# Project on WHO Dataset: Analysing factors affecting life expectancy

# **Motivation for the study:**

Motivation for the study is what factors can influence life expectancy. The dataset is taken from the World Health Organization (WHO) and United Nations website for the year 2012 and takes into account 183 countries. The variables included in the dataset include social and economic indicators as well as the effects of diseases and their respective rates of vaccinations. The results of the study would help us understand the major factors influencing life expectancy for different countries and can help determine specific policy actions.

# **Variable Description**

- 1. life\_expectancy: Life Expectancy in years
- 2. adult\_mortality: Adult Mortality Rates of both sexes (probability of dying between 15 and 60 years per 1000 population)
- 3. infant\_deaths: Number of Infant Deaths per 1000 population
- 4. alcohol: Alcohol, recorded per capita (15+) consumption (in liters of pure alcohol)
- 5. percentage expenditure: Expenditure on health as a percentage of Gross Domestic Product per capita(%)
- 6. hepatitis b: Hepatitis B (HepB) immunization coverage among 1-year-olds (%)
- 7. bmi: Average Body Mass Index of entire population
- 8. under\_five\_deaths: Number of under-five deaths per 1000 population
- 9. Total\_expenditure: General government expenditure on health as a percentage of total government expenditure (%)
- 10. Diphtheria: Diphtheria tetanus toxoid and pertussis (DTP3) immunization coverage among 1-year-olds (%)
- 11. HIV/AIDS: Deaths per 1000 live births HIV/AIDS (0-4 years)
- 12. GDP: Gross Domestic Product per capita (in USD)
- 13. Schooling: Number of years of Schooling(years)
- 14. Polio: Polio immunization coverage among 1-year-olds (%)

- 15. Africa
- 16. Oceania
- 17. Americas
- 18. Europe
- 19. Asia
- 20. Country
- 21. Continent

# **Question 1**

# Part (a): Boruta Algorithm

# In [119]: pip install BorutaShap

```
Looking in indexes: https://pypi.org/simple, (https://pypi.org/simple,) https://us-python.pkg.dev/colab-wheels/public/simple/ (https://us-python.pkg.dev/colab-wheels/public/simple/)
Requirement already satisfied: BorutaShap in /usr/local/lib/python3.7/dist-packages (1.0.16)
Requirement already satisfied: tqdm in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (4.64.1)
Requirement already satisfied: shap>=0.34.0 in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (0.41.0)
Requirement already satisfied: numpy in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (1.21.6)
Requirement already satisfied: pandas in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (1.3.5)
Requirement already satisfied: statsmodels in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (0.12.2)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (3.2.2)
Requirement already satisfied: seaborn in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (0.12.2)
```

Requirement already satisfied: scikit-learn in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (1.0.2)

Requirement already satisfied: scipy in /usr/local/lib/python3.7/dist-packages (from BorutaShap) (1.7.3)

Requirement already satisfied: cloudpickle in /usr/local/lib/python3.7/dist-packages (from shap>=0.34.0 ->BorutaShap) (1.5.0)

Requirement already satisfied: slicer==0.0.7 in /usr/local/lib/python3.7/dist-packages (from shap>=0.34 .0->BorutaShap) (0.0.7)

Requirement already satisfied: numba in /usr/local/lib/python3.7/dist-packages (from shap>=0.34.0->Boru taShap) (0.56.4)

Requirement already satisfied: packaging>20.9 in /usr/local/lib/python3.7/dist-packages (from shap>=0.3 4.0->BorutaShap) (21.3)

Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/python3.7/dist-packages (from packaging>20.9->shap>=0.34.0->BorutaShap) (3.0.9)

Requirement already satisfied: python-dateutil>=2.1 in /usr/local/lib/python3.7/dist-packages (from mat plotlib->BorutaShap) (2.8.2)

Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.7/dist-packages (from matplo tlib->BorutaShap) (1.4.4)

Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.7/dist-packages (from matplotlib->BorutaShap) (0.11.0)

Requirement already satisfied: typing—extensions in /usr/local/lib/python3.7/dist—packages (from kiwiso lver>=1.0.1—>matplotlib—>BorutaShap) (4.1.1)

Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-packages (from python-dateutil >=2.1->matplotlib->BorutaShap) (1.15.0)

Requirement already satisfied: setuptools in /usr/local/lib/python3.7/dist-packages (from numba->shap>= 0.34.0->BorutaShap) (57.4.0)

Requirement already satisfied: importlib-metadata in /usr/local/lib/python3.7/dist-packages (from numba ->shap>=0.34.0->BorutaShap) (4.13.0)

Requirement already satisfied: llvmlite<0.40,>=0.39.0dev0 in /usr/local/lib/python3.7/dist-packages (fr om numba->shap>=0.34.0->BorutaShap) (0.39.1)

Requirement already satisfied: zipp>=0.5 in /usr/local/lib/python3.7/dist-packages (from importlib-meta data->numba->shap>=0.34.0->BorutaShap) (3.10.0)

Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.7/dist-packages (from pandas->Bor utaShap) (2022.6)

Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.7/dist-packages (from sci kit-learn->BorutaShan) (3.1.0)

Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.7/dist-packages (from scikit-lear

n->BorutaShap) (1.2.0)

Requirement already satisfied: patsy>=0.5 in /usr/local/lib/python3.7/dist-packages (from statsmodels-> BorutaShap) (0.5.3)

# In [120]: pip install RegscorePy

Looking in indexes: https://pypi.org/simple, (https://pypi.org/simple,) https://us-python.pkg.dev/colab-wheels/public/simple/ (https://us-python.pkg.dev/colab-wheels/public/simple/)

Requirement already satisfied: RegscorePy in /usr/local/lib/python3.7/dist-packages (1.1)

Requirement already satisfied: numpy in /usr/local/lib/python3.7/dist-packages (from RegscorePy) (1.21. 6)

Requirement already satisfied: pandas in /usr/local/lib/python3.7/dist-packages (from RegscorePy) (1.3. 5)

Requirement already satisfied: python-dateutil>=2.7.3 in /usr/local/lib/python3.7/dist-packages (from p andas->RegscorePv) (2.8.2)

Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.7/dist-packages (from pandas->Reg scorePy) (2022.6)

Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-packages (from python-dateutil >=2.7.3->pandas->RegscorePy) (1.15.0)

## In [121]: pip install simple-colors

Looking in indexes: https://pypi.org/simple, (https://pypi.org/simple,) https://us-python.pkg.dev/colab-wheels/public/simple/ (https://us-python.pkg.dev/colab-wheels/public/simple/)

Requirement already satisfied: simple-colors in /usr/local/lib/python3.7/dist-packages (0.1.5)

```
In [222]: import pandas as pd
          import statsmodels.stats.api as sms
          from simple colors import *
          import matplotlib.pyplot as plt
          import numpy as np
          import statsmodels.formula.api as smf
          import statsmodels.api as sm
          import scipy
          import seaborn as sns
          from scipy.stats import bootstrap
          from BorutaShap import BorutaShap
          from RegscorePy import mallow
          import itertools
          from sklearn.linear model import LinearRegression
          from sklearn.model selection import cross val score
          import statsmodels
          from scipy import stats
          import matplotlib.pyplot as plt
```

## In [122]:

## **Importing the Dataset**

```
In [123]: # pd.read_csv('/content/sample_data/430_Data_Cleaned.xlsx') # starbucks data is still in here if you'd
```

```
In [124]: df = pd.read_excel("/content/sample_data/430_Data_Cleaned.xlsx", index_col = "Country")
# to add data to this colab, click on the folder icon on the left. Click on the '...' icon and then 'upl
df2 = pd.read_csv("/content/sample_data/countryContinent.csv", encoding = "ISO-8859-1")

df = pd.merge(df, df2[['country', 'continent']], left_on='Country', right_on='country')

df = df.join( pd.get_dummies(df['continent']) ) # add dummy vars for continent
print(df.columns)
df.head()

Index(['life_expectancy', 'adult_mortality', 'infant_deaths', 'alcohol',
```

0+	[12/1
uul	[124]

	life_expectancy	adult_mortality	infant_deaths	alcohol	percentage_expenditure	hepatitis_b	bmi	under_five_deaths	polio	total_expenditur
0	59.5	272	69	0.01	78.184215	67.0	17.6	93	67	8.5
1	76.9	86	0	5.14	412.443356	99.0	55.8	1	99	5.5
2	75.1	113	21	0.66	555.926083	95.0	56.1	24	95	6.1
3	56.0	358	72	8.24	256.122524	75.0	21.5	110	75	3.3
4	75.9	134	0	8.18	2156.229842	98.0	45.7	0	97	5.3

5 rows × 21 columns

```
In [125]: df.isnull().any()
Out[125]: life_expectancy
                                      False
          adult mortality
                                      False
          infant deaths
                                      False
          alcohol
                                       True
           percentage expenditure
                                      False
                                       True
           hepatitis b
                                       True
           bmi
          under_five_deaths
                                      False
           polio
                                      False
          total expenditure
                                       True
          diphtheria
                                      False
          hiv aids
                                      False
                                       True
          qdp
           schooling
                                       True
          country
                                      False
                                      False
           continent
          Africa
                                      False
           Americas
                                      False
                                      False
           Asia
                                      False
          Europe
          Oceania
                                      False
          dtype: bool
```

We see that there are NA values present in the dataset. Therefore, in order to impute them, we add the median values instead. The reason for not replacing them with the mean values is because there are extreme values present in the dataset. Due to there being extreme values, median seemed like the more appropriate tool to use in order to impute NA values.

```
In [126]: df = df.fillna(df.median())
df = df.replace(0,df.median())
```

df.isnull().sum()	
life_expectancy	0
adult_mortality	0
infant_deaths	0
alcohol	0
percentage_expenditure	0
· —	0
	0
	0
•	0
<del>-</del> •	0
-	0
_	0
= -	0
_	0
	0
	0
	0
	0
	0
•	0
	0
utype: Into4	
	life_expectancy adult_mortality infant_deaths alcohol

Now, we see that all the NA and zero values have been imputed.

```
In [128]: # regressions for Harvey Collier test for linearity
          ols fit1 = smf.ols(formula = 'life expectancy ~ adult mortality', data =df).fit()
          ols fit2 = smf.ols(formula = 'life expectancy ~ infant deaths', data =df).fit()
          ols fit3 = smf.ols(formula = 'life expectancy ~ alcohol', data =df).fit()
          ols fit4 = smf.ols(formula = 'life expectancy ~ percentage expenditure', data =df).fit()
          ols fit5 = smf.ols(formula = 'life expectancy ~ hepatitis b', data =df).fit()
          ols fit6 = smf.ols(formula = 'life expectancy ~ bmi', data =df).fit()
          ols fit7 = smf.ols(formula = 'life expectancy ~ under five deaths', data =df).fit()
          ols fit8 = smf.ols(formula = 'life expectancy ~ polio', data =df).fit()
          ols fit9 = smf.ols(formula = 'life expectancy ~ total expenditure', data =df).fit()
          ols fit10 = smf.ols(formula = 'life expectancy ~ diphtheria', data =df).fit()
          ols fit11 = smf.ols(formula = 'life expectancy ~ hiv aids', data =df).fit()
          ols fit12 = smf.ols(formula = 'life expectancy ~ qdp', data =df).fit()
          ols fit13 = smf.ols(formula = 'life expectancy ~ schooling', data =df).fit()
In [129]: # results of Harvey Collier test for linearity
          name = ["t-stat", "p-value"]
          test = sms.linear harvey collier(ols fit1)
          print(blue("Linearity Test Results for adult mortality:", ['bold']))
          print(list(zip(name, test)))
          print("\n")
          name = ["t-stat", "p-value"]
          test = sms.linear harvey collier(ols fit2)
          print(blue("Linearity Test Results for infant deaths:", ['bold']))
          print(list(zip(name. test)))
          print("\n")
          name = ["t-stat", "p-value"]
          test = sms.linear harvey collier(ols fit3)
          print(blue("Linearity Test Results for alcohol:", ['bold']))
          print(list(zip(name, test)))
```

name = ["t-ctat" "n-value"]

print("\n")

```
name - L c scae , p vacae j
test = sms.linear harvey collier(ols fit4)
print(blue("Linearity Test Results for percentage expenditure:", ['bold']))
print(list(zip(name. test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear harvey collier(ols fit5)
print(blue("Linearity Test Results for hepatitis b:", ['bold']))
print(list(zip(name, test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear harvey collier(ols fit6)
print(blue("Linearity Test Results for bmi:", ['bold']))
print(list(zip(name, test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear harvey collier(ols fit7)
print(blue("Linearity Test Results for under five deaths:", ['bold']))
print(list(zip(name, test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear_harvey_collier(ols_fit8)
print(blue("Linearity Test Results for polio:", ['bold']))
print(list(zip(name. test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear_harvey_collier(ols_fit9)
print(blue("Linearity Test Results for total_expenditure:", ['bold']))
print(list(zip(name, test)))
print("\n")
```

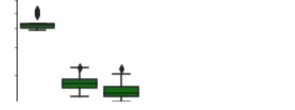
```
name = ["i-stat", "p-vatue"]
test = sms.linear harvey collier(ols fit10)
print(blue("Linearity Test Results for diphtheria:", ['bold']))
print(list(zip(name, test)))
print("\n")
# name = ["t-stat", "p-value"]
# test = sms.linear harvey collier(ols fit11)
# print(blue("Linearity Test Results for hiv/aids:", ['bold']))
# print(list(zip(name, test)))
# print("\n")
name = ["t-stat", "p-value"]
test = sms.linear harvey collier(ols fit12)
print(blue("Linearity Test Results for gdp:", ['bold']))
print(list(zip(name, test)))
print("\n")
name = ["t-stat", "p-value"]
test = sms.linear harvey collier(ols fit13)
print(blue("Linearity Test Results for schooling:", ['bold']))
print(list(zip(name, test)))
print("\n")
Linearity Test Results for adult mortality:
[('t-stat', -1.2623610028178232), ('p-value', 0.20853780627075733)]
Linearity Test Results for infant deaths:
[('t-stat', -0.7980353862773476), ('p-value', 0.4259568098087615)]
Linearity Test Results for alcohol:
[('t-stat', 0.09217335961688396), ('p-value', 0.9266681977785822)]
```

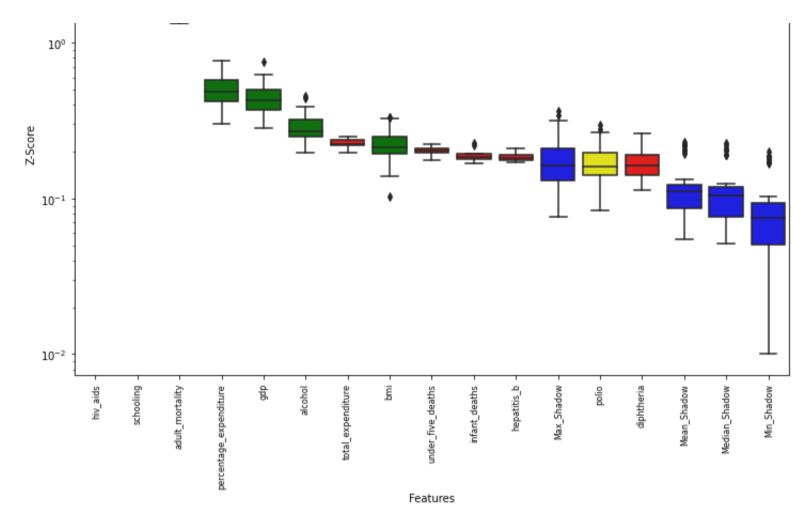
```
Linearity Test Results for percentage expenditure:
[('t-stat', -0.5610465243102468), ('p-value', 0.5755002730904755)]
Linearity Test Results for hepatitis b:
[('t-stat', -0.5601640915966886), ('p-value', 0.5761006250943638)]
Linearity Test Results for bmi:
[('t-stat', -0.5908060496540367), ('p-value', 0.5554304461731521)]
Linearity Test Results for under five deaths:
[('t-stat', -0.7399417466671597), ('p-value', 0.4603500032783766)]
Linearity Test Results for polio:
[('t-stat', -0.6216965456529443), ('p-value', 0.534969385826874)]
Linearity Test Results for total expenditure:
[('t-stat'. -0.9248083972233752). ('p-value'. 0.3563688148380044)]
Linearity Test Results for diphtheria:
[('t-stat', -0.8027315310717292), ('p-value', 0.42324420253692163)]
Linearity Test Results for adp:
[('t-stat', -0.6730158475131821), ('p-value', 0.5018457270461104)]
Linearity Test Results for schooling:
[('t-stat', 0.15713172741719925), ('p-value', 0.8753264093640525)]
```

Using the Harvey-Collier test, we check the linearity of the model. The p-values are all higher than the alpha (0.05) so there is support for the model being linear.

#### **Running the Boruta Algorithm**







According to Boruta's Algorithm, the top 10 predictors of life expectancy are, hiv\_aids, alcohol, percentage\_expenditure, adult\_mortality, total\_expenditure, schooling, bmi, under\_five\_deaths, diphtheria, polio

#### Mallow's CP

```
In [132]: subdat = df[['adult mortality', 'alcohol', 'bmi', 'total expenditure', 'percentage expenditure', 'hiv ai
In [133]: # mallow's cp narrowing to at least 5 predictors
          model = smf.ols(formula='life_expectancy ~ adult_mortality +alcohol + bmi +total_expenditure + percentage
          y = df['life expectancy']
          y_hat=model.fittedvalues
          storage cp = pd.DataFrame(columns = ["Variables", "CP"])
          k = 14 \# number of parameters in orginal model (includes y-intercept)
          for L in range(5, len(subdat.columns[1:]) + 1):
              for subset in itertools.combinations(subdat.columns[1:], L);
                  # join the strings in the data together
                  formula1 = 'v~'+'+'.join(subset)
                  # get the cp
                  results = smf.ols(formula=formula1, data = df).fit()
                  y sub = results.fittedvalues
                  p = len(subset)+1 # number of parameters in the subset model (includes y-intercept)
                  cp = mallow.mallow(y, y hat, y sub, k, p)
                  # add to the dataframe
                  storage cp = storage cp.append({'Variables': subset, 'CP': cp}, ignore index = True)
```

In [134]: # mallow's cp narrowing to at least 5 predictors storage\_cp.sort\_values(by = "CP")

## Out[134]:

	Variables	СР
2637	(alcohol, bmi, percentage_expenditure, hiv_aid	30.402871
2928	(bmi, percentage_expenditure, hiv_aids, diphth	30.618291
2927	(bmi, percentage_expenditure, hiv_aids, diphth	30.715454
3092	(alcohol, bmi, percentage_expenditure, hiv_aid	31.336299
3093	(alcohol, bmi, percentage_expenditure, hiv_aid	31.586341
574	(total_expenditure, percentage_expenditure, di	464.315028
144	(alcohol, total_expenditure, percentage_expend	473.777527
411	(bmi, total_expenditure, Americas, Asia, Oceania)	515.798774
593	(total_expenditure, percentage_expenditure, Am	533.903383
648	(total_expenditure, diphtheria, Americas, Asia	572.241943

3302 rows × 2 columns

According to Mallow's CP, the best performing model includes 8 of the 10 predictors suggested by Boruta's Algorithm: alcohol,bmi,percentage\_expenditure,hiv\_aids,diphtheria,schooling,Africa,Oceania

```
In [135]: pref_model = 'life_expectancy ~ alcohol + bmi + percentage_expenditure + hiv_aids + diphtheria + sch
alt_pref_model = 'life_expectancy ~ alcohol + bmi + total_expenditure + percentage_expenditure + hiv_aid
```

#### THE SUBSET OF VARIABLES FROM MALLOW'S CP

1.life expectancy ( Y-Variable)

2.BIM

3.HIV/AIDS

4.Percentage expenditure

5.Diptheria

6.Schooling

7.Alcohol

8.Africa

9.0ceania

# **Question 2**

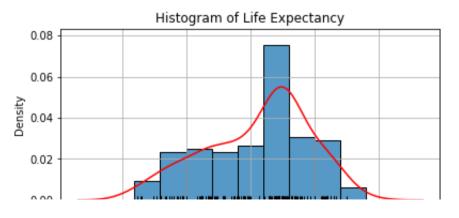
# **Univariate analysis**

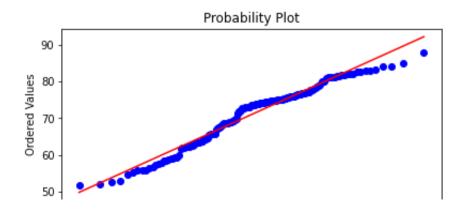
# Part(a) and (b)

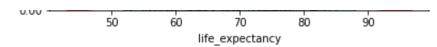
In [136]: import scipy.stats as stats

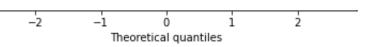
In [224]:

```
plt.figure(figsize = (14,10))
plt.subplot(3,2,1)
plt.title("Histogram of Life Expectancy")
sns.histplot(df.life_expectancy, stat = "density")
sns.kdeplot(df.life_expectancy, color = "red")
sns.rugplot(df.life_expectancy, color = "black")
plt.grid()
plt.subplot(3,2,2)
stats.probplot(df.life_expectancy, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.subplot(3,2,3)
sns.boxplot(data = df['life_expectancy'])
```

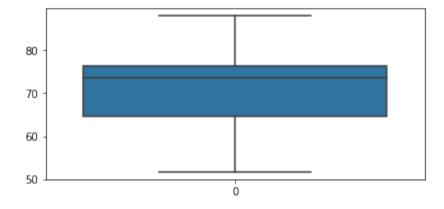






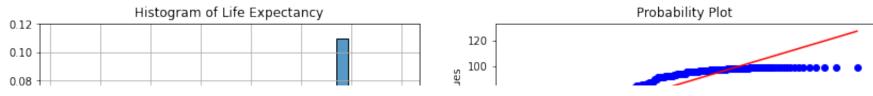


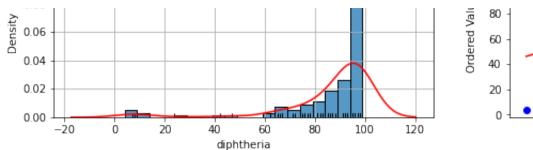
Out[224]: <matplotlib.axes.\_subplots.AxesSubplot at 0x7f391799ff50>

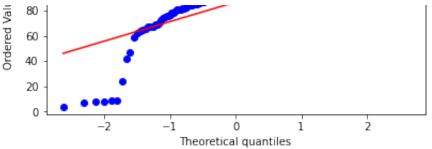


# In [212]:

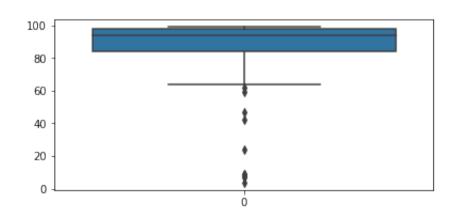
```
plt.figure(figsize = (14,10))
plt.subplot(3,2,1)
plt.title("Histogram of Life Expectancy")
sns.histplot(df.diphtheria, stat = "density")
sns.kdeplot(df.diphtheria, color = "red")
sns.rugplot(df.diphtheria, color = "black")
plt.grid()
plt.subplot(3,2,2)
stats.probplot(df.diphtheria, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.subplot(3,2,3)
sns.boxplot(data = df['diphtheria'])
plt.subplot(3,2,4)
plt.scatter(df["diphtheria"],df["life expectancy"])
plt.xlabel("diphtheria")
plt.ylabel("Life expectancy")
plt.title("Realtionship between diphtheria and life expectancy ")
```

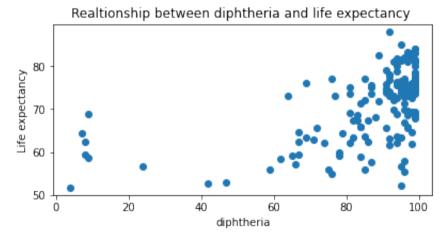






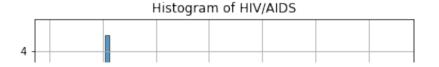
Out[212]: Text(0.5, 1.0, 'Realtionship between diphtheria and life expectancy ')



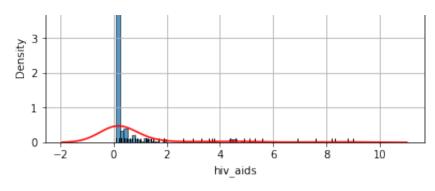


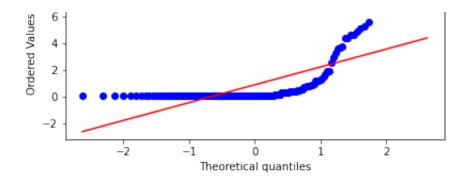


```
plt.figure(figsize = (14,10))
plt.subplot(3,2,1)
plt.title("Histogram of HIV/AIDS")
sns.histplot(df.hiv_aids, stat = "density")
sns.kdeplot(df.hiv aids, color = "red")
sns.rugplot(df.hiv aids, color = "black")
plt.grid()
plt.subplot(3,2,2)
stats.probplot(df.hiv_aids, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.subplot(3,2,3)
sns.boxplot(data = df['hiv_aids'])
plt.subplot(3,2,4)
plt.scatter(df["hiv_aids"],df["life_expectancy"])
plt.xlabel("hiv aids")
plt.ylabel("Life expectancy")
plt.title("Realtionship between hiv_aids and life expectancy ")
```

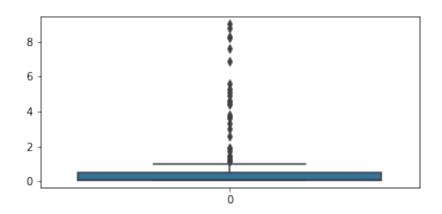


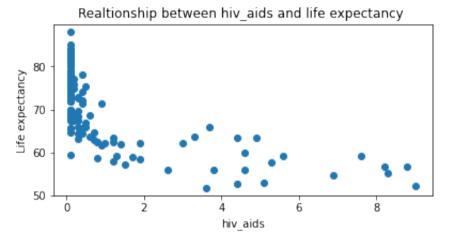






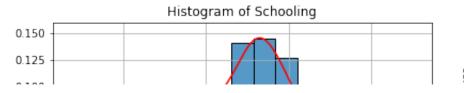
Out[213]: Text(0.5, 1.0, 'Realtionship between hiv\_aids and life expectancy ')

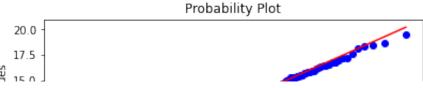


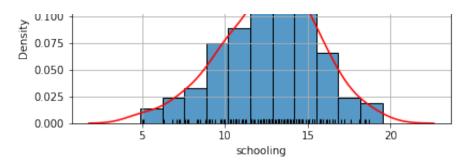


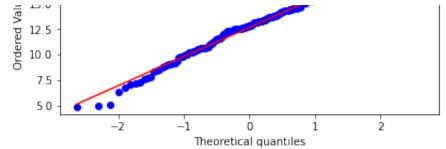
In [215]:

```
plt.figure(figsize = (14,10))
plt.subplot(3,2,1)
plt.title("Histogram of Schooling")
sns.histplot(df.schooling, stat = "density")
sns.kdeplot(df.schooling, color = "red")
sns.rugplot(df.schooling, color = "black")
plt.grid()
plt.subplot(3,2,2)
stats.probplot(df.schooling, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.subplot(3,2,3)
sns.boxplot(data = df['schooling'])
plt.subplot(3,2,4)
plt.scatter(df["schooling"],df["life_expectancy"])
plt.xlabel("schooling")
plt.ylabel("Life expectancy")
plt.title("Realtionship between schooling and life expectancy ")
```

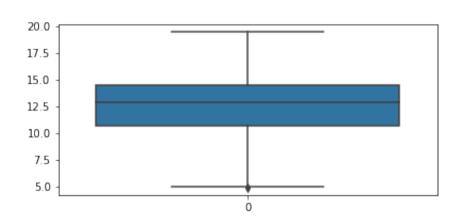


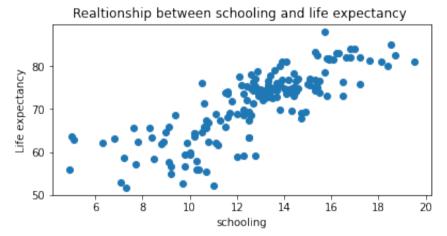






Out[215]: Text(0.5, 1.0, 'Realtionship between schooling and life expectancy ')





```
In [216]: plt.figure(figsize = (14,10))
    plt.subplot(3,2,1)

plt.title("Histogram of BMI")
    sns.histplot(df.bmi, stat = "density")
    sns.kdeplot(df.bmi, color = "red")
    sns.rugplot(df.bmi, color = "black")

plt.grid()
```

```
plt.subplot(3,2,2)

stats.probplot(df.bmi, dist="norm", plot=plt)
plt.show()

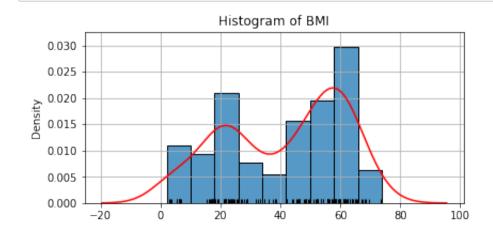
plt.figure(figsize = (14,10))

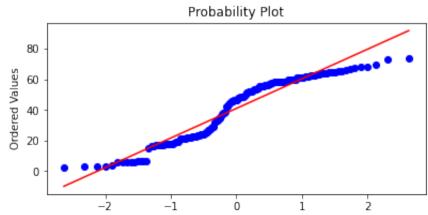
plt.subplot(3,2,3)

sns.boxplot(data = df['bmi'])

plt.subplot(3,2,4)

plt.scatter(df["bmi"],df["life_expectancy"])
plt.xlabel("bmi")
plt.ylabel("Life expectancy")
plt.title("Realtionship between bmi and life expectancy ")
```

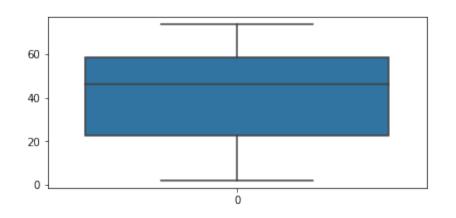


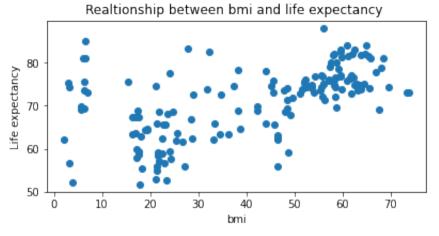


bmi

Theoretical quantiles

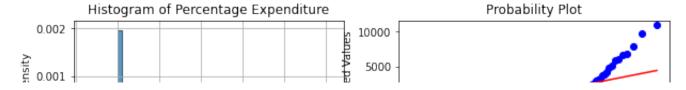
Out[216]: Text(0.5, 1.0, 'Realtionship between bmi and life expectancy ')

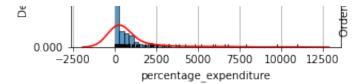


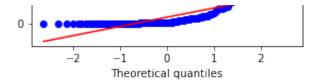


In [226]:

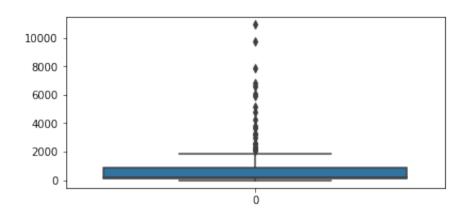
```
plt.figure(figsize=(10,6))
plt.subplot(3,2,1)
plt.title("Histogram of Percentage Expenditure")
sns.histplot(df.percentage expenditure, stat = "density")
sns.kdeplot(df.percentage expenditure, color = "red")
sns.rugplot(df.percentage expenditure, color = "black")
plt.grid()
plt.subplot(3,2,2)
stats.probplot(df.percentage_expenditure, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.subplot(3,2,3)
sns.boxplot(data = df['percentage_expenditure'])
plt.subplot(3,2,4)
plt.scatter(df["percentage_expenditure"],df["life_expectancy"])
plt.xlabel("Percentage Expenditure")
plt.ylabel("Life expectancy")
plt.title("Realtionship between Percentage Expenditure and life expectancy ")
```

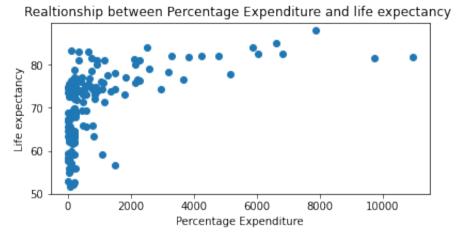






Out[226]: Text(0.5, 1.0, 'Realtionship between Percentage Expenditure and life expectancy ')





```
In [221]: plt.figure(figsize = (14,10))
    plt.subplot(3,2,1)

    plt.title("Histogram of alcohol")
    sns.histplot(df.alcohol, stat = "density")
    sns.kdeplot(df.alcohol, color = "red")
    sns.rugplot(df.alcohol, color = "black")

plt.grid()
```

```
stats.probplot(df.alcohol, dist="norm", plot=plt)
plt.show()

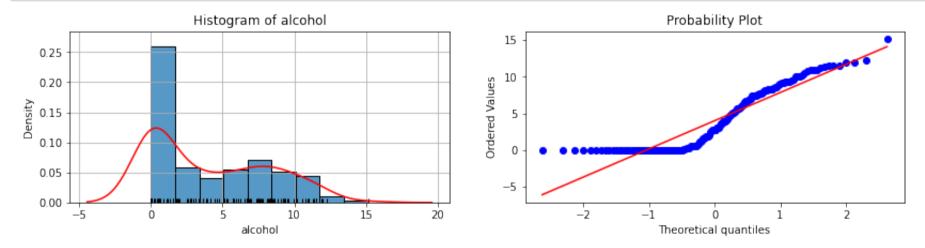
plt.figure(figsize = (14,10))

plt.subplot(3,2,3)

sns.boxplot(data = df['alcohol'])

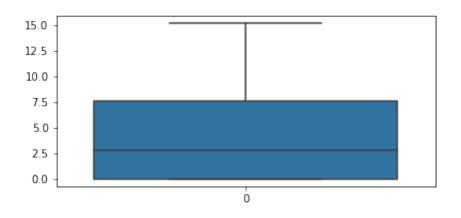
plt.subplot(3,2,4)

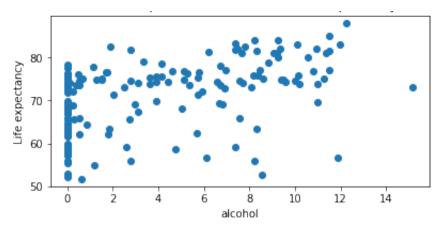
plt.scatter(df["alcohol"],df["life_expectancy"])
plt.xlabel("alcohol")
plt.ylabel("Life expectancy")
plt.title("Realtionship between alcohol and life expectancy")
```



Out[221]: Text(0.5, 1.0, 'Realtionship between alcohol and life expectancy ')

Realtionship between alcohol and life expectancy



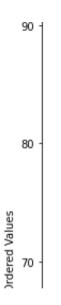


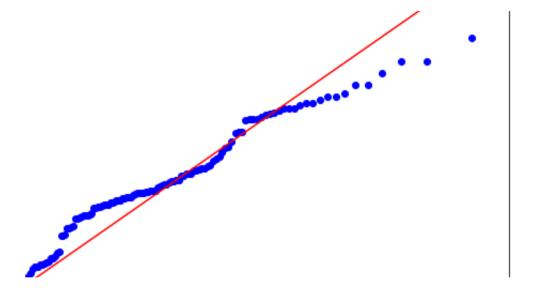
## **Quantile Plots**

In [144]:

```
plt.figure(figsize = (14,10))
stats.probplot(df.life expectancy, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
stats.probplot(df.diphtheria, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
stats.probplot(df.hiv aids, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
stats.probplot(df.percentage_expenditure, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
stats.probplot(df.schooling, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
plt.figure(figsize = (14,10))
stats.probplot(df.bmi, dist="norm", plot=plt)
plt.show()
plt.figure(figsize = (14,10))
stats.probplot(df.alcohol, dist="norm", plot=plt)
plt.show()
```

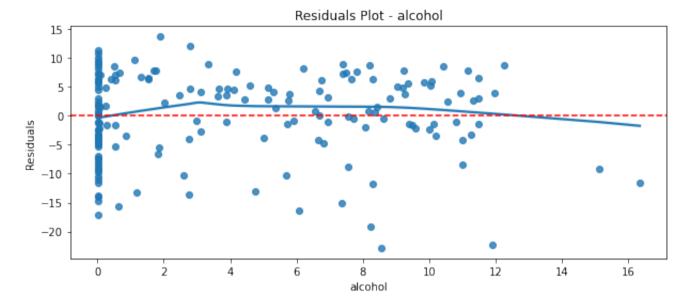
Probability Plot



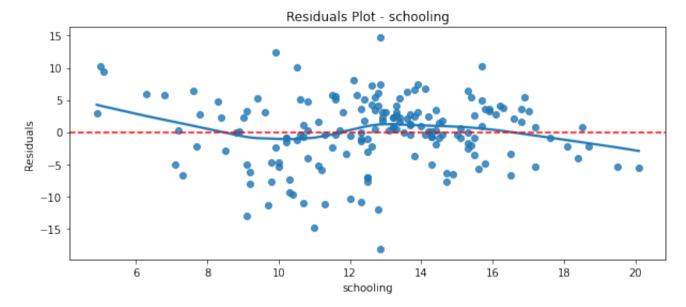


## **Residual Plots**

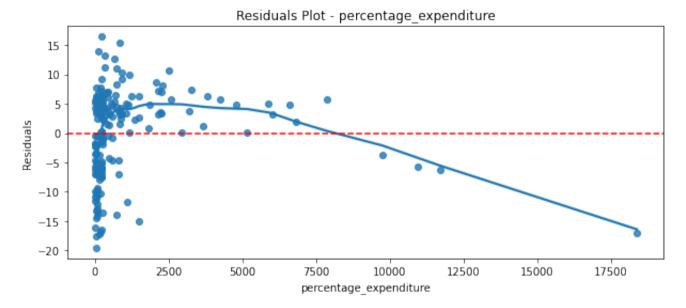
```
In [145]: reg = smf.ols('life_expectancy ~ alcohol', data = df).fit()
plt.figure(figsize = (10, 4))
sns.regplot(x = df.alcohol, y = reg.resid, lowess = True)
plt.axhline(0, linestyle = '--', color = "red")
plt.ylabel("Residuals")
plt.title("Residuals Plot - alcohol")
plt.show()
```



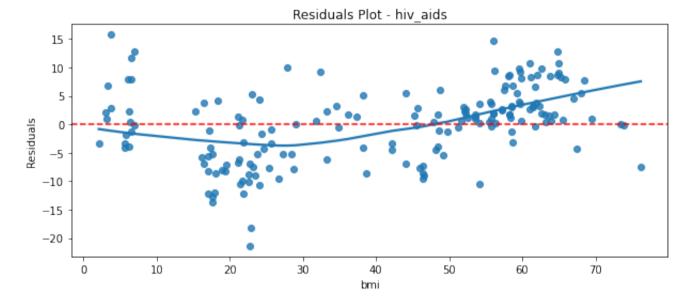
```
In [146]: reg = smf.ols('life_expectancy ~ schooling', data = df).fit()
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.schooling, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - schooling")
    plt.show()
```



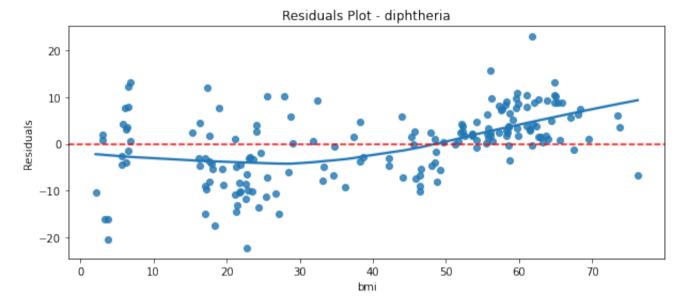
```
In [147]: reg = smf.ols('life_expectancy ~ percentage_expenditure', data = df).fit()
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.percentage_expenditure, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - percentage_expenditure")
    plt.show()
```



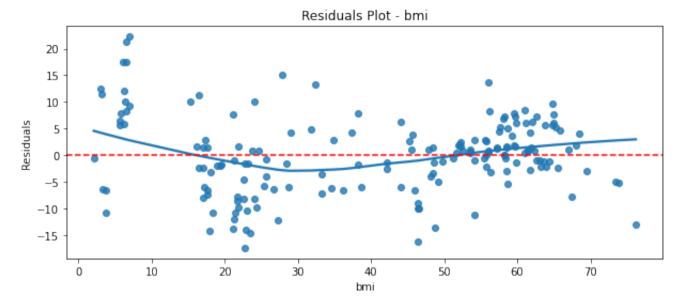
```
In [148]: reg = smf.ols('life_expectancy ~ hiv_aids', data = df).fit()
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.bmi, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - hiv_aids")
    plt.show()
```



```
In [149]: reg = smf.ols('life_expectancy ~ diphtheria', data = df).fit()
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.bmi, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - diphtheria")
    plt.show()
```



```
In [150]: reg = smf.ols('life_expectancy ~ bmi', data = df).fit()
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.bmi, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - bmi")
    plt.show()
```



The residuals seem to be randomly distributed about the residual line (=0). Since the residuals are randomly scattered around the residual = 0, it means that a linear model approximates the data points well without favoring certain inputs. In such a case, we conclude that a linear model is appropriate. \*\*\*\*

```
In [151]: ##
plt.figure(figsize=(10,10))

ols_mod11 = smf.ols(formula = 'life_expectancy ~ hiv_aids', data =df)

ols_fit11 = ols_mod11_fit()
```

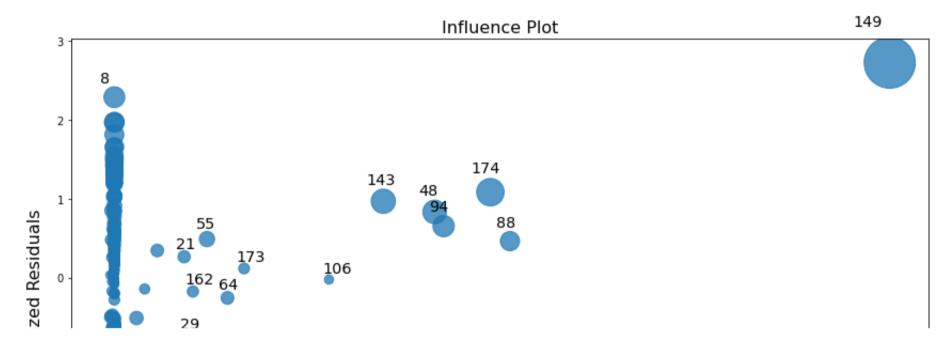
```
OCO ITCIT - OCO MONITEITC//
# For hiv/aids
figd, ax = plt.subplots(figsize=(12.8))
figd = sm.graphics.influence plot(ols fit11, ax = ax, criterion="DFFITS")
figd.tight lavout(pad=1.0)
fige, ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence_plot(ols_fit11, ax = ax, criterion="cooks")
fige.tight layout(pad=1.0)
##
plt.figure(figsize=(10,10))
ols mod13 = smf.ols(formula = 'life expectancy ~ schooling', data =df)
ols fit13 = ols mod13.fit()
# For schooling
figd, ax = plt.subplots(figsize=(12.8))
figd = sm.graphics.influence plot(ols fit13, ax = ax, criterion="DFFITS")
figd.tight layout(pad=1.0)
fige, ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence plot(ols fit13, ax = ax, criterion="cooks")
fige.tight lavout(pad=1.0)
plt.figure(figsize=(10,10))
ols mod9 = smf.ols(formula = 'life expectancy ~ diphtheria', data =df)
ols fit9 = ols mod9.fit()
# For total expenditure
plt.title
figd, ax = plt.subplots(figsize=(12.8))
figd = sm.graphics.influence plot(ols fit9, ax = ax, criterion="DFFITS")
figd.tight lavout(pad=1.0)
fige, ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence plot(ols fit9, ax = ax, criterion="cooks")
```

```
Tige.tignt layout(pag=1.0)
plt.figure(figsize=(10,10))
ols mod1 = smf.ols(formula = 'life expectancy ~ bmi', data =df)
ols fit1 = ols mod1.fit()
#For adult mortality
figd, ax = plt.subplots(figsize=(12,8))
figd = sm.graphics.influence plot(ols fit1, ax = ax, criterion="DFFITS")
figd.tight layout(pad=1.0)
fige. ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence plot(ols fit1, ax = ax, criterion="cooks")
fige.tight layout(pad=1.0)
##
plt.figure(figsize=(10,10))
ols mod3 = smf.ols(formula = 'life expectancy ~ alcohol', data =df)
ols fit3 = ols mod3.fit()
# For alcohol
figd, ax = plt.subplots(figsize=(12,8))
figd = sm.graphics.influence plot(ols fit3, ax = ax, criterion="DFFITS")
figd.tight layout(pad=1.0)
fige, ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence_plot(ols_fit3, ax = ax, criterion="cooks")
fige.tight layout(pad=1.0)
plt.figure(figsize=(10,10))
ols mod1 = smf.ols(formula = 'life expectancy ~ percentage expenditure', data =df)
ols fit1 = ols mod1.fit()
#For adult mortality
figd, ax = plt.subplots(figsize=(12,8))
```

```
figd = sm.graphics.influence_plot(ols_fit1, ax = ax, criterion="DFFITS")
figd.tight_layout(pad=1.0)

fige, ax = plt.subplots(figsize=(12,8))
fige = sm.graphics.influence_plot(ols_fit1, ax = ax, criterion="cooks")
fige.tight_layout(pad=1.0)
##
```

<Figure size 720x720 with 0 Axes>

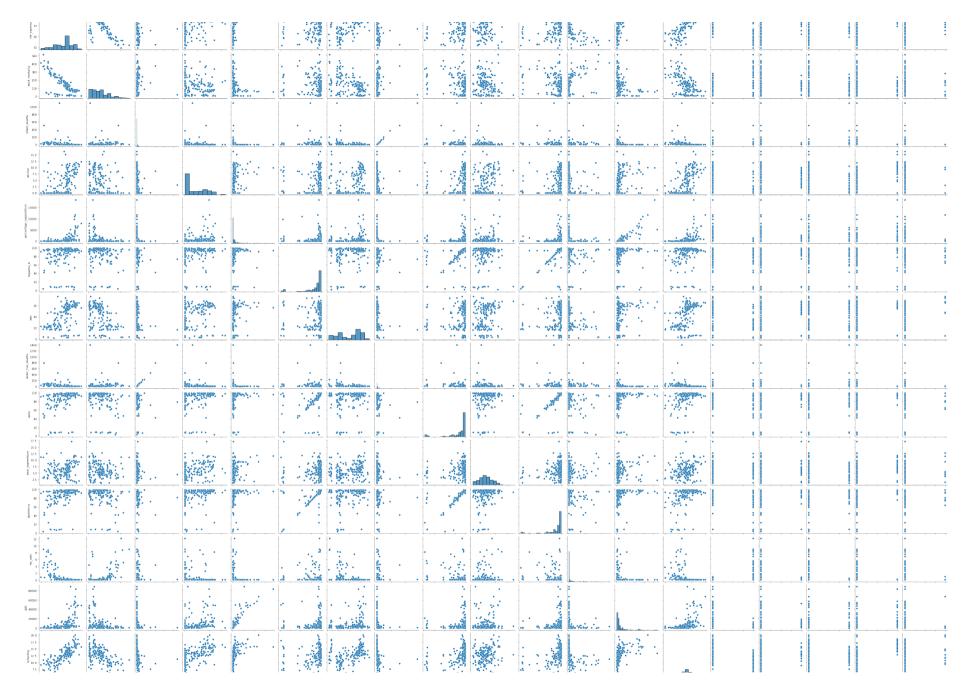


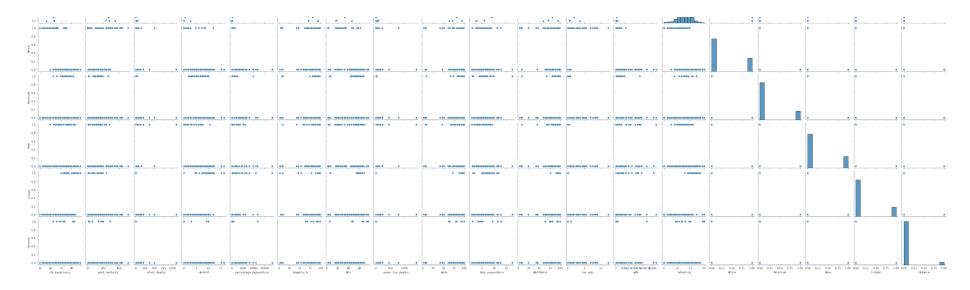
### **Pairplot**

In [152]: sns.pairplot(df)

Out[152]: <seaborn.axisgrid.PairGrid at 0x7f391c84cc90>



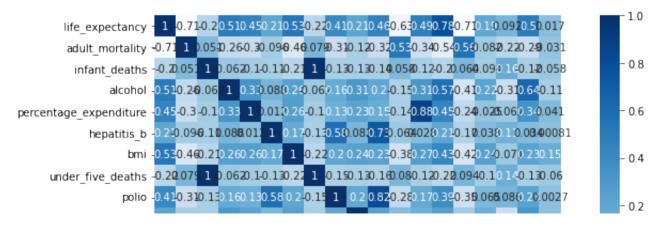


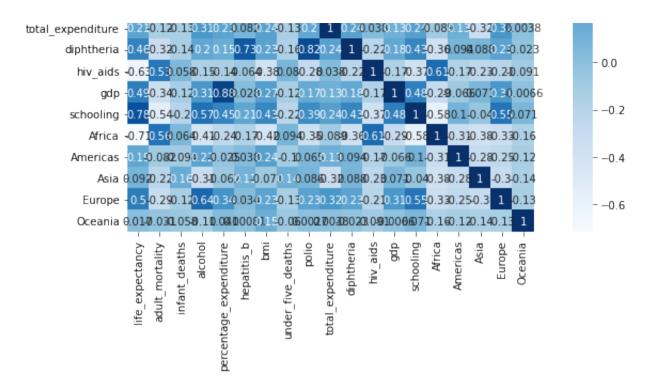


#### **Correlation Matrix**

```
In [153]: plt.figure(figsize=(13,7))
    c= df.corr()
    sns.heatmap(c,cmap="Blues",annot=True,square = True)
```

Out[153]: <matplotlib.axes.\_subplots.AxesSubplot at 0x7f3914f69fd0>





#### **Scatter Plots**

In [154]:

```
plt.figure(figsize = (15, 8))

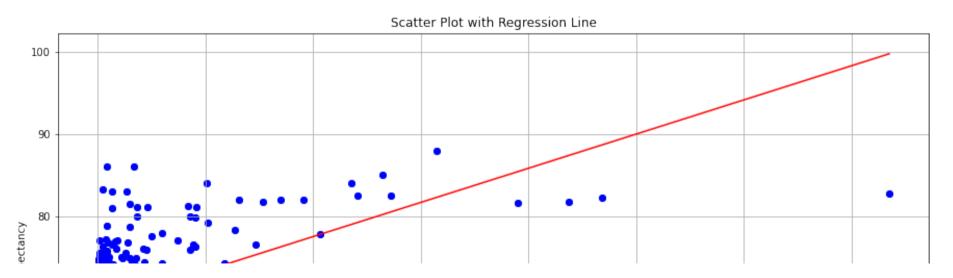
# Create scatterplot between two variables
plt.scatter(df["percentage_expenditure"], df["life_expectancy"])

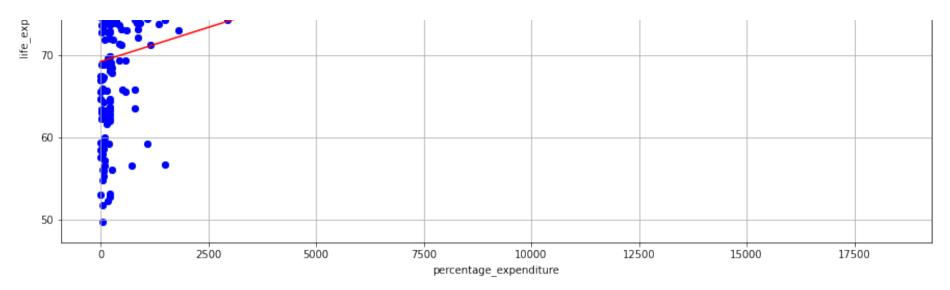
# Create regression line
m,b = np.polyfit(df["percentage_expenditure"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(0, df.percentage_expenditure.max(), 100)

# combining the two plots
plt.scatter(df["percentage_expenditure"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.xlabel("percentage_expenditure")
plt.xlabel("percentage_expenditure")
plt.grid()
```

The slope of the regression line is: 0.001662229374351855 The Intercept is: 69.20308722093486





In [155]: df.adult\_mortality.max()

Out[155]: 513

In [156]:

```
plt.figure(figsize = (15, 8))

# Create scatterplot between two variables
plt.scatter(df["alcohol"], df["life_expectancy"])

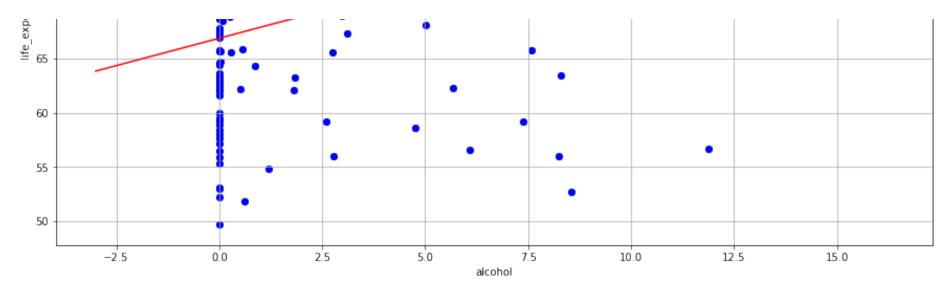
# Create regression line
m,b = np.polyfit(df["alcohol"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(-3, df.alcohol.max(), 100)

# combining the two plots
plt.scatter(df["alcohol"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.xlabel("alcohol")
plt.grid()
```

The slope of the regression line is: 1.0123386295715437 The Intercept is: 66.89219159960423





# In [157]:

```
plt.figure(figsize = (15, 8))

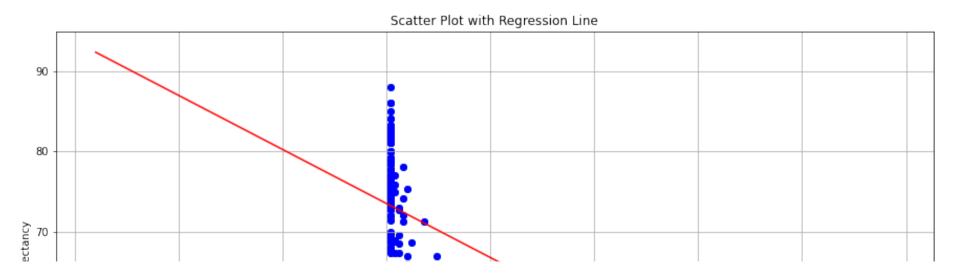
# Create scatterplot between two variables
plt.scatter(df["hiv_aids"], df["life_expectancy"])

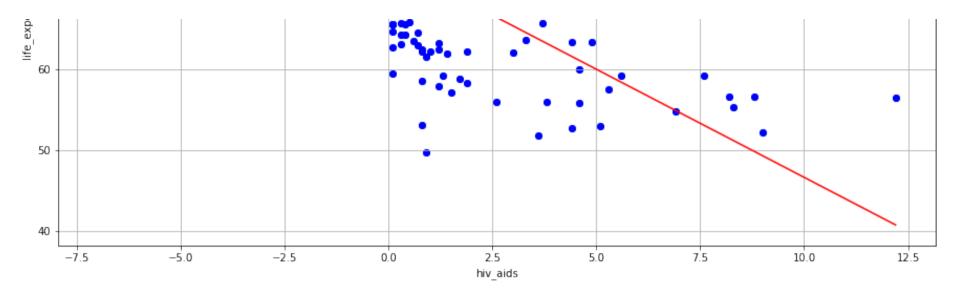
# Create regression line
m,b = np.polyfit(df["hiv_aids"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(-7, df.hiv_aids.max(), 100)

# combining the two plots
plt.scatter(df["hiv_aids"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.xlabel("hiv_aids")
plt.grid()
```

The slope of the regression line is: -2.688032124614532 The Intercept is: 73.4975292218568





# In [158]:

```
plt.figure(figsize = (15, 8))

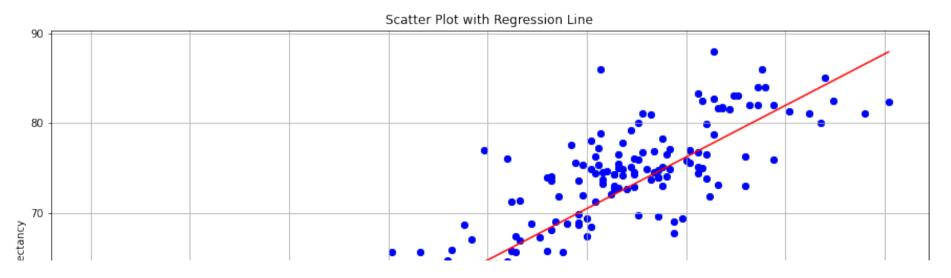
# Create scatterplot between two variables
plt.scatter(df["schooling"], df["life_expectancy"])

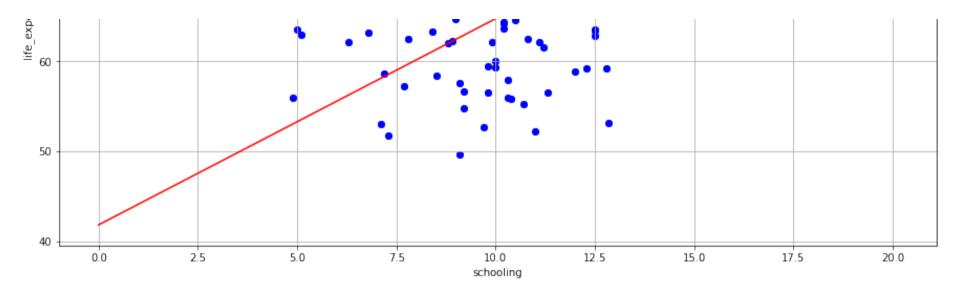
# Create regression line
m,b = np.polyfit(df["schooling"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(0, df.schooling.max(), 100)

# combining the two plots
plt.scatter(df["schooling"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.xlabel("schooling")
plt.xlabel("schooling")
plt.grid()
```

The slope of the regression line is: 2.293071348669156 The Intercept is: 41.824255771892695





# In [159]:

```
plt.figure(figsize = (15, 8))

# Create scatterplot between two variables
plt.scatter(df["diphtheria"], df["life_expectancy"])

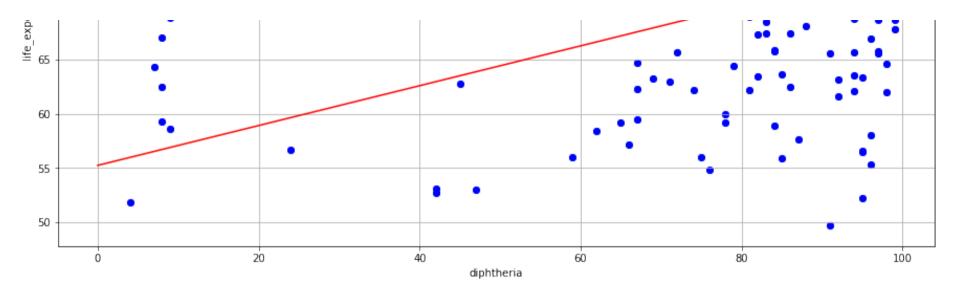
# Create regression line
m,b = np.polyfit(df["diphtheria"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(0, df.diphtheria.max(), 100)

# combining the two plots
plt.scatter(df["diphtheria"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.xlabel("diphtheria")
plt.grid()
```

The slope of the regression line is: 0.18402114107160206 The Intercept is: 55.24079470968922





# In [160]:

```
plt.figure(figsize = (15, 8))

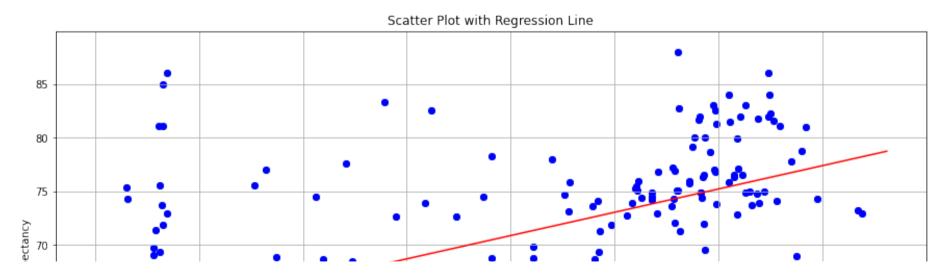
# Create scatterplot between two variables
plt.scatter(df["bmi"], df["life_expectancy"])

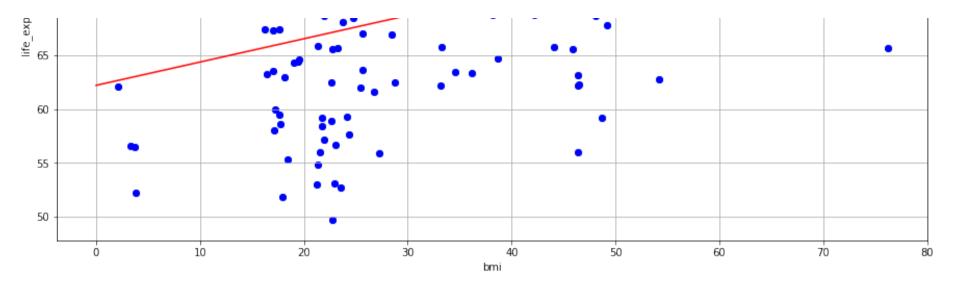
# Create regression line
m,b = np.polyfit(df["bmi"], df["life_expectancy"], deg = 1)
print("The slope of the regression line is:", m, "The Intercept is:", b)

# Create a series of equaly spaced values
x_range = np.linspace(0, df.bmi.max(), 100)

# combining the two plots
plt.scatter(df["bmi"], df["life_expectancy"],color = "blue")
plt.plot(x_range, m*x_range+b, color = "red")
plt.title("Scatter Plot with Regression Line")
plt.ylabel("life_expectancy")
plt.ylabel("bmi")
plt.grid()
```

The slope of the regression line is: 0.21681306642119505 The Intercept is: 62.21881040446138





Part (c) Linearities with the variables

As per the Harvey-Collier Test performed above, there are no non-linearities within our model and therefore require no transformations.

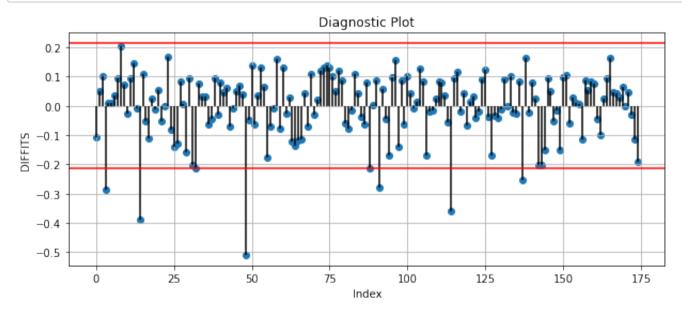
However, if we do include non-linear variables in the regression model, we break the linearity assumption of the Ordinary Least Squares regression model. Due to this, our parameter estimates would be biased, the model would have poor performance and the results will be inaccurate.

### Part (d)

Finding, if there are outliers in the model.

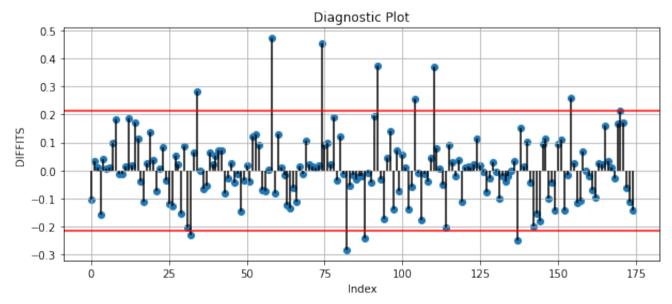
### **DFFITS**

```
In [161]: reg = smf.ols('life_expectancy ~ alcohol', data = df)
    alc = reg.fit()
    dffits, threshold = alc.get_influence().dffits
    plt.figure(figsize = (10,4))
    plt.scatter(df.index, dffits)
    plt.axhline(threshold, color = 'red')
    plt.axhline(-threshold, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = dffits)
    plt.xlabel('Index')
    plt.ylabel('DIFFITS')
    plt.title("Diagnostic Plot")
    plt.grid()
```



```
In [162]:
```

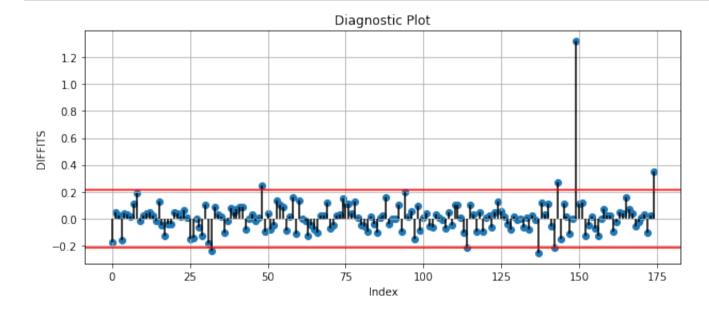
```
reg = smf.ols('life_expectancy ~ bmi', data = df)
bmi = reg.fit()
dffits, threshold = bmi.get_influence().dffits
plt.figure(figsize = (10,4))
plt.scatter(df.index, dffits)
plt.axhline(threshold, color = 'red')
plt.axhline(-threshold, color = 'red')
plt.vlines(x = df.index, ymin = 0, ymax = dffits)
plt.xlabel('Index')
plt.ylabel('DIFFITS')
plt.title("Diagnostic Plot")
plt.grid()
```



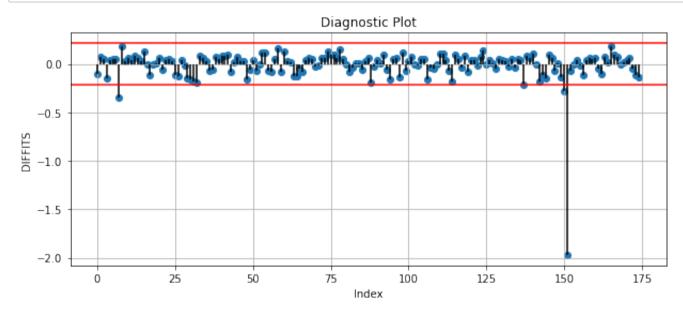
plt.grid()

```
In [163]:
    reg = smf.ols('life_expectancy ~ hiv_aids', data = df)
    hiv = reg.fit()
    dffits, threshold = hiv.get_influence().dffits
    plt.figure(figsize = (10,4))
    plt.scatter(df.index, dffits)
    plt.axhline(threshold, color = 'red')
    plt.axhline(-threshold, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = dffits)
    plt.xlabel('Index')
    plt.ylabel('DIFFITS')
```

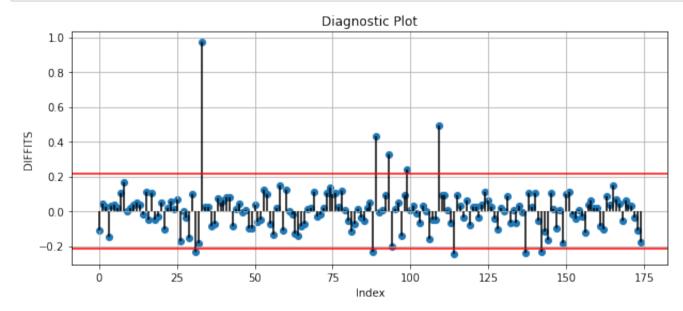
plt.title("Diagnostic Plot")



```
In [164]: reg = smf.ols('life_expectancy ~ percentage_expenditure', data = df)
    per = reg.fit()
    dffits, threshold = per.get_influence().dffits
    plt.figure(figsize = (10,4))
    plt.scatter(df.index, dffits)
    plt.axhline(threshold, color = 'red')
    plt.axhline(-threshold, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = dffits)
    plt.xlabel('Index')
    plt.ylabel('DIFFITS')
    plt.title("Diagnostic Plot")
    plt.grid()
```

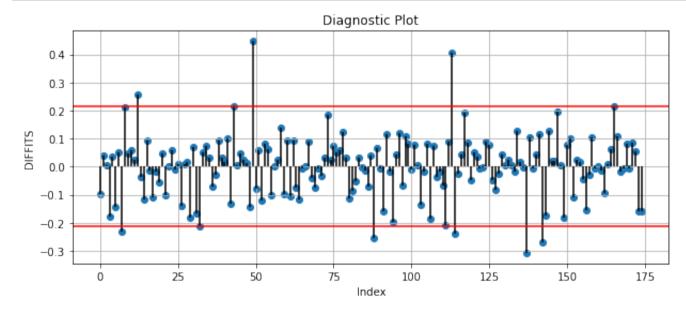


```
In [165]: reg = smf.ols('life_expectancy ~ diphtheria', data = df)
dip = reg.fit()
    dffits, threshold = dip.get_influence().dffits
    plt.figure(figsize = (10,4))
    plt.scatter(df.index, dffits)
    plt.axhline(threshold, color = 'red')
    plt.axhline(-threshold, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = dffits)
    plt.xlabel('Index')
    plt.ylabel('DIFFITS')
    plt.title("Diagnostic Plot")
    plt.grid()
```

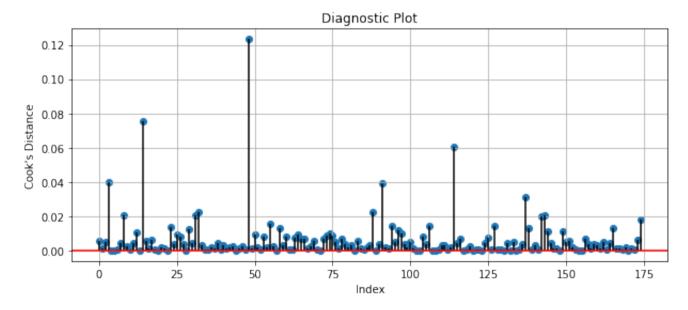


```
In [166]:
```

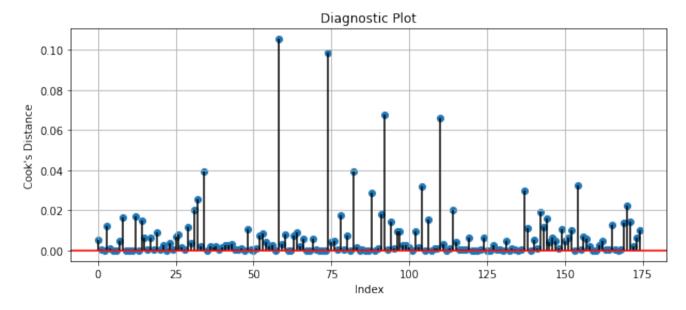
```
reg = smf.ols('life_expectancy ~ schooling', data = df)
school = reg.fit()
dffits, threshold = school.get_influence().dffits
plt.figure(figsize = (10,4))
plt.scatter(df.index, dffits)
plt.axhline(threshold, color = 'red')
plt.axhline(-threshold, color = 'red')
plt.vlines(x = df.index, ymin = 0, ymax = dffits)
plt.xlabel('Index')
plt.ylabel('DIFFITS')
plt.title("Diagnostic Plot")
plt.grid()
```



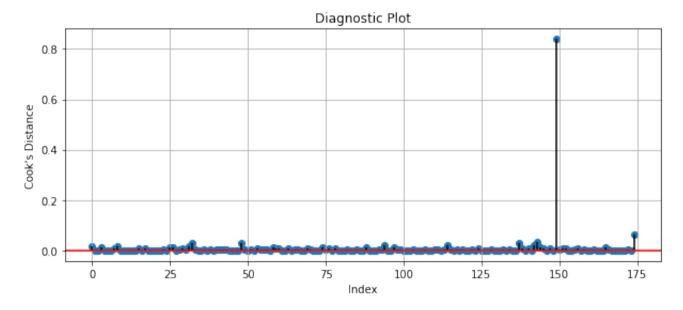
```
In [167]: reg = smf.ols('life_expectancy ~ alcohol', data = df)
    alc = reg.fit()
    cooks_distance = alc.get_influence().cooks_distance
    plt.figure(figsize = (10, 4))
    plt.scatter(df.index, cooks_distance[0])
    plt.axhline(0, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
    plt.xlabel('Index')
    plt.ylabel('Cook\'s Distance')
    plt.title("Diagnostic Plot")
    plt.grid()
```



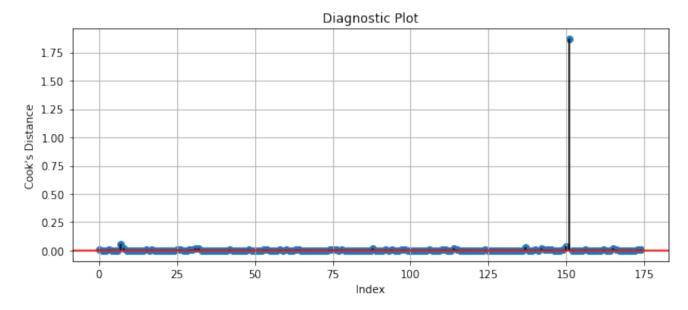
```
In [168]: reg = smf.ols('life_expectancy ~ bmi', data = df)
bmi = reg.fit()
cooks_distance = bmi.get_influence().cooks_distance
plt.figure(figsize = (10, 4))
plt.scatter(df.index, cooks_distance[0])
plt.axhline(0, color = 'red')
plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
plt.xlabel('Index')
plt.ylabel('Cook\'s Distance')
plt.title("Diagnostic Plot")
plt.grid()
```



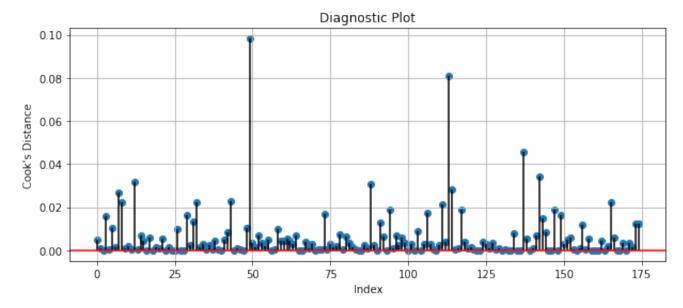
```
In [169]: reg = smf.ols('life_expectancy ~ hiv_aids', data = df)
hiv = reg.fit()
cooks_distance = hiv.get_influence().cooks_distance
plt.figure(figsize = (10, 4))
plt.scatter(df.index, cooks_distance[0])
plt.axhline(0, color = 'red')
plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
plt.xlabel('Index')
plt.ylabel('Cook\'s Distance')
plt.title("Diagnostic Plot")
plt.grid()
```

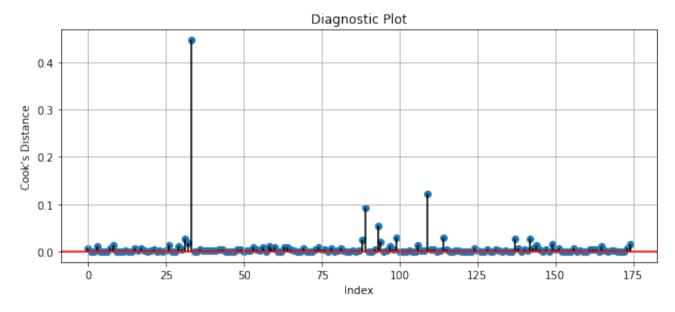


```
In [170]: reg = smf.ols('life_expectancy ~ percentage_expenditure', data = df)
    per = reg.fit()
    cooks_distance = per.get_influence().cooks_distance
    plt.figure(figsize = (10, 4))
    plt.scatter(df.index, cooks_distance[0])
    plt.axhline(0, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
    plt.xlabel('Index')
    plt.ylabel('Cook\'s Distance')
    plt.title("Diagnostic Plot")
    plt.grid()
```



```
In [171]: cooks_distance = school.get_influence().cooks_distance
    plt.figure(figsize = (10, 4))
    plt.scatter(df.index, cooks_distance[0])
    plt.axhline(0, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
    plt.xlabel('Index')
    plt.ylabel('Cook\'s Distance')
    plt.title("Diagnostic Plot")
    plt.grid()
```





Using Cook's Distance as our test for influentail observations, we removed outliers using the estimated cutoff.

This has been done in Question 3.

```
In [173]: df.diphtheria
Out[173]: 0
                  67
                  99
                  95
                  75
                  98
          170
                  81
          171
                  97
          172
                  67
          173
                  78
           174
                  95
          Name: diphtheria, Length: 175, dtype: int64
```

We see that there are NA values present in the dataset. Therefore, in order to impute them, we add the median values instead. The reason for not replacing them with the mean values is because there are extreme values present in the dataset. Due to there being extreme values, median seemed like the more appropriate tool to use in order to impute NA values.

This has been done above as well.

### **Question 3**

Part (e)

### **Evaluating Transformations**

From the Harvey-Collier test performed above, we see that our model is linearly specified and does not require any transformation.

Furthermore, we see from the Ramsey-Reset test below that our model does not require any interaction variables as well.

```
In [174]: import statsmodels.formula.api as smf
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm
import statsmodels
```

### **Test for Multicollinearity**

```
In [175]: ## Selected model
reg = smf.ols(pref_model, data = df)
results1 = reg.fit()
```

In [176]: results1.summary()

Out[176]:

**DLS Regression Results** 

```
0.813
    Dep. Variable:
                     life_expectancy
                                           R-squared:
          Model:
                               OLS
                                                           0.804
                                      Adj. R-squared:
         Method:
                      Least Squares
                                           F-statistic:
                                                           90.41
           Date: Tue, 15 Nov 2022 Prob (F-statistic): 1.75e-56
           Time:
                                      Log-Likelihood:
                           02:07:36
                                                        -474.49
No. Observations:
                               175
                                                           967.0
                                                 AIC:
                               166
                                                 BIC:
                                                           995.5
    Df Residuals:
       Df Model:
                                  8
Covariance Type:
                         nonrobust
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	52.5525	2.013	26.103	0.000	48.578	56.527
alcohol	0.1489	0.086	1.730	0.086	-0.021	0.319
bmi	0.0458	0.016	2.838	0.005	0.014	0.078
percentage_expenditure	0.0005	0.000	3.547	0.001	0.000	0.001
hiv_aids	-1.2715	0.185	-6.874	0.000	-1.637	-0.906
diphtheria	0.0431	0.015	2.846	0.005	0.013	0.073
schooling	1.1041	0.153	7.220	0.000	0.802	1.406
Africa	-3.2557	0.935	-3.484	0.001	-5.101	-1.411
Oceania	-2.7551	1.275	-2.161	0.032	-5.273	-0.238

Omnibus: 9.564 Durbin-Watson: 2.112

Prob(Omnibus): 0.008 Jarque-Bera (JB): 13.254

**Skew:** -0.335 **Prob(JB):** 0.00132

**Kurtosis:** 4.170 **Cond. No.** 1.87e+04

## Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.87e+04. This might indicate that there are strong multicollinearity or other numerical problems.

By just visually inspecting the summary statistics, we see that the p-values of our predictors are all below 0.05 i.e. they are all statistically significant. Furthermore, the adjusted R^2 is pretty good as well, standing at approximately 0.869 which means that about 87% of the the regression is explained by our model.

Both these observations show that there might not be multicollinearity present in the dataset. However, we will conduct further checks for the same.

## Out[177]:

	Intercept	alcohol	bmi	percentage_expenditure	hiv_aids	diphtheria	schooling	Africa	Oceania
0	1.0	0.01	17.6	78.184215	0.1	67.0	9.8	0.0	0.0
1	1.0	5.14	55.8	412.443356	0.1	99.0	14.2	0.0	0.0
2	1.0	0.66	56.1	555.926083	0.1	95.0	14.4	1.0	0.0
3	1.0	8.24	21.5	256.122524	2.6	75.0	10.3	1.0	0.0
4	1.0	8.18	45.7	2156.229842	0.2	98.0	13.8	0.0	0.0
170	1.0	6.70	6.4	217.212579	0.1	81.0	14.1	0.0	0.0
171	1.0	4.12	15.3	217.212579	0.1	97.0	12.2	0.0	0.0
172	1.0	0.04	38.7	217.212579	0.1	67.0	9.0	0.0	0.0
173	1.0	2.59	21.7	196.915250	5.6	78.0	12.3	1.0	0.0
174	1.0	6.09	3.3	92.602336	8.8	95.0	9.8	1.0	0.0

175 rows × 9 columns

Variance Inflation Factors (VIF)

```
In [178]: # Pull the number of regressors (+ intercept)
k = X.shape[1]

# create an empty matrix to store results
VIF = np.empty(k)

# Loop for each regressor (+ intercept)
for i in range(k):

# calculate the VIF for each
VIF[i] = smo.variance_inflation_factor(X.values, i)
print('VIF:', VIF)
```

VIF: [50.73807078 1.66062147 1.36288559 1.28283261 1.68830389 1.28617184 2.40583405 2.25755832 1.09639045]

By using Variance Inflation Factor (VIF), we try and check if there is any multicollinearity among the variables. The VIF method shows how much the variance increases due to the presence of correlation among predictors. It measures the variance by regressing predictor on predictor i.e. running an auxilliary regression. This way, we get dufferent values of R^2 from the regressions.

By rule of thumb, we take the threshold of VIF values as either 4 or 10 depending on how the values are. In our instance, the threshold we take is 4. As we can see, all the values are below 4, hence we can again conclude that there seems to be no presence of multicollinearity in the model.

# Heteroskedasticity

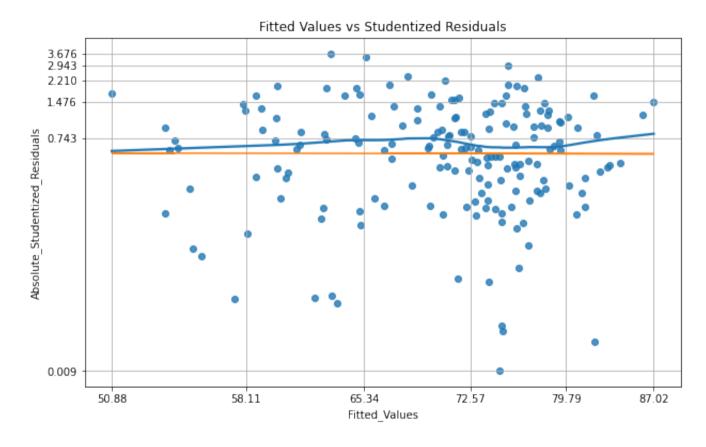
```
In [179]:
    def spread_level(model, data):
        df_copy = df.copy()
        # Get the studentized residuals
```

```
df copy["Absolute Studentized Residuals"] = (np.abs(model.get influence().resid studentized))
df copv["Fitted Values"] = (model.fittedvalues)
# run regression to get slope of fitted vs resid, rlm is a robust linear model used by R
slreg = smf.rlm("np.log(Absolute Studentized Residuals) ~ np.log(Fitted Values)", df copy).fit()
slope = slreq.params[1]
# plot values
fig, ax = plt.subplots(figsize = (10, 6))
ax.set title("Fitted Values vs Studentized Residuals")
sns.regplot(x = "Fitted Values", y = "Absolute Studentized Residuals", data = df copy, lowess = True
ax.plot(df copy.Fitted Values.values, np.exp(slreg.fittedvalues).values)
# Set to the logarithmic scale
ax.set yscale('log')
ax.set xscale('log')
# convert froms scientific notation to scalar notation
ax.vaxis.set major formatter(ScalarFormatter())
ax.xaxis.set major formatter(ScalarFormatter())
# Resolve overlapping label bug
ax.minorticks off()
# Set tick labels automatically
ax.set xticks(np.linspace(df copy["Fitted Values"].min(),df copy["Fitted Values"].max(), 6))
ax.set yticks(np.linspace(df copy["Absolute Studentized Residuals"].min(),
                          df copy["Absolute Studentized Residuals"].max(), 6))
ax.grid()
# return a suggested power transform of your y-variable that may correct heteroscedastcity
# The transform is just one minus the slope of the reegression line of your fitted values vs residua
print("Suggested Power Transformation:", 1-slope)
```

```
In [180]: import statsmodels.formula.api as smf
from matplotlib.ticker import ScalarFormatter
```

```
In [181]: model = smf.ols('life_expectancy ~ hiv_aids + schooling + alcohol + percentage_expenditure + diphtheria
results = model.fit()
spread_level(results, df)
```

Suggested Power Transformation: 1.0285380866401956



By visually inspecting the Spread-Level Plot above, we see that there does not seem to be a any evidence of increasing variance as the fitted values get larger. This suggests that there seems to be no presence of heteroskedasticity in our model.

We further conduct tests to verify our intuition. To further verify our results, we are conducting BP test, White test. For indicator variables, we are conducting GQ test.

### **Breush-Pagan Test**

```
In [182]:
          df["res3"] = results1.resid**2
          # try to predict the squared residuals using a linear combination of our variables
          aux reg = smf.ols('res3 ~ hiv aids + schooling + alcohol + percentage expenditure + diphtheria + bmi + A
          # Get the regression f-statistic (f-test version)
          f = aux req.fvalue
          fp = aux req.f pvalue
          print("The F-Statistic for the Auxiliary Regression is: "+ str(f) +" and the P-Value is: "+ str(fp))
          The F-Statistic for the Auxiliary Regression is: 1.4859372199670122 and the P-Value is: 0.1657530961291
          3972
In [183]:
          sm.stats.diagnostic.het breuschpagan(results1.resid, X)
Out[183]: (11.694538031917157,
           0.1653620031268073,
           1.4859372199670084,
           0.1657530961291411)
```

The Breush-Pagan test tells about any heteroskedasticity present in our model. From the above function, we see that the p-value is about 0.16, which is greater than 0.05. This means that we fail to reject the null. The null, in this case, being that there is no heteroskedasticity.

Therefore, we can say that there is no evidence of heteroskedasticity in the model, as per the BP-Test

#### White-Test

In [184]: sm.stats.diagnostic.het\_white(results1.resid, X)

Out [184]: (40.35598544723332, 0.4990798673459415, 0.9722740372555461, 0.5263256231787328)

The white test also gives us a p-value of 0.52, which tells us that we fail to reject the null, ultimately showing that there is no presence of heteroskedasticity in the model.

**Godlfield-Quant test** 

```
In [185]: # manual implementation
    data1 = df[df.Africa == 1]
    data0 = df[df.Africa == 0]

# run regs on different groups
    reg1 = smf.ols('life_expectancy ~ hiv_aids + schooling + alcohol + percentage_expenditure + diphtheria +
    reg0 = smf.ols('life_expectancy ~ hiv_aids + schooling + alcohol + percentage_expenditure + diphtheria +

# pull out the residuals of each regression
    df1 = reg1.df_resid
    df0 = reg0.df_resid

# Get the variance of each regression
    sig1squared = reg1.scale
    sig0squared = reg0.scale

fstat = sig1squared/sig0squared

# calculate critical calue for right side test
    stats.f.ppf(.95, df1, df0)
```

Out[185]: 1.4814643707028299

In [186]: fstat

Out[186]: 1.3049587910521845

We see that the critical value is greater than the fstat, which means that we fail to reject the null. The null, is this case, is that the variance of of the two groups created above are the same. This means that the variance has remained constant. Hence, this further proves that there is no presence of Heteroskedasticity in our model.

```
In [187]: # manual implementation
          data1 = df[df.Oceania == 1]
          data0 = df[df.Oceania == 0]
          # run regs on different groups
          reg1 = smf.ols('life expectancy ~ hiv aids + schooling + alcohol + percentage expenditure + diphtheria +
          reg0 = smf.ols('life expectancy ~ hiv aids + schooling + alcohol + percentage expenditure + diphtheria +
          # pull out the residuals of each regression
          df1 = req1.df resid
          df0 = req0.df resid
          # Get the variance of each regression
          sig1squared = reg1.scale
          sig0squared = reg0.scale
          fstat = sig1squared/sig0squared
          # calculate critical calue for right side test
          stats.f.ppf(.95, df1, df0)
Out[187]: 2.662196418824825
In [188]: fstat
Out[188]: 0.5537932759452712
In [189]: | df.Oceania.sum()
Out[189]: 10
```

We see that the critical value is greater than the fstat, which means that we fail to reject the null. The null, is this case, is that the variance of of the two groups created above are the same. This means that the variance has remained constant. Hence, this further proves that there is no presence of Heteroskedasticity in our model.

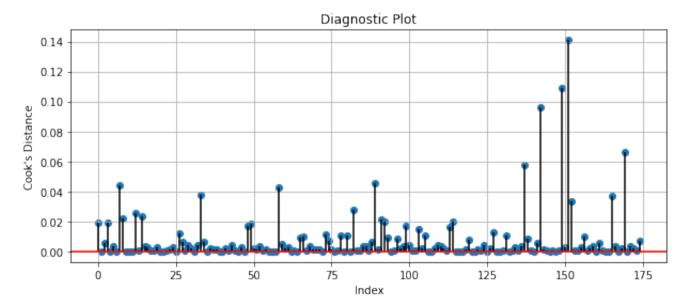
## Testing if the model is misspecified using Reset test

```
In [190]: ### Reset ramsey for model misspecification
    reset_out = statsmodels.stats.outliers_influence.reset_ramsey(res = results1, degree = 2)
    reset_out
```

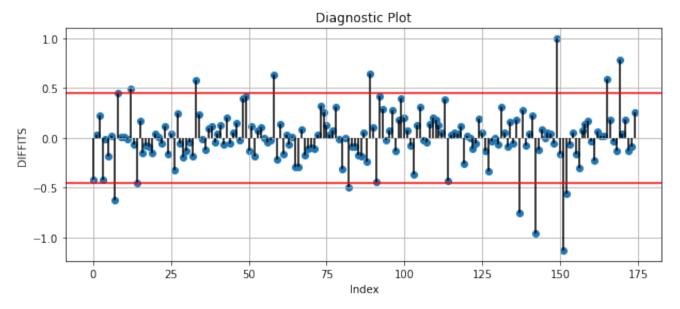
We see that the p-value is greater than 0.05. This means that we fail to reject to null, which here, is that the model is correctly specified. Hence, we can say that our model is correctly specified does not require any interaction variables and hence is correctly specified.

```
In [191]: import patsy as pt
import statsmodels.api as sm
```

```
In [192]: # Cook's Distance plot
    cooks_distance = results1.get_influence().cooks_distance
    plt.figure(figsize = (10, 4))
    plt.scatter(df.index, cooks_distance[0])
    plt.axhline(0, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = cooks_distance[0])
    plt.xlabel('Index')
    plt.ylabel('Cook\'s Distance')
    plt.title("Diagnostic Plot")
    plt.grid()
```



```
In [193]: dffits, threshold = results1.get_influence().dffits
    plt.figure(figsize = (10,4))
    plt.scatter(df.index, dffits)
    plt.axhline(threshold, color = 'red')
    plt.axhline(-threshold, color = 'red')
    plt.vlines(x = df.index, ymin = 0, ymax = dffits)
    plt.xlabel('Index')
    plt.ylabel('Index')
    plt.ylabel('DIFFITS')
    plt.title("Diagnostic Plot")
    plt.grid()
```



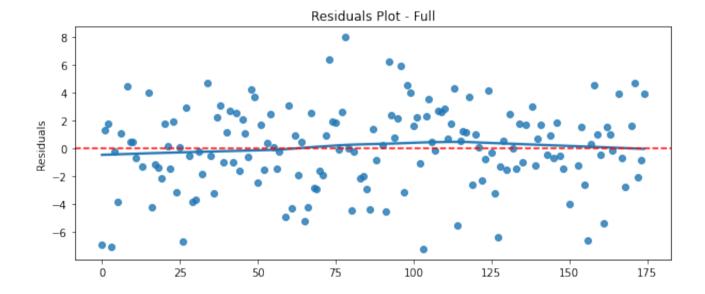
```
In [194]: df = df[cooks_distance[0]<4/len(cooks_distance[0])]</pre>
```

Removing any unusual variables present in the multivariate regression using cooks distance.

#### **Residual Plot**

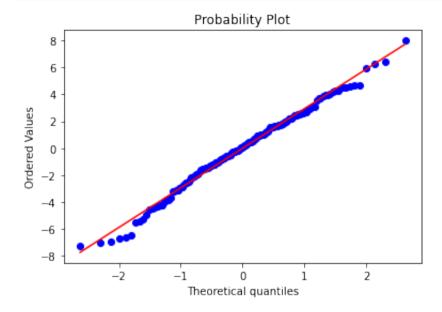
plt.show()

```
In [195]:
    reg = smf.ols('life_expectancy ~ hiv_aids + schooling + alcohol + percentage_expenditure + diphtheria +
    plt.figure(figsize = (10, 4))
    sns.regplot(x = df.index, y = reg.resid, lowess = True)
    plt.axhline(0, linestyle = '--', color = "red")
    plt.ylabel("Residuals")
    plt.title("Residuals Plot - Full")
```



QQ Plot

In [227]: stats.probplot(results1.resid, dist="norm", plot=plt)
 plt.show()



As we can see through the QQ plot as well, we can reinforce the result that our residuals are approximately normally distributed.

# Comparing competing models using AIC and BIC

We have chosen an alternative model from running the Mallow CP when indicator variables were not included in the model in order to compare top models with and without indicator variables.

In [197]:

```
## An alternate model we are comparing our models with ****
alt_pref_model
reg = smf.ols(alt_pref_model, data = df)
results_alt = reg.fit()
results_alt.summary()
```

## Out[197]:

OLS Regression Results

Dep. Variable:	life_expectancy	R-squared:	0.865
Model:	OLS	Adj. R-squared:	0.859
Method:	Least Squares	F-statistic:	139.9
Date:	Tue, 15 Nov 2022	Prob (F-statistic):	3.63e-63
Time:	02:08:55	Log-Likelihood:	-403.68
No. Observations:	161	AIC:	823.4
Df Residuals:	153	BIC:	848.0
Df Model:	7		
Covariance Type:	nonrobust		

coef	std err	t	P> t	[0.025	0.975]
49.7458	1.566	31.766	0.000	46.652	52.840
0.1663	0.074	2.258	0.025	0.021	0.312
0.0489	0.014	3.436	0.001	0.021	0.077
0.1247	0.100	1.250	0.213	-0.072	0.322
0.0008	0.000	5.046	0.000	0.000	0.001
-1.7508	0.149	-11.722	0.000	-2.046	-1.456
0.0502	0.015	3.450	0.001	0.021	0.079
	49.7458 0.1663 0.0489 0.1247 0.0008 -1.7508	49.7458       1.566         0.1663       0.074         0.0489       0.014         0.1247       0.100         0.0008       0.000         -1.7508       0.149	49.7458       1.566       31.766         0.1663       0.074       2.258         0.0489       0.014       3.436         0.1247       0.100       1.250         0.0008       0.000       5.046         -1.7508       0.149       -11.722	49.7458       1.566       31.766       0.000         0.1663       0.074       2.258       0.025         0.0489       0.014       3.436       0.001         0.1247       0.100       1.250       0.213         0.0008       0.000       5.046       0.000         -1.7508       0.149       -11.722       0.000	49.7458       1.566       31.766       0.000       46.652         0.1663       0.074       2.258       0.025       0.021         0.0489       0.014       3.436       0.001       0.021         0.1247       0.100       1.250       0.213       -0.072         0.0008       0.000       5.046       0.000       0.000         -1.7508       0.149       -11.722       0.000       -2.046

Omnibus: 1.740 Durbin-Watson: 1.981

Prob(Omnibus): 0.419 Jarque-Bera (JB): 1.320

**Skew:** -0.182 **Prob(JB):** 0.517

**Kurtosis:** 3.254 **Cond. No.** 1.31e+04

#### Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.31e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
In [198]: Model1_bic = results1.bic
Model2_bic = results_alt.bic
```

In [199]: print('model 1 BIC:',Model1\_bic)

model 1 BIC: 995.4637064747459

In [200]: print('Alternate model BIC:',Model2\_bic)

Alternate model BIC: 848.0126368102004

In [202]: Model1\_aic = results1.aic
 Model2\_aic = results\_alt.aic

```
In [201]: print('model 1 AIC:', Model1_aic)
    model 1 AIC: 966.9806327094343
```

```
In [203]: print('Alternate model AIC:',Model2_aic)
```

Alternate model AIC: 823.3614018903247

On comparing competing models, the perefered model (model 1) is the best model for our analysis because the prefered model has a lower AIC and BIC. This model is life\_expectancy ~ alcohol + bmi + percentage\_expenditure + hiv\_aids + diphtheria + schooling + Africa + Oceania.

```
In [204]: reg = smf.ols(pref_model, data = df)
results1 = reg.fit()
```

#### **Bootstrapping**

```
plt.figure()
for _ in range(n_boots):
# sample the rows, same size, with replacement
    sample df = df.sample(n=n points. replace=True)
# fit a linear regression
    ols model temp = smf.ols(formula = 'life expectancy ~ hiv aids + schooling + alcohol + percentage ex
    results temp = ols model temp.fit()
# append coefficients
    boot interc.append(results temp.params[0])
    boot slopes1.append(results temp.params[1])
    boot slopes2.append(results temp.params[2])
    boot slopes3.append(results temp.params[3])
    boot slopes4.append(results temp.params[4])
    boot slopes5.append(results temp.params[5])
    boot slopes6.append(results temp.params[6])
    boot slopes7.append(results temp.params[7])
    boot slopes8.append(results temp.params[8])
    boot adjR2.append(results temp.rsquared adj)
```

<Figure size 432x288 with 0 Axes>

```
In [206]: ols_model_temp = smf.ols(formula = 'life_expectancy ~ hiv_aids + schooling + alcohol + percentage_ex
    results_temp = ols_model_temp.fit()
    results_temp.summary()
```

## Out [206]:

JLS Regression Results

```
Dep. Variable:life_expectancyR-squared:0.876Model:OLSAdj. R-squared:0.870Method:Least SquaresF-statistic:134.8Date:Tue, 15 Nov 2022Prob (F-statistic):5.02e-65
```

Time:	02:09:23	Log-Likelihood:	-394.46
No. Observations:	161	AIC:	806.9
Df Residuals:	152	BIC:	834.7
Df Model:	8		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	53.5080	1.741	30.730	0.000	50.068	56.948
hiv_aids	-1.3930	0.168	-8.276	0.000	-1.726	-1.060
schooling	0.9333	0.124	7.514	0.000	0.688	1.179
alcohol	0.0766	0.072	1.058	0.292	-0.067	0.220
percentage_expenditure	0.0008	0.000	5.478	0.000	0.000	0.001
diphtheria	0.0391	0.014	2.864	0.005	0.012	0.066
bmi	0.0729	0.014	5.198	0.000	0.045	0.101
Africa	-2.7990	0.821	-3.408	0.001	-4.422	-1.176
Oceania	-1.0435	1.296	-0.805	0.422	-3.604	1.517

 Omnibus:
 0.733
 Durbin-Watson:
 2.148

 Prob(Omnibus):
 0.693
 Jarque-Bera (JB):
 0.417

 Skew:
 -0.087
 Prob(JB):
 0.812

 Kurtosis:
 3.179
 Cond. No.
 1.70e+04

Notes:

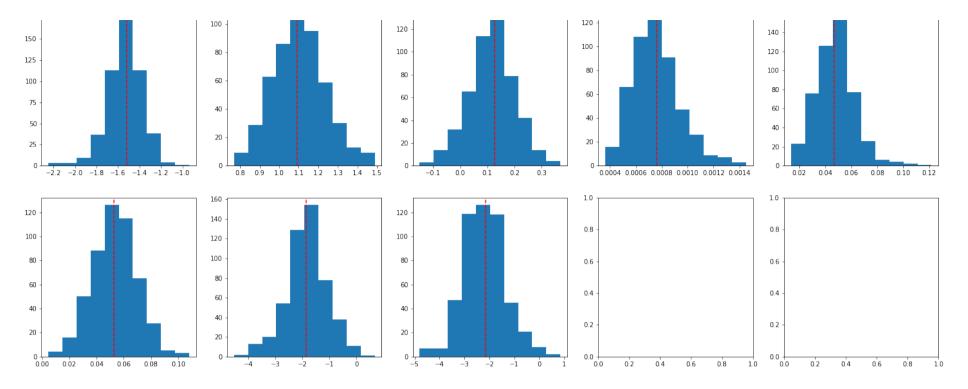
- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.7e+04. This might indicate that there are strong multicollinearity or other numerical problems.

In [207]:	

```
# histogram of bootstrapped estimates
fig, ax = plt.subplots(2, 5)
fig.set size inches(25, 10)
ax[0,0].set title('')
ax[0, 0].hist(boot slopes1) #row=0, col=0
ax[0, 0].axvline(x=est.params[1],color='red', linestyle='--')
ax[0, 1].hist(boot slopes2) #row=0, col=0
ax[0, 1].axvline(x=est.params[2],color='red', linestyle='--')
ax[0, 2].hist(boot slopes3) #row=0, col=0
ax[0, 2].axvline(x=est.params[3],color='red', linestyle='--')
ax[0, 3].hist(boot slopes4) #row=0, col=0
ax[0, 3].axvline(x=est.params[4],color='red', linestyle='--')
ax[0.4].hist(boot slopes5) #row=0. col=0
ax[0, 4].axvline(x=est.params[5],color='red', linestyle='--')
ax[1, 0].hist(boot slopes6) #row=0, col=0
ax[1, 0].axvline(x=est.params[6],color='red', linestyle='--')
ax[1. 1].hist(boot slopes7) #row=0. col=0
ax[1, 1].axvline(x=est.params[7],color='red', linestyle='--')
ax[1, 2].hist(boot slopes8) #row=0, col=0
ax[1, 2].axvline(x=est.params[8],color='red', linestyle='--')
```

Out[207]: <matplotlib.lines.Line2D at 0x7f391381f510>

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**K-Fold Cross Validation** 

5-Fold CV RMSE Scores: [-3.31313465 -2.72832905 -3.62525063 -2.68887013 -2.75749281]

```
In [209]: 3/df.life_expectancy.mean()
```

Out[209]: 0.04224317374801029

Across our five folds of out of sample predicitons of life-expectancy, the average RMSE is approximately 3 years. This is approximately a 4% error, which seems fairly good for predicting the life expectancy of each country.

```
In [210]: # Test Out of Sample Performance

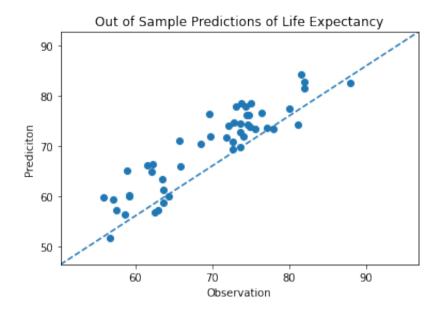
# Define Model Vars
# x = df[['hiv_aids', 'adult_mortality' , 'schooling' , 'alcohol' , 'total_expenditure' , 'polio' , 'gdp
# y = df[['life_expectancy']]
```

```
# Split the data into train (70%)/test(30%) samples:
x train, x test, y train, y test = train test split(x, y, test size=0.3, random state=0)
# Train the model:
regr = LinearRegression()
regr.fit(x_train, y_train)
# Make predictions based on the test sample
y pred = regr.predict(x test)
# Evaluate Performance
print(rear.coef )
print('MAE:', metrics.mean_absolute_error(y_test, y_pred))
print('MSE:', metrics.mean squared error(y test, y pred))
print('RMSE:', np.sqrt(metrics.mean squared error(y test, y pred)))
res comp = pd.merge(y test.reset index(), pd.DataFrame(y pred), left index=True, right index=True)
# plt.scatter(res comp.iloc[:,1], res comp.iloc[:,2])
plt.title('Out of Sample Predictions of Life Expectancy')
plt.scatter(y_test, y_pred)
plt.xlabel('Observation')
plt.vlabel('Prediciton')
xmax, xmin = y test.max() * 1.10, y test.min() * 0.90
ymax, ymin = y pred.max() * 1.10, y pred.min() * 0.90
plt.xlim([xmin[0],xmax[0]])
plt.vlim([vmin.vmax])
plt.plot(np.linspace(xmin[0],xmax[0]), np.linspace(ymin,ymax), linestyle ='--')
```

```
[[ 6.28973427e-02  9.45391207e-02  5.80520455e-04 -1.33288899e+00  5.96828716e-02  1.17946223e+00 -2.53896376e+00 -2.18916736e+00]]
MAE: 2.8332260249467427
```

MSE: 11.724483888342926 RMSE: 3.4241033699850427

Out[210]: [<matplotlib.lines.Line2D at 0x7f3913600f50>]



## **Marginal Effect**

Because there are no quadratic or interaction terms in our model, the marginal effects are the estimated coefficients from the OLS regression results.

# **Conclusion**

Of the 21 predictor variables we initially selected for analysis, we finalized an OLS regression model which is life expectancy = HIV and AIDs + schooling + alcohol + percentage and expenditure + diphtheria + BMI + Africa + Oceania using AIC, BIC and CP Mallow. Our model has low VIF score indicating that there is no multicollinearity present between predictors. Additionally, 83 percent of the variation in y is explained by the model. Using the Harvey collier test, the test indicates that all our parameters are linear. This means that none of our parameters need any transformation, thus a one unit increase in each of the quantitative variables would lead to a beta increase/decrease in expected life expectancy on average. On using white test, BP test and the GQ test we also identify that our variable are not heteroskedastic, and after using the Ramsey reset test, we find that our preferred model is not mis-specified. It is important to also note that the indicator variables which include Africa and Oceania, have a negative relationship between life expectancy and the indicator variables. Further, on the continent of Africa, life expectancy decreases on average by 3.25 and in Oceania, the life expectancy decreases by 2.75 on average. There is a negative relationship between HIV/AIDS deaths per thousand people and life expectancy. We would also like to conclude that in a univariate analysis, the number of HIV/AIDS deaths per thousand people is the variable is ranked the highest when conducting the boruta algorithm.

## References

- 1. Statistical Analysis on factors influencing Life Expectancy: (<a href="https://www.kaggle.com/datasets/kumarajarshi/life-expectancy-who">https://www.kaggle.com/datasets/kumarajarshi/life-expectancy-who</a> (<a href="https://www.kaggle.com/datasets/kumarajarshi/life-expectancy-who">https://www.kaggle.co
- 2. Econ 430 lecture material: Professor Randall R. Rojas
- 3. Econ 442A lecture material: Professor Nathan Kunz

