But Hall electric field produced in the rod

$$E_H = -\frac{1}{ne} J_x H_z$$

Hence $\quad E_H = -\frac{1}{ne}\frac{I}{bd} H_z$

$\therefore$ Hall voltage $V_{Hall} = E_H \times d$

$$= -\frac{1}{ne}\frac{I}{bd} H_z d = R_{Hall}\frac{IH_z}{b}$$

Hence Hall coefficient

$$R_{Hall} = \frac{V_{Hall}\, b}{IH_z} \qquad \text{....(3·26)}$$

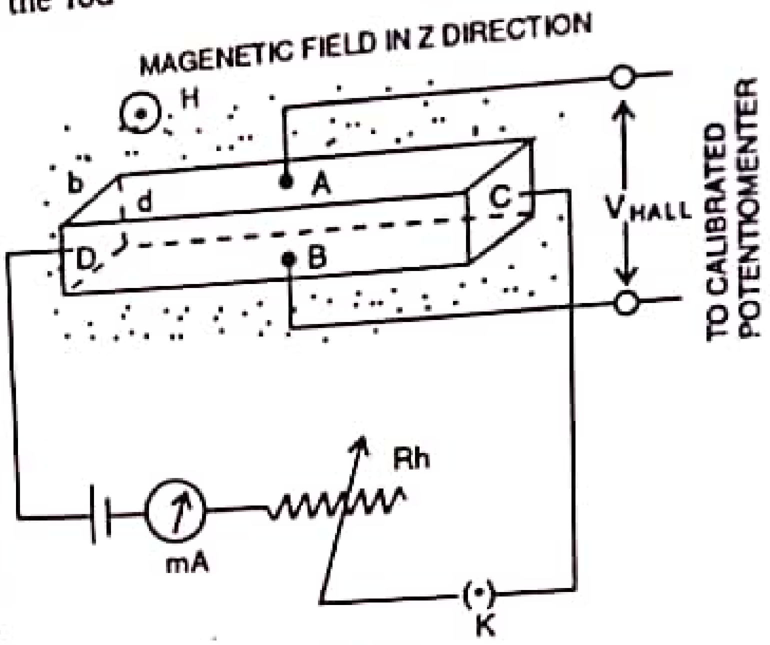


Fig. 3·17

Thus knowing the Hall voltage $V_{Hall}$, width of the rod $b$, current flowing in the rod $I$, and the magnetic field $H_z$, we can calculate the Hall coefficient $R_{Hall}$ from the above expression.

**Errors and their Removal :** (i) When current flows in the metallic rod, a temperature gradient is produced. For the removal of the error due to it, first we determine the Hall voltage by passing current in one direction and then we determine the Hall voltage by passing current in the opposite direction. Then the mean value is obtained.

(ii) In absence of the magnetic field also, some voltage is developed between the points $A$ and $B$ due to imperfect allignment. To avoid this error, the Hall voltage is measured once by applying the magnetic field in one direction and then in the opposite direction. Then its mean value is obtained.

## 3·7. Super Conductivity

The flow of electrons in some specific metals and alloys without any resistance is called super conductivity. It was discovered in 1911, by Kamerlingh-Onnes. They in their experiment found that the d.c. electric resistance of mercury suddenly falls to zero at a temperature below a particular temperature (4·2K) *i.e.*, its conductivity becomes infinite. In other words, at temperatures below 4·2K, electrons flow in mercury without any resistance. They named this phenomenon as super conductivity. Actually this property is found in many metals and alloys at low temperatures.

Following are the important properties of material in the super conducting state.

**(i) Zero Resistance and Critical Temperature :** The temperature below which the resistivity of a metal (or alloy) becomes zero is called the critical temperature ($T_c$) of that metal. The critical temperature of some metals and alloys are given in the following table :

| Metal | Critical temperature (in K) | Alloys | Critical temperature (in K) |
|---|---|---|---|
| Al | 1·96 | $BaBi_3$ | 5·69 |
| Cd | 0·56 | $Bi_2Pt$ | 0·16 |
| Hg | 4·153 | $CoSi_2$ | 1·4 |
| In | 3·407 | $Nb_3Sn$ | 18·07 |
| Nb | 9·25 | $Nb_3Ge$ | 23·2 |
| Pb | 7·175 | $ErRb_4B_4$ | 8·7 |
| [illegible] | 3·74 | | |

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From the above table, it is clear that the maximum critical temperature is alloy $Nb_3Ge$, but this temperature (= 23.2K) is also very low as compared to the temperature, hence the super conducting substances can not be practically used at room temperature.

Fig. 3.18 shows the variation in resistivity of mercury (Hg) with temperature from which it is clear that at 4.2K, the resistivity of mercury falls abruptly to zero.

It may be mentioned here that in the super conducting state, the meaning of resistivity (ρ) of the substance to be zero is that the electric field ($E$) inside the substance becomes zero (from Ohm's law $E = \rho.J$, if $\rho = 0$, $E = 0$)

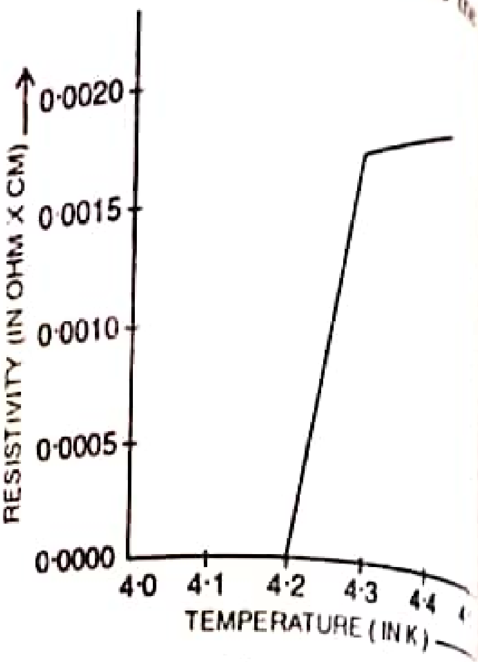


Fig. 3.18

**(2) Persistent Currents :** Experimentally it is found that if in a ring of a super conducting substance d.c. current of several ampere is induced several times, it remains flowing without any loss for several years. Thus flow of persistent current (i.e., zero resistance) is a characteristic property of super conductivity.

**(3) Critical Magnetic Field :** Experimentally it is found that the property of super conductivity disappears when a magnetic field of particular intensity is applied. This particular magnetic field is called the critical magnetic field $H_c$. Its value depends on temperature as follows :

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right] \quad ...(3)$$

where $H_0$ is the critical field at absolute zero, which is fixed for each metal, $H_c$ is critical field at absolute temperature $T$, and $T_c$ is the critical temperature in absence of magnetic field.

Obviously if a magnetic field of strength greater than the critical field $H_c$ is applied on the super conducting substance, its resistance is restored.

**(4) Meissner Effect :** In 1633, Meissner experimentally found that a substance in the super conducting state shows the behaviour of perfect diamagnetic substance i.e. if a magnetic field is applied on the substance in the super conducting state either by an external magnet or by passing current in the coil wound round the substance, the substance is repelled. It means that the value of magnetic induction ($B$) inside a super conducting substance is zero. This is called Meissner effect.

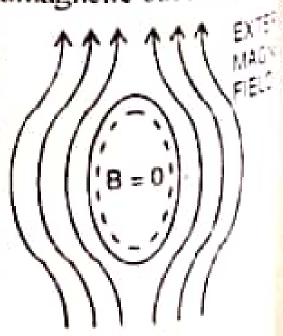


Fig. 3.19

The magnetic susceptibility of the substance in the super conducting state is negative.

Fig. 3.19 shows the behaviour of super conducting substance in an external magnetic field.

On the basis of this effect, the super conducting substances can be classified in two categories (i) Type-I or soft super conductors, and (ii) Type-II or hard super conductors.

Those super conductors in which the magnetic induction inside the substance becomes zero below the critical magnetic field $H_c$ and the super conductivity of substance abruptly vanishes at the critical magnetic field $H_c$ are called type-I or soft super conductors. On the other hand, those super conductors in which there are critical fields $H_{c_1}$ and $H_{c_2}$ and at the magnetic field below the lower value $H_{c_1}$, the substance is diamagnetic and then with increase of magnetic field the super conductivity of the substance vanishes between the lower critical field $H_{c_1}$ to the upper critical field $H_{c_2}$, are called type-II or hard super conductors.

**(5) Entropy :** Experimentally it is found that on cooling the super conductors below the critical temperature $T_c$, their entropy decreases. It means that the super conducting state of the substance is more ordered as compared to its normal state.

**(6) Josephson Tunnelling :** If a junction is made by joining an insulator in between two different super conductors, then super current can be made to flow through it if the thickness of insulating layer is very thin. This is called Josephson tunnelling.

Super current means the following two currents : (i) d.c. current which flows through the junction in absence of external magnetic field or electric field, and (ii) a.c. which flows on applying d.c. voltage at the ends of the junction.

## SOLVED EXAMPLES

**Ex. 1. Calculate the ground state energy and density of energy states of free electron in a mono-atomic one dimension metallic wire of length 1 cm.**

**Sol.** Given, $L = 1$ cm $= 10^{-2}$ m

Energy of electron in $n^{th}$ level, $E_n = \dfrac{n^2h^2}{8mL^2}$

∴ Energy of electron in ground state ($n = 1$),

$$E_1 = \frac{(1)^2 \times (6.6 \times 10^{-34})^2}{8 \times (9.1 \times 10^{-31}) \times (10^{-2})^2}$$
$$= 5.98 \times 10^{-34}\ \text{J}$$
$$= 3.74 \times 10^{-15}\ \text{eV}$$

Density of states of $n^{th}$ energy level

$$g_{(E_n)} = \frac{4L}{h}\sqrt{\frac{m}{2E_n}}$$

∴ Density of states in ground state ($n = 1$),

$$g_{(E_1)} = \frac{4 \times 10^{-2}}{6.6 \times 10^{-34}}\sqrt{\frac{9.1 \times 10^{-31}}{2 \times 5.98 \times 10^{-34}}}$$
$$= 1.67 \times 10^{33}\ \text{per Joule}$$

**Ex. 2. If number of electrons present in 1 cm length of a metal (work funtion =2.1 eV) is $4 \times 10^7$, find the Fermi energy, the Fermi temperature and the depth of potential well of that metal.**

**Sol.** Given, $L = 1$ cm $= 10^{-2}$ m, $N = 4 \times 10^7$, $\phi = 2.1$ eV

Fermi energy, $E_F = \dfrac{N^2h^2}{32mL^2} = \dfrac{(4 \times 10^7)^2 \times (6.6 \times 10^{-34})^2}{32 \times 9.1 \times 10^{-31} \times (10^{-2})^2}$

$$= 2.39 \times 10^{-19}\ \text{J} = 1.5\ \text{eV}$$

Fermi temperature, $T_F = \dfrac{E_F}{k} = \dfrac{2.39 \times 10^{-19}}{1.38 \times 10^{-23}} = 1.73 \times 10^4$ K

Depth of potential well $E_s = E_F + \phi = 1.5 + 2.1 = 3.6$ eV

**Ex. 3. Calculate the total number of electrons present in 1 cm length of a mono-atomic one dimensional copper wire (Fermi temperature = $8.1 \times 10^4$ K).**

**Sol.** Given, Fermi temperature $T_F = 8.1 \times 10^4$K

∴ Fermi energy, $E_F = kT_F = (1.38 \times 10^{-23}) \times (8.1 \times 10^4) = 1.12 \times 10^{-18}$ J

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# Super Conductivity

## Super Conductivity :-

**Intro:-** Certain substances like mercury resistivity suddenly drops to zero at very low temp. typically near boiling point of Helium. This phenomenon is known as superconductivity.

**Example** - alloys, ceramics, metal doped semiconductors

Temp at which resistivity drops to zero is called critical temp for mercury it is 4.2K. Below this temp mercury is in superconducting state above this temp it behaves as a normal conductor.

Different super conducting materials have diff critical temp but the nature of variation of resistivity with temp remains more or less similar.

- Resistivity of metal ↑ses linearly with temperature.
- Resistivity of semiconductor decreses non linearly with ↑ of temp.
- This phenomenon was discoverd by 'Kannerlingh Onnes' in 1911 on Mercury.

Example of super conductors :- silver, lead, gallium, iridium etc.

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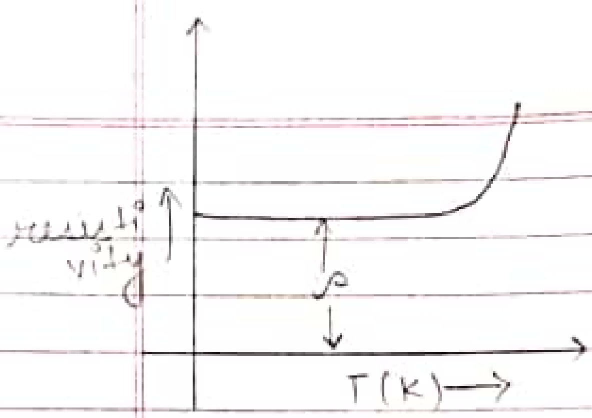
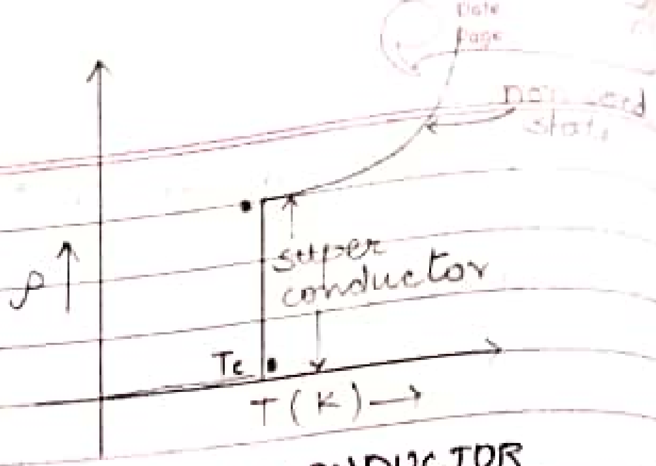



METAL




SUPERCONDUCTOR

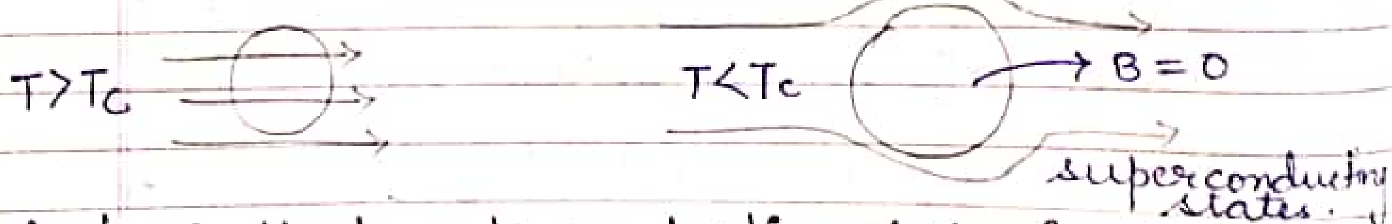
$\rho = 0 \ \forall\ T < T_c$

## Props of superconductors :-

(1) Zero electrical Resistance

(2) MEISNER EFFECT –

when a specimen is placed in a weak magnetic field and cooled below the critical temp, the magnetic flux present in the specimen is ejected out from the specimen this is called meisner effect. Effect is reversible.




(Q) Show that superconducting state indicates perfect dimagnetization.

we know that

$$B = \mu_0(H+M)$$

external applied magnetic field

magnetization in the specification

when $T < T_c \Rightarrow B = 0$

i.e. for superconductivity

$$\Rightarrow \mu_0 (H+M) = 0.$$

$$H = -M \qquad M/H = -1 = \text{susceptibility } (\chi)$$

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Hence we can say that meissner effect is the condition for perfect diamagnetisation.

(b) Meissner effect can't be explained by assuming it a perfect conductor with zero electrical ~~conductivety~~ resistance.

we know that

$$E = \frac{V}{L} \Rightarrow E = \frac{I \cdot R}{L}$$

$\boxed{I/A = J}$

$R = \rho \frac{l}{A}$

multipling and dividing by A

$\rho = \frac{RA}{l}$

$\rho = 0$ ↓ $i = \infty$ ↓ $J =$ infinite

$$E = \frac{IRA}{LA} = \rho J$$

If $\rho = 0$ then $J \to$ finite $\Rightarrow E = 0$

from maxwell equation we know that

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

as $E = 0$

$\Rightarrow B =$ constant

therefore it implies, in a conductor the magnetic flux not change on cooling below the critical temp, which contradicts the meissner effect acco. to which flux must change (below or reduced) to zero. Hence superconductor is not just a perfect conductor.

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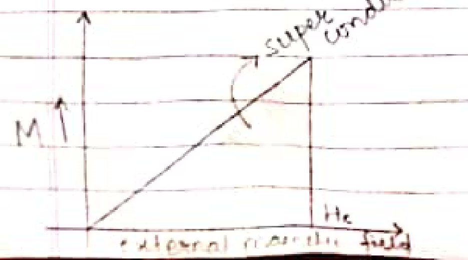
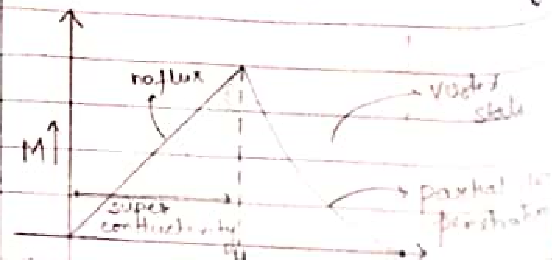
Types of Superconductors :- Based on diff. in magnetisation exhibited by them.

| TYPE 1 (soft super conductors) | TYPE 2 (Hard super conductors). Incomplete Meissner effect |
| --- | --- |
| (1) Exibits complete meissner effect | (1) shows complete meissner effect below critical magnetic field $H_c(1)$ and allows the flux to penetrate b/w $H_c(1)$ and $H_c(2)$, the region is called mixed state b/w $H_c(1)$ and $H_c(2)$ for a material |
| (2) Above the critical field $H_c$ the super conductor becomes conductor | (2) B/w $H_c(1)$ and $H_c(2)$ the superconductor exist in mixed state also called vortex state and above $H_c(2)$ comes in normal state (conductor state). |
| (3) The critical field $H_c$ is relatively low, they can generate field about 100 to 1000 ~~gauss~~ guass. | (3) The value of $H_c(2)$ is very large |
| Example :- Al, Zn, Ga etc | Example – Pb-indium aloy |
|  |  |

What is vortex state?

(1) Material is in the magnetically mixed state but electrically it is a superconductor.

(2) In this state there is a flux penetration yet the material retains zero electrical resistance prop. and still it is a superconductor.

Dielectric polarisation :-

In certain substances called insulators which do not have free electrons when potential difference is applied, it is observed that the behavior of insulator gets changed that the modified behaviour comes and insulator conductors. Then such an insulator whose behaviour gets modified on application of potential diff is called dielectrics.

Dielectrics.

Isotropic → change is in the direct<sup>n</sup> of electric field

anisotropic → change is in the opp direction of electric field

When the +ve and -ve part of the dielectric in the presence of electric field are displaced for there ~~eq~~ equilibrium

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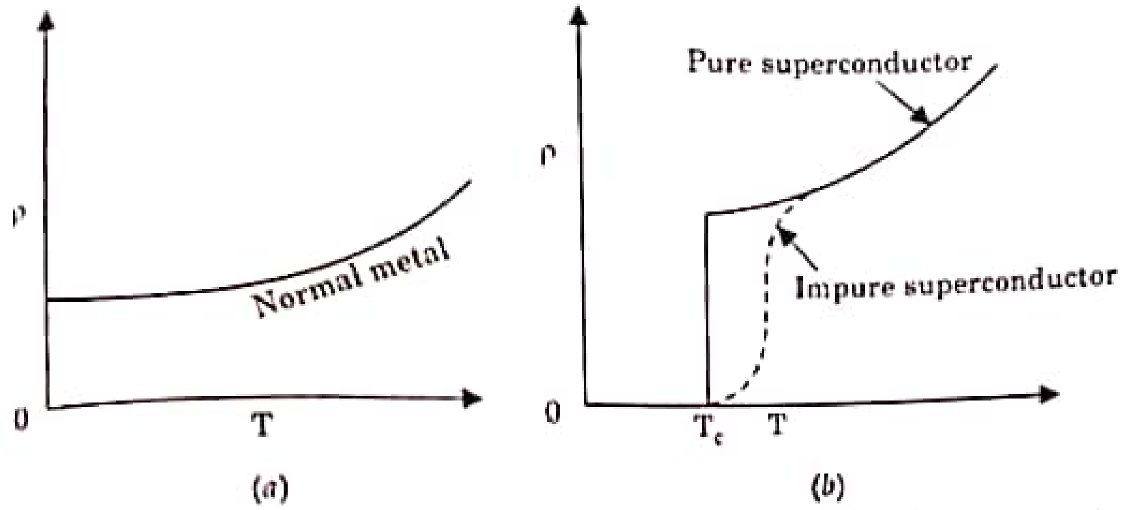


Fig. 4.46. Schematic representation of the resistivity of the (*a*) Normal metal, and (*b*) Pure and impure superconductor as a function of temperature *T*.

## 4.20.1 Superconductivity : The Free Electron Model

According to free electron model of metals, the resistivity of the metal may be written as :

$$\boxed{\rho = \frac{m}{ne^2\tau}} \quad ...(68)$$

where '$m$' is the mass of the electron

'$n$' is the number of electrons per unit volume

'$\tau$' is average time of collision.

From eq. (68) it is clear that resistivity decreases as temperature is lowered because as temperature decreases the lattice vibrations begin to "freeze" and hence the scattering of electron diminishes. This results in a larger '$\tau$' and hence a smaller resistivity $\rho$. If average time of collision $\tau$ becomes infinite at sufficiently low temperature, then the resistivity $\rho$ vanishes entirely, which is observed in case of superconducting materials.

In these materials, a fraction of electrons have infinite collision time, hence these electrons do not undergo scattering even though the substance may contain some impurities and defects. It is these electrons which are responsible for superconductivity.

## 4.20.2 Critical Field

*"The minimum applied magnetic field necessary to destroy superconductivity and restore the normal resistivity is called the* ***critical field*** *$H_c$".* When the applied magnetic field exceeds the critical value $H_c$, the superconducting state is destroyed and the material goes into the normal state.

$H_c$ depends on the temperature. Fig. 4.47 shows the critical field $H_c$ as a function of temperature. A specimen is superconducting below the curve and normal above the curve.

For a given substance, value of $H_c$ decreases as temperature increases from $T = 0$ K to $T_c$ (critical temperature).

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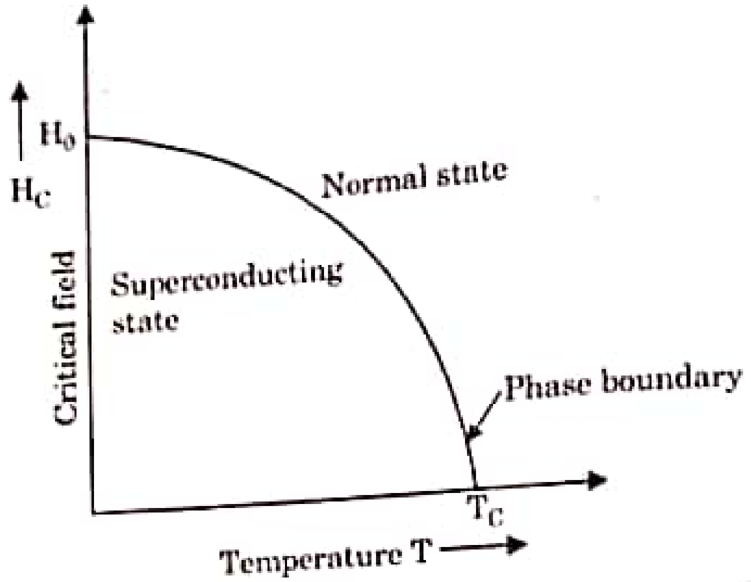


**Fig. 4.47.** Schematic representation of the critical field $H_c$ as a function of temperature.

The curve is *nearly parabolic* and can be represented as :

$$\boxed{H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right)} \quad ...(69)$$

where $H_0$ is the critical field at 0 K.

Thus the field has its maximum value $H_0$ at $T = 0$ K

At $T = 0$ K

$$H_c = H_0\left(1 - \frac{0}{T_c^2}\right) = H_0$$

At $T = T_c$

$$H_c = H_0\left(1 - \frac{T_c^2}{T_c^2}\right) = 0$$

Eqn. (69) is the phase boundary between the normal and superconducting state.

**EXAMPLE 4.10 :**

*A superconducting Sn has a critical temperature of 3.7 K in zero magnetic field and critical field of 0.0306 T at 0 K. Find the critical field of 2 K.*

**Solution.** $T_c = 3.7$ K, $H_0 = 0.0306\ T$ (at $T = 0$ K)

$$H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right) = 0.0306\left[1 - \left(\frac{2}{3.7}\right)^2\right]$$

$$= 0.0306\,[1 - (0.54)^2] = 0.0306 \times 0.7078$$

$$= \mathbf{0.02166\ Tesla.}$$

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**EXAMPLE 4.11 :**

*The critical field for niobuim is $1 \times 10^5$ A/m at 8 K and $2 \times 10^5$ A/m at 0 K. Calculate the critical temperature of the material.*

**Solution.**
$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$\therefore \quad \frac{H_c}{H_0} = 1-\left(\frac{T}{T_c}\right)^2$$

$$\frac{T}{T_c} = \left(1-\frac{H_c}{H_0}\right)^{1/2} \quad \text{or} \quad T_c = \frac{T}{\left[1-\frac{H_c}{H_0}\right]^{1/2}}$$

$$T_c = \frac{8\text{ K}}{\left[1-\frac{1\times10^5\text{ A/m}}{2\times10^5\text{ A/m}}\right]^{1/2}} = \mathbf{11.31\text{ K.}}$$

**EXAMPLE 4.12 :**

*The transition temperature for Pb is 5.26 K. The maximum critical field for the material is $8 \times 10^5$ A/m. Pb has to be used as a superconductor subjected to a magnetic field of $4 \times 10^4$ A/m. What precaution will have to be taken ?*

**Solution.**
$$T = T_c\left[1-\frac{H_c}{H_0}\right]^{1/2}$$

$$= 7.26\left[1-\frac{4\times10^4\text{ A/m}}{8\times10^5\text{ A/m}}\right]^{1/2}$$

$$= \mathbf{7.08\text{ K}}$$

Hence the temperature of the material should be held below 7.08 K.

**EXAMPLE 4.13 :**

*For a specimen of $V_3Ga$, the critical fields are respectively $1.4 \times 10^5$ A/m and $4.2 \times 10^5$ A/m for 14 K and 13 K. Calculate the transition temperature and critical fields at 0 K and 4.2 K.*

**Solution.**
$$H_c = 1.4 \times 10^5 \text{ A/m} \quad \text{at } T = 14\text{ K}$$
$$H_c = 4.2 \times 10^5 \text{ A/m} \quad \text{at } T = 13\text{ K}$$

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$1.4 \times 10^5 = H_0\left[1-\left(\frac{14}{T_c}\right)^2\right] \qquad \text{...(i)}$$

$$4.2 \times 10^5 = H_0 \left[1 - \left(\frac{13}{T_c}\right)^2\right]$$

Dividing eqn. (*i*) by equation (*ii*), we have

$$\frac{1.4}{4.2} = \frac{1 - \frac{196}{T_c^2}}{1 - \frac{169}{T_c^2}} \quad \text{or} \quad 1 - \frac{196}{T_c^2} = 0.33 - \frac{56.33}{T_c^2}$$

$$\therefore \quad T_c^2 = 208.46$$

$$T_c = \mathbf{14.43\ K.}$$

$$1.4 \times 10^5 = H_0 \left[1 - \left(\frac{14}{14.43}\right)^2\right]$$

$$\therefore \quad H_0 = \frac{1.4 \times 10^5}{0.0586} = \mathbf{23.8 \times 10^5\ A/m.}$$

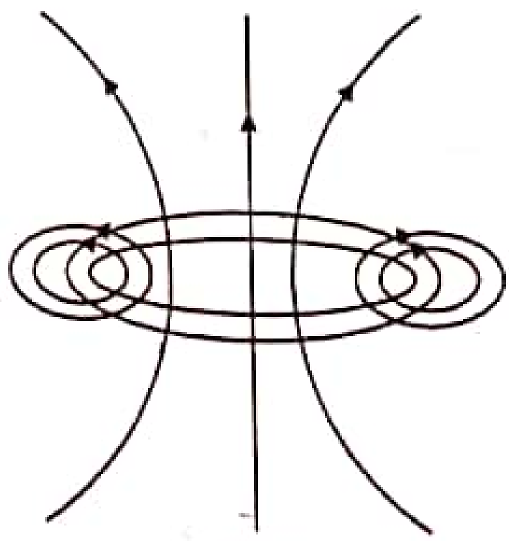
$$H_c = 23.8 \times 10^5 \left[1 - \left(\frac{4.2}{14.43}\right)^2\right]$$

$$= 23.8 \times 10^5 [1 - 0.0847]$$

$$= \mathbf{21.78 \times 10^5\ A/m.}$$

## 4.20.3 Persistent Current

In superconducting material, current will continue to flow in a closed loop as long as loop is held below the critical temperature $T_c$ (Fig. 4.48). Such a steady current which fl with undiminishing strength is called a ***persistent current.***



$T < T_c$

**Fig. 4.48.**

**Critical Current.** *"The minimum current that can be passed in a sample with destroying its superconductivity is called critical current I."*

If $P$ and $H_0$ are held constant

Then entropy $\quad S = -\left(\frac{\partial G}{\partial T}\right)_{P, H_0}$

Also
$$G_S = G_N - \frac{H_c^2 - H_0^2}{2\mu_0} \qquad ...(76)$$

($G_S$ and $G_N$ are the Gibb's free energies for superconducting and normal state respectively)

$$S_S - S_N = \frac{d}{dT}\left(\frac{H_c^2 - H_0^2}{2\mu_0}\right)$$

$$= \frac{2H_c}{2\mu_0}\frac{dH_c}{dT} = \frac{H_c}{\mu_0}\frac{dH_c}{dT} \qquad ...(77)$$

Heat capacity $\quad C = T\frac{dS}{dT}$

$$C_S - C_N = T\frac{d}{dT}\left(\frac{H_c}{\mu_0}\frac{dH_c}{dT}\right)$$

$$= \frac{T}{\mu_0}\left[\left(\frac{dH_c}{dt}\right)^2 + H_c\frac{d^2H_c}{dT^2}\right] \qquad ...(78)$$

when $T = T_c$; $H_c = 0$

$$(C_S - C_N)_{T_c} = \frac{T_c}{\mu_0}\left(\frac{dH_c}{dT}\right)^2 \qquad ...(79)$$

This equation is known as ***Ruger's Formula.***

## 4.24 APPLICATIONS OF SUPERCONDUCTORS

(1) These materials are used for producing very strong magnetic fields of about 50 Tesla, which is much larger than the field obtainable from an electromagnet.

(2) High current densities with zero resistance properties of super-conducting materials make useful for strong electromagnets for example, in MRI (magnetic resonance imaging) devices, used in medicine.

(3) In superconducting materials, heating loss is zero [$I^2R = 0$], therefore power can be transmitted through superconducting cables without loss.

(4) These materials can be used to perform logic and storage functions in computers.

(5) Type II superconducting materials are mainly utilized for super-conducting solenoids.

(6) These are also used in high speed leviated trains (Maglev).

(7) SQUIDs are used in the field of medicine, it measures the very weak fields generated by heart and brain.

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## 4.23 CHARACTERISTICS OF SUPERCONDUCTORS

### 4.23.1 Empirical Criteria

Some observations about the material to show superconductivity are as follows :

(i) The materials (metallic substance) whose number of valence electrons $Z$ lies between 2 to 8, generally show the superconductivity.

(ii) Critical temperature $T_c$ of the superconducting materials shows maximum value for $Z$ = 3, 5 and 7 (as shown in Fig. 4.53).

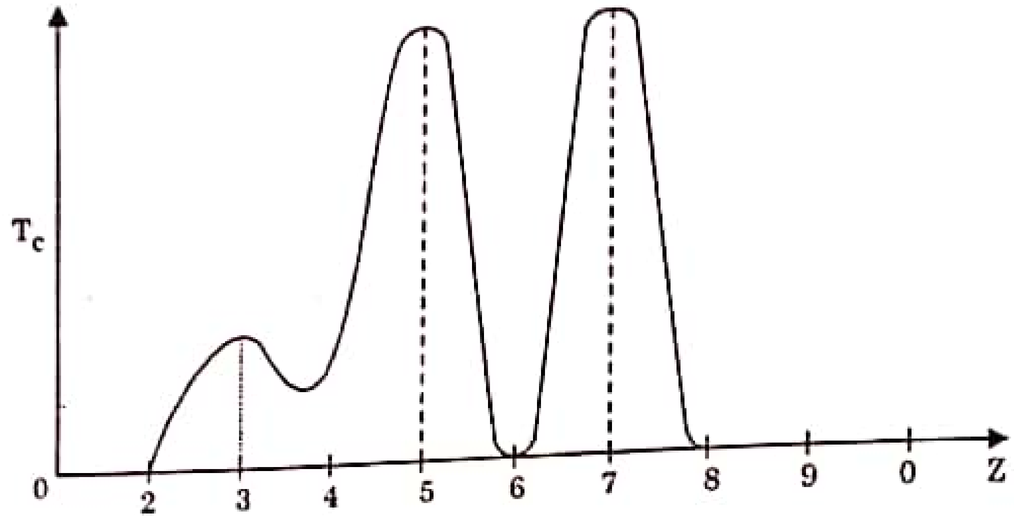


Fig. 4.53. Critical temperature $T_c$ versus $Z$.

(iii) Graph between critical temperature $T_c$ and $Z^2$ is a straight line.

(iv) Critical temperature $T_c$ varies inversely as the atomic mass of the superconducting materials.

(v) The materials having high normal resistivities show the superconductivity.

### 4.23.2 Properties of Superconductors

(i) At room temperature, the resistivity ρ of superconducting materials are greater than other elements (as shown in Fig. 4.54).

(ii) All thermoelectric effects disappear in superconducting state.

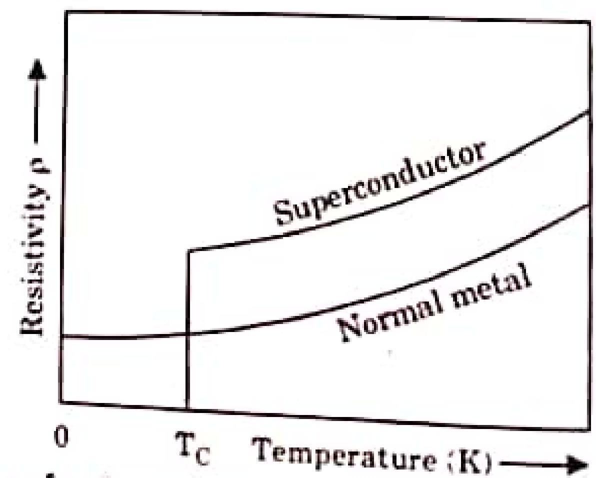


Fig. 4.54. Comparison of r of superconductor and normal metal at room temperature.

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(iii) When a sufficient strong magnetic field is applied to superconductor below critical temperature $T_c$, its superconducting property is destroyed.

(iv) When current is passed through the superconducting materials, the heating loss ($I^2R$) is zero.

As resistivity, $\rho = \frac{RA}{l}$

as $\rho \rightarrow$ Very small (zero at $T_c$)

$\therefore$ $R = 0$

Hence no heating losses.

## 23.3 Thermal Characteristics of Superconductors

***Heat Capacity.*** The transition of a metal from its normal state to a superconducting state ...s not involve a change of crystallographic structure. No ferromagnetic, ferrimagnetic or ...ferromagnetic transition occurs. Only thermodynamic phase transition takes place where the ...cific heat changes discontinuously at the transition temperature $T_c$.

Specific heat varies linearly with temperature in normal conductor and exponentially in a ...perconductor, as shown in Fig. 4.55 and it is given by

$$C_v = e^{-\frac{AT_c}{T}} \qquad \text{...(73)}$$

...ere $A$ is a constant.

From equation (73) it is clear that specific heat in superconductors decreases exponentially.

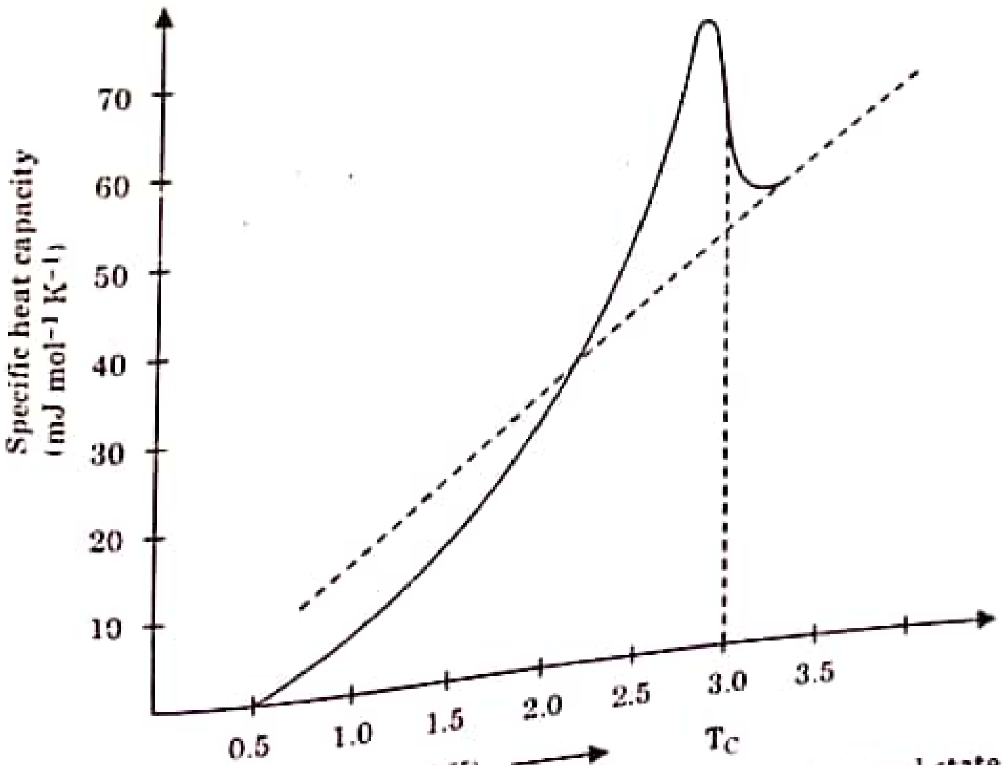


Fig. 4.55. The heat capacity in superconducting state and normal state. (...conductor, $N \rightarrow$ Normal conductor)

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