Who Plays Video Games?

Joshua Castro, Kevin Elkin, Zunlin Lin, Sneheil Saxena, Yuka Sukamaki, Tony Wu

Abstract -

I. INTRODUCTION

Each year approximately 3,000 - 4,000 students enroll in statistics courses at the University of California, Berkeley. In order to further support the education and instruction for students in statistics, a committee consisting of both faculty and students are designing a series of lab computers that will allow for increased learning and efficiency. The new computer labs are being built with the intent of helping students have a more interactive and hands-on learning experience regarding the subject of statistics.

It has been accepted that video games have been linked to computer labs; thus, with the intent of improving the productivity and learning in labs for students, the committee conducted a survey to find the extent in which students play video games along with which aspects of video games students like and dislike the most. This will, in turn, allow the committee to design a lab based around student preferences of video games.

Students in the advanced statistics course conducted the study. The students in advanced statistics surveyed Statistics 2, Section 1 course during Fall 1994 containing 314. Each student enrolled in Statistics 2, Section 1 was assigned a number between 1 and 314; using a pseudo random number generator 95 students (numbers between 1 - 314) were selected (without replacement) to participate in the survey. To promote and encourage honest and accurate responses, students were told that their identity and anonymity would be preserved. The students in the advanced statistics who gathered the survey data would attend the Tuesday and Thursday sections the week after the exam of those students who were selected to participate in the study. If the students were unable to be reached during the section then they would be located on Friday during lecture. A total of 91 student responses were recorded.

II. DATA

The data set we are given contains observations gathered through surveys given to 95 of the 314 students enrolled in University of California, Berkeley's Statistics 2, Section 1 of Fall 1994 - 1995. Since this data is derived from a survey, most of the variables are categorical. However, we are also given discrete variables such as student age, student year, time student spent working prior to the survey, and the number of hours the student spent playing game prior to survey.

It is worth noting that the data from the first survey is not fully complete. Students who did not play video games were asked to skip portions of the surveys. Hence, the dataset only represents 91 surveys filled out in full.

We also use data given from a followup survey, in which students gave more insight regarding their preferences for type of games, reason for playing games, and other important factors to consider. It is also worth noting that we have four missing student surveys for this dataset, which we must consider during our analysis.

TABLE I: A table containing the type of variables and possible responses for students who participated in the survey to select from an fill out

Variable Name	Variable Type	Variable Description
Time	Numerical	Number of hours played in the week prior to survey
Like to Play	Ordinal	How much a student likes to play video games: ● 1 = never played ● 2 = very much ● 3 = somewhat ● 4 = not really ● 5 = not at all
Where Play	Categorical	Location a student plays video games: ● 1 = arcade ● 2 = home system ● 3 = home computer ● 4 = arcade and either home computer of system ● 5 = home computer and system ● 6 = all three
How Often	Ordinal	Frequence student plays video games: ● 1 = daily ● 2 = weekly ● 3 = monthly ● 4 = semesterly
Play if Busy	Categorical	Does a student play if they are busy: ● 1 = yes ● 0 = no
Playing Educational	Categorical	Does a student perceive their game as educational: ● 1 = yes ● 0 = no
Sex	Categorical	The gender of the player ● 1 = male ● 0 = female
Age	Numerical	Students age in years
Computer at Home	Categorical	Does a student have a computer at home: ● 1 = yes ● 0 = no

Hate Math	Categorical	Does a student dislike math: ● 1 = yes ● 0 = no
Work	Numerical	Number of hours a student worked during the week prior to the survey
Own PC	Categorical	Does a student own a PC: ● 1 = yes ● 0 = no
PC has CD-Rom	Categorical	Does a students PC have CD-Rom: ● 1 = yes ● 0 = no
Have Email	Categorical	Does a student have an email: ● 1 = yes ● 0 = no
Grade Expected	Ordinal	The grade a student expects: ● 4 = A ● 3 = B ● 2 = C ● 1 = D ● 0 = F

*TABLE II: A table showing the proportion of students that play a specific genre of video game(s). *Note: up to three game genres are allowed to be selected per student*

Game Type	Percentage
Action	50%
Adventure	28%
Simulation	17%
Sport	39%
Strategy	63%

*TABLE III: A table showing the reasons students like playing video games. *Note: up to three reasons are allowed to be selected per student*

The reasons students like video game	Percentage
Graphics/Realism	26%
Relaxation	66%
Eye/Hand Coordination	5%
Mental Challenge	24%
Feeling of Mastery	28%
Bored	27%

*TABLE IV: A table showing the reasons students don't like playing video games. *Note: up to three reasons are allowed to be selected per student*

The reasons students don't like video game	Percentage
Too much time	48%
Frustrating	26%
Lonely	6%
Too much rules	19%
Costs too much	40%
Boring	17%
Friends don't play	17%
It is Pointless	33%

III. BACKGROUND

As we investigate the relationship of video games preferences with the work and study characteristics of the Berkeley students, we must first understand prior studies regarding the intersections of video games and educational productivity. A study conducted in 1980 investigated how the characteristics of certain video games intrinsically motivated individuals to find these video games entertaining and how these motivations could be used to incorporate the practice of playing video games or the principles video games used to motivate players into possible education curriculum. This study found two main conclusions. First, they found that video game preferences vary among different identities and personalities, specifically between the male-identified and female-identified participants of the study. What was perceived as intrinsically motivating for one participant did not mean it was the same case for the rest of the participants, which should be taken into account when incorporating video games into education. Second, it has been found that video games which stimulate and satisfy curiosity have the potential of being both captivating and educational. [Thomas] A video game that can cater to the preferences of the individual and maintain interest by cycling through creating curiosity and satisfying it has the potential of effectively convey educational messages.

As an additional investigation, we wish to examine how the busyness of each student can potentially affect the performance of the student in the class. We are able to categorize the student population by their busy-ness as given in the dataset, allowing us to check whether or not students who are busy perform as well as or better than students who are not busy. A previous study [Festini, McDonough, Park] claimed that being busy helped improve cognition. The mentioned study concluded with finding that busyness did provide a positive relationship with cognition, as their procedures consistently revealed that busyness improved processing speed, working memory, episodic memory, crystallized knowledge, and reasoning. Though this study was conducted with an elderly population of ages ranging from 50-89 years old, the study also found that age did not affect the cognition of the participants. The study found a similar relationship between the busyness of the participant and their cognition across the full age range, prompting us to see whether or not the busyness of the Berkeley students could be used as an indicator of their performance in the class. While the busyness of the student is not directly stated, we would like to investigate if students who still played video games while busy received higher grades than those who did not.

In conducting our study, we will use the given dataset to find patterns and trends which could indicate relationships between video games and education or busyness and education. Since our dataset is derived from a relatively small sample population, we will use the bootstrap technique and an analysis of statistical characteristics of the bootstrapped data, including such as skewness and kurtosis, to further our investigations. We must also note that this dataset is not i.i.d. (independent, identically distributed), so we must account for this in our methods.

IV. HYPOTHESIS

In our investigation, we formulate the following hypotheses:

I. Will the results from the survey provide useful information about the students to the designers of the computer lab?
   A. We believe that the results from the survey will provide useful information, as we will demonstrate in our investigations.
II. Do students who play video games while they are busy perform relatively worse in the class?
   A. We believe that the students who play video games while busy will still perform relatively well in the class.

V. SCENARIO I

We begin by providing an estimate for the fraction of students who played a video game in the week prior to the survey and an interval estimate as well as a point estimate for this proportion as follows.

We have our point estimate, which is the value predicting the parameter of our population, which is the proportion of students who played a video game in the week prior to the survey. To obtain this value, we calculated the amount of students who played a video game in the sample (calculated to be 34 students) divided by the number of students in the sample (91 students in the sample), so we see our point estimate is $\overline{X} = \frac{34}{91} \approx 0.3736$. With this, we will compute a 95% confidence interval to obtain an interval estimate of this proportion. We see that our data will be approximately normal by the Central Limit Theorem as our sample size holds a sufficient proportion of the population.

1) Standard Confidence Interval

Using a standard confidence interval, we see that our margin of error is $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, calculated to be 0.09939. So, our confidence interval is $(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\ ,\ \overline{X} +\ z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) = (0.2742, 0.4730)$.

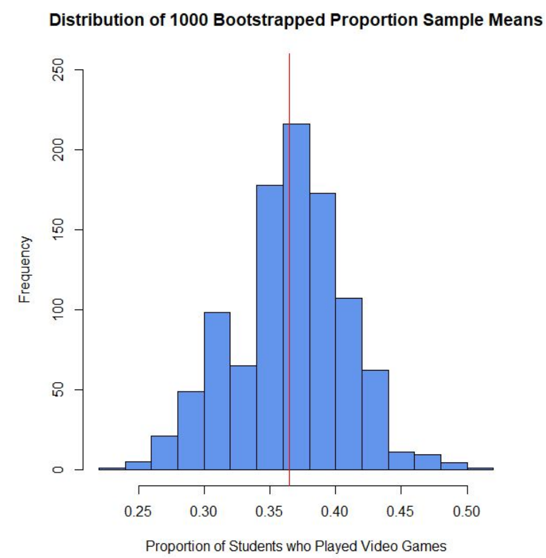
However, we must note that our survey data is non i.i.d., so we must account for the difference in variance, thus affection our result.

2) Confidence Interval using finite population correction factor

With non i.i.d. data, we note that our variance is $\frac{1}{n}\sigma^2\frac{N-n}{N-1}$, so our margin of error is now $z_{\alpha/2}\sqrt{\frac{\overline{X}\,(1-\overline{X})}{n-1}\frac{N-n}{N}}$, calculated to be 0.08423. Thus, our confidence interval using a finite population correction factor is (0.2894, 0.4579). Thus, we are 95% confident that the proportion of students who played video games the week prior to the survey lies in the interval (0.2894, 0.4579).

We note here that our margin of error is smaller using a finite population correction factor than a standard margin of error. The correction factor is defined to be $\frac{N-n}{N}$ for variances, calculated to be 0.7102 and $\sqrt{\frac{N-n}{N}}$ for standard deviations, calculated to be 0.8427. Thus, the width of our interval becomes smaller due to the correction factor, otherwise the variances of the confidence intervals would be the same.

Fig. 1: A histogram showing the distribution of 1000 bootstrapped sample means pertaining to the proportion of student video game players



To further the findings made in the previous step, a bootstrap of 1000 sample means was constructed to find the average proportion of students who played video games 1 week prior to the exam. Fig. 1 above shows a distribution of 1000 bootstrapped sample means of the proportion of students who played video games the week prior to the exam; the above distribution above has a meanline drawn at 0.365 suggesting that the average proportion of 1000 sample means is slightly lower as compared to our point estimate of 0.3736 . Based on the 1000 bootstrap sample above, we can see that on average 36.5% of students played video games the week prior to the exam.

VI. SCENARIO 2

We now wish to examine the amount of time spent playing games in the week prior to the survey, comparing this to the reported frequency of play.

Fig. 2: A boxplot modeling the four distributions of students who played based on the frequency they selected on the survey and the number of hours they played during the week prior to the survey.

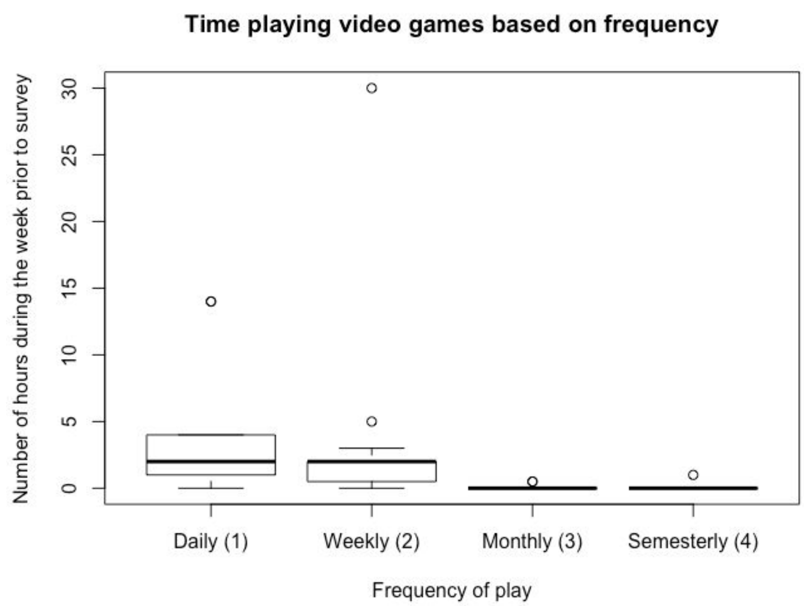


Fig. 2 above shows the amount of time students spent on playing video games based on their frequency prior to the survey; it is shown that the median of groups 1 and 2 (daily and weekly) are higher than groups 3 and 4 (monthly and semesterly). This box plot suggests that the students who play video games more frequently (groups 1 and 2) will have a higher median number of hours played during the week prior to the survey as compared to those who play at less frequent rates (groups 3 and 4).

TABLE V: A table showing the average time students spend playing video games during the week based on the given frequency

Frequency	**Average Time Spent Playing During the Week Prior to Survey (Hours)**
1: Daily	4.444
2: Weekly	2.539
3: Monthly	0.055
4: Semesterly	0.043

Using the frequency of play groupings given from the data, we can see that the average amount of hours playing during the week prior to the survey are what we expect them to be: the population of those who stated they played video games daily have the highest average amount of time spent playing video games, and the averages decrease as the frequency of play grows in respect to intervals of time as noted in Table V.

TABLE VI: A table showing the average time students spend playing video games during the week based on the given frequency. This table further segments the students based on if they play video games while they are/are not busy

Frequency of Play	Avg Hours Played for Non-busy Students Gamers	Number of students who play when Non-busy	Avg Hours Played for Busy Students Gamers	Number of students who play when Busy
1: Daily	1.00	4	7.20	5
2: Weekly	1.59	17	4.00	11
3: Monthly	0.06	17	0.00	1
4: Semesterly	0.00	22	0.00	0

TABLE VII: A table showing the proportion of students who play video games when they are busy

Frequency of Play	Proportion of students who play when busy
1: Daily	0.56
2: Weekly	0.39
3: Monthly	0.06
4: Semesterly	0.00

From Table VI, we can see direct comparisons of the average number of hours spent playing video games per frequency grouping demographics between the population of students who played when they were busy and those who did not. The numbers from this table suggest that the average number of hours spent playing video games by those who did not play when they were busy is generally less than that of the students who played when they were busy despite the size of the population of students who did not play video games while they were busy is larger than its counterpart.

To add, from Table VII we see that those who do play video games even though they are busy tend to play daily, as this frequency grouping represents half of the population of these students.

Fig. 3 Graphical representation of the average number of hours playing video games spent by those who did not play when they are busy.

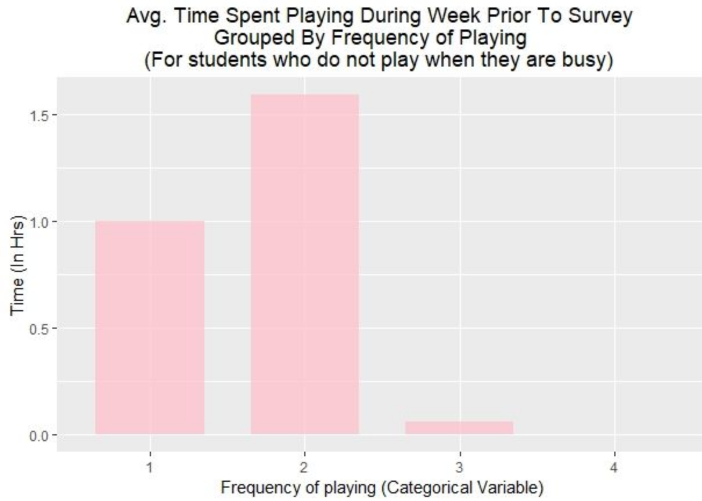
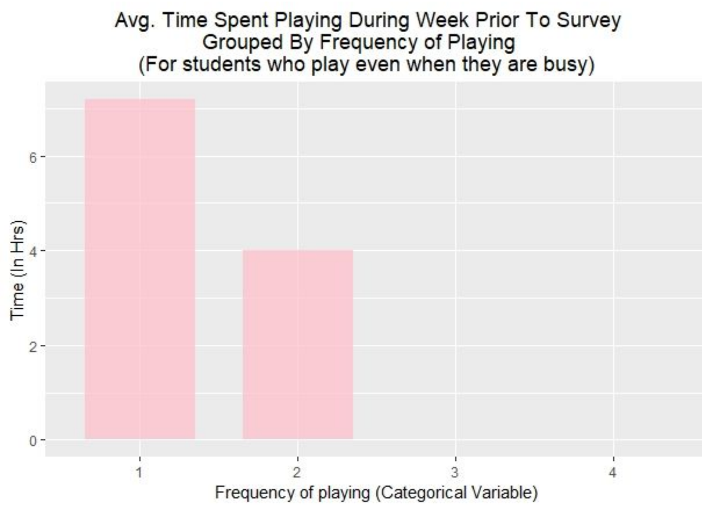


Fig. 4 Graphical representation of the average number of hours playing video games spent by those who played when they are busy.



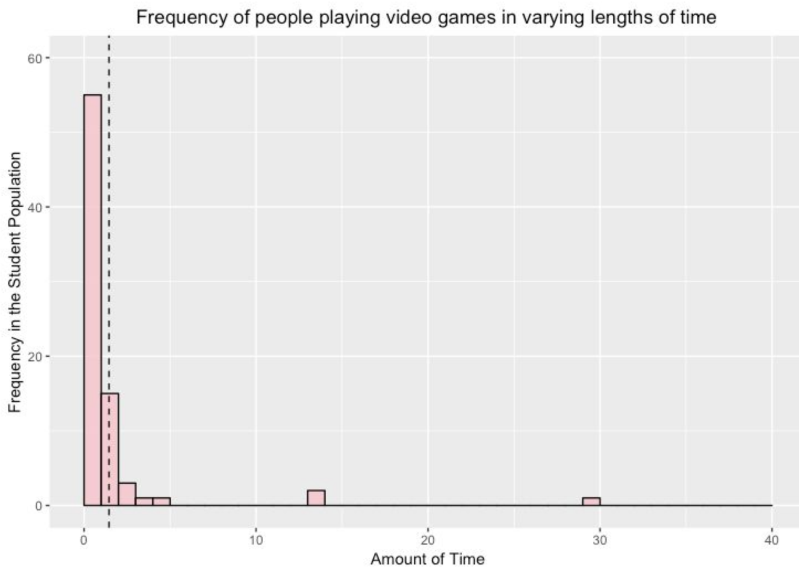
In examining Fig. 3 and Fig. 4, we can see that Fig. 3, which denotes the distribution of the population of students who did not play video games when they were busy, is more skewed right but remains balanced in distribution compared Fig. 4 which is more dense toward the left of the distribution. This means that students who played while they are busy still play more hours on average than those who did not play when they were busy. This suggests that adding video games into the design lab may be detrimental to the productivity of the students as this population of students who played video games though they were busy may spend more of their time playing the provided video games in the lab than studying. This is back by the findings described in Table VII.

These numbers may have been affected by the upcoming exam, as students who normally play outside of class could have delegated more time towards studying rather than playing video games, influencing them to denote a lesser amount of time spent playing video games. In order to further investigate how the presence of an exam can affect these numbers, we must rethink how we interpret Table VI. The students who could have been affected by the presence of the exam are those who both played daily or weekly and played even though they are busy. Due to an impending exam, these students may have reported lower numbers than their true tendencies. If these students were to have reported these hours while there was no incoming exam, the average of this category could be higher. Graphically, this possible error could mean that the results described in Fig. 4 could be more skewed right.

VII. SCENARIO 3

We now wish to provide and interval estimate for the average amount of time spent playing video games in the week prior to the survey using the bootstrap simulation study.

Fig. 5: A histogram of the amount of time (in hours) students played video games the week prior to the survey



The average amount of hours that students played video games prior to the survey is $\overline{X} = 1.2429$. We will use a confidence interval to estimate the amount of time spent playing video games.

1) Standard Confidence Interval

Using a standard confidence interval, we see that our margin of error is $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, calculated to be 0.7760. So, our confidence interval is $(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}) = (0.4668, 2.0189)$.

2) Confidence Interval using finite population correction factor

With non i.i.d. data, we previously noted that our variance is affected by a correction factor, so our margin of error is now $z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{n-1}\frac{N-n}{N}}$, calculated to be 0.6540. Thus, our confidence interval using a finite population correction factor is (0.5889, 1.8968).

However, we see from Fig. 4 that the distribution is heavily skewed right, so we cannot take an accurate confidence interval, because our data is non-normal. From Table VIII, we see that skewness and kurtosis of the amount of time played prior to the survey of the survey data is heavily skewed and not normal. Therefore, we used the bootstrap method to normalize our data.

TABLE VIII: *A table comparing the skewness and kurtosis values amount of time played prior to the survey of the survey and the bootstrap*

	Survey	Bootstrap
Skewness	5.78857	0.2478847
Kurtosis	40.69921	2.77392

Fig. 6: A histogram to represent the skewness of the bootstrapped samples

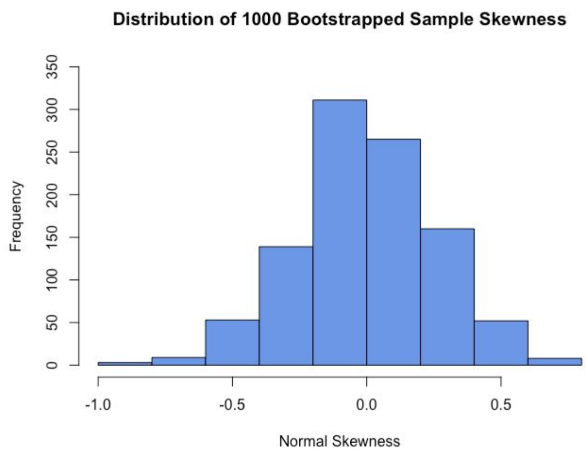
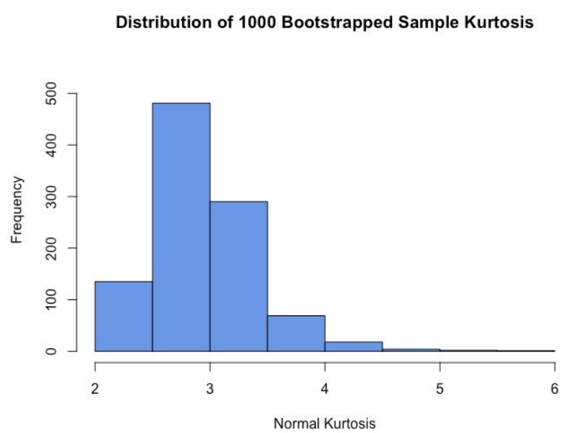
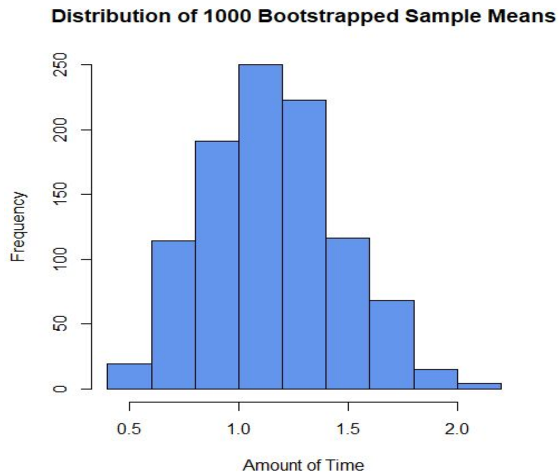


Fig. 7: A histogram to represent the kurtosis of the bootstrapped samples



We see from Fig. 8, that the distribution is approximately normal. Also, we see that the mean of the distribution of skewness in Fig. 6 is close to zero, as well as the mean of the distribution of kurtosis in Fig. 7 is close to three, thus we have strong evidence that the distribution of the bootstrapped data is normal. To confirm we have normal data, we used the Kolmogorov-Smirnov test to compare the normal distribution against the amount of time played of the survey data and the bootstrapped means. In Table IX, we see that we have a low p-value for the survey data, where we reject that distribution of the survey data is normal and the p-value of the bootstrapped means is sufficiently high, so we fail to reject that the distribution of the bootstrapped data is normal. Therefore, we can conclude that the bootstrapped data is normal, thus we can take a confidence interval of the amount of time played prior to the survey.

Fig.8: A histogram of 1000 bootstrapped sample means of the amount of time played prior to the survey.



We bootstrapped the amount of time played prior to the survey and obtained a mean of $\bar{\hat{X}} = 1.1543$. Because our data is normal and bootstrapped, our data is i.i.d.,, thus we can take a standard confidence interval of the bootstrapped data. So, we see that a 95% confidence interval of the bootstrapped data is (1.07995, 1.20656). Therefore, we are 95% confident that the average amount of time that students played video games the week prior to the survey is in between the interval 1.07995 hours to 1.20656 hours.

TABLE IX: Kolmogorov-Smirnov test against the normal distribution

Two-sample Kolmogorov-Smirnov Test	**Survey**	**Bootstrap**
Test Statistic D	0.40659	0.093121
p-value	5.855e-07	0.4646

VIII. SCENARIO 4

We would now like to examine whether or not students like to play video games in general. In order to do so, we refer back to the tables given in the data section, which is seen again below:

*TABLE III: A table showing the reasons students like playing video games. *Note: up to three reasons are allowed to be selected per student*

The reasons students like video game	**Percentage**

Graphics/Realism	26%
Relaxation	66%
Eye/Hand Coordination	5%
Mental Challenge	24%
Feeling of Mastery	28%
Bored	27%

*TABLE IV: A table showing the reasons students don't like playing video games. *Note: up to three reasons are allowed to be selected per student*

The reasons students don't like video game	Percentage
Too much time	48%
Frustrating	26%
Lonely	6%
Too much rules	19%
Costs too much	40%
Boring	17%
Friends don't play	17%
It is Pointless	33%

Let us first examine what characteristics of the video games make them appealing to students. Using the information given in Table III, we see that 66% of the students use video games to relax. This means that students generally enjoy video games because it allows them to relax. It is also worth noting that the two biggest reasons why students do not like video games, according to Table IV, is because it takes too much time or because it costs too much. These two reasons do not necessarily have to do with the characteristics of the actual video games, but may have to do with outside factors such as social pressures or monetary issues hindering them from enjoying video games. We can also see that from Table IV, only 17% of students say they dislike video games because they are boring which is much less than the general trends shown. From these general statements, we can see that students do enjoy video games in general.

In a shortlist of reasons according to Table III, the students may enjoy video games because:

1. Video games could allow students to get their minds off their educational stresses and workload.
2. Video games could fill in free time between assignments, classes, or other breaks.
3. Video games provide students another medium to appreciate visual aesthetics.

4. Video games could give students another objective to accomplish outside of academia.

In a shortlist of reasons according to Table IV, students may not enjoy video games because:

1. In the time spent playing video games, students could be taking steps to prepare for their future careers.
2. With the money spent on video games, students could allocate their funds elsewhere or invest their money into companies or products they believe in.
3. Students may not see a point in investing their time into games with no true accolades or physical returns.

IX. SCENARIO 5

Lastly, we want to analyze the differences between the population of students who like to play video games and their counterparts (i.e. dislike playing video games). We would like to see if there is some sort of pattern between the two populations.

In order to accurately represent the likes and dislikes of students who play video games we must first eliminate the students who have never played video games before; this is due to the fact that this study only seeks to determine results from students who *have* played video games at some point in their lifetime. Thus, if a student answered ‘1’ (meaning that they never played) under the like category, then they were removed from the data.

We collapsed the “very much” and “somewhat” categories into “Like to Play”, and “not really” and “not at all” categories into “Dislike”, to make it easier to compare.
We then made bar graphs and cross tabulations for seeing how those who like to play video games and those who don’t are distributed amongst the following sets of categories:

1) Sex
2) Working during the week prior
3) Ownership of a PC

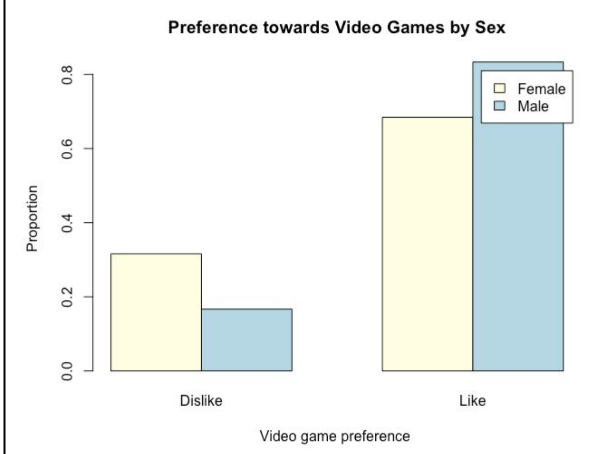
For the cross tabulations, the cell contents are as follows:

```
   Cell Contents
|-------------------------|
|                       N |
| Chi-square contribution |
|           N / Row Total |
|           N / Col Total |
|         N / Table Total |
|-------------------------|
```

The chi-square test gives us a p-value that gives us information about the independence of the row and column variable. For the chi-square test, a p-value of greater than 0.05 (most commonly used alpha value) indicates dependence between the two variables (which gets stronger as the p-value gets closer to 1).

Fig. 9: A cross table showing how the observations are distributed between the four categories (male, female, like, dislike). A bar graph showing the proportion of males who like/dislike and proportion of females who like/dislike video games

```
               | like.data$sex
like.data$like |    Female |      Male | Row Total |
---------------|-----------|-----------|-----------|
       Dislike |        12 |         8 |        20 |
               |     1.132 |     0.896 |           |
               |     0.600 |     0.400 |     0.233 |
               |     0.316 |     0.167 |           |
               |     0.140 |     0.093 |           |
---------------|-----------|-----------|-----------|
          Like |        26 |        40 |        66 |
               |     0.343 |     0.272 |           |
               |     0.394 |     0.606 |     0.767 |
               |     0.684 |     0.833 |           |
               |     0.302 |     0.465 |           |
---------------|-----------|-----------|-----------|
  Column Total |        38 |        48 |        86 |
               |     0.442 |     0.558 |           |
---------------|-----------|-----------|-----------|
```



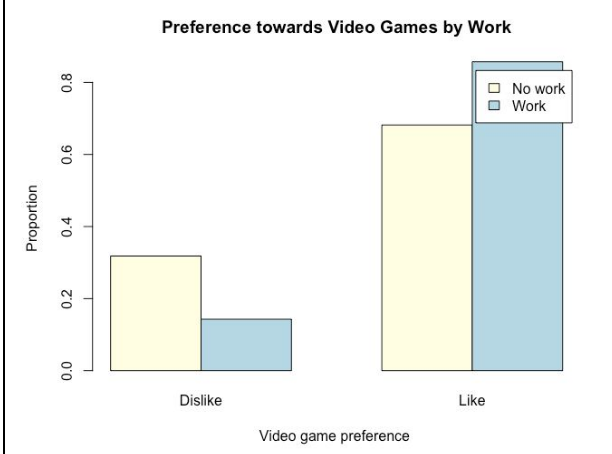

```
Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  2.642637     d.f. =  1     p =  0.1040307

Pearson's Chi-squared test with Yates' continuity correction
------------------------------------------------------------
Chi^2 =  1.873142     d.f. =  1     p =  0.1711157
```

We graphed the proportions of males and females who like/dislike video games, and we can see that most people seem to like video games. By looking at the graph we can see that in general, males like video games more than females, which thus implies (and as we can see from the bar graph) that females dislike them more. The p-value is greater than 0.05, thus implying that sex and preference for video games are independent.

Fig. 10: A cross table showing how the observations are distributed between the four categories (work, no work, like, dislike). A bar graph showing the proportion of working students who like/dislike and proportion of no working students who like/dislike video games

```
                | like.data$work
like.data$like  |   No work |      Work | Row Total |
----------------|-----------|-----------|-----------|
        Dislike |        14 |         6 |        20 |
                |     1.387 |     1.453 |           |
                |     0.700 |     0.300 |     0.233 |
                |     0.318 |     0.143 |           |
                |     0.163 |     0.070 |           |
----------------|-----------|-----------|-----------|
           Like |        30 |        36 |        66 |
                |     0.420 |     0.440 |           |
                |     0.455 |     0.545 |     0.767 |
                |     0.682 |     0.857 |           |
                |     0.349 |     0.419 |           |
----------------|-----------|-----------|-----------|
   Column Total |        44 |        42 |        86 |
                |     0.512 |     0.488 |           |
----------------|-----------|-----------|-----------|
```



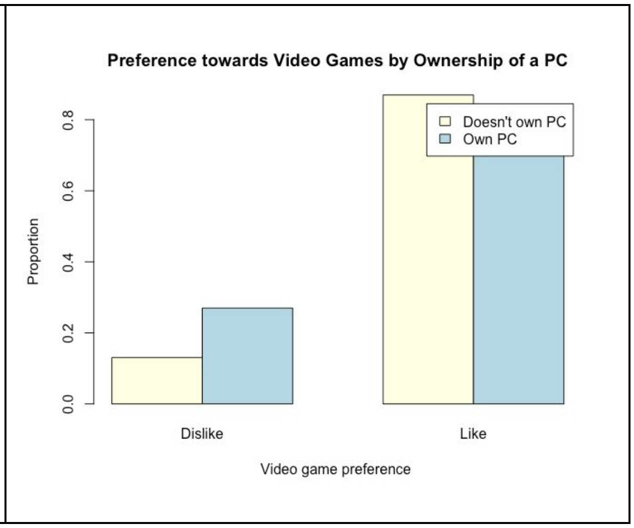

```
Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  3.700945     d.f. =  1     p =  0.05438168

Pearson's Chi-squared test with Yates' continuity correction
------------------------------------------------------------
Chi^2 =  2.783782     d.f. =  1     p =  0.09522308
```

We graphed the proportions of workers and nonworkers who like/dislike video games, and once again, we can see that most people like video games. By looking at the graph we can see that in general, those who work like video games more than those who don't. The p-value is greater than 0.05, thus implying that working for pay and preference for video games is independent.

Fig. 11: A cross table showing how the observations are distributed between the four categories (has PC, doesn't have PC, like, dislike). A bar graph showing the proportion of PC owners who like/dislike and proportion of non PC owners who like/dislike video games

like.data$like	like.data$own Does not own PC	Own PC	Row Total
Dislike	3	17	20
	1.031	0.377	
	0.150	0.850	0.233
	0.130	0.270	
	0.035	0.198	
Like	20	46	66
	0.313	0.114	
	0.303	0.697	0.767
	0.870	0.730	
	0.233	0.535	
Column Total	23	63	86
	0.267	0.733	




```
Pearson's Chi-squared test
------------------------------------------------------------
Chi^2 =  1.834674     d.f. =  1     p =  0.1755765

Pearson's Chi-squared test with Yates' continuity correction
------------------------------------------------------------
Chi^2 =  1.136712     d.f. =  1     p =  0.2863482
```

Once again, from the bar graph, we can see that most people like video games. By looking at the graph we can see that in general, those who don't own PCs like video games more than those who don't.

The p-value is greater than 0.05, thus implying that PC ownership and preference for video games are independent.

X. SCENARIO 6

For fun, we wish to investigate how the expected grade of the students match the target grade distribution of 20% A's, 30% B's, 40% C's, and 10% D's or lower.

Fig 12: A barplot comparing the targeted and expected grade distributions

We see that from Fig 12, the expected grade distribution is skewed right with the density of the population of students expecting A's and B's is much higher than that of the target distribution. From this, we can see that the disparity between the expected and targeted grade distribution is quite apparent. More students are expecting themselves to pass with A's and B's than the targeted grade distribution denotes.

If the population of students who did not respond to the survey were to have failed the class, this would still not fit the target grade distribution. Those students would fill the 10% requisite failing students, and adding more students into the total population will help bring down the proportion of students of students expecting A's and B's. It is worth noting, however, that the population of students expecting to receive C's is still much lower than the target proportion of students to receive C's. With that, the addition of these failing students will not help the expected grade distribution match the targeted grade distribution.

XI. ADDITIONAL INVESTIGATION

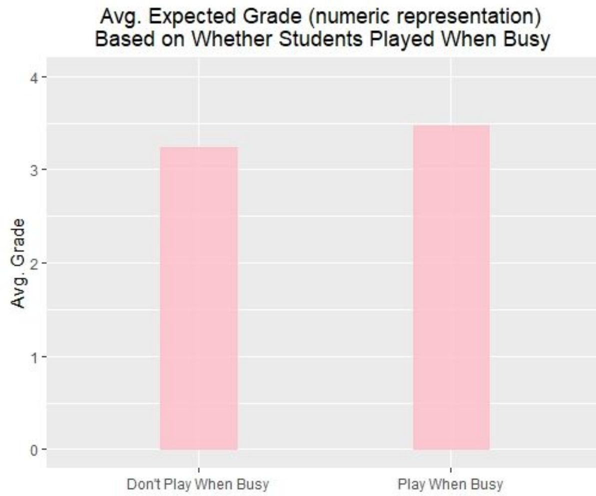
We are trying to see if students who play even when they are busy score lower than students who do not play when they are busy. To do this, we can look at the expected grades of the students who played during the week when they had an exam.

We can compute the average of the expected grades for the two categories of students and see which one is higher. There is however a potential issue with using this random variable; by using it, we're making an assumption that the grades that the students received afterwards were indeed close to the expected grades. We're assuming that the estimates for the expected grades were made in a reasonable and logical manner, without too much inflation or deflation. One reason to believe that that assumption is probably true is that these estimates were made in the formal context of a survey and not just in an informal setting.

The averages are as follows:

- 3.471, for students who play when they are busy
- 3.238, for students who do not play when they are busy

Fig. 13: A bar graph depicting the average grade for students who played video games when busy and students who did not play video games when they were busy. The rankings are as follows: (A = 4, B = 3, C = 2, D = 1, F = 0)



The difference between the two is 0.232493, which isn't that much. More importantly, the expected grade for students who play when they're busy is actually higher than students who do not play when they're busy, which is a surprising result.

Of course, another way to interpret this result could be that students who play when they're busy *think* that they'll do well on exams, but this interpretation might perhaps be a little biased.

XII. DISCUSSION AND CONCLUSION

We go back through each scenario in order to discuss our investigations.

From our investigations conducted in Scenario 1, we see that, on average, around 36.5% of the sampled population play video games. So if the designers were to move forward with installing video game system into the new computer labs, they should expect roughly 36.5% of the student population to participate in the activities.

As we move into Scenario 2, we see that the busyness of the students has influence on the amount of time spent playing these video games. The students who spend the most time playing video games are those who play when they are busy, but this population is quite small being a group of 5 students. It is worth noting that students who refrain from playing video games when they are busy are the largest population in this study. With that being said, designers of the new labs should avoid holding labs with video games when there are upcoming exams.

From Scenario 3, we found that students are likely to play video games for 1.07995 hours to 1.20656 hours, with a 95% confidence interval. With this in mind, lab designers should consider having a time limit of an hour to an hour and a half allotted per student or party to play these video games.

As discussed in Scenario 4, we found that students generally like to play video games to relax and those who do not like to play video games only generally attribute this to monetary or time inconveniences, not necessarily the characteristics of the video games. So lab designers should expect students to participate in these video games during more stressful weeks. In order to combat students overindulging in these video games, since the general majority seems to favor playing video games, designers should consider placing some sort of time limit or game limit per student or party, preferably according to the limit stated in Scenario 3.

From Scenario 5, we have determined that males, those who work for pay, and non PC-owners like video games more than females, non-workers, and PC owners. Additionally, from the chi-squared test, we believe that sex, working for pay, and PC ownership are not signficantly correlated to liking video games.

In our extra and fun Scenario 6, we found that the expected grade distribution of the class does not match the targeted grade distribution, as the proportion of students expecting B's is immensely greater than the described proportion of the targeted B- range distribution, and the proportion of students expecting C's is extremely sparse compared to the targeted C-range distribution. Adding the students who did not respond to the survey to the failing distribution will not help the expected distribution match the target distribution since it does not help the C-range meet its quota.

Due to the nature of how the data was collected (survey), there are limiting factors that could have skewed the results of this study. The nature of a survey presents challenges such as respondents skipping questions, lack of understanding/interpretation, dishonesty, lack of conscientious responses, etc. This can skew the data and thus get a reading that is off from the actual and accurate result. This notion has been takin into account while conducting this study.

In conclusion, we see that the information gathered through each scenario is more than sufficient in helping to decide whether or not lab designers would be able to extract information from the surveys. Each point of the above discussions gives easy guidelines extracted from the data to answer questions regarding incorporating video games into lab curriculum. As for our additional investigation, we find that the average expected grades between students who play when they are busy and those who do not are relatively similar, so we are not able to distinguish any significant relationship between expected grade and whether or not the student plays video games despite being busy.

XIV. METHODS & THEORIES

1. Sample statistics

Dependent sample

→ The case that is related to the elements which consists of two populations.

Independent sample

→ The case that is not related to the elements which consists of two populations.

When two specimens are related (Dependent sample), we can compare the result with sample under two conditions. We let data from first condition be X_i and data from second condition be Y_i, then consider the difference $D_i = X_i - Y_i$ and use t-test.

When two specimens are unrelated (Independent sample), we can calculate independently how the average value distributes is divided into two populations, so the discussion becomes easy. As with a single population, we can distribute with two parts to discuss,①variance is known and ②variance is unknown. Especially, when variance is unknown, we can take five steps below;

a. We calculate the unbiased estimators of the variances of the two populations.
b. And we find the estimate of the variance of the two populations.
c. We estimate the variance of the distribution followed by the difference in mean.
d. And determine statistics according to t distribution.
e. Find the rejection area from the significance level and make a conclusion.

2.Unbiased estimator

Unbiasedness

A point estimator $\widehat{\theta} = \widehat{\theta}_n$ is said to be an unbiased estimator of θ if $E(\widehat{\theta}) = \theta$ for every θ. If $\widehat{\theta}$ is not unbiased, the difference $E(\widehat{\theta}) - \theta$ is called the bias of $\widehat{\theta}$.

Let $X_1, X_2, ..., X_n$ be an iid sample with mean μ and variance σ^2.
First, calculate

$$E\overline{X_n} = ES_n/n = (EX_1, X_2, ..., EX_n)/n = \mu.$$

So the sample mean (aka empirical mean, sample average) is an unbiased estimator of μ.
Next, recall that the MLE of σ^2 (for normal population) is

$$\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X_n})^2.$$

Note that

$$\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i^2 - 2\overline{X_n}X_i + \overline{X_n^2})$$
$$= \frac{1}{n}(\sum_{i=1}^{n} X_i^2 - \frac{2\overline{X_n}}{n}\sum_{i=1}^{n} X_i + \frac{1}{n}\sum_{i=1}^{n}\overline{X_n^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \overline{X_n^2}.$$

Taking the expectation, we have $E\sigma^2 = EX_1^2 - E\overline{X_n^2}$, where

$$EX_1^2 = var(X_1) + (EX_1)^2 = \sigma^2 + \mu^2,$$
$$E\overline{X_n^2} = var(\overline{X_n}) + (E\overline{X_n})^2 = \sigma^2/n + \mu^2.$$

Therefore, $E\widehat{\sigma^2} = \frac{n-1}{n}\sigma^2$.
In conclusion, $\widehat{\sigma^2}$ is a biased estimator of σ^2 with $bias = -\sigma^2/n$.
Define the bias- corrected sample variance estimator

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X_n})^2,$$

and S will be referred to as the sample standard deviation.

Remarks:
① Note that $S^2 = \frac{n}{n-1}\widehat{\sigma^2}$, so $ES^2 = \frac{n}{n-1}E\widehat{\sigma^2} = \sigma^2$, i.e. S^2 is an unbiased estimator of σ^2.
② S itself, as an estimator of σ, is not unbiased because $ES \le (ES^2)^{1/2} = \sigma$ where "=" holds if and only if S is a constant almost surely.

Example:
Suppose that $X_1, ..., X_n$ are iid from $Unif(0, \theta)$. We use that MLE^{*1} of θ is $\widehat{\theta} = max\{X_1, ..., X_n\}$. Then, we obtain that $E\widehat{\theta} = \frac{n}{n+1}\theta$. Hence, $\widehat{\theta}$ is biased with $bias = -\theta/(n+1)$.
The density of $\widehat{\theta}$ is

$$f_{\hat{\theta}}(x) = \begin{cases} \frac{n}{\theta^n}x^{n-1} & \text{if } 0 \le x \le \theta, \\ 0 & \text{otherwise}. \end{cases}$$

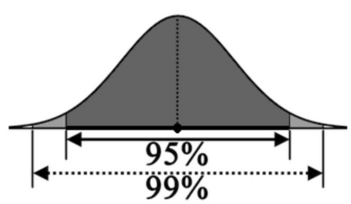
Calculate $E\widehat{\theta} = \theta^{-n}\int_0^{\theta} nx^n dx = n\theta/(n+1)$.
On the other hand, the MOM^{*2} estimator is unbiased.
MLE^{*1} … Maximum Likelihood estimator
MOM^{*2} … Method of Moment

Mean Squared Error(MSE)
The mean squared error of an estimator $\widehat{\theta}$ is defined as $MSE(\widehat{\theta}) = E(\widehat{\theta} - \theta)^2$.
If $\widehat{\theta}$ is unbiased, then $MSE(\widehat{\theta}) = var(\widehat{\theta})$.
To see this,

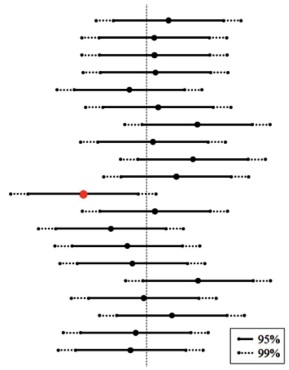
$$\begin{aligned} MSE(\widehat{\theta}) &= E(\widehat{\theta} - E\widehat{\theta} + E\widehat{\theta} - \theta)^2 \\ &= E(\widehat{\theta} - E\widehat{\theta})^2 + E\{(\widehat{\theta} - E\widehat{\theta})(E\widehat{\theta} - \theta)\} + E(E\widehat{\theta} - \theta)^2 \\ &= E(\widehat{\theta} - E\widehat{\theta})^2 + (E\widehat{\theta} - \theta)E(\widehat{\theta} - E\widehat{\theta}) + E(E\widehat{\theta} - \theta)^2 \\ &= E(\widehat{\theta} - E\widehat{\theta})^2 + E(E\widehat{\theta} - \theta)^2 \\ &= var(\widehat{\theta}) + \{bias(\widehat{\theta})\}^2 . \end{aligned}$$

Note:
When choosing among several different estimators, first we would like to select one that is unbiased. However, in the previous case, even though the MLE $\widehat{\theta}$ is biased, the bias is very small and is negligible in the long run, i.e. when n is large.

3.Confidence Intervals/ Normal approximation / Estimators for Standard Error
Confidence Intervals
When estimating an interval using a sample extracted from a population, a criterion for determining the width of that interval is necessary. The criterion for determining the width of the confidence interval is what is called "confidence level". Confidence level, also called confidence factor.There are the following two standards that are routinely used as a reliability conventionally, ***Reliability***: 95%, 99%.



If there is a criterion of this reliability, for example, if it is a normal distribution as shown in the right figure, the width of the interval centered on the average is determined.
If the reliability is 95%, the significance level will be 5%. The significance level is what is also called the dangerous rate, which is what proportionally gives wrong answers. For example, the significance level of 5% is 5% = 1/20, so if you do the same thing twenty times it shows the extent that you get the wrong answer about once. If the significance level is 1%, 1% = 1/100, so if you do it 100 times, it will be about 1 time to make a mistake.



According to the above figure, the vertical dotted line represents the true average of the population, and the result of interval estimation in one sample extraction with one horizontal bar is shown. The central point of each 20 horizontal bars is the sample mean, the solid line the confidence interval of 95% confidence, the dotted line the confidence interval of 99% confidence. The higher the reliability, the wider the confidence interval. In this figure, you can see that there is a time (true red in the figure) where the true value is not included in the 95% confidence interval only once when the same sample extraction is done 20 times.

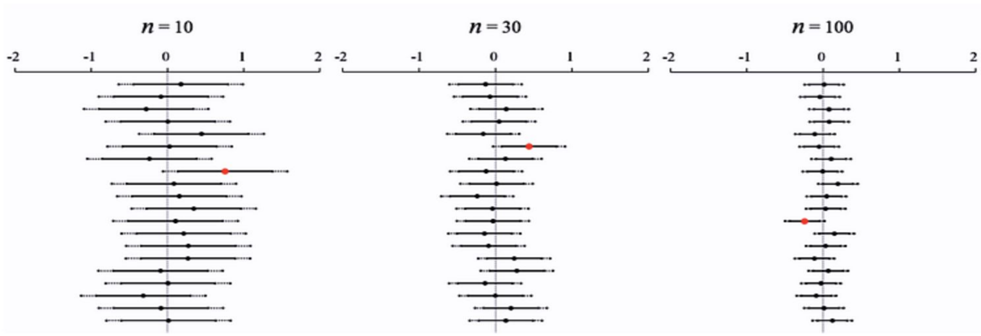
As described above, the confidence interval means that when the interval width is estimated on the basis of the reliability, if sampling is repeated several times under the same condition, the number of times the true value is included in the interval is this degree It shows the width to be done.

For example, ten specimens are extracted from the population according to the standard normal distribution N(0,1) to estimate the sample mean and confidence interval, and when the true value enters or exits the confidence interval I will see if there is a degree.

When obtaining n samples from a population according to the normal distribution $N(\mu, \sigma^2)$, it is known that the distribution of the sample average follows the normal distribution $N(\mu, \sigma^2 / n)$. In other words, the width of the confidence interval of the sample average can be obtained from the standard deviation which is the positive square root of this variance σ^2 / n, and the confidence interval is multiplied by this multiple of the standard deviation depending on the reliability You will be asked. The standard deviation of the distribution of the sample mean which is the basis of the width of this confidence interval is called "standard error (SE)" in particular.

Therefore, in the simulation, n = 10 random numbers Xi according to the standard normal distribution N(0, 1) are generated, from which the average estimated value and the confidence interval calculated by the following expression are displayed, and in the average of the population You will see the relationship with some $\mu = 0$.
· Estimated average: Σ Xi / 10.
 Standard error: The square root of the variance ($\sigma^2 / n = 1/10$).
· 95% confidence interval: average ± 1.96 × standard error.
· 99% confidence interval: average ± 2.58 × standard error.

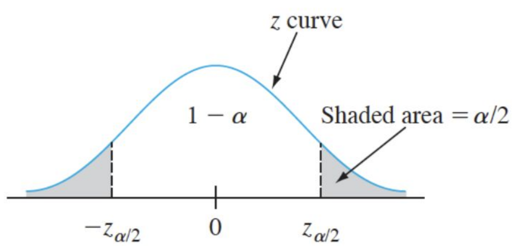
The figure below shows the simulation result with the same reliability and sample size changed to n = 10, 30, 100. Although the extent to which the confidence interval deviates from the true value does not change, the range of variation becomes narrow near the true value, and the larger the sample size is, the better the estimation accuracy is improved. In this way, by visually seeing the simulation results, we can better understand the meaning of the confidence intervals.



Normal approximation

Suppose $X_1, X_2, ..., X_n$ are iid from $N(\mu, \sigma^2)$, where σ is known. We can find $1-\alpha$ confidence interval for μ. First, we estimate μ by $\overline{X_n} = (X_1 + ... + X_n)/n$. By normality and independence, $\overline{X_n} \sim N(\mu, \sigma^2)$, or equivalently, $\frac{\overline{X_n}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$.

Next, we use the definition of $Z_{\alpha/2}$, the upper α/2 quantile of $N(0, 1)$.



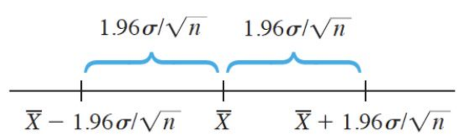
Then,

$$1-\alpha = P(-Z_{\alpha/2} \le \frac{\overline{X_n}-\mu}{\sigma/\sqrt{n}} \le Z_{\alpha/2}) = P(-Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \overline{X_n} - \mu \le Z_{\alpha/2}\frac{\sigma}{\sqrt{n}})$$

$$= P(\overline{X_n} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X_n} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}})$$

Then, a $1-\alpha$ CI for μ is given by $(\overline{X_n} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \overline{X_n} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}})$.

In particular, $z_{0.05} = 1.645$, $z_{0.025} = 1.96$, $z_{0.005} = 2.58$.



A 95% CI centered at $\bar{X}_n$

Estimators for Standard Error

Standard error (SE) is the standard deviation divided by $\sqrt{N}$.

$$Standard\ error\ (SE) = \frac{Standard\ Deviation}{\sqrt{N}}$$

Standard error is always smaller than standard deviation. Because of this nature, it seems that there is less variation in data when you create a graph using standard error. Indeed, it seems that standard errors are used in

many papers. The reason for this is that we can estimate the interval of the mean value of the population by standard error.
Therefore, according to the section for Normal approximation, we could find the $1-\alpha$ CI for μ, which is
$\overline{X_n} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X_n} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ ... ❋
Above ❋ implies population mean μ is that the probability between $\overline{X_n} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ and $\overline{X_n} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is 95%. In other words, population mean μ lies on between $\pm 2SE$ with 95% possibility. Hence, an interval estimation of population mean μ can be performed by standard error.
The standard deviation was an indicator of the variation of data. The probability that data will fall within the range of $\overline{X}$ ± 2SD is 95%.
So, if you want to show variations in data or compare, you should use standard deviation. For example, you want to show things like data becoming uneven depending on a factor.
On the other hand, the standard error was the interval estimator of population mean μ. The probability that the population mean μ is within the range of $\overline{X}$ ± 2SE is 95%. Therefore, when you want to estimate population mean μ or want to compare, you can show standard error. In many cases, it is the population mean μ that you want to know through experiments, so many papers show standard errors.

4.Bootstrap
The bootstrap is a method to estimate the property of population inference by computing the properties when sampling by approximate distribution. As an approximate distribution, it may be used for empirical distribution or hypothesis test obtained from measured values. In cases where the hypothesized distribution is suspicious, or where a parametric hypothesis is not impossible but requires a very complicated calculation, it is used instead of an estimate based on a parametric hypothesis.

The advantage of bootstrap method is very simple compared to analytic method. To obtain standard error and confidence intervals for complicated estimation functions of complex parameters of population distribution (percentile point, distribution, odds ratio, correlation coefficient, etc...), it is only necessary to apply a single bootstrap sample .

On the other hand, as a disadvantage of the bootstrap method, when asymptotically matching, finite samples are not guaranteed and tend to be optimistic.

Modern computers not only produce fast solutions to old problems, but also provide new avenues of approach. One such set of new methods goes under the general name of resampling. One important piece of resampling is known as ***bootstrapping***. This technique is useful when classical inference is impossible. Bootstrapping is useful in many different ways. Here we discuss one particular example of its application to estimating the ***standard error*** (SE).

The SE of an estimator $\widehat{\theta}_n$ is its standard deviation, i.e. $Var(\widehat{\theta}_n)^{1/2}$.
The SE, or an approximation of it, is an essential part of constructing confidence intervals.
In simple parametric models, we can derive an expression for the SE. In the normal case $N(\mu, \sigma^2)$, the SE of $\overline{X_n}$ is $\sigma/\sqrt{n}$; in the Bernoulli case $Ber(p)$, the SE of $\widehat{p} = \overline{X_n}$ is $\sqrt{p(1-p)/n}$.

In general, estimating the SE may not be so straightforward. It can be difficult to compute the Fisher information analytically. In this case, the bootstrap provides a feasible way to find an approximation of the SE, which can be used to construct confidence intervals.

To demonstrate the idea, let us start with an easy case.

Let $X_1, X_2, ..., X_n$ be an iid sample from X with PDF $f(x;\theta) = \frac{xe^{-x/\theta}}{\theta^2} I_{[0,\infty)}(x)$.

The MLE of θ is $\frac{1}{2}\overline{X_n}$. Its variance is $var(\overline{X_n}/2) = var(\overline{X_n})/4 = var(X)/(4n) = \theta^2/(2n)$. So, we know that the SE is $\theta/\sqrt{2n}$.

To understand the use of bootstrapping for estimating the SE, let us go through the following empirical demonstration.

Step 1. Generate a sample of size 20 from Gamma (true θ is 10):

27.1812 26.5997 17.8107 5.4840 16.6618 28.4702 3.2963
45.9762 13.5940 60.5345 19.4844 6.3704 26.5075 16.0415
16.2537 15.9309 32.7601 12.4935 12.8734 12.4788

Step 2. Using the MLE, we get an estimate $\widehat{\theta_{mle}}$ =10.4201 .

Step 3. Pretend that we don't know the truth, which is $\theta = 10$.

Given the estimate $\widehat{\theta_{mle}}$ =10.4201 from the data, use computer to generate 500 random samples (each has size 20) from $Gamma(2, \widehat{\theta_{mle}})$, denoted by

$$\begin{array}{c} X_1^{(1)}, \ldots, X_{20}^{(1)} \\ X_1^{(2)}, \ldots, X_{20}^{(2)} \\ \vdots \ , \ldots, \ \vdots \\ X_1^{(500)}, \ldots, X_{20}^{(500)} \end{array}$$

Step 4. From each random sample that we just generated, we can compute a maximum likelihood estimate

$$\widehat{\theta_{mle}^{(b)}} = \frac{1}{2*20}\sum_{i=1}^{20} X_i^{(b)}, for\ b = 1, ..., 500$$

Step 5. Now we have created 500 maximum likelihood estimates. Calculate the sample standard deviation of these 500 numbers, which gives 1.6292. This is the bootstrap estimate of the standard error of the MLE $\frac{1}{2}\overline{X_n}$.

Now, recall that the actual standard error is

$$\frac{\theta}{\sqrt{2}} = \frac{10}{\sqrt{2*20}} = 1.5811$$

The bootstrap estimate 1.6292 is fairly close to the actual value 1.5811.

5. Kolmogorov-Smirnov

The Kolmogorov-Smirnov test is a type of hypothesis tests in statistics, and based on a finite number of samples, whether the probability distributions of the two populations are different or whether the probability distribution of the population is presented as a null hypothesis It is used to investigate whether it is different from the distributed distribution. Often abbreviated as KS test.

One sample Kolmogorov-Smirnov tests compares the empirical distribution with the cumulative distribution function shown in the null hypothesis. The main application is fitness test for normal distribution and uniform distribution. As for the test on the normal distribution, a slight improvement by the lift force is known. In the case of normal distribution, the Shapiro - Wilk test and Anderson - Darling test are generally more powerful methods than the Lily force test.

Two sample Kolmogorov-Smirnov tests is one of the most effective and general nonparametric methods to compare two specimens. This is because this approach depends on both the position and the shape of the empirical distribution for the two specimens.

Empirical distributions Fn for samples $y_1, y_2, ..., y_n$ are given as follows.

$$F_n(x) = \frac{\#\{\, 1 \le i \le n \mid y_i \le x \,\}}{n}$$

Let F(x) be the distribution presented in the null hypothesis or the other empirical distribution, then the two one-sided KS test statistics are given by

$$D_n^+ = \sup_x (F_n(x) - F(x))$$
$$D_n^- = \sup_x (F(x) - F_n(x))$$

Assuming that the null hypothesis that the two distributions are equal is not rejected, the probability distribution that the above two statistics should obey is not dependent on the shape of the distribution as long as the distribution presented by the hypothesis is a continuous distribution.
In the one sample Kolmogorov-Smirnov test, when the number of samples n is sufficiently large, the distribution of the examination amount under the assumption that the empirical distribution Fn (x) follows the null hypothesis is

$$\mathrm{Prob}(\sqrt{n} D_n \le x) = 1 - 2\sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2x^2} = \frac{\sqrt{2\pi}}{x} \sum_{i=1}^{\infty} e^{-(2i-1)^2\pi^2/(8x^2)}$$

Therefore, when the significance level is α, the verification amount Dn is $n\sqrt{D_n} > K_\alpha$ (where K_α is the number satisfied with $Prob(\ n\sqrt{D_n} > K_\alpha) = 1 - \alpha$) is satisfied, the null hypothesis is rejected and it is suggested that the empirical distribution Fn (x) is different from the distribution F (x) presented in the null hypothesis.

XV. ACKNOWLEDGEMENTS

Thomas W. Malone, “*What Makes Things Fun to Learn? A Study of Intrinsically Motivating Computer Games*” *Cognitive And Instructional Sciences Series*, August 1980,
https://hcs64.com/files/tm%20study%20144.pdf

Festini SB, McDonough IM and Park DC(2016), *“The Busier the Better: Greater Busyness is Associated with Better Cognition”* Front. Aging Neurosci. 8:98. Doi: 10.3389/fnagi.2016.00098
https://www.frontiersin.org/articles/10.3389/fnagi.2016.00098/full

“Bias of an estimator.” *Wikipedia*, Wikimedia Foundation, 19 Jan. 2019, en.wikipedia.org/wiki/Bias_of_an_estimator

“Unbiased estimation of standard deviation.” *Wikipedia*, Wikimedia Foundation, 17 Feb. 2019, en.wikipedia.org/wiki/Unbiased_estimation_of standard_deviation

“Mean squared error.” *Wikipedia*, Wikimedia Foundation, 28 Jan. 2019, en.wikipedia.org/wiki/Mean_squared_error

“Normal approximation.” *Wikipedia*, Wikimedia Foundation, 24 Nov. 2018, en.wikipedia.org/wiki/Normal_Approximation

“Confidence intervals.” *Wikipedia*, Wikimedia Foundation, 15 Feb. 2019, en.wikipedia.org/wiki/Confidence_intervals

“Bootstrapping (statistics).” *Wikipedia*, Wikimedia Foundation, 8 Feb. 2019, en.wikipedia.org/wiki/Bootstrapping_(statistics)

“Standard error.” *Wikipedia*, Wikimedia Foundation, 17 Jan. 2019, en.wikipedia.org/wiki/Standard_error

“Kolmogorov–Smirnov test.” *Wikipedia*, Wikimedia Foundation, 7 Feb. 2019, en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_testr