Question 6

Integration using Monte Carlo

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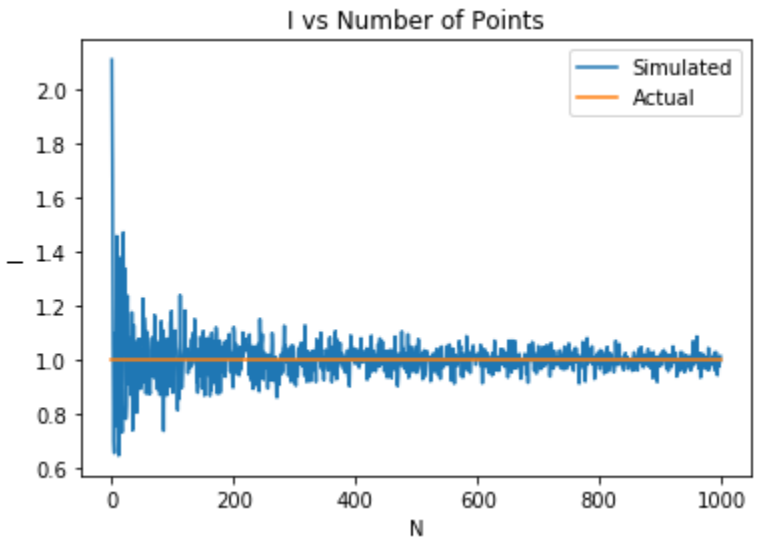
Monte Carlo simulations are used to obtain numerical results through repeated random sampling. We can apply this principle to calculate the definite integrals.

Case 1:

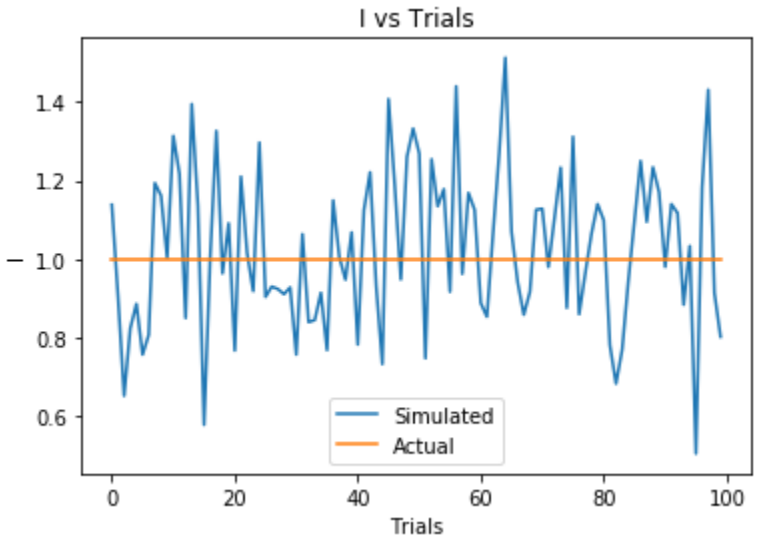
$$I = \int_0^1 3x^2 \, dx$$

Plotting I vs Number of points:



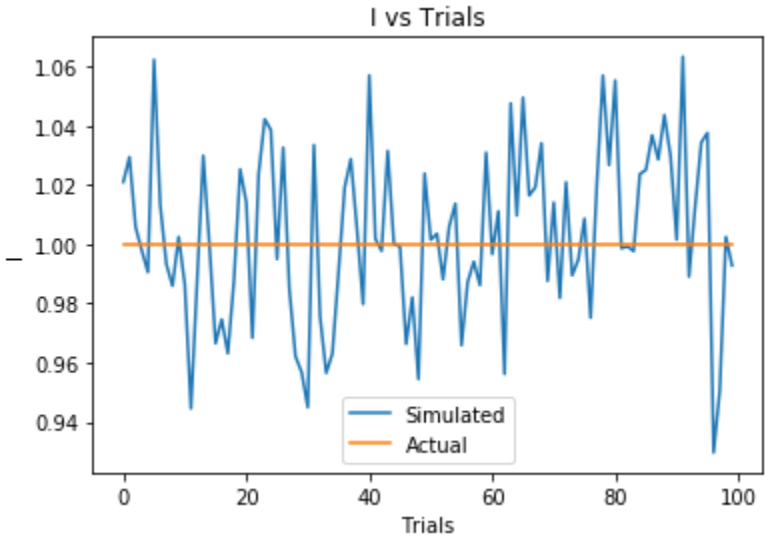
For N=20, Trials=100

Standard deviation: 0.19978270592802957

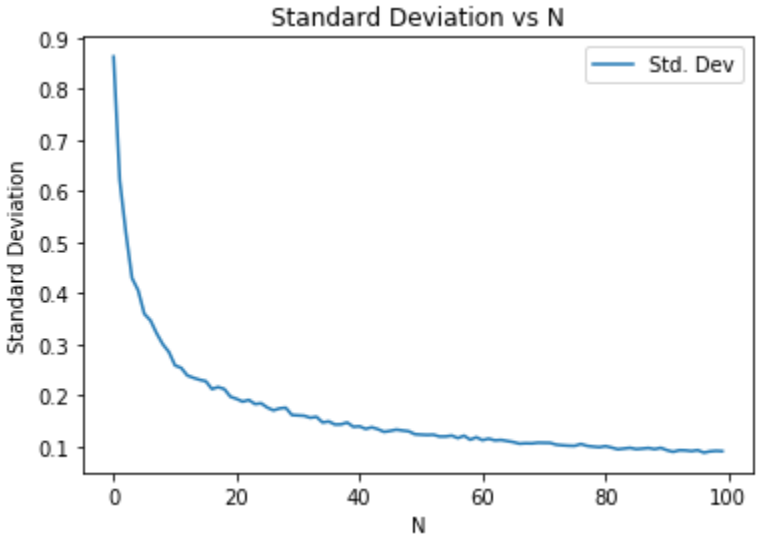


For N=1000, Trials=100

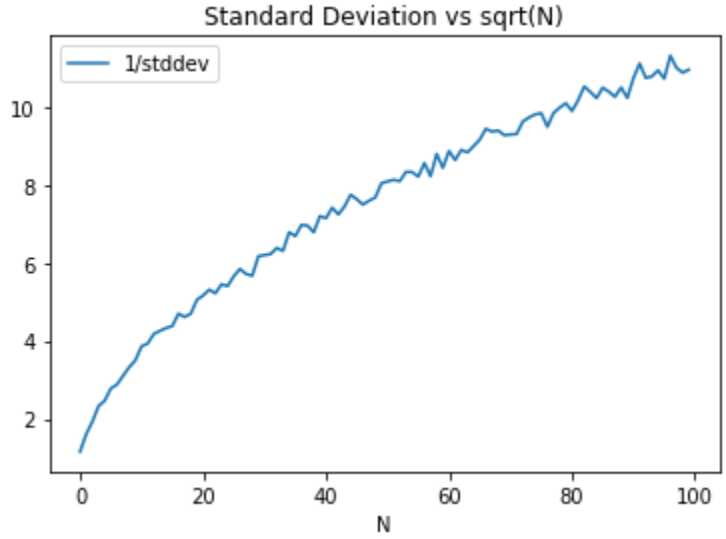
Standard deviation: 0.0286288538549087



Plotting Standard Deviation vs N:



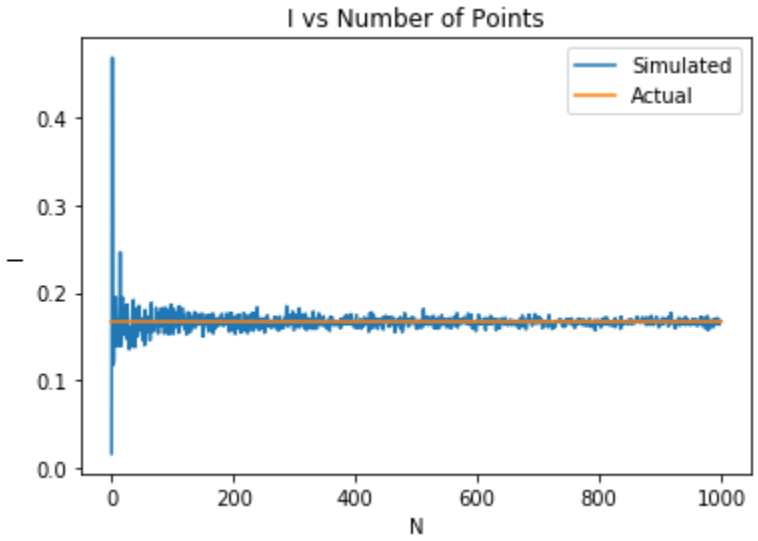
Plotting Standard Deviation inverse:



Case 2:

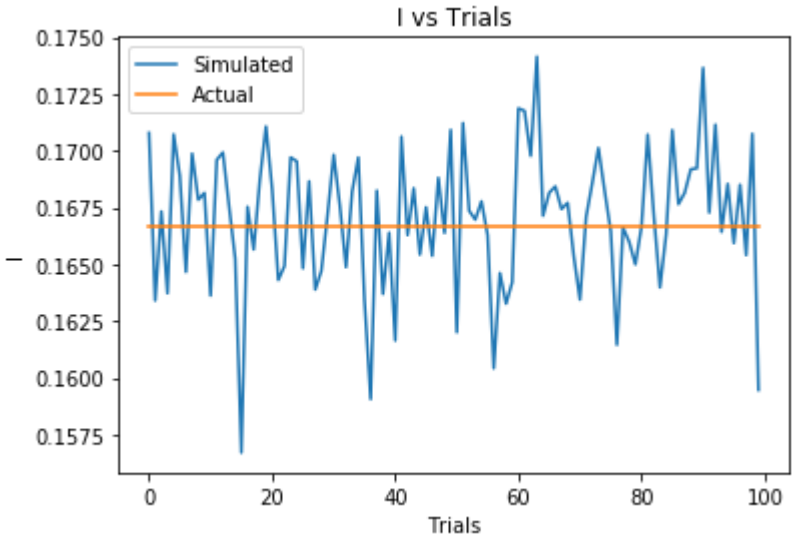
$$I = \int_0^1 \int_0^1 x^2 y \; dxdy$$

Plotting I vs Number of points:



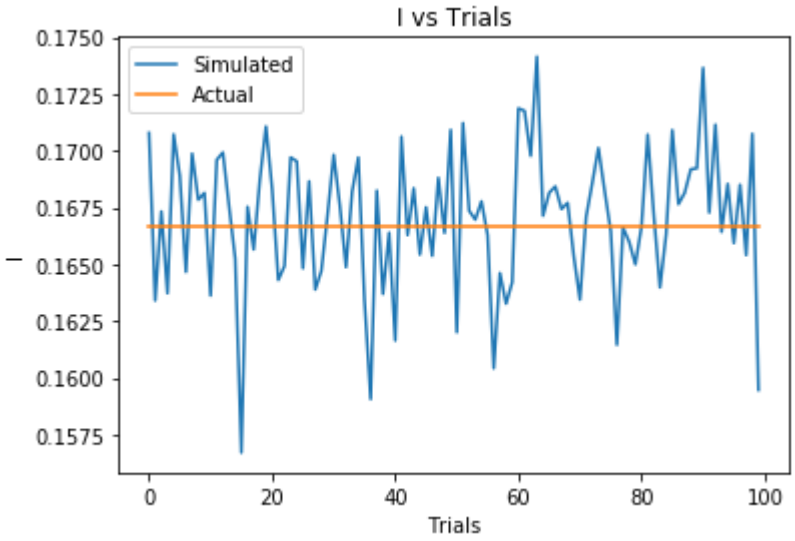
For N=20, Trials=100

Standard deviation: 0.024555639141231926

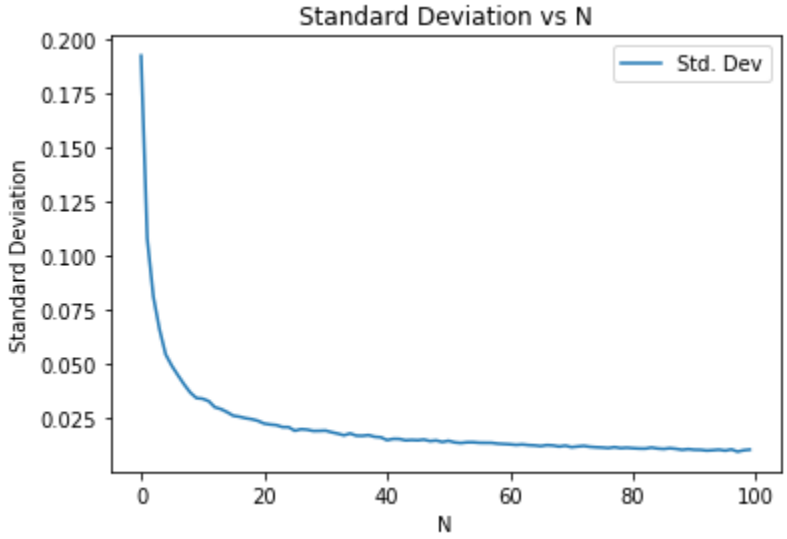


For N=1000, Trials=100

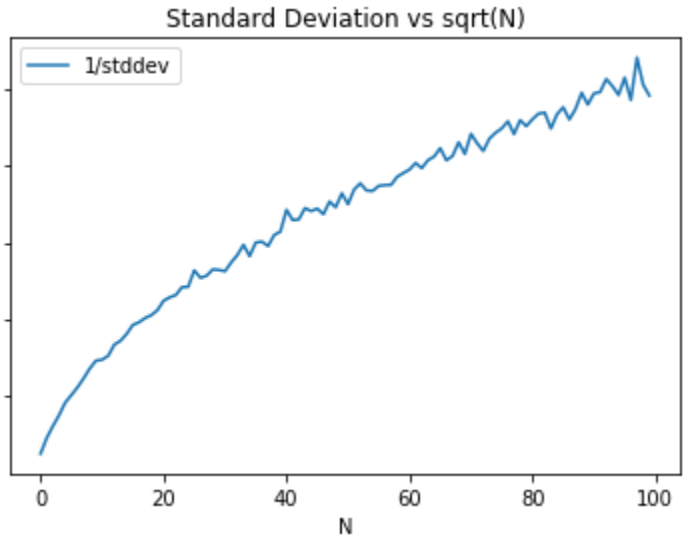
Standard deviation: 0.0030616586302177054



Plotting Standard Deviation vs N:



Plotting Standard Deviation inverse:



From the plot we can see that the standard deviation is proportional to sqrt(N) by its inverse.

In both integrals, we see that the number of points used in the Monte Carlo reduces the standard deviation by a tenth which is true as the likelihood of the outcome increases with more points.