Module1: Set Theory, Functions, Relations

20. Define the cartesian product of two sets and give an example.

21. If the relation R is defined on the set of real numbers by the rule $a\ R\ b$ hold if $|a-b| \leq \frac{1}{2}$ then prove that R is not equivalence relation.

22. If A, B, C are sets, prove algebraically that

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

23. Define a relation on a set and give an example.

24. If R is the relation from $A = \{1,2,3,4\}$ to $B = \{2,3,4,5\}$, find the elements in R defined by $a\ R\ b$, if a and b are both odd. Write also the domain and range of R.

25. Give an example of a relation that is both symmetric and anti-symmetric.

26. In the set $S = \{0,1,2,3,4,5,.........,15\}$ the relation R is defined by $2x + 3y = 30$ where $x, y \in S$. Find whether R is transitive, reflexive, symmetric and anti-symmetric.

27. Give an example of a relation that is symmetric and transitive but not reflexive.

28. (a) If $A = \{1,2,3,4\} \times \{1,2,3,4\}$ and the relation R is defined on A by $(a,b)\ R\ (c,d)$ if $a + b = c + d$, verify that R is an equivalence relation on A.

42. Examine whether the following define a function :

(i) $A=\{3,1,4,5\}$, $B=\{2,4,8,9\}$ and $f(3)=2$, $f(1)=4$, $f(4)=9$, $f(5)=8$, $f(5)=2$.

(ii) $A=\{1,2,3,4\}$, $B=\{1,4,9,16\}$ and $f(x)=x^2$ where $x\in A$.

(iii) $X=\{5,6,8\}$, $Y=\{2,0,1,4\}$ and $f=\{(5,0),(6,1),(8,1)\}$.

43. If $f:x\rightarrow$ highest prime factor of x and the domain of f is {18, 13, 21, 15, 16, 17}, find the range set of f.

44. Show that the function f is injective but not surjective

(i) $f:N\rightarrow N$ defined by $f(x)=x+1, x\in N$

(ii) $f:Z\rightarrow Q$ defined by $f(x)=2^x$, $x\in Z$.

45. Discuss the nature of the following function :

(i) $f:R\rightarrow R$ defined by $f(x)=x^3-x, x\in R$

(ii) $f:R\rightarrow R$ defined by $f(x)=|x|$, $x\in R$

(iii) $f:R\rightarrow R$ defined by $f(x)=\sin x$, $x\in R$.

46. Determine whether the following function is one-to-one and/ or onto :

$f:R\rightarrow R$ given by $f(x)=3x^3-x$.

Ex1. Find which of the followings relations are partial order:

(i) The relation '$<$' on Z^+

(ii) The relation ρ on Z defined by $a\rho b$ means $a \le b$

Solution.

(i) Since $a < a$ does not hold good for all $a \in Z^+$, so the relation '$<$' is not reflexive. Thus the relation '$<$' is not partial order on Z^+

(ii) Since $a\rho a$, as $a \le a \forall a \in Z$

$\therefore$ ρ is reflxive.

Again

$\therefore$ $a\rho b$ and $b\rho a \Rightarrow a \le b$ and $b \le a$ for all $a, b \in Z$

$\Rightarrow a = b$ $\therefore \rho$ is antisymmetric.

Lastly $a\rho b$ and $b\rho c$

$\Rightarrow a \le b$ and $b \le c$.

$\Rightarrow a \le c$.

$\Rightarrow a\rho c$

$\therefore \rho$ is transitive.

Hence the relation ρ is partial order

Ex.2. If $S = \{1,2,3\}$, then draw the Hasse diagram of the poset $(P(S), \subseteq)$ where $P(S)$ is the power set of S [W.B.U.T.2014]

54. Write down all possible partial order relation of the set $\{a,\ b\}$.

55. Prove that the set $Y = \{1, 2, 3, 4, 6, 9\}$

forms a PO set w.r.t the 'divide' relation.

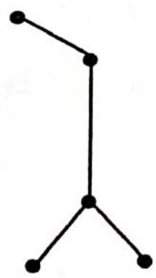
Draw the Hasse diagram for each.

56. Draw the Hasse diagram for the PO set $(A,\ /)$ where / stand for divisibility and (i) $A = \{2, 3, 6, 12, 24, 36\}$ [*W.B.U.T. 2006*]

(ii) $A = \{3, 6, 12, 24, 48\}$

57. Let $X = \{a, b, c, d, e\}$. Determine the relation represented by the following Hasse diagram

(i)



(ii)

Ex. 1. *Give an example of relation which is reflexive but is neither symmetric nor transitive.*

Let $A=\{1,2,3\}$ and R be the relation defined as $R=\{(1,1),(1,2),\ (2,2),(2,3),(3,3)\}$. Then R is reflexive as $(x,x)\in R\ \forall\ x\in A$ but is not symmetric as $(1,2)\in R$ and $(2,1)\notin R$. Again $(1,2)\in R$ and $(2,3)\in R$ but $(1,\ 3)\notin R$. Hence R is not transitive.

So the relation R is reflexive but is neither symmetric nor transitive.

Ex. 2. *Give an example of a relation which is reflexive and transitive but not symmetric.*

Let N be the set of all natural numbers and R be the relation defined as

$R=\{(x,y): x,y\in N \text{ and } x \text{ is a divisor of } y\}$

As x is a divisor of x, so $(x,x)\in R$.

Therefore R is reflexive.

Again, if x is a divisor of y, then y cannot be a divisor of x. Thus $(x,y)\in R \not\Rightarrow (y,x)\in R\ \ \forall\ x,y\in N$.

Hence R is not symmetric.

18. Let $A = \{a, b, c, d\}$ and consider the relation

$$R = \{(a,a), (a,b), (a,c), (b,b), (c,b), (c,c), (d,b), (d,c), (d,d)\}.$$

Show that R is a PO relation. Draw its Hasse diagram.

19. Prove that the sets

$$X = \{2, 3, 5, 30, 60, 120, 180, 360\}$$

forms a PO set w.r.t the 'divide' relation.

Draw the Hasse diagram for each. Find the maximal and minimal element; greatest and least element; *u.b.*, *l.b.*

Supremum and Infimum of each of the set X.

20. Let $X = \{24, 18, 12, 9, 8, 6, 4, 3, 2, 1\}$ be ordered by the relation 'x divides y'. Find the Hasse diagram.

21. (a) Draw the Hasse diagram for the P.O.Set $(A, /)$ where $/$ stand divisibility

(i) $A =$ set of all factors of 30 (including 1 and 30)

(ii) $A =$ set of all factors of 17