

Module 2: Propositional Logic

Ex.20 Without using truth table, prove that

(i) $p \rightarrow q \equiv p \rightarrow (p \wedge q)$

(ii) $(p \vee r) \rightarrow q \equiv (p \rightarrow q) \wedge (r \rightarrow q)$

(iii) $p \rightarrow (q \rightarrow r) \equiv p \wedge q \rightarrow r$ [W.B.U.T.2]

(iv) $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$

(v) $(p \rightarrow q) \rightarrow q \equiv p \vee q$

(vi) $(q \rightarrow (p \wedge \sim p)) \rightarrow (r \rightarrow (p \wedge \sim p)) \equiv r \rightarrow q$

14. Find the Truth table of the following :

(i) $p \vee \sim q$

(ii) $(p \vee \sim q) \wedge p$

(iii) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$

(iv) $(q \vee r) \wedge p$

(v) $\sim p \vee q \rightarrow \sim q$

(vi) $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$

(vii) $q \vee r \leftrightarrow p \wedge \sim r$

15. Write down the converse, inverse and contrapositive of the following statements :

(i) If today is independence-day, then tomorrow is Monday.

(ii) If ABC is a right triangle then $|AB|^2 + |BC|^2 = |AC|^2$.

(iii) If P is a rectangle then it is a parallelogram.

(iv) If n is prime, then n is 2 or n is odd.

(v) If a triangle is not isosceles, then it is not equilateral.

16. Write the converse, contrapositive and inverse of the followings :

(i) If it is raining, the grass is wet

(ii) It is raining if it is cloudy

(iii) Rain is sufficient for it to be cloudy

(iv) Rain is necessary for it to be cloudy.

17. Show by a truth table the inverse of $p \rightarrow q$ is equivalent to converse of $p \rightarrow q$.

18. Prove that the following propositions are Tautology :

(i) $(p \wedge q) \rightarrow p$

(ii) $\sim p \rightarrow (p \rightarrow q)$

(iii) $[p \wedge (p \rightarrow q)] \rightarrow q$

(iv) $(p \wedge q) \rightarrow (p \rightarrow q)$

(v) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ [W.B.U.T.2014]

(vi) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

(vii) $p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$

(viii) $\{p \wedge (p \leftrightarrow q)\} \rightarrow q$

(ix) $(p \wedge q) \rightarrow (p \vee q)$

Ex-22. Write a CNF (Conjunctive normal form) of the following statements :

(i) $p \wedge (p \rightarrow q)$

(ii) $\{q \vee (p \wedge r)\} \wedge \sim \{(p \vee r) \wedge q\}$

Solution. (i) $p \wedge (p \rightarrow q) \equiv p \wedge (\sim p \vee q)$ which is the required CNF

(ii) $\{q \vee (p \wedge r)\} \wedge \sim \{(p \vee r) \wedge q\}$

$$\equiv \{q \vee (p \wedge r)\} \wedge \{\sim (p \vee r) \vee \sim q\} \text{ by D' Morgan's Law}$$

$$\equiv \{q \vee (p \wedge r)\} \wedge \{\sim (p \wedge \sim r) \vee \sim q\} \text{ by D' Morgan's law.}$$

$$\equiv (q \vee p) \wedge (q \vee r) \wedge \{(\sim p \vee \sim q) \wedge (\sim r \vee \sim q)\}$$

$$\equiv (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim r \vee \sim q) \text{ which is required CNF.}$$

Ex.23. Find the CNF of the following statement :

$$\neg (p \vee q) \leftrightarrow (p \wedge q)$$

Solution.

$$\neg (p \vee q) \leftrightarrow (p \wedge q) \equiv \sim (p \vee q) \leftrightarrow (p \wedge q)$$

$$\equiv \{\sim (p \vee q) \rightarrow (p \wedge q)\} \wedge \{(p \wedge q) \rightarrow \sim (p \vee q)\}$$

$$\equiv \{\sim \sim (p \vee q) \vee (p \wedge q)\} \wedge \{\sim (p \wedge q) \vee \sim (p \vee q)\}$$

$$[\because p \rightarrow q \equiv \sim p \vee q]$$

$$\equiv \{(p \vee q) \vee (p \wedge q)\} \wedge \sim \{(p \wedge q) \wedge (p \vee q)\} \quad \dots (1)$$

by D'Morgan's law

$$\text{Now, } (p \vee q) \vee (p \wedge q) \equiv \{(p \vee q) \vee p\} \wedge \{(p \vee q) \vee q\}$$

$$\equiv \{(q \vee p) \vee p\} \wedge \{p \vee (q \vee q)\}$$

$$\equiv \{q \vee (p \vee p)\} \wedge \{p \vee q\}$$

$$\equiv (q \vee p) \wedge (p \vee q) \text{ using Idempotent law}$$

$$\equiv (p \vee q) \wedge (p \vee q) \text{ using Commutative law}$$

$$\equiv p \vee q \text{ again using idempotent law.}$$

19. Show that

$$(i) \quad p \wedge (\sim q \vee q) \equiv p$$

$$(ii) \quad p \vee (p \wedge q) \equiv p$$

$$(iii) \quad (p \wedge q) \vee (p \wedge \sim q) \equiv p$$

$$(iv) \quad \sim \{p \vee (\sim p \wedge q)\} \equiv \sim p \wedge q$$

$$(v) \quad p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p) \equiv p \wedge \{(q \rightarrow r) \wedge (r \rightarrow q)\} \vee \{(r \rightarrow p) \wedge (p \rightarrow r)\}$$

$$(vi) \quad p \wedge \{(\sim q \vee r) \wedge (\sim r \vee q)\} \equiv p \wedge (q \leftrightarrow r)$$

$$(vii) \quad \sim (p \wedge q) \wedge (p \vee q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$$

$$(viii) \quad (p \rightarrow q) \vee \sim (p \vee q) \equiv (p \rightarrow q) \wedge (p \leftrightarrow q)$$

$$(ix) \quad \sim \{p \vee (\sim p \wedge q)\} \equiv \sim p \wedge \sim q$$

20. Show that the following propositions are contradiction

$$(i) \quad p \wedge \sim q \quad (ii) \quad (p \vee q) \wedge (\sim q) \wedge (\sim p)$$

$$(iii) \quad (p \wedge q) \wedge \{\sim (p \vee q)\}$$

21. Find the DNF of the following propositions

$$(i) \quad \sim \{p \rightarrow (q \wedge r)\} \quad (ii) \quad (\sim p \rightarrow r) \wedge (p \leftrightarrow q)$$

22. Obtain the CNF of the following propositions :

$$(i) \quad \sim \{(p \vee \sim q) \wedge \sim r\}$$

$$(ii) \quad \sim (p \vee q) \leftrightarrow (p \wedge q)$$