Question 3

1D Harmonic Oscillator

3. 1D Harmonic oscillator.

$$H(x,p) = \frac{1}{2}kx^2 + \frac{p^2}{2m}$$

Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m} \quad \text{--- (1)}$$

$$\dot{p} = -\frac{\partial H}{\partial x} = -kx \quad \text{--- (2)}$$

From (1), (2),

$$\frac{dp}{dt} = -kx \quad , \quad \frac{dx}{dt} = \frac{p}{m}$$

$$\Rightarrow \frac{d(m\dot{x})}{dt} = -kx \Rightarrow m\ddot{x} = -kx$$

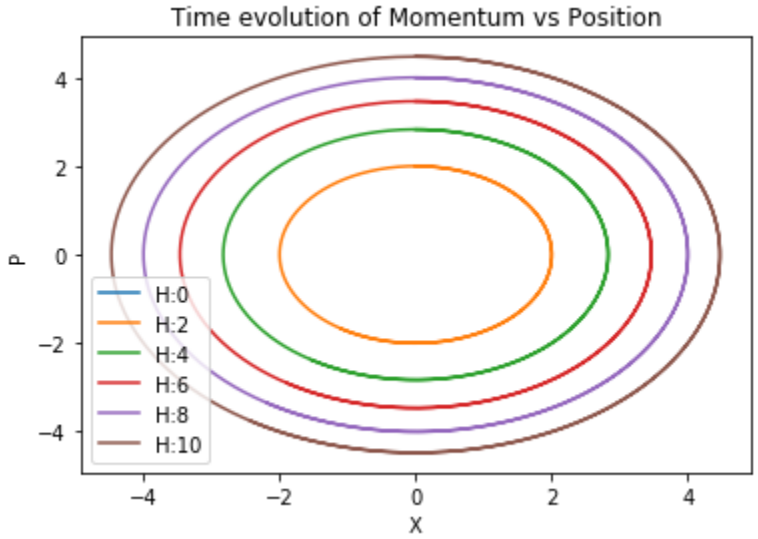
Time evolution of x, P:

$$x(t) = A \cos(\omega t + \delta)$$

$$P(t) = Am\omega \sin(\omega t + \delta)$$

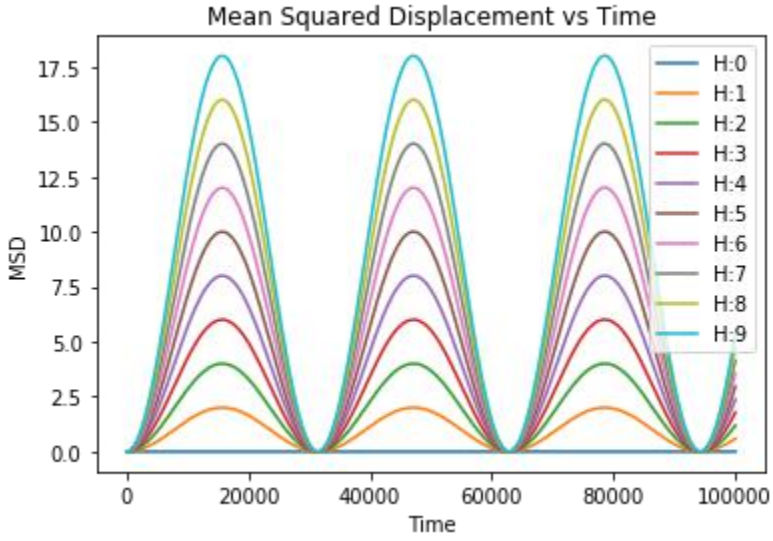
After getting the Hamilton's equations from the equation of motion, we get the time evolution of position and momentum. For different values of H (determined by values of k,m)

Plotting the momentum vs position:



The trajectory is elliptical in nature as seen from the plot and the equation.

Plotting the Mean Squared Displacement:



The mean squared displacement is oscillatory in nature.