(5) Convex hull :

→ It is the smallest convex polygon containing all the given points.

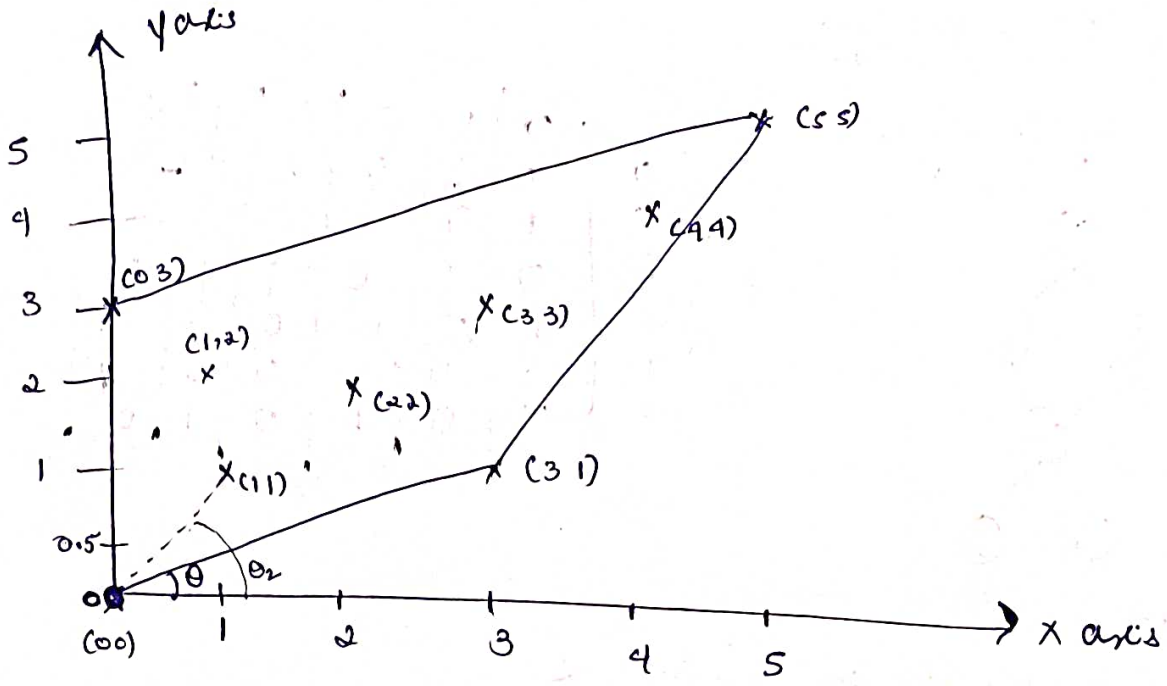
Example :

Consider the below points

(0 0) (1 1) (2 2) (3 3) (4 4) (5 5)

(0 3) (3 1) (1 2)

plot it



(0 0)
(3 1)
(5 5)
(0 3)

⇒ Convex Hull points.

Tracing of Algorithm:

(a) Select the smallest y-coordinate point.

(0,0)

(b) Sort the points according to the angle (θ) (shown in fig) relative to the bottom most point & x axis

(step (a))

⇒ Sorted list:

(0 0) (3 1) (1 1) (2 2) (3 3) (4 4) (5 5)

(1 2) (0 3)

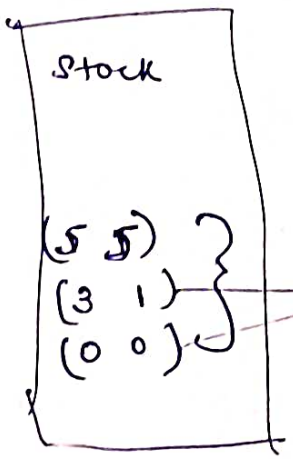
(c) Iterate the sorted order.

→ If it makes a anti-clockwise turn relative to the previous 2 points on the stack

⇒ push into the stack.

→ If it makes clock wise angle

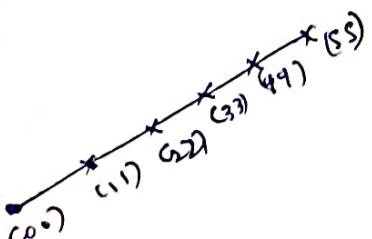
⇒ pop from the stack.



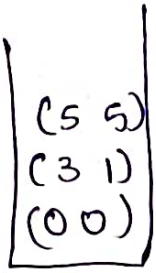
initially push 2 points from the sorted order.

iterate (1 1) (2 2) (3,3) (4 4) (5 5)

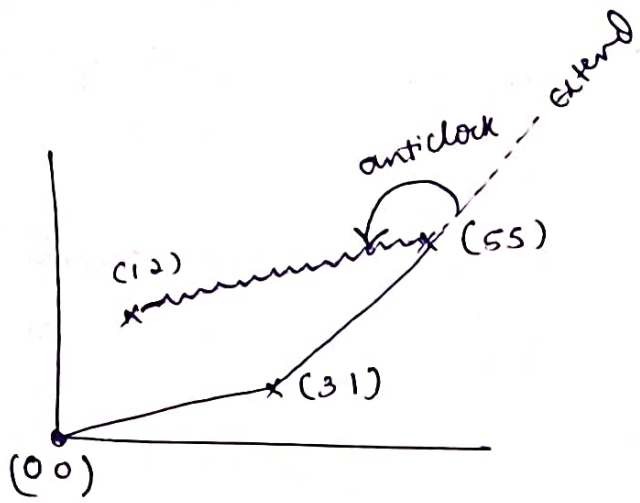
Since all these points are co-linear, pick the farthest co-linear point & place on the stack.



Stock :

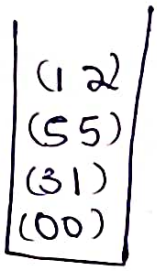


(1 2)

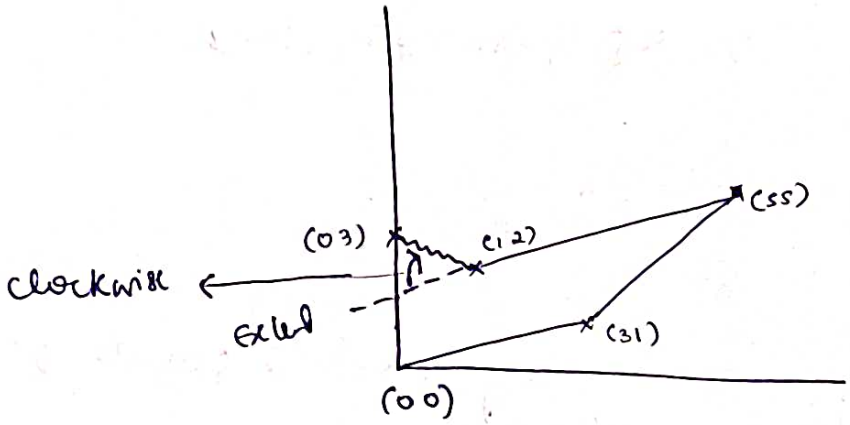


(1,2) is forming anti-clock wise so push it on the stock.

⇒

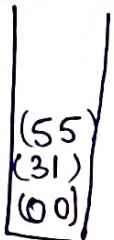


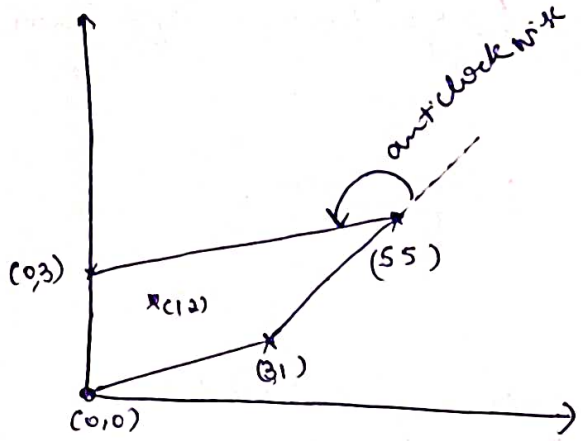
consider next point (03)



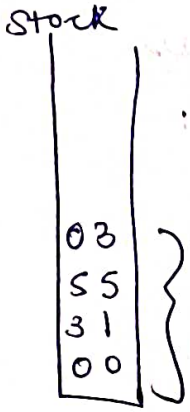
Since (0,3) is forming clock-wise turn, pop an element from stock.

⇒





Since (0,3) is forming anti-clockwise turn. push it into the stack.



⇒ convex Hull

completed sorted elements tracing.

Time complexity :-

$$\left[\begin{array}{l}\text{Total Time}\\ \text{complexity}\end{array}\right] = \left[\begin{array}{l}\text{sort all}\\ \text{the points}\end{array}\right] + \left[\begin{array}{l}\text{travel each point}\\ \text{to either push/pop}\\ \text{into stack}\end{array}\right]$$

$$= n\log n + n$$

(high)

$$\Rightarrow O(n\log n)$$

———— X ————