Scenario 1

Introduction

To calculate the number of heads (N_h) and length of consecutive number of heads (L_h) in 50 coin flips. Both theoretical and simulation models are used for this scenario. We take input for N_h and L_h from the user and then compare it with simulated results. Here *randi* function is used to generate random 0 or 1.

Theoretical model

Consider an experiment of flipping coin 5 times.

Total number of outcomes for 5 flips = 2^5 = 32

So, we can say that probability for number of heads x=0,1,2,3,4,5 is given by

$$P(x=0) = \frac{5C0}{32} = \frac{1}{32} = P(x=5)$$

$$P(x=1) = \frac{5C1}{32} = \frac{5}{32} = P(x=4)$$

$$P(x=2) = \frac{5C2}{32} = \frac{10}{32} = P(x=3)$$

Plotting the above values in a graph of probability v/s possible number of heads as shown in fig 1.1.

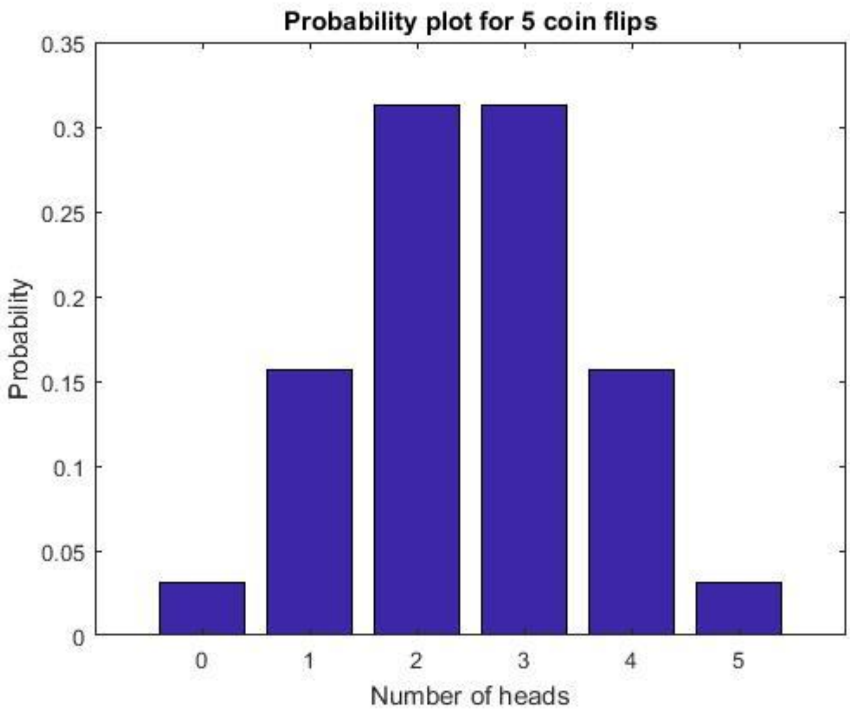

Fig 1.1 Probability plot for 5 coin flips

Generalising the formula for calculating probability for any n number of flips and implementing in MATLAB code.

```
n = input('Enter the number of coin flips: ');
x = 0:n;
y = zeros(1,n+1);
for i = 1:n+1
    y(i)= factorial(n)/((2^n)*(factorial(i-1))*(factorial(n-i+1)));
end
bar(x,y)
xlabel('Number of heads')
ylabel('Probability')
title(['Probability plot for ' num2str(n) ' coin flips'])
```

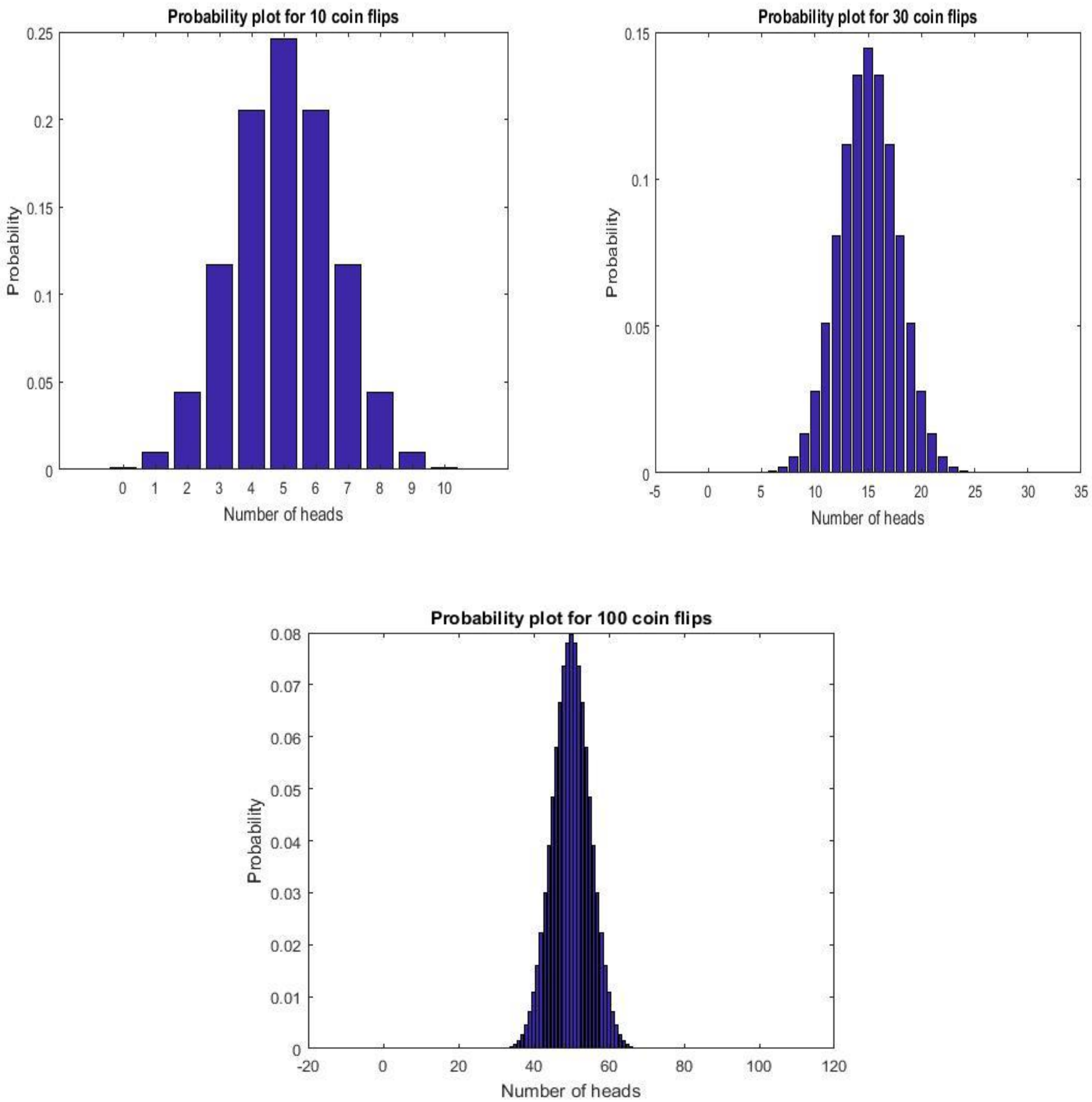

Fig 1.2 Probability plot for 15,40,100 coin flips

As shown in fig 1.2 as the number of coin flips increases from 10 to 30 and 30 to 100, the probability plot approaches the shape of the bell curve. Hence, per the law of large numbers the probability of number of heads converges as the flips increases. Now let’s see if the results of the theoretical model match with the simulation model.

Simulation model

Below is the MATLAB code to calculate number of heads N_h and length of consecutive number of heads L_h for multiple times.

Code:

```
% Matlab program to simulate scenario_1 the number of times
% required by the user

%-----------------Initializing variables-----------------%
n_times = input ('Enter the number of simulation runs: ');
nh=zeros(1,n_times);
lh=zeros(1,n_times);

%-----------------Logic to run scenario multiple times and calculate number
% of heads and length of consecutive heads-----------------%
for i=1:n_times
    X = randi([0,1],1,50);          % Function to generate a vector of
    n_heads = 0;                    %1x50 containing random 0 and 1
    lh_old = 0;
    lh_new = 0;
    for id=1:50
        if X(id) == 1               % Indicates heads
            n_heads = n_heads + 1;
            lh_new = lh_new + 1;
            if lh_new > lh_old
                lh_old = lh_new;
            end
        else
            lh_new = 0;
        end
    end
    nh(i) = n_heads;
    lh(i) = lh_old;
end

%-----------------Plotting the data obtained-----------------%
subplot(3,1,1)
histogram(nh)
xlim([0 50])
xlabel('Number of heads')
ylabel('Number of outcomes')
title(['Histogram for number of heads in ' num2str(n_times) ' runs'])
subplot(3,1,2)
tb=tabulate(nh);
plot(tb(:,2))
xlim([0 50])
xlabel('Number of heads')
ylabel('Number of outcomes')
subplot(3,1,3)
x = 0:50;
y = binopdf(x,50,0.5);            % Gives the binomial probability
plot(x,y*n_times)                 % distribution for 50 coin flips
xlabel('Number of heads')
ylabel('Probalility of outcome')
title('Theoretical probalility curve for 50 coin flips')

%-----------------Display Ferquency Distributin of Lh-----------------%
tabulate(lh)
```

Now, let us run the experiment 100, 1000, 5000 times and compare the results.

Output:

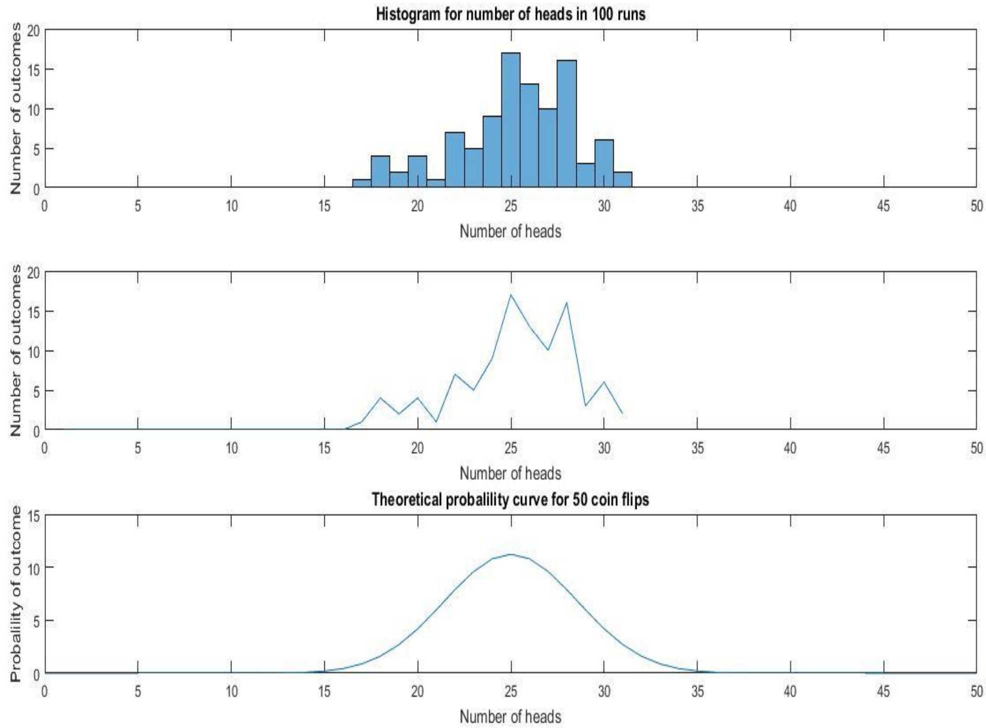

Fig 1.3 Simulation results for 100 runs

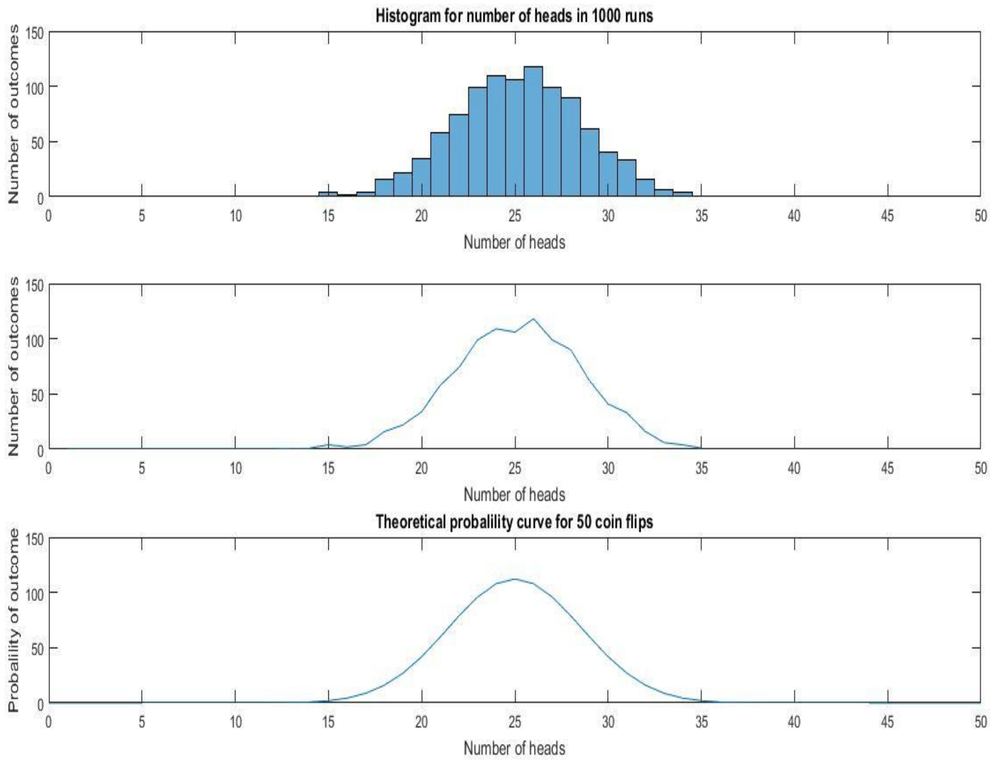

Fig 1.4 Simulation results for 1000 runs

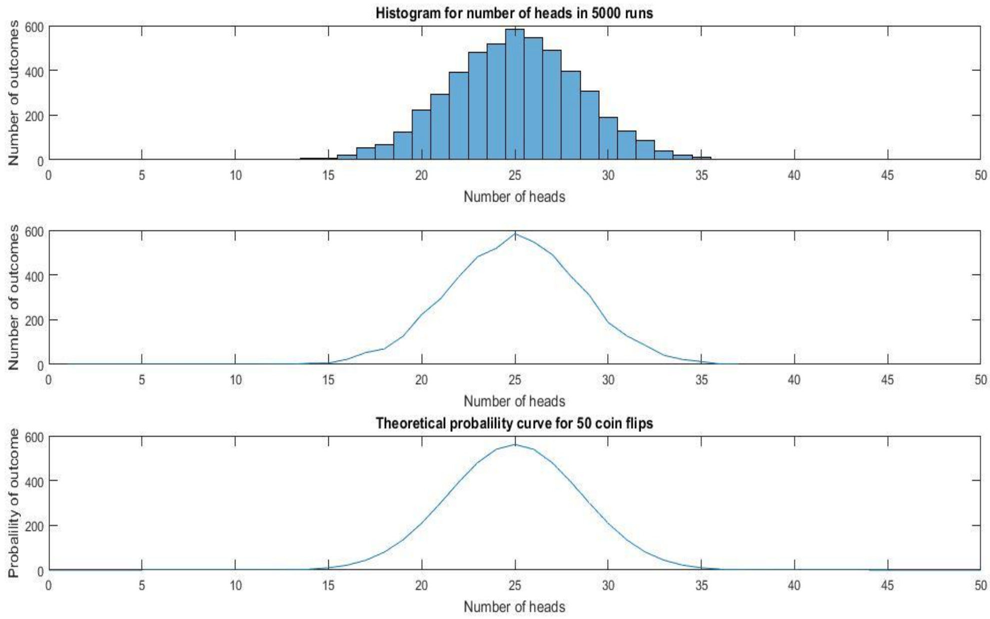

Fig 1.5 Simulation results for 5000 runs

From fig 1.3,1.4 and 1.5 as the number of runs increases the results becomes more and more normalised. For 100 runs the probability curve has only the outline of the theoretical curve, in 1000 runs it becomes more normalize with the theoretical more and in 5000 runs it takes the shape like that of the theoretical probability curve. Hence, the law of large numbers also holds true for the simulation model. So, the results of the theoretical model and simulation model for the experiment of flipping coins are similar.

Let us look at the data obtained for L_h in the above runs.

100 runs			1000 runs			5000 runs		
Value	Count	Percent	Value	Count	Percent	Value	Count	Percent
1	0	0.00%	1	0	0.00%	1	0	0.00%
2	3	3.00%	2	17	1.70%	2	186	1.86%
3	13	13.00%	3	153	15.30%	3	1521	15.21%
4	39	39.00%	4	293	29.30%	4	2790	27.90%
5	20	20.00%	5	236	23.60%	5	2370	23.70%
6	14	14.00%	6	151	15.10%	6	1468	14.68%
7	6	6.00%	7	73	7.30%	7	866	8.66%
8	4	4.00%	8	33	3.30%	8	404	4.04%
9	1	1.00%	9	19	1.90%	9	198	1.98%
			10	16	1.60%	10	107	1.07%
			11	6	0.60%	11	48	0.48%
			12	2	0.20%	12	23	0.23%
			13	1	0.10%	13	10	0.10%
						14	3	0.03%
						15	5	0.05%
						16	0	0.00%
						17	1	0.01%

Table 1.1 Frequency distribution of L_h for 100, 1000 and 10000 runs

From table 1.1 we can see that at least for 80 % of the runs the value of L_h is 3 or 4 or 5 or 6. So, here the probability for the value of L_h does not depends on the number of runs, it almost remains constant for any number of runs.

Scenario 2

Introduction

To calculate the number of flips until we get the desired sequence viz. H, HH, HHH and HHHH.

Theoretical model

For sequence H, the probability of the number of flips is given by

Number of flips	Possible ways	Count	Probability
1	H	1	0.5
2	TH	1	0.25
3	TTH	1	0.125
4	TTTH	1	0.0625

For sequence HH, the probability of the number of flips is given by

Number of flips	Possible ways	Count	Probability
1	-	0	0
2	HH	1	0.25
3	THH	1	0.125
4	TTHH, HTHH	2	0.125
5	TTTHH, THTHH, HTTHH	3	0.09375
6	TTTTHH, HTTTHH, THTTHH, TTHTHH, HTHTHH	5	0.078125
7	TTHTTHH, TTTHTHH, TTTTTHH, HTTTTHH, HTTHTHH, HTHTTHH, THTHTHH, THTTTHH	8	0.0625

For sequence HHH, the probability of the number of flips is given by

Number of flips	Possible ways	Count	Probability
1	-	0	0
2	-	0	0
3	HHH	1	0.125
4	THHH	1	0.0625
5	HTHHH, TTHHH	2	0.0625
6	HTTHHH, HHTHHH, THTHHH, TTTHHH	4	0.0625
7	THHTHHH, THTTHHH, TTHTHHH, TTTTHHH, HHTTHHH, HTHTHHH, HTTTHHH	7	0.0546875

For sequence HHHH, the probability of the number of flips is given by

Number of flips	Possible ways	Count	Probability
1	-	0	0
2	-	0	0
3	-	0	0
4	HHHH	1	0.0625
5	THHHH	1	0.03125
6	TTHHHH, HTHHHH	2	0.03125
7	TTTHHHH, THTHHHH, HTTHHHH, HHTHHHH	4	0.03125
8	THHTHHHH, THTTHHHH, TTHTHHHH, TTTTHHHH, HHHTHHHH, HHTTHHHH, HTHTHHHH, HTTTHHHH	8	0.03125

Table 2.1 Possible ways and Probability for number of flips for H, HH, HHH, HHHH

From table 2.1, it can be inferred that for sequence H the number of possible ways for number of flips is arithmetic series 1,2,3,4, … Similarly, for sequence HH: Fibonacci series 0,1,1,2,3,5,8, … For sequence HHH: the next term is obtained by adding previous three terms 0,0,1,1,2,4,7, … For sequence HHHH: the next term is obtained by adding previous four terms 0,0,0,1,1,2,4,8**.**

Simulation model

The below MATLAB code is used to simulate the scenario and calculate S1, S2, S3, S4 for sequence H, HH, HHH, HHHH respectively. In this program, we are also calculating the theoretical count for S1, S2, S3 and S4. As the display results we are comparing the simulation count with the theoretical count.

Code:

```
% Matlab program to simulate scenario_2 the number of times
% required by the user

%-----------------Initializing variables-----------------%
n_times = input ('Enter the number of simulation runs: ');
s1_nways = zeros(n_times,1);
s2_nways = zeros(n_times,1);
s3_nways = zeros (n_times,1);
s4_nways = zeros (n_times,1);
s3_nways(1:7)= [0,0,1,1,2,4,7];
s4_nways(1:7)= [0,0,0,1,1,2,4];
s1=zeros(1,n_times);
s2=zeros(1,n_times);
s3=zeros(1,n_times);
s4=zeros(1,n_times);

%-----------------Logic to run scenario 2 multiple times-----------------%
for i=1:n_times
    flag = 0;
    n_flips=0;
    sn=zeros(4,1);
    counter=0;
    for n=1:4
        flag=0;
        while flag==0
            x = randi([0,1],1,1);
            n_flips = n_flips + 1;
            if x == 1                    % Indicates head
                counter=counter+1;
            else
                counter=0;
            end
            if counter == n
                sn(n)=n_flips;
                flag=1;
            end
        end
    end
    s1(i)=sn(1);
    s2(i)=sn(2);
    s3(i)=sn(3);
    s4(i)=sn(4);
end

%---------Logic to count number of ways we can get the sequence---------%
for i=1:n_times
    s1_nways(i)= 1/2^i;
    if i>1
        s2_nways(i) = fibonacci(i-1);
    end
    if i>7
        s3_nways(i)= s3_nways(i-1)+s3_nways(i-2)+s3_nways(i-3);
```

```
        s4_nways(i)= s4_nways(i-1)+s4_nways(i-2)+s4_nways(i-3)+s4_nways(i-
4);
    end
end

%-----Logic to calculate theoretical probability for number of flips------%
for i=1:n_times
    s1_nways(i)= 1/2^i;
    s2_nways(i)=s2_nways(i)/2^i;
    s3_nways(i)=s3_nways(i)/2^i;
    s4_nways(i)=s4_nways(i)/2^i;
end

%-----------------Displaying results-----------------%
tb1=tabulate(s1);
tb2=tabulate(s2);
tb3=tabulate(s3);
tb4=tabulate(s4);

table(tb1(:,2),s1_nways(1:size(tb1,1),1)*n_times)
table(tb2(:,2),s2_nways(1:size(tb2,1),1)*n_times)
table(tb3(:,2),s3_nways(1:size(tb3,1),1)*n_times)
table(tb4(:,2),s4_nways(1:size(tb4,1),1)*n_times)
```

Output:

The theoretical count is obtained my multiplying the probability of individual number of flips with the number of runs i.e. 5000. From the table 2.2, we can clearly see that count of simulation results matches with the theoretical count.

Sequence H		
No. of flips	Simulation Count	Theoretical count
1	2525	2500
2	1198	1250
3	647	625
4	335	312.5
5	142	156.25
6	75	78.125
7	46	39.0625
8	20	19.53125
9	2	9.765625
10	5	4.882813
11	3	2.441406
12	1	1.220703
13	1	0.610352

Sequence HH		
No. of flips	Simulation Count	Theoretical count
1	0	0
2	1263	1250
3	590	625
4	631	625
5	491	468.75
6	370	390.625
7	286	312.5
8	283	253.90625
9	222	205.078125
10	151	166.015625
11	142	134.2773438
12	129	108.6425781
13	80	87.890625
14	68	71.10595703
15	58	57.52563477
16	42	46.53930664
17	36	37.65106201
18	48	30.46035767
19	22	24.64294434
20	14	19.93656158
21	10	16.12901688
22	12	13.04864883
23	9	10.55657864
24	7	8.5
25	6	7.5

Sequence HHH		
No. of flips	Simulation count	Theoretical count
1	0	0
2	0	0
3	638	625
4	297	312.5
5	321	312.5
6	296	312.5
7	268	273.4375
8	237	253.90625
9	257	234.375
10	221	214.84375
11	214	197.7539063
12	164	181.8847656
13	197	167.2363281
14	146	153.8085938
15	128	141.4489746
16	140	130.0811768
17	117	119.6289063
18	114	110.0158691
19	101	101.1753082
20	81	93.04523468
21	74	85.56842804
22	62	78.69243622
23	70	72.36897945
24	67	66.55365229
25	65	61.20562553
26	55	56.28734827
27	51	51.76428705
28	58	47.60468379
29	57	43.77933219
30	35	40.26137292
31	44	37.02610498
32	32	34.05081225
33	30	31.31460398
34	21	28.79826818
35	24	26.48413662
36	22	24.35596085
37	21	22.3987981
38	24	20.59890634
39	20	18.9436478
40	20	17.42140025
41	18	16.02147537

Sequence HHHH		
No. of flips	Simulation count	Theoretical count
1	0	0
2	0	0
3	0	0
4	306	312.5
5	147	156.25
6	166	156.25
7	151	156.25
8	148	156.25
9	133	146.484375
10	142	141.6015625
11	121	136.71875
12	148	131.8359375
13	100	126.953125
14	136	122.3754883
15	107	117.9504395
16	117	113.6779785
17	113	109.5581055
18	100	105.5908203
19	109	101.7665863
20	112	98.08063507
21	90	94.52819824
22	95	91.10450745
23	74	87.80479431
24	90	84.62458849
25	85	81.55956864
26	87	78.60556245
27	74	75.75854659
28	78	73.01464677
29	77	70.37012838
30	68	67.82139186
31	69	65.36496803
32	74	62.99751345
33	61	60.71580574
34	62	58.51673923
35	51	56.39732073
36	46	54.35466548
37	55	52.38599319
38	30	50.48862426
39	60	48.65997616
40	46	46.89755988
41	42	45.1989659
42	48	43.5619143
43	45	41.98414479
44	44	40.46352054
45	48	38.99797179
46	43	37.58550377
47	39	36.22419395
48	31	34.91218943
49	37	33.64770441
50	29	32.42901779
51	39	31.2544708
52	24	30.12246474
53	27	29.03145882
54	28	27.97996805
55	31	26.96656125
56	33	25.98985904
57	15	25.04853201
58	26	24.14129892
59	30	23.26692492
60	22	22.42421988
61	23	21.61203679
62	25	20.82927016
63	23	20.07485457
64	16	19.34776317
65	16	18.6470063
66	12	17.97163015
67	17	17.32071545
68	16	16.69337625
69	18	16.08875865
70	19	15.5060397
71	19	14.94442626
72	22	14.4031539
73	12	13.8814859
74	14	13.37871219
75	9	12.89414845
76	14	12.42713513
77	9	11.97703657
78	11	11.54324013
79	8	11.12515538

Table 2.2 Comparison of simulation counts with theoretical counts