GATE Question Paper-2009

EC Question Number 36

If $X = 1$ in the logic equation:

$$\left[X + Z\left(\overline{Y} + (\overline{Z} + X\overline{Y})\right)\right]\left(\overline{X} + \overline{Z}(X + Y)\right) = 1$$

then,

(A) $Y = Z$

(B) $Y = \overline{Z}$

(C) $Z = 1$

(D) $Z = 0$

Step-by-Step Solution:

Step 1: Understand the problem,We are given a logic equation and told that $X = 1$. We need to find the condition on Y and Z that makes the equation true.

Step 2: Substitute $X = 1$ in the equation When $X = 1$, the equation becomes:

$$\left[1 + Z\left(\overline{Y} + (\overline{Z} + \overline{Y})\right)\right]\left(0 + \overline{Z}(1 + Y)\right) = 1$$

Since $\overline{1} = 0$, the equation simplifies to:

$$\left[1 + Z\left(\overline{Y} + (\overline{Z} + \overline{Y})\right)\right]\left(\overline{Z}(1 + Y)\right) = 1$$

Step 3: Simplify the first part, Using the property that $1 + (\text{anything}) = 1$, we can simplify the first part:

$$1 \times \overline{Z}(1 + Y) = 1$$

So, the equation reduces to:

$$\overline{Z}(1 + Y) = 1$$

Step 4: Analyze the condition For $\overline{Z}(1 + Y) = 1$ to hold true, both conditions must be satisfied:

- $\overline{Z} = 1 \Rightarrow Z = 0$
- $(1 + Y) = 1$ is always true because anything ORed with 1 is 1.

Since both conditions must hold, the only solution is $Z = 0$.

Step 5: Why other options do not work

- If $Z = 1$, then $\overline{Z} = 0$, making the entire expression $0 \times (1 + Y) = 0$, which does not satisfy the equation.
- Other conditions do not meet both requirements simultaneously.

Step 6: Conclusion
The only way this equation holds true is if $Z = 0$.

Final Answer: (D) $Z = 0$

Reducing the equation using karnaugh map:

$$\left[X + Z\left(\overline{Y} + (\overline{Z} + X\overline{Y})\right)\right]\left(\overline{X} + \overline{Z}(X + Y)\right) = 1$$

The above equation is multiplied and If $X = 1$ in the logic equation:

$$F = (X + Z\overline{Y} + \overline{Z} + X\overline{Y}Z)(\overline{X} + \overline{Z}(X + Y)) = 1$$

Step 1: We are given a logic equation and told that $X = 1$. We need to find the condition on Y and Z that makes the equation true.

Step 2: Substitute $X = 1$ in the equation. When $X = 1$, the equation becomes:

$$F = (1 + Z\overline{Y} + \overline{Z} + \overline{Y}Z)(\overline{1} + \overline{Z}(1 + Y))$$

Since $\overline{1} = 0$, the equation simplifies to:

$$F = (1 + Z)(\overline{Z}(1 + Y))$$

Step 3: Simplify the expression using Boolean algebra:
$1 + Z = 1$ (since 1 OR anything is 1)
So,

$$F = \overline{Z}(1 + Y)$$

$1 + Y = 1$ (anything ORed with 1 is 1)

$$F = \overline{Z}$$

Step 4: Find the condition for $F = 1$ For $F = 1$, $\overline{Z} = 1$, which means:

$$Z = 0$$

Step 5: Final simplified expression using XOR. We observed through simplification that the function F can also be written as:

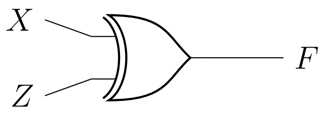
$$F = X \oplus Z$$

When $X = 1$, for $F = 1$ to hold, Z must be 0.

Expression:

$$F = X \oplus Z$$

Logic Gate Diagram:



Final Answer: (D) $Z = 0$