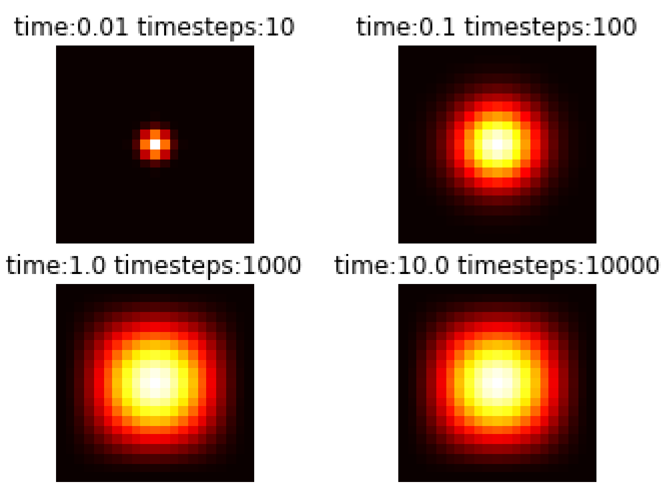
Diffusion equation for 2D:

$$\frac{\partial P(x,y,t)}{\partial t} = D_x \frac{\partial^2 P(x,y,t)}{\partial x^2} + D_y \frac{\partial^2 P(x,y,t)}{\partial y^2}$$

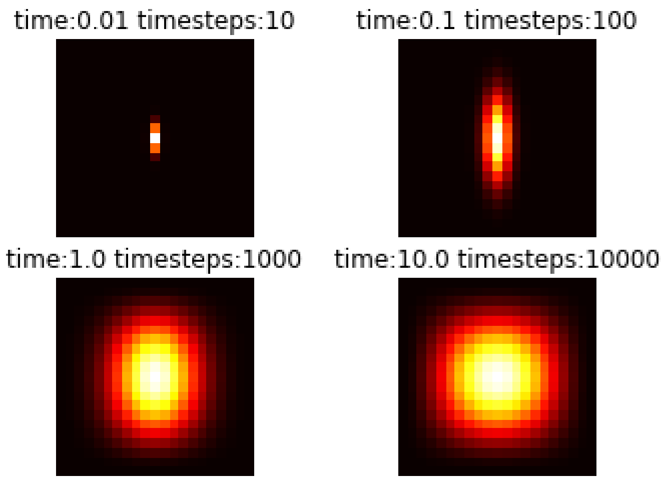
Where Dx, Dy are diffusion constants for x,y respectively. This equation allows us to talk about the statistical movements of randomly moving particles in 2 dimensions.

The density plots for different timesteps (n=10,100,1000,10000) for:

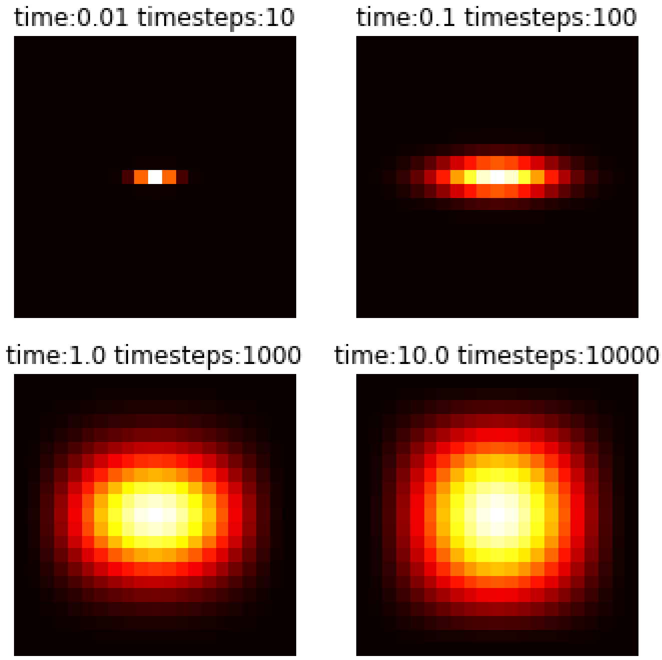
I) Dx = Dy



Ii) Dx > Dy



Iii) Dx < Dy


time:0.01 timesteps:10
time:0.1 timesteps:100
time:1.0 timesteps:1000
time:10.0 timesteps:10000