SCIENCE-II

CLASS-I

SNEHAL KUMAR

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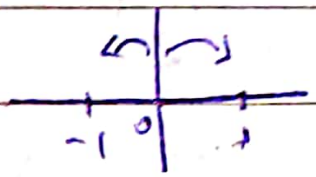
Q.1. Probability of 2 drunks meeting after 'N' steps.

$x_i = \{-1, 1\}$ (i^{th} step left or right)



$x_i = -1$

$P(\quad) = \frac{1}{2} = P(x_i = 1)$

Let $X = \sum_{i=1}^{N} x_i$

For $X = K$, after N steps :

a = no of right steps, b = no of left steps

$a + b = N$

$a - b = K$.

$2a = N + K \Rightarrow a = \frac{N+K}{2}$

$b = \frac{N-K}{2}$

$P(X = K) = {}^{N}C_{\left(\frac{N+K}{2}\right)} \left(\frac{1}{2}\right)^{\left(\frac{N+K}{2}\right)} \left(\frac{1}{2}\right)^{\left(\frac{N-K}{2}\right)}$

$P(X = K) = {}^{N}C_{\left(\frac{N+K}{2}\right)} \left(\frac{1}{2}\right)^{N}$

Both drunks will have same PDF(x) = $\begin{cases} \frac{1}{2}, & \text{same direction} \\ \frac{1}{2}, & \text{opp direction} \\ 0, & \text{otherwise} \end{cases}$

$P(E) = \sum_{j=-N}^{N} P(X = j)^2$:c !

$$P(E) = \sum_{j=-N}^{N} P(x=j)^2$$

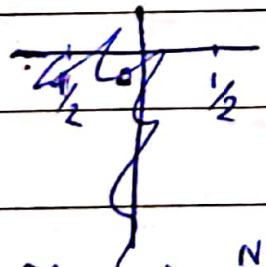
$$= \left(\frac{1}{2}\right)^{2N} \left[\left({}^{N}C_0\right)^2 - \ldots \left({}^{N}C_N\right)^2 \right]$$

$$= \left(\frac{1}{2}\right)^{2N} \left({}^{2N}C_N\right)$$

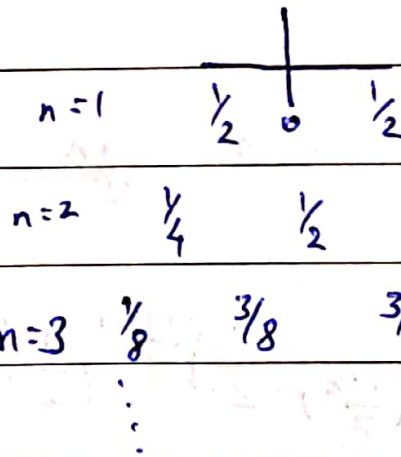
$$P(E) = {}^{2N}C_N \left(\frac{1}{2^{2N}}\right)$$

Q.2. Probability of drunk returning to origin:

Here $K=0$ after N steps.

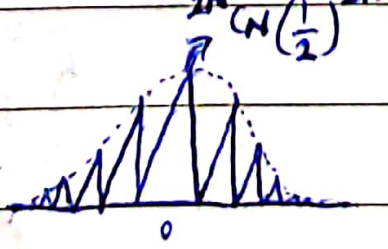


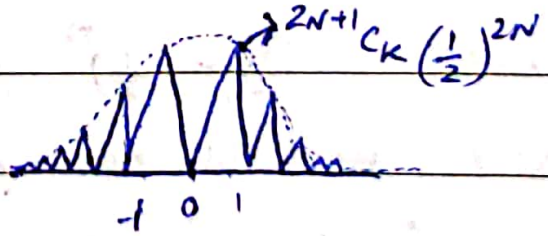
$$P(x=0) = {}^{N}C_{N/2} \left(\frac{1}{2}\right)^{N}$$



if N is odd $\Rightarrow P(x=0) = 0$.

N even $\Rightarrow$





(Bell shaped distribution)

Probability : $P(x) = \begin{cases} {}^{2n}C_x \left(\frac{1}{2}\right)^{2n}, & x \text{ is even} \\ 0, & x \text{ is odd} \end{cases}$

Mean Displacement:

$E[x] = E[\Sigma x_i] = \Sigma E[x_i]$

$x_i : \{-1, 1\}$

$E[x] = (1)\left(\frac{1}{2}\right) + (-1)\frac{1}{2} = 0$

Mean Squared Displacement:

$E[x^2] = E[(\Sigma x_i)^2] = E[\Sigma x_i^2] + 2 \cdot E[\sum_{i \neq j} x_i x_j]$

$E[x^2] = \frac{1}{2} + (-1)^2\left(\frac{1}{2}\right) = n \times 1 = n$

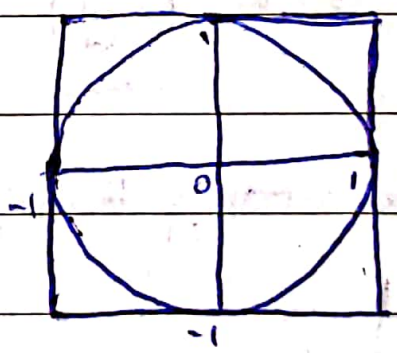
Q.3 Find value of π

Favourable area $= \pi r^2$

$r = 1, \Rightarrow \pi r^2 = \pi$

Total area $= s \times s$, $(s = 2)$

$= 4$



Probability of pebble in circle $= \frac{\pi}{4}$

Let x be number of pebbles in circle out of n pebbles:

$\frac{x}{n} = \frac{\pi}{4}$

$\pi = 4 \times \left(\frac{x}{n}\right)$

can be simulated for large n for accurate result.