



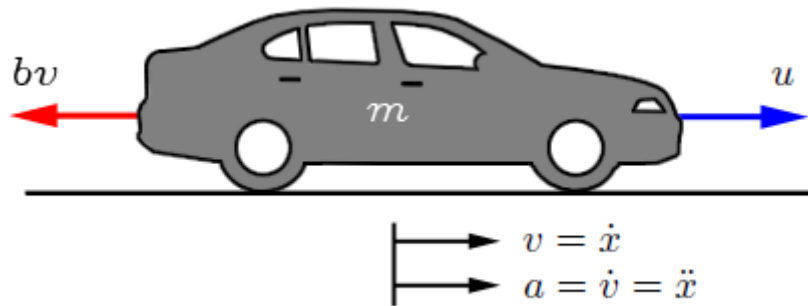
# PID CONTROLLER MODELLING

TEAM - 12

## Problem B

*Cruise control using PID controller*

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### Problem B

(*m*) vehicle mass *m* kg

(*b*) damping coefficient *b* N.s/m

When an external force(*u*) is applied the car starts it accelerates from rest and runs at a speed of *S* m/s in the presence of resistive forces by the road(*bv*). From the figure, along with the aforementioned information we get

$$m\dot{v} + bv = u$$

We want it to reach the speed within *c* s and once it reaches that speed that changes in the speed be less than *a* % of the desired speed. Once the the vehicle started, before it reaches the steady state running condition any possible overshoot in speed should be less than *b* %. Design a PID controller for this purpose.

#### Parameters (Group-B2): -

$$m = 1500$$

$$S = 20$$

$$a = 1$$

$$b = 2$$

$$c = 2$$

Our objective is to manipulate force *u*(*t*) such that the vehicle reaches speed 20 m/s within 2 s and the changes in speed should be less than 1% of 20, that is  $\pm 0.2$  m/s. This means *v*(*t*) should be lie within  $20 \pm 0.2$ , i.e., between 19.8 m/s to 20.2 m/s in steady state. Also, before the vehicle reaches the steady state, the maximum possible overshoot should be less than 2% of 20, that is 0.4 m/s. This means *v*(*t*) should not exceed limit of  $20 + 0.4$ , i.e., it must always lie below 20.4 m/s before reaching steady state. We achieve is using a PID controller.

$$m\dot{v} + bv = u$$

$$\frac{m dv(t)}{dt} + bv(t) = u(t)$$

On taking Laplace transform of the functions, to change from time domain to complex 's' domain.

$$msV(s) - v(0) + bV(s) = U(s)$$

As the car starts from rest, velocity is 0 m/s at time 0 s.

$$v(0) = 0$$

On substituting the value of  $v(0)$ , we get

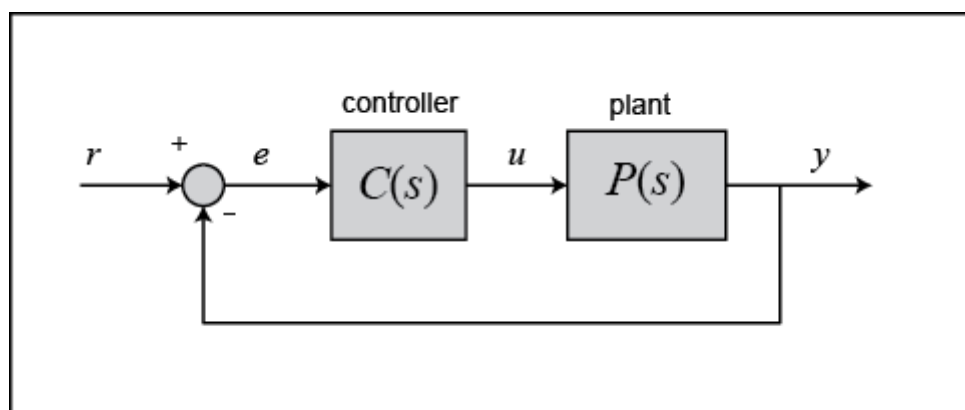
$$msV(s) + bV(s) = U(s)$$

$$V(s) \cdot (ms + b) = U(s)$$

$$V(s) = \frac{U(s)}{(ms + b)}$$

$$V(s) = H(s) * U(s)$$

$$H(s) = \frac{1}{ms + b}$$



This diagram shows the basic modelling of a PID controlled system. Here 'r' represents our step function of step value  $S = 20$ . 'e' represents the error function  $E(s)$ .

$$E(s) = S - V(s)$$

$$E(s) = 20 - V(s)$$

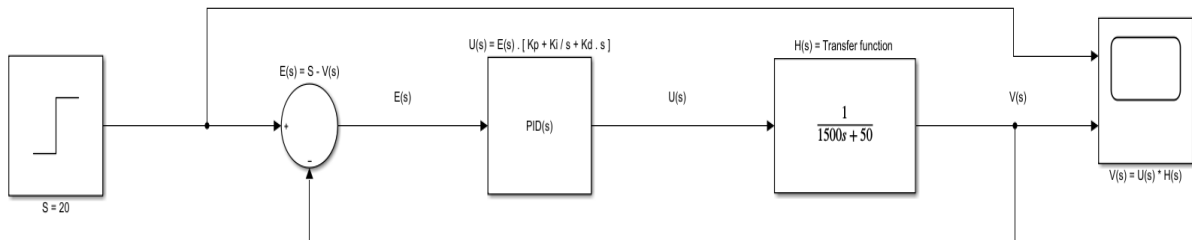
Controller  $C(s) = [Kp + Ki/s + Kd.s]$  or  $U(s)/E(s)$ . Plant  $P(s) = H(s)$  or Transfer function. Output  $y = V(s)$ .

$$U(s) = E(s) \cdot [Kp + Ki/s + Kd.s]$$

$$H(s) = \frac{1}{1500s + 50}$$

$$V(s) = H(s) \cdot U(s)$$

$$V(s) = \frac{1}{1500s + 50} \cdot U(s)$$



The magnitude of  $Kp$  controls the initial rise rate, to reach the required speed in given time. The magnitude of  $Ki$  controls constant steady state error, to stabilize speed in given parameters and reduce the error to zero. The magnitude of  $Kd$  controls the overshoot, to keep it within the limits.

After much trial and error with many different values, we found these values which seems to fit well within the given parameters.

$$Kp = 7000$$

$$Ki = 300$$

$$Kd = 7000$$

Our model crosses the 20 m/s mark in slightly less than 1.5 seconds. The overshoot before 2 s is under 20.05 m/s. The variation is steady state lies within 19.95 m/s to 20.05 m/s. It satisfies the given parameters with a good margin.

