## Problem Statement

Let $X_i$ be iid exponentially distributed random variables with $E[Xi] = \frac{1}{\lambda} = 1$. Define the random variable $Sn = \sum_{i=1}^{n} Xi$.

We need to do following simulations for $X_i$ and $S_n$:

A. Generate samples of $X_i$ and use them to generate the series of samples for $S_n$. Compute the sample mean and sample variance for n = 1 and n = 5 and compare to analytical value. Also plot the histogram of pdf of Sn for n = 1 and n = 5.
B. Generate samples of Sn by using $Sn = \frac{-\ln(\prod_{i=1}^{n} Ui)}{\lambda}$, where $U_i$ is uniform (0, 1). Compute the sample mean and sample variance for n = 5 and compare to analytical value. Also plot the histogram of pdf of Sn for n = 5.

## Theoretical Computations

The random variable $S_n$ is sum of iid exponential random variables, so its mean is given by

$$E[Sn] = E[\sum_{i=1}^{n} Xi]$$

$$\therefore E[Sn] = E[X1 + X2 + \cdots + Xn]$$

$$\therefore E[Sn] = nE[X1] = \frac{n}{\lambda}$$

And variance is given by

$$Var(Sn) = var(\sum_{i=1}^{n} Xi)$$

$$\therefore Var(Sn) = nVar(X1) = \frac{n}{\lambda^2}$$

In our case λ = 1 so E[Sn] = n and var(Sn) = n.

For n = 1, distribution of Sn converges to exponential distribution and for n = 5, distribution of Sn converges to normal distribution.

## Simulation

### Part A

In this part, we generate $X_i$ a sequence of iid exponential random variables with mean 1 from uniform random variable distributed between 0 and 1. From $X_i$ we compute $S_n$. Then we compute sample mean and sample variance of $S_n$. We plot histogram of pdf of $S_n$.

**Code:**

```
% Value of n (1 or 5)
n=1;
% Value of lamda parameter of Xi
lamda=1;
% Number of samples
```

```
N=50000;
S=zeros(1,N);
for i=1:n
    X=-log(rand(1,N)); % Generate exponential RV from uniform RV
    S=S+X; % Generate samples of Sn
end
% Plot probability distribution function of Sn
histogram(S,'Normalization','probability')
% Mean of Sn
meanS=mean(S)
% Variance of Sn
varianceSn=var(S)
```

**Output:**

For n = 1, λ = 1, N = 50000:

| | Theoretical | Simulation |
|---|---|---|
| **Mean** | 1 | 1.0001 |
| **Variance** | 1 | 0.9892 |

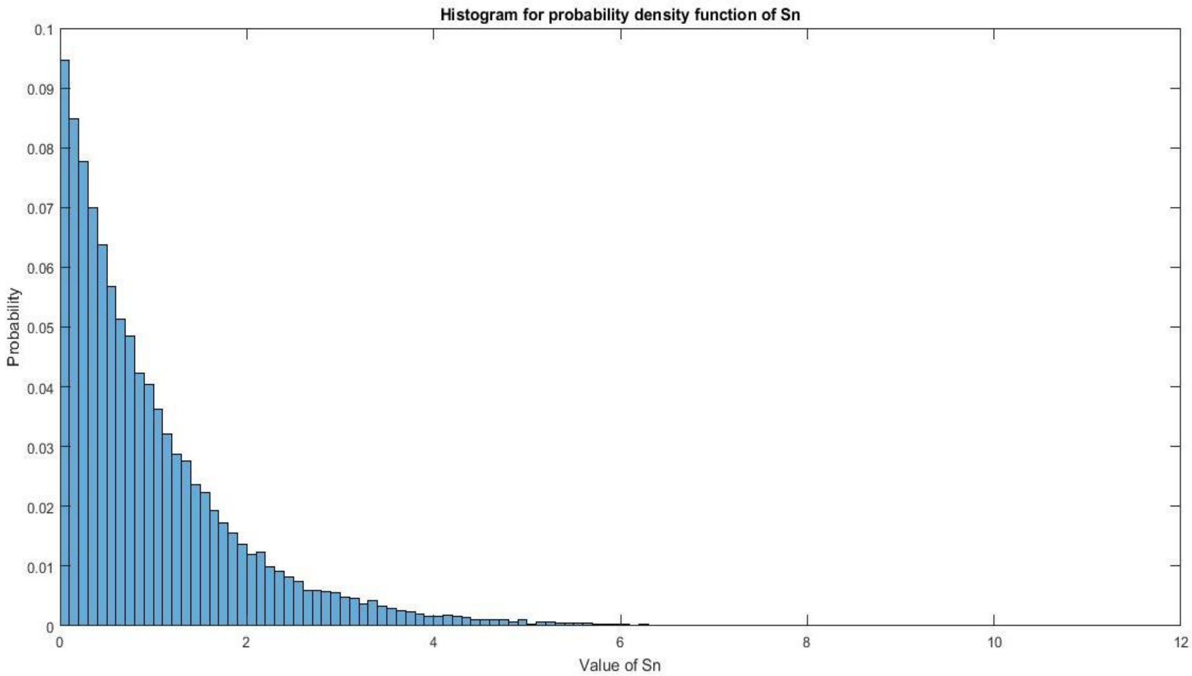


*Figure 1 Histogram for probability density function of S1*

For n = 5, λ = 1, N = 50000:

| | Theoretical | Simulation |
|---|---|---|
| **Mean** | 5 | 4.9872 |
| **Variance** | 5 | 4.9483 |

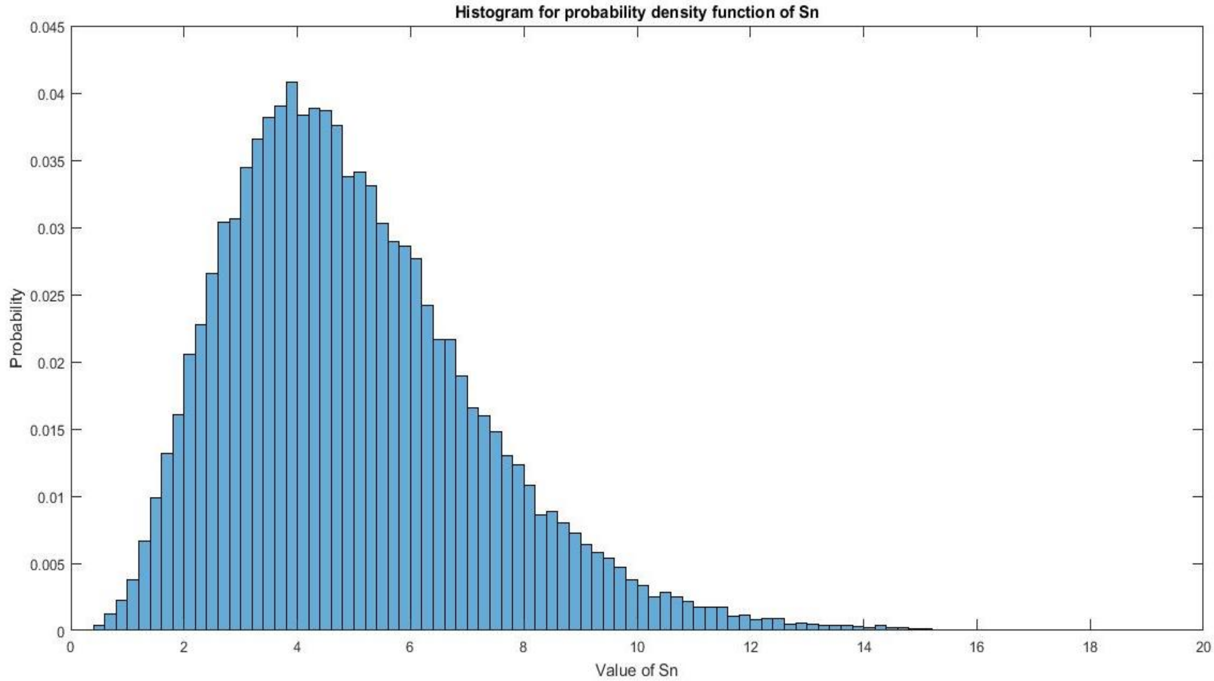


*Figure 2 Histogram for probability density function of S5*

For n=1, the probability distribution function for $S_n$ is exponentially distributed and for n=5 it is normally distributed.

## Part B

In this part, we generate samples of $S_n$ by using the function of uniform random variable and then compute sample mean and sample variance for Sn.

**Code:**

```
% Value of n
n=5;
% Value of lamda parameter of Xi
lamda=1;
% Number of samples
N=50000;
S=zeros(1,N);
temp=rand(n,N); % Generate marix of uniform RVs between 0 and 1
X=temp(1,:);
if n~=1
    for i=2:n
        X=X.*temp(i,:); % Multiply samples of uniform RVs
    end
end
S=-log(X)/lamda; % Generate samples of Sn from uniform RV
% Plot probability distribution function of Sn
histogram(S,'Normalization','Probability')
% Mean of Sn
meanS=mean(S)
% Variance of Sn
varianceSn=var(S)
```

**Output:**

For n=5, the time taken for part A = 0.829 seconds and the time taken for part B = 0.458 seconds.

For n = 5, λ = 1, N = 50000:

| | Theoretical | Simulation |
|---|---|---|
| **Mean** | 5 | 4.9990 |
| **Variance** | 5 | 4.9957 |

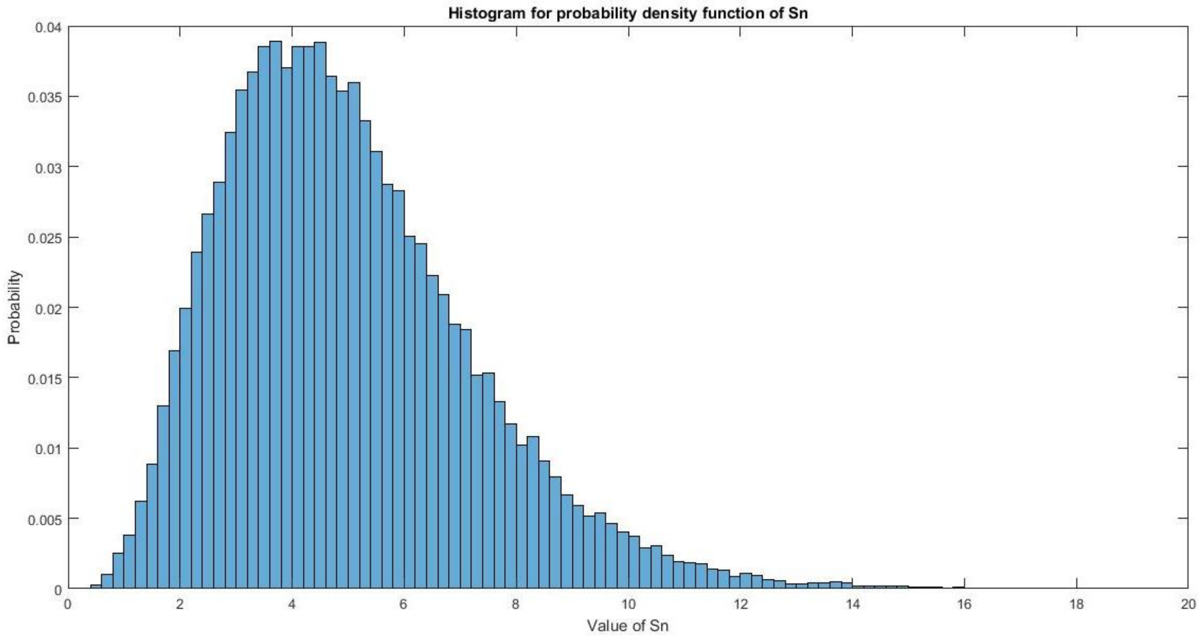


*Figure 3 Histogram for probability density function of S5*