Fundamentals of Logic.

Introduction:-

logic:- It is a method of Reasoning, logic is expressed in a Symbolic language, is called mathematical logic.

Propositions

A proposition is a statement (declaration) which in a given context, can be said to be either true or false but not both. All sentences are not a proposition.

Ex:- 1. Bangalore is in Karnataka. True

2. 7 is divisible by 3 false

3. $xy = yx$ (not a proposition)

* Propositions are represented by small letters p, q, r, s....
* The truth value of proposition is the truth or falsity of that proposition.
* If the truth Value is true then denote as 1.

If the truth Value is false then denote as 0.

Logical Connectives

The words like not, and, if then, if and only if such words are called as logical Connectives

Compound Proposition:-

The new proposition that ~~obtained~~ obtained by Combaining the given two or more propositions.

Simple Proposition :-

Propositions which doesnot contains any logical Connectives.

Negation:- The proposition obtained by adding 'not' at an appropriate place in a given proposition denoted by '~'.

Eg:- p : 3 is a prime number. (1)

~p : 3 is not a prime number (0)

Truth Table

p	~p
0	1
1	0

Conjunction [AND] '∧' : A compound proposition obtained by inserting the word 'and'.

The Conjunction of p and q is denoted by $p \wedge q$

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Disjunction '$\vee$' OR The Compound proposition is obtained by inserting the 'OR' b/w two propositions

$\Rightarrow p \vee q$

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Conditional '$\rightarrow$' (If then) denoted as $p \rightarrow q$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Biconditional ($\leftrightarrow$) Iff A compound proposition is obtained by inserting if and only if. [$p \leftrightarrow q$]

$$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Exclusive Disjunction : ($\veebar$) : The Exclusive disjunction of two propositions P and q is denoted by $p \veebar q$ read as either P (or) q.

P	q	$P \veebar q$
0	0	0
0	1	1
1	0	1
1	1	0

1. Let P: A circle is a Conic
   q: $\sqrt{5}$ is a real number
   r: Exponential Series is Convergent

Express the following Compound propositions in words
(i) $P \wedge (\sim q)$ (ii) $(\sim P) \vee q$ (iii) $P \veebar (\sim q)$ (iv) $q \rightarrow (\sim P)$
(v) $P \rightarrow (q \veebar r)$ (vi) $\sim P \leftrightarrow q$

Solⁿ:

(i) $P \wedge (\sim q)$

A circle is Conic and $\sqrt{5}$ is not a real number.

(ii) $(\sim P) \vee q$

A circle is not a conic or $\sqrt{5}$ is a real number

(iii) $P \veebar (\sim q)$

Either a circle is conic or $\sqrt{5}$ is not a real number.

(iv) $q \rightarrow (\sim p)$

If $\sqrt{5}$ is a real number, then a circle is not a Conic

(v) $p \rightarrow (q \vee r)$

If A circle is a Conic, than either $\sqrt{5}$ is a real number or Experential Series is Convergent.

(vi) $\sim p \leftrightarrow \sim q$

If a circle is not a Conic then $\sqrt{5}$ is a real number and if $\sqrt{5}$ is a real number than a circle is not a Conic.

2) Construct the truth tables for the following

(i) $p \wedge (\sim q)$ (ii) $(\sim p) \vee q$ (iii) $p \rightarrow (\sim q)$ (iv) $(\sim p) \veebar (\sim q)$

p	q	$\sim p$	$\sim q$	$p \wedge (\sim q)$	$(\sim p) \vee q$	$p \rightarrow (\sim q)$	$(\sim p) \veebar (\sim q)$
0	0	1	1	0	1	1	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	1	0	0

3) Construct the truth table for the following

(i) $(p \wedge q) \rightarrow \sim r$ (ii) $q \wedge (\sim r \rightarrow p)$

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \rightarrow \sim r$	$\sim r \rightarrow p$	$q \wedge (\sim r \rightarrow p)$
0	0	0	0	1	1	0	0
0	0	1	0	0	1	1	0
0	1	0	0	1	1	0	0
0	1	1	0	0	1	1	1
1	0	0	0	1	1	1	0
1	0	1	0	0	1	1	0
1	1	0	1	1	1	1	1
1	1	1	1	0	0	1	1

Tautology The Compound Proposition which is True for all possible truth values of its Components is called a tautology.

Contradiction [Absurdity]
The Compound Proposition which is always false.

Contingency: The Compound proposition which is neither a tautology nor a Contradition.

1. Prove that $\{P \to (q \to r)\} \to \{(P \to q) \to (P \to r)\}$

P	q	r	$P \to q$ A	$P \to r$ B	$q \to r$ B	$P \to (q \to r)$ C	$A \to B$ $(P \to q) \to (P \to r)$	$C \to (A \to B)$ $(P \to (q \to r)) \to ((P \to q) \to (P \to r))$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

∴ Given Proposition is Tautology

2) Prove that $\{(P \vee q) \wedge (P \to r) \wedge (q \to r)\} \to r$ is a tautology.

P	q	r	$P \vee q$ A	$P \to r$ B	$q \to r$ C	$A \wedge (B \wedge C)$ X	$X \to r$ $\{(P \vee q) \wedge (P \to r) \wedge (q \to r)\} \to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

∴ Given Compound Proposition is a tautology

3) P.T $[((P \vee q) \wedge \sim(\sim P \wedge (\sim q \vee \sim r))] \vee [(\sim P \wedge \sim q) \vee (\sim P \wedge \sim r)]$ is a tautology

4) Show that $P \wedge (\sim p \wedge q)$ is a Contradiction

p	q	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
0	0	1	0	0
0	1	1	1	0
1	0	0	0	0
1	1	0	0	0

5) P.T $(P \veebar q) \vee (P \leftrightarrow q)$ is a tautology

p	q	$P \veebar q$	$P \leftrightarrow q$	$P \rightarrow q$	$(P \veebar q) \vee (P \leftrightarrow q)$
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	0	0	1
1	1	0	1	1	1

6) P.T $(P \veebar q) \wedge (P \leftrightarrow q)$ is a Contradiction

7) P.T $(P \veebar q) \wedge (P \rightarrow q)$ is a Contingency

Logical Equivalence ($\Leftrightarrow$)

Two Compound Proposition x and y are Said to be logically Equivalance if their Corresponding truth Values are Same. we denote as $x \Leftrightarrow y$ if not $x \not\Leftrightarrow y$.

1. Show that the following Proposition is logically Equivalent. $(p \to q) \Leftrightarrow (\sim p \vee q)$

(Extra) not required

p	q	$\sim p$	$p \to q$ (x)	$\sim p \vee q$ (y)	$(p \to q) \leftrightarrow (\sim p \vee q)$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	0	1	1	1

$\therefore x \Leftrightarrow y$ also $x \leftrightarrow y$ is a tautology

2) $\{(p \vee q) \to r\} \Leftrightarrow \{\sim r \to \sim(p \vee q)\}$ and also verify Given Compound Proposition is tautology or not

Let $x = (p \vee q) \to r$ $\quad y = [\sim r \to \sim(p \vee q)]$

p	q	r	$p \vee q$	x $(p \vee q \rightarrow r)$	$\sim r$	$\sim (p \vee q)$	y $\sim r \rightarrow \sim (p \vee q)$	$x \leftrightarrow y$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	1	1
0	1	0	1	0	1	0	0	1
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	1	1

$x \Leftrightarrow y$ and also tautology.