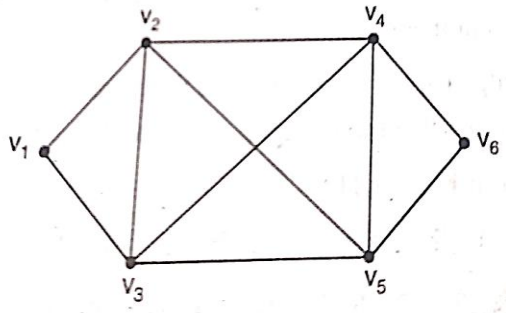
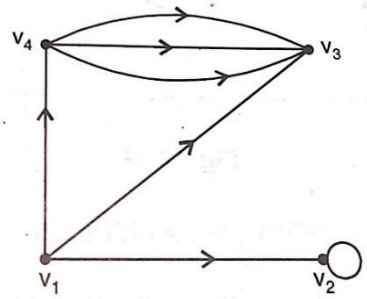
**Example 2.** Draw the graphs of the chemical molecules of

(*a*) methane ($CH_4$) (*b*) propane ($C_3H_8$)

***Example 3.*** *Find the degree of each vertex of the following graph.*



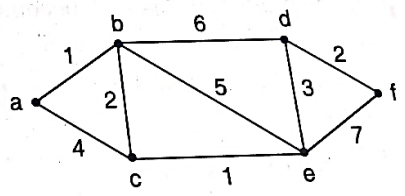

***Example 4.*** Find the in degree out degree and of total degree of each vertex of the following graph.



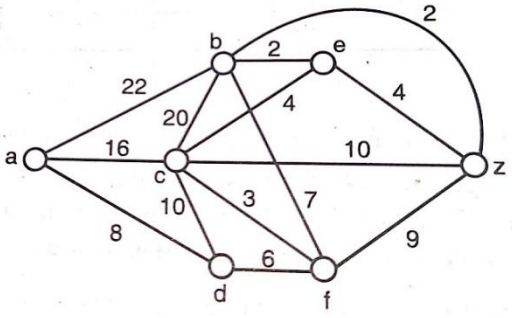

**Example 11.** Show that $C_6$ is a bipartite graph.

**Example 5.** Find all spanning trees of the graph $G$ shown in Fig. 16.5

**Example 36.** Apply Dijkstra's algorithm to the graph given below and find the shortest path from $a$ to f

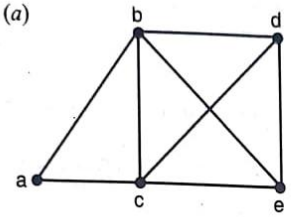



***Example 37 :*** Determine a shortest path between the vertices $a$ to $z$ as shown below.

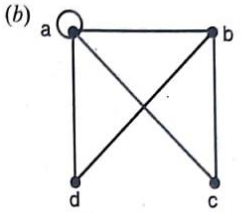


3. Consider the following graphs and determine the degree of each vertex.

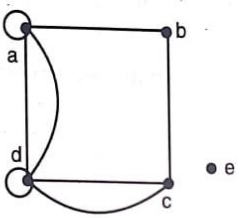
(*a*)



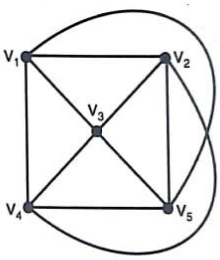

(*b*)




(*c*)

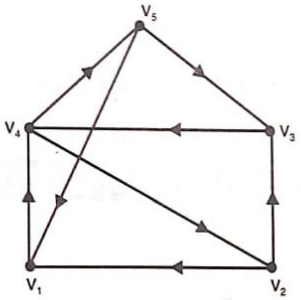



(*d*)

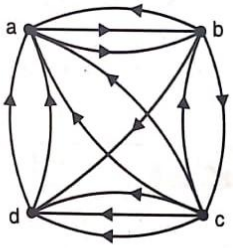



4. Find the in-degree and out-degree of each vertex of the following directed graphs

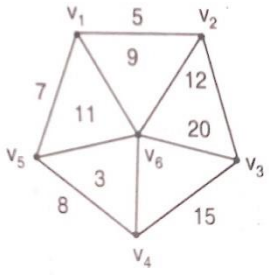
(*a*)




(*b*)

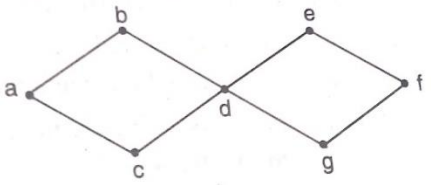



**Example 14.** Using Kruskal's algorithm find a spanning tree with minimum weight from the graph below. Also calculate the total weight of spanning tree.



5. Draw a graph having the given properties or explain why no such graph exists.
   (*a*) Graph with four vertices of degree 1,1,2 and 3.
   (*b*) Graph with four vertices of degree 1,1,3 and 3.
   (*c*) Simple graph with four vertices of degree 1,1,3 and 3.
   (*d*) Graph with six vertices each of degree 3
   (*e*) Graph with six vertices and four edges.
   (*f*) Graph with five vertices of degree 3, 3, 3, 3, 2
   (*g*) Graph with five vertices of degree 0, 1, 2, 2, 3
6. How many vertices do the following graphs have if they contain
   (*a*) 16 edges and all vertices of degree 2.
   (*b*) 21 edges, 3 vertices of degree 4 and others each of degree 3.
7. Suppose a graph has vertices of degree 0,2,2,3 and 9. How many edges does the graph have ?
8. How many vertices and how many edges do the following graphs have ?
   (*a*) $K_n$ (*b*) $C_n$ (*c*) $W_n$ (*d*) $K_{m,n}$ (*d*) $Q_n$

**Example 10.** Find a spanning tree of the graph of Fig. 16.11 using Depth–first search algorithm.




**Example 13.** Show how Kruskal's algorithm finds a minimal spanning tree of the graph of Fig. 16.15.

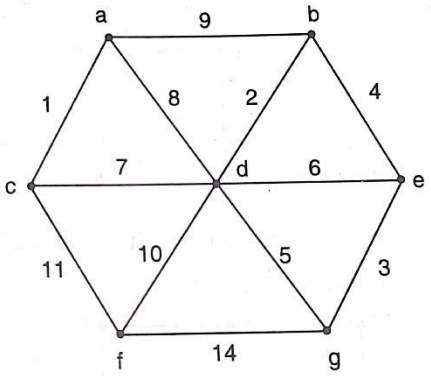
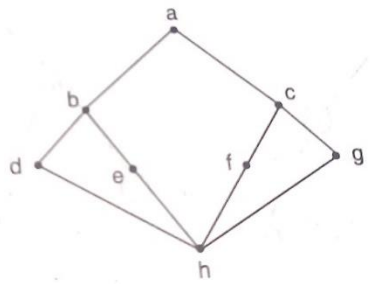



**Fig. 16.15**

**16.** Find the spanning trees of the graph shown using depth. First search and breadth-firsr search.




**Example 15 (*a*).** Find the minimal spanning tree of the weighted graph of Fig. 16.16 using Prim's algorithm.

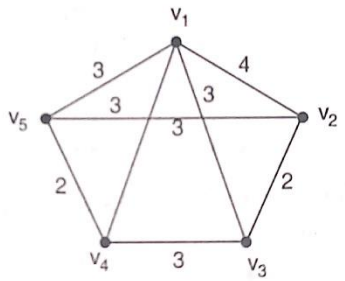
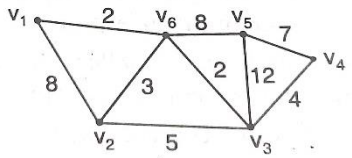
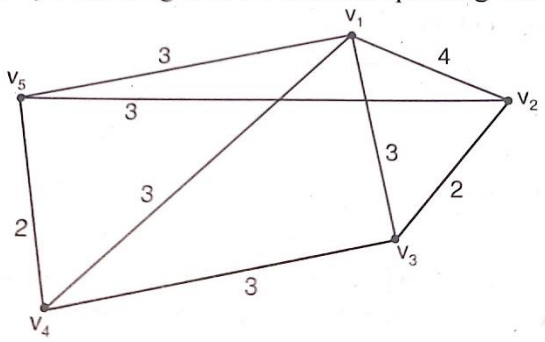



Fig. 16.16

**Example 16 (*a*).** Describe Prim's algorithm and use this to find out the minimal spanning tree of the following graph.




***Example 16 (b).*** Find by Prim's algorithm a minimal spanning tree from the following graph



**Example 17.** What are the left and right children of $b$ shown in Fig. 16.18? What are the left and right subtrees of a?

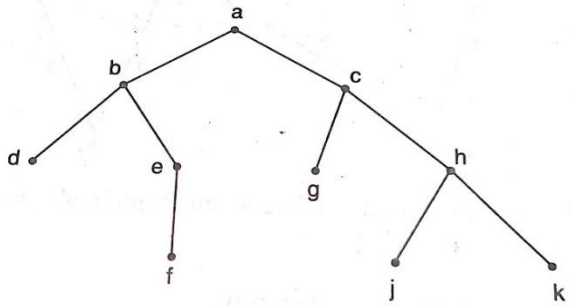



Fig. 16.18

**Example 29.** Find by Prim's algorithm a minimal spanning tree from the following connected weighted graph:

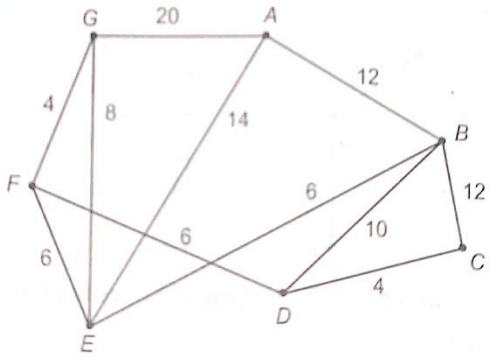
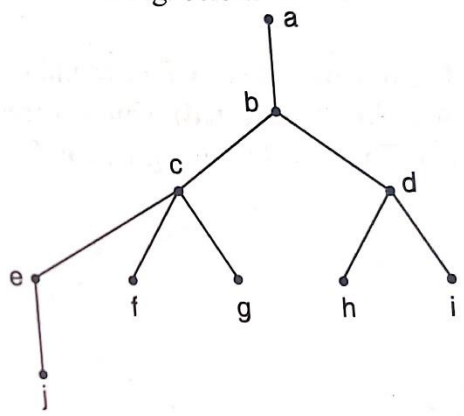



Fig. 16.41

2. Find all the trees with six vertices.
3. Either draw a graph with the given specification or explain why no such graph exists.
   (*a*) tree with nine vertices and nine edges
   (*b*) tree with six vertices and having total degree 14.
   (*c*) tree with five vertices and having total digree 8
   (*d*) graph, connected, six vertices, five edges has a nontrivial circuit.
   (*e*) tree, six vertices having degrees 1, 1, 1, 1, 3, 3
   (*f*) tree, four internal vertices, six terminal vertices.
   (*g*) tree with all vertices of degree 2
4. Draw three distinct rooted trees that have 4 vertices.

**5.** Given the tree with root at a as shown in Fig. below.




(*a*) find the parents of *c* and *h*

(*b*) find the children of *d* and *e*

(*c*) find the descendents of *c* and *e*

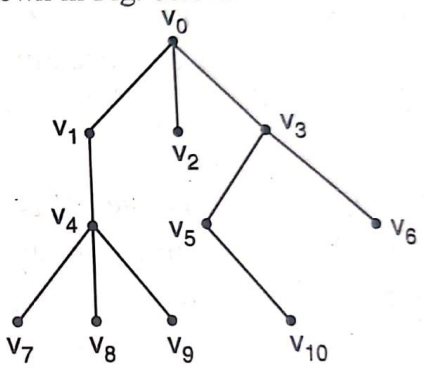
(*d*) find the siblings of f and h

(*e*) find the leaves

(*f*) find the internal vertices

(g) Draw the subtree rooted at *c*.

**6.** Consider the tree with root $v_0$ shown in Fig. below.




(*a*) what are the levels of $v_0$ and $v_4$?

(*b*) what are the children of $v_3$?

(*c*) what is the height of this rooted tree?

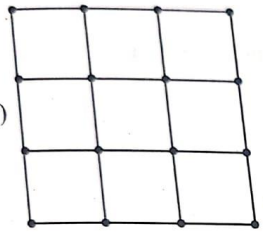
(*d*) what is the parent of $v_5$?

(*e*) what are the siblings of $v_7$?

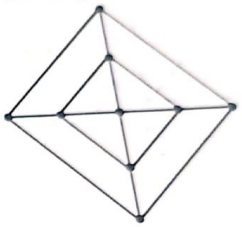
(*f*) what are the descendants of $v_3$?

9. Find a spanning tree for each of the graphs shown by removing edges

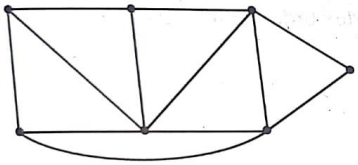
(*a*)



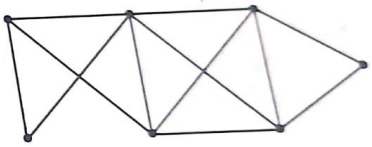
(*b*)



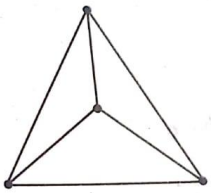
(*c*)



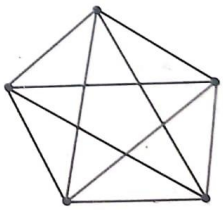
(*d*)



(*e*)

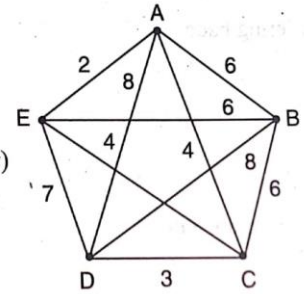


(*f*)

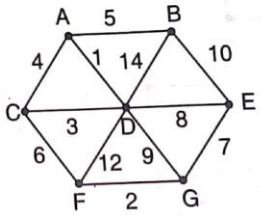


15. Use Prim's algorithm to find a minimum spaning tree for the given weighted graphs.

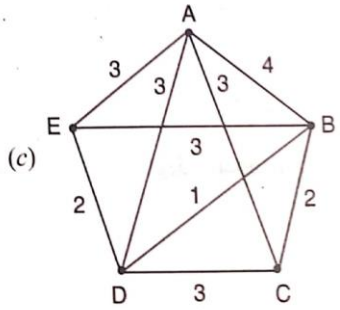
(*a*)



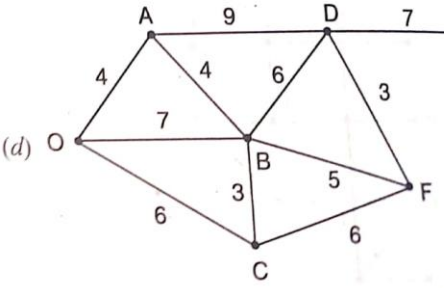

(*b*)




(*c*)



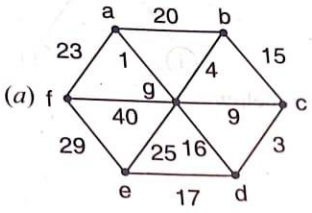

(*d*)



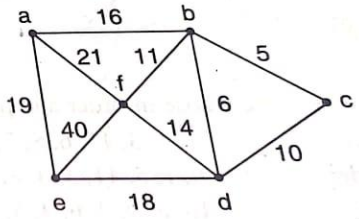
h

**17.** Use Kruskal's algorithm to find a minimum spanning tree for the given weighted graphs

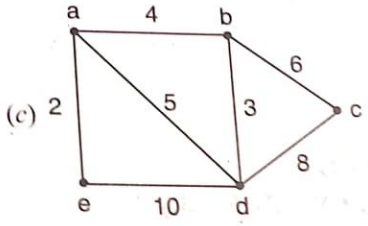
(*a*)




(*b*)




(*c*)

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