Name: Sneha Shinde

Quantum Computing – Assignment 1

Problem 1 - Quantum Teleportation (Bell State)

PART B (CONCEPTS)

1) What do you observe in the histogram after applying $U^\dagger$? Is Bob measuring $|0\rangle$ with 100% probability?
Answer: After running the full circuit — including Bell state preparation, the Bell-measurement circuit (without performing intermediate measurements), the coherent correction using `cx(1,2)` and `cz(0,2)`, and finally applying $U^\dagger$ to Bob's qubit before measuring qubit 2 — I observe that the histogram shows only a single outcome: `0`.

In my notebook run, the measurement counts were:

$$\{'0' : 5000\}$$

out of 5000 shots. The histogram therefore contains only one bar corresponding to `0`, which means Bob measures $|0\rangle$ in every single shot.

From this result, I conclude that Bob is measuring $|0\rangle$ with effectively 100% probability in the simulator.

In an ideal noiseless quantum simulator, this probability is theoretically exactly 1, since the circuit evolution is perfectly unitary and free from noise. The assignment also states that Bob should measure $|0\rangle$ with probability approximately 100% if the teleportation and correction steps are implemented correctly.

However, if this experiment were performed on a real quantum device, small deviations from exactly 100% could occur due to gate errors, decoherence, or measurement noise. In such a case, a very small number of `1` outcomes might appear. Nevertheless, theoretically — and in the ideal simulation — Bob measures $|0\rangle$ with probability 1 after applying $U^\dagger$.

2) Why does applying $U^\dagger$ verify teleportation? (Use $U^\dagger U = |0\rangle$ in your explanation.)
Answer: Applying $U^\dagger$ verifies teleportation because it maps Bob's qubit back to the computational basis state $|0\rangle$ if and only if Bob's qubit actually contains the state

$$|\psi\rangle = U|0\rangle.$$

Therefore, measuring $|0\rangle$ with certainty after applying $U^\dagger$ confirms that Bob's qubit was indeed in the state , which means teleportation was successful.

Step-by-step reasoning:

State preparation:
In my circuit, Alice prepares an unknown quantum state $|\psi\rangle$ on qubit 0 by applying a unitary U to the initial state $|0\rangle$:

$$|\psi\rangle = U|0\rangle.$$

Teleportation process:
After performing the teleportation circuit — which includes Bell pair creation, the Bell-measurement circuit (without intermediate measurement in this coherent version), and the appropriate correction step on Bob's qubit — Bob's qubit should ideally be in the state $|\psi\rangle$.

In other words, after teleportation:

$$\rho_{\text{Bob}} = |\psi\rangle\langle\psi|.$$

so Bob's qubit is exactly the same state that Alice originally prepared.

Applying the inverse $U^\dagger$:
then apply $U^\dagger$ to Bob's qubit. Since unitaries satisfy:

$$U^\dagger U = I,$$

we have:

$$U^\dagger|\psi\rangle = U^\dagger(U|0\rangle) = (U^\dagger U)|0\rangle = I|0\rangle = |0\rangle.$$

Thus, if Bob's qubit truly contains $|\psi\rangle$, applying $U^\dagger$ deterministically converts it back to $|0\rangle$.

Measurement in the computational basis:
When I now measure Bob's qubit in the $\{|0\rangle, |1\rangle\}$ basis, the result must be `0` with probability 1.

Observing `0` with near-certainty therefore verifies that Bob's qubit was exactly $|\psi\rangle$
before applying $U^\dagger$.
This confirms that teleportation has successfully transferred the quantum state from Alice to Bob.

Why this acts as a verification step
If teleportation had failed and Bob's qubit was not equal to $|\psi\rangle$, then applying would not map it deterministically to $|0\rangle$.
In that case, the measurement histogram would show a non-zero probability of measuring `1`.

Therefore, the fact that I observe only `0` outcomes after applying $U^\dagger$ is direct evidence that Bob's qubit was exactly $U|0\rangle$ prior to inversion. This makes $U^\dagger$ a practical verification tool for confirming that teleportation worked correctly.

3) WHY WERE WE ABLE TO AVOID SENDING CLASSICAL BITS IN THIS ASSIGNMENT? HOW DOES THE FULLY QUANTUM CORRECTION DIFFER FROM THE REAL PROTOCOL?

Answer: In this assignment, classical bits were avoided because the correction step was implemented in a fully quantum (coherent) manner instead of using measurement and classical feed-forward.

WHAT WAS DONE IN THE CIRCUIT

Instead of measuring Alice's two qubits and using the two classical bits to decide which Pauli corrections Bob should apply, the entire circuit was kept unitary and coherent controlled operations were applied:

```
qc.cx(1, 2)
qc.cz(0, 2)
```

These gates conditionally apply the required corrections to Bob's qubit based on the state of Alice's qubits, but without measuring them. Since no measurement is performed at this stage, the quantum state does not collapse. The corrections are applied coherently, meaning they act in superposition.

Mathematically, these controlled gates implement the same correction that would normally be applied using classical bits, but they do so entirely within the quantum circuit.

WHY THIS IS POSSIBLE IN THE SIMULATION

In the notebook simulation, all three qubits (Alice's two qubits and Bob's qubit) exist inside the same quantum device (the simulator). Because of this, multi-qubit controlled gates can be directly applied between them.

Since everything is local within one quantum circuit, there is no need to measure Alice's qubits and send classical bits to Bob. The simulator allows direct implementation of `cx` and `cz` gates between Alice's and Bob's qubits, effectively replacing classical communication with coherent quantum control.

The entire evolution remains unitary, and the global state of the system stays pure throughout the process.

HOW THIS DIFFERS FROM THE REAL TELEPORTATION PROTOCOL

Although this coherent approach works in simulation, it differs from the actual distributed teleportation protocol in several important ways:

1) Locality and physical separation

In the real protocol, Alice and Bob are spatially separated. Alice performs measurements on her qubits, obtains two classical bits, and sends those bits to Bob through a classical communication channel. Bob then applies the appropriate Pauli corrections locally.

In contrast, in this assignment, controlled gates (`cx(1,2)` and `cz(0,2)`) are directly applied between Alice's qubits and Bob's qubit. If Alice and Bob were physically far apart, such nonlocal controlled gates would not be possible without transmitting quantum information. Therefore, the assignment models everything within one device rather than two distant parties.

2) Measurement and irreversibility

In the real teleportation protocol, Alice measures her qubits. This collapses the quantum state and produces classical information. The process is irreversible.

In the coherent implementation, no measurement occurs during the correction stage. The corrections are applied coherently, and the overall evolution remains reversible. The system stays in a pure quantum state throughout.

3) Role of classical communication and causality

In an actual distributed setting, classical communication is necessary to preserve causality. Without sending classical bits, Bob would not know which correction to apply, and instantaneous state transfer would appear possible, violating relativistic constraints.

In this simulation, classical bits are avoided only because all qubits are inside a single quantum circuit. Classical communication is effectively replaced by controlled quantum gates. However, this does not remove the need for classical communication in a physically realistic teleportation scenario.

FINAL CONCLUSION

Classical bits were avoided in this assignment because the entire protocol was implemented within a single quantum device, allowing coherent controlled corrections instead of measurement-based feed-forward.

Although this approach is mathematically equivalent to the standard teleportation protocol, it is operationally different from the real distributed version. In practice, when Alice and Bob are physically separated, classical communication is essential, since nonlocal controlled gates between remote qubits cannot be implemented without transmitting quantum information.

Problem 2: Superdense Coding (Bell State)

PART B (CONCEPTS)

1) What happens if an eavesdropper intercepts the qubit sent from Alice to Bob?
Answer: In superdense coding, Alice and Bob initially share an entangled Bell state:

$$|\Phi^+\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

Alice encodes her 2-bit classical message by applying a single-qubit Pauli operation (I, X, Z, or XZ) on her qubit before sending it to Bob.
If an eavesdropper (Eve) intercepts the qubit that Alice sends:

(A) THE INTERCEPTED QUBIT ALONE CONTAINS NO READABLE INFORMATION

The key idea is that each individual qubit of an entangled pair is in a maximally mixed state.
Mathematically, if we trace out Bob's qubit, Alice's reduced density matrix becomes:

$$\rho = \frac{I}{2}$$

This means the intercepted qubit looks completely random — it is equally likely to be measured as 0 or 1.
Therefore, Eve cannot determine which of the four possible messages (00, 01, 10, 11) was encoded by measuring the intercepted qubit alone.

(B) MEASUREMENT DESTROYS ENTANGLEMENT

If Eve tries to measure the intercepted qubit:

- The entangled Bell state collapses.
- The shared quantum correlations between Alice and Bob are destroyed.
- Bob's decoding operation will no longer produce the correct 2-bit message.

Instead of obtaining a deterministic result (e.g., `{'11': 1000}` in simulation), Bob would observe a disturbed or random distribution.

Thus, interception either:

- Gives Eve no useful information, or

- Introduces detectable disturbance.

CONCLUSION

If an eavesdropper intercepts the transmitted qubit:

- They cannot extract the encoded classical message from that qubit alone.
- Any attempt to measure it will destroy entanglement.
- The disturbance can be detected because Bob's decoding results will be incorrect.

This highlights an important security feature of entanglement-based quantum communication.

2) Give one application or implication of superdense coding.
Answer: One important application of superdense coding is the increase of classical communication capacity using quantum entanglement.

Superdense coding allows 2 classical bits to be transmitted using only 1 qubit, provided that Alice and Bob share a pre-established entangled Bell pair. This demonstrates that entanglement acts as a resource that enhances the effective capacity of a communication channel.

In the implemented protocol:

- Alice and Bob first share a Bell state.
- Alice encodes the 2-bit classical message "11" by applying Pauli-X followed by Pauli-Z on her qubit.
- She then sends only one qubit to Bob.
- Bob performs Bell-basis decoding (CNOT followed by Hadamard and measurement).
- After measurement, Bob successfully recovers the full 2-bit message.

The simulation confirms this result with deterministic output:

$$\{'11' : 1000\}$$

THIS IS SIGNIFICANT:

Classically, transmitting information follows a one-to-one relationship:
1 classical bit requires 1 physical bit (or signal carrier) to be sent through the channel.

However, in superdense coding, 2 classical bits are transmitted by sending only 1 qubit, provided that Alice and Bob share prior entanglement. The entangled Bell pair acts as a pre-shared resource that allows the encoding of additional information into the quantum correlations between the two qubits.

This demonstrates a fundamental principle of quantum information theory:
entanglement functions as a communication resource. It effectively increases the classical channel capacity beyond what is possible using classical systems alone.

BROADER IMPLICATIONS

Superdense coding has important implications for:

- **Quantum communication networks** – enabling more efficient data transfer between nodes that share entanglement.
- **Quantum internet architectures** – where pre-distributed entangled states can enhance communication efficiency.

- **Resource-efficient data transmission** – reducing the number of physical qubits that need to be transmitted.
- **Entanglement-assisted classical channel capacity** – forming the basis for theoretical models that quantify how entanglement improves communication rates.

Foundations of quantum information theory – demonstrating how quantum correlations fundamentally change information-processing limits.
Overall, superdense coding shows that entanglement can enhance classical communication efficiency beyond classical limits, highlighting the transformative role of quantum resources in information transmission.