Week 04 Homework Report

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Problem 1

P0 = 100

For simulation, I used Mu=0, Sigma=0.1 to sample the returns

	Mu (Sim)	Sigma (Sim)	Mu (Expected)	Sigma (Expected)
Brownian Motion	100.00	0.10	100.00	0.10
Arithmetic Returns	100.06	9.92	100.00	10.00
Geometric Brownian Motion	100.55	10.02	100.50	10.08

Derivation of Expected Mu, Sigma

Problem 1

Brownian Motion

$P_t = P_{t-1} + r_t$

$E(P_t \mid P_{t-1} = x] = E[P_{t-1} + r_t]$

$= E[P_{t-1}] + E[r_t]$

$= P_{t-1} + 0$ because μ is 0

$\boxed{E = P_{t-1}}$

$Var(P_t \mid P_{t-1}) = Var(P_{t-1} + r_t)$ P_{t-1} treated as constant

$= \boxed{Var(r_t) = \sigma^2}$

Arithmetic Return

$P_t = P_{t-1}(1 + r_t)$

$E[P_t \mid P_{t-1} = x] = E[P_{t-1} + P_{t-1} \cdot r_t]$

$= E[P_{t-1}] + P_{t-1} E[r_t]$

$P_{t-1} + P_{t-1}(0) = \boxed{P_{t-1}}$

$Var(P_t \mid P_{t-1}) = Var(P_{t-1} + P_{t-1} r_t)$ P_{t-1} is constant

$= Var(P_{t-1} r_t)$

$= (P_{t-1})^2 \, var(r_t)$

$= \boxed{(P_{t-1})^2 \sigma^2}$

Geometric Brownian Motion

$P_t = P_{t-1} e^{r_t}$

$E(P_t \mid P_{t-1}) = E[P_{t-1} e^{r_t}]$

$= (P_{t-1}) E[e^{r_t}]$

$(P_{t-1}) e^{(\mu + \frac{\sigma^2}{2})}$ $\mu = 0$

$\boxed{E = (P_{t-1}) e^{\frac{\sigma^2}{2}}}$

$Var(P_t \mid P_{t-1}) = Var(P_{t-1} e^{r_t})$

$= (P_{t-1})^2 var(e^{r_t})$

$(P_{t-1})^2 (e^{\sigma^2} - 1) e^{(2\mu + \sigma^2)}$

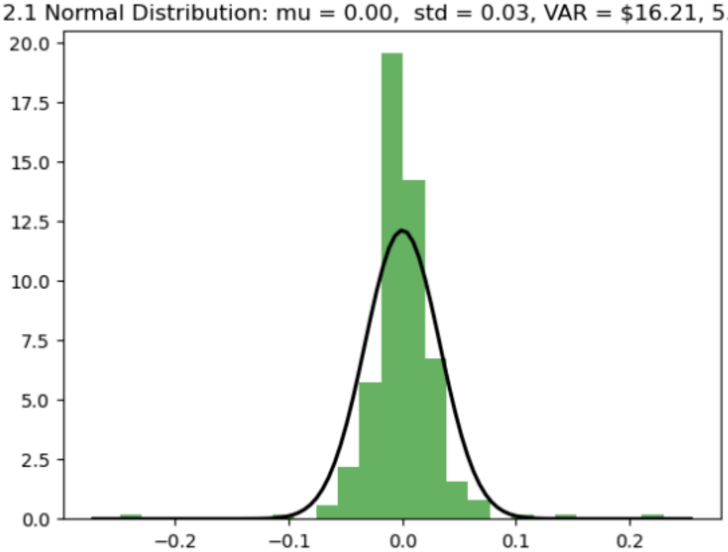
$\boxed{(P_{t-1})^2 (e^{2\sigma^2} - e^{\sigma^2})}$ $\mu = 0$

$(P_{t-1})^2 (e^{2\sigma^2} - e^{\sigma^2})$

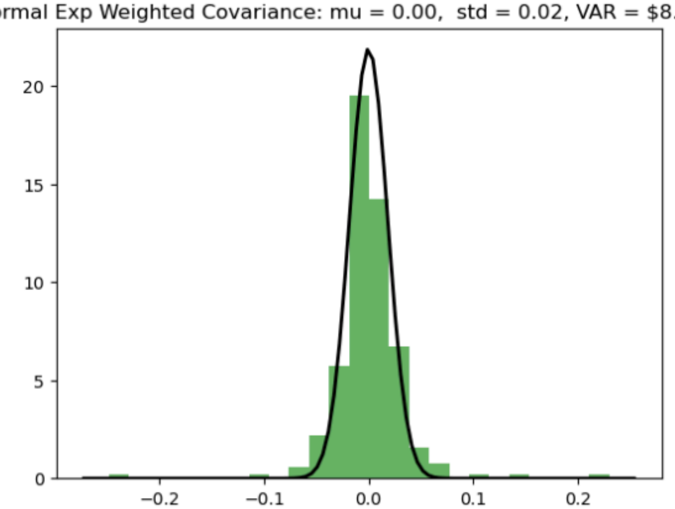
Problem 2

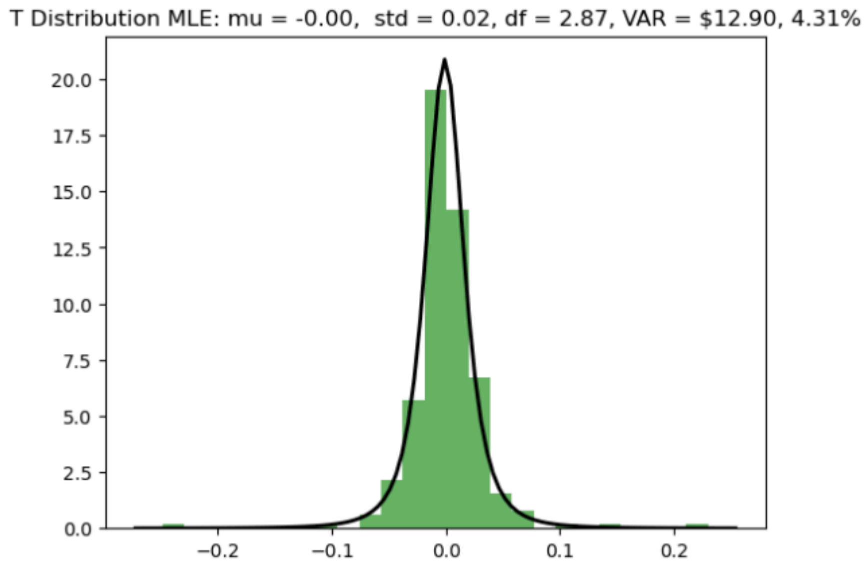
Model	VAR ($)	VAR (%)
Normal	$16.21	5.42%
Normal w/ Exp Weighted Covariance	$8.97	3.00%
T w/ MLE	$12.90	4.31%
AR1 Process	$16.07	5.37%
Historical Simulation	$12.19	4.08%

2.1 Normal Distribution: mu = 0.00, std = 0.03, VAR = $16.21, 5.42%



2.2 Normal Exp Weighted Covariance: mu = 0.00, std = 0.02, VAR = $8.97, 3.00%



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                               SARIMAX Results
==============================================================================
Dep. Variable:                   META   No. Observations:                  265
Model:                 ARIMA(1, 0, 0)   Log Likelihood                 528.712
Date:                Sun, 25 Feb 2024   AIC                          -1051.423
Time:                        22:32:29   BIC                          -1040.684
Sample:                             0   HQIC                         -1047.108
                                - 265
Covariance Type:                  opg
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==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const       -1.14e-06      0.002     -0.001      1.000      -0.004       0.004
ar.L1          0.0461      0.081      0.569      0.569      -0.113       0.205
sigma2         0.0011   4.17e-05     25.987      0.000       0.001       0.001
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):              4463.63
Prob(Q):                              0.99   Prob(JB):                         0.00
Heteroskedasticity (H):               0.20   Skew:                             0.00
Prob(H) (two-sided):                  0.00   Kurtosis:                        23.11
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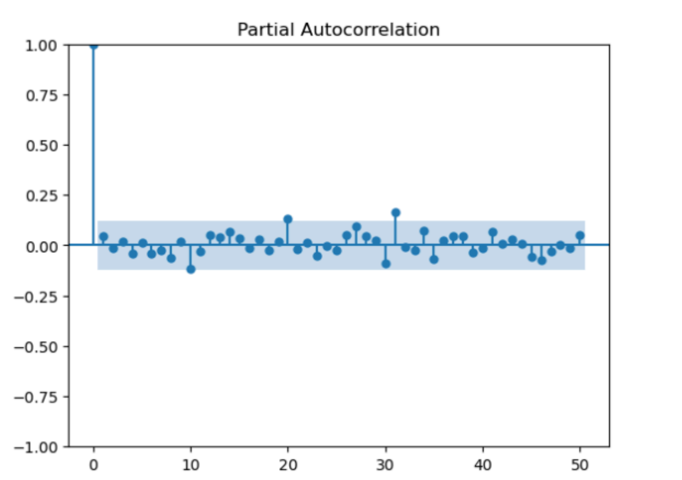

Partial Autocorrelation graph and SARIMAX AR1 fit results on left.

Problem 3

Using Normal Distribution with Exp Weighted Covariance, lambda=0.94

10,000 rounds of simulation

Portfolio	currentValue	VaR95
A	1.089316E+06	15324.602412
B	5.745424E+05	8012.230757
C	1.387410E+06	18016.012036
Total	3.051268E+06	38634.047417

Using Normal Distribution with Exp Weighted Covariance, lambda=0.98

Portfolio	currentValue	VaR95
A	1.089316E+06	15230.641488
B	5.745424E+05	8415.222566
C	1.387410E+06	19650.669187
Total	3.051268E+06	39881.073972

Using Normal Distribution with Exp Weighted Covariance, lambda=0.90

Portfolio	currentValue	VaR95
A	1.089316E+06	16674.402572
B	5.745424E+05	8455.675841
C	1.387410E+06	18065.393668
Total	3.051268E+06	41110.960432

Here I experiment with varying the lambda to determine how much weight is given to the recent data. This reveals each of the portfolios sensitivity to recent data. Some portfolios have increased VaR with greater lambda and other have the opposite effect. Still others are stable despite change to lambda.